An Exploratory Study Into Secondary Mathematics Technology Enhanced Formative Assessment

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Abstract

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Computer-based technological tools can be an efficient and effective way to enhance mathematics classroom activities like formative assessment. Whilst there are a number of theoretical frameworks and standards to help pre- and in-service teachers understand the individual complexities of mathematics pedagogy, technology integration, and formative assessment, there are no clear definitions of what encompasses the integration of these knowledge and skills known as Technology Enhanced Formative Assessment (TEFA) Literacy. Moreover, there are no standardized or easily replicable assessment tools that can evaluate teachers’ attainment of TEFA Literacy.

This dissertation seeks to fill these gaps by, first, developing and validating a theoretical framework for Secondary Mathematics Technology Enhanced Formative Assessment (TEFA) Literacy and corresponding rubric, and then, by designing a performance-based instrument aimed at eliciting secondary mathematics preservice teachers’ TEFA Literacy. The resulting framework represents an interaction of three main components composed of seven essential elements of Secondary Mathematics TEFA Literacy. The corresponding rubric provides definitions for the essential elements, with an analytic rubric design containing qualitative descriptors for ascending levels of performance. The performance-based instrument – the TEFA Literacy Test for Secondary Mathematics – contains a number of secondary mathematics
classroom scenarios, each followed by a set of questions intended to elicit evidence of participants’ TEFA Literacy. Using qualitative case study methodology, the study collected and analyzed secondary mathematics preservice teachers’ responses to the *TEFA Literacy Test for Secondary Mathematics*. The results of analysis of preservice teachers’ responses showed that the participants excelled at analyzing and selecting computer-based technologies that aligned to the instructional goals. However, they were challenged by using the technology to engage students in the feedback process.
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A.L.M.
Chapter 1: Introduction

Need for the Study

Assessment and Formative Assessment

Assessment has become an integral part of education and teacher professionalism, with research showing that effective assessment practices lead to an increase in students’ academic achievement, motivation, and interest (Brookhart, 1999; Khal et al., 2013; National Council of Teachers of Mathematics [NCTM], 2000; Xu & Brown, 2016). Assessment is generally categorized as either summative, an end of learning evaluation of student achievement (Black & Wiliam, 2009), or formative, classroom practices that elicit evidence about student achievement that is then interpreted and used by teachers and students “to make decisions about the next steps in instruction” (Black & Wiliam, 2009, p. 9). The goals of formative assessment are, ultimately, to promote further learning by using assessment-based evidence to help both teachers and students understand what students have, or have not, mastered (Black & Wiliam, 2009; Popham, 2008). The focus of this study is on formative assessment, in particular, the use of educational technology to enhance formative assessment in secondary mathematics classrooms.

Educational Technology and Formative Assessment

In 21st Century classrooms, educational technology has been praised for its ability to capture and streamline vital elements that make formative assessment more efficient (Looney, 2010; Mitten et al., 2017; Spector et al., 2016). Educational technology, for the purpose of this study, includes technological tools that can be accessed through a computer, laptop, tablet, or smartphone. Technology Enhanced Formative Assessments (TEFAs) are defined as those computer-based technological tools that are used by teachers and/or students to support the formative assessment process (Beatty et al., 2008; Looney, 2010). TEFAs, therefore, are
computer-based technological tools that are able to perform such actions as eliciting evidence of students’ current knowledge and understanding, collecting data about students’ performance, or providing feedback. Moreover, to ensure that TEFAs are upholding the goals of formative assessment, the technological tools should be able to collect data on students’ conceptual understanding rather than just computational or procedure-based skills often associated with multiple-choice questions (Leung, 2013; Olsher et al., 2016; Stacey & Wiliam, 2013).

The responsibility of how well TEFAs are able to elicit students’ conceptual understanding lies partially with the technological tools, but also with the teacher selecting or designing the TEFAs. Given the unique features of mathematics as a discipline and the diversity of mathematics educational technologies available, secondary teachers must develop a particular set of knowledge and skill for implementing TEFAs. As Hollebrands describes, “when adding technology to mathematical pedagogical activities it is important for teachers to think about how the use of technology influences representations of mathematics and how the use of technology influences pedagogy” (2017, p. 82). Specifically, pre- and in-service mathematics teachers must develop combined competencies and skills in mathematics pedagogy, technology integration, and formative assessment (Brookhart, 2011; Dalby & Swan, 2019; Hollebrands, 2017; Looney, 2010; NCTM, 2000; Zelkowski et al. 2013). Consequently, to ensure that future mathematics teachers are prepared to enact these skills and knowledge, teacher preparation programs must incorporate TEFA-focused course work as well as develop methods to evaluate mastery of TEFA implementation (Brookhart, 1998; Gotch & French, 2014; Khal et al., 2013). The focus of this study is on the second function, evaluating secondary mathematics preservice teachers (PSTs) mastery of TEFA implementation.

**Background Standards and Theory**
Evaluating secondary mathematics PSTs’ understanding and skill in implementing TEFAs requires that we first discern what it is, which can be gained by analyzing the origins of TEFA. Understanding and skill in assessment, often referred to as *assessment literacy* (Stiggins, 1991), is a multidimensional concept that constitutes educators’ ability to identify and design goal-oriented assessments that will produce valid and unbiased data on student achievement (Stiggins, 1991, 1995). Developing assessment literacy has long been considered to be a critical part of teacher education, (McMillan et al., 2002; Mertler, 2005; Stiggins, 1991, 1995; Xu & Brown, 2016) mastery of which is often measured against teacher certification or standards documents (Gotch & French, 2014). One of the most notable sets of assessment standards used to help evaluate assessment literacy are the *Standards for Teacher Competence in Educational Assessment of Students*, published in 1990 by the American Federation of Teachers (AFT), the National Council on Measurement in Education (NCME), and the National Education Association (NEA).

Advances in assessment theory and practice since the publication of the 1990 standards include, but are not limited to, formative assessment, content specific assessment, and technology-based assessment. Consider, for example, Brookhart’s (2011) *Educational Assessment Knowledge and Skills for Teachers*, which pays special attention to formative assessment; The Association of Mathematics Teacher Educators’ (AMTE) 2017 *Standards for Preparing Teachers of Mathematics*, which include mathematics and technology specific assessment standards; and, the International Society for Technology in Education’s (ISTE) *National Educational Technology Standards for Teachers: Preparing Teachers to Use Technology* (2002; updated in 2020) which provide guidance on how to effectively incorporate technology into all teaching and learning activities. Alas, there are currently no standards that
specifically address the essential understandings and skills secondary mathematics teachers need to implement TEFAs. That being the case, what it means for a teacher to possess TEFA Literacy, a multidimensional concept that constitutes teachers’ ability to identify and design TEFAs that will produce valid data and feedback to promote further learning, has yet to be clearly expounded.

Beatty and Gerace (2009) did attempt to provide guidelines for teachers to enact TEFAs in the classroom with the TEFA pedagogical framework. Their conceptual framework is described as an approach to teaching mathematics and science with the assistance of a classroom response system (CRS). Grounded in “several educational research traditions and perspectives” (Beatty & Gerace, 2009, p. 149), the conceptual framework outlines a set of four principles based on “question-driven instruction, dialogical discourse, formative assessment, and meta-level communication” (p. 146). Unfortunately, the principles do not align directly to secondary mathematics nor, as Beatty and Gerace (2009) mention, does their conception of the TEFA pedagogical framework actually require the use of technology.

Lack of a well-defined framework for Secondary Mathematics TEFA Literacy (i.e., TEFA Literacy that is specific to secondary mathematics teachers) has resulted in many other researchers (e.g., Bush, 2020; Dalby & Swan, 2019; Lee et al., 2012; Oshler et al., 2016) relying on a compilation of multiple, separate theories and frameworks. For instance, Black and Wiliam’s (2009) framework for formative assessment, NCTM’ (2014) Principles to Actions, and Mishra and Koehler’s (2006) Technological Pedagogical Content Knowledge (TPACK) framework are all frequently referenced. Such a strategy may suggest some of the knowledge and skills teachers need to possess, but it lacks continuity and consideration of how each of these concepts must be interwoven. This presents the first purpose of this study, to address the gap in
theory and propose a theoretical framework that clarifies and defines Secondary Mathematics TEFA Literacy.

Although standards and theory do supply much of the foundation for how Secondary Mathematics TEFA Literacy may be defined, still, it is important to analyze existing research and study. Not only will this help to provide further insight into the essential knowledge and skills related to Secondary Mathematics TEFA Literacy, it may also help to highlight gaps in existing research.

**Research in TEFA Literacy**

Teacher preparation programs serve as the foundation of secondary mathematics PSTs’ pedagogical knowledge, which encompasses their development of TEFA knowledge and skills. Research related to TEFA Literacy in secondary mathematics teacher preparation programs, however, is limited and often disjointed. Many studies tend to concentrate separately on development of general assessment literacy (Clark, 2005; Frykholm, 1999a; Herrington et al., 1998; Wallace & White, 2015; Widiati et al., 2020), formative assessment skills (Ayalon & Wilkie, 2020a; Lee & Lim, 2020), or technology integration skills (Huang, 2018; Karatas et al., 2017; Mangram & Sun, 2021; McCulloch et al., 2021; Ozcakir, 2019).

The limited number of studies that directly address the concept of TEFA Literacy generally focus on elementary PSTs’ and investigate opinions and perceptions of TEFA rather than mastery of TEFA Literacy. For example, Karaoglan-Yilmaz et al. (2020) investigated elementary mathematics PSTs’ “opinions about online formative assessment” (p. 12). Similarly, Bennett and Cunningham (2009) explored if elementary PSTs found technological tools “useful for conducting regular formative assessment” and if they “recognize the value of formative assessment” (p. 99). Though both researchers were able to collect valuable data on the
elementary PSTs’ opinions and perceptions of TEFA, they did not address their attainment of the knowledge and skills necessary to implement TEFAs. This signals a need for more research in secondary mathematics teacher preparation programs that specifically focuses on development or evaluation of TEFA Literacy.

According to Darling-Hammond (2010), there are a number of research-based practices that make evaluation and assessment in teacher education more successful, including the use of performance-based assessments. Performance-based assessments are designed to test higher-order thinking by measuring one’s ability to apply a learned skill or knowledge through the use of open-ended questions or discussion, or by developing a product, such as a lesson plan or portfolio (Darling-Hammond, 2010; Khal et al., 2013). Performance-based assessments are judged to be better measures of PSTs’ knowledge of and skills in assessment (Khal et al., 2013), and can help teacher educators to evaluate and address PSTs’ needs while building on their strengths (Darling-Hammond, 2010; DeLuca & Volante, 2016).

Consider Wallace and White’s (2015) case study of secondary mathematics PSTs that focused on the learning and implementation of general assessment practices. Data collection included PSTs’ lesson outlines with descriptions of assessment practices as well as observations of the PSTs in the classroom as they engaged in assessment related activities. Evaluation of these data provided insight into PSTs’ knowledge of assessment practices and their ability to apply that knowledge. Nevertheless, replicating the processes employed by these researchers could prove to be both challenging and time consuming for teacher educators. This suggests the need for a more standardized, performance-based assessment that can be used to efficiently and effectively evaluate PSTs’ Secondary Mathematics TEFA Literacy.
Currently, there are no standardized methods or instruments that are designed specifically to evaluate Secondary Mathematics TEFA Literacy. To address gaps like these, past researchers have often chosen to adapt or modify existing instruments to suit their own needs (Adamson, 2010; Mertler & Campbell, 2005). The majority of existing instruments that could be adapted or modified to Secondary Mathematics TEFA Literacy, however, primarily use quantitative, multiple-choice questionnaires (Adamson, 2020; Mertler & Campbell, 2005; Plake et al., 1993). This presents the second purpose of this study, to design a performance-based instrument that can efficiently and effectively evaluate PSTs’ Secondary Mathematics TEFA Literacy.

**Purpose of the Study**

First, this study aims to build on existing frameworks and standards to propose a theoretical framework for Secondary Mathematics TEFA Literacy and develop a performance-based instrument for evaluating Secondary Mathematics TEFA Literacy. Second, a case study approach will be used to field test this performance-based instrument for evaluating secondary mathematics PSTs’ TEFA literacy. This study is guided by the following research questions:

1. What are the components and essential elements of the theoretical framework for Secondary Mathematics TEFA Literacy?
2. How can Secondary Mathematics TEFA Literacy be evaluated through a performance-based instrument?
3. What are the TEFA Literacy levels of secondary mathematics preservice teachers enrolled in a graduate level initial certification program at a private urban university as assessed by this instrument?
Procedures of the Dissertation

The dissertation will encompass three main parts. The first part will address the first research question and will include the systematic development of a Theoretical Framework for Secondary Mathematics TEFA Literacy. This part will also include the development of a rubric with qualitative descriptors for the essential elements of the theoretical framework. The second part will address the second research question to develop a performance-based instrument, titled the TEFA Literacy Test for Secondary Mathematics. The third part will address the third research question and will include an evaluation of a sample of secondary mathematics PSTs’ responses to the TEFA Literacy Test for Secondary Mathematics.

Part 1: Development of Theoretical Framework

Cai et al. (2019) broadly define a theoretical framework as “the set of assumptions, theories, hypotheses, and claims (as well as the relationships between them) that guide a researcher’s thinking about the phenomenon” (p. 219). Using Jabareen’s (2009) phases of developing a theoretical framework as a guide, I developed the framework in eight phases:

1. Identifying and mapping literature sources
2. Initial categorization of sources
3. Identifying and naming concepts
4. Deconstructing and categorizing the concepts
5. Integrating concepts
6. Synthesis of concepts into initial framework
7. Validating the framework
8. Rethinking the framework
In Phase 6, an analytic rubric was developed that introduced qualitative descriptors for the essential elements of the theoretical framework. To assist in the validation process, in Phases 7 and 8, this rubric was examined by external experts and used by external raters to evaluate responses to the *TEFA Literacy Test for Secondary Mathematics*.

**Part 2: Development of the TEFA Literacy Test for Secondary Mathematics**

The development of the *TEFA Literacy Test for Secondary Mathematics* and evaluation method was adapted from DeVellis’ (2003, 2012) steps for instrument development, and organized into three steps:

1. Identify the construct to be evaluated.
2. Develop an initial item pool and evaluation method.
3. Review by experts.

**Part 3: Evaluation of PSTs’ TEFA Literacy Levels**

The third draft of the *TEFA Literacy Test for Secondary Mathematics* was administered to five secondary mathematics preservice teachers (PSTs) to produce responses that were evaluated for their levels of TEFA Literacy. Consistent with other studies of assessment literacy in preservice teacher education programs (Ayalon & Wilkie, 2020; DeLuca et al., 2013; Huang, 2018; O’Sullivan & Johnson, 1993) participants were selected from the same course within their teacher education program.

**Context and Participants**

The test was administered to PSTs at a graduate-level program for initial certification in mathematics grades 7-12 program at a private university in an urban setting in the Northeastern United States. Data was collected from participants in the course *Student Teaching in Mathematics* required for all PSTs. Prior to enrolling in student teaching, PSTs are required to
complete the pedagogy course *Mathematics in Secondary Schools*. Both of these courses are designed to address the multiple aspects of teaching mathematics. While the topic of TEFA is not explicitly addressed in the syllabus, the PSTs often discuss concepts and questions related to TEFA. A convenience sample was recruited from those PSTs currently enrolled. Five agreed to participate.

**Data Collection**

The *TEFA Literacy Test for Secondary Mathematics* was administered electronically through Qualtrics at a date and time convenient to the participants. On average, participants took one hour to complete the test.

Three participants were asked to participate in an unstructured, follow-up interview to seek additional information because they did not respond to a question or they responded, but did not address the whole question. Two agreed. In their follow-up interviews, they were asked to complete their responses. Their responses were recorded, transcribed, and added to their responses on the test. On average, the interview took 20 minutes.

**Data Analysis**

*A priori* coding is a pre-specified coding system which is often used in qualitative research to test “theory against empirical data” (Elliott, 2018, p. 2855). The *TEFA Literacy Test for Secondary Mathematics* was designed with the intent that each question would elicit one or more of the essential elements on the evaluation rubric. This meant that each question had “Applicable Rubrics” that served as an *a priori* code. By using the qualitative descriptors in the evaluation rubric participants’ responses to each question were analyzed by:

1. Grouping responses by the essential element the question intended to elicit (i.e., the “Applicable Rubric”).
2. Coding words and phrases by essential elements, evaluating for a preliminary level of proficiency, and assigning a corresponding Numerical Level.

3. Determining an overall level of proficiency for each essential element based on the average of the Numerical Levels (rounded to a whole number), and converted back to a categorical level.

These overall levels of proficiency served as the participants’ finalized levels of proficiency for each essential element.
Chapter 2: Literature Review

Introduction

Figure 1

Analysis of Literature for Secondary Mathematics TEFA Literacy

In this chapter, I will present an analysis of the literature related to the concept of secondary mathematics technology enhanced formative assessment (TEFA) literacy. Specifically, I will present the theoretical frameworks and standards that help to shape what it means for a secondary mathematics preservice teacher (PST) to have TEFA Literacy.

Throughout this first part of the analysis, I will suggest how these concepts can help to clarify what secondary mathematics TEFA Literacy is, as well as propose definitions for related fields of literacy. In the second part of the literature review, I will outline how literacy in these concepts have typically been evaluated and suggest how these methods could be applied to the evaluation of secondary mathematics TEFA Literacy. I will finish each section by noting the gaps in the literature that this study aims to fill (see the green boxes in Figure 1).
Theoretical Frameworks and Standards

An extensive review of literature has revealed that there is not a clearly defined theoretical framework for Secondary Mathematics TEFA Literacy. Nonetheless, there are a number of independent frameworks that can serve as the basis for the conceptualization and development of such a framework. First, I begin by analyzing three key concepts that serve as the bases for Secondary Mathematics TEFA Literacy: (1) Technology, (2) Formative Assessment, and (3) Mathematics Pedagogy. Each of these concepts have been defined by well-developed theoretical and conceptual frameworks that have helped teachers and teacher educators to understand the underlying knowledge and skills that are necessary to ensure their effective integration and, ultimately, student achievement. Additionally, each of these concepts have their own set of standards for practice. These standards for practice provide further guidance on research-based pedagogical actions in the classroom that have been shown to increase student success. More recently, with regard to integration of technology in the mathematics classroom, there are theories and standards that are designed for the specific use by mathematics educators.

Technology

What we consider to be educational technology today varies greatly from its initial conception. With the introduction of the personal computer in the late 1970s new technological tools began to develop at an almost exponential rate, infiltrating both the daily lives of people and the classroom. And, with the new capabilities of educational technology, has come new research, standards for teaching and learning, and teacher training that all seek to develop the most efficient and effective way to use technology in the classroom.
In the early 2000s a number of researchers began to discuss the complex web of interactions between teachers’ technological knowledge and Shulman's (1986) theory of pedagogical content knowledge (PCK). Pierson (2001), for example, conducted a case study to analyze teachers’ pedagogical expertise, as well as their perceptions, routines, and strategies for technology integration. Pierson’s (2001) findings of the study suggested that teachers’ ability to effectively integrate technology relied on the relationships between their content knowledge, pedagogical knowledge, and technological knowledge. Pierson (2001) called the intersection of these three types of knowledge “true technology integration” (p. 427) and labeled it as technological-pedagogical-content knowledge.

In 2003, Zhao contested that the question of technology integration switched from whether teachers should know about technology to what they should know. Zhao (2003) noted three elements essential to teachers’ knowledge and use of technology: “(a) knowledge of problems that can be solved by technology, (b) knowledge of a technology that can solve their problems, and (c) knowledge of how technology can solve their problems” (p. 4). Similar to Pierson (2001), Zhao (2003) suggested that technology knowledge be considered “as an integrated part of teachers pedagogical knowledge and pedagogical content knowledge” (p. 8).

In Angeli and Valanides (2005) study of preservice elementary teachers' use of educational technology, they came to define five principles for information and communication technology (ICT)-related PCK: (1) Identifying topics that can be taught with and enhanced by ICT; (2) Identifying ICT representations that make content more understandable; (3) Identifying teaching strategies and classroom activities that are enhanced or made possible by ICT; (4) Selecting appropriate ICT tools; and (5) Infusing “ICT activities in the classroom” (p.294). Niess’ (2005) research, likewise, recognized that in order for technology to be truly integrated
into the classroom, “teachers must also develop an overarching conception of their subject matter with respect to technology and what it means to teach with technology – a technology PCK (TPCK)” (p. 510).

These ideas were expanded upon by Mishra and Koehler (2006) and context was added, resulting in their theoretical framework, known as Technological Pedagogical Content Knowledge (TPACK). TPACK “attempts to capture some of the essential qualities of teacher knowledge required for technology integration in teaching, while addressing the complex, multifaceted, and situated nature of this knowledge” (Mishra & Koehler, 2006, p.1017). Their TPACK framework describes seven domains of teacher knowledge that include Content Knowledge (CK), Pedagogical Knowledge (PK), and Technological Knowledge (TK) (Figure 2), and domains formed by the interaction of CK, PK, and TK, and focuses more on teacher knowledge of how to effectively integrate technology rather than just on what technology to integrate:

Figure 2

*Mishra and Koehler's (2006) TPACK Framework*

In the conceptualization of this framework, Mishra and Koehler (2006) describe TPACK as “the basis of good teaching with technology” (p. 1029) which encapsulates a combined understanding of using technology in the classroom, effective teaching practices, and the intertwined relationship between these two. In defining the seven domains, Mishra and Koehler (2006) emphasize that the domains should not be considered as separate entities, rather, they should be conceptualized as existing “in a state of dynamic equilibrium” (p. 1029) so that if a change is made in one, the others must also be adjusted. For instance, if the content is changed to mathematics, teachers will have to consider specific types of mathematical technology and pedagogical strategies that are founded in mathematics.

This framework provides valuable information on the complexities of the technology knowledge that must be a part of Secondary Mathematics TEFA Literacy, as well as a method by which to present a theoretical framework for Secondary Mathematics TEFA Literacy (which will be presented in the following chapter).

Standards for educational technology can also serve as a resource for analyzing what it means for teachers to be literate in technology and, therefore, TEFA. The most predominant technology education standards are the International Society for Technology in Education (ISTE) standards that provide research-based guidance for integrating technology. The ISTE 2020 revision of the standards now includes individual guides for students, educators, education leaders, and coaches. Since this study is focused on the perspective of the preservice teachers, I will refer to the educators’ standards (see Table 1).

Table 1

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
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</thead>
</table>

\[16\]
<table>
<thead>
<tr>
<th>2.1 Learner</th>
<th>Educators continually improve their practice by learning from and with others and exploring proven and promising practices that leverage technology to improve student learning.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2 Leader</td>
<td>Educators seek out opportunities for leadership to support student empowerment and success and to improve teaching and learning.</td>
</tr>
<tr>
<td>2.3 Citizen</td>
<td>Educators inspire students to positively contribute to and responsibly participate in the digital world.</td>
</tr>
<tr>
<td>2.4 Collaborator</td>
<td>Educators dedicate time to collaborate with both colleagues and students to improve practice, discover and share resources and ideas, and solve problems.</td>
</tr>
<tr>
<td>2.5 Designer</td>
<td>Educators design authentic, learner-driven activities and environments that recognize and accommodate learner variability.</td>
</tr>
<tr>
<td>2.6 Facilitator</td>
<td>Educators facilitate learning with technology to support student achievement of the ISTE Standards for Students.</td>
</tr>
<tr>
<td>2.7 Analyst</td>
<td>Educators understand and use data to drive their instruction and support students in achieving their learning goals.</td>
</tr>
</tbody>
</table>


The phrasing of the standards outlines the actions teachers should take to improve their own understanding of integrating technology in the classroom which should translate into effective and successful technology enhanced lessons or activities. These standards can further support the framing of what it means for a secondary mathematics teacher to be literate in technology integration and, ultimately, literate in TEFA. Applying the TPACK framework and the ISTE standards, I will define *technology integration literacy* as the knowledge and skills of an educator that are necessary to successfully integrate technology into the teaching and learning process so that student learning is enhanced.

TEFA literacy does not rely solely on teachers’ knowledge and skills with technology, it also encapsulates their understanding of how to effectively use technology to enhance the formative assessment process.
Formative Assessment

Assessments, in general, are used by educators to gather and interpret data on students’ achievement and understanding of concepts (Brookhart, 1999a). Assessments are also used by educators and educational systems to survey how well students and schools meet educational standards, provide feedback to students, or serve as part of a certification or selection process (Broadfoot, 2009). Assessment is commonly categorized into summative, an end of learning evaluation of student achievement, or formative, classroom practices that elicit evidence about student achievement that is then “interpreted, and used by teachers, learners, or their peers” (Black & Wiliam, 2009, p. 9) to promote further learning. The focus of this study is on formative assessment. Many of the initial frameworks and standards for assessment, however, did not delineate between the two and, instead, focused on assessment practices in general.

The 1990 Standards for Teacher Competence in Educational Assessment of Students, presented by the American Federation of Teachers (AFT), National Council on Measurement in Education (NCME), and the National Education Association (NEA), was one of the initial steps taken by the education community to develop a resource for teachers and teacher educators to use as guidelines for effective assessment practices. The seven standards, which will be referred to as the “1990 Standards” heretofore, in this document outline the skills necessary for choosing, developing, administering, scoring, interpreting and communicating results, and making ethical decisions about assessments (AFT et al., 1990), see Table 2.

Table 2

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
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</table>

1990 Standards for Teacher Competence in Educational Assessment of Students
<table>
<thead>
<tr>
<th>Standard 1</th>
<th>Teachers should be skilled in <em>choosing</em> assessment methods appropriate for instructional decisions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard 2</td>
<td>Teachers should be skilled in <em>developing</em> methods appropriate for instructional decisions.</td>
</tr>
<tr>
<td>Standard 3</td>
<td>The teacher should be skilled in administering, scoring and interpreting the results of both external-produced and teacher-produced assessment methods.</td>
</tr>
<tr>
<td>Standard 4</td>
<td>Teachers should be skilled in using assessment results when making decisions about individual students, planning teaching, developing curriculum, and school improvement.</td>
</tr>
<tr>
<td>Standard 5</td>
<td>Teachers should be skilled in developing valid pupil grading procedures which use pupil assessments.</td>
</tr>
<tr>
<td>Standard 6</td>
<td>Teachers should be skilled in communicating assessment results to students, parents, other lay audiences, and other educators.</td>
</tr>
<tr>
<td>Standard 7</td>
<td>Teachers should be skilled in recognizing unethical, illegal, and otherwise inappropriate assessment methods and uses of assessment information.</td>
</tr>
</tbody>
</table>

*Note.* Adapted from “Standards for Teacher competence in Educational Assessment of Students”, by AFT, NCME, and NEA, 1990.

Around the same time as the 1990 Standards were published, Richard Stiggins began to write extensively on what he called *assessment literacy*. Stiggins (1991) defines assessment literacy as a multidimensional concept that constitutes educators’ ability to identify and design goal oriented assessments that will produce valid and unbiased data on student achievement. Stiggins (1995) later developed five standards of assessment literacy:

1. Starting with clear purposes.
2. Focusing on achievement targets.
3. Selecting proper achievement methods.
4. Sampling student achievement.
5. Avoiding bias and distortion. (p. 240-242)
Both the 1990 Standards and those presented by Stiggins (1995) are reflective of the basic skills and knowledge that teachers must have as they engage in assessment practices. In this capacity, they can serve as the groundwork for what it means for a secondary mathematics teacher to be literate in formative assessment. Unfortunately, they do not attend to the more specialized knowledge and skills associated with many of the concepts within formative assessment. For this, we must use a different framework and a revised set of standards.

One of the first uses of the terms “formative” and “summative” was in Michael Scriven’s 1967 essay, which delineated the differences between these forms of assessment. Since then, a number of researchers have investigated the application and effectiveness of formative assessment in the classroom. More recently, educational researchers have been looking at ways to clearly define and conceptualize formative assessment into a framework. Educational researcher Valeria Otero (2006), for example, related Vygotsky’s theory of concept formation to build a model for formative assessment. In her conceptualization, Otero (2006) describes a “formative assessment cycle” which starts with eliciting students’ knowledge and, then, using this “prior knowledge to make instructional decisions that lead to the development of intermediate objectives, feedback, and relevant instruction” (p. 252), ultimately, returning to eliciting students’ knowledge. Otero’s (2006) cycle strengthens the fact that formative assessment can and should be used almost continuously during the learning process, relying on students’ understanding and prior knowledge to serve as a guide for instructional decisions.

Popham (2008) synthesized the work of the 2006 Formative Assessment for Students and Teachers – State Collaborative on Assessment and Student Standards (FAST-SCASS) to define formative assessment as a process “in which teachers or students use assessment-based evidence to adjust what they’re currently doing” (p. 6). This definition integrates Otero’s (2006)
concept of formative assessment as a cycle, while also emphasizing that students should be active participants who are encouraged to utilize formative assessment to evaluate their own learning (Popham, 2008).

Otero’s (2006) cycle also has many similarities to one of the most well-known formative assessment frameworks published by researchers Paul Black and Dylan Wiliam in 2009. Synthesizing their and their colleagues’ extensive work on formative assessment, Black and Wiliam (2009) defined five key strategies to conceptualize what formative assessment should look like:

1. Clarifying and sharing learning intentions and success criteria;
2. Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
3. Providing feedback that moves learners forward;
4. Activating students as instructional resources for one another; and
5. Activating students as the owners of their own learning. (p. 8)

In Otero’s (2006) cycle, Popham’s (2008) process, and Black and Wiliam’s (2009) framework there is an emphasis on eliciting students' knowledge and providing feedback. Black and Wiliam (2009), however, also incorporate the concept of clarifying success criteria and ensuring that students are active participants in the process, both of which have been shown to increase student motivation and interest. While Popham (2008) notes that a key, but understated, element of formative assessment is that of well-designed tests that generate the desired evidence of data. This relates back to the 1990 Standards and Stiggins (1991,1995) conceptualization of assessment literacy, in which teachers must have a knowledge of what constitutes a high-quality assessment that will elicit valid and reliable data.
Analyzing the work of these researchers, formative assessment can be considered as a broad set of classroom activities, ranging from quizzes and tests to classroom discussions. Well-designed formative assessments are ones that ensure students’ thinking is made visible by eliciting responses that focus on explanation and justification of ideas (Collins, 2011). Formative assessments that help to do this include open ended questions, activities such as peer- and self-assessment, or comment-only marking, in which the teacher provides written feedback to students without a formal letter or numerical grade (Black & Wiliam, 2009).

By narrowing the scope and refining the concept of assessment literacy, each of these researchers present key ideas and details about what it means for a teacher to be literate in formative assessment. However, there is still more to consider. Given these theories and the numerous, ongoing reforms of educational practices, a number of additional assessment standards, competencies lists, and principles have been released since the 1990 Standards (McMillan, 2000; National Board for Professional Teaching Standards [NBPTS], 2001; Stiggins, 2009; Stiggins & Duke, 2008). For example, Brookhart’s (2011) *Educational Assessment Knowledge and Skills for Teachers* presents a research-based list of eleven competencies “of knowledge and skills teachers need to perform the assessment-related aspects of their work in a competent and professional manner”:

I. Teachers should understand learning in the content area they teach.

II. Teachers should be able to articulate clear learning intentions that are congruent with both the content and depth of thinking implied by standards and curriculum goals, in such a way that they are attainable and assessable.

III. Teachers should have a repertoire of strategies for communicating to students what achievement of a learning intention looks like.
IV. Teachers should understand the purposes and uses of the range of available assessment options and be skilled in using them.

V. Teachers should have the skills to analyze classroom questions, test items and performance assessment tasks to ascertain the specific knowledge and thinking skills required for students to do them.

VI. Teachers should have the skills to provide effective, useful feedback on student work.

VII. Teachers should be able to construct scoring schemes that quantify student performance on classroom assessments into useful information for decisions about students, classrooms, schools, and districts. These decisions should lead to improved student learning, growth, or development.

VIII. Teachers should be able to administer external assessment and interpret their results for decisions about students, classrooms, schools, and districts.

IX. Teachers should be able to articulate their interpretations of assessment results and their reasoning about the educational decisions based on assessment results to the educational populations they serve (student and his/her family, class, school, community).

X. Teachers should be able to help students use assessment information to make sound educational decisions.

XI. Teachers should understand and carry out their legal and ethical responsibilities in assessment as they conduct their work (p. 7).

