Unforeseen Asymmetries

A Contemporary Mathematical Reading of Kant's Second Antinomy

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Introduction

In the *Critique of Pure Reason*, Kant proposes transcendental idealism as our reality's metaphysical and epistemological framework, holding that all objects of possible experience have no existence outside of thought, but rather are mere appearances for cognizers. His project is to limit cognition "to make room for faith" by showing that boundless reason and transcendental realism—the notion that representations of our world are things in themselves—ultimately drives cognizers to contradictory pseudo-rational assertions (Kant, Bxxx). The Second Antinomy is one such case in which Kant demonstrates that unbounded reason and realism results in contradiction as a way to propose his own transcendental idealism as a solution. In the Second Antinomy, Kant works through how demands of reason drive cognizers to disprove both (defeasibly) exclusive and exhaustive options for a mereological account of the world, namely, atomism or infinite complexity of composite substances. Kant offers a thesis that every composite substance in the world is made up of simple parts, and an antithesis that no composite thing in the world is made up of simple parts, both of which he will reduce to absurdity in order to disprove transcendental realism. Yet through his arguments against the thesis and antithesis, Kant's mereological account of space and bodies and conceptualization of reason's infinite regress raises its own questions. These arguments on which the Second Antinomy rely, I will argue, suppose a decomposition operation that leaves open the possibility that the composite whole be *continuous* prior to the infinite regress of division of the body. Yet at the same time, the composition of particular parts can only result in a *discrete*, or at best, *dense*, whole.[1] These mathematical distinctions regarding cardinalities of infinity and transfinite numbers were still

[1] These notions of a whole being continuous, discrete, or dense will be defined and discussed at length later in this paper, but at this point, it is enough to note that continuity, discreteness, and density are not equivalent notions in contemporary mathematics.

largely controversial while Kant was writing the *Critique*,[2] and thus, led him to mistake these mereological notions of a whole. Yet for our purposes, contemporary mathematical definitions can clarify the unforeseen asymmetry that Kant's mistake creates in the Second Antinomy. This paper will use these contemporary mathematical distinctions to demonstrate that when Kant uses the concepts of decomposition to disprove the antithesis and composition to disprove the thesis, his asymmetric presuppositions of the structure of the body we are dealing with stifle his ability to use these two opposing assertions as a *reductio* of transcendental realism. In fact, if the proposed asymmetry holds, the entire project of the Second Antinomy collapses.

In the paper, I will first offer an analysis of the arguments of the thesis and antithesis in the Second Antinomy, parsing through the *reductio ad absurdum* on both sides, which is supposed to give way for Kant's transcendental idealism. Part of this analysis will be working through Kant's reflections and solutions to the cosmological problems in the rest of the Transcendental Dialectic in which he raises the language about the *quantum continuum*, infinite divisibility, and progress *in infinitum* that justifies this contemporary mathematical reading. I will propose a first asymmetry in the Second Antinomy that arises from Kant's discussion of space in the antithesis, as an implicit assumption seems to suppose *mereological harmony*, a notion that is itself controversial. I will then offer a reframing of Kant's mereological questions, composition and decomposition operations, and weak definitions of infinity by introducing contemporary mathematical explanations that were sharpened in the 20th century, after Kant. By doing so, I plan to show a second asymmetry at greater length: Kant proposes an asymmetrical argument between his discussion of composition and decomposition, and thereby, the proposed opposition

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[2] It was not until the second half of the 19th century that modern set theory developed distinct sizes and cardinalities of infinite sets with Cantor's theorem in 1891 and Dedekind cuts in 1872, as will be discussed in Section IV; (Hosch, 2016); (Enderton, 2011).

between the thesis and antithesis entirely breaks down. I will analyze whether Kant's proposed transcendental idealism, and his move from things in themselves to mere appearances, solves any of this asymmetry in the Second Antinomy. Lastly, I will ask for potential solutions to these proposed asymmetries that would allow the Second Antinomy to still provide its intended function of acting as a *reductio* of transcendental realism in the *Critique of Pure Reason*.[3]

I. Thesis and Antithesis

Let us first work through the arguments of the Second Antinomy. The thesis—that every composite body is constituted by simple parts—stands opposed to the antithesis—that no composite body has simple parts. In order to prove the atomist thesis, Kant assumes the opposite, namely that composite bodies are not made up of simpler parts, or as Van Cleve calls it, composite substances are infinitely complex. Kant's move is to remove all composition in thought, which Van Cleve calls total decomposition, namely "bringing it about that no two parts of it that formerly composed a larger whole any longer do so" (Van Cleve, 482). After this removal of composition, then there would be no composite part left, since composition itself was removed. Furthermore, there would also be no simple parts because we assumed that there are no simple parts. Then there would be nothing at all left, yet this for Kant is impossible, given that

[3] It is worth mentioning that in the antinomies, Kant purports to be working in the realm of transcendental realism in order to demonstrate that the alternative, namely, transcendental idealism, would dispel of these pseudo-rational conflicts. However, in the discussions and reflections of the antinomies throughout the rest of the Transcendental Dialectic, Kant uses not only the realist *de re* assertions of things in themselves (independent objects that bear properties), but also discusses appearances (objects of sensible experience) in the scope of idealism. As Kant establishes in the Transcendental Aesthetic, an empirical object affects the mind of the cognizer through sensibility to given a representation, which is further made sense of through space and time as the form of the sensible intuition, the object of which is an appearance. While a controversial relation, there is a sense in which these appearances (objects of sensible experience) are representations, yet also objects of representation, and therefore in a certain sense separate from the representations. For our purposes, Kant's use of both things in themselves and appearances, his different relations of appearances with representations, and his engagement in both realist and idealist scopes often interfere with the structure of the arguments in the antinomies. Parsing through the validity of each realist or idealist comment lies outside the scope of this paper, but I will do my best to clarify the realms in which Kant seems to be operating in particular parts.

composition is just the accidental unity of parts. So, removing composition could not result in nothing at all being left. Therefore, it must be the case that either it is impossible to remove all composition in thought or there must remain the simple. Since composition in substances is merely an accidental unity, substance could very well stand on its own, so it is possible to totally decompose a substance by removing all composition in thought. We are left to choose the alternative option. Kant thereby proves that there must remain the simple.

