Analyzing the National College Entrance Mathematics Examinations in China


Yihua Shen

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Abstract


Yihua Shen

This research examined the Chinese National College Entrance Examination (NCEE) in mathematics before and after the Great Proletarian Cultural Revolution, specifically covering the periods 1952–1965 and 1977–1984. The central focus was on the organization, structure, and content of the examinations, as well as their influence on and interaction with Chinese people and society. A mixed methodology approach was employed, primarily comprising three steps: (1) scrutinizing the sources and coding the information into structured formats, (2) organizing the data and tracking trends and changes, and (3) synthesizing the findings to formulate conclusions.

Key findings included: (1) An increase in the number of items from 1979 to 1984, attributed to the introduction of new question formats following international collaboration between China and the United States. (2) A shift in topic coverage from traditional to modern subjects after 1976, reflecting curriculum concerns raised by Chinese mathematicians who advocated for educational content to evolve with societal and human development. (3) A decrease in item difficulty during the post-war and post-revolutionary periods of 1952, 1953, and 1977, reflecting the education system’s recovery from disruption and generally lower quality of teachers and students. (4) A shift toward an exam-oriented approach in teaching and learning, with its negative ramifications leading to criticisms from Chinese society and the eventual abolition of the NCEE system in 1966.
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Chapter I
INTRODUCTION

Need for the Study

Academic assessment, widely considered an equitable way to allocate educational resources and classify students based on individual merit rather than socioeconomic status, has become a popular practice since the middle of the twentieth century (Moses & Nanna, 2007). Among different assessment methods, high-stakes examinations have received considerable attention from policymakers and scholars around the world because of their broad implications. On the one hand, they help to measure curriculum impact, evaluate teacher performance, assess student achievement, and motivate teachers and students to work harder (e.g., Evers & Walberg, 2013; Nichols et al., 2005; Oakland & Hambleton, 1995). On the other hand, they are suggested to narrow the curriculum, constrain the teachers, and stress the students (e.g., Amrein & Berliner, 2002; Booher-Jennings, 2005; Cizek, 2001; Jones et al., 2003).

In the field of mathematics education, high-stakes mathematics examinations have similarly captured the interest of mathematics educators around the world. Previous studies have recommended several valuable paths to explore these examinations. First, conducting a historical analysis helps gain insights into a nation’s social views and changes, thus contributing to an understanding of the culture and politics of an era (Karp, 2007). Second, examining the evolution of high-stakes mathematics examinations elucidates the effects of policy and curriculum changes in a country (Morgan & Sfard, 2016). Third, evaluating the alignment between assessment, curriculum, and instruction assists assessment developers, policymakers, and instructors in refining their work so that the education system can set priorities more effectively (Webb, 1997)
and students can better demonstrate their achievement (Martone & Sireci, 2009). Fourth, analyzing the item difficulty in high-stakes mathematics examinations provides significant insights into curriculum interpretation, examination design, and instruction (Jacobs et al., 2014). Fifth, investigating the objectives and methods of a national examination in mathematics offers implications for other nations’ practices (Karp, 2003; Wu, 1993). Overall, research suggests that studying high-stakes mathematics examinations is both beneficial in practice and contributes meaningfully to the historical record.

Compared to the high-stakes examinations in Western civilizations, the high-stakes testing in China has spanned a much longer history and, therefore, attracts attention from academics across the world. According to Suen and Yu (2006), the Keju examination system—the historical civil service examination system of China, which lasted a total of 1,298 years—was considered by them the only examination system that could contribute to the understanding of certain long-lasting chronic problems associated with high-stakes and large-scale examinations. The current notable high-stakes examination system in China is the National College Entrance Examination (NCEE). This large-scale annual academic assessment was first administered in 1949 and has continued to the present day, though it was abolished during the Great Proletarian Cultural Revolution from 1966 to 1976. The NCEE typically comprises five separate examinations: three subject tests in Chinese, mathematics, and English; one comprehensive liberal arts test that covers history, politics, and geography; and one comprehensive science test that covers physics, chemistry, and biology. Students can opt to take either the comprehensive liberal arts test or the comprehensive science test, but all students are required to take the subject tests in Chinese, mathematics, and English. Even though the NCEE has a much shorter history than the Keju examination system, it serves as the gatekeeper to Chinese higher education.
institutions, significantly influencing Chinese students’ life choices and opportunities. Perhaps for this reason, academics worldwide consider it a valuable research topic (e.g., Cai, 1994; Hua, 2017; Kirkpatrick & Zang, 2011; Li, 2016; Ross & Wang, 2010, 2013).

As one of the three tests that all examinees must take, the NCEE subject test in mathematics has garnered interest from scholars globally. A review of existing literature from the Gottesman Libraries at Teachers College—one of the world’s top libraries for educational research—and the China National Knowledge Infrastructure (CNKI) database—one of China’s largest library systems for academic studies—using the keywords “the NCEE mathematics examinations” in both English and Chinese revealed several focal points. These included studies on the format and contents of the NCEE mathematics examinations (e.g., Ding, 2008; Shao & Li, 2010; Zhang et al., 2016), their impact on the teaching and learning of mathematics (e.g., Ding, 2017; Huang, 2005; Zhao & Jiang, 2013), comparative analyses between the NCEE mathematics examinations and those in other countries (e.g., Jiang & Huang, 2009; Liu et al., 2012; Zhou, 2011), and their evolution over time (e.g., Chen, 2013; Liu, 2017; Mei, 2013; Yu, 2005).

While there existed some research on the evolution of the NCEE mathematics examinations, the information available remained limited. Specifically, only three studies had investigated the historical periods preceding 1985 (i.e., Ren & Chen, 2017; Tian, 1998; Zheng & Chen, 2017). Notably, both Ren and Chen (2017) and Tian (1998) focused their research on examination syllabi rather than specific examination questions. Zheng and Chen (2017), on the other hand, based their analysis on actual examinations, but their analysis was brief, spanning only seven pages. Additionally, the primary focus of their research centered on the organization and structure of the examinations and the context of the test questions, leaving aspects like topic coverage and item difficulty unexplored. Furthermore, none of these studies had examined the
reasons behind the evolution of the NCEE mathematics examinations or the mutual influences between the examinations and Chinese society. This research aimed to bridge these gaps, offering a thorough examination of the NCEE mathematics examinations before 1985.

An investigation into the historical evolution of NCEE mathematics examinations prior to 1985 not only bridged gaps in academia but was also valuable and intriguing. While numerous historical events and policies in Chinese history impacted the NCEE, one particularly notable event occurred before 1985, namely, the Great Proletarian Cultural Revolution. This sociopolitical movement, which spanned from 1966 to 1976, was marked by its anti-intellectual stance. During this period, schools were closed, examinations were abolished, and both teachers and students faced persecution (Swetz & Chi, 1983). Influenced by this event, the NCEE system experienced substantial and varied changes, including denunciation, abolition, restoration, and reformation. Therefore, studying the evolution of the NCEE mathematics examinations around the time of the Cultural Revolution was both insightful and meaningful. Such an investigation could help elucidate not only how these examinations responded to societal shifts in China, but also the societal attitudes embedded within them. As Karp and Furinghetti (2016) stated, “the life of a society is reflected in many different spheres of its activity, and for long periods, mathematics education was thought to be among the more important of such spheres” (p. 4).

Most importantly, examining the history of mathematics examinations can deepen our understanding of the interplay between the development of mathematics and broader societal, political, economic, and cultural shifts. This can be invaluable for future research in mathematics education. Additionally, such a study could contribute to a better understanding of the history of mathematics education in China and, more broadly, the historical and developmental trajectory of the country itself.
Purpose of the Study

The purpose of this study is to explore the changes that occurred in the NCEE mathematics examinations during two specific historical periods: before the Great Proletarian Cultural Revolution (i.e., from 1952 to 1965) and after it (i.e., from 1977 to 1984). As previously mentioned, the NCEE system was abolished from 1966 to 1976. The years 1952 and 1984 marked the first and final years when a uniform examination was used across the entire country.

The following questions guided this research:

1. How did the organization and structure of the NCEE mathematics examinations evolve during the years 1952–1965 and 1977–1984?
2. How did the content of the NCEE mathematics examinations change during the periods 1952–1965 and 1977–1984?
3. What were the mutual influences between the NCEE and Chinese society during the years 1952–1965 and 1977–1984?

Procedures of the Study

The procedures for this research comprised three main steps:

2. Scrutinizing the collected documents, coding the information, and organizing the data into integrated and meaningful clusters; and
3. Utilizing the coded information to analyze the evolution of the NCEE mathematics examinations in terms of their organization and structure, mathematical content, and influence on and interaction with Chinese people and society.
In the first step, the quality of the collected documentation will significantly impact the quality of this research. Historical records are not always reliable and may contain information gaps, errors, or biases (Fraenkel et al., 2011; Simonton, 2003). Therefore, it is crucial to gather a diverse range of documents to obtain a relatively representative array of opinions from various stakeholders. Additionally, comparing these opinions from multiple perspectives is essential for constructing a more objective narrative. The documents for this study included, but were not limited to, the NCEE mathematics examinations, their solution manuals, examination syllabi, newspaper articles, official documents, scholarly books, as well as commentaries and personal memoirs from educators, parents, and students.

The objective of the second step is to prepare and sort the information for analyzing the evolution of the NCEE mathematics examinations. The actual analysis takes place in the third step. To answer the first research question, the analysis of the organization and structure of the NCEE mathematics examinations focused on attributes such as the duration of the examinations, the number of questions they contained, and the types of questions they included. A spreadsheet was created in Excel to facilitate this analysis, with each column representing a single year’s examination and each row representing one of the attributes described above. Comparing the data in successive columns revealed changes that had occurred in the NCEE mathematics examinations over the years. If patterns and trends were identified, relevant information was sought from the collected sources to help explain why certain changes occurred at specific times.

To answer the second research question, the analysis of the content in the NCEE mathematics examinations focused on topic coverage and item difficulty. In analyzing topic coverage, each examination question was assigned to one or more mathematics topics based on the topics required for solving them. To record the frequencies of each topic, another Excel
spreadsheet was created, with each column representing a single year’s examination and each row representing a different mathematical topic. Subsequently, comparing the data in successive columns revealed changes in topic coverage over the years.

In analyzing item difficulty, each examination question was categorized using Bao’s (2002) model based on the following criteria: (1) whether solving the question requires rote memorization of mathematical concepts, application of mathematical knowledge, or construction of mathematical models; (2) whether the question is purely mathematical or placed in real-world contexts; (3) the number of computational steps; (4) the number of reasoning steps; and (5) the number of different mathematical concepts involved. Another Excel spreadsheet was created, with each column representing a single year’s examination and each row representing a difficulty level. Comparing the data in successive columns revealed changes in item difficulty over time.

Finally, if patterns and trends were identified, the researcher investigated the reasons behind them using the collected sources and drew reasoned conclusions about how both topic coverage and item difficulty in the NCEE mathematics examinations had evolved over time.

Lastly, to address the third research question regarding the mutual influences between the NCEE and Chinese society, the analysis primarily drew on commentaries and reflections from scholars, teachers, parents, and students. These insights were compiled from leading newspapers and personal memoirs. Initially, the researcher scrutinized the collected sources and documented relevant information, such as shifts in societal views and behaviors. Subsequently, the researcher organized these historical facts into coherent and meaningful clusters. Finally, he synthesized and summarized the clustered evidence to draw well-founded conclusions about past events.
This chapter provides background information on the history of examinations in China. It begins with a brief introduction to past examination systems (e.g., the Keju examination system), continues with a detailed discussion of the current examination system (i.e., the NCEE system), focusing on the years 1949–1965 and 1977–1984, and concludes with a concise summary of the NCEE system’s development after 1984. The history of examinations in China is presented chronologically in the following sections:

- Keju examination system,
- Examination system during the Republic of China period,
- NCEE system before the Cultural Revolution (1949–1965),
- Great Proletarian Cultural Revolution (1966–1976),
- NCEE system after the Cultural Revolution (1977–1984), and
- NCEE system after 1984.

It should be pointed out that this chapter cites two books extensively, i.e., Yang’s (2007) *A Review of the History of the National College Entrance Examination in China, 1949–1999* and Jiang’s (2008a, 2008b, 2008c) *The History of the National College Entrance Examination in China*. The reasons for extensively citing these two works are as follows: Firstly, due to the paucity of existing literature on the history of the NCEE, the author had limited choices. Secondly, these books provide a comprehensive and detailed study of the NCEE’s history, covering almost every aspect. Thirdly, the authors and editors of these works are leading experts in the relevant field, which assures the quality of these studies. Lastly, these works have received
nationwide recognition from Chinese scholars, enhancing the reliability and validity of the information. The following paragraphs provide further details about these two publications.

The first book, *A Review of the History of the National College Entrance Examination in China, 1949–1999*, was the achievement of a research project launched by China’s Ministry of Education (Yang, 2007). This book has a total of 654 pages. The author, Yang Xuewei, who dedicated over 30 years to the Ministry of Education and its examination center, specialized in the management, reform, and research of Chinese examinations (Liu, 2008). According to a book review by Liu (2008), this work has become an essential reference for many theses and academic papers, and it serves as an important resource for understanding the history of examinations in China and their historical reforms.

The second book, *The History of the National College Entrance Examination in China*, is a comprehensive work spanning four volumes and approximately 2.8 million Chinese words ("Zhongguo," 2023). The chief editor, Jiang Chao, dedicated significant time to the development and research of the NCEE at the examination center of the Ministry of Education. Alongside him, a diverse group of experts in political science and education contributed to the book’s writing or editing. Notable contributors included Zhang Xiaojing, a political science professor at Renmin University of China, and Li Songlin, a doctor in education from Beijing Normal University ("Zhongguo," 2023). The book received wide acclaim from esteemed Chinese scholars, such as He Yaomin, a professor of economics at Renmin University of China, Zhang Baosheng, vice-president and law professor at China University of Political Science and Law, and Wang Puqu, the vice dean of social sciences at Macao Polytechnic University ("Zhongguo," 2023).
It is noteworthy that the field of the history of examinations in China is both rich and diverse, encompassing a wide range of scholarly interpretations. It is important to acknowledge that some perspectives in this area are subject to debate and controversy. Although alternative viewpoints exist and contribute to a broader understanding of the subject, this chapter primarily aligns with the traditional interpretations as presented in the works of Yang (2007) and Jiang (2008a, 2008b, 2008c). These interpretations are widely recognized and frequently referenced in academic discourse on this topic. However, readers should be aware that these views represent only partial aspects of a complex and multifaceted historical narrative.

**History of Examinations in China**

**Keju Examination System**

According to Yang (2007), the earliest documentary evidence of examinations in China dates back to around 2000 BC: the ancient Emperor Yao was getting old and sought to select his successor. Four courtiers recommended Shun, prompting Yao to devise a series of real-life tasks to test Shun’s moral character and governance skills. This story was chronicled by Sima Qian (91 BC/2018) in the *Records of the Grand Historian* (*Shiji* in Chinese), a monumental work that documents the history of China from the legendary Yellow Emperor era to 122 BC, spanning approximately 3000 years (Library of Congress, 2023).

The method of using real-life tasks to evaluate candidates’ personal characteristics and professional abilities remained prevalent in ancient China until the Western Han Dynasty (202 BC–8 AD). During this period, Liu Heng (Emperor Wen of Han) and Liu Che (Emperor Wu of Han) introduced *Duice* (written examinations) and *Cewen* (oral examinations) (Yang, 2007). Both examinations featured questions derived from major governance challenges and were,
therefore, more cost-efficient than Yao’s method in terms of time, money, and human resources (Yang, 2007). However, anyone wishing to take the civil service examinations had to secure a recommendation from the royal family, noble families, or wealthy families, leading to widespread political corruption and rigid social stratification (Yang, 2007). The growing tension between the aristocracy and the common people led successive rulers, such as Cao Cao and Cao Pi, to recognize the need for an impartial examination and selection system (Yang, 2007). In 605 AD, Yang Guang (Emperor Yang of Sui) implemented such a system, known as the Keju examination system (Yang, 2007).

Compared to previous forms of examination in China, the Keju examination system featured two major improvements: (1) common people could nominate themselves to take the civil service examination, and (2) test scores became the primary determinant for selecting government officials (Yang, 2007). From these criteria, it is evident that under the new system, individuals from lower social classes could, through their own efforts, obtain government positions and participate in state management. Fairbank and Reischauer (1989) commented that the opportunities provided by the Keju examination system for common people to ascend to the upper class helped to mitigate the conflicts between the aristocracy and the common people, thus aiding in maintaining the social stability in ancient China.

Spanning 1298 years, the Keju examination underwent numerous reforms in the subjects it tested. A significant reform took place during the Ming Dynasty (1368–1644). Prior to this, a wide array of subjects was tested in the Keju examination, including Jinshike (political science and philosophy), Mingfake (legal studies), Mingzike (calligraphy), Mingsuanke (mathematics), and Yixueke (medical studies) (Jiang, 2008a). Notably, candidates only needed to excel in one subject to secure a government position. Historical studies indicate that more than 120 different
subjects were tested during the Tang Dynasty (618–907 AD) (Jiang, 2008a). However, by the Ming Dynasty, Jinshike (political science and philosophy) had become the only subject of the examination.

The primary content of the Jinshike examination was the Confucian classics, which remained largely unchanged for a span of 1,298 years (Yang, 2007). Its examination format, on the other hand, underwent a major reform during the Ming Dynasty. Before this reform, the examination primarily featured four types of questions: (1) Tiejing, which tested rote memorization of the Confucian classics; (2) Moyi and Wenyi, questions that examined the basic understanding of the Confucian classics; (3) Duice and Cewen, originated from the Han Dynasty, now included questions assessing candidates’ ability to apply the Confucian classics to solving state affairs; and (4) Shifu, which evaluated students’ capacity to compose classical Chinese poetry using content from the Confucian classics (Yang, 2007).

After the reform, the eight-legged essay emerged as the primary test format for the examination (Yang, 2007). This essay was a highly formulaic, eight-part response to examination questions, based on Confucian thought (Elman, 2009). It required balanced clauses and phrases and was restricted to a limited number of characters (Elman, 2009). Yang (2007) noted, this new test format effectively synthesized the requirements of previous formats, including understanding the Confucian classics, applying Confucian thought, and creating classical writing.

The historical facts presented above reveal a significant trend in the development of the Keju examination: its evolution from a diversified to a uniform style. Several scholars have pointed out that the uniformity of the examination in terms of subject, content, and format was instrumental in promoting ideological unity and, more importantly, social stability in ancient
China. Jiang (2008a) suggested that by solely testing Confucian thought in the examinations, examinees were compelled to study Confucian classics, thereby spreading Confucianism in Chinese society, and making it the dominant ideology in ancient China. Yang (2007) contended that ideological unity within a nation can reduce cultural conflicts, enhance ethnic unity, and maintain social stability. These perspectives are also supported by researchers such as MacFarquhar et al. (1987) and Fairbank and Reischauer (1989).

However, this uniform testing also had its drawbacks. The primary disadvantage of the Jinshike examination, or more accurately, of Confucianism, was its disregard for disciplines such as mathematics, science, and technology (Yang, 2007). Students in the Ming and Qing (1644–1912) dynasties devoted their lives to studying the Confucian classics, leaving fields like mathematics, science, and technology largely unexplored (Yang, 2007). Even more concerning, students who later became government officials often expressed contempt for these disciplines when dealing with national affairs (Yang, 2007). For instance, the advanced artillery and wagons, received in 1792 when the British were attempting to establish diplomatic relations with the Qing Dynasty government, remained untouched in the Old Summer Palace until the year 1900 (Yang, 2007). According to Peyrefitte (2013), Chinese officials at the time were convinced that China was superior to all other nations and the Confucian classics could help them solve any difficult problems. Elman (2009) commented that China’s neglect of science and technology, stemming from the Keju examination system, contributed to its cultural stagnation and economic backwardness in the nineteenth century.

It was not until China faced setbacks in the Opium Wars (1839–1842; 1856–1860), the Sino-Japanese War (1894–1895), and the Siege of the International Legations (1900) that Chinese ministers and intellectuals began to recognize the advancements in Western
mathematics, science, and technology, as well as the limitations of the Keju examination system (Yang, 2007). In 1905, the Guangxu Emperor abolished the Keju examination system. Yang (2007) noted that as human society progressed from an agricultural civilization to an industrial one, the Keju examination system and the Confucian classics could no longer address the demands of social and economic development. As a result, the abolition of the Keju examination system became an inevitable outcome.

**Examination System during the Republic of China Period**

In ancient China, the Keju examination system served not only as a civil service selection mechanism but also functioned as an education system (Jiang, 2008a). In this system, becoming a government official was the ultimate goal of study, a focus that led to China’s scientific and technological stagnation (Jiang, 2008a). To avoid the same issue, the government of the Republic of China implemented two separate examination systems: one for civil service and another for school education (Jiang, 2008a).

The civil service examination during the Republic of China period, first administered in 1916, was intended to be held every two years. However, it was cancelled from 1921 to 1930 due to political reasons, such as wars (Jiang, 2008a). All examinees were required to have received an undergraduate degree from an accredited college or university by the time of the examination (Jiang, 2008a). Given that only a small segment of the Chinese population could afford postsecondary education during that period, it is hypothesized that most candidates taking the civil service examination came from affluent families.

The categories of the examination varied depending on job categories, including general, financial, and educational administration; legal counsel; foreign affairs; economic development; public health; accounting and statistics; and civil, electrical, and chemical engineering (Jiang,
2008a). The subject tests covered Chinese language, forms and formulas of official documents, theories and opinions of the Chinese Nationalist Party, history, geology, the constitution, politics, economics, and specialized tests tailored to each job category (Jiang, 2008a). These classifications indicate that the civil service examination during the Republic of China period began to emphasize science and technology.

Jiang (2008a) highlighted that the primary improvement in the civil service examination system during the Republic of China period was its standardization and legalization. This was guaranteed by a government agency called the “Examination Yuan,” the first government agency in Chinese history responsible for validating the qualifications of civil servants, and by the Examination Law, the first law in Chinese history to regulate civil service examinations and personnel management. By 1948, the civil service examination came to a halt due to impending governmental changes (Jiang, 2008a).

In ancient China, there was no established examination system linked to formal schooling. Consequently, the government of the Republic of China adopted its school examination system, as well as its broader educational structure, entirely from Western society (Jiang, 2008a). This included two years of kindergarten, six years of elementary school, three years of middle school, three years of high school, four to six years of undergraduate studies, an indefinite period for graduate studies, and two to six years of vocational school, available at both secondary and post-secondary levels (Jiang, 2008a).

Before 1938, higher education institutions in the Republic of China held their entrance examinations independently, leading to issues such as corruption, inequality, and imbalanced educational development across various regions and majors (Jiang, 2008a; Zheng & Yang, 2003). To address these challenges, promote educational equity, and maintain quality, the
government of the Republic of China implemented specific laws and regulations to oversee college admissions (Jiang, 2008a). For instance, in an effort to tackle corruption and inequality in college admissions, the government abolished the enrollment policy that allowed admissions based on recommendations without requiring college entrance examinations. To cope with imbalanced development across different majors, the government adjusted enrollment quotas for science disciplines, aiming to discover and nurture talent in STEM fields (Jiang, 2008a). Starting in 1940, all public higher education institutions in the Republic of China adopted a uniform entrance examination (Jiang, 2008a).

The uniform examination typically consisted of three common tests: citizenship, Chinese language, and English language. Additionally, students were required to take several specialized tests tailored to their intended fields of study, encompassing subjects such as mathematics, physics, chemistry, biology, history, and geology (Yang, 2005). The content of the mathematics examination varied depending on students’ prospective fields of study. For example, students aiming for careers in science and engineering were required to take a test covering advanced algebra, analytic geometry, and trigonometry. In contrast, those pursuing the arts, law, business, medicine, and agriculture were tested on advanced algebra, plane geometry, and trigonometry (Yang, 2005). This examination format remained largely unchanged until 1949.

**NCEE System Before the Cultural Revolution (1949-1965)**

On October 1, 1949, the People’s Republic of China was founded. During the postwar period from 1949 to 1952, the Communist Party of China (CPC) and the Chinese government guided the Chinese people in repairing the economic damage inflicted by the Second Sino-Japanese War (1937–1945) and the Chinese Civil War (1927–1937; 1945–1949) (Yang, 2007). In the course of this recovery, party leaders and government officials came to recognize that
talent shortage was the most critical issue hindering the reconstruction and development of the country (National Institute of Education Sciences, 1984). As a result, talent selection and development were highlighted as major priorities in the first Five-Year Plan of China (1953–1957) (Yang, 2007). The Five-Year Plan of China is a series of economic and social development guidelines issued every five years by the Communist Party of China, focusing on setting goals and directions for the national economic development (“Five-Year Plan,” 2003).

One of the first decisions the Ministry of Education had to make at that time was whether universities should adopt a uniform entrance examination (Yang, 2007). Historical experiences suggested that independent examinations could meet the specialized recruiting needs of individual universities. However, such examinations could also lead to problems such as corruption, inequality, and imbalanced educational development across different regions and majors (Jiang, 2008a; Zheng & Yang, 2003). In contrast, a uniform examination could reduce the time, money, and opportunity costs associated with taking multiple university entrance examinations. Additionally, it could more efficiently distribute students between developed and less developed areas (Yang, 2003). For instance, students who were not accepted into Peking University could be reallocated to less competitive institutions based on their scores from the uniform examination. After years of discussions and trials, a consensus was reached in 1957: all universities should adopt a uniform entrance examination (Yang, 2007).

Actually, the first attempt to administer a uniform entrance examination after the period of the Republic of China occurred in 1952 (Yang, 2007). However, due to a lack of experience, the new government faced numerous challenges that took years to resolve (Yang, 2007). A significant issue was the low attendance rates of students who, upon not gaining admission to their desired universities or programs, were redirected to institutions in less developed regions or
to programs they had minimal interest in (Yang, 2007). To address this concern, the government enhanced ideological education in secondary schools, encouraging students to accept the reallocations that would benefit the country’s development (Yang, 2007). In 1957, a comprehensive system of rules and regulations was established to formalize the procedures of the National College Entrance Examination (NCEE) (Yang, 2007).

At this time, the NCEE was divided into three categories: students aiming for degrees in science and engineering took examinations in Chinese, politics, mathematics, physics, and chemistry; those pursuing degrees in arts, law, or business were assessed in Chinese, politics, history, and geology; and students with interests in medicine or agriculture were required to take examinations in Chinese, politics, biology, chemistry, and physics (Yang, 2003).


In 1957, during the Third Plenary Session of the 8th Central Committee of the Communist Party of China (CCCPC), Mao Zedong, the founding father and first chairman of the People’s Republic of China, asserted that efforts should be made to prevent the potential restoration of capitalism in China, given its occurrence in other countries (Yang, 2007).
As the supreme leader of the People’s Republic of China, Chairman Mao’s views and statements had a profound impact on Chinese society. In 1945, the Communist Party of China established Mao Zedong Thought as its guiding principle (“Mao Zedong Thought,” 2023). Mao Zedong Thought encompasses a series of political, military, and developmental theories developed by Mao Zedong and the Communist Party of China. These theories are grounded in Marxism-Leninism and its application in China’s socialist revolution (“Mao Zedong Thought,” 2023). Mao’s statements were frequently published in newspapers, encouraging the masses to study them (“Chairman Mao’s Quotations,” 2023). In 1964, a compilation of Mao’s speeches and writings was released as a pamphlet titled *Quotations from Chairman Mao Tse-tung*. At that time, this pamphlet, widely recognized by its red cover, was not only carried by almost every Chinese but also gained prominence internationally, becoming known as the “little red book” (“Chairman Mao’s Quotations,” 2023).

Consequently, when Mao Zedong asserted that efforts should be made to prevent the potential restoration of capitalism in China, this statement had a direct impact on Chinese society, including the NCEE system. In 1958, *the People’s Daily*, the official newspaper of the Central Committee of the Communist Party of China, raised concerns about the NCEE system. Firstly, universities and admissions offices operated without the direct oversight of the Communist Party of China (“Jiaqiang,” 1958). This posed a problem because many admission officers, having received their education in a bourgeois society, were perceived by Mao as “bourgeois intellectuals” (CCCPC Party Literature Research Office, 1999a). Therefore, there was a concern that these officers might limit the enrollment of students from working-class backgrounds (“Jiaqiang,” 1958). Secondly, admission decisions were solely based on test scores. This was also problematic because working-class children often lacked access to formal
education, leading to lower test scores ("Jiaqiang," 1958). To address these problems, the Ministry of Education introduced two significant changes to the admission process: (1) party members or non-party left-wingers would supervise the execution of the NCEE; and (2) admission decisions would take into consideration students’ political backgrounds and social relations (Yang, 2003).

In the same year, Mao launched the Great Leap Forward campaign in China. The primary objective of this campaign was to boost the country’s agricultural and industrial production (CCCPC Party Literature Research Office, 1999a). In the educational sphere, Mao advocated for the cultivation of China’s own cadre of proletarian intellectuals over the next decade. This included roles such as professors, instructors, scientists, journalists, writers, and artists (CCCPC Party Literature Research Office, 1999a). In alignment with Mao’s vision, the Ministry of Education employed every feasible measure to boost college enrollment, even if it meant reducing the minimum test score for admission (Yang, 2003). From 1957 to 1960, the number of admitted students surged from 106,000 to 323,000, marking an increase of more than 200 percent (Yang, 2007). During this period, the university admission process prioritized quantity over quality (Yang, 2003).

In 1960, the Great Leap Forward campaign was considered a mistake by the Communist Party of China (CPC) and brought to a halt (Jiang, 2008a). Within the realm of education, the emphasis then shifted toward the quality of education (Yang, 2007). Zhou Enlai, the first premier of the People’s Republic of China, played a significant role in this transition.

As a member of the Central Committee of the Communist Party of China, Zhou Enlai was also an influential political figure in China. His contributions to the country include, but are not limited to, (1) emphasizing economic construction as a top priority in socialist construction,
and (2) underscoring the role that science, technology, and intellectuals play in socialist construction (“Zhou Enlai,” 2023). Zhou Enlai’s position on intellectuals differed from that of Mao Zedong. In 1962, during the Third Session of the 2nd National People’s Congress, Zhou remarked that, in the preceding years, “bourgeois intellectuals” had contributed positively to the community under the leadership of the CPC (National Institute of Education Sciences, 1984). As a result, he suggested they be recognized as allies and be approached with trust and consideration (National Institute of Education Sciences, 1984). Zhou’s care and support for the intellectuals stemmed from his perception of intellectuals being an indispensable and significant force for the socialist construction (“Zhou Enlai,” 2023).

In 1961 and 1962, the Communist Party of China and the Chinese government formulated and implemented a series of reform plans and policies to address the problems arising from the Great Leap Forward campaign (Jiang, 2008a; Yang, 2007). In the domain of education, the primary focuses included improving the quality of teaching, enhancing college enrollment standards, respecting teachers and intellectuals, and cultivating scientific researchers (Jiang, 2008a). However, these reforms were short-lived.

On September 24, 1962, during the Tenth Plenary Session of the 8th CCCPC, Mao Zedong revisited concerns about the potential restoration of capitalism in China (Yang, 2007). Following this meeting, discussions concerning the dynamics between the bourgeoisie and the proletariat became more pronounced in Chinese society (Jiang, 2008a; Yang, 2007). Meanwhile, criticisms and scrutiny of the education sector and various artistic domains, such as drama, music, painting, dance, film, poetry, and literature, intensified (CCCPC Party Literature Research Office, 1996a, 1996b). These developments eventually led to the onset of the Great Proletarian Cultural Revolution (CCCPC Party Literature Research Office, 1983).
**Great Proletarian Cultural Revolution (1966-1976)**

On May 16, 1966, Mao launched the Great Proletarian Cultural Revolution in China with the aim of purging remnants of capitalism and traditional elements from Chinese society (Yang, 2007). Within education, the primary effort made to prevent the restoration of capitalism in China was to diminish the distinctions between intellectual and manual work (Yang, 2003). Given the concern that university graduates were often assigned to intellectual work (deemed as “bourgeois work” at the time), future graduates might find themselves placed in various settings, ranging from research centers and government agencies to factories and farms (Yang, 2003). It was anticipated that all students would adapt to their assigned roles and possess the ability to take on working-class jobs (Yang, 2007).

On May 28, the CCCPC established the Central Cultural Revolution Group (CCRG), positioning it as the leading group for the Great Proletarian Cultural Revolution (Yang, 2007). The CCRG primarily consisted of radical supporters of Mao Zedong, including Chen Boda, Mao’s wife Jiang Qing, Kang Sheng, Yao Wenyuan, and Zhang Chunqiao (“Cultural Revolution Group,” 2023).

By May 31, this group took control of *the People’s Daily* (the official newspaper of the CCCPC) and subsequently published a series of articles criticizing the NCEE system (Yang, 2003). Among their arguments was the view that the NCEE system mirrored the Keju examination system, which, from their perspective, was designed to nurture successors for the ruling class while restricting educational access for the working class. Consequently, they advocated for its abolition (Yang, 2003). These newspaper articles mobilized the masses, escalating the criticism toward the NCEE system to a national scope (Jiang, 2008b).
In response to the criticisms, the CCCPC and the State Council decided to discontinue the NCEE system from 1966 onwards and adopt a referral-based system for college admissions as proposed by the group (Yang, 2003). In this new system, aspects such as an applicant’s political background, health status, and practical experience became primary determinants for college entry (Yang, 2003). However, without standardized measures to assess academic skills and knowledge, there were indications of a decline in the academic readiness of new entrants. For example, some newly admitted students had difficulties understanding basic mathematics concepts, such as 1/2 being greater than 1/4, and others had limited exposure to disciplines like geometry, physics, or chemistry (Yang, 2007).

During this period, Zhou Enlai voiced different opinions. He expressed concerns about the decline in student quality and emphasized the significance of talent selection and training as well as scientific and technological research and development (Jiang, 2008b; Yang, 2007). Specifically, Zhou argued that depriving certain students of educational opportunities was not conducive to diverse thought. He further suggested that intellectuals be assessed individually rather than being stereotyped broadly as a uniform group (CCCPC Party Literature Research Office, 1998). Zhou’s perspective resonated with many in the public and intellectual circles, but it was ignored by the Cultural Revolution Group and the Gang of Four (Yang, 2003).

The Gang of Four was a political faction later convicted of implementing harsh policies during the Great Proletarian Cultural Revolution (“Gang of Four,” 2023). The members of the gang included Jiang Qing, Zhang Chunqiao, Wang Hongwen, and Yao Wenyuan, three of whom were members of the Cultural Revolution Group. During the revolution, they were promoted to higher positions within the party and controlled areas such as intellectual education, party
policies regarding intellectuals, and mass media and propaganda outlets (“Gang of Four,” 2023). In 1976, the Gang of Four was arrested and sentenced to imprisonment (“Gang of Four,” 2023).

In 1971, the Cultural Revolution’s influence on Chinese society seemed to wane. However, Chairman Mao did not take this opportunity to correct the mistakes of the revolution. He maintained a distance from Premier Zhou’s views and continued to express concerns over ultra-right ideologies in the country (Yang, 2007). This stance was leveraged by the Gang of Four, who took actions against those they deemed as rightists, intellectuals, and occasionally against Premier Zhou (Yang, 2007).

In 1975, during the First Session of the 4th National People’s Congress, Premier Zhou, despite facing health challenges, reemphasized the country’s goal of achieving the Four Modernizations: strengthening the fields of agriculture, industry, national defense, and science and technology (Yang, 2007). Deng Xiaoping, then the Vice President of the Central Military Commission, articulated that achieving these Four Modernizations would require enhancing teachers’ motivation to teach, stimulating students’ enthusiasm for learning, and elevating the standard of education (CCCPC Party Literature Research Office, 2004).

Deng Xiaoping is generally considered the supreme leader of the People’s Republic of China following Mao Zedong’s presidency (1949–1976) (“Deng Xiaoping,” 2023). His primary contributions to the country include, but are not limited to, the “ Reform and Opening-Up” policy and Deng Xiaoping Theory (“Deng Xiaoping,” 2023). The “ Reform and Opening-Up” policy refers to China’s opening up of the country to foreign investment and permission for entrepreneurs to start businesses, leading to significant economic growth for the country (“Chinese Economic Reform,” 2023). Deng Xiaoping Theory encompasses a series of political, economic, and military theories developed primarily by Deng Xiaoping. These theories are
considered to be an adaptation of Marxism-Leninism and Mao Zedong Thought to the existing socioeconomic conditions of China (“Deng Xiaoping Theory,” 2023). In 1997, the Communist Party of China established Deng Xiaoping Theory as one of its guiding principles (“Deng Xiaoping Lilun,” 2023).

On January 8, 1976, Zhou Enlai passed away. A significant public gathering, later referred to as the April 5 Tiananmen Incident (distinct from the more widely known 1989 Tiananmen Square Protests), took place at Tiananmen Square, where large crowds gathered to mourn Zhou and express their disapproval of the Gang of Four (Yang, 2007). Deng Xiaoping, who was perceived to have influenced the gathering, was relieved of his duties within the party. Yang (2007) commented that although Zhou and Deng’s efforts against the Gang of Four and the broader Cultural Revolution were unsuccessful at the current stage, events like the Tiananmen Incident highlighted a growing public awareness of the devastation inflicted by the Cultural Revolution on Chinese society. This awareness later contributed to the circumstances leading to the downfall of the Gang of Four (Yang, 2007).

On September 9, 1976, Mao Zedong passed away. Soon after, the Gang of Four was arrested by the Political Bureau of the Central Committee of the Communist Party of China (CCCPC). They were convicted of imposing harsh policies against individuals who held different views from theirs, with the aim of usurping the party’s leadership (“Gang of Four,” 2023; Yang, 2007). This marked the end of the Great Proletarian Cultural Revolution.

**NCEE System After the Cultural Revolution (1977-1984)**

In 1977, Deng Xiaoping resumed all of his party duties during the Third Plenary Session of the 10th CCCPC. One of his primary objectives was to revitalize and foster advancements in science, technology, and education in China (Yang, 2007).
Deng started by addressing several misconceptions that had hindered educational development in China. He stressed the importance of respecting knowledge, talents, and intellectuals. He emphasized that those engaged in intellectual work are also contributors to the socialist workforce and should not be stereotypically perceived as enemies of the proletariat (CCCPC Party Literature Research Office, 1994). Additionally, Deng sought to elucidate Mao Zedong Thought, particularly focusing on Mao’s stance on intellectuals. Deng conveyed that Mao’s intent was to motivate intellectuals to improve themselves for better participation in socialist production and construction. Unfortunately, his perspective had been distorted and utilized by the Gang of Four to persecute intellectuals (CCCPC Party Literature Research Office, 1994). These clarifications by Deng played a significant role in reshaping public attitudes toward intellectuals and instigating educational reforms, including changes in the examination system (Yang, 2007).

On October 12, 1977, the State Council announced a new college admission policy. The referral approach was phased out in favor of reinstating the previous NCEE system (Yang, 2007). That year, more than 5.7 million individuals registered for the examination, a substantial number of whom had been disqualified from participating during the Cultural Revolution (Yang, 2007).

In 1978, Deng delivered a classic speech on the National College Entrance Examination (NCEE), and many of the views he expressed in this speech became the guiding principles for future research on and reforms of the NCEE system (Yang, 2007). Yang (2007) summarized Deng’s points as follows:
1. While the negative impacts of the NCEE on society are inevitable, its positive effects should not be disregarded, as was done during the Cultural Revolution, nor should the system be abolished entirely.

2. The NCEE should not serve as the sole criterion for university admissions.

3. The content and organization of the NCEE need continuous reforms.

4. Examination reforms should proceed in three stages: basic research, applied research, and dissemination.

5. The aim of these reforms is to maximize the positive effects of the NCEE while mitigating its negative impacts.

6. The conclusion of the NCEE is not a definitive end. Subsequent efforts are equally essential, such as supporting and motivating students who did not achieve favorable results.

The first bullet point suggests that Deng held a more moderate view on the NCEE system, favoring reforms over abolition. The subsequent points provide more specific guidance on designing and implementing reforms.

Between 1979 and 1982, the State Council and the Ministry of Education introduced several reforms to improve the NCEE system (Yang, 2007):

1. A preliminary examination was introduced to manage the substantial number of candidates, a backlog resulting from the Cultural Revolution era.

2. Scrutiny of candidates’ political backgrounds was relaxed.

3. University preparatory courses were made available in economically less developed regions and to minority ethnic communities.
4. Separate examinations were organized for candidates from Hong Kong, Macao, Taiwan, and other overseas regions.

5. An English language test became mandatory for all candidates.

6. Candidates were no longer allowed to review their graded examinations.

7. The minimum test scores required for admission were determined based on candidates’ collective performance rather than being predetermined.

These reforms indicate that Chinese leaders and policymakers became more open-minded about candidates’ political backgrounds and more open to the rest of the world.

On September 1, 1982, at the 12th National Congress of the Communist Party of China, Deng introduced the idea of “building socialism with Chinese characteristics.” This concept, which later evolved into a comprehensive theoretical framework, set the direction for the CPC’s guiding principles (Yang, 2007). Subsequently, in 1983, the State Council and the Ministry of Education, under the guidance of “building socialism with Chinese characteristics,” pronounced that the ultimate objective for the NCEE system reforms was to “establish a socialist higher education enrollment system with Chinese characteristics” (Yang, 2007).

**NCEE System After 1984**

Starting in 1984, the Ministry of Education emphasized scientific research related to the university enrollment system (Yang, 2003). Any major adjustments to the NCEE system were subject to rigorous research, followed by pilot studies, before potential nationwide implementation (Yang, 2007). For example, the process to standardize the NCEE system involved initial research discussions conducted at both national and international levels, pilot testing in Guangdong province, and eventual nationwide application after pilot testing proved
successful (Yang, 2007). Several key reforms of the NCEE system were identified in Jiang’s (2008c) and Yang’s (2007) publications:

First, as China transitioned from a planned to a market economy, the existing university enrollment system required updates to cater to the growing educational needs of students and the increasing talent demands of enterprises (Yang, 2001). Thus, in 1987, the Ministry of Education issued a new policy stating that university enrollment would occur in three forms: state-planned, enterprise-commissioned, and self-financed (Yang, 2003).

Second, historically, the central decision-making for university admissions resided with the Ministry of Education and provincial educational departments, limiting universities’ autonomy (Yang, 2007). To address this, the State Education Commission issued a policy change in 1987, setting only minimum score criteria at the provincial level and allowing universities to establish specialized selection criteria (Yang, 2003).

Third, the emphasis on examination-oriented education has historically led to academic imbalances, such as favoring either arts or sciences at the expense of the other, resulting in psychological, educational, and social issues (Kirkpatrick & Zang, 2011; Yang, 2003). To counter this, a high school achievement test system was introduced in 1990, assessing broader subjects not covered in the NCEE and incorporating these results into college admissions (Yang, 2007).

Fourth, reforms on the subjects tested in the NCEE by students pursuing different fields of study have long been a research topic. In 1991, the “Three Nan” system, named after the three provinces where the pilot study first occurred (i.e., Hunan, Hainan, and Yunnan), divided the NCEE into four testing groups. The major flaw of this system was the lack of common subjects between groups (Yang, 2007). In 1992, the “3 + 2” system divided the NCEE into an arts group
featuring Chinese, mathematics, English, history, and politics and a science group featuring Chinese, mathematics, English, physics, and chemistry. However, the omission of biology and geology led to dissatisfaction among scholars in these fields (Yang, 2007). A new system, termed the “3 + X,” was introduced in 1999. It comprised three compulsory subjects—Chinese, mathematics, and English—and six optional subjects—physics, chemistry, biology, politics, history, and geology—allowing students to select one or more to take (Yang, 2007).

Last but not least, these reforms witnessed a remarkable change in the NCEE system: an increasing number of regions were permitted to administer independent examinations. Initially, independent examinations were designed for scientific research purposes. For instance, the standardization reform was piloted in Guangdong province, while the achievement test reform was piloted in Shanghai (Jiang, 2008c). Over time, as research, reforms, and improvements to the NCEE system continued, the Chinese government and policymakers began to recognize the benefits of implementing independent regional examinations. These benefits included:

1. Motivating local governments to be more proactive in educational management and development,
2. Fostering the reform of educational and examination systems,
3. Optimizing the selection and cultivation of students in ways that aligned with regional development goals, and
4. Reducing the risk of test content leakage from one region to another (Jiang, 2008c).

Ultimately, the authorities decided that while maintaining a uniform national examination system—with consistent examination dates, times, and admission process—certain regions would be allowed to determine their examination subjects and content independently. In 2004, nine provinces embarked on the pilot study. By 2006, the number of provinces conducting
independent examinations had increased to 16 (Jiang, 2008c). This significant shift toward allowing independent regional examinations signified China’s move toward less rigid, more open, and more flexible educational and examination policies in the early 21st century. Although these changes fall outside the time range of this study, they remain important and warrant further exploration.

Overall, after 1984, China’s educational and examination reforms have become more scientific, systematic, diversified, and in tune with the times, making China’s educational development full of vitality.

In conclusion, as Davey et al. (2007) once noted, the evolution of the NCEE system is deeply linked with other transformations in higher education. These include increased diversification, a shift from serving political needs to catering to economic and social development, decentralization from government control to autonomy for individual universities, the end of government decisions on student recruitment and job assignment, and enhanced international educational exchange and collaboration (Zhou, 1994; Hannum & Park, 2006). These changes and developments provide fertile ground for further academic study.
Chapter III
LITERATURE REVIEW

This chapter presents a comprehensive literature review on three key topics relevant to this study: historical research in mathematics education, educational systems in socialist countries, and the evolution of high-stakes mathematics examinations. The first section introduces the field of historical research, emphasizing its particular applications in mathematics education. The second section describes the characteristics of educational systems in socialist countries, detailing how they respond to economic, political, and social development demands. The third and final section provides a summary of the literature on the evolution of high-stakes mathematics examinations, highlighting the analysis of their organization, structure, topic coverage, and item difficulty, as well as their correlations to societal and political changes.

Historical Research in Mathematics Education

General Historical Research

Historical research is a systematic investigation based on historical materials (Elana et al., 2009). Historians first collect data through a methodical and comprehensive study of both primary and secondary sources (Elana et al., 2009). Primary sources are documents or artifacts created by witnesses or participants who experienced the events in question, such as diaries, speeches, letters, contemporary newspaper articles, official records, meeting notes, and photographs (Harvard Library, 2021a). Secondary sources include books and articles produced by researchers who have interpreted and analyzed primary sources, such as journal articles, criticisms, commentaries, and encyclopedias (Harvard Library, 2021b). Historians then
categorize and arrange scattered facts into integrated and meaningful clusters (Simiand, 1985). Finally, they evaluate and synthesize all evidence to draw conclusions about past events (Walliman, 2017). In addition to establishing facts and reassessing beliefs about the past, historical research can also offer constructive solutions for contemporary challenges and inform present and future trends (Walliman, 2017).

On the one hand, the overall quality of historical research depends heavily on researchers’ analytic and integrative skills, as well as their understanding of the historical context in which the data were collected (Bhattacherjee, 2012). On the other hand, the quality of historical research is also closely related to the reliability of source material and the authenticity and credibility of the historical evidence (Mcdowell, 2002). Triangulation, a strategy used in qualitative research that involves employing various data collection methods, comparing multiple data sources, and consulting different perspectives or theories, can help verify and validate historical analysis (Patton, 1999). Other techniques to enhance the quality of historical analysis include testing rival explanations, looking for negative cases, maintaining data context, cross-referencing, and document examination (Mcdowell, 2002; Patton, 1999; Walliman, 2017).

**Research in Mathematics Education**

Historical research in mathematics education has a well-established tradition. According to Schubring (2014), the earliest known study dates back to 1843, when an assessment was conducted on the evolution of the Prussian Gymnasium’s mathematics curriculum, tracing its development from Enlightenment-inspired reforms to subsequent changes in the Prussian Gymnasium (Fisch, 1843). In the USA, the first two doctoral dissertations in mathematics education focused on its history (Jackson, 1906; Walker Stamper, 1909).
Despite an increasing number of publications in this area (e.g., Bidwell & Clason, 1970; Karp & Vogeli, 2010, 2011; Schmidt, 1991; Schubring, 1984; Stanic & Kilpatrick, 2003), this field has not been systematically developed. This lack of systematic development is likely because previous research efforts were mainly individual initiatives that lacked international communication (Schubring, 2006; 2014). However, the landscape significantly changed in 2004 when the field became internationally institutionalized, specifically being designated as Topic Study Group 29 at the 10th International Congress on Mathematics Education (ICME 10) (Schubring, 2014). Importantly, the first journal solely dedicated to this field, the International Journal for the History of Mathematics Education, published ten volumes from 2006 to 2016 (COMAP, 2016).

In China, while historians and mathematicians have traditionally studied the history of mathematics to understand its ancient development and contributions to human civilization, few have focused on the educational implications, often considering it to have limited academic value (Wang, 2017). However, the history and pedagogy of mathematics (HPM) has recently gained popularity among younger researchers in China, thanks to pioneering efforts from scholars such as Ji Zhigang, Wang Xiaoqin, and Zhang Dianzhou (Wang, 2017).

As an interdisciplinary field of study, historical research in mathematics education intersects significantly with various other disciplines, including history, the history of education, sociology, and the history of mathematics (Schubring, 2014). Specifically:

- Researchers use historical methods to locate, interpret, and analyze sources.
- The history of education is crucial for understanding the broader development of educational systems where mathematics education takes place.
• Sociology offers multiple methods and concepts for studying the history of mathematics education, especially since mathematics serves as a bridge between general education and professional training, thereby linking its history of teaching to the labor market.

• The history of mathematics and the history of mathematics education often explore the same documents and figures but from different angles. This relationship is no longer simply one of production and reproduction; rather, teaching has become a key factor in the advancement of mathematics itself (Schubring, 2001, 2014).

Additionally, the history of teaching and learning mathematics also connects with other fields such as psychology, philosophy, politics, economics, and religious and cultural studies (Cotton & Hardy, 2004; Fauvel & Van Maanen, 2006; Karp, 2014; Schoenfeld, 2010).

Schubring (2006) noted that researchers often treat the historical dimension of mathematics education superficially. In line with this view, Karp (2014) posited, “Any topic of contemporary research (for instance, any topic mentioned in the name of a topic study group at international congresses on mathematics education) may be studied from a historical point of view” (p. 12). The following research topics are recommended by contemporary researchers in the field:

• The history of the development and formation of any school mathematics subject and associated teacher education.

• The history of changes in beliefs and attitudes toward mathematics and its teaching and learning.

• Characteristics specific to a country’s approach to mathematics teaching and learning, as well as the interactions between different countries and cultural exchanges.
• Programs of study, teaching aids, and administrative (legislative) decisions governing the process of mathematics education.

• Biographies, training experiences, and commentaries from educators and planners in mathematics education, along with public perceptions of the field (Karp & Schubring, 2014; Karp & Furinghetti, 2016).

To explore these topics, researchers can consult various “relics” and “narratives” related to mathematics education or, more broadly, to the lives of individuals devoted to the field’s development (Karp, 2014). These may include concrete objects such as teaching manipulatives, blackboards, models, computers, and calculators, as well as official documents like textbooks, state examinations, and internal reviews. Unofficial records, such as personal diaries, memoirs, newspaper articles, and even poems and songs composed by teachers and students, can also be valuable (Karp, 2014).

The notion of creating an exhaustive list of sources in the history of mathematics education is impractical. Sometimes, seemingly irrelevant diaries can yield surprising insights (Karp, 2014). At other times, the clearest and most accommodating texts and archaeological discoveries may turn out to be invalid testimony (Bloch, 2004). Consequently, a thorough examination of any potential sources before using them is both necessary and significant.

Last but not least, scholars in the field have highlighted the significance of studying the history of teaching and learning mathematics. From the perspective of teacher education, understanding the historical emergence and development of their profession, as well as the challenges overcome to establish effective teaching, can better equip mathematics teachers to address problems in their professional lives (Schubring, 2006). In terms of student learning, gaining access to courses related to the history of mathematics teaching and learning can deepen
students’ understanding of mathematical concepts and their interconnectedness across different cultures and societies (Fauvel & Van Maanen, 2006). As for disciplinary development, insights into past successes, failures, strategies, and findings can help researchers identify the root causes of current challenges, leading to practical solutions that benefit the field of mathematics education (Karp & Furinghetti, 2016). Overall, as Fauvel and Van Maanen (2006) noted, the historical dimension encourages teachers, students, and researchers to view mathematics as “a continuous process of reflection and improvement over time, rather than as a defined structure composed of irrefutable and unchangeable truths” (p. 64). This perspective is sure to bring fresh and innovative insights into mathematics education.

In summary, historical research in mathematics education is a rich and multidisciplinary field that has evolved considerably over time. It intersects with various disciplines, offers a wide range of research topics, and can be approached through multiple types of sources. The insights gained from this type of research are invaluable for improving teacher education, enriching student learning, and advancing the discipline itself. As previously mentioned, the history of education is crucial for understanding the broader development of educational systems in which mathematics education occurs (Karp & Schubring, 2014). In the following section, a historical review of educational systems in socialist countries is presented to help readers understand the broader educational, political, and cultural contexts where the NCEE takes place.

**Educational Systems in Socialist Countries**

In 1978, Frank J. Swetz published a book called *Socialist Mathematics Education*. This book examines the policies and practices of mathematics education in seven socialist countries, including the Union of Soviet Socialist Republics (U.S.S.R.), the German Democratic Republic,
the People’s Republic of China, Yugoslavia, Sweden, Hungary, and Tanzania. Despite being somewhat dated, this work likely serves as the only book that offers a relatively comprehensive understanding of socialist mathematics education. Consequently, it is used as a guideline for reviewing socialist educational systems in this section.

Notably, Swetz’s (1978) contributions have been influential but also subject to criticism, particularly regarding his generalization of “socialist mathematics education” as a unified concept. Critics such as Howson (1980) pointed out that Swetz’s investigation did not include all socialist countries and that the educational objectives and conditions in these countries differed significantly. The researcher acknowledged these critiques and understood that the notion of a uniform “socialist mathematics education” may oversimplify a complex reality. This research did not aim to resolve these disputes but rather to focus on some aspects of socialist educational systems highlighted by Swetz’s work, specifically those observable in Chinese society from 1949 to 1984:

1. Universal and uniform education provided to all children,
2. Education offered to farmers, workers, and soldiers,
3. Securing an adequate number of properly trained teachers,
4. Integrating school instruction with workplace practice, and
5. Strict control and monitoring of education.