Brookhart (2011) contends that these eleven competencies not only capture the depth and detail of the 1990 Standards, but also reflect modern reform concepts of formative assessment and standardized assessments. Specifically, competencies II and III attend to “Clarifying and sharing
learning intentions and success criteria” (Black & Wiliam, 2009, p.8); competencies VI and VII attend to “Providing feedback that moves learners forward” (Black & Wiliam, 2009, p.8); and competency X attends to “Activating students as the owners of their own learning” (Black & Wiliam, 2009, p.8).

Most recently, the Joint Committee on Standards for Education Evaluation (JCSEE) (2015) published *The Classroom Assessment Standards for PreK-12 Teachers*, which align closely to the 1990 Standards, but also incorporate important concepts of formative assessment (Table 3).

**Table 3**

*JCSEE (2015) Classroom Assessment Standards for PreK-12 Teachers*

<table>
<thead>
<tr>
<th>FOUNDATIONS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F.1 Assessment Purpose</td>
<td>Classroom assessment practices should have a clear purpose that supports teaching and learning.</td>
</tr>
<tr>
<td>F.2 Learning Expectation</td>
<td>Learning expectations should form the foundation for aligning classroom assessment practices with appropriate instruction and learning opportunities for each student.</td>
</tr>
<tr>
<td>F.3 Assessment Design</td>
<td>The types and methods of classroom assessment used should clearly allow students to demonstrate their learning.</td>
</tr>
<tr>
<td>F.4 Student Engagement in Assessment</td>
<td>Students should be meaningfully engaged in the assessment process and use of the assessment evidence to enhance their learning.</td>
</tr>
<tr>
<td>F.5 Assessment Preparation</td>
<td>Adequate teacher and student preparation in terms of resources, time, and learning opportunities should be part of classroom assessment practices.</td>
</tr>
<tr>
<td>F.6 Informed Students and Parents/Guardians</td>
<td>The purposes and uses of classroom assessment should be communicated to students and, when appropriate, parents/guardians.</td>
</tr>
</tbody>
</table>
U.1 Analysis of Student Performance  | The methods for analyzing evidence of student learning should be appropriate for the assessment purpose and practice.
---|---
U.2 Effective Feedback  | Classroom assessment practices should provide timely and useful feedback to improve student learning.
U.3 Instructional Follow-Up  | Analysis of student performance should inform instructional planning and next steps to support ongoing student learning.
U.4 Grades and Summary Comments  | Summative grades and comments should reflect student achievement of the learning expectations.
U.5 Reporting  | Assessment reports should be based on a sufficient body of evidence and provide a summary of a student’s learning in a clear, timely, accurate, and useful manner.

| **QUALITY** |
|---|---|
Q.1 Cultural and Linguistic Diversity  | Classroom assessment practices should be responsive to and respectful of the cultural and linguistic diversity of students and their communities.
Q.2 Exceptionality and Special Education  | Classroom assessment practices should be appropriately differentiated to meet the specific educational needs of all students.
Q.3 Unbiased and Fair Assessment  | Classroom assessment practices and subsequent decisions should be free from all factors unrelated to the intended purposes of the assessment.
Q.4 Reliability and Validity  | Classroom assessment practices should provide consistent, dependable, and appropriate information that supports sound interpretations and decisions about each student’s knowledge and skills.
Q.5 Reflection  | Classroom assessment practices should be monitored and revised to improve their overall quality.

*Note.* Adapted from *Classroom Assessment Standards for PreK-12 Teachers*, by D. Klinger, P. McDivitt, B. Howard, T. Rogers, M. Munoz, & C. Wylie, 2015, Kindle E Book. Copyright 2015 by the Joint Committee on Standards for Educational Evaluation.

As with Brookhart’s (2011) competencies, aspects of formative assessment are evident in a number of the JSCEE (2015) standards. In particular, there is direct reference to engaging
students as active participants in F.4, a connection to the feedback process in U.2 and U.4, and the importance of using assessments to move learning forward in U.3.

Combining the groundwork of the 1990 Standards and Stiggins conceptualization of assessment literacy, the theories for formative assessment (Black & Wiliam, 2009; Otero, 2006; Popham, 2008), and these revised assessment standards (Brookhart, 2011; JCSEE, 2015), we have established a basis for what it means for a teacher to be literate in formative assessment. I, thus, define *formative assessment literacy* as the knowledge and skills of an educator to effectively engage students in a formative assessment process with strategically planned assessments (formal, or informal, peer, and self) that elicit students’ understanding and the exchange of feedback to help the teacher and student to regulate learning and move it forward.

The combination of the previous two concepts, along with their theories and standards, give rise to the concept of TEFA literacy. Given that TEFA Literacy is a particularly specialized form of pedagogy that relies on both technology integration literacy and formative assessment literacy, the current research into a theory for defining TEFA Literacy is limited and, to my knowledge, there are currently no specific set of standards or practices. There are, however, definitions and theories as to what TEFA Literacy could or should look like in the classroom; In the next section I will discuss some of the existing definitions and theories.

*Technology Enhanced Formative Assessment*

The use of educational technology in assessment, both formative and summative, has helped to streamline everything from the development of test items (Striewe, 2019) and automated feedback (van der Stappen & Baartman, 2019), to data collection, tracking, and analysis (Stacey & William, 2013). Special attention has been given to how new educational technological tools can enhance the formative assessment process with “frequent feedback,
creating immersive learning environments that highlight problem-solving processes and make student thinking visible, and by providing opportunities for independent and collaborative learning” (Looney, 2010, p. 2-3). Regardless of the capabilities of the technological tools, implementing technology enhanced assessments in the classroom requires knowledge and skills that teachers must be trained in (Mishra & Koehler, 2006; Pachler et al., 2010). In the following section, I will review some of the literature that focused on defining TEFA Literacy.

In 2008, Beatty and colleagues designed a “pedagogy for teaching with classroom response technology” (2008, p. 1), which they called Technology-Enhanced Formative Assessment, or TEFA. Beatty and Gerace (2009) further refined this pedagogy into four principles to guide teachers’ application of TEFA in the classroom:

1. The teacher should motivate and focus student learning with question-driven instruction.
2. The teacher should work to develop students’ understanding and scientific fluency with dialogical discourse.
3. The teacher should inform and adjust their teaching and learning decisions using the feedback from formative assessments.
4. The teacher should strive to help students develop metacognitive skills and cooperate in the learning process with meta-level communication (p. 146-162).

Beatty and Gerace’s (2009) four principles reinforce many of the theories for formative assessment. For example, the third principle aligns with the recommendations of Black and Wiliam (2009), Popham (2008, 2009), and Otero (2006), to use feedback to guide instruction. Unfortunately, upon further analysis of the principles, Beatty and Gerace (2009) determined that the principles did not truly require the use of technology to be fulfilled and could be applied to
any type of formative assessment. Thus, while the principles may not provide a specific
definition of TEFA Literacy, they can still support the notion of what it means for teachers to
have formative assessment literacy.

In addition to the four principles, Beatty and Gerace (2009) also provided guidelines for
how to engage students in a TEFA “question cycle” with a classroom response system (CRS):

1. Pose the question or problem.
2. Allow students time to think individually or in groups.
3. Use the CRS to collect responses and display results.
4. Ask students to explain or justify their responses.
5. Engage students in a discussion.
6. Provide a summary or transition to a new question.

The question cycle presents a specific guide on how and when teachers can integrate CRSs into
TEFAs. Although the question cycle does provide more of an understanding of the process and
skills teachers should apply when implementing TEFAs, there are some major limitations.
Specifically, that the technology must be a CRS and the formative assessment must be completed
within a question cycle that, presumably, happens in the classroom. Considering there are a
number of educational technological tools that can engage students in formative assessment
(Looney, 2010), it would seem counterintuitive to limit TEFAs to just CRSs. Moreover, the
formative assessment process does not always have to be in the classroom, nor as part of a
question cycle. Teachers can utilize homework or quizzes as a form of formative assessment
(Popham, 2008) and students can engage in peer- and self-assessment (Black & Wiliam, 2009),
neither of which require the use of a question cycle.
Consequently, the pedagogy presented by Beatty and Gerace (2009) only provides some of the basis for defining TEFA Literacy. Therefore, I will expand upon these ideas, as well as those discussed in the previous sections on technology integration literacy and formative assessment literacy, to broadly define TEFA Literacy as the knowledge and skills necessary to effectively use technological tools to engage students in the formative assessment process.

The goal of this study, however, is to clearly define Secondary Mathematics TEFA Literacy. And, just as with the TPACK framework, when a specific content matter is introduced, the particulars of the strategies and processes will need to be adapted for content specific pedagogical practices. Thus, in order to get a clearer picture of what it means for secondary mathematics teachers to have TEFA Literacy, I consider research-based strategies for teaching and learning mathematics in the next section.

Mathematics Pedagogy

The teaching and learning of mathematics have been somewhat controversial, with a multitude of viewpoints on what constitutes “good” teaching, and a number of reform efforts to try to improve students’ achievement in mathematics. For the purpose of this research, I reflect on two commonly used and well-researched perspectives of effective mathematics pedagogy. First, the work of the National Research Council and the Mathematics Learning Study Committee: Adding It Up: Helping Children Learn Mathematics (Kilpatrick et al., 2001). And, second, the numerous standards and positions published by the National Council of Teachers of Mathematics (NCTM).

Kilpatrick and colleagues’ (2001) report sought to help inform those within the mathematics education community “of the mathematics children need to learn, how they learn it, and how it might be taught to them effectively” (p. XIV). By analyzing and synthesizing a
A plethora of research Kilpatrick et al. (2001) conceptualized the term *mathematical proficiency* to describe the complex competencies, knowledge, and facilities that lead to successful learning of mathematics. Mathematical proficiency is composed of five intertwined strands (see Table 4), that teachers must help students to develop.

**Table 4**

*Kilpatrick et al.’s (2001) Five Strands of Mathematical Proficiency*

<table>
<thead>
<tr>
<th>Strand</th>
<th>Description</th>
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<tbody>
<tr>
<td>Conceptual Understanding</td>
<td>An integrated and functional grasp of mathematical ideas.</td>
</tr>
<tr>
<td>Procedural Fluency</td>
<td>Knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.</td>
</tr>
<tr>
<td>Strategic Competence</td>
<td>The ability to formulate mathematical problems, represent them, and solve them.</td>
</tr>
<tr>
<td>Adaptive Reasoning</td>
<td>The capacity to think logically about the relationships among concepts and situations.</td>
</tr>
<tr>
<td>Proactive Dispositions</td>
<td>The tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics payoff, and to see oneself as an effective learner and doer of mathematics.</td>
</tr>
</tbody>
</table>


The main goal is for mathematics teachers to provide students with the opportunities to develop each of these strands and, ultimately, mathematical proficiency. This can be done with intentionally planned lessons and activities that help students to make connections between mathematical concepts, ask them to reason and problem solve, or prompt them to make conjectures and estimate results (Kilpatrick et al., 2001).
The concept of mathematical proficiency and how to help students develop it, can be used to identify the elements of technological tools used for TEFAs that make it effective for a mathematics classroom. For example, a TEFA that only asks multiple choice questions may provide limited or no opportunities for students to reason, problem solve, or make conjectures. While a TEFA that allows students to manipulate representations or asks open-ended questions may provide more of these opportunities. Many of these same ideas, providing students opportunities to learn with understanding, are core to the National Council of Teachers of Mathematics’ (NCTM’) standards and positions.

The NCTM was founded in 1920 and is recognized as the largest and one of the most prominent resources for mathematics teachers. In addition to their training, advocacy, journal publications, and more, the NCTM is known for its well-researched and commonly referenced Principles and Standards for School Mathematics (2000). With the vision to transform mathematics classroom to spaces “where all students have access to high-quality, engaging mathematics instruction” (NCTM, 2000, p.3), the NCTM presented six principles for school mathematics (Table 5):

Table 5

<table>
<thead>
<tr>
<th>Principle</th>
<th>Description</th>
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<tbody>
<tr>
<td>Equity</td>
<td>Excellence in mathematics education requires equity, high expectations and strong support for all students.</td>
</tr>
<tr>
<td>Curriculum</td>
<td>A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades.</td>
</tr>
<tr>
<td>Teaching</td>
<td>Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.</td>
</tr>
<tr>
<td>Learning</td>
<td>Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.</td>
</tr>
</tbody>
</table>
Assessment | Assessment should support the learning from experience and prior knowledge.
---|---
Technology | Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.


It is clear that there are many similarities between the concept of mathematical proficiency (Kilpatrick et al., 2001) and the NCTM’s (2000) principles. In particular, the Learning principle highlights that “procedural fluency and conceptual understanding can be developed through problem solving, reasoning, and argumentation” (NCTM, 2000, p. 21). In alignment with many of the ideas expressed by Kilpatrick et al., (2001), the NCTM (2000) stresses that intentionally planned lessons and activities should be focused on meaningful mathematical tasks, encourage discourse, appropriately use tools and resources (e.g., technology) to enhance learning, and provide opportunities for reflection in the search for improvement.

When these principles are analyzed alongside the theories and standards for technology integration and formative assessment, it begins to become clearer what it might mean for a mathematics teacher to have TEFA Literacy. Importantly, it becomes evident that teachers must be selecting and analyzing technologies for formative assessments that will enhance students’ ability to develop mathematical proficiency, as well as engage students in meaningful mathematics, discourse, and reflection. To explore more into how mathematics teachers can successfully select or analyze TEFAs, in the following section I outline the theories and standards for teaching mathematics with technology, as well as how they have evolved.

*Teaching Mathematics with Technology*
Given that the types and use of educational technology varies greatly among content areas (Niess, 2015; Zelkowski et al., 2013), the pedagogical judgment that goes into deciding what technological tool to use will rely greatly on teachers’ understanding of the content. In mathematics, educational organizations like the NCTM, and the Common Core State Standards Initiative (CCSSI) acknowledge the importance of educational technology integration in mathematics. As noted previously, the NCTM’ (2000) *Principles and Standards* marks technology as the sixth principle, contending that technology serves as an “essential” resource for teaching and learning mathematics, while acknowledging that its effectiveness depends greatly on how well a teacher can integrate technology into learning. The Common Core State Standards for Mathematics (CCSSM) (2010) also emphasize the use of technology in helping students visualize, create, and analyze different mathematical concepts. However, some of the most extensive work on technology integration in mathematics classrooms is that of Margaret Niess. In 2005, Niess conducted a study in which she amended the four essential components of Schulman’s (1986) concept of pedagogical content knowledge (PCK) to describe what preservice teachers must develop in order to effectively integrate technology into mathematics teaching and learning:

1. An overarching conception about the purposes for incorporating technology in teaching mathematics;
2. Knowledge of students’ understandings, thinking, and learning of mathematics with technology;
3. Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics;

Based on Niess’ (2005) work in technology pedagogical content knowledge (TPCK), the Association of Mathematics Teacher Educators (AMTE) formalized these concepts into a set of standards for teaching mathematics with technology known as the Mathematics TPACK Framework (2009) (Table 6).

**Table 6**

**AMTE’ (2009) Mathematics TPACK Framework**

<table>
<thead>
<tr>
<th>Guideline</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>Design and develop technology-enhanced mathematics learning environment and experience. Educators use their knowledge of technology, pedagogy, and content to design and develop learning environments and experiences to maximize learning.</td>
</tr>
<tr>
<td>II.</td>
<td>Facilitate mathematics instruction with technology as an integrated tool. Educators implement curricular plans that integrate appropriate technology to maximize mathematical learning and creativity.</td>
</tr>
<tr>
<td>IV.</td>
<td>Engage in ongoing professional development to enhance technological pedagogical content knowledge. Educators seek, identify, and use technology to enhance their knowledge, productivity, and professional practice.</td>
</tr>
</tbody>
</table>

Note. Adapted from “Mathematics TPACK (Technological Pedagogical Content Knowledge) Framework” by the Association of Mathematics Teacher Educators, 2009. Copyright 2009 by the Association of Mathematics Teacher Educators.

The Mathematics TPACK Framework (AMTE, 2009) provides more detail on how mathematics educators can effectively integrate technology in their classrooms. Thus, it is unsurprising that it contains many of the same concepts as the ISTE (2002, 2020) standards.
Nonetheless, it does help to narrow the scope of these resources to focus on some of the specialized practices and knowledge that mathematics teachers need to have technology integration literacy. The AMTE (2009) particularly notes that the technology must be evaluated for its ability to enrich and enhance mathematics. Understanding how certain technological tools can or cannot do this has been extensively researched.

Building on research of mathematics teaching and learning with technology, Hollebrands (2017) introduced the “didactic tetrahedron” (see Figure 3), to help explain the relationships between technology, teachers, students, and mathematics.

**Figure 3**

Hollebrands’ (2017) Didactic Tetrahedron for Teaching Mathematics With Technology
Hollebrands (2017) contends that when mathematics teachers are appraising different technological tools, they must consider three factors: (1) mathematical fidelity, (2) amplification or reorganization, and (3) representation. The mathematical fidelity of a technology refers to the accurate representation of mathematical concepts, such as the correct depiction of the slopes of positive and negative lines (Hollebrands, 2017). Technologies can be considered amplifiers, in that they allow students and teachers to complete tasks “more precisely, quickly, and efficiently” (Pea 1985, 1987, as cited in Hollebrands, 2017, p. 83) than they would if they were doing so by hand. A reorganizer technology takes “advantage of the representation and actions afforded by the tools” (Pea 1985, 1987, as cited in Hollebrands, 2017, p. 83). For example, a dynamic geometry technological tool like Geometry Expressions, that allows students to manipulate digital images of geometric solids, could be used as an amplifier for its ability to precisely and quickly draw the geometric solids. Geometry Expressions could also be considered a reorganizer, as it can instantaneously recalculate measurements (e.g., lengths, surface area, and volume) as students manipulate the geometric solids. Lastly, representation refers to technology’s ability to produce multiple representations of mathematics, such as the algebraic and graphic representation of functions (Hollebrands, 2017). Based on these three important factors, Hollebrands’ (2017) framework for teaching mathematics with technology provides lists of questions teachers can ask themselves as they plan for and implement technology enhanced
activities. In particular, she prompts teachers to consider the limitations and extensions that technological tools offer to enhance student learning, productive mathematical discourse in the classroom, questioning techniques, or assessment (Hollebrands, 2017).

With the work of Niess, the AMTE, and Hollebrands, the scope of TPACK has been further narrowed to focus specifically on mathematics content and provide yet another resource to help define what it means for mathematics teachers to have technology integration literacy. Yet, the element of how this relates to the instructional goal of formative assessment still remains to be addressed. The NCTM, once again, serves as a useful resource for helping to describe this aspect of Secondary Mathematics TEFA Literacy.

**Formative Assessment in Mathematics**

Though there are no standards specifically for formative assessment in mathematics, the NCTM has provided guidance on general assessment and evaluation methods in mathematics. The NCTM’s (1995) Assessment Standards highlight the importance of assessments to elicit and evaluate the “mathematics that students need to know and are able to do” (p. 12) with six standards: (1) *The Mathematics Standard*, (2) *The Learning Standard*, (3) *The Equity Standard*, (4) *The Openness Standard*, (5) *The Inferences Standard*, and (6) *The Coherence Standard*. Unlike other assessment standards, the NCTM (1995) chose to use guiding questions to help teachers reflect on their assessment practices. Within *The Mathematics Standard*, for example, the NCTM asks teachers to think about:

1. What mathematics is reflected in the assessment?
2. What efforts are made to ensure that the mathematics is significant and correct?
3. How does the assessment engage students in realistic and worthwhile mathematical activities?
4. How does the assessment elicit the use of mathematics that is important to know and be able to do?

5. How does the assessment fit within the framework of mathematics to be assessed?

6. What inferences about students’ mathematical knowledge, understanding, thinking processes, and dispositions can be made from the assessment? (p.12)

The NCTM’ (1995) mathematics assessment standards, like other assessment standards, help to formulate the groundwork of what it is for a mathematics teacher to have assessment literacy. Some of those aspects of effective formative assessment practices are evident within The Learning Standard, which encourages teachers to “help students to become independent self-assessors … [and] … active participants in assessment” (NCTM, 1995, p. 14). Yet, to help define and clarify what it means for a mathematics teacher to have TEFA Literacy, the scope must be narrowed to that of just formative assessment.

To further examine formative assessment specifically for the mathematics classroom, the NCTM recently published a guide: Using Classroom Assessment to Improve Student Learning (Collins, 2011). Within this guide, Collins (2011) outlines key aspects of the roles that mathematics teachers and students play in formative assessment (see Table 7), and provides a toolkit of strategies to “gather evidence about your students’ understanding, misconceptions, and challenges” (Collins, 2011, p. 16-17).

Table 7

Collins’ (2007) Roles in Formative Assessment

<table>
<thead>
<tr>
<th>Teachers’ Roles</th>
<th>Students’ Roles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers must be proficient in and knowledgeable of…</td>
<td>Students must …</td>
</tr>
</tbody>
</table>
• The mathematical underpinning of formulas and procedures that explain why they work.
• The future mathematics for which current mathematics lays the foundation.
• Using many different representations of concepts and procedures.
• Reorganizing and encouraging deep thinking in classroom discussions (p.11).

• Be actively engaged in the process of criteria and goal setting.
• Maintain a record of their performance through self-reflection and self-evaluation.
• Be encouraged to describe strategies and explain reasoning, evaluate feedback, and collaborate with peers (p. 13).

In addition to the NCTM’s standards and guides, there is at least one theory for formative assessment specifically for the mathematics classroom. Ginsberg’s (2009) *Conceptual Framework for Formative Assessment* contends that “formative assessment is built on a foundation of mathematical knowledge and understanding of three kinds of trajectories” (p. 115). These include: (1) the *normative information* teachers collect about students’ mathematical performance and how this information compares to typical student performance; (2) the *cognitive trajectory* of students’ ability to progress from memorization of facts, to detection of patterns, to generation of ideas, to, finally, conceptual understanding; and (3) the *trajectory of mathematical ideas*, or the logical development of mathematical concepts that build upon one another, and the connections between them (Ginsberg, 2009). Ginsberg’s (2009) framework for formative assessment highlights key ideas of how mathematics teachers should be thinking about the relationship between students’ prior knowledge, what they are currently learning, and what they still need to learn. Ginsberg (2009) also discusses some common obstacles that may prevent students from achieving learning objectives. These include access to and appropriate use of
teaching tools and materials, such as technology; the teacher’s pedagogical skills and knowledge of effective teaching strategies; and the complexity of mathematics itself (Ginsberg, 2009).

Speaking directly to mathematics educators, Collins’ (2011) guide and Ginsberg’s (2009) framework for formative assessment attend to aspects that the previously mentioned theories of formative assessment did not. In particular, they bring up two fundamental facts that support effective implementation of formative assessment in mathematics: (1) teachers must have a deep understanding of mathematics, how it can be learnt and how it can be taught; and (2) the teaching and learning of mathematics must be done in a logical and coherent manner so that students can apply and make connections to prior knowledge as they are introduced to new topics.

The NCTM’s assessment standards and formative assessment guide, along with Ginsberg’s (2009) framework, provide a narrowed focus of formative assessment literacy for the mathematics teacher. Thus, I now have the essential building blocks to clarify and define Secondary Mathematics TEFA Literacy.

**Secondary Mathematics TEFA Literacy.** As stated at the beginning of this chapter, there is no clearly defined framework or theory to describe Secondary Mathematics TEFA Literacy. However, by analyzing and synthesizing the three main concepts of: (1) Technology, (2) Formative Assessment, and (3) Mathematics Pedagogy, one could create this framework. Accomplishing this task will be the focus of Chapter 4. Additionally, in an effort to further validate the contents of the framework, in Chapter 4 I will design an instrument by which to evaluate Secondary Mathematics TEFA Literacy. Thus, in the next section I will review and analyze how other educational researchers have evaluated literacy in the past.
Evaluating Literacy

The goal of teacher education programs is to prepare PSTs for the classroom. In addition to methodology courses and field work (e.g., student teaching), PSTs knowledge and skill can be increased through effective assessment methods (Khal et al., 2013). In particular, performance-based assessments have been lauded to have more efficacy and provide PSTs with a better method by which to demonstrate their understanding and skill (Darling-Hammond, 2010).

Educational researchers and teacher educators have devised a wide variety of methods by which to evaluate pre- and in-service teachers’ technology integration literacy, formative assessment literacy, and mathematics pedagogy, not all of which employ performance-based measures. In the following section, I review the literature associated with the development and use of different instruments that have been used to measure, evaluate, or assess literacy. Further, I analyze the content of these instruments for their ability to be used in a performance-based assessment of Secondary Mathematics TEFA Literacy.

Evaluating Literacy in TPACK

Following the development of the TPACK framework (Mishra & Koehler, 2006) over 1000 publications have analyzed the role of educational technology in a plethora of settings and content areas (Harris et al., 2018). In their review of 66 TPACK studies that employed the use of an instrument by which to evaluate pre- and in-service teachers’ TPACK competencies, Koehler et al. (2012) identified five types of measures: self-reported measures, performance assessments, open-ended questionnaires, interviews, and observations.

Schmidt and colleagues (2009) conducted a study “to develop and validate an instrument to measure preservice teachers’ self-assessment of their Technological Pedagogical Content Knowledge (TPACK) and related knowledge domains” (p. 123). The survey consisted of 75
Likert scale items prompting 124 participating PSTs to self-report their TPACK knowledge. Results showed varying degrees of correlation between the different subscales of TPACK, and, more importantly, that the instrument could provide valuable data that could “examine and support preservice teachers’ development of TPACK” (Schmidt et al., 2009, p. 137). On the other hand, the instrument was designed specifically as a self-assessment and does not ask PSTs to demonstrate their knowledge and skills through performance-based measures. Moreover, various studies attempting to verify the construct validity of the survey produced mixed results (Shinas et al., 2013), leaving its reported result questionable. Thus, while the types of questions posed in this survey may help with phrasing and ideas about what to ask on a test for Secondary Mathematics TEFA Literacy, it is not the best model for an instrument that is intended to evaluate PSTs’ knowledge and skills.

Researchers Angeli and Valanides (2009) TPACK study, on the other hand, focused on performance-based tasks. Elementary PSTs enrolled in an Instructional Technology course were prompted to create two information and communication technologies (ICT) enhanced learning activities (one early on in the course and one near the end). Based on Angeli and Valanides’ (2009) framework for ICT-TPCK knowledge, the ICT-enhanced learning activities were analyzed using five criteria:

1. Identification of suitable topics to be taught with technology.
2. Identification of appropriate representation to transform content.
3. Identification of teaching strategies difficult to be implemented by traditional means.
4. Selection of appropriate tools and appropriate pedagogical uses of their affordances.
5. Identification of appropriate integration strategies (p. 154-168).
Angeli and Valanides (2009) were able to use this framework to find a statistically significant difference in PSTs’ learning activities from early on in the course to the one designed later in the course. Following a similar model, using performance-based tasks evaluated quantitatively, teacher educators could track PSTs growth in Secondary Mathematics TEFA Literacy. Nonetheless, the way in which Angeli and Valanides (2009) analyzed PSTs data may prove to be challenging for teacher educators without a deep understanding of how to differentiate between ICT-TPCK performance levels. Thus, something like an analytic rubric with detailed, qualitative descriptors may prove more transferable and useful to teacher educators.

Based on existing TPACK theories (e.g., Niess, 2005; Mishra & Koehler, 2006) Harris et al. (2010) designed and validated an analytic rubric to assess technology integration “along with the ‘fit’ of selected content, teaching strategies, and technologies” (Harris et al., 2010, p. 324) in the PSTs’ written lesson plans. The process Harris et al. (2010) applied to the development of their rubric could serve as a guide for the development of a rubric to assess PSTs’ secondary mathematics TEFA literacy. Moreover, the organizational structure, and the terminology within the qualitative descriptors could serve as a guide for the contents of a TEFA literacy rubric.

Around the same time, Lyublinskaya and Tournaki (2012) developed the TPACK Levels Rubric to “assess in-service teachers’ levels of TPACK teaching algebra with TI-Nspire technology” (p. 302). By operationalizing Niess’ (2005) qualitative descriptors of the TPACK developmental levels within the four components of TPACK, Lyublinskaya and Tournaki (2012) introduced the first quantitative instrument for external assessment of teacher’s progressive development of TPACK. Lyublinskaya and Tournaki (2012) were able to use their rubric with a variety of data sources, including lesson plans and classroom observations. Recently,
Lyublinskaya and Kaplon-Schilis (2022) further revised and validated the rubric on a set of 175 lesson plans and videos of mathematics lessons taught by elementary school teachers (Figure 4).

**Figure 4**

*Example of one Component on Lyublinskaya and Kaplon-Schilis' (2022) TPACK Rubric*

<table>
<thead>
<tr>
<th>TPACK Levels Rubric for Instructional Technology Applications</th>
<th>TPACK Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPACK Components</td>
<td>Recognizing (1)</td>
</tr>
<tr>
<td>An overarching conception about the purposes for incorporating technology in teaching subject matter topics.</td>
<td>Instructional technology is used for motivation, rather than actual subject matter development. All learning of new ideas presented by the teacher must be without technology. Technology-based activities do not include inquiry tasks. Technology procedures concentrate on drills and practice only.</td>
</tr>
</tbody>
</table>


This work demonstrates how to operationalize the qualitative descriptors of a rubric for a specific content, while remaining versatile enough to be applicable to various data sources and technology tools. Such a methodology could be similarly applied to the development of a rubric for Secondary Mathematics TEFA Literacy.

Other replicable and adaptable methods to measure and evaluate TPACK include open-ended questionnaires, like the one designed by So and Kim (2009). Their questionnaire is intended to “identify participants’ understandings, misconceptions, and difficulties” in TPACK (p. 107) by asking the participating preservice teachers to analyze technology integrated tasks.
And, although they did not employ the use of a standardized tool like a rubric to evaluate responses, the researchers were able to identify themes within responses that helped to discern the PSTs’ strengths and weaknesses.

Lachner and colleagues (2021) also created open-ended questionnaires to “measure pre-service teachers’ subject-specific knowledge about technology integration (TPACK)” (p. 9). Their test items included subject-specific vignettes about an encounter or problem that a teacher might have in the classroom followed by questions that asked for the PSTs to decide “whether and how educational technology could help to solve the … problem and to justify their answer” (Lachner et al., 2021, p.9). Additionally, the team created a scoring rubric with qualitative descriptors used to evaluate the PSTs’ responses for pedagogy, technology use, and justifications. The combination of an open-ended questionnaire and a standardized rubric may serve as not only a more efficient method by which to measure or evaluate TPACK, but also one that can be replicated in teacher education courses.

Analyzing a number of methods by which researchers have measured, evaluated, and assessed both pre- and in-service teachers’ TPACK reveals a number of different methods that may be modified or applied to the evaluation of Secondary Mathematics TEFA Literacy. In particular, it appears that performance-based assessments provide rich data, while qualitative rubrics serve as efficient means to assess TPACK. A method by which to measure, evaluate, or assess Secondary Mathematics TEFA Literacy, however, must incorporate more than just TPACK. Thus, in the next section I review and analyze a variety of methods researchers have used to evaluate assessment literacy.
Evaluating Literacy in Formative Assessment

Some of the initial instruments designed to measure assessment literacy did not focus specifically on formative assessment, but the design and contents may provide insight for developing an instrument to evaluate TEFA Literacy. One of the first instruments that measured assessment literacy was Plake and colleagues’ (1992) *Teacher Assessment Literacy Questionnaire* (TALQ), a 35 multiple choice questionnaire sent out as part of a large scale quantitative survey to measure teachers’ competence in the 1990 Standards. The questionnaire was designed so that each of the seven competencies from the 1990 Standards is addressed by five questions. Questions ranged from assessing understanding of assessment related vocabulary, as in Example 1 (see Table 8), to situational questions that challenge participants to think about appropriate assessment strategies, as in Example 2 (see Table 8).