Alternatively, in order to prove the antithesis that there are no simple parts, Kant first assumes the opposite, namely that a composite thing as substance is made up of simple parts. Kant starts with the notion that everything exists in space by occupying it. Since all composition—that is, the manifold of elements external to one another—is only possible in space and all parts exist in space, then space must be constituted by as many parts as the composite has. Yet, space is not built up from simple parts, but rather space is made up of space. Everything that occupies space, therefore, everything in general, has a manifold of extensive constituents. As Bayle puts it, "every extension, no matter how small it may be, has a right and a left side" (Bayle, 360). In other words, no matter the size of the part in space, it always has a manifold of constituents external to one another, making it composite. Just by occupying space, every part of a composite is itself a composite of substances. In space, there cannot be simple parts. Kant has thereby offered a *reductio ad absurdum* of both the thesis that everything composite is made up of simple parts and the antithesis that the simple does not exist.

II. First Asymmetry: Mereological Harmony

In order to prove that there are no simple parts in the antithesis, Kant argues that everything occupies space and since space is not made up of simple parts, but rather always has a manifold of elements in space external to it, everything in space is itself a composite. In other words, the mereological structure of everything in space reflects the structure of space itself (and vice versa), in the way that a substance in space always has constituents external to one another that make the substance a composite. This brings us to our first asymmetry in the Second Antinomy, regarding with the way in which the structure of an object reflects the structure of the space it occupies. More specifically, Kant implicitly supposes what is today known as *mereological harmony* by utilizing the structure of space in his argument for the structure of a substance in the antithesis. Mereological harmony is the view that a body is atomistic if and only if the space it occupies is atomistic, and a body is infinitely complex if and only if the space it occupies is infinitely complex, with no smallest part. That is, an object's mereological complexity and structure is necessarily the same as the mereological complexity and structure of the space that object takes up, and there is mirroring between the mereological structure of material bodies and the mereological structure of their locations (in a region of space, for example), and vice versa. For example, we say my right hand is part of my body insofar as the space my right hand occupies is part of the space my body occupies, and we say that my right hand has no part in common with my left hand because the space the right hand occupies itself has no part in common with the space my left hand occupies. Every mereological entity has a perfect counterpart: space has the body and the body has the space. In other words, mereological harmony holds that there is perfect isomorphism, or one-to-one mapping of points and their relations, between an object and the space it occupies. Yet this is a controversial notion.

Those who reject mereological harmony may accept a kind of *parthood misalignment*, namely, cases in which the mereological world's structure does not align with the spaciotemporal world, such as the notion that it is possible "for x to be a part of y without x's location being a part of y's location, and vice versa" (Leonard, 1950). Those who reject mereological harmony may also take the view that there exist *internal disparities*, namely, disparities concerning the internal structure of a body and its location, such as "mereologically complex material objects that are located at mereologically simple regions, gunky material objects that are located at non-gunky regions, and non-gunky material objects that are located at gunky regions" (Saucedo, 227).[4] These are a few cases in which the structure of a body does not necessarily reflect the structure of the space in which it is located, or vice versa.

Turning back to Kant's antithesis argument, it becomes clear that Kant implicitly assumes mereological harmony in which the mereological structure of a body cannot be composed of atoms because space itself is not made up of simple parts. In other words, Kant presupposes that because the space that a body occupies is not composed of smallest parts, atoms, the body itself does not have simple parts. Yet if one accepts internal disparities, for example, maintaining that it is possible for a material body to be an extended simple—that is, an atom occupying a mereologically infinitely complex region—then one can argue that even if space *isn't* made up of simple parts, it doesn't follow that the substance in space *cannot* itself be simple. That is, if one rejects mereological harmony, then the leap from a substance occupying space—which is itself composed of space, rather than simple parts—to that substance not having any simple parts is unconvincing. Furthermore, Kant's notion that substances in space must have extensive constituents, and thus, be composite substances, does not necessarily follow from the claim that space is not simple, especially if one pushes back on mereological harmony; even Democritus argued for an "infinite variety of shapes and sizes" of atoms (Varzi, 1022). That is, if one rejects the theory of mirroring by arguing that a substance in space cannot be simple merely

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[4] A gunky body is a body whose every part itself has a proper part, and a gunky region is a region whose every subregion itself has a proper subregion. Atomless gunk as "an individual whose parts all have further proper parts" was introduced by David Lewis in his 1991 work *Parts of Classes* (20).

because the space is not simple, one can also argue for the possibility of extended simples, which would unravel Kant's argument that everything in space must be composite.

Kant's disproval of simple parts, therefore, rests on this mirroring assertion that every body must be a non-simple, infinitely complex composite insofar as it occupies space, which, for Kant, has no smallest (indivisible, hence, atomic) part. Note that the argument for the thesis, on the other hand, can seemingly still stand without introducing the concept of space and mirroring. Recall that in the thesis, Kant argues that everything is composed of simple parts by first assuming that composite matter is not made up of simple parts. If one removed all composition in thought, then there would be both no simple parts (by assumption) and no composition, so there would be no matter left at all. This is impossible, however, because "any substances that are so related as to compose a larger whole," that is, the composite's parts, "could exist without composing that," that is, they could exist independent of the composite whole (Van Cleve, 482). Since removing all composition in thought would leave nothing left at all, yet removing composition in thought should be possible since composition is merely an accidental unity, then there must be simple parts. Kant's argument for the thesis that there are simple parts, therefore, relies on the accidental unity of composition and more importantly for our purposes here, never invokes space, and thereby the notion of mirroring, in the argument. Kant discusses substance and composition without regard to the isomorphism between that substance and the space it occupies. In fact, the argument for the thesis stands whether or not one takes mereological harmony to be the case; whether the mereological structure of a substance mirrors the mereological structure of its space seems to have no bearing on the validity of the argument that if one removed all composition in thought, then there would be nothing left, so composite substances must be made up of simple parts.

If one rejects the controversial notion of mereological harmony in its many forms, one can argue that the antithesis and thesis are asymmetrical when it comes to the mereological structure of the body in space. In using the concept of space and mereological harmony in the antithesis yet not the thesis, Kant sets himself up for controversy when it comes to the relation of an object's mereological structure and the space it occupies. The proposed symmetry of the thesis and antithesis, which is needed in order to stand in true opposition in an antinomy, already breaks down. I mean to point out this first asymmetry and bracket this worry in order to further parse through the reflections of the Second Antinomy.