**Universal and Uniform Education Provided to All Children**

A primary focus of socialist education is the universalization and uniformization of the educational system. Regarding universalization, Swetz (1978) observed that a broad-based education was established for all children in socialist countries, a statement confirmed by the studies of Holmes (1991), Johnston (2011), Lauwerys (2011), and Suchodolski (2011).
Specifically, within the field of mathematics education, Karp and Vogeli (2010) noted that mathematics courses, which had previously been available to only a few students, now became accessible to millions of students in the Soviet Union.

In terms of uniformization, Birzea (1995) provided a model that outlines the socialist chain of education from one party to one ideology, which then leads to one nation, one education system, one curriculum, and one textbook, ultimately shaping a new individual. In some countries, this uniformization even extends to lesson plans, homework, and grading systems (Cox, 2011). In China, such uniformization also extends to the examination system, specifically the National College Entrance Examination (NCEE) system.

Researchers have suggested some advantages of the universalization and uniformization of the educational system. Firstly, a nationwide uniform education system is a more effective instrument for disseminating socialist ideology (Cox, 2011). Secondly, a uniform ideology can contribute to reducing cultural conflicts, fostering ethnic unity, and maintaining social stability (Yang, 2007). Thirdly, universal education helps to reduce illiteracy in a country, cultivating a more educated populace, which is beneficial to the country’s development (Jiang, 2008a).

Data indicate that the universalization of the educational systems in China and the Soviet Union led to a remarkable reduction in their illiteracy rates (Cox, 2011; Lauwerys, 2011). Specifically, in China, between 1954 and 1965, the number of illiterate people decreased by approximately 95.713 million (China Education Yearbook Editorial Department, 1984). Notably, this figure includes not only children and youth but also adults. This highlights another feature of the socialist education system at the time: the universalization of education was also aimed at adults, especially those who lacked access to education when they were young, such as farmers, workers, and soldiers.
Education Offered to Farmers, Workers, and Soldiers

Adult education is a significant component of socialist educational systems, especially during a country’s early years when a substantial number of illiterate adults existed. Suchodolski (2011) observed that, in Poland, individuals across various ages, social strata, and professions were keen to expand their professional knowledge and educational scope. In China, the establishment of adult education programs targeting workers, farmers, and soldiers was a focal point for party and state leaders.

Specifically, Chairman Mao Zedong advocated for the establishment of educational programs aimed at enhancing the academic qualifications of individuals from worker or peasant backgrounds (CCCPC Party Literature Research Office, 1999b). Premier Zhou Enlai emphasized the significance of developing intellectuals from these backgrounds, which would align with the country’s governance nature that the proletariat holds state power (China National Academy of Educational Sciences, 1984). Education Minister Ma Xulun suggested that educating these groups would facilitate their effective participation in socialist production and political life (Liu, 1993).

Guided by these stands, China established a variety of specialized schools that allowed workers and farmers to attend during their free time. Statistical data show that the number of adult learners increased substantially, from 13.268 million in 1949 to 51.08 million in 1951 (Jiang, 2008a). Additionally, the NCEE system introduced more flexible admission policies, such as lowering admission standards for students from worker and farmer backgrounds who had been unable to complete high school due to financial limitations (Yang, 2003). At the time, the goal of “equipping peasants and workers with cultural and scientific knowledge” became a widely recognized objective throughout the country (Lauwerys, 2011).
Securing an Adequate Number of Properly Trained Teachers

A shortage of adequately trained educators was a common challenge faced by socialist countries upon establishing their regimes (Lauwerys, 2011). To address this problem, socialist countries frequently implemented short-term emergency programs to expedite the training of new educators (Johnston, 2011; Kola, 2014).

Specifically, in China, the insufficient number of teachers with adequate training presented a significant challenge to educational development during the country’s early years (Yao, 2009). Feng (2002) identified two major factors contributing to this shortage: the slow progress in teacher education before the founding of the People’s Republic of China and the rapid expansion of basic education afterward. At the time, the rate of teacher training struggled to match the speed of educational development. For instance, in 1954, there was a demand for over 28,000 additional secondary school teachers, but only about 7,600 were available for placement (Feng, 2002). To address this gap, China, like other socialist nations, introduced a variety of accelerated teacher training programs and short-term courses (Feng, 2002). According to Chen (1953), in 1952, more than 23,200 new students enrolled in teacher training institutions, with 45% participating in these expedited training programs.

Integrating School Instruction with Workplace Practice

Swetz (1978) observed that most socialist countries highlighted a close relationship between classroom-acquired theory and workplace practice. This relationship was established in polytechnic institutions, which combined school instruction with practical skills training and industrial production studies. The curriculum in polytechnic schools covered a broad range of subjects, such as mathematics, physics, economics, industrial work, and farming, aimed at preparing students for productive roles in the workforce (Fraser, 2011; King, 2011; Price, 2017).
Following the establishment of the People’s Republic of China in 1949, the Chinese government adapted the Soviet polytechnic system for its higher education sector (Hayhoe & Zha, 2010). Prestigious universities like Tsinghua University and Zhejiang University were required to discontinue their historic programs in arts and basic sciences. Additionally, they were expected to shift their focus to applied fields, retaining only the basic science and mathematics courses to ensure good technological standards (Hayhoe & Zha, 2010). According to data, these reforms aligned well with the country’s economic objectives; the proportion of college students enrolling in engineering programs, which were considered vital for socialist production and construction, increased considerably from 15% to 36.5% by 1960 (Hayhoe & Zha, 2010).

**Strict Control and Monitoring of Education**

Another noteworthy feature of socialist educational systems is their strict control and monitoring of course content, teaching materials, instructional approaches, teacher training, student behaviors, extracurricular activities, educational research, and other educational details (Birzea, 1995; Karp & Vogeli, 2010; King, 2011).

From a classroom teaching perspective, the course content was expected to align with the socialist worldview (Pachocinski, 1993), and instructors were required to adhere strictly to established lesson structures (Karp & Schubring, 2014). In practice, rigorous monitoring was administered to ensure that the instructional methodology and procedures were carried out smoothly. For example, Karp and Vogeli (2010) noted that, in the Soviet Union, school principals and government officials continuously monitored the teachers, observing hundreds of classes over the course of a year.

From a student learning angle, rote memorization and adherence to teachers were generally emphasized (Cox, 2011), with teachers rigorously monitoring the students in a similar
manner (Karp & Vogeli, 2010). Party-sponsored youth organizations, such as the Little Octobrists, the Young Pioneers, and the Komsomols, also played important roles in moral-political education (Price, 2017). They formed continuous, personalized mentoring relationships between older and newer members (King, 2011) and contributed to the cultivation of faithful party members and future leaders (Ross, 1960).

Lastly, in terms of curriculum, socialist parties and governments had significant control over course selection, textbook adoption, and examination criteria (Adams, 2011). They limited courses such as social sciences and religious studies (Birzea, 1995; Pardała, 2010), closely monitored subjects like history, literacy, and visual arts (Gaworek, 1977; King, 2011), and introduced new courses that aligned with socialist ideology (Dumbraveanu, 2007).

**Summary**

Overall, socialist educational systems shared several common characteristics, and these features were tailored to satisfy specific economic, political, and social needs of the country. For example, by providing universal and uniform education to all children, establishing adult education for the working class, initiating short-term teacher training programs, and adopting polytechnic education, socialist countries aimed to accelerate the training of a generation skilled in science and technology, involved in productive labor, and prepared for socialist construction (Suchodolski, 2011). By exercising strict oversight over all aspects of education, they aimed to shape public opinion about civil society and governing bodies from a young age, benefiting national unity and social cohesion (Cox, 2011; Yang, 2007).

In conclusion, socialist countries have shaped their education systems to meet the states’ economic, political, and social demands (Cox, 2011; Fraser, 2011). While some critics and scholars describe this system as extremely rigid, monolithic, and inflexible (Adams, 2011;
United States. Education Mission to the USSR, 1959), suggesting that their emphasis on speed and efficiency may sacrifice freedom (Lauwerys, 2011), others highlight its advantages. Centralization is perceived as essential for rapidly developing countries that aim to catch up with more developed nations (Lauwerys, 2011; Suchodolski, 2011). Furthermore, some policymakers and researchers argue that within its specific framework, socialist education is continually evolving to address the changing needs of teachers and students (Adams, 2011; United States. Education Mission to the USSR, 1959). Such a centralized system is also considered capable of implementing educational reforms across regions more swiftly than a decentralized one (United States. Education Mission to the USSR, 1959).

This section aimed to help readers understand the broader educational, political, and cultural contexts within which the NCEE resides, facilitating the interpretation of its evolution. The next section provides a detailed summary of existing literature on the evolution of high-stakes college entrance mathematics examinations in varying political and social environments, offering valuable insights into the study of the NCEE mathematics examinations in China.

**The Evolution of High-stakes Mathematics Examinations**

Over the last few decades, education researchers have become increasingly aware of the factors that closely correlate with high-stakes examinations. The notion that “what tests test is what teachers teach,” as evidenced by multiple studies (e.g., Pellegrino, 1999; Polesel et al., 2014), illustrates a clear connection between testing and teaching practices. Some scholars have suggested that this phenomenon narrows the curriculum and constrains both teachers and students (e.g., Cizek, 2001; Koretz, 2008). Others contend that it actually expands the curriculum and motivates teachers and students (e.g., Amrein & Berliner, 2002; Nichols et al., 2005).
Au’s (2007) study provided a sound explanation for these conflicting findings: the nature of high-stakes-test-induced curricular control depends on whether the alignment between tests and the curriculum takes the form of content contraction or content expansion. The alignment study mentioned is a methodology commonly used by academics to evaluate the connections between examinations, curriculum, and teaching practice (e.g., Ananda, 2003; Roach et al., 2008; Squires, 2012; Ziebell & Clarke, 2018). The results of such studies can help policymakers, test developers, and educators refine their work, enabling the curriculum, assessment, and instruction to better complement each other in fulfilling students’ learning goals (La Marca et al., 2000).

Apart from factors interacting with high-stakes examinations within the education system, researchers have also grown increasingly concerned about external forces affecting the examination landscape. For example, Pellegrino (1999) noted in his sixth annual William H. Angoff Memorial Lecture that “assessment practice is the product of multiple streams of influence, including social policy and societal goals, theories of the mind, and computational capacities” (p. 7). Sacks (2000) commented that political incentives, rather than legitimate educational considerations, were driving the state’s use of standardized examinations in schools. Moses and Nanna (2007) argued that factors such as administrative utility, profit motives, political ideology, and a pervasive testing culture contributed to the widespread adoption of high-stakes examination reforms.

Within the field of mathematics education, mathematics educators and researchers have also developed an interest in the relationship between high-stakes mathematics examinations and social and political forces. One branch of this research investigates the historical changes in high-stakes mathematics examinations and their correlation with social and political changes. Provided below are some examples of such research.
Research on Examinations in Western Countries

Lawrence et al. (2003) conducted a historical analysis of the SAT, a standardized test widely used for college admissions in the United States, from its inception to the present. They examined the verbal and mathematical portions of the SAT separately and discovered that the test had undergone several rounds of changes over the years. In terms of the mathematics section, it featured only free-response questions from 1930 to 1935 and exclusively multiple-choice questions from 1942 to 1959. Data sufficiency questions were introduced in 1959 and were later replaced by quantitative comparison questions in 1974. Student-produced response questions were added in 1994 (Lawrence et al., 2003). Overall, they concluded that the mathematical portion of the SAT has evolved from being a measure of students’ ability to apply rules and formulas to a measure of their capacity to think through and solve problems (Lawrence et al., 2003). Additionally, Lawrence et al. explored the fundamental reasons behind these changes, citing “fairness issues, scaling issues, cost, public perception, face validity, changes in the test-taking population, changes in patterns of test preparation, and changes in the college admissions process” (p. 6).

Dossey (1996) conducted a comparative analysis of mathematics examinations taken by college-bound students in England and Wales, France, Germany, Japan, Sweden, and the United States during the years 1991 and 1992. Notably, the U.S. examinations used for comparison were from the years 1988 and 1993. Dossey (1996) examined various aspects of the examinations, such as their general structure (e.g., test lengths, number of scorable events), item characteristics (e.g., question types, use of diagrams, graphs, and tables), topics covered, and expectations regarding student performance. One significant finding was that nearly all the examinations lacked connections between mathematics and real-world contexts (Dossey, 1996). Dossey
posited that this disconnect could be attributed to the prevailing perception of school mathematics as a subject characterized mainly by the recognition and repetition of definitions and theorems, as well as by the performance of symbol manipulation procedures. Another noteworthy finding was that the examinations in different countries reflected their respective educational objectives and curricula for school mathematics (Dossey, 1996). For example, in Japan, the examination was designed to rank and identify outstanding students, while in the U.S., it aimed to assess the minimum concepts and skills required for college admission.

**Research on Examinations in Socialist Countries**

In addition to research on high-stakes mathematics examinations in Western countries, Karp (2007) examined the history of Russian graduation mathematics examinations in algebra from 1890 to the mid-1950s. To conduct this historical analysis, Karp utilized a diverse array of sources, including official documents such as administrative reports and examination results, memoirs from former teachers and students, contemporary journalism, and methodological articles, speeches, manuals, and problem books. Through these documents, Karp (2007) sought to uncover issues related to the composition and administration of past examinations, as well as the historical debates these issues engendered. Among the key findings were the following: First, after 1917, all examinations were canceled in Russia due to public sentiment that viewed testing as both meaningless and harmful; however, this decision was later deemed a mistake, and examinations were reinstated with their former status after 1932. Second, although official instructions called for stringent conduct during the administration of examinations, anecdotal evidence from students’ memoirs painted a different picture, indicating that cheating was rampant. Third, students were required to not only solve problems but also justify each step of their solutions in detail, sparking debates among Russian mathematicians and educators about
the requirements for writing solutions on examinations (Karp, 2007). Karp (2007) contended that these findings could help educators become more familiar with historical practices and challenges in examination administration and contribute to broader understandings of the culture, politics, society, and individuals of a particular era in Russia.

In China, Zheng and Chen (2017) conducted a review of the National College Entrance Examination (NCEE) in mathematics from 1949 to 2017. They aimed to provide insights into the upcoming high school mathematics curriculum reforms, particularly in the areas of topic selection and assessment. Their analysis primarily focused on three areas: the total number and types of questions, the contexts in which the questions were framed, and the nature of the questions (i.e., open-ended or not) (Zheng & Chen, 2017). Their data revealed the following: (1) multiple-choice, fill-in-the-blank, and free-response questions constituted the question types; (2) the total number of questions varied from year to year, ranging from 8 to 28; (3) examinations prior to the 1990s primarily featured questions based in pure mathematics, whereas applied mathematics has gained more attention in subsequent years; (4) reading comprehension and cultural contexts have become increasingly prominent in recent years; and (5) open-ended questions were introduced in 1999 but still require further exploration and refinement (Zheng & Chen, 2017). In summary, Zheng and Chen’s (2017) study provides detailed insights into the organization and structure of the examinations and the context of the test questions, however, it does not delve into areas like topic coverage and item difficulty.

Tian’s (1998) research focused on the NCEE mathematics examinations in China from 1978 to 1997. The primary source was the examination syllabi rather than the real examinations. The analysis subdivided this period into four distinct stages, each characterized by unique features. The initial stage (1978–1983) occurred immediately after the Great Proletarian Cultural
Revolution; during this time, examination policies were flawed and imperfect, leading to inconsistencies between the school curriculum and the examination questions. The exploration stage (1984–1988) was marked by debates among policymakers and scholars about whether the examination should assess students’ memorization skills or problem-solving abilities. During the formation stage (1989–1993), the State Education Commission revised the examination syllabus and issued new guidelines. Lastly, the transformation stage (1994–1997) saw a shift in China’s education system from being examination-centric to quality-oriented (Tian, 1998). Tian argued that Chinese policymakers and educators have continually reformed their examination system and developed new test questions in response to rapid economic, political, and social changes.

Similarly, Ren and Chen (2017) examined the NCEE mathematics examinations in China from 1978 to 2017, subdividing this period into five distinct phases and relying on examination syllabi for their analysis. In the first period (1978–1982), examinations emphasized the assessment of fundamental mathematical knowledge and skills. The second period (1983–1988) shifted focus toward assessing problem-solving abilities. During the third period (1989–2000), standardization reforms in the NCEE system led to clearer specifications for test content, question types, and difficulty levels in the examination syllabus. The fourth period (2001–2007) introduced significant changes to examination content, incorporating modern mathematical concepts such as limits, derivatives, integrals, and vectors. The fifth period (2008–2017) further updated the examination content to include topics like algorithms, projections of three-dimensional figures, and flowcharts in programming. This period also emphasized skills in data processing, spatial imagination, and practical application (Ren & Chen, 2017). Ren and Chen concluded that over these 40 years, the NCEE mathematics examinations have consistently undergone reform and innovation. This has been instrumental in identifying the talent most
needed during each specific historical period and has also positively influenced high school curricula and teaching practices (Ren & Chen, 2017).

**Research on Item Difficulty in Examinations**

When analyzing high-stakes examinations, a key focus is often on item difficulty. Various theories and models have been developed for this purpose. In psychometrics, Classical Test Theory (CTT) and Item Response Theory (IRT) are the two primary approaches widely used for item analysis. Although these theories differ in substance and complexity, they both require student performance data to quantify item difficulty. For instance, in CTT, item difficulty is measured by the percentage of students who answer a test item correctly; in IRT, item difficulty is represented by the level of ability at which a test-taker has a 50% chance of answering the item correctly.

However, some scholars, including Ginsburg et al. (2005), Heinze et al. (2004), and Nohara and Goldstein (2001), critiqued these approaches. They argued that CTT and IRT primarily measure item difficulty based on student performance, overlooking the inherent characteristics of the test items themselves. To address this, these researchers developed new models specifically designed to assess the difficulty of mathematics examinations by focusing on item characteristics rather than student responses.

For example, Ginsburg et al. (2005) characterized the item difficulty of mathematics problems using three distinct criteria:

- The approximate number of steps required for a solution;
- The necessity of solving for an unknown intermediate variable; and
- The requirement of either a routine application of a formula or definition, or a nonroutine strategy.
These criteria were developed for the following reasons: (1) “Problems that require more steps demand greater reasoning skills” (p. 73); (2) solving for an intermediate unknown requires students to derive the missing information as part of developing the solution; and (3) nonroutine problems do not have obvious solutions based on definitions and formulas.

Heinze et al. (2004) divided test items into three levels of competency:

- Basic competency (applying facts and rules, calculations);
- Argumentative competency with one-step argument; and
- Argumentative competency that combines several arguments.

They empirically validated this classification through a statistical analysis. The analysis revealed that low-achieving students struggled significantly with third-level questions, whereas high-achieving students generally demonstrated satisfactory performance.

Nohara and Goldstein (2001) identified four factors that could potentially influence mathematics question complexity: the response type, the question context, the need for multi-step reasoning, and the extent of required computation. They justified these difficulty factors as follows: First, they noted that certain response types are associated with higher-order thinking skills. For instance, although multiple-choice questions may involve advanced reasoning and extended-response questions can be straightforward, multiple-choice questions are generally less complex. Second, questions situated in a real-world context typically demand more thinking than those presented directly. Third, a distinction is commonly made between basic skills, such as recalling facts or using routine procedures, and higher-order thinking skills, such as developing multi-step solutions for nonroutine problems. Last, they highlighted that computation, a common element in school mathematics, can add to the complexity of a question.
Finally, Bao (2002) categorized the item difficulty of mathematics problems into five factors, each with multiple levels:

- **Discovery**: knowing, understanding, and investigating
- **Context**: none, personal life, public affairs, and scientific scenarios
- **Computation**: none, numerical, simple symbolic, and complex symbolic
- **Reasoning**: none, simple, and complex
- **Topic coverage**: single, two, and three or more

In the *discovery* factor, *knowing* refers to the ability to recall mathematical facts and execute standard mathematical procedures. This level emphasizes rote memorization of mathematical concepts. *Understanding* involves the ability to apply mathematical concepts, formulas, rules, and theories, as well as to use standard methods to obtain definite results. It also entails the capacity to justify and demonstrate mathematical facts. This level focuses on the application of mathematical knowledge. *Investigating* denotes the ability to formulate mathematical conjectures and develop mathematical models innovatively. This level centers on mathematical modeling and encourages creativity in problem-solving. Typical questions at this level are often open-ended, non-routine, and exploratory. *Investigating* questions are considered the most difficult, followed by *understanding* questions, with *knowing* questions being the simplest.

In the *context* factor, different levels signify varying degrees of relevance to the students’ everyday lives. In other words, these levels can be interpreted as the frequency with which students might encounter these contexts. *Personal life* includes scenarios such as household chores and school activities, which students experience frequently. *Public affairs* encompass contexts like interest rates and sales, which students encounter at a medium frequency. *Scientific*
scenarios involve contexts such as physical experiments, to which students are only occasionally exposed. Lastly, none refers to questions rooted purely in mathematical theory. Scientific scenarios questions are deemed the most difficult, followed by public affairs and personal life questions, with purely mathematical questions being the simplest.

In the computation factor, levels are determined by the complexity of the operations involved. The first two levels, none and numerical, are self-explanatory. The simple symbolic level involves computations that require one or two steps, while the complex symbolic level involves computations that require three or more steps. According to Bao, numerical computation problems are easier than symbolic ones, and questions requiring no computation are the simplest.

In the reasoning factor, levels are classified based on the number of procedures involved: none, simple (involving one or two steps), and complex (involving three or more steps). Questions requiring more reasoning steps are considered more difficult.

Lastly, in the topic coverage factor, different levels correspond to the number of mathematical topics needed to solve each item. Questions requiring more topics are more challenging.

Several researchers (e.g., Wu, 2015; Zhao & Sun, 2021; Zhou, 2012) adopted Bao’s model to investigate the evolution of mathematics examinations over time. Zhou (2012) employed Bao’s model in a longitudinal comparative study of the NCEE mathematics examinations from 2001 to 2011. She analyzed the changes and developmental trends for each type of question (i.e., multiple-choice, fill-in-the-blank, and free-response) across the five dimensions outlined in Bao’s model. Her findings included the following: (1) The overall difficulty levels for multiple-choice, fill-in-the-blank, and free-response questions were relatively
low, medium, and high, respectively; (2) There was an overall trend toward emphasizing the
evaluation of students’ creativity skills, learning potential, and historical and cultural awareness
(Zhou, 2012).

Similarly, Wu (2015) applied Bao’s model to examine the structure, features, and changes in high school entrance mathematics examinations in China from 2006 to 2011. His findings revealed that (1) the examinations emphasized computation, reasoning, and relations among various mathematics topics but neglected the application of mathematics in real-world contexts; (2) there was a general trend toward including more process-oriented and real-world context questions; and (3) regional differences were observed, which could lead to equity issues that warrant serious consideration (Wu, 2015).

Summary

In conclusion, the study of high-stakes mathematics examinations and their evolution over time has attracted the attention of researchers worldwide. The research summarized in this section revealed that each redesign of these examinations was influenced by specific socio-political decisions within a given country. In China, much of the existing research has focused on the structural and content-related aspects of examinations, often neglecting their broader socio-political context. Additionally, there is a research gap concerning the NCEE mathematics examinations during the periods 1952–1965 and 1977–1984. Echoing the sentiments of Karp (2007), understanding the history of examinations is invaluable for identifying potential challenges and contradictions that may arise in their future development. This research aimed to fill that gap by examining the history of the NCEE mathematics examinations during these periods, thus informing ongoing reforms and potentially benefiting their future development.
Chapter IV

METHODOLOGY

This chapter describes the methodology of this study. Historical research generally involves three procedures: (1) collecting historical data through a methodical and comprehensive study of primary and secondary sources (Elana et al., 2009); (2) organizing scattered facts into integrated and meaningful clusters (Simiand, 1985); and (3) evaluating and synthesizing all evidence to draw conclusions about past events (Walliman, 2017). Guided by these procedures, this chapter explains in detail what the primary and secondary sources of this study are, why the selected documents are reliable and trustworthy, and most importantly, how these archives were investigated and why the selected methods of investigation are sound and valid. Based on the general procedures of historical research and the particular purpose and research questions of this study, this chapter is arranged into the following sections:

- Sources of the study,
- Analysis of the organization and structure,
- Analysis of the mathematics content, and
- Analysis of the bidirectional influences.

Sources of the Study

Historical research recognizes two types of sources: primary and secondary. Primary sources consist of documents or artifacts created by witnesses or participants who experienced the events, such as newspaper articles, diaries, government documents, letters, and photographs (Harvard Library, 2021a). Secondary sources comprise works created by researchers who have
interpreted and analyzed primary sources, such as commentaries, criticisms, journal articles, bibliographies, and encyclopedias (Harvard Library, 2021b). This study investigated both types of sources. In addition to the traditional dichotomy of primary and secondary sources, this research also consulted Karp’s (2007) descriptions of sources on the history of mathematics examinations. Karp divided sources into three groups:

1. Official documents about examinations, including reports on their administration and results;
2. Methodological articles, speeches, manuals, and problem books, which analyze the items and solutions of the examinations; and
3. Literature reflecting public discussions or personal memoirs about the examinations.

The sources examined in this study included:


The materials in Source 1 constitute the core of this study. These documents were collected using Baidu, the largest Chinese search engine, which functions as Google’s counterpart in China. The reliance on an online search engine was primarily due to the absence of well-preserved, relevant historical documents. Many were believed to have been destroyed during the Cultural Revolution or lost over time. For example, the researcher could not find any administrative information related to the 1953 NCEE mathematics examination on official websites, such as the Chinese Ministry of Education’s site, or in published books and documents. Ultimately, relevant information was identified in a picture of an admission ticket for the examination, sourced via Baidu. To ensure the reliability and validity of these online sources, each piece of information gathered was cross-verified from multiple websites. Appendix A includes a comprehensive list
of the translated NCEE mathematics examinations from 1952–1965 and 1977–1984, along with selected administrative information, their solutions, and links to the original Chinese versions. The researcher’s translations have been verified by two native Chinese speakers who received bachelor’s degrees in applied mathematics from accredited institutions in the United States.


Source 2 was published by the National Education Examinations Authority (NEEA) (http://www.neea.edu.cn/), an institution supervised and appointed by the Chinese Ministry of Education. The NEEA is tasked exclusively with conducting educational examinations and exercises some administrative authority. The 2019 syllabus was chosen because the NEEA’s database only contained three versions of the syllabus, specifically those from 2017, 2018, and 2019. The researcher elected to use the 2019 syllabus primarily because it was the most recent of the available options. This source was used to generate a reference list of mathematical concepts for the analysis of the examinations’ topic coverage. Therefore, the impact of discrepancies between syllabi from different years on the analysis was considered to be tangential at best.

3. 2020 Shanghai High School Entrance Mathematics Examination Syllabus.

4. UC-Berkeley Undergraduate Course Requirements for Mathematics.

When the researcher first attempted to create the reference list, it became evident that certain topics appearing on the examinations were missing from Source 2. These missing topics were drawn from middle school and undergraduate mathematics courses. Consequently, the researcher consulted Source 3, published by the Shanghai Education Examinations Authority (http://www.91zhongkao.com/), and Source 4, published by the UC-Berkeley Mathematics Department (https://math.berkeley.edu/), to encompass a broader range of topics. The selection of these two additional sources was influenced by the researcher’s educational background: he
had attended schools both near Shanghai and at UC Berkeley and was therefore acquainted with the mathematics curricula of these two places. Moreover, Shanghai is an internationally renowned city, and UC Berkeley is an internationally renowned university with widely recognized courses and curricula. Therefore, referencing mathematics topics from these two locations should be generally acceptable. A comprehensive list of translated mathematics topics is presented in Table 1. This list comprises 11 general topics, each further divided into various detailed topics. The accuracy of the translations was verified by two native Chinese speakers who are fluent in English and have academic experience in mathematics.

**Table 1**

*Math Topics Tested on the NCEE Math Exams*

<table>
<thead>
<tr>
<th>General Topic</th>
<th>Detailed Topic</th>
<th>General Topic</th>
<th>Detailed Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers &amp; Expressions</td>
<td>Real</td>
<td>2D-Geometry</td>
<td>Point, Line, &amp; Angle</td>
</tr>
<tr>
<td></td>
<td>Complex</td>
<td></td>
<td>Triangle</td>
</tr>
<tr>
<td></td>
<td>Absolute Value</td>
<td></td>
<td>Pythagorean Theorem</td>
</tr>
<tr>
<td></td>
<td>Exponential &amp; Logarithmic</td>
<td></td>
<td>Congruence &amp; Similarity</td>
</tr>
<tr>
<td></td>
<td>Radical &amp; Rational</td>
<td></td>
<td>Quadrilateral</td>
</tr>
<tr>
<td></td>
<td>Trigonometric</td>
<td></td>
<td>Polygon</td>
</tr>
<tr>
<td></td>
<td>Binomial</td>
<td></td>
<td>Circle</td>
</tr>
<tr>
<td></td>
<td>Factorization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mathematical Logic</td>
<td>Set &amp; Subset</td>
<td>3D-Geometry</td>
<td>Line &amp; Plane</td>
</tr>
<tr>
<td></td>
<td>Union &amp; Intersection</td>
<td></td>
<td>Prism &amp; Pyramid</td>
</tr>
<tr>
<td></td>
<td>Conditional Statement</td>
<td></td>
<td>Cylinder &amp; Cone</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Sphere</td>
</tr>
<tr>
<td></td>
<td>Linear</td>
<td></td>
<td>Polyhedron</td>
</tr>
<tr>
<td></td>
<td>Quadratic</td>
<td></td>
<td>Dihedral Angle</td>
</tr>
<tr>
<td></td>
<td>Polynomial</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equations &amp; Functions</td>
<td>Exponential &amp; Logarithmic</td>
<td>Analytic Geometry</td>
<td>Point &amp; Line</td>
</tr>
<tr>
<td></td>
<td>Radical &amp; Rational</td>
<td></td>
<td>Distance</td>
</tr>
<tr>
<td></td>
<td>Trigonometric</td>
<td></td>
<td>Polygon</td>
</tr>
<tr>
<td></td>
<td>Piecewise</td>
<td></td>
<td>Conic Section</td>
</tr>
<tr>
<td></td>
<td>Parametric</td>
<td></td>
<td>Polar System</td>
</tr>
<tr>
<td></td>
<td>System</td>
<td></td>
<td>Ratio</td>
</tr>
<tr>
<td></td>
<td>Inequality</td>
<td></td>
<td>Identity</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Law of Sines &amp; Cosines</td>
</tr>
</tbody>
</table>

The *People’s Daily* is the official newspaper of the Central Committee of the Communist Party of China (CCCPC). Source 5 consists of archived issues of the *People’s Daily*, specifically covering the years 1952–1965 and 1977–1984. These archives were accessible through the website of the National Library of China (http://www.nlc.cn/).


Source 6 is a book that chronicles significant events in the history of China. It was published by the Party History and Literature Research Office of the CCCPC.


Source 7 is a compilation detailing significant educational milestones in China from 1949 to 2014. This resource was published by the China Education and Research Network on September 28, 2015 (https://www.edu.cn/).


Source 8 is a book published by Hubei People’s Press. It was the achievement of a research project launched by China’s Ministry of Education (Yang, 2007). The book has a total of 654 pages. The author, Yang Xuewei, who dedicated over 30 years to the Ministry of Education and
its examination center, specialized in the management, reform, and research of Chinese examinations (Liu, 2008). According to a book review by Liu (2008), this work has become an essential reference for many theses and academic papers, and it serves as an important resource for understanding the history of examinations in China and their historical reforms.


Source 9 is a book published by China Yan Shi Press, a publishing house sponsored by the State Council Research Office. It is a comprehensive work on the history of the NCEE, spanning four volumes and approximately 2.8 million Chinese words (“Zhongguo,” 2023). The chief editor, Jiang Chao, dedicated significant time to the development and research of the NCEE at the examination center of the Ministry of Education. Alongside him, a diverse group of experts in political science and education contributed to the book’s writing or editing. Notable contributors included Zhang Xiaojing, a political science professor at Renmin University of China, and Li Songlin, a doctor in education from Beijing Normal University (“Zhongguo,” 2023). The book also received wide acclaim from esteemed Chinese scholars, such as He Yaomin, a professor of economics at Renmin University of China, Zhang Baosheng, vice-president and law professor at China University of Political Science and Law, and Wang Puqu, the vice dean of social sciences at Macao Polytechnic University (“Zhongguo,” 2023).


Source 10 is a book published by Fujian People’s Press. The first author, Gao Junfeng, was a professor at Zhengzhou University, and the second author, Yao Runtian, was the dean of history and social sciences at Shangqiu Normal University. While this work may not be as well-known
as Source 8 and Source 9, it provides alternative perspectives on the NCEE’s history and a cross-reference to the findings presented in the other two books.


Source 11 comprises additional personal memoirs, commentaries, news articles, and magazine pieces, all of which were searched for using Baidu. As Karp (2007) noted, informal literature is no less important than formal sources and the real picture of examinations “can be reconstructed only by taking into account their reception by ‘ordinary’ people and not just professional mathematics educators” (p. 41). The informal literature pieces investigated in this research can be categorized as follows:

1. Articles discussing how students prepare for and take the NCEE (e.g., Doldentate1, 2020; Le, 2022; Li, 2021; Liang, 1998; Zhai, 2017; Zhu, 2019);

2. Works on how teachers and schools assist students in preparing for the NCEE (e.g., Bei et al., 2019; Cui, 2013; Hui, 2022; Li, 2007; M. Liu, 2018; Su, 2018; Tian, 2001; Wang, 2020; Wang, 2021; Xiang, 2022; Zhang, 2019);

3. Literature focusing on the impact of family members and environment on NCEE candidates (e.g., Ben, 1983; Chen, 2001; Du, 2015; Fan, 2020; Jiu, 2023; Li, 2003; Liu, 2013; Ma, 2017; Mei, 2022; Qian, 2019; Shen & You, 2014; Sun, 2014; Wu & Zhao, 2014; Ye, 2020; Yi, 2022; Zhuang, 2008a, 2008b); and

4. Those depicting the impact of the community and social environment on NCEE candidates (e.g., Lu, 2008; Wang, 2022; Xin, 2017; Yan, 2023; Yuan, 2020; Zheng, 2017).
While these articles may not possess the scientific rigor of formal literature, they provide invaluable insights not typically covered in academic sources. They offer alternative perspectives on the development and evolution of the NCEE and the mutual influences between the NCEE and Chinese society. Notably, despite including some very recent articles, all materials focus on the years 1952–1984, recalling and reflecting upon experiences of individuals who participated in the NCEE during 1952–1965 and 1977–1984.

Overall, when collecting the existing documentation regarding the history of NCEE mathematics examinations, the researcher kept in mind that historical records are not always reliable and may contain information gaps, errors, or biases (Fraenkel et al., 2011; Simonton, 2003). To resolve these potential problems, the researcher adopted the method of triangulation, a strategy used in qualitative research to increase the credibility and validity of historical analysis (Cohen et al., 2002). Standard techniques include employing different data collection methods, comparing various data sources, and consulting multiple perspectives or theories (Patton, 1999). As for the application of triangulation in this study, the researcher reviewed multiple websites to verify the data collected, examined diverse data sources to develop a comprehensive understanding of the materials, and consulted different scholars’ perspectives on the subject matter. In sum, the sources collected in this step should be able to provide a reliable, valid, and diversified basis for the subsequent analysis of the NCEE mathematics examinations.

**Analysis of the Organization and Structure**

The analysis of the organization and structure of the NCEE mathematics examinations focused on the following attributes: test length, number of questions, and number of questions of each type, including multiple-choice, fill-in-the-blank, and free-response questions.
While reviewing the literature on the evolution of high-stakes mathematics examinations, the researcher identified several studies analyzing the organization and structure of examinations (e.g., Dossey, 1996; Karp, 2003; Lawrence et al., 2003; Zheng & Chen, 2017). After comparing these studies, the researcher decided to use Dossey’s (1996) “Mathematics Examinations” and Zheng and Chen’s (2017) “Analysis and Prospect of the NCEE Mathematics Examinations” to guide this part of the analysis.

Dossey’s (1996) research was chosen because it analyzed mathematics examinations in seven countries, including many with examination organizations and structures similar to those of the NCEE. For this reason, Dossey’s (1996) analytical method should be applicable to this study. Zheng and Chen’s (2017) research was selected because their analysis also focused partly on the NCEE’s organization and structure. Consequently, their experience could provide valuable insights for the analysis in this research.

Guided by these studies, the researcher created the following procedures for the analysis:

1. Recording information into an Excel spreadsheet,
2. Conducting a descriptive statistical analysis of the data,
3. Creating line graphs and column charts to explore trends and changes, and
4. Summarizing the findings and identifying the reasons behind them.

Notably, while the use of tables and column charts was informed by Dossey’s (1996) and Zheng and Chen’s (2017) studies, the decision to incorporate line graphs was based on the researcher’s analytical experience that they more clearly show data changes and trends over time.

A more detailed explanation of the methodology used to analyze the organization and structure of the NCEE mathematics examinations from 1952–1965 and 1977–1984 is presented below, along with concrete examples.
Duration of the Examinations

Analyzing the duration of the examinations was straightforward. Dossey (1996) simply recorded the data in a table and compared them afterward. In this research, a spreadsheet was created using Excel, with each column representing a single year’s examination and a row representing the duration of the examination. To see more clearly the data changes and trends over different time periods, a line graph was created accordingly, with the horizontal axis representing the years and the vertical axis representing the duration of the examination.

Total Number of Questions

The approach used to analyze the total number of items in the examinations was identical to the one employed to analyze the duration of the examinations. The same spreadsheet was reused here. However, another row representing the total number of items in the examination was added. Likewise, a line graph was generated to inspect the data changes and trends.

To avoid ambiguity, examination items composed of a set of mini questions required special attention during the analysis. In this research, mini questions were counted as individual questions because they were graded separately. In the following example (Item 9 in 1956):

9. Prove: (1) If three angles of a triangle form an arithmetic sequence, then one of its angles must be $60^\circ$. (2) If, in addition, three sides of this triangle form a geometric sequence, then all its angles must be $60^\circ$.

Two mini questions were counted as individual questions.

Types of Questions

The analysis of question types, also referred to as item characteristics in some studies (e.g., Dossey, 1996; Gjoci, 2017), was slightly different. Three rows—representing the total number of multiple-choice, fill-in-the-blank, and free-response questions—were added to the
spreadsheet described above. When analyzing item characteristics, researchers such as Dossey (1996) were more interested in the weight of each item category within the examinations. Consequently, instead of a line graph, a clustered column chart was constructed to show the percentage distribution of each item type on the NCEE mathematics examinations across different years and their trends over time (see Figure 1 for an illustration).

Figure 1

Percentage Distribution of Item Types in 1977-1984 NCEE Math Exams

Note. The horizontal axis represents the years of the examinations, with the years abbreviated (e.g., “1977” abbreviated to “77”). The vertical axis indicates the percentage of multiple-choice, fill-in-the-blank, or free-response questions in the examination.

After creating the line graphs and column charts described above, the researcher analyzed the evolution of the examinations’ organization and structure by identifying patterns and changes in the data. For example, in Figure 1, there are clear changes in the percentage distribution of item types in 1983 and 1984. More detailed descriptions are presented in the following chapter.
Notably, the test duration and total question count were analyzed together because they are closely interrelated. For instance, if both increased simultaneously, the amount of time students could allocate to each question might remain the same, potentially leaving no direct impact on test-taking.

Finally, the analysis used narrative and descriptive approaches to interpret the changes and development trends identified in the tables and figures. The researcher consulted various sources, searching for descriptions related to the changes in the examinations’ organization and structure from significant events in Chinese history (Source 6 and Source 7), scholarly books (Source 8, Source 9, and Source 10), and public discussions and personal memoirs on various online platforms (Source 5 and Source 11). The focus was on the years in which changes occurred and patterns emerged. After evaluating and synthesizing the various pieces of evidence related to the changes and trends, the researcher drew reasoned conclusions about how the organization and structure of the NCEE mathematics examinations had evolved over time.

**Analysis of the Mathematics Content**


The rationale for selecting Dossey’s (1996) research has been stated in the previous section. The reason for choosing Bao’s (2006) study was that the composite difficulty model introduced in his research was employed to examine the item difficulty in this study. Wu’s
(2015) research was chosen because it successfully applied Bao’s (2006) model to the analysis of Chinese high school entrance mathematics examinations. Thus, Wu’s (2015) approach to coding test items and summarizing data could provide useful guidelines for the current analysis.

Guided by Dossey’s (1996) study, the researcher created the following procedures for the analysis of topic coverage:

1. Coding test questions according to the mathematical topics listed in Table 1 (p. 58).
2. Recording the coded information into Excel spreadsheets.
3. Organizing the data into integrated and meaningful clusters.
4. Conducting a descriptive statistical analysis of the data.
5. Identifying patterns and changes in the data.
6. Summarizing the findings and exploring the reasons behind them.

Guided by the studies from Bao (2006) and Wu (2015), the researcher created the following procedures for the analysis of item difficulty:

1. Coding test questions according to Bao’s model.
2. Recording the coded information into Excel spreadsheets.
3. Organizing the data into integrated and meaningful clusters.
4. Conducting a descriptive statistical analysis of the data.
5. Identifying patterns and changes in the data, and if necessary, creating line graphs to facilitate the analysis.
6. Summarizing the findings and exploring the reasons behind them.

A more detailed explanation of the methodology used to analyze the content of the NCEE mathematics examinations for the years 1952–1965 and 1977–1984 is provided below, illustrated with concrete examples.
**Topic Coverage**

Dossey (1996) analyzed topic coverage in two complementary ways: focusing on the breadth, reporting all general categories covered by examinations (e.g., functions, relations, and equations), and assessing the depth by identifying five detailed subtopics most emphasized within examinations (e.g., logarithmic and exponential equations). This study adopted the same methodology but applied it to different examinations, i.e., the National College Entrance Examination (NCEE) for the years 1952–1965 and 1977–1984.

Unlike previous analyses, which treated each examination as a single unit for analysis, this portion considered each examination item as an individual unit. To determine the topics covered by an item, a step-by-step inspection of the item’s solution was conducted, as the knowledge assessed in each item is not always explicitly stated in the question itself. Consider, for example, item 6 in the 1952 NCEE mathematics examination:

6. Two circles have a common radius of 4, and one circle passes through the center of the other. What is the length of their common chord?

Answer: Let the length of their common chord be L, then \( \left(\frac{L}{2}\right)^2 + 2^2 = 4^2 \rightarrow L = 4\sqrt{3} \).

**Figure 2**

*Graph for Item 6 in the 1952 NCEE Math Exam*
The question asks about circles and their chords. However, the solution requires students to understand: (1) the definition of a circle, including its radius, center, and chord; (2) the properties of chords, isosceles triangles, or rhombuses; (3) the Pythagorean Theorem; and (4) how to solve quadratic equations.

This approach may give rise to disputes because a problem could have multiple solutions. To minimize inconsistencies, the researcher relied on the suggested solutions provided in examination solution manuals, where each question typically has a single solution. This does not imply that the question can be approached in only one way. Rather, the suggested solution was the method commonly taught in schools and required by the curriculum standard.

**Coding Procedure**

A step-by-step analysis of an item from the 1952 NCEE mathematics examination is provided below to illustrate the coding procedure. In brief, the following steps guided the coding process:

1. Truncating the solution at each step,
2. Assigning to each truncated part a general mathematics topic and a detailed mathematics topic (indicated in bold italics),
3. Summing up the general and detailed topics for each item while removing any repeated information, and
4. Recording the data (i.e., a topic’s frequency in each year’s examination) into Excel spreadsheets.

Example from the 1952 NCEE Mathematics Examination:

1. Factorizing the expression: $x^4 - y^4$.

Answer: $x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) = (x + y)(x - y)(x^2 + y^2)$. 
Firstly, the solution was divided into two steps, as shown below:

\[ x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) \]
\[ = (x + y)(x - y)(x^2 + y^2) \]

Secondly, each step was categorized under a general mathematics topic and a detailed mathematics topic based on the list of topics tested in the NCEE mathematics examinations, as outlined in Table 1 (p. 58). In this question, both steps fall under the general topic of *Numbers & Expressions* and the detailed topic of *Factorization*:

\[ x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) \text{ [Numbers & Expressions; Factorization]} \]
\[ = (x + y)(x - y)(x^2 + y^2) \text{ [Numbers & Expressions; Factorization]} \]

Thirdly, after compiling all the general and detailed topics and removing any duplicates, it was concluded that the topic coverage for this question is:

**General Topics:** Numbers & Expressions.

**Detailed Topics:** Factorization.

Afterward, the researcher repeated the above coding procedures for each test item.

Finally, the researcher recorded the frequency of each topic for a given year in two separate tables: one for general topics and another for detailed topics. The columns of these tables represented different years of the examinations, while the rows represented general and detailed topics. For instance, in the 1952 NCEE mathematics examination, there were 3 questions categorized under the general topic of *Numbers & Expressions*, resulting in a frequency of 3 being entered in the column for the year 1952 and the row for *Numbers & Expressions*.

**Reliability Check**

The coding procedure for this research is subjective, as it involves the coder’s personal interpretation of mathematical concepts and procedures. Different coders may produce different
results even when using the same rubric. To validate the reliability of the coding procedure, the researcher invited four individuals with diverse backgrounds and expertise to conduct the coding:

- Coder A holds a doctorate in mathematics education.
- Coder B is a doctoral student in the field of social science.
- Coder C is an in-service high school mathematics teacher.
- Coder D is a high school student who will take the NCEE next year (i.e., 2024).

Given the extensive time demands of the coding procedure, these individuals coded a subset of the examinations, specifically the NCEE mathematics examinations for the years 1952–1956, which is approximately one-fourth of the total. Afterwards, the researcher compared his own coding results with those of the four coders and had one-on-one discussions with them about any discrepancies. The discussions concluded with both parties reaching an agreement on the coding results. Finally, the researcher coded the remaining examinations based on a refined understanding of the subject.

While they were coding, the coders raised several concerns about the procedure. Firstly, a step in the solution may require understanding multiple mathematical concepts. Take item 6 from the 1952 NCEE mathematics examination as an example:

6. Two circles have a common radius of 4, and one circle passes through the center of the other. What is the length of their common chord?

Answer: Let the length of their common chord be \( L \), then \( \left( \frac{L}{2} \right)^2 + 2^2 = 4^2 \rightarrow L = 4\sqrt{3} \).

In deriving the equation \( \left( \frac{L}{2} \right)^2 + 2^2 = 4^2 \), multiple mathematical concepts are required, such as the definition of a radius and the properties of chords, isosceles triangles, and rhombuses.

Secondly, the concepts required to derive a step in the solution may not be listed in Table 1. Consider item 10 from the 1952 NCEE mathematics examination:
10. How many times the surface area of a sphere is the area of its great circle?

Answer: \( S_{\text{sphere}} = 4\pi R^2, S_{\text{circle}} = \pi R^2 \). The solution is 4.

In deriving the solution 4, the knowledge of ratio and division is used. These concepts are typically introduced in elementary school mathematics classes. Given that the standards of these mathematical concepts are far below those expected for college entrance examinations, they were deemed trivial and were not documented.

Based on these concerns, the researcher revised the second coding procedure as follows: each step of the solution is to be assigned one or more general and detailed mathematics topics based on the knowledge it requires. If the knowledge required is not listed in Table 1 and is considered well below the standards for college entrance examinations, no topics should be assigned.

The inter-rater reliability—that is, the percentage of questions on which the two raters agreed in their coding—is 98.5% between Coder A and the researcher. The discrepancy occurred in item 6 from the 1952 NCEE mathematics examination:

6. Two circles have a common radius of 4, and one circle passes through the center of the other. What is the length of their common chord?

Answer: Let the length of their common chord be \( L \), then \( \left( \frac{L}{2} \right)^2 + 2^2 = 4^2 \rightarrow L = 4\sqrt{3} \).

In deriving the equation \( \left( \frac{L}{2} \right)^2 + 2^2 = 4^2 \), the researcher used the properties of chords and isosceles triangles, whereas Coder A employed the properties of rhombuses. In the end, the researcher decided to include all three topics in the coding, indicating that this question covers all these topics. Since this discrepancy involves only one question, its influence on the analysis was considered to be tangential.
The inter-rater reliability between Coder B and the researcher is 97.0%. In addition to item 6 from the 1952 NCEE mathematics examination, another discrepancy occurred in item 8 from the 1953 NCEE mathematics examination:

8. The length, width, and height of a rectangular prism are 12, 3, and 4, respectively. Find the length of its diagonal.

Answer: \( \sqrt{12^2 + 3^2 + 4^2} = \sqrt{169} = 13. \)

Coder B believed that solving this item required only the Pythagorean Theorem, while the researcher argued that an understanding of the definition of a rectangular prism was also necessary. Eventually, Coder B agreed with the researcher.

The inter-rater reliability between Coder C and the researcher is 98.5%. Like Coder A, Coder C also used the properties of rhombuses to derive the equation \( \left( \frac{1}{2} \right)^2 + 2^2 = 4^2. \)

The inter-rater reliability between Coder D and the researcher is 95.5%. A distinct discrepancy occurred in item 18 from the 1952 NCEE mathematics examination:

18. If the origin is on the circle, and the circle’s center is (3, 4). Find the circle’s equation.

Answer: \( R = \sqrt{3^2 + 4^2} = 5. \) The equation is \( (x - 3)^2 + (y - 4)^2 = 25. \)

Coder D initially coded item 18 under the general topic of two-dimensional geometry by mistake. During the discussion with the researcher, he realized that it should be coded under the general topic of analytical geometry instead.

Overall, the inter-rater reliabilities between the four coders and the researcher fall within an acceptable range. When coding the remaining examinations, the researcher paid more attention to whether alternative methods existed for deriving each step in the solutions.

After finishing the coding, the researcher investigated the evolution of the examinations’ topic coverage by organizing and manipulating the data in the two frequency tables:
1. The frequencies of each topic for 1952–1965 and 1977–1984 were summed to compare the topic coverage before and after the Cultural Revolution.

2. Topics were rearranged in descending order based on their total frequency to identify the most emphasized mathematical topics in both pre- and post-revolutionary NCEE mathematics examinations.

3. The percentage of each topic on the examinations was calculated to show its relative weight.

4. Changes and developmental trends in the data were identified, and if necessary, descriptive statistical analysis was conducted or graphs were created to facilitate the analysis.

Finally, the researcher scrutinized the collected sources to understand the historical reasons behind these changes. This involved searching for descriptions related to the changes in the examinations’ topic coverage. After evaluating and synthesizing all evidence, reasoned conclusions were drawn about the evolution of topic coverage in the NCEE mathematics examinations over time.

**Item Difficulty**

This research utilized Bao’s (2002) composite difficulty model as the foundational framework for analyzing item difficulty. While Classical Test Theory and Item Response Theory are more commonly used, these approaches proved unsuitable for this study due to the absence of data on student performance for the NCEE mathematics examinations during the periods under investigation. Although various alternative models have been proposed to evaluate item difficulty in mathematics examinations (e.g., Ginsberg et al., 2005; Heinze et al., 2004; Nohara & Goldstein, 2001), the researcher opted for Bao’s (2002) model. This decision was influenced
by the model’s comprehensive nature, which integrates elements from other models, including those mentioned in the parentheses above (see pages 50-53 for a more in-depth discussion).

Bao’s (2002) model comprises five factors, each with different levels:

- Discovery (knowing, understanding, and investigating),
- Context (none, personal life, public affairs, and scientific scenarios),
- Computation (none, numerical, simple symbolic, and complex symbolic),
- Reasoning (none, simple, and complex), and
- Topic coverage (single, two, and three or more).

**Discovery Factor**

In the discovery factor, knowing refers to the ability to recall mathematical facts and execute standard mathematical procedures. This level emphasizes the rote memorization of mathematical concepts. Understanding involves the ability to apply mathematical concepts, formulas, rules, and theories, as well as to use standard methods to obtain definite results. It also entails the capacity to justify and demonstrate mathematical facts. This level focuses on the application of mathematical knowledge. Investigating denotes the ability to formulate mathematical conjectures and develop mathematical models innovatively. This level centers on mathematical modeling and encourages creativity in problem-solving. Typical questions at this level are often open-ended, non-routine, and exploratory.

The discovery factor levels may be understood through an analogy. Imagine when someone asks you, “Do you know what a piano is?” and “Do you know how to play the piano?” Many of us know what a piano is but probably do not know how to play the instrument, illustrating the difference between the knowing level and the understanding level.
Questions at the *knowing* level stress the rote memorization of mathematical knowledge. Consider item 9 from the 1952 NCEE mathematics examination:

9. What is the value of $\pi$?

Answer: $3.14159265 \ldots$, $\frac{22}{7}$, or $\frac{355}{113}$.

This question only requires students to recall the value of $\pi$. Students do not need to demonstrate how this value is derived or how it can be utilized mathematically.

Questions falling under the *understanding* level emphasize the application of mathematical knowledge in problem-solving. Consider item 3 from the 1952 NCEE mathematics examination:

3. The three roots of the equation $x^3 + bx^2 + cx + d = 0$ are $1, -1$, and $\frac{1}{2}$. Find the value of $c$.

Answer: By Vieta’s formulas, we have $c = 1 \times (-1) + 1 \times \frac{1}{2} + (-1) \times \frac{1}{2} = -1$.

This question requires students not only to know Vieta’s formulas but also to apply them to a specific problem. This example shows that applying knowledge often presupposes knowing the knowledge as a foundation. Thus, questions at the *understanding* level are often considered more demanding than those at the *knowing* level.

To understand the *investigating* level, let us return to our piano analogy. Suppose you have played some pieces in front of your friends, demonstrating that you know how to play the piano. Then someone asks, “I know you can play pieces composed by Beethoven. Can you give an improvised performance or play some pieces you have composed yourself?” This shows that the *investigating* level requires some further creativity in the application process.

Questions at the *investigating* level have an exploratory nature and stress the ability to make mathematical conjectures and build up mathematical models in an innovative way. Consider the following items from the 1964 NCEE mathematics examination:
8. In the figure, \(ABCD\) is a square with a side length of 1. In the square, \(\odot O\) and \(\odot O'\) are externally tangent to each other; \(\odot O\) is tangent to the sides \(AB\) and \(AD\); \(\odot O'\) is tangent to the sides \(CB\) and \(CD\).