Table 8

*Examples of Questions from Plake et al.'s (1992) TALQ*

<table>
<thead>
<tr>
<th>Example 1</th>
<th>When scores from a standardized test are said to be reliable, what does it imply?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Student scores from the test can be used for a large number of educational decisions.</td>
</tr>
<tr>
<td></td>
<td>B. If a student retook the same test, he or she would get a similar score on each retake.</td>
</tr>
<tr>
<td></td>
<td>C. The test score is a more valid measure than teacher judgments.</td>
</tr>
<tr>
<td></td>
<td>D. The test score accurately reflects the content of instruction in classes where the test is administered.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Example 2</th>
<th>Several students in Ms. Atwell's class got low scores on her end-of-unit test in doing multi-step story problems in mathematics. She wanted to know which students were having similar problems so she could group them for instruction. Which assessment strategy would be best for her to use for grouping students?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A. Use the test provided in the &quot;teacher's guide.&quot;</td>
</tr>
<tr>
<td></td>
<td>B. Have the students take a test that has separate items for each step of the process.</td>
</tr>
<tr>
<td></td>
<td>C. Look at the student's records and standardized test scores to see which topics the students had not performed well on previously.</td>
</tr>
</tbody>
</table>
Questions like these, while not performance-based, provide a basis for which other questions about assessment and assessment strategies could be formed or built upon.

Plake and Impara (1997) later validated the instrument with an extensive review by two separate panels with 10 measurement specialists each, as well as a review by members of the AFT, NCME, and the NEA. Specialists reviewed “item-to-competency standard alignment” (Plake & Impara, 1997, p. 55) as well as the clarity and appropriateness of test items. After surveying 555 teachers across the United States with the validated instrument, Plake and Impara (1997) were able to pinpoint common strengths and weaknesses, as well as demonstrate that experience and training correlated to “higher overall knowledge” (p.55) of assessment. Their process for instrument validation can serve as a model, further, their findings may also serve as a comparison to TEFA Literacy.

O'Sullivan and Johnson (1993) chose to extend the TALQ by incorporating eight additional “performance-based tasks” to examine the assessment literacy of PSTs. The PSTs, who were enrolled in a course on educational measurement, were pre- and post-tested with the TALQ. Additionally, PSTs were instructed to complete a number of performance-based tasks that included analyzing standardized tests, designing a multiple choice test, and designing grading rubrics (O'Sullivan & Johnson, 1993). By incorporating a pre- and post-test model with the TALQ, O’Sullivan and Johnson (1993) sought to track growth in PSTs’ assessment literacy and, by incorporating the additional performance-based tasks, to gain a deeper understanding of PSTs’ competencies and skills in assessment. The researchers found that PSTs significantly increased their scores on the TALQ; however, given that there was no pre- and post-evaluation for the performance-based tasks, O’Sullivan and Johnson (1993) were unable to track progress
based on those data. Nonetheless, ideas from O’Sullivan and Johnson’s (1993) performance-based tasks could be used in other performance-based assessments, such as asking students to analyze test questions, design their own questions, or determine appropriate grading schemas.

When discussing the measurement and evaluation of assessment literacy, it is important to note one of the most commonly used instruments to measure assessment literacy, the Assessment Literacy Inventory (ALI) by Mertler and Campbell (2005). Closely aligned to the TALQ, the ALI is also a 35 multiple choice questionnaire based on the 1990 standards. Mertler and Campbell (2005) contend that the ALI addresses the fact that the TALQ was “difficult to read, extremely lengthy, and contained items that were presented in a decontextualized way” (p. 9). Thus, the main redevelopment within the ALI was the use of five scenarios to add context to each of the questions (Mertler & Campbell, 2005). Consider, for example, Scenario #1 and two subsequent questions from the ALI:

Ms. O’Connor, a math teacher, questions how well her 10th grade students are able to apply what they have learned in class to situations encountered in their everyday lives. Although the teacher’s manual contains numerous items to test understanding of mathematical concepts, she is not convinced that giving a paper-and-pencil test is the best method for determining what she wants to know.

1. Based on the above scenario, the type of assessment that would best answer Ms. O’Connor’s question is called a/an
   A. performance assessment.
   B. authentic assessment.
   C. extended response assessment.
   D. standardized test.

2. In order to grade her students’ knowledge accurately and consistently, Ms. O’Connor would be well advised to
   A. identify criteria from the unit objectives and create a scoring rubric.
   B. develop a scoring rubric after getting a feel for what students can do.
   C. consider student performance on similar types of assignments.
   D. consult with experienced colleagues about criteria that has been used in the past.
Mertler and Campbell (2005) were able to determine that the ALI had good overall psychometric properties (i.e., satisfactory reliability, effectiveness, and item discrimination). Moreover, they contend that their use of five scenarios may have reduced the “cognitive overload associated with reading 35 unrelated items” (Mertler & Campbell, 2005, p. 16). Thus in the development of an instrument to evaluate Secondary Mathematics TEFA Literacy it may be important to consider using a limited number of scenarios with connected questions.

Although the TALQ and ALI both offer an efficient and replicable means by which to measure assessment literacy, they lack the depth of performance-based evaluations like those used by O’Sullivan and Johnson. Moreover, multiple choice tests, like the TALQ and ALI, cannot accurately determine if the participant chose or just guessed correctly. Thus, a combination of these methods, such as a standardized open-ended questionnaire that provides the participant with the opportunity to demonstrate their knowledge and skills, may provide more meaningful data on Secondary Mathematics TEFA Literacy.

Engaging students in formative assessment requires a more specific set of assessment competencies, as was evidenced by Brookhart’s (2011) reform of the 1990 Standards. Specifically, formative assessment relies much more heavily on eliciting and building on students’ prior knowledge, engaging them in feedback, and encouraging them to be active participants in the assessment process (Black & Wiliam, 2009, Otero, 2006; Popham, 2009). Thus, just as the 1990 Standards needed to be revised to address specialized formative assessment skills and knowledge, methods of measuring assessment literacy also need to be reevaluated and revised. The ALI (Mertler & Campbell, 2005), for example, includes aspects of formative assessment, such as asking participants to identify “the primary purpose of conducting
formative assessment” (p. 5), but rarely touches on student participation in the formative assessment process nor the procedures and purposes of providing students with feedback.

Recently, Adamson (2020) sought to address these gaps in the ALI by using both the 1990 Standards and the updated JCSEE (2015) standards to develop additional questions for the ALI that attend to both formative assessment and feedback. Adamson’s (2020) Modified Assessment Literacy Inventory (MALI) sought to measure both assessment and formative assessment literacy of in-service teachers. Consider, for example, the additional questions for Scenario #1 from the MALI:

### Additional Q1: Ms. O’Connor decides to administer a brief formative assessment at the beginning of a new topic. What should Ms. O’Connor do to make the formative assessment effective?

A. Enter the assessment data into her grade book.
B. Use the assessment data to inform instructional decisions and practice.
C. Allow her students to work on the assessment in groups.
D. Assign the formative assessment for homework and revisit it the next day as a class.

### Additional Q2: Ms. O’Connor is about to analyze her students’ performance on an assessment near the end of a reporting period. Which of the following should she always keep in mind?

A. The purpose of the assessment.
B. How many absences her students had.
C. The group work each student took part in.
D. The number of late assignments each student had.

The MALI, however, presents a number of issues. Given that Adamson’s (2020) study had a limited sample size (n=10) and that the additional questions were not tested for validity, the formative assessment items may not be as rigorous or valid as those on the ALI. Further, the MALI poses a similar problem to both the TALQ and the ALI, in that it does not engage participants in performance-based tasks, nor does it address content specific assessment standards. Therefore, while the questions and prompts in the MALI that specifically address
formative assessment may serve as a guide for an evaluation of Secondary Mathematics TEFA Literacy, they should be applied with caution.

Besides Adamson’s (2020) MALI, the measurement and evaluation of formative assessment literacy tends to employ qualitative measures to analyze growth or development of formative assessment literacy. For example, Lamberg et al. (2020) used qualitative measures to track the growth in teachers’ perceptions and conceptions of formative assessment in mathematics. Lamberg and colleagues’ (2020) study was designed around a professional development (PD) for elementary and secondary mathematics teachers that sought to change how teachers thought about and used formative assessment in their classrooms. Collecting data from teachers’ journals, field notes of classroom observations, and video recordings of the PD, Lamberg et al. (2020) analyzed and coded the data to find themes within teachers’ conceptions. While this methodology produced rich data that helped Lamberg et al. (2020) to track shifts in conceptions of formative assessment during the PD, it would prove difficult for teachers and researchers to efficiently replicate. On the other hand, elements of the tasks that mathematics teachers had to complete, as well as the journal prompts, may serve as resources for the development of an instrument to evaluate PSTs’ Secondary Mathematics TEFA Literacy.

In another study, Lee and Lim (2020) analyzed changes in mathematics PSTs’ written feedback throughout a learning module on “the nature and uses of constructive formative feedback” (p. 4). Based on research about formative assessment and feedback (Hattie & Timperley, 2007), Lee and Lim (2020) devised an analytic framework to code PSTs feedback; Much like a rubric, this framework included qualitative descriptors to help categorize the qualitative data into different quantitative levels. Although the coding framework was not tested for validity, Lee and Lim (2020) were able to demonstrate reliability by achieving 95%
agreement with a third coder. Analysis of PSTs’ responses before and after a learning module on “the nature and uses of constructive formative feedback” (Lee & Lim, 2020, p. 4) showed significant gains in the PSTs’ ability to provide more constructive, written feedback.

Lee and Lim’s (2020) research utilizes many measurement and evaluation strategies lauded by other researchers, such as the use of scenarios (Mertler & Campbell, 2005) in performance-based tasks (Khal et al., 2013; Lachner et al., 2021; O’Sullivan & Johnson, 1993), that are evaluated by a coding framework resemblant of a rubric (e.g., Harris et al., 2010; Lyublinskaya & Tournaki, 2012). Following a similar model of assessment, researchers and teacher educators may be able to gain insight into secondary mathematics PSTs’ TEFA literacy and, therefore, be able to identify strengths and weaknesses. Further, each of these studies present a methodology for validation and reliability checking that need to be considered when developing the instrument for evaluating PSTs’ Secondary Mathematics TEFA Literacy. To this point, all of the evaluation methods mentioned have not considered the influence of technology on formative assessment. In the next section, I will discuss the existing literature on methods to evaluate TEFA Literacy.

**Evaluating TEFA Literacy**

After an extensive review of literature, I was unable to find a standardized instrument that was specifically intended to measure pre- or in-service secondary mathematics teachers’ TEFA Literacy. Much of the research on TEFA Literacy focuses on how to support both mathematics and science, pre- and in-service teachers to develop strategies, methods, or resources for integrating technology into formative assessment (Aldon & Panero, 2020; Bennett & Cunningham, 2009; Dalby & Swan, 2019; Cusi et al., 2017; Feldman & Capobianco, 2008; Mitten et al., 2017; Moreno & Pineda, 2020). Additional research has drawn attention to the
relationships between mathematics teachers and TEFAs (Not TEFA Literacy), investigating the role perceptions and beliefs play in teachers’ acceptance and integration of TEFA (Lee et al., 2012), the role TEFAs may play in reforming mathematics instruction (Bush, 2020; Olsher et al., 2016), or PSTs opinions about TEFAs (Elmahdi et al., 2018; Karaoğlan-Yılmaz et al., 2020). And while many of these studies may provide more context for defining concepts related to Secondary Mathematics TEFA Literacy and what to evaluate, they do not present an evaluation methodology.

This study seeks to address this gap in the literature, particularly for secondary mathematics PSTs, by developing a performance-based instrument to evaluate Secondary Mathematics TEFA Literacy. To ensure that the instrument accounts for the particulars of evaluating Mathematics TEFA Literacy, the next section will review measurement and evaluation methods that are frequently used for mathematics pedagogy.

Evaluating Literacy in Mathematics Pedagogy

The selection, analysis, and design of TEFAs will depend greatly on teachers’ knowledge of mathematics as well as their beliefs and training in mathematics pedagogy. Thus, when designing an instrument by which to evaluate PSTs’ Secondary Mathematics TEFA Literacy, it is integral to also consider how teachers’ competencies in mathematics pedagogy are measured and evaluated.

One of the most prominent ways that secondary mathematics and other PSTs are evaluated for their pedagogical knowledge and skill is through the use of comprehensive portfolios of their field work (i.e., student teaching). For example, the educative Teacher Performance Assessment (edTPA) developed by the Stanford Center for Assessment, Learning, and Equity (SCALE) includes a number of tasks that ask PSTs to design their own lessons and
assessments, implement these, and, then, reflect upon their work. The edTPA for secondary mathematics, in particular, relies heavily on the NCTM (2000) principles and standards, as well as the concepts of conceptual understanding, procedural fluency, mathematical reasoning and problem-solving skills from Kilpatrick et al. (2001). To evaluate these portfolios, the edTPA provides educators, PSTs, and professional edTPA graders with an extensive analytic rubric. In spite of the fact that this performance-based measure provides rich data to evaluate PSTs pedagogy, it is incredibly time consuming and may not be the most efficient method by which to gain insight into secondary mathematics PSTs’ TEFA Literacy. On the other hand, the use of an analytic rubric does appear to help efficiently standardize the assessment of PSTs’ mathematics pedagogy and should be considered in the development of an instrument to evaluate PSTs’ Secondary Mathematics TEFA Literacy.

In addition to portfolios, researchers have suggested a number of strategies and tasks that may help to elicit and, therefore, evaluate mathematics PSTs’ pedagogical knowledge. These recommendations have included asking PSTs to analyze mathematics problems for their alignment to “a specific mathematical conception” (Patterson et al., 2020, p. 813), asking PSTs to select or generate appropriate feedback for students’ questions, mistakes, or misconceptions (Ball, 1990; Hill et al., 2008), or having PSTs complete a self-assessment of their mathematics pedagogical knowledge (Moh’d et al., 2021). Given the broad nature of these strategies, they could be modified or adapted to be included in a performance-based evaluation of Secondary Mathematics TEFA Literacy.

Other researchers have employed the use of open-ended, situational questionnaires, such as Turnuklu and Yesildere’s (2007) study of primary mathematics PSTs in Turkey. Based on the frameworks of Fennema and Franke (1992) and Shulman (1995), Turnuklu and Yesildere (2007)
designed four situational problems that included examples of student-teacher dialogue and student's work. In each, PSTs had to analyze the problem for students’ thinking, possible misunderstanding or misconceptions, prior knowledge, as well as how to give appropriate feedback or how to elicit more data from students (Turnuklu & Yesildere, 2007). To evaluate the PSTs’ responses, Turnuklu and Yesildere’s (2007) further devised a quantitative rubric in which response could be marked 1-Insufficient, 2-Mediocre, or 3-Excellent. Although this instrument was not validated, the premise of incorporating questions that specifically address students’ thinking, as well as common mistakes and misconceptions in mathematics should be considered in the development of an instrument to evaluate PSTs’ Secondary Mathematics TEFA Literacy.

It may be noted that there are some common themes among the methods for eliciting and evaluating literacy in mathematics pedagogy, technology integration, and formative assessment. In particular, the analysis of classroom scenarios, the application of skills to tasks that require participants to design their own tests, problems, or solutions, as well as the use of a rubric to analyze open-ended responses have all consistently appeared. Consequently, in the development of an instrument to evaluate the Secondary Mathematics TEFA Literacy of PSTs, it would be reasonable to incorporate similar methods.

**Evaluating Secondary Mathematics TEFA Literacy**

Just as there is no clear framework for what it means for a secondary mathematics teacher to have TEFA Literacy, there is also no formalized method by which to evaluate this. This gap in the literature will, therefore, be addressed in the following chapters.
Chapter 3: Methodology

In this chapter, I present the methodology for this study. The study was completed in three parts: (1) The development of the theoretical framework for Secondary Mathematics TEFA Literacy and rubric, (2) The development of the *TEFA Literacy Test for Secondary Mathematics*, and (3) The evaluation of levels of TEFA Literacy of a sample of secondary mathematics PSTs based on their responses to the *TEFA Literacy Test for Secondary Mathematics*.

**Part 1: Development of the Theoretical Framework**

The goal of the first part of the study was to answer the first research question: What are the components and essential elements of the theoretical framework for Secondary Mathematics TEFA Literacy? Guided by Jabareen’s (2009) recommendations for framework development, I developed the framework in eight phases:

1. Identifying and mapping literature sources
2. Initial categorization of sources
3. Identifying and naming concepts
4. Deconstructing and categorizing the concepts
5. Integrating concepts
6. Synthesis of concepts into initial framework
7. Validating the framework
8. Rethinking the framework

It is important to note that Phase 7 and Phase 8 were conducted in an iterative process, going back and forth between the two phases before producing the final Theoretical Framework for Secondary Mathematics TEFA Literacy.
Phase 1: Identifying and Mapping Literature Sources

Phase 1 consisted of “an extensive review of the multidisciplinary texts” (Jabareen, 2009, p. 53), to include a spectrum of literature related to Secondary Mathematics TEFA Literacy. The initial pool of texts included theories, frameworks, standards, research studies, news articles, as well as websites and blogs. Merriam and Tisdell (2016) recommend that a review of literature begins with checking texts “that reference specific aspects of a topic” (p. 93). Therefore, the first literature sources were identified by conducting a broad online search (e.g., Google and Google Scholar, library collections from various universities/colleges, ResearchGate, ProQuest, etc.) using various combinations of the keywords and phrases related to the topic of Secondary Mathematics TEFA Literacy. These keywords included, but were not limited to: mathematics, secondary mathematics, mathematics pedagogy, mathematics assessment, educational technology, technology integration, TPACK, teaching with technology, assessment, assessment literacy, formative assessment, teacher education, teacher training, and/or preservice teachers. Additional literature sources were identified using the strategy of “citation chasing”, in which the researcher tracks down the references from the existing literature sources.

Phase 2: Initial Categorization of Sources

Phase 2 consisted of categorizing the initial pool of literature “by discipline and by a scale of importance and representative power” (Jabareen, 2009, p. 54). The literature was first sorted into six categories:

1. mathematics pedagogy,
2. technology,
3. formative assessment,
4. mathematics pedagogy and formative assessment,
5. formative assessment and technology, and
6. technology and mathematics pedagogy.

From there, each literature source was ranked to denote its importance and representative power (see Table 9):

**Table 9**

*Determining Factors for Ranking the Importance and Representative Power of Literature Sources*

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Determining Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The literature source provides information or data directly related to its assigned category. The literature source is considered reliable. For example, the piece is research-based, fact-based, information can be verified by other reliable sources, and/or the author(s) is affiliated with a reputable university or organization.</td>
</tr>
<tr>
<td>2</td>
<td>The literature source provides some new information or data directly related to its assigned category, but this information is similar to those ranked as a “1” The literature source is considered reliable.</td>
</tr>
<tr>
<td>3</td>
<td>The literature source provides little to no new information or data related to its assigned category. The reliability of the literature source is questionable. For example, the piece is more opinion-based, lacks an author affiliation, and/or lacks references.</td>
</tr>
<tr>
<td>4</td>
<td>The literature source does not provide new information or data related to its assigned category. The literature source is not reliable. For example, the literature source is purely opinion-based, lacks an author affiliation, has no references, and/or cannot be verified by other reliable sources.</td>
</tr>
</tbody>
</table>

Literature sources assigned a rank of “4” were removed from consideration. Literature sources assigned a rank of “3”, were closely compared to those assigned a rank of “1” or “2” and were removed if they did not add any new information to what was already identified from these
sources. If sources ranked as a “3” provided additional information, they were reassigned a rank of “2”. Only literature sources with rankings of “1” or “2” were used in the next phase.

**Phase 3: Identifying and Naming Concepts**

Phase 3 was dedicated to reading and rereading the literature sources ranked “1” or “2” to create “a list of numerous competing and sometimes contradictory concepts” (Jabareen, 2009, p. 54). Concepts, such as theories, definitions, standards, studies of practice, or examples of practice were identified, highlighted, and named by their category (see Phase 2). For example, if a journal article in the “technology” category presented a list of standards for integrating technology and a definition for the term “technology integration”, the list of standards and the definition would both be identified as a “technology concept”. This resulted in each of the six categories (see Phase 2) consisting of a number of identified concepts. These concepts would be further analyzed in the following phase.

**Phase 4: Deconstructing and Categorizing the Concepts**

In Phase 4, each concept was analyzed to “identify its main attributes, characteristics, assumptions, and role” (Jabareen, 2009, p. 54). This included identifying:

- The type of concept: e.g., theory, definition, standard.
- The intended purpose or application: e.g., provide a model or strategy, set a standard, improve teaching, report findings.
- The potential role the concept could play in the theoretical framework: Directly incorporated, supporting, or underlying concept.
Phase 5: Integrating Concepts

“The aim in this phase is to integrate and group together concepts that have similarities to one new concept” (Jabareen, 2009, p. 54) and, therefore, drastically reduce the number of concepts. Using an iterative process, the concepts were sorted and filtered (see Figure 5).

Figure 5

Phase 5: Process to Integrate and Reduce Concepts

As concepts had already been grouped into the six categories (noted in Phase 2), in Iteration 1, the concepts were first sorted and grouped by “type”. For example, the “technology concepts” that were identified as a type of standard were all grouped together. Within each of these groups, common themes were identified. Common themes were established from repeated words or phrases, repeated intended purposes or strategies, or patterns among potential roles the concepts could play in the theoretical framework. For example, in the “technology concepts”, a common theme between two different sets of standards was: Teachers need to have a knowledge of
technological tools and the ability to use them. Thus, the two concepts were integrated into this new one. A new, shorter list of concepts was then created.

Iteration 2 consisted of identifying common themes and integrating concepts, the newest list of concepts was compared across types. For example, the concepts in the standards and the concepts in theories of “technology category” were compared, if there were common themes, these concepts were integrated into one, new concept. Finally, in Iteration 3, the remaining concepts were compared across categories. For example, a common theme in the “technology category” was: Teachers should be able to choose technology that is appropriate for the content and the instructional goals. Similarly, in the “technology and mathematics pedagogy category” a common theme was: Teachers should be able to choose technologies that accurately reflect the mathematics and enhance the pedagogical activity. These two themes were integrated into one, new concept that focused on: Teachers’ ability to select technology that is aligned to the mathematics content and the instructional goals. The final list of new, integrated concepts was used in the next phase.

**Phase 6: Synthesis of Concepts into Initial Framework**

Jabareen (2009), recommends that Phase 6 is one that is “iterative and includes repetitive synthesis and resynthesis until the researcher recognizes a general theoretical framework that makes sense” (Jabareen, 2009, p. 54). Synthesis is described as “the combination of ideas to form a theory or system” (Oxford University Press, n.d.). Therefore, the final list of concepts (i.e., the ideas) from Phase 5 were combined to formulate the essential competencies that a secondary mathematics teacher must have in order to successfully implement TEFAs (i.e., the theory).
Through an analysis of the final list of concepts from Phase 5, it became clear that there were a number of “essential elements” that described the different knowledge and skills secondary mathematics teachers must have in order to successfully implement TEFAs. These essential elements were established through an iterative process that included grouping, analyzing, and re-grouping concepts to form a cohesive group that focused on one fundamental idea. For example, concepts that highlighted the different types of feedback teachers should provide, how and when feedback should be provided, and how students should participate in giving and receiving feedback were grouped together. These formed the essential element that focused on teachers’ knowledge and skill in the fundamental idea of the Feedback Process.

Once the essential elements had been established and named, they were classified into “components”. In a similar process of grouping, analyzing, and re-grouping, the components were developed to group together essential elements with a shared overarching concept. For instance, the essential elements: Feedback Process (FP), Eliciting Student Understanding (ESU), Moving Learning Forward (MLF) are all associated with formative assessment and were grouped into a Formative Assessment component.

This initial framework was named the Theoretical Framework for Secondary Mathematics TEFA Literacy and consisted of three components that were composed of eight essential elements. Finally, other theoretical frameworks (Angeli & Valanides, 2009; Cochran et al., 1993; Hollebrands, 2017; Mishra & Koehler, 2006; Xu & Brown, 2016) were analyzed to serve as guides for the development of a visual representation of the initial Theoretical Framework for Secondary Mathematics TEFA Literacy.

**Rubric Development**
Once the initial Theoretical Framework for Secondary Mathematics TEFA Literacy was designed, I developed an analytic rubric to define different levels of TEFA Literacy for each essential element. This rubric provided performance-based qualitative descriptors of each level for each essential element of the initial Theoretical Framework for Secondary Mathematics TEFA Literacy. In Phases 7 and 8, the rubric was examined by the external experts who provided feedback on the qualitative descriptors of the essential elements as part of the validation of the theoretical framework. The rubric also served as a tool for evaluation of pre-service teachers’ responses to the *TEFA Literacy Test for Secondary Mathematics* (see Part 2 and Part 3).

The structure of the rubric was developed based on the analysis of other rubrics that have been used to evaluate qualitative data collected on pre- and in-service teachers’ assessment, formative assessment, technology integration, and/or mathematics pedagogical knowledge and skills (e.g., Bangert & Kelting-Gibson, 2006; Lyublinskaya & Kaplon-Schilis, 2022; Lyublinskaya & Tournaki, 2012; Niess, 2013; Schmitt, 2007). In line with these examples, the rubric was designed to include four levels of proficiency: Advanced, Proficient, Partially Proficient, and Novice, for each essential element in each component of TEFA Literacy. The qualitative descriptors in the rubric were developed first for the Advanced levels, which defined the essential elements of the Theoretical Framework for Secondary Mathematics TEFA Literacy. The qualitative descriptors of Novice levels were developed next, and were designed to underscore the most basic demonstration of the skills and understandings at the Advanced levels or, for certain essential elements, to stand contrary to the descriptors in the Advanced levels. The descriptors for Proficient and Partially Proficient levels were developed last, highlighting the distinction of the Proficient level from the Advanced level and distinction of the Partially Proficient level from the Novice level. The Proficient and Partially Proficient
levels were then compared to determine if there was an observable difference between the highlighted skills and understandings that needed to be demonstrated. These descriptors were then placed into a graphic organizer to form the first draft of the rubric (see Appendix A).

**Phase 7 and Phase 8: Validating and Rethinking the Framework**

“A theoretical framework … will always be dynamic and may be revised according to new insights, comments, literature, and so on” (Jabareen, 2009, p. 55). Thus, validating and rethinking of the Theoretical Framework for Secondary Mathematics TEFA Literacy and corresponding rubric, was completed in an iterative process. This process consisted of multiple stages, each followed by rethinking of the framework and revisions to the rubric before proceeding to the next stage (see Figure 6).

**Figure 6**

*Iterative Process for Phase 7 and Phase 8: Development of the Theoretical Framework*

![Diagram of the iterative process for Phase 7 and Phase 8](image)

**Stage 0: Researcher Validation**

Jabareen contends that “validating a theoretical framework is a process that starts with the researcher, who then seeks validation among ‘outsiders’” (2009, p. 54). For the purposes of
this study, validation at the researcher level was considered a “Stage 0”. In this stage I, the researcher, “go back and compare the scheme against the raw data” (Corbin & Strauss, 2012, p. 113). Stage 0, therefore, involved the comparison of the components and essential elements (i.e., the scheme) to the list of concepts (i.e., the raw data).

**Stage 1: Content Validity with Expert Reviewers**

In the first stage, content validity was sought with two experts in the field. Expert Reviewer 1 is a university professor and UNESCO Chair specializing in learning innovation and teaching excellence. With a Doctorate of Education in Leadership and Policy studies, Expert Reviewer 1 has multiple publications in topics such as teacher preparation, STEM activities for students, TEFA, network-based assessment. Expert Reviewer 2 is a senior research fellow and faculty member at a European school of education. Expert Reviewer 2 has experience as a secondary and post-secondary mathematics teacher and has multiple publications in the fields of functional skills in mathematics, professional development in education, TEFA in mathematics, and mathematics education policy.

In Stage 1, the rubric, along with a set of “Review Directions” (Appendix C) were sent to the Expert Reviewers. In the “Review Directions”, the reviewers were asked open-ended questions about the rubric that guided them to comment on the content, clarity, potential redundancies, concepts that could be added or eliminated, noted ambiguities, syntax, etc. Reviewers were also given the opportunity to provide additional, general comments they believed might help in refining the qualitative descriptors in the rubric.

The reviewers’ responses to the open-ended questions and additional comments on the rubric were first categorized as a type of “direct feedback”, such as pointing out an issue like a redundancy, or a type of “interpretive feedback”, such as asking a question or proposing a new
idea. The direct feedback was first checked for accuracy, ensuring the comment reflected an actual issue. Then, if considered accurate, the direct feedback was used to make revisions of the rubric. The interpretive feedback was first analyzed alongside literature. This included reading the interpretive feedback and looking for a connection to the list of concepts from Phase 5. When no connection could be made, new literature was identified and used to help analyze the interpretive feedback. Based on this analysis, more revisions were made to the qualitative descriptors on the rubric. The rubric revisions were then analyzed for how they might affect the theoretical framework by asking questions such as:

- Do these revisions mean that the essential elements need to be modified or changed?
- If the essential elements do need to be modified or changed, how should this be done?
- Do these revisions mean the components need to be modified or changed?
- If the components do need to be modified or changed, how should this be done?

Stage 1 resulted in a revised theoretical framework and a second draft of the rubric (see Appendix D).

**Stage 2: Inter-rater Reliability Analysis of Mock Responses with External Raters**

In the second stage, inter-rater reliability was sought with External Raters. External Rater A is a licensed secondary mathematics teacher with a Bachelor of Science in Mathematics, a Master of Education in Curriculum and Instruction, and pursuing a Doctorate of Education in Mathematics Education. External Rater B is a post-secondary mathematics instructor with a Bachelor of Science in Mathematics, a Master of Arts in Applied mathematics, and pursuing a Doctorate of Education in Mathematics Education.

In Stage 2, External Raters were provided with a copy of the second draft of the rubric, a copy of the second draft of the *TEFA Literacy Test for Secondary Mathematics* (see Part 2) with
Mock Responses (Appendix E), and the “Review Directions”. The External Raters were first asked to use the rubric to evaluate the Mock Responses following the instructions for the use of the rubric. Following their evaluation, the External Raters were then directed to provide responses to the open-ended questions on the “Review Directions”.

Inter-rater reliability procedures sought to reach 80% agreement (Billups, 2021) in evaluation of levels of proficiency in each essential element (see evaluation method in Part 2). Given the qualitative nature of both the participant’s responses and the descriptors on the rubric, as well as the inherent subjectivity in the interpretation of these descriptors, a certain amount of error needed to be accounted for. Consequently, agreement in levels of proficiency included an exact match (e.g., all raters evaluating at the same level), and a +/-1 match (e.g., raters evaluating within one level of proficiency).

Given the External Raters only reached 71.4% agreement in their evaluation of the Mock Responses, they were asked to participate in an unstructured follow-up interview, that also served as a debriefing session. In the interview, External Raters were asked to comment on their understanding of the qualitative descriptors in the rubric and how those were, were not, or could be demonstrated in the Mock Responses. External Raters were also asked to comment on their understanding of the evaluation method (see Part 2). Further, the External Raters were asked to clarify or confirm any of the responses on the “Review Directions” that had been categorized as “interpretive feedback”. In the same process as described in the Expert Review, the feedback from the interview was categorized, analyzed, and appropriate revisions were made to the qualitative descriptors on the rubric. This process resulted in a final version of the rubric that had 90.6% inter-rater agreement in evaluation of responses to one of the study participants to the test (see Stage 3).
Stage 3: Inter-rater Reliability Analysis Based on Data From Field Testing

In Stage 3, field testing, the third draft of the rubric (Appendix F) was used to evaluate a sample of secondary mathematics PSTs’ responses to the third draft of the TEFA Literacy Test for Secondary Mathematics (Appendix G, see Part 2 for description of the test development process). Methodology for the field testing, including participant selection and context is provided in Part 3.

For the purpose of measuring the inter-rater reliability of the third draft of the rubric, both External Raters were asked to evaluate the responses to the TEFA Literacy Test for Secondary Mathematics by one of the study participants. Hallgren (2012) notes that when evaluation of data is “costly and/or time intensive, selecting a subset of subjects … may be more practical” (p. 25). A single participant was, therefore, selected for the External Raters to evaluate. The participant, Heather, was selected based on the fact that she had the most complete responses and did not leave any questions blank.

To ensure that the External Raters followed the evaluation methodology (see Part 2), they were provided with the “External Rater Evaluation Guide” (Appendix H) that included improved directions and a graphic organizer to report their evaluations. Inter-rater reliability procedures were the same as those described in Stage 2 above. Stage 3 resulted in the finalized theoretical framework (see Figure 9) and rubric (see Appendix F).