III. Regress of Reason

Kant argues that the antinomies arise from the demands of reason, which maintains that "if the conditioned is given, the entire sum of conditions, and consequently the absolutely unconditioned… is also given" (Kant, B436). In other words, when a cognizer is met with something through experience, say matter in space, then reason demands that all conditions corresponding to that matter must also be given. Reason thereby ascends up the chain of conditions in order to consequently attempt to grasp the absolutely unconditioned, that is, the condition that makes all following conditions possible. Yet Kant distinguishes between two different accounts of the unconditioned that reason seeks to reach, namely that the unconditioned is the totality of conditions or the unconditioned is the beginning of the conditions. If the unconditioned is the totality of the conditions, there is an infinite regress of the conditions, the whole of which is the unconditioned, but if the unconditioned is the beginning of the conditions, the unconditioned is the simple. This distinction will be further explored later, but for now, notice that whether the unconditioned is the totality of the conditions or the beginning of the conditions, reason still demands a regressive synthesis of all other conditions associated with the conditioned.

In the context of the Second Antinomy, reason's demand for an ascent in conditions to the unconditioned means that reason demands the decomposition of matter in thought in order to reach the simple or to continue to infinitely divide. Space itself is always conditioned because its limits rely on another part of space, another condition. Therefore, everything in space is also conditioned. It follows that because reality in space, namely, matter, is conditioned, "its internal conditions are its parts, and the parts of these parts its remote conditions" (Kant, A413). Thus, reason demands synthesis that regresses until one reaches the absolute totality by a "completed division in virtue of which the reality of matter vanishes either into nothing or into what is no longer matter—namely, the simple" (Kant, A413). In other words, when a cognizer is met with matter in space, the parts of which are its conditions, reason attempts to reach the totality of the matter's conditions by dividing the matter's parts into parts. Through this division as an advance to the unconditioned, the cognizer is left with either nothing or with the simple. If ascending through the matter's conditions by dividing the matter's parts leaves the cognizer with nothing, this implies that the unconditioned is the totality of the series of matter's conditions. That is, reason demands an infinite regress by which the unconditioned consists of the entire series in which all members are conditions and only the totality of them is absolutely unconditioned. This infinite regress of reason to reach the absolutely unconditioned as the totality of the series of conditions itself would amount to dividing the matter's parts infinitely until it is left with no matter at all. Alternatively, our cognizer can be left with the simple in an advance to the unconditioned, meaning that the unconditioned is itself the simple, namely, the first condition in the series of all matter's conditions. The unconditioned, in this case, is itself part of the series, a part of the series "to which the other members are subordinated, and which does not itself stand under any other condition" (Kant, A418). This successive regress reaching the unconditioned as the beginning of the series of conditions amounts to matter's division reaching the first condition of matter, namely, simple parts.

The Second Antinomy then becomes a conflict of how the unconditioned is reached, namely, as the totality of the entire series of conditions themselves or as the first part of the series that all conditions are subordinated under. This is because if the conditioned is given, then cognizers are tasked with a regress through the series of all its conditioned, to the unconditioned. As Alan Wood clarifies, the question of the antinomies becomes "does the series of conditioned conditions terminate in a first member of the series that is utterly unlike the other members in needing no further condition, or does the series go to infinity, with every member presupposing further conditioned conditions without end?" (Wood, 248).

Kant goes on to argue, however, that even though all possible perceptions and experiences are always involved in the conditioned, in virtue of their relationship with space and time, possible perception is met with nothing unconditioned that requires us to determine whether it is "located in an absolute beginning of synthesis, or in an absolute totality of a series that has no beginning" (Kant, A483). That is, the distinction between whether the unconditioned as the simple or as the totality of the infinite regress is never demanded of us by possible experience, and only by reason itself. Kant puts it another way when he says, "the whole of division…with all questions as to whether it is brought about through finite synthesis or through a synthesis requiring infinite extension, have nothing to do with any possible experience" (Kant, A483). Only reason, not possible experience, demands an explanation of the whole, division, and the nature of the unconditioned. The antinomies are meant to show that limitless pure reason is an illusion leading to mutually conflicting pseudo-rational assertions. Objects of experience, rather than things in themselves, are only given in experience and for Kant, "have no existence outside it" (Kant, B521).[5] Decomposition then becomes not a material operation, but a regress of reason.

Because objects of experience have no existence outside of experience, the question of whether the regress towards the unconditioned is infinite or finite disappears when we turn to transcendental idealism, the idea that the world is not a thing in itself. The question changes from "how great this series of conditions may be in itself, whether it can be finite or infinite" since it is nothing in itself to "how we are to carry out the empirical regress, and how far we should continue it" (Kant, A514). In other words, transcendental idealism allows us to ask not if the unconditioned is the first of the conditions or the totality of the conditions but about the normative dimension of reason's limits.

IV. Infinitude and the Decomposition Operation

For our purposes, which are not to directly question Kant's limits of reason, but to understand Kant's decomposition of substance, let us turn back to the notion of the regress. When discussing the unconditioned as the totality of an infinite regress, we took for granted Kant's notion of infinitude, but Kant expounds on the extension of the regress later in the Transcendental Dialectic. In particular, a regress can be infinite or indeterminate. When taken in the context of the division of a body, that is, "a portion of matter given between certain limits", the matter is given as a whole totality and therefore with all its parts as conditions (Kant, B541). Each part, as a condition of the body, is itself conditioned with parts of its own. The regress of

[5] This would prima facie solve the antinomy because appearances would not demand to have conditions outside experience. The issue becomes dialectic opposition rather than analytic contradiction. Whether this provides a solution to the proposed asymmetries, on the other hand, is a different issue that will be discussed at length in Section VII.

decomposition through the series of conditions is never met with the unconditioned, nor is it met with any “empirical ground for stopping in the division”, and the next members are themselves empirically given prior to continuing the division (Kant, B541). The resulting parts are always themselves empirically given as a whole conditioned by its own parts, driving reason to continue the division. This means that the division of a limited body goes on in infinitum, that is, there is always a continuation of division of a whole given in totality (Kant, B541). In other words, “because the whole is given as an intuition enclosed within boundaries, and the parts are contained… within it… we can assert that the regress continues to infinity (*in infinitum*)” (Sutherland, 112).

Kant then differentiates this infinite regress from a regress that extends only indeterminately, that is, one that ascends successively for an indeterminate number of steps. This is constituted by the whole never at once being accessible, so just the process of iteration indefinitely progressing. Since members of an ancestry, for example, are never given in their absolute totality at once, the regress can only possibly go on indefinitely. In other words, “because the world in its totality ‘is not given to me through any intuition, hence its magnitude is not given at all prior to the regress’ (A519/B547), we cannot say that the regress is a regress to infinity (*regressus in infinitum*), but ‘only an indeterminately continued regress (*in indefinitum*)’ (A518/B546)” (Sutherland, 112). The distinction, then, between infinity and indeterminacy, rests upon whether the totality of conditions can be given at once.