1) Find the sum of the radii of two circles.

2) For what values of two radii, the sum of the areas of two circles reaches the minimum and maximum values? Justify your results.

**Figure 3**

*Graph for Item 8 in the 1964 NCEE Math Exam*

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**Answer:**

1) In \(\triangle O'O'S\), \(OO' = \sqrt{2}OS \Rightarrow R + r = \sqrt{2}(1 - R - r) \Rightarrow R + r = \frac{\sqrt{2}}{1 + \sqrt{2}} = 2 - \sqrt{2}.

2) \(S = \pi \left[R^2 + (2 - \sqrt{2} - R)^2\right] = 2\pi \left[R^2 - (2 - \sqrt{2})R + 3 - 2\sqrt{2}\right] = 2\pi \left[(R - \frac{2 - \sqrt{2}}{2})^2 + \frac{3 - 2\sqrt{2}}{2}\right].\) When \(R = \frac{2 - \sqrt{2}}{2}\) and \(r = \frac{2 - \sqrt{2}}{2}, S\) is the minimum.

Since \(\frac{2 - \sqrt{2}}{2} \leq R \leq \frac{1}{2}\), when \(R = \frac{1}{2}, r = \frac{3}{2} - \sqrt{2}, S\) is the maximum.

9. If the square in Q8 is changed into a rectangle, what results will you get and why?

10. If the square in Q8 is changed into a cube with a side length of 1, and the circles are changed into spheres, what results will you get and why?
Items 8, 9, and 10 together form a composite problem. Among them, Item 8 serves as the base question, providing problem-solving ideas and hints for Items 9 and 10. Item 8 itself is an understanding-level question, with a clear question stem and illustrative figure. Students can solve it by applying knowledge of squares, triangles, circles, the Pythagorean theorem, and quadratic equations, as illustrated in its solution.

Unlike item 8, the question stems and illustrative figures for Items 9 and 10 are not fully provided. Students need to make mathematical conjectures based on the experience and conclusions they obtained when solving Item 8, create new mathematical models to meet new requirements, and then solve the problems. This problem-solving process has an exploratory nature and complies with other definitions of the investigating level, so Items 9 and 10 were classified as investigating-level questions. Since questions at the investigating level require additional activities in the application process, such as making mathematical conjectures and creating mathematical models, they are considered more demanding than those at the understanding level.

**Context Factor**

The four difficulty levels in the context factor can be interpreted more broadly as two categories: real-world and purely mathematical, a criterion also employed by Nohara and Goldstein (2001) to assess the item difficulty of mathematics examinations. Bao (2002) further classified the real-world context into personal life, public affairs, and scientific scenarios.

Nohara and Goldstein (2001) argued that real-world questions typically require more cognitive effort than those presented in a purely mathematical context. Bao (2002) seemed to concur, considering questions at the purely mathematical level—or the none computational level
in his study—as less demanding than those in the other three real-world-context levels. Their notion is exemplified by the following two items:

1. When the angle of elevation is 30 degrees, the length of the tower’s shadow is 5. What is the height of the tower? (Item 15 in 1952)

2. In triangle $ABC$, $\angle A = 30^\circ$, $\angle C = 90^\circ$, $AC = 5$. Find the length of $BC$.

Item 2 is a mathematical translation of Item 1, illustrating how this additional step can add some complexity to the problem-solving process.

However, some may contend that mathematical models derived from real-life scenarios, such as Item 2, are often simpler than questions originally posed in a purely mathematical setting, such as some geometric proof questions.

The researcher found merit in both views and acknowledged that the perceived difficulty of context may differ among individuals. Consequently, this research recognized question context—whether real-world or purely mathematical—as a factor in measuring item difficulty, without stating which context is more challenging, allowing readers to form their own conclusions.

The categorization of the real-world context into personal life, public affairs, and scientific scenarios by Bao (2002) is based on each context’s degree of relevance to students’ everyday lives. In other words, these levels can be interpreted as the frequency with which students might encounter these contexts.

The personal life level includes scenarios such as household chores and school activities, which students experience frequently. The following is an example from the 1981 examination:
2. How many ways are there to select a class president and a vice president from among four candidates, A, B, C, and D? List all possible outcomes. How many ways are there to select three members for the class committee? List all possible outcomes.

The public affairs level encompasses contexts like interest rates and sales, which students encounter at a medium frequency. The following is an example from the 1979 examination:

7. In the U.S., the price index increased from 100 in 1939 to 500 in 1979 over a span of 40 years. If the annual price growth rate remained the same, what was the percentage increase in price each year? \( e = 2.718 \ldots, \ln(1 + x) \approx x, \lg 2 = 0.3, \ln 10 = 2.3 \)

The scientific scenarios level involves contexts such as physical and chemical experiments, to which students are only occasionally exposed. The following is an example from the 1979 examination:

3. Containers A and B hold \( v_1 \) and \( v_2 \) kilograms of alcohol, respectively. In container A, the ratio of alcohol to water is \( m_1 \) : \( n_1 \), while in container B, the ratio is \( m_2 \) : \( n_2 \). What is the ratio of alcohol to water when the contents of A and B are mixed?

According to Bao (2002), scientific scenarios questions are deemed the most difficult, followed by public affairs questions, with personal life questions being the simplest. Recognizing that the perceived difficulty of these contexts may vary among individuals, this research refrained from specifying which context is more demanding, leaving the judgment to the readers.

**Computation Factor**

Nohara and Goldstein (2001) noted that computation is often a component in all areas of school mathematics, adding an extra layer of difficulty for students in problem-solving. The researcher had reservations about the claim that computation is a component in all areas of
school mathematics but agreed that computation is a significant aspect in test taking, contributing to its difficulty.

Measuring students’ computational ability is one of the primary focuses of the NCEE. Test candidates are not permitted to use calculators, a rule that is still being used today. This restriction makes computation a challenging point in taking the NCEE. In a time-limited testing environment, extensive calculations can increase the difficulty of the examination, as students often make mistakes when calculating at a fast pace. Moreover, even a small mistake can lead to errors in the final answer, affecting the examination results.

In Bao’s (2002) model, the computation factor levels are determined by the complexity of the operations involved. Questions that fall under the none computational level do not require any numerical or symbolic calculations. Instead, they focus on memorization, graphing, conceptual understanding, or logical reasoning. Questions at the numerical level involve calculations dealing directly with numerical values. The following is an example from the 1953 examination:

4. Calculate $\log_{\frac{300}{7}} + \log_{\frac{700}{3}} + \log 1$.

Answer: $\log_{\frac{300}{7}} + \log_{\frac{700}{3}} + \log 1 = \log \left(\frac{300 \cdot 700 \cdot 1}{7 \cdot 3 \cdot 1}\right) = \log 10000 = 4$.

Questions at the symbolic level involve calculations requiring the manipulation of mathematical symbols. The simple symbolic level involves computations that require one or two steps (Example I below), while the complex symbolic level involves computations that require three or more steps (Example II below).

Example I: Item 2 in 1953.

2. If two roots of the equation $3x^2 + kx + 12 = 0$ are equal, find the value of $k$.

Answer: $k^2 - 4 \cdot 3 \cdot 12 = 0$ (Setting up the equation based on the formula),
\[ k^2 = 144 \text{ (The 1st step in the calculation),} \]
\[ k = \pm 12 \text{ (The 2nd step in the calculation).} \]

Example II: Item 2 (2) in 1982.

2. (2) Find the derivative of \( y = \cos^2 \frac{x}{3} \).

Answer: (2) \( y' = 2 \left( \cos \frac{x}{3} \right) \left( \cos \frac{x}{3} \right)' \) (The 1st step in the calculation),
\[ = -2 \cos \frac{x}{3} \sin \frac{x}{3} \left( \frac{x}{3} \right)' \text{ (The 2nd step in the calculation),} \]
\[ = -\frac{2}{3} \cos \frac{x}{3} \sin \frac{x}{3} \text{ (The 3rd step in the calculation),} \]
\[ = -\frac{1}{3} \sin \frac{2x}{3} \text{ (The 4th step in the calculation).} \]

For this study, the *complex symbolic* level had been further subdivided into *medium symbolic* (involving three or four steps) and *complex symbolic* (involving five or more steps), because approximately 70% of the examination items in the NCEE mathematics examinations for the years 1952–1965 and 1977–1984 required more than three steps to solve. This additional distinction helped to clarify the varying degrees of item difficulty within this large group of problems.

According to Bao (2002), numerical computation problems are considered easier than symbolic ones, while questions that require no computation are the simplest. The researcher concurred with this ranking, as symbolic calculations, typically introduced after students have mastered numerical computations, are often viewed as more challenging in school mathematics. It was acknowledged that some may view non-computational questions, such as proof questions, as more difficult than computational ones. The researcher understood that the perceived difficulty of the question types—whether proof or computational—can be subjective. However,
in the context of assessing item difficulty with a focus on computation, such subjectivity is reduced.

Last but not least, Bao (2002) posited that computational problems that require more steps demand greater reasoning skills. Therefore, the complex symbolic level is considered more difficult than the simple symbolic level. It may be argued that a basic calculus question requiring 5 steps to solve is not necessarily less demanding than a high-school arithmetic problem requiring 10 steps. Sometimes, more steps may make a question more tedious rather than more difficult. Nonetheless, in general, questions in the NCEE that require more steps tend to be more challenging. Additionally, questions with tedious steps also contribute to the difficulty in the NCEE because they take longer to solve in a time-limited setting.

**Reasoning Factor**

Reasoning factors are commonly used by mathematics educators (e.g., Bao, 2006; Heinze et al., 2004; Nohara & Goldstein, 2001) to assess item difficulty of mathematics examinations. This indicates that most scholars concur with the notion that the difficulty of a question is associated with the amount of reasoning required to solve it.

In Bao’s (2002) study, the reasoning factor levels are determined by the number of reasoning steps required to solve each question: None reasoning for questions that require no reasoning steps, simple reasoning for one or two steps, and complex reasoning for three or more steps. In the current research, the complex reasoning level was further divided into medium reasoning for three or four steps and complex reasoning for five or more steps, given that many examination items require more than three steps to solve. The complex reasoning level was considered the most difficult, followed by the medium and simple reasoning levels, with the none reasoning level being the simplest. Examples from each level are presented below:
Example I: The Simple Reasoning Level (Item 7 in 1952).

7. If the area of $\Delta ABC$ is 60, $M$ is the midpoint of $AB$, $N$ is the midpoint of $AC$, what is the area of $\Delta AMN$?

Answer: $\frac{S_{\Delta AMN}}{S_{\Delta ABC}} = \left(\frac{MN}{BC}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$ (The 1st reasoning step),

$S_{\Delta AMN} = \frac{60}{4} = 15$ (Computation; no reasoning involved).

Example II: The Medium Reasoning Level (Item 4 in 1957).

4. In tetrahedron $ABCD$, $AC = BD$, $P$, $Q$, $R$, $S$ are the midpoints of $AB$, $BC$, $CD$, $DA$. Prove:

$PQRS$ is a rhombus.

**Figure 4**

*Graph for Item 4 in the 1957 NCEE Math Exam*

Answer: $PS \parallel BD \parallel QR$, $PS = \frac{1}{2}BD = QR$ (The 1st reasoning step),

Thus, $PQRS$ is a parallelogram (The 2nd reasoning step),

$PS = \frac{1}{2}BD = \frac{1}{2}AC = SR$ (The 3rd reasoning step),

Thus, $PQRS$ is a rhombus (The 4th reasoning step).

Example III: The Complex Reasoning Level (Item 7 in 1982).

7. Quadrilateral $ABCD$ resides in a 3D space. $AB = BC$, $CD = DA$, and $M$, $N$, $P$, $Q$ are midpoints of $AB$, $BC$, $CD$, $DA$. Prove: $MNPQ$ is a rectangle.

**Figure 5**

*Graph for Item 7 in the 1982 NCEE Math Exam*
Answer: $MN \parallel AC \parallel QP$, $MQ \parallel BD \parallel NP$ (The 1st reasoning step),

Thus, $MNPQ$ is a parallelogram (The 2nd reasoning step),

Let $K$ be the midpoint of $AC$, connect $BK$ and $DK$ (No reasoning involved),

$\rightarrow BK \perp AC$, $DK \perp AC$ (The 3rd reasoning step),

$\rightarrow AC \perp$ plane $BKD$, $AC \perp BD$ (The 4th reasoning step),

$\rightarrow MQ \perp QP$, $\angle MQP = 90^\circ$ (The 5th reasoning step),

$\rightarrow MNPQ$ is a rectangle (The 6th reasoning step).

It is important to note that in the coding of the reasoning steps, some minor steps had been consolidated. For instance, the first reasoning step in Example III could be further broken down into four smaller steps: $MN \parallel AC$, $QP \parallel AC$, $MQ \parallel BD$, and $NP \parallel BD$. Given that these steps involve repeated applications of the same concept (the triangle’s midsegment theorem), separating them does not necessarily reflect an increase in reasoning difficulty. Additionally, it might complicate the ranking of reasoning difficulty due to a wider range of reasoning steps. Consequently, in this research, steps that employ the same mathematical principle for reasoning in a repeated and continuous manner had been combined.

A major dispute concerning the reasoning factor levels is whether a mathematical problem can require no reasoning steps at all. While it is widely acknowledged that every mathematical procedure is based on some form of reasoning, this study considered three types of
questions in the NCEE mathematics examinations for the years 1952–1965 and 1977–1984 do not require reasoning steps.

The first category includes questions that can be answered through rote memorization of mathematical concepts, such as recalling the value of $\pi$ (Item 9 in 1952), naming all Platonic solids (Item 12 in 1952), or writing down the definition of a dihedral angle (Item 4 in 1955). Students respond to these questions by recalling the information stored in their memory, with no reasoning procedures involved.

The second category comprises pure calculation problems, such as the following question (Item 4 in 1952):

4. Solve the equation $\sqrt{x^2 + 7} - 4 = 0$.

Answer: $\rightarrow x^2 + 7 = 16$

$\rightarrow x^2 = 9$

$\rightarrow x = \pm 3$.

Verification confirms that both roots are correct.

In item 4, students derive the solution by adhering to routine mathematical procedures—methods frequently practiced in class and during exercises—thus bypassing the need for any additional reasoning skills.

The third category encompasses questions solvable through the direct application of mathematical formulas, as exemplified by the following question (Item 3 in 1953):

3. Find the value of $\begin{vmatrix} 3 & -1 & 1 \\ 2 & 4 & 6 \\ 7 & 0 & 5 \end{vmatrix}$.

Answer: $\begin{vmatrix} 3 & -1 & 1 \\ 2 & 4 & 6 \\ 7 & 0 & 5 \end{vmatrix}$

$= 3 \times 4 \times 5 + (-1) \times 6 \times 7 + 1 \times 2 \times 0 - 3 \times 6 \times 0 - (-1) \times 2 \times 5 - 1 \times 4 \times 7$
In this item, students obtain the solution by recalling the determinant formula ingrained in their memory, hence no reasoning skills are required.

In summary, questions categorized under the none reasoning level emphasize memorization of knowledge or formulas and the execution of routine procedures.

**Topic Coverage Factor**

In the topic coverage factor, different levels reflect the number of mathematical topics needed to solve each item. Generally, questions involving more topics demand greater thought and effort, making them more challenging.

Based on the range of topics needed for questions in the NCEE mathematics examinations, the researcher adapted Bao’s (2002) three-level structure (consisting of single, two, and three or more topics) to a four-level system: single topic, two topics, three topics, and four or more topics. Examples from each level are presented below:


12. Simplify $\sqrt{12} + 4\sqrt{90000} + \frac{6}{\sqrt{27}}$.

Answer: $= 2\sqrt{3} + 10\sqrt{3} + \frac{2}{3}\sqrt{3} = \frac{166}{15}\sqrt{3}$.

In this item, students need to utilize the rules of radicals to solve the question.

Example II: The Two-Topic Level (Item 11 in 1952).

11. If the base radius of a right cone is 3, and its slant height is 5, what is its volume?

Answer: $h = \sqrt{5^2 - 3^2} = 4, V = \frac{1}{3}\pi r^2 h = 12\pi$.

In this item, students need to utilize the formula for the volume of a cone and the Pythagorean Theorem to find the solution.
Example III: The *Three-Topic* Level (Item 9 (1) in 1956).

9. Prove: (1) if three angles of a triangle form an arithmetic sequence, then one of its angles must be 60°.

Answer: (1) Let three angles be $A$, $B$, and $C$, then $B - A = C - B \rightarrow 2B = A + C$.

$A + B + C = 180° \rightarrow 3B = 180° \rightarrow B = 60°$.

In this item, students must engage with three mathematical topics: the sum of the interior angles of a triangle, the arithmetic sequence, and the linear system.

Example IV: The *Four-or-More-Topic* Level (Item 2 in 1962).

2. Find the real part in the expanded form of the expression $(1 - 2i)^5$.

Answer: $(1 - 2i)^5 = 1 - \binom{5}{1}2i + \binom{5}{2}(2i)^2 - \binom{5}{3}(2i)^3 + \binom{5}{4}(2i)^4 - (2i)^5$.

The real part is $1 + \binom{5}{2}(2i)^2 + \binom{5}{4}(2i)^4 = 1 + 10 \times 4 \times (-1) + 5 \times 16 \times 1$.

$= 1 - 40 + 80 = 41$.

In this item, students first need to use the binomial theorem to find the binomial expansion of the expression $(1 - 2i)^5$, and then calculate the real part using the knowledge of combinations, exponents, and the complex numbers. A total of four mathematical topics are employed.

Summary

In summary, this research applied the following five criteria, adapted from Bao’s (2002) model, to analyze the item difficulty of the NCEE mathematics examinations:

1. *Discovery* factor, encompassing *knowing*, *understanding*, and *investigating* levels.

2. *Context* factor, with levels *none*, *personal life*, *public affairs*, and *scientific scenarios*.

3. *Computation* factor, including *none*, *numerical*, *simple symbolic*, *medium symbolic*, and *complex symbolic* levels.

4. *Reasoning* factor, categorized as *none*, *simple*, *medium*, and *complex*.
5. *Topic coverage* factor, defined as *single, two, three,* and *four or more topics.*

**Coding Procedure**

A step-by-step analysis of an item from the 1952 NCEE mathematics examination is presented below to demonstrate the coding procedure. In brief, the following steps guided the coding process:

1. Truncating the solution at each step,
2. Analyzing the item based on the five criteria described above, and
3. Recording the data (i.e., the frequency) into Excel spreadsheets.

Example from the 1952 NCEE Mathematics Examination:

1. Factorizing the expression: \(x^4 - y^4\).

   Answer: \(x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) = (x + y)(x - y)(x^2 + y^2)\).

First, the solution was divided into two steps as indicated below:

\[
x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) = (x + y)(x - y)(x^2 + y^2)
\]

Second, regarding the *discovery* factor, this question requires an application of the Difference of Two Squares formula to the difference of two fourth powers. Therefore, it was categorized under the *understanding* level, which emphasizes the application of mathematical formulas.

Third, regarding the *context* factor, this question contains no contexts; thus, it was categorized under the *none* level, denoting a purely mathematical problem.

Fourth, regarding the *computation* factor, this question requires a two-step symbolic calculation and was therefore assigned to the *simple symbolic* level.

Fifth, regarding the *reasoning* factor, this question requires two steps of reasoning. Students first need to deduce that \(x^4 - y^4\) could be written as \((x^2)^2 - (y^2)^2\), which allows them to use the
Difference of Two Squares formula. They then need to apply the same formula for $x^2 - y^2$.

Therefore, this question was categorized under the *simple reasoning* level.

Sixth, regarding the *topic coverage* factor, based on the topic coverage analysis from the previous section, this question was categorized under the *single-topic* level.

Finally, the researcher concluded that the difficulty levels for this item are as follows:

**Discovery**: Understanding Level  
**Context**: None (Purely Mathematical) Level  
**Computation**: Simple Symbolic Level  
**Reasoning**: Simple Reasoning Level  
**Topic Coverage**: Single Topic Level

Subsequently, the researcher repeated the above coding procedures for each test item.

Finally, the researcher recorded the frequency of each factor level for a given year in a frequency table, with the columns representing different years of the NCEE mathematics examinations and the rows representing the five factors and their specific levels. For instance, in the 1952 NCEE mathematics examination, there were 23 questions classified as purely mathematical, resulting in a frequency of 23 being entered in the column for the year 1952 and the row for purely mathematical.

**Reliability Check**

The same four coders helped to validate the reliability of coding item difficulty. Firstly, these individuals coded a subset of the examinations, specifically the NCEE mathematics examinations for the years 1952–1956, which represents approximately one-fourth of the total. Afterwards, the researcher compared his own coding results with those of the four coders and had one-on-one discussions with them about any discrepancies. The discussions concluded with
both parties reaching an agreement on the coding results. Finally, the researcher coded the
remaining examinations based on a refined understanding of the subject.

Concerning the discovery factor, the inter-rater reliabilities for Coders A, B, and C, when
compared with the researcher’s coding, are 100% each. For Coder D, the inter-rater reliability is
98.5%. A discrepancy occurred with item 17 from the 1952 NCEE mathematics examination:

17. A straight line passes through the point (2, 3) with a slope of $-1$. Find its equation.
Answer: $y - 3 = -(x - 2)$.

Coder D noted that, as he read the question, the solution seemed obvious and did not appear to
require any thought. He did not realize that the point-slope formula for linear equations was
applied. So, he categorized it as a knowing-level question. In the end, Coder D agreed with the
other coders that this question belongs to the understanding level.

Regarding the context factor, the inter-rater reliabilities between the four coders and the
researcher are all 100%.

In terms of the computation factor, the inter-rater reliabilities of Coders A, B, C, and D
compared with the researcher’s coding are 98.5%, 91.0%, 98.5%, and 98.5%, respectively.
Discrepancies resulted from different interpretations of what constitutes symbolic computation.
Consider Item 11 in the 1952 NCEE mathematics examination:

11. If a right cone’s base radius is 3, and its slant height is 5, what is its volume?
Answer: $h = 4$, $V = \frac{1}{3} \pi R^2 h = 12\pi$.

The researcher initially treated $\pi$ as a numerical value, in contrast to the other coders who
viewed it as a mathematical symbol similar to the variable $x$. Eventually, the researcher
concurred with the other coders.

Another illustrative example would be Item 5 in the 1953 examination:
5. Find the value of $\tan 870^\circ$.

Answer: $\tan 870^\circ = \tan (720^\circ + 150^\circ) = \tan 150^\circ = - \tan 30^\circ = - \frac{\sqrt{3}}{3}$

Coder B interpreted the tangent of a definite value as symbolic computation because the notation “$\tan$” seemed symbolic to him. The researcher clarified that $\tan 870^\circ$, like $\sqrt{870}$, is a complex operation, not a symbol. Coder B ultimately agreed with this interpretation.

As for the *reasoning* factor, the inter-rater reliabilities for Coders A, B, C, and D, when compared with the researcher’s coding, are 94.0%, 91.0%, 92.5%, and 95.5%, respectively. The primary source of discrepancies comes from the categorization of questions as *none-reasoning*. As previously introduced, this research identified three types of questions as *none-reasoning*:

1. Questions that can be answered through rote memorization of mathematical concepts.
   No coding discrepancies were observed regarding this category.

2. Pure calculation problems.
   Coders A and C contended that not all calculation problems are straightforward and proficiently mastered by students. They pointed to two questions as examples:

4. Solve the equation $\sqrt{x^2 + 7} - 4 = 0$. (Item 4 in 1952)
   Answer: $\rightarrow x^2 + 7 = 16 \rightarrow x^2 = 9 \rightarrow x = \pm 3$.

9. Solve the equation $\frac{1 + \tan x}{1 - \tan x} = 1 + \sin 2x$. (Item 9 in 1954)
   Answer: $\frac{\cos x + \sin x}{\cos x - \sin x} = (\cos x + \sin x)^2 \rightarrow \cos x + \sin x = (\cos x + \sin x)^2 (\cos x - \sin x) \rightarrow 
   (\cos x + \sin x)(1 - \cos^2 x + \sin^2 x) = 0 \rightarrow 2(\cos x + \sin x) \sin^2 x = 0. \cos x + \sin x = 0$ or $
   \sin^2 x = 0. \cos x + \sin x = 0 \rightarrow \tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}, k \in \mathbb{Z}; \sin^2 x = 0 \rightarrow x = k\pi, k \in \mathbb{Z}$. 


According to their view, while high-achieving students might consider both questions to be routine and easy to solve, low-achieving students might encounter difficulty in solving the second. Finding this argument convincing, the researcher gave more consideration in the subsequent coding of calculation problems.

3. Questions solvable through the direct application of mathematical formulas.

Coder A argued that some formulas are complicated and, thus, require reasoning skills to apply. Coder B mentioned having forgotten many formulas learned in high school. Coder D noted that some formulas have been removed from the current curriculum. These comments suggested that the mastery of mathematical formulas is subjective and varies among individuals. To minimize inconsistency in coding, the researcher carefully considered each formula’s complexity and its commonality in high school mathematics classes during the subsequent coding process.

Concerning the topic coverage factor, the inter-rater reliabilities of Coders A, B, C, and D compared with the researcher’s coding are 98.5%, 97%, 98.5%, and 97%, respectively. The first three percentages are consistent with those found in the reliability check for the topic coverage analysis in this study. However, the value for Coder D differs because the mistake of categorizing analytical geometry as two-dimensional geometry did not change the number of topics and, therefore, had no impact on the topic coverage factor analysis.

Overall, the inter-rater reliabilities between the four coders and the researcher fall within an acceptable range. When coding the remaining examinations, the researcher paid more attention to the coding issues that had been addressed in the discussions with the four coders.

After completing the coding, the researcher analyzed the evolution of the item difficulty by conducting the following procedures:

1. Conducting descriptive statistical analysis.
2. Constructing line graphs to illustrate data changes over the years.

3. Identifying changes and developmental trends in the data.

Finally, the researcher scrutinized the collected sources to understand the historical reasons behind these changes. This involved searching for descriptions related to the changes in item difficulty concerning the five criteria. After evaluating and synthesizing all the evidence, reasoned conclusions were drawn about the evolution of item difficulty in the NCEE mathematics examinations over time.

**Analysis of the Bidirectional Influences**

The analysis of the bidirectional influences between the NCEE and Chinese society followed a historical approach. The researcher examined historical data extracted from both primary and secondary sources and developed a narrative exposition of the findings. The guiding literature for this part of the analysis was Karp’s (2007) “Exams in Algebra in Russia: Toward a History of High Stakes Testing.” Karp’s research was chosen because it employed a historical approach and there were many similarities between the teaching policies and educational philosophies of China and Russia during the historical periods under investigation.

Karp (2007) explored the following aspects of algebra examinations in Russia, with the aim of uncovering issues related to the composition and administration of past examinations:

1. General facts about examinations,
2. Goals of examinations and their role in ensuring the quality of education,
3. Procedures for developing and administering examinations,
4. Instances of rule-breaking,
5. Subject matter and structure of examination problems, and
6. Requirements for writing solutions on examinations.

Although the subject matter and research direction of Karp’s (2007) study differ from those of the current research, the organization, structure, and writing styles employed in his study provided valuable insights for this research.

This study aimed to investigate the following aspects concerning the mutual influences between the NCEE and Chinese society from 1952–1965 and 1977–1984:

1. Influence on student attitudes and behaviors,
2. Influence on teacher attitudes and behaviors,
3. Influence on parental attitudes and behaviors,
4. Influence on social relationships, and
5. Influence on the examinations.

The analysis followed grounded theory, a systematic methodology commonly employed in qualitative research, and was guided by the following procedures:

1. Scrutinizing the collected documentation (i.e., Sources 5 through 11) and documenting relevant information regarding various forms of influence,
2. Organizing and grouping related historical facts into integrated and meaningful clusters, and
3. Synthesizing and summarizing the clustered evidence to draw reasoned conclusions about past events.

In the first procedure, the researcher examined the collected sources one by one, taking notes on the bidirectional influences between the NCEE and Chinese society. Specifically, he looked for keywords that describe societal changes or the evolution of the examinations, including, but not limited to, “change,” “adjustment,” “social views,” “social behaviors,” “new
policies,” and “mathematics examinations.” Special attention was paid to changes in the attitudes and behaviors of students, teachers, parents, and other stakeholders.

In the second procedure, the researcher reviewed the notes and organized them into meaningful clusters based on specific themes, such as “Student Behaviors: Predicting Test Questions,” “Teacher Behaviors: Preparing Test Study Materials,” and “Parental Attitudes and Behaviors: Opposing Children Taking the NCEE.”

Finally, in the third procedure, the researcher revisited the historical evidence within each cluster, reorganized and synthesized it into convincing narratives, and then drew reasoned conclusions about the mutual influences between the NCEE and Chinese society.

In conclusion, this chapter outlined the research methodology. The researcher collected sources following the general historical approach and using Karp’s (2007) categorization of sources on the history of mathematics examinations. The analyses of the organization, structure, topic coverage, and item difficulty of the NCEE mathematics examinations, as well as the mutual influences between the NCEE and Chinese society were structured based on invaluable insights provided by multiple seminal works. These included Dossey’s (1996) “Mathematics Examinations,” Zheng and Chen’s (2017) “Analysis and Prospect of the NCEE Mathematics Examinations,” Bao’s (2006) “A Comparative Study of Mathematics Tests in China and UK,” Wu’s (2015) “The Examination System in China: The Case of Zhongkao Mathematics,” and Karp’s (2007) “Exams in Algebra in Russia: Toward a History of High Stakes Testing.” Given that the methodology of this research was guided by a rich and diversified range of scientific studies published in well-recognized books, journals, and electronic databases, it should provide a solid foundation for analyzing the NCEE mathematics examinations.
Chapter V

RESULTS FOR ORGANIZATION, STRUCTURE, AND CONTENT

The results of this research are presented in two chapters. Chapter V shows the results for the organization, structure, and content of the NCEE mathematics examinations for the years 1952–1965 and 1977–1984. It aims to answer the first two research questions of this study:

1. How did the organization and structure of the NCEE mathematics examinations evolve during the years 1952–1965 and 1977–1984?
2. How did the content of the NCEE mathematics examinations change during the periods 1952–1965 and 1977–1984?

Organization and Structure of the Examinations

Data

The general structure and organization exhibited similarities across all the NCEE mathematics examinations conducted both before and after the Great Proletarian Cultural Revolution. Table 2 (shown on the next page) presents data pertaining to the organization and structure of these examinations for the years 1952–1965 and 1977–1984.

In this table, columns represent the years the examinations took place, and rows indicate various attributes related to their organization and structure. For illustration, entries in the column for the year 1952 indicate that the 1952 NCEE mathematics examination had a duration of 100 minutes and consisted of 24 questions. Specifically, the numbers of multiple-choice, fill-in-the-blank, and free-response questions for that year were 0, 0, and 24, respectively.
Table 2

*Organization and Structure of the NCEE Math Exams Before and After the Cultural Revolution*

<table>
<thead>
<tr>
<th>Year of the examination (19XX)</th>
<th>52</th>
<th>53</th>
<th>54</th>
<th>55</th>
<th>56</th>
<th>57</th>
<th>58</th>
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<td>0</td>
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<tr>
<td>Number of fill-in-the-blank questions</td>
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<td>0</td>
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<table>
<thead>
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<th>84</th>
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<tbody>
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<td>120</td>
<td>120</td>
<td>120</td>
<td>120</td>
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<tr>
<td>Total number of questions</td>
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<td>12</td>
<td>13</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>22</td>
</tr>
<tr>
<td>Number of multiple-choice questions</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Number of fill-in-the-blank questions</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Number of free-response questions</td>
<td>16</td>
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<td>12</td>
<td>13</td>
<td>16</td>
<td>17</td>
<td>13</td>
<td>11</td>
</tr>
</tbody>
</table>

*Note.* The top table presents data for pre-revolutionary examinations, while the bottom table covers post-revolutionary examinations.

The first rows indicate the years of the examinations, which have been abbreviated, e.g., “1952” as “52.”
Descriptive Statistical Results

A descriptive statistical analysis was conducted to analyze the data in Table 2. Table 3 displays the descriptive statistics for the organization and structure of the NCEE mathematics examinations for the years 1952–1965 and 1977–1984.

Table 3

Descriptive Statistics for the Org. and Struct. of the NCEE Math Exams

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
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<td>Duration of the examination (min)</td>
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<td>119.09</td>
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<td>Total number of questions</td>
<td>22</td>
<td>8</td>
<td>24</td>
<td>13.64</td>
<td>4.03</td>
</tr>
<tr>
<td>Number of multiple-choice questions</td>
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<td>0.45</td>
<td>1.47</td>
</tr>
<tr>
<td>Number of fill-in-the-blank questions</td>
<td>22</td>
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<td>6</td>
<td>0.27</td>
<td>1.28</td>
</tr>
<tr>
<td>Number of free-response questions</td>
<td>22</td>
<td>8</td>
<td>24</td>
<td>12.91</td>
<td>3.44</td>
</tr>
</tbody>
</table>

As Table 2 and Table 3 reveal:

1. Except for the 1952 examination, which lasted 100 minutes, all other examinations lasted 120 minutes.

2. The majority of examinations had a total number of questions concentrated around the value of 14 (M = 13.64, SD = 4.03), with the minimum value of 8 occurring in 1955 and the maximum value of 24 occurring in 1952.

3. Before the Cultural Revolution, all test items were free-response questions; after the Cultural Revolution, multiple-choice and fill-in-the-blank questions began to appear, first in 1983 and then in 1984, respectively.

Graphical Analysis

To examine the evolution of the organization and structure of the NCEE mathematics examinations, Figure 6 and Figure 7 present the data from Table 2 in a line chart and a clustered column chart. This helps to visualize the changes and trends over different time periods.
Figure 6

*Duration & Total Number of Items of the 1952-1984 NCEE Math Exams*

*Note.* The horizontal axis represents the years of the examinations, with the years abbreviated (e.g., “1952” abbreviated to “52”). The notation “CR” within the years signifies the Cultural Revolution period. The vertical axis indicates either the duration of the examination (in hours) or the total number of questions in the examination.

Figure 6 compares the test length and the total number of questions of the NCEE mathematics examinations across different years. As the figure shows, while the duration of the examination remained almost constant, the total number of questions constantly changed and exhibited three distinct characteristics. Firstly, there was a notable and sustained decline in the total number of questions between 1952 and 1955.

One possible explanation could be that when the nation first administered a uniform examination in 1952, test-makers lacked an accurate understanding of the overall mathematical competency of test-takers across the country. It took several years for them to become familiar with the regional differences and adjust the test length and number of questions accordingly.
This argument is supported by Gao and Yao’s (2009) study, which identified a clear difference in student quality between areas with abundant educational resources (e.g., east China) and areas with fewer educational resources (e.g., northwest China) at that time.

Another possible explanation could be the intentional lowering of examination standards to improve college enrollment. This was inferred from a 1954 government report, which stated, “This year’s enrollment requirement must be fulfilled without any reduction…the student quality cannot be lowered” (Yang, 2003, pp. 67-69). This directive implies a stringent enrollment requirement at the time, and the caution against lowering student quality hints at a precious decline. Therefore, the researcher speculated that to satisfy enrollment quotas, test-makers might have reduced the number of questions, inadvertently leading to a decline in student quality.

Comparing these two explanations, the second appears to be more plausible because, in socialist countries, the centralized educational system allows for a quick introduction of reforms and policy changes. As a result, it is unlikely that test-makers would need three years to become familiar with regional differences and adjust accordingly. Furthermore, as suggested by the government report, pre-determined goals were expected to be met strictly at that time. Thus, lowering the examination standards to ensure adequate college enrollment would be a logical move. Nonetheless, the decrease in the total number of questions may result from a combination of these explanations, or there may be other unidentified factors requiring further exploration.

Secondly, between 1955 and 1979, the total number of questions slightly fluctuated around the value of 12, indicating a relatively stable structure of the examinations.

Thirdly, the total number of questions began to increase steadily after 1979. The initial increase from 1979 to 1982 was considered stable, as the total number of questions is still within one standard deviation from the mean (M = 13.64, SD = 4.03). However, the counts in 1983 and
1984 began to exceed this range. In these two years, multiple-choice and fill-in-the-blank questions were introduced to the examinations for the first time, respectively. Therefore, the researcher attributed the increase in the total number of questions to the addition of new question types in the examinations. Zheng and Chen’s (2017) study corroborates this view, suggesting that the total number of questions continued to grow after 1984, accompanied by an increase in the number of multiple-choice and fill-in-the-blank questions, eventually stabilizing at around 25 questions when the proportion of question types reached equilibrium.

**Figure 7**

*Percentage Distribution of Item Types in 1952-1984 NCEE Math Exams*

![Graph showing percentage distribution of item types](image)

*Note.* The horizontal axis represents the years of the examinations, with the years abbreviated (e.g., “1952” abbreviated to “52”). The notation “CR” within the years signifies the Cultural Revolution period. The vertical axis indicates the percentage of each question type.

Figure 7 displays the percentage distribution of question types in the NCEE mathematics examinations across different years. The data clearly indicates that free-response questions have consistently been the primary question type in these examinations. Prior to 1983, free-response
questions were the sole type of question included in the test. In 1983 and 1984, free-response questions still accounted for at least 50 percent of the total number of questions.

The integration of multiple-choice and fill-in-the-blank questions into the test may have resulted from communication and collaboration between Chinese educators and experts from the Educational Testing Service (ETS), the world’s largest private nonprofit educational testing and assessment organization. The standardization process of the NCEE, as introduced in Gao and Yao’s (2009) study, can be summarized into the following timeline:

In 1981, ETS sent representatives to China.

In 1982, representatives from the Chinese Ministry of Education visited ETS, signing a contract regarding the implementation of the TOEFL exam in China. Gao and Yao noted that this event marked the introduction of standardized testing in China.

Between 1981 and 1985, Chinese scholars engaged in multiple discussions about standardizing the NCEE.

In 1985, the NCEE’s standardization reform was piloted in Guangdong, focusing, among other things, on the structure of the examination and, particularly, the distribution of question types.

Given that the inclusion of multiple-choice and fill-in-the-blank questions in the NCEE mathematics examinations coincided with China’s engagement with ETS and preceded the more official standardization reform of the NCEE, the researcher posited that the adoption of new question types was influenced by this international collaboration on education.

This change also suggests that during this period, the relationship between China and the United States was more relaxed, and China demonstrated more openness to adopt new ideas and methods from Western countries.
Summary

In conclusion, the following list summarizes the key findings regarding the organization and structure of the NCEE mathematics examinations from 1952 to 1984:

1. The duration of the examinations remained constant from 1953 to 1984.

2. The total number of questions exhibited a notable decline from 1952 to 1955. This decline is possibly attributable to test-makers overlooking regional differences in students’ mathematical competencies when designing a uniform examination for the first time or to their intentional lowering of the examination standards to ensure adequate college enrollment.

3. There was a notable increase in the total number of questions from 1979 to 1984. The increase in 1983 and 1984 can be attributed to the inclusion of multiple-choice and fill-in-the-blank questions to the examinations.

4. Free-response questions were the dominant question type in the NCEE mathematics examinations from 1952 to 1982.

5. The introduction of multiple-choice and fill-in-the-blank questions after 1982 may have resulted from international collaboration with ETS.

Mathematics Content of the Examinations

Topic Coverage

General Mathematics Topics

Data. Table 4, shown on the next page, presents the frequencies and percentage distributions of general topics covered in the NCEE mathematics examinations for the years 1952–1965 and 1977–1984.
<table>
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<tr>
<th>Topic/Year</th>
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Table 4
General Topics’ Frequency and Percentage on the NCEE Math Exams
Note. The first row indicates the years of the examinations, which have been abbreviated, e.g., “1952” as “52.” The notation “Σ” within the years signifies summation. The entries under the first “Σ” represent the sum of the entries from 1952 to 1965. The entries under the second “Σ” represent the sum of the entries from 1977 to 1984. The entries in a row under the name of a general mathematics topic indicate the number of questions in a given year’s examination that falls under that topic. The entries in a row under the name of “Pct.” represent the percentage of questions in a given year’s examination that falls under the general mathematics topic above that row. The abbreviations in the table are defined as follows: “No. & Expr.” means “Numbers and Expressions;” “Eqn. & Func.” means “Equations and Functions;” “Analyt. Geom.” means “Analytical Geometry;” “Seqnc. & Ser.” means “Sequences and Series.”
Descriptive Analysis. As indicated by Table 4, calculus was only included in the examinations administered after the Cultural Revolution, whereas all other topics were featured in examinations both before and after this period. Additionally, analytical geometry and sequences and series, which were infrequently featured in pre-revolutionary examinations, were included in every examination administered after the revolution.

When the topics were rearranged in descending order based on the sum of their frequencies within each period, the five most emphasized topics in the pre-revolutionary examinations were equations and functions, two-dimensional geometry, trigonometry, numbers and expressions, and three-dimensional geometry. In the post-revolutionary examinations, the five most emphasized topics were equations and functions, numbers and expressions, trigonometry, two-dimensional geometry, and analytical geometry. These topics are highlighted in bold and italics for both periods in the table.

Another noteworthy aspect revealed by the table is that the post-revolutionary examinations—particularly those conducted after 1980—covered a broader array of general mathematics topics compared to their pre-revolutionary counterparts. This trend could be explained by one of the findings in the previous section. It was found that the total number of questions began to increase steadily after 1979 and surpassed the pre-revolutionary levels after 1981, which provided extra space for a wider range of topics. The two outliers—the 1952 and 1953 NCEE mathematics examinations—also support this argument, as they contained a greater number of questions and simultaneously covered a broader spectrum of mathematics topics. Gao and Yao’s (2009) research further corroborates this argument, noting that the introduction of multiple-choice and fill-in-the-blank questions in 1983 and 1984 enabled the inclusion of a wider range of topics in the examinations.
**Descriptive Statistical Analysis.** A descriptive statistical analysis was conducted to analyze the percentage distributions of the most emphasized six general topics covered in the NCEE mathematics examinations for both the pre- and post-revolutionary periods. The choice to analyze percentage distributions rather than frequencies of general topics was made because percentage distributions better convey the weight that each topic carries in the examinations. Table 5 displays the descriptive statistics for the percentage distributions of the most emphasized six general topics included in both the pre- and post-revolutionary NCEE mathematics examinations.

**Table 5**

*Descriptive Statistics for the Pct. Dist. of General Topics on the NCEE Math Exams*

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<th>Topic</th>
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The statistical results reveal that, on average, the percentages of equations and functions on the pre- and post-revolutionary NCEE mathematics examinations were 43.86% and 43.13%, respectively. Notably, both percentages were the highest within each period. The percentage of two-dimensional geometry had a mean value (M = 39.79%) comparable to that of equations and functions before the Cultural Revolution but decreased to 22.38% after the revolution. Similarly,
the percentage of three-dimensional geometry decreased from 17.21% on the pre-revolutionary examinations to 8.50% on the post-revolutionary examinations. In contrast, the percentage of analytical geometry increased from 2.71% to 13.13%. Finally, the percentages of numbers and expressions and trigonometry, which had moderate mean values both on the pre-revolutionary examinations (M = 24.00% & 28.64%) and post-revolutionary examinations (M = 27.13% & 21.75%), did not change much over the years.

These findings suggest that the general content emphasis of the NCEE mathematics examinations changed considerably after the Cultural Revolution, with a decrease in the percentages of two- and three-dimensional geometry and an increase in the percentage of analytical geometry. However, the percentages of equations and functions, numbers and expressions, and trigonometry remained relatively stable over time, with equations and functions being the most emphasized general topics in both the pre- and post-revolutionary NCEE mathematics examinations.

*Detailed Mathematics Topics*

Data. The detailed mathematics topics covered on the NCEE mathematics examinations prior to and after the Cultural Revolution exhibited a little more variation. Table 6, displayed on the subsequent four pages, shows the frequencies of detailed topics covered in the NCEE mathematics examinations for the years 1952–1965 and 1977–1984.
### Table 6

**Detailed Topics’ Frequency on the NCEE Math Exams**

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*Note.* The first row indicates the years of the examinations, which have been abbreviated, e.g., “1952” as “52.” The notation “Σ” within the years signifies summation. The entries under the first “Σ” represent the sum of the entries from 1952 to 1965. The entries under the second “Σ” represent the sum of the entries from 1977 to 1984. The entries in a row under the name of a detailed mathematics topic indicate the number of questions in a given year’s examination that falls under that topic. The abbreviations in the table are defined as follows: “Abs. Value” means “Absolute Value;” “Exp. & Log.” means “Exponential and Logarithmic;” “Rdi. & Rtn.” means “Radical and Rational;” “Trig.” means “Trigonometric;” “Un. & Int.” means “Union and Intersection;” “Pt, Ln, & Agl.”
**Descriptive Analysis.** Table 6 shows that quadratic equations and functions were the only topics featured in every single examination. Triangles, circles, and trigonometric ratios followed closely, with each topic missing from one examination.

When ordered by the sum of their frequencies for each period, the six most emphasized topics in the pre-revolutionary examinations were quadratic equations and functions, trigonometric ratios, triangles, circles, factorization, and the Pythagorean theorem, all highlighted in bold and italics in the table. In the post-revolutionary examinations, the most emphasized topics were quadratic equations and functions, trigonometric ratios, inequality, factorization, conic sections, and trigonometric identities, also highlighted in bold and italics.

Merely presenting statistical data under the names of mathematics topics does not adequately help readers gain a comprehensive understanding of the questions on the NCEE mathematics examinations. In the following paragraphs, while analyzing the evolution of detailed mathematics topics, the researcher provides detailed descriptions of all the questions categorized under each topic. These are supplemented with concrete examples from actual examinations.

**Numbers and Expressions.** In the category of numbers and expressions, factorization was the most frequently tested topic, followed by exponential and logarithmic expressions. Almost every test included at least one question that required students to understand these concepts to solve the problems. Factorization primarily involved two types of questions: the common type involved factorizing quadratic expressions, such as $x^2 + x - 2$ (Item 2 in 1957), which could also involve more than one variable, such as $2x^2 + 5xy - 12y^2$ (Item 11 in 1953); the less common type involved factorizing polynomials, such as $x^4 + 5x^3 - 7x^2 - 8x - 12$ (Item 21 in 1952). Exponential and logarithmic expressions also included two main types of
questions. One type involved multiple applications of properties of exponents or logarithms in calculations, such as \[ \left[ (a^{-\frac{3}{2}}b^2)^{-1} (ab^{-3})^2 \left( b^2 \right)^{\frac{1}{3}} \right] = \left( a^{\frac{3}{2}}b^{-\frac{1}{2}} - \frac{3}{2} \right)^{\frac{1}{3}} = (a^2b^0)^{\frac{1}{3}} = a^{\frac{2}{3}} \] (Item 1 in 1954). The other type involved using the binomial theorem to find a particular term in the algebraic expansion of powers of a binomial, such as finding the constant term in the expanded form of the expression \((2x^3 + \frac{1}{x})^{12}\) (Item 13 in 1953).

**Mathematical Logic.** In the category of mathematical logic, sets and their operations and conditional statements were the detailed subtopics. Only 10 questions fell into this category and 8 of them appeared on the post-revolutionary examinations. These were often relatively simple questions that can be solved in one or two steps, such as finding the union and intersection of rational and irrational numbers (Item 1 in 1981) and whether \(a = 3\) is sufficient, necessary, both sufficient and necessary, or neither sufficient nor necessary for \(|a| = 3\) (Item 3 in 1981). Perhaps because of their simplicity, they usually appeared in multiple-choice and fill-in-the-blank questions.

**Equations and Functions.** In the category of equations and functions, quadratic equations and functions were the only topics that were present in all examinations, indicating their significance on the NCEE mathematics examinations. There were three main types of questions. The first type involved solving quadratic equations, such as \(2x^2 - 5x + 2 = 0\) (Item 1 in 1960), and quadratic systems, such as \[ \begin{cases} x^2 - 2xy + 3y^2 = 9 \\ 4x^2 - 5xy + 6y^2 = 30 \end{cases} \] (Item 11 in 1953). The second type involved exploring various properties of quadratic functions, such as the following question (Item 7 in 1977):

7. Given: quadratic function \(y = x^2 - 6x + 5\).

   (1) Find the vertex and axis of symmetry of this function;
(2) Draw its graph;
(3) Find its $x$-intercept and $y$-intercept.

The third type involved constructing quadratic equations based on various formulas, such as the Pythagorean theorem ($a^2 + b^2 = c^2$), the circle’s area ($A = \pi r^2$), and the law of cosines ($c^2 = a^2 + b^2 - 2ab \cdot \cos \theta$), and then solving them. For example, in item 10 from 1953, students had to set up a quadratic equation based on the formula for the surface area of a sphere and then solve for the radius of the sphere:

$$4\pi R^2 = 36\pi \rightarrow R = 3.$$  

Similar to quadratic equations and functions, exponential, logarithmic, radical, rational, and trigonometric equations and functions, which had moderate frequencies in both the pre- and post-revolutionary NCEE mathematics examinations, can be classified into the same three types of questions: directly solving equations, exploring different properties of functions, and establishing equations and then solving them.

Inequalities also had moderate frequencies in both the pre- and post-revolutionary NCEE mathematics examinations. Except for directly solving inequalities, such as $2x^2 - 5x < 3$ (Item 3 in 1959), two additional types of questions were often seen in the examinations. One type involved determining the domain of radical functions, such as $y = \sqrt{\lg (2 + x)}$ (Item 3 in 1978). The other type involved solving for the conditions for the number of roots in quadratic equations, such as finding the values of $k$ when $x^2 - 2(k + 3)x + 3k^2 + 1 = 0$ has real roots (Item 10 in 1961).

System of equations, which had at least one item in 13 out of 14 pre-revolutionary examinations, appeared only 3 times in the post-revolutionary examinations. In contrast,
piecewise functions and parametric equations only showed up in the post-revolutionary examinations.

**Two-Dimensional Geometry.** In the category of two-dimensional geometry, triangles and circles were the two leading topics, and almost every test had at least one question that required knowledge of them to be solved. Out of 39 questions related to triangles, 21 were proof questions, such as “ΔABC’s altitudes are AD, BE, and CF. H is the orthocenter. Prove: HD bisects ∠EDF” (Item 14 in 1953). The remaining questions focused on finding areas, angle measures, and side lengths of triangles, such as “if two angles of ΔABC are 45° and 60°, and the side between them has a length of 1, find the shortest side length and the area of ΔABC” (Item 15 in 1953). As concepts closely related to triangles, trigonometric ratios and formulas were also frequently tested in the NCEE mathematics examinations. Due to their high frequencies on the tests and being a branch of mathematics themselves, they were analyzed separately under the category of trigonometry. On the other hand, the Pythagorean theorem, congruence, and similarity problems had moderate frequencies before the revolution but decreased afterwards.

Circles primarily involved three types of questions. Out of 37 questions related to circles, 18 were proof questions, 12 involved solving for angle measures (e.g., inscribed angles) and segment lengths (e.g., chords), and 7 were related to the areas of circles.

**Three-Dimensional Geometry.** In the category of three-dimensional geometry, lines and planes was the most frequently tested topic, both before and after the revolution. Out of 17 questions related to lines and planes, 14 were proofs questions that required students to explore the relationship between lines and planes, such as the following question (Item 5 in 1957):
5. $a$ and $b$ are two skew lines, and their common perpendicular line is $EF$. $\pi$ is a plane passing through the midpoint of $EF$ and parallel to $a$ and $b$. Let $M$ be a point on $a$ and $N$ be a point on $b$. Prove that plane $\pi$ bisects the segment $MN$. (See Figure 8 as an illustration).

**Figure 8**

*Graph for Item 5 in the 1957 NCEE Math Exam*

The remaining three questions required students to find angle measures or segment lengths in three-dimensional space.

Following the lines and planes, three-dimensional figures, including prisms, pyramids, cylinders, cones, and spheres, were tested from time to time in the pre-revolutionary examinations but rarely showed up in the post-revolutionary examinations. Notably, all related questions were surface area and volume problems.

**Analytical Geometry.** In the category of analytical geometry, all subtopics barely appeared in the pre-revolutionary examinations. After the revolution, points, lines, and conic sections, including circles, ellipses, parabolas, and hyperbolas, became more frequently tested in the examinations. There were two main types of questions. The first type involved solving for equations of conic sections and lines, such as finding the equation of a circle with the center at $(3, 4)$ and the origin on its circumference (Item 18 in 1952). The second type involved finding
intersections and exploring the relationships between conic sections and lines, such as the following question (Item 5 in 1965):

5. The coordinates of point $P$ are $(4, -2)$, the equation of line $l$ is $y - x + 5 = 0$, and the equation of curve $C$ is $\frac{(x+1)^2}{2} + \frac{(y-1)^2}{4} = 1$. Find the coordinates of the intersection point of the line passing through point $P$ and perpendicular to line $l$, and the curve $C$.

**Trigonometry.** In the category of trigonometry, trigonometric ratios, identities, and the laws of sines and cosines were the three most frequently tested topics in descending order, both before and after the revolution. As mentioned previously, trigonometry is closely related to triangles. Out of 68 questions related to trigonometry, 36 involved solving for angles measures, side lengths, and areas of triangles. The remaining 32 questions can be divided into three types. The first type involved finding values of trigonometric functions, such as the value of $\cos 165^\circ$ (Item 4 in 1959). The second type involved solving trigonometric functions, such as $\sin^3 x - \sin x + \cos 2x = 0$ (Item 6 in 1963). The third type were proof questions that required justification of two expressions being identical, such as proving $\cos x \cdot \cos 2x \cdot \cos 4x = \frac{\sin 8x}{8 \sin x}$ (Item 2 in 1958).

**Sequences and Series.** In the category of sequences and series, arithmetic and geometric sequences and series were commonly tested in both the pre-and post-revolutionary NCEE mathematics examinations. After the revolution, non-typical sequences and series began to appear in the examinations, and every post-revolutionary examination included at least one question related to the sequences and series. There were two main types of questions: calculation and proof. Calculation questions often required students to find the sum of arithmetic and geometric series, such as $\left(1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots\right) = 1 \div \left(1 - \frac{1}{4}\right) = \frac{4}{3}$ (Item 7 in 1965). Proof questions often required students to prove a given sequence to be arithmetic or geometric, such
as proving that the logarithms of the terms of a geometric sequence form an arithmetic sequence (Item 3 in 1960).