Part 2: Development of the TEFA Literacy Test for Secondary Mathematics

The goal of the second part of this study is to answer the second research question: How can Secondary Mathematics TEFA Literacy be evaluated through a performance-based instrument? The development of the TEFA Literacy Test for Secondary Mathematics was
adapted from DeVellis’ (2003, 2012) model for instrument development and completed in three steps:

1. Identify the construct to be evaluated.
2. Develop an initial item pool and evaluation method.
3. Review by experts.

**Step 1: Identify the Construct**

The TEFA Literacy Test for Secondary Mathematics is aimed at eliciting evidence of Secondary Mathematics TEFA Literacy. The construct for the test was based on the components composed of the essential elements from the Theoretical Framework for Secondary Mathematics TEFA Literacy as defined by the rubric.

**Step 2: Item Pool and Evaluation Method**

Based on the recommendations of Darling-Hammond (2010) and Khal et al. (2013), the items of a performance-based test were developed to contain open-ended questions that would prompt participants to demonstrate their secondary mathematics TEFA Literacy. Similar to the test designs of several researchers (Adamson, 2020; Mertler & Campbell, 2005; Lachner et al., 2021; Plake & Impara, 1993; Turnuklu & Yesildere, 2007) secondary mathematics classroom scenarios were used to prompt responses that demonstrate participants’ TEFA Literacy.

The initial pool of items consisted of 12 secondary mathematics classroom scenarios, each followed by two to three open-ended questions. Each scenario and subsequent questions were developed in three steps: First, I used the Common Core State Standards for Mathematics (2010) to determine the secondary mathematics topic that the scenario would be based on. Next, I used the descriptions of the essential elements on the rubric to determine the content and aim of
the questions that would follow the scenario. And, finally, I selected a computer-based technological tool to be used in the scenario and/or subsequent questions.

The mathematical standard determined the course and the learning objective for the scenario. For example, if the standard was on rigid transformation, the scenario would be based in a high school Geometry class, with a learning objective focused on students’ understanding of the three rigid transformations. The selected essential element(s) from the rubric were then used to determine what additional information may be needed in the scenario and to frame how and what the questions attempted to elicit. For example, if the essential element was Feedback Process (FP) the scenario may contain an example of the automated feedback provided on a TEFA and the subsequent questions might prompt the participants for an analysis of the quality of this feedback, how to improve it, or how to supplement it. As with other scenario-based tests (e.g., Plake et al., 1993; Mertler & Campbell, 2005), individual questions could not address every aspect of TEFA Literacy and, therefore, only aimed to elicit a limited number of the essential elements per question. Finally, a technological tool was selected. The chosen technologies were selected from widely used open access secondary mathematics educational technological tools. Based on the mathematical standard and the essential element(s), the technological tools were incorporated in a variety of ways. For instance, if the mathematical standard was about graphing quadratics, the technological tool would be one that has graphing capabilities. It is important to note that in some instances a technological tool was not selected. This was because the essential elements were focused on eliciting evidence of the participants’ knowledge of existing technologies and how to operate them.

After each scenario and subsequent questions were developed, they were re-analyzed for how well they captured the components composed of the essential elements from the Theoretical
Framework for Secondary Mathematics TEFA Literacy. This was followed by a final analysis of the scenarios questions to ensure appropriateness of length, reading level difficulty, proper grammar usage, and the removal of potential ambiguities (DeVellis, 2003, 2012; Hathcoat et al, 2016). By the end of Step 2, the first version of the TEFA Literacy Test for Secondary Mathematics had been drafted and included 12 scenarios with 35 questions.

Evaluation Method

As in other educational research (e.g., Angeli & Valanides, 2009; Lee & Lim, 2020; Lyublinskaya & Kaplon-Schilis, 2012; Lyublinskaya & Tournaki, 2012; Niess, 2013), a rubric has often served as a useful tool to evaluate qualitative data. As the rubric had already been developed (see Part 1) at the time of test development, the “evaluation method” consisted of directions for how to use the rubric to evaluate participants’ responses on the test and determine their levels of proficiency in each essential element. Additionally, the test identified the “Applicable Rubrics” for each question. For example, if a question intended to elicit evidence of the essential elements Mathematical Proficiency (MP) and Student Learning (SL), the “Applicable Rubrics” were MP and SL. Directions for how to evaluate responses to the TEFA Literacy Test for Secondary Mathematics were added to the beginning of the rubric and the “Applicable Rubrics” were added to the test.

Step 3: Review by Experts

Review of the TEFA Literacy Test for Secondary Mathematics was conducted in two iterations with subsequent revisions. Each review of the test was conducted at the same time as the reviews of the rubric (see Part 1), with the same Expert Reviewers and External Raters.

Expert Review
A first draft of the *TEFA Literacy Test for Secondary Mathematics* (Appendix B) was sent to the two Expert Reviewers along with the “Review Directions” (Appendix C). Expert Reviewers were asked to comment on the clarity, content, and ability of the test to elicit responses that are adequate and measurable. The Expert Reviewers were also directed to provide any additional comments on the test itself.

The Expert Reviewers’ response to the “Review Directions” concerning the test and the comments they made on the test (see Appendix I) were categorized, analyzed, and incorporated into revisions to the test using the same methodology as described in Part 1.

This analysis and consequent revisions resulted in a second draft of the *TEFA Literacy Test for Secondary Mathematics* (Appendix E).

**Review by External Raters**

The second draft of the *TEFA Literacy Test for Secondary Mathematics* and the “Review Directions” were sent to the two External Raters. The External Raters were also asked to comment on the clarity, content, and ability of the test to elicit responses that are adequate and measurable.

The External Raters’ responses to the “Review Directions” concerning the test and the comments they made on the test (see Appendix J) were also categorized, analyzed, and incorporated using the same procedures as described in the Expert Review. It is important to note that any feedback categorized as “interpretive” on the “Review Directions” was clarified and/or confirmed with the External Raters during their follow-up interview (see Part 1). The follow-up interview also included collecting feedback on the External Raters’ understanding of the evaluation method.
This analysis and consequent revisions resulted in the final *TEFA Literacy Test for Secondary Mathematics* and evaluation method.

**Part 3: Evaluation of PSTs’ TEFA Literacy Levels**

Part 3 seeks to answer the third research question: What are the TEFA Literacy levels of secondary mathematics preservice teachers enrolled in a graduate level initial certification program at a private urban university as assessed by this instrument? To do this, the final *TEFA Literacy Test for Secondary Mathematics* was administered to a sample of secondary mathematics PSTs and evaluated using the finalized rubric.

**Context and Participant Selection**

Teacher education programs are where most teachers obtain their foundational pedagogical knowledge. In traditional teacher education programs, PSTs are required to take teaching methodology courses which are designed to address the many aspects of teaching and learning. Additionally, PSTs must participate in student teaching, in which they work under the mentorship of a certified teacher at a participating school.

A convenience sample of participants was selected from a graduate-level, initial certification in secondary mathematics teacher education program at a private university in an urban setting in the Northeastern United States. Within this particular program, PSTs enrolled in the course *Student Teaching in Mathematics* are seeking initial certification in mathematics grades 7-12. Prior to enrolling in *Student Teaching in Mathematics*, PSTs are required to complete the pedagogy focused course *Mathematics in the Secondary School*. Both of these courses are designed to address the multiple aspects of teaching and learning mathematics. While the topic of TEFA is not explicitly addressed in the syllabus of either course, the PSTs often bring up concepts and questions related to TEFA during seminar discussions, and are
consistently engaging in different forms of TEFA in their student teaching classrooms. Moreover, Kilpatrick et al.’s (2001) book *Adding It Up* on the five strands of mathematical proficiency – which served as an important literature source for the development of the theoretical framework – is a required reading for the course.

Consistent with other studies of assessment literacy in preservice teacher education programs (Ayalon & Wilkie, 2020; DeLuca et al., 2013; Huang, 2018; O’Sullivan & Johnson, 1993) and to ensure similar experiences and knowledge base, participants were recruited from the same class. Of the 12 potential participants, five agreed to participate in the study.

**Research Design**

This part of the study employed a qualitative case study methodology. Qualitative research seeks to provide meaning and understanding to a particular phenomenon through the collection of rich data (Merriam & Tisdell, 2016). Case studies in qualitative research are used to conduct in-depth analyses of bounded systems “to develop a highly detailed description and gain a better, more thorough understanding of the individual entity” (Mertler, 2019, p. 83). By bounding the entity to that of secondary mathematics PSTs enrolled in the same class, I can ensure that their prior knowledge and experiences are relatively the same and, therefore, identify common strengths and weaknesses that may be a result of these shared experiences.

**Data Collection**

Research indicates that there are a number of factors that increase response rates to questionnaires, including having a researcher available to answer any questions while the participant is taking the questionnaire as well as an online/electronic delivery (Boyton, 2004). Administration of the final *TEFA Literacy Test for Secondary Mathematics* was conducted with five secondary mathematics preservice teachers (PSTs) using Qualtrics, an online survey tool.
Each participant agreed to a date and time to virtually meet via a video conferencing platform and take the *TEFA Literacy Test for Secondary Mathematics*. At the beginning of the meeting, I provided the participants with a link to the Qualtrics version of the *TEFA Literacy Test for Secondary Mathematics* and my contact information in case of any issues. Participants were also provided a reference sheet with an excerpt from *Adding It Up* (Kilpatrick et al., 2001) with a summary of the five strands of mathematical proficiency, as a number of questions specifically asked for the participants to use these terms in their responses. Participants were then directed to take the assessment, and to contact me via the video conferencing platform if they had any questions. Participants took an average of 60 minutes to complete the *TEFA Literacy Test for Secondary Mathematics*.

Initial review of the participants’ responses to the test indicated that three participants (Mitch, Brad, and Megan) did not complete or did not answer some of the questions. In their unstructured individual follow-up interviews, Mitch and Brad (Megan did not agree to participate), were asked to complete their responses. The interview took approximately 20 minutes to complete. Mitch and Brad’s responses were transcribed and added to their initial responses to their tests.

**Data Analysis**

Participants’ responses to the *TEFA Literacy Test for Secondary Mathematics* were evaluated using an a priori coding system based on the final version of the rubric without the MLF essential element (Appendix F). First, all questions that were intended to elicit an essential element (labeled as “Applicable Rubric”) were grouped together by that essential element. For example, Scenario 1, Question (b) (noted as 1b), 2a, 3a, 4a, 4c, 5c, 6c, 7b, 8c*, and 10b were all intended to elicit evidence of the participant’s Mathematical Proficiency (MP) Literacy. Thus all
of these questions were grouped together under Mathematical Proficiency. Given the open-ended nature of the test, some of the questions had the potential to elicit evidence of essential elements, but did not directly prompt for them. These questions were marked with an asterisk (*). For example, in 8c*, the participants are asked to analyze the technology Delta Math for its “benefits and drawbacks”. In this instance, the question does not directly prompt for MP, but a participant may incorporate it in their analysis.

Then, using the qualitative descriptors in the evaluation rubric, the words and phrases in each response were coded by the applicable essential elements and evaluated for a preliminary level of proficiency. For example, if a question was intended to elicit evidence of the essential element Mathematical Proficiency (MP), the code MP was applied to all words and phrases that showed evidence of MP and, then, those coded words and phrases were evaluated as either Advanced, Proficient, Partially Proficient, or Novice, according to the descriptors on the rubric.

Finally, each grouping was analyzed to determine an overall level of proficiency (i.e., Advanced, Proficient, Partially Proficient, or Novice) for each essential element. This was done by assigning each preliminary level of proficiency a “Numerical Level”:

- Advanced = 3
- Proficient = 2
- Partially Proficient = 1
- Novice = 0

The values of these “Numerical Levels” were then averaged, rounded to a whole number using standard rounding, and converted back into a categorical value to determine the overall level of proficiency (i.e., Advanced, Proficient, Partially Proficient, or Novice).
Chapter 4: Results

In this chapter, I will present the results in three parts: (1) The Theoretical Framework for Secondary Mathematics TEFA Literacy and rubric, (2) The *TEFA Literacy Test for Secondary Mathematics*, and (3) The evaluation of the participating PSTs’ TEFA Literacy levels.

**Part 1: The Theoretical Framework for Secondary Mathematics TEFA Literacy**

Part 1 seeks to answer the first research question: What are the components and essential elements of the theoretical framework for Secondary Mathematics TEFA Literacy? The Theoretical Framework for Secondary Mathematics TEFA Literacy was developed in eight phases (see Chapter 3, Part 1), with the framework and rubric taking their initial shape in Phase 6 and undergoing validation through an iterative process in Phases 7 and 8. In this part, I will begin by outlining the developments from Phases 6, 7, and 8. Next, I will present the final Theoretical Framework for Secondary Mathematics TEFA Literacy and corresponding rubric.

**Phase 6: Results of the Synthesis of Concepts into Initial Framework**

The initial version of the Theoretical Framework for Secondary Mathematics TEFA Literacy consisting of three components and eight essential elements (see Figure 7).

**Figure 7**

*Initial Version of the Theoretical Framework for Secondary Mathematics TEFA Literacy*
In the following sections I will describe the synthesis of the concepts and how these resulted in the above components and essential elements.

*Mathematics Pedagogy Component*

Each of the components of Secondary Mathematics TEFA Literacy will, inevitably, be intrinsically linked to secondary mathematics teaching and learning. Teachers’ knowledge and skill in mathematics pedagogy will be one of the driving forces in the selection and use of computer-based technologies for TEFAs in different mathematical concepts. Based on the extensive research by the National Research Council (2001), one of the most successful ways for students to learn mathematics is for teachers to consistently strive for students to attain the five strands of mathematical proficiency (see Table 4). This means that TEFAs should be geared towards eliciting evidence of and assessing these strands. Additionally, knowledge of mathematics pedagogy will play a key role in ensuring TEFAs are appropriately challenging, taking into consideration what students have or have not learned and where they typically make mistakes (NCTM, 2001, 2014). The Mathematics Pedagogy component is focused on the knowledge and skill it takes to ensure TEFAs are supporting students’ mathematical proficiency and are developed with the typical student learning process in mind.

It is important to note that teachers’ knowledge of mathematics content will play an influential role in their ability to successfully implement TEFAs in their classrooms (Collins, 2011; Ginsberg, 2009; Hill et al., 2008). Nonetheless, teachers’ mathematical competencies were determined to be outside of the scope of this framework, and will not be directly addressed in the components and essential elements.

*Mathematical Proficiency.* The Mathematical Proficiency (MP) essential element is about teachers’ knowledge and skill in selecting and designing TEFAs that promote the five
strands of mathematical proficiency (Kilpatrick et al., 2001, see Table 4). As each of these strands focus on different skills, teachers need to be skilled in analyzing pre-designed TEFAs for their alignment to the five strands, but also be able to design their own questions or prompts that promote the different strands. This means that the questions and prompts on the TEFAs should present different ways for students to demonstrate that they are making connections between mathematical concepts, efficiently carrying out procedures, problem solving, explaining and justifying their reasoning, and understanding the sense and use of mathematics (Collins, 2011; Kilpatrick et al., 2001; NCTM, 2000, 2014). Clearly, to know and be able to design and analyze TEEFA questions that are aligned to the five strands, teachers must be familiar with what the strands are, how they interact, and how they build one another. For example, how does a question promote procedural fluency? How can it also promote strategic competence? Teachers without an understanding of the strands of mathematical proficiency may struggle to answer questions like these.

**Student Learning.** The essential element of Student Learning (SL) is related to Schulman’s (1986) concept of PCK, Hill et al.’s (2008) theory for knowledge of content and students (KSC), and NCTM’ (2000, 2014) theories and standards on the use of assessment to support the learning of mathematics. This knowledge includes an understanding of how to analyze and design TEFAs that align to the ways in which students typically learn and understand mathematics. Given that mathematics is a subject that builds on itself, teachers must be knowledgeable of the particular mathematical skills and competencies necessary for TEFAs to assess the desired student understanding. Consequently, teachers must be cognizant of students’ prior knowledge to build TEFAs that are appropriately challenging. Moreover, as TEFAs serve as tools to help teachers to identify strengths and weaknesses in student understanding, teachers
must be knowledgeable of common mathematical misconceptions or mistakes students might make. A teacher with a deep understanding of SL will also have knowledge as to why such misconceptions and mistakes happen. Teachers’ knowledge and understanding of SL will help them to select and design TEFAs that are valid, as well as accurately analyze student data.

Consider, for example, an algebra teacher wants to use a TEFA to determine if students were able to make the connection between algebraically solving a system of two equations in two variables and the graphical representation of this solution. The teacher can demonstrate their SL knowledge by first considering the prior mathematical knowledge students need to have to complete the TEFA, such as graphing linear functions, identifying points of intersection, and algebraically solving systems of two equations in two variables. Then, they can demonstrate their MP knowledge by considering what types of questions they need to ask to provide students the opportunity to demonstrate different strands of mathematical proficiency, such as the conceptual understanding of the algebraic solution representing the intersection of the linear graphs. Concurrently, they should anticipate the most common mistakes or misconceptions students may have and why. For instance, students may make computational mistakes in their algebraic solution due to a lack of understanding of substitution or how to isolate a variable. They may, then, consider intentionally isolating these skills within the TEFA, or ensuring that they look for these mistakes when analyzing students’ responses. The teacher can demonstrate a culmination of both MP and SL knowledge and skill by applying these considerations to selecting an appropriate pre-designed TEFA or creating their own.

**Technology Integration Component**

The Technology Integration Component – later called the Technology Component – of Secondary Mathematics TEFA Literacy focuses on mathematics PSTs’ ability to select, analyze,
and design TEFAs using computer-based technology tools that are aligned to the instructional
goal of formative assessment and mathematical learning objectives. Educational technology can
be interpreted as a variety of tools, something as basic as a white board, to a complex statistics
computer software that requires coding. For the purpose of this framework, I will be limiting the
concept of educational technology to that of computer-based technology, meaning that students
and teachers must interact with the technology through a computer, laptop, tablet, or smartphone.
Educational technologies, even when limited to computer-based, serve many different purposes
in the teaching and learning of a variety of content areas. Teachers, therefore, must have an
understanding of how different educational technologies align to the formative assessment
process and content specific learning objectives (Dick & Hollebrands, 2011; Hollebrands, 2017;
Niess, 2005; Mishra & Koehler, 2006; Zelkowski et al., 2013). Underlying this understanding, is
the teachers’ ability and skill in operating and helping students to operate the technological tools

**Technology and Instructional Goals.** The essential element of Technology and
Instructional Goals (TIG) – later called Alignment to Instructional Goals (AIG) – includes
knowledge of computer-based technologies that are appropriate for the instructional goal of
formative assessment, as well as the mathematical learning objective(s). Closely related to
Hollebrands’ (2017) framework, this involves the teachers’ ability to analyze computer-based
technologies pedagogical fidelity, cognitive fidelity, and mathematical fidelity for a TEFA and
select the most appropriate one. Teachers need to know how well a technological tool facilitates
students to engage in the formative assessment processes and, therefore, are designed or can be
modified to elicit student understanding, provide feedback, and collect data to help move
learning forward (Black & Wiliam, 2009; Otero, 2006; Popham 2009). This includes teachers’
knowledge of how well a technological tool is at staying “faithful to students' cognitive process” (Dick, 2008, p. 339) so that their responses to TEFA questions resembles and/or enhances how they might have answered without the technological tool. Further, teachers must be able to analyze technological tools that are best suited for the specific mathematics topics being assessed. This includes an understanding of how the technology works to ensure “what is represented in the tools are accurate representation of the mathematics” (Hollebrands, 2017, p. 83).

**Identifying Classroom Technologies and Operating Classroom Technologies.**

Fundamental to mathematics teachers' ability to analyze and design TEFAs that are aligned to formative assessment and the mathematics learning objectives, is their knowledge of the technologies that are available to them. The essential elements Identifying Classroom Technologies (ICT) and Operating Classroom Technologies (OCT) – later called Technology Knowledge (TK) – are related to Mishra and Koehler’s (2006) framework, and incorporate teachers’ knowledge and skill in operating computer-based technologies to formatively assess mathematics. Teachers should know what resources are necessary (i.e., hardware and software), how questions and prompts are generated or designed, how students interact with the technology, how feedback is generated or designed, and how student data are accessed within the tool (Barana et al., 2019; ISTE, 2002, 2020; Looney, 2010). Teachers should also have an understanding and skill in working within the capabilities of different types of technological tools that can be used for TEFAs in mathematics. TK, consequently, includes a knowledge of a variety of technological tools that are able to facilitate TEFAs for different mathematical concepts.
Consider an Algebra I class that has been investigating the slopes of linear graphs, and suppose the teacher wants to use a TEFA to gauge students’ current understanding. The teacher can demonstrate their TK by first identifying a number of different technological tools that may be appropriate for a formative assessment of slopes of linear graphs. They can then demonstrate their AIG knowledge by analyzing the different tools and deciding which is best for this particular TEFA. They will need to consider that they are creating a formative assessment, so the tools should allow for students to submit responses that will demonstrate their understanding and collect data on their understanding. Concurrently, they will need to consider which of the technological tools will maintain the mathematical and cognitive fidelity associated with slopes of linear graphs; which tools will enable or enhance students’ ability to demonstrate their understanding of slopes of linear graphs. Lastly, the teacher can further demonstrate their TK by implementing the TEFA with students, which may include explaining to students how to engage with the technology as well as accessing data after students submit responses.

It is important to note two substractive concepts of the technology component: (1) that the teacher also understands what capabilities the technology does not have (ISTE, 2002, 2020). For example, Delta Math © does not allow for the manipulation of geometric features.; And, (2) teachers’ ability to troubleshoot technology issues as needed or know how to seek appropriate help (ISTE, 2002, 2020; Zelkowski et al., 2013). Such as teachers’ ability to solve student login issues themselves or contact technical support staff at the school for help.

**Formative Assessment Component**

Given the main instructional goal of a TEFA is to engage students in the formative assessment process, teachers must also have a thorough understanding of what that means and how to do it. As has been previously defined, “practice in a classroom is formative to the extent
that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction” (Black & Wiliam, 2009, p. 9). TEFAs serve as the tools to elicit the evidence of student achievement and, in many cases, produce feedback on their achievement. Moreover, the data collected during TEFAs play an essential role in the teachers’ and students’ ability to make the decisions about where learning should go next. The Formative Assessment Component, therefore, focuses on the knowledge and skills that teachers must have to ensure TEFAs are designed to effectively elicit students’ understanding, engage them in the feedback process, and move learning forward.

**Elicitation of Student Understanding.** The Elicitation of Student Understanding (ESU) essential element is about the knowledge and skill used to elicit evidence of students’ current understandings that will help determine what students know and what they still need to learn. Determining what students already know or still need to learn requires that teachers engage in formative assessments that encourage students to share their thoughts, ideas, strategies, or problem solving processes (Lamberg et al., 2020). This includes an understanding of how to select or design formative assessment questions and prompts that emphasize reasoning, explanation, and justification (Lamberg et al., 2020). Moreover, this involves knowledge and skill in providing students with a variety of ways to demonstrate their understanding algebraically, graphically, geometrically, in words, etc. Consequently, a mathematics teacher with a mastery of how to elicit student understanding should also grasp that problems which utilize memorization or computational procedures are counterproductive to eliciting understanding.

**Feedback Process.** The Feedback Process (FP) essential element is consistent with Hattie and Timperley’s (2007) constructs of effective feedback practices. This knowledge
includes knowing what types of feedback are most effective as well as the skill to ensure students are receiving and processing this feedback. FP is concerned with feedback that targets specific errors or misconceptions, rather than utilizing correct and incorrect responses that may lead students to believe that they have “a total lack of understanding” (Hattie & Timperley, 2007, p.82). This knowledge also includes the teaching strategies associated with ensuring students are processing the feedback, such as allowing them to comment on each other’s work, having peer discussions about problems or tasks, or using success criteria (e.g., a rubric or grading schema) to formatively assess each other’s work. In many cases the technological tools will aid in the feedback process (Spector et al., 2016). For example, many pre-made quizzes on sites like Delta Math, MathIsFun.com, and Khan Academy provide automated feedback that displays a correct answer and an explanation of how to arrive at the answer. Consequently, secondary mathematics teachers need to possess the knowledge and skill necessary to analyze automated feedback for its efficacy. As such, FP knowledge requires an understanding of what constitutes sufficient automated feedback, when it is necessary to supplement it, and the best strategies for supplementing it.

Consider, for example, a Geometry class that has recently finished a unit on congruence and similarity in triangles to prove relationships and solve problems with geometric figures. The teacher can demonstrate their ESU knowledge by selecting or designing their own TEFA problems that elicit students’ ability to use proof to explain why two triangles are similar or to justify why two triangles are not congruent. Additionally, the teacher may want to present students with a problem that asks them to apply their knowledge of similarity and congruence to solve an unfamiliar problem. Depending on the computer-based technological tool the teacher has decided would be best (i.e., using their AIG and TK), they will demonstrate their FP
knowledge by monitoring and, when necessary, supplementing automated feedback. Further, they will demonstrate more FP knowledge by applying different strategies to ensure students are processing the feedback they receive. This may include having them pair off to share their responses to the TEFA and discuss the feedback they received.

**Moving Learning Forward.** The Moving Learning Forward (MLF) essential element is concerned with the knowledge and skills teachers need to analyze the data collected during TEFAs and make informed decisions about the next instructional steps. To do this, Popham (2008) recommends that teachers ask themselves a two-part question: “Is an adjustment needed, and, if so, what should that adjustment be?” (p. 23). This means teachers must possess the ability to interpret the data collected by the TEFAs and accurately identify students’ strengths and deficits. This includes an understanding of what it looks like for a student to reach, or not reach, the mathematical learning objective within the confines of the TEFA. Teachers must also possess the knowledge and skill to make decisions, based on the data analysis, about the most appropriate strategies for addressing the identified deficits. MLF, therefore, encompasses knowledge of teaching strategies that are specifically meant to target gaps or deficits in student learning, and the skill to implement them.

Consider a statistics teacher who has recently used a Desmos Quiz as a TEFA to assess students’ understanding of the measures of central tendency (i.e., mean, median, mode). The Desmos Quiz can collect students’ responses and put them into a report for the teacher to view. The teacher can demonstrate their MLF knowledge and skill by successfully identifying and analyzing which measures of central tendency students struggled with. This may include noting a specific question that most students did not respond to correctly and ascertaining what led to incorrect responses. The teacher’s MLF knowledge is further reflected in a decision that
continues to move the students’ learning forward, and helps them to attain the learning objective(s). The teacher may decide that the identified deficit requires re-teaching of all the measures of central tendency, or they may decide to continue instruction as planned and simply incorporate problems that address the deficit into future activities. In either case, this decision should be based on the analysis of the data collected during the TEFA.

An analytic rubric was developed next. The rubric, through the use of qualitative descriptors in ascending levels of proficiency, is intended to highlight the fundamental qualities from each of the essential elements.

**Rubric Development**

The first draft of the rubric (see Appendix A) with qualitative descriptions of the essential elements was then designed. The first draft of the rubric contained four levels of proficiency – Advanced, Proficient, Partially Proficient, and Novice – for each of the essential elements from the Theoretical framework for Secondary Mathematics TEFA Literacy. Each level contained a qualitative descriptor of how a teacher could demonstrate their literacy in the essential element. Figure 8 presents an example of the levels of progression for the Eliciting Student Understanding (ESU) essential element from the first draft of the rubric.

**Figure 8**

*Example of First Draft of Rubric: Elicitation of Student Understanding Essential Element*
At the Advanced level of ESU, the first criteria aim to express that a teacher should be focused on TEFA questions and prompts that will elicit information about students’ understanding of the mathematical concept(s) being assessed. The second criteria aim to provide more detail about what those questions or prompts should look like, by providing “students with a variety of ways to demonstrate their thinking”.

The Novice Level stands contrary to both of these criteria, with TEFA questions or prompts that do not elicit information about students’ understanding, by focusing on memorization or computational procedures. Questions that “focus on memorization and computational procedures” are noted to produce very little evidence of students’ understanding of mathematical concepts (NCTM, 2000).

The Proficient and Partially Proficient levels aim to highlight decreasing ESU Literacy by noting differences in frequency of teacher actions. The frequency at which the teacher selects or designs TEFA questions that elicit understanding has decreased from “consistently” in the Advanced level, to “often” in the Proficient level. Then, in Partially Proficient, the frequency decreases again to “sometimes”. Finally, in the Novice level, the frequency of eliciting understanding has gone down to none and, instead, the teacher demonstrates a Novice level by consistently focusing on memorization and computation. For the second criteria, the frequency of utilizing “memorization and computational procedures” increases from “few” in Proficient, to “some” in Partially Proficient. It is important to note that the quantifiers of “few” and “some” were scrutinized by the Expert Reviewers in the following Phases (see below) for their potential subjectivity and were removed from the final version of the rubric.
Phase 7 and 8: Results of Validating and Rethinking the Framework

As described in Chapter 3, Phase 7 and Phase 8 were completed in an interactive process. In the following sections, I will provide examples of the feedback collected at each stage and how analysis of this feedback resulted in revisions to the theoretical framework and rubric.

Stage 1: Results of Content Validity with Expert Reviewers

Following the process described in Chapter 3, the Expert Reviewers’ feedback resulted in a more precise Theoretical Framework for Secondary Mathematics TEFA Literacy. Expert Reviewer 2, for instance, provided feedback on the clarity of the essential elements, which led to the merging of two essential elements of the Technology Component (Table 10).

Table 10

Example of Expert Reviewer 2’s Feedback and Resulting Revisions to the Initial Draft of the Theoretical Framework

<table>
<thead>
<tr>
<th>Prompt on Review Directions</th>
<th>Expert Reviewer 2’s Response</th>
<th>Revision to Framework</th>
<th>How Revision Addressed Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is the wording of the rubric clear and concise? Do you understand what is being evaluated? If not, how can it be improved?</td>
<td>It would be worth considering whether you need all these criteria. Perhaps you could reduce some of these down to less criteria that still capture the most important indicators</td>
<td>The essential elements Identifying Classroom Technologies (ICT) and Operating Classroom Technologies (OCT) were integrated into one: Technology Knowledge (TK). The essential element Technology and Instructional Goals (TIG) was rephrased to: Align to Instructional Goals (AIG).</td>
<td>Categorized as “interpretive feedback” and analyzed alongside the Technology concepts, these revisions addressed the overlapping concepts within ICT and OCT and highlighted the most important indicator of both of them: a knowledge of technology. Changing TIG to AIG helped to make it clearer what the essential element was attempting to highlight: the knowledge and skill in selecting technological tools that are...</td>
</tr>
</tbody>
</table>
As a result of revisions based on expert’s feedback, the Theoretical Framework for Secondary Mathematics TEFA Literacy included three components composed of seven essential elements (Figure 9).

**Figure 9**

*Components Composed of Essential Elements from the Theoretical Framework for Secondary Mathematics TEFA Literacy*

**Rubric Development.** The rubric was first revised to align to the structure of the theoretical framework. This also necessitated the revision of the corresponding qualitative descriptors, which was guided by analysis of the Expert Reviewers’ feedback. For example, both Expert Reviewers recommend changes to the qualitative descriptors to be more precise (see Table 11).
### Table 11

**Example of Revisions to the First Draft of the Rubric Based on Expert Reviewers’ Feedback**

<table>
<thead>
<tr>
<th>Original Text in First Draft</th>
<th>Expert Reviewer 2’s Feedback</th>
<th>Revision on Second Draft</th>
<th>How Revision Addressed Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Component - Student Learning Element - Advanced Level</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher is consistently selecting, analyzing or designing mathematical tasks on TEFAs that align with students’ prior knowledge and typical learning processes. Mathematics content on the TEFA consistently prompts students to make connections, conjecture, and reason mathematically</td>
<td>Are these two sides of the same coin? Do you need both?</td>
<td>The teacher demonstrates that they are effectively using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs. The teacher is able to identify all common mistakes and misconceptions in student responses to TEFA questions; they demonstrate that they are thinking deeply about why these mistakes and misconceptions happen. <em>NOTE: The teacher must accomplish both criteria to be at that level, otherwise they are at the next lower level</em></td>
<td>Categorized as “interpretive feedback” and analyzed alongside the Mathematics Pedagogy concepts, these revisions addressed the feedback with: An additional note to clarify what is necessary to obtain a level of proficiency. Qualitative descriptors that better reflect two of the main factors within the essential element of Student Learning: prior knowledge and identifying mistakes/misconceptions. The elimination of a redundancy in the descriptors on the rubric between Student Learning and Eliciting Student Understanding.</td>
</tr>
<tr>
<td>Technology Integration Component - Operating Classroom Technologies - Partially Proficient Level</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| For at least one mathematics specific technology, the teacher is able to explain the necessary materials/resources for operating the technology **and at least one** of the following: - how teachers interact with the technology - how students interact with the technology The teacher cannot explain how to make modification to pre-designed TEFAs (when applicable) | This [referring to the second paragraph] doesn’t really help me with assessing the evidence. Perhaps it could be left out. [In general …] the wording of the rubric is clear but the quantity of description gives the reader a lot of information to digest and process about one category. It would be worth trying to shorten the wording in places. … it is not always immediately clear whether all of these need to be evidenced or only one. The teacher can identify 2-3 different technologies that are appropriate for formative assessments of 2-3 different topics in mathematics. The teacher attempts to, but struggles to explain how to use these different technologies from both the student and teacher perspective. | The first comment was categorized as “direct feedback” and was incorporated by eliminating the second paragraph. The second comment was categorized as “interpretive feedback” and analyzed alongside the Technology concepts, these revisions addressed the feedback by reducing the length of the descriptors into one simplified version that highlights the breadth and depth of necessary Technology Knowledge (TK).