This decomposition as a division of parts is a process of regress in the series of conditions. Parts, as conditions of the body, are themselves contained in the starting conditioned body. Thereby, the parts are given together with the conditioned, making it impossible for the regress to be merely indefinite. Yet because this body’s parts themselves are conditioned parts, the absolute totality of the series of conditions can never be reached. For Kant, if all parts in a "continuously progressing decomposition are themselves again divisible, the division, that is, the regress from the conditioned to its conditions, proceeds in infinitum" (Kant, A523). Only if the regress reached simple parts, the unconditioned being the first member of the series, would it be possible for the absolute totality of the series to be accessed. Thus, Kant defines infinite divisibility of a body.

Even though a body in space is infinitely divisible, we are not entitled to say that the whole is made up of infinitely many parts because while the parts are contained in the intuition of the whole, the whole division itself is not contained in the whole. That is, the division is constituted only by the regress through conditions, the "continuous decomposition…whereby the series first becomes actual" (Kant, A524). Not only is this decomposition operation merely a function of reason's regress, rather than something empirically contained in the body, division is also not contained in the whole because it is "a successive infinite and never whole" (Kant, A524). That is, because the infinite division is by Kant's definition never completed, the division cannot exhibit any infinite multiplicity in a whole. One cannot say the whole contains infinitely many parts because the whole of the infinite division will never be completed, and thus, will never be contained. The decomposition operation is distinct from the decomposed parts.

In order to further understand how Kant conceptualizes this decomposition operation, we turn to more of his reflections on the cosmological ideas. In the argument for the thesis, Kant relies on the notion of compositeness being "thought away" in order to show the impossibility of compositeness without simple parts. Kant argues that decomposition could never remove all compositeness from space because this would leave nothing self-subsistent in space, implying space would stop being space. This, for Kant, is impossible. In other words, thinking away composite entities from space would leave nothing else that is itself a substance, and since there would be no subsisting thing in space, space, which is itself not a self-subsisting thing, would thereby cease to be space, which is impossible. Yet Kant adds that being left with nothing after removing compositeness in thought is not compatible with the concept of substance, substance being the subject of all compositeness that persists in the elements of the composite. Even without the substances' connection to space, "whereby they constitute a body", the substance still must persist in the parts of the composite (Kant, A525). This infinite decomposition, leaving us with either no space or no substance, is only possible for a whole without already distinguished parts. That is, this infinite regress of decomposition could not be applied to a whole in which "already, as given, the parts are so definitely distinguished off from one another" (Kant, A526). Kant calls this whole whose parts are already clearly given as parts a "quantum discretum" (A526). This body has particular parts that are "articulated, structured, or jointed" (Sutherland, 113). Here Kant distinguishes between actually divided parts of space, that is, quantum discretum, and "merely potential and as-yet-undivided parts of space that are nevertheless given in the whole intuition", that is, a quantum continuum (Sutherland, 112). Kant's definition of continuity of a body rests on the ability to divide infinitely without reaching its smallest parts.

Kant uses this to show that even if a whole can be infinitely divided, that is, is organized to infinity, the whole is not already divided. This whole is organized to infinity, so to speak, because the number of parts in the whole depend on "how far we care to advance in the regress of the division" (Kant, A526). The divisibility of an appearance, and thus the number of parts determined in the division, is a function of our regress, which, when the whole is given, is infinite rather than indeterminate. In this mereological account, the parts in an infinitely organized whole are only given through the division and decomposition operation, while the parts in a quantum discretum are already given as distinguished off entities. This is important because Kant is trying to further prove that the decomposition operation—that infinite regress that produces the whole's parts—is distinct from the decomposed parts, and that even if matter is infinitely divisible, there are not infinitely many parts. Kant discusses the whole, while infinitely divisible, not already divided because he wants to say that this is impossible.

Kant suggests that if the infinite parts determined by division were both organized in infinitum through our division but also already represented as divided into parts, this yields us "prior to all regress, a determinate and yet infinite number of parts" (Kant, A527). To Kant, this simultaneity of determinate (completed) yet infinite (never completed) parts is self-contradictory. "This infinite involution," that is, the decomposition operation, is thought about as an infinite "that is, never to be completed" series, yet simultaneously as "completed in a [discrete] complex" (Kant, A527). Further, he argues that only a "quantum continuum" whose whole is "inseparable from the occupation of space" is infinitely divisible, while a quantum discretum's parts are determined and "in every case equal to some number" (Kant, A527). So, if the infinitely divisible whole's parts were in fact already given in the whole, this would leave us with what Kant believes to be a contradiction: a completed, determined whole containing infinitely many parts. This leaves us with the following: decomposition can be progressed to infinity, but this does not mean that the whole is given to us as already divided and therefore infinite in the composite.

To outline what we have worked through so far: The division of a whole body is infinite because it is extended only through space, which is itself infinitely divisible. A body's parts are the conditions of the whole and are all contained in the whole. Yet, the whole infinite division cannot be contained in the whole, but only consists in the continuous decomposition process, an infinite regress of the unconditioned as totality. This is to say that just because a body is *capable* of being divided infinitely by this decomposition operation, it does not follow that there is an actual infinity of parts of which the body is antecedently composed. That is, the possibility of infinite division does not entail the actuality of infinite parts. Rather, because this infinite division can never be whole or completed, a body can never contain this infinite multiplicity. For appearances, infinite divisibility belongs to *quantum continuum*, rather than *quantum discretum* in which the number of units is determined. Decomposition can be progressed to infinity but this does not mean that the whole is given as already divided, that is, infinitely organized in the composite.

The decomposition of an appearance—an object of sensible experience related to intuition, namely, space—is decided by the principle of reason that drives the cognizer to search for the unconditioned in the series of an appearance's conditions. Appearances as objects of intuition are never met with unconditionedness because they are always conditioned by space and time as forms of intuition. Reason prescribes that "the empirical regress, in conformity with the nature of appearance [as itself never unconditioned], be never regarded as absolutely completed" (Kant, A527). The question of the extent of the regress and of decomposition for an object of intuition is not a question of experience, but rather of the nature of reason. That is, the organization of a body does not meet us in experience, but only through the regress of reason. Reason by nature leads cognizers to these antinomies.