**Combinatorics.** In the category of combinatorics, combinations were the most frequently tested topic, both before and after the revolution. Out of 9 questions related to combinations, 6 were finding the terms in the binomial expansion, such as \( T_4 = \binom{5}{3}(2x)^3 = \frac{20}{2} \cdot 8x^3 = 80x^3 \) in the expanded form of \((1 + 2x)^5\) (Item 1 in 1958). The remaining three questions were finding the number of ways to select objects from a set, such as the ways to select three class committees from four candidates (Item 2 in 1981).

**Calculus.** In the category of calculus, all subtopics were absent from the pre-revolutionary examinations. After the revolution, derivatives were the leading topic presented in the tests, followed by the limit. Continuity and integrals were topics that only appeared in the 1977 examination. Perhaps because calculus was a newly added category in the NCEE mathematics examinations, most questions were fundamental calculation problems, such as finding the derivative of \( y = \cos^2 \frac{x}{3} \) (Item 2 in 1982) and the limit \( \lim_{n \to \infty} \frac{1-2^n}{3^n+1} \) (Item II-5 in 1984).

**Linear Algebra.** In the category of linear algebra, determinants of matrices were the only topic that was tested on both pre- and post-revolutionary examinations.

In summary, the data discussed above suggests that the emphasis on the more detailed subtopics in the NCEE mathematics examinations has evolved over time. Before the Cultural Revolution, topics like system of equations, the Pythagorean theorem, and three-dimensional geometric figures were more frequently tested. However, these topics saw diminished attention in examinations conducted post-revolution. Conversely, certain topics generally perceived as more advanced—such as inequalities, conic sections, piecewise functions, parametric equations, non-typical sequences and series, and various calculus concepts—were seldom addressed in pre-
revolutionary examinations but gained prominence post-revolution. This transition indicates a shift in content emphasis from foundational mathematics topics toward more advanced ones. Notably, some topics, such as factorization, quadratic equations and functions, triangles, circles, and trigonometric ratios and formulas, have maintained a relatively stable presence in both pre- and post-revolutionary NCEE mathematics examinations.

**Historical Reasons Behind the Changes**

The evolution in content emphasis within the NCEE mathematics examinations can be attributed in part to the influence of mathematics researchers and educators in China. A notable individual in this context is Guan Zhaozhi, recognized as one of China’s eminent mathematicians. Serving as a scientist and an academician of the Chinese Academy of Sciences (the national academy for natural sciences in China), he contributed significantly to areas like artificial satellite orbit design and determination, missile guidance, and submarine inertial navigation, among others. Beyond his research contributions, Guan was a celebrated mathematics educator, holding professorships at prestigious universities such as Peking University and Beijing Normal University. He mentored a wide range of students, from undergraduates to young researchers. Moreover, his editorial contributions spanned several scientific journals, including but not limited to *Science China*, *Science Bulletin*, *Acta Mathematica Sinica*, and *Journal of Systems Science and Mathematical Sciences* (“Guan,” 2022).

Given his stature in the mathematics community, Guan’s (1958) perspectives were influential. In 1958, he penned an article in *People’s Daily* offering a critique of the prevailing mathematics curriculum. He observed that the mathematics knowledge emphasized by the curriculum, such as two- and three-dimensional geometry, had been discovered by
mathematicians before the 17th century. While he acknowledged their significance, he viewed these topics as not entirely aligned with the evolving demands of socialist construction and development. Guan advocated for the integration of more advanced mathematics topics, such as trigonometry, analytical geometry, set theory, and calculus, to better align with the contemporary needs (Guan, 1958). Guan’s views exemplify a growing sentiment among some mathematicians and educators of that era in China who believed in modernizing the existing curriculum. His effort to disseminate this viewpoint through a platform like People’s Daily, the official newspaper of the Central Committee of the Communist Party of China, showcases his intent to reach a wider audience.

The mathematics curriculum subsequently became a focal point of discussion during the Second National Congress of the Chinese Mathematical Society (CMS), held in Shanghai in 1960, with the participation of 128 mathematicians. CMS is an academic social group that aims to unite Chinese mathematicians, promote the development of mathematics, enhance Chinese science and technology, foster talent growth, and contribute to revitalizing the Chinese economy and accelerating socialist modernization (Chinese Mathematical Society, n.d.). The conference focused on two major issues: the future trajectory of mathematics and necessary reforms in mathematics education (Gong, 1960).

During the conference, Guan delivered a speech regarding the future trajectory of mathematics. He pointed out that the purpose of studying mathematics is to utilize it in the service of socialist construction, while the mathematics education at that time had long been detached from practical applications. He emphasized the need to align the direction of mathematical learning and research with the practical demands of socialist construction and scientific and technological development (Gong, 1960).
Subsequently, Ding Ersheng, a representative from Beijing Normal University, spoke about necessary reforms in mathematics education. He advocated for a shift away from excessive focus on elementary mathematics, such as Euclidean geometry, and proposed a greater emphasis on learning the foundations of modern mathematics that have extensive applications in contemporary socialist construction and cutting-edge science and technology. This includes analytic geometry, calculus, differential equations, probability theory, and computational mathematics (Gong, 1960).

Guan and Ding’s presentations prompted active discussions among the conference attendees, and their perspectives garnered widespread agreement (Gong, 1960). From this, one might deduce that the idea of reforming the mathematics curriculum to include modern mathematical concepts resonated with many Chinese mathematicians and mathematics educators of the time. However, fully actualizing this vision remained an ongoing endeavor.

In 1961, one year after the conference, the Chinese Ministry of Education assigned the People’s Education Press (PEP) to begin the process of rewriting textbooks for primary and secondary schools (“Xin,” 2021). The primary discussion during the development of the new textbooks was whether to include modern mathematics topics, such as derivatives (“Xin,” 2021). After researching materials from both ancient and modern times, as well as from domestic and foreign sources, and soliciting opinions from teachers and students nationwide, the PEP decided not to include advanced mathematics concepts like derivatives in the new curriculum standards temporarily (“Xin,” 2021). This decision was made because, on average, the quality of teachers and students at that time could not support the teaching and learning of these new contents (“Xin,” 2021). After two years of research and compilation, the new edition of the textbooks was officially put into use in 1963. Just three years later, in 1966, the Cultural Revolution broke out
in China. During this period, schools were closed, examinations were abolished, and both teachers and students were persecuted by revolutionaries (Swetz & Chi, 1983). As a result, the implementation of plans to incorporate modern mathematical topics into the curriculum was further postponed until the end of the revolution.

In 1977, the first year after the Cultural Revolution, Chairman Deng Xiaoping emphasized in a symposium on science and education that “textbooks should reflect the advanced level of modern scientific culture… and it is necessary to educate the youth with the latest scientific knowledge” (“Xin,” 2021, para. 3). In the same year, the Ministry of Education hired eight mathematicians, including Guan Zhaozhi and Ding Ersheng, as consultants for the compilation of mathematics textbooks. Consequently, topics such as sequences and series, limits, derivatives, and integrals were officially included in the high school mathematics curriculum (“Xin,” 2021). These changes aligned with the observed changes in the content emphasis of the NCEE mathematics examinations.

In summary, the evolution of the topic coverage in the NCEE mathematics examinations has been a dynamic process influenced by a combination of social, political, and technological factors in China. Central to this development has been the contributions of Chinese mathematicians and educators, notably Guan Zhaozhi and Ding Ersheng, who advocated for more advanced and updated mathematics topics in the curriculum to meet the changing demands of Chinese society. The Second National Congress of the Chinese Mathematical Society became a pivotal forum where these ideas were widely discussed and endorsed, leading to a consensus on the reform of the mathematics curriculum in China. However, the implementation of these proposals was not immediate, facing delays due to the quality of teachers and students and significant political events like the Cultural Revolution. Nevertheless, the commitment to
modernize mathematics education in China remained, leading to significant curriculum reforms after the Cultural Revolution, as evidenced by the changes in content emphasis of the NCEE mathematics examinations.

Summary

In conclusion, the following list provides a summary of the findings on the topic coverage of the NCEE mathematics examinations from 1952 to 1984:

1. The five most emphasized general topics before the revolution were equations and functions, two-dimensional geometry, trigonometry, numbers and expressions, and three-dimensional geometry;
2. The five most emphasized general topics after the revolution were equations and functions, numbers and expressions, trigonometry, two-dimensional geometry, and analytical geometry;
3. There were remarkable changes in the general content emphasis of NCEE mathematics examinations, with a decrease in the percentages of two-and three-dimensional geometry and an increase in the percentage of analytical geometry;
4. The six most emphasized detailed topics before the revolution were quadratic equations and functions, trigonometric ratios, triangles, circles, factorization, and the Pythagorean theorem;
5. The six most emphasized detailed topics after the revolution were quadratic equations and functions, trigonometric ratios, inequality, factorization, conic sections, and trigonometric identities;
6. There were remarkable changes in the detailed content emphasis of NCEE mathematics examinations, with a shift from elementary mathematics topics, such as
system of equations, the Pythagorean theorem, and three-dimensional geometric figures, to more advanced topics, such as piecewise functions, parametric equations, inequalities, conic sections, sequences and series, and various concepts in calculus;

7. The post-revolutionary examinations covered a broader range of topics, likely due to the increased total number of questions; and

8. The evolution of the topic coverage in the NCEE mathematics examinations has been a dynamic process influenced by a combination of social, political, and technological factors in China, including Chinese mathematicians and educators, the Second National Congress of the Chinese Mathematical Society, and the Great Proletarian Cultural Revolution.

**Item Difficulty**

**Data**

The item difficulty of the NCEE mathematics examination is analyzed based on the following five factors according to Bao’s (2006) model: the *discovery* factor, the *context* factor, the *computation* factor, the *reasoning* factor, and the *topic coverage* factor.

Table 7 displays the coding results for all the NCEE mathematics examinations. In this table, columns represent the years of the examinations, while rows indicate various factor levels. For illustration, entries in the column for the year 1952 under the *discovery* factor indicate that the number of questions categorized under the *knowing*, *understanding*, and *investigating* levels in the 1952 NCEE mathematics examinations were 4, 20, and 0, respectively. Similarly, entries in the column for the year 1952 under the *reasoning* factor indicate that the same 24 questions in the 1952 NCEE mathematics examinations, when categorized under the *none*, *simple*, *medium*, and *complex reasoning* levels, received outcomes of 19, 4, 0, and 1, respectively.
Table 7

*Bao’s Model Factors’ Levels’ Frequency in the NCEE Math Exams*

<p>| Year | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 | 61 | 62 | 63 | 64 | 65 | 77 | 78 | 79 | 80 | 81 | 82 | 83 | 84 |
|------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| <strong>Discovery Factor</strong> |     |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Knw. | 4  | 4  | 0  | 1  | 0  | 1  | 1  | 2  | 0  | 0  | 0  | 0  | 0  | 0  | 3  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |    |
| Und. | 20 | 11 | 10 | 7  | 10 | 10 | 8  | 10 | 13 | 10 | 12 | 15 | 9  | 10 | 13 | 11 | 11 | 13 | 15 | 16 | 18 | 22 |    |
| Invs.| 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 2  | 2  | 0  | 2  | 0  | 0  | 1  | 1  | 0  | 0  |    |    |    |
| <strong>Context Factor</strong> |     |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| None | 23 | 15 | 10 | 8  | 10 | 11 | 9  | 12 | 9  | 6  | 11 | 15 | 9  | 10 | 14 | 14 | 9  | 13 | 13 | 17 | 17 | 21 |    |
| Per. | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 3  | 3  | 0  | 0  | 2  | 1  | 1  | 0  | 1  | 0  | 1  | 0  | 1  | 1  |    |    |
| Pub. | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 1  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 2  | 0  | 0  |    |    |
| Sci. | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 1  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 1  |    |    |
| <strong>Computation Factor</strong> |     |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| None | 4  | 2  | 1  | 2  | 3  | 4  | 3  | 2  | 0  | 0  | 2  | 2  | 1  | 0  | 3  | 2  | 1  | 1  | 6  | 7  | 4  | 7  |    |
| Nm. | 10 | 8  | 1  | 0  | 3  | 2  | 0  | 4  | 4  | 3  | 2  | 6  | 1  | 0  | 4  | 1  | 1  | 1  | 2  | 1  | 2  | 2  |    |
| Sim. | 5  | 3  | 2  | 2  | 2  | 1  | 4  | 2  | 5  | 5  | 5  | 3  | 2  | 1  | 3  | 2  | 0  | 5  | 0  | 1  | 3  | 3  |    |
| Med. | 3  | 1  | 4  | 2  | 1  | 0  | 1  | 0  | 1  | 2  | 1  | 1  | 3  | 3  | 3  | 3  | 3  | 3  | 1  | 1  | 1  | 2  | 1  |
| Com. | 2  | 1  | 2  | 2  | 1  | 4  | 1  | 4  | 3  | 0  | 2  | 3  | 4  | 8  | 3  | 6  | 7  | 5  | 7  | 7  | 7  | 9  |    |
| <strong>Reasoning Factor</strong> |     |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| None | 19 | 13 | 6  | 5  | 3  | 4  | 3  | 6  | 4  | 4  | 2  | 3  | 1  | 0  | 12 | 5  | 2  | 3  | 1  | 4  | 4  | 3  |    |
| Sim. | 4  | 1  | 3  | 1  | 2  | 3  | 0  | 4  | 6  | 3  | 3  | 9  | 3  | 3  | 2  | 3  | 4  | 3  | 8  | 7  | 3  | 8  |    |
| Med. | 0  | 1  | 0  | 2  | 3  | 1  | 5  | 0  | 2  | 2  | 5  | 1  | 4  | 5  | 2  | 1  | 2  | 1  | 2  | 0  | 3  | 2  |    |
| Com. | 1  | 0  | 1  | 0  | 2  | 3  | 1  | 2  | 1  | 1  | 2  | 2  | 3  | 4  | 0  | 5  | 4  | 6  | 5  | 6  | 8  | 9  |    |
| <strong>Topic Coverage Factor</strong> |     |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| Sing. | 13 | 7  | 3  | 2  | 2  | 3  | 0  | 3  | 4  | 0  | 3  | 6  | 0  | 1  | 8  | 4  | 3  | 5  | 1  | 6  | 10 | 6  |    |
| Two  | 7  | 3  | 2  | 3  | 2  | 2  | 4  | 2  | 2  | 2  | 3  | 3  | 6  | 2  | 7  | 2  | 3  | 4  | 10 | 4  | 1  | 4  |    |</p>
<table>
<thead>
<tr>
<th>Year</th>
<th>52</th>
<th>53</th>
<th>54</th>
<th>55</th>
<th>56</th>
<th>57</th>
<th>58</th>
<th>59</th>
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<th>62</th>
<th>63</th>
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<th>65</th>
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<td>0</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>8</td>
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</tr>
<tr>
<td>Fr.+</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>3</td>
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<td>4</td>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
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</tr>
</tbody>
</table>

**Descriptive Statistical Analysis**

A descriptive statistical analysis was conducted to analyze the data in Table 7. This analysis helps to reveal the central tendency and dispersion of the number of questions categorized under each specific factor level. Table 8, which is shown on the next page, presents the descriptive statistics for the frequencies of questions categorized under the factor levels in Bao’s model.

For illustration, the data results for the discovery factor, which are shown in the upper left corner of the table, indicate that the number of questions in the NCEE mathematics examinations for the years 1952–1965 and 1977–1984, categorized under the *knowing* level, had a mean value of 0.82 and a standard deviation of 1.30. The minimum and maximum values were 0 and 4, respectively. Similarly, the central tendency and dispersion for the questions categorized under the *understanding* level were 12.45 and 3.84, respectively, with a minimum value of 7 and a maximum value of 22. The central tendency and dispersion for the questions categorized under the *investigating* level were 0.36 and 0.73, respectively, with a minimum value of 0 and a maximum value of 2.
Table 8

Descriptive Statistics for the Frequency of Bao’s Model Factors’ Levels

<table>
<thead>
<tr>
<th>Discover. Factor</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>Context Factor</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
<th>Comp. Factor</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Know.</td>
<td>0</td>
<td>4</td>
<td>0.82</td>
<td>1.30</td>
<td>None</td>
<td>6</td>
<td>23</td>
<td>12.55</td>
<td>4.23</td>
<td>None</td>
<td>0</td>
<td>7</td>
<td>2.59</td>
<td>2.06</td>
</tr>
<tr>
<td>Under.</td>
<td>7</td>
<td>22</td>
<td>12.45</td>
<td>3.84</td>
<td>Person.</td>
<td>0</td>
<td>3</td>
<td>0.68</td>
<td>0.95</td>
<td>Num.</td>
<td>0</td>
<td>10</td>
<td>2.64</td>
<td>2.57</td>
</tr>
<tr>
<td>Invest.</td>
<td>0</td>
<td>2</td>
<td>0.36</td>
<td>0.73</td>
<td>Public.</td>
<td>0</td>
<td>2</td>
<td>0.27</td>
<td>0.55</td>
<td>Sim.</td>
<td>0</td>
<td>5</td>
<td>2.68</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Scienti.</td>
<td>0</td>
<td>1</td>
<td>0.14</td>
<td>0.35</td>
<td>Med.</td>
<td>0</td>
<td>4</td>
<td>1.73</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reason. Factor</td>
<td>Min.</td>
<td>Max.</td>
<td>Mean</td>
<td>SD</td>
<td>Topic Cover. Factor</td>
<td>Min.</td>
<td>Max.</td>
<td>Mean</td>
<td>SD</td>
<td></td>
<td>N = 22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>None</td>
<td>0</td>
<td>19</td>
<td>4.86</td>
<td>4.42</td>
<td>Single</td>
<td>0</td>
<td>13</td>
<td>4.09</td>
<td>3.34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sim.</td>
<td>0</td>
<td>9</td>
<td>3.77</td>
<td>2.39</td>
<td>Two</td>
<td>1</td>
<td>10</td>
<td>3.55</td>
<td>2.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Med.</td>
<td>0</td>
<td>5</td>
<td>2.00</td>
<td>1.60</td>
<td>Three</td>
<td>0</td>
<td>8</td>
<td>3.14</td>
<td>1.88</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Com.</td>
<td>0</td>
<td>9</td>
<td>3.00</td>
<td>2.58</td>
<td>Four +</td>
<td>0</td>
<td>5</td>
<td>2.86</td>
<td>1.36</td>
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</tr>
</tbody>
</table>

**Discovery Factor.** According to Table 7 and Table 8, in the *discovery* factor, the *understanding* level dominated all examinations with a mean value of 12.45 (SD = 3.84). The *knowing* and *investigating* levels had mean values of 0.82 (SD = 1.30) and 0.36 (SD = 0.73), respectively. This indicates that the vast majority of questions in each year’s examination required students to apply mathematical concepts, formulas, rules, and theories, use standard methods to obtain definite results, and justify and demonstrate mathematical facts.

Having few items at the *knowing* level implies that the NCEE mathematics examinations required more than mere memorization of mathematical facts. This makes intuitive sense as purely memorizing mathematical facts is a requirement well below the general standard for college admissions. Furthermore, the presence of items in the past examinations, such as recalling the value of $\pi$ (Item 9 in 1952) and naming all Platonic solids (Item 12 in 1952), which are rarely seen in recent examinations, raises questions about their inclusion.

Table 7 reveals that the presence of these questions was concentrated in the years 1952 ($f = 4$), 1953 ($f = 4$), and 1977 ($f = 3$), corresponding to the post-war and post-Cultural Revolution periods in China. Historical sources, including books by Jiang (2008a, 2008c) and Yang (2007), indicate that student quality during these periods was unsatisfactory. Therefore, it was speculated that the inclusion of *knowing*-level questions in these years was intended to accommodate the lower quality of students resulting from the impacts of the wars and the Cultural Revolution.

Another possible explanation is that, during the post-war or post-revolutionary periods, the government sought comprehensive data on students’ mathematical competency to inform the development of curriculum standards and assessment methods. This necessitated a broader range of topic levels in the examination. This hypothesis is supported by evidence showing that during these years, the NCEE mathematics examinations included not only *knowing*-level questions but
also those at the college level, such as finding the determinant of matrices (in 1952 and 1953) and calculus-related questions (in 1977). While the researcher found this second hypothesis to be more plausible than the first, the actual reasons may be more complex.

The NCEE mathematics examinations also had few items at the *investigating* level, a feature that could possibly be explained by the following reasons. Firstly, *investigating*-level questions require students to make mathematical conjectures, create mathematical models, and engage in mathematical explorations. These are more demanding compared to *knowing*-level questions, which can be solved through rote memorization, and *understanding*-level questions, which are solved within given mathematical contexts and models. Such questions are typically used to identify students who excel in mathematics. However, considering the generally lower student quality during the historical periods under investigation, there was less need to include *investigating*-level questions in the examinations.

A major flaw in the first argument is that the often-cited low student quality in historical sources lacked concrete data support. Therefore, it was unclear what level of student quality aligned with *investigating*-level questions. Consequently, the researcher proposed a second explanation.

According to Zheng and Chen (2017), China’s initial exposure to exploratory mathematics questions, often categorized under the *investigating*-level, can be traced to Sawada’s (1980) research article about open-ended questions. Building on Sawada’s research, Professor Dai Zaiping of Zhejiang International Studies University conducted extensive research in this area and compiled five books of practice problems that had been widely circulated across the country. These books contributed to the growing popularity of open-ended questions in China, leading to the relevant reforms of the NCEE mathematics examination after 1999.
From the information gathered, the researcher inferred that between 1952 and 1984, the concept of exploratory and open-ended mathematics questions was not yet widely recognized in China. The primary approach to mathematics teaching and learning in the country focused on applying concepts, formulas, rules, and theories, using standard methods to obtain definite results, and justifying and demonstrating mathematical facts. This methodology likely shaped the NCEE mathematics examinations during this period, making understanding-level questions the primary question type in the examinations.

In summary, regarding the discovery factor, the NCEE mathematics examinations predominantly featured understanding-level questions, emphasizing the application of mathematical knowledge rather than rote memorization. The inclusion of few investigating-level questions reflects a more traditional view of mathematics as a discipline focused on solving equations and justifying facts, rather than encouraging exploration.

**Context Factor.** In the context factor, the none-context level, representing purely mathematical questions, yielded a mean value of 12.55 (SD = 4.23). In stark contrast, the personal life, public affairs, and scientific scenarios levels had pronounced lower mean values of 0.68 (SD = 0.95), 0.27 (SD = 0.55), and 0.14 (SD = 0.35), respectively. This discrepancy suggests that the NCEE mathematics examinations predominantly consisted of non-contextual, purely mathematical questions. Among real-world-context questions, those featuring personal life scenarios, which students are most familiar with, were the most common, while questions at the public-affairs and scientific-scenarios levels, which students are less familiar with, were less frequent.

The statistical results presented above confirm and support the findings of Zheng and Chen (2017), who stated that before the 1990s, the NCEE mathematics examinations lacked an
emphasis on applying mathematical knowledge to real-world scenarios. However, their study lacked sufficient quantitative data to support these claims, a gap that this research filled.

Few questions placed in a real-world context during the years 1952–1965 and 1977–1984 make the subdivisions of *personal-life, public-affairs,* and *scientific-scenarios* levels less meaningful. This is partly because Bao’s (2002) model was developed when real-world-context questions were more prevalent in examinations.

The following narrative details a pivotal change in the context of questions in the NCEE mathematics examinations, occurring in 1992. Although this event falls outside the time range of this research, examining it provides valuable insights for a more comprehensive understanding of the historical development of the NCEE mathematics examinations.

Ren and Chen (2017) observed that the emphasis on testing purely mathematical knowledge in examinations, while neglecting practical applications, led to a phenomenon known as “high scores but low abilities” (“gao fen di neng” in Chinese). This term describes students excelling in examinations but struggling with practical applications of their knowledge. For instance, students might perform well in physics examinations but poorly in physics labs. Mathematician Zhang Dianzhou (2009) also noted similar observations reported by both domestic and international scholars. Zhang suggested that this discrepancy could be resulted from the NCEE’s emphasis on theoretical rather than practical applications.

To address the “high scores but low abilities” phenomenon, mathematicians Zhang Dianzhou, Yan Shijian, and Su Shidong recommended to the National Education Examinations Authority in 1992 that the NCEE mathematics examinations should include more application-based questions (Zheng & Chen, 2017). Since then, there has been a growing number of context-
rich questions in the NCEE mathematics examinations, with contexts becoming more flexible, diverse, and reflective of real-life scenarios (Zheng & Chen, 2017).

The researcher found the historical development of including more real-world questions in the NCEE mathematics examinations, as outlined by the above researchers, convincing. This is due to the consistency in the people, times, and phenomena referenced across various studies.

Last but not least, the predominance of non-contextual, purely mathematical questions in the NCEE mathematics examinations during the years 1952–1965 and 1977–1984 reflects a more traditional view of mathematics as a discipline focused on theoretical work rather than real-world applications in Chinese society during these years.

In summary, regarding the context factor, the NCEE mathematics examinations primarily focused on non-contextual, purely mathematical questions. Questions involving personal life, public affairs, and scientific scenarios contexts were also incorporated, but these were considerably fewer. Several mathematicians and educators critiqued this heavy emphasis on purely mathematical knowledge, arguing that it led to a “high scores but low abilities” phenomenon among Chinese students, who often struggled with practical applications. In response to these criticisms, the examinations started to incorporate more context-rich and application-based questions from 1992 onwards, following recommendations from Chinese mathematicians. This shift reflects a gradual transition away from a purely mathematical approach toward one that values real-world applications in mathematics in Chinese society.

**Computation Factor.** When it came to the computation factor, a distinct pattern emerged. The complex symbolic level exhibited the highest mean value (M = 4, SD = 2.62), followed by the simple symbolic level (M = 2.68, SD = 1.62), the numerical level (M = 2.64, SD = 2.57), and the none-computation level (M = 2.59, SD = 2.06). The level with the lowest mean
value was the *medium symbolic* level (M = 1.73, SD = 1.12). These statistical results suggest that most questions in the NCEE mathematics examinations incorporated symbolic computations. If one treats the three symbolic levels as a unified entity, their combined mean value would be 8.41. Of the remaining questions, the number involving numerical computations and those requiring no computations were almost identical.

In contrast to the *discovery* and *context* factors, no single level within the *computation* factor consistently dominated the NCEE mathematics examinations. Therefore, examining changes across different time periods could be both valuable and interesting. Figure 9, presented in a line chart format, displays the frequency changes of both the *numerical* and *complex symbolic* levels from 1952 to 1984. Frequencies for the other three levels are not included in the graph due to their irregular variation and lack of discernible patterns.

**Figure 9**

*Computation Factor Levels’ Frequency in 1952-1984 NCEE Math Exams*

![Line chart showing frequency changes of numerical and complex symbolic levels from 1952 to 1984.](image)

*Note.* The horizontal axis represents the years of the examinations, with the years abbreviated (e.g., “1952” abbreviated to “52”). The notation “CR” within the years signifies the Cultural
Revolution period. The vertical axis indicates the number of questions categorized under the numerical or complex symbolic level.

In Figure 9, the frequency of the numerical level generally exhibited a decreasing pattern, decreasing from 10 (in 1952) to 2 (in 1984). In contrast, the frequency of the complex symbolic level generally displayed an increasing pattern, rising from 2 (in 1952) to 9 (in 1984). These trends indicate a shift in emphasis from numerical to symbolic calculations in the NCEE mathematics examinations. Moreover, in general, symbolic computation questions are considered more difficult than numerical ones. Thus, it was hypothesized that the computational difficulty of the NCEE mathematics examinations has gradually increased over the historical periods under investigation.

Several arguments have been formed to explain this shift. First, it is likely that the gradual improvement in student quality in China necessitated an increase in examination difficulty to ensure their effectiveness in selecting students for college admissions. While no direct evidence from the periods under investigation was found to support this, recent news articles (e.g., Guan, 2022; Lei, 2022) suggest that contemporary high school and middle school students often find past NCEE mathematics examination questions easy, indicating a long-term improvement in student quality in China.

Second, as shown in the topic coverage section, the content emphasis of the NCEE mathematics examinations was shifting from elementary mathematics topics to more advanced ones, which often involve more complicated calculations. This shift could be another reason for the increased computational difficulty of the NCEE mathematics examinations.

Beyond the general trends, two distinct features stand out in Figure 9. Firstly, the frequency of the numerical level was unusually high in 1952 (f = 10) and 1953 (f = 8). Secondly,
in 1977, there was an abrupt increase in the *numerical* level’s frequency, rising from 0 to 4, accompanied by a decrease in the *complex symbolic* level’s frequency, falling from 8 to 3. These occurrences deviated from their regular developmental trends.

The unusual patterns might be attributed to the fact that these years followed the post-war and post-revolutionary periods in China, a time characterized by low general education and student quality. This suggests that the NCEE mathematics examinations would likely favor simpler computational questions over those that require more knowledge and skills. Lauwerys’ (2011) study supports this interpretation, describing post-war China as having poorly maintained school buildings, dilapidated textbooks, a shortage of adequately trained educators, and a generation of uneducated pupils and illiterate young adults. Furthermore, Yang’s (2007) research corroborates this argument, indicating that during the Cultural Revolution, student quality dropped dramatically, to the extent that some students struggled to understand basic mathematics concepts, such as why 1/2 is greater than 1/4.

In summary, regarding the computation factor, the NCEE mathematics examinations predominantly featured symbolic-computation questions, particularly at the *complex* and *simple symbolic* levels. The investigation also revealed an evident shift from numerical to symbolic calculations over the examined periods from 1952 to 1984, indicating an increasing emphasis on more sophisticated computational skills in the examinations. This shift aligned with the changes in the content emphasis and the rising difficulty of the NCEE mathematics examinations, possibly reflecting an improved quality of Chinese students.

**Reasoning Factor.** When investigating the reasoning factor, the results were a bit more complex. The *none-reasoning* level exhibited the highest mean value (M = 4.86, SD = 4.42). However, this is due to the presence of three extreme values that occurred in the years 1952 (f =
19), 1953 (f = 13), and 1977 (f = 12). Following this, the simple, complex, and medium reasoning levels had mean values of 3.77 (SD = 2.39), 3 (SD = 2.58), and 2 (SD = 1.60), respectively, and their combined mean value is 8.77. These statistical results suggest that the NCEE mathematics examinations primarily favored questions that require reasoning skills over those that do not. However, in some years, the situation was the opposite.

Figure 10 illustrates the frequency changes for both the none-reasoning level and the complex reasoning level from 1952 to 1984, providing a clearer depiction of the evolution of the NCEE mathematics examinations’ reasoning difficulty. The frequency changes for the other two levels are not included in the graph due to their negligible variations.

**Figure 10**

*Reasoning Factor Levels’ Frequency in 1952-1984 NCEE Math Exams*

Note. The horizontal axis represents the years of the examinations, with the years abbreviated (e.g., “1952” abbreviated to “52”). The notation “CR” within the years signifies the Cultural Revolution period. The vertical axis indicates the number of questions categorized under the none reasoning or complex reasoning level.
In Figure 10, the frequency of the *non-reasoning* level exhibited a notable downward trend, decreasing from 19 (in 1952) to 3 (in 1984). Conversely, the frequency of the *complex reasoning* level showed a gradual upward trend, rising from 1 (in 1952) to 9 (in 1984). These changes suggest a shift in emphasis from questions requiring minimal reasoning to those necessitating more complex reasoning skills. This trend indicates an increase in the reasoning difficulty of the NCEE mathematics examinations over the periods under investigation.

Similar to Figure 9, two unique features are notable in Figure 10. Firstly, the frequency of the *non-reasoning* level was exceptionally high in 1952 (f = 19) and 1953 (f = 13). Secondly, in 1977, there was a sudden increase in the frequency of the *non-reasoning* level, rising from 0 to 12. At the same time, there was a decrease in the frequency of the *complex reasoning* level, falling from 4 to 0.

Since Figure 9 and Figure 10 exhibit similar patterns, their developmental trends can be interpreted similarly. The years 1952 and 1953 marked the post-war period following the founding of the People’s Republic of China. The Chinese government led the entire nation in efforts to repair the economic damage wrought by the Second Sino-Japanese War (1937–1945) and the Chinese Civil War (1927–1937; 1945–1949) (Yang, 2007). These conflicts significantly damaged the Chinese educational system. Lauwerys (2011) depicts the post-war China, highlighting the educational challenges faced, including poorly maintained school buildings, dilapidated textbooks, a shortage of adequately trained educators, and a generation of uneducated pupils and illiterate young adults.

Given these circumstances, it is reasonable that the NCEE mathematics examinations favored simpler, non-reasoning questions that emphasize calculation and memorization skills rather than reasoning questions that require a higher quality of education. Such an approach
would have been more effective in screening students for college admissions during that period and in assessing the overall mathematical competence of students nationwide. This could provide crucial data to inform future reforms and developments in the mathematics curriculum and examinations.

Similarly, Swetz and Chi (1983) noted that in 1977, due to the outbreak of the Cultural Revolution from 1966 to 1976, schools in China were closed, examinations were abolished, and teachers and students were persecuted by the revolutionaries. Although the impact of the revolution was not as severe as that of the wars, it led to a sharp decline in the mathematical competence of Chinese students. Yang (2007) observed that many students at that time could not comprehend why 1/2 is greater than 1/4. Therefore, it is understandable that after 11 years of stagnation and regression in the educational system, the NCEE mathematics examination in 1977 reverted to focusing on simple, non-reasoning questions.

In summary, with respect to the reasoning factor, the NCEE mathematics examinations generally favored reasoning-based questions over non-reasoning ones. However, non-reasoning questions took the lead during the early post-war period and in 1977, influenced by the state of the educational system following wars and the Cultural Revolution. Overall, there was a long-term shift from non-reasoning to reasoning-based questions from 1952 to 1984, indicating a gradual increase in the reasoning difficulty of the NCEE mathematics examinations.

**Topic Coverage Factor.** Finally, when analyzing the *topic coverage* factor in the NCEE mathematics examinations, a more balanced pattern emerged. The *single-topic* level exhibited the highest mean value (M = 4.09, SD = 3.34), followed by the *two-topic* level (M = 3.55, SD = 2.18), the *three-topic* level (M = 3.14, SD = 1.88), and the *four-or-more-topic* level (M = 2.86, SD = 1.36). These relatively close mean values indicate that the NCEE mathematics
examinations did not favor any level in the *topic coverage* factor. Instead, all topic levels were equally important in the examinations.

In general, as the number of mathematical topics required to solve a question increases, the difficulty of the question also increases because it requires students to integrate and apply more mathematical knowledge to derive the solution. This rationale helps to explain the developmental trends in the *topic coverage* factor levels discussed below. Figure 11 displays the frequency changes for both the low *topic coverage* level (combining the *single-topic* level and *two-topic* level) and the high *topic coverage* level (combining the *three-topic* level and *four-or-more-topic* level). The decision to combine these topic levels was made due to the chaotic and indiscernible trends observed when analyzing the four levels separately.

**Figure 11**

*Topic Coverage Factor Levels’ Frequency in 1952-1984 NCEE Math Exams*

![Graph showing frequency changes for low and high topic coverage levels from 1952 to 1984](image)

*Note.* The horizontal axis represents the years of the examinations, with the years abbreviated (e.g., “1952” abbreviated to “52”). The notation “CR” within the years signifies the Cultural
Revolution period. The vertical axis indicates the number of questions categorized under the low
*topic coverage* level or high *topic coverage* level.

In Figure 1, there is no remarkable increasing or decreasing pattern in the general
developmental trends of the high and low *topic coverage* levels. However, two prominent
features, similar to those observed in Figures 9 and 10, are evident in the graph. First, the
frequency of the low *topic coverage* level was unusually high in 1952 (*f* = 20). Second, in 1977,
the frequency of the low *topic coverage* level increased sharply from 3 to 15, while that of the
high *topic coverage* level decreased from 9 to 1. These unusual trends could be explained by the
same reasons affecting the developmental trends of the *computation* and *reasoning* factors in
Figures 9 and 10.

As previously noted, the years 1952 and 1977 were, respectively, the early post-war and
post-revolutionary periods in China. During these times, the educational system in China was
severely disrupted and the quality of teachers and students was notably poor. Consequently, the
choice of simpler questions that require fewer mathematical topics to solve in the NCEE
mathematics examinations was a logical response to the prevailing educational conditions in
China at that time.

In summary, regarding the *topic coverage* factor, all *topic coverage* levels, ranging from
single to four or more, were equally important in the NCEE mathematics examinations.
However, during the early post-war and post-revolutionary periods, the low *topic coverage* level
(combining the *single* and *two-topic* levels) briefly dominated the NCEE mathematics
examinations. This shift was a response to the deteriorated state of the education system resulting
from the wars and the Cultural Revolution. In the long run, there was no remarkable increase or
decrease in the development trends of both the high and low *topic coverage* levels from 1952 to
1984, indicating a relatively stable topic coverage difficulty in the NCEE mathematics examinations.

**Summary**

In conclusion, a detailed investigation of the NCEE mathematics examinations from 1952 to 1984, based on Bao’s (2006) model, revealed an evolving trend toward increased difficulty, complexity, and diversity. Over time, each factor—discovery, context, computation, reasoning, and topic coverage—has undergone distinctive shifts in response to historical and social changes, academic criticisms, and the evolving capabilities of students. These shifts have collectively influenced the overall difficulty of the examination. Furthermore, the consistency of data results across different factors lends additional credibility to these conclusions.

The following list provides a summary of the findings on the item difficulty of the NCEE mathematics examinations from 1952 to 1984:

1. For the discovery factor, the examinations predominantly featured understanding-level questions that encouraged the application of mathematical knowledge rather than rote learning. However, there was a limited presence of investigating-level questions, presumably due to the traditional view of mathematics in China, which focuses more on solving equations and justifying facts rather than encouraging exploration.

2. Regarding the context factor, the examinations initially focused on purely non-contextual mathematical questions. However, following criticisms of the "high scores but low abilities" phenomenon among students—a term describing students excelling in examinations but struggling with practical applications of their knowledge—the examinations started incorporating more context-rich and application-based questions.
from 1992 onwards. This shift increased the real-world relevance and complexity of the examination.

3. In terms of the computation factor, there was a noticeable shift from numerical to symbolic calculations between 1952 and 1984. This shift indicates an increasing emphasis on complex computational skills and reflects a rise in the computational difficulty of the examination, potentially in response to improved curriculum standards and the quality of Chinese students.

4. As for the reasoning factor, reasoning-based questions were generally more prevalent. However, non-reasoning questions gained prominence following chaotic historical periods such as wars and revolutions. Nevertheless, a steady shift from non-reasoning to reasoning-based questions from 1952 to 1984 suggests a gradual increase in the reasoning difficulty of the examination.

5. Concerning the topic coverage factor, while all topic coverage levels were equally significant most of the time, there were temporary periods where lower topic levels dominated due to disruptions caused by wars and the Cultural Revolution. Overall, the difficulty level in terms of the topic coverage factor remained relatively stable from 1952 to 1984.
Chapter VI

RESULTS FOR BIDIRECTIONAL INFLUENCES

This chapter presents the results of the bidirectional influences between the Chinese National College Entrance Examination (NCEE) and Chinese society during the periods 1952–1965 and 1977–1984. It aims to answer the third research question of this study:

3. What were the mutual influences between the NCEE and Chinese society during the years 1952–1965 and 1977–1984?

Influence on Chinese Society

The influence of the NCEE on Chinese society was organized into the following themes: student behaviors, student attitudes, teacher behaviors, parental attitudes and behaviors, and social relationships. It should be noted that political, societal, and scholarly discussions about the societal impact of the NCEE often consider the examination as a whole, rather than focusing solely on the mathematics examination. Consequently, the evidence, statements, and conclusions presented in this section may be more general than those in the previous sections. Nonetheless, as a mathematics educator, the researcher has supplemented the narrative with numerous examples specifically related to mathematics examinations.

Student Behaviors

The descriptions of student behaviors in the collected sources were grouped into three clusters: Predicting Test Questions, Immersion in Practice Problems, and “Mimeographed Materials Flying All Over the Sky”—a literal translation of a Chinese phrase indicating an excessive use of test study materials.
Predicting Test Questions

“Predicting Test Questions” refers to students making educated guesses about potential questions on upcoming examinations based on actual questions from previous years. Successful predictions are often highlighted as anecdotes or mysteries in news articles or personal memoirs. Such stories were identified in some sources for this study (e.g., Le, 2022; Liang, 1998). Specifically, Le (2022) noted in a news article that while preparing for the 1980 NCEE mathematics examination, he anticipated a theorem-proof question based on its presence in the 1979 examination. His prediction proved accurate, as a question on proving the Change of Base formula appeared in the 1980 NCEE mathematics examination. Having reviewed this concept in advance, Le easily scored 10 points on the test.

A significant reason for the accurate prediction of questions by many students seemed to be the recurrence of similar questions in the examinations. For instance, Item 13 in 1953, Item 1 in 1958, and Item 1 in 1961 all posed similar mathematical questions—finding terms in the algebraic expansion of powers of a binomial. This repetition of similar questions in the NCEE might have contributed to the widespread practice of “predicting test questions” among students.

Immersion in Practice Problems

The term “immersion in practice problems” was adapted from a Chinese phrase to make it more understandable for English-speaking readers. A more literal translation might be “the tactic of problems sea” (akin to the idiom “people mountain people sea”), suggesting that students needed to tackle a sea of practice questions to succeed in the NCEE. This concept was found in existing literature, such as Jiang’s book (2008a, 2008b, 2008c) and Zhu’s (2019) blog, where students are often portrayed as studying overtime or staying up late to complete as many practice problems as possible. Specifically, Li (2021) described in his memoir how some of his
classmates even forwent family reunions during Chinese New Year, choosing instead to stay at school and study for the NCEE.

This intense focus on studying led to two unintended consequences for students: deteriorating health conditions and insufficient participation in productive labor. According to Yang (2007), the first issue sometimes resulted in students being unable to attend college or being forced to drop out midway. The second issue was viewed as a necessary activity that the Communist Party of China (CPC) believed students should engage in at the time. This view is supported by government documents, which highlighted the need for special attention to student health and their participation in productive labor, as illustrated in Yang’s book.

**Mimeographed Materials Flying All Over the Sky**

The phrase “mimeographed materials flying all over the sky,” while exaggerated, vividly depicts the excessive use of test study materials by students. Several pieces of evidence have been identified to support the prevalence of this student behavior during the time periods under investigation. Doldentate1 (2020) mentioned in his memoir that both teachers and students at his school often stayed up late to produce mimeographed study materials. In an interview conducted by Zhai (2017), Ge reported that students in his city were eagerly seeking test study materials, rushing to mimeograph copies as soon as a school released new ones. Liu Zhixin (2018) observed that the mimeographed study materials at his school were even thicker than textbooks.

This widespread reliance on test study materials has led to a significant social and educational concern: an exam-oriented educational system—a system that emphasizes scoring high on examinations and pursuing university enrollment rates, often at the expense of aligning with the actual needs of human and societal development. Historical evidence indicates that Chinese educators and government officials were already concerned about this educational issue
during the periods under investigation. For instance, many higher education institutions reported that their newly admitted students, while adept at acing examinations, lacked innovative skills and struggled with practical problem-solving (Yang, 2007). Similar concerns have been raised by government officials, who worried about whether the quality of these students can meet the standards of socialist production and construction (Yang, 2007).

Summary

This section discussed three student behaviors that were prevalent in Chinese society under the influence of the NCEE and their broader societal impacts during the years 1952–1965 and 1977–1984. “Predicting test questions” refers to the practice of forecasting questions on upcoming tests based on previous examinations. “Immersion in practice problems” entails relentless studying, leading to deteriorating health conditions and a lack of participation in productive labor. The phrase “mimeographed materials flying all over the sky” illustrates the overuse of test study materials, which in turn contributes to an exam-oriented education system. These practices drew criticism from educators and government officials of that era.

Student Attitudes

The historical evidence identified in the sources of this study revealed a notable shift in student attitudes, moving from a preference for liberal arts studies to a preference for science studies within the periods under investigation.

A Preference for Liberal Arts Studies

Firstly, there is a correlation between students’ participation in the NCEE and their preferences for either liberal arts or science studies. Although the mathematics examination is mandatory for all, students have the option to choose between a comprehensive liberal arts test
covering history, politics, and geography, and a comprehensive science test covering physics, chemistry, and biology. This choice allowed the identification of a shift in student preferences.

As introduced in Chapter II, influenced by Confucianism, which traditionally disregards disciplines such as mathematics, science, and technology, Chinese society historically valued liberal arts studies more than science studies (Yang, 2007). This societal value persisted even in the days before the founding of the People’s Republic of China. Supporting evidence was found in a 1932 document stating that, at that time, more than 50% of students were studying law, politics, and liberal arts, 6% were studying education, 11.5% were studying engineering, and less than 10% were studying natural sciences (Fei, 1998). Jiang (2008a) noted that even by 1946, engineering students accounted for only 18.9% of all college students in China.

**The Shift Toward Science Studies**

However, after the establishment of the People’s Republic of China, this traditional value underwent a major shift. According to Gao and Yao (2009), the student distribution, which favored liberal arts over sciences, could not meet the urgent needs of the new China for scientists and engineers to carry out socialist production and construction. Consequently, the Communist Party of China (CPC) and the Chinese government actively promoted the importance of studying science and technology across the country. Evidence of this was found in *People’s Daily*, the official newspaper of the CPC. Hua Luogeng, a renowned Chinese mathematician, wrote articles highlighting the significance of mathematics for national development (Hua, 1953a, 1953b). Yang (2003) also noted in his book that high school teachers at that time were instructed to educate their students about prioritizing national needs over personal aspirations (e.g., their desired majors) and complying with national allocation (e.g., majors that would benefit the construction and development of their motherland).
Years of concerted efforts led to a marked shift in educational priorities. An increasing numbers of Chinese students began choosing science-based majors that aligned with national needs. Notable examples include Wang Yongzhi, who later became a prominent aerospace scientist (Liao, 2004), Qu Qinyue, who emerged as a renowned astrophysicist (Mu, 2008), and Yuan Longping, globally recognized as the “Father of Hybrid Rice” (Liu, 2008). This shift was not confined to students alone; parents, too, began to favor science and engineering majors for their children. For example, Ma (2017) recalled in a blog post that his parents, despite having backgrounds in liberal arts, “looked down upon liberal arts, worshipped sciences” (para. 16) and encouraged their children to study science or engineering.

This shift is also illustrated by a notable change in the distribution of majors chosen by college students. According to the data, in 1955, students studying liberal arts, law, business, and education accounted for only 34.9% of the total, while those studying science, engineering, agriculture, and medicine made up 65.1% (Jiang, 2008a). Over the years, the proverb “If you have studied mathematics, physics, and chemistry well, you can go anywhere without fear” gained widespread popularity (Lu, 2008), and it is still often heard to this day.

**Summary**

This section illustrated how traditional Chinese values, which prioritized liberal arts over science studies, shifted dramatically within the period from 1952 to 1984. Before this transition, less than 30% of all college students in China studied natural sciences and engineering. Recognizing that this scarcity failed to meet the country’s growing need for scientists and engineers, the Chinese government and the Communist Party of China (CPC) initiated a national campaign to promote the importance of science and technology. Years of persistent effort led to a remarkable shift in educational attitudes and choices. By 1955, the distribution of student
majors had substantially changed, with 65.1% of students studying science, engineering, agriculture, and medicine, surpassing those studying liberal arts, law, business, and education. Within approximately ten years, the CPC and the Chinese government successfully reversed the educational attitudes and choices of Chinese students, receiving a student population that is beneficial for the country’s construction and development.

**Teacher Behaviors**

Teacher behaviors are intrinsically linked to those of their students. In the collected sources, descriptions of teacher behaviors were categorized into three clusters: Predicting Test Questions, Preparing Test Study Materials, and Employing an Exam-Oriented Teaching Style.

**Predicting Test Questions**

The practice of “predicting test questions” was not limited to students; teachers also engaged in forecasting questions for the upcoming NCEE to help their students achieve high scores. Historical evidence indicates that teachers approached this task in a more professional and systematic manner compared to students. For instance, some experienced teachers made educated guesses based on their years of teaching experience and a comprehensive understanding of the core knowledge in their subject area (M. Liu, 2018). Other teachers employed statistical methods to analyze past NCEEs and forecast questions based on their analyses (e.g., Cui, 2013). Interestingly, once the test designers were appointed each year, which often included university professors, high school teachers, and scientific researchers, the books they read and published, and even the words they spoke in daily life, became subjects of analysis for teachers and students trying to predict test questions (Cui, 2013; Zhang, 2019).

Teachers’ professionalism is illustrated by their higher success rate in accurately predicting NCEE questions compared to students (e.g., Bei et al., 2019; Hui, 2022). For instance,
Hui (2022) once recalled, “Teachers in Haidian were exceptional. The major questions in the political science examination and the essay question in the Chinese examination matched their predictions. The questions in the mathematics, physics, and chemistry examinations were also very similar to their predictions” (para. 2).

**Preparing Test Study Materials**

“Preparing test study materials” was a crucial task for senior high school teachers instructing grade 12 students. At that time, it was common for schools to allocate their most proficient teaching resources to grade 12, aiming to improve college enrollment rates. Numerous documents confirm that schools assigned their most competent teachers to grade 12 to establish teaching and research groups, design and compile test study materials, and deliver lessons (e.g., Gao & Yao, 2009; Li, 2021; Wang, 2021). In some areas, local governments even concentrated their best teachers into a single school to prepare students for the NCEE, with the goal of enhancing the region’s reputation and influence, should the students achieve high scores. For example, Su (2018) recounted that the education bureau in his region gathered 18 distinguished teachers from various high schools to collaboratively compile test study materials for the NCEE.

Typically, teachers would first analyze the syllabus for the upcoming year’s examination and then select practice problems from textbooks, existing test study materials, and past examinations (Su, 2018; Zhu, 2019). Subsequently, they would categorize these questions into different clusters based on question types or concepts tested, and finally compile the materials into workbooks for students (Su, 2018; Zhu, 2019).

In regions with fewer educational resources, teachers also made considerable efforts to prepare test study materials. Teacher Xiang (2022) described in his memoir how he was dispatched to areas with high test scores to study effective teaching methods and collect test
study materials prepared by local teachers. He would then bring these materials back for his own students to use.

Literature indicates that this centralized approach to test preparation had some unintended consequences. In 1963, the Chinese Ministry of Education noted that concentrating the best teaching resources on grade 12 and engaging in intensive review sessions for the NCEE within one grade could lead students to neglect regular learning in other grades. This approach might foster a habit of last-minute cramming and promoting an opportunistic mindset (Bian, 2019).

**Employing an Exam-Oriented Teaching Style**

The practice of “teaching with an exam-oriented style” was not limited to 12th-grade teachers but was widespread across all high school levels. One hypothesized reason for the prevalence of exam-oriented teaching in China from 1952 to 1984 is as follows. According to Yang (2007) and Jiang (2008a), the higher education institutions in China during those years had limited enrollment quotas, which were insufficient for an ever-increasing number of test candidates. Therefore, it is hypothesized that in an effort to help students distinguish themselves in the high-stakes NCEE and secure admission to their desired universities, teachers gradually adopted an exam-oriented teaching style.

Teachers’ exam-oriented teaching behaviors identified from the sources for this study were further categorized into three clusters:

First, Tian (2001) noted in his study that many teachers selectively omitted topics outlined in the teaching syllabus but absent from the examination syllabus. This practice aligned with the major division system in place at the time. As previously mentioned, students taking the NCEE could choose between a comprehensive liberal arts test and a comprehensive science test. Consequently, many schools prematurely divided students into liberal arts and science tracks.
Liberal arts students often skipped learning physics, chemistry, and biology, while science students avoided history and geography (Li & Wang, 2000). Li and Wang observed that this early specialization led to students being extremely weak in and lacking fundamental knowledge of some subjects, resulting in imbalanced personal development.

Second, teachers commonly accelerated the teaching pace to finish the 12th-grade curriculum ahead of schedule, using the remaining time to help students review for the NCEE. For example, Li (2021) remembered that in 1961, his grade completed the curriculum for the entire academic year by the end of the fall semester, dedicating the entire spring semester to test preparation. An extreme example is documented in Li’s (2007) article, where Huanggang High School selected 23 high-performing, newly admitted students to complete the full high school curriculum in one year, allowing them to spend the remaining years preparing for the NCEE. Impressively, all 23 students were admitted to prestigious universities, with 13 gaining acceptance to top-tier institutions such as Tsinghua University and Peking University (Li, 2007).

Third, teachers frequently emphasized rote learning—such as memorization of concepts and completion of numerous practice problems—over meaningful learning, which involves applying knowledge in real-life situations. Evidence of this is drawn from Wang’s (2020) memoir, in which he recounted: “In mathematics classes, teachers focused solely on defining concepts without providing any practical background or insight into the history of mathematics. For me, learning was a painful experience” (para. 16). Tian (2001) argued that this teaching philosophy could negatively affect Chinese students by contributing to increased stress and emotional strain, potentially leading to issues like school weariness.

Overall, while exam-oriented education might assist students in improving their test scores and gaining admission to their desired schools, its drawbacks attracted the attention of
Chinese scholars and policymakers of that era. In 1983, the Chinese Ministry of Education released a document emphasizing that schools should not overlook students’ moral and physical education, nor the cultivation of practical abilities, in a blind pursuit of higher university enrollment rates (Gao & Yao, 2009).

**Summary**

This section outlined three primary teacher behaviors associated with the significant societal issue of test preparation in China from 1952 to 1984 and their broader societal impacts. These behaviors included predicting test questions, preparing test study materials, and teaching to the test. The practice of “predicting test questions,” though similar to that of students, tended to be more professional and systematic when performed by teachers, leading to a higher likelihood of accurate forecasts for the NCEE. The task of “preparing test study materials” was particularly vital for 12th-grade teachers—who were often the most proficient in their respective subjects—to boost college enrollment rates. Nonetheless, this approach has been criticized for promoting last-minute cramming and fostering an opportunistic mindset among students. Finally, “teaching with an exam-oriented style” was prevalent across all high school grades and led to behaviors such as focusing only on test-relevant topics, accelerating the teaching pace to allow time for review, and emphasizing rote learning over meaningful understanding. While these practices may enhance test performance, they have also attracted criticism and concern from both educators and the government in China.