**NOTE:** This description is for the new, integrated essential element: Technology Knowledge (TK). See Table 10.

Revisions resulted in the second draft of the rubric (Appendix D) which now contained qualitative descriptors for the seven essential elements.

**Stage 2: Results of Inter-rater Reliability Analysis of Mock Responses with External Raters**

Analysis of evaluations of the Mock Responses by two independent External Raters indicated that there were three out of the seven *overall levels* of proficiency that had exact matches, and two out of the seven *overall levels* of proficiency that were different by only one
level. Thus, the inter-rater agreement was only 71.4%. Based on analysis of the External Raters’ comments in the “Review Directions” and follow-up interviews, four of the qualitative descriptors on the rubric received major editing, such as restructuring the wording. The other three qualitative descriptors on the rubric received minor editing, such as moving or replacing a word with a synonym. For example, in the follow-up interview, External Rater A commented that not all of the descriptors for the Eliciting Student Understanding (ESU) essential element were a clear explanation of measurable skills. The ESU descriptors were then revised to reflect this feedback (see Table 12).

Table 12

Example of Revisions to the Second Draft of the Rubric Based on External Rater A’s Feedback

<table>
<thead>
<tr>
<th>Formative Assessment Component – Eliciting Student Understanding – Advanced Level</th>
<th>Original Text in Second Draft</th>
<th>External Rater A’s Feedback</th>
<th>Revision on Third Draft</th>
<th>How Revision Addressed Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that provide students with a variety of ways to demonstrate their understanding. i.e.: There is a strong emphasis on explanation, justification, and problem solving; <strong>no</strong> questions utilize memorization and computational procedures.</td>
<td>The i.e. part, to me, seems like a better description … I used that to help me evaluate the ESU problems more so than the first part [referring to the first paragraph].</td>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that strongly emphasize explanation, justification, and problem solving; <strong>no</strong> questions utilize memorization and computational procedures.</td>
<td>Categorized as “interpretive feedback” and analyzed alongside the Formative Assessment concepts, this revision addresses the feedback with a qualitative descriptor that focuses more on the language used in the “i.e. part”. Further, aligned with many of the concepts that helped to form the ESU element, the qualitative descriptor eliminates the ambiguity of the</td>
<td></td>
</tr>
</tbody>
</table>
It is important to note that following the External Raters’ feedback, the Moving Learning Forward (MLF) essential element was found to be irrelevant to the test (see Part 2 below). However, I made a decision to keep the MLF element in the theoretical framework.

**Stage 3: Results of Inter-rater Reliability Analysis Based on Data From Field Testing**

Since inter-rater reliability was not established during evaluation of the Mock Responses to the test, the third draft of the rubric was used to evaluate Heather’s responses to the *TEFA Literacy Test for Secondary Mathematics*. This time the evaluation was completed independently by the researcher and by the External Rater B using the evaluation method as described in Chapter 3, Part 3, Data Analysis. A comparison of my evaluation of Heather’s responses and External Rater B’s evaluation, using the procedures described in Chapter 3, Part 1, Stage 2, produced 90.6% agreement (see Appendix K) and therefore, the rubric (see Appendix F) was validated.

**Part 2: The TEFA Literacy Test for Secondary Mathematics and Evaluation Method**

Part 2 focuses on answering the second research question: How can Secondary Mathematics TEFA Literacy be evaluated through a performance-based instrument? In the following section, I will summarize the results for each step of development of the *TEFA Literacy Test for Secondary Mathematics*, provide examples, and finish by presenting the final version of the test.
Step 1: Identified Construct

DeVellis describes this step as determining “clearly what it is you want to measure” (2012, p. 73). In the case of the TEFA Literacy Test for Secondary Mathematics, what I wanted to measure was PSTs’ proficiency in each of the components and their essential elements from the Theoretical Framework for Secondary Mathematics TEFA Literacy.

Step 2: Results for the Initial Item Pool and Evaluation Method

Item Pool

The results of Step 2 were an initial pool of test items that included 12 classroom scenarios, each followed by two to three open-ended questions. As described in Chapter 3, Part 2, Step 2, the CCSSM (2010), selected essential elements, and selected technological tools guided the development of the classroom scenarios and following questions. An example of a scenario with questions from the first draft of the test is shown in Table 13.

Table 13

Example of Development of a Scenario and Questions from the First Draft of the TEFA Literacy Test for Secondary Mathematics

<table>
<thead>
<tr>
<th>Standards Guiding Mathematics Content:</th>
<th>Mr. O’Connor, an Algebra 1 teacher, is working with students to graph linear functions in slope-intercept and point-slope form. Students need to be able to graph the function on the x-y-axis, identify the x- and y-intercepts, and calculate the slope.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CCSS.MATH.CONTENT.8.F.A.2</td>
<td>a) Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions.</td>
</tr>
<tr>
<td>CCSS.MATH.CONTENT.8.F.B.4</td>
<td>b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.</td>
</tr>
<tr>
<td>Most applicable class: Algebra 1 (either eighth or ninth grade)</td>
<td>c) Given the question/prompt that you designed in part (b), what might you be able to infer about students’ learning?</td>
</tr>
<tr>
<td>Essential Elements to be Addressed:</td>
<td></td>
</tr>
<tr>
<td>• Identifying Classroom Technologies (ICT)</td>
<td></td>
</tr>
<tr>
<td>• Technology and Instructional Goals (TIG)</td>
<td></td>
</tr>
<tr>
<td>• Eliciting Student Understanding (ESU)</td>
<td></td>
</tr>
<tr>
<td>• Student Learning (SL)</td>
<td></td>
</tr>
<tr>
<td>Mathematical Proficiency (MP)</td>
<td></td>
</tr>
</tbody>
</table>
Beginning with the mathematical content to be addressed, the classroom scenario was
developed first. Looking at the CCSM (2010), concept(s) were selected from seventh grade to
twelfth grade level. This aligns to the grade range of most secondary teaching licenses. In the
example above, the CCSSM (2010) concepts are most closely aligned to an Algebra 1 course
with a learning objective about graphing linear functions.

Next, the essential elements from the theoretical framework guided the development of
the scenarios and their subsequent questions. In some instances, part of the scenario included an
element of a computer-based technological tool that participants’ have to analyze to elicit
evidence of their literacy in the essential elements. For example, to elicit evidence of their MP
literacy, a participant may be provided with an example of a pre-fabricated TEFA question and
asked to analyze its ability to promote the different strands of mathematical proficiency.
However, as this scenario and its questions intended to elicit evidence of the participants’ ability
to identify an appropriate technological tool (i.e., ICT), no tool was included in the scenario. In
the above example, the essential elements to be addressed by the question included: ICT, TIG,
ESU, SL, and MP. Asking the participant to “identify a computer-based technology” in Question
(a) was intended to prompt for evidence of their literacy in the essential elements ICT and TIG.
In Question (b), asking the participant to use the technological tool to provide an example of a
question they might ask students was intended to prompt for evidence of the participants’ literacy
in the essential elements SL and ESU. In particular, the question participants’ design should,
according to the first draft of the rubric (Appendix A), prompt “students to make connections,
conjecture, and reason mathematically” and provide them with “a variety of ways to demonstrate
their thinking.” Lastly, Question (c) intended to elicit evidence of the participants' MP by asking them to analyze the question for what they could “infer about students’ learning?”, specifically prompting with terms from the five strands (Kilpatrick et al., 2001) like “procedural fluency, reasoning”.

The first draft of the TEFA Literacy Test for Secondary Mathematics had a total of 12 scenarios each followed by two or three questions resulting in a total of 35 questions (see Appendix B).

**Evaluation Method**

Individual questions on the test were not intended to elicit responses that would address every essential element within the rubric. Thus, a key, known as the “Applicable Rubrics” was added to the test. For example, the questions following Scenario 11 (see Table 14) on the first draft of the test were intended to elicit responses that could be evaluated by ICT, OCT, TIG, ESU, SL, and MP essential elements, and these are listed as the “Applicable Rubrics”.

Additional instructions for how to evaluate the test were also added to the beginning of the rubric. These instructions were continuously improved upon, as will be detailed in the following sections.

**Table 14**

*Example of Applicable Rubrics for Evaluating Responses to Questions Following Scenario 11 on the First Draft of the TEFA Literacy Test for Secondary Mathematics*

<table>
<thead>
<tr>
<th>Scenario 11</th>
<th>Applicable Rubrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ms. Fischer and her students are working on graphing quadratic functions, and understanding how the coefficients in standard form transform the graph of a parabola (i.e., how different values of a,b,c in $y = ax^2 + bx + c$ transform the graph).</td>
<td>ICT</td>
</tr>
<tr>
<td>a) Describe a technology enhanced formative assessment activity Ms. Fisher could engage students in?</td>
<td>OCT</td>
</tr>
</tbody>
</table>
b) What resources and materials will Ms. Fisher and her students need to complete this activity?
c) Provide an example of a question or prompt you might give students. Explain what you intend to measure about students’ understanding with this question/prompt (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?)

Applying the Evaluation Method. Using the “Applicable Rubrics”, the evaluator can determine the participant’s preliminary levels of literacy. As an example, consider the questions following Scenario 3 from the final TEFA Literacy Test for Secondary Mathematics with the “Applicable Rubrics”: MP, FP, and TK (see Figure 10).

Figure 10

TEFA Literacy Test for Secondary Mathematics: Example Responses to Scenario 3 Questions

Scenario 3:
Mx. Knope is working with students on the properties of exponents, both whole number and rational, and decided to use a Khan Academy © quiz to formatively assess students. One of the questions asks:

MP

| MP | a) What mathematical concepts is this question assessing? What mathematical proficiencies (e.g., conceptual understanding, procedural fluency, strategic |

Neil tried to rewrite the expression $\frac{s^{-5}}{s^{-3}}$:

- $s^{-4}$
- $s^{-2}$
- $\frac{1}{s^2}$

Step 1: $s^{-4}$
Step 2: $s^{-2}$
Step 3: $\frac{1}{s^2}$

Did Neil make a mistake? If so, in which step?

Choose 1 answer:

- Neil did not make a mistake.
- Neil made a mistake in Step 1.
- Neil made a mistake in Step 2.
- Neil made a mistake in Step 3.
competence, adaptive reasoning, and/or productive dispositions) is this question prompting? Explain your responses.

This question is assessing students’ knowledge of exponent rules (i.e. how to proceed when there is an exponent in the numerator and an exponent with the same base in the denominator). This question does test conceptual understanding, because error assessment can be a valuable tool in seeing how comfortable a student is with a concept. However, the fact that it is multiple choice and does not ask students to explain their thinking means that it is more difficult to assess conceptual understanding, because students could simply make a good guess. This question also addresses procedural fluency, because it is asking students to think about the process involved in solving problems of this type. Therefore, I would also say this question addresses strategic competence, because it forces students to think about how they would solve this problem.

b) This quiz provides students with correct/incorrect feedback. What additional feedback (if any) would you provide students if they made a mistake? Explain your response.

Since correct/incorrect feedback is not necessarily helpful, I would, in an ideal situation, want to ask each student why they chose the answer that they did. In explaining their thinking (which this question does not require), perhaps they would see their mistake. However, if this isn’t possible, I would provide each student with feedback about each step (i.e. Step 1 is done correctly because...) This way, they could go step by step through the problem and truly comprehend each element of it.

c) To the best of your knowledge, explain to Mx. Knope how they can access a report on students’ performance on this Khan Academy © quiz.

Although I have never used a Khan Academy quiz, I believe that the teacher has access to their roster of students. Each student and each question is displayed in a grid, showing which questions which students answered correctly. If you click on the individual student, you can see their individual work, or you can look at the class as a whole to see areas of difficulty.

Next, consider the qualitative descriptors for the essential element MP from the final rubric in Figure 11.

**Final Rubric: Mathematical Proficiency (MP) Qualitative Descriptors**

<table>
<thead>
<tr>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Proficiency (MP)</td>
<td>The teacher is able to analyze, select, and design questions/prompts that promote all of the 5 Strands of Mathematical Proficiency. Teacher can analyze students' responses for their alignment to all of the 5 Strands of Mathematical Proficiency.</td>
<td>Some evidence that the teacher is able to analyze, select, or design questions/prompts that promote 3 or 4 of the 5 Strands of Mathematical Proficiency. Teacher struggles to analyze students' responses for their alignment to some of the 5 Strands of Mathematical Proficiency.</td>
<td>Little to no evidence that the teacher is able to analyze, select, or design questions/prompts that promote the 5 Strands of Mathematical Proficiency. Teacher is unable to analyze students' responses for their alignment to the 5 Strands of Mathematical Proficiency.</td>
</tr>
</tbody>
</table>

**NOTE:** The teacher must accomplish both criteria to be at that level, otherwise they are at the next lower level.
Using the descriptors on the rubric for MP (Figure 11) to evaluate the response to Question (a) (Figure 10), an evaluator might note that the participant is analyzing the TEFA for its ability to develop the strands of mathematical proficiency, including: conceptual understanding, procedural fluency, and strategic competence. This in-depth analysis is therefore at the Proficient Level.

Next, consider the qualitative descriptors for essential element FP from the final rubric in Figure 12.

**Figure 12**

*Final Rubric: Feedback Process (FP) Qualitative Descriptors*

<table>
<thead>
<tr>
<th>Feedback Process (FP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher is able to analyze, select, or design TEFA that provide targeted feedback and plans for supplemental feedback.</td>
</tr>
<tr>
<td>The teacher plans for students to be active participants in the feedback process in a variety of ways (e.g., discussions, guided commentary, peer use of rubric/assessment criteria, etc.).</td>
</tr>
<tr>
<td>The teacher is able to analyze, select, or design TEFA that provide targeted feedback, but does not plan for supplemental feedback.</td>
</tr>
<tr>
<td>The teacher shows evidence of being active participants in the feedback process primarily through class/peer discussions.</td>
</tr>
</tbody>
</table>

NOTE: The teacher must accomplish both criteria to be at that level, otherwise they are at the next lower level.

Looking at the example response to Question (b) (Figure 10), the participant first noted that the use of correct and incorrect feedback is “not necessarily helpful”. Then, they go on to give two examples of how to provide students with feedback. The first example involves students in the feedback process by asking them questions, while the second provides targeted feedback with step-by-step explanations. Based on the qualitative descriptors in the rubric, the evaluator would most likely evaluate the response at a Proficient level for targeted feedback that involves a plan for discussion.

Finally, consider the qualitative descriptors for the essential element TK on the final rubric in Figure 13.
Examining the example response for Question (c) (Figure 10), the evaluator would most likely assign a *preliminary level* of Partially Proficient. The participant admitted to a lack of experience with Khan Academy quizzes, but still attempted to provide an explanation of how to use the technology. This explanation demonstrates that they have knowledge of other, similar, technological tools and how to use them to access reports on students’ performance.

**Step 3: Results of the Review by Experts**

As described in Chapter 3, the review of the test was completed in two stages. First, the Expert Reviewers examined the test along with the framework and the rubric and provided their recommendations. Then, the External Raters provided feedback about the test items after they evaluated Mock Responses to the test using the rubric. In this section, I will provide examples of how the analysis of the Expert Reviewers’ and External Raters’ feedback led to revisions to the test and its evaluation method.

**Expert Review**

Based on the Expert Reviewers’ feedback (see Appendix I), the context of the scenarios on the test did not need any significant revisions. The wording of the subsequent questions,
however, was changed to better elicit appropriate and measurable responses (see Table 15). No scenarios or questions were removed at this point.

**Table 15**

*Examples of Revisions to First Draft of the Test Based on Expert Reviewer 2's Feedback*

<table>
<thead>
<tr>
<th>Original Text on First Draft</th>
<th>Expert Reviewer 2 Feedback</th>
<th>Revision on Second Draft</th>
<th>How Revision Addressed Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 6 - Question (c)</strong></td>
<td>This sounds rather vague. I wouldn’t be clear whether for example “verbally” was sufficient as an answer. I suspect you would like more than this?</td>
<td>Given the most common mistake on students’ projects was confusing the correlation coefficient (R) with the coefficient of determination (R^2), what feedback might you give students? Describe how you could use the platform/method you chose in part (a), if possible, to provide this feedback. (If not possible, describe an alternative way to provide feedback.)</td>
<td>Categorized as “interpretative feedback” and analyzed alongside the Technology and Formative Assessment concepts: The question now specifically asks for the participant to either use the technology to provide feedback or to use an alternative method. This eliminates the possibility of participants not knowing how to appropriately format their response. Further, it will elicit more data on the participants’ knowledge of the technology they selected in part (a).</td>
</tr>
</tbody>
</table>

| **Scenario 12 - Question (a)** | “Analyze” sound rather broad. What depth of analysis and level of detail are you expecting? Could you | Do you think this is an appropriate use of technology for the given learning | Categorized as “direct feedback”: The word “analyze” is replaced by asking about the |
**Evaluation Method.** In Part 1, Expert Reviewers’ feedback helped to contribute to the simplification of the essential elements. Specifically the ICT and OCT essential elements were simplified into one, TK, and TIG was rephrased to AIG. Thus, following the changes to the wording of questions on the first draft of the test, see above examples, I updated the “Applicable Rubrics” on the second draft of the test to reflect the new codes.

Lastly, the Expert Reviewer 1 provided feedback on the proposed evaluation method from the first (see Table 16).

**Table 16**

*Example of Revisions to the Evaluation Method Based on Expert Reviewer 1's Feedback*

<table>
<thead>
<tr>
<th>Original Text on First Draft</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Each question on the test corresponds to most, but not all of the elements within each Component. Each question has “Applicable Rubrics” noted on the test.</td>
<td></td>
</tr>
<tr>
<td>• Literacy Levels for each element will be determined by simple majority.</td>
<td></td>
</tr>
<tr>
<td>Expert Reviewer 1 Feedback</td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td></td>
</tr>
<tr>
<td>The proposal that the simple majority wins might not be a good separatrix in every circumstance...e.g. if it’s a 5-4 decision out of nine items, because there could be weighting issues on each item in a cluster for a specific context in which there are multiple rubrics that would sway a judge to award a higher or lower overall score based on what else is happening to the scores.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Revision on Second Draft</th>
</tr>
</thead>
</table>
| Each question on the test corresponds to most, but not all of the elements within each Component. Therefore, each rubric notes the “Applicable Questions”  
  - For example: The element “Mathematical Proficiency (MP)” is applicable to Questions 1a, 2a, 3a, 4a, 4c, …  
    - The grader should use MP to evaluate all applicable questions and then determine an overall level of proficiency in that element (i.e., Advanced, Proficient, Partially Proficient, or Novice).  
  - Pay attention to the “NOTES” on the left-hand side of the rubric.  
  - An * indicates that the “Applicable Question” may be applicable, but it will depend if the participant includes this in their answers  
    - For example: In question 5b, the participant has the option to “not use technology” which would, therefore, make the Technology element TK not applicable.  
  - Final “Proficiency Levels” are recorded on the last page. |

<table>
<thead>
<tr>
<th>How Revision Addressed Feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>The revisions are meant to better guide the evaluation of responses and determination of an overall proficiency level for each element. Most importantly, the directions have eliminated the concept of a simple majority and, instead, ask that the evaluator consider all applicable responses in their determination of a proficiency level. Additionally, several “NOTES” were added within the rubric to help evaluators have a more concrete method by which to decide between levels of proficiency.</td>
</tr>
</tbody>
</table>

In the second draft of the review directions (Table 16), a few of the “Applicable Rubrics” may or may not be elicited by the question. For example in Scenario 5, Question (b) on the
second draft of the test (see Appendix E), the participant has the option to not use technology. If the participant does select a new computer-based technological tool, it would not be possible to elicit evidence of their TK literacy. It is important to note that these potentially “Applicable Rubrics” were scrutinized by the External Raters following their evaluation of the Mock Responses and were revised prior to the field testing.

**Review by External Raters**

External Raters’ reviews contributed to a number of revisions to the content of the *TEFA Literacy Test for Secondary Mathematics*, including the removal of Scenario 6. In creating the Mock Responses, I noticed that the time it took to complete the 35 questions on the test was over an hour. Studies show that questionnaires and surveys that are over 30 questions long and take more than 30 minutes may result in a lower response rate as well as participants spending less time on each question (Chudoba, 2023; Sharma, 2022). Thus the External Raters were also asked to analyze the scenarios to determine if one could be removed. External Rater A suggested Scenario 6 be removed from the test, commenting that there were “too many stats questions”. External Rater B suggested that Scenario 8 be removed from the test, noting they did “not see how technology would make a difference” and the questions would need to focus more on why the technology “really facilitates formatively assessing”. Based on analysis of this feedback and that from the interviews, Scenario 6 was removed and the questions following Scenario 8 were revised (see Table 17).

**Table 17**

*Example of Revisions to Scenario 8 Based on External Rater B's Feedback*

<table>
<thead>
<tr>
<th>Original Text on Second Draft</th>
<th>External Rater B’s Feedback</th>
<th>Revision on Final Test</th>
<th>How Revision Addressed Feedback</th>
</tr>
</thead>
</table>

105
Scenario 8 (now Scenario 7 due to removal of Scenario 6)

<table>
<thead>
<tr>
<th>a) What other prior knowledge might students need to answer this question?</th>
</tr>
</thead>
<tbody>
<tr>
<td>I do not see how technology would make a difference … [the question] needs to tie into the use more, and why it really facilitates formatively assessing the student.</td>
</tr>
<tr>
<td>a) What other prior knowledge might students need to answer this question?</td>
</tr>
<tr>
<td>What are some common misconceptions or mistakes students might make on a problem like this?</td>
</tr>
<tr>
<td>b) How well do you think this problem assesses students’ understanding (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions)? Explain your response.</td>
</tr>
<tr>
<td>b) How well do you think this problem assesses students’ understanding (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions)? Explain your response.</td>
</tr>
<tr>
<td>c) Provide an example question/prompt that you believe would assess students’ achievement of the learning objective. Be specific about the technology you would choose and why you would choose it. If you would not use technology, explain why not.</td>
</tr>
<tr>
<td>c) Provide an example question/prompt that you believe would assess students’ achievement of the learning objective. Be specific about the technology you would choose and why you would choose it. If the task is NOT accomplishable via technology, explain why not.</td>
</tr>
<tr>
<td>Categorized as “interpretive feedback:” and analyzed alongside Mathematics Pedagogy, Technology, and Formative Assessment concepts:</td>
</tr>
<tr>
<td>Part a) was elaborated on to better address the descriptors of the Student Learning essential element on the rubric.</td>
</tr>
<tr>
<td>Part b) was rephrased to specifically prompt participants to address the descriptor: “analyze” from the Mathematical Proficiency and Eliciting Student Understanding essential elements on the rubric.</td>
</tr>
<tr>
<td>Part c) was rephrased to be more direct about providing an explanation as to why the question/prompt they designed was not accomplishable via technology, rather than just asking why they would choose not to use technology. This is intended to focus more on the use</td>
</tr>
</tbody>
</table>
Lastly, it was determined that the Moving Learning Forward (MLF) essential element was irrelevant to the test and questions that sought to elicit evidence of MLF were revised or removed. This decision was based on the following reasoning: During a follow-up interview, External Rater A commented that they “struggled the most” with the assessment of the MLF element. In particular, they struggled to see how the test could adequately elicit this competency. Upon additional comparison to the formative assessment concepts, it was noted that the decisions teachers make about how and when to move students’ learning forward during an in-class formative assessment often have to be made in the moment (Black & Wiliam, 2009; Otero, 2006). If the TEFA is producing real-time evidence that students need to re-learn or move-forward, the teacher would need to make a decision, without much time for contemplation, about the appropriate next move. Asking participants to provide a response on a written test, in which they have time to think, may not capture their true literacy in MLF. For TEFAs that are completed asynchronously, teachers’ analysis of the data collected by the technological tool would most likely include the analysis of the whole class as well as data that has been previously collected on students’ understanding of related content. Consequently, it may prove extensively challenging to provide a classroom scenario on the *TEFA Literacy Test for Secondary Mathematics* that provides enough detail and context to adequately elicit a teacher’s literacy in the MLF essential element in a reasonable amount of time. Suggestions for how to elicit evidence of the essential element MLF and evaluate it using the rubric will be presented in Chapter 5.
The final version of the test contained 11 Scenarios, each with three questions, for a total of 33 questions.

**Evaluation Method.** External Rater A commented that the evaluation method was “a long process” and they would like to have numbers to “somehow average to get an overall score”. This led to the development of the “External Rater Evaluation Guide” (see Appendix H) which helps to organize the evaluation process. The guide also included the introduction of a “Numerical Level” for each proficiency level, which facilitated calculations of the overall levels of proficiency in each essential element as the average of the preliminary levels. Importantly, this “External Rater Evaluation Guide” seeks to ensure that External Raters follow the same a priori coding process as described in Chapter 3, Part 3.

Following changes to the test, each question was reassessed for its alignment to the intended essential elements and the “Applicable Rubrics” were updated prior to the administration of the test to the participants in the field.

**Part 3: Evaluation of PSTs’ TEFA Literacy Levels**

Part 3 seeks to answer the third research question: What are the TEFA Literacy levels of secondary mathematics preservice teachers enrolled in a graduate level initial certification program at a private urban university as assessed by this instrument? In this part I will provide an example of the evaluation of a response to the *TEFA Literacy Test for Secondary Mathematics*. I will conclude with a summary of the analysis of the participants’ TEFA Literacy levels.

**Example Evaluation of TEFA Literacy Levels: Mitch**

As described in Chapter 3, participants' responses to the final version of the *TEFA Literacy Test for Secondary Mathematics* (Appendix G) were evaluated following an a priori coding method that used the final version of the rubric (Appendix F).
In order to demonstrate the process of evaluation of test responses, I selected Mitch’s response to Question (a) in Scenario 1.

**Scenario 1:**
Mr. O’Connor, an Algebra 1 teacher, is working with students to graph linear functions in slope-intercept and point-slope form. The learning objective is that: *Students will be able to graph the function on the x-y-axis, identify the x- and y-intercepts, and calculate the slope.*

**Question (a):**
Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions.

**Mitch’s Response:**
Mr. O’Connor and his students could first *utilize the graphing calculator on Desmos* and the *slider function to see how the slope and y-intercept affect the line on the coordinate plane.* Students will identify the x-intercepts as the point where the y-coordinate is equal to zero and the y-intercept as the point where the x-coordinate is equal to zero. The slope can then be calculated by observing the change in y values divided by the change in x values.

(Emphasis is added by the researcher)

For this question the “Applicable Rubrics” were Alignment to Instructional Goals (AIG) and Technology Knowledge (TK). In the Mitch’s response, “utilize the graphing calculator on Desmos” was coded as AIG, “Desmos” was coded as TK, and “slider function to see how the slope and y-intercept affect the line on the coordinate plane” was coded as both AIG and TK. The last two sentences of the response were not coded since they did not analyze or explain how to use the technology (Desmos) to perform these actions.

Using the qualitative descriptors for AIG and TK on the rubric (see Appendix F), the coded words and phrases were then evaluated for their *preliminary levels* of literacy and assigned a “Numerical Level” as shown in Table 18.
### Table 18

*Example of Evaluating Mitch's Response to Scenario 1, Question (a): Preliminary Levels*

<table>
<thead>
<tr>
<th>Applicable Rubrics</th>
<th>Coded Words and Phrases</th>
<th>Preliminary Level</th>
<th>Numerical Level</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIG</td>
<td>utilize the graphing calculator on Desmos slider function to see how the slope and y-intercept affect the line on the coordinate plane.</td>
<td>Advanced</td>
<td>3</td>
<td>The selection of the graphing calculator on Desmos is directly aligned to the mathematical learning objective about graphing and identifying important features of a linear function. The description of how students would use the slider tool shows that Mitch may be selecting this technology specifically for its mathematical fidelity in relation to graphing linear functions. Desmos has a number of built-in tools that can be used for formative assessment.</td>
</tr>
<tr>
<td>TK</td>
<td>Desmos slider function to see how the slope and y-intercept affect the line on the coordinate plane.</td>
<td>Proficient</td>
<td>2</td>
<td>For the first criteria of TK: Desmos is an appropriate technology for this mathematics learning objective and for formative assessment. For the second criteria of TK: Mitch is providing a basic explanation of the features of Desmos, but is not providing detailed or advanced directions for how to use the technology.</td>
</tr>
</tbody>
</table>

Mitch’s *overall level* of proficiency in AIG and TK was determined by averaging the “Numerical Levels”, rounding to a whole number, and converting back into the corresponding categorical proficiency level. See Table 19 for an example of how this was done for Mitch in the AIG essential element.
Table 19

*Example of Determining Mitch's Overall Level of Proficiency Level in AIG*

<table>
<thead>
<tr>
<th>Scenario, Question</th>
<th>Words and Phrases Coded as AIG</th>
<th>Preliminary Level</th>
<th>Numerical Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (a)</td>
<td>- utilize the graphing calculator on Desmos</td>
<td>Advanced</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>- slider function to see how the slope and y-intercept affect the line on the coordinate plane.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 (b)</td>
<td>- students can calculate the rates of change between the two consecutive points from the table.</td>
<td>Advanced</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>- students plotting points onto desmos and looking to see if a linear or exponential regression would be appropriate.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 5 (b)              | *Interview Transcripts*  
- I would use the graphing calculator  
- I’ve done some regression on excel, but not nearly enough to where i’d be comfortable doing it without some prep work  
- TI-84 | Novice            | 0               |
| 6 (a)              | - show their students a video of a two-dimensional cross-section on Youtube  
- ask students to find the area of a cross section or volume | Novice            | 0               |
| 7 (c)*             | *did not select a new technological tool - AIG not applicable* | N/A              | Ignored         |
| 8 (c)*             | - the benefit is the instant response to see whether students go a solution correct  
- the disadvantage seems to be that teachers cannot see how students arrived a their solution, making it difficult to provide glow and grows | Advanced          | 3               |
| 10 (a)             | - the graphing calculator on Desmos | Advanced          | 3               |
the slider tool to change the values of one of these values and students can see how it affects the graph

- Kahoot! is limited by its inability to provide meaningful feedback that considers possible misconceptions the students have and concrete steps on how to improve

<table>
<thead>
<tr>
<th>Essential Element</th>
<th>Mathematical Proficiency</th>
<th>Student Learning</th>
<th>Alignment to Instructional Goals</th>
<th>Technology Knowledge</th>
<th>Elicitation of Student Understanding</th>
<th>Feedback Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mitch</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
</tbody>
</table>

Mitch was devalued as having an overall Proficient level of literacy in AIG. This was evidenced by the fact that while Mitch was able to consistently analyze technology for its alignment to formative assessment and mathematical learning objectives, he sometimes struggled to select technology that was aligned to the instructional goal of formative assessment. For example, in Scenario 6, Question (a), Mitch selected the technology “Youtube” to share a “video of two-dimensional cross-section” with students. This describes an instructional activity using a conveyance technology (Dick & Hollebrands, 2011; Hollebrands, 2017) rather than a formative assessment activity that would elicit data pertaining to students’ understanding of the mathematical concept.

**Results for All Participants**

Using the same process for all participants, I determined participants’ overall levels of proficiency for each essential element in the rubric (see Table 20).