What is important for our purposes is Kant's notions of infinitude and continuity, both of appearances and of synthesis. Kant suggests that we can only perform infinite division, what I call the decomposition operation, if there is a whole with no simple parts. Kant defines appearance's continuity "as the fact that they have no smallest parts" because "the division or decomposition can continue ad infinitum" (Sutherland, 114). This, recall, is distinct from a body having particular parts, forming a quantum discretum. But when it comes to how cognizers synthesize, Kant argues that the parts that make the representation of the determinate, extensive whole possible are themselves not represented clearly as parts. Rather, these parts are only "potentially determinately represented" in the same way that "a continuously drawn line only indeterminately represents all the parts of the line" (Sutherland, 118). The representation of all parts is necessary to understand and to present the whole itself.

The body's parts are themselves first indeterminate, that is, not an indeterminate number, but rather indeterminate representations.[6] The multiplicity of parts is first given as "absolutely indeterminate", that is, not yet determined and "merely potential", which is the feature that in fact allows parts to count as a manifold capable of being synthesized (Sutherland, 113, 112). Because there is no determinate representation prior to synthesis, there is no first part of synthesis. Rather, successive synthesis is itself continuous and therefore "runs through" all the indeterminate parts that "could potentially be determinately represented" (Sutherland, 114). In other words, Kant does not have in mind that the successive synthesis starts with the representation of one particular part and onto the next and next until the whole is determinately represented. Rather, the process of synthesis is itself continuous that runs through all the indeterminate representations of the parts in order to then have them determinately represented.

Kant discusses that magnitudes of continuous space and time can also be called "flowing, since the synthesis (of the productive imagination) in their generation is a progress in time, the

[6] Note that Kant clarifies earlier the distinction between indeterminate regress and infinite regress when it came to divisibility. That indeterminacy of division in regress is again distinct from this indeterminacy of parts. Here, we discuss a feature of the parts as not being yet articulated. We are not discussing an indeterminate number of parts.

continuity of which is customarily designated by the expression 'flowing'" (Kant, A170). Sutherland argues that here Kant appeals to Newton's notion of *fluxion*, which flows or elapses. This notion of magnitudes "generated by a continuous motion" was introduced so Newton could avoid difficulties of Leibniz's infinitesimals without asserting something as composed of parts (Sutherland, 116). Sutherland argues Kant uses the notion of flowing of successive synthesis for a similar purpose; Kant wants to allow for continuous quanta to have no smallest parts while also allowing for synthesis of a multiplicity that "in some sense has parts", namely, those indeterminate representations of parts that make it a multiplicity capable of being synthesized (Sutherland, 116). In other words, using motion to explain continuity allows Kant to discuss a whole with no smallest parts yet simultaneously containing indeterminate representations of parts prior to synthesis.

V. Hierarchies of Infinitude

In parsing through the arguments of the Transcendental Dialectic as they relate to atomism, decomposition, and infinitude, it becomes clear that Kant's language begs for a mathematical reading to help make sense of it. An account of modern set theoretic hierarchies in infinities is in order, insofar as it exposes a destructive asymmetry in his decomposition and composition operations; Kant's decomposition operation requires the composite whole be *continuous* prior to the infinite regress of division of the body, while his composition operation of particular parts can only result in a *discrete*, or at best, *dense*, whole. This results in, I will argue, asymmetry and thereby a collapse of the antinomy as a whole.

To introduce these hierarchies of infinitude, let first us consider a path through space, that is, a line. Say this line has points only at the natural numbers, that is, the positive whole numbers, so the line resembles the number line most learn in grade school. This line would be *discrete* because each point of that line is preceded by a unique point and also succeeded by a unique point. Yet of course, it seems naïve to regard the line being composed of only the points {1, 2, 3, …, 10}. Rather, it seems like a natural step to notice that between any of these two points, there is always a third. This would mean this line is *dense*. That is, the line does not have just the points at 1 and 2 associated with discreteness, but the line also contains a point between these points, namely 1/2. In fact, this operation—finding a point between any two points—can be extended further, say, between 1 and the point just found at 1/2, resulting in another point at 1/4. And so on. This is what it is for the line to be dense. The segment of our line, our path through space, between any two points can always be divided to give us a third point. In this way, the natural numbers and integers—positive and negative whole numbers and zero—can be extended to construct the rational numbers, that is, a number of the form of a ratio of two integers. In terms of our line or path through space, one can clearly distinguish between a discrete line, in which one unique and determinate point succeeds another unique point, and a dense line, in which a point always lies between two other points.

Yet our line can be not just discrete and not just dense, but rather *continuous*. In this case, the line would be cut at a certain point to produce two sets such that every point on our defined number line is a part of the first set or the second set but not both. Say, for example, we cut our number line of positive rational numbers at point like √2, an irrational number that is equal to 1.4142…. Cutting our number line at √2 leaves us with two sets: one containing all positive rational and irrational numbers up to and containing √2 itself, and the other containing all positive rational and irrational numbers after √2. That is, only one of the sets after the cut, namely, the first set, would contain all positive real—both rational and irrational—numbers up to √2 and √2 itself, while the other set would have no first point, but rather contain the values on the number line *after* √2. We can express this with the following: the starting set is expressed [1,10] while the first set after the cut at √2 is expressed [1, √2] and the second set is expressed (√2,10].[7] This number line would be *continuous*, because if after a cut producing two sets X, Y, there is a last point of X but no first point of Y, *or* there is no last point of X but a first point of Y.[8] This is precisely the way that Dedekind formulated the arithmetic distinction between the rational numbers and irrational, and thus, real, numbers in the 19th century through Dedekind cuts (Dedekind, 1901). So, if our path through space, shown as a line, is continuous, it is isomorphic to the set of the real numbers.

The aforementioned distinctions can be extended to a space or body more generally. For the body to be *discrete*, the body would need to have articulated, unique building blocks that precede and stack atop of one another to compose the whole body. As mentioned, this body would be isomorphic to the set of natural numbers. In order for the body to be *dense*, the body's articulated building blocks could always be cut further, as if cutting each block in half again and again. For each cut of the building block, there will always be a last point of the left part and a first point of the right part. This body would be isomorphic to the set of rational numbers. Yet for it to be *continuous*, the building blocks would need to be cut in order for there to be either a last point of the left part but no first point of the right part, *or* no last point of the left part but a first point of right side. This body would be isomorphic to the set of real numbers.