**Parental Attitudes and Behaviors**

The attitudes of parents toward test preparation in China from 1952 to 1984, as described in the sources of this study, were categorized into three types: those opposing their children
taking the examination, those indifferent to their children’s participation, and those supporting their children in taking the examination.

**Opposing Children Taking the Examination**

The first parental attitude, opposition to their children’s participation in the NCEE, is evidenced in only a few documents. According to Chen (2001) and Fushun News (n.d.), this attitude was predominantly observed in large, less affluent families where parents could not afford to support all their children’s higher education. Consequently, they opposed their children taking the college entrance examination.

Additionally, Mrs. Li recounted that her parents disagreed with her decision to take the NCEE because they believed that girls should learn to farm and marry well rather than attend university (Liu, 2013). This perspective was not unique to Mrs. Li’s case; Qian (2019) noted that even in the more progressive city of Shanghai, most working-class parents still held similar beliefs regarding their daughters’ life goals.

Lastly, an article published in *People’s Daily* reported an extreme case of parents actively preventing their child from taking the NCEE: Xu’s parents, fearing their son would attend a university far from home, hid his examination admission ticket and locked Xu in his room, causing him to miss the NCEE geology examination (Ben, 1983).

In summary, from 1952 to 1984, parental opposition to children taking the NCEE was influenced by various factors such as the number of children in the family, the family’s socioeconomic status, the children’s gender, and the distance of the university from home.

Despite facing opposition and obstruction from their parents, the students mentioned earlier persisted in taking the NCEE secretly and ultimately gained acceptance into their desired universities. In Xu’s case, the presidents at Fudan University, upon learning the reasons for his
absence, carefully reviewed his other NCEE scores and school performance. They eventually decided to make an exception and admit him (Ben, 1983). This story was published in *People’s Daily* with the aim of educating parents in society who prioritize personal interests. The presidents of the Fudan University were quoted in the newspaper as saying, “Admitting this young man exceptionally serves as an education for those in our society who prioritize short-term personal interests and neglect the country’s long-term needs. As parents, they should support their children in acquiring knowledge and skills and encourage them to work where the country needs them most” (Ben, 1983).

Later, with China’s rapid economic growth in the 20th and 21st centuries, Chinese living standards and public transportation have improved substantially. Additionally, China has also become more progressive with women’s rights in society. In other words, factors contributing to parental opposition to children taking the NCEE, such as the family’s socioeconomic status, the children’s gender, and the distance of the university from home, have gradually disappeared. Consequently, this form of parental opposition has become increasingly rare in Chinese society.

*Indifferent Toward Children Taking the Examination*

The second parental attitude, indifference toward their children’s participation in the NCEE, was more prevalent from 1952 to 1984. Many test-takers of that era recalled that taking the NCEE was largely a personal matter, as parents were often preoccupied with their own work and showed limited concern for their children’s education (e.g., Yi, 2022). One most direct and widely discussed manifestation of this attitude involved parents accompanying their children to test centers and waiting outside for the entire duration of the examinations—behaviors often seen nowadays but rarely observed in the past. Many former test-takers, who now have children taking the NCEE, frequently compare past and present experiences in their memoirs.
According to Fan (2020), Jiu (2023), Sun (2014), and Ye (2020), students in the past traveled to the test centers and returned home unaccompanied. Parents neither escorted them to the test centers nor waited outside. In contrast, it has become almost routine for modern parents to escort their children to test centers. Some even rent hotels near the centers to reduce travel fatigue for their children. During the examination, these parents often wait outside the test centers for hours until the examination concludes. These contrasting behaviors suggest that, from 1952 to 1984, parents overall did not place significant emphasis on the NCEE.

Yang and Wu (2018) offered some plausible explanations for this marked difference. They posited that families in the past frequently had multiple children, which might have diluted the level of attention, love, and care each child received. Nowadays, due in part to the One-Child Policy, each family typically has only one child who garners the entire family’s attention. Moreover, the underdeveloped economy in China from 1952 to 1984 meant that many parents had to devote the majority of their energy to work, leaving little time or attention for their children’s education (Yi, 2022). Consequently, this second parental attitude has become less prevalent in Chinese society, influenced by both the implementation of the One-Child Policy in 1982 and the substantial economic development in China.

Supporting Children Taking the Examination

The third parental attitude, support for their children’s participation in the NCEE, has become increasingly prevalent in recent times, as evidenced by parents accompanying their children to test centers, a behavior discussed in the preceding paragraphs. This attitude was also present from 1952 to 1984. According to Zhuang (2008a), Luo Dongjin’s father, whose own college studies were interrupted by wars, deeply regretted not completing his education and thus encouraged his son to attend university. Similarly, Li and Yang, who were the children of
teachers and soldiers, respectively, were educated by their parents since childhood to study hard, pursue higher education, and serve the country well (Li, 2003; Zhuang, 2008b). Moreover, Du (2015) recounted how her mother took her to a class reunion after her initial failure in the NCEE, where everyone encouraged her to retake it. While the parents mentioned above provided ideological and emotional support to their children, other parents offered material assistance.

Journalists Shen and You (2014) and Wu and Zhao (2014) conducted interviews with contemporary parents who participated in the NCEE from 1952 to 1984. They reported that right before the NCEE, Liang and Guo received packages of cotton clothes, eggs, and corn from their parents, aiming to help them improve nutrition and combat cold weather. Wen’s mother saved money over a long time to provide her with chicken soup daily in the month leading up to the NCEE (Wu & Zhao, 2014). Additionally, Mei (2022) mentioned that her mother used the only ingredients available at home to make nutritious meals for her and her sister, both of whom were preparing for the NCEE, while other family members subsisted on coarse grains.

These behaviors suggest that during an era when families in China were not affluent, providing warm clothing and nutritious food was considered as significant support for their children taking the NCEE. Nowadays, with improved living standards and greater economic stability, more diverse forms of support—such as escorting children to test centers and renting nearby hotels—have gained prominence in China. In brief, as the Chinese economy and living standards have improved, supporting children’s participation in the NCEE has increasingly become mainstream in Chinese society, and its manifestations have also become more diverse.

Summary

This section analyzed three types of parental attitudes in China toward their children’s participation in the NCEE from 1952 to 1984: opposition, indifference, and support. Opposition
was primarily observed in large, less affluent families and was motivated by factors such as financial constraints, gender norms, and concerns about children studying far from home. Indifference was more prevalent, as parents were often preoccupied with work, leaving little time or energy to be concerned with their children’s education. Supportive attitudes were also present, manifested through various forms of assistance, including informational support (such as recommendations to pursue higher education), emotional support (as seen in consolation and encouragement to retake the examination despite an initial failure), and material support (such as providing warm clothing and nutritious food). With the implementation of the One-Child Policy in 1982 and China’s remarkable economic growth, the first two attitudes have become less common in Chinese society, while supportive attitudes have become mainstream. Overall, the evolution of parental attitudes toward the NCEE from 1952 to 1984 in China reflected broader social, economic, and political changes, shifting from opposition and indifference to support.

**Social Relationships**

Beyond their natural close contacts, such as parents and teachers, existing literature suggests that examinees also developed closer relationships with other community members, including neighbors and colleagues. This observation highlights increased social connections influenced by the NCEE during the years 1952–1965 and 1977–1984.

For instance, Wang (2022) noted in his memoir that, in addition to parents, siblings, and teachers, his friend’s uncle and his mother’s classmate provided him with various handwritten study materials. Shi also recalled that upon learning he would be taking the NCEE, his boss voluntarily granted him three months of paid leave to review for the examination; a colleague delivered test study materials to him late at night; a neighbor lent him old mathematics textbooks; and a former classmate sent him review materials for liberal arts (Yan, 2023).
Alongside test preparation materials, test candidates also received life assistance, as Zheng (2017) emphasized in his memoir, mentioning the delicious meals prepared by his neighbor, Auntie Zhang, during the two-day NCEE. These examples show that the NCEE led to enhanced interactions and support networks between students and those around them.

Similar supportive behaviors were described earlier, where parents offered both tangible and intangible support, such as advice and consolation. Following are examples of colleagues and neighbors providing intangible support to examinees. Yuan (2020) recounted how one of his colleagues became his chemistry tutor, and when he was accepted into college, the entire factory staff held a celebration party for him. Xin (2017) mentioned that her neighbor introduced her to a highly experienced mathematics teacher, significantly improving her test performance and increasing her interest in mathematics (Xin, 2017).

In general, these examples suggest that NCEE test-takers from 1952 to 1984 engaged in closer interactions with individuals beyond their parents and teachers to obtain study materials and educational resources. As Wang (2022) vividly described: “One person takes the test, everyone around helps” (paras. 108).

However, many of the supportive behaviors mentioned above have become less common in recent decades, owing to China’s rapid advancements in economy, education, science, and technology, along with improved living standards. For instance, the widespread availability of bookstores offering diverse test preparation materials eliminates the need for students to seek help from others. Similarly, the prevalence of restaurants and food delivery services makes it easier for students to access nutritious meals. Numerous tutoring companies staffed by experienced teachers further reduce the need for students to rely on community support for their academic needs. These examples do not suggest that supportive behaviors between neighbors
and colleagues no longer exist; rather, they indicate that Chinese people now have additional options. Overall, China’s economic and technological progress has enabled Chinese people to fulfill many of their needs independently, thereby reducing the impetus to seek help from others.

**Summary**

This section highlighted a historical increase in social communications and interactions among Chinese students and their community members, stemming from their participation in the NCEE from 1952–1965 and 1977–1984. In an era marked by a scarcity of educational resources, students received a wide array of both tangible and intangible support from those around them. This support included test study materials, meals, and tutoring, all of which contributed to enriched social interactions and communications within China. The phrase, “One person takes the test, everyone around helps,” encapsulates this communal effort. However, China’s rapid economic, educational, and technological growth in recent decades has altered this trend. The increased availability of educational resources, food services, and tutoring businesses has reduced the need for students to seek assistance from others.

**Influence on the Examinations**

An investigation into the sources of this study revealed that historically, Chinese leaders and political factions held diverse attitudes toward the NCEE. These attitudes significantly influenced the Chinese people’s views toward the NCEE, playing crucial roles in the abolition and restoration of the NCEE system, and more broadly, in the onset and conclusion of the Cultural Revolution. Based on these historical facts, this section is structured to emphasize these differing perspectives, specifically focusing on the influence of Mao Zedong, the Gang of Four, Zhou Enlai, and Deng Xiaoping.
As the supreme leader of the People’s Republic of China, Chairman Mao’s views and statements had a profound impact on Chinese society. In 1945, the Communist Party of China established Mao Zedong Thought as its guiding principle (“Mao Zedong Thought,” 2023). Mao Zedong Thought encompasses a series of political, military, and developmental theories developed by Mao Zedong and the Communist Party of China. These theories are grounded in Marxism-Leninism and its application in China’s socialist revolution (“Mao Zedong Thought,” 2023). Mao’s statements were frequently published in newspapers, encouraging the masses to study them (“Chairman Mao’s Quotations,” 2023). In 1964, a compilation of Mao’s speeches and writings was released as a pamphlet titled Quotations from Chairman Mao Tse-tung. At that time, this pamphlet, widely recognized by its red cover, was not only carried by almost every Chinese but also gained prominence internationally, becoming known as the “little red book” (“Chairman Mao’s Quotations,” 2023).


To forestall a resurgence of capitalism in China, it was particularly important to stabilize the mood of the masses and strengthen their determination for socialist construction. This made
the educational sector a focus for political leaders due to its function of influencing public opinion about civil society and governing bodies from a young age (Cox, 2011).

Since 1956, Mao Zedong frequently gave speeches on educational matters. His views on the educational system, curriculum, and examination methods were summarized as follows. Firstly, Mao believed that the protracted school system limited students’ exposure to the real world, hindered their personality development, and prevented them from growing into well-rounded individuals (Jiang, 2008a). Secondly, Mao expressed that the extensive curriculum content imposed both physical and mental burdens on students, restricting their participation in productive labor and social practice (Jiang, 2008a). Thirdly, Mao contended that the primary obstacle to reducing curriculum content and simplifying instruction lay in the examination system. In his view, when education becomes exam-centric, the examinations dictate both the instructional approach and the teaching content (Jiang, 2008a). These perspectives suggest a critical view held by Mao toward China’s education and examination systems at the time.

Mao’s critique of the examination system is further illustrated through the following quotations from him. The first quotation is from an educational work seminar held on February 13, 1964. Attendees included Mao Zedong, Liu Shaoqi, Deng Xiaoping, and 13 other committee members of the Communist Party of China, as well as leading figures from the education sector. During the meeting, Mao stated:

The examinations we have now treat our students as enemies, not compatriots, by posing bizarre and off-topic questions. They are analogous to the eight-legged essays of ancient China. I do not support this form of testing and believe it needs complete reform. I advocate making the examination questions public, allowing students to research and refer to books for their answers. For example, if there are 20 questions and a student can
answer 10 exceptionally well with creativity, he or she should receive a full 100 points. Another student who answers all 20 questions correctly but merely copies answers from textbooks or parrots the teacher’s words might only receive 50 or 60 points. (Jiang, 2008a, p. 548)

In this statement, Mao described the NCEE as the enemy of the students, reflecting his strong disapproval of examination methods at the time. Additionally, it reveals Mao’s vision for a comprehensive reform of the examination system. This vision differed fundamentally from that of the Gang of Four, which will be discussed shortly afterward.

The second quotation comes from a conversation on July 5, 1964, between Mao Zedong and his nephew, Mao Yuanxin. Mao Zedong stated:

The current education system always urges students to strive for every point in examinations. However, some students see beyond the test scores and bravely pursue what interests them most. It is said that there is a student in a certain university who does not take notes in class and only scores three or four points in examinations, yet his graduation thesis is the best in the class. Being top in examinations does not necessarily mean being top in work… Do not overemphasize scores. Instead, focus on cultivating the ability to analyze and solve practical problems. Do not blindly follow instructors and lose the initiative to learn just for the sake of chasing those bizarre and off-topic questions for an extra 5 points in examinations. (CCCPC Party Literature Research Office, 1996b, p. 96)

This conversation reflects Mao’s criticism of the exam-oriented student learning behaviors that were prevailing in Chinese society at the time.
Mao Zedong’s remarks and criticisms of the education and examination systems quickly spread across the entire country through various conferences, symposia, and seminars. A major consequence of this widespread dissemination was that an increasing number of government officials, educators, and students joined in the criticism of these systems, ultimately elevating it to a national scale. Notably, this quick and effective dissemination was inseparable from Mao’s strong political image in Chinese society. On May 16, 1966, Mao launched the Great Proletarian Cultural Revolution. The primary intent of the revolution was to critique bourgeois ideologies in academia, education, journalism, the arts, and publishing, and to assume leadership in these sectors (Yang, 2007).

The following criticisms are drawn from the minutes of a 1966 symposium that discussed college admissions, reflecting a national-scale discussion of the issues in the examination system. The symposium’s attendees included representatives from the Ministry of Higher Education, the Ministry of Education, the Ministry of Health, the Commission for Science, Technology, and Industry for National Defense, education bureaus of ten provinces and cities, ten universities, and eight high schools.

Firstly, test scores had become the primary standard for assessing student quality and for determining their post-graduation job placements, resulting in enormous pressure on students. The pressure manifested in various fears: fear of public opinion, fear of parental complaints, fear of teacher dissatisfaction, fear of repeating grades or dropping out, fear of not gaining college admission, and fear of adverse job placements (Yang, 2003).

Secondly, the best high schools were located in major cities. They led the pursuit of high college enrollment rates and discouraged students from engaging in productive labor in rural areas. These schools were viewed as nurturing successors of the ruling class (Yang, 2003).
Thirdly, the grading of the NCEE was dominated by academic experts with bourgeois backgrounds, who taught the lessons, designed the examinations, assigned the scores, and thus exercised extensive control over students’ futures (Yang, 2003).

Upon receiving similar feedback from various regions, the Ministry of Higher Education drafted a report in 1966 addressing various reforms regarding the NCEE system. According to the report, students who enrolled that year would be assigned roles not limited to intellectual work but could also include manual labor upon graduation (Yang, 2003). This aimed to challenge the assumption that university attendance guaranteed a career solely in intellectual work. Moreover, the examination that year would emphasize assessing students’ understanding and application of knowledge, rather than solely relying on memorization. Creative answers would receive bonus points (Yang, 2003). This change seemed to respond to Mao’s suggestion, as illustrated in the first quotation. It is important to note that the proposal from the Ministry of Higher Education pertained solely to reforms of the NCEE system, not to its abolition.

**The Influence of the Gang of Four**

The Central Cultural Revolution Group (CCRG) was the leading group for the Great Proletarian Cultural Revolution (Yang, 2007). The CCRG primarily consisted of radical supporters of Mao Zedong, including Chen Boda, Mao’s wife Jiang Qing, Kang Sheng, Yao Wenyuan, and Zhang Chunqiao (“Cultural Revolution Group,” 2023).

The Gang of Four was a political faction later convicted of implementing harsh policies during the Great Proletarian Cultural Revolution (“Gang of Four,” 2023). The members of the gang included Jiang Qing, Zhang Chunqiao, Wang Hongwen, and Yao Wenyuan, three of whom were members of the Cultural Revolution Group. During the revolution, they were promoted to higher positions within the party and controlled areas such as intellectual education, party
policies regarding intellectuals, and mass media and propaganda outlets ("Gang of Four," 2023). In 1976, the Gang of Four was arrested and sentenced to imprisonment ("Gang of Four," 2023).

The suggested reforms did not meet the expectations of the CCRG, the command group overseeing the Cultural Revolution, whose core members included the Gang of Four. Under the guise of students, they fabricated a letter addressed to the Central Committee of the Communist Party of China (CCCPC) and Chairman Mao on June 6, 1966, calling for the abolition of the NCEE system. The letter stated:

The NCEE system is a continuation of the Keju examination system that persisted for thousands of years in the old feudal society. Its aim is to nurture successors for the ruling class while depriving the working class of educational opportunities. It should, therefore, be abolished immediately. (Yang, 2003, pp. 616-619)

On June 11, they drafted another proposal, posing as revolutionary teachers and students, and addressed it to all teachers and students. This proposal called for the immediate termination of the NCEE system and suggested that high school graduates work directly with workers, peasants, and soldiers (Yang, 2003). They proclaimed, “What we aim to shatter is not just an examination system, but also the cultural norms that have constrained society for thousands of years—the very foundation that produces intellectual aristocrats” (Yang, 2003, pp. 619-621).

In these letters, the Gang of Four depicted the NCEE as an instrument to “nurture successors for the ruling class,” deprive the working class of educational opportunities, and “constrain society” (Yang, 2003, pp. 616-621). Their words and sentences were carefully crafted to provoke the audience’s resistance toward the NCEE system. Influenced by these letters, the CCCPC and the State Council decided to postpone the 1966 NCEE by six months.
To further advance their agenda of abolishing the NCEE system, the Gang of Four published both letters in the People’s Daily, the official newspaper of the CCCPC. This strategy, a common propaganda technique of the era, sought to cultivate a nationwide atmosphere conducive to revolutionary change (Jiang, 2008b). The approach quickly proved effective; an increasing number of the public expressed dissatisfaction with the education and examination systems through various channels, including letters, posters, and publications (Jiang, 2008b).

The strategy later became a prominent revolutionary tactic during the Cultural Revolution. Jiang (2008b) noted that this grassroots form of activism negated the need for centralized leadership and successfully galvanized public sentiment. In conversations with foreign delegates, Chairman Mao Zedong expressed enthusiasm for this method, stating, “We have finally discovered a way to mobilize the masses from the bottom up, enabling them to comprehensively expose the darker aspects of society” (Song, 2018, sec. 26). Responding to growing public calls for the abolition of the NCEE system, the CCCPC and the State Council announced the official discontinuation of the NCEE system on July 23, 1966.

The Influence of Zhou Enlai and Deng Xiaoping

As a member of the Central Committee of the Communist Party of China, Zhou Enlai was also an influential political figure in China. His contributions to the country include, but are not limited to, (1) emphasizing economic construction as a top priority in socialist construction, and (2) underscoring the role that science, technology, and intellectuals play in socialist construction (“Zhou Enlai,” 2023). Zhou Enlai’s position on intellectuals differed from that of Mao Zedong. In 1962, during the Third Session of the 2nd National People’s Congress, Zhou remarked that, in the preceding years, “bourgeois intellectuals” had contributed positively to the community under the leadership of the CPC (National Institute of Education Sciences, 1984). As
a result, he suggested they be recognized as allies and be approached with trust and consideration (National Institute of Education Sciences, 1984). Zhou’s care and support for the intellectuals stemmed from his perception of intellectuals being an indispensable and significant force for the socialist construction (“Zhou Enlai,” 2023).

Deng Xiaoping is generally considered the supreme leader of the People’s Republic of China following Mao Zedong’s presidency (1949–1976) (“Deng Xiaoping,” 2023). His primary contributions to the country include, but are not limited to, the “Reform and Opening-Up” policy and Deng Xiaoping Theory (“Deng Xiaoping,” 2023). The “Reform and Opening-Up” policy refers to China’s opening up of the country to foreign investment and permission for entrepreneurs to start businesses, leading to significant economic growth for the country (“Chinese Economic Reform,” 2023). Deng Xiaoping Theory encompasses a series of political, economic, and military theories developed primarily by Deng Xiaoping. These theories are considered to be an adaptation of Marxism-Leninism and Mao Zedong Thought to the existing socioeconomic conditions of China (“Deng Xiaoping Theory,” 2023). In 1997, the Communist Party of China established Deng Xiaoping Theory as one of its guiding principles (“Deng Xiaoping Lilun,” 2023).

In a period surrounded by criticisms toward the education and examination systems, Zhou Enlai and Deng Xiaoping expressed reservations about the Cultural Revolution. In 1971, Zhou remarked, “People have nowhere to study now. This is a complete ideological monopoly, not socialist democracy. Most intellectuals accept the leadership of the CPC and serve socialism. We must analyze the political backgrounds of teachers and students individually, rather than stereotype an entire group” (CCCPC Party Literature Research Office, 1997, pp. 268, 292). His statements, however, were largely ignored by the Gang of Four.
By 1975, Deng Xiaoping commented on the decline in educational standards, saying, “Teachers lack enthusiasm, and students are not focused on their studies; the quality of teaching has diminished. How are we supposed to achieve the Four Modernizations?” (CCCPC Party Literature Research Office, 2004, p. 91). He also observed, “Our scientific research teams have been significantly weakened” (CCCPC Party Literature Research Office, 1994, p. 32). He further clarified Mao’s stance on education, adding, “What Mao opposed was the detachment of study from society, the masses, and labor. He did not discourage studying; instead, he promoted better ways to study” (CCCPC Party Literature Research Office, 1994, pp. 36-37).

Zhou and Deng’s words somehow awakened the masses, leading them to reassess their attitudes toward the Cultural Revolution. Detecting a trend against their will, the Gang of Four resumed their old strategies to mobilize public sentiment, particularly among student groups, against the acquisition of cultural knowledge and the study of scientific theories (Jiang, 2008b). However, the efficacy of these tactics seemed to wane as the public, encouraged by the words of Zhou and Deng, began to realize the negative ramifications of the Cultural Revolution on Chinese society (Yang, 2007). This shift in public sentiment became especially noticeable during the Tiananmen Incident in 1976, an event distinct from the more widely-known 1989 Tiananmen Square Protests. Following the death of Zhou Enlai in 1976, large crowds gathered in Tiananmen Square to mourn Zhou and express their disapproval of the Gang of Four (Yang, 2007). The Cultural Revolution finally came to an end on October 6, 1976, with the arrest of the Gang of Four.

In 1977, during the Third Plenary Session of the 10th CCCPC, Deng Xiaoping was reinstated to all his former positions within the Party. His primary objective was to rectify the misconceptions propagated during the Cultural Revolution that had hampered the development
of China’s education system (Yang, 2007). He frequently delivered speeches, urging the masses to “respect knowledge and respect talents” (CCCPC Party Literature Research Office, 1994, p. 40). He particularly emphasized that, “those who engage in intellectual work are also socialist laborers and should not be stereotyped as enemies of the proletariat” (CCCPC Party Literature Research Office, 1994, p. 41). He also clarified misinterpretations of Mao Zedong’s thoughts, asserting, “Mao’s criticism of intellectuals was intended to encourage them to elevate themselves, enrich their thoughts, and better serve socialist society. However, this was distorted by the Gang of Four to persecute intellectuals” (CCCPC Party Literature Research Office, 1994, pp. 42-47). Deng’s statements effectively altered the public’s perception of intellectuals and led the way for reforms in China’s education and examination systems (Yang, 2007).

In 1978, Deng Xiaoping delivered a seminal speech on examinations. Many of the viewpoints he expressed in this speech later served as guiding principles for subsequent research and reforms of the NCEE system. Yang (2003) summarized Deng’s primary points as follows:

1. While the negative impacts of the NCEE on society are inevitable, its positive effects should not be disregarded, as was done during the Cultural Revolution; nor should the system be abolished entirely.

2. The NCEE should not serve as the sole criterion for university admissions.

3. The content and organization of the NCEE need continuous reforms.

4. Examination reforms should proceed in three stages: basic research, applied research, and dissemination.

5. The aim of these reforms is to enhance the positive effects of the examination while mitigating its negative impacts.
6. The conclusion of the NCEE does not signify a definitive end. Subsequent efforts are equally essential, such as supporting and motivating students who did not achieve favorable results.

Deng’s words provided the Chinese with new insights and solutions to address the social ramifications of the NCEE system. Although the system has produced some adverse effects, such as promoting an exam-oriented educational culture, it would be imprudent to abolish it entirely. A more balanced approach would involve ongoing reforms to enhance its strengths and mitigate its weaknesses. For instance, to address the tendency for students to rely excessively on test preparation materials, examination creators could avoid designing similar questions for future examinations. Schools could also enhance their supervision of teachers and students, educating them about the pitfalls of exam-oriented learning. In summary, there are numerous approaches to address the societal challenges posed by the NCEE system without resorting to its complete abolition.

After 1984, despite the ongoing challenges presented by the NCEE system, government officials and educational specialists began to adopt a more constructive approach. Through sustained reforms and innovations, such as the introduction of the “3 + 2” system in 1992 and the “3 + X” system in 1999, efforts have been underway to develop a more refined examination system for China.

Summary

This section revealed that different views toward the NCEE system held by Mao Zedong, the Gang of Four, Zhou Enlai, and Deng Xiaoping played significant roles in the abolition and restoration of the NCEE system, and more broadly, in the onset and conclusion of the Cultural Revolution.
Initially, Mao Zedong’s criticisms of the education and examination systems, widely disseminated through speeches and meetings, facilitated the onset of the Cultural Revolution and the subsequent reforms of the NCEE. However, the reforms alone were not sufficient for the Gang of Four. By disguising themselves as teachers and students, forging letters, and mobilizing the masses, they managed to abolish the NCEE system by 1976.

During the revolution, significant counter-voices emerged. Zhou Enlai openly criticized the ideological monopoly, emphasizing the importance of intellectuals for socialism. Deng Xiaoping shed light on Mao’s generally positive stance on education, which the Gang of Four had distorted for their own purposes. Public perception, influenced by these contrasting viewpoints, underwent a significant transformation. Initially swayed by the Gang of Four’s propaganda, the public later grew more discerning, especially after witnessing the disasters caused by the Cultural Revolution.

By 1977, the downfall of the Gang of Four and the conclusion of the Cultural Revolution marked a turning point. Deng Xiaoping’s reinstatement and his 1978 speech underscored guiding principles for the NCEE system, emphasizing its significance and advocating continual reform rather than outright abolition. From 1984 onwards, despite ongoing challenges, the government and educational specialists pursued more constructive efforts to improve the NCEE system, introducing new methodologies and structures to better serve the needs of Chinese society.

**Conclusion**

In conclusion, from 1952 to 1984, the NCEE system significantly influenced Chinese society. Both student and teacher behaviors increasingly shifted toward an exam-oriented approach. While this led to improved test scores, it also raised concerns about educational
quality. Over this period, societal values transitioned from traditional views favoring the liberal arts to a preference for science and engineering. Parental attitudes varied, ranging from opposition and indifference to support, with a discernible shift toward the latter over time. The broader community also rallied behind the students, offering various forms of assistance. Criticisms of the negative societal impacts of the NCEE system contributed to the onset of the Cultural Revolution. These critiques were amplified by the Gang of Four to abolish the system. The revolution concluded with the gang’s downfall and Deng Xiaoping’s return, offering new perspectives on the NCEE system and advocating for reform rather than abolition.

Overall, the NCEE from 1952 to 1984 served as both a reflection and an influencer of Chinese society. Their impacts extended beyond the academic realm, reaching into the social, political, and cultural fabric of the nation. As China continues to develop, the lessons from this period provide crucial insights into the delicate balance between educational priorities, social demands, and political ideologies.
Chapter VII

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

This research examined the Chinese National College Entrance Examination (NCEE) in mathematics from 1952–1965 and 1977–1984. The impetus for this study stemmed from the broad implications of high-stakes mathematics examinations and an existing gap in scholarly literature regarding the evolution of older NCEE mathematics examinations. The historical period under investigation was interrupted by the Great Proletarian Cultural Revolution. This sociopolitical movement from 1966 to 1976 was marked by its anti-intellectual stance and had significant ramifications for China’s educational and examination systems. The central focus of this research revolved around the organization, structure, and content of these examinations, as well as the mutual influences between these examinations and Chinese society.

The major sources for this study included NCEE mathematics examinations and their administrative details and solution manuals; examination syllabi; newspaper articles; official documents; scholarly books; personal memoirs; and online commentaries.

The analysis of the organization and structure focused on the test length and the total number of questions, including the breakdown of each type, such as multiple-choice, fill-in-the-blank, and free-response questions.

The analysis of the mathematics content focused on topic coverage and item difficulty. Topic coverage was examined by analyzing both the general and detailed mathematics topics included. Item difficulty was assessed using the five criteria outlined in Bao’s (2002) composite difficulty model.
The analysis of the mutual influences examined changes in student, teacher, and parental attitudes and behaviors, as well as shifts in social relationships and movements.

This research employed a mixed methodology approach, primarily comprising three procedures: (1) scrutinizing the sources and coding the information into Excel spreadsheets, (2) organizing the data into meaningful clusters and tracking trends and changes through tables, graphs, or textual descriptions, and (3) synthesizing the findings to formulate reasoned conclusions about the historical developments of examinations.

The following section presents the findings of this research.

**Conclusions**

1. How did the organization and structure of the NCEE mathematics examinations evolve during the years 1952–1965 and 1977–1984?

Test Length: Except for the 1952 examination, which had a duration of 100 minutes, all remaining examinations lasted 120 minutes.

Number of Questions: (1) A marked reduction was identified between 1952 (with 24 questions) and 1955 (with 8 questions). This decline could be due to the test-makers’ initial oversight of regional differences when designing a uniform examination for the first time, or an intentional lowering of examination standards to improve college enrollment. (2) Conversely, there was a notable increase from 1979 (with 12 questions) to 1984 (with 22 questions), which could be attributed to the introduction of multiple-choice and fill-in-the-blank questions in the examinations.

Question Types: Prior to 1983, free-response questions were the only format used. In 1983, multiple-choice questions were introduced, followed by fill-in-the-blank questions in
1984. The introduction of these new formats was a result of the international collaboration between Chinese scholars and representatives from the Educational Testing Service (ETS).

2. How did the content of the NCEE mathematics examinations change during the periods 1952–1965 and 1977–1984?

General Topic Coverage: Before 1966, the focus was on equations and functions, two-dimensional geometry, trigonometry, numbers and expressions, and three-dimensional geometry. After 1976, the focus shifted to equations and functions, numbers and expressions, trigonometry, two-dimensional geometry, and analytical geometry.

Detailed Topic Coverage: Before 1966, the emphasis was on quadratic equations and functions, trigonometric ratios, triangles, circles, factorization, and the Pythagorean theorem. After 1976, the emphasis shifted to quadratic equations and functions, trigonometric ratios, inequalities, factorization, conic sections, and trigonometric identities.

Changes and Reasons: After 1976, there was a shift in topic coverage from traditional ones, such as the Pythagorean theorem and three-dimensional geometric figures, toward more modern ones, such as inequalities, conic sections, combinatorics, and calculus. This shift aligned with the curriculum concerns raised by Chinese mathematicians at the time, who advocated for educational content to evolve in tandem with societal and human development.

Item Difficulty: From 1952 to 1984, the examinations generally favored application-based, purely mathematical, symbolic-computation, and reasoning-based questions. There was a notable shift during this period from numerical to symbolic computation and from non-reasoning to reasoning-based questions. This shift could be attributed to long-term improvements in student quality and the introduction of more advanced topics in the examinations. Notably, in 1952, 1953, and 1977, numerical computation and non-reasoning-based questions were more prevalent,
and there was a narrower range of topics required for solving questions. These years marked the post-war and post-revolutionary periods, during which the education system was recovering from disruption, and the quality of teachers and students was generally lower and less satisfactory.

3. What were the mutual influences between the NCEE and Chinese society during the years 1952–1965 and 1977–1984?

Influence on People and Society: Teaching and learning became exam-oriented. Societal attitudes shifted from favoring the liberal arts to emphasizing science and engineering. Parental attitudes and behaviors toward their children taking the NCEE manifested in three distinct ways: opposition, indifference, and support. Community attitudes and behaviors were generally supportive, indicating strengthened social connections.

Influence on the NCEE: Since 1956, Chairman Mao Zedong had expressed strong criticisms toward the examination system. In 1966, the political faction known as the Gang of Four took more radical approaches, ultimately managing to abolish the NCEE system that year. In contrast, political leaders Zhou Enlai and Deng Xiaoping advocated for more constructive approaches, suggesting reforms rather than complete abolition. Their efforts culminated in the restoration of the NCEE system in 1977. These differing stances on the NCEE system played influential roles in both the onset and conclusion of the Cultural Revolution.

**Recommendations**

The following are some recommendations for improvement and future studies:

Firstly, fellow researchers considering a similar line of inquiry should recognize the challenges associated with obtaining historical documents, especially those from historically
tumultuous periods. The discovery of additional documents could substantiate the analysis conducted in this study and lend greater credence to the research findings.

Secondly, the item difficulty model employed in this study, while the best option available, may warrant further justification. Some examiners of this research argued that the model measures item complexity rather than difficulty. Others contended that using question solutions to analyze item difficulty might lead to inconsistent and non-replicable results due to variability in problem-solving approaches. Discovering new models or improving existing ones could benefit future analyses of item difficulty in mathematics examinations.

Thirdly, the coding procedures implemented in this research could be further justified. Given the considerable time demands of the coding process, this research enlisted four individuals to code only partial examinations. Comprehensive coding by a larger team could enhance the study’s overall reliability.

Fourthly, educators and students should be aware of the negative ramifications of an exam-oriented educational approach. Specifically, students should understand that this approach can limit the development of essential practical skills, such as critical thinking, problem-solving, creativity, communication, and teamwork. Meanwhile, teachers should be mindful of the mental stress and emotional strain imposed by extensive test preparation on students.

Fifthly, examination designers should avoid creating similar items repeatedly, as this promotes strategic prediction among teachers and students and leads to an overreliance on test preparation materials. The appendix of this research provides a collection of mathematics examinations available for various practical uses, including scientific research, examination design, and teaching and learning practices.
Lastly, policymakers should be aware of the unintended consequences associated with the abolition of established examination and educational systems. Proactively pursuing continuous reforms to address emerging challenges could be a more effective strategy. It is hoped that the Chinese experience, as presented in this study, can offer valuable insights for such decisions.
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Appendix A
National College Entrance Mathematics Examinations

This appendix includes the 1952-1965 and 1977-1984 NCEE mathematics examinations with administrative information and solutions, administered by the National Education Examinations Authority. (Source: Baidu.com)

1952 National College Entrance Examination

Mathematics

Date and time: Aug 15, 9:30-11:10

1. Factoring the expression $x^4 - y^4$.
   Answer: $x^4 - y^4 = (x^2 - y^2)(x^2 + y^2) = (x + y)(x - y)(x^2 + y^2)$.

2. Solve the equation $\log 2x = 2 \log x$.
   Answer: $\log 2x = 2 \log x \rightarrow \log 2x = \log x^2 \rightarrow 2x = x^2 \rightarrow x = 2 \text{ or } x = 0 \text{ (eliminate)}$.

3. Three roots of the equation $x^3 + bx^2 + cx + d = 0$ are 1, $-\frac{1}{2}$, and $\frac{1}{2}$. Find $c$’s value.
   Answer: By Vieta’s formula, we have $c = 1 \times (-1) + 1 \times \frac{1}{2} + (-1) \times \frac{1}{2} = -1$.

4. Solve the equation $\sqrt{x^2 + 7} - 4 = 0$.
   Answer: $\sqrt{x^2 + 7} - 4 = 0 \rightarrow x^2 + 7 = 16 \rightarrow x^2 = 9 \rightarrow x = \pm 3$.
   Verifying both roots reveals that both roots are correct.

5. Find the value of $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 3 & 2 & 1 \end{vmatrix}$.
   Answer: $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 0 \\ 3 & 2 & 1 \end{vmatrix} = 5 + 24 - 45 - 8 = -24$. 

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6. Two circles have a common radius of 4, and one circle passes through the center of the other. What is the length of their common chord?

Answer: Let the length of their common chord be L, then \((L/2)^2 + 2^2 = 4^2 \rightarrow L = 4\sqrt{3}\).

7. If the area of \(\Delta ABC\) is 60, \(M\) is the midpoint of \(AB\), \(N\) is the midpoint of \(AC\), what is the area of \(\Delta ANM\)?

Answer: \(\frac{S_{\Delta ANM}}{S_{\Delta ABC}} = \left(\frac{AM}{AB}\right)^2 = \frac{1}{4}, S_{\Delta ANM} = \frac{60}{4} = 15\).

8. What is the measure of a regular decagon’s interior angle.

Answer: \((10 - 2) \times 180^\circ \div 10 = 144^\circ\).

9. What is the value of \(\pi\).

Answer: 3.14159265\cdots, \(\frac{22}{7}\), or \(\frac{355}{113}\).

10. How many times the surface area of a sphere is the area of its great circle?

Answer: \(S_{\text{sphere}} = 4\pi R^2, S_{\text{circle}} = \pi R^2\). The solution is 4.

11. If a right cone’s base radius is 3, and its slant height is 5, what is its volume?

Answer: \(h = 4, V = \frac{1}{3} \pi R^2 h = 12\pi\).

12. How many different regular polyhedrons in total and what is their names?

Answer: 5. Tetrahedron, Cube, Octahedron, Dodecahedron, and Icosahedron.

13. If \(\sin \theta = \frac{1}{3}\), what is the value of \(\cos 2\theta\).

Answer: \(\cos 2\theta = 1 - 2\sin^2 \theta = \frac{7}{9}\).

14. Find the general solution of the equation \(\tan 2x = 2\).

Answer: \(2x = k\pi + \frac{\pi}{4}, x = \frac{k\pi}{2} + \frac{\pi}{8} (k \epsilon \Z)\).

15. When the angle of elevation is 30 degrees, the length of tower’s shadow is 5. What is the height of the tower?
Answer: \( x = 5 \tan 30^\circ = \frac{5\sqrt{3}}{3} \).

16. In \( \triangle ABC \), \( b = 3, c = 4, \angle A = 30^\circ \). Find the area of \( \triangle ABC \).
Answer: \( S_{\triangle ABC} = \frac{1}{2} bc \sin A = 3 \).

17. A straight line passes through the point (2, 3) with a slope of \(-1\). Find its equation.
Answer: \( y - 3 = -(x - 2) \).

18. If the origin is on the circle, and the circle’s center is (3, 4). Find the circle’s equation.
Answer: \( R = \sqrt{3^2 + 4^2} = 5 \). The equation is \((x - 3)^2 + (y - 4)^2 = 25\).

19. Find the distance between the origin and the equation \( 3x + 4y + 1 = 0 \).
Answer: \( d = \frac{|3 \times 0 + 4 \times 0 + 1|}{\sqrt{9 + 16}} = \frac{1}{5} \).

20. Find the vertex of the quadratic equation \( y^2 - 8x + 6y + 17 = 0 \).
Answer: \( (y + 3)^2 = 8(x - 1) \). The solution is \((1, -3)\).

21. Solving the equation \( x^4 + 5x^3 - 7x^2 - 8x - 12 = 0 \).
Answer: Let \( f(x) = x^4 + 5x^3 - 7x^2 - 8x - 12 = 0 \).
We have \( f(2) = 0 \) and \( f(-6) = 0 \), therefore,
\[
x^4 + 5x^3 - 7x^2 - 8x - 12 = (x - 2)(x + 6)(x^2 + x + 1) = 0
\]
Solving the equation, we get \( x_1 = 2, x_2 = -6, x_{3,4} = \frac{-1 \pm \sqrt{3}i}{2} \).

22. In \( \triangle ABC \), the exterior bisector of \( \angle A \) intersects the circumscribed circle of \( \triangle ABC \) at \( D \).
Prove: \( BD = CD \).
Answer: \( \angle 1 = \angle 2 \) (AD is the Angle Bisector)
\( \angle 2 = \angle 3 \) (Vertical Angles)
\( \angle 1 = \angle 5 \) (Inscribed Angle Theorem)
\( \angle 3 = \angle 4 \) (Inscribed Quadrilateral Theorem)
\[ \angle 4 = \angle 5 \text{ (Transitive Property)} \]

\[ BD = CD \text{ (Isosceles Triangle Theorem)} \]

**Figure A1**

*Graph for Item 22 in the 1952 NCEE Math Exam*

23. In \( \triangle ABC \), \( a = 4 \), \( b = 5 \), and \( c = 6 \). Find the values of \( \cos C \), \( \sin C \), \( \sin B \), and \( \sin A \). Whether \( \angle A \), \( \angle B \), and \( \angle C \) are acute angles or obtuse angles?

**Answer:**

\[
\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{16 + 25 - 36}{40} = \frac{1}{8}, \quad \angle C \text{ is an acute angle.}
\]

Since \( C \) is the biggest angle, \( A \) and \( B \) are both acute angles.

\[
\sin C = \sqrt{1 - \cos^2 C} = \frac{3\sqrt{7}}{8}
\]

Then by Law of Sines \( \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \), we get \( \sin B = \frac{5\sqrt{7}}{16} \) and \( \sin A = \frac{\sqrt{7}}{4} \).

24. An ellipse passes through \((2, 3)\) and \((-1, 4)\). Its center is the origin, and its major and minor axes are on the coordinate axes. Find its foci and the lengths of its major and minor axes.

**Answer:**

Let the equation of the ellipse be \( \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \).

Since it passes through \((2, 3)\) and \((-1, 4)\), we get \( \frac{4}{b^2} + \frac{9}{a^2} = 1 \) and \( \frac{1}{b^2} + \frac{16}{a^2} = 1 \).

Solving the equations, we get \( a = \frac{\sqrt{165}}{3} \) and \( b = \frac{\sqrt{385}}{7} \). Therefore,

Major axis \( 2a = \frac{2\sqrt{165}}{3} \) and Minor axis \( 2b = \frac{2\sqrt{385}}{7} \).
\[ c = \sqrt{a^2 - b^2} = \frac{2\sqrt{1155}}{21}. \] Hence, focuses’ coordinates are \((0, \pm \frac{2\sqrt{1155}}{21})\).

The link to the Chinese Version:

https://mbd.baidu.com/newspage/data/dtlandingsuper?nid=dt_450992477929296668

1953 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30

1. Solve the equation \(\frac{x+1}{x-1} + \frac{x-1}{x+1} = \frac{10}{3}\).

Answer: \(\frac{x+1}{x-1} + \frac{x-1}{x+1} = \frac{10}{3} \rightarrow 3(x+1)^2 + 3(x-1)^2 = 10(x^2 - 1) \rightarrow 6x^2 + 6 = 10x^2 - 10 \rightarrow 4x^2 = 16 \rightarrow x = \pm 2\) (verified, both are solutions).

2. If two roots of the equation \(3x^2 + kx + 12 = 0\) are equal, find the value of \(k\).

Answer: \(k^2 - 4 \cdot 3 \cdot 12 = 0 \rightarrow k^2 = 144 \rightarrow k = \pm 12\).

3. Find the value of \(\begin{vmatrix} 3 & -1 & 1 \\ 2 & 4 & 6 \\ 7 & 0 & 5 \end{vmatrix}\).

Answer: \(\begin{vmatrix} 3 & -1 & 1 \\ 2 & 4 & 6 \\ 7 & 0 & 5 \end{vmatrix} = 60 - 42 + 10 - 28 = 0\).

4. Calculate \(\log \frac{300}{7} + \log \frac{700}{3} + \log 1\).

Answer: \(\log \frac{300}{7} + \log \frac{700}{3} + \log 1 = \log \left(\frac{300}{7} \cdot \frac{700}{3} \cdot 1\right) = \log 10000 = 4\).

5. Find the value of \(\tan 870^\circ\).

Answer: \(\tan 870^\circ = \tan (720^\circ + 150^\circ) = \tan 150^\circ = -\tan 30^\circ = -\frac{\sqrt{3}}{3}\).

6. If \(\cos 2x = \frac{1}{2}\), find the value of \(x\).

Answer: \(2x = 2k\pi \pm \frac{\pi}{3} \rightarrow x = k\pi \pm \frac{\pi}{6} (k \in \mathbb{Z})\).
7. What are the conditions for two triangles to be similar? (Write everything you know).

Answer: AA, SAS, SSS, R-HS.

8. The length, width, and height of a rectangular prism are 12, 3, and 4, respectively. Find the length of its diagonal.

Answer: \( \sqrt{12^2 + 3^2 + 4^2} = \sqrt{169} = 13 \).

9. The height of a triangular prism is 6. The side lengths of its base are 3, 4, and 5. Find its volume.

Answer: Base is a right triangle. \( V = \frac{1}{2} \cdot 3 \cdot 4 \cdot 6 = 36 \).

10. The surface area of a sphere is \( 36\pi \). Find its volume.

Answer: \( 4\pi R^2 = 36\pi \rightarrow R = 3. V = \frac{4}{3}\pi R^3 = 36\pi \).

11. Solve \( \begin{cases} x^2 - 2xy + 3y^2 = 9 \\ 4x^2 - 5xy + 6y^2 = 30 \end{cases} \).

Answer: \( \begin{cases} 10x^2 - 20xy + 30y^2 = 90 \\ 12x^2 - 15xy + 18y^2 = 90 \end{cases} \rightarrow 2x^2 + 5x - 12y^2 = 0 \rightarrow (x + 4y)(2x - 3y) = 0 \rightarrow x + 4y = 0 \text{ or } 2x - 3y = 0; \)

Solve \( \begin{cases} x^2 - 2xy + 3y^2 = 9 \\ x + 4y = 0 \end{cases} \), get \( \begin{cases} x_1 = \frac{4\sqrt{3}}{3} \\ y_1 = -\frac{\sqrt{3}}{3} \end{cases} \) \( \begin{cases} x_2 = -\frac{4\sqrt{3}}{3} \\ y_2 = \frac{\sqrt{3}}{3} \end{cases} \);

Solve \( \begin{cases} x^2 - 2xy + 3y^2 = 9 \\ 2x - 3y = 0 \end{cases} \), get \( \begin{cases} x_3 = 3 \\ y_3 = 2 \end{cases} \) \( \begin{cases} x_4 = -3 \\ y_4 = -2 \end{cases} \).

12. Simplify \( \sqrt{\frac{12}{25}} + \sqrt[4]{90000} + \sqrt[6]{\frac{64}{27}} \).

Answer: \( = \frac{2\sqrt{3}}{5} + 10\sqrt{3} + \frac{2\sqrt{3}}{3} = \frac{166\sqrt{3}}{15} \).

13. Find the constant term in the expanded form of the expression \( (2x^3 + \frac{1}{x})^{12} \).
Answer: By the binomial theorem, \( T_{k+1} = \binom{12}{k}(2x^3)^{12-k} \left( \frac{1}{x} \right)^k = \binom{12}{k}2^{12-k}x^{36-4k} \).

Let \( 36 - 4k = 0 \), \( k = 9 \). The constant term is \( \binom{12}{9}2^{12-9} = 1760 \).

14. \( \triangle ABC \)'s altitudes are \( AD, BE, \) and \( CF \). \( H \) is the orthocenter. Prove: \( HD \) bisects \( \angle EDF \).

Answer: \( AD \perp BC, BE \perp CA \rightarrow A, B, D, E \) on one circle \( \rightarrow \angle ADE = \angle ABE \)

\( BE \perp CA, CF \perp AB \rightarrow F, B, C, E \) on one circle \( \rightarrow \angle FBE = \angle FCE \)

\( AD \perp BC, CF \perp AB \rightarrow C, A, F, D \) on one circle \( \rightarrow \angle FCA = \angle FDA \)

\( \angle ABE = \angle FBE \) and \( \angle FCE = \angle FCA \rightarrow \angle ADE = \angle FDA \)

Therefore, \( HD \) bisects \( \angle EDF \).

**Figure A2**

*Graph for Item 14 in the 1953 NCEE Math Exam*

15. If two angles of \( \triangle ABC \) are \( 45^\circ \) and \( 60^\circ \), and the side between them has a length of 1, find the shortest side length and the area of \( \triangle ABC \).

Answer: The third angle is \( 75^\circ \). Hence, the shortest side is opposite to the \( 45^\circ \) angle.

The shortest side = \( \frac{1 \cdot \sin 45^\circ}{\sin 75^\circ} = \frac{\sqrt{2}}{\sqrt{2}(\sqrt{2} + 1)} = \sqrt{3} - 1 \).

The area = \( \frac{1}{2} \cdot 1 \cdot (\sqrt{3} - 1) \cdot \sin 60^\circ = \frac{1}{4}(3 - \sqrt{3}) \).
1954 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30

1. Simplify the expression \[ \left( a^{\frac{3}{2}} b^2 \right)^{-1} (ab^{-3})^\frac{1}{2} \left( b^{\frac{1}{2}} \right)^7 \]

Answer: \[ = \left( a^{3/2} b^{2-3} b^{7/2} \right)^{1/3} = (a^2 b^0)^{1/3} = a^{2/3}. \]

2. Solve the equation \( \frac{1}{6} \log x = \frac{1}{3} \log a + 2 \log b + \log c. \)

Answer: \[ \log x^{\frac{1}{6}} = \log a^{\frac{1}{3}} b^2 c \rightarrow x^{\frac{1}{6}} = a^{\frac{1}{3}} b^2 c \rightarrow x = \left( a^{\frac{1}{3}} b^2 c \right)^6 = a^2 b^{12} c^6. \]

3. Calculate \((3.02)^4\) using the binomial theorem with error less than 0.001.

Answer: \[ = (3 + 0.02)^4 = 3^4 + 4 \times 3^3 \times 0.02 + 6 \times 3^2 \times 0.02^2 + 4 \times 3 \times 0.02^3 + 0.02^4; \]

Since the 4th and 5th terms less than 0.0001, it is sufficient to calculate first three terms; Thus, \((3.02)^4 = 81 + 2.16 + 0.0216 = 83.182. \)

4. Prove that the area of the semicircle on the hypotenuse of a right triangle equals to the sum of the areas of the semicircles on the two legs.

Answer: Let the hypotenuse be \( c \) and two legs be \( a \) and \( b \). The area of the semicircle on the hypotenuse \( \frac{1}{2} \pi \left( \frac{c}{2} \right)^2 = \frac{1}{2} \pi \frac{c^2}{4} = \frac{1}{2} \pi \frac{a^2 + b^2}{4} \) (by the Pythagorean Theorem) \[ + \frac{1}{2} \pi \left( \frac{b}{2} \right)^2 = \text{the sum of the areas of the semicircles on the two legs}. \]

5. The radius of a sphere is \( r \). Find the volume of its inscribed cube.

Answer: \( 3a^2 = 4r^2 \rightarrow a = \frac{2\sqrt{3}}{3} r. V = a^3 = \frac{8\sqrt{3}}{9} r^3. \)
6. A side of a triangle is $a$, and it is between two angles $\beta$ and $\gamma$. Find the length of another side $b$.

Answer: \[ \frac{a}{\sin(180^\circ - \beta - \gamma)} = \frac{b}{\sin \beta} \to b = \frac{a \sin \beta}{\sin(\beta + \gamma)}. \]

7. Draw the function $y = 3x^2 - 7x - 1$. Find $x$ such that $y > 0$ and $y < 0$.

Answer: Let $3x^2 - 7x - 1 = 0$, $x = \frac{7 \pm \sqrt{61}}{6}$. $x$-intercepts are $\left(\frac{7 - \sqrt{61}}{6}, 0\right)$. Hence, when $y > 0$, $x \in (-\infty, \frac{7 - \sqrt{61}}{6}), (\frac{7 + \sqrt{61}}{6}, +\infty)$; when $y < 0$, $x \in (\frac{7 - \sqrt{61}}{6}, \frac{7 + \sqrt{61}}{6})$.

**Figure A3**

*Graph for Item 16 in the 1954 NCEE Math Exam*

8. Two circles are externally tangent to each other. The line segment constructed by connecting the centers of the two circles is the diameter of a third circle. Prove that the third circle is tangent to the common tangent lines of previous two circles.

Answer: Construct midpoint $O$, and $OA \perp A_1A_2$. $OA = \frac{1}{2} (O_1A_1 + O_2A_2) = \frac{1}{2} O_1O_2$.

$A$ is on the $\bigcirc O$ and $\bigcirc O$ is tangent to $A_1A_2$ at $A$.

**Figure A4**

*Graph for Item 17 in the 1954 NCEE Math Exam*
9. Solve the equation \( \frac{1 \tan x}{1 - \tan x} = 1 + \sin 2x \).

Answer: \( \frac{\cos x + \sin x}{\cos x - \sin x} = (\cos x + \sin x)^2 \rightarrow \cos x + \sin x = (\cos x + \sin x)^2 (\cos x - \sin x) \rightarrow (\cos x + \sin x)(1 - \cos^2 x + \sin^2 x) = 0 \rightarrow 2(\cos x + \sin x) \sin^2 x = 0. \cos x + \sin x = 0 \) or \( \sin^2 x = 0. \cos x + \sin x = 0 \rightarrow \tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}, k \in \mathbb{Z}; \sin^2 x = 0 \rightarrow x = k\pi, k \in \mathbb{Z}. \)

10. A right cone and cylinder have the same base and height. \( a \) and \( a' \) are the surface areas of the cone and cylinder. Find the ratio of the cone’s height to its slant height.