**Table 20**

*Participants’ Overall Levels of Proficiency in Each Essential Element*
<table>
<thead>
<tr>
<th>Name</th>
<th>Proficient</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Proficient</th>
<th>Advanced</th>
<th>Proficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heather</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Proficient</td>
</tr>
<tr>
<td>Brad</td>
<td>Proficient</td>
<td>Partially Proficient</td>
<td>Advanced</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Partially Proficient</td>
</tr>
<tr>
<td>Megan</td>
<td>Partially Proficient</td>
<td>Partially Proficient</td>
<td>Proficient</td>
<td>Partially Proficient</td>
<td>Partially Proficient</td>
<td>Novice</td>
</tr>
<tr>
<td>Michael</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Proficient</td>
</tr>
</tbody>
</table>
Chapter 5: Discussion and Conclusion

In this chapter, I will first summarize the findings of the three parts of the study. Next, I will reflect on and discuss the evolution of the theoretical framework and rubric, the design of the TEFA Literacy Test for Secondary Mathematics, and the evaluation of participants’ responses to the test. Finally, I will examine limitations of this study and provide recommendations for further research and practice.

Summary

This exploratory study began with the aspiration to determine what secondary mathematics PSTs knew about and can do when it came to TEFA. Through an extensive review of literature, it was found that there was no clear definition of Secondary Mathematics TEFA Literacy, nor was there a standardized method to determine one’s TEFA Literacy levels. Thus, the exploratory study needed to, first, develop a theoretical framework that defines Secondary Mathematics TEFA Literacy and, then, develop a standardized instrument (i.e., TEFA Literacy Test for Secondary Mathematics) that could be used to elicit evidence of Secondary Mathematics TEFA Literacy in PSTs. The Theoretical Framework for Secondary Mathematics TEFA Literacy, corresponding rubric, and the TEFA Literacy Test for Secondary Mathematics were developed alongside each other, undergoing numerous stages of revisions that led to final validated Theoretical Framework for Secondary Mathematics TEFA Literacy, corresponding rubric, and the TEFA Literacy Test for Secondary Mathematics. The developed instruments were then used in a case study that examined TEFA Literacy of secondary mathematics PST.

The Theoretical Framework for Secondary Mathematics TEFA Literacy and corresponding rubric were developed first. The theoretical framework outlines the components composed of the essential elements that make up one’s TEFA Literacy. The rubric gives context
to the essential elements, providing qualitative descriptors in ascending levels of proficiency for the ways in which teachers can demonstrate their TEFA Literacy. Closely following the development of the theoretical framework, the TEFA Literacy Test for Secondary Mathematics sought to elicit evidence of Secondary Mathematics TEFA Literacy that can be evaluated by the rubric. Though separated into parts for the purposes of describing the methodology and findings, the development of the theoretical framework, rubric, and test were closely linked. A revelation or change to the essential elements of the theoretical framework necessitated changes to the rubric which, in turn, led to revisions to the test.

The final part of the exploratory study, Part 3, was intended to fulfill the original aspiration of determining secondary mathematics PSTs’ TEFA Literacy. The case study of the five secondary mathematics PSTs gave insights into their TEFA Literacy levels. In particular, the PSTs’ responses to the test revealed to both myself and the participants their strengths and deficits. As one participant noted: “It was a really good survey of all the different technologies that are out there. And I felt fortunate in that I recognized almost all of them … I might not be able to use many of these platforms yet, but at least I know where to start.”

Conclusions

This study sought to answer three research questions:

1. What are the components and essential elements of the theoretical framework for Secondary Mathematics TEFA Literacy?

2. How can Secondary Mathematics TEFA Literacy be evaluated through a performance-based instrument?
3. What are the TEFA Literacy levels of secondary mathematics preservice teachers enrolled in a graduate level initial certification program at a private urban university as assessed by this instrument?

Research Question 1: The Theoretical Framework for Secondary Mathematics TEFA Literacy

In this section, I will reflect on the evolution of the Theoretical Framework for Secondary Mathematics TEFA Literacy. Starting with a survey and an analysis of literature sources related to mathematics pedagogy, technology integration, and formative assessment, the Theoretical Framework for Secondary Mathematics TEFA Literacy began as a broad and a lengthy list of concepts.

One early, but notable development, was the elimination of a potential essential element: Mathematical Content Knowledge (MCK). Ball et al. (2008) delineates the necessary *mathematical knowledge for teaching* with a number of domains, including teachers’ MCK as it pertains to how students learn and make mistakes in mathematics, as well as how teachers use MCK to appropriately sequence content for students. Thus, while MCK does play an essential role in teachers’ Mathematics Pedagogy (the first component of the theoretical framework), it serves more as the foundational knowledge necessary for a teacher to successfully develop students’ mathematical proficiencies, as well as plan for and understand how students typically learn mathematics. The decision not to include MCK as an essential element in the theoretical framework is consistent with other mathematics related frameworks, like AMTE’ (2009) Mathematics TPACK Framework, that do not specifically address teachers’ MCK. The NCTM’ *Principles and Standards for School Mathematics*, similarly, do not list MCK as a separate
standard, rather, it is noted that “To be effective, teachers must know and understand deeply the mathematics they are teaching” (2000, p. 17).

The Theoretical Framework for Secondary Mathematics TEFA Literacy encompasses both the knowledge and skills necessary for secondary mathematics teachers to effectively engage students in TEFAs. These knowledge and skills have been divided into the three key components of Mathematics Pedagogy, Technology, and Formative Assessment, each with a subset of essential elements (see Figure 9). And, while separating each of the components was helpful for analysis and defining of the theoretical framework, educators must understand that these components really exist in a careful balance with each other. No one component or essential element can stand alone, with the level of knowledge and skill in each serving as a determining factor of how well another can be accomplished.

As Mishra and Koehler (2006) noted in the TPACK Framework, integrating technology into instruction and assessment “requires developing a nuanced understanding of the complex relationships between technology, content, and pedagogy” (p. 1029). A teacher will need to apply their knowledge and skill in mathematics pedagogy and formative assessment to select or design appropriately challenging questions that successfully elicit the desired student understanding. At the same time, they will need to apply knowledge of the computer-based technological tools that will effectively assist in the eliciting of student understanding as well as the skill to implement the TEFAs with students.

As teacher educators work with secondary mathematics PSTs, they must take into consideration how these components relate to one another and how they should be presented to PSTs. The interconnectedness of the components and their essential elements is central to a true understanding of TEFAs and the ability to implement them. Secondary mathematics PSTs should
be aware of how their levels of literacy in one essential element may impact their level in another, looking for possibilities as well as constraints.

**Rubric Development**

Following the initial design of the Theoretical Framework for Secondary Mathematics TEFA Literacy, I developed the analytic rubric with qualitative descriptors for each of the essential elements. The rubric, examined by the Expert Reviewers, served to validate the components composed of the essential elements from the Theoretical Framework for Secondary Mathematics TEFA Literacy (see Figure 9). The Expert Reviewers provided valuable feedback that assisted in the simplification of the initial essential elements. In particular, Expert Reviewer 2’s feedback that led to the consolidation of essential elements ICT and OCT into the more meaningful essential element: TK. The TK essential element, as with Mishra and Koehler’s (2006) description of technology knowledge, places an emphasis on the necessity of knowing of different technological tools (ICT) *and* being able to operate them (OCT). It is not enough for a secondary mathematics teacher to simply be able to identify a type of technology, they must be able to use it in the classroom. This also aligns with the ISTE Standards (2020) for educators, that emphasize teachers’ responsibility “to discover and use new digital resources and diagnose and trouble technological issues” (2.4 Collaborator section).

The qualitative descriptors on the final rubric (Appendix F) give context to the knowledge and skills teachers need to successfully implement TEFAs in the secondary mathematics classroom. Importantly, the rubric highlights the interconnectedness of the components and their essential elements from the Theoretical Framework for Secondary Mathematics TEFA Literacy. For example, in the MP essential element, a teacher must be able to analyze, select, and design TEFA questions that promote the five strands of mathematical proficiency (Kilpatrick et al., 2001). This is connected to the ESU essential element, in which a
teacher should demonstrate they are emphasizing explanation, justification, and problem solving in these questions. Success in both of these essential elements relies on the teacher’s proficiency in TK and AIG, with their ability to select a computer-based technological tool that has the appropriate pedagogical, cognitive, and mathematical fidelity.

Implications

From both a practical and a research perspective, the Theoretical Framework for Secondary Mathematics TEFA Literacy and corresponding rubric serve as a conceptual lens of the foundational knowledge and skills that secondary mathematics teachers need to successfully plan and implement TEFAs.

Practical Implications. Theoretical frameworks, like this one, can help teachers and teacher educators develop an understanding of a phenomenon, leading to the ability to apply the concepts to their profession. The Theoretical Framework for Secondary Mathematics TEFA Literacy and corresponding rubric can support teacher educators in determining the types of pedagogical knowledge and skills they need to incorporate into their course work. Further, the theoretical framework and rubric can help to guide teacher educators towards lessons that focus on integration of these skills, rather than separating them into unrelated tasks. Teacher educators, for instance, may consider a lesson on computer-based technological tools that examines the capabilities of the tool not just for mathematics learning, but also for formative assessment, data collection, and providing feedback to students.

For teachers, both pre- and in-service, the theoretical framework and rubric provide a basis for how they can improve the ways in which they plan, implement, and use the data collected from TEFAs to inform their practice. The theoretical framework itself provides the concept, while the rubric descriptions for each essential element provide guidelines for putting
the theory into practice. For instance, the qualitative descriptors for the AIG and FP essential elements can help teachers to make a more informed decision about the computer-based technological tools they choose to use for TEFAs, looking for those that have both mathematical and pedagogical fidelity. Moreover, teachers can use the theoretical framework and rubric to reflect on and, potentially, make changes to their current TEFA related classroom practices. They may notice, for example, that they are not effectively applying their FP knowledge and skills during TEFAs and, consequently, are not reaping the full benefits of the feedback process to help students understand and correct their mistakes or misconceptions.

**Research Implications.** The Theoretical Framework for Secondary Mathematics TEFA Literacy and corresponding rubric provide educational researchers with a number of tools to discuss, analyze, assess, and make predictions. Much like the descriptions of assessment literacy (Stiggins, 1995, 1999) and TPACK (Niess, 2005, 2013; Mishra & Koehler, 2006), the descriptions of the components composed of the essential elements put forth in this study furnish educational researchers with some of the language necessary to discuss Secondary Mathematics TEFA Literacy. Moreover, the rubric can serve as a basis for other educational researchers to explore TEFA Literacy in different educational contexts.

Educational researchers may also use the Theoretical Framework for Secondary Mathematics TEFA Literacy and rubric as the basis for their analysis of teacher education programs or in-service professional development focused on TEFA Literacy. Not only would researchers be able to identify potential pitfalls, they may also be able to make predictions about indicators of success. Secondary mathematics teachers’ effectiveness in planning and implementing TEFAs is dependent on the combined competencies in each of the components
and their essential elements. A course or workshop, for instance, that ignores this interconnectedness may not properly equip teachers.

**Research Question 2: The TEFA Literacy Test for Secondary Mathematics**

In this section, I will reflect on the process of developing the *TEFA Literacy Test for Secondary Mathematics* and its evaluation method. Following the recommendations put forth by research and the feedback from Experts and the External Reviewers, the final *TEFA Literacy Test for Secondary Mathematics* (Appendix G) consists of 11 classroom scenarios, each followed by three questions. These questions intend to elicit evidence of participant’s Secondary Mathematics TEFA Literacy. The evaluation method, also improved upon by the External Raters, provides a systematic process to evaluate responses to the test using the rubric (Appendix F).


The structure of the test was based on the structure of other instruments that have been used to assess pre- and in-service teachers’ attainment of pedagogical, technology integration, and formative assessment skills. The recurring theme in many of these assessments was the use of classroom scenarios. While the use of classroom scenarios on a test will never substitute the types of data that can be collected through real-world classroom observations, they do provide a structure to elicit the ways in which teachers think about and plan for classroom activities. Classroom scenarios provide the opportunity for teachers to apply their understanding of TEFAs to analyze situations and make judgments. Further, presenting a variety of classroom scenarios,
at different levels of mathematics with different mathematical content, prompts teachers to consider mathematical content outside of the course(s) they may typically teach. This, in turn, may help teachers and teacher educators identify potential gaps in knowledge or skill associated with unfamiliar mathematics content.

The language used in the scenarios and questions on the test aligned with much of the language in concepts used to develop the Theoretical Framework for Secondary Mathematics TEFA Literacy. Still, I had to be considerate of how academic and technical the language on the test was to ensure that the “reading difficulty level” (DeVellis, 2012) was on par with that of an adult in a teacher education program. For instance, a number of the synthesized literature sources from Phase 1 of framework development included technical terms such as “pedagogical fidelity, cognitive fidelity, and mathematical fidelity” to explain the knowledge associated with the Alignment to Instructional Goals (AIG) essential element. Although some PSTs may be familiar with these terms, they are rather technical and may require a more extensive background in educational research. Instead of using these terms on the TEFA Literacy Test for Secondary Mathematics, participants were asked to “explain why” they chose particular technologies and were prompted to “explain why” certain technologies were effective or not.

The test reviews by Experts and External Raters proved to be essential in further refining the language and content on the test, as well as in creating a more reliable evaluation method. The Expert Reviewers provided the perspectives of veteran researchers and educators. In particular, their feedback helped to refine the phrasing of the questions on the test to better elicit the desired knowledge and skill. Expert Reviewer 2, for example, prompted me to think about the wording of questions by noting when they were “rather vague”, “rather broad”, or “a bit tricky”. This type of feedback helped me to think about the language I was using in the questions.
and how it could be improved to be more specific and diminish room for interpretation or assumptions.

The External Raters provided the perspectives of a secondary and a postsecondary teacher, as well as that of novice mathematics education researchers. It often appeared that the External Raters were reflecting on their own classroom experiences with computer-based technological tools. Their feedback, consequently, contributed to revisions that included asking for more directions and explanations about how to use the technological tools, as well as the addition of a scenario that incorporated a new computer-based technological tool. The External Raters, following their first-hand experience evaluating the Mock Responses, also supplied valuable feedback that helped to refine the evaluation method. In particular, they helped to develop the use of “Numerical Levels” which simplified the process for determining participants' overall levels of proficiency.

**Implications**

The *TEFA Literacy Test for Secondary Mathematics* was designed with the intention that it be used in teacher education and in research on teacher education. At the pre-service level, teacher educators may want to consider using the test as a baseline assessment to determine the strengths and deficits of secondary mathematics PSTs. This information can, then, guide teacher educators in making decisions about what to teach in their courses. The test may also serve as a self-assessment, giving secondary mathematics pre- and in-service teachers feedback on their own strengths or gaps in knowledge that they can address through individual self-improvement. In research, the test may be used in a variety of ways, such as the tool in a pretest-posttest, experimental research design, or as a tool in action research to evaluate and reflect on the
effectiveness of a course or professional development that specifically focuses on Secondary Mathematics TEFA Literacy.

It is important to emphasize that while the *TEFA Literacy Test for Secondary Mathematics* is based on the Theoretical Framework for Secondary Mathematics TEFA Literacy and corresponding rubric, they are independent of each other. The test may be modified or updated to account for more generalized or newer computer-based technological tools, but the theoretical framework and rubric would remain the same.

**Evaluation Method.** The *TEFA Literacy Test for Secondary Mathematics* is designed to produce qualitative responses that do not necessarily have a single correct answer. This, in turn, has made the evaluation of participants’ responses time-consuming. And while the improved evaluation method (Appendix H), did help to streamline the process, during the case study, I found that it would take me between 45 and 60 minutes to evaluate a single participant’s responses to the test. With a greater number of participants, the evaluation of responses may take a considerable amount of time. To help reduce the time it takes to evaluate the test, teacher educators may consider engaging PSTs in self- or peer-assessment practices, where they either evaluate their own responses or their peers’. Each of these activities may serve as learning opportunities for future educators, helping them to better understand such things as how to use a rubric to evaluate qualitative responses and how to successfully engage students in peer-assessment.

In research, self-assessment may not be viable, as uncontrolled testing could lead to skewed or invalid results and it would be unreasonable to have participants evaluate their own responses. Researchers may, instead, choose to use a modified and shortened version of the *TEFA Literacy Test for Secondary Mathematics*. They may also consider developing a modified
version of the test that employs either a Likert Scale or multiple choice. Such modifications will, inevitably, change the type of data (i.e., qualitative versus quantitative) and require field testing prior to confirming results.

**Elicitation of the MLF Essential Element.** Following the External Raters’ review of the Mock Responses to the *TEFA Literacy Test for Secondary Mathematics*, the MLF essential element was removed as it proved to be adversely challenging to elicit evidence of PSTs’ literacy in MLF in a test format. Given the nature of how teachers can demonstrate their MLF literacy, it may prove more effective to use the qualitative descriptors of MLF on the rubric to evaluate observations and reflections of secondary mathematics teachers’ engaging students in TEFAs. For example, a teacher educator or researcher may observe a secondary mathematics teacher as they implement a TEFA, making notes of real-time instructional decisions that appear to be a result of the teacher’s analysis of students’ data. This should be followed-up with a member-check (Billups, 2021) to confirm that those decisions were in fact a result of the teacher’s analysis of student data collected on the TEFA. Additionally, the teacher may be asked to reflect on how they analyzed the students’ data and how this led them to make adjustments to their instruction.

**Research Question 3: Evaluating PSTs’ TEFA Literacy**

Evaluation of the secondary mathematics PTSs’ responses to the *TEFA Literacy Test for Secondary Mathematics* revealed not only the TEFA Literacy levels of the participants, but also gave insights into their individual strengths and deficits. In this section, I will reflect on the evaluation of participants’ responses to the test.

**Mathematical Proficiency**
Given the specialized knowledge of the participants in the concept of Mathematical Proficiency (MP) and its’ five strands (Kilpatrick et al., 2001), it was not surprising that four out of five were evaluated as Proficient in this level. Participants seemed most adept at analyzing, selecting, or designing questions that prompted Conceptual Understanding and Procedural Fluency. Adaptive Reasoning was the next most frequently evidenced strand, followed by Strategic Competence. The primary reason for the participants’ being unable to obtain an Advanced level in MP was the lack of evidence of the strand of Productive Dispositions in their responses. Kilpatrick et al. (2001) describe Productive Dispositions as the “habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (p. 5). Megan was the only participant to explicitly use the term in their response to Scenario 4, Question (c), noting for “adaptive reasoning / productive disposition I would give students tables of values and ask them to justify whether they think it is linear or exponential.” Having students “justify” their response may help them “to see mathematics as sensible” (Kilpatrick et al., 2001, p. 5), but Megan’s response is lacking the depth and details to ensure that students will be prompted to develop “a belief in diligence and one’s own efficacy” (Kilpatrick et al., 2001, p. 5).

**Student Learning**

Heather was the only participant to be evaluated as advanced in Student Learning (SL). Heather consistently demonstrates that she was thinking about the prior knowledge students needed to complete a TEFA, as well as about the types and origins of common mistakes and misconceptions. Mitch and Michael, evaluated as Proficient in SL, provided sufficient evidence that they could effectively use student’s prior knowledge to analyze, select or design TEFAs, as well as identify possible mistakes and misconceptions. Mitch and Michael, however, did not
consistently demonstrate they were thinking deeply about the origin of the mistakes or misconceptions. For example, while in Scenario 10, question (c), Michael provided evidence of thinking deeply about the origin of mistakes or misconception, stating “Students might not fully make connections between a, b, and c without formal understanding of vertex formulas, intercepts, and the quadratic formula.” Yet, in Scenario 1, Question (c), Michael only mentions that “some possible misconceptions students might have would be mixing up x- and y-intercepts” without further explanation of why this is a common misconception. Brad, evaluated as Partially Proficient in SL, was proficient at using students’ prior knowledge, but struggled to demonstrate that he was thinking deeply about why students have misconceptions, often only listing mistakes students might make. Megan, evaluated as Partially Proficient in SL, did not respond to half of the questions that were intended to elicit evidence of SL. For those SL questions that Megan did respond to, there was evidence that they were using students’ prior knowledge and able to identify common mistakes and misconceptions. For example, in Scenario 7, Question (a), Megan noted three different concepts students would need to have prior knowledge in, as well as two different common misconceptions. Thus, while Megan could not consistently provide evidence of her SL knowledge, she was able to provide “some evidence”.

Alignment to Instructional Goals

Brad and Michael, evaluated as Advanced in Alignment to Instructional Goals (AIG), were both able to consistently analyze and select technology that aligned to the instruction goals of both the mathematics objectives and formative assessment. Heather, Mitch, and Megan, evaluated as Proficient in AIG, were able to provide analyses of technologies for their alignment to the mathematics objectives and formative assessment, but sometimes struggled to select appropriate technologies. For example, in Scenario 6, Question (a), Heather noted that she was
“Not too sure about a computer-based technology”, to use for the given mathematical learning objective. And, in Scenario 10, Question (a), both Mitch and Megan provided ample explanations of why and how their selected technological tools would be appropriate for a TEFA of graphing quadratic functions. On the other hand, in Scenario 5, Question (b), Mitch did not provide a technology to use for the TEFA and Megan did not respond to the prompt.

Technology Knowledge

All of the participants, excluding Megan, had Proficient Technology Knowledge (TK). While Mitch, Heather, Brad, and Michael were all able to demonstrate they had knowledge of at least four different technological tools appropriate for mathematics TEFAs, they could not be considered Advanced as their responses lacked detail and/or depth in the explanations of how teachers and students could use these tools for TEFAs. Throughout the test, all of the participants provided evidence they knew how to use Desmos and GeoGebra, with all of them being able to analyze and/or design TEFAs that utilize these technological tools. The responses to the questions following Scenario 5 produced a wide variety of TK. For Question (a), Megan was the only participant to not provide evidence of knowledge of spreadsheet technology, while all others demonstrated some understanding of the basic use for either Google Sheets or Excel. For Question (b), Heather introduced the computer-based technological tool, Google Forms, as a way to collect data on students’ understanding. Mitch, on the other hand, selected a TI-84 graphing calculator as a tool for the task, but did not provide evidence for knowledge of a computer-based technological tool that would engage students in a formative assessment. In Scenario 8, only Brad and Michael demonstrated knowledge of Delta Math. Brad provided evidence of an Advanced understanding by supplying detailed descriptions of the benefits and limitations of Delta Math from both the teacher and student perspective. For Scenario 11, Megan struggled
once again, and was also the only participant who did not demonstrate knowledge of Kahoot!, while all the others provided evidence of their understanding of this computer-based technological tool.

**Elicitation of Student Understanding**

Heather and Michael, who were evaluated as Advanced for Elicitation of Student Understanding (ESU), both consistently demonstrated they could design TEFA questions that focused on ESU, often using “how” and “why” questions. Heather and Michael also demonstrated an ability to critique TEFAs, like the Desmos Quiz in Scenario 7, for its inability to effectively elicit students’ understanding. Mitch and Brad, who were evaluated as Proficient in ESU, both demonstrated that they could design TEFA questions that emphasized more than memorization or computation. However, they both designed a few questions that lacked an emphasis on explanation, justification, or problem solving. For example, in Scenario 7, Question (c), Mitch designed a task that asked students to “Write the equations of two perpendicular lines which intersect at the point (3,11)”. A prompt like this does require application of knowledge, but lacks explanation for how the students arrive at the answer or justification for why their response is correct. Megan, who was evaluated as Partially Proficient in ESU, did not provide any evidence of an ability to analyze, select, or design TEFA questions in five out of the nine applicable questions. Nonetheless, in Scenario 4, question (b), Megan excelled, asking students to “justify” their thinking, analyze word problems, provide their own examples “of a real life situation”, and to come up with a definition “in their own words.” Thus, while Megan could not consistently demonstrate ESU literacy, there was some evidence that she could design TEFA questions that elicit student understanding.

**Feedback Process**

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Mitch, Heather, and Michael, evaluated as Proficient in Feedback Process (FP), were all able to consistently analyze, select, or design TEFAs that provided targeted and supplemental feedback. Mitch provided some evidence that he would make plans for students to be active participants in the feedback process, though this mainly consisted of class or peer discussions. For instance, in Scenario 9, Question (c), Mitch made plans to actively engage students in the feedback by putting them into groups “to engage with and discuss the solutions” by sharing “their solutions and the feedback they received”. Heather and Michael, similarly, planned for students to actively engage in class, small group, or peer discussions, but did not attend to other methods of student engagement, therefore not proceeding to an Advanced level. Brad, who was evaluated as Partially Proficient in FP, demonstrated the ability to analyze, select, and design TEFAs that emphasized targeted feedback, but showed limited evidence of providing supplemental feedback. Brad made no clear plans for students to be active participants in the feedback process, only noting that he would “review” concepts with students. Megan was evaluated as Novice in FP, primarily due to the fact she either did not respond or responded “not sure” for six out of the eight applicable questions. In the two questions she responded to, Megan was able to provide some evidence that she could analyze the deficiencies in the feedback given by the technology in Scenario 2, Question (b), and provide targeted and supplemental feedback for Scenario 3, Question (b). Nonetheless, there was no evidence that Megan made plans for students to actively participate in the feedback process, and, therefore, could not be evaluated at a higher level.

**Implications**

The participants in the field test demonstrated relatively high levels of TEFA Literacy, with some of them even demonstrating Advanced levels in two of the essential elements. There
are a variety of factors that could contribute to these results, but it is difficult to make any
generalizations given the small sample size. What can be gleaned, is that all participants may
benefit from further instruction on and experience with secondary mathematics TEFAs.

Limitations and Recommendations

There are a number of limitations to the proposed theoretical framework, rubric, and the
TEFA Literacy Test for Secondary Mathematics. In the following section I will describe these
limitations and put forth some recommendations for future research and development.

All types of research are subject to some form of bias (Merriam & Tisdell, 2016). There
are many concepts that other educational researchers and teacher educators may consider “more
important” or “essential” to Secondary Mathematics TEFA Literacy. Although I sought to
minimize subjectivity and bias with an extensive review of literature and a systematic
methodology for the development of the Theoretical Framework for Secondary Mathematics
TEFA Literacy and rubric, my personal experiences, perspectives, or biases may have influenced
the selection of resources and the definitions of components and essential elements. The TEFA
Literacy Test for Secondary Mathematics may also have been influenced by these same biases.
In particular, the language used, the mathematical concepts addressed, and the computer-based
technological tools selected may have been impacted by my personal experiences in mathematics
education.

It is recommended that the Theoretical Framework for Secondary Mathematics TEFA
Literacy and corresponding rubric be tested in other settings and contexts. For example, using the
framework and rubric as a lens for evaluating classroom observations of teachers implementing
TEFAs, teachers’ lesson plans for TEFAs, interviews with PSTs, in-service teachers, and teacher
educators that explore their perceptions and beliefs about TEFAs, or surveys similar to those
developed by Zelkowski et al. (2013) and Schmidt et al. (2009). It may also prove useful to develop other tests, like the TEFA Literacy Test for Secondary Mathematics, that are specifically designed to elicit evidence of secondary mathematics teachers’ TEFA Literacy.

A notable limitation to the contents of the TEFA Literacy Test for Secondary Mathematics lies in the types of computer-based technological tools presented for participants to analyze and critique. Given the rate at which technology is advancing, many of these technological tools may be obsolete within five years. The contents of the test will, therefore, need to be regularly updated to the most relevant and commonly used technological tools. These modifications would primarily consist of replacing the images and names of the technological tools with updated information. Correspondingly, teacher educators and educational researchers may consider creating a more generalized version of the test, that excludes the mention of specific technological tool. For example, instead of asking a participant to analyze the feedback capabilities of Kahoot!, the test may ask the participant to provide their own example of a “game-based, multiple-choice quiz that is accessed through a web browser” and analyze it for its feedback capabilities.

The evaluation method is most limited by the inherent subjectivity derived from individuals’ understanding of Secondary Mathematics TEFA Literacy. Others’ ability to appropriately apply the evaluation method and use the rubric to evaluate responses to the TEFA Literacy Test for Secondary Mathematics may be limited by their own understanding of the components, their essential elements, and the fidelity of computer-based technological tools. Individual understandings of these concepts may also lead to a certain degree of subjectivity when evaluating participants' responses. It is recommended that educational researchers and teacher educators intending to use the test and evaluation method ensure they have a thorough
understanding of each of the essential elements and how they can be demonstrated. Further, they should have knowledge of the types of computer-based technological tools presented on the test, as well as others that participants may select.

The evaluation of the five secondary mathematics PSTs’ responses to the *TEFA Literacy Test for Secondary Mathematics* is most notably limited by the small sample size. Although the results produced rich and valuable information about the participants’ TEFA Literacy levels, they cannot be used to identify trends among PSTs in this course or within the teacher education program. Future research should implement the *TEFA Literacy Test for Secondary Mathematics* and evaluation method with larger, diverse groups of secondary mathematics pre-service teachers. Further, extending the pool of participants to both pre- and in-service teachers may help to identify common strengths and deficits in Secondary Mathematics TEFA Literacy across different levels of experience.
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Appendix A: First Draft of Rubric

This rubric is used to evaluate responses on the TEFA Literacy Test for Secondary Mathematics.

Directions:
- Each question on the test corresponds to most, but not all of the elements within each Component. Each question has “Applicable Rubrics” noted on the test.
- Literacy Levels for each element will be determined by simple majority.
  - For example, if a preservice teacher is determined to be “Partially Proficient” 4 times and “Proficient” 6 times out of 10 ICT applicable questions, they will be considered “Proficient” in ICT.
  - In the case of a “tie”, the preservice teacher will be considered at the lower level. For example, if they are “Partially Proficient” 5 times and “Novice” 5 times out of 10 MLF applicable questions, they will be considered “Novice” in MLF.