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[7] The inclusive [x,y] notation means the set contains all elements between x and y, including both x and y. The exclusive (y,z] notation means the set contains all elements between y and z, including z but excluding y.

[8] So, the previous number line example could also be continuous if the first set had no last element, but the second set had a first element, namely, √2. Then, the starting set is expressed [1,10] while the first set after the cut at √2 is expressed [1, √2) and the second set is expressed [√2,10].

At this point, one might notice that the number of points on both a dense line, path through space, or body and a continuous line, path through space, or body is infinite. In both cases, one can always make further cuts. Furthermore, one can even have an infinite number of points of a discrete line if it is extended boundlessly. It would seem as though the infinite structure of the dense line would be most related to the infinite structure of the continuous line, rather than the discrete line. Yet as it turns out, the infinity of points correlated with density has a lesser cardinality, that is, a smaller numerical size, than the infinity of points correlated with continuity, yet has the same cardinality as the infinity of points correlated with discreteness. That is, discreteness and density are bijectable—can be put in one-to-one correspondence—with each other, while discreteness and density are not bijectable with continuity. Furthermore, any discrete set and any dense set can be put in one-to-one correspondence with the set of natural numbers ($\mathbb{N}$), while any continuous set can be put in one-to-one correspondence not with $\mathbb{N}$, but rather with the set of real numbers ($\mathbb{R}$). In fact, in the late 19th century, the mathematician Georg Cantor proved in the late 19th century that certain infinite sets were larger, that is, had larger cardinality, than others. Specifically, Cantor proved that the cardinality of the positive real numbers, like our continuous line, was greater than the cardinality of the positive natural numbers, like our discrete line. Furthermore, sets with a cardinality "equal to that of the natural numbers (like the integers and the rationals) are said to be *countably infinite* or *denumerable*," while infinite sets with a cardinality of the real numbers "are said to be *uncountable*" (Easwaran, 2021). This means that the discrete line of natural numbers has the same cardinality as the dense line of rational numbers, namely, both are countably infinite or denumerable. The continuous line of real numbers, containing rational and irrational numbers, however, has a larger cardinality than the discrete and dense sets, as it would be considered uncountably infinite. In fact, the set of irrational numbers is itself said to be uncountably infinite, since the irrational numbers make the difference between the countably infinite rationals and the uncountably infinite reals; it is adding the irrational numbers through Dedekind cuts in the set of merely rational numbers that makes the set uncountably infinite, rather than just countably infinite.

Maybe surprisingly, the dense set is isomorphic to the discrete set, rather than the continuous set. It would not be valid to treat the structure of a dense set as equivalent to the structure of a continuous set, since, as we have seen, these are not bijectable and thus have distinct cardinalities. Yet this is exactly what Kant mistakes. Kant's decomposition and composition operations bring to the surface his confusion regarding the distinct structures of infinity. In particular, Kant's decomposition operation leaves open the possibility that the composite whole be continuous prior to the infinite regress of division of the body, while the composition of particular parts can only result in a discrete, or at best, dense, whole.

VI. Second Asymmetry: Cardinality of Decomposition

Turning back to Kant's Transcendental Dialectic, recall that Kant distinguishes between a quantum continuum—a whole that is infinitely divisible in virtue of its occupying space—and a quantum discretum—whose whole has already determined and articulated parts. That is, Kant's definition of a quantum continuum, and thus, continuity, rests on the ability to divide infinitely without reaching its smallest parts and assumes mereological harmony, as discussed in Section II, when it regards this infinite divisibility as a feature of a body's occupation of space, which is itself infinitely divisible.[9] Space's and the body's infinite divisibility, namely, the decomposition operation discussed in Section IV, can only be performed if the whole has no simple parts

[9] As we discuss the regress continuing without bound, recall the distinction between decomposition *ad infinitum* and decomposition that proceeds *indefinitely* in Section IV. Here, we are focused on the former, which requires the whole to be given, the parts of which are not yet determinate, rather than the latter, which is concerned with a boundless ascension through conditions when the totality is itself not given.

because only then, according to Kant, would the decomposition be able to continue ad infinitum. As Kant puts it, "although this rule of progress in infinitum undoubtedly applies to the subdivision of an appearance, viewed as a mere filling of space [with indetermined parts], it cannot be made to apply to a whole in which already, as given, the parts are so definitely distinguished off from one another that they constitute a quantum discretum" (Kant, A526).

Recall that the decomposition operation is used in the antithesis in order to describe the infinite divisibility of space, and thereby, a body, given that if all parts in a "continuously progressing decomposition are themselves again divisible, the division, that is, the regress from the conditioned to its conditions, proceeds in infinitum" (Kant, A523).[10] The divisibility of outer appearances within limits, namely, bodies, is grounded in the divisibility of space. Space, as Kant outlines in the Transcendental Aesthetic, can only be represented as one space, rather than a composite of unique parts of space. "Space is essentially one," and to speak about different spaces is to speak about parts of space that thought of "only as *in* it" (Kant, A25). This is because space is regarded as "not *compositum* but *totum*, since its parts are possible only in the whole, not the whole through the parts" (Kant, A438). That is, space's parts "cannot precede the one all-embracing space" as constituents that form a composite space, but rather, parts of space depend solely on "[the introduction of] limitations" (Kant, A25). It follows from the notion that no parts that precede and compose space that there cannot be any simple parts of space, which, as Kant suggests, is equivalent to being infinitely divisible. From the presupposition of mereological harmony, as discussed in Section II, this entails the infinite divisibility and atomlessness of the body in space, on which one can operate this decomposition ad infinitum.

[10] Of course, here, Kant's use of "continuously" is not exactly the continuity that was defined in Section V.

The criterion for decomposition and infinite divisibility, then, is the notion of a part itself always having a proper part, and in terms of conditions, the part itself always conditioned by its parts. As long as a body exists as a portion of matter given in certain limits of space, the division of it can proceed infinitely. This body, in virtue of being given as a whole and a totality, is given with "all its possible parts, in empirical intuition" and as such, its conditions are given in the totality (Kant, B541). As such, "since the condition of this whole is its part, and the condition of this part is the part of the part, and so on, and since this regress of decomposition an unconditioned (indivisible) member of this series of conditions is never met with, not only is there never any empirical ground for stopping in the division, but the further members of any continued division are themselves empirically given prior to the continuation of the division" (Kant, B541). In other words, a body given in certain special limits is conditioned by its parts, each part always conditioned with its own parts, and as such, the decomposition never reaches an unconditioned member, so the division always has more to divide. For a body in space given as a whole, the decomposition operation continues infinitely.