Answer: Let the base radius, height, and slant height be \( R, h, \) and \( L, \) respectively.

\[
\frac{a}{a'} = \frac{\pi R(R + L)}{2\pi R(R + h)} = \frac{R + L}{2(R + h)} \rightarrow 2a(R + h) = a'(R + L). \] Since \( R = \sqrt{L^2 - h^2}, 2a(\sqrt{L^2 - h^2} + h) = a'(\sqrt{L^2 - h^2} + L) \rightarrow (2a - a')\sqrt{L^2 - h^2} = a'L - 2ah. \) Divide both side by \( L, \) it becomes

\[
(2a - a')\sqrt{1 - \left(\frac{h}{L}\right)^2} = a' - 2a\frac{h}{L}. \] Squaring both sides, it becomes a quadratic equation about \( \frac{h}{L}, \)

\[
(8a^2 - 4aa' + a'^2)\left(\frac{h}{L}\right)^2 = 4aa'\frac{h^2}{L} + (4aa' + a'^2) = 0. \] Using quadratic formula to solve it, the two real roots are the solutions \( \frac{h}{L} = \frac{2aa' \pm 2(2a - a')\sqrt{a(2a - a')}}{4a^2 + (2a - a')^2}. \)

The link to the Chinese Version:

http://www.360doc.com/content/17/0812/04/682382_678666722.shtml
1. Use the square of the two roots of \( x^2 - 3x - 1 = 0 \) to create a quadratic equation.

Answer: Let two roots be \( \alpha \) and \( \beta \), \( \alpha + \beta = 3 \), \( \alpha \beta = -1 \). \( \alpha^2 + \beta^2 = 9 - (-2) = 11 \), \( \alpha^2 \beta^2 = 1 \). Hence, the new function is \( x^2 - 11x + 1 = 0 \).

2. The legs of an isosceles triangle are four times the length of its base. Find the cosines of the angles of this isosceles triangle.

Answer: Let vertex angle be \( \alpha \), base angles be \( \beta \), base length be \( x \).

\[
\cos \alpha = \frac{(4x)^2 + (4x)^2 - x^2}{2 \cdot 4x \cdot 4x} = \frac{31}{32}, \quad \cos \beta = \frac{0.5x}{4x} = \frac{1}{8}.
\]

3. A square pyramid has a base length of \( a \), and the angle between the lateral edge and the base is 45°. Find the height of this square pyramid.

Answer: \( \frac{a}{\sqrt{2}} = \frac{\sqrt{2}a}{2} \).

4. What is the definition of a dihedral angle?

Answer: A dihedral angle is the angle between two intersecting planes or half-planes.

5. Find the values of \( b \), \( c \), and \( d \) such that the polynomial \( x^3 + bx^2 + cx + d \) satisfies the following three conditions: (1) it is divisible by \( x - 1 \); (2) when it is divided by \( x - 3 \), the reminder is 2; (3) when it is divided by \( x + 2 \) and \( x - 2 \), the reminders are equal.

Answer: \( 1 + b + c + d = 0 \); \( 27 + 9b + 3c + d = 2 \); \( -8 + 4b - 2c + d = 8 + 4b + 2c + d \).

\[
c = -4, \quad b + d = 3, \quad 9b + d = -13, \quad b = -2, \quad d = 5.
\]

6. Construct a perpendicular line through point \( D \), a point on one leg of a right triangle, to the hypotenuse \( AB \), intersecting \( AB \) at \( E \), the other leg (extended) at \( F \), and the circumscribed circle at \( Q \). Prove that \( EQ \) is the geometric mean between \( EA \) and \( EB \), and \( ED \) and \( EF \).
Answer: Connect QA and QB, \( \angle AQB = 90^\circ \). In right \( \triangle AQB \), EQ is the altitude on the hypotenuse, hence, \( EA:EQ = EQ:EB \), \( EQ^2 = EA \cdot EB \). \( \angle EAD = \angle EFB \), \( \triangle EAD \sim \triangle EFB \), \( EA:EF = ED:EB \), \( ED \cdot EF = EA \cdot EB = EQ^2 \).

**Figure A5**

*Graph for Item 6 in the 1955 NCEE Math Exam*

7. Solve the equation \( \cos 2x = \cos x + \sin x \).

Answer: \( \cos^2 x - \sin^2 x = \cos x + \sin x \rightarrow (\cos x + \sin x)(\cos x - \sin x) = \cos x + \sin x \rightarrow (\cos x + \sin x)(\cos x - \sin x - 1) = 0 \rightarrow \cos x + \sin x = 0 \rightarrow \tan x = -1 \rightarrow x = k\pi - \frac{\pi}{4}, k \in \mathbb{Z} \); \( \cos x - \sin x - 1 = 0 \rightarrow \cos x - \sin x = 1 \rightarrow \frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x = \frac{\sqrt{2}}{2} \rightarrow \cos \left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \rightarrow x + \frac{\pi}{4} = 2k\pi \pm \frac{\pi}{4} \rightarrow x = 2k\pi, k \in \mathbb{Z} \) or \( x = 2k\pi - 0.5\pi, k \in \mathbb{Z} \).

8. Three sides of a triangle form an arithmetic sequence. If the triangle’s perimeter is 12, and its area is 6. Prove that the triangle is a right triangle.

Answer: Let the three side lengths be \( x - d \), \( x \), and \( x + d \). \( x - d + x + x + d = 12 \rightarrow x = 4 \).

By the Heron’s formula, \( 6 = \sqrt{6[6 - (4 - d)](6 - 4)[6 - (4 + d)]} \rightarrow 36 = 12(2 + d)(2 - d) \rightarrow d^2 = 1 \rightarrow d = \pm 1 \). Thus, three side lengths are 3, 4, and 5, and they form a right triangle.
1. Use logarithmic properties to calculate $\log 2^5 + \log 2 \cdot \log 50$.

Answer: $= \log 2^5 + \log 2 (\log 5 + 1) = \log 5 (\log 5 + \log 2) + \log 2 = \log 5 + \log 2 = 1$.

2. $m$ is a real number. Prove: the roots of $2x^2 - (4m - 1)x - m^2 - m = 0$ are real.

Answer: $b^2 - 4ac = [-(4m - 1)]^2 - 4 \cdot 2 \cdot (-m^2 - m) = 24m^2 + 1 > 0$.

3. $M$ is the midpoint of $AC$ in $\Delta ABC$. Construct a straight line passing through $M$ and intersecting $AB$ at $E$. Construct another straight line passing through $B$, parallel to $ME$, and intersecting $AC$ at $F$. Prove that $\Delta AEF$’s area equals to half the $\Delta ABC$’s area.

Answer: $S_{\Delta AEF} = S_{\Delta AEM} + S_{\Delta MEF} = S_{\Delta AEM} + S_{\Delta MEB} = S_{\Delta ABM} = \frac{1}{2} S_{\Delta ABC}$

(Triangles on the same base & between the same parallel lines)

Figure A6

Graph for Item 3 in the 1956 NCEE Math Exam

4. Three side lengths of a triangle are 3, 4, and $\sqrt{37}$. Find the largest angle’s measure.

Answer: $\cos A = \frac{3^2 + 4^2 - \sqrt{37}^2}{2 \cdot 3 \cdot 4} = -\frac{1}{2}$, $\angle A = 120^\circ$. 
5. Let \( \tan \alpha \) and \( \tan \beta \) be the two roots of the equation \( x^2 + 6x + 7 = 0 \). Prove: \( \sin(\alpha + \beta) = \cos(\alpha + \beta) \).

Answer: \( \tan \alpha + \tan \beta = -6 \), \( \tan \alpha \cdot \tan \beta = 7 \). \( \tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = 1 \), \( \sin(\alpha + \beta) = \cos(\alpha + \beta) \).

6. Solve \( \begin{cases} 7\sqrt{x + y} - x - y = 12 \\ x^2 + y^2 = 136 \end{cases} \).

Answer: \( 7\sqrt{x + y} - x - y = 12 \rightarrow (x + y) - 7\sqrt{x + y} + 12 = 0 \rightarrow (\sqrt{x + y} + 3)(\sqrt{x + y} - 4) = 0 \). \( \sqrt{x + y} + 3 = 0 \rightarrow x + y = 9 \rightarrow y = 9 - x \). \( \sqrt{x + y} - 4 = 0 \rightarrow x + y = 16 \rightarrow y = 16 - x \). \( x^2 + (9 - x)^2 = 136 \rightarrow x = \frac{9 \pm \sqrt{191}}{2} \), \( y = \frac{9 \pm \sqrt{191}}{2} \). \( x^2 + (16 - x)^2 = 136 \rightarrow x = 6, y = 10 \) or \( x = 10, y = 6 \).

7. Let \( P \) be a point on the circumscribed circle of equilateral \( \triangle ABC \). Prove that \( PA^2 = AB^2 + PB \cdot PC \).

Answer: In \( \triangle ABP \) and \( \triangle ADB \), \( \angle BAP = \angle DAB \), \( \angle APB = \angle ACB = \angle ABD = 60^\circ \), \( \triangle ADB \sim \triangle ABD \). Thus, \( AB^2 = PA \cdot AD \). Similarly, \( \triangle BPD \sim \triangle APC \), \( PB \cdot PD = PA \). Thus, \( PB \cdot PC = PA \cdot PD \). Therefore, \( AB^2 + PB \cdot PC = PA \cdot AD + PA \cdot PD = PA^2 \).

**Figure A7**

*Graph for Item 7 in the 1956 NCEE Math Exam*
8. In the figure, the bases of a quadrilateral prism are rhombuses. \( \angle A'AB = \angle A'AD \). Prove that plane \( A'ACC' \) is perpendicular to plane \( ABCD \).

Answer: Let \( O \) be the rhombus’ center, connect \( A'D \), \( A'O \), and \( A'B \). \( A'A = A'A, \angle A'AB = \angle A'AD, AB = AD \rightarrow \triangle A'AB \cong \triangle A'AD \). Thus, \( A'B = A'D, \rightarrow \triangle A'BD \) is isosceles, \( A'O \perp DB \).

Also, \( DB \perp AC \), \( DB \) is perpendicular to \( A'ACC' \). Since \( DB \) is on the plane \( ABCD \), plane \( A'ACC' \) is perpendicular to plane \( ABCD \).

**Figure A8**

*Graph for Item 8 in the 1956 NCEE Math Exam*
9. Prove: (1) if three angles of a triangle form an arithmetic sequence, then one of its angles must be 60°; (2) if, in addition, three sides of this triangle form a geometric sequence, then all its angles must be 60°.

Answer: (1) Let three angles be \( A, B, \) and \( C \), then \( B - A = C - B \rightarrow 2B = A + C \).

\[
A + B + C = 180° \rightarrow 3B = 180° \rightarrow B = 60°.
\]

(2) Let three sides be \( a, b, \) and \( c \), then \( b^2 = ac \).

Also, \( b^2 = a^2 + c^2 - 2ac \cos B \), thus, \( a^2 + c^2 - 2ac \cos 60° = ac \rightarrow a^2 + c^2 - 2ac = 0 \rightarrow (a - c)^2 = 0 \rightarrow a = c \). Therefore, this triangle is equilateral.

The link to the Chinese Version:

http://www.360doc.com/content/20/1105/06/38683567_944170607.shtml

1957 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30

1. Simplify the expression \( \left( 2 \frac{7}{9} \right)^{\frac{1}{2}} + 0.1^{-2} + \left( 2 \frac{10}{27} \right)^{\frac{2}{3}} \).

Answer: \( \frac{5}{3} + 100 + \frac{9}{16} = 102 \frac{11}{48} \).

2. Solve the inequality \( x^2 + x < 2 \).

Answer: \( x^2 + x - 2 < 0 \rightarrow (x + 2)(x - 1) < 0 \rightarrow x + 2 < 0, x - 1 > 0 \) or \( x + 2 > 0, x - 1 < 0 \).

\( x + 2 < 0, x - 1 > 0 \rightarrow x < -2, x > 1 \), no solutions; \( x + 2 > 0, x - 1 < 0 \rightarrow -2 < x < 1 \).

3. Prove: \( \frac{1}{\tan 22°30'} = 1 + \sqrt{2} \).

Answer: \( \cot 22°30' = \cot \frac{45°}{2} = \frac{1 + \cos 45°}{\sin 45°} = \frac{1 + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1 + \sqrt{2} \).
4. In tetrahedron $ABCD$, $AC = BD$, $P, Q, R, S$ are the midpoints of $AB, BC, CD, DA$. Prove:

$PQRS$ is a rhombus.

Answer: $PS \parallel BD \parallel QR$, $PS = \frac{1}{2}BD = QR$. Thus, $PQRS$ is a parallelogram. $PS = \frac{1}{2}BD = \frac{1}{2}AC = SR$. Thus, $PQRS$ is a rhombus.

**Figure A9**

*Graph for Item 4 in the 1957 NCEE Math Exam*

5. $a$ and $b$ are two skew lines, and their common perpendicular line is $EF$. $\pi$ is a plane passing through the midpoint of $EF$ and parallel to $a$ and $b$. Let $M$ be a point on $a$ and $N$ be a point on $b$. Prove that plane $\pi$ bisects the $MN$.

Answer: Let $C$ be the midpoint of $EF$, $MN$ intersects plane $\pi$ at $A$. Connect $EF$ intersecting plane $\pi$ at $B$. Connect $BC$. $b \parallel \pi \rightarrow b \parallel BC \rightarrow BC \parallel FN$. In $\triangle EFN$, $C$ is the midpoint of $EF$, $B$ is the midpoint of $EN$. Connect $BA$. Follow the same reasoning, $A$ is the midpoint of $MN$. Since $A$ in plane $\pi$, the plane $\pi$ bisects the $MN$.

**Figure A10**

*Graph for Item 5 in the 1957 NCEE Math Exam*
6. Solve \( \begin{cases} \lg(2x + 1) + \lg(y - 2) = 1 \\ 10^{xy} = 10^x \cdot 10^y \end{cases} \).

Answer: \( \begin{cases} \lg c = \lg 10 \\ 10^{xy} = 10^{x+y} \end{cases} \rightarrow \begin{cases} (2x + 1)(y - 2) = 10 \\ xy = x + y \end{cases} \rightarrow \begin{cases} 2xy - 4x + y - 12 = 0 \\ xy - x - y = 0 \end{cases} \rightarrow \begin{cases} \frac{2x+12}{3} = x = \frac{-7\pm\sqrt{145}}{4}, y = \frac{17\pm\sqrt{145}}{6} \end{cases} \).

7. The radius of the inscribed circle of \( \triangle ABC \) is \( r \). Prove that the triangle’s altitude \( AD \) on the side \( BC \) equals to \( \frac{2r \cos \frac{B}{2} \cos \frac{C}{2}}{\sin \frac{A}{2}} \).

Answer: \( \angle A + \angle B + \angle C = 180^\circ \rightarrow \frac{A+B}{2} = 90^\circ - \frac{C}{2} \rightarrow \sin \frac{A+B}{2} = \cos \frac{C}{2} \), \( AE = r \cot \frac{A}{2} \), \( EB = r \cot \frac{B}{2} \rightarrow AD = AB \sin B = (AE + EB) \sin B = r \left( \cot \frac{A}{2} + \cot \frac{B}{2} \right) \sin B = \frac{2r \sin \frac{B}{2} \cos \frac{B}{2}}{2} \left( \frac{\cos \frac{A}{2}}{\sin \frac{A}{2}} + \frac{\cos \frac{B}{2}}{\sin \frac{B}{2}} \right) = 2r \sin \frac{B}{2} \cos \frac{B}{2} \sin \frac{A}{2} \cos \frac{A}{2} \sin \frac{B}{2} \sin \frac{B}{2} = 2r \cos \frac{B}{2} \frac{\cos \frac{C}{2}}{\sin \frac{A}{2}} = \frac{2r \cos \frac{B}{2} \cos \frac{C}{2}}{\sin \frac{A}{2}} \).

**Figure A11**

*Graph for Item 7 in the 1957 NCEE Math Exam*
8. ΔABC is an acute triangle. Construct a circle with BC as the diameter, a tangent line passing through point A and intersecting the circle at D, a point E on AB such that AE = AD, and a perpendicular line passing through point E and intersecting the extended AC at F. Prove: (1) \( \frac{AE}{AB} = \frac{AC}{AF} \); (2) ΔABC’s area = ΔAEF’s area.

Answer: (1) Assume AB intersects the circle at G, connect CG, \( \angle BGC = 90^\circ \), CG \( \parallel EF \), ΔAGC ∼ ΔAEF, \( \frac{AG}{AE} = \frac{AC}{AF} \). \( AE^2 = AD^2 = AG \cdot AB \rightarrow \frac{AE}{AB} = \frac{AC}{AF} \). (2) \( S_{\Delta ABC} = \frac{1}{2} AB \cdot AC \cdot \sin A = \frac{1}{2} AE \cdot AF \cdot \sin A = S_{\Delta AEF} \).

Figure A12

Graph for Item 8 in the 1957 NCEE Math Exam

9. (1) Prove that the equation \( x^3 - (\sqrt{2} + 1)x^2 + (\sqrt{2} - q)x + q = 0 \) has a root of 1. (2) Let the three roots of this equation be the \( \sin A \), \( \sin B \), and \( \sin C \) of a triangle ABC. Find the measures of A, B, and C, and the value of q.

Answer: (1) \( 1 - (\sqrt{2} + 1) + (\sqrt{2} - q) + q = 0 \rightarrow 1 \) is a root. (2) \( x^3 - (\sqrt{2} + 1)x^2 + (\sqrt{2} - q)x + q = x^2(x - 1) - \sqrt{2}x(x - 1) - q(x - 1) = (x - 1)(x^2 - \sqrt{2}x + q) = 0 \).

\( x - 1 = 0 \) or \( x^2 - \sqrt{2}x + q = 0 \). Assume \( \sin A \) is the root to \( x - 1 = 0 \), \( \sin A = 1, A = 90^\circ \), \( B + C = 90^\circ \). Then, \( \sin B \) and \( \sin C \) are the roots to \( x^2 - \sqrt{2}x + q \). \( \sin B + \sin C = \sqrt{2} \),
\[
\sin B \cdot \sin C = -q \cdot \sqrt{2} = \sin B + \sin C = 2 \sin \frac{B+C}{2} \cos \frac{B-C}{2} = 2 \sin \frac{90^\circ}{2} \cos \frac{B-C}{2} = \cos \frac{B-C}{2} = 1 \to \frac{B-C}{2} = 0^\circ \to B = C = 45^\circ.
\]

1. Find the coefficient of the \(x^3\) in the expanded form of the expression \((1 + 2x)^5\).

Answer: By the binomial theorem, \(T_{k+1} = \binom{5}{k}(1)^{5-k}(2x)^k = \binom{5}{k}(2x)^k\). Let \(k = 3\). Then, \(T_4 = \binom{5}{3}(2x)^3 = \frac{20}{2} \cdot 8x^3 = 80x^3\).

2. Prove: \(\cos x \cdot \cos 2x \cdot \cos 4x = \frac{\sin 8x}{8 \sin x}\).

Answer: \(\sin 8x = 2 \sin 4x \cos 4x = 4 \sin 2x \cos 2x \cos 4x = 8 \sin x \cos x \cos 2x \cos 4x \)
\[
\to \frac{\sin 8x}{8 \sin x} = \cos x \cdot \cos 2x \cdot \cos 4x.
\]

3. \(AB\) and \(AC\) are two chords of a circle. \(D\) and \(E\) are the midpoints of arc \(AB\) and arc \(AC\).

Construct segment \(DE\) intersecting \(AB\) at \(M\) and \(AC\) at \(N\). Prove: \(AM = AN\).

Answer: Connect \(AD\) and \(AE\). \(\angle AMN = \angle DAM + \angle MDA, \angle ANM = \angle EAN + \angle NEA. \text{arc} AD = \text{arc} DB, \angle DAB = \angle AED, \text{arc} AE = \text{arc} EC, \angle ADE = \angle EAC, \angle AMN = \angle ANM. AM = AN\).

**Figure A13**

*Graph for Item 3 in the 1958 NCEE Math Exam*
4. Prove that the opposite edges of a regular tetrahedron are perpendicular.

Answer: All faces of a regular tetrahedron are equilateral triangles. Construct $AE \perp BC$, connect $DE$, $DE \perp BC$, $DE$ is perpendicular to Plane $AED$. $AD$ is on the plane, $DE \perp AD$. The rest follows the same reasoning.

**Figure A14**

*Graph for Item 4 in the 1958 NCEE Math Exam*

5. Solve the equation $\sin x = \sqrt{3} \cos x$.

Answer: $\tan x = \sqrt{3}$, $x = k\pi + \frac{\pi}{3}, k \in \mathbb{Z}$.

6. Solve the system of equations:

$$\begin{cases} \sqrt{x + \frac{1}{y}} + \sqrt{x + 2y - 1} = 4 \\ 2x + 2y + \frac{1}{y} = 9 \end{cases}$$
Answer: Let \( \sqrt{x + \frac{1}{y}} = u, \sqrt{x + 2y - 1} = v \), then \( \sqrt{x + \frac{1}{y}} + \sqrt{x + 2y - 1} = 4 \to u + v = 4; \)

\[
2x + 2y + \frac{1}{y} = 9 \to x + \frac{1}{y} + x + 2y - 1 = 8 \to u^2 + v^2 = 8.\ u = 2, v = 2.
\]

\[
\sqrt{x + \frac{1}{y}} = 2 \to x + \frac{1}{y} = 4; \sqrt{x + 2y - 1} = 2 \to x + 2y - 1 = 4.\ y_1 = 1, y_2 = -\frac{1}{2}.
\]

\[
x_1 = 3, y_1 = 1; x_2 = 6, y_2 = -\frac{1}{2}.
\]

7. Two circles are concentric with radii of \( R \) and \( r \) \( (R > r) \). Construct a radius \( OA \) that intersecting the large circle at \( A \) and small circle at \( A' \). Construct a perpendicular line passing through \( A \) and intersecting the diameter \( BC \) of the large circle at \( D \). Construct another perpendicular line passing through \( A' \) and intersecting \( AD \) at \( E \). It is known that \( \angle OAD = \alpha \).

Find the length of \( OE \).

Answer: In \( \triangle OAD \), \( OD = R \sin \alpha, AD = R \cos \alpha \). In \( \triangle A'AE \), \( AE = (R - r) \cos \alpha \). \( DE = AD - AE = r \cos \alpha \). \( OE = \sqrt{OD^2 + DE^2} = \sqrt{R^2 \sin^2 \alpha + r^2 \cos^2 \alpha} \).

Figure A15

*Graph for Item 7 in the 1958 NCEE Math Exam*

8. In \( \triangle ABC \), draw a circle through point \( A \) and the midpoint \( M \) of \( AB \), and is tangent to \( BC \).

Given: \( M \) is the midpoint of \( \triangle ABC \). Prove: there exists a circle passing through points \( A \) and \( M \) and is tangent to \( BC \).
Answer: Construct segment such that \( A'B' = AB, B'M' = BM \). Construct circle using diameter \( A'M' \). Construct \( B'P' \perp A'M' \). Construct \( BP = B'P' \). Construct circle through points \( A, M, P \).

Proof: \( B'P'^2 = A'B' \cdot B'M' \rightarrow BP^2 = AB \cdot BM \rightarrow \) circle is tangent to \( BC \) at \( P \).

**Figure A16**

*Graph for Item 8 in the 1958 NCEE Math Exam*

9. The lengths of the hypotenuse and the altitude on the hypotenuse are 2 and \( \frac{\sqrt{3}}{2} \) in a right triangle. Prove that the measures of the two acute angles of this right triangle are the roots of the equation \( \sin^2 x - \frac{1+\sqrt{3}}{2} \sin x + \frac{\sqrt{3}}{4} = 0 \).

Answer: Let the length of \( AD \) be \( k \). \( DB = 2 - k \). \( \left( \frac{\sqrt{3}}{2} \right)^2 = k(2 - k) \rightarrow k^2 - 2k + \frac{3}{4} = 0 \rightarrow k = \frac{3}{2} \) or \( \frac{1}{2} \). If \( AD = \frac{3}{2} \), \( \tan A = \frac{\sqrt{3}}{3} \), \( A = 30^\circ \), and \( B = 60^\circ \). If \( AD = \frac{1}{2} \), \( \tan A = \sqrt{3} \), \( A = 60^\circ \), and \( B = 30^\circ \). When \( x = 30^\circ \), \( \sin^2 30^\circ - \frac{1+\sqrt{3}}{2} \sin 30^\circ + \frac{\sqrt{3}}{4} = \left( \frac{1}{2} \right)^2 - \frac{1+\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 0 \). When \( x = 60^\circ \), \( \sin^2 60^\circ - \frac{1+\sqrt{3}}{2} \sin 60^\circ + \frac{\sqrt{3}}{4} = \left( \frac{\sqrt{3}}{2} \right)^2 - \frac{3+\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 0 \). Therefore, \( 30^\circ \) and \( 60^\circ \) are the roots of the equation.

**Figure A17**

*Graph for Item 9 in the 1958 NCEE Math Exam*
1959 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30

1. Given: \( \lg 2 = 0.3010, \lg 7 = 0.8451 \). Calculate \( \lg 35 \).

   Answer: \( \lg 35 = \lg \frac{10 \times 7}{2} = \lg 10 + \lg 7 - \lg 2 = 1 + 0.8451 - 0.3010 = 1.5441 \).

2. Simplify the expression \( \frac{(1-i)^3}{1+i} \).

   Answer: \( \frac{(1-i)^3}{1+i} = \frac{1-3i+3i^2-i^3}{1+i} = \frac{1-3i-3+i}{1+i} = \frac{-2i}{1+i} = -2. \)

3. Solve the inequality \( 2x^2 - 5x < 3 \).

   Answer: \( 2x^2 - 5x - 3 < 0 \rightarrow (2x - 1)(x + 3) < 0 \rightarrow 2x - 1 > 0, x + 3 < 0 \) or \( 2x - 1 < 0, x + 3 > 0 \).

   \( 0.2x - 1 > 0, x + 3 < 0 \rightarrow x > 0.5, x < -3 \), no solutions. \( 2x - 1 < 0, x + 3 > 0 \rightarrow -3 < x < 0.5 \).

4. Find the value of \( \cos 165^\circ \).

   Answer: \( \cos 165^\circ = \cos(180^\circ - 15^\circ) = -\cos 15^\circ = -\cos(45^\circ - 30^\circ) = -\left( \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \right) = -\frac{\sqrt{3}+1}{2\sqrt{2}} = -\frac{\sqrt{6}+\sqrt{2}}{4} \).

5. In a truncated cone, the area of the top surface is \( 25\pi \) \( cm^2 \), the diameter of the base is \( 20 \) \( cm \), and the slant height is \( 10 \) \( cm \). Find its lateral surface area.
Answer: \( S_L = \pi(R + r)L = \pi \cdot (10 + 5) \cdot 10 = 150cm^2 \).

6. Given: three parallel lines \( a, b, \) and \( c \) are not in the same plane. \( AB \) is a fixed segment on \( a \).

Construct points \( C \) and \( D \) on the lines \( b \) and \( c \). Prove that the volume of the tetrahedron \( ABCD \) is fixed wherever the two points are constructed.

Answer: \( a \parallel b \rightarrow \) distance from \( a \) to \( b \) is fixed \( \rightarrow S_{ABD} \) is fixed. \( a \parallel b \rightarrow a \) and \( b \) form a plane \( ABD. c \parallel a \) and \( c \parallel b \rightarrow c \parallel ABD \rightarrow \) distance from \( c \) to \( ABD \) is fixed \( \rightarrow \) tetrahedron’s height is fixed \( \rightarrow \) volume is fixed.

**Figure A18**

*Graph for Item 6 in the 1959 NCEE Math Exam*

7. Three numbers form an arithmetic progression. Three times the sum of the first two numbers equals to twice the third number. If the second number is subtracted by two (would still be the second term), the three numbers become a geometric progression. Find the original three numbers.

Answer: Let three numbers be \( x - y, x, \) and \( x + y \). \( \begin{cases} 3[(x - y) + x] = 2(x + y) \\ (x - 2)^2 = (x - y)(x + y) \end{cases} \)

\[ \begin{cases} 4x = 5y \\ y^2 - 4x + 4 = 0 \rightarrow y^2 - 5y + 4 = 0 \rightarrow (y - 1)(y - 4) = 0 \rightarrow y_1 = 1 \text{ or } y_2 = 4. \end{cases} \]

\[ x_1 = \frac{5}{4} \text{ or } x_2 = 5. x_1 - y_1 = \frac{1}{4}, x_1 + y_1 = \frac{9}{4}; x_2 - y_2 = 1, x_2 + y_2 = 9. \text{ Thus, three numbers are } \frac{1}{4}, \frac{5}{4}, \text{ and } \frac{9}{4} \text{ or } 1, 5, \text{ and } 9. \]

8. In \( \triangle ABC \), \( \angle B = 60^o \) and \( AC = 4 \). The area of \( \triangle ABC \) is \( \sqrt{3} \). Find \( AB \) and \( BC \).
Answer: \(x^2 + y^2 - 2xy \cos 60^\circ = 4^2, \frac{1}{2} xy \sin 60^\circ = \sqrt{3} \rightarrow x^2 + y^2 - xy = 16,\)

\(xy = 4 \rightarrow x^2 + y^2 = 20, \quad 2xy = 8. \quad (x + y)^2 = 28, \quad (x - y)^2 = 12.\)

\(x + y = 2\sqrt{7}, \quad x - y = \pm 2\sqrt{3}. \quad x = \sqrt{7} \pm \sqrt{3}, \quad y = \sqrt{7} \mp \sqrt{3}.\)

**Figure A19**

*Graph for Item 8 in the 1959 NCEE Math Exam*

9. \(A, B,\) and \(C\) are three points on the line \(l,\) and \(AB = BC = a.\) \(P\) is a point outside the line, and \(\angle APB = 90^\circ\) and \(\angle BPC = 45^\circ.\) Find

(1) The sine, cosine, and tangent of \(\angle PBA.\)

(2) The length of \(PB.\)

(3) The distance from \(P\) to \(l.\)

Answer: Let \(\angle PBA = \theta\) and \(PB = x.\) In \(\triangle APB,\) \(x = a \cos \theta.\) In \(\triangle BPC,\) \(\frac{a}{\sin 45^\circ} = \frac{a \cos \theta}{\sin (\theta - 45^\circ)} \rightarrow \sqrt{2} (\sin \theta \cos 45^\circ - \cos \theta \sin 45^\circ) = \cos \theta \rightarrow \sin \theta - \cos \theta = \cos \theta \rightarrow \tan \theta = 2.\) Since \(\theta\) is acute, \(\sin \theta = \frac{2}{\sqrt{5}}, \quad \cos \theta = \frac{1}{\sqrt{5}}, \quad x = a \cos \theta = \frac{a}{\sqrt{5}}, \quad h = x \sin \theta = \frac{2}{5} a.\)

**Figure A20**

*Graph for Item 9 in the 1959 NCEE Math Exam*
10. Two chords $AB$ and $CD$ of a circle are extended and intersect at a point $E$ outside the circle.

Construct a line passing through point $E$, parallel to $AD$, and intersecting the extended $BC$ at point $F$. Construct another line passing through point $F$ and tangent to the circle at point $G$.

Prove: $EF = FG$.

Answer: $EF \parallel AD \Rightarrow \angle FEB = \angle BAD$. $\angle BAD = \angle BCD \Rightarrow \angle FEB = \angle BCD$. $\angle EFB = \angle EFC \Rightarrow \triangle EFB \sim \triangle CFE \Rightarrow \angle FE\overline{B} = \angle FC\overline{E}$. $FE = FB; FE \rightarrow FE^2 = FB \cdot FC$. $FG^2 = FB \cdot FC \Rightarrow EF = FG$.

**Figure A21**

*Graph for Item 10 in the 1959 NCEE Math Exam*

The link to the Chinese Version:

http://www.360doc.com/content/12/0121/07/682382_678620468.shtml

1960 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30

1. Solve the equation $\sqrt{2x^2 - 5} - \sqrt{5x - 7} = 0$ (in the real domain).

Answer: $2x^2 - 5 = 5x - 7 \Rightarrow 2x^2 - 5x + 2 = 0 \Rightarrow x_1 = \frac{1}{2}$ or $x_2 = 2$.

2. There are 5 groups of basketball teams, 6 teams in each group. First, each group plays a round-robin tournament, and then the champions of each group play a round-robin tournament. How many games played in total?

Answer: $5\left(\binom{6}{2}\right) + \binom{5}{2} = 75 + 10 = 85$. 

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3. Prove that the logarithms of the terms of a geometric sequence form an arithmetic sequence

(all terms in the geometric sequence are positive numbers).

Answer: Let the first term of the geometric sequence be \(a\) \((a > 0)\), and the common ratio be \(q\) \((q > 0)\), i.e., \(a, aq, aq^2, \ldots\). Take the logarithms of the sequence, it becomes \(\lg a\), \(\lg a + \lg q\), \(\lg a + 2 \lg q\), \(\ldots\), i.e., an arithmetic sequence with the first term \(\lg a\), and the common difference \(\lg q\).

4. Find the intervals of \(x\) such that \(\sqrt{1 - \sin^2 \frac{x}{2}} = \cos \frac{x}{2} \left(0^\circ \leq x \leq 720^\circ\right)\).

Answer: \(\cos \frac{x}{2} \geq 0 \rightarrow \frac{x}{2}\) is in the first or fourth quadrant. \(0^\circ \leq \frac{x}{2} \leq 90^\circ\) or \(270^\circ \leq \frac{x}{2} \leq 360^\circ\).
Thus, \(0^\circ \leq x \leq 180^\circ\) or \(540^\circ \leq x \leq 720^\circ\).

5. The figure shows how to use a steel ball to measure the diameter of a small hole on the machine’s body. The center of the steel ball is \(O\), and its diameter is 12 \(mm\). The steel ball is placed on the hole, and the distance between the upper end of the steel ball and the surface of the machine \(CD\) is 9 \(mm\). Find the diameter \(AB\) of the hole.

Answer: Connect \(AB\), \(CD\), and \(OB\).

\[
OC = OB = 6 \text{ mm}, \ OD = CD - OC = 9 - 6 = 3 \text{ mm}. \ BD = \sqrt{BO^2 - OD^2} = \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3} \text{ mm}. \ AB = 2 \cdot BD = 6\sqrt{3} \text{ mm}.
\]

**Figure A22**

*Graph for Item 5 in the 1960 NCEE Math Exam*
6. In square pyramid $P-ABCD$, $PA$ is perpendicular to the base and $PA = 3 \text{ cm}$. The distance from $P$ to $BC$ is $5 \text{ cm}$. Find the length of $PC$.

Answer: $ABCD$ is a square $\rightarrow AB \perp BC$. $PA \perp$ plane $ABCD$, $PB \perp BC$. In $\triangle PAB$, $AB = \sqrt{PB^2 - PA^2} = \sqrt{5^2 - 3^2} = 4 \text{ cm}$. In $\triangle PBC$, $PC = \sqrt{PB^2 + BC^2} = \sqrt{5^2 + 4^2} = \sqrt{41} \text{ cm}$.

**Figure A23**

*Graph for Item 6 in the 1960 NCEE Math Exam*

7. A right cylinder’s height is $20 \text{ cm}$, base radius is $5 \text{ cm}$, and inscribed rectangular cuboid’s volume is $800 \text{ cm}^3$. Find the length and width of this rectangular cuboid.

Answer: Let the length and width of this rectangular cuboid be $x \text{ cm}$ and $y \text{ cm}$.

\[
\begin{align*}
20xy &= 800 \\
x^2 + y^2 &= (2 \times 5)^2 \\
x^2 + y^2 &= 100
\end{align*}
\rightarrow \begin{align*} xy &= 40 \\
(x + y)^2 &= 180 \rightarrow x + y &= 6\sqrt{5}; (x - y)^2 &= 20, \\
x - y &= 2\sqrt{5}. x &= 4\sqrt{5}, y &= 2\sqrt{5}.
\end{align*}
\]
8. From a boat, one can see a lighthouse on the sea in the direction 30° east of south. The boat sails to the southeast at a speed of 30 miles per hour. After half an hour, the lighthouse is due west of the boat. Find the current distance between the lighthouse and the boat (rounded to the nearest tenth miles).

Answer: Let $O$ be the original boat’s position and $A$ be the lighthouse’s position. Since $A$ is due west of $B$, $BC \perp OC$, $\angle OBC = \angle BOC = 45^\circ$. Thus, $OC = BC = OB \cdot \sin 45^\circ = 30 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$, $CA = OC \cdot \tan 30^\circ = \frac{15\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{3} = \frac{5\sqrt{6}}{4}$. $AB = CB - CA = \frac{15\sqrt{2}}{2} - \frac{5\sqrt{6}}{2} = \frac{5\sqrt{2}}{2}(3 - \sqrt{3}) \approx 4.5$.

Figure A24

Graph for Item 8 in the 1960 NCEE Math Exam

9. To cut a rectangular glass window in the wall with the perimeter of 6 meters.

(1) Find $y$, the area of the window as a function of $x$, one side length of the rectangle;

(2) Find the vertex’s coordinates and axis of symmetry’s equation of this function;

(3) Draw the graph of this function and find its domain.

Answer: (1) $y = x \cdot (3 - x) = -x^2 + 3x$. (2) $y = -(x - \frac{3}{2})^2 + \frac{9}{4}$. The vertex is $\left(\frac{3}{2}, \frac{9}{4}\right)$; axis of symmetry is $x = \frac{3}{2}$, $0 = -x^2 + 3x \rightarrow x = 0$ or $x = 3$. $0 < x < 3$.

Figure A25

Graph for Item 9 in the 1960 NCEE Math Exam
10. The two roots of the function $2x^2 - 4x \cdot \sin \theta + 3 \cos \theta = 0$ are equal and $\theta$ is an acute angle. Find the values of $\theta$ and two roots of the function.

Answer: $b^2 - 4ac = (-4 \sin \theta)^2 - 4 \cdot 2 \cdot 3 \cos \theta = 0$. $16(1 - \cos^2 \theta) - 24 \cos \theta = 0$

$\rightarrow 2 \cos^2 \theta + 3 \cos \theta - 2 = 0 \rightarrow \cos \theta = \frac{1}{2}$ or $\cos \theta = -2$ (NP). $\theta$ is acute = 60°.

$2x^2 - 2\sqrt{3}x + \frac{3}{2} = 0 \rightarrow x = \frac{\sqrt{3}}{2}$.

11. For what values of $a$, the solution of the following linear system is positive?

\[
\begin{align*}
2x + ay &= 4 \\
x + 4y &= 8
\end{align*}
\]

Answer: $(8 - a)y = 12 \rightarrow y = \frac{12}{8-a}, x = \frac{16-8a}{8-a}, \frac{12}{8-a} > 0 \rightarrow 8 - a > 0 \rightarrow a < 8$.

$\frac{16-8a}{8-a} > 0 \rightarrow 16 - 8a > 0 \rightarrow a < 2$. $a < 2 \cap a < 8 = a < 2$.

The link to the Chinese Version:


1961 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30

1. Find the coefficient of the $x^7$ in the expanded form of the expression $(2 - x)^{10}$. 

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By the binomial theorem, $T_{k+1} = \binom{10}{k}(2)^{10-k}(-x)^k$. Let $k = 7$. Then, $T_6 = \binom{10}{7}(2)^{10-7}(-x)^7 = \frac{720}{6} \cdot 8 \cdot -x^7 = -960x^3$.

2. Solve the equation $2 \log x = \log(x + 12)$.

Answer: $\log x^2 = \log(x + 12) \rightarrow x^2 = x + 12 \rightarrow (x - 4)(x + 3) = 0 \rightarrow x = 4$ or $x = -3$

$x$ cannot be $-3$.

3. Find the domain of the function $y = \frac{\sqrt{x-1}}{x-5}$.

Answer: $x - 1 \geq 0$ and $x - 5 \neq 0$, $x \geq 1$ and $x \neq 5$.

4. Find the value of $\sin \frac{\pi}{12} \cdot \sin \frac{5\pi}{12}$.

Answer: $\sin \frac{\pi}{12} \cdot \cos \frac{\pi}{12} = \frac{1}{2} \sin \frac{\pi}{6} = \frac{1}{4}$.

5. A horizontally placed cylindrical water pipe has an inner radius of $12 \text{ cm}$. The arc of the submerged part on the cross-section of the pipe (see the figure) has a measure of $150^\circ$. Find the area of the submerged part on the cross-section of the pipe ($\pi = 3.14$).

Answer: $\text{arc} ACB = 2\pi \cdot 12 \cdot \frac{150^\circ}{360^\circ} = 10\pi \text{ cm}$. Area of the sector $O-ACB = \frac{1}{2} \times 12 \times 10\pi = 60\pi \text{ cm}^2$. $\text{Sector } OAB = \frac{1}{2} \times 12^2 \times \sin 150^\circ = 36 \text{ cm}^2$. The area of the submerged part $= 60\pi - 36 = 152.4 \text{ cm}^2$.

**Figure A26**

*Graph for Item 5 in the 1961 NCEE Math Exam*
6. The side $BC$ of $\triangle ABC$ is in the plane $M$. Construct a perpendicular line from $A$ to the plane $M$, intersecting $M$ at $A_1$. Let the area of $\triangle ABC$ be $S$, and the dihedral angle between the $\triangle ABC$ and the plane $M$ is $\alpha$ ($0^\circ < \alpha < 90^\circ$). Prove that the area of $\triangle A_1BC = S \cdot \cos \alpha$.

Answer: $S_{\triangle A_1BC} = \frac{1}{2} \cdot BC \cdot A_1D = \frac{1}{2} \cdot BC \cdot AD \cdot \cos \alpha = S \cdot \cos \alpha$.

Figure A27

*Graph for Item 6 in the 1961 NCEE Math Exam*

7. The three-year production plan of a machine factory aimed to produce the same increasing number of machines each year. If 1k more machines were produced in the third year, the annual increase rate would be the same for each year. The number of machines produced in the third year is the same as the half of the total number of machines originally planned to be produced in three years. How many machines were originally planned to be produced at each year?

Answer: Let the machines planned to be produced in three years be $x$, $x + y$, and $x + 2y$. 
1. \[
\begin{aligned}
\left\{
\frac{(x+y)-x}{x} &= \frac{(x+2y+1)-(x+y)}{x+y} \\
x + 2y + 1 &= \frac{1}{2}[x + (x + y) + (x + 2y)]
\end{aligned}
\]

\[
\rightarrow \quad y^2 - x = 0 \quad x - y - 2 = 0 \quad \rightarrow y_1 = 2 \quad \text{or} \quad y_2 = -1 \quad x_1 = 4 \quad \text{or} \quad x_2 = 1.
\]

Machines planned to be produced are 4k, 6k, and 8k.

2. There is a piece of annular iron with an inner radius of 45 cm and an outer radius of 75 cm.

Use one-fifth of it (the shaded part in the picture) to create a truncated-cone-shaped bucket.

What is the volume of the bucket in cubic centimeters?

Answer: \( C_{\text{bot}} = \frac{1}{5} \cdot 2\pi \cdot 45 = 18\pi \text{ cm} \).

\( C_{\text{top}} = \frac{1}{5} \cdot 2\pi \cdot 75 = 30\pi \text{ cm} \).

\( R_{\text{bot}} = \frac{18\pi}{2\pi} = 9 \text{ cm} \),

\( R_{\text{top}} = \frac{30\pi}{2\pi} = 15 \text{ cm} \).

Slant height = 75 - 45 = 30 cm. Height = \( \sqrt{30^2 - (15 - 9)^2} = 12\sqrt{6} \text{ cm} \).

Volume = \( \frac{1}{3} \cdot \pi \cdot h \cdot (R_t^2 + R_b^2 + R_tR_b) = \frac{1}{3} \cdot \pi \cdot 12\sqrt{6} \cdot (15^2 + 9^2 + 15 \cdot 9) = 1764\sqrt{6}\pi \text{ cm}^3 \).

**Figure A28**

*Graph for Item 8 in the 1961 NCEE Math Exam*

9. There are two points A and B on the flat ground. A is in the east of the mountain. B is in the southeast of the mountain and away from A at a distance of 300 meters and a direction of 65° south of west. The elevation angle to the top of the mountain measured at A is 30°. Find the height of the mountain (rounded to the nearest ten meters, \( \sin 70° = 0.94 \)).
Answer: In $\triangle ABC$, $\angle ABC = 180^\circ - 65^\circ - 45^\circ = 70^\circ$. $AC = \frac{AB \sin 70^\circ}{\sin 45^\circ} = 300 \sqrt{2} \sin 70^\circ$. In $\triangle ACD$, $CD = AC \tan 30^\circ = 300 \sqrt{2} \sin 70^\circ \cdot \tan 30^\circ = 100 \sqrt{3} \sin 70^\circ = 100 \times 2.45 \times 0.94 = 230\, m$.

Figure A29

*Graph for Item 9 in the 1961 NCEE Math Exam*

10. Choose one of the following questions to answer:

(1) For what values of $k$, the function $x^2 - 2(k + 3)x + 3k^2 + 1 = 0$ has real roots.

(2) Two roots of the $8x^2 - (8 \sin \alpha)x + 2 + \cos 2\alpha = 0$ are the same. Find $\alpha$.

Answer: (1) $b^2 - 4ac = [-2(k + 3)]^2 - 4(3k^2 + 1) \geq 0 \rightarrow k^2 - 3k + 4 \leq 0$. $-1 \leq k \leq 4$.

(2) $b^2 - 4ac = [-(8 \sin \alpha)]^2 - 4 \cdot 8 \cdot (2 + \cos 2\alpha) = 0 \rightarrow 2 \sin^2 \alpha - \cos 2\alpha - 2 = 0$.

$\sin^2 \alpha = \frac{3}{4}$, $\sin \alpha = \pm \frac{\sqrt{3}}{2}$, $\alpha = k\pi \pm \frac{\pi}{3}, k \in \mathbb{Z}$.

The link to the Chinese Version:


1962 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30
1. The third year’s output of a factory increased by 21% compared with the first year’s output. What is its average annual percentage increase? What percentage of the third year’s output is the first year’s output (rounded to the nearest 1%)?

Answer: Let the average annual percentage increase be \( x \). 
\[
\left(1 + \frac{x}{100}\right)^2 = \frac{121}{100}, \quad x = 10.
\]
\[
1 + \frac{121}{100} = \frac{100}{121} \approx 0.83.
\]
2. Find the real part in the expanded form of the expression \( (1 - 2i)^5 \).

Answer: \((1 - 2i)^5 = 1 - \binom{5}{1}2i + \binom{5}{2}(2i)^2 - \binom{5}{3}(2i)^3 + \binom{5}{4}(2i)^4 - (2i)^5\). The real part is \(1 + \binom{5}{2}(2i)^2 + \binom{5}{4}(2i)^4 = 1 - 40 + 80 = 41\).
3. Solve the equation \( \lg(x - 5) + \lg(x + 3) - 2 \lg 2 = \lg(2x - 9) \).

Answer: \(\lg \frac{(x-5)(x+3)}{4} = \lg(2x - 9) \rightarrow (x - 5)(x + 3) = 4(2x - 9) \rightarrow x^2 - 10x + 21 = 0 \rightarrow (x - 3)(x - 7) = 0 \rightarrow x_1 = 3 \text{ or } x_2 = 7. \lg(3 - 5) \text{ not possible.} \)
4. Find the value of \( \sin(2\arcsin \frac{4}{5}) \).

Answer: Let \( \arcsin \frac{4}{5} = a \) \( (0 < a < \frac{\pi}{2}) \). \( \sin a = \frac{4}{5} \), \( \cos a = \frac{3}{5} \). \( \sin \left(2\arcsin \frac{4}{5}\right) = \sin 2a = 2 \sin a \cos a = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25} \).
5. Prove: (1) Circles’ inscribed parallelograms are rectangles; (2) Circles’ circumscribed parallelograms are rhombuses.

Answer: (1) \( \angle A = \angle C, \angle A + \angle C = 180^\circ, \angle A = \angle C = 90^\circ. ABCD \) is a rectangle. (2) \( AE = AH, BE = BF, CG = CF, DG = DH. AB + CD = AD + BC. AB = CD, AD = BC. AB = BC. ABCD \) is a rhombus.

Figure A30

Graph for Item 5-(1) in the 1962 NCEE Math Exam
6. Solve the system \( \begin{cases} y^2 - 4x - 2y + 1 = 0 \\ y = x + a \end{cases} \) and discuss for what values of \( a \), the linear system has two different real roots; two same real roots; no real roots.

**Answer:**

\[ y - a = y^2 - 4(y - a) - 2y + 1 = 0 \Rightarrow y^2 - 6y + 4a + 1 = 0. \]

\[ y = 3 \pm \sqrt{9 - (4a + 1)} = 3 \pm 2\sqrt{2 - a}. \]

\[ x = 3 - a \pm 2\sqrt{2 - a}. \]

\( a < 2, a = 2, a > 2. \)

7. \( D \) is a point in \( \Delta ABC. AB = AC = 1, \angle BAC = 63^\circ, \angle BAD = 33^\circ, \angle ABD = 27^\circ. \) Find the length of \( DC \) (rounded to the nearest hundredth, \( \sin 27^\circ \approx 0.4540 \)).

**Answer:**

In \( \Delta ABD, \angle BDA = 180^\circ - 33^\circ - 27^\circ = 120^\circ. \)

\[ \frac{AD}{\sin 27^\circ} = \frac{1}{\sin 120^\circ}. \]

\[ AD = \frac{2}{\sqrt{3}} \sin 27^\circ. \]

\[ \angle DAC = 63^\circ - 33^\circ = 30^\circ. \]

In \( \Delta ADC, CD^2 = 1 + AD^2 - 2AD \cos 30^\circ = 1 + \frac{4}{3} \sin^2 27^\circ - 2 \sin 27^\circ = 0.3669. \)

\[ DC = 0.60. \]
8. \(ABCD\) and \(A'B'C'D'\) are two squares. Points \(A', B', C',\) and \(D'\) divide the \(AB, BC, CD,\) and \(DA\) into a ratio of \(m:n.\) Let \(AB = 1.\)

(1) Find the area of \(A'B'C'D'.\)

(2) Prove that the area of \(A'B'C'D'\) is no less than \(\frac{1}{2}\).

Answer: \(AA' = \frac{m}{m+n}, A'B = \frac{n}{m+n}. A'B' = \sqrt{A'B^2 + BB'^2} = \sqrt{A'B^2 + AA'^2} = \sqrt{\frac{m^2+n^2}{(m+n)^2}}.\)

The area of \(A'B'C'D' = A'B'^2 = \frac{m^2+n^2}{(m+n)^2} \cdot \frac{m^2+n^2}{(m+n)^2} - \frac{1}{2} = \frac{2(m^2+n^2)-(m+n)^2}{2(m+n)^2} = \frac{(m-n)^2}{2(m+n)^2} \geq 0.\)

Figure A33

9. In cube \(ABCD-A_1B_1C_1D_1,\) construct a perpendicular line through point \(A,\) intersecting the diagonal \(A_1C\) of the cube at point \(E.\) Prove: \(A_1E:EC = 1:2.\)
Answer: $A_1A \perp AC$. $AC = \sqrt{1^2 + 1^2} = \sqrt{2}$. In right triangle $A_1AC$, $AE \perp A_1C$, $A_1A^2 = A_1E \cdot A_1C$, $1 = A_1E \cdot A_1C$. Similarly, $AC^2 = EC \cdot A_1C$, $2 = EC \cdot A_1C$. $A_1E: EC = 1:2$.

Figure A34

Graph for Item 9 in the 1962 NCEE Math Exam

10. Prove: four lines intersecting each other not at the same point must lie in the same plane.

Answer: Assume two lines $a$ and $b$ intersect at a point $P$. At least one of the two other lines, e.g., $c$, does not pass through the point $P$. $c$ intersects $a$ and $b$ at two different points. Thus, $c$ must lie in the plane determined by the lines $a$ and $b$. The fourth line $d$ intersects the lines $a$, $b$, and $c$ at at least two different points. Thus, $d$ must also lie in the plane determined by the lines $a$ and $b$.

The link to the Chinese Version:


1963 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30

1. Given: $\tan \theta = \sqrt{2}$. Find the value of $\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta}$.

Answer: $\cos \theta \neq 0$. $\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} = \frac{1 + \tan \theta}{1 - \tan \theta} = \frac{1 + \sqrt{2}}{1 - \sqrt{2}} = -3 - 2\sqrt{2}$. 

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2. Given: complex number $1 + \sqrt{3}i$. (1) Find its modulus and argument; (2) Find a new complex number by rotating $1 + \sqrt{3}i$ 150° counterclockwise in the complex plane.

Answer: $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$; \(\tan \theta = \sqrt{3}\), \(\theta = 60°\), argument: \(k \cdot 360° + 60°, k \in \mathbb{Z}\).

\(60° + 150° = 210°\).

\(2 \cdot (\cos 210° + i \cdot \sin 210°) = -\sqrt{3} - i\).

3. \(AB\) is the diameter of the semicircle, \(CD \perp AB\), \(AB = 1\), \(AC:CB = 4:1\). Find \(CD\).

Answer: \(AC = \frac{4}{5}\), \(CB = \frac{1}{5}\), \(CD = \sqrt{\frac{4}{5} \times \frac{1}{5}} = \frac{2}{5}\).

**Figure A35**

*Graph for Item 3 in the 1963 NCEE Math Exam*

4. A point is inside a dihedral angle. Construct two perpendicular lines from the point to the two planes of the dihedral angle. Prove: the dihedral angle’s edge is perpendicular to the plane formed by the two perpendicular lines.

Answer: \(PC \perp \text{Plane } M \rightarrow PC \perp AB\). \(PD \perp \text{Plane } N \rightarrow PD \perp AB\). \(AB \perp \text{Plane } PCD\).

**Figure A36**

*Graph for Item 4 in the 1963 NCEE Math Exam*
5. Find the value of $23.28^{-1.1}$ using the logarithmic and anti-logarithmic tables.

Answer: $\lg 23.28^{-1.1} = -1.1 \times \lg 23.28 = -1.1 \times 1.3670 = -1.5073 = 2.4963$

$23.28^{-1.1} = 0.03135$.