Component #1: Technology Integration

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identifying Classroom Technologies (ICT)</td>
<td>Demonstrated knowledge of a variety of computer-based technologies. The knowledge includes that of a variety of mathematics specific technologies and the types of mathematics they can support.</td>
<td>Demonstrated knowledge of computer-based technologies. This knowledge includes that of some mathematics specific technologies and the types of mathematics they can support.</td>
<td>Some evidence of knowledge of computer-based technologies. This knowledge is limited to general computer-based technologies and does NOT include mathematics specific technologies.</td>
<td>Little to no evidence of knowledge of computer-based technologies.</td>
</tr>
<tr>
<td>Operating Classroom Technologies (OCT)</td>
<td>For a number of different mathematics specific technologies, the teacher is able to explain all of the following: - the necessary material/resources for operating the technology</td>
<td>For more than one mathematics specific technology, the teacher is able to explain all of the following: - the necessary material/resources for operating the technology</td>
<td>For at least one mathematics specific technology, the teacher is able to explain the necessary materials/resources for operating the technology and at least one of the following:</td>
<td>Little to no evidence of how to operate computer-based technologies, both non-mathematics and mathematics specific</td>
</tr>
<tr>
<td>Technology and Instructional Goals (TIG)</td>
<td>The teacher is able to analyze, select, and design TEFAs that align to the purposes of formative assessment and given mathematical learning objectives</td>
<td>The teacher is able to analyze, select, and design TEFAs that align to the purposes of formative assessment and given mathematical learning objectives</td>
<td>The teacher is able to analyze and select TEFAs that align to the purpose of formative assessment and given mathematical objectives, but often struggle to design their own</td>
<td>Little to no demonstrated understanding of how to align the TEFA to the instructional goal of formative assessment or mathematics.</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Component #2: Formative Assessment</td>
<td>Advanced</td>
<td>Proficient</td>
<td>Partially Proficient</td>
<td>Novice</td>
</tr>
<tr>
<td>Eliciting Student Understanding (ESU)</td>
<td>The teacher consistently selects and designs questions/prompts that elicit information on student understanding. Questions/prompts consistently provide students with a variety of ways to demonstrate their thinking.</td>
<td>The teacher often selects or designs questions/prompts that elicit information on student understanding. Questions/prompts provide students with a variety of ways to demonstrate their thinking; few questions utilize memorization and computational procedures.</td>
<td>The teacher sometimes selects or designs questions/prompts that elicit information on student understanding. Some questions/prompts focus on memorization and computational procedures.</td>
<td>The teacher consistently selects or designs TEFA questions/prompts that elicit little to no information on student understanding. Questions/prompts focus on memorization and computational procedures.</td>
</tr>
<tr>
<td>Feedback Process (FP)</td>
<td>computational procedures</td>
<td>Feedback Process (FP)</td>
<td>computational procedures</td>
<td></td>
</tr>
<tr>
<td>-----------------------</td>
<td>---------------------------</td>
<td>-----------------------</td>
<td>---------------------------</td>
<td></td>
</tr>
<tr>
<td>The teacher consistently plans for TEFAs that provide more than “correct” or “incorrect” feedback, and plans for targeted and supplemental feedback.</td>
<td>The teacher plans for TEFAs that provide more than “correct” or “incorrect” feedback, but does not plan for providing more targeted or supplemental feedback.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, and provide clear alternatives</td>
<td>The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, alternatives are vague or missing</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher makes clear plans for students to be active participants in the feedback process</td>
<td>The teacher makes clear plans for students to be active participants in the feedback process</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher plans for TEFAs that provide more than “correct” or “incorrect” feedback, and plans for targeted or supplemental feedback.</td>
<td>The teacher plans for TEFAs that provide more than “correct” or “incorrect” feedback, but does not plan for providing more targeted or supplemental feedback.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, and provide clear alternatives</td>
<td>The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, but do not provide alternatives</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher might say they encourage students to be active participants, but do not make clear plans</td>
<td>The teacher might say they encourage students to be active participants, but do not make clear plans</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moving Learning Forward (MLF)</th>
<th>Moving Learning Forward (MLF)</th>
<th>Moving Learning Forward (MLF)</th>
<th>Moving Learning Forward (MLF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher consistently analyzes students’ data to determine what students know in relation to students’ attainment of the learning objectives.</td>
<td>The teacher demonstrates that they can analyze students’ data to determine what students know, it is not always related to students’ attainment of the learning objective(s), but may not do so consistently</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher consistently makes clear plans for instructional adjustments based on students’ feedback.</td>
<td>The teacher demonstrates that they can plan for instructional adjustments based</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The teacher attempts to analyze students’ data to determine what students know.</td>
<td>The teacher demonstrates that they want to use student-feedback to</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher demonstrates that they do not plan to use student-feedback to make adjustments to instruction</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: The teacher must accomplish all 3 criteria to be at that level, otherwise they are at the next lower level.
Component #3: Mathematics Pedagogy

<table>
<thead>
<tr>
<th>Student Learning (SL)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher is consistently selecting, analyzing or designing mathematical tasks on TEFAs that align with students’ prior knowledge and typical learning processes. Mathematics content on the TEFA consistently prompts students to make connections, conjecture, and reason mathematically</td>
<td>The teacher is often selecting, analyzing or designing mathematical tasks on TEFAs that align with students’ prior knowledge and typical learning processes. Mathematics content on the TEFA often prompts students to make connections and/or reason mathematically</td>
<td>The teacher is sometimes selecting, analyzing or designing mathematical tasks on TEFAs that align with students’ prior knowledge and typical learning processes. Mathematics content on the TEFA sometimes prompts students to make connections or reason mathematically</td>
<td>Little to no evidence that the teacher is selecting, analyzing or designing mathematical tasks on TEFAs that align with students’ prior knowledge and typical learning processes. Mathematics content on the TEFA does not prompt students to make connections or reason mathematically</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mathematical Proficiency (MP)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher is able to select and design TEFAs that help to develop students’ mathematical proficiency. All of the strands are consistently addressed.</td>
<td>The teacher is able to select or design TEFAs that help to develop students’ mathematical proficiency. Most of the strands are addressed. The teacher can analyze TEFAs</td>
<td>Some evidence that the teacher is able to select or design TEFAs that help to develop students’ mathematical proficiency; however, this is limited to primarily procedural</td>
<td>Little to no evidence that the teacher is able to select or design TEFAs that help to develop students’ mathematical proficiency</td>
<td></td>
</tr>
</tbody>
</table>
The teacher can consistently and accurately analyze TEFAs for their ability to develop the different strands of mathematical proficiency. Some evidence that the teacher can analyze TEFAs for their ability to develop the different strands of mathematical proficiency, and is mostly accurate for fluency and/or conceptual understanding.
Appendix B: First Draft of the TEFA Literacy Test for Secondary Mathematics

In all scenarios, please assume that the teacher and students are working in a secondary mathematics classroom that is aligned to the Common Core State Standards for Mathematics (2010).

<table>
<thead>
<tr>
<th>Q #</th>
<th>Scenario</th>
<th>Applicable Rubrics</th>
</tr>
</thead>
</table>
| 1   | Mr. O’Connor, an Algebra 1 teacher, is working with students to graph linear functions in slope-intercept and point-slope form. Students need to be able to graph the function on the x-y-axis, identify the x- and y-intercepts, and calculate the slope.  
   a. Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions.  
   b. Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.  
   c. Given the question/prompt that you designed in part (b), what might you be able to infer about students’ learning? (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?) | ICT  
   TIG  
   ESU  
   SL  
   MP |
| 2   | Students in Ms. Glow’s Geometry class have recently learned about the AA and SSS conjectures for similar triangles, and have begun to use two-column proofs. Following one of the lessons, Ms. Glow created a formative assessment on GeoGebra ©. One of the problems asked: | MLF  
   FP  
   SL  
   MP  
   ESU |
a. Sam provided the following answer:

**The triangles are not similar because they do not have matching angles**

What can you determine about Sam’s understanding of similar triangles?

b. GeoGebra © provided the following feedback to Sam (in green):

In your opinion, is this feedback useful for Sam? Explain.

c. Provide an example of how you might expand upon or change this GeoGebra © problem to help make students’ understanding of similar triangles more visible. Be specific about any technological changes.

3 Mx. Knope is working with students on the properties of exponents, both whole number and rational, and decided to use a Khan Academy © quiz to formatively assess students. One of the questions asks:
a. What mathematical concepts is this question assessing? What mathematical proficiencies (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) is this question prompting? Explain your responses.

b. The computer only offers students correct/incorrect feedback. What additional feedback (if any) would you provide students if they made a mistake? Explain your response.

c. To the best of your knowledge, explain to Mx. Knope how they can access a report on students’ performance on this Khan Academy © quiz.

---

Valeria turned in her homework on linear and exponential growth on Desmos ©. The first question asked her to analyze the table and determine if they demonstrated linear or exponential growth.
160

a. What can you determine about Valeria’s understanding of this topic?
b. To the best of your knowledge, how else could you use Desmos © to formatively assess students’ understanding about the difference between linear and exponential growth? Be as detailed as possible.
c. Based on your response to part (b), what do you anticipate you will be able to determine about a student’s understanding?

5 Ms. Zhang is teaching a Mathematical Modeling class and is introducing her students to finding the curve of best fit using spreadsheet technology (e.g., Excel, Google Sheets).

a. What do you think Ms. Zhang will need to teach students about the spreadsheet technology before they begin the project?

After initial instruction, students begin working in pairs to create their first curves of best fit for a given data set.

b. How should Ms. Zhang collect formative assessment data on students’ learning process while they are working?

c. Given your response in (b), what mathematical proficiencies (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) will the formative assessment measure?

6 Back in Ms. Zhang’s class, students have finished their first drafts of their modeling projects on finding the curve of best fit. They now have to give a short, digital presentation of their findings.

a. What platform/method would you use to give the digital presentations? Explain your choice.
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b.</td>
<td>How would you encourage students to give peer-feedback during the presentations? Be specific.</td>
</tr>
<tr>
<td>c.</td>
<td>Given the most common mistake on students’ projects was confusing the correlation coefficient (R) with the coefficient of determination (R^2), what feedback might you give students? Describe how you would provide this feedback.</td>
</tr>
</tbody>
</table>
| 7 | Mr. Kanumba’s Statistics class is working on the learning objective:  
*Students will be able to use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*  
   a. Identify a computer-based technology that Mr. Kanumba could use to help students achieve this objective.  
   b. Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.  
   c. Given the question/prompt that you designed in (b), what might you be able to infer about students’ learning? (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?) |
| 8 | Students in Ms. Swanson’s class have demonstrated that they know:  
(1) how to find the equation of a line parallel to a given line that passes through a given point and  
(2) that perpendicular lines must intersect at a 90 degree angle.  
The learning objective for today’s lesson is:  
*Students will be able to find the equation of a line perpendicular to a given line that passes through a given point.*  
Ms. Swanson asked students the following question on Desmos ©:  
What is the equation of the line perpendicular to x+3y=6 that travels through the point (1,5)?  
- y=2x+1  
- y=2/3x+6  
- y=3x+2  
- y=6x-3  
   a. What other prior knowledge might students need to answer this question?  
   b. Given this question, what might she be able to infer about students’ understanding of finding the equation of a line perpendicular to a given line that passes through a given point? |
c. Provide an example question/prompt that you believe would assess students’ achievement of the learning objective. Be specific about the technology you would choose and why you would choose it. If you would not use technology, explain why not.

9 Mr. Kanumba would like to use Delta Math to review solving linear equations and inequalities in one variable, but has never done so before.

   a. To the best of your knowledge, explain to Mr. Kanumba the steps he needs to take to create the Delta Math quiz.
   
   b. To the best of your knowledge, explain what types of questions/prompts and feedback Mr. Kanumba can design on Delta Math.
   
   c. In your opinion, what are the benefits and drawbacks of using Delta Math when the topic is on solving linear equations and inequalities in one variable? Explain your response.

ICT 
OCT 
FP 
SL 
MP

10 Julian was completing an online GeoGebra © formative assessment for his Trigonometry class. After incorrectly responding to the question, he received the following automated feedback (outlined in red):

```
Trigonometric ratios of general angles and basic angles
Express cos 510° in terms of the trigonometric ratio of its basic angle.
a. cos 30°  b. cos 150°  c. -cos 150°  d. -cos 30°
Your answer: d □  Hint  Next Question
510° lies in quadrant 2, and cosine of the angle should be negative. Also, 150° is > 90°, so it is not the basic angle.
```

   a. Explain how this feedback does or does not help Julian understand his mistake or misconception?

When Julian selected “Hint”, he received the diagram below:

```
Trigonometric ratios of general angles and basic angles
Express cos 510° in terms of the trigonometric ratio of its basic angle.
a. cos 30°  b. cos 150°  c. -cos 150°  d. -cos 30°
Your answer: b □  Hint  Next Question
510° lies in quadrant 2, and cosine of the angle should be negative. Also, 150° is > 90°, so it is not the basic angle.
```

FP 
SL 
MP
b. Explain how this feedback does or does not help Julian understand his mistake or misconception?
c. Explain what adjustments would you make (if any) to this feedback?

<table>
<thead>
<tr>
<th>Ms. Fischer and her students are working on graphing quadratic functions, and understanding how the coefficients in standard form transform the graph of a parabola (i.e., how different values of a,b,c in (ax^2+bx+c) transform the graph).</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Describe a technology enhanced formative assessment activity Ms. Fisher could engage students in?</td>
</tr>
<tr>
<td>b. What resources and materials will Ms. Fisher and her students need to complete this activity?</td>
</tr>
<tr>
<td>c. Provide an example of a question or prompt you might give students. Explain what you intend to measure about students’ understanding with this question/prompt (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mr. Flint has the learning objective that:</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Students will be able to explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.</em></td>
</tr>
<tr>
<td>Using this learning objective Mr. Flint designed the following question in their Google Classroom:</td>
</tr>
<tr>
<td>a. Analyze Mr. Flint’s use of technology. Explain what changes (if any) you would make to this technology enhanced formative assessment.</td>
</tr>
<tr>
<td><strong>b.</strong> Analyze how well the question aligns to the learning objective. Explain what changes (if any) you would make to this question.</td>
</tr>
</tbody>
</table>
Appendix C: Review Directions

Thank you for agreeing to review the TEFA Literacy Test for Secondary Mathematics and corresponding Rubric. Your feedback will be invaluable to the development of a valid tool to be used in my dissertation study on TEFA Literacy for Secondary Mathematics.

1) Please provide how you would like your name and affiliation to appear in the acknowledgements section of my dissertation:

<table>
<thead>
<tr>
<th>Name</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Affiliation</td>
<td></td>
</tr>
</tbody>
</table>

2) Please provide a review of the test with regard to the following. You are encouraged to make tracked changes on the document and provide general comments here:

<table>
<thead>
<tr>
<th>Topic</th>
<th>General Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Clarity:</strong> Are the questions clear? Do you understand what they are asking? If not, how can they be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Content:</strong> Do the questions address common skills and knowledge that would be associated with TEFA Literacy for Secondary Mathematics? If not, how can they be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Eliciting:</strong> Do you think the questions will elicit responses that are adequate and measurable? If not, how can they be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Additional comments:</strong> This could be about any redundancies, questions you might eliminate, ambiguities that need to be addressed, potential leading questions, syntax, etc.</td>
<td></td>
</tr>
</tbody>
</table>
3) Please provide a review of the rubric with regard to the following. You are encouraged to make tracked changes on the document and provide general comments here:

<table>
<thead>
<tr>
<th>Topic</th>
<th>General Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Directions &amp; Use:</strong> Do the scoring directions make sense? If not, how can they be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Clarity:</strong> Is the wording of the rubric clear and concise? Do you understand what is being evaluated? If not, how can it be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Content:</strong> Does the content of the rubric align to the components/elements being measured? If not, how can it be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Progression:</strong> Does the progression of levels make sense? Are there too big or too small of differences between one level and the next?</td>
<td></td>
</tr>
<tr>
<td><strong>Additional comments:</strong> This could be about any redundancies, elements you might eliminate/add, ambiguities that need to be addressed, syntax, etc.</td>
<td></td>
</tr>
</tbody>
</table>

4) **Alignment:** Please comment on the alignment between the test and the rubric. Are there any instances of misalignment? Unclear connections? Improvements to be made?

5) **Additional Comments:** Please provide any additional comments that may help in the development of the test and rubric below.
Appendix D: Second Draft of Rubric

This rubric is used to evaluate responses on the TEFA Literacy Test for Secondary Mathematics and determine participants’ levels of literacy. The results should be used to identify strengths and weaknesses and, therefore, guide future instruction.

Directions:

- Each question on the test corresponds to most, but not all of the elements within each Component. Therefore, each rubric notes the “Applicable Questions”
  - For example: The element “Mathematical Proficiency (MP)” is applicable to Questions 1b, 2a, 3a, 4a, 4c, …
    - The grader should use MP to evaluate all applicable questions and then determine an overall level of proficiency in that element (i.e., Advanced, Proficient, Partially Proficient, or Novice).
- Pay attention to the “NOTES” on the left-hand side of the rubric.
- An * indicates that the “Applicable Question” may be applicable, but it will depend if the participant includes this in their answers
  - For example: In question 5b, the participant has the option to “not use technology” which would, therefore, make the Technology element TK not applicable.
- Final “Proficiency Levels” are recorded on the last page.

Component #1: Mathematics

<table>
<thead>
<tr>
<th>Applicable Questions</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
</table>
| **Mathematical Proficiency (MP)**

  *NOTE: All 5 strands do NOT need to be addressed in every question/prompt. Evaluate applicable questions as a whole.*

  The teacher is able to analyze, select, and design questions/prompts on TEFAs that measure students’ mathematical proficiency. The questions/prompts address:
  - procedural fluency
  - conceptual understanding
  - strategic competence
  - adaptive reasoning
  - productive disposition
  
  The teacher is able to analyze, select, or design questions/prompts on TEFAs that measure most of students’ mathematical proficiency. The questions/prompts address:
  - procedural fluency
  - conceptual understanding
  - and at least 1 more strand

  Some evidence that the teacher is able to analyze, select, or design questions/prompts on TEFAs that measure some of students’ mathematical proficiency. The questions/prompts address:
  - procedural fluency
  - conceptual understanding

  Little to no evidence that the teacher is able to analyze, select, or design questions/prompts on TEFAs that measure the 5 Strands of Mathematical Proficiency

  1b
  2a,b
  3a
  4a,c
  5c
  7b,c
  8b,c
  9c
  11a,b
<table>
<thead>
<tr>
<th>Component #2: Technology</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Align to Instructional Goals (AIG)</strong></td>
<td>The teacher is able to analyze, select, and design TEFAs that align to the purposes of formative assessment and given mathematical learning objectives</td>
<td>The teacher is able to analyze and select TEFAs that align to the purposes of formative assessment and given mathematical learning objectives, but they struggle to design their own.</td>
<td>The teacher struggles to select or analyze TEFAs that align to the instructional goal of formative assessment or mathematics. They cannot design their own.</td>
<td>Little to no demonstrated understanding of how to align the TEFAs to the instructional goal of formative assessment or mathematics.</td>
<td>1a 2c 4b 5b 6a, 7a 8c* 9c 11a</td>
</tr>
</tbody>
</table>

**Student Learning (SL)**

*NOTE: The teacher must accomplish both criteria to be at that level, otherwise they are at the next lower level*

<table>
<thead>
<tr>
<th>Level</th>
<th>Criteria</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Novice</td>
<td>All criteria not met</td>
<td>Little to no evidence that the teacher is using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs. Little to no evidence that the teacher is able to identify mistakes or misconceptions in student responses to TEFAs.</td>
</tr>
<tr>
<td>Partially Proficient</td>
<td>One criterion met</td>
<td>There is some evidence that the teacher is using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs, but this is not always done effectively. The teacher is able to identify some common mistakes, but struggles to identify common misconceptions. They struggle to demonstrate why these mistakes and misconceptions happen.</td>
</tr>
<tr>
<td>Proficient</td>
<td>Two criteria met</td>
<td>The teacher demonstrates that they are using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs. The teacher is able to identify most common mistakes and misconceptions. They demonstrate basic understanding as to why these mistakes and misconceptions happen.</td>
</tr>
<tr>
<td>Advanced</td>
<td>Both criteria met</td>
<td>The teacher demonstrates that they are effectively using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs. The teacher is able to identify all common mistakes and misconceptions; they demonstrate that they are thinking deeply about why these mistakes and misconceptions happen.</td>
</tr>
</tbody>
</table>

**Component #2: Technology**

- **Align to Instructional Goals (AIG)**: The teacher is able to analyze, select, and design TEFAs that align to the purposes of formative assessment and given mathematical learning objectives. The teacher can analyze and select TEFAs that align to the purposes of formative assessment and given mathematical learning objectives, but they struggle to design their own. The teacher struggles to select or analyze TEFAs that align to the instructional goal of formative assessment or mathematics. They cannot design their own. Little to no demonstrated understanding of how to align the TEFAs to the instructional goal of formative assessment or mathematics.

**Questions**

- 1c 2a 5a 6c 8a 9b 10a,b 11c 12a, b
<table>
<thead>
<tr>
<th>Technology Knowledge (TK)</th>
<th><strong>Advanced</strong></th>
<th><strong>Proficient</strong></th>
<th><strong>Partially Proficient</strong></th>
<th><strong>Novice</strong></th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher can identify <em>more than 5</em> different technologies that are appropriate for formative assessments of <em>more than 5</em> different topics in mathematics. The teacher can provide detailed explanations on how to use these different technologies from both the student and teacher perspective.</td>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that provide students with a variety of ways to demonstrate their understanding. i.e.: There is a strong emphasis on explanation, justification, and problem solving; <strong>no</strong> questions utilize memorization and computational procedures.</td>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that provide students with more than 1 way to demonstrate their understanding. i.e.: There is more emphasis on explanation and justification and almost no questions utilize memorization and computational procedures.</td>
<td>There is some evidence that the teacher is analyzing, selecting, or designing questions/prompts on TEFAs that provide students with a way to demonstrate their understanding. i.e.: There is some emphasis on explanation and justification, although many questions still utilize memorization and computational procedures.</td>
<td>Little to no evidence of knowledge of computer-based technologies or how to operate them.</td>
<td>1a, 2c, 3c, 4b, 5a,b*, 6a,c*, 7a, 8c, 9a,c, 11a</td>
</tr>
<tr>
<td>The teacher can identify <em>4-5</em> different technologies that are appropriate for formative assessments of <em>4-5</em> different topics in mathematics. The teacher can explain basic use of these different technologies from both the student and teacher perspective, but does not provide detailed or advanced directions.</td>
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<tr>
<td>The teacher can identify <em>2-3</em> different technologies that are appropriate for formative assessments of <em>2-3</em> different topics in mathematics. The teacher attempts to, but struggles to explain how to use these different technologies from both the student and teacher perspective.</td>
<td></td>
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<tr>
<td>Little to no evidence of knowledge of computer-based technologies or how to operate them.</td>
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</tr>
</tbody>
</table>

**Component #3: Formative Assessment**

<table>
<thead>
<tr>
<th><strong>Eliciting Student Understanding (ESU)</strong></th>
<th><strong>Advanced</strong></th>
<th><strong>Proficient</strong></th>
<th><strong>Partially Proficient</strong></th>
<th><strong>Novice</strong></th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that provide students with a variety of ways to demonstrate their understanding. i.e.: There is a strong emphasis on explanation, justification, and problem solving; <strong>no</strong> questions utilize memorization and computational procedures.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1b, 2c, 4b, 5b, 7b, 8c, 9c, 11b</td>
</tr>
<tr>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that provide students with more than 1 way to demonstrate their understanding. i.e.: There is more emphasis on explanation and justification and almost no questions utilize memorization and computational procedures.</td>
<td></td>
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</tr>
<tr>
<td>There is some evidence that the teacher is analyzing, selecting, or designing questions/prompts on TEFAs that provide students with a way to demonstrate their understanding. i.e.: There is some emphasis on explanation and justification, although many questions still utilize memorization and computational procedures.</td>
<td></td>
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</tr>
<tr>
<td>Little to no evidence of knowledge of computer-based technologies or how to operate them.</td>
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<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>i.e.: Questions/prompts focus on memorization and computational procedures.</td>
</tr>
</tbody>
</table>
### Feedback Process (FP)

**NOTE:** The teacher must accomplish both criteria to be at that level, otherwise they are at the next lower level.

<table>
<thead>
<tr>
<th>Component #1: Mathematics</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Proficiency (MP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Learning (SL)</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

### Moving Learning Forward (MLF)

<table>
<thead>
<tr>
<th>Teacher demonstrates that they are able to analyze students’ data to determine what students know and use this data to make appropriate adjustments to instruction.</th>
<th>Teacher demonstrates that they are able to analyze students’ data to determine what students know and use this data to make adjustments to instruction. These adjustments may not always be aligned to the student data.</th>
<th>The teacher may struggle to appropriately analyze students’ data to determine what students know (i.e., Not focused on attainment of the learning objective(s), but on semantics or presentation), but they use this data to make adjustments to instruction.</th>
<th>Teacher demonstrates that they do not analyze student feedback, not do they plan to use student feedback to make adjustments to instruction.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2b</td>
<td>3b</td>
<td>5b*</td>
<td>6b,c</td>
</tr>
<tr>
<td>9c*</td>
<td>10a,b,c</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Proficiency Levels

For each Rubric, mark an “X” for the proficiency level attained by the participant. You may include additional comments in that section if you like.
### Component #2: Technology

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Align to Instructional Goals (AIG)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology Knowledge (TK)</td>
<td></td>
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</tbody>
</table>

### Component #3: Formative Assessment

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eliciting Student Understanding (ESU)</td>
<td></td>
<td></td>
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<tr>
<td>Feedback Process (FP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moving Learning forward (MLF)</td>
<td></td>
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</tbody>
</table>
Appendix E: Second Draft of the TEFA Literacy Test for Secondary Mathematics with Mock Responses

Thank you for participating in the TEFA Literacy Test for Secondary Mathematics. The goal of this test is to measure your knowledge and skills in TEFA for the secondary mathematics classroom. Please answer each question to the best of your ability.

In all scenarios, please assume that the teacher and students are working in a secondary mathematics classroom that is aligned to the Common Core State Standards for Mathematics (2010).

<table>
<thead>
<tr>
<th>Applicable Rubrics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 1:</strong> Mr. O’Connor, an Algebra 1 teacher, is working with students to graph linear functions in slope-intercept and point-slope form. The learning objective is that: Students will be able to graph the function on the x-y-axis, identify the x- and y-intercepts, and calculate the slope.</td>
</tr>
<tr>
<td><strong>AIG TK</strong> a. Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions. They could use Desmos</td>
</tr>
<tr>
<td><strong>MP ESU</strong> b. Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment. Given the graph of the linear function (I would use Desmos to provide the graph), find the equation of the line in a. slope-intercept form: b. point-slope form: c. What do you notice about these two equations?</td>
</tr>
<tr>
<td><strong>SL</strong> c. Given the question/prompt that you designed in part (b), what prior knowledge would students need in order to answer this question? They would have to have knowledge about the coordinate plane, linear graphs, how to identify/plot points, how to find/plot slopes. They would also need to know the difference between slope-intercept and point-slope form.</td>
</tr>
</tbody>
</table>

Scenario 2:
Students in Ms. Glow’s Geometry class have recently learned about the AA and SSS conjectures for similar triangles, and have begun to use two-column proofs. Following one of the lessons, Ms. Glow created a formative assessment on GeoGebra ©. One of the problems was:

![Diagram of two triangles with angle measures: Triangle on the left has angles A = 74.64°, B = 78.73°, C = 16.63°. Triangle on the right has angles D = 44.39°, E = 78.75°, F = 66.89°.]

Sam provided the following answer:

The triangles are not similar because they do not have matching angles

a. What can you infer about Sam’s understanding of similar triangles? Do you think Sam has any misconceptions about this concept? If so, what?

Based on Sam’s answer, it does seem like he knows that the triangles are not similar and it is because of the angle measures. He does not use the AAA conjecture or a formal proof, which may mean he does not understand what is meant by “justify your response”. Since I do not know more of the structure of the class, I cannot determine if this is because of a misconception or a lack of teaching.

GeoGebra © provided the following feedback to Sam (in green):

b. In your opinion, is this feedback useful for helping Sam to better understand similar triangles? Explain.

This feedback only provides the student with a partially correct answer and, considering the class is working on two-column proofs, this feedback is only partially helpful. It does not prompt the student to think about what they did
wrong, we usually try to ask them a question to make them try to come up with the correct answer.

<table>
<thead>
<tr>
<th>AIG TK ESU</th>
<th>c. Provide an example of how you might expand upon or change this GeoGebra problem to help make students’ understanding of similar triangles more visible. Be specific about any technological changes. I think I would change the directions of the problem to ask for a proof that uses the conjectures students have learned, instead that of just saying “justify”. I think I might also include the side lengths of the triangles so they can use those to further prove that they are not proportional.</th>
</tr>
</thead>
</table>

### Scenario 3:

Mx. Knope is working with students on the properties of exponents, both whole number and rational, and decided to use a Khan Academy quiz to formatively assess students. One of the questions asks:

$$\frac{5^{-6}}{5^{-4}}$$

Neil tried to rewrite the expression.

\[
\begin{align*}
5^{-6} & \quad \text{Step 1} \\
\frac{1}{5^2} & \quad \text{Step 3}
\end{align*}
\]

Did Neil make a mistake? If so, in which step?

**Choose 1 answer:**

- **A** Neil did not make a mistake.
- **B** Neil made a mistake in Step 1.
- **C** Neil made a mistake in Step 2.
- **D** Neil made a mistake in Step 3.

| MP | a. What mathematical concepts is this question assessing? What mathematical proficiencies (conceptual understanding, procedural fluency, strategic... |
competence, adaptive reasoning, and/or productive dispositions) is this question prompting? Explain your responses.

This is assessing procedural fluency since the student must look at the steps (procedures) that Neil is taking and decide if they are correct or not.

| FP | b. The computer provides students with correct/incorrect feedback. What additional feedback (if any) would you provide students if they made a mistake? Explain your response.  
In this problem, Neil did not make a mistake, but if Neil had made a mistake I would provide additional feedback of asking them to correct the mistake and evaluate that work as well. This would help students to think more deeply about the kinds of mistake that can be made in problems like these and, hopefully not make them. |
| TK | c. To the best of your knowledge, explain to Mx. Knope how they can access a report on students’ performance on this Khan Academy © quiz.  
I am not sure – I have not used Khan Academy like this before. |

**Scenario 4:**
Valeria turned in her homework on linear and exponential growth on Desmos ©. The first question asked her to analyze the table and determine if they demonstrated linear or exponential growth.

<p>| MP | a. What can you infer about Valeria’s understanding of this topic? |</p>
<table>
<thead>
<tr>
<th><strong>Scenario 5:</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ms. Zhang is teaching a Mathematical Modeling class and is introducing her students to finding the curve of best fit using spreadsheet technology (e.g., Excel, Google Sheets).</strong></td>
<td></td>
</tr>
</tbody>
</table>

| **AIG** | **b.** To the best of your knowledge, how else could you use Desmos © to formatively assess students’ understanding about the difference between linear and exponential growth? Be as detailed as possible. You could change the prompt of this question to ask the student to explain how they know. Desmos also allows graphing (point and lines) which means students could be asked to generate their own linear and exponential graphs and say what is different. Similarly, they could compare and contrast the already made graphs of linear and exponential, using math vocabulary to explain distinct features.  |
| **TK** |  |
| **ESU** |  |

| **MP** | **c.** Based on your response to part (b), what do you anticipate you will be able to determine about a student’s understanding? If they are asked to explain how they know it is linear or exponential, I will be able to determine if they have more of a conceptual understanding about the differences. If they generate their own graphs, I will be able to determine if they are able to go between algebraic and graphic representations. Compare and contrast also gives me more information about their conceptual understanding.  |
| **SL** | **a.** Assuming this is the first time students have used spreadsheet technology in her class, what do you think Ms. Zhang will need to teach students about the spreadsheet technology before they begin the project? If this is the first time, and I am going to assume they are using google sheets, they will need to know how to log on, how to label and input values, how to generate graphs (scatter plots) from these values, how to use sheets to generate a curve of best fit, how to use sheets to determine other things like the R^2 and residuals (if that is how far Ms. Zhang is going with the topic).  |
| **TK** |  |

| **AIG** | **b.** After initial instruction, students begin working in pairs to create their first curves of best fit for a given data set.  |
| **TK** | **b.** Provide an example of a question/prompt you would incorporate as students are working to formatively assess students’ understanding and progress.  |
| **ESU** | **FP** |
**Scenario 6:**
Back in Ms. Zhang’s class, students have finished their first drafts of their modeling projects on finding the curve of best fit. They now have to give a short, digital presentation of their findings.

<table>
<thead>
<tr>
<th><strong>AIG</strong></th>
<th><strong>MP</strong></th>
<th><strong>FP</strong></th>
<th><strong>SL</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a.</strong> What platform/method would you use to give the digital presentations? Explain how your choice might help with formative assessment.</td>
<td>c. Given your response in (b), what mathematical proficiencies (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) will the formative assessment measure? This would tell me a little about procedural fluency (are they able to create the graphs and equations). The discussion we would have would give me information about their conceptual understanding as well as their productive dispositions.</td>
<td>b. How would you encourage students to give peer-feedback during the presentations? Be specific about the methods and resources you would use. Like I said in the last questions, in Google Sheets students could comment on each other’s work, so I would have them do that. I would make sure to explain to them that they need to be commenting on the mathematics and not just the look or neatness of the presentation.</td>
<td>c. Given the most common mistake on students’ projects was confusing the <em>correlation coefficient</em> (R) with the <em>coefficient of determination</em> (R^2), what feedback might you give students? Describe how you could use the platform/method you chose in part (a), if possible, to provide this feedback. (If not possible, describe an alternative way to provide feedback.)</td>
</tr>
</tbody>
</table>
This mistake may be because of the similarity of the names and symbols for the terms and I think I would identify the mistake and then ask students to compare and contrast the two terms. On Google Sheets, I could leave a comment on their work.

### Scenario 7:

Mr. Kanumba’s Statistics class is working on the learning objective:

*Students will be able to use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*

<table>
<thead>
<tr>
<th>AIG TK</th>
<th>a. Identify a computer-based technology that Mr. Kanumba could use to formatively assess students on this objective. Possibly Desmos, but I am not really sure since I don’t have much experience teaching statistics.</th>
</tr>
</thead>
</table>
| MP ESU | b. Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment. Again, I am not really sure if I can use Desmos for statistics, but if I can, I would ask something like: Given the two histograms (and have two histograms displayed) identify the a. Mean  
b. Median  
c. Interquartile range  
d. Standard deviation  
What is the same in these two data sets? What is different? How do you know? |
| MP     | c. Given the question/prompt that you designed in (b), what might you be able to infer about students’ learning? (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?) I would be able to determine their procedural fluency in finding the mean, median, interquartile range, and sd of a data set. I would also be able to see some reasoning and conceptual understanding in their ability to compare the two data sets. |

### Scenario 8:

Students in Ms. Swanson’s class have demonstrated that they know:
(1) how to find the equation of a line parallel to a given line that passes through a given point and
(2) that perpendicular lines must intersect at a 90 degree angle.