Yet after defining distinct cardinalities of infinity in the previous section, it becomes evident that Kant's decomposition ad infinitum is weakly defined. Asserting atomlessness—or put another way, gunk—and infinite divisibility does not provide a clear enough mereological picture defined in terms of the decomposition's cardinality. In fact, it is contentious if there is an upper limit to the cardinality that atomlessness and the infinite divisibility of gunk. For example, there may even be what Daniel Nolan calls *hypergunk*, namely, "gunk such that, for any set of its parts, there is a set of its parts of strictly greater cardinality" (Cotnoir & Varzi, 155). While gunk's having a proper class of parts is a messy business, these definitions serve to elucidate the ambiguity of atomlessness and how decomposition functions on an atomless composite.[11] That is, a weak definition of infinite divisibility, like the one Kant assumes, leaves unanswered the question of that decomposition's cardinality, which became a sharpened issue after Cantor and Dedekind in the 20th century. For our purposes, a weak definition of infinite divisibility leaves open the question of whether the decomposition ad infinitum is a merely dense or rather continuous operation, nor is it clear that Kant's quantum continuum really is continuous, just by virtue of its infinite divisibility.

Given the aforementioned ambiguity regarding atomlessness—and the possibility of gunk and even hypergunk that arises from it—of Kant's decomposition operation, leaves open the possibility that the division is one of a whole that is not merely dense—each part being further divided, with a third point between any two boundaries—but further, continuous—each cut results in one part with a last point and the other part with no first point. In the same way that a dense body was distinguished from a continuous body in Section V, so too can the decomposed whole be defined; the whole may be dense with parts that can always be cut further, each cut resulting in one part with an ending boundary and the other part with a beginning boundary, or the whole may be continuous with parts that can always be cut further, each cut resulting in one part with an ending boundary and the other part with *no* beginning boundary. Yet the aforementioned criteria of infinite divisibility that Kant provides are not sharp enough to delineate between the mereological structure of the whole we are operating on. It is evident how, given Kant's notion that every part always has a part and decomposition is never met with a part that itself has no part, this mereological structure is not entirely clear; this body could be gunk,

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[11] A class is a collection of objects (say, sets, or here, parts) corresponding to certain properties. In Zermelo–Fraenkel set theory, a proper class is such a class that is not a set. Informally, a proper class is so large that it could not be a set, otherwise paradoxes would arise.

hypergunk, or perhaps another atomless structure, which could require the decomposition operation to be not just dense, but further continuous, or even of a strictly greater cardinality. The decomposition operation integral in Kant's arguing for the antithesis is not *necessarily* an operation on a merely dense, countably infinite whole. Rather, the cardinality of the whole on which the decomposition operates *could* be strictly greater than that of the countable set of natural and rational numbers, that is, that of the uncountable set of real numbers.

The thesis, on the other hand, that every composite body is constituted by simple parts, utilizes a composition operation in which the composite is as it were, built up from simples, leaving something discrete or, at best, dense. This composition is "that accidental unity of the manifold, which, given as *separate* (at least in thought), is brought into a mutual connection, and thereby constitutes a unity" (Kant, A438). That is, composition operates on independent parts that, although are able to stand on their own, reach a completed, determinate unity. Recall that a discrete body would have articulated, unique building blocks that precede and stack atop of one another to compose the whole body. It is evident that the composition operation would be able to bring these articulated parts into a unity of a completed composite. But I propose to take the composition operation also a step further. Because composition results in a determinate and completed unity, there is a sense in which, even if the body *were* infinitely complex, the composition operation would reach an end as it reaches a unity of the manifold, that is, the whole composite. For a body to be *dense*, the body's articulated building blocks could always be cut further, as if cutting each block in half again and again, leaving our parts always be a last point of the left part and a first point of the right part. There is a sense in which this dense body has articulated parts which could be brought into unity and constitute a composite. Furthermore, the dense body is said to be countable, which would seem to suggest an end to the "building up" of the body, leaving us with a completed, determinate unity of the composite. To try to extend this composition operation to a continuous body would seem impossible, as this body is regarded as uncountable and would fail to result in a completed composite unity.

This means that at best—and not necessarily—the composition could be operated on a dense body, insofar as its infinite complexity is countable, suggesting an end to the operation that results in a determined composite. Yet recall that perhaps surprisingly, the infinite cardinality of discreteness and the infinite cardinality of density are both equivalent to the cardinality of the set of natural numbers, and thus, also to each other. Although it is not necessarily the case, it seems as though the composition operation Kant puts forward could in principle also operate on a dense whole, that, although infinitely complex, is countable. This is why we say the composition operation operates on a discrete and *at best*, dense whole.

Moreover, when this composition is removed in thought for the purpose of the thesis argument, which as Van Cleve calls it, is a "total decomposition", by which one "[brings] it about that no two parts of it that formerly composed a larger whole any longer do so" (Van Cleve, 482). That is, since composition is a mere accidental unity, by which multiple parts create a whole of multiplicity, "any substances that are so related as to compose a larger whole could exist without composing that whole" (Van Cleve, 482). This total decomposition, then, would in principle would leave parts that are able to stand independently without the accidental unity of composition. These parts, existing without composition and left after the total decomposition, could not be atomless.

This is exactly Kant's thesis; the total decomposition, or as he puts it "removing composition in thought", cannot be operated on an atomless whole (Kant, A434). If an infinitely complex substance were totally decomposed, it would leave either the simple or no composite part whatsoever. But because there are no simples in the thesis "*ex hypothesi*" and because the decomposition is supposed to be total, leaving no composition at all, then no part—even, no matter whatsoever—would remain, per the second step in Kant's thesis argument as outlined in Section I (Van Cleve, 482). In order to remove composition in thought, every composite whole must have simple parts. The decomposition that the thesis deals with, therefore, cannot be operated on an atomless whole, but rather requires a discrete whole, whose cardinality is that of the set of natural numbers. The composition and total decomposition operations in the thesis thereby deal necessarily with the cardinality of the set of natural numbers, while the decomposition operation in the antithesis leaves open the possibility of working with the cardinality of the set of real numbers. That is, the thesis's decomposition can only be countably infinite, yet the antithesis's decomposition may be uncountably infinite.