6. Solve the equation $\sin^3 x - \sin x + \cos 2x = 0$.

Answer: $2 \cos 2x \sin x + \cos 2x = 0 \rightarrow \cos 2x (2 \sin x + 1) = 0 \rightarrow \cos 2x = 0, 2x = 2n\pi + \frac{\pi}{2}$,

$x = n\pi + \frac{\pi}{4}, n \in \mathbb{Z}$. $2 \sin x + 1 = 0 \rightarrow \sin x = -\frac{1}{2}, x = 2n\pi - \frac{\pi}{6}, x = 2(n + 1)\pi + \frac{\pi}{6}, n \in \mathbb{Z}$.

7. Use 1, 2, 3, 4, 7, 9 to form a five-digit number without repeating numbers.

(1) How many such five-digit numbers are there in total?

(2) How many of these five-digit numbers are even?

(3) How many of these five-digit numbers are multiples of 3?

Answer: (1) $6 \times 5 \times 4 \times 3 \times 2 = 720$; (2) $5 \times 4 \times 3 \times 2 \times 2 = 240$; (3) $5 \times 4 \times 3 \times 2 \times 1 = 120$.

8. Solve the system \[
\begin{align*}
x^2 - 2xy - y^2 &= 1, \\
\sqrt{xy + 3} &= x
\end{align*}
\] (in the real domain).

Answer: \[
\begin{align*}
3x^2 - 6xy - 3y^2 &= 3, \\
x^2 &= 2x^2 - 5xy - 3y^2 = 0 \rightarrow (2x + y)(x - 3y) = 0.
\end{align*}
\]

$2x + y = 0$ or $x - 3y = 0$. \[
\begin{align*}
xy + 3 &= x^2 \\
2x + y &= 0 \rightarrow x_1 = 1, y_1 = -2; x_2 = -1, y_2 = 2.
\end{align*}
\]
\[ \begin{align*}
xy + 3 &= x^2 \\
x - 3y &= 0 
\end{align*} \]

\[ x_3 = \frac{3\sqrt{2}}{2}, \quad y_3 = \frac{\sqrt{2}}{2}; \quad x_4 = -\frac{3\sqrt{2}}{2}, \quad y_4 = -\frac{\sqrt{2}}{2}. \]

\( x_1, y_1; x_3, y_3 \) are solutions.

9. In the figure, \( CD \) intersects the circle \( O \) at points \( A \) and \( B \), and \( AC = BD \). \( CE \) and \( DF \) are tangent to the circle at points \( E \) and \( F \). Prove: \( \triangle OEC \cong \triangle OFD \).

Answer: \( OA = OB, \angle OAB = \angle OBA, \angle OAC = \angle OBD. AC = BD, \triangle OAC \cong \triangle OBD. OC = OD. \)

\( \angle OEC = \angle OFD = 90^\circ. OE = OF, \triangle OEC \cong \triangle OFD. \)

**Figure A37**

*Graph for Item 9 in the 1963 NCEE Math Exam*

10. In the figure, the sphere has a radius of 1 and is inscribed in a cone. The angle between the slant height of the cone and the base is \( 2\theta \).

   (1) Prove that the sum of the slant height and the base’s radius is \( \frac{2}{\tan \theta (1 - \tan^2 \theta)} \). \( \frac{2\pi}{\tan^2 \theta (1 - \tan^2 \theta)}; \)

   (2) Prove that the surface area of the cone is \( \frac{2\pi}{\cos^2 \theta - \sin^2 \theta} \). \( \frac{2\pi}{\cos^2 \theta - \sin^2 \theta} \). \( \frac{2\pi}{\cos^2 \theta - \sin^2 \theta} \). \( \frac{2\pi}{\cos^2 \theta - \sin^2 \theta} \).

   (3) For what values of \( \theta \), the surface area of the cone is the minimum.

Answer: (1) Let \( C \) be the intersection between slant height and sphere. \( OA = OC = 1, OB = OB, \)

\( \angle OAB = \angle OCB = 90^\circ, \triangle OAB \cong \triangle OCB. \angle OBA = \angle OBC = \theta. \) Let the radius be \( r \) and the slant height be \( l. \)

\( r = \cot \theta, l = \frac{\cot \theta}{\cos 2\theta} \cdot r + l = \cot \theta \left( \frac{1}{\cos 2\theta} + 1 \right) = \frac{1}{\tan \theta} \cdot \frac{1 + \cos 2\theta}{\cos 2\theta} = \frac{\tan \theta}{\tan \theta} \).

\[ \frac{2 \cos^2 \theta}{\cos^2 \theta - \sin^2 \theta} = \frac{1}{\tan \theta} \cdot \frac{2}{1 - \tan^2 \theta} \cdot (2) \] \( \frac{2\pi r l + \pi r^2}{\pi r (l + r)} = \pi \cdot \cot \theta \cdot \frac{2}{\tan \theta (1 - \tan^2 \theta)} = \frac{2\pi}{\tan^2 \theta (1 - \tan^2 \theta)}. \) (3) For \( S_{\text{cone}} \) to be the minimum, \( \tan^2 \theta \cdot (1 - \tan^2 \theta) \) needs to be the
maximum. \( \tan^2 \theta \cdot (1 - \tan^2 \theta) = -\tan^4 \theta + \tan^2 \theta = -\left(\tan^2 \theta - \frac{1}{2}\right)^2 + \frac{1}{4} \cdot \tan^2 \theta - \frac{1}{2} = 0. \)

\( \tan^2 \theta = \frac{1}{2}, \theta = \arctan \frac{\sqrt{2}}{2}. \)

Figure A38

Graph for Item 10 in the 1963 NCEE Math Exam

The link to the Chinese Version:


1964 National College Entrance Examination

Mathematics

Date and time: Aug 20, 7:30-9:30

1. Simplify the expression \( \frac{3\left(\frac{3}{2}(\sqrt{3})^3\right)^3}{(\sqrt{3}-1)^2}. \)

Answer: \( = \frac{\frac{3}{2}(\sqrt{3})^3}{4-2\sqrt{3}} = \frac{\sqrt{3}}{4-2\sqrt{3}} = \frac{\sqrt{3}(2+\sqrt{3})}{2} = \frac{\sqrt{3} + \frac{3}{2}}{2}. \)

2. People A and B are at place D on the south bank of the river, and there is a building P in the due north across the river. A goes due east and B goes due west. A walks \( a \) meters more than B every minute. After 10 minutes, A looks at P in the direction \( \alpha^\circ \) west of north, and B looks at P in the direction \( \beta^\circ \) east of north. Find the distance between D and P.
Answer: Let \( A \) and \( B \)'s velocity be \( v + a \) and \( v \). In \( \Delta PBD \), \( PD = 10v \cdot \cot \beta \). In \( \Delta PAD \), \( PD = 10(v + a) \cdot \cot \alpha \). \( 10v \cdot \cot \beta = 10(v + a) \cdot \cot \alpha \) → \( v = \frac{a \cot \alpha}{\cot \beta - \cot \alpha} \).

\[
\frac{10a \cot \alpha \cdot \cot \beta}{\cot \beta - \cot \alpha} = \frac{10a}{\tan \alpha \cdot \tan \beta}
\]

**Figure A39**

*Graph for Item 2 in the 1964 NCEE Math Exam*

3. Solve the equation \( x^4 + 1 = 0 \) and prove that the four points in the plane that represent the roots of this equation are the vertices of a square.

Answer: \( x^4 = -1 = \cos(2k + 1)\pi + i \sin(2k + 1)\pi \). \( x = \cos \frac{2k + 1}{4} \pi + i \sin \frac{2k + 1}{4} \pi \), \( k = 0, 1, 2, 3 \). \( x_1 = \cos \frac{1}{4} \pi + i \sin \frac{1}{4} \pi = \sqrt{2} (1 + i) \); \( x_2 = \sqrt{2} (-1 + i) \); \( x_3 = \sqrt{2} (-1 - i) \); \( x_4 = \sqrt{2} (1 - i) \). The moduli (e.g., \( |x_1| \)) are all 1 and the arguments’ difference (e.g., \( x_2 - x_1 \)) are all \( \frac{\pi}{2} \). Thus, four vertices form a square.

4. \( A, B, \) and \( C \) are interior angles of a triangle. Prove: \( \cos A = \frac{\sin^2 B + \sin^2 C - \sin^2 A}{2 \sin B \sin C} \).

Answer: \( \frac{\sin^2 B + \sin^2 C - \sin^2 A}{2 \sin B \sin C} = \frac{b^2 + c^2 - a^2}{2bc} = \cos A \).

5. The sum of the squares of the three roots of the equation \( x^3 + mx^2 - 3x + n = 0 \) is 6, and the equation has two equal positive roots. Find the values of \( m \) and \( n \).
Answer: $2\alpha^2 + \beta^2 = 6, \alpha^2 + 2\alpha\beta = -3, 4\alpha^2 + 4\alpha\beta + \beta^2 = 0 \rightarrow (2\alpha + \beta)^2 = 0 \rightarrow \beta = -2\alpha.
2\alpha^2 + (-2\alpha)^2 = 6 \rightarrow \alpha = \pm 1, \alpha = -1 (NP). \beta = -2. m = -(2\alpha + \beta) = 0,
\begin{align*}
n &= -\alpha^2\beta = 2.
\end{align*}

6. The radius of the mouth of a truncated-cone-shaped iron bucket is 15 cm; the radius of its base is 10 cm, and the length of the slant height is 30 cm. Cut the side of the iron bucket along a slant height and lay it flat to get a fan-shaped iron sheet ABCD as shown in the picture. Find the distance between the points A and B.

Answer: $arcAB = 2\pi \cdot 15 = 30\pi; arcCD = 2\pi \cdot 10 = 20\pi; AD = 30$. Let OD be $x$ and $\angle AOB = \theta, (x + 30) \cdot \theta = 30\pi, x \cdot \theta = 20\pi, 30 \cdot \theta = 10\pi. \theta = \frac{\pi}{3}, x = \frac{20\pi}{\theta} = 60^\circ, \triangle OAB$ is an equilateral triangle, $AB = OA = 90$ cm.

**Figure A40**

*Graph for Item 6 in the 1964 NCEE Math Exam*

7. Four points $A, B, C,$ and $D$ are outside the planes $M$ and $N$. The projections of $A, B, C,$ and $D$ in plane $M$ are $A_1, B_1, C_1,$ and $D_1$, and in plane $N$ are $A_2, B_2, C_2,$ and $D_2$. Given that $A_1, B_1, C_1,$ and $D_1$ are on a straight line, and $A_2B_2C_2D_2$ is a parallelogram. Prove that $ABCD$ is also a parallelogram.

Answer: Let the straight line be $l, A_1A$ and $l$ form a plane $P$. 

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$P \perp M$, $B_1$ is in $P$ and $BB_1 \perp M$, $B$ is in $P$. For the same reasons, $C$ and $D$ are in $P$. $AA_2 \parallel DD_2$, $A_2B_2 \parallel D_2C_2$, plane $AA_2B_2 \parallel$ Plane $DD_2C_2$. $P$ intersects these two planes at $AB$ and $CD$, $AB \parallel CD$. For the same reasons, $BC \parallel AD$. Thus, $ABCD$ is a parallelogram.

**Figure A41**

*Graph for Item 7 in the 1964 NCEE Math Exam*

8. In the figure, $ABCD$ is a square with a side length of 1. In the square, $\odot O$ and $\odot O'$ are externally tangent to each other; $\odot O$ is tangent to the sides $AB$ and $AD$; $\odot O'$ is tangent to the sides $CB$ and $CD$. (1) Find the sum of the radii of two circles; (2) For what values of two radii, the sum of the areas of two circles reaches the minimum and maximum values? Justify your results.

Answer: (1) In $\triangle O'O'S$, $O'O' = \sqrt{2}OS \to R + r = \sqrt{2}(1 - R - r)$. $R + r = \frac{\sqrt{2}}{1 + \sqrt{2}} = 2 - \sqrt{2}$. (2) $S = \pi \left[ R^2 + \left( 2 - \sqrt{2} - R \right)^2 \right] = 2\pi \left[ R^2 - (2 - \sqrt{2})R + 3 - 2\sqrt{2} \right] = 2\pi \left[ \left( R - \frac{2 - \sqrt{2}}{2} \right)^2 + \frac{3 - 2\sqrt{2}}{2} \right]$. $R = \frac{2 - \sqrt{2}}{2}, r = \frac{2 - \sqrt{2}}{2}, S$ is the minimum. $\frac{2 - \sqrt{2}}{2} \leq R \leq \frac{1}{2}, R = \frac{1}{2}, r = \frac{3}{2} - \sqrt{2}, S$ is the maximum.

**Figure A42**

*Graph for Item 8 in the 1964 NCEE Math Exam*
9. If the square in Q8 is changed into a rectangle, what results will you get and why?

Answer: isosceles right triangle $OO'S$ becomes a non-isosceles right triangle. Let rectangles’ length be $a$ and width be $b$. $OS = a - R - r$, $O'S = b - R - r$, $OO' = R + r$. By the Pythagorean Theorem, $OS^2 + O'S^2 = OO'^2$, $R + r = a + b - \sqrt{2ab}$. $S = \pi (R^2 + r^2) = \frac{\pi}{2} [(R + r)^2 + (R - r)^2]$. $R = r = \frac{1}{2} (a + b - \sqrt{2ab})$, $S$ is the minimum. $R = \frac{b}{2}, r = a + \frac{b}{2} - \sqrt{2ab}$, $S$ is the maximum.

10. If the square in Q8 is changed into a cube with a side length of 1, and the circles are changed into spheres, what results will you get and why?

Answer: $R + r = \sqrt{3}(1 - R - r)$. $R + r = \frac{3 - \sqrt{3}}{2}$. $SA = 4\pi (R^2 + r^2) = 2\pi [(R + r)^2 + (R - r)^2]$. $R = r = \frac{3 - \sqrt{3}}{4}$, $SA$ is the minimum. $R = \frac{1}{2}, r = \frac{2 - \sqrt{3}}{2}$, $SA$ is the maximum. $V = \frac{4}{3} \pi (R^3 + r^3) = \frac{4}{3} \pi (R + r)(R^2 - Rr + r^2) = \frac{\pi}{3} (R + r)[(R + r)^2 + 3(R - r)^2]$. $R = r = \frac{3 - \sqrt{3}}{4}$, $V$ is the minimum. $R = \frac{1}{2}, r = \frac{2 - \sqrt{3}}{2}$, $V$ is the maximum.

The link to the Chinese Version:

http://www.360doc.com/document/22/0513/22/40122662_1031218061.shtml

1965 National College Entrance Examination
Mathematics

Date and time: Aug 20, 7:30-9:30

1. What is the solid represented by the front and top views shown in the figure? Find its volume.

Answer: regular hexagonal pyramid. \( h = \sqrt{3}a \). Base area is \( 6 \cdot \frac{\sqrt{3}}{4}a^2 = \frac{3\sqrt{3}}{2}a^2 \). Volume is \( \frac{1}{3}hA = \frac{1}{3} \cdot \sqrt{3}a \cdot \frac{3\sqrt{3}}{2}a^2 = \frac{3}{2}a^3 \).

Figure A43

Graph for Item 1 in the 1965 NCEE Math Exam

2. Ship X at point A measured that ship Y was heading due north at a speed of 22 miles per hour at point B, a place in the direction 49°48′ west of north from point A. Ship X immediately set off from A and sailed in the direction \( \alpha^\circ \) west of north at a speed of 26 miles per hour to chase ship X. For what values of \( \alpha \), ship X can meet ship Y after some time without changing directions? (\( \lg 2.2 \approx 0.3424, \lg 2.6 \approx 0.4150 \))

Answer: Let the time it takes for two ships to meet be \( t \). \( BC = 22t, AC = 26t \). \( \frac{26t}{\sin(180^\circ - 49^\circ 48')} = \frac{22t}{\sin(49^\circ 48' - \alpha)} \rightarrow \sin(49^\circ 48' - \alpha) = \frac{22}{26} \sin 49^\circ 48'. \lg \sin(49^\circ 48' - \alpha) = \lg 22 - \lg 26 + \lg \sin 49^\circ 48' = 1.3424 - 1.4150 + 1.8830 = 1.8104. \alpha = 9^\circ 33' \) using a natural sine table.
3. Consider the earth as a sphere with a radius of $R$. Assume the latitudes of two places $A$ and $B$ are the same ($\alpha^\circ$), and their longitudes differ by $\beta^\circ$ ($0^\circ < \beta^\circ \leq 180^\circ$). Find the distance between $A$ and $B$ (the length of the great-circle arc between them).

Answer: Let the distance between $A$ and $B$ be $x$, and the central angle of the great-circle arc $AB$ be $\theta$, $x = \frac{\pi \theta}{180^\circ} R$. Let the center of circle of latitude $\alpha$ be $P$ with a radius of $r$, then $\angle APB = \beta$. In $\Delta PAB$, $AB = 2r \sin \frac{\beta}{2}$. In right $\Delta OAP$, $\angle OAP = \alpha$, $r = R \cos \alpha$, $AB = 2R \cos \alpha \sin \frac{\beta}{2}$. In $\Delta OAB$, $AB = 2R \sin \frac{\theta}{2}$. In right $\Delta OAP$, $\angle OAP = \alpha$, $r = R \cos \alpha$, $AB = 2R \cos \alpha \sin \frac{\beta}{2}$.

$\theta = \arcsin(\cos \alpha \sin \frac{\beta}{2})$. $x = \frac{\pi R}{180^\circ} \arcsin(\cos \alpha \sin \frac{\beta}{2}) = \frac{\pi R}{90^\circ} \arcsin(\cos \alpha \sin \frac{\beta}{2})$.

4. (1) Prove that for any values of $x$, $|\sin 2x| \leq 2|\sin x|$.

(2) Given that $n$ is any positive integer. Use mathematical induction to prove that for any values of $x$, $|\sin nx| \leq n|\sin x|$.

Answer: (1) $|\sin 2x| = 2|\sin x \cos x| = 2|\sin x| \cdot |\cos x| \leq 2|\sin x|$ (since $|\cos x| \leq 1$).

(2) For $n = 1$, $|\sin x| = |\sin x|$, the inequality holds. Assume for $n = k$, the inequality holds. For $n = k + 1$, $\sin(k + 1)x = \sin kx \cos x + \cos kx \sin x$, $|\sin(k + 1)x| \leq |\sin kx| \cdot |\cos x| + \ldots$
\[|\cos kx| \cdot |\sin x|. \text{ Since } |\cos x| \leq 1, \ |\cos kx| \leq 1, \text{ and the hypothesis, } |\sin (k + 1)x| \leq k|\sin x| + |\sin x| = (k + 1)|\sin x|.\]

5. The coordinates of a point \(P\) are \((4, -2)\), the equation of a line \(l\) is \(y - x + 5 = 0\), the equation of a curve \(C\) is \(\frac{(x+1)^2}{2} + \frac{(y-1)^2}{4} = 1\). Find the coordinates of the intersection of the line passing through the point \(P\) and perpendicular to the line \(l\) and the curve \(C\) and draw the simple graphs.

Answer: The line through point \(P\) and perpendicular to the line \(l\) is \(y + 2 = -(x - 4) \rightarrow x + y - 2 = 0\). \(y = -x + 2.\) \(2(x + 1)^2 + (-x + 2 - 1)^2 = 4 \rightarrow 3x^2 + 2x - 1 = 0 \rightarrow (3x - 1)(x + 1) = 0 \rightarrow x_1 = \frac{1}{3} \text{ or } x_2 = -1.\) \(y_1 = \frac{5}{3} \text{ or } y_2 = 3.\)

**Figure A45**

*Graph for Item 5 in the 1965 NCEE Math Exam*

6. For what values of \(p\), the functions \(x^2 + px - 3 = 0\) and \(x^2 - 4x - (p - 1) = 0\) have a common root?

Answer: Let the common root be \(a.\) \(a^2 + pa - 3 = 0\) and \(a^2 - 4a - (p - 1) = 0.\)

\[p = a^2 - 4a + 1.\] \(a^2 + (a^2 - 4a + 1)a - 3 = 0 \rightarrow a^3 - 3a^2 + a - 3 = 0 \rightarrow (a - 3)(a^2 + 1) = 0.\) \(a = 3.\) \(p = -2.\)

7. Given: parabola \(y^2 = 2x.\)
(1) Pick any two points on the parabola $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$. Construct a line through the midpoint of $P_1P_2$, parallel to the axis of symmetry of the parabola, and intersects the parabola at point $P_3$. Prove that the area of $\Delta P_1P_2P_3$ is $\frac{1}{16}|y_1 - y_2|^3$.

(2) Construct lines through the midpoints of $P_1P_3$ and $P_2P_3$, parallel to the axis of symmetry of the parabola, and intersect the parabola at points $Q_1$ and $Q_2$. Express the sum of areas of $\Delta P_1P_3Q_1$ and $\Delta P_2P_3Q_2$ in terms of $y_1$ and $y_2$.

(3) Following the same procedure in (2), four smaller triangles can be made, and so on, a series of triangles can be made. Try to find the area of the figure enclosed by the line segment $P_1P_2$ and the parabola.

Answer: (1) Let $P_3(x_3, y_3)$. $\Delta P_1P_2P_3$’s area is half the absolute value of $\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix}$.

\[
\begin{vmatrix} 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \\ 1 & x_3 & y_3 \end{vmatrix} = \begin{vmatrix} 1 & \frac{1}{2}y_1^2 & y_1 \\ 1 & \frac{1}{2}y_2^2 & y_2 \\ 1 & \frac{1}{2}y_3^2 & y_3 \end{vmatrix} = \frac{1}{2}[y_3^2(y_1 - y_2) + y_3(y_2^2 - y_1^2) + y_1y_2(y_1 - y_2)].
\]

$y_3 = \frac{y_1 + y_2}{2} \cdot \frac{1}{2}[y_3^2(y_1 - y_2) + y_3(y_2^2 - y_1^2) + y_1y_2(y_1 - y_2)] = -\frac{1}{8}(y_1 - y_2)^3$. The area is $\frac{1}{2} \cdot \left|\frac{1}{8}(y_1 - y_2)^3\right| = \frac{1}{16}|y_1 - y_2|^3$. (2) Similarly, the area of $\Delta P_1P_3Q_1$ is $\frac{1}{16}|y_1 - y_3|^3$.

the area of $\Delta P_2P_3Q_2$ is $\frac{1}{16}|y_3 - y_2|^3$. $|y_1 - y_3| = |y_3 - y_2| = \frac{1}{2}|y_1 - y_2|$. the areas of $\Delta P_1P_3Q_1$ and $\Delta P_2P_3Q_2$ is $\frac{1}{16} \cdot \frac{1}{2}|y_1 - y_2|^3$. The sum is $\frac{1}{4} \cdot \frac{1}{16} |y_1 - y_2|^3$. (3) Let the area of $\Delta P_1P_2P_3$ be $A_1$, the sum of the areas in (2) be $A_2$, and so on. Then $A_1 + A_2 + A_3 + A_4 + \cdots = A_1 \left(1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots\right) = A_1 \cdot \frac{1}{1 - \frac{1}{4}} = \frac{1}{12}|y_1 - y_2|^3$. 

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8. (1) \( a, b, \) and \( c \) are real numbers. Prove that the necessary and sufficient conditions for \( a, b, \) and \( c \) to be positive numbers are \[
\begin{align*}
& a + b + c > 0 \\
& ab + bc + ca > 0; \\
& abc > 0
\end{align*}
\]

(2) The three roots \( \alpha, \beta \) and \( \gamma \) of \( x^3 + px^2 + qx + r = 0 \) are real numbers. Prove that the necessary and sufficient conditions for \( \alpha, \beta \) and \( \gamma \) to be three sides of a triangle are

\[
\begin{align*}
& p < 0, q > 0, r < 0 \\
& p^3 > 4pq - 8r.
\end{align*}
\]

Answer: (1) if \( a, b, \) and \( c \) are positive numbers, then \( a + b + c > 0, ab + bc + ca > 0, \) and \( abc > 0; a, b, \) and \( c \) are roots of the function \( x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc = 0, abc > 0, \) \( x \) cannot be 0. For \( x \) to be any negative values, the left side of the equation must be less than 0. Thus, \( x \) must be greater than 0. (2) For \( \alpha, \beta \) and \( \gamma \) to be three sides of a triangle, \( \alpha > 0, \beta > 0 \) and \( \gamma > 0, \) and \( \alpha + \beta > \gamma, \beta + \gamma > \alpha \) and \( \gamma + \alpha > \beta. \) From (1), \( \alpha > 0, \beta > 0 \) and \( \gamma > 0 \rightarrow \alpha + \beta + \gamma > 0, \alpha \beta + \beta \gamma + \gamma \alpha > 0, \) and \( \alpha \beta \gamma > 0 \rightarrow -p > 0, q < 0, \) and \( -r > 0. \)

\( \alpha + \beta > \gamma, \beta + \gamma > \alpha \) and \( \gamma + \alpha > \beta \rightarrow \alpha + \beta - \gamma > 0, \beta + \gamma - \alpha > 0 \) and \( \gamma + \alpha - \beta > 0. \)

Since \( \alpha + \beta + \gamma = -p, \alpha + \beta - \gamma = -p - 2\gamma, \beta + \gamma - \alpha = -p - 2\alpha \) and \( \gamma + \alpha - \beta = -p - 2\beta. \) \( \alpha + \beta > \gamma, \beta + \gamma > \alpha \) and \( \gamma + \alpha > \beta \rightarrow -p - 2\gamma > 0, -p - 2\alpha > 0, \) and \( -p - 2\beta > 0. \)

From (1) \((-p - 2\alpha) + (-p - 2\beta) + (-p - 2\gamma) > 0, (p + 2\alpha)(p + 2\beta) + (p + 2\beta)(p + 2\gamma) + (p + 2\gamma) \)

\( (p + 2\alpha) > 0, -(p + 2\alpha)(p + 2\beta)(p + 2\gamma) > 0. -p > 0. 3p^2 - 4p^2 + 4q > 0 \rightarrow p^2 < 4q. \)

\(-p^3 + 2p^3 - 4pq + 8r > 0 \rightarrow p^3 > 4pq - 8r. \)

The link to the Chinese Version:


1977 National College Entrance Examination

Mathematics
1. Solve the equation \( \sqrt{x - 1} = 3 - x \).

Answer: \( x - 1 = (3 - x)^2 \rightarrow x^2 - 7x + 10 = 0 \rightarrow (x - 2)(x - 5) = 0 \rightarrow x = 2 \) or \( x = 5 \). \( x = 2 \) is not possible.

2. Calculate \( 2^{-\frac{1}{2}} + \frac{2^0}{\sqrt{2}} + \frac{1}{\sqrt{2} - 1} \).

Answer: \( \frac{2}{\sqrt{2}} + \frac{1}{\sqrt{2} - 1} = \sqrt{2} + \sqrt{2} + 1 = 2\sqrt{2} + 1 \).

3. Given: \( \lg 2 = 0.3010, \lg 3 = 0.4771 \). Find the value of \( \lg \sqrt{45} \).

Answer: \( \frac{1}{2} \lg \frac{3^2 \times 10}{2} = \frac{1}{2} (2 \lg 3 + \lg 10 - \lg 2) = 0.8266 \).

4. Prove: \( (1 + \tan \alpha)^2 = \frac{1 + \sin 2\alpha}{\cos^2 \alpha} \).

Answer: \( (1 + \tan \alpha)^2 = \left( \frac{\cos \alpha + \sin \alpha}{\cos \alpha} \right)^2 = \frac{\cos^2 \alpha + 2 \sin \alpha \cos \alpha + \sin^2 \alpha}{\cos^2 \alpha} = \frac{1 + \sin 2\alpha}{\cos^2 \alpha} \).

5. Find the equation of the line passing through point \( (1, 1) \) and the intersection of the lines \( x + y - 7 = 0 \) and \( 3x - y - 1 = 0 \).

Answer: \( 4x = 8, x = 2, y = 5. y - 1 = 4(x - 1) \rightarrow y = 4x - 3 \).

6. The output value of a factory in July this year was 1,000 \( k \) yuan. After that, the monthly output value increased by 20% each month. What was the total output value of this factory from July to October this year?

Answer: \( 1000 + (1 + 20\%)1000 + (1 + 20\%)^21000 + (1 + 20\%)^31000 = \frac{1000 \times [(1.2)^4 - 1]}{1.2 - 1} = \frac{1000 \times 1.0736}{0.2} = 5368k \text{ yuan} \).

7. Given: quadratic function \( y = x^2 - 6x + 5 \).

(1) Find the vertex and axis of symmetry of this function;

(2) Draw its graph;
(3) Find its x-intercept and y-intercept.

Answer: (1) (3, −4), x = 3; (2) See figure; (3) (x − 5)(x − 1) = 0, (1, 0), (5, 0), (0, 5).

**Figure A46**

*Graph for Item 7 in the 1977 NCEE Math Exam*

8. A ship is sailing due east at 20 miles per hour. At a point A, the ship sees a lighthouse B in the direction 45° east of north. After an hour, the ship sees the lighthouse at point C in the direction 15° east of north. Find the distance CB between the ship and the lighthouse.

Answer: $AC = 20. \angle BAC = 45^\circ, \angle ABC = 30^\circ. CB = \frac{AC \cdot \sin A}{\sin B} = \frac{20 \sqrt{2}}{1/2} = 20 \sqrt{2}$ miles.

**Figure A47**

*Graph for Item 8 in the 1977 NCEE Math Exam*
9. A circle has an inscribed triangle ΔABC. The angle bisector of ∠A intersects the side BC at D and the circle at E. Prove: \( AD \cdot AE = AC \cdot AB \).

Answer: In ΔABD and ΔAEC, \( \angle BAD = \angle EAC \), \( \angle ABD = \angle AEC \), ΔABD ~ ΔAEC. \( AD \cdot AE = AC \cdot AB \).

**Figure A48**

*Graph for Item 9 in the 1977 NCEE Math Exam*

10. For what values of \( m \), the straight line \( y = x + m \) and the ellipse \( \frac{x^2}{16} + \frac{y^2}{9} = 1 \) has one intersection, two intersections, and no intersection? Draw the graph when they have one intersection.
Answer: \(\frac{x^2}{16} + \frac{(x+m)^2}{9} = 1\) \(\rightarrow\) \(25x^2 + 32mx + (16m^2 - 144) = 0\). \(b^2 - 4ac = (32m)^2 - 4 \cdot 25 \cdot (16m^2 - 144) = 576(25 - m^2)\). \(m = \pm 5\), one intersection; \(m < 5\), two intersections; \(|m| > 5\), no intersection.

**Figure A49**

*Graph for Item 10 in the 1977 NCEE Math Exam*

11. (1) Find the derivative of the function \(f(x) = \begin{cases} x^2 \sin \frac{\pi}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases} \); 
   
   (2) Find the volume of the solid by rotating the ellipse \(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\) around the \(x\)-axis.

   Answer: (1) \(x \neq 0\), \(f'(x) = 2x \sin \frac{\pi}{x} + x^2 \cos \frac{\pi}{x} \left(\frac{-\pi}{x^2}\right) = 2x \sin \frac{\pi}{x} - \pi \cos \frac{\pi}{x}. x = 0\), \(f'(0) = \lim_{\Delta x \to 0} \frac{f(\Delta x + 0) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x^2 \sin \frac{\pi}{x} \Delta x}{\Delta x} = \lim_{\Delta x \to 0} \Delta x \sin \pi \frac{\Delta x}{\Delta x} = 0\). (2) \(V = \int_{-a}^{a} \pi y^2 dx = \int_{-a}^{a} \pi b^2 (1 - \frac{x^2}{a^2}) dx = \frac{4}{3} \pi ab^2\).

12. (1) Use \(\varepsilon-\delta\) language to describe the definition of a function \(f(x)\) to be continuous at the point \(x = x_0\);
   
   (2) Prove: if \(f(x)\) is continuous at \(x = x_0\), and \(f(x_0) > 0\), then there exists a \(\delta\) such that \(f(x) > 0\) in the interval \((x_0 - \delta, x_0 + \delta)\).
Answer: (1) \( f \) is continuous at \( x_0 \) if for every \( \varepsilon > 0 \), there exists \( \delta > 0 \) such that \( |x - x_0| < \delta \) implies \( |f(x) - f(x_0)| < \varepsilon \). (2) Let \( \varepsilon > 0 \) such that \( f(x_0) - \varepsilon > 0 \) (for instance, \( \varepsilon = \frac{f(x_0)}{2} \)). Since \( f \) is continuous, there exists \( \delta > 0 \) such that \( |x - x_0| < \delta \) implies \( |f(x) - f(x_0)| < \varepsilon \).

\[ \forall x \in (x_0 - \delta, x_0 + \delta), f(x) > f(x_0) - \varepsilon > 0. \]

The link to the Chinese Version: https://zhuanlan.zhihu.com/p/513640251

1978 National College Entrance Examination

Mathematics

Date and time: July 21, 7:30-9:30

1. Factoring the expression \( x^2 - 4xy + 4y^2 - 4z^2 \).

Answer: \( = (x - 2y)^2 - (2z)^2 = (x - 2y - 2z)(x - 2y + 2z) \).

2. A square has a side length of \( a \). Find the volume of the right cylinder whose lateral surface area equals to the square’s area and height equals to the square’s side length.

Answer: Let the base radius be \( r \). \( 2\pi ra = a^2 \rightarrow r = \frac{a}{2\pi} \). \( V = \pi r^2 a = \pi \left( \frac{a}{2\pi} \right)^2 a = \frac{a^3}{4\pi} \).

3. Find the domain of the function: \( y = \sqrt{\log(2 + x)} \).

Answer: \( \log(2 + x) \geq 0 \rightarrow 2 + x \geq 1 \rightarrow x \geq -1 \).

4. Without referring to the trigonometric ratio table, find the value of \( \cos 80^\circ \cos 35^\circ + \cos 10^\circ \cos 55^\circ \).

Answer: \( = \sin 10^\circ \cos 35^\circ + \cos 10^\circ \sin 35^\circ = \sin(10^\circ + 35^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2} \).

5. Simplify the expression \( \left( \frac{1}{4} \right)^{\frac{1}{2}} \left( \frac{\sqrt{4ab^{-1}}}{(0.1)^2(a^3b^{-4})^\frac{3}{2}} \right) \).

Answer: \( = \left( 2^{-2} \right)^{\frac{1}{2}} \left( \frac{2^2ab^{-1}}{(10^{-1})^2(a^3b^{-4})^\frac{3}{2}} \right) = \frac{2^2 \cdot 2}{10^2a^2b^{-2}} = \frac{4}{25} \cdot a^0 \cdot b^2 = \frac{4}{25} b^2 \).
6. Given: \( kx^2 + y^2 = 4, k \in \mathbb{R} \). For different values of \( k \), state the type of the graph represented by the equation, and draw the graph reflecting its characteristics.

Answer: \( k > 0, \frac{x^2}{k} + \frac{y^2}{4} = 1 \) is ellipse whose center is origin; \( k > 1 \), major axis is on \( y \), \( a = 2 \), \( b = \frac{2}{\sqrt{k}} \); \( k = 1 \), circle with radius \( r \); \( k < 1 \), major axis is on \( x \), \( a = \frac{2}{\sqrt{k}} \), \( b = 2 \); \( k = 0, y^2 = 4 \), the graph is two lines \( y = \pm 2 \) parallel to the \( x \)-axis; \( k < 0, -\frac{x^2}{|k|} + \frac{y^2}{4} = 1 \) is hyperbola whose center is origin. Graphs are omitted.

7. In the picture, \( AB \) is the diameter of the semicircle, \( C \) is a point on the semicircle, \( MN \) is tangent to the semicircle at \( C \), \( AM \perp MN, BN \perp MN, CD \perp AB \).

Prove: (1) \( CD = CM = CN \), (2) \( CD^2 = AM \cdot BN \).

Answer: (1) Connect \( CA \) and \( CB \), \( \angle ACB = 90^\circ \). \( \angle ACM = \angle ABC, \angle ACD = \angle ABC, \angle ACM = \angle ACD \). \( \triangle ACM \cong \triangle ACD \). \( CM = CD \). For the same reasons, \( CN = CD \). Thus, \( CD = CM = CN \).

(2) \( CD \perp AB, \angle ACB = 90^\circ, CD^2 = AD \cdot DB \). \( AM = AD, BN = BD, CD^2 = AM \cdot BN \).

Figure A50

Graph for Item 7 in the 1978 NCEE Math Exam

8. Given: \( \log_{18} 9 = a(a \neq 2), 18^b = 5 \). Find the value of \( \log_{36} 45 \).

Answer: \( \log_{18} 5 = b \). \( \log_{36} 45 = \frac{\log_{18} 45}{\log_{18} 36} = \frac{\log_{18} 5 + \log_{18} 9}{\log_{18} 18 + \log_{18} 2} = \frac{a + b}{1 + \log_{18} 18 - \log_{18} 9} = \frac{a + b}{2 - a} \).
9. The interior angles of $\Delta ABC$ form an arithmetic progression, $\tan A \tan C = 2 + \sqrt{3}$. Find the magnitudes of three angles. Also, the height on the opposite side $c$ of vertex $C$ is $4\sqrt{3}$. Find the lengths of three sides $a, b, c$. (Hint: $(1 + \sqrt{3})^2 = 4 + 2\sqrt{3}$)

Answer: $A + B + C = 180^\circ, 2B = A + C, B = 60^\circ, A + C = 120^\circ. \tan(A + C) =
\frac{\tan A + \tan C}{1 - \tan A \tan C} \to \tan A + \tan C = (1 - \tan A \tan C) \tan(A + C) = (-1 - \sqrt{3}) \cdot (-\sqrt{3}) = 3 + \sqrt{3}.$

$\tan A$ and $\tan C$ are the roots of the equation $x^2 - (3 + \sqrt{3})x + 2 + \sqrt{3} = 0. (x - 1)[x - (2 + \sqrt{3})] = 0. x_1 = 1 \text{ or } x_2 = 2 + \sqrt{3}. \tan A = 1, A = 45^\circ. C = 120^\circ - A = 75^\circ. a = \frac{4\sqrt{3}}{\sin 60^\circ} = 8; b = \frac{4\sqrt{3}}{\sin 45^\circ} = 4\sqrt{6}; c = AD + DB = b \cos 45^\circ + a \cos 60^\circ = 4\sqrt{3} + 4.$

Figure A51

Graph for Item 9 in the 1978 NCEE Math Exam

10. Given: $\alpha$ and $\beta$ are acute angles, $3 \sin^2 \alpha + 2 \sin^2 \beta = 1, 3 \sin 2\alpha - 2 \sin 2\beta = 0.$

Prove: $\alpha + 2\beta = \frac{\pi}{2}.$

Answer: $3 \sin^2 \alpha = 1 - 2 \sin^2 \beta = \cos 2\beta, 6 \sin \alpha \cos \alpha = 2 \sin 2\beta.$ Since $\alpha$ and $\beta$ are acute angles, $\sin \alpha, \cos \alpha, \sin 2\beta$ cannot be zero. Thus, $\frac{3 \sin^2 \alpha}{6 \sin \alpha \cos \alpha} = \frac{\cos 2\beta}{2 \sin 2\beta}, \tan \alpha = \cot 2\beta, \tan \alpha = \tan\left(\frac{\pi}{2} - 2\beta\right). 0 < \beta < \frac{\pi}{2}, -\frac{\pi}{2} < -\beta < \frac{\pi}{2} - 2\beta < \frac{\pi}{2}. 0 < \alpha < \frac{\pi}{2}, \alpha = \frac{\pi}{2} - 2\beta.$

11. Given: $y = x^2 + (2m + 1)x + m^2 - 1, m \in \mathbb{R}.$
(1) Find the values of $m$ when the extreme value of $y$ is 0.

(2) Prove: For all $m$, the vertices of the function $y$ are on the same straight line $l_1$.

Draw the graphs of the function when $m = -1, 0, 1$ to verify this statement.

(3) For the lines parallel to $l_1$, which intersect the parabolas, and which do not?

Prove: For all lines parallel to $l_1$ and intersect the parabolas, the lengths of the line segments cut by each parabola are the same.

Answer: (1) $y = \left(x + \frac{2m+1}{2}\right)^2 - \frac{4m+5}{4} - \frac{4m+5}{4} = 0 \rightarrow m = -\frac{5}{4}$.

(2) The vertices are \(-\frac{2m+1}{2}, -\frac{4m+5}{4}\), $x = -\frac{2m+1}{2} = -m - \frac{1}{2}$,

$y = -\frac{4m+5}{4} = -m - \frac{5}{4}, x - y = \frac{3}{4}$. Since the equation does not include $m$, the vertices are always on the line $x - y = \frac{3}{4}$ regardless of $m$.

(3) Let $x - y = a$ be any line parallel to $l_1$, $x - a = x^2 + (2m + 1)x + m^2 - 1 \rightarrow (x + m)^2 = 1 - a$. For $1 - a \geq 0$, $a \leq 1$, the line and the parabola have intersections. For $1 - a < 0$, $a > 1$, the line and the parabola do not have intersections. When $a \leq 1, x = -m \pm \sqrt{1-a}$, the $x$-coordinates of the intersections. Since the slope of the line is 1, the angle of inclination is $45^\circ$. Thus, the lengths of the line segments cut by parabolas are $\left[\left(-m + \sqrt{1-a}\right) - \left(-m - \sqrt{1-a}\right)\right] \cdot \sqrt{2} = 2\sqrt{2(1-a)}$. Since the equation does not include $m$, the lengths are the same.

Figure A52

*Graph for Item 11 in the 1978 NCEE Math Exam*
1. If \((z - x)^2 - 4(x - y)(y - z) = 0\), prove that \(x, y, z\) form an arithmetic progression.

Answer: \((x + z)^2 - 2 \cdot 2y \cdot (x + z) + 4y^2 = (x + z - 2y)^2 = 0, x + z = 2y\).

2. Simplify the expression \(\frac{1}{1 - \frac{1}{1 - \csc^2 x}}\).

Answer: \(\frac{1}{1 - \cot^2 x} = \frac{1}{1 - \tan^2 x} = \frac{1}{1 - \csc^2 x} = \frac{1}{\sin^2 x} = \csc^2 x\).

3. Containers A and B are filled with \(v_1\) and \(v_2\) kilograms of alcohol, respectively. The ratio of the alcohol to water in A is \(m_1 : n_1\); The ratio of the alcohol to water in B is \(m_2 : n_2\). What is the ratio of the alcohol to water when A is mixed with B?

Answer: The alcohol after the mixture is \(\frac{m_1v_1}{m_1+n_1} + \frac{m_2v_2}{m_2+n_2} = \frac{m_1v_1(m_2+n_2)+m_2v_2(m_1+n_1)}{(m_1+n_1)(m_2+n_2)}\).

The water after the mixture is \(\frac{n_1v_1(m_2+n_2)+n_2v_2(m_1+n_1)}{(m_1+n_1)(m_2+n_2)}\). Ratio is \(\frac{m_1v_1(m_2+n_2)+m_2v_2(m_1+n_1)}{n_1v_1(m_2+n_2)+n_2v_2(m_1+n_1)}\).
4. State and prove the Pythagorean Theorem.

Answer: In right triangles, the sum of the areas of the two squares on the legs \((a \text{ and } b)\) equals the area of the square on the hypotenuse \((c)\). Both squares have a side length of \(a + b\). Thus, their areas must be equal. Left square: \(4 \times \frac{1}{2}ab + c^2\). Right square: \(a^2 + b^2 + 4 \times \frac{1}{2}ab\). Left equals to Right: \(c^2 = a^2 + b^2\).

Figure A53

*Graph for Item 4 in the 1979 NCEE Math Exam*

5. Foreign vessels, except those authorized, are not allowed to enter the sea area within \(D\) nautical miles from the coastline of our country. Let A and B be our observatories, the distance between A and B is \(S\) nautical miles, and the coastline is a straight line across A and B. A foreign vessel is at point P. The angle measured at A \((\angle BAP)\) is \(\alpha\). The angle measured at B \((\angle ABP)\) is \(\beta\). What simple trigonometric inequalities are satisfied by \(\alpha\) and \(\beta\), the vessel should be warned and ordered to withdraw?

Answer: construct \(PC \perp AB\), and assume \(PC = d\). In right \(\triangle PAC\), \(AC = d \cdot \cot \alpha\). In right \(\triangle PBC\), \(BC = d \cdot \cot \beta\). \(S = AC + BC = d \cdot \cot \alpha + d \cdot \cot \beta = d(\cot \alpha + \cot \beta)\). \(d \leq D \rightarrow \cot \alpha + \cot \beta \geq \frac{S}{D}\).

Figure A54
6. In triangular pyramid $VABC$, $\angle AVB = \angle BVC = \angle CVA = 90^\circ$. Prove: $\triangle ABC$ is acute.

Answer: Let $VA = a, VB = b, VC = c, AB = p, BC = q, CA = r$. $p^2 = a^2 + b^2, q^2 = b^2 + c^2, r^2 = c^2 + a^2$. $\cos \angle CAB = \frac{a^2+b^2+c^2-(b^2+c^2)}{2\sqrt{a^2+b^2}\sqrt{c^2+a^2}} = \frac{a^2}{\sqrt{a^2+b^2}\sqrt{c^2+a^2}} > 0$. Thus, $\angle CAB$ is acute. For the same reasons, $\angle ABC$ and $\angle BCA$ are also acute.

7. The price in the U.S. increased from 100 in 1939 to 500 in 1979 after 40 years. If the annual price growth rate stayed the same, what was the price’s percentage increase in each year.

(Note: $e = 2.718 \ldots, \ln(1 + x) \approx x, \lg 2 = 0.3, \ln 10 = 2.3$)

Answer: Let the percentage increase be $x$, $100(1 + x)^{40} = 500 \rightarrow (1 + x)^{40} = 5 \rightarrow 40 \ln(1 + x) = \ln 5$. $\lg 5 = 1 - 0.3 = 0.7, \ln 5 = \ln 10 \cdot \lg 5 = 1.61$. $x \approx \frac{\ln 5}{40} \approx 4\%$.

8. Given: $CEDF$ is an inscribed rectangle of a circle. A line tangent to the circle at D intersect the extended CE at point A and extended CF at point B. Prove: \( \frac{BF}{AE} = \frac{BC^3}{AC^3} \).

Answer: connect $CD, \angle CFD = 90^\circ, CD$ is the diameter. $CD \perp AB$. In right $\triangle ABC, \angle ACB = 90^\circ, AC^2 = AD \cdot AB, BC^2 = BD \cdot BA$. $\frac{BD}{AD} = \frac{BC^2}{AC^2}, AD^2 = AC \cdot AE, BD^2 = BC \cdot BF$. $\frac{BD^2}{AD^2} = \frac{BC \cdot BF}{AC \cdot AE}$

Thus, $\frac{BF}{AE} = \frac{BC^4}{AC^4} \rightarrow \frac{BF}{AE} = \frac{BC^3}{AC^3}$.

Figure A55
9. Given the sequence: \( \lg 100, \lg \left( 100 \sin \frac{\pi}{4} \right), \lg \left( 100 \sin^2 \frac{\pi}{4} \right), \ldots, \lg \left( 100 \sin^{n-1} \frac{\pi}{4} \right) \). Find (1) the value of \( n \) when the sum of the first \( n \) terms achieves the maximum value; (2) the maximum value in (1). (Note: \( \lg 2 = 0.301 \)).

Answer: (1) the \( k \)th term in the sequence is \( a_k = \lg (100 \sin^{n-1} \frac{\pi}{4}) = 2 - \frac{1}{2} (k - 1) \log 2 \). The sequence is a decreasing arithmetic sequence with the first term 2. To find its maximum value, the first \( k \) terms must be positive or zero, and the \((k + 1)\)th term is negative. Thus, \( 2 - \frac{1}{2} (k - 1) \log 2 \geq 0 \), \( 2 - \frac{1}{2} (k - 1) - 1) \log 2 < 0 \). 13.2 < \( k \) ≤ 14.2. \( k = 14 \). (2) \( S = \frac{a_1 + a_{14}}{2} \times 14 = 14.30 \).

10. Let isosceles \( \Delta OAB \)’s vertex angle be \( 2\theta \) and height be \( h \).

(1) The distance from a moving point \( P \) to OA, OB, and AB are \( |PD|, |PF|, \) and \( |PE| \), and satisfy the equation \( |PD| \cdot |PF| = |PE|^2 \). Find the trajectory of point \( P \).

(2) Find the coordinates of \( P \) when \( |PD| + |PE| = |PF| \).

Answer: (1) Let \( OC \) be the altitude on the base of isosceles \( \Delta OAB \) and \( \angle POC = \alpha \).

Assume \( P \)'s coordinates are \((x, y)\). Then, \( |OP| = \sqrt{x^2 + y^2} \). \( |PD| = |OP| \cdot \sin(\theta - \alpha) = |OP| \cdot (\sin \theta \cos \alpha - \cos \theta \sin \alpha) = x \sin \theta - y \cos \theta \). \( |PF| = |OP| \cdot \sin(\theta + \alpha) = |OP| \cdot (\sin \theta \cos \alpha + \cos \theta \sin \alpha) = x \sin \theta + y \cos \theta \). \( |PD| \cdot |PF| = |PE|^2 \rightarrow x^2 \sin^2 \theta - y^2 \cos^2 \theta = (h - x)^2 \rightarrow x^2 \cos^2 \theta - 2hx + y^2 \cos^2 \theta + h^2 = 0 \). Since \( \cos^2 \theta \neq 0 \), \( x^2 - \frac{2h}{\cos^2 \theta} x + y^2 + \frac{h^2}{\cos^2 \theta} = 0 \). \( (x - \frac{h}{\cos^2 \theta})^2 + y^2 = \left( \frac{h \sin \theta}{\cos^2 \theta} \right)^2 \). The trajectory of point \( P \) part of the circle with
center \( \left( \frac{h}{\cos^2 \theta}, 0 \right) \) and radius \( h \frac{\sin \theta}{\cos^2 \theta} \) inside the isosceles \( \Delta OAB \). (2) \(|PD| + |PE| = |PF| \to x \sin \theta - y \cos \theta + h - x = x \sin \theta + y \cos \theta \to x - 2y \cos \theta = h. x^2 \sin^2 \theta - y^2 \cos^2 \theta = (h - x)^2 \to x^2 \sin^2 \theta - y^2 \cos^2 \theta = 4y^2 \cos^2 \theta \). Thus, \( 5y^2 \cos^2 \theta = x^2 \sin^2 \theta, y = \frac{1}{\sqrt{5}} \tan \theta \cdot x. \)

Since \( |PD| + |PE| = |PF|, y > 0. x \left( 1 + \frac{2}{\sqrt{5}} \sin \theta \right) = h, x = \frac{\sqrt{5}h}{\sqrt{5} + 2 \sin \theta}, y = \frac{h \tan \theta}{\sqrt{5} + 2 \sin \theta}. \)

**Figure A56**

*Graph for Item 10 in the 1979 NCEE Math Exam*

The link to the Chinese Version:

https://wenku.baidu.com/view/57e841806337ee06eff9aef8941ea76e59fa4a7c?aggId=0b262fd385c24028915f804d2b160b4e777f81dc&wktid=1703624630998&bdQuery=1978+2010+%E6%95%B0%E5%AD%A6%E6%95%B0%E9%AB%98%E8%80%83

1980 National College Entrance Examination

Mathematics

Date and time: July 8, 8:00-10:00

1. Factoring the polynomial \( x^5y - 9xy^5 \) with respect to the following domains:

   (1) rational numbers, (2) real numbers, (3) complex numbers.

Answer: (1) \( x^5y - 9xy^5 = xy(x^4 - 9y^4) = xy(x^2 + 3y^2)(x^2 - 3y^2) \); (2) \( x^5y - 9xy^5 \)
\[ xy(x^2 + 3y^2)(x + \sqrt{3}y)(x - \sqrt{3}y); \]
\[ x^5 y - 9x y^5 = xy(x + \sqrt{3}yi)(x - \sqrt{3}yi) \cdot (x + \sqrt{3}y)(x - \sqrt{3}y). \]

2. Three circles with radii 1, 2, and 3 are externally tangent to each other.

Prove: the centers of these three circles form a right triangle.

Answer: Let the radii of \( O_1, O_2, \) and \( O_3 \) be 1, 2, and 3, respectively. Since they are externally tangent to each other, \( O_1 O_2 = 3, O_2 O_3 = 5, O_1 O_3 = 4. \)

\[ 0_1 O_2^2 + O_1 O_3^2 = 3^2 + 4^2 = 5. \]

3. Use coordinate geometry to prove that three altitudes of a triangle are concurrent.

Answer: Let \( A, B, \) and \( C \) be \((0, a), (b, 0), \) and \((c, 0)\). The equation of \( AB \) is \( \frac{x}{b} + \frac{y}{a} = 1 \), with a slope of \(-\frac{a}{b}\); The equation of \( AC \) is \( \frac{x}{c} + \frac{y}{a} = 1 \), with a slope of \(-\frac{a}{c}\). Thus, \( CE \)'s equation is \( y = \frac{b}{a}(x - c) \); \( BD \)'s equation is \( y = \frac{c}{a}(x - b) \). \((b - c)x = 0, x = 0. \) The \( x \)-coordinate of \( BE \) and \( CF \)'s intersection is 0, meaning it is on the altitude \( AO \).

**Figure A57**

*Graph for Item 3 in the 1980 NCEE Math Exam*

4. Prove the change of base formula: \( \log_b N = \frac{\log_a N}{\log_a b}, a, b, N \in \mathbb{R}^+, a, b \neq 1. \)
Answer: Let \( \log_b N = x \). Then, \( b^x = N \). \( \log_a b^x = \log_a N \rightarrow x \log_a b = \log_a N \). Since \( b \neq 1 \), 
\( \log_a b \neq 0 \), \( x = \frac{\log_a N}{\log_a b} = \log_b N \).

5. The orthogonal projection of a point \( P \) on the helicopter on ground plane \( M \) is \( A \). To look
   from point \( P \) at an object \( B \) on the ground, the line \( PB \) is perpendicular to the plane of a
   helicopter window \( N \). Prove: plane \( N \) must intersect plan \( M \) and their intersection line \( l \) is
   perpendicular to line \( AB \).
   Answer: Assume plane \( N \) is parallel to plan \( M \), then \( PA \perp N \), \( PA \) and \( PB \) coincide, \( B \) and \( A \) must
   coincide, contradiction. Thus, plane \( N \) must intersect plan \( M \) at a line and let it be \( l \). Since \( PA \perp M \), \( PA \perp l \). \( PB \perp N \rightarrow PB \perp l \). Thus, \( l \) is perpendicular to plane \( PAB \). \( l \perp AB \).