The learning objective for today’s lesson is:

*Students will be able to find the equation of a line perpendicular to a given line that passes through a given point.*

Ms. Swanson asked students the following question on Desmos ©:

![Desmos Question]

<table>
<thead>
<tr>
<th>SL</th>
<th>a. What other prior knowledge might students need to answer this question?</th>
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</thead>
<tbody>
<tr>
<td></td>
<td><strong>What is means for lines to be parallel and perpendicular, the relationship between the slopes of parallel and perpendicular lines, slope-intercept and point-slope forms for lines, basic arithmetic</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MP</th>
<th>b. Given this question, what might she be able to infer about students’ understanding of finding the equation of a line perpendicular to a given line that passes through a given point?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Not much – this is multiple choice, so we cannot know if the student guessed right or actually knew the right answer.</strong></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>MP</th>
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</thead>
<tbody>
<tr>
<td>AIG</td>
</tr>
<tr>
<td>TK</td>
</tr>
<tr>
<td>ESU</td>
</tr>
<tr>
<td>c. Provide an example question/prompt that you believe would assess students’ achievement of the learning objective. Be specific about the technology you would choose and why you would choose it. If you would not use technology, explain why not.</td>
</tr>
</tbody>
</table>

I would want to do a few different questions on paper to really understand their learning. For one, I would ask students to find the equation of the line perpendicular to [equation] that travels through a point [point] and give them space to show their work instead of just a multiple choice. I think it would just be easier for students to show their work on a problem like this on paper rather than having to type it out on a computer.

**Scenario 9:**
Mr. Kanumba would like to use Delta Math to formatively assess students understanding of solving linear equations and inequalities in one variable, but has never done so before.

| TK | a. To the best of your knowledge, explain to Mr. Kanumba the steps he needs to take to create the Delta Math quiz. First he needs to create an account, next he needs to search for the topic, and then Delta Math will automatically generate questions for him. |
| SL MLF | b. After your instructions, Mr. Kanumba begins using Delta Math and has students works on the following problem: Carson has $560 to spend at a bicycle store for some new gear and biking outfits. Assume all prices listed include tax. - He buys a new bicycle for $283.84. - He buys 2 bicycle reflectors for $8.55 each and a pair of bike gloves for $15.96. - He plans to spend some or all of the money he has left to buy new biking outfits for $47.96 each. What is the greatest number of outfits Carson can buy with the money that's left over? Answer: 
| MP AIG TK ESU FP* | c. In your opinion, what are the benefits and drawbacks of using Delta Math when the topic is on solving linear equations and inequalities in one variable? Explain your response. I haven’t used Delta Math very much, but it seems like what students can input is just an answer, so I would not be able to see their work to identify any mistakes or misconceptions. I do think it gives automated feedback with the correct answer, so |
this would prevent me from trying to have them figure out the correct answer on their own.

**Scenario 10:**

Julian was completing an online GeoGebra © formative assessment for his Trigonometry class. After incorrectly responding to the question, he received the following automated feedback (outlined in red):

<table>
<thead>
<tr>
<th>SL</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. What might be Julian’s mistake or misconception? Explain how this feedback does or does not help Julian understand his mistake or misconception?</td>
<td></td>
</tr>
<tr>
<td>It’s hard to tell what his mistake is since this is multiple choice, but by selecting (b0 it shows he subtracted 360 from 510 correctly and selected the one that had 150 in it. The feedback is pretty helpful, it is directing him to think about where the angle lies while not giving away the answer.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SL</th>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>When Julian selected “Hint”, he received the diagram below:</td>
<td></td>
</tr>
<tr>
<td>b. Explain how this feedback does or does not help Julian understand his mistake or misconception?</td>
<td></td>
</tr>
</tbody>
</table>
This helps Julian to visualize what the hint was saying while still not completely giving away the answer. It may help him remember which way we measure angles.

c. Explain what adjustments would you make (if any) to this feedback?
I think I would change the “510 lies in quadrant 2, and cosine of the angle should be negative” to “510 lies in quadrant 2, what does this mean about the value of cosine?” Doing this makes the student have to think more instead of just giving the answer away.

Scenario 11:
Ms. Fischer and her students are working on graphing quadratic functions, and understanding how the coefficients in standard form transform the graph of a parabola (i.e., how different values of a, b, c in \( ax^2 + bx + c \) transform the graph).

MP AIG TK
a. Describe a technology enhanced formative assessment activity Ms. Fisher could engage students in?
I would use a Desmos Quiz

MP ESU
b. Provide an example of a question or prompt you might give students. Explain what you intend to measure about students’ understanding with this question/prompt (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?)
I would give students the graph of a quadratic and ask them to figure out the equation in standard form. This would assess their procedural fluency in identifying the values in a graph that correspond to the values in the equation. It would also assess their conceptual understanding about this relationship.

SL
c. Given your response in part (b), what mistake might students make? Why might they make these mistakes?
Students will most likely struggle with finding the b-value from the graph, this is not as straightforward as the a and c values as you have to do more algebraic manipulation after you identify the roots in the graph.

Scenario 12:
Mr. Flint has the learning objective that:

Students will be able to rewrite expressions involving radicals and rational exponents using the properties of exponents.
<table>
<thead>
<tr>
<th>SL</th>
<th>a. What prior knowledge must students have to complete this learning objective? Knowledge of fractions, order of operations, radicals, rational numbers, exponents, properties of exponents, and how to simplify exponents</th>
</tr>
</thead>
<tbody>
<tr>
<td>SL</td>
<td>b. What are some possible mistakes or misconceptions that might arise on a question like this? Why might they arise?</td>
</tr>
<tr>
<td>SL</td>
<td>I could see a mistake in just reading the problem, the first radical has a root three and a square, which some students might miss. I could also see a mistake in order of operations in this problem. Students may also struggle with the fractions that will occur as this always seems to be a weakness.</td>
</tr>
<tr>
<td>SL</td>
<td>c. Suppose the majority of Mr. Flint’s students select “4” as their answer, what would your next instructional steps be? Explain your response.</td>
</tr>
<tr>
<td>MLF</td>
<td>I would have students work out the problem with a partner and try to find the right answer. I would then have a few students share their responses and reasoning. If the majority of students missed it, it means that there is a problem, by working with groups they may be able to identify their own mistakes and by sharing correct answers with reasoning they should be able to solidify what they know.</td>
</tr>
</tbody>
</table>
## Mathematics Pedagogy Component

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Proficiency (MP)</td>
<td>The teacher is able to analyze, select, and design questions/prompts that promote all of the 5 Strands of Mathematical Proficiency.</td>
<td>The teacher is able to analyze, select, or design questions/prompts that promote 3 or 4 of the 5 Strands of Mathematical Proficiency.</td>
<td>Some evidence that the teacher is able to analyze, select, or design questions/prompts that promote 1 or 2 of the 5 Strands of Mathematical Proficiency.</td>
<td>Little to no evidence that the teacher is able to analyze, select, or design questions/prompts that promote the 5 Strands of Mathematical Proficiency</td>
<td>1b 2a 3a 4a 5c 6c 7b 8c* 10b</td>
</tr>
<tr>
<td><strong>NOTE:</strong> The teacher must accomplish both criteria to be at that level, otherwise they are at the next lower level</td>
<td>Teacher can analyze students’ responses for their alignment to all of the 5 Strands of Mathematical Proficiency.</td>
<td>Teacher can analyze students’ responses for their alignment to some of the 5 Strands of Mathematical Proficiency.</td>
<td>Teacher struggles to analyze students’ responses for their alignment to the 5 Strands of Mathematical Proficiency.</td>
<td>Teacher is unable to analyze students’ responses for their alignment to the 5 Strands of Mathematical Proficiency.</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** All 5 strands do not need to be addressed in every response. Evaluate applicable questions as a whole.

<table>
<thead>
<tr>
<th></th>
<th>The teacher demonstrates that they are effectively using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs.</th>
<th>The teacher demonstrates that they are using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs, but this is not always done effectively.</th>
<th>There is some evidence that the teacher is using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs.</th>
<th>Little to no evidence that the teacher is using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs.</th>
<th>1c 2a 5a 7a 8b 9a 10c 11a</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NOTE:</strong> The teacher must accomplish both criteria to be at that level, otherwise they are at the next lower level</td>
<td>The teacher is able to identify all common mistakes and misconceptions; they demonstrate that they are thinking deeply about why these mistakes and misconceptions happen.</td>
<td>The teacher is able to identify most common mistakes and misconceptions. They demonstrate basic understanding as to why these mistakes and misconceptions happen.</td>
<td>The teacher is able to identify some common mistakes, but struggles to identify common misconceptions. They struggle to demonstrate why these mistakes and misconceptions happen.</td>
<td>Little to no evidence that the teacher is able to identify mistakes or misconceptions in student responses to TEFA questions.</td>
<td></td>
</tr>
</tbody>
</table>

184
## Technology Component

<table>
<thead>
<tr>
<th></th>
<th><strong>Advanced</strong></th>
<th><strong>Proficient</strong></th>
<th><strong>Partially Proficient</strong></th>
<th><strong>Novice</strong></th>
<th><strong>Applicable Questions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Alignment to</strong></td>
<td>The teacher is able to <strong>analyze and select</strong> technology that is aligned to</td>
<td>The teacher is able to <strong>analyze</strong> technology for its alignment to the</td>
<td>The teacher struggles to <strong>analyze</strong> technology for its alignment to</td>
<td>Little to no demonstrated understanding of how to align the technology to</td>
<td>1a 4b 5b 6a 7c* 8c* 10a 11c*</td>
</tr>
<tr>
<td><strong>Instructional</strong></td>
<td>the instructional goals of formative assessment and given mathematical</td>
<td>the instructional goals of formative assessment and given mathematical</td>
<td>the instructional goal of formative assessment and given mathematical</td>
<td>the instructional goal of formative assessment and given mathematical</td>
<td></td>
</tr>
<tr>
<td><strong>Goals (AIG)</strong></td>
<td>learning objectives.</td>
<td>learning objectives, but they struggle to <strong>select</strong> technology that is</td>
<td>learning objectives.</td>
<td>learning objectives.</td>
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<td></td>
<td></td>
<td>that is aligned to the instructional goal of formative assessment or</td>
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<td></td>
<td></td>
<td>mathematical learning objectives.</td>
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<tr>
<td></td>
<td><strong>Technology</strong></td>
<td>The teacher is knowledgeable of at least 4 different technologies that</td>
<td>The teacher is knowledgeable of at least 3 different technologies that</td>
<td>Little to no evidence of knowledge of computer-based technologies or how</td>
<td>1a 2c 3c 4b 5a,b 6a 7c* 8a,c*</td>
</tr>
<tr>
<td>Knowledge (TK)</td>
<td><strong>The teacher must accomplish both criteria to be at that level, otherwise</strong></td>
<td>are appropriate for formative assessments in mathematics.</td>
<td>are appropriate for formative assessments in mathematics.</td>
<td>to operate them.</td>
<td></td>
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<tr>
<td></td>
<td><strong>they are at the next lower level</strong></td>
<td>The teacher can provide detailed explanations on how to use technology</td>
<td>The teacher attempts to, but struggles to explain how to use technology</td>
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<tr>
<td><strong>NOTE: The teacher</strong></td>
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<td>from both the student and teacher perspective.</td>
<td>from both the student and teacher perspective.</td>
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<tr>
<td><strong>must accomplish both</strong></td>
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<tr>
<td><strong>criteria to be at that level, otherwise they are at the next lower level</strong></td>
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</tr>
<tr>
<td></td>
<td><strong>Technology</strong></td>
<td>The teacher is knowledgeable of at least 3 different technologies that</td>
<td>The teacher attempts to, but struggles to explain how to use technology</td>
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<td></td>
<td><strong>The teacher</strong></td>
<td>are appropriate for formative assessments in mathematics.</td>
<td>from both the student and teacher perspective.</td>
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<td></td>
<td></td>
<td>The teacher can explain basic use technology from both the student and</td>
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<td></td>
<td></td>
<td>teacher perspective, but is unable to provide detailed or advanced</td>
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<td>directions.</td>
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</table>
## Formative Assessment Component

<table>
<thead>
<tr>
<th>Elicitation of Student Understanding (ESU)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that strongly emphasize explanation, justification, and problem solving; no questions utilize memorization and computational procedures.</td>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that emphasize explanation and justification, and almost no questions utilize memorization and computational procedures.</td>
<td>There is some evidence that the teacher is analyzing, selecting, or designing questions/prompts on TEFAs that emphasize explanation and justification, although many questions still utilize memorization and computational procedures.</td>
<td>The teacher selects or designs questions/prompts on TEFAs that primarily focus on memorization and computational procedures.</td>
<td>1b, 2c, 4b, 5b, 6b, 7b,c, 8e*, 10a*, b</td>
<td></td>
</tr>
</tbody>
</table>

**Feedback Process (FP)**

**NOTE: The teacher must accomplish both criteria to be at that level, otherwise they are at the next lower**

<table>
<thead>
<tr>
<th>Feedback Process (FP)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher is able to analyze, select, or design TEFAs that provide targeted feedback and plans for supplemental feedback. The teacher plans for students to be active participants in the feedback process in a variety of ways (e.g. discussions, guided commenting, peer use of rubric/success criteria, etc.).</td>
<td>The teacher is able to analyze, select, or design TEFAs that provide more than “correct” or “incorrect” feedback, although this feedback may not be targeted. The teacher might state they encourage students to be active participants, but do not make clear plans for how to do this.</td>
<td>The teacher is able to analyze, select, or design TEFAs that provide targeted feedback, but does not plan for supplemental feedback. The teacher plans for students to be active participants in the feedback process primarily through class/peer discussions.</td>
<td>The teacher selects or designs TEFAs that primarily provide “correct” or “incorrect” feedback. The teacher does not plan for students to be active participants in the feedback process.</td>
<td>2b, 3b, 5b, 8b*, c*, 9a,b,c, 11b,c</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moving Learning Forward (MLF)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher demonstrates that they are able to analyze students’ data to determine what students know and use this data to make adjustments to</td>
<td>The teacher demonstrates that they are able to analyze students’ data to determine what students know and use this data to make adjustments to</td>
<td>The teacher may struggle to appropriately analyze students’ data to determine what students know (i.e., Not focused on attainment of the</td>
<td>Teacher demonstrates that they do not analyze student feedback, not do they plan to use student-feedback to make N/A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>appropriate adjustments to instruction.</td>
<td>instruction. These adjustments may not always be aligned to the student data.</td>
<td>learning objective(s), but on semantics or presentation), but they use this data to make adjustments to instruction.</td>
<td>adjustments to instruction.</td>
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</tr>
</tbody>
</table>
Appendix G: Final TEFA Literacy Test for Secondary Mathematics

Thank you for participating in the TEFA Literacy Test for Secondary Mathematics. The goal of this test is to measure your knowledge and skills in TEFA for the secondary mathematics classroom.

In this test you will be presented with 11 different scenarios, each followed by 3 questions. There is not right or wrong answer – Please answer each question to the best of your ability.

In all scenarios, please assume that the teacher and students are working in a secondary mathematics classroom that is aligned to the Common Core State Standards for Mathematics (2010).

<table>
<thead>
<tr>
<th>Applicable Rubrics</th>
<th>Scenario 1:</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Mr. O’Connor, an Algebra 1 teacher, is working with students to graph linear functions in slope-intercept and point-slope form, with the learning objective: <em>Students will be able to graph the function on the x-y-axis, identify the x- and y-intercepts, and calculate the slope.</em></td>
</tr>
</tbody>
</table>

| AIG TK | a) Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions. |
|        | b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment. |
| SL     | c) Given the question/prompt that you designed in part (b), what prior knowledge would students need in order to answer this question? What are some possible misconceptions student might have? |
**Scenario 2:**

Students in Ms. Glow’s Geometry class have recently learned about the AA and SSS conjectures for similar triangles, and have begun to use two-column proofs. Following one of the lessons, Ms. Glow created a formative assessment on GeoGebra ©. One of the problems was:

![Similar Triangles - Part 1](image)

**a)** What mathematical proficiencies (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) is Sam demonstrating? Do you think Sam has any misconceptions about this concept? If so, what?

**FP**

GeoGebra © provided the following feedback to Sam (in green):

![Feedback](image)

b) In your opinion, is this feedback useful for helping Sam to better understand similar triangles? Explain.

c) Provide an example of how you would use the technology available within GeoGebra © to help make students’ understanding of similar triangles more visible. Include directions for how to make your adjustments in GeoGebra.
**Scenario 3:**

Mx. Knope is working with students on the properties of exponents, both whole number and rational, and decided to use a Khan Academy © quiz to formatively assess students. One of the questions asks:

Neil tried to rewrite the expression $\frac{5^{-6}}{5^{-4}}$.

\[
\frac{5^{-6}}{5^{-4}} = 5^{-6-(-4)} \quad \text{Step 1} \\
= 5^{-2} \quad \text{Step 2} \\
= \frac{1}{5^2} \quad \text{Step 3}
\]

Did Neil make a mistake? If so, in which step?

**Choose 1 answer:**

- **A** Neil did not make a mistake.
- **B** Neil made a mistake in Step 1.
- **C** Neil made a mistake in Step 2.
- **D** Neil made a mistake in Step 3.

| MP | a) What mathematical concepts is this question assessing? What mathematical proficiencies (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) is this question prompting? Explain your responses. |
| FP | b) This quiz provides students with correct/incorrect feedback. What additional feedback (if any) would you provide students if they made a mistake? Explain your response. |
c) To the best of your knowledge, explain to Mx. Knope how they can access a report on students’ performance on this Khan Academy © quiz.

**Scenario 4:**
Valeria turned in her homework on linear and exponential growth on Desmos ©. The first question asked her to analyze the table and determine if they demonstrated linear or exponential growth.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2.36</td>
</tr>
<tr>
<td>2</td>
<td>5.57</td>
</tr>
<tr>
<td>3</td>
<td>13.14</td>
</tr>
<tr>
<td>4</td>
<td>31.02</td>
</tr>
<tr>
<td>5</td>
<td>73.21</td>
</tr>
</tbody>
</table>

Determine if the following table demonstrates LINEAR or EXPONENTIAL growth:

- **a)** What mathematical proficiencies (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) is Valeria demonstrating? What can you infer about her understanding?

- **b)** To the best of your knowledge, how else could you use Desmos © to formatively assess students’ understanding about the difference between linear and exponential growth? Be as detailed as possible.

- **c)** Based on your response to part (b), what do you anticipate you will be able to determine about a student’s understanding (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions)?

**Scenario 5:**
Ms. Zhang is teaching a Mathematical Modeling class and is introducing her students to finding the curve of best fit using spreadsheet technology (e.g., Excel, Google Sheets).
### Scenario 6:
Mr. Kanumba’s Geometry class is working on the learning objective:

*Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>SL TK</td>
<td>a) Assuming this is the first time students have used spreadsheet technology in her class, what do you think Ms. Zhang will need to teach students about the spreadsheet technology before they begin the project?</td>
</tr>
<tr>
<td>AIG TK ESU FP</td>
<td>After initial instruction, students begin working in pairs to create their first curves of best fit for a given data set. b) Provide an example of how you could use technology to formatively assess students as they are working on this project. Be specific about the type of technology and how students would engage in the feedback process.</td>
</tr>
<tr>
<td>MP</td>
<td>c) Given your response in (b), what mathematical proficiencies (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) will the formative assessment measure? Be sure to explain your reasoning.</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 6:</td>
<td></td>
</tr>
<tr>
<td>Mr. Kanumba’s Geometry class is working on the learning objective:</td>
<td></td>
</tr>
</tbody>
</table>

*Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.*

| AIG TK | a) Identify a computer-based technology that Mr. Kanumba could use to formatively assess students on this objective. |
| ESU | b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment focused on this learning objective. |
| MP | c) Given the question/prompt that you designed in (b), what might you be able to infer about students’ learning? (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) |
### Scenario 7:

Students in Ms. Swanson’s class have demonstrated that they know:

1. how to find the equation of a line parallel to a given line that passes through a given point and
2. that perpendicular lines must intersect at a 90 degree angle.

The learning objective for today’s lesson is:

_Students will be able to find the equation of a line perpendicular to a given line that passes through a given point._

Ms. Swanson asked students the following question on Desmos ©:

![Question Image]

- What is the equation of the line perpendicular to \(x + 3y = 6\) that travels through the point (1,5)?
  - \(y = 2x + 1\)
  - \(y = \frac{2}{3}x + 6\)
  - \(y = 3x + 2\)
  - \(y = 8x - 3\)

<table>
<thead>
<tr>
<th>SL</th>
<th>a) What other prior knowledge might students need to answer this question? What are some common misconceptions or mistakes student might make on a problem like this?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>b) How well do you think this problem assesses students’ understanding (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions)? Explain your response.</td>
</tr>
<tr>
<td>AIG* TK*</td>
<td>c) Provide an example question/prompt that you believe would assess students’ achievement of the learning objective. Be specific about the technology you would choose and why you would choose it. If the task is NOT accomplishable via technology, explain why not.</td>
</tr>
</tbody>
</table>

### Scenario 8:

Mr. Kanumba would like to use Delta Math to formatively assess students understanding of solving linear equations and inequalities in one variable, but has never done so before.

| TK | a) To the best of your knowledge, explain to Mr. Kanumba the steps he needs to take to create the Delta Math quiz. Be as detailed as possible. |
After your instructions, Mr. Kanumba begins using Delta Math and has students work on the following problem:

Carson has $560 to spend at a bicycle store for some new gear and biking outfits. Assume all prices listed include tax.
- He buys a new bicycle for $283.84.
- He buys 2 bicycle reflectors for $8.55 each and a pair of bike gloves for $15.96.
- He plans to spend some or all of the money he has left to buy new biking outfits for $47.96 each.

What is the greatest number of outfits Carson can buy with the money that’s left over?

b) Mr. Kanumba notices students are struggling to begin, how might you help students? Provide as many details as possible.

c) In your opinion, what are the benefits and drawbacks of using Delta Math when the topic is on solving linear equations and inequalities in one variable? Explain your response.

Scenario 9:

Julian was completing an online GeoGebra © formative assessment for his Trigonometry class. After incorrectly responding to the question, he received the following automated feedback (outlined in red):

a) What might be Julian’s mistake or misconception? Explain how this feedback does or does not help Julian understand his mistake or misconception?
When Julian selected “Hint”, he received the diagram below:

**Trigonometric ratios of general angles and basic angles**

Express \( \cos 510^\circ \) in terms of the trigonometric ratio of its basic angle.

- a. \( \cos 30^\circ \)
- b. \( \cos 150^\circ \)
- c. \(-\cos 150^\circ \)
- d. \(-\cos 30^\circ \)

Your answer: [b] ✅ Hint [Next Question]

510° lies in quadrant 2, and cosine of the angle should be negative. Also, 150° is > 90°, so it is not the basic angle.

b) What adjustments would you make to this feedback and why? If you would not make any adjustments, explain why not.

c) Suppose a number of other students also struggled with this problem. How would you engage students to address their mistake/misconception? Be as detailed as possible.

---

**Scenario 10:**

Ms. Fischer and her students are working on graphing quadratic functions, and understanding how the coefficients in standard form transform the graph of a parabola (i.e., how different values of \( a, b, c \) in \( ax^2 + bx + c \) transform the graph).

<table>
<thead>
<tr>
<th>AIG</th>
<th>TK</th>
<th>ESU*</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Identify a technology Ms. Fisher could use to formatively assess students on this topic. Explain why you chose this technology and how the teacher and students can use it.</td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>MP</th>
<th>ESU</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) Provide an example of a question or prompt you might give students. Explain what you intend to measure about students’ understanding with this question/prompt (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions).</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>SL</th>
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<tbody>
<tr>
<td>c) Given your response in part (b), what mistake might students make? What misconceptions might they have? Explain your response.</td>
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</tbody>
</table>
**Scenario 11:**
Mr. Flint has the learning objective that:

*Students will be able to rewrite expressions involving radicals and rational exponents using the properties of exponents.*

| SL | a) What prior knowledge must students have to complete this learning objective? What might be some common misconceptions or mistakes? |
| FP | Suppose Mr. Flint uses a Kahoot! to assess students’ learning, and one of the questions was: |
| TK | b) Suppose the majority of Mr. Flint’s students select “4” as their answer, how would you engage students in the feedback process? Be specific about how students would receive this feedback. |
| AIG* | c) To the best of your knowledge, what kind of feedback does Kahoot! provide? Do you think this feedback is adequate? Explain why or why not. |
Appendix H: External Rater Evaluation Guide

On the following pages you will find a table that I have used to help organize my thoughts as I evaluate the test.

I go from RUBRIC to TEST, meaning that I look at one rubric element and then evaluate all applicable questions. For example,
1) I look at the Mathematical Proficiency element, and then I go to each applicable question.
2) I start with question 1b, circle words/phrases and make a note about anything that may show evidence of Mathematical Proficiency. I then assign a “Preliminary Level” of proficiency for that question based on the description in the rubric.

<table>
<thead>
<tr>
<th>Component 1: Mathematics Pedagogy</th>
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</thead>
<tbody>
<tr>
<td>Mathematical Proficiency</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Notes/Evidence</th>
<th>Preliminary Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1b</td>
<td>Procedural fluency – graphing</td>
<td>Proficient</td>
</tr>
<tr>
<td></td>
<td>Conceptual understanding – given equation, graph</td>
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</tbody>
</table>

3) After I have reviewed all applicable questions, I assign each “Preliminary Level” a “Numerical Level”
   a) Novice = 0
   b) Partially Proficient = 1
   c) Proficient = 2
   d) Advanced = 3

4) Finally, I determine an “Overall Level” by averaging and rounding the “Numerical Levels”
   a) Add all the Numerical Levels and divide by the total number (ignore those marked NA)
   b) Round to a whole number using standard rounding procedures
   c) Assign an “Overall Level” by converting the whole number back to Novice, Partially Proficient, Proficient, or Advanced.

NOTES given the open-ended nature of the test …
- There are questions noted with an asterisk (*) - this indicates that the question may elicit that data, but may not. If you do not find evidence of the rubric element in the question with an asterisk, you can write “NA”
### Mathematical Proficiency

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Notes/Evidence</th>
<th>Preliminary Level</th>
<th>Numerical Level</th>
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</thead>
<tbody>
<tr>
<td>1b</td>
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<td>2a</td>
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<td>4a</td>
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<td>4c</td>
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<td>5c</td>
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<td>8c*</td>
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<td>10b</td>
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</table>

Average of Numerical Levels rounded to whole number

Overall proficiency level

### Student Learning

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Notes/Evidence</th>
<th>Preliminary Level</th>
<th>Numerical Level</th>
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<tbody>
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</table>
## Technology Component

### Alignment to Instructional Goals

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Notes/Evidence</th>
<th>Preliminary Level</th>
<th>Numerical Level</th>
</tr>
</thead>
<tbody>
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Average of Numerical Levels rounded to whole number ↓

Overall proficiency level

### Technology Knowledge

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<th>Question Number</th>
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Average of Numerical Levels rounded to whole number ↓

Overall proficiency level
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Average of Numerical Levels rounded to whole number

Overall proficiency level

Formative Assessment Component

Elicitation of Student Understanding
<table>
<thead>
<tr>
<th>Question Number</th>
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Average of Numerical Levels rounded to whole number \( \downarrow \)

Overall proficiency level
Appendix I: Expert Review

Review of the Rubric and TEFA Literacy Test for Secondary Mathematics

Expert Reviewer 1

NOTE: Expert Reviewer 1 did not fill out the Review Directions, they only commented on the Test and Rubric

Expert Reviewer 1’s Comments on the First Draft of the Test

<table>
<thead>
<tr>
<th>Q #</th>
<th>Scenario</th>
<th>Applicable Rubric</th>
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| 1   | Mr. O’Connor, an Algebra I teacher, is working with students to graph linear functions in slope-intercept and point-slope form. Students need to be able to graph the function on the x-y-axis, identify the x- and y-intercepts, and calculate the slope.  
   a) Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions.  
   b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.  
   c) Given the question/prompt that you designed in part (b), what might you be able to infer about students’ learning? (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?) | ICT  
   TIG  
   ESU  
   SL  
   MP |
| 2   | Students in Ms. Glow’s Geometry class have recently learned about the AA and SSS conjectures for similar triangles, and have begun to use two-column proofs. Following one of the lessons, Ms. Glow created a formative assessment on GeoGebra ©. One of the problems asked: | MLF  
   FP  
   SL  
   MP  
   ESU |
a) Sam provided the following answer:

The triangles are not similar because they do not have matching angles.

What can you determine about Sam’s understanding of similar triangles?

b) GeoGebra® provided the following feedback to Sam (in green):

Justify your response above:

1. The triangles are not similar because they do not have matching angles.
   Possible answer: The corresponding angles are not congruent.

In your opinion, is this feedback useful for Sam? Explain.

c) Provide an example of how you might expand upon or change this GeoGebra® problem to help make students’ understanding of similar triangles more visible. Be specific about any technological changes.

3  Ms. Knoppe is working with students on the properties of exponents, both whole number and rational, and decided to use a Khan Academy® quiz to formatively assess students. One of the questions asks:

\[
\frac{5^4}{5^2} = \frac{5^4}{5^2} = 5^{4-2} = 5^2 = 25
\]

Did she make a mistake? If so, in which step?

Choose an answer:

- She did not make a mistake.
- She made a mistake in Step 1.
- She made a mistake in Step 2.
- She made a mistake in Step 3.

a) What mathematical concepts is this question assessing? What mathematical proficiencies (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) is this question prompting? Explain your responses.

b) The computer only offers students correct/incorrect feedback. What additional feedback (if any) would you provide students if they made a mistake? Explain your response.

c) To the best of your knowledge, explain to Ms. Knoppe how they can access a report on students’ performance on this Khan Academy® quiz.
Valeria turned in her homework on linear and exponential growth on Desmos ©. The first question asked her to analyze the table and determine if they demonstrated linear or exponential growth.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
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<tbody>
<tr>
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<td>3</td>
<td>8.08</td>
</tr>
<tr>
<td>4</td>
<td>16.16</td>
</tr>
</tbody>
</table>

a) What can you determine about Valeria’s understanding of this topic?

b) To the best of your knowledge, how else could you use Desmos © to formatively assess students’ understanding about the difference between linear and exponential growth? Be as detailed as possible.

c) Based on your response to part (b), what do you anticipate you will be able to determine about a student’s understanding?

Ms. Zhang is teaching a Mathematical Modeling class and is introducing her students to finding the curve of best fit using spreadsheet technology (e.g., Excel, Google Sheets).

a) What do you think Ms. Zhang will need to teach students about the spreadsheet technology before they begin the project?

After initial instruction, students begin working in pairs to create their first curves of best fit for a given data set.

b) How should Ms. Zhang collect formative assessment data on students’ learning process while they are working?

c) Given your response in (b), what mathematical proficiencies (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) will the formative assessment measure?

Back in Ms. Zhang’s class, students have finished their first drafts of their modeling projects on finding the curve of best fit. They now have to give a short, digital presentation of their findings.

a) What platform/method would you use to give the digital presentations? Explain your choice.

b) How would you encourage students to give peer-feedback during the presentations? Be specific.

c) Given the most common mistake on students’ projects was confusing the correlation coefficient (R) with the coefficient of determination (R²), what feedback might you give students? Describe how you would provide this feedback.

Mr. Kanumba’s Statistics class is working on the learning objective:

*Students will be able to use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.*

a) Identify a computer-based technology that Mr. Kanumba could use to help students achieve this objective.

b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.

c) Given the question/prompt that you designed in (b), what might you be able to infer about students’ learning? (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?)

Students in Ms. Swanson’s class have demonstrated that they know:

1) how to find the equation of a line parallel to a given line that passes through a given point and
2) that perpendicular lines must intersect at a 90 degree angle.

The learning objective for today’s lesson is:

*Students will be able to find the equation of a line perpendicular to a given line that passes through a given point.*

Ms. Swanson asked students the following question on Desmos ©:
a) