Integration Into the Mathematics Community: An Exploratory Interpretative Phenomenological Study Based on Mathematics High School Research Mentorships

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Abstract

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This dissertation examines how high school mathematics research mentorships shape students' integration into mathematics through the development of self-efficacy, identity, and values. Using the Tripartite Integration Model of Social Influence (TIMSI) as a theoretical framework, this study investigates the mechanisms through which mentorship experiences foster these developmental constructs. Through in-depth interviews analyzed using Interpretative Phenomenological Analysis (IPA), the research reveals how adaptive scaffolding, professional skill building, collaborative dynamics, and fostering belonging interact to support mentees' growth.

The findings highlight the importance of domain-specific mentoring strategies, thoughtful selection of research questions, and real-world applications in fostering engagement. The study also reveals significant equity concerns, as most participants attended well-resourced schools with established research cultures. Recommendations include expanding access through virtual mentorships and regional initiatives. While limitations include sample diversity and retrospective design, these constraints inform future research directions in mentorship scalability and longitudinal impact.

This research extends TIMSI to the high school mathematics context while providing actionable recommendations for mentorship program design. The findings inform strategies for expanding access to research mentorships, particularly for students in underserved communities, while maintaining program quality and effectiveness. This study addresses critical gaps in

understanding how early research experiences shape students' mathematical development and integration into the field.

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Dedication

This dissertation is dedicated to my mother, Malina Delgado, who, despite extraordinary challenges, gave me the tools to accomplish this dream, and to my wife, Yulu Tian, whose unwavering support and the countless days we spent studying together made this difficult journey not only tolerable but deeply meaningful. I also dedicate this work to the many mentors who shaped my path, including Sylvain Cappell, Mike Gargano, Louis V. Quintas, and Martin Lewinter.

Chapter 1 Introduction

1.1 Need for Study

The United States faces a significant challenge in attracting and retaining individuals in STEM careers, particularly in mathematics (Kiazad et al., 2024; National Academies of Sciences, Engineering, & Medicine, 2018). The field of mathematics is crucial for addressing global challenges such as climate change and pandemics (Dabreo et al., 2022; Jouve, 2023). Despite the growing demand for mathematics professionals, with math-related occupations projected to double by 2031, the number of graduates entering the field is decreasing (National Science Board, 2020; U.S. Bureau of Labor Statistics, 2021). This decline underscores the urgent need for effective strategies to cultivate a robust mathematical workforce.

One potential strategy to combat this issue is effective mentoring. In this study, mentorship is defined as a dynamic, multifaceted relationship in which a more experienced individual (the mentor) provides guidance, support, and knowledge to a less experienced individual (the mentee), fostering the mentee's personal and professional development (Nuis et al., 2023; Allen & Eby 2007). While mentorships can vary substantially in their execution and efficacy (Eby et al., 2008), the National Academies of Sciences, Engineering, and Medicine (2019) reports that "effective mentorship has an overall positive effect on academic achievement, retention, and degree attainment, as well as on career success, career satisfaction, and career commitment" (p. 6). However, despite its critical role in supporting students, mentorship often receives less evaluation, analysis, and recognition compared to other aspects of professional development, such as teaching or research (National Academies of Sciences and Medicine, 2019).

A form of mentorship regarded as a high-impact practice in higher education is a research mentorship (Kuh, 2008; Osborn & Karukstis, 2009; Lopatto, 2007). Typically aimed at supporting college students, this type of mentorship focuses on engaging students in a research project alongside a mentor. Research mentorships share the general advantages of traditional mentorships and offer unique benefits such as the development of research skills. Numerous studies underscore the benefits of participation in undergraduate research. These advantages include increased self-confidence, development of strong mentor relationships, improved communication, critical thinking, problem-solving skills, refinement of career and educational objectives, and better preparation for careers or graduate studies (Laursen et al., 2010; Lopatto, 2004, 2010; Walkington et al., 2019). One study indicates that "intensive undergraduate research experiences (UREs) are one of the few strategies shown to improve longitudinal student interest and persistence in STEM-related career pathways" (Hernandez et al., 2018, p.1).

Given the decline in the number of mathematics professionals and graduate students in the United States (National Academies of Sciences, Engineering, & Medicine, 2018; National Science Foundation, 2022), it is crucial to explore the impact of research mentorships during earlier educational stages, including high school. Graham et al. (2013) emphasize the need for early research engagement, contrasting with typical university experiences like senior projects. This early involvement can lead to more students identifying as science or mathematics scholars before graduation, enhancing retention in these fields (Estrada et al., 2011). Compared to college students, high school students represent an important population for these research mentorships due to their higher numbers and earlier developmental stage. Research mentorships during this period offer a unique opportunity to shape academic trajectories, potentially fostering students' pursuit of degrees and careers in mathematics (Qua et al., 2020). However, literature on high school research mentorships, especially in mathematics, is sparse. There is a need to investigate how mentoring at this earlier stage supports sustained engagement in mathematics. High school represents a formative stage in students' academic trajectories, and research shows that early exposure to STEM opportunities significantly shapes long-term interest and success. For instance, Maltese and Tai (2011) found that high school is a pivotal period when most students' interests in STEM careers are solidified, making it an exceptionally critical timeframe for interventions like research mentorships to influence their long term-academic and career trajectories.

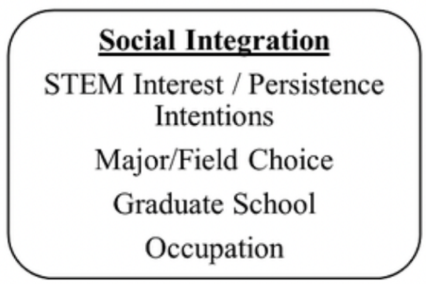
Although research mentorships at the high school level are less prevalent than at the undergraduate level, there is evidence of their long-term impact through the achievements of participants in prominent competitions such as the Regeneron Science Talent Search and the Westinghouse Science Talent Search. Typically, the research projects submitted to these prestigious competitions are the outcomes of high school research mentorships. Alumni of the Regeneron Science Talent Search have received notable recognition in mathematics and science, securing over 100 prestigious awards including the Fields Medal, Nobel Prize, and National Medal of Science (Society for Science, 2023). Kaye (2001) found that 91% of male and 74% of female Westinghouse finalists went on to earn doctoral degrees, demonstrating the long-term success of students who participated in the Westinghouse Science Talent Search. The requirement for students to complete a research project under a mentor's guidance in these competitions presents a unique opportunity to study the developmental impact of these early experiences on sustained engagement in STEM, including in mathematics.

Although these high school research competitions have a high percentage of producing distinguished alumni, the underlying processes that enable students to achieve such high levels of success remain less understood (Hernandez et al., 2018). These remarkable high success rates, alongside the gap in understanding the underlying mechanisms, warrant an investigation into the experiences of high school students who participated in mathematics research mentorships. While existing literature on research mentorships primarily focuses on measuring outcomes, there's a need for in-depth exploration of the developmental processes within these experiences (Adedokun et al., 2014; Fakayode et al., 2014; Fuchs et al., 2016; Hernandez et al., 2018; Lopatto, 2007). A more in-depth exploration of these processes could shed light on the pathways and mechanisms that enhance persistence in academic trajectories (National Academies of Sciences, Engineering, and Medicine, 2017).

High school is a critical period when students' mindsets and interests can be significantly influenced, making them more receptive to the development of key affective components, such as self-efficacy and identity, across various academic domains, including mathematics (Flowers & Banda, 2016; Maltese & Tai, 2010). In STEM, these constructs—self-efficacy (students' confidence in their ability to succeed in science/mathematics) and identity (how they see themselves and are recognized as scientists/mathematicians)—are particularly vital for fostering long-term engagement and success (Estrada et al., 2018; Byars-Winston & Rogers, 2019). The development of these constructs during research mentorship experiences have been highlighted as important social influence processes to students' long-term engagement in STEM fields (Hernandez et al., 2018). Additionally, Robnett (2015) highlights the importance of initiating research mentorships earlier in the academic pipeline, suggesting that "the association between research experience, self-efficacy, and identity may unfold over the course of months or potentially years" (p. 861), underscoring the case for implementing research mentorships earlier at the high school level. However, fewer studies examine how these mentorship processes unfold at earlier educational stages, particularly during high school, when students are forming key attitudes, skills, and identities.

Recent theoretical advancements in sociology and STEM education have introduced a framework that elucidates the connection between mentorship experiences and positive outcomes, such as career choices and degree attainment, by examining changes in cognitive processes among undergraduate students (National Academies of Sciences, Engineering, and Medicine, 2017). "The Science of Effective Mentorship in STEMM," presented by the National Academies of Sciences, Engineering, and Medicine (2019), highlights the Tripartite Integration Model of Social Influence (TIMSI) as one of six major theoretical frameworks for examining STEM research mentorships at the undergraduate level. TIMSI could be illuminating as a guiding framework for mathematics at the high school level because of its emphasis on process. It has the potential for deeper theoretical insights into how these mentorship experiences foster the development of self-efficacy, identity, and values (the perceived importance and purpose of mathematics' in one's life) within this specific context, as well as influence students' long-term academic and professional trajectories.

TIMSI, derived from Kelman's model of social influence (1958), categorizes integration into a community through three dimensions: rules (self-efficacy), roles (identity), and values. Hernandez et al. (2018) applied the TIMSI framework to investigate the contextual features and psychological processes linking undergraduate research experiences (UREs) to student integration into STEM careers. Their longitudinal study revealed that quality mentorship and sustained research experiences positively influence scientific self-efficacy, identity, and values. Given the interdisciplinary nature of STEM, which encompasses both science and mathematics, the successful application of the TIMSI framework to mathematics research mentorships indicates a promising potential for fostering similar positive outcomes in mathematics education. While most applications of TIMSI have concentrated on STEM fields like biology and biomedical sciences, its principles are equally relevant to mathematics (Estrada, 2014). Within the TIMSI framework, "integration into the mathematics community" can signify having an interest in and persistence with mathematics, majoring in the field, earning a degree in a math-centric domain, and pursuing a career in mathematics or a closely related field (see Figure 1). This integration is essential for addressing the shortage of graduate students and professionals in mathematics within the United States.



(Hernandez et al., 2020, p. 4)

Figure 1: TIMSI Framework Integration into STEM

While quantitative methodologies dominate existing research on TIMSI and research mentorships (Chemers et al., 2011; Estrada et al., 2011; Estrada et al., 2018; Hernandez et al., 2020; Schultz et al., 2011), they do shed light on some of the mechanisms underlying mentorship outcomes, such as the development of self-efficacy, identity, and values. However, there is a need for more qualitative studies to provide deeper insights into the development of these constructs and their relationship among each other and other factors through students' experiences and perceptions.

1.2 Purpose

The purpose of this exploratory qualitative study is to investigate the lived experiences of individuals who participated in high school mathematics research mentorships and subsequently integrated into the mathematics community. This research aims to uncover the factors and developmental processes that influenced participants' mathematical self-efficacy, identity, and values, as well as their long-term academic and professional trajectories in mathematics. Towards this end, this study addresses the following research questions:

1. What factors influenced participants' decisions to engage in high school mathematics research mentorships, and what personal meaning do they assign to these decisions?
2. How do participants describe and make sense of their lived experiences during their high school mathematics research mentorships and their integration into the mathematics community? Specifically, what mentorship processes and interactions shape participants' self-efficacy, identity, and values, and how do these experiences facilitate their integration into mathematics?
3. What additional factors, beyond self-efficacy, identity, and values, do participants identify as significant from their high school mathematics research experiences, and how do these factors contribute to their integration into mathematics?
4. What factors beyond the mentorship do participants perceive as most significant in their integration into mathematics, and how do they interpret the role of these factors in shaping their academic and professional trajectories? How, and to what extent, do these interpretations align with the Tripartite Integration Model of Social Influence?

1.3 Procedures of the Study

This study employs Interpretative Phenomenological Analysis (IPA) to explore the lived experiences of individuals who participated in high school mathematics research mentorships and subsequently integrated into the mathematics community. IPA is well-suited for this research as it allows for an in-depth examination of how participants make sense of their personal and social world, and the meanings they attach to their experiences (Smith & Osborn, 2015).

By using IPA, this study remains open to an inductive exploration, capturing the nuances of high school research mentorship processes and potential factors beyond the constructs of self-efficacy, identity, and values. This flexibility ensures a balance between grounding in established theory and uncovering new insights from participants' experiences.

While IPA serves as the primary methodology, the study also incorporates exploratory elements to investigate factors beyond mentorship that contributed to participants' integration into mathematics. Given the complexity of participants' trajectories, elements of case study methodology are integrated to examine the broader context in which participants' research mentorships occurred (Yin, 2014). Each participant's story is treated as a unique case, and cross-case analysis is employed to identify patterns and themes across participants. This approach allows for a comprehensive examination of how various external factors interacted with mentorship experiences to shape participants' journeys (Yin, 2014).

Data Collection

The IPA methodology will guide the data collection and analysis processes (Smith et al., 2009). Smith and Fieldsend (2021) propose having 8 to 10 participants for a doctoral study using IPA. Purposeful sampling was used to recruit 8 to 10 participants who meet specific criteria: previous participation in a high school math research mentorship, attainment of an advanced mathematics degree, and a professional career in mathematics. Data on semi-finalists and finalists from the Westinghouse, Intel, and Regeneron Science Talent Searches were supplied to me under conditions ensuring confidentiality and the ethical use of the data for academic purposes. This data allowed for me to identify potential participants who completed mathematical research projects. Additionally, MIT PRIMES, a prominent organization supporting high school mathematics research mentorships, provided me with lists of mentors and mentees, particularly those who have received awards for their mathematics projects. Additionally, established relationships with mathematics research mentors will be leveraged to identify additional potential participants and facilitate snowball sampling.

Google searches allowed me to locate their LinkedIn profiles or academic/professional web pages. While this method may not identify every potential participant, it yielded a substantial subset central to this study. To minimize recall bias, priority was given to participants with more recent high school mentorship experiences.

Data Analysis

The research questions will be structured chronologically. The first question will focus on the period leading up to the high school mathematics research mentorship. The second and third questions will explore participants' experiences during the research mentorship. Finally, the fourth question will provide an overview of participants' mathematical journeys, taking a holistic approach to understand their progression into professional roles in math-related fields.

To address Research Question 1, the study will analyze responses to interview questions that probe participants' initial interest in mathematics and their subsequent decision-making processes leading to participation in mentorship programs. This examination aims to contextualize the participants' motivations and early experiences within the broader trajectory of their integration into the field of mathematics. The methodology will involve using Interpretative Phenomenological Analysis (IPA) to identify and elaborate on emergent themes that illustrate these early decision-making processes. Additionally, a cross-case analysis will be conducted to provide a holistic view of these experiences across participants, thereby enriching the understanding of their journeys into the mathematical community. This approach aims to capture nuanced factors influencing their initial engagement with mathematics, setting the stage for deeper insights into their developmental pathways.

To investigate Research Question 2, this study employs Interpretative Phenomenological Analysis (IPA) to examine how high school mathematics research mentorships influenced participants' mathematical self-efficacy, identity, and values. Through semi-structured interviews, participants' reflections on their mentorship experiences will be analyzed to understand how specific mentorship processes and characteristics shaped these three psychological constructs. This analysis will then delve into how these processes and experiences facilitated participants' integration into the mathematical community.

To address Research Question 3, this study employs Interpretative Phenomenological Analysis (IPA) to examine additional factors, beyond self-efficacy, identity, and values, that participants gained through their high school mathematics research mentorships. Through semi-structured interviews, participants' reflections will be analyzed to identify key developmental outcomes that contributed to their integration into mathematics. The analysis will proceed from identifying Personal Experiential Themes (PETs) in individual interviews to developing Group Experiential Themes (GETs) that represent shared experiences across participants. This systematic approach allows for an examination of developmental factors that participants attributed to their mentorship experiences, while maintaining distinctions from the psychological constructs (self-efficacy, identity, and values). By focusing on participants' own interpretations of significant factors gained through their mentorships, this analysis aims to uncover additional dimensions of how these additional contributed to their integration into mathematics.

To address Research Question 4, which explores the factors participants identify as most influential in their integration into the mathematics community, I will employ a comparative analysis of the PETs I obtained from the interview data. This analysis will focus on distilling key elements outside of the mentorship, critical educational experiences, and major personal and professional milestones as described by the participants. By anchoring the analysis in their reflections, it ensures that the findings are deeply rooted in their lived experiences and personal reflections, offering a participant-centered perspective. The analysis will not only highlight the current role of mathematics in participants' lives but also delve into their reflective journeys and the dynamics that influenced their paths. These emergent themes will be systematically contextualized within the Tripartite Integration Model of Social Influence (TIMSI) framework, providing a detailed understanding of the factors deemed crucial by participants for their successful integration into the mathematics community.

In weaving together these individual cases with overarching thematic threads, this study aims to contribute meaningfully to the discourse surrounding high school mathematics research mentorship, highlighting participants' firsthand experiences and their integration into the mathematics community.

Chapter 2: Literature Review

This literature review explores how high school mathematics research mentorships address the significant shortage of American students in graduate mathematics programs. With growing concerns over global challenges and scientific progress, such mentorships are pivotal interventions for cultivating future mathematicians (Etingof et al., 2015; Gerver et al., 2017). While the significance of research mentorships at advanced academic stages is well-documented, there remains limited understanding of these experiences at the high school level. The literature reveals a notable gap in examining how early research experiences contribute to long-term engagement in mathematics, particularly regarding the developmental processes that occur during adolescence. Current research primarily focuses on undergraduate and graduate mentorships, leaving the potential impact of high school mathematics research experiences relatively unexplored. Given the limited direct literature on high school mathematics research mentorship, this review synthesizes findings from interdisciplinary research and applicable theoretical frameworks to offer insights into the developmental processes that these mentorships can foster.

The initial section of this review focuses on the general role of mentorship in mathematics, highlighting its significance in fostering students' academic and professional development. It examines high school mathematics research mentorship literature, supplemented by findings from mathematics undergraduate-level studies, to draw parallels and identify developmental continuities. The subsequent section further explores broader high school STEM research mentorships, analyzing their operational dynamics, students' developmental processes, and the crucial role of socio-emotional support in enhancing STEM education outcomes. This broader perspective helps to contextualize mathematics mentorships within the larger framework of STEM education. The review then broadens its scope to focus on undergraduate mentorship programs within mathematics and STEM fields, where the literature is more robust. This section draws from diverse theoretical perspectives and highlights effective mentoring practices, particularly underscoring the affective components, such as beliefs, self-efficacy, and identity, which can significantly shape students' educational trajectories and engagement.

This literature review then delves into the concept of self-efficacy. This section discusses the theoretical foundations of mathematics self-efficacy and its pivotal role in boosting students' mathematical achievement and sustained interest. It then places self-efficacy within the context of research mentorships, examining how these experiences influence students' long-term educational and professional trajectories. The following section intertwines the constructs of mathematics self-efficacy and identity, drawing on sociocultural and psychological theories as well as mathematics education literature to understand their joint impact on students' mathematical journeys. This synthesis aims to uncover how educational experiences, such as research mentorships, contribute to shaping a robust mathematics identity.

Finally, this review outlines the theoretical frameworks underpinning this study, utilizing the Tripartite Integration Model of Social Influence (TIMSI) as a broad framework. It focuses on theoretical perspectives related to mathematics self-efficacy, mathematics identity formation, and mathematics value. These frameworks provide a comprehensive perspective for examining cognitive and social processes influencing successful careers in mathematics through high school research mentorships. This structured approach not only prepares for empirical investigation but also aligns with the study's broader aim: understanding how high school research mentorships support future mathematics professionals' development.

2.1 The Role of Mentorship and Research Mentorships in Mathematics

Mentorships are increasingly recognized as vital strategies for enhancing student engagement and commitment at advanced educational levels (Crisp & Cruz, 2009). Although the quality and approach of these mentorships can vary greatly (Eby et al., 2008), effective mentorship has the potential to profoundly inspire students and promote vital connections within mathematics and broader STEM communities (National Academies of Sciences, Engineering, and Medicine, 2017). However, it is crucial to recognize that negative mentorship experiences can be detrimental, potentially leaving mentees in a less advantageous position than before their mentorship (Ragins et al., 2000). Eby et al. (2000) found that just one poor mentoring experience can deter students from continuing their education. Despite the significant role of mentorship in student development, they often do not receive a different level of analysis and attention than other aspects of professional development, such as teaching or research (National Academies of Sciences, Engineering, and Medicine, 2019). A notable disconnect exists between the established knowledge of effective mentoring and its actual implementation in educational settings (National Academies of Sciences, Engineering, and Medicine, 2019). Consequently, there is a need for a thorough evaluation and refinement of mentorship practices in mathematics education to optimize their effectiveness and impact.

Quality mentorships and research experiences have been particularly effective in increasing persistence in mathematics, especially among underrepresented groups (Valantine & Collins, 2015). These initiatives diversify the field and contribute to broader societal and technological advancements. Additionally, they are pivotal in retaining talent within the discipline, ensuring a diverse and capable mathematics workforce that is essential for future innovations (Committee on STEM Education of the National Science & Technology Council, 2018). While the positive impact of undergraduate research mentorships (URMs) on STEM pathways is well-documented (Fuchs et al., 2016; Pfund et al., 2016; Robnett et al., 2018), research on the role of high school research mentorships, particularly in mathematics, remains sparse (National Science Board, 2010; Qua et al., 2020).

Despite their diverse approaches, high school mathematics research mentorships are particularly impactful, but remain insufficiently studied. These mentorships can profoundly influence participants, many achieving significant acclaim, securing prestigious awards such as the Fields Medal and Nobel Prizes, and advancing to become leading figures in their fields (Society for Science, 2023).

Early engagement with research during high school can cultivate critical mathematical skills and interests, aligning with broader strategies for promoting STEM fields, as recommended by the President's Council of Advisors on Science and Technology (PCAST, 2012). The existing literature on high school mathematics research mentorships, although limited, highlights programs that have significantly impacted students' engagement with and understanding of mathematics through original research. The scarcity of studies specifically addressing this level of mentorship emphasizes the uniqueness and critical need for further exploration in this research area (National Science Board, 2010; Qua et al., 2020). However, there remains a scarcity of research specifically examining high school-level mentorships, which points to a critical gap in the literature.

Mentorship has been shown to support the development of academic pathways that enhance persistence in STEM fields (National Academies of Sciences, Engineering, and Medicine, 2017). By focusing on the interaction between mentors and mentees, the literature suggests that effective mentorship can foster long-term engagement and success in mathematics, contributing to the broader goal of building a diverse and resilient mathematical workforce (National Academies of Sciences, Engineering, and Medicine, 2019).

High School Mathematics Research Mentorship Literature

High school mathematics research programs, such as those at North Shore High School (Gerver et al., 2017) and MIT PRIMES (Etingof et al., 2015), represent two contrasting approaches to engaging students in mathematical research. While both programs demonstrate the feasibility of high school mathematics research, their documentation primarily focuses on program structures, selection processes, and outcomes.

The North Shore program exemplifies an inclusive model of mathematics research mentorship. Initiated in 1991, it expanded from ten to forty students over 25 years through a comprehensive engagement strategy emphasizing research skills, technical writing, and presentations. The program's success is evidenced by its 95% retention rate, production of over 600 research papers, and consistent participation in competitions (Gerver et al., 2017).

In contrast, MIT PRIMES represents a selective model targeting exceptional students through rigorous entrance requirements. While this approach ensures high-quality research output, it potentially limits accessibility to students with privileged educational backgrounds. The program emphasizes carefully structured research projects with specific criteria: accessible starting points, flexibility to explore related questions, computer-assisted exploration opportunities, and connection to broader mathematical contexts. PRIMES particularly values projects that develop students' ability to pose their own questions, fostering ownership and independence in mathematical research. Despite these accessibility limitations, PRIMES has demonstrated remarkable success in fostering advanced mathematical research among high school students, with participants producing publishable results in fields ranging from combinatorics to representation theory (Etingof et al., 2015).

Both programs illustrate essential elements of successful mathematics research mentorships - from North Shore's broader accessibility to PRIMES' selective intensity. However, the existing literature primarily focuses on program structures and outcomes rather than the developmental processes and psychological mechanisms that facilitate student success (Eby et al., 2008; Lopatto, 2007; Nelson & Cutucache, 2017). While these articles demonstrate positive outcomes in terms of program success and mathematical achievement, they provide limited insight into how these outcomes are achieved.

Gerver's (2017) additional contribution, "Writing Math Research Papers: A Guide for High School Students and Instructors," provides valuable practical guidance but similarly focuses on structural rather than developmental aspects of research mentorship. This guide has influenced program design and implementation, as evidenced by its application at SUNY Old Westbury's Institute of Creative Problem Solving. However, like the broader literature, it predominantly focuses on structural aspects rather than the underlying processes that cultivate sustained engagement in mathematics.

This gap in the literature underscores the need for further research into the developmental pathways and psychological mechanisms that enable high school students to engage meaningfully with mathematics. Understanding these mechanisms is crucial for scaling successful interventions and fostering equitable access to mentorship opportunities (DuBois et al., 2011). To bridge this gap, this literature review extends its scope to include research initiatives at the undergraduate level, providing a continuum of insight into mentorship practices across educational stages.

Undergraduate Mathematics Research Literature

Reflecting on the evolution of undergraduate research in mathematics, Joseph A. Gallian's (2012) article "Undergraduate Research in Mathematics Has Come of Age" underscores the profound transformation ignited by the NSF Research Experience Undergraduate (REU) programs over the past quarter-century. Gallian characterizes this shift as a paradigmatic change, debunking the once-prevalent myth that undergraduates cannot conduct mathematical research. This change, catalyzed mainly by the establishment and growth of REU programs, is evidenced by the significant increase in undergraduate participation in mathematical research and presentations at academic conferences. For instance, the number of posters presented by undergraduates at the AMS-MAA joint meetings soared from 12 in 1991 to 310 in 2012, highlighting a broader acceptance and integration of undergraduate research within the mathematics community (Gallian, 2012). Gallian's insights chronicle this transformative period and anticipate continued growth and recognition of undergraduate contributions to mathematical research. The article serves as both a retrospective examination and a forward-looking perspective on the growing role of undergraduate research in mathematics, emphasizing its importance in nurturing the next generation of mathematicians.

In his exploration of mentoring undergraduate research in mathematical modeling, Ledder (2022) draws from over three decades of experience to offer a framework for guiding the next generation of mathematicians. This paper articulates the nuanced differences between undergraduate and professional research, advising on mentorship approaches, research dissemination, and the intricacies of mathematical modeling for novices. Ledder posits that while professional research prioritizes product, undergraduate endeavors should focus on the process—emphasizing the generation and communication of knowledge. He champions the inclusion of top-tier students and those at the outset of their academic journeys or those not at the peak of their class (‘mid-tier’), underscoring the transformative power of early research experiences with varying mathematics students. Through examples from his career and the RUTE Summer Scholars program, Ledder (2022) illustrates the feasibility and impact of engaging undergraduates in research, regardless of their prior mathematical background. His reflections provide a roadmap for educators looking to navigate the terrain of undergraduate research mentorship and advocate for a broader, more inclusive approach to nurturing budding mathematicians (Ledder, 2022).

In "Mentoring Undergraduate Research in Statistics: Reaping the Benefits and Overcoming the Barriers," Nolan et al. (2020) explore the landscape of undergraduate research experiences (UREs) within the field of statistics, underscoring both the substantial benefits and notable barriers associated with their implementation. This comprehensive analysis is anchored by insights derived from a 2017 survey of statistics faculty, reflecting on the collective experiences and perspectives of students, faculty mentors, and academic institutions. The study identifies the multi-faceted advantages of UREs, including enhanced student engagement with material, skill development, and significant preparation for graduate studies or professional careers. Moreover, it emphasizes the unique opportunities within statistics education for active, hands-on learning that mirrors real-world research dynamics.

However, the path to integrating UREs into statistics education has its challenges. Nolan et al. (2020) detail several obstacles, such as time constraints for faculty, insufficient student preparation, and institutional undervaluation of undergraduate research, that can hinder effective mentorship and engagement in these programs. Despite these hurdles, Nolan et al. (2020) offer strategic recommendations for leveraging the distinct attributes of the statistics discipline to surmount these barriers. It suggests fostering a culture that values UREs through institutional support, financial incentives, faculty recognition, and practical advice for faculty to incorporate research into their workload effectively.

This insightful work by Nolan et al. (2020) reflects on the current state of undergraduate research in statistics and a call to action. It encourages stakeholders across the educational spectrum to recognize and harness the intrinsic value of UREs, thereby enriching the statistics and mathematics education landscape and empowering the next generation of mathematicians, statisticians, and data scientists.

Self-Directed Learning in Mathematics Research Mentorships

Self-directed learning is widely acknowledged as a cornerstone of effective research mentorships, particularly in mathematics and science (Pfund et al., 2006; Sadler et al., 2010). This pedagogical approach emphasizes empowering students to take ownership of their learning processes (Knowles, 1975), enabling them to independently navigate complex mathematical inquiries. Research consistently demonstrates that fostering self-directed learning enhances problem-solving abilities, deepens engagement with mathematical concepts, and cultivates critical skills such as goal-setting, method selection, and progress evaluation—skills foundational to success in mathematical research (Amidi et al., 2021; Schunk & Zimmerman, 2012; Shoemaker et al., 2016).

Within mathematics research mentorships, self-directed learning operates as both a prerequisite and an outcome of effective mentorship experiences. Students gradually acquire the ability to analyze mathematical literature, explore advanced concepts, and pursue novel lines of inquiry that extend beyond the confines of formal coursework. Shoemaker et al. (2016) provide a compelling example through the Mentorship Program at the North Carolina School of Science and Mathematics (NCSSM), which deliberately scaffolds students' self-directed learning through structured curricular phases. These phases begin with preparatory activities—such as guided readings of research articles, professional skill-building exercises, and mentor-matching assignments—and progress toward independent data collection and analysis. Throughout this process, students face challenges such as interpreting dense mathematical literature, formulating research questions, and maintaining perseverance in the face of ambiguity and setbacks (Baker, 2017; Zimmerman, 2002). Mentors play an essential role in supporting students' development, offering structured guidance in the early stages and gradually transferring responsibility as mentees build confidence and competence (Gerver et al., 2017; Sadler et al., 2010; Shoemaker et al., 2016). This phased approach not only fosters self-efficacy but also prepares students to engage with authentic research projects in a meaningful way.

Despite its transformative potential, self-directed learning poses significant challenges for students with limited prior research experience or confidence (Baker, 2017; Pfund et al., 2006; Sadler et al., 2010; Zimmerman, 2002). These challenges often manifest as difficulty interpreting technical terminology, formulating meaningful research questions, or persevering through initial failures, which students may misinterpret as personal inadequacy rather than opportunities for growth (Baker, 2017). Addressing these challenges requires mentors to provide timely, constructive feedback and to reframe failure as a natural and productive component of the research process. Such practices are particularly critical in mathematics, where problem-solving frequently involves navigating ambiguity and uncertainty (Amidi et al., 2021).

Effective self-directed learning environments also depend on the availability of well-designed resources and robust support systems. Shoemaker et al. (2016) emphasize the value of providing students with tailored tools, such as templates for analyzing scholarly articles, glossaries for technical terminology, and structured research journals that promote reflection and planning. These resources help students overcome barriers to self-directed inquiry by providing clear frameworks for navigating complex academic content and developing independent research capabilities. Mentorship programs that incorporate such tools not only enhance students' immediate learning experiences but also equip them with the skills needed for long-term success in mathematical research. Given that a majority of students enter science and mathematics pathways during high school (Maltese & Tai, 2011), providing effective mentorship and support at this stage is critical to fostering future engagement in mathematics.

The literature further highlights that the success of self-directed learning hinges on students' self-efficacy and motivation (Schunk & Zimmerman, 2012). However, as Gafoor and Kurukkan (2016) argue, these traits are not uniformly distributed among students, necessitating a delicate balance between mentor support and autonomy. Mentors who provide structured guidance early on, while progressively ceding control, help students build resilience and confidence in their ability to tackle the inherent challenges of mathematical research (Etingof et al., 2015; Pfund et al., 2006; Shoemaker et al., 2016). These strategies align with broader educational theories that underscore the role of self-regulation in promoting student success, particularly in STEM fields (Zimmerman, 2002).

Ultimately, the literature underscores the dynamic interplay between self-directed learning and the development of self-efficacy in mathematics (Boaler, 2016; Schunk & Zimmerman, 2012). However, gaps remain in understanding specific aspects of this relationship, particularly in high school mentorship contexts. For instance, limited research explores how self-directed learning in high school mentorships impacts students' transition to undergraduate research. Additionally, the mechanisms through which mentorship fosters self-efficacy in mathematics—including the roles of specific instructional strategies or mentor-mentee interactions—require further exploration. Finally, there is a lack of longitudinal studies investigating how early exposure to self-directed learning influences long-term career persistence and success in mathematics-related fields. Addressing these gaps will provide deeper insights into designing effective mentorship programs that cultivate a strong foundation for academic and professional success in mathematics.

2.2 STEM High School Research Mentorships

This section broadens the discussion from high school and college mathematics research mentorships to encompass the broader scope of high school STEM research mentorships. It shifts focus from the well-documented positive outcomes of research mentorships to delve into the intricate processes and less-explored dimensions that drive these successes. Despite the clear benefits, there remains a critical need for an in-depth examination of the operational dynamics and fundamental mechanisms within these mentorships. This exploration aims to uncover the developmental and theoretical aspects of STEM research mentorships in high school settings, particularly emphasizing their implications for mathematics research mentorships. Through a discussion of key findings and identifying gaps in the current literature, this study seeks to pave the way for a comprehensive investigation into the effectiveness and intricacies of mathematics mentorships at the high school level.

The limited studies in this domain, such as Shoemaker et al. (2016) and Kim (2021), have highlighted the benefits of comprehensive curricula and practical applications within STEM mentorship programs. These findings reinforce the effectiveness of mentorship in enriching students' educational experiences and preparing them for the challenges they face in STEM.

Beyond academic outcomes, some studies have explored the socio-emotional dimensions of STEM mentorship programs. Qua et al. (2020) examined the Near Peer Mentoring Program (NPMP), which pairs high school students from underrepresented minority groups with medical research students. This program addressed a wide range of topics, including college preparation, career aspirations, social issues, and health and wellness, while simultaneously fostering identity development and value alignment within the STEM community. Notably, the NPMP's emphasis on structured yet open dialogue allowed mentees to engage in meaningful discussions about their aspirations and well-being, supported by relatable mentors who served as role models. The program’s success highlights the importance of mentorship strategies that address both socio-emotional and academic needs, particularly for students navigating intensive STEM environments.

Wilson et al. (2023) explored the effects of near-peer mentorship on high school students’ attitudes and identities in mathematics. While not focused on research mentorships, their study demonstrated significant positive shifts in students’ enjoyment and perceived value of mathematics. However, gains in self-confidence were less pronounced, pointing to the need for mentorship programs to better address mathematical self-efficacy, a critical factor for sustained engagement and achievement in the discipline (Pajares & Miller, 1994). Drawing on Anderson’s (2007) four-face model of identity, Wilson et al. analyzed how mentorship influenced students’ perceptions of themselves and their abilities in mathematics. They found that connecting students with relatable mentors enhanced identification with mathematics and increased engagement. These results underscore the value of diverse mentors who can relate to students’ lived experiences, particularly for underrepresented groups, as a way to foster shared understanding and relational trust in mathematics education.

The findings from Qua et al. (2020) and Wilson et al. (2023) contribute to an emerging body of literature emphasizing the socio-emotional and relational aspects of mentorship. Near-peer mentorship, in particular, has been shown to support students' identity development and sense of belonging in STEM fields, offering a promising model for fostering engagement. However, these programs primarily focus on socio-emotional dimensions and attitudinal shifts, with limited attention to the advanced cognitive and developmental processes central to mathematics research mentorships. While these insights are valuable, they highlight a gap in the literature on how mentorship programs cultivate the deeper skills, confidence, and identity necessary for long-term integration into the mathematics community.

Unlike broader mentorship models, research-specific mentorships provide opportunities for students to engage in advanced mathematical inquiry, develop problem-solving abilities, and navigate complex cognitive and affective challenges. The significance of affective factors such as emotions, beliefs, and identity within mathematics education has been well-documented (Maasz & Schlöglmann, 2009). Positive emotional experiences in mathematics have been shown to significantly enhance student engagement, motivation, and perseverance, shaping their academic trajectories and career choices (Hannula, 2012; McLeod, 1992; Pekrun et al., 2017). While near-peer mentorship offers valuable strategies for fostering socio-emotional support and relational trust, this study emphasizes the distinct role of research mentorships in shaping students' cognitive, affective, and professional growth in mathematics.

2.3 Undergraduate STEM Research Mentorships

Expanding the discussion to undergraduate STEM research mentorships, the body of literature becomes more substantial, providing in-depth analyses into the impact of these affective components (Atkins, 2020; Palmer, 2015). This literature highlights their profound impact on enhancing students' self-efficacy, identity, and internalization of disciplinary values (Estrada et al., 2011; Hernandez et al., 2018; Syed et al., 2019). These research experiences shape students' perceptions of their capabilities and foster a strong sense of belonging within their fields, significantly influencing their academic and professional trajectories (Chemers et al., 2011). The comprehensive analysis of undergraduate STEM research mentorships provides helpful frameworks for examining the potential benefits and underlying mechanisms of high school mathematics research mentorships, suggesting essential considerations for future high school-level research.

There is a substantial and detailed body of literature on undergraduate research mentorships in contrast to the scant literature on high school research mentorships. These studies have consistently reported various advantages, including increased self-confidence, stronger mentor-mentee relationships, enhanced communication skills, and refined problem-solving abilities (Laursen et al., 2010; Walkington et al., 2019). Undergraduate research mentorship experiences also play a crucial role in shaping student's career aspirations and educational pathways, preparing them more effectively for future professional endeavors or advanced studies (Lopatto, 2004, 2010). Both governmental and private funding bodies allocate substantial financial resources to support undergraduate research mentorship programs (National Science Foundation, 2023; Research Corporation for Science Advancement, 2020). This extensive support and investment in undergraduate research underscore its perceived importance in fostering education and training, highlighting the consensus on the effectiveness of research mentorships in nurturing the next generation of scientists and mathematicians.

Like high school research mentorships, most undergraduate research mentorship literature focuses on outcomes. However, recent advancements in theoretical research within undergraduate STEM research mentorships offer valuable insights for understanding the developmental processes of students (Hernandez et al., 2018; National Academies of Sciences, Engineering, and Medicine, 2017). Due to the limited literature at the high school level, examining these theoretical perspectives in STEM undergraduate research mentorships reveals significant insights that can inform my study of high school mathematics research mentorships. These insights particularly highlight the roles of self-efficacy, identity, values, and the impact of research mentorship on students' academic and professional paths.

Thiry and Laursen’s (2011) qualitative investigation into the dynamics of student-advisor interactions within undergraduate research (UR) mentorships highlighted mentors' multifaceted role in fostering students' academic and professional growth. By employing situated learning theory, this seminal study elucidated how these interactions serve as a conduit for conducting scientific knowledge and as a critical mechanism for integrating students into the professional science community. This integration process involves professional socialization, intellectual support, and personal/emotional support, highlighting research mentors' dual role in advancing scientific inquiry and nurturing the next generation of scientists. The emphasis on the specific practices employed by research mentors to support undergraduate mentees illuminates the complex interplay of factors contributing to developing a robust professional identity and value system among students. Thiry and Lauren's findings revealed the importance of mentors in guiding students through their journey from novice to experienced researchers, a path marked by increasing integration into the scientific community and alignment with its norms and values. These findings illustrate the role of social influence processes of mentorships in integrating students into a professional community. These insights from Thiry and Laursen's study are instrumental in highlighting the critical role of mentorship in student development within STEM fields. By examining the developmental processes and socio-emotional aspects of student experiences in undergraduate research mentorships, this study laid the groundwork for exploring similar mechanisms.

Estrada et al. (2011) reinforced this perspective by showing how STEM research experiences can enhance students' commitment to STEM careers through increased science self-efficacy, science identity, and internalization of scientific community values. They used structural equation modeling to analyze how these factors predict scientific integration for undergraduates. Their study found that while self-efficacy is essential for attracting students to STEM fields, a lasting commitment relies more on developing their science identity and adopting the community's values. These results emphasize the critical roles of identity and values in sustaining a career in science, suggesting that effective integration into the scientific community requires more than just self-efficacy; it also involves a deeper alignment with the community's norms, values, and one's identity.

Aalderen-Smeets (2018) and Syed et al. (2019) both underscored the critical influence of self-efficacy and identity in guiding students toward STEM careers. They argued that students' self-perceptions and beliefs significantly shape their career intentions, with self-efficacy as a key mediator. These findings align with the understanding that experiences in research can bolster students' commitment to STEM through enhancing self-efficacy and identity.

Hernandez et al. (2020) highlighted the pivotal role of faculty mentors in integrating students into STEM communities, facilitating their internalization of STEM values. This study revealed that early social integration within STEM domains exerts a reciprocal, longitudinal impact on students' future interactions with social influence agents, such as research mentors. Specifically, students who experience earlier and greater initial integration are more likely to pursue additional mentorship and engagement opportunities, fostering a cycle of deeper integration. This finding showcases the potential of research mentorships at the high school level. Furthermore, this study emphasized the critical role of social influence processes—including science self-efficacy, identity, and the internalization of community values—in facilitating students' integration into STEM communities and careers. Such processes, shaped significantly by interactions with mentors and other social influence agents, seem crucial to sustaining long-term commitment and persistence within STEM disciplines.

These influential works draw on insights from social cognitive theory, situated learning theory, and related frameworks to provide a more comprehensive understanding of how mentorships can cultivate the next generation of mathematicians and STEM professionals. These theories highlight the critical influence of self-efficacy, identity, and values in STEM students' educational and professional trajectories, enriching our comprehension of community integration and how STEM research mentorships impact students' STEM pathways. Building on these insights, my work employs elements of these theoretical frameworks to inform the application of the Tripartite Integration Model of Social Influences (TIMSI) as a lens for analyzing high school mathematics research mentorships. By examining these processes at the high school level, this study will provide insights into how early research experiences can initiate and strengthen these social influence processes, potentially informing strategies for earlier and more effective integration into mathematics.

2.4 Theoretical Framework

The literature underscores the critical roles of self-efficacy, identity, and values in the success of STEM and mathematics mentorships. This thesis integrates multiple theoretical perspectives to form a comprehensive framework for understanding how high school mathematics research mentorships can facilitate students' integration into the mathematics community. The Tripartite Integration Model of Social Influences (TIMSI) serves as the central theoretical framework of this study. In a mathematics context, it emphasizes that the development of mathematics self-efficacy, identity, and values is essential for successful integration into the mathematics community. Drawing on the literature, I will also incorporate theoretical frameworks that address mathematics self-efficacy, mathematics identity, and values in mathematics. Additionally, Social Cognitive Career Theory (SCCT) will be integrated to complement TIMSI by examining how individuals develop career interests, make educational choices, and achieve professional success (Lent et al., 1994). SCCT posits that career development is influenced by the complex interaction between self-efficacy beliefs, outcome expectations, and personal goals, while accounting for environmental supports and barriers that can facilitate or hinder career progression (Lent et al., 1994). This theoretical framework is particularly relevant as it helps explain how early experiences, such as high school research mentorships, can shape academic and career trajectories through their impact on self-efficacy and other psychological constructs (Lent et al., 1994). This combined approach ensures a nuanced analysis of how mentorship experiences and contextual factors interact to shape students' long-term engagement and success in mathematics. This combined approach provides a more nuanced analysis of how mentorship experiences and contextual factors interact to shape students' long-term engagement and success in mathematics.

2.5 The Tripartite Integration Model of Social Influences

The Tripartite Integration Model of Social Influence (TIMSI) provides a comprehensive framework for understanding how social influences facilitate integration into professional communities. The model posits that successful integration occurs through the development of three interconnected psychological constructs: self-efficacy, identity, and values (Estrada et al., 2011). Research in STEM fields has demonstrated the dynamic interplay of these constructs in supporting long-term engagement and success. For instance, self-efficacy often drives initial engagement, while sustained commitment relies more heavily on identity development and value internalization (Estrada et al., 2011; Hernandez et al., 2020). These findings align with the broader literature in mathematics education, which emphasizes the importance of affective factors such as confidence, identity, and values in shaping student persistence and achievement (Boaler, 2002; Bishop, 2012).

The Evolution of the Tripartite Integration Model of Social Influence

The Tripartite Integration Model of Social Influence (TIMSI) emerged from the foundational work of Kelman (1958, 1961, 2006), whose theoretical model of social influence provided a nuanced understanding of how attitudes, beliefs, and behaviors are shaped within social systems. Kelman's seminal contributions conceptualized three distinct processes—compliance, identification, and internalization—which together form a continuum of influence ranging from superficial behavioral changes to profound internal shifts in values and identity. These processes form the bedrock of TIMSI and offer insights into the mechanisms through which mentorship can shape students' integration into academic and professional communities.

Kelman's Social Influence Theory

Kelman (1958) introduced compliance as the initial level of influence, wherein individuals adjust their public behavior to gain approval or avoid sanctions, often without altering private beliefs. This stage is typically transient, driven by external motivators, and serves as a gateway to deeper forms of influence. As individuals move toward identification, they adopt behaviors and beliefs to establish or maintain meaningful relationships, such as with a mentor or community. Here, social connection and belonging become key motivators, marking a partial internalization of external expectations. Internalization, the most profound level of influence, represents a genuine alignment of attitudes, beliefs, and behaviors with an individual's values. At this stage, the external influence is fully integrated into the individual's core identity and worldview (Kelman, 1961).

Building on this foundational model, Kelman (2006) extended his framework to examine how these processes operate within larger social systems, such as organizations, societies, or academic communities. He introduced the corresponding concepts of rule orientation, role orientation, and value orientation, which map onto compliance, identification, and internalization, respectively. These refinements highlight the applicability of his theory to complex systems and emphasize how individuals navigate and internalize the norms, roles, and values of their social environments.

Adapting Kelman's Framework to TIMSI

Building on Kelman's (1958, 2006) work, Estrada et al. (2011) developed the Tripartite Integration Model of Social Influence (TIMSI) to analyze the integration of college students into the scientific community following research mentorships. Estrada et al. (2011) adapted Kelman's theoretical constructs to align with the specific dynamics of research mentorships in STEM fields. In this adaptation:

(1) Compliance was reframed as science self-efficacy, reflecting the belief in one's ability to perform scientific tasks. (2) Identification was reconceptualized as science identity, representing how individuals perceive themselves as scientists and are recognized as such by others. (3) Internalization became internalization of the values of the scientific community, indicating a deep alignment with the norms and values of the scientific discipline.

This adaptation has been widely applied to undergraduate STEM research mentorships to explain how students integrate into the scientific community, enhancing their self-efficacy, professional identity, and alignment with the values of their discipline (Byars-Winston et al., 2015; Estrada et al., 2018). Longitudinal studies have validated TIMSI's emphasis on the interaction of these constructs in fostering persistence in STEM fields, highlighting their relevance for understanding the mechanisms of professional community integration (Estrada et al., 2018; Hernandez et al., 2017; Robnett et al., 2018).

Extending TIMSI to High School Mathematics

While TIMSI has been extensively applied to undergraduate research mentorships in STEM, its extension to high school mathematics mentorships represents a significant gap in the literature. High school students are at a formative stage in their academic development, making them particularly receptive to mentorship processes that foster self-efficacy, identity, and values alignment (Estrada, 2014). However, mathematics as a discipline presents unique challenges compared to other STEM fields. Its reliance on abstract reasoning, iterative problem-solving, and extended intellectual uncertainty requires tailored mentorship strategies that align with these characteristics (Oehrtman & Lawson, 2007).

Adapting TIMSI to high school mathematics contexts involves reinterpreting its constructs to align with the specific needs and dynamics of the discipline. Figure 2 illustrates this adaptation, emphasizing the developmental pathways through which mathematics research mentorships can foster mathematics self-efficacy, mathematics identity, and mathematics values, ultimately supporting students' integration into the mathematics community.

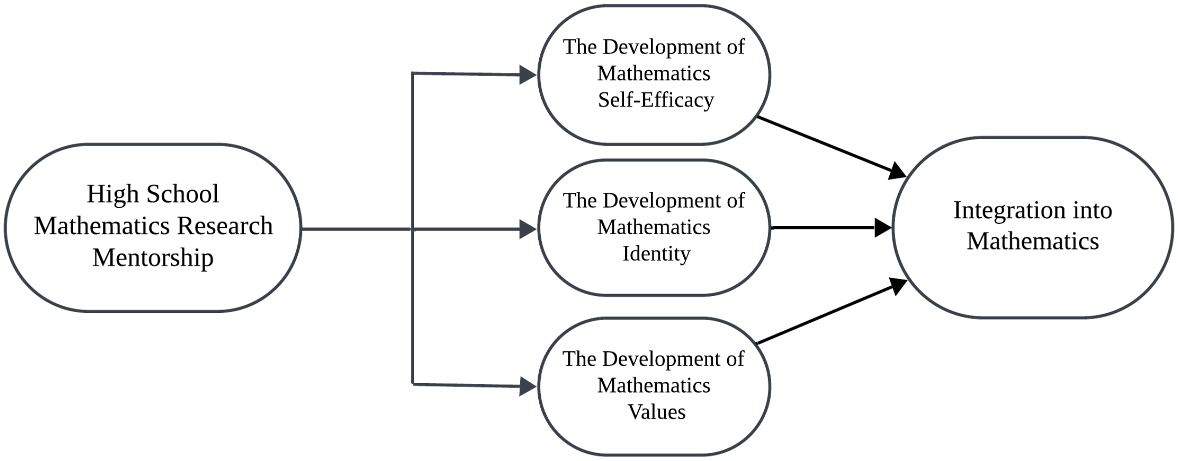


Figure 2: Pathways from High School Mathematics Research Mentorship to Integration through TIMSI Constructs

The constructs of self-efficacy, identity, and values are particularly salient in mathematics education and have been widely discussed in the literature. However, there are ongoing scholarly debates about the precise meanings and boundaries of these constructs (Darragh, 2016; Graven & Heyd-Metzuyanim, 2019).

Self-efficacy, as conceptualized by Bandura (1997), refers to an individual's belief to perform tasks and overcome challenges. Mathematics self-efficacy specifically relates to a person's perceived capability to perform mathematical tasks and overcome challenges in mathematics (Hackett & Betz, 1989).

Mathematics identity, is a multifaceted construct that encompasses an individual's self-perception, social recognition, and experiences within the field of mathematics. It includes both personal narratives about one's history with mathematics and the performative aspects of engaging with the subject, such as self-assessments of being a "math person" and the influence of recognition by peers and educators (Anderson, 2007; Cribbs et al., 2022; Graven & Heyd-Metzuyanim, 2019; Sfard & Prusak, 2005). This identity is understood to be dynamic, evolving over time through continuous experiences and beliefs about competence and innate ability (Anderson, 2007; Darragh, 2016; Wenger, 1998). These constructs—self-efficacy and identity—are critical as they influence students' motivation and persistence in challenging academic contexts (Boaler & Greeno, 2000; Cobb et al., 2009; Estrada et al., 2011; Hernandez et al., 2018; Schukajlow et al., 2017; Yıldız et al., 2019a). Recent research highlights the interplay between these constructs, suggesting that self-efficacy can significantly impact the development of mathematics identity (Alexander, 2015; Bohrnstedt et al., 2021; Hannula et al., 2014; Radišić et al., 2024). Values, relate to the perceived importance of mathematics in one's life (Dede et al., 2024). Values in mathematics education are not static; they evolve based on the context in which mathematics is taught and learned, and they profoundly impact student engagement, motivation, and overall learning outcomes (Tambunan & Yang, 2022; Yang et al., 2023).

The constructs of mathematics self-efficacy, identity, and values are deeply interconnected. A strong sense of self-efficacy enhances a student's confidence in engaging with mathematics and significantly predicts achievement across all grade levels (Yang et al. 2024). Identity influences how students see themselves within the mathematical community, with early experiences playing crucial roles in shaping mathematical self-perception (Anderson, 2007; Bishop, 2012). Both are shaped by the values they internalize through education and experiences, particularly through interactions with mentors and other social influence agents (Estrada et al., 2011; Hernandez et al., 2018). These constructs work together to build a framework for understanding how high school mathematics research mentorships can foster long-term integration.

Addressing Gaps in the Literature

Despite TIMSI’s extensive application to undergraduate STEM mentorships, there is limited research on its relevance to high school contexts, particularly in mathematics education. Longitudinal studies have highlighted the importance of TIMSI constructs in sustaining engagement and success in STEM fields (Estrada et al., 2018; Byars-Winston et al., 2015), but their developmental pathways in younger students remain underexplored. High school mathematics mentorships, with their potential to bridge the gap between classroom learning and integration into professional communities, offer a critical opportunity to extend this framework (Boaler, 2002; Bishop, 2012). Exploring how mentorship fosters the development of mathematics self-efficacy, identity, and values at this stage could provide valuable insights into the socio-emotional and cognitive processes underlying students’ long-term engagement and success.

Given the focus on mathematics rather than science, further delineation of TIMSI’s constructs within a mathematical context is essential. Mathematics education literature provides valuable insights into the development of mathematics self-efficacy, mathematics identity, and mathematics values. While Kelman's original theory and subsequent TIMSI research presents self-efficacy, identity, and values as distinct pathways to integration, mathematics education research suggests more complex interactions between these constructs (Alexander, 2015; Bohrnstedt et al., 2020; Hannula et al., 2016). This tension between discrete versus interconnected development of these constructs warrants further investigation, particularly in mathematics contexts. Examining these theoretical tensions through the lens of high school mathematics research mentorships, could contribute to a more nuanced understanding of how these constructs develop and interact in supporting mathematical integration. Accordingly, the next sections of this literature review will delve into these constructs, refining their definitions and exploring their implications for student development and mentorship in mathematics.

2.6 Mathematics Self-Efficacy

Self-efficacy, a concept pioneered by Bandura (1977), is crucial in understanding educational achievement. Self-efficacy fundamentally influences how individuals approach goals, tasks, and challenges (Bandura, 1977, 1982, 1994). In mathematics education, self-efficacy significantly impacts persistence, effort, and resilience in facing mathematical difficulties (Pajares & Miller, 1994; Zimmerman, 2000). Previous research has established self-efficacy as a pivotal determinant of mathematics achievement, correlating strongly with enhanced outcomes and educational success (Hackett, 1985; Hackett & Betz, 1989; Lent et al., 1997; McLeod, 1992; Pajares & Graham, 1999; Pajares & Miller, 1994; Pietsch et al., 2003). High self-efficacy in mathematics is linked to greater persistence, higher achievement, and an increased likelihood of pursuing STEM careers, emphasizing its critical role in students' success (Diseth, 2011; Honicke & Broadbent, 2016; Komarraju & Nadler, 2013).

The importance of self-efficacy in mathematics extends beyond just task performance. It is a multidimensional construct that varies depending on the specific mathematical content, task difficulty, and situational factors, making it a dynamic and context-sensitive belief system (Street et al., 2024). For instance, research highlights that task-specific self-efficacy is particularly relevant in educational settings that emphasize reform-oriented instructional practices, such as those promoted by the National Council of Teachers of Mathematics (NCTM), which focus on conceptual understanding and inquiry-based learning (Cribbs et al., 2015; Zakariya et al., 2020). Such practices are crucial for fostering self-efficacy in creative and conceptual mathematical tasks, including those involved in mathematics research (Boaler, 2002).

Self-efficacy is considered an affective component of students' academic experience and motivation. Research shows that positive affective factors such as interest, motivation, and beliefs about the usefulness and importance of mathematics are strongly correlated with improved performance and future engagement in the subject (Grootenboer & Marshman, 2016; Schukajlow et al., 2019; Hannula, 2014). These factors are interrelated, leading some researchers to consider the simultaneous impacts of multiple affective factors (Alexander, 2015; Schindler & Bakker, 2020). Schindler and Bakker (2020) introduced the concept of an "affective field," comprising an individual's full set of affective factors, arguing that students' affects are more appropriately viewed holistically rather than individually. This perspective aligns with the understanding that self-efficacy is intertwined with various motivational and emotional processes, serving as a key determinant of how students interpret and respond to their educational experiences (Street et al., 2024).

Self-efficacy, rooted in Bandura's Social Cognitive Theory, refers to one's beliefs in their capabilities to organize and execute the actions required to manage prospective situations (Bandura, 1997). Unlike self-esteem or general self-confidence, self-efficacy is domain-specific, requiring separate consideration of mathematics self-efficacy when examining educational outcomes (Bandura, 1997). This domain-specific nature underscores the need for precise interventions and instructional practices that address students' unique challenges in mathematics, particularly in high-stakes or complex mathematical tasks such as research (Boaler, 2002; Pajares & Miller, 1994).

Bandura (1977) identified four primary sources of self-efficacy: mastery experiences, vicarious experiences, social persuasion, and physiological states. Among these, mastery experiences are particularly influential in boosting students' confidence and competence, as

direct success strengthens self-efficacy beliefs (Usher & Pajares, 2006). While these sources are distinct, they often overlap and interact, particularly in academic settings where complex tasks like mathematics can simultaneously engage multiple sources of self-efficacy. For example, observing peers succeed—vicarious experiences—can enhance a student's belief in their own abilities, especially when those peers serve as relatable models (Pajares, 2002; Schunk, 1987). Research underscores the dominance of mastery experiences over other sources, particularly in academic contexts like mathematics, where prior successes significantly contribute to higher self-efficacy (Lent et al., 1991; Lopez & Lent, 1992). Özdemir and Pape (2013) highlight how mastery experiences are crucial in fostering adaptive self-efficacy, especially for students confronting varied levels of challenge. Similarly, Yildiz et al. (2019b) found that mastery experiences, alongside social persuasion, are instrumental in shaping math self-efficacy among elementary students, with less emphasis on physiological states or vicarious experiences.
The development of self-efficacy is shaped by task difficulty, effort, and situational conditions. Success in challenging tasks strengthens self-efficacy, whereas excessive assistance or external obstacles can weaken it (Bandura, 1997; Street et al., 2024). Moreover, witnessing others succeed—particularly those viewed as relatable—enhances self-efficacy by offering social models and reducing uncertainty (Bandura, 1997). Social persuasion, in the form of specific and positive feedback, reinforces belief in one's abilities, while physiological and emotional states, such as stress and anxiety, also play critical roles in influencing self-efficacy beliefs (Yildiz et al., 2019a; Pajares, 2002).

Self-efficacy regulates behavior through cognitive, motivational, affective, and selection processes. Higher self-efficacy leads to setting challenging goals, persistence despite obstacles, and effective coping strategies (Bandura, 1994). These processes directly and indirectly influence performance by enhancing motivation, goal-setting, and resilience (Schunk, 2012). Motivational processes derived from attribution theory, expectancy-value theory, and goal theory are moderated by self-efficacy beliefs. Individuals with higher self-efficacy attribute failures to effort rather than ability, set challenging goals, and sustain motivation through proactive goal adjustment (Bandura, 1994).

The task-specific nature of self-efficacy (Liu et al., 2020b) highlights its importance in academic achievement, where tailored instructional practices, such as those recommended by the NCTM, promote conceptual understanding and inquiry-based learning (Sowder, 1989). These practices foster task-specific self-efficacy, which is particularly critical in creative and conceptual mathematical tasks like mathematics research (Boaler, 2002). Research indicates that self-efficacy, influenced by sustained collaborative and inquiry-based learning, significantly contributes to students' confidence in reform-oriented tasks (Bicer et al., 2020; Schindler & Bakker, 2020). Moreover, Liu et al. (2020a) demonstrate how academic self-efficacy influences not only performance but also procrastination, underscoring the complex relationship between self-belief and academic behavior in various contexts, including mathematics.

Mathematics self-efficacy and identity are closely linked, influencing students' engagement, performance, and persistence in mathematics (Alexander, 2015, Berry, 2008; McGee, 2015). Research suggests that self-efficacy moderates mathematics identity development and mediates relationships between students' learning experiences and long-term career interests (Alexander, 2015; Syed et al., 2019). For example, Alexander (2015) found that students' self-efficacy beliefs informed their pathways toward mathematics identity development, specifically by moderating students' senses of school belonging, which significantly predicted shifts in mathematics identity.

Given the foundational impact of self-efficacy on educational outcomes, this study advocates for further exploration into how various educational practices can influence these critical beliefs. Such research can offer invaluable insights into designing effective interventions that enhance mathematics self-efficacy, thereby fostering a solid foundation for students' success in mathematics and beyond. The intricate dynamics of personal, behavioral, and environmental factors play a crucial role in the development of self-efficacy, reflecting the core principles of Bandura's (1986) Social Cognitive Theory (SCT). This theory elucidates how these interconnected elements collectively influence an individual's belief in their capabilities, providing a comprehensive framework for understanding the development of self-efficacy within educational contexts.

Research Self-Efficacy (RSE)

Research Self-Efficacy (RSE) is an individual's confidence in their ability to execute the tasks required for conducting research effectively (Forester et al., 2004). This confidence is a foundational element influencing career decisions in the STEM research domain (Livinți et al., 2021). Within the framework of Social Cognitive Career Theory (SCCT), RSE has been investigated both as an outcome influenced by contextual factors, such as the quality of the research training environment (Chumwichan & Siriparp, 2016), research mentorship experiences (Hollingsworth & Fassinger, 2002), and as an antecedent that predicts key outcomes, such as research productivity (Morrison & Lent, 2014; Quimbo & Sulabo, 2014), expectations of research outcomes (Bard et al., 2000), research interest (Eam & Seng, 2017; Lambie et al., 2014), intentions toward a research career (Frantz et al., 2017; Epstein & Fischer, 2017), engagement in future research activities (Eam & Seng, 2017; Eke et al., 2012), aspirations for graduate education (Tate et al., 2015), and researcher identity (Albold, 2011).

The construct of RSE is critical for my research on mentorships in high school mathematics, as it shifts the focus from general mathematical confidence to a more targeted confidence in conducting mathematical research. This nuanced view of self-efficacy could profoundly affect a student's path in the field of mathematics. Livinți et al. (2021) illustrate that a well-developed sense of RSE is strongly associated with favorable research outcomes, ranging from increased productivity to enhanced involvement in scientific discovery. Moreover, this research emphasizes the importance of fostering RSE early in students’ academic careers, suggesting that targeted mentorship programs that build RSE can lead to long-term benefits in students’ professional trajectories (Morrison & Lent, 2014; Quimbo & Sulabo, 2014).

Incorporating RSE into the framework of high school mathematics research mentorships can provide a robust foundation for students to develop their mathematical skills and the confidence necessary to pursue advanced research in STEM fields. This connection between RSE and future career outcomes highlights the potential impact of well-structured mentorship programs on students' long-term success in mathematics-related careers, particularly in fostering the resilience and perseverance required to navigate the challenges of mathematical research (Hollingsworth & Fassinger, 2002; Livinți et al., 2021).

Further research is needed to explore how different types of mentorship models can be optimized to enhance RSE among high school students, particularly those from underrepresented groups in STEM fields (Robnett et al., 2018). Understanding the specific mentorship practices that most effectively build RSE could inform the design of interventions aimed at increasing diversity and inclusion in mathematics and other STEM disciplines (Tate et al., 2015; Frantz et al., 2017). This line of inquiry is crucial for developing strategies that support the growth of a diverse, confident, and capable generation of future mathematicians and researchers.

2.7 Mathematics Identity

This study merges several theoretical perspectives to examine how mathematics identity and self-efficacy interact to influence students' mathematical journeys. By analyzing the development of mathematics identity through sociocultural and psychological theories, I seek to uncover how educational experiences, particularly in secondary school mathematics, contribute to forming a robust and positive mathematics identity. Such insights are crucial for designing educational interventions that enhance mathematical skills and nurture a sense of belonging and competence among students, thereby supporting a diverse and resilient pipeline of talent in mathematics and related disciplines (Alexander, 2015; Estrada et al., 2011; Hernandez et al., 2020).

Mathematics Self-Efficacy and Mathematics Identity

Mathematics identity and self-efficacy are deeply intertwined, with each influencing the other and collectively shaping students' engagement, performance, and persistence in mathematics (Boaler & Greeno, 2011; Cobb et al., 2009). Research by Bishop (2012), along with contributions from Boaler and Greeno (2000), Cobb et al. (2009), and Martin (2006), underscores the significant impact that an individual's identity in mathematics has on their learning processes and achievement levels. These studies collectively highlight how students' perceptions of themselves as capable mathematics learners are crucial in shaping their engagement, persistence, and success in mathematical contexts. Understanding how mathematics identity is developed and influenced becomes paramount in fostering students' mathematical achievements. This exploration into mathematics identity will reveal how students' perceptions of themselves, influenced by their experiences and achievements in mathematics, contribute to their continued interest and success in the field. The interplay between self-efficacy and mathematics identity underscores a complex web of cognitive and affective factors that educational interventions must address to support students' holistic development in mathematics (Sfard & Prusak, 2005; Cobb et al., 2009).

Building upon the foundation of self-efficacy in mathematics, it is essential to delve into the concept of mathematics identity—a construct that encapsulates how students see themselves as participants in the world of mathematics. These identities are not static; they are influenced by a complex web of cognitive and social processes, individual self-beliefs, and the broader educational ecosystem (Martin, 2000; Boaler, 2002). Mathematics identity is a deeply held belief and recognition of oneself as a capable participant in the mathematical community, shaped by the interaction of personal experiences, social influences, and educational practices within mathematical contexts (Bishop, 2012; Op't Eynde & Hannula, 2006).

Research Identity in STEM

In addition to the concept of mathematics identity, the notion of research identity has become increasingly important in STEM fields. Research identity is defined as the totality of traits such as confidence, logical thinking, the ability to design experiments, interpret results, and the desire to succeed in authentic research (Marvasi et al., 2019). Developing a strong research identity, much like mathematics identity, is crucial for students' persistence and success in STEM careers. While research identity often remains elusive in early academic stages, authentic research experiences are pivotal in fostering this identity, particularly through hands-on learning and collaborative problem-solving (Jorgensen & Duncan, 2015; Marvasi et al., 2019; Murakami-Ramalho et al., 2008).

In the context of mathematics education, integrating authentic research experiences in secondary school mathematics mentorships could similarly foster a mathematics research identity. This identity, like general research identity in STEM, may develop through students' active engagement in research activities that mirror the practices of professional mathematicians. Such experiences can help students build confidence in their mathematical abilities, enhance their logical and critical thinking skills, and strengthen their self-concept as future contributors to the field of mathematics (Aikens et al., 2016; Pratt et al., 2006; Thompson & Jensen-Ryan, 2018).

Identity Theories and Perspectives

To effectively examine the specific ways in which high school mathematics research mentorships influence students' perceptions of themselves as mathematics learners, it is crucial to ground this discussion within a broader theoretical understanding of identity formation. This approach not only enriches our comprehension of the complex dynamics at play in mathematics education but also highlights the transformative potential of mentorship in shaping positive mathematics identities (Darragh & Radovic, 2019; Martin, 2006; Wilson et al., 2023).

This study draws on the foundational psychosocial development theory of Erikson (1950), which posits adolescence as a critical period for identity formation, emphasizing the role of mentorship in fostering positive identity development within the realm of mathematics. Gee's (2001) discourse on identity within social contexts provides a nuanced framework for examining how mentorships may validate and reinforce students' identities within the mathematical community, highlighting the importance of recognition and belonging. Complementing this, the situated learning framework proposed by Lave and Wenger (1998) conceptualizes mentorships as communities of practice, where learning is a social endeavor that significantly contributes to the shaping of mathematical identities through participation.

The integration of Pratt's (2006) professional identity development model, particularly the concepts of work-learning and identity-learning cycles, into the study of mathematics identity further enriches our understanding. Similar to early career researchers in science who engage in these cycles to assess their identity as researchers (Carlone & Johnson, 2007; Hazari et al., 2010), students in mathematics mentorship programs navigate these cycles to develop their mathematics identity. Through authentic engagement in mathematical practices, students continuously assess the alignment of their work with their identity as mathematicians, influenced by the recognition they receive from mentors and peers (Cribbs et al., 2015).

Collectively, these theoretical perspectives underscore the complex interplay between individual and communal factors in the development of mathematical identities, providing a rich scaffold for understanding how high school mathematics research mentorships can serve as pivotal experiences in integrating students into the mathematics community, thereby influencing their self-efficacy, identity, and values in profound ways (Anderson, 2007; Grootenboer & Edwards-Groves, 2018; Wenger, 1998).

Mathematics Identity and the Role of Context

The construct of identity, foundational to our understanding of students' experiences in mathematics, offers a lens through which we can explore how individuals come to see themselves as part of the mathematical community. Identity formation is a pivotal aspect of human development, involving a complex process that encompasses an individual's values, abilities, strengths, weaknesses, aspirations, and motivations, all while being influenced by external social factors such as family, community, and society (Beijaard, Meijer & Verloop, 2004; Cieciuch & Topolewska, 2016; Erikson, 1950; Marcia, 1960).

Scales and Leffert (2004) conceptualize identity as an amalgamation of self-concept, beliefs, capacities, roles, and personal history, suggesting that identity formation is not merely an internal process but unfolds through dynamic interactions with a broader social and cultural context. This framework underscores the significance of family dynamics, peer influences, educational experiences, and wider societal norms in molding and continuously reshaping an individual's sense of self (Holland et al., 1998). This perspective highlights the interconnectedness of various social dimensions, including the impact of cultural narratives, social expectations, and institutional practices on the development of personal and social identities.

Transitioning to mathematics identity, this broader conceptualization of identity serves as a vital backdrop for understanding mathematics identity within a continuum of social identity formation. It underlines the importance of the socialization processes that students undergo in mathematics classrooms (Boaler & Greeno, 2011). Mathematics identity thus encompasses not only students' self-perceptions as learners of mathematics but also their navigation and negotiation of their place within the mathematical community, mediated by classroom interactions, societal expectations, personal mathematical experiences, and aspirations (Boaler & Greeno, 2011; Graven & Heyd-Metzuyanim, 2021).

As we explore the development of mathematics identity among high school students, the frameworks provided by general identity theories offer invaluable insights. They illuminate the ways in which identities are constantly being formed and reformed through engagement in mathematical practices, the influence of institutional and societal expectations, and the power of imagination in shaping one's mathematical trajectory (Anderson, 2007; Sfard & Prusak, 2005; Wenger, 1998).

Mentorship in high school mathematics research plays a crucial role in shaping students' mathematics identities. The theoretical frameworks discussed above, including Erikson's theory of identity formation, Gee's discourse analysis, and Sfard and Prusak's narrative identity, provide a rich scaffold for understanding how these mentorships influence students' integration into the mathematical community. By fostering a strong mathematics identity, these mentorships can significantly impact students' self-efficacy, engagement, and persistence in the field (Boaler & Greeno, 2011).

Research indicates that positive recognition from mentors and peers within mathematics mentorship programs significantly reinforces students' identities as mathematicians. Conversely, the absence of such recognition can impede their identity development (Aikens et al., 2016; Thompson & Jensen-Ryan, 2018). Therefore, effective mentorship should encompass strategies that not only advance students' mathematical skills but also actively acknowledge and validate their contributions, thereby fostering a strong and enduring mathematics identity.

In synthesizing these theoretical perspectives, this study constructs a comprehensive framework to understand how mathematics identity is cultivated and sustained through various educational practices, with a particular emphasis on mentorship programs. By investigating the complex interplay between identity theories, contextual factors, and the crucial role of mentorship, this research seeks to identify the key elements of successful high school mathematics mentorships that contribute to the cultivation of a robust, positive mathematics identity among students. Such interventions are expected to significantly enhance students' engagement, persistence, success, and integration into the mathematical community.

Understanding the formation and development of mathematics identity provides crucial insights into how students perceive themselves and their roles within the mathematical community. However, to fully grasp the factors that influence students' engagement and persistence in mathematics, it is equally important to examine the values they associate with the subject. Values play a pivotal role in shaping attitudes, motivations, and behaviors, ultimately influencing how students relate to mathematics and envision their future pathways within the field.

The relationship between mathematics identity and values is intrinsically linked. As students develop their mathematical identities, they also cultivate a set of values that guide their decision-making processes, commitment to learning, and perceptions of the utility and relevance of mathematics in their lives. Values such as the importance of rigorous logical reasoning, the role of research in advancing knowledge, and the intellectual satisfaction derived from mathematical discoveries become integral to their identity as mathematicians (Boaler, 2002; Chapman & Lim, 2013; Schoenfeld, 1992).

2.8 Values of the Mathematics Community

By exploring these values, we can gain a deeper understanding of the factors that contribute to students' sustained interest and success in the subject. This exploration will provide insights into how values influence students' mathematical journeys, guiding their engagement, persistence, and the formation of a strong mathematics identity. The literature highlights various faucets of values, including the importance of rigorous logical reasoning and proof-based discussions in developing new mathematical theories (Boaler, 2002; Schoenfeld, 1992), the role of mathematical research in advancing knowledge and solving complex problems (NCTM, 2000), the excitement and intellectual satisfaction derived from mathematical discoveries (Polya, 1945; Lockhart, 2009), the potential of mathematical research to address and solve real-world challenges (Steen, 2001; Dallas, 2023), the appreciation of the beauty and elegance of mathematical concepts and solutions (Hardy, 1940), and a genuine love and passion for mathematics as an intellectual pursuit (Lockhart, 2009).

Values in mathematics education are not static or uniform; they vary significantly depending on the context in which mathematics is taught and learned. Bishop (2008) highlights how values are crucial components of classroom environments, influencing student engagement with mathematics. He identifies six value clusters—rationalism, objectivism, control, progress, openness, and mystery—that structure the values associated with mathematics education. These clusters emphasize the importance of logical reasoning, the development of knowledge, and the intellectual joy derived from mathematical inquiry, aligning with the values essential to students in research mentorships. Notably, Bishop points out that while both mathematics and science share some values, mathematics places a stronger emphasis on rationalism and control, whereas science values empiricism and human intuition.

Further, the literature on values in mathematics education underscores their profound impact on student engagement, motivation, and overall learning outcomes (Tambunan & Yang 2022). Values, often implicit in teaching, permeate all levels of mathematics education, influencing both pedagogical approaches and students' perceptions of the subject. For instance, some educators view mathematics as a core discipline within STEM, essential for understanding and solving problems in science, technology, and engineering. This "core" orientation, as identified by Ortiz-Revilla et al. (2020), positions mathematics at the heart of STEM education, where it is seen as fundamental to the development of problem-solving skills and logical reasoning. On the other hand, the "peripheral" orientation sees mathematics as a supporting discipline, often overshadowed by other STEM fields. This perspective can diminish the perceived value of mathematics, potentially affecting student engagement and achievement (Shaughnessy, 2013).

Seah (2019) expands on this by introducing the Tripartite Model of the Human Mind, which integrates cognition (thinking), affect (feeling), and conation (willing). According to Seah (2019), values play a critical role as part of the conative component, driving students' motivation and persistence in mathematical tasks. This model emphasizes how these components work together in the learning process, underscoring the importance of values in shaping student engagement with mathematics (Eccles et al., 1983; Schwartz, 2012). Seah also proposes a structured approach to developing these values within the classroom, suggesting that explicit discussions about values can significantly enhance students' internalization of them. This aligns with the broader educational framework, which sees values as not only cognitive or affective but also conative, deeply influencing students' motivation (Seah & Wong, 2012).

The role of values in fostering student engagement is further explored by Dede et al. (2024), who argue that understanding the intrinsic value of mathematics can lead to greater student appreciation and interest in the subject. This perspective is particularly relevant in high school research mentorships, where students are often exposed to advanced mathematical concepts and research practices for the first time. By helping students see the broader value and application of mathematics, mentors can enhance their engagement and inspire them to pursue further studies in the field (Atweh & Wee, 2008).

The literature on values in mathematics education underscores the critical role these values play in shaping students' mathematical identities and engagement. In high school mathematics research mentorships, these values are not merely abstract concepts but are fundamental elements that influence how students approach mathematical problems, interact with mentors, and perceive their potential within the field of mathematics. Internalizing these values can lead to more meaningful and sustained engagement, ultimately contributing to the formation of a strong mathematical identity and the pursuit of advanced studies or careers in mathematics (Seah, 2019; Fan, 2021).

While much existing research has focused on quantitatively measuring factors like mathematics self-efficacy and its correlation with academic achievement, there is a notable gap in the literature regarding the qualitative exploration of these concepts. Specifically, few studies have examined how the interactions between values, self-efficacy, and identity formation unfold within the context of mentorship. As Yetkin Özdemir and Pape (2013) pointed out, most studies have relied on quantitative methods to establish correlations between self-efficacy and traditional measures of achievement. However, such approaches may overlook the nuanced and complex ways in which values, self-efficacy, and identity are experienced and expressed by students within the mentorship environment.

The interactions between students and mentors in mathematics research mentorships are complex and deeply embedded in the development of self-efficacy, identity, and values. Research suggests that qualitative methods are particularly effective in capturing the nuances of such interpersonal dynamics, allowing for a deeper exploration of subjective experiences in educational contexts (Creswell & Poth, 2017). In the context of STEM education, frameworks like TIMSI offer a valuable lens for understanding how mentorship can support students' long-term engagement with mathematics and science (Hernandez et al., 2018). Despite its potential, TIMSI has been underutilized in studies of mathematics education and qualitative research. The current literature largely focuses on quantitative measures of self-efficacy, identity, and values in

STEM fields, leaving a gap in understanding the processes through which mentorship fosters these outcomes (Estrada et al., 2011; NASEM, 2017).

The following chapter details the methodology of this study, outlining the qualitative research design that will be employed to explore these complex interactions and processes within the mentorship context.

Chapter 3: Methodology

3.1 Introduction

This chapter outlines the methodology used in this dissertation, which investigates the lived experiences of participants during their high school mathematics research mentorships and their subsequent integration into the mathematics community. The study employed Interpretative Phenomenological Analysis (IPA) as its primary methodological approach, complemented by exploratory elements to capture the multifaceted nature of participants' developmental journeys. The goal of this study is to investigate the factors and developmental processes most influential in the long-term integration of individuals into the mathematics profession, focusing on students who participated in high school mathematics research mentorships. By examining both the strengths and limitations of these mentorship programs, alongside other significant personal, educational, and professional factors, this research aims to address the shortage of U.S. mathematics graduate students by uncovering mechanisms that foster sustained engagement with mathematics. Additionally, this study seeks to advance theoretical understanding by applying and extending the Tripartite Integration Model of Social Influence (TIMSI) framework within the context of high school mathematics mentorships. This theoretical contribution aims to enhance our understanding of how self-efficacy, identity, and values interact and develop within mentorship experiences to facilitate integration into the mathematics community.

This chapter outlines the research design, participant selection, data collection methods, analysis strategies, and the study's limitations and credibility measures.

The study is guided by the following research questions:

1. What factors influenced participants' decisions to engage in high school mathematics research mentorships, and what personal meaning do they assign to these decisions?

2. How do participants describe and make sense of their lived experiences during their high school mathematics research mentorships and their integration into the mathematics community? Specifically, what mentorship processes and interactions shape participants' self-efficacy, identity, and values, and how do these experiences facilitate their integration into mathematics?

3. What additional factors, beyond self-efficacy, identity, and values, do participants identify as significant from their high school mathematics research experiences, and how do these factors contribute to their integration into mathematics?

4. What factors beyond the mentorship do participants perceive as most significant in their integration into mathematics, and how do they interpret the role of these factors in shaping their academic and professional trajectories? How, and to what extent, do these interpretations align with the Tripartite Integration Model of Social Influence?

Pilot Study

The primary purpose of my pilot study was to conduct a qualitative case study on a mathematics professor with over three decades of high school research mentoring experience in New York. Through interviews, observations, surveys, and artifact analysis, this investigation yielded three key methodological insights that shaped the design of the current study:

1. The observation that students with varying levels of mathematical ability benefited from mentorships, aligning with Ledder's (2022) research on the potential of 'mid-tier' students in mathematics research, led to developing an interview protocol that explored participants' mathematical backgrounds prior to their mentorship experiences.
2. Students' increased engagement and enthusiasm during mentorship sessions, despite fatigue from school, suggested the importance of examining lived experiences and affective components of the mentorship. These observations influenced the decision to

employ Interpretative Phenomenological Analysis (IPA) as the methodological approach and informed the selection of TIMSI as the theoretical framework, as both allow for examination of psychological and developmental processes.

3. The mentor's approach to fostering research competencies through research techniques, collaborative writing, and presentations informed Research Question 3, which investigates these broader developmental factors and their role in shaping participants' long-term engagement with mathematics, while also enabling a more nuanced examination of mentorship differences and approaches among participants.

These insights informed both the methodological approach and theoretical framework of the current study, enabling a more comprehensive examination of how mentorships influence students' integration into mathematics.

3.2 Executive Summary of Methodology

1. Research Design

This study employs Interpretative Phenomenological Analysis (IPA) to examine how high school mathematics research mentorships influence participants' long-term integration into the mathematics community. The investigation focuses on eight participants (aged 23-27) who completed high school mathematics research mentorships and subsequently established careers in mathematics-related fields.

2. Participant Selection and Recruitment

Through purposeful sampling, participants were recruited via two primary channels: the Society for Science's records of Science Talent Search participants (2015-2019) and MIT PRIMES program alumni. Additional participants were identified through established mentor networks using snowball sampling, ensuring a diverse participant pool while maintaining focus on individuals who successfully integrated into the mathematics community.

3. Data Collection

The primary data collection method centered on semi-structured interviews (45-75 minutes) exploring participants' experiences chronologically: pre-mentorship, during mentorship, and post-mentorship. Following Smith et al.'s (2009) recommendations for IPA doctoral studies, one participant underwent an in-depth case study with additional data collection methods, including document analysis and a mentor interview.

4. Data Analysis

Analysis followed IPA's systematic approach, beginning with detailed transcript validation and immersive reading. The process progressed through three levels: descriptive comments capturing content, linguistic comments examining language patterns, and conceptual comments exploring deeper meanings. This led to the identification of Personal Experiential Themes (PETs) for each participant, followed by cross-case analysis to develop Group Experiential Themes (GETs).

5. Quality Assurance and Findings

Quality assurance measures included member checking, peer review, and continuous reflexive practice.

3.3 Methodological Approach

The phenomenon under investigation is the lived experiences of students during high school mathematics research mentorships and how these experiences, along with the developmental processes participants underwent, shaped their academic and professional trajectories. This research defines 'lived experiences and developmental processes' as the participants' personal reflections and interpretative meaning-making that influenced their development of mathematics self-efficacy, identity, and values—constructs that the literature highlights as important processes for integration into the field of mathematics (Boaler & Greeno, 2000; Cobb et al., 2009; Estrada et al., 2011; Hernandez et al., 2018; Schukajlow et al., 2017). Given the focus on subjective experiences and personal interpretations, IPA provides a robust framework for exploring not only the participants' lived experiences but also how they make sense of these experiences in relation to their evolving self-efficacy, identity, values, and integration into the mathematics community.