Not only do these different decomposition operations require distinct cardinalities, but furthermore, the antithesis's decomposition operation is not the mere "undoing" of the composition operation discussed in the thesis. It would seem that when the accidental unity of composition is removed in the thesis's total decomposition operation, the process is instantaneous and even atemporal, in which one merely separates these independent parts to stand by themselves. Because the composition is a merely *accidental* unity, these parts subsist on their own, and their participation in the composite has no bearing on their existence. Yet the antithesis's decomposition operation necessitates a sense of progress and temporality when it is regarded as "the regress from the conditioned to its conditions, [which] proceeds in infinitum" (A523). Further, as discussed in Section IV, this infinitely divisible body on which the decomposition operates on has parts that are not yet determined prior to the regress, and this decomposition is that which itself makes the series of conditions actual and renders these divided parts determinate. Yet still through the regress, the decomposition operation is distinct from the decomposed parts. Because the infinite division is, for Kant, never completed, the division cannot exhibit any infinite multiplicity in a whole, so we cannot regard the infinitely divisible body as containing infinitely many parts. That is, the decomposition operation functions on a whole that cannot contain its own infinitely many parts, distinguished off in a temporal regress, yet composition operates on self-subsistent, independent parts and their unity can be removed in thought instantaneously.

On the grounds of cardinality, temporality, and mereological requirements, the decomposition operation on which the antithesis argument rests is distinct from, even incompatible with, the composition operation—and total decomposition—that the thesis requires. This asymmetry in essential mechanisms in Kant's arguments is destructive for the alleged equivalent opposition in the Second Antinomy as a whole.

VII. Does It Really Count?

Let us situate this conflict within Kant's larger project. The Second Antinomy, put in terms of the regress of reason, reveals the question of how the unconditioned is reached: as the totality of conditions themselves or as the first condition under which all other conditions fall. It is because all possible perceptions always remain involved in conditions, whether in space or in time as the forms of intuition, that one is drawn by reason to come upon the unconditioned, thereby "requiring us to determine whether this unconditioned is to be located in an absolute beginning of synthesis, or in an absolute totality of a series that has no beginning" (Kant, A483). That is, the unconditioned being the totality of the series of conditions would lead us to the antithesis, where an infinite regress of reason through the decomposition operation would result in an infinitely divisibly composite that is not composed of simple parts, which as Kant puts it is a quantum continuum. Yet, if the unconditioned is the beginning of the series of conditions, decomposing a composite would result in simple parts, as in the thesis. It is *reason*, not experience itself, that leads cognizers to this antinomy. That is, when reason is extended without bound in its attempt to ascend through the series of conditions to reach the unconditioned, it leads cognizers to "mutually conflicting pseudo-rational assertions" (Kant, A490). The Second Antinomy acts as a confirmation of the limits of pure reason—one might say a critique of pure reason—by showing how two equivalent yet opposing arguments can each be reduced to absurdity. Yet clearly there are issues here.

So, I ask on two fronts: does it really count? This question first applies to the decomposition operation: is the decomposition ad infinitum associated with merely countable infinity or does the criterion for infinite divisibility allow for decomposition at the level of uncountable infinity? The question further extends to the validity of the argument as a whole: if the antithesis's mechanism rests on the requirement of a continuous whole while the thesis deals with an at best dense whole, an asymmetry of distinct cardinalities of infinity in the antithesis and thesis, can the Second Antinomy really stand as a valid opposition of allegedly equivalent sides? Moreover, after revealing this destructive asymmetry, can the Second Antinomy really count as proof for transcendental idealism?

Through the course of parsing through the Second Antinomy, the first ground on which I ask if it really counts, namely, if the antithesis's decomposition operation is necessarily countable, answers in the negative. Kant's criterion for infinite divisibility of space and a body, which proves to be weak after introducing contemporary mathematical definitions, leaves open the possibility of an operation on an uncountably infinite whole. If this is the case, it would leave Kant with a destructive asymmetry in his argument; the decomposition that the argument of the antithesis rests upon would not be isomorphic to the composition that the thesis requires. In the process, the requirement of mereological harmony for the antithesis's argument revealed another asymmetry in the Second Antinomy, which we bracketed for the sake of addressing the question of infinite divisibility, which inextricably rests upon the body's mereological structure mirroring of the structure of the space it occupies.

This brings us to the second question: after elucidating these asymmetries, can the Second Antinomy remain a valid argument through which Kant can prove transcendental idealism? It is clear that it cannot. The Antinomy is meant to demonstrate that transcendental realism—the notion that representations are things in themselves—is a common faults assumption that leaves us with two exhaustive and exclusive, yet opposing arguments. As Kant puts it, "if two opposed judgments presuppose an inadmissible condition, then in spite of their opposition…both fall to the ground inasmuch as the condition, under which alone either of them can be maintained, itself falls" (Kant, A503). That is, proving both the thesis and the antithesis with a reductio ad absurdum is meant to demonstrate that objects of experience cannot be things in themselves, thereby refuting transcendental realism. In order to do this, the arguments must be opposing in the same framework, yet it is clear that the thesis and the antithesis of the Second Antinomy have distinct and asymmetric mereological requirements and decomposition operations. The Second Antinomy, then, cannot stand as a true antinomy.

Kant means to solve the antinomy by proposing transcendental idealism—the notion that all objects of possible experience have no existence outside of thought and rather are mere appearances for cognizers—as the critical solution to this cosmological conflict of the totality of composition. While transcendental realism would require that the world, as a thing in itself, be either simple or atomless, transcendental idealism relieves us of this worry; if the world is not given as a thing in itself, then appearances in general are nothing outside of cognizers' representations. It follows from the assertion that appearances and the world are mere representations that the world need not be simple or atomless, since this is never given through experience but rather always through the regress of reason. This shift to transcendental idealism prima facie solves the issue of a failing thesis and antithesis in the Second Antinomy, yet does not vindicate the issue of asymmetry itself.

This paper reveals destructive asymmetries in the Second Antinomy, so while Kant proposes a solution to the conflict that arises from the antinomies, what we have been interested in is if Kant can even say he arrived at a conflict through symmetrical arguments in the first place. That is, we ask whether these asymmetrical arguments, operations, and mereological requirements even *arrive* to a point at which we *can* propose transcendental idealism. The Second Antinomy thereby cannot stand in Kant's project as a proof of transcendental idealism insofar as the proposed asymmetries of harmony, cardinality, and mechanisms indeed hold. In fact, a case can be made that the Second Antinomy's collapse in virtue of these asymmetries stifles Kant's ability to fully prove, rather than merely propose, his framework of transcendental idealism in the *Critique of Pure Reason*. I will leave this for the reader to ponder.

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