Figure A58

*Graph for Item 5 in the 1980 NCEE Math Exam*

6. Let the trigonometric function be \( f(x) = \sin \left( \frac{kx}{5} + \frac{\pi}{3} \right), k \neq 0 \).

   (1) State the maximum value \( (M) \), the minimum value \( (m) \), and the period \( (T) \).

   (2) Find the smallest positive integer \( k \), such that when the independent variable \( x \) varies
   between any two integers, the function \( f(x) \) has at least one \( M \) and one \( m \).

   Answer: (1) \( M = 1, m = -1 \), and \( T = \frac{5 \times 2\pi}{|k|} = \frac{10\pi}{|k|} \). (2) \( T = \frac{10 \pi}{|k|} \leq 1 \), \( |k| \geq 10 \pi \). \( k = 32 \).
7. CD is the altitude on the hypotenuse of a right triangle ABC. Given: the areas of
\( \Delta ADC, \Delta CBD, \) and \( \Delta ABC \) form a geometric progression. Find the measure of \( \angle B \) (expressed by inverse trigonometric functions).

Answer: Let \( CD = h, AB = c, BD = x, AC = c - x \). 
\[
S_{\Delta ACD} = \frac{1}{2} h(c - x); S_{\Delta CBD} = \frac{1}{2} hx;
\]
\[
S_{\Delta ABC} = \frac{1}{2} hc. \left(\frac{1}{2} hx\right)^2 = \frac{1}{2} h(c - x) \cdot \frac{1}{2} hc \rightarrow x^2 = c(c - x). \ x = \frac{-c + \sqrt{c^2 + 4c^2}}{2}. \text{ A side cannot be negative. \( AC^2 = AD \cdot AB = c(c - x) = x^2 \). Thus, \( AC = x = DB = \frac{\sqrt{5} - 1}{2} c \).}
\]
In right \( \Delta ABC \), \( \sin B = \frac{AC}{AB} = \frac{\sqrt{5} - 1}{2} \). \( \angle B = \arcsin \frac{\sqrt{5} - 1}{2} \).

\[ \text{Figure A59} \]

\[ \text{Graph for Item 7 in the 1980 NCEE Math Exam} \]

8. Given: \( 0 < \alpha \leq \pi \). Prove: \( 2 \sin 2\alpha \leq \cot \frac{\alpha}{2} \), and discuss when does the equality hold?

Answer: \( 2 \sin 2\alpha \leq \frac{1 + \cos \alpha}{\sin \alpha} \). Since \( \sin \alpha > 0 \), \( 2 \sin \alpha \sin 2\alpha \leq 1 + \cos \alpha \cdot 2 \sin \alpha \sin 2\alpha \)
\[
= 4 \sin^2 \alpha \cos \alpha = 4(1 - \cos \alpha)(1 + \cos \alpha) \cos \alpha. \ 2 \sin \alpha \sin 2\alpha \leq 1 + \cos \alpha \rightarrow
\]
\[
4(1 - \cos \alpha)(1 + \cos \alpha) \cos \alpha \leq 1 + \cos \alpha \rightarrow (1 + \cos \alpha)[4(1 - \cos \alpha) \cos \alpha - 1] \leq 0
\]
\[
\rightarrow (1 + \cos \alpha)\left[-4 \left(\cos \alpha - \frac{1}{2}\right)^2\right] \leq 0. \ \text{The equality holds when} \ \cos \alpha - \frac{1}{2} = 0, \ \alpha = 60^\circ.
\]

9. The equation of a parabola is \( y^2 = 2x \). The center of a circle whose radius is 1 is moving along the \( x \)-axis. Find the position of the circle when the tangents of the circle and the
parabola at the intersection are perpendicular to each other. (Note: Let $P(x_0, y_0)$ be a point on the parabola $y^2 = 2px$, the slope of the tangent at $P$ is $\frac{P}{y_0}$.)

Answer: Let the circle’s equation be $(x - k)^2 + y^2 = 1$ and one intersection be $P(x_0, y_0)$. The tangent’s slope at $P$ of the parabola is $\frac{1}{y_0}$; The radius’ slope at $P$ of the circle is $\frac{y_0}{x_0-k}$. Since the radius of a circle is perpendicular to its tangents, $\frac{1}{y_0} = \frac{y_0}{x_0-k}$.

Also, $P$ is an intersection $\rightarrow y_0^2 = 2x_0$, $(x_0 - k)^2 + y_0^2 = 1$, $x_0 = -k$. $4k^2 - 2k - 1 = 0$, $k = \frac{1 \pm \sqrt{5}}{4}$. Since the parabola opens to the right, $k$ is negative. The equation of the circle is $\left(x - \frac{1 - \sqrt{5}}{4}\right)^2 + y^2 = 1$.

10. Let the parametric equation of the line $l$ be \begin{align*}
 x &= t \\
 y &= b + mt
\end{align*} (t is the parameter) and the ellipse $E$ be \begin{align*}
 x &= 1 + a \cos \theta \ (a \neq 0) \\
 y &= \sin \theta
\end{align*} ($\theta$ is the parameter). What conditions should $a$ and $b$ satisfy so that for any value of $m$, the line $l$ and the ellipse $E$ always have a common point?

Answer: Eliminating the parameter to get $y = b + mx$ and $\frac{(x-1)^2}{a^2} + y^2 = 1$. Eliminating the $y$ to get $(1 + a^2m^2)x^2 + 2(a^2mb - 1)x + a^2b^2 - a^2 + 1 = 0$. $b^2 - 4ac \geq 0 \rightarrow (a^2 - 1)m^2 - 2bm + (1 - b^2) \geq 0$. For the inequality to hold, $a^2 - 1 > 0$, $b^2 - (a^2 - 1)(1 - b^2) \leq 0$, or $a^2 - 1 = 0, b = 0$. Solving the system to get $|a| \geq 1$, $\frac{-\sqrt{a^2 - 1}}{|a|} \leq b \leq \frac{\sqrt{a^2 - 1}}{|a|}$.

The link to the Chinese Version:
https://wenku.baidu.com/view/57e841806337ee06eff9aef8941ea76e59fa4a7e?agglId=0b262fd385c24028915f804d2b160b4e777f81dc&_wkts_=1703624630998&bdQuery=1978+2010+%E6%95%B0%E5%AD%A6%E9%AB%98%E8%80%83

282
1981 National College Entrance Examination
Mathematics

Date and time: July 8, 8:00-10:00

1. Let A and B be the sets of rational and irrational numbers. Find $A \cup B$ and $A \cap B$.

Answer: $A \cup B = \mathbb{R}$, $A \cap B = \emptyset$.

2. How many ways are there to select a class president and a vice president from A, B, C, and D four candidates? List all possible outcomes. How many ways to select three class committees? List all possible outcomes.

Answer: Twelve. AB, AC, AD, BA, BC, BD, CA, CB, CD, DA, DB, DC. Four. ABC, ABD, ACD, BCD.

3. In the table below, determine whether the statements A are sufficient, necessary, both sufficient and necessary, or neither sufficient nor necessary for the statements B.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$ABCD$ is a parallelogram</td>
<td>$ABCD$ is a rectangle</td>
<td>N</td>
</tr>
<tr>
<td>2</td>
<td>$a = 3$</td>
<td>$</td>
<td>a</td>
</tr>
<tr>
<td>3</td>
<td>$\theta = 150^\circ$</td>
<td>$\sin \theta = \frac{1}{2}$</td>
<td>S</td>
</tr>
<tr>
<td>4</td>
<td>Points $(a, b)$ on the circle $x^2 + y^2 = R^2$</td>
<td>$a^2 + b^2 = R^2$</td>
<td>S &amp; N</td>
</tr>
</tbody>
</table>

4. State the law of cosines (one formula is sufficient) and prove it.

Answer: $a^2 = b^2 + c^2 - 2bc \cos A$. If $A$ is acute, $a^2 = CD^2 + DB^2 = (b \sin A)^2 + (c - b \cos A)^2 = b^2 + c^2 - 2bc \cos A$. If $A$ is right, $\cos A = 0, a^2 = b^2 + c^2 = b^2 + c^2 -$
2bc \cos A. If A is obtuse, 
\[ a^2 = CD^2 + DB^2 = [b \sin(180° - A)]^2 + [c + b \cos(180° - A)]^2 = b^2 + c^2 - 2bc \cos A. \]

**Figure A60**

*Graph for Item 4 in the 1981 NCEE Math Exam*

5. Solve the inequality (x is the unknown variable):

\[
\begin{vmatrix}
  x - a & b & -c \\
  a & x - b & c \\
  -a & b & x - c
\end{vmatrix} > 0
\]

Answer:
\[
\begin{vmatrix}
  x - a & b & -c \\
  a & x - b & c \\
  -a & b & x - c
\end{vmatrix} = x^2 \begin{vmatrix}
  1 & 1 & 0 \\
  0 & 1 & 1 \\
  -a & b & x - c
\end{vmatrix} = x^2(x - a - b - c) > 0. x \neq 0, x > a + b + c.
\]

6. Use mathematical induction to prove the equation:

\[
\cos \frac{x}{2} \cdot \cos \frac{x}{2^2} \cdot \cos \frac{x}{2^3} \cdot \cdots \cdot \cos \frac{x}{2^n} = \frac{\sin x}{2^n \sin \frac{x}{2^n}}, n \in \mathbb{N}
\]

Answer: When \( n = 1 \), left side = \( \cos \frac{x}{2} \), right side = \( \frac{\sin x}{2 \sin^2 \frac{x}{2}} = \frac{2 \sin^2 \frac{x}{2} \cos \frac{x}{2}}{2 \sin^2 \frac{x}{2}} = \cos \frac{x}{2} \).

Assume when \( n = k \), the equation holds. Then for \( n = k + 1 \),

\[
\cos \frac{x}{2} \cdot \cos \frac{x}{2^2} \cdot \cos \frac{x}{2^3} \cdot \cdots \cdot \cos \frac{x}{2^{k+1}} = \frac{\sin x}{2 \sin^2 \frac{x}{2^k}} \cdot \cos \frac{x}{2^{k+1}} = \frac{\sin x \cdot \cos \frac{x}{2^{k+1}}}{2^{k+1} \sin^2 \frac{x}{2^{k+1}} \cos \frac{x}{2^{k+1}}} = \frac{\sin x}{2^n \sin^2 \frac{x}{2^n}}.
\]

7. Assume that the population of our country at the end of 1980 was one billion.
(1) If the annual population growth rate was 2%, what would be the population of our
country at the end of 2000?

(2) If the country’s population at the end of 2000 does not exceed 1.2 billion, what would be
the highest average annual growth rate?

Table A2

Table for Item 7 in the 1981 NCEE Math Exam

<table>
<thead>
<tr>
<th>Reference Table</th>
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</thead>
<tbody>
<tr>
<td>log 1.0087 = 0.00377</td>
</tr>
<tr>
<td>log 1.0096 = 0.00417</td>
</tr>
<tr>
<td>log 1.2000 = 0.07918</td>
</tr>
<tr>
<td>log 1.4568 = 0.16340</td>
</tr>
<tr>
<td>log 1.5157 = 0.18060</td>
</tr>
</tbody>
</table>

Answer: (1) Let the population be \( x \), then
\[
x = 10 \times (1.02)^{20}, \quad \lg x = 1 + 20 \lg 1.02 = 1.1720,
\]
\[x = 14.859.\]
(2) Let the highest growth rate be \( y\%\), then
\[
10 \times (1 + y\%)^{20} \leq 12, \quad (1 + y\%)^{20} \leq 1.2, \quad 20 \lg(1 + y\%) = \lg 1.2, \quad \lg(1 + y\%) = 0.00396, \quad 1 + y\% \leq 1.0092, \quad y\% \leq 0.0092.
\]

8. Points \( A \) and \( B \) are in the planes \( P \) and \( Q \) of a 120° dihedral angle with an edge \( a \). The
distances from \( A \) and \( B \) to \( a \) are 2 and 4, respectively. The length of \( AB \) is 10.

(1) Find the measure of the angle formed by \( AB \) and \( a \).

(2) Find the measure of the angle formed by \( AB \) and \( Q \).

Answer: (1) \( \angle ADC = 120^\circ, AD = 2, BCDE \) is a rectangle, \( CD = BE = 4. AC^2 = AD^2 +
\]
\[
CD^2 - 2AD \cdot CD \cdot \cos 120^\circ = 28, AC = 2\sqrt{7}. AD \perp a, CD \perp a \rightarrow a \perp \text{plane} ACD. BC \parallel a,
\]
\[BC \perp \text{plane} ACD \rightarrow BC \perp AC.\] In right \( \triangle ABC \),
\[
\sin \angle ABC = \frac{AC}{AB} = \frac{\sqrt{7}}{5} \rightarrow \angle ABC = \arcsin \frac{\sqrt{7}}{5}. (2)
\]
plane \( ACD \perp \text{plane} Q, AF \perp \text{plane} Q. \) In \( \triangle ADF, \angle ADF = 60^\circ, AD = 2, AF = 2 \sin 60^\circ = \sqrt{3}.
\]

In right \( \triangle ABF \),
\[
\sin \angle ABF = \frac{AF}{AB} = \frac{\sqrt{3}}{10} \rightarrow \angle ABF = \arcsin \frac{\sqrt{3}}{10}.
\]

Figure A61
9. Given hyperbola: \( x^2 - \frac{y^2}{2} = 1 \).

(1) A straight-line \( l \) passing through point \( A (2, 1) \) intersects the hyperbola at points \( P_1 \) and \( P_2 \). Find the equation of the trajectory of \( P_1P_2 \)’s midpoint \( P \).

(2) Is it possible to construct a straight-line \( m \) passing through point \( B (1, 1) \), such that \( m \) intersects the hyperbola at points \( Q_1 \) and \( Q_2 \), and \( B \) is the midpoint of \( Q_1Q_2 \)? If \( m \) exists, find its equation; if \( m \) does not exist, explain the reasons.

Answer: (1) Let \( l \) be \( y = k(x - 2) + 1 \), then \( x^2 - \frac{[k(x-2)+1]^2}{2} = 1 \rightarrow (2 - k^2)x^2 + (4k^2 - 2k)x - 4k^2 + 4k - 3 = 0 \). Let \( P_1(x_1, y_1), P_2(x_2, y_2), P(\bar{x}, \bar{y}) \). \( x_1 \) and \( x_2 \) must be two real roots of the quadratic equation. Thus, \( x_1 + x_2 = \frac{4k^2 - 2k}{k^2 - 2} \) \((k^2 - 2 \neq 0)\). Since
\[
\bar{x} = \frac{x_1 + x_2}{2}, \quad \bar{x} = \frac{2k^2 - 2k}{k^2 - 2}, \quad \bar{y} = k(\bar{x} - 2) + 1 = \frac{2(2k-1)}{k^2 - 2}. \quad \bar{y}^2 = \bar{y} + 2(2 - \bar{x}) = 0 \text{ is the equation. }
\]
(2) Let \( l \) be \( y = k(x - 1) + 1 \), then \( (2 - k^2)x^2 + (2k^2 - 2k)x - k^2 + 2k - 3 = 0 \). Let \( Q_1(\bar{x}_1, \bar{y}_1), Q_2(\bar{x}_2, \bar{y}_2), \bar{x}_1 \) and \( \bar{x}_2 \) must be two real roots of the quadratic equation. \( \bar{x}_1 + \bar{x}_2 = \frac{2k^2 - 2k}{k^2 - 2} \). If \( B \) is the midpoint of \( Q_1Q_2 \), \( \frac{x_1 + x_2}{2} = 1 \rightarrow \frac{2k^2 - 2k}{k^2 - 2} = 2 \rightarrow k = 2 \). But \( k = 2 \) does not satisfy the hyperbola’s equation, so \( m \) does not exist.
10. *CDEF* is an inscribed square (side length = 1) of a semicircle with diameter *AB* as shown in the picture below. Let *AC* = *a*, *BC* = *b*, and construct the sequence *u*₁ = *a* − *b*, *u*₂ = *a*² − *ab* + *b*², *u*₃ = *a*³ − *a*²*b* + *ab*² − *b*³, ..., *u*ₖ = *a* *k* − *a*⁻¹ *b* + *a*⁻² *b*² − ... + (−1)*k* *b*; 

Prove: *u*ₙ = *u*ₙ₋₁ + *u*ₙ₋₂ (n ≥ 3).

Answer: *u*ₖ = *a* *k* − *a*⁻¹ *b* + *a*⁻² *b*² − ... + (−1)*k* *b* = \( \frac{a^{k+1} - (-1)^{k+1} b^{k+1}}{a + b} \). *a* − *b* = *FC* = 1.

\[ ab = CD^2 = 1. \text{ } u_{n-2} = \frac{a^{n-1} - (-1)^{n-1} b^{n-1}}{a + b}, \text{ } ab = \frac{a^n b - (-1)^n a b^n}{a + b}, \text{ } u_{n-1} = \frac{a^{n-1} - (-1)^n b^n}{a + b}, \text{ } (a - b) = \frac{a^{n+1} - a^n b - (-1)^n a b^n - (-1)^{n+1} b^{n+1}}{a + b}. \text{ } u_{n-1} + u_{n-2} = \frac{a^{n+1} - (-1)^{n+1} b^{n+1}}{a + b}. \]

**Figure A62**

*Graph for Item 10 in the 1981 NCEE Math Exam*

The link to the Chinese Version:
[https://wenku.baidu.com/view/57e841806337ee06eff9ae8941ea76e59fa4a7c?agglId=0b262fd385c24028915f804d2b160b4e777f81dc&_wkts_=1703624630998&bdQuery=1978+2010+%E6%95%B0%E5%AD%A6%E9%AB%98%E8%80%88](https://wenku.baidu.com/view/57e841806337ee06eff9ae8941ea76e59fa4a7c?agglId=0b262fd385c24028915f804d2b160b4e777f81dc&_wkts_=1703624630998&bdQuery=1978+2010+%E6%95%B0%E5%AD%A6%E9%AB%98%E8%80%88)
Table for Item 1 in the 1982 NCEE Math Exam

<table>
<thead>
<tr>
<th></th>
<th>Function</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y = \sqrt{-x^2} )</td>
<td>( x = 0 )</td>
</tr>
<tr>
<td>2</td>
<td>( y = \sqrt{(-x)^2} )</td>
<td>( \mathbb{R} )</td>
</tr>
<tr>
<td>3</td>
<td>( y = \arcsin(\sin x) )</td>
<td>( \mathbb{R} )</td>
</tr>
<tr>
<td>4</td>
<td>( y = \sin(\arcsin x) )</td>
<td>(</td>
</tr>
<tr>
<td>5</td>
<td>( y = 10^{\lg x} )</td>
<td>( x &gt; 0 )</td>
</tr>
<tr>
<td>6</td>
<td>( y = \lg 10^x )</td>
<td>( \mathbb{R} )</td>
</tr>
</tbody>
</table>

2. (1) Write out the 15th term in the expanded form of the expression \((-1 + i)^{20}\).

(2) Find the derivative of \( y = \cos^2 \frac{x}{3} \).

Answer: (1) \( T_{15} = \binom{20}{14}(-1)^6 i^{14} = -(\binom{20}{14}) = -38760 \). (2) \( y' = 2 \left( \cos \frac{x}{3} \right) \left( \cos \frac{x}{3} \right)' = -2 \cos \frac{x}{3} \sin \frac{x}{3} \left( \frac{x}{3} \right)' = -2 \cos \frac{x}{3} \sin \frac{x}{3} = -\frac{1}{3} \sin \frac{2x}{3} \).

3. What types of curves are represented by the following equations. Draw them out in the given coordinate planes.

\[
\begin{vmatrix}
2x & 1 & 1 \\
-3y & 2 & 3 \\
6 & 3 & 4
\end{vmatrix} = 0;
\]

\[
\begin{cases}
x = 1 + \cos \varphi \\
y = 2 \sin \varphi
\end{cases}
\]

Answer: (1) \( 2x - 3y - 6 = 0 \); (2) \((x - 1)^2 + \frac{y^2}{4} = 1\).

Figure A63

Graph for Item 3-(1) in the 1982 NCEE Math Exam
4. A cone has a base radius of \( R \) and a height of \( H \). Find the height \( h \) of the cone’s inscribed cylinder whose volume is the largest.

Answer: Let the cylinder’s radius be \( r \).

\[
H - \frac{H-h}{H} = \frac{r}{R} \Rightarrow r = \frac{R}{H}(H - h).
\]

\[
V(h) = \pi r^2 h = \frac{\pi R^2}{H^2} (H - h)^2 h.
\]

\[
V'(h) = \frac{\pi R^2}{H^2} (H - h)(H - 3h). \quad V'(h) = 0 \Rightarrow h = \frac{1}{3}H.
\]

5. Given: \( 0 < x < 1, a > 0, a \neq 1 \). Comparing the quantities \( |\log_a(1 - x)| \) and \( |\log_a(1 + x)| \) with an explanation.

Answer: When \( a > 1 \), \( |\log_a(1 - x)| = -\log_a(1 - x); |\log_a(1 + x)| = \log_a(1 + x). \)

\[
|\log_a(1 - x)| - |\log_a(1 + x)| = -\log_a(1 - x^2). \text{ Since } a > 1 \text{ and } 0 < 1 - x^2 < 1,
\]
\[-\log_a(1 - x^2) > 0 \rightarrow |\log_a(1 - x)| > |\log_a(1 + x)|.\] When \(0 < a < 1\), \(|\log_a(1 - x)| = \log_a(1 - x);|\log_a(1 + x)| = -\log_a(1 + x).\ |\log_a(1 - x)| - |\log_a(1 + x)| = \log_a(1 - x^2).

Since \(0 < a < 1\) and \(0 < 1 - x^2 < 1\), \(\log_a(1 - x^2) > 0\)

\(\rightarrow |\log_a(1 - x)| > |\log_a(1 + x)|.\)

6. Given: \(\angle AOB = 2\alpha\), a moving point \(P\) is inside the angle, \(PM \perp OA,\ PN \perp OB\), and the area of the quadrilateral \(PMON\) is \(c^2\). Let \(O\) be the pole and the angle bisector \(OX\) be the polar axis. Find the polar equation of \(P\)’s trajectory and state the curve’s type.

Answer: Let \(P\) be \((\rho, \theta)\), \(\angle POM = \alpha - \theta, \angle NOP = \alpha + \theta, OM = \rho \cos(\alpha - \theta), PM = \rho \sin(\alpha - \theta), ON = \rho \cos(\alpha + \theta), PN = \rho \sin(\alpha + \theta)\). The area of Quadrilateral \(PMON\) is \(\frac{1}{2} OM \cdot PM + \frac{1}{2} ON \cdot PN = \frac{\rho^2}{2} [\cos(\alpha - \theta) \sin(\alpha - \theta) + \cos(\alpha + \theta) \sin(\alpha + \theta)]\). The polar equation is \(\frac{\rho^2}{2} [\cos(\alpha - \theta) \sin(\alpha - \theta) + \cos(\alpha + \theta) \sin(\alpha + \theta)] = c^2\). Simplify the equation to get \(\frac{\rho^2}{2} \sin 2\alpha \cos 2\theta = c^2 \rightarrow \rho^2 \cos 2\theta = \frac{2c^2}{\sin 2\alpha}\). Since \(x = \rho \cos \theta\) and \(y = \rho \sin \theta\), the equation is a hyperbola \(x^2 - y^2 = \frac{2c^2}{\sin 2\alpha}\). The trajectory is part of hyperbola inside the angle.

**Figure A65**

*Graph for Item 6 in the 1982 NCEE Math Exam*
7. Quadrilateral $ABCD$ resides in a 3D space. $AB = BC, CD = DA$, and $M, N, P, Q$ are midpoints of $AB, BC, CD, DA$. Prove: $MNPQ$ is a rectangle.

Answer: In $\triangle ABC$, $AM = MB$, $CN = NB \rightarrow MN \parallel AC$. In $\triangle ABC$, $QP \parallel AC$. Thus, $MN \parallel QP$.

For the same reasons, $MQ \parallel NP$. $MNPQ$ is a parallelogram. $K$ is the midpoint, connect $BK$ and $DK$. $BK \perp AC$, $DK \perp AC \rightarrow AC \perp plane$ $BKD \rightarrow BD \perp AC$. Since $MQ \parallel BD$ and $QP \parallel AC$, $MQ \perp QP \rightarrow \angle MQP = 90^\circ$. $MNPQ$ is a rectangle.

Figure A66

Graph for Item 7 in the 1982 NCEE Math Exam

8. Two sides of an inscribed triangle of the parabola $y^2 = 2px$ are tangent to the parabola $x^2 = 2qy$. Prove: the third side of the triangle is also tangent to $x^2 = 2qy$.

Answer: without loss of generality, let $p > 0$ and $q > 0$, and the three vertices of the inscribed triangle be $A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3)$. Since the tangent line of the $x^2 = 2qy$ at the origin is the axis of symmetry of the $y^2 = 2px$, the three vertices cannot be $(0,0)$. Since $A_1A_2$ is tangent to the $x^2 = 2qy, x_1 \neq x_2, y_1 \neq -y_2$. $A_1A_2$’s equation is $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1), y_2^2 - y_1^2 = (y_2 - y_1)(y_2 + y_1) = 2p(x_2 - x_1), y = \frac{2p}{y_1+y_2}x + \frac{y_1y_2}{y_1+y_2}$. The $A_1A_2$ and $x^2 = 2qy$’s intersection’s $x$-coordinate satisfy $x^2 - \frac{4pq}{y_1+y_2}x + \frac{2y_1y_2}{y_1+y_2} = 0$. $b^2 - 4ac =$

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\[
\left(-\frac{4pq}{y_1+y_2}\right)^2 - 4 \left(\frac{2y_1y_2}{y_1+y_2}\right) = 0 \rightarrow 2p^2q + y_1y_2(y_1 + y_2) = 0.
\]
For the same reasons, \(2p^2q + y_2y_3(y_2 + y_3) = 0\). Solving the system to get \(y_1 + y_2 + y_3 = 0\), \(y_3 \neq -y_1\). \(y_2 = -y_1 - y_3 \rightarrow 2p^2q + y_3y_1(y_3 + y_1) = 0\), meaning the two intersections of \(A_1A_3\) and \(x^2 = 2qy\) coincide, i.e., \(A_1A_3\) is also tangent to \(x^2 = 2qy\).

**Figure A67**

*Graph for Item 8 in the 1982 NCEE Math Exam*

9. Given sequences: \(a_1, a_2, \ldots, a_n, \ldots\) and \(b_1, b_2, \ldots, b_n, \ldots\). \(a_1 = p, b_1 = q, a_n = pa_{n-1},\) \(b_n = qa_{n-1} + rb_{n-1}\) \((n \geq 2)\) \((p, q, r\) are constants, and \(q \neq 0, p > r > 0\)).

(1) Use \(p, q, r, n\) to represent \(b_n\), and prove it with mathematical induction.

(2) Find \(\lim_{n \to \infty} \frac{b_n}{\sqrt{a_n^2 + b_n^2}}\).

**Answer:** (1) \(a_1 = p, a_n = pa_{n-1} \rightarrow a_n = p^n. b_1 = q, b_2 = qa_1 + rb_1 = q(p + r), b_3 = q(p^2 + pr + r^2), \ldots, b_n = \frac{q(p^n - n)}{p-r}.\) When \(n = 2, b_2 = \frac{q(p^2 - r^2)}{p-r},\) the equation holds. Assume when \(n = k,\) the equation holds. For \(n = k + 1, b_{k+1} = qa_k + rb_k = qp^k + \frac{rq(p^{k+1} - r^{k+1})}{p-r}\), the equation holds. (2) \(\lim_{n \to \infty} \frac{b_n}{\sqrt{a_n^2 + b_n^2}} = \lim_{n \to \infty} \frac{q(p^n - n)}{p-r} - \frac{q(p^{n+1} - n+1)}{p-r}\).
Since $p > r > 0$, 
\[ \lim_{n \to \infty} \frac{q(p^n-r^n)}{\sqrt{p^{2n}(p-r)^2+q^2(p^n-r^n)^2}} = \lim_{n \to \infty} \frac{q \left[ 1 - \left( \frac{r}{p} \right)^n \right]}{\sqrt{(p-r)^2+q^2 \left[ 1 - \left( \frac{r}{p} \right)^n \right]^2}}. \]

Since $0 < \frac{r}{p} < 1$, as $n \to \infty$, $\frac{r}{p} \to 0$. The limit becomes
\[ \frac{q}{\sqrt{(p-r)^2+q^2}}. \]

The link to the Chinese Version:
https://wenku.baidu.com/view/57e841806337ee06eff9aef8941ea76e59faa7c?aggId=0b262fd385c24028915f804d2b160b4e777f81dc&_wkt=_1703624630998&bdQuery=1978+2010+%E6%95%B0%E5%AD%A6%E9%AB%98%E8%80%83

1983 National College Entrance Examination

Mathematics

Date and time: July 16, 8:00-10:00

Part I: Multiple Choice.

1. Two skew lines are (   D   ).
   A. Two lines that do not intersect.
   B. Two lines that are in two different planes.
   C. A line in a plane and a line outside this plane.
   D. Two lines that are not in the same plane.

2. The graph represented by the equation is (   A   ).
   A. Two intersecting lines.
   B. Two parallel lines.
   C. Two coincident lines.
   D. A point.

3. The necessary and sufficient conditions for three numbers $a, b, c$ to be not all zero are (   D   ).
A. None of the $a, b, c$ is zero.

B. At most one of the $a, b, c$ is zero.

C. Only one of the $a, b, c$ is zero.

D. At least one of the $a, b, c$ is zero.

4. Let $\alpha = \frac{4\pi}{3}$, then $\arccos(\cos \alpha) = ( \text{ C } )$.

A. $\frac{4\pi}{3}$

B. $-\frac{2\pi}{3}$

C. $\frac{2\pi}{3}$

D. $\frac{\pi}{3}$

5. The order of three numbers $0.3^2$, $\log_2 0.3$, $2^{0.3}$ are ( C ).

A. $0.3^2 < 2^{0.3} < \log_2 0.3$

B. $0.3^2 < \log_2 0.3 < 2^{0.3}$

C. $\log_2 0.3 < 0.3^2 < 2^{0.3}$

D. $\log_2 0.3 < 2^{0.3} < 0.3^2$

Part II: Free response.

1. (1) In the same coordinate plane, draw the graphs of the functions $y = -\sqrt{x}$ and $x = \sqrt{-y}$, and write down their intersection’s coordinates.

(2) In the polar coordinate plane, what curve does the equation $\rho = 5 \cos \theta$ represent? Draw the graph out.

Answer: (1) $(0,0), (1, -1)$; (2) A circle.

**Figure A68**

*Graph for Item II-1-(1) in the 1983 NCEE Math Exam*
2. (1) Given: \( y = e^{-x} \sin 2x \), Find \( dy \).

(2) There are 10 students in a group, of which 4 are female and 6 are male. How many ways to choose three representatives from the group, including at least one female student?

Answer: (1) \( dy = [e^{-x} \cos 2x (2x)' + e^{-x}(-x)' \sin 2x] \, dx = 2e^{-x} \cos 2x - e^{-x} \sin 2x \).

(2) \( \binom{4}{1} \binom{6}{2} + \binom{4}{2} \binom{6}{1} + \binom{4}{3} = 60 + 36 + 4 = 100 \).

3. Calculate the determinant (simplify the result).

\[
\begin{vmatrix}
\sin \alpha & \cos(\alpha + \varphi) & \cos \alpha \\
\cos \beta & \sin(\beta - \varphi) & \sin \beta \\
\sin \varphi & \cos 2\varphi & \cos \varphi \\
\end{vmatrix}
\]
\[
\begin{vmatrix}
\sin \alpha & \cos(\alpha + \varphi) + \sin \alpha \sin \varphi - \cos \alpha \cos \varphi & \cos \alpha \\
\cos \beta & \sin(\beta - \varphi) + \cos \beta \sin \varphi - \sin \beta \cos \varphi & \sin \beta \\
\sin \varphi & \cos 2\varphi + \sin^2 \varphi - \cos^2 \varphi & \cos \varphi
\end{vmatrix} = \\
\begin{vmatrix}
\sin \alpha & \cos(\alpha + \varphi) - \cos(\alpha + \varphi) & \cos \alpha \\
\cos \beta & \sin(\beta - \varphi) - \sin(\beta - \varphi) & \sin \beta \\
\sin \varphi & \cos 2\varphi - \cos 2\varphi & \cos \varphi
\end{vmatrix} = \\
\begin{vmatrix}
\sin \alpha & 0 & \cos \alpha \\
\cos \beta & 0 & \sin \beta \\
\sin \varphi & 0 & \cos \varphi
\end{vmatrix} = 0.
\]

4. (1) Prove: for any real number \( t \), the complex number \( z = \sqrt{|\cos t|} + \sqrt{|\sin t|}i \)'s modulus \( r = |z| \) satisfies \( r \leq \sqrt{2} \).

(2) Find the values of \( t \) when the complex number \( z = \sqrt{|\cos t|} + \sqrt{|\sin t|}i \)'s principal argument \( \theta \) satisfies \( 0 \leq \theta \leq \frac{\pi}{4} \).

Answer: (1) \( r = \sqrt{\left(\sqrt{|\cos t|}\right)^2 + \left(\sqrt{|\sin t|}\right)^2} = \sqrt{|\cos t| + |\sin t|} \). Since \( |\cos t|^2 + |\sin t|^2 = 1 \),

Let \( |\cos t| = \cos \phi, |\sin t| = \sin \phi \), then \( |\cos t| + |\sin t| = \cos \phi + \sin \phi = \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos \phi + \frac{\sqrt{2}}{2} \sin \phi\right) = \sqrt{2} \left(\sin \frac{\pi}{4} + \phi\right) \leq \sqrt{2} \cdot 1 = \sqrt{2} \).

5. In triangular pyramid \( S-ABC \), the projection of \( S \) on the base lies on the base’s height \( CD \); \( M \) is on the lateral edge \( SC \) such that the dihedral angle formed by the cross-section \( MAB \) and the base equals to \( \angle NSC \). Prove: \( SC \) is perpendicular to \( MAB \).

Answer: Since \( AB \perp NC \), the projection \( SC \) of on the base \( ABC \), \( AB \perp SC \).

\( AB \perp DC \rightarrow AB \perp \) plane \( SDC \). Since \( DM \) is inside plane \( SDC \), \( AB \perp DM \). Thus, the dihedral angle mentioned equals to \( \angle MDC, \angle MDC = \angle NSC \). In \( \triangle MDC \) and \( \triangle NSC \), \( \angle DCS \) is the common angle \( \rightarrow \angle DMC = \angle SNC = 90^\circ \). Thus \( DM \perp SC \) and \( SC \perp \) plane \( MAB \).

**Figure A70**

*Graph for Item II-5 in the 1983 NCEE Math Exam*
6. Given: an ellipse’s major axis $|A_1A_2| = 6$, foci $|F_1F_2| = 4\sqrt{2}$. Constructing a straight line passing through $F_1$ and intersecting the ellipse at $M$ and $N$. Let $\angle F_2F_1M = \alpha \ (0 \leq \alpha < \pi)$.

Find the values of $\alpha$ when $|MN|$ equals to the length of ellipse’s minor axis.

Answer: $a = 3$, $c = 2\sqrt{2}$, $b = 1$, $e = \frac{2\sqrt{2}}{3}$, distance from Center to Directrix $= \frac{9\sqrt{2}}{4}$, distance from Focus to Directrix $p = \frac{\sqrt{2}}{4}$. The polar equation of the ellipse is $\rho = \frac{ep}{1-e \cos \theta} = \frac{1}{3-2\sqrt{2} \cos \theta}$. Thus,

$$|F_1M| = \rho_1 = \frac{1}{3-2\sqrt{2} \cos \alpha}, \quad |F_1N| = \rho_2 = \frac{1}{3+2\sqrt{2} \cos \alpha}, \quad |MN| = \rho_1 + \rho_2 = \frac{6}{9-8 \cos^2 \alpha}.$$ Let

$$\frac{6}{9-8 \cos^2 \alpha} = 2, \quad \cos \alpha = \pm \frac{\sqrt{3}}{2}, \quad \alpha = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}.$$

**Figure A71**

*Graph for Item II-6 in the 1983 NCEE Math Exam*

7. Given: sequence $\{a_n\}$’s first term $a_1 = b \ (b \neq 0)$, and the sum of the first $n$ terms $S_n = a_1 + a_2 + \cdots + a_n \ (n \geq 1)$. $S_1, S_2, \ldots, S_n, \ldots$ forms a geometric progression with a common ratio $p \ (p \neq 0 \text{ and } |p| < 1)$.

Prove: (1) $a_2, a_3, \ldots, a_n, \ldots$ forms a geometric progression.
(2) Let \( W_n = a_1 S_1 + a_2 S_2 + a_3 S_3 + \cdots + a_n S_n \) \((n \geq 1)\). Find \( \lim_{n \to \infty} W_n \) (with \( b \) and \( p \)).

Answer: (1) \( S_1 = a_1 = b \). \( S_n = S_1 p^{n-1} = b p^{n-1} \) \((n \geq 1)\). When \( n \geq 2 \), \( S_n = a_1 + a_2 + \cdots + a_n = S_n - S_{n-1} = b p^{n-1} - b p^{n-2} = b p^{n-2}(p - 1)(n \geq 2) \).

\[
\frac{a_{n+1}}{a_n} = \frac{b p^{n-1}(p - 1)}{b p^{n-2}(p - 1)} = p \quad (n \geq 2). \text{ Thus, } a_2, a_3, \cdots, a_n, \cdots \text{ forms a geometric progression.}
\]

(2) When \( n \geq 2 \),
\[
\frac{a_{n+1} S_{n+1}}{a_n S_n} = \frac{b p^{n-1}(p - 1)b p^n}{b p^{n-2}(p - 1)b p^{n-1}} = p^2 < 1.
\]

Thus, \( a_2 S_2, a_3 S_3, \cdots, a_n S_n, \cdots \) is an infinite geometric sequence with common ratio \( p^2 < 1 \). \( \lim_{n \to \infty} a_2 S_2 + a_3 S_3 + \cdots + a_n S_n = \frac{a_2 S_2}{1 - p^2} = \frac{b^2 (p - 1)}{1 - p^2} \).

8. (1) \( a \) and \( b \) are real numbers, and \( e < a < b \), \( e \) is Euler’s number. Prove: \( a^b > b^a \).

(2) If positive real numbers \( a \) and \( b \) satisfy \( a^b = b^a \), and \( a < 1 \). Prove: \( a = b \).

Answer: (1) Consider function \( y = \frac{\ln x}{x} \) \((0 < x < +\infty)\). When \( x > e \), \( y' = \frac{1 - \ln x}{x^2} < 0 \). \( y \) is a decreasing function in the interval \((e, +\infty)\). Since \( e < a < b \), \( \frac{\ln a}{a} > \frac{\ln b}{b} \), \( b \ln a > a \ln b \), \( a^b > b^a \). (2) Since \( a^b = b^a \), from (1), one can get \( b \ln a = a \ln b \), \( \frac{\ln a}{a} = \frac{\ln b}{b} \). Consider function \( y = \frac{\ln x}{x} \) \((0 < x < +\infty)\), \( y' = \frac{1 - \ln x}{x^2} \). In the interval \((0, 1)\), \( y' > 0 \), thus, \( y \) is increasing. Since \( 0 < a < 1, b > 0 \), \( a^b < 1 \), \( b^a < 1 \), \( a > 0 \)

\( b < 1 \). \( 0 < a < 1 \), \( 0 < b < 1 \), if \( a \neq b \), \( \frac{\ln a}{a} \neq \frac{\ln b}{b} \), \( a^b \neq b^a \), contradiction.

The link to the Chinese Version:

https://wenku.baidu.com/view/57e841806337ee06eff9aef8941ea76e59fa4a7c?aggId=0b262fd38 5c24028915f804d2b160b4e777f81dc&_wkts_=1703624630998&bdQuery=1978+2010+%E6%95%B0%E5%AD%A6%E9%AB%98%E8%80%83 5%B0%E5%AD%A6%A6%E9%AB%98%E8%80%83

1984 National College Entrance Examination
Mathematics

Date and time: July 8, 8:00-10:00

Part I: Multiple Choice.

1. The relationship between set $X = \{(2n + 1)\pi, n \in \mathbb{Z}\}$ and set $Y = \{(4k + 1)\pi, k \in \mathbb{Z}\}$ is

   (   C   ).

   A. $X \subset Y$
   B. $X \supset Y$
   C. $X = Y$
   D. $X \neq Y$

2. If circle $x^2 + y^2 + Gx + Ey + F = 0$ is tangent to the $x$-axis at origin, then (   C   ).

   A. $F = 0, G \neq 0, E \neq 0$
   B. $E = 0, F = 0, G \neq 0$
   C. $G = 0, F = 0, E \neq 0$
   D. $G = 0, E = 0, F \neq 0$

3. If $n$ is a positive integer, then the value of $\frac{1}{8}[1 - (-1)^n](n^2 - 1)$ is (   B   ).

   A. Zero
   B. Even numbers
   C. Integers
   D. Possibly not an integer

4. The necessary and sufficient conditions for $\arccos(-x) > \arccos(x)$ is (   A   ).

   A. $x \in (0,1]$
   B. $x \in (-1,0)$
   C. $x \in [0,1]$
D. \( x \in \left[0, \frac{\pi}{2}\right] \)

5. If \( \theta \) is in the second quadrant and satisfies the equation \( \cos \frac{\theta}{2} - \sin \frac{\theta}{2} = \sqrt{1 - \sin \theta} \), then \( \frac{\theta}{2} \) is
   (B).

   A. In the first quadrant
   B. In the third quadrant
   C. In the first quadrant or the third quadrant
   D. In the second quadrant

Part II: Fill in the blank.

1. A cylinder’s lateral surface is a rectangle with length of 4 and width of 2. Find its volume.

   Answer: \( V = \pi \left(\frac{2}{\pi}\right)^2 \times 2 = \frac{8}{\pi} \) or \( V = \pi \left(\frac{1}{\pi}\right)^2 \times 4 = \frac{4}{\pi} \).

2. Find the intervals on which \( \log_{0.5}(x^2 + 4x + 4) \) is increasing.

   Answer: \( 0.5 < 1 \), the function is increasing when \( x^2 + 4x + 4 \) is increasing. \((-\infty, -2)\).

3. Solve the equation \( (\sin x + \cos x)^2 = \frac{1}{2} \).

   Answer: \( \sin 2x = -\frac{1}{2}, \quad x = n\pi + \frac{7\pi}{12} \) or \( n\pi - \frac{\pi}{12}, \quad n \in \mathbb{Z} \).

4. Find the constant term in the expansion of \( \left(|x| + \frac{1}{|x|} - 2\right)^3 \).

   Answer: \( \left(|x| + \frac{1}{|x|} - 2\right)^3 = \left(\sqrt{|x|} - \frac{1}{\sqrt{|x|}}\right)^6, \quad T_{r+1} = \binom{6}{r}|x|^\frac{6-r}{r}(-1)^r|x|^{-\frac{r}{2}} = (-1)^r\binom{6}{r}|x|^{3-r} \),

   when \( r = 3, \quad -\binom{6}{3} = -20 \).

5. Find \( \lim_{n \to \infty} \frac{1 - 2^n}{3^n + 1} \).

   Answer: \( \lim_{n \to \infty} \frac{1 - 2^n}{3^n + 1} = \lim_{n \to \infty} \frac{\frac{1}{3^n} - \frac{2^n}{3^n}}{1 + \frac{1}{3^n}} = \frac{0-0}{1+0} = 0 \).
6. To arrange a performance list with 6 singing programs and 4 dance programs, any two dance programs must not be adjacent to each other. How many different arrangements are possible?

(Write out the formula only)

Answer: \[ A_6^6 A_4^4 = A_7^6 \times 6! \].

Part III: Free response.

1. (1) Let \( H(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases} \). Draw the graph of \( y = H(x - 1) \).

(2) Draw the graph of the polar equation \( (\rho - 2) \left( \theta - \frac{\pi}{4} \right) = 0, \rho > 0 \).

Answer: (1) See Figure A71 (2) See Figure A72

**Figure A72**

*Graph for Item III-1-(1) in the 1984 NCEE Math Exam*

![Graph](image1)

**Figure A73**

*Graph for Item III-1-(2) in the 1984 NCEE Math Exam*

![Graph](image2)
2. Given: three planes intersect each other, and there are three intersecting lines. Prove: these three intersecting lines intersect at one point or are parallel to each other.

Answer: Let three planes be $\alpha$, $\beta$, and $\gamma$, and $\alpha \cap \beta = c$, $\alpha \cap \gamma = b$, and $\beta \cap \gamma = a$. $\alpha \cap \beta = c$ and $\alpha \cap \gamma = b \to c \subset a$ and $b \subset a \to c \parallel b$ or they intersect at a point. If $b$ and $c$ intersect at a point $P, P \subset c$ and $c \subset \beta \to P \subset \beta$. Similarly, $P \subset \gamma$. $P \subset \beta \cap \gamma = a$. Thus, three lines intersect at the same point. If $c \parallel b, b \subset \gamma \to c \parallel \gamma$. Also, $c \subset \beta, \beta \cap \gamma = a \to c \parallel a$. Thus, three lines are parallel to each other.

3. $c, d$, and $x$ are real numbers, $c \neq 0$, and $x$ is an unknown variable. Discuss when the function

$$\log_{(cx + \frac{d}{x})} x = -1$$

has solutions and find them.

Answer: $(cx + \frac{d}{x})^{-1} = x \ (x > 0), x(cx + \frac{d}{x}) = 1, cx^2 + d = 1$. Since $c \neq 0, x^2 = \frac{1-d}{c} > 0 \to c > 0, d < 1$ or $c < 0, d > 1$. $cx + \frac{d}{x} \neq 1$ and $x(cx + \frac{d}{x}) = 1 \to x \neq 1$.

$x > 0$ and $x^2 = \frac{1-d}{c} \to c \neq 1 - d$. Thus, when $c > 0, d < 1, c \neq 1 - d$ or $c < 0, d > 1, c \neq 1 - d$, the function has solutions and $x = \sqrt{\frac{1-d}{c}}$.

4. (1) The equation $z^2 - 2pz + q = 0 \ (p \neq 0)$ has real coefficients and two imaginary roots $z_1$ and $z_2$. The corresponding points of $z_1$ and $z_2$ in the complex plane are $Z_1$ and $Z_2$. Find the length of the major axis of the ellipse with $Z_1$ and $Z_2$ as the foci and passing through the origin.

(2) Find the trajectory equation of the left vertex of the ellipse with an eccentricity of $\frac{1}{2}$, passing through the point $M(1,2)$, and taking the y-axis as the directrix.
Answer: \((1) \(-2p)^2 - 4q < 0, q > p^2 > 0\). \(z_1\) and \(z_2\) are symmetry about \(x\)-axis. Thus, the minor axis is on the \(x\)-axis. Since the ellipse passes through the origin, the origin is one vertex.

\[
2b = |z_1 + z_2| = 2|p|, 2c = |z_1 - z_2| = \sqrt{(z_1 + z_2)^2 - 4z_1z_2} = 2\sqrt{q - p^2},
\]

\[
2a = 2\sqrt{b^2 + c^2} = 2\sqrt{q}.\]

(2) Let the left vertex of the ellipse be \(A(x, y)\). Since \(e = \frac{1}{2}\), the left focus’ coordinates are \(F\left(\frac{3x}{2}, y\right)\). Let \(d\) be the distance from point \(M\) to \(y\)-axis, \(d = 1\). Since

\[
|MF| = \frac{1}{2} \left(\frac{3}{2}x - 1\right)^2 + (y - 2)^2 = \left(\frac{1}{2}\right)^2 \rightarrow 9 \left(x - \frac{2}{3}\right)^2 + 4(y - 2)^2 = 1.
\]

5. In \(\Delta ABC\), \(\angle A, \angle B,\) and \(\angle C\) are opposite to the sides \(a, b,\) and \(c, c = 10,\) and \(\cos A = \frac{b}{c} = \frac{4}{3}\). \(P\)

is a point on the inscribed circle of \(\Delta ABC\). Find the maximum and minimum value of the sum of squares of the distances from point \(P\) to points \(A, B,\) and \(C\).

Answer: \(\frac{\cos A}{\cos B} = \frac{\sin B}{\sin A}\) (by Law of Sines), \(\sin A \cos A = \sin B \cos B, \sin 2A = \sin 2B\). Since \(A \neq B, 2A = \pi - 2B\), \(A + B = \frac{\pi}{2}\). Thus, \(\Delta ABC\) is a right triangle, \(a = 6, b = 8, AD + DB + EC = 12.\)

\(AD + DB = c = 10, EC = r = 2\). The circle’s equation is \((x - 2)^2 + (y - 2)^2 = 4\). Let \(P’\)s coordinates be \((x, y)\), \(S = |PA|^2 + |PB|^2 + |PC|^2 = (x - 8)^2 + y^2 + x^2 + (y - 6)^2 + x^2 + y^2 = 3x^2 + 3y^2 - 16x - 12y + 100 = 3[(x - 2)^2 + (y - 2)^2] - 4x + 76 = 88 - 4x, 0 \leq x \leq 4. S_{max} = 88, S_{min} = 72.\)

Figure A74

Graph for Item III-5 in the 1984 NCEE Math Exam
6. Given the sequence \( \{x_n\} \), \( x_1 = a \) \((a > 2)\), \( x_{n+1} = \frac{x_n^2}{2(x_n-1)} \) \((n = 1, 2, \cdots)\).

Prove: (1) \( x_n > 2 \), and \( \frac{x_{n+1}}{x_n} < 1 \) \((n = 1, 2, \cdots)\):

(2) If \( a \leq 3 \), then \( x_n \leq 2 + \frac{1}{2^{n-1}} \) \((n = 1, 2, \cdots)\);

(3) If \( n > 3 \), then when \( n \geq \frac{\lg a}{\lg \frac{3}{4}} \), \( x_{n+1} < 3 \).

Answer: (1) when \( n = 1 \), \( x_1 = a > 2 \); Assume the inequality holds when \( n = k \), then for \( = k + 1 \), \( x_{k+1} > 2 \leftrightarrow x_k^2 - 4x_k + 4 > 0 \leftrightarrow (x_k - 2)^2 > 0 \), which holds by assumption. Since \( x_n > 2 \),

\[
\frac{x_{n+1}}{x_n} < 1 \leftrightarrow \frac{x_n}{2(x_n-1)} < 1 \leftrightarrow x_n > 2. \text{ Thus, } \frac{x_{n+1}}{x_n} < 1. \text{ (2) when } n = 1, x_1 = a \leq 3; \text{ Assume the inequality holds when } n = k \text{, then for } = k + 1, x_k > 2 \rightarrow x_{k+1} \leq 2 + \frac{1}{2^k} \leftrightarrow x_k^2 \leq 2(x_k - 1) \left( 2 + \frac{1}{2^k} \right) x_k + 2 \left( 2 + \frac{1}{2^k} \right) \leq 0 \leftrightarrow (x_k - 2) \left[ x_k - \left( 2 + \frac{1}{2^k-1} \right) - \frac{1}{2^k-1} \right] \leq 0 \text{, which holds by assumption. (3) } \frac{x_{k+1}}{x_k} = \frac{1}{2} \left( 1 + \frac{1}{x_k-1} \right) < \frac{1}{2} \left( 1 + \frac{1}{3-1} \right) = \frac{3}{4}. \text{ If } n \geq \frac{\lg a}{\lg \frac{3}{4}}, \text{ } x_{n+1} \geq 3, \text{ from (1), } x_1 > x_2 > \cdots > x_n > x_{n+1} \geq 3. \text{ Since } x_1 = a, \text{ } 3 \leq x_{n+1} = x_1 \cdot x_2 \cdot x_3 \cdots \cdot x_n < a \left( \frac{3}{4} \right)^n \rightarrow n < \frac{\lg a}{\lg \frac{3}{4}}, \text{ contradiction, } \rightarrow x_{n+1} < 3.\]

7. A circle with center \( O \) and radius 1 is known to be tangent to line \( l \) at point \( A \). When a moving point \( P \) moves to the right from point \( A \) along the line \( l \), construct arc \( AC \) with the length of \( \frac{2}{3} AP \). The line \( PC \) and the line \( AO \) intersect at the point \( M \). It is also known that when \( AP = \frac{3\pi}{4} \), \( P \)'s velocity is \( v \). Find the velocity of \( M \).

Answer: Construct \( CD \perp AM \), let \( AP = x, AM = y, \angle COD = \theta \). \( \arccos = \frac{2}{3} AP = \frac{2}{3} x, OC = 1 \rightarrow \theta = \frac{2}{3} x, x \in (0, \pi). \text{ Since } \Delta APM \sim \Delta DCM, \frac{AM}{AP} = \frac{DM}{DC} \). \( DM = y - \left( 1 - \cos \frac{2}{3} x \right) \), \( DC = \)
\[
\sin \frac{2}{3} x \rightarrow \frac{y}{x} = \frac{y - (1 - \cos \frac{2}{3} x)}{\sin \frac{2}{3} x} \rightarrow y = \frac{x(1 - \cos \frac{2}{3} x)}{x - \sin \frac{2}{3} x}, \quad \frac{dy}{dt} = \left\{ \left[ (x - \sin \frac{2}{3} x) \left( 1 - \cos \frac{2}{3} x + \frac{2}{3} x \sin \frac{2}{3} x \right) - \\
(1 - \cos \frac{2}{3} x) \left( 1 - \frac{2}{3} \cos \frac{2}{3} x \right) \right] / (x - \sin \frac{2}{3} x)^2 \right\} \frac{dx}{dt}, \text{ when } x = \frac{3}{4}, \frac{dx}{dt} = v, \frac{dy}{dt} = \frac{2(3\pi^2 - 4\pi - 8)}{(3\pi - 4)^2} v.
\]

**Figure A75**

*Graph for Item III-7 in the 1984 NCEE Math Exam*

The link to the Chinese Version:

https://wenku.baidu.com/view/57e841806337ee06eff9aef8941ea76e59fa4a7c?aggId=0b262fd385c24028915f804d2b160b4e777f81dc&_wkts_=1703624630998&bdQuery=1978+2010+%E6%95%B0%E5%AD%A6%E9%AB%98%E8%80%83