IPA is a qualitative research methodology that focuses on understanding how individuals make sense of their lived experiences, particularly in relation to significant life events or processes (Larkin et al., 2006). IPA is particularly well-suited for this study for several reasons: (1) IPA leverages in-depth interviews to capture participants' personal reflections on their mentorship experiences. Its idiographic approach emphasizes the unique meanings participants assign to their experiences, allowing for nuanced insights into how mentorship influenced their self-efficacy, identity, and values (Smith et al., 2009; Moses & Knutsen, 2019). This ensures participants' perspectives remain central to the analysis.

(2) IPA employs double hermeneutics, where both participants and the researcher engage in the sense-making process (Smith, 2007). While participants interpret their experiences, I, the researcher, interprets these interpretations, providing a dual-layered understanding that underpins the entire analytical process. This phenomenological approach shifts the focus away from generalizable findings to understanding how participants' unique experiences contribute to their long-term academic and professional outcomes (Smith et al., 2009).

(3) IPA's flexible coding approach focuses on meaningful sections of text rather than rigid line-by-line coding (Smith et al., 2022), enabling nuanced analysis of participants' sense-making processes. Peoples (2020) claims IPA is best suited for researchers seeking to understand a particular experience and how individuals make sense of that experience in their own context.

While the study primarily uses IPA, it also incorporates exploratory elements to investigate factors beyond the mentorship experience that contributed to participants' integration into mathematics. This approach allows for a comprehensive examination of the developmental processes participants underwent, recognizing that the high school research mentorship is part of a larger context influenced by various factors such as family, school experiences, and personal characteristics.

Given the complexity of participants' trajectories, elements of case study methodology are integrated to examine the broader context in which the research mentorship occurred (Yin, 2014). Each participant’s story is treated as a unique case, and cross-case analysis, consistent with IPA's approach, is employed to identify patterns and themes across participants. This dual-layered approach allows the study to explore not only the individual impacts of mentorship but also how various factors—whether pre-mentorship, during, or post-mentorship—interacted with the mentorship experience to shape participants’ integration into the mathematics community. Although the literature emphasizes the importance of these research mentorships (Hernandez et al., 2018; Robnett et al., 2015; Pfund et al., 2016), I remained open to whatever participants expressed as significant. This approach acknowledges that a range of influences beyond the immediate mentorship experience may play a crucial role in shaping long-term integration into the field.

Research Design

The research design of this study is based on Interpretative Phenomenological Analysis (IPA), a qualitative methodology that seeks to explore how individuals make sense of their personal experiences (Larkin et al., 2006). IPA informs the entire research process, encompassing both data collection and analysis (Smith et al., 2009).

The primary data collection method was in-depth semi-structured interviews, which provided participants the opportunity to articulate their experiences and personal meanings attributed to their high school mathematics research mentorships and their integration into the mathematics community (Smith et al., 2009).

The essence of IPA's analysis lies in its emphasis on exploring how individuals interpret their lived experiences. IPA employs a structured yet flexible approach that transitions from specific instances to shared experiences and from descriptive accounts to interpretative analyses (Smith et al., 2022). This process allows for a rich and nuanced understanding of participants' experiences and their meaning-making within specific contexts (Reid et al., 2005).

The following are Smith et al.'s (2022) eight-step guidelines for data analysis:

1. **Reading and Re-reading**: Each transcript was reviewed multiple times while simultaneously listening to interview recordings. This step ensured accuracy and captured nuances such as tone, inflection, and emotional context, which were crucial for understanding participants' lived experiences.
2. **Initial Noting**: Exploratory notes were made to capture descriptive, linguistic, and conceptual observations. Descriptive comments focused on participants' accounts of their mentorship experiences, linguistic comments analyzed language use and patterns, and conceptual comments emerged through iterative readings, offering interpretative insights

into the deeper meanings within participants' narratives. These conceptual comments explored underlying patterns, theoretical connections, and implicit meanings that weren't immediately apparent in participants' explicit statements.

3. **Developing Experiential Statements (Emergent Themes)**: Themes were generated by synthesizing the exploratory notes into coherent ideas that reflected the essence of participants' experiences. These emergent themes (Personal Experiential Themes, or PETs) captured the most significant aspects of each participant's narrative.
4. **Searching for Connections**: Themes were organized into a cohesive narrative for each participant by employing IPA techniques such as abstraction (grouping similar themes), polarization (highlighting contrasts), contextualization (placing themes within specific contexts), and numeration (tracking theme frequency).
5. **Visualization Techniques**: Visual aids such as thematic maps and tables were used to identify relationships between themes, helping to articulate how mentorship experiences influenced participants' developmental trajectories.
6. **Rigorous Interpretation**: Continuous supervision and member-checking were employed to validate interpretations, ensuring that the analysis remained grounded in participants' accounts while incorporating thoughtful, interpretative insights.
7. **Crafting a Narrative**: Detailed narratives were constructed to guide readers through the analytic process, with illustrative quotes from participants highlighting key themes.
8. **Engaging in Reflective Practice**: Regular reflection on analytical biases and processes ensured the authenticity and credibility of the findings, enhancing the study’s reliability.

Following Smith et al.'s (2009) recommendations for PhD studies, I structured the research as a series of three interconnected investigations:

1. An in-depth case study of one participant, including two interviews and multiple data sources (mentor interviews, document analysis, multimodal analysis).
2. Analysis of three participants from the same high school who shared the same mentor.
3. Analysis of additional participants with different mentors from various different locations throughout the United States.

This design allowed for both detailed individual analysis and broader comparison across different mentorship contexts while maintaining IPA's idiographic emphasis.

The study included eight participants, aligning with methodological recommendations for IPA doctoral research (Smith & Fieldsend, 2021). This sample size balanced the need for deep individual analysis with the ability to identify patterns across cases.

3.4 Participant Selection

In line with the research design, purposeful sampling was employed to select participants who meet three specific criteria: (1) All participants must have participated in a high school mathematics research mentorship; (2) All participants must have attained advanced degrees in mathematics or a closely related field; and (3) All participants must have established professional careers in mathematics or a closely related field.

To minimize recall bias and ensure relevance, the study focuses on participants aged 23-27. This age range balances retaining clear memories of high school experiences with having had sufficient time to establish professional careers. The targeted selection allows for capturing meaningful insights into how high school mathematics research mentorships influence long-term academic and career trajectories in mathematics. The study also explored participants' motivations for engaging in these mentorships and their subsequent integration into the mathematics community. By focusing on individuals who appeared to have benefited from these experiences—evidenced by their pursuit of advanced degrees and careers in mathematics—this approach aims to provide detailed accounts of their journeys. While high school mathematics research mentorships could likely be a significant factor contributing to their success, this is not definitively clear. This study also seeks to uncover the role these experiences played alongside other influences. By doing so, it aims to inform strategies for improving mentorship programs and identifying additional practices that can better cultivate and support students' integration into mathematics, ultimately encouraging more students to pursue advanced studies and careers in the field.

Recruitment

Building on previously established relationships, I leveraged connections with three mathematics research mentors based in New York to assist with snowball sampling. These mentors recommended mentees and other potential mentors whose experiences aligned with the study's criteria (Creswell, 2017).

Outside of my network, recruitment was guided by outreach to two key sources:

1. The Society for Science, which administers prestigious national STEM research competitions such as the Regeneron Science Talent Search.
2. MIT PRIMES, a leading high school mathematics research mentorship organization.

After securing IRB approval, I accessed confidential records from the Society for Science, which included data on the top 300 semi-finalists and finalists from 1992 to 2023. I narrowed the focus to participants from 2015–2019 to balance the ability to recall mentorship experiences with sufficient time for career establishment. Using project titles, I identified approximately 1,500 projects for initial review and flagged 43 as potentially mathematics-related. To verify the relevance of these projects, I reviewed associated research papers and consulted two research mathematicians for ambiguous cases, ensuring only mathematical research mentorships were included.

Similarly, MIT PRIMES provided a list of mentors and mentees who participated in their program and won awards for mathematics research projects between 2011 and 2024. I identified 114 mentees from the 2015–2019 cohort for potential recruitment. These combined efforts ensured a diverse participant pool, with projects spanning a range of topics and mentorship contexts.

Outreach and Participant Verification

Outreach prioritized recent participants, starting with the 2019 cohort and working backward. Lists from the Society for Science and MIT PRIMES were organized alphabetically, and I alternated outreach between the two sources. To maintain manageable communication, I contacted approximately 10 potential participants per week. Additional referrals from my network of mathematics mentors yielded 15 more mentees for outreach. Once I secured responses from seven participants, I reduced outreach to five emails per week until the final participant was confirmed, resulting in a total of eight participants—aligning with the recommended range for IPA studies (Smith & Fieldsend, 2021). Having contacted 65 potential participants in total, I concluded the recruitment process once the desired number of participants was reached.

Potential participants were contacted through LinkedIn or academic/work emails. Initial communication verified their participation in a high school mathematics research mentorship and subsequent integration into mathematics through advanced degrees or careers. Once alignment with the study's objectives was confirmed, participants received detailed information about the study, an IRB-approved consent form, and assurances of confidentiality. To protect their privacy, pseudonyms were assigned to all participants for use throughout the study.

Researcher's Role

My background in mathematics research and education provides relevant experience for this study. Having been both a mentee and mentor in mathematics research, I understand mentorship dynamics from multiple perspectives. My experiences range from a high school STEM research mentorship, which highlighted the challenges of ineffective mentoring, to transformative undergraduate mathematics research opportunities that shaped my academic trajectory. Additionally, mentoring high school students in mathematics research at schools in New York has deepened my practical understanding of the challenges and opportunities inherent in these relationships. This multifaceted experience informs the study design and enabled the analysis of participants' mentorship experiences.

3.5 Data Collection

The primary method of data collection in this study was semi-structured interviews with each participant, designed to capture the full scope of their experiences during their high school mathematics research mentorships. These interviews were organized chronologically into three key phases: pre-mentorship, during the mentorship, and post-mentorship. This structure allowed for a more comprehensive and in-depth analysis of the high school research mentorship phenomenon by embedding participants' mentorship experiences within the broader context of their mathematical development.

The interviews began by exploring what initially sparked participants' interest in mathematics, followed by tracing how this interest evolved and what motivated their decision to engage in a high school research mentorship. This phase offered valuable insights into the mindset, motivations, and formative experiences that shaped participants' early engagement with mathematics and their decision to pursue the high school mathematics research mentorship.

Next, the interview focused on the participants' high school mathematics research mentorship experience itself. Questions were designed to elicit detailed descriptions of the mentorship environment, challenges encountered, mentor interactions, and specific tasks or roles undertaken. This phase also probed into how these experiences influenced participants' engagement with mathematics, their perceptions of the subject, their affective domains (beliefs, attitudes, values, emotions), and the impact on their academic persistence and career choices. The aim is to gain a deep understanding of how the mentorship shaped participants' paths within the field of mathematics.

The final phase of the interview shifted to participants' current engagements and future aspirations. Participants were asked to reflect on what they considered to be the most significant factors in their integration into mathematics. These reflections provided insights into both the long-term impact of the high school research mentorship and potential solutions to the broader challenge of encouraging more students to pursue mathematics.

This before, during, and after structured approach not only helped identify key life events and turning points in participants' mathematical journeys (Merriam & Grenier, 2019), but also facilitated a deeper exploration of how their mathematical identity, self-efficacy, and values have evolved over time, with particular emphasis on the impact of the high school research mentorship on these constructs and their overall mathematical journey.

To provide a clearer understanding of the participants' backgrounds, the chart below summarizes key details such as their current careers, research topics during the high school mathematics research mentorship (MRM), and the type of high school they attended:

Table 1: Participants Profiles

Participants	Current Career	MRM Research Topic	HS and Year of MRM
RC	Data Scientist	Number Theory	Private school Illinois 2017
CM	Theoretical Computer Scientist	Combinatorics	Public HS in California 2018
KN	AI Research	Fluid Dynamics and PDES	Private HS in New Jersey 2018
TV	AI Research	Mathematical Modeling, Graph Theory, and Probability Theory	Public HS in Texas 2019
JB	Mechanical Engineer	Graph Theory	Public HS in New York 2016
DB	Mathematics Professor	Graph Theory	Public HS in New York 2015
BH	Finance	Number Theory	Public HS in New York 2017
NM	Machine Learning Researcher	Number Theory	Private K-12 School in Massachusetts. 2019

This table complements the interview data by highlighting the diversity of participant experiences and their subsequent career paths. It serves as a snapshot that allows readers to quickly grasp the context and scope of the high school mathematics research mentorships, as well as participants' successful integration into mathematics-related careers. Visual tools like this can be valuable in qualitative research for effectively conveying complex data (Miles et al., 2014).

By focusing on participants' career paths, mentorship experiences, research topics, and educational backgrounds, this study offers a nuanced exploration of the impact of high school mathematics research mentorships on their broader mathematical journeys. This approach provides a holistic view of how mentorship experiences contribute to long-term engagement in mathematics, aligning with qualitative research principles that emphasize understanding phenomena through detailed, context-specific experiences (Merriam & Tisdell, 2015).

Additional Data Collection Methods

In keeping with Smith et al.'s (2009) guidelines for an IPA PhD study, a case study was conducted on one participant (DB) involving two interviews and multiple sources of data collection, including interviews with their mentor, document analysis, and a review of multimodal media. Three additional participants also had supplementary data collected, though semi-structured interviews remained the primary method of data collection.

Document Analysis

The analysis of relevant documents such as awards, project reports, recommendation letters, and mentor emails during or after the mentorship period serves as an important component of this study. These documents provided tangible evidence of participants' achievements and allowed for a deeper understanding of how their experiences within high school mathematics research mentorships contributed to their academic and professional development. By examining these materials, the study gained insight into how mentorship shaped participants' identities and sense of self-efficacy, as well as how external recognition, such as awards or mentor recommendations, reinforced their commitment to mathematics. The documents also helped illuminate the nature of the mentorships themselves, shedding light on the specific tasks and projects participants engaged in. This provided a contextualized view of the participants' experiences, enabling a richer analysis of the developmental processes they underwent during the mentorship. Furthermore, these materials offered opportunities for triangulating findings from interviews, ensuring that the insights derived from participants' reflections are supported by concrete evidence of their engagement with mathematics.

Multimodal Media Review

Participants' public communications, including social media posts and other publicly available content, added another layer of understanding to their identities and the long-term impact of mentorship. Public outputs reflected how participants chose to present their experiences and self-perceptions, offering insights into how their mentorships influenced both their academic identities and broader social engagement with mathematics. This method broadened the scope of the study by capturing how participants framed their experiences in ways that transcended personal reflection.

Mentor Interview

To enrich the perspective on mentorship dynamics, an interview was conducted with a research mentor who guided three participants. This mentor provided valuable insights into the goals, challenges, and strategies underpinning the mentorship experience. By highlighting the mentor's role in fostering self-efficacy, identity development, and values alignment, this perspective complemented participants' reflections and revealed developmental processes participants may not have fully articulated. The mentor's input offered a critical external viewpoint that added depth and balance to the study.

In sum, the study collected nine in-depth interviews, each lasting 45 to 75 minutes, alongside additional documents and artifacts. All interviews were transcribed and analyzed following IPA's framework. Supplemental data, such as letters from mentors and social media posts, provided real-time insights into participants' reflections on their mentorship experiences. The mentor interview offered an external perspective on participants' development, further enriching the analysis. Together, these methods provided a comprehensive and multifaceted exploration of how high school mathematics research mentorships shape participants' academic and professional integration into the mathematics community.

3.6 Data Analysis

I began my data analysis by reading each transcript while simultaneously listening to the corresponding interview recordings, which allowed me to validate and correct any transcription errors and capture important nuances such as tone, emphasis, and inflection. This initial step ensured the accuracy of the data and helped establish the emotional tone and voice inflection points of the participants. Once all the transcripts were validated, I immersed myself in the data by reading and re-reading them multiple times—a crucial step in Interpretative Phenomenological Analysis (IPA) for engaging deeply with participants' lived experiences (Smith et al., 2009).

This initial engagement with the data was followed by detailed exploratory noting, which involved making descriptive, linguistic, and conceptual observations (Smith et al., 2009). Descriptive comments focused on what participants shared about their experiences, while Linguistic comments examined how they conveyed those experiences—such as their tone, language patterns, and repetition. Conceptual comments, in turn, offered deeper interpretations by exploring potential underlying meanings and connections within participants' narratives.

These conceptual comments typically emerged after multiple readings of the transcripts, allowing for a more nuanced understanding of the participants' experiences as patterns and insights became clearer over time. This multi-layered approach facilitated a comprehensive understanding of their experiences within the mentorship context.

Rather than coding every line, I adhered to IPA’s flexible approach to coding. Depending on the relevance of the content to their lived experience, I sometimes focused on coding individual sentences or entire paragraphs (Smith et al., 2022). This flexibility allowed me to capture the subtleties in participants' narratives, ensuring that the codes remained closely aligned with their personal interpretations of their experiences. I began by highlighting key passages that stood out as significant, reflecting on what these moments meant within the participant's own world. I then recorded my initial thoughts and reflections on the text. Upon revisiting the transcripts, I often added a second layer of comments (usually conceptual), informed by a more comprehensive understanding of the participant's overall narrative.

As the analysis progressed, I began developing emergent themes, also known as personal experiential themes (PETs), based on these exploratory comments (Smith et al. 2022). These themes reflected the core aspects of participants' experiences and gradually revealed deeper insights into their developmental processes over the course of the mentorship. This iterative process allowed me to capture the evolving nature of participants’ experiences while staying grounded in their own words and perspectives.

Once the emergent themes (Personal Experiential Themes, or PETs) were established for all participants, I created a table for each participant to map out connections among the themes. I employed IPA techniques such as abstraction (grouping similar themes), polarization (highlighting contrasting themes), subsumption (allowing one theme to encompass others), contextualization (placing themes within particular life or mentorship contexts), numeration (tracking the frequency of themes), and function (examining the role of themes within participants' narratives) (Smith et al., 2022). This rigorous approach enabled me to capture the complexity of participants' experiences while remaining closely grounded in their personal narratives.

After completing this process for each participant, I conducted a cross-case analysis to map out patterns and connections across all participants. This step revealed both commonalities and unique experiences in how participants integrated into the mathematics community through the mentorship program. By working with PETs across different cases, I was able to identify and develop Group Experiential Themes (GETs) that represented shared experiences among participants (Smith et al., 2022).

Finally, I organized these themes and interpretations into a structured table, providing a clear and comprehensive view of each participant's journey. This structured presentation of the data offered a clear and comprehensive overview of each participant's experience, showing how PETs, GETs, and the coding process contributed to understanding their academic and professional trajectories. Below is a snapshot of this data analysis process:

Stage 1: Initial Noting

Quote: "I didn't think I belonged doing math research. I was so nervous."

Descriptive Comment: The participant expresses feeling out of place and nervous at the beginning of their mentorship, indicating doubts about their research capabilities.

Stage 2: Second Layer of Comments

Linguistic Comment: Repeated use of terms like "nervous" and "overwhelmed" throughout the interview underscores their sense of intimidation and uncertainty.

Conceptual Comment: These expressions of self-doubt are closely linked to a lack of prior experience in research mathematics and low research mathematics self-efficacy. However, as the participant gains more exposure and advances through the mentorship, their self-efficacy strengthens, leading to a gradual reduction in self-doubt.

Stage 3: Emergent Themes

Theme 1: Initial Self-Doubt and Lack of Belonging in Research

Theme 2: Growth in Confidence and Self-Efficacy through Mentorship

Stage 4: Cross-Case Analysis

Cross-Case Comparison: Similar patterns of early self-doubt transitioning to later confidence were observed across participants, indicating a common developmental trajectory in self-efficacy facilitated by the research mentorship. This recurring theme across multiple cases contributed to the formation of a Group Experiential Theme (GET) which I labeled “Low Mathematics Research Self-Efficacy.”

This analytical approach facilitated a deeper exploration of participants' developmental processes during the high school research mentorship by identifying moments where their research mathematics self-efficacy showed improvement. These specific instances and quotes were further analyzed to identify patterns and distinctions across participants, shedding light on the evolution of their mathematics research self-efficacy throughout the mentorship experience. This analysis contributed to the development of additional Group Experiential Themes (GETs), which I labeled “Evolving Mathematics Research Self-Efficacy,” encapsulating the shared journey of growing confidence in mathematical research.

Addressing Research Questions

To address the first research question, I began with a detailed examination of participants' responses to questions about what initially sparked their interest in mathematics and the significant moments, influences, or factors that propelled them toward the mentorship program. I assigned initial notes to capture descriptions of these early influences, such as family encouragement, teacher support, or key events that heightened their interest in mathematics. These notes helped identify key motivational factors, including an intrinsic interest in mathematics, influential experiences during earlier schooling, and external encouragement or recognition. I then focused on identifying the internal and external factors that influenced participants to decide to engage in the mentorship, such as early mathematical experiences, interactions with teachers, or exposure to advanced concepts.

Next, I analyzed participants' reflections on why they chose to engage in the mentorship program, including their expectations, self-perceived abilities, and what they hoped to gain from the experience. This phase was critical for understanding the transitional processes that shifted participants from an initial interest in mathematics to a more deliberate pursuit of research mentorship. Codes like "wanting to be competitive for college" or "desire to explore advanced mathematics" captured the range of motivations driving participants' decisions. By tracing their narratives, I was able to map how personal aspirations, academic goals, and social influences—such as peers engaging in similar programs—combined to shape their involvement in the mentorship.

Through an inductive, iterative coding process, I identified experiential statements that reflected participants' deeper motivations and developmental trajectories. These PETs were refined through several rounds of coding to capture the nuances of participants' experiences and motivations. After this iterative process and comparing cases, several key group themes emerged

GETS. These themes encapsulated both individual and collective factors that guided participants toward the mentorship.

Through cross-case analysis, I developed a network of interconnected themes illustrating the complex interplay of factors leading to participants' involvement in mentorship programs. This thematic network highlighted not only the individual motivational factors but also how these factors collectively contributed to participants' evolving decisions and mathematics identities. The analysis further explored how participants' initial motivations and expectations aligned—or, in some cases, diverged—from their pre-mentorship self-assessments. For instance, some participants initially joined the mentorship to enhance their college applications but later discovered their love for mathematics deepened through the mentorship, leading to a shift in their primary motivation to pursue mathematics. Other participants who joined the program because their peers were doing it (peer influences) eventually developed a deeper academic commitment or a stronger sense of belonging within the mathematical community.

This cross-case analysis revealed a network of interconnected themes that were not isolated but intertwined, collectively shaping participants' evolving decisions, identities, and values. It provided valuable insights into the internal transformations and developmental processes that occurred during the high school mathematics research mentorship, offering a comprehensive understanding of the personal significance participants attached to their decision-making. Additionally, it illuminated how their motivations evolved and how these mentorship experiences influenced the development of their mathematical identities and values over time.

I addressed research questions 2 and 3 by employing the data analysis guidelines of IPA outlined by Smith et al. (2022). Research Question 2 investigates the developmental processes through which high school mathematics research mentorships shape participants' self-efficacy, identity, and values, while Research Question 3 examines additional factors and experiences gained through these mentorships that facilitated participants' integration into mathematics.

In addition to IPA, I employed process coding to focus on participants' actions and the dynamic processes they engaged in during the mentorship. Process coding, which is particularly suited for capturing developmental shifts, involves the use of gerunds (e.g., “collaborating,” “writing,” “presenting”) to highlight participants’ ongoing actions and interactions (Saldana, 2013). This technique provided further insights into how constructs such as mathematical self-efficacy, identity, and value internalization—recognized in the literature as important developmental processes—evolved over time (Estrada et al., 2011; Hernandez et al., 2018). Process coding allowed me to trace how participants’ actions and interactions contributed to their integration into mathematics, focusing not only on the outcomes but also on the processes that drove personal growth. This approach enriched the analysis by revealing how participants engaged with key aspects of their mentorship, such as tackling research tasks, collaborating with mentors, or overcoming setbacks. These actions were crucial for understanding how participants developed self-efficacy and identity over the course of the program.

Following some of the guidelines by Smith et al. (2022) laid the groundwork for identifying Personal Experiential Themes (PETs), which captured key aspects of each participant's experience. I then grouped related PETs into Group Experiential Themes (GETs), which reflected common experiences shared across participants. These themes were grounded in participants’ own language and interpretations, ensuring that the analysis remained true to their lived experiences. I then applied IPA techniques such as abstraction (grouping similar themes), polarization (identifying contrasting themes), and subsumption (using overarching themes to encompass smaller ones) to connect these themes and develop a coherent understanding of participants' experiences.

Once emergent themes were established for each participant, I performed a cross-case analysis to compare themes across the dataset, identifying both commonalities and unique variations. This step was critical in ensuring that the analysis captured broader patterns while still respecting the individuality of each participant's experience. While mathematical self-efficacy, identity, and value internalization were central to answering research question 2, additional themes relevant to research question 3 also emerged, providing insights into factors beyond these constructs.

IPA's inductive approach enabled the emergence of both anticipated and unexpected themes through iterative analysis. While themes from the mentorship relating to mathematical self-efficacy, identity, and value internalization naturally appeared, aligning with Research Question 2, the analysis remained open to additional developmental factors beyond these constructs. This methodological flexibility allowed for the identification of distinct themes addressing Research Question 3, such as the development of research competencies and professional skills, thereby providing a comprehensive understanding of participants' mentorship experiences. The dual focus on theoretically-aligned and emergent themes ensured that the analysis captured both the psychological constructs central to TIMSI and other significant factors influencing participants' integration into mathematics.

By organizing my analysis around Personal Experiential Themes (PETs) and Group Experiential Themes (GETs), I was able to map the developmental trajectories of individual participants while also identifying broader, shared experiences. This methodical approach to coding and theme development provided a framework for understanding how participants' engagement in the high school research mentorship influenced their academic and professional growth.

To address Research Question 4, which seeks, what factors participants perceive as most significant in their integration into mathematics, I tailored my analysis around participants' responses to the question, 'What factor or key factors do you attribute most to your integration into the field of mathematics? Why?' Utilizing the IPA framework (Smith et al. 2022), which includes thorough reading and re-reading of transcripts, making exploratory notes, and developing Personal Experiential Themes (PETs), I gained a nuanced understanding of each participant's journey into the field of mathematics which allowed me better to interpret their responses to the interview question above. While the high school mathematics research mentorship was central to this study, the depth of IPA allowed me to probe beyond this experience to uncover initial interests in mathematics and subsequent professional paths. Furthermore, I created PETs on every participants' response to the interview question above along with my interpretation. I then created a table of all these PETs in regard to the key factors they attributed most to their integration. This table allowed for cross case analysis on these factors as well as the creation of GETs.

After identifying these PETs and GETs, I evaluated the degree to which the participants' experiences aligned with the TIMSI framework. The process involved analyzing how their self-efficacy grew over time, how their identity as mathematicians evolved, and how they adopted the values of the mathematical community. I also looked for factors beyond TIMSI's constructs that may have played a significant role in their development but are not explicitly covered by the model. This cross-case analysis enabled me to determine both commonalities and divergences in how participants' interpretations aligned with TIMSI.

3.7 Validity and Reliability

Ensuring that data is trustworthy is critical to the validity and reliability of qualitative research (Lincoln & Guba, 1985). This study incorporated several strategic measures:

Triangulation

To deepen the validity of the findings, this study utilized data triangulation, drawing insights from diverse data sources such as interviews, documents, and multimodal media artifacts. This method corroborated evidence from various angles, enhancing the study's reliability and providing a robust basis for the conclusions (Creswell & Poth, 2017; Merriam, 1998).

Multiple and Consensual Coding

To enhance the integrity and reliability of the data analysis, a multi-coder approach was employed (Saldaña, 2016). Initially, I conducted the analysis independently, but to safeguard against potential biases and to deepen the analysis, I engaged in discussions with academic peers. These discussions helped to challenge and refine coding decisions, ensuring that interpretations are not solely my own but are validated through scholarly dialogue. Furthermore, to enhance the reliability of the data, coding was undertaken by four independent coders, each bringing distinct expertise to the analysis. This approach ensures a more rigorous and unbiased analysis, contributing to the study's overall trustworthiness. The coding team includes:

- Myself, drawing on experience in mathematics research mentoring and education research.
- A mathematics professor with over 30 years of research high school mentorship experience, bringing a deep understanding of mathematical concepts, mentoring nuances, and theories.

- A seasoned professor of mathematics education with over 50 years of research and teaching experience, deeply knowledgeable in both mathematics and contemporary pedagogical theories.
- An expert in gifted education, who provides insights into the nuances of high-achieving students in mathematics.

This strategy of involving multiple coders from diverse academic backgrounds is strongly supported in the literature. Scholars (Guest et al., 2012) advocate for employing several coders in qualitative research to compare and reconcile coding discrepancies. This collaborative approach significantly enhanced the trustworthiness of the analysis by ensuring that multiple perspectives scrutinize the data (Guest et al., 2012). Furthermore, this method aligned with established best practices in qualitative research, ensuring a comprehensive and validated interpretation of the data collected (Creswell & Poth 2017). This multifaceted coding strategy not only bolstered the validity of the findings but also enriched the analytical process, providing a robust foundation for the research conclusions.

Iterative Analysis Process

The analytical process involved continuous engagement with the data through repeated readings and interpretations, aligning with IPA's commitment to deep phenomenological understanding (Smith et al., 2022). This iterative approach enabled the refinement of themes while maintaining close connection to participants' lived experiences. As new insights emerged, earlier interpretations were revisited and refined, ensuring that the analysis captured the full complexity of participants' mathematical journeys. This systematic process of returning to the data strengthened the credibility of interpretations and deepened understanding of how high school mathematics research mentorships influenced participants' development.

Member Checking

To ensure the authenticity of the findings, member checking was employed, a critical process in qualitative research for validating the accuracy and credibility of the data (Lincoln & Guba, 1985; Birt et al., 2016). Preliminary findings were shared with participants via email, inviting them to review and affirm or refine the interpretations made from their data. This process allowed participants to suggest modifications, provide additional information, or confirm the findings, thereby enhancing the trustworthiness of the analysis.

Reflexive Practices

My reflections on how my background and perspectives might have influenced the interpretation have been documented, in line with the reflexive practices recommended in qualitative research (Finlay, 2002). This reflexive stance was crucial for acknowledging and mitigating potential biases, ensuring that the analysis remains as objective as possible.

Peer Debriefing

Regular discussions and email exchanges with the coding team about the coding and theme development process served as an external check on the research approach. Peer debriefing is a recognized method in qualitative research for enhancing the credibility and consistency of the analysis (Spall, 1998). These discussions helped to ensure that the analysis remained objective, rigorous, and consistent with the data.

This chapter has detailed the methodological strategies employed to conduct a rigorous and trustworthy analysis of the lived experiences of participants in high school mathematics research mentorships. The methods for ensuring validity and reliability are fundamental in establishing the credibility of the findings, which are thoroughly examined in the subsequent chapters of this dissertation. Although it is impossible to achieve a completely bias-free analysis, the steps outlined here are carefully designed to minimize bias and ensure an accurate representation of the participants' experiences.

The findings, presented in Chapter 4, provide a comprehensive examination of how high school mathematics research mentorships influence participants' development of mathematical self-efficacy, identity, and values, while Chapter 5 offers implications for mentorship practices and recommendations for future research.

Chapter 4: Findings

This chapter presents the findings from the semi-structured interviews, document analysis, and multimodal media reviews conducted with participants. The primary goal is to understand and interpret the participants' lived experiences with high school mathematics research mentorships and their integration into the mathematics community, in line with the Interpretative Phenomenological Analysis (IPA) approach of this study.

The findings are structured to reflect each participant's comprehensive journey of integration into mathematics. Each participant's story is treated as a unique case, allowing for an in-depth exploration of their individual experiences while also enabling cross-case analysis. Section 1 presents detailed idiographic accounts for each participant, chronicling their mathematical journey from:

1. Their initial interest and engagement with mathematics
2. Factors leading to their participation in high school mathematics research mentorships
3. Their experiences during the research mentorship
4. Their subsequent academic and professional paths in mathematics

These accounts aim to capture the participants' own sense-making of their experiences, while also incorporating the researcher's interpretations, in keeping with IPA methodology.

Section 2 transitions into a cross-case analysis, identifying convergences and divergences in participants' experiences across their mathematical journeys. This section aims to address the research questions and explores how participants' mathematical self-efficacy, identity, and values were shaped throughout their experiences, with a particular focus on the role of high school research mentorships. Rather than a thematic analysis, this section follows IPA's approach of looking for patterns across cases while maintaining the idiographic focus. Throughout the chapter, the analysis remains inductive and interpretative, in line with IPA methodology. The findings are presented with a focus on the participants' lived experiences and the researcher's interpretations of these experiences, rather than fitting them into predetermined themes. This approach allows for the emergence of unexpected insights and maintains the phenomenological focus on exploring the essence of the participants' experiences. By combining in-depth individual case analyses with cross-case examination, this chapter aims to provide a comprehensive understanding of the impact of high school mathematics research mentorships within the broader context of participants' mathematical journeys. This approach not only offers rich, contextualized accounts of individual experiences but also illuminates broader patterns that contribute to a deeper understanding of participants' integration into the mathematics community.

4.1 Participants' Integrations into Mathematics

Participant RC

RC is currently a data scientist at a major retail company, holding an M.S. in Computational and Applied Mathematics and a B.A. in Statistics with a specialization in Machine Learning. He attributes his mathematical foundation to his father's early influence: "I will always attribute the start of my love for math with my dad." His mathematical journey began during childhood summers in India, where his father "would incentivize us like, 'Okay, if you can solve like X amount of problems...you'll get like 50 rupees.'" This early engagement with mathematics set him on a path of academic excellence.

By middle school, RC was enrolled in an honors program, and upon entering high school, he tested into an advanced track: "I was two years ahead of the normal curriculum...instead of algebra freshman year...I was taking trigonometry." His involvement in math team competitions exposed him to a community of "mathletes," where "hanging around in like groups...you hear things, like what other people are doing." This exposure led him to the Young Scholars Program at the University of Chicago, where he encountered "more different, quote unquote, types of abstract math" that transformed his understanding of mathematics.

Without formal mentorship opportunities at his high school, RC sought guidance independently, eventually finding a mentor through an online forum. This mentor guided him through his first research project on elliptic curve cryptography, which culminated in a submission to the Intel Science Talent Search. Reflecting on this experience, RC shares, “It was intimidating…you don’t know what you don’t know. But my mentor helped me ask the right questions, which gave me the momentum I needed.” The mentorship not only improved his technical competencies, such as reading advanced mathematical literature and writing research papers, but also built his confidence as a researcher. “I didn’t think I belonged in math research, but those small breakthroughs really solidified my pursuit of math,” he says.

The mentorship experience, though primarily conducted through email correspondence, proved transformative. RC's research on elliptic curve cryptography gained recognition, with "news outlets that used my paper for headlines...like potentially a new algorithm that can be used by the NSA." Reflecting on the impact of his mentorship experience, RC notes, "I'm a prime example of how mentorship can drastically affect your life...looking back, I've never really performed this exercise of like, thinking retrospectively on like, how these impacts have affected my life." From his early puzzle-solving days to his involvement in high school research, RC's integration into mathematics reflects a journey shaped by family encouragement and proactive pursuit of opportunities. His experience demonstrates how mentorship can "open many doors" and significantly impact academic and professional trajectories, even when conducted primarily through remote interactions.

Participant CM

CM is a theoretical computer science researcher with a PhD from a prestigious West Coast university. Her mathematical journey began in elementary school through an after-school math club that hosted informal competitions. "There's like an after-school club like math thing. And they would have like these Mini contests," she recalled, noting these early experiences sparked her interest in mathematics.

Her engagement with mathematics deepened through competitive mathematics: "I really got into them in eighth grade," she reflected, describing how contests became central to her mathematical development. This passion for problem-solving extended beyond formal competitions: "I love logic puzzles...even before that, and well into like middle school and such. And even now."

In eleventh grade, CM participated in the MIT PRIMES program for mathematics research. While she "really liked thinking about a math problem for a long period of time," she encountered challenges with problem selection. "Now I know in hindsight that it was just an extremely hard problem... it's a problem that I've literally heard as an open problem in classes in college," she reflected, adding "I think I would have appreciated a little bit more mentorship... in like how to select a problem that's actually solvable."

Her perspective on research evolved significantly during her undergraduate years. At a Research Experience for Undergraduates (REU) program, she worked on "a pretty niche conjecture that my mentor had stumbled across," which proved more approachable. This experience led to meaningful results: "I legitimately figured out something about the area that no one had before." This evolution shaped her perspective on research, leading her to pursue a research career in theoretical computer science. She explained, “Theoretical computer science has a very similar feel to combinatorics... I do it because I like this type of math.”

CM notes the natural progression from her combinatorics background: "Theoretical computer science is quite literally like okay, let's choose a math problem that... sounds like we could build an algorithm for the situation and solve it... it's very much just like math research. It's like the same thing." Her journey from contest enthusiast to researcher reflects a deep commitment to mathematical problem-solving and theoretical exploration.

In addition to her research, CM is deeply committed to creating a supportive and inclusive mathematical community. She actively leads initiatives for underrepresented groups in mathematics, co-organizing summer programs and coaching for international competitions. Her journey illustrates a progression from early success in contests to impactful research, underpinned by a commitment to fostering broader engagement in mathematics.

Participant TV

TV is an AI researcher who applies advanced mathematical knowledge to innovate in artificial intelligence. He holds an M.S. in Mathematics with minors in Computer Science and Biology from a prestigious Northeast technical institute, where he graduated with the highest possible GPA. His professional contributions focus on machine learning and computational infrastructure at a leading technology company.

TV’s mathematical journey began in elementary school when an extra math class in fourth grade introduced him to puzzles and problem-solving. Reflecting on this experience, he shared, “I thought that was really fun,” noting how it sparked his interest in mathematics and laid the foundation for his developing mathematical identity.

While not exceptionally gifted initially, TV noted, "I wasn't bad at it before, but like I hadn't specifically been like learning extra math on the side." His engagement with mathematics deepened through contests in middle school: "I was also participating in like some math contests in middle school, partially as a result of that class, because, like part of the class focus was like we're going to give you some problems from these contests."

A pivotal moment in TV's development came during his preparation for MIT PRIMES. Faced with a linear algebra problem on the admission test without prior knowledge of the subject, he took initiative: "I don't want to like not get into this program because I didn't learn linear algebra. So I'm just going to try to learn it in like the next month." This experience proved transformative: "that was probably my first time...learning something that wasn't in the school curriculum, and that like I didn't actually know how to learn...that probably gave me more confidence that, like a lot of other math, is learnable, even if I don't have like a teacher for it."

His research focused on modeling refugee migration using mathematical and physics-inspired models. The experience changed his perception of mathematics research: “I realized a lot of research isn’t about inventing new things but applying existing ideas to new problems.” His research mentorship experience taught him that research mathematics is often incremental: "before that I had always thought of like, oh, doing math research as like proving like new theorems, or like new results...This program helped me realize that actually, a lot of research is more incremental." The mentorship process involved weekly online meetings where "the first few months at least...he would just like, send me a bunch of papers to read...that was helpful, because I, at that point didn't have any experience with like finding papers and figuring out which ones are important to read."

TV emphasized the mentorship's role in shaping his research skills, particularly in navigating academic papers and structuring research outputs. “My mentor helped me understand which papers were important and guided me through the process of writing a research paper,” he shared. Despite their largely professional relationship, TV appreciated his mentor's understanding when high school commitments occasionally limited his progress.

Reflecting on the mentorship's impact, TV acknowledged its influence on his academic and professional trajectory. “It helped me figure out my interests and avoid getting stuck in projects I wasn't as interested in,” he explained. The mentorship, combined with his self-driven learning and early engagement with mathematics, shaped his path toward applied mathematics and ultimately a career in artificial intelligence.

Participant KN

KN is currently an AI researcher working with large language models after completing studies in mathematics. He has aspirations of contributing to the field through a mathematics Ph.D. or further AI development. His mathematical journey began with a transformative documentary experience in middle school about topology and higher dimensions: "There was one documentary that I watched...about topology and geometry of higher dimensions...I thought it was very amazing that people are thinking about these higher dimensions and getting these useful and meaningful constructions." This early inspiration motivated him to explore mathematics further, particularly geometry and algebraic geometry, which he later described as "beautiful" and intellectually captivating.

Despite starting as an average math student with no extracurricular involvement in middle school, KN's passion for the subject led him to excel. His transition to a private high school in the U.S. exposed him to advanced mathematical courses. "My high school was atypical; it offered courses like topology, and I had a teacher willing to teach me abstract algebra," he explained, underscoring the role his educational environment played in nurturing his mathematical growth. This unique environment exposed him to advanced mathematical concepts early, including "the basics of the point set topology" and "basic abstract algebra."

Despite being in a competitive environment where "every year we have people who do very well like in the U.S. big competitions," KN pursued his own path through research programs like MIT PRIMES and RSI. These experiences introduced him to mathematical research and problem-solving. At MIT PRIMES, he worked on modeling traffic flow with partial differential equations (PDEs), simulating the effects of autonomous vehicles. "The project involved learning PDEs and simulating equations in Octave," he recounted, crediting his mentor's guidance and patience for his progress. Reflecting on the process, he noted, "I almost didn’t feel much distinction between research and a really long, hard homework problem," highlighting the incremental nature of his research experiences.

KN’s mentorship experiences were instrumental in shaping his mathematical trajectory. His high school mentor not only guided him through complex research but also provided support and encouragement, exemplified by their patience and willingness to explain concepts multiple times. "Being hands-on and kind goes a long way," KN reflected, emphasizing the importance of the mentorship’s human elements. His mentor also contributed a recommendation letter, which KN noted as "very helpful" for his college applications.

Beyond high school, KN’s mathematical identity deepened in college, where he immersed himself in algebraic geometry. He described the subject as combining his love for algebra and geometry, stating, "It takes a lot of preparation to feel comfortable with abstract objects, but once I got there, the innate beauty of the subject drew me in." His admiration for mathematician Alexander Grothendieck further inspired his dedication. "He’s a very impressive person... his work in scheme theory and his sense of responsibility to the world motivated me," he shared.

His mathematical journey was shaped more by intellectual curiosity than career considerations: "I grew up in a privileged situation where my family doesn't give me any pressure to do a career that will pay me well...I just really enjoyed this stuff." This freedom allowed him to explore mathematics purely for its intrinsic interest, leading to his current work in AI research.

Participant NM

NM is currently a machine learning researcher after completing her mathematics education. She has an interest in pursuing a PhD in applied fields such as probability, statistics, and machine learning. Her mathematical journey began in China's highly competitive educational environment, where "the education atmosphere there is very competitive. From an early age we all try to get into a good middle school and then a good high school." Initially, she wasn't naturally gifted at mathematics, recalling "getting zero scores on practice exams" in elementary school.

A transformative moment occurred in middle school when she developed an intense dedication to mathematics competitions: "I trained intensely for like months before that competition... I would just like wake up super early like five or six a.m. And then I would just like do a bunch of practice problems before school." This self-driven commitment led to unexpected success: "I ended up placing in the top five in my province and Beijing and I was first among all girls." Her success shifted others' perceptions and expectations: "People started noticing like my name... a lot of my classmates, parents or whatever would come up to me and be like, oh congratulations, like you are like so talented." This recognition, while validating, also created new pressures: "I started telling myself, okay, I need to keep doing well at these competitions and I started giving myself a little pressure."

NM transitioned to the U.S. for high school to escape the rigid educational system in China, attending a private school in Massachusetts. There, NM explored pathways beyond competitions, such as summer programs and research opportunities. "I think when I did all these things in high school it was primarily motivated by just the objective of getting into a good college with math and finding the best way to do it."

A notable experience was NM's participation in the MIT PRIMES research program, where NM engaged in group research on commutative algebra. Reflecting on the mentorship experience, NM stated, "Collaborating with my peers really helped me resolve lingering questions...It made me care about [math] more." However, NM acknowledged the challenges of maintaining motivation, noting, "I didn’t care about the research problem or solution...I cared more about not pulling my weight in the group."

The mentorship also provided insights into the incremental nature of mathematical research, contrasting with NM's preference for the definitive solutions offered by competition problems. NM observed, "Math research is often incremental...I struggled with the open-endedness of it."

Despite these challenges, NM recognizes the value of these experiences in shaping a broader mathematical perspective and career aspirations. NM's journey illustrates the evolution from intense competition-driven engagement to a more exploratory approach, balancing intrinsic interests in problem-solving with practical career considerations. As NM reflects, "Ultimately, my passion is problem-solving...logical puzzles in general, not just math problems." This passion continues to guide NM's academic and professional pursuits.

Participant DB

DB is an assistant professor in a mathematics department at a public college, a role inspired and shaped by his early experiences with high school mathematics research mentorships. His mathematical journey began in middle and high school, where an accelerated curriculum and support from his father sparked a serious interest in the subject. Reflecting on these formative years, DB shared, "My father taught me a whole bunch of high school math and then connected me with Professor L at P University. It was sometimes difficult to work with my father, and this [mentorship] exposed me to something different."

DB's mentorship experience focused on graph theory and extended across multiple projects. The mentorship experience profoundly shaped DB's mathematical development. "The biggest thing was just learning to think about math in a creative way and learning to think about math that I'd never seen before," he reflected. Beyond technical skills, he gained crucial experience in mathematical communication: "I also realized that the experience of writing and presenting math was a huge goal that I'm very indebted to Professor L for.

DB's research achievements included presenting at conferences and publishing multiple papers, accomplishments that profoundly influenced his self-perception as a mathematician. Despite being "completely terrified" before talks, DB learned to manage his anxiety through preparation: "Professor L taught me to really practice it over and over again, and I was able to get through it thanks to his help and support." He recalled the excitement of seeing his name in print: "That was extremely exciting—it made me feel like a real mathematician." Additionally, his participation in prestigious competitions, such as the Intel Science Talent Search where he was a semifinalist, which he described as "a very significant accomplishment" provided further validation and encouragement. "These successes were nice boosts of confidence," he said, reflecting on the value of external recognition.

This mentorship experience proved decisive in DB's career choice: "I could point to as the single moment in my life where I said, 'this is what I want to become, a professor.' It was really that dramatic." Now as an educator himself, DB emphasizes the importance of "investing in it and putting time into it... devoting the time to the student, and always being available to address whatever concerns they have."

DB’s journey underscores the transformative impact of mentorship, demonstrating how early exposure to research can foster a lifelong commitment to mathematics. His story highlights the profound influence of mentorship on academic and professional trajectories, shaping not only technical skills but also a deep sense of identity and purpose within the mathematics community.

Participant BH

BH is a research engineer at a crypto hedge fund, where he develops tools and software to support quantitative researchers in analyzing trading opportunities. His mathematical journey began in early childhood, where he was naturally drawn to the subject's clarity and objectivity: "I liked that there was always a right answer to problems, and you know, a correct way to do things. I found a lot of other subjects to be much more subjective.” His early aptitude for mathematics was evident: "Even at a relatively early age, it just came pretty easily to me." This foundation paved the way for his growing interest in middle school, where programs like Kumon and the guidance of family and teachers nurtured his mathematical identity.

In eighth grade, BH's mother and math teacher introduced him to a mentor, Professor L, who worked at the high schools in his area. Professor L would become instrumental in shaping his mathematical journey. Reflecting on this, BH shared, "It was less of my initiative and more due to my mom and teachers pushing me in the right direction."

This mentorship allowed BH to explore advanced topics in graph theory and number theory, sparking his creativity and critical thinking. The mentorship evolved from basic mathematical concepts to collaborative research: "At the beginning we looked at comparably basic mathematics, basic graph theory and number theory," he recalls. Later, it became "more of a collaborative sessions where we would take a problem, maybe something that had come up in his research that he was interested in, and kind of brainstorm ideas together." This approach fostered both technical skills and creative problem-solving abilities. He recalled, "We would take a problem—something that came up in his research—and brainstorm ideas together," emphasizing the collaborative and exploratory nature of their sessions.

BH noted that his self-directed learning—studying math independently outside of school or Kumon—developed as a result of his mentorship experience. He shared, "That mainly came about as an interest in things Professor L and I worked on during our sessions. That was more enjoyable to work on than schoolwork or the like." This newfound enjoyment of mathematics beyond structured coursework reinforced his identity as a budding mathematician and cultivated his intrinsic motivation to explore mathematical ideas further. BH particularly valued the mentorship's relaxed, exploratory nature: "It was much more of a fun and exploratory experience than it was you know, we need to deliver research results for some reason." This informal atmosphere "definitely differentiated itself from school at the time where it was all about assignments and due dates."

The mentorship also provided BH with his first opportunity to present research at an academic conference. He described this experience as "a new and challenging experience," which boosted his confidence and sense of identity within the mathematics community.

Throughout high school and college, BH continued to build on the skills he developed during the mentorship. He majored in mathematics and earned a master's degree in applied mathematics, crediting the mentorship with solidifying his confidence and confirming his interest in pursuing mathematics at an advanced level. "The mentorship certainly confirmed that mathematics was a path I was interested in and capable of pursuing in college," he reflected.

Currently, BH applies his mathematical background in his role building tools for quantitative researchers, noting that "there's still quite a bit of graph theory in the work that I do." He credits his mentorship experience for developing his creative problem-solving abilities: "I think probably the thing that it's helped with most is creativity for some of these problems...especially on the creative side of problem solving, I think the mentorship has played a very large part in that." His journey illustrates the profound impact of early mentorship in fostering a lifelong connection to mathematics, creativity, and professional success.

Participant JB

JB is an industrial operations engineer, and the CEO of a manufacturing company based in the Northeast. JB's path to mathematics was influenced by his father, an engineer who encouraged focus on science and math. While naturally "decent in math," JB's early interests were divided, particularly with soccer: He reflected, "I wasn't so focused on math or school...I really wanted to be a professional soccer player."

A pivotal moment came in ninth grade when JB joined a science research program, following his brothers' footsteps: "I applied for science research in the 9th grade as my brothers did it." Initially uncertain about his research direction, he recalls, "I had no idea what I wanted to do. I thought I wanted to do something around like global warming, or something with the environment."

Initially uncertain about his research direction, JB was inspired by Professor L's engaging teaching style after a classroom presentation. What began as casual mathematics discussions transformed into a formal research mentorship. JB appreciated the interactive nature of their sessions: "I wasn't just sitting there being lectured to... I enjoyed trying to solve these problems." He described their mentorship as pivotal, noting that it allowed him to "learn math in a new way" by actively solving complex problems rather than passively absorbing lectures. JB recalled, "I enjoyed doing puzzles as a kid...trying to solve these problems with Professor L felt like that same kind of fun.

Under Professor L's guidance, JB explored graph theory, delving into problems that required both creativity and rigorous analysis. Their research process was highly collaborative: "We would start messing around right? We would start... play around. Just drawing dots and lines and looking trying to make something to happen." JB valued his mentor's approach to questions: "I would ask stupid questions that... a normal professor of mathematics, or a scientist of math or theorist would be like, 'oh, that's silly or stupid,' but Professor L would never do something like that. He would take what you say and build on it." JB noted, "It was always rewarding when my interactions with him led to deeper questions."

The mentorship also introduced JB to academic writing and presentation. Co-authoring a paper with Professor L taught him "how to write math lingo" and formalize his ideas in a technical format. Presenting their research at conferences boosted his confidence and reinforced his mathematical identity. JB credited the mentorship with fostering essential skills like perseverance, critical thinking, and problem-solving, which he continues to apply in his professional career.

Reflecting on the impact of his mentorship, JB acknowledged its role in shaping his academic and career trajectory. While his work as an industrial operations engineer doesn’t directly involve graph theory, the problem-solving mindset he developed remains central to his approach. He explained, "The ability to minimize or maximize something, thinking theoretically, is certainly helpful in what I do."

From his early interest in soccer to his eventual integration into mathematics, JB’s story underscores the transformative power of mentorship in uncovering and nurturing potential. He remarked, "The mentorship certainly made me more math-oriented," highlighting the lasting influence of those early experiences on his career and personal development.

4.2 Pathways, Processes, and Factors in Mathematics Research Mentorships

RQ1: Motivational Pathways: Factors Influencing Participants' Decision to Join the High School Mathematics Research Mentorship

This section addresses Research Question 1: “What factors influenced participants' decisions to engage in high school mathematics research mentorships, and what personal meaning do they assign to these decisions?”

Through Interpretative Phenomenological Analysis (IPA) and the examination of Personal Experiential Themes (PETs), which led to the identification of Group Experiential Themes (GETs), my analysis revealed four interconnected themes that collectively shaped participants' decisions to engage in high school mathematics research mentorships: influence of supportive environments, academic and career aspirations, intellectual curiosity and personal growth, and accessibility and visibility of opportunities. These themes were not isolated but deeply intertwined, collectively influencing participants' evolving decisions and mathematical identities.

Influence of Supportive Environments

Family support, school culture, and peer networks emerged as critical factors in motivating participants to pursue mentorship opportunities. These environmental influences manifested through three key channels:

Family Influence

Several participants' mathematical journeys began with strong parental support, particularly from fathers who actively engaged in their mathematical development. DB's path into advanced mathematics started with his father's direct involvement

DB attributed his “serious interest” in mathematics to his father’s active involvement and early teaching: “I started having pretty serious interest in math, when I was in middle school and high school. And started having accelerated curriculum …because my father taught me a whole bunch of high school math.”

RC experienced similar pivotal parental influence, with his father creating early mathematical engagement through incentivized learning: “My dad incentivized me to solve puzzles and challenges, which set me on this path.”

This early support accelerated both participants academically. RC noted being "two years ahead of the normal curriculum," which led him to pursue additional mathematical opportunities that “strong math students do” like the Young Scholars Program, math team, and eventually the research mentorship. Similarly, DB's advanced preparation through his father's teaching enabled him to engage with university-level mathematics earlier than his peers.

Family influence in both cases fostered a nurturing environment that not only instilled early confidence and a sense of identity as accelerated and "strong math students," but also heightened their intrinsic motivation and values surrounding the importance of mathematics in their lives. This foundational support facilitated access to advanced mathematical opportunities and reinforced their engagement with the subject. Additionally, the recognition they received from peers and teachers for their accelerated abilities and mathematical achievements further bolstered their confidence and deepened their commitment to mathematics. Together, these factors played a pivotal role in shaping their trajectories and decisions to pursue high school mathematics research mentorships.

For BH, family support also extended to identifying resources and mentors. BH credited his mother and middle school math teacher for facilitating his connection with Professor L, which ultimately led to his participation in a high school research mentorship:

> It was less of my doing my initiative and more of, you know, my parents and teachers kind of seeking out an extracurricular activity...My mom, in conjunction with my 8th-grade math teacher, identified Professor L as a potential mentor and tutor.

School Culture

All participants, apart from RC and NM, attended schools with established research programs that actively promoted and facilitated mathematics research opportunities. These programs created environments where research engagement was both normalized and celebrated. For JB, his school's structured program played a pivotal role in his decision to engage in research:

> I applied for science research in the 9th grade...for science research, the school had different mentors present and that's how I started working with my mentor. I remember I liked his presentation and understood the math and thought he was funny...he was working with other students at my school, and getting results. So I was seeing it in action as I was getting started myself.

KN similarly highlighted how his school's research-oriented environment shaped his trajectory:

> My high school definitely had a culture where doing research was seen as something prestigious. It was something they encouraged a lot, especially in math and science. I saw older students presenting their research and winning competitions, and that made me want to get involved. I think having that kind of environment made a big difference—it made it seem normal to do research and even exciting.

This institutional support not only provided clear pathways to research mentorship opportunities but also fostered an environment where research engagement was both normalized and celebrated. For participants like JB and KN, their schools' established research programs provided clear pathways to mentorship opportunities while fostering a culture that valued mathematical research. The visibility of peer success and mentor accessibility within their schools made research participation feel both attainable and desirable.

Opportunities to Observe Success.

Schools with established research cultures often showcased research accomplishments through presentations and competitions, inspiring students to envision their own potential. JB reflected on how observing his mentor's work with other students encouraged him: "He was working with other students at my school and getting results. So, I was seeing it in action as I was getting started myself. That kind of pushed me over the edge."

Similarly, BH was motivated by witnessing older students' successes: "I saw students in my school winning competitions and presenting their projects. It showed me that research was something I could do too, and it gave me the confidence to try." BH also noted how teachers reinforced this culture by actively encouraging him: "My teachers were always pushing me to pursue math research. They really believed in my abilities and made me think that I could do something impactful in research." This highlights that some of the teachers at BH's high school played a critical role in reinforcing the school's research culture and encouraging students to participate.

For TV, peer influence played a key role:

> I had friends who had done it in previous years who are older than me. That was a big part of it. I also just knew that I wanted to try some kind of research. And this is actually like the only like high school math mentorship program I was aware of at the time. I think it was mostly that I knew people who had done the program, and they said, good things about it.

Structured Pathways to Participation

For many participants, their school environments not only showcased the potential of research but also provided structured pathways to make engagement possible for interested students. BH described his school's systematic approach: "There was this whole infrastructure in place...where students would start in their 10th grade, identify something that we're interested in, find a mentor, and then work on research." This structured support combined with the presence of successful peer role models helped participants envision themselves as capable of engaging in mathematical research. It motivated them to pursue mentorship opportunities, reinforcing the importance of a supportive and well-organized school culture in fostering advanced mathematical engagement.

The combination of an encouraging culture and clearly defined pathways was crucial in making these opportunities tangible for students. While structured pathways are closely tied to accessibility and visibility (explored further in the next section), their integration into the school's broader culture helped participants feel capable of engaging in research and motivated them to pursue these high school research mentorships.

Peer Networks

Peer networks emerged as a significant influence in shaping participants' decisions to engage in high school mathematics research mentorships, particularly for participants like RC and NM who attended schools without formal mentorship programs. For these individuals, peers provided both inspiration and critical guidance, helping them identify and navigate opportunities outside the structures of their schools.

Observing and Learning from Peers.

NM observed the trajectories of peers with similar mathematical backgrounds, using their experiences as a roadmap for her own aspirations. She reflected: "I started researching and seeing what people would share with like a similar background as me were doing...they all seem to be doing the same things...they all did competitions, Olympiads, and they all applied to the same research programs." For NM, this pattern not only inspired her but also informed her strategy for building a strong academic profile for elite universities. By modeling her decisions on those of successful peers, she identified research mentorships as a crucial step in her journey.

Similarly, RC's participation was shaped by insights he gained from his math team and the Young Scholars Program. He noted how hearing about peers' involvement in mentorships introduced him to the idea: "When you're just in these competitions, you hear things, like what other people are doing... that concept of mentorship sort of stuck." These peer networks provided RC with the knowledge that mentorships were both possible and valuable, even though his high school did not offer a formal program.

Motivated by Peer Influence.

For both NM and RC, peers did more than provide information—they also served as a motivating factor. Seeing peers engage in mentorships made these experiences feel attainable and desirable, pushing them to seek out opportunities independently. RC explained that his father's general guidance, combined with peer encouragement, led him to find a mentor externally:

> My dad had a background in math, but he was kind of removed from math like, he doesn't really like practice anything. But, he was able to give me like general guidance on like, oh, you should look for a mentor online. That's how I found Professor S. And he was the one who I worked with just via email… He was the one who sponsored my application in Science Talent the [research] competition I was in.

This proactive approach highlights the role of peer influence in normalizing research engagement and inspiring students like RC to pursue mentorships despite the absence of school-based opportunities.

Transition to Academic and Career Aspirations

For NM, peer influence was closely tied to her academic aspirations. She viewed mentorship as a stepping-stone toward her goal of attending elite universities: "I looked at these people, and they all started off doing competitions and then pivoted to research… I thought, okay, this is the path I need to follow."

Similarly, RC reflected that while his initial motivation was not explicitly tied to college, the broader influence of peers and academic culture shaped his perception of research as a valuable step: "I feel like most are motivated by, you know, how does this help my, like, college prospects? ... I can do a research paper and this will just help my high school career."

This interplay between peer networks and academic motivations underscores the interconnected nature of participants' decisions. Peers provided both a sense of possibility and a framework for understanding the value of research mentorships, setting the stage for participants to pursue these opportunities with a clear sense of purpose.

Bridging to Academic and Career Aspirations.

The influence of peer networks not only introduced participants to the concept of mentorship but also set the stage for their broader academic and career motivations. For participants like NM and RC, peers were a critical link between aspiration and action, providing the guidance and encouragement needed to pursue mentorships as a pathway to both personal and professional growth. This interconnectedness between peer influence and academic aspirations will be further explored in the next section.

Academic and Career Aspirations

Participants viewed research mentorships as pathways to academic and professional advancement, though their motivations varied in nature and intensity.

College Preparation

For some participants, the mentorship experience was directly tied to their aspirations of strengthening their candidacy for elite universities. NM explicitly linked her decision to these goals: "I wanted to go to MIT, Harvard, University too... I looked at these people and I started to notice this pattern, and I just tried to do everything they did." NM strategically modeled her actions on peers who had successfully used mentorships as stepping-stones to prestigious institutions. By following a clear pattern of engagement, she sought to align her experiences with what she perceived as the requirements for academic success.

In addition to helping RC with his "college prospects" he noted how the mentorship fit into his broader academic development. Reflecting on his decision, he shared:

> I was thinking of it like a stepping-stone. Both like affirmation for myself... I could do a research paper and submit it to competitions. It gave me something concrete to show for what I was learning ... it really opened up many opportunities.

This highlights how RC saw the mentorship as an opportunity to strengthen and validate his mathematical abilities while building a tangible accomplishment that supported his academic trajectory.

For both NM and RC, research mentorships were more than just academic experiences; they were seen as pivotal steps toward achieving their higher education goals, providing tangible outputs like research papers and competition submissions that bolstered their credentials.

Professional Exploration and Career Development

For some participants, mentorships provided clarity about potential career pathways and affirmed their commitment to mathematics. DB, who wanted to become a professor, described his mentorship as a transformative experience that defined his professional aspirations: "I could point to as the single moment in my life where I said, 'this is what I want to become, a professor.' It was really that dramatic."

Similarly, TV found that his mentorship experience helped him refine his academic interests and focus on a specific area of mathematics:

> I got lucky with my mentor. If I hadn't done this research project, I could have potentially gotten stuck in projects I wasn't as interested in... this project helped a lot. It helped me narrow my focus on what I actually wanted to specialize in math.

These mentorships were not only a means of exploring potential career paths but also a way of aligning participants' academic trajectories with their long-term professional aspirations.

Building Professional Skills

Participants recognized that mentorships offered opportunities to develop critical skills necessary for academic and professional success. For many participants, mentorships provided an introduction to the rigorous and iterative processes involved in mathematical research, equipping them with skills that extended into their professional and academic lives. CM, who later pursued theoretical computer science research, described the parallels between the two fields and how her mentorship experience laid the foundation for her approach to research:

> In math research, you're basically trying to solve a problem that no one's ever solved before. It's the same thing in theoretical computer science. You don't know if you'll get anywhere, but you try different approaches, and over time, you get better at figuring out what might work.

DB highlighted the importance of gaining experience in presenting research findings, a skill that required overcoming significant challenges: "Learning how to present my work was a very challenging thing. Certainly, every time I gave a talk, I was completely terrified. But my mentor taught me to really practice it over and over again."

These experiences equipped participants with essential competencies, such as persistence, communication, and problem-solving, that extended beyond the scope of the mentorship itself and prepared them for future academic and professional endeavors.

Interconnected Motivations

The interplay between college preparation, career exploration, and skill-building illustrates the multifaceted motivations driving participants to engage in research mentorships. For some, the mentorship was a stepping-stone toward academic and professional goals, while for others, it served as a catalyst for personal growth and career affirmation. The following section will explore how intellectual curiosity and personal growth further enriched participants' decisions and experiences, highlighting the intrinsic motivations that often underpinned their engagement with mathematics research mentorships.

Intellectual Curiosity and Personal Growth

Participants' decisions to pursue high school mathematics research mentorships were deeply rooted in their intellectual curiosity and desire for personal growth beyond the confines of traditional classroom experiences. These motivations stemmed from a love of mathematical exploration, a drive to experience non-traditional mathematics and a commitment to challenging themselves academically and personally.

Love for Mathematical Exploration

Several participants described being drawn to mathematics for its intrinsic qualities. KN recalled being captivated by mathematical concepts through a documentary:

> There was one documentary that I watched...about topology and geometry of higher dimensions...I thought it was very amazing that people are thinking about these higher dimensions and getting these useful and meaningful constructions.

TV got into mathematics from an alternate math class he took in middle school, this led to a curiosity and love of math that stayed with him throughout his whole life, leading him to participate in extracurricular math activities such as the research mentorship:

> so I got into math because my elementary school in like fourth or fifth grade had a program for people who were good at math and seemed to like it. They took an alternate math class. And that class was more focused on puzzles and problem solving, and you learn some more advanced math. And I thought that was really fun.

Desire to Experience Non-traditional Mathematics

Participants frequently mentioned the desire to challenge themselves and grow intellectually as a key driver for joining the mentorships. TV exemplified this drive as well as his love for mathematics when he recounted how he prepared to meet the MIT PRIMES's exam requirements:

> I don't want to like not get into this program because I didn't learn linear algebra. So I'm just going to try to learn it in like the next month...that probably gave me more confidence that, like a lot of other math, is learnable.

CM expressed curiosity about authentic mathematical research compared to competition mathematics:

> I’ve been really curious to try it...it sounds insane to me that people can solve problems that literally no one solve before, because I only worked in math contests, which are all constructed problems that people have solved before...I thought it was like a weird concept. And that’s largely why I wanted to try it.

Participants sought experiences that transcended standard coursework. BH appreciated how the mentorship offered: "breadth of knowledge about mathematics beyond the very...rigid curriculum that is taught K through 12."

This intrinsic motivation and desire for intellectual growth played a crucial role in participants' decisions to pursue research mentorships, complementing their academic and career aspirations. DB valued the creative aspects of mathematical research: "Learning to think about math in a creative way and learning to think about math that I'd never seen before...that was really exciting." As JB summarized: "I wasn't just sitting there being lectured to...I enjoyed trying to solve these problems...it was more about exploring and discovering."

The interplay of intellectual curiosity and personal growth was evident across all participants, as they sought mentorships not only to deepen their understanding of mathematics but also to challenge themselves and grow as individuals. These motivations, intertwined with their academic and career aspirations, highlight the multifaceted value of the research mentorship experience.

Accessibility and Visibility of Opportunities

Participants’ decisions to pursue high school mathematics research mentorships were strongly influenced by the visibility and accessibility of these opportunities. While school environments and peer networks played roles in normalizing research engagement, the structural mechanisms that provided tangible access to mentorships were pivotal for many participants.

This theme focuses on how participants encountered and navigated pathways to mentorships, particularly when these opportunities were not readily available through their schools.

Structured Programs and External Pathways

For participants like JB, BH, TV, CM, DB, and KN, structured school programs played a key role in making research mentorships accessible. These programs offered clear guidance on how to connect with mentors and engage in research. BH described how his school's infrastructure simplified the process: "There was this whole infrastructure in place...where students would start in their 10th grade, identify something that we're interested in, find a mentor, and then work on research."

This formalized approach ensured that students with an interest in research could engage without needing to navigate complex systems independently. JB similarly noted the ease of accessing mentorships through his school: "The school had different mentors present, and that's how I started working with mine...it was all set up for us, so we didn't have to go looking."

For others, like RC and NM, accessing mentorships required individual initiative and external resources, as their schools did not offer formal programs. RC reflected on his journey to find a mentor: "My dad encouraged me to look online, and that's how I found Professor S. He was the one who I worked with just via email...he reviewed my work and sponsored my application to the Science Talent competition."

RC's exposure to peers who had participated in research mentorships introduced him to the concept, which he noted "stuck" with him. This initial awareness became a catalyst for his proactive efforts to seek out mentorship opportunities independently. His journey underscores the importance of external platforms, such as Math Stack Exchange, in making research mentorships visible and accessible to students, particularly those whose schools lacked formal mentorship programs.

Influence of Outreach and Visibility

The visibility of mentorship opportunities through presentations, competitions, and peer networks also played a significant role. BH emphasized how school presentations made research feel attainable: "There were presentations about research opportunities, and teachers were always pushing us to apply. That made it easy to know what was available and how to get started."

For TV, visibility came from hearing about programs like MIT PRIMES through older peers: "I had friends who had done it in previous years who are older than me...I knew people who had done the program, and they said good things about it."

Efforts to highlight and promote research opportunities ensured that students became aware of mentorships and were encouraged to pursue them, even in schools without fully integrated mathematics research programs. Many schools celebrated and emphasized the achievements of students who participated in prestigious STEM research competitions, such as Regeneron, especially when they achieved notable success, such as becoming semifinalists, finalists, or winners.

Bridging Gaps through Initiative

Participants like NM and RC, who lacked direct access to mentorships through their schools, often relied on self-guided research and strategic planning to identify opportunities. NM described how she mapped out her pathway by observing the trajectories of successful peers:

> I started researching what people with similar backgrounds as me were doing...they all applied to these programs and did research, and they all ended up in these colleges I wanted to go to. So I thought, okay, this is what I need to do too.

RC similarly relied on a combination of peer networks and family guidance to navigate external mentorships: "It wasn't something my school offered, so I had to figure it out on my own. Hearing what other students were doing made me realize this was something I needed to pursue."

This resourcefulness underscores the importance of both individual determination and the visibility of mentorship pathways in enabling students to access opportunities that might otherwise be out of reach.

Navigating Institutional and External Resources

While some participants benefited from institutional structures that directly connected them to mentors, others relied on a patchwork of resources, including online platforms, Research Science Institute (RSI), MIT PRIMES, and personal networks. These varied approaches highlight the uneven accessibility of mentorship opportunities and the critical role of visibility in leveling the playing field. As TV noted: "This was the only high school math mentorship program I was aware of at the time...so it felt like my only option."

While the supportive culture of schools with established research programs played a key role in normalizing mentorship engagement, the accessibility and visibility of these opportunities were equally crucial in enabling participation. This theme focuses specifically on the mechanisms—both institutional and external—that provided tangible pathways to mentorships, complementing the broader cultural influences explored earlier.

RQ2: Developmental Processes in High School Mathematics Research Mentorships.

This section addresses Research Question 2: "How do participants describe and make sense of their lived experiences during their high school mathematics research mentorships and their integration into the mathematics community? Specifically, what mentorship processes and interactions shape participants' self-efficacy, identity, and values, and how do these experiences facilitate their integration into mathematics?”

Description of Structure and Format of Mentorships

Participants described consistent structural elements across their mentorships, typically meeting with mentors weekly for sessions lasting one to two hours. These mentorships operated predominantly on a one-on-one basis, creating intimate and personalized environments conducive to deep engagement with mathematical research. Participants DB, JB, and BH, engaged in individual mentorships at informal spaces, as BH recalled: "Usually in a very casual place, Starbucks or a casual café of sorts." While most participants met in person, either at informal locations or university offices/classrooms, NM collaborated virtually with two other students under their mentor's guidance through MIT PRIMES, while KN participated in person with another student and their mentor at a university setting. These formats, whether individual or small group, provided tailored guidance while fostering meaningful interactions that supported participants' mathematical development.

While most participants engaged in productive mentorships, the quality and impact of these experiences varied significantly. CM, for example, encountered challenges with her mentorship, particularly in selecting an approachable research problem. She reflected: "The problem I ultimately settled on, like, now I know in hindsight that it was just an extremely hard problem... I would have appreciated a little bit more guidance in problem selection."

Despite these difficulties, CM’s strong foundational love for mathematics and her deep-seated values as a math enthusiast enabled her to persist. She emphasized: "I still really enjoyed thinking about the problem...it made me better at thinking about math. The whole point of high school is to learn, and I definitely felt like I learned."

Her passion for mathematics and commitment to growth drove her to seek additional opportunities beyond her initial subpar experience. By college, she participated in a Research Experiences for Undergraduates (REU) program, which provided a more structured and supportive mentorship. CM described the difference in this experience:

> I think my first REU experience was a really, really good one. My mentor gave me a problem that was actually approachable, and I felt like I made legitimate progress—like I figured out something about the area that no one had before. That was a really proud moment for me.

CM's experience underscores how intrinsic motivation, a strong mathematical identity, and cultivated values can act as protective factors, enabling participants to persist despite challenges. These qualities allowed her to navigate a difficult initial mentorship and pursue more positive opportunities later, contributing to her integration into the mathematical community.

Similarly, TV reflected on the importance of his research mentorship in shaping his mathematical trajectory, though his circumstances differed. He expressed gratitude for the alignment between his interests and his mentorship experience:

> I got lucky with my mentor. If I hadn't done this research project, I could have potentially gotten stuck in other projects I wasn't as interested in...this project helped a lot. It helped me narrow my focus on what I actually wanted to specialize in math.

While TV's mentorship was more supportive, his comments suggest that, like CM, his pre-existing mathematical identity and passion for the field were critical factors in his ability to make the most of his research experience. Even if his mentorship had been less aligned with his interests, his intrinsic motivation and love of mathematics would likely have driven him to seek other opportunities to grow and explore his passion.

This variation in experiences highlights how participants' pre-existing mathematical values and identity often helped them navigate challenges and seek additional opportunities for growth. For those with less optimal mentorship experiences, their intrinsic motivation and love for mathematics drove them to pursue subsequent positive mathematical experiences that ultimately contributed to their integration into the field.

Emergence of Developmental Processes

Through analysis of participant interviews, four interconnected processes emerged as central to their mathematical development and integration into the mathematics community: adaptive scaffolding, professional skill-building, collaborative dynamics, and fostering a sense of belonging within the field. These processes evolved organically, progressing from structured guidance to more collaborative and independent research activities. As BH explained:
"My role in it kind of evolved over time...at the beginning we looked at comparably basic mathematics...later on, it was more of collaborative sessions."

These developmental processes not only cultivated research-specific self-efficacy and strengthened mathematical identities but also encouraged participants to internalize the values of the mathematics community. BH captured this evolution: "I think probably the thing that it's helped with most is creativity for some of these problems...especially on the creative side of problem solving, I think the mentorship has played a very large part in that."

The mentorship experience was characterized by mentors' ability to adapt to participants' needs, introduce them to professional mathematical practices, foster collaboration, and guide the development of their identities within the mathematics community. Together, these processes provided a transformative experience that extended beyond technical skill-building to shape participants' broader understanding of mathematics as a discipline.

Adaptive Scaffolding: Building Research Self-Efficacy and Career Awareness (Early Development Stage)

Participants consistently described their mentors' use of adaptive scaffolding, which enabled them to tackle challenging research problems incrementally. Mentors provided structured support while tailoring guidance to participants' evolving needs, creating an environment that balanced challenge and encouragement.

Early Stages of Guidance

Mentors played a crucial role in introducing participants to research processes, often starting with foundational concepts. TV recounted how his mentor began by assigning foundational papers: "The first few months were just like, he would send me a bunch of papers to read...that was helpful because I didn't have any experience with finding papers and figuring out which ones are important to read."

KN also reflected:

> My mentor gave me a textbook and said, 'Read these chapters.' Then he'd give me a problem, and I'd think really hard about it. I didn't feel much distinction between research and a long, hard homework problem—it just felt like something to work on.

This reframing of research as an achievable endeavor, rather than an insurmountable task, normalized the research process and contributed to KN's evolving self-concept as a capable mathematician.

TV similarly described how mentorship strategies encouraged incremental progress:

> The first like maybe four or five months were super slow and incremental. It was like actually kind of hard to tell if we were making progress at all. But then there was one time where we tried a small change to some model, and actually, it got a lot better. That was probably the only real breakthrough.

These small but meaningful milestones, facilitated by the mentor's guidance, reinforced participants' belief in their ability to contribute to mathematical research, which is a foundational component of self-efficacy. For TV, these experiences not only validated his abilities but also sparked a long-term interest in applied mathematics.

Tailoring to Participant Needs

Participants consistently highlighted the importance of guidance and adaptability in navigating the unfamiliar territory of mathematics research. Many entered the mentorship with high general mathematics self-efficacy but low self-efficacy in research-specific mathematics. The adaptive nature of the mentorship was crucial in sustaining participants' engagement when faced with challenges. Mentors responded by employing a combination of structured approaches and adaptive strategies to support participants' growth. Professor L noted, "Sometimes we'd go to some special cases that were a little easier for the student to deal with," emphasizing how he tailored his guidance to individual needs.

Mentors often adapted their strategies to sustain engagement and overcome challenges. BH emphasized the flexibility in his experience: "It was much more of a fun and exploratory experience than it was, you know, we need to deliver research results...it definitely differentiated itself from school."

Professor L described his approach to scaffolding: "Sometimes I simplified the problem when it looked like we weren't getting results. Sometimes I even switched to a different but related problem. At times, I solved the problem and asked for consequences of the theorems I proved."

Through these incremental tasks and flexible approaches, participants developed research-specific confidence. By cultivating confidence in research processes, mentors often planted the seeds for participants' career aspirations. This adaptability enabled participants to experience early successes and develop a stronger sense of research-specific self-efficacy. JB explained that observing his mentor simplify complex problems or adjust direction helped him feel more capable of achieving results in areas he initially found intimidating: "Seeing my mentor simplify complex problems made me feel like, okay, I can actually do this." Through adaptive scaffolding, participants not only developed technical skills but also began to see themselves as future mathematicians or researchers, setting the stage for continued academic and professional engagement.

Professional Skill-Building: Preparing for Academic and Career Success (Middle Developmental Stage)

Research mentorships exposed participants to professional practices, including writing, presenting, and publishing their work. These experiences not only enhanced their research skills but also fostered their professional development and developed their confidence in pursuing a career in mathematics.

Developing Presentation Skills

Research mentorships equipped participants with professional skills that extended far beyond their high school experiences, shaping their academic and career aspirations. DB's mentorship included intensive practice in presenting research findings, a skill he described as transformative:

> Learning how to present my work was very challenging. Certainly, every time I gave a talk, I was completely terrified. But my mentor taught me to really practice it over and over again, and I was able to get through it.

DB shared video footage of his first conference presentation at Graph Theory Day 72 at Pace University, along with recordings of subsequent presentations at various conferences he delivered during high school. These videos illustrate his remarkable progression as a mathematics speaker and presenter, showcasing his growth in presentation skills over time.

These presentation skills prepared DB for future academic environments, where clear communication of complex ideas is essential. DB went on to study mathematics at a prestigious university, crediting his high school mentorship as pivotal in building the confidence needed for his academic journey.

BH similarly highlighted how presenting at conferences built his confidence: "I also did get to present my work at various conferences...it was terrifying at first, but over time, I learned how to communicate my ideas more effectively."

Developing Writing Skills

KN described the mentorship as particularly supportive through small, manageable tasks: "The mentor was super helpful... I hadn't done any of that before, so it was my first time seeing how it worked." His mentor provided both structure and incremental tasks, initially creating a high-level outline to guide the project. KN shared, "He would give me little assignments, like, could you write up this example?" These manageable tasks allowed KN to contribute gradually: "I wrote most or all of the sections, and then he would edit them."

As participants gained familiarity with research practices, mentors adjusted their guidance as TV continued:

> And then after that [reading research papers the mentor assigned], we moved into like more of the specifics of what we had already read from most of the papers, and we're just like trying to figure out what specific ones [mathematical models] we wanted to emulate and like build on top of. And then we started like writing our own models based on that.

Exposure to Research Practices (Advanced Development/Integration Stage)

Participants valued their mentors' guidance in navigating the research process, from conducting simulations to writing and submitting papers. TV explained: "After we read most of the papers, we started writing our own models...my mentor was super helpful for that because he just knew how to write [research] papers well and come up with the right figures."

This guidance extended to the publication process, as DB shared: "Seeing my name in print was extremely exciting. It really made me feel like a real mathematician. That thrill is something I carry with me even now." This tangible recognition of his contributions reinforced DB's sense of belonging within the mathematics community, encouraging him to explore further opportunities in research and academia.

Collaborative Dynamics: Transforming Identity Through Partnership

Mentorships often marked participants' first exposure to collaborative research, shifting their perception of mathematics from an individual pursuit to a community-based endeavor.

Building Research Partnerships

Mentors treated participants as collaborators rather than assistants, fostering a sense of partnership. DB emphasized this shift:

> My mentor always treated me as a collaborator and not as an assistant. It was never, 'Here, I solve this, you go write it up.' It was always us speaking together, saying, 'Hey, here's an idea,' and going back and forth.

Similarly, Professor L noted: "I tried very hard to treat my students as research equals. They slowly started to feel that we were two collaborating researchers, and that made a big difference in how they saw themselves."

Peer Collaboration

In some mentorships, collaboration extended to peers, offering participants a new perspective on the communal nature of mathematics. KN, who worked on a joint project, reflected: "There was another high school student with me, and we worked on the same project together. It felt like a group effort, which was different from anything I'd done before."

Similarly, NM described how her experience with MIT PRIMES reshaped her perception of mathematics research:

> Before I joined MIT PRIMES, I thought math research was like competitions—an independent effort where I worked alone, communicating only with my mentor until I found a solution. However, working with S and B under Dr. F in a group project completely changed my perspective. Collaborating with them really helped me resolve lingering questions and gave me new perspectives on our project.

Through these collaborations, participants learned to rely on the strengths of their peers while contributing their own insights. NM elaborated on the dynamics of teamwork in her mentorship: "We bonded over revising each other's work and shared the frustrations of slow progress. It was eye-opening to see how much collaboration could enhance the research process."

These collaborative experiences not only fostered teamwork and mutual support but also reinforced participants' identities as emerging mathematicians. The shared challenges and triumphs of working together demonstrated the inherently communal nature of mathematics research, encouraging participants to see themselves as part of a broader academic community.

Fostering a Sense of Belonging (Developing Mathematics Identity and Values)

Mentorship processes played a critical role in shaping participants' mathematical identities, helping them see themselves as members of the professional mathematics community.

Internalizing Research Values

Through mentorship, participants developed an appreciation for the creative and iterative nature of mathematical research. BH reflected: If you have an interest in math beyond just another school subject, you can see the creativity and beauty in the proofs or problems...it's a great way to expand your capabilities.

RC described how publishing his work shifted his understanding of research's broader impact:

> I'd say after submitting, only then did I realize that you know, I'm contributing to the literature and like this, maybe useful to some extent. Like there were a couple of news outlets that used my paper for headlines like the NSA, like, oh, like, potentially a new algorithm that can be used by the NSA, for elliptic curve cryptography. Stuff like that. So I mean, it's buzzwordy, but when I saw that I was like, oh, maybe this may have more impact than what I did find like yeah, I mean, I was pretty naive. And if I'm being honest, I didn't really know like, the broad sense of my work, I didn't really know like, what effect it could have. I just did it for like a research competition. If I'm being honest.

Many research mentorships, particularly those in applied mathematics, also instilled a deeper appreciation for how math can address real-world problems, thereby reinforcing participants' perceptions of its importance in their lives. This firsthand experience of adding to the mathematical literature not only strengthened participants' mathematical values and their perception of the importance of mathematics but also contributed to their identities as individuals capable of advancing mathematics and demonstrating its value to the world.

Career Aspirations and Long-Term Support

Mentors also provided career guidance, helping participants envision their future in mathematics. BH shared:

> Having those conversations with my mentor gave me a better sense of the mathematics world. He'd talk about his experiences, what it took to get where he was, and it gave me better insight into the whole grad school process and helped me know what classes I should take, the quals, and sort of helped me envision what the road would be like. And it seemed doable.

Similarly, DB claimed:

> It wasn't until I started working with my mentor that I said, well, this could actually be a career in and of itself, and, you know, this is something that I could pursue. And so that really became my goal from when I first started working with him and it was the first time I really realized, oh I could actually do this [graph theory research].

This explicit support demystified the pathways to advanced study, inspiring participants to consider long-term careers in mathematics or related fields.

Professor L described how he supported students' long-term development:

> After seeing a student, once a week for maybe several months. We would also have a drop of small talk, you know, how was your day. You can get to those students a little bit, and you get to you get the ability to make an assessment that the student might need a little bit more encouragement in general. So, I started to realize that my role was more than just the research director, but I had to keep my student going, and to keep the student excited about the project when maybe enthusiasm lags a little bit. And once in a while, we'd have a discussion about what mathematicians do, and I explained to the students you're doing what mathematicians do. Congratulations, you are a budding mathematician, and sometimes students will ask me questions about how hard is it to get a math PhD, for example. Now I'll explain with diligence and daily work. While it's very difficult. But, let the student know you can do it. And it's very rewarding. They asked me how I like teaching, how I like the research, the pressure, etc. So herein we would chat a little bit about what a career in mathematics entails, the joy of it, the difficulties. So yeah, eventually I realized that I was a little bit of a career counselor, a little bit of a guidance counselor, a little bit of friend, and of course, very much a research mentor.

This combination of technical preparation and personalized career guidance gave participants the tools and confidence to navigate academic transitions and professional challenges, solidifying their integration into the mathematics community.

Summary: Processes Supporting Integration and Growth

Participants' high school mathematics research mentorships facilitated their integration into the mathematics community through key processes, including adaptive scaffolding, professional skill-building, collaborative dynamics, and the development of mathematical identity and values. These experiences equipped participants with the confidence and skills to navigate mathematical research, while fostering a sense of belonging within the discipline. By blending technical development with personal and professional growth, these mentorships served as a critical foundation for participants' future trajectories in mathematics and beyond.

RQ3: Key Insights and Skills Gained through High School Mathematics Research Mentorships

This section addresses Research Question 3: "What additional factors, beyond self-efficacy, identity, and values, do participants identify as significant from their high school mathematics research experiences, and how do these factors contribute to their integration into mathematics?"

Through analysis of participants' experiences, three key factors emerged as significantly influential aspects of their high school mathematics research experiences, extending beyond the constructs of self-efficacy, identity, and values. These factors include: self-directed learning, insights into the research process, and professional networks.

Self-Directed Learning

Participants frequently emphasized the role of self-directed learning as a skill they cultivated during their high school research mentorships. For many, this marked the first time they engaged in independent study beyond the structured confines of school curricula. TV recounted how preparing for the MIT PRIMES application test required him to teach himself linear algebra, a topic he had not yet encountered in school:

> One of the problems was this linear algebra problem. And I hadn't learned linear algebra yet...I had like two months from when they released the application to when I had to submit it. So I was like, 'I don't want to not get into this program because I didn't learn linear algebra.' So I just decided to try to learn it in the next month.

Although he admitted not mastering the subject entirely, TV successfully taught himself enough to solve the problem, describing the experience as transformative:

> That was probably my first time learning something that wasn't in the school curriculum, and that I didn't actually know how to learn. But I just tried to learn it really quickly...it was pretty fun. That probably gave me more confidence that a lot of other math is learnable, even if I don't have a teacher for it.

For other participants, their mentorships fostered a natural curiosity that extended beyond formal sessions. BH reflected on how working with his mentor encouraged him to explore mathematical topics independently:

> I first started reading, studying, or doing math independently outside of schoolwork or Kumon mainly came about as an interest in things Professor L and I worked on during our sessions. That was more enjoyable to work on than schoolwork or the like.

These experiences fostered participants' ability to take initiative, manage their learning independently, and persist in solving complex problems, skills that extended well beyond the research mentorship and into their academic and professional lives. Self-directed learning became a cornerstone of their ability to approach unfamiliar challenges with confidence and resilience.

Insights into the Research Process

Participants developed a deeper understanding of the nature and practice of mathematics research, reshaping their preconceived notions about the field. This process helped them build foundational research skills, including navigating long-term projects, interpreting existing literature, and presenting findings. DB described the iterative nature of research: "Research is about starting with questions that might not even have answers, but those questions lead to discoveries. It taught me to think creatively."

For TV, the experience transformed his perspective on what mathematics research entails and his capacity to contribute:

> My first time doing math research, and specifically, my project was more like applied in computational math. There is not that much like new theoretical contributions. And I guess, before that I had always thought of doing math research as like proving new theorems, or new results whatever coming up with new things. And this program helped me realize that actually, a lot of research is more incremental or like you're not necessarily inventing a new thing on your own. It's a lot of applying stuff that other people have done to a new problem. For example. I would say, that's how my view of

> math changed, or that's like the biggest part of it. How it changed and how I view myself…beforehand. I think I was just not sure I didn't really have any conception of what math research was. I would guess, probably not really. But afterwards, like realizing that the research is more incremental and less about necessarily always coming up with big new ideas. Then, I realized, yeah, it's probably doable.

JB reflected on the practical skills he gained, particularly in writing and problem-solving:

> I learned how to write a theory and use math lingo to write a technical paper. There's a certain way in which you write it. That was a skill I picked up. Problem-solving was a skill I already had, but the mentorship worked it from a different angle.

Participants noted that these experiences often had direct implications for their academic and professional trajectories. For example, BH, now a research engineer, highlighted how graph theory from his mentorship carried into his career:

> There's still quite a bit of graph theory in the work that I do, particularly different graphs and directed acyclic graphs (DAGs) for processing and analyzing lots of financial data…whatever career success I've had has been a result of my skills in the underlying areas. And that, I do attribute in large part to my overall education, background, and the mentorship a great deal as well.

Similarly, CM described how her early exposure to combinatorics aligned with her current role:

> Now, I do theoretical computer science research, which is like algorithms and such. But honestly, theoretical computer science has a very similar feel to combinatorics in terms of the type of math. My job now is basically doing combinatorics research.

While JB's work as an industrial operations engineer does not directly involve graph theory, he emphasized how the skills he developed in the research mentorship—particularly thinking theoretically and problem-solving—shaped his studies and career:

> The number theory and graph theory I did with Professor L was a lot of maximizing, minimizing, and playing at those limits. I don't use the specifics of what I learned in graph theory, but I do take the problem-solving mindset into my work, especially thinking theoretically about efficiency and optimization.

These insights into the research process not only demystified mathematics research but also empowered participants to see themselves as capable contributors. The mentorships provided them with essential tools for approaching complex problems, navigating the iterative nature of research, and engaging in academic and professional communities.

Professional Networks and Community Integration

High school research mentorships provided participants with invaluable connections to broader mathematical communities through conferences, presentations, and collaborative work. These experiences helped participants build professional networks and envision their place within the mathematics field. DB described how presenting his work at conferences enhanced his confidence and professional exposure:

> I also did get to present my work at various conferences, mostly the graph theory day where they have keynote speakers during the day and there are the short 15-minute sessions and learning how to present my work was a very, you know, challenging thing. Certainly, every time I gave a talk, I was completely terrified. But, Professor L taught me to really practice it over and over again, and I was able to get through it. Thanks to his help and support.

DB further noted, "These conferences gave me opportunities to meet many different graph theorists and mathematicians who studied various research topics in math, which excited me and made me want to be like them."

Professor L highlighted the importance of fostering professional relationships, stating:

> I must say I did assist students in becoming mathematicians, so by teaching them the research ropes. They, they slowly started to get the feeling that we were two collaborating researchers, and I tried very hard to make that happen as opposed to just Hello, Hello students, but I started to treat these students as research equals, and I think they felt that, and most of my former proteges went on to get masters or PhD in mathematics. Some are professors today, and some are very successful ƒs working in industry on Wall Street.

Participants often described how these relationships and experiences helped them envision their place within the professional mathematics community. BH reflected:

> Having those conversations with my mentor gave me a better sense of the mathematics world. He'd talk about his experiences, what it took to get where he was, and it gave me better insight into the whole grad school process and helped me know what classes I should take, the quals, and sort of helped me envision what the road would be like. And it seemed doable.

While mentors provided essential guidance for navigating academic pathways, the professional networks participants built through their mentorships often translated into tangible opportunities for career development. As RC noted: "The research paper got my foot in the door... it was something I had in my back pocket for college applications, internships... it really opened up many opportunities."

These professional connections not only offered practical benefits, such as facilitating admissions and internships, but also helped participants envision themselves as legitimate members of the mathematics community, further supporting their long-term integration into the field.

Summary of Additional Factors

The additional outcomes identified—self-directed learning, insights into the research process, and professional networks—enhanced participants' high school mathematics research mentorship experiences by providing them with practical skills, valuable connections, and a deeper understanding of the mathematical field. These factors complemented the constructs of self-efficacy, identity, and values by bridging psychological growth with tangible academic and professional preparation.

RQ4: Beyond the Mentorship: Influential Factors in Participants' Integration into Mathematics

This section addresses Research Question 4: "What factors beyond the mentorship do participants perceive as most significant in their integration into mathematics, and how do they interpret the role of these factors in shaping their academic and professional trajectories? How, and to what extent, do these interpretations align with the Tripartite Integration Model of Social Influence?"

Through IPA analysis, participants identified diverse influences that shaped their mathematical integration. While each pathway was unique, several shared experiential themes emerged, revealing how these factors collectively supported participants' long-term engagement in mathematics. This section explores both personal experiential themes (PETS) relating to individual pathways and shared group experiential themes (GETS), highlighting their alignment with TIMSI.

Early Family Support and Academic Acceleration

RC emphasized his father as having the most significant role in cultivating his mathematical discipline and advanced abilities: "My dad for sure. Just starting from a young age that got me into the discipline aspect of it...He got me to a more advanced place compared to my peers, and everything followed from there."

Similarly, DB credited his father for fostering early mathematical development: "I started having pretty serious interest in math, when I was in middle school and high school. And started having accelerated curriculum…because my father taught me a whole bunch of high school math."

For BH, his family—particularly his mother—played a critical role in connecting him with mentorship opportunities: "It was my mom in conjunction with my 8th grade math teacher, who identified Professor L as a potential mentor."

These quotes demonstrate the variety of family influences: (1) RC with direct teaching, curriculum acceleration, and discipline, (2) DB with direct teaching and curriculum acceleration and (3) BH with facilitation of opportunities.

TIMSI Analysis: Family Support and Academic Acceleration

The role of family support aligns with TIMSI's three components, reflecting its multifaceted impact on participants' mathematical integration:

Self-Efficacy.

Participants gained early confidence in their mathematical abilities through direct teaching, encouragement, and access to opportunities provided by their families. For instance, RC's father accelerated his math curriculum, helping him enter high school at the level of being "two years ahead of the normal curriculum" compared to his peers, while DB credited his father's teaching for sparking his "serious interest" in math.

Identity.

By cultivating a disciplined and supportive environment, families fostered participants' identification with mathematics. For example, RC saw himself as a "strong math student" early on due to his father's teaching, and BH's mother played an active role in connecting him to mentorship opportunities, reinforcing his belonging in the mathematics community.

Values.

Families embedded the importance of mathematics as a meaningful pursuit, whether through direct engagement, as in the case of RC and DB, or by facilitating access to advanced opportunities, as in BH's case. These values were internalized by participants, motivating them to pursue advanced studies and professional opportunities in mathematics.

This analysis highlights how familial influences not only provided participants with practical support but also shaped their perceptions of themselves as capable and dedicated mathematicians, reinforcing their integration into the field.

Competition-Driven Engagement

Math competitions emerged as the most significant influence for both CM and NM, though their motivations and experiences differed markedly. For CM, competitions sparked genuine enjoyment and intrinsic interest:

> Yeah, so that the main thing that got me into math was the contests...I really like thinking about the problems. But I also, I'm sure that, like, you know, I pursued it because I was really good at it...even before I did contests like I know that there are all these little kid like logic puzzles...I would really like them when I was a kid.

This intrinsic enjoyment of problem-solving, combined with the reinforcement of her abilities through competitive success, solidified her identity within the mathematical community. These experiences also laid the groundwork for her future pursuits in combinatorics and theoretical computer science. CM noted how contests shaped her mathematical values: "I think the math contests had a big impact on me pursuing research and on me pursuing math."

In contrast to CM's intrinsic enjoyment of mathematics, NM's path was initially shaped by external pressures and cultural context. Growing up in China's competitive educational environment, NM explained:

> I'm from Hebei province in China, so the education atmosphere there is very competitive. From an early age we all try to get into a good middle school and then a good high school. There's a lot of pressure to do well academically, especially in subjects like math because it's a big part of the middle school and high school entrance exams.

This systemic pressure was reinforced by clear consequences, as NM recalled: "My parents and some teachers stressed that if you don't perform well in school, you're likely to end up working long hours in low-paying factory jobs." Despite not starting as a strong mathematics student, NM experienced a transformative moment through intense personal dedication:

> I actually wasn't good at math at all! In elementary school, I remember getting zero scores on practice exams...But I think a huge turning point was in middle school...I remember I trained intensely for like months before that competition...I'd literally spend maybe like 12 to 15 hours a day just like doing nothing but like math problems. It was like I was obsessed.

This led to an unexpected success which reshaped her mathematics identity: "Nobody was expecting this...I ended up placing in the top five in my province and Beijing and I was first among all girls." However, this achievement created new pressures: "Once I started getting recognized, there was this pressure to keep performing well...I saw myself as one [strong mathematics student/competitor] too and I didn't want to disappoint anyone or myself."

NM's mathematical integration ultimately became strategic, viewing mathematics as a pathway to elite universities: "I knew I wanted to go to a good college so I kept studying hard." Unlike CM's organic progression from competitions to research, NM approached additional mathematical opportunities, including research mentorships, as calculated steps toward her academic goals, particularly admission to institutions like "MIT or Harvard University."

TIMSI Analysis: Shared Impact of Mathematics Competitions Through Different Lenses

The impact of math competitions aligns with all three components of the Tripartite Integration Model of Social Influence (TIMSI):

Self-Efficacy.

Both participants gained confidence in their mathematical abilities through the recognition and success achieved in competitions.

Identity.

CM's sustained enjoyment reinforced her identity as a "math person," while NM's competitive successes shaped how she and others perceived her within the mathematical community.

Values.

For CM, contests deepened her appreciation for problem-solving. For NM, they became a means to achieve broader life goals, reflecting a practical valuation of mathematical skills. This contrast illustrates the varied pathways through which competitions influence mathematical integration, highlighting both intrinsic and extrinsic motivations as drivers of sustained engagement.

Early Mathematical Experiences

For TV and KN, early mathematical experiences emerged as the most significant influences on their integration into mathematics. These formative moments sparked a passion and curiosity for the subject that persisted through challenges and setbacks.

TV's Early Engagement Through Enrichment Programs

TV described a pivotal moment during elementary school when he participated in an advanced math class designed for students with a natural aptitude for the subject: I got into math because my elementary school in fourth or fifth grade had a program for people who were good at math...that class was more focused on puzzles and problem solving, and I thought that was really fun.

This program not only introduced him to advanced concepts but also emphasized creative problem-solving, which he found particularly enjoyable. As TV progressed through middle school, participation in math contests further fueled his interest, reinforcing his confidence and skills: "Part of the class focus was like, we're going to give you some problems from these contests. And so part of it was like doing better on those contests. Part of it was just learning more math."

Despite initial trepidation about advanced coursework like calculus, TV's independent exploration of mathematical topics deepened his understanding and love for the subject:

> I picked up a really good calculus textbook that covered it more fundamentally...when I actually took the class the next year, it was super easy. That gave me confidence that a lot of math is learnable, even if I don't have a teacher for it.

KN's Inspiration from Media and Exploration of Advanced Topics

KN traced his interest in mathematics to a moment of inspiration sparked by a documentary on topology and higher dimensions:

> There was one documentary that I watched in late middle school...it talked about things like higher dimensional objects and Hopf fibrations. I thought it was amazing that people could think about these things and get meaningful constructions and results out of them. That's when I thought, maybe I could be a mathematician and explore these things myself.

While KN initially saw himself as an average math student, this documentary ignited a curiosity that motivated him to explore challenging mathematical concepts. He pursued this passion through high school, participating in competitions and advanced coursework, even though his competition results were not always stellar: "I didn't do super well [in competitions], so then I tried to push through and do projects in math. Those experiences introduced me to harder math, and I enjoyed the research."

KN's journey highlights the transformative power of intellectual curiosity, as his exposure to advanced topics in high school cultivated a love for geometry and algebraic geometry, which he pursued further in university and graduate school.

Distinctive Contributions to Integration

For both TV and KN, these early experiences fostered a persistent passion for mathematics that helped them navigate challenges and setbacks. Their intrinsic curiosity, coupled with opportunities to explore advanced concepts, strengthened their mathematical identity and resilience. These foundational experiences aligned with TIMSI in the following ways:

Self-Efficacy.

Participation in enrichment programs and independent exploration gave TV and KN the confidence to tackle increasingly complex mathematical challenges.

Identity.

Both participants internalized a sense of belonging in the mathematical community, as they began to see themselves as a "math person."

Values.

Their early experiences instilled an appreciation for the beauty and creativity of mathematics, which they valued and carried forward in their academic journeys.

This analysis underscores how early 'fun' engagement with mathematics can play a pivotal role in shaping long-term integration into the field, providing the foundation for sustained interest and achievement.

Multiple Mentoring Relationships

For some participants, the influence of multiple mentors played a pivotal role in their integration into mathematics, especially outside the high school research mentorship experience. DB particularly highlighted how the guidance of various mentors shaped his journey: "Having professors who were willing to give their time and share their experiences and thoughts in math really helped me picture my future. They offered help with research, guidance, and encouragement that helped me stay grounded."

These mentoring relationships spanned high school and college, providing DB with both academic support and personal encouragement during critical moments. Reflecting on his mentorship journey, DB noted that one of the most valuable aspects of having multiple mentors was the ongoing advice and collaboration, which extended beyond the boundaries of individual projects.

Similarly, JB described the cumulative impact of diverse mentors throughout his academic and personal development: "Looking back, it was a variety of things. Each mentor I had, my dad, business partners...all those small moments of advice added up."

JB's experience underscores how mentorship is often an evolving, multifaceted process. Even outside formal mentorship settings, JB found value in the guidance he received from various sources, which collectively influenced his academic choices and career trajectory.

BH also credited his career success to the combination of his educational background and mentorships. He stated: "I think whatever career success I've had has been a result of my skills in the underlying areas...And that I do attribute in large part to my overall education, background, and the mentorship a great deal as well."

This synthesis of mentoring relationships highlights how these experiences collectively provided participants with technical knowledge, personal confidence, and practical insights into navigating their academic and professional paths.

TIMSI Analysis: Multiple Mentors as Sustained Sources of Influence

Self-Efficacy.

By working with multiple mentors, participants built confidence in their mathematical abilities through consistent encouragement and support. These relationships helped them navigate challenges, reinforcing their belief in their capacity to succeed.

Identity.

The guidance participants received from mentors reinforced their perception of themselves as mathematicians. Mentors often treated them as collaborators, which strengthened their sense of belonging within the mathematics community.

Values.

Mentors instilled an appreciation for the creative and collaborative aspects of mathematics. Through their relationships, participants internalized values of persistence, intellectual curiosity, and a commitment to contributing to the field.

The narratives of DB, JB, and BH illustrate how mentorship is not a singular experience but rather a network of influences that extend across time and contexts. These relationships not only provided immediate academic support but also shaped participants' long-term trajectories in mathematics.

Chapter 5 Summary, Conclusions, and Recommendations

This study explored the experiences of individuals who participated in high school research mentorships and their subsequent integration into the mathematics community. Specifically, it aimed to understand the factors influencing participants' decisions to engage in high school research mentorships, the processes within these mentorships that shaped their mathematical self-efficacy, identity, and values, and the factors beyond the mentorships that participants identified as most significant in their academic and professional trajectories. Using Interpretative Phenomenological Analysis (IPA), I analyzed semi-structured interviews with eight participants, ranging in age from 23 to 27, who had completed high school research mentorships and are now established in mathematics-related careers or academic pursuits. The IPA approach allowed for an in-depth exploration of their lived experiences, with a focus on understanding the personal and social meanings they assigned to their high school research mentorships and their integration into the mathematics community.

This study identified key factors shaping participants' journeys into mathematics, including intrinsic interest in the subject, peer and school influences, and mentorship structures. It also highlighted the role of high school research mentorships in fostering mathematical self-efficacy, identity, and values while identifying additional contextual and structural factors, such as institutional support and career-oriented motivations, that contributed to participants' sustained engagement in mathematics.

The findings extend the Tripartite Integration Model of Social Influence (TIMSI) by applying it to high school-level mentorships in mathematics and exploring how self-efficacy, identity, and values interact with other factors to influence long-term integration into the mathematics community. This research contributes to a deeper theoretical understanding of how high school research mentorships impact participants' trajectories and how these experiences shape foundational elements critical to their sustained engagement in the field.

The findings of this study reveal distinct patterns in how participants engaged with and navigated their mathematical journeys through high school research mentorships. Beginning with their initial decisions to participate, the analysis sheds light on the interplay of factors that influenced their entry into these programs.

5.1 Summary of Research Questions

Research Question 1: **What factors influenced participants' decisions to engage in high school mathematics research mentorships, and what personal meaning do they assign to these decisions?**

Participants' decisions to engage in high school mathematics research mentorships were shaped by a combination of personal, social, and structural factors, revealing four interconnected themes: supportive environments, academic and career aspirations, intellectual curiosity and personal growth, and the accessibility and visibility of opportunities. Supportive environments, including family encouragement, school culture, and peer networks, provided foundational motivation and facilitated pathways to mentorships. Academic and career aspirations often intertwined with participants' goals to enhance their educational profiles and explore professional trajectories. Intellectual curiosity and a desire for personal growth drove participants to pursue mentorships as opportunities to engage with mathematics beyond traditional classroom settings. Finally, the accessibility and visibility of mentorship programs, through structured pathways or individual initiative, played a pivotal role in enabling participants to identify and seize these opportunities. Collectively, these factors underscore the multifaceted motivations driving participants' engagement with high school mathematics research mentorships.

Research Question 2: How do participants describe and make sense of their lived experiences during their high school mathematics research mentorships and their integration into the mathematics community? Specifically, what mentorship processes and interactions shape participants' self-efficacy, identity, and values, and how do these experiences facilitate their integration into mathematics?

Participants' high school mathematics research mentorships facilitated their integration into the mathematics community through four key processes: adaptive scaffolding, professional skill-building, collaborative dynamics, and fostering a sense of belonging within the field. These mentorships were typically structured as weekly one-to-two-hour sessions, either in person or via virtual platforms, and provided personalized guidance through one-on-one or small-group formats. While experiences ranged from positive to transformative, one participant reported a less optimal mentorship that was still valuable in fostering mathematical growth.

Adaptive scaffolding emerged as a central process, enabling participants to tackle challenging research problems incrementally. Mentors tailored their guidance to participants' evolving needs, balancing structured support with opportunities for independent exploration. Professional skill-building further prepared participants for academic and career success, as they developed competencies in presenting, writing, and conducting authentic mathematical research. Collaborative dynamics shifted participants' perceptions of mathematics from an individual pursuit to a community-oriented discipline, deepening their engagement and sense of belonging.

These processes collectively supported the development of self-efficacy, identity, and values, equipping participants with the confidence and skills to navigate mathematical research and envision long-term careers in mathematics. Through scaffolded learning, professional preparation, and collaborative engagement, participants transitioned from learners to active contributors within the mathematics community, setting a pathway for their academic and professional trajectories.

Research Question 3: What additional factors, beyond self-efficacy, identity, and values, do participants attribute to their high school mathematics research mentorships, and how have these factors contributed to their academic and professional development in mathematics?

Participants' high school mathematics research mentorships fostered growth beyond self-efficacy, identity, and values through three key additional factors: self-directed learning, insights into the research process, and professional networks. These factors played pivotal roles in shaping their academic trajectories and integrating them into the mathematics community.

Self-Directed Learning: The mentorship experience fostered independent learning capabilities as participants navigated unfamiliar mathematical territory beyond traditional curricula. These experiences developed crucial skills in self-regulated study, time management, and problem-solving that extended well beyond the mentorship period.

Insights into Research Process: Participants gained deeper understanding of mathematical research practices, shifting their perceptions from viewing mathematics as theorem-proving to appreciating its incremental and collaborative nature. This understanding demystified research processes and helped participants envision themselves as capable contributors to mathematical knowledge.

Professional Networks: The mentorship experience connected participants to broader mathematical communities through conferences, presentations, and collaborative work. These professional networks provided both immediate opportunities and long-term pathways for career development, helping participants establish themselves within the mathematics community.

Together, these additional factors complemented the development of self-efficacy, identity, and values by providing practical tools and connections that supported participants' sustained engagement in mathematics. The combination of self-directed learning capabilities, research understanding, and professional networking opportunities created a robust foundation for continued mathematical engagement and career development.

Research Question 4: What factors beyond the mentorship do participants perceive as most significant in their integration into mathematics, and how do they interpret the role of these factors in shaping their academic and professional trajectories?

Participants identified several factors beyond the mentorships that were instrumental in their integration into mathematics, highlighting the multifaceted influences shaping their academic and professional trajectories. These factors included family support, early mathematical experiences, math competitions, and multiple mentoring relationships, each playing distinct but interrelated roles in fostering self-efficacy, identity, and values.

Family support emerged as a foundational influence for some participants, providing early academic acceleration, encouragement, and access to mentorship opportunities. Others credited formative mathematical experiences, such as advanced classes or exposure to stimulating content, with sparking their curiosity and solidifying their mathematical identity.

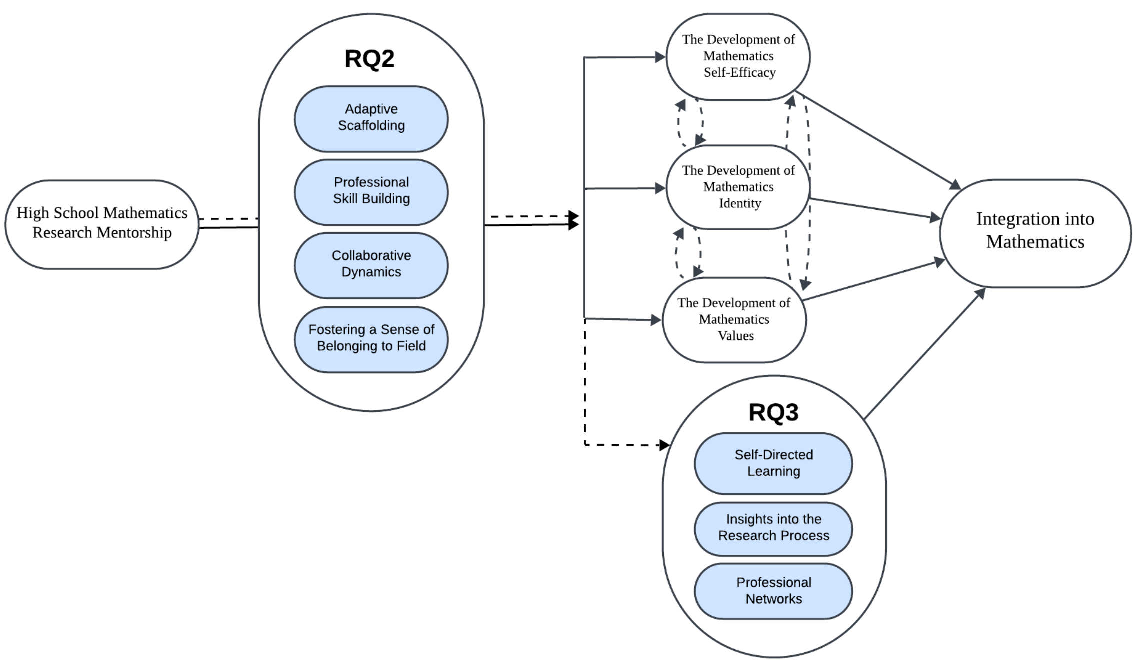
Math competitions also played a pivotal role, offering both intrinsic enjoyment and extrinsic motivation through success and recognition, though participants' motivations ranged from passion-driven engagement to strategic academic goals.

The influence of multiple mentors extended beyond high school mentorships, offering participants long-term guidance and professional connections. These relationships provided technical knowledge, encouragement, and practical advice, shaping participants' confidence and their ability to navigate academic challenges. Together, these factors aligned with TIMSI's components by fostering participants' self-efficacy, strengthening their mathematical identities, and embedding values of persistence and intellectual curiosity, ensuring their sustained integration into the mathematics community.

5.2 Theoretical Contributions of the Study

This study advances understanding of how high school mathematics research mentorships foster integration into mathematics by identifying four developmental processes that contribute to the TIMSI constructs of self-efficacy, identity, and values. While prior research on the Tripartite Integration Model of Social Influence (TIMSI) emphasizes the importance of these constructs in STEM integration (Estrada et al., 2011; Hernandez et al., 2018), this study focuses on the mechanisms by which these constructs develop within high school mathematics mentorship contexts.

Figure 3: Developmental Processes and Integration Pathways in High School Mathematics Research Mentorships



Key Developmental Processes

As shown in Figure 3, this study identifies four core processes that support participants' integration into the mathematics community: adaptive scaffolding, professional skill-building, collaborative dynamics, and fostering a sense of belonging within the field. These four key processes make unique yet interconnected contributions to the development of TIMSI constructs, supporting students' integration into mathematics

Adaptive Scaffolding and Self-Efficacy.

Adaptive scaffolding plays a fundamental role in developing mathematics self-efficacy through structured support and progressive challenges. Research shows that when mentors carefully calibrate task difficulty, students develop stronger beliefs in their mathematical capabilities through mastery experiences (Bandura, 1997; Hackett & Betz, 1989). The process begins with structured guidance on foundational skills, gradually increasing task complexity as students demonstrate readiness. This progressive approach allows students to build confidence through successful experiences while developing resilience in facing mathematical challenges (Boaler, 2016). Additionally, mentors' verbal encouragement and constructive feedback serve as social persuasion, another critical source of self-efficacy (Bandura, 1997).

Professional Skill-Building and TIMSI Constructs.

Research on mathematics education highlights professional skill-building as a key mechanism for developing TIMSI's core constructs—self-efficacy, identity, and values. This process enables students to engage authentically in mathematical research while fostering their confidence, self-perception, and alignment with disciplinary norms.

Mathematics Self-Efficacy through Mastery Experiences.

Professional skill-building enhances mathematics self-efficacy by providing students with opportunities for mastery experiences. As students acquire research skills, technical writing skills, and presentation capabilities, they build confidence in their ability to tackle mathematical challenges. Bandura (1997) emphasizes that successful task completion is a cornerstone of self-efficacy development, a point supported by studies showing that hands-on engagement in authentic research reinforces domain-specific confidence (Causapin, 2012). For example, completing a research project or delivering a presentation allows students to see tangible outcomes of their efforts, solidifying their belief in their mathematical abilities.

Mathematics Identity through Legitimate Participation.

Professional skill-building is equally integral to mathematics identity development. Through legitimate peripheral participation in mathematical practices, students begin to see themselves—and be recognized by others—as emerging mathematicians. Cribbs et al. (2022) note that participation in authentic research activities, such as writing papers and attending conferences, facilitates this recognition process, which is essential for identity formation. As students accumulate experiences where they perform as mathematicians, their self-perception evolves to align with this role. Mentorship plays a critical role in this process by validating students' contributions and providing opportunities for them to engage with the broader mathematical community.

Values Internalization through Disciplinary Immersion.

Professional skill-building also facilitates the internalization of mathematical values. By immersing students in disciplinary practices such as collaborative research, conference presentations, and knowledge dissemination, mentorship helps mentees appreciate the collaborative and systematic nature of mathematics. Tambunan and Yang (2022) highlight that engaging in these activities fosters a deeper understanding of mathematics as a tool for innovation and inquiry. Additionally, studies show that when mentors support mentees in areas that align with their personal and professional goals, students are more likely to adopt the values and norms of the discipline (Baker & Griffin, 2010; National Academies of Sciences, Engineering, and Medicine, 2019).

Synergistic Development of TIMSI Constructs.

The interconnected nature of these developments underscores professional skill-building as a pathway that strengthens self-efficacy, identity, and values simultaneously. For instance, gaining technical skills enhances students' confidence (self-efficacy) while also providing opportunities for recognition (identity). Participating in presentations and research dissemination fosters a sense of belonging, further integrating mentees into the mathematical community and reinforcing their alignment with its values.

Mentorship further amplifies this process. Research has shown that quality mentoring experiences improve students' sense of inclusion and integration into their disciplines (Estrada et al., 2018; Hernandez et al., 2018, 2020). By equipping students with professional skills and validating their contributions, mentors create an environment where students feel empowered to take ownership of their roles as mathematicians, driving the development of TIMSI's core constructs.

Collaborative Dynamics and Identity/Values

Collaborative dynamics play a crucial role in developing both mathematics identity and values through authentic engagement in mathematical practices. When students participate in joint problem-solving activities, they begin to see themselves as legitimate participants in the mathematics community, strengthening their mathematical identity (Cribbs et al., 2022). As mentees transition from guided instruction to equal partnership in research activities, they receive recognition as emerging mathematicians, a critical component of identity development in mathematics education (Sfard & Prusak, 2005).

The collaborative nature of mathematical inquiry also facilitates the internalization of disciplinary values. Through sustained engagement in authentic research partnerships, students adopt the mathematics community's values of precision, creativity, and systematic investigation (Tambunan & Yang, 2022). This process of value internalization occurs naturally as students work alongside mentors, engaging in mathematical practices that emphasize both individual contribution and collective knowledge building (Hernandez et al., 2018). The collaborative environment provides opportunities for students to observe, practice, and ultimately internalize these fundamental mathematical values while developing their identity as contributing members of the mathematics community.

Fostering Belonging Across TIMSI Constructs

As students become integrated into the mathematics community through professional activities, peer recognition, and active contributions, they experience a deepened sense of connection to the field. This deepened sense of belonging enhances mathematics self-efficacy through positive social feedback and validation of their contributions to the field.

The development of mathematics identity is particularly strengthened through community integration. As students gain recognition as legitimate participants in mathematical practices, they begin to see themselves—and are seen by others—as emerging mathematicians (Cribbs et al., 2022; Sfard & Prusak, 2005). This recognition process is fundamental to identity formation within mathematical communities, as students transition from peripheral participants to active contributors.

This sense of belonging also facilitates the internalization of mathematical values through immersion in authentic mathematical practices. When students participate in meaningful mathematical activities within a supportive community, they develop deeper appreciation for the discipline's values and norms (Tambunan & Yang, 2022). This process of value alignment occurs naturally as students engage with the mathematics community, leading to long-term integration and commitment to the field (Estrada et al., 2018).

Process Interactions and Synergies

These developmental processes are not independent pathways but operate as a cohesive system that collectively strengthens the TIMSI constructs of self-efficacy, identity, and values. This study reveals the deeply interconnected nature of these constructs within high school mathematics research mentorships, emphasizing their dynamic and reciprocal relationships. As participants mastered challenging research tasks, their growing self-efficacy fueled a stronger sense of belonging and alignment with mathematical values such as persistence and intellectual curiosity. Concurrently, their evolving identity as emerging mathematicians reinforced their commitment to these values, creating a positive feedback loop that supported sustained engagement. For example, professional skill-building activities, such as presenting at conferences, simultaneously bolstered self-efficacy and solidified participants' sense of identity within the mathematics community.

These interactions are also evident within the processes themselves. Adaptive scaffolding complements collaborative dynamics, enabling students to take on increasingly complex tasks collaboratively, fostering both self-efficacy and value internalization. Professional skill-building strengthens identity development and value alignment by equipping students with the tools and social validation necessary to see themselves as mathematicians and active contributors to the field. Together, these processes form a cohesive support system that holistically nurtures self-efficacy, identity, and values. For instance, students who gain confidence in their mathematical abilities through adaptive scaffolding are more likely to engage deeply in collaborative dynamics, where they further internalize mathematical values. Similarly, professional skill-building often leads to opportunities for peer and mentor recognition, enhancing belonging and reinforcing identity.

To represent these findings, this study proposes adding dotted lines to the TIMSI framework to reflect the bidirectional and synergistic influences among self-efficacy, identity, and values. This adjustment offers a more nuanced depiction of TIMSI by highlighting how growth in one construct often catalyzes development in others. By detailing the connections between these processes and TIMSI constructs, this study contributes to a deeper understanding of how high school mathematics research mentorships facilitate integration into mathematics. It highlights the interdependence of mentorship processes and constructs, providing additional insights into the mechanisms that support students' growth and integration into the mathematics community.

Extending TIMSI: Additional Integration Factors

The study extends TIMSI by identifying three additional factors that support integration into mathematics: self-directed learning capabilities, research process insights, and professional network development. These processes and factors work together to facilitate sustained engagement with mathematics, providing a comprehensive framework for understanding how mentorships support integration into mathematics: (1) Self-Directed Learning Capabilities: With some initial guidance from their mentors, participants eventually cultivated autonomy and persistence by managing their own learning of research outside structured curricula. (2) Insights into the Research Process: Mentorships demystified the nature of research, helping participants understand the incremental and collaborative aspects of mathematical inquiry. (3) Professional Network Development: Mentorships connected participants to broader academic communities through conferences, presentations, and mentor relationships, helping them envision their place in mathematics.

Together, these findings refine TIMSI by highlighting the specific mentorship mechanisms that foster self-efficacy, identity, and values while introducing additional integration factors. These contributions provide a more nuanced understanding of how mentorship experiences facilitate participants' sustained engagement with mathematics, offering a comprehensive framework for future mentorship design and implementation.

Integration with Broader Theoretical Frameworks

This study situates TIMSI within a broader theoretical context, integrating insights from developmental psychology, social cognitive theory, and communities of practice theory. By doing so, it highlights the dynamic interplay between individual growth and social context during adolescence.

Developmental Psychology (Erikson, 1968)

The findings align with Erikson's theory that adolescence is a critical period for identity formation, emphasizing the role of mentorship in shaping participants' academic identities and aspirations.

Social Cognitive Theory (Bandura, 1997)

The study supports Bandura's emphasis on mastery experiences as key drivers of self-efficacy, illustrating how mentorships provide opportunities for students to build confidence through authentic research tasks.

Communities of Practice (Wenger, 1998)

By engaging in collaborative research and interacting with mentors and peers, participants became part of a community of practice, internalizing the norms and values of the mathematics community.

This integration enriches TIMSI by framing its constructs within a broader developmental and sociocultural framework, emphasizing the importance of context in shaping students' experiences.

5.3 Implications for Practice

This study highlights the transformative impact of high school mathematics research mentorships in fostering mentees' self-efficacy, identity, and values. These mentorships not only inspire sustained engagement with mathematics but also serve as a bridge between education and professional integration. Based on the findings, the following suggestions are offered to guide the design and implementation of effective mentorship programs.

Integration of TIMSI Constructs through Developmental Processes

While the literature identifies self-efficacy, identity, and values as essential constructs for integration into the mathematics community, my study highlights the importance of the developmental factors and processes that underpin their growth. Adaptive scaffolding, professional skill-building, collaborative dynamics, and fostering belonging are not merely peripheral activities but integral mechanisms that shape the TIMSI constructs. For instance, adaptive scaffolding provides structured yet flexible support, enabling students to build confidence (self-efficacy) while navigating mathematical challenges. Similarly, collaborative dynamics create opportunities for recognition and validation, reinforcing students' sense of identity and belonging within the mathematics field.

These findings suggest that mentorship programs should prioritize the implementation of processes that indirectly foster the TIMSI constructs. By embedding activities such as tailored skill-building sessions, structured collaboration, opportunities for meaningful community engagement, and supports for self-directed learning, mentorship programs can address the multifaceted needs of mentees while creating pathways for sustained integration into mathematics.

Structured Mentorship Frameworks with Flexibility

My research suggests that mentorship programs might benefit from implementing structured frameworks that evolve with mentee development. The findings indicate that a phased approach could enhance mentee growth: beginning with foundational research methods and mathematical concepts, transitioning to collaborative problem-solving, and culminating in mentee-directed research projects. This progression appears to support the development of both confidence and autonomy while allowing mentors to adjust their support based on individual needs.

The initial phase might be integrated into existing educational structures, potentially leveraging classroom teachers or group settings to build foundational skills before pairing students with research mentors. However, maintaining flexibility within these structures seems crucial, as the findings highlight how successful mentorships adapt to accommodate diverse student backgrounds and the inherent uncertainties of mathematical research.

Branch-Specific Scaffolding Approaches

Beyond flexible frameworks, effective mentorship also requires domain-specific approaches to address the unique challenges of various branches of mathematics. To enhance the effectiveness of mathematics research mentorships, programs may benefit from implementing branch-specific scaffolding approaches that cater to the unique characteristics of various mathematical domains. For instance, participants who engaged in research projects in number theory often had mentors who focused on developing proof techniques and abstract reasoning skills. This approach typically involved starting with guided readings of foundational theorems, which progressed to proof construction and eventually led students toward independent conjecture development.

In contrast, participants working on graph theory projects experienced mentorship that emphasized a more visualization-based approach. Mentors began with concrete examples and computational explorations before transitioning to theoretical aspects. This method allowed students to engage with visual representations of graphs, fostering a deeper understanding of complex concepts.

Applied mathematics mentorships required yet another tailored approach, emphasizing real-world applications and computational methods. Mentors in these areas often started with practical problems that illustrated the relevance of mathematical concepts, gradually advancing to more sophisticated modeling techniques. For example, one participant worked on modeling refugee population dynamics, starting with basic population growth models before advancing to sophisticated partial differential equations. Another participant worked on an epidemiological modeling research project initially starting with basic compartmental models (e.g., SIR models) and advancing to the incorporation of stochastic elements using real-world datasets.

The progression of research questions should align with these domain-specific approaches. In number theory, questions may evolve from exploring patterns in specific number sequences to developing and proving general conjectures. Graph theory projects could progress from analyzing properties of specific graph families to investigating broader structural patterns.

Applied mathematics questions often advance from simplified models of real phenomena to more comprehensive mathematical representations.

By adopting these domain-specific scaffolding strategies, mentorship programs can maintain flexibility to accommodate individual student development while preserving the rigor and depth characteristic of each mathematical field. Regular assessment of student progress allows mentors to adjust the pace and complexity of assignments, ensuring sustained engagement and growth regardless of the mathematical domain. These domain-specific strategies not only address the unique demands of each mathematical field but also establish a foundation for independent learning by addressing the unique challenges of each mathematical domain.

Supporting Self-Directed Learning

Building on this foundation, mentorship programs might aim for structured approaches to develop self-directed learning capabilities, beginning with progressive research assignments that build independence. These assignments could start with guided reading of specific paper sections and clear summarization tasks, then advance to independent literature searches within defined topic areas, and ultimately culminate in student-directed research question formulation. Regular check-ins would be essential to monitor progress and adjust support levels as needed. These practices worked well for participant KN and seem to be applicable for all branches of mathematics.

Programs could also offer templates for analyzing mathematical papers, including strategies for understanding theorems, proofs, and key concepts. Teaching specific approaches for navigating technical language and notation, supported by domain-specific glossaries and annotation guides, helps students develop crucial analytical skills.

The scaffolding reduction timeline represents another critical element of fostering independence. To do so, programs might begin with highly structured weekly tasks and detailed feedback, then gradually increase time between check-ins as students demonstrate growing competence. This progression allows for a natural transition from specific instructions to broader research goals, ultimately enabling students to set their own intermediate objectives while maintaining long-term guidance.

To support independent exploration, programs could develop comprehensive resources including curated databases of accessible research problems organized by mathematical domain. Additional support tools might include guides for mathematical software usage, self-assessment rubrics for progress monitoring, and access to online mathematical communities. These resources create a supportive environment for independent learning while maintaining connection to the broader mathematical community.

This structured approach to developing self-directed learning aligns with findings about how participants developed crucial skills in time management and independent study during their mentorships, while providing concrete strategies for implementing these insights in practice.

Addressing Equity Regional Implementation Strategies

While this study's participants primarily came from well-resourced schools in areas like Westchester, NY, the findings suggest strategies for expanding access to mathematics research mentorships. Regional initiatives could help bridge resource gaps between affluent suburban districts and urban areas through collaborative networks. These initiatives could include organizing mini-conferences where students from different districts present their research, fostering collaboration and peer learning opportunities.

Regional mentor networks could connect experienced mentors across institutions, sharing resources and expertise while supporting schools developing new programs. Small group research teams, structured across schools within a region, could provide peer support while maximizing the impact of limited mentor resources. Virtual components could complement these regional structures, allowing students from resource-constrained schools to participate regardless of location. This hybrid approach of regional and virtual collaboration could help democratize access to mathematics research experiences while maintaining program quality.

Encouraging Peer Collaboration and Support

Mentorship programs could integrate peer interaction opportunities to enhance collaboration, mutual support, and a sense of belonging. Regular cohort meetings or team-based projects can expose mentees to group dynamics and provide platforms for sharing progress and challenges. Such interactions not only cultivate teamwork and professional connections but also extend mentorship opportunities to more students, making programs more inclusive. This aligns with findings from the study, which showed that participants who engaged with peers reported enriched experiences and stronger connections to the mathematics community.

Enhancing Professional Skill Development

Professional skill-building components could be embedded within mentorship programs to prepare mentees for future academic and professional success. For instance, programs might include workshops on technical writing, effective presentation, and navigating academic publishing. Participants could also benefit from training in practical tools commonly used in mathematical research, such as LaTeX, Python, or MATLAB. These opportunities not only build self-efficacy through mastery experiences but also enhance identity formation as mentees begin to see themselves as capable contributors to the mathematics field.

The Role of Research Questions in Mathematics Research Mentorship Success

The findings indicate that selection and framing of research questions emerged as a pivotal factor in determining the success of mathematics research mentorship experiences. Participants consistently highlighted the importance of research problems that balanced difficulty with feasibility, offering opportunities for growth without leading to frustration or disengagement. For instance, CM encountered challenges when assigned overly ambitious problems that exceeded her capacity, while RC attributed the success of his mentorship experience to the thoughtful selection of a feasible and engaging research question by his mentor. These contrasting experiences underscore the necessity of aligning research topics with mentees' abilities and interests to foster meaningful engagement and sustained motivation. To address these challenges, mentorship programs could benefit from prioritizing the development of resources, such as curated databases of research problems tailored to different branches of mathematics and varying skill levels. Similar to the structured frameworks used in mathematics competitions or age-specific math team problems, such databases would enable mentors to select projects that balance complexity with accessibility. These resources could help ensure that research questions align with mentees' abilities, fostering a sense of accomplishment and sustained engagement.

Additionally, mentorship programs should consider integrating regular feedback mechanisms to allow mentors to monitor mentees' progress and adjust the complexity of tasks as needed. Such mechanisms would enable mentors to identify moments where mentees might feel overwhelmed or under-challenged, providing timely interventions that build confidence and maintain motivation. By carefully framing research questions and balancing support with challenge, mentorship programs can create environments that inspire curiosity, foster resilience, and cultivate a deeper appreciation for mathematical inquiry.

The Importance of Real-Life Applications

Building on the importance of well-framed research questions, mentorship programs can further enhance engagement by selecting projects with real-world significance. Participants in this study reported higher levels of motivation and satisfaction when their projects addressed tangible issues, such as mathematical modeling of refugees or graph theory applications in computer science. These findings suggest that selecting research topics with practical relevance could help students see their work as meaningful contributions to society. Additionally, incorporating interdisciplinary skills and involving multiple mentors could enrich mentorship experiences, particularly by highlighting the diverse applications of mathematics in fields like biology, physics, and computer science.

Building Connections with the Broader Mathematical Community

Programs could also explore ways to integrate mentees into the wider mathematics community. Encouraging mentees to present at conferences, participate in professional societies, or engage with mathematicians in other institutions could deepen their understanding of the field's values and provide opportunities for networking. These experiences may strengthen mentees' mathematics identity by situating them within a professional community and fostering recognition of their contributions.

By focusing on these areas, mentorship programs can foster mentees' integration into the mathematics community, helping them develop the confidence, skills, and values necessary for sustained success.

5.4 Limitations and Future Research Directions

This study provides critical insights into the role of high school mathematics research mentorships in fostering participants' self-efficacy, identity, and values. However, several limitations highlight the need for further research to deepen and broaden our understanding of these mentorship experiences.

Temporal and Methodological Constraints

The retrospective design of this study, which relies on participants reflecting on mentorship experiences from several years prior, presents inherent limitations. While retrospective interviews offered valuable insights into long-term outcomes, they limit the ability to establish direct causality between mentorship experiences and career trajectories (Maxwell, 2013). The potential for recall bias may affect the accuracy of participants' accounts, particularly regarding nuanced aspects of their mentorships. Future longitudinal studies could address these limitations by tracking mentees' development in real-time, capturing how self-efficacy, identity, and values evolve over the course of the mentorship and beyond. Employing mixed methods approaches, including surveys, interviews, and observational data, would provide a richer and more dynamic understanding of these processes.

Sample Characteristics and Generalizability

The study's sample predominantly consisted of participants from well-resourced schools with strong research cultures, raising concerns about the generalizability of the findings. The mentorship experiences of students from underfunded or underserved schools remain largely unexplored, as do the perspectives of those who may not have pursued careers in mathematics. These gaps suggest important avenues for future research, such as investigating how mentorship outcomes vary across different demographic groups, socioeconomic settings, and educational environments. Including participants from a broader range of backgrounds and those who did not follow traditional career trajectories in mathematics would provide a more comprehensive picture of mentorship impacts and challenges.

Future research should explicitly compare mathematics research mentorship experiences across different socioeconomic and geographic contexts. For instance, studies could examine how mentorship programs function in urban environments like Chicago versus suburban areas like Westchester, NY, or compare established programs like MIT PRIMES with emerging regional initiatives. Such comparative studies could identify effective strategies for adapting successful mentorship models to diverse educational settings while maintaining program quality. Additionally, research examining virtual and hybrid mentorship models could inform efforts to scale these opportunities beyond well-resourced schools.

Domain-Specific Considerations

Future studies could investigate how different branches of mathematics influence participant experiences, engagement, and outcomes. Research could also focus on developing domain-specific mentorship strategies, exploring how mentors scaffold and support projects across mathematical disciplines. For example, mentors specializing in applied fields like partial differential equations might prioritize modeling and problem-solving, whereas those in theoretical areas like number theory may emphasize abstract reasoning and proof development. Future investigations could explore how the unique characteristics of different mathematical domains influence mentorship outcomes, identify best practices, investigate different research scaffolding approaches, and develop frameworks for designing appropriate research questions across these different mathematical domains.

Research Question Development

The selection and framing of research questions emerged as a pivotal factor in mentorship success. Participants highlighted the importance of research problems that balanced challenge with feasibility, offering opportunities for growth without overwhelming mentees. Despite its significance, this area remains underexplored. Future research should examine methods for creating curated databases of scalable research problems tailored to varying skill levels and mathematical domains. Additionally, frameworks for matching research questions to mentees' abilities and interests warrant further investigation. By addressing these gaps, mentorship programs can create environments that foster intellectual growth, resilience, and a deeper appreciation for mathematical inquiry.

Broader Access and Scalability

The study also highlighted disparities in access to research mentorships, as most participants attended high-SES schools with established research programs. This inequity underscores the need to scale mentorship opportunities and democratize access for students from underserved communities. Virtual mentorship models offer a promising solution, as they can connect students from resource-constrained schools with expert mentors regardless of geographic or economic barriers. Future research should evaluate the effectiveness of these models, identifying best practices for delivering impactful remote mentorship experiences. Additionally, collaborative initiatives involving schools, universities, and non-profits could help establish sustainable mentorship programs in underfunded regions. Examining strategies for scaling small, localized mentorship efforts into broader district-wide or virtual programs could further enhance equity in access to these transformative opportunities.

5.5 Conclusion

High school mathematics research mentorships demonstrate significant potential for fostering integration into mathematics through the development of self-efficacy, identity, and values. While prior articles, such as Etingoff et al. (2015) and Gerver et al. (2017), have provided valuable insights into program structures, frameworks, and outcomes of mathematics high school research mentorships, they do not delve into the nuanced psychological mechanisms or developmental processes experienced by individual students. This distinction highlights the complementary nature of my research. Whereas Etingoff et al. (2015) emphasize the accessibility of advanced research opportunities and the impressive scholarly outputs of participants, and Gerver et al. (2017) document the organizational evolution of a mathematics research mentorship program at North Shore High School, my work offers a micro-level exploration of how specific mentorship processes—such as adaptive scaffolding, professional skill-building, and fostering belonging—interact with mentees' personal journeys to facilitate integration into the mathematics community.

Many papers focus on the macro-level impact and organization of mentorship programs, showcasing the accessibility and success of advanced research opportunities. My study, in contrast, highlights how these opportunities translate into the development of self-efficacy, identity, and values within mentees. Participants in my research explicitly attributed transformative experiences, like conducting research for the first time or engaging in authentic collaborative problem-solving, to this unique mentorship context—experiences they indicated would not have been available through their high schools alone. For instance, one participant noted that it was their experience of doing graph theory research with a mentor that helped them envision mathematics as a career, catalyzing their deeper integration into mathematics.

Integrating insights from both articles and this study allows for a more comprehensive understanding of high school mathematics research mentorships. The macro-level frameworks and outcomes documented by Etingoff et al. (2015) and Gerver et al. (2017) are underscored by the micro-level processes identified in my research, providing a clearer picture of how and why these programs succeed. Together, these perspectives contribute to the growing discourse on how structured, yet flexible mentorship programs can foster both exceptional mathematical outcomes and deeper personal integration into the discipline.

By illuminating these processes, this study expands the current understanding of how mentorships cultivate not only the skills and confidence necessary for academic success but also the deeper identity formation and value alignment that underpin sustained engagement in mathematics. The findings contribute to the theoretical refinement of the TIMSI framework by emphasizing the interconnected nature of self-efficacy, identity, and values, offering a more nuanced view of how these constructs interact in mathematics research mentorship contexts.

While this study sheds light on the mechanisms driving mentorship success, it also raises critical questions about equity, scalability, and domain-specific strategies. Addressing these questions through continued research will be essential for designing mentorship programs that are not only effective but also accessible and inclusive. By advancing our understanding of mentorship practices, we can ensure that their benefits reach a broader and more diverse population, inspiring the next generation of mathematicians and innovators.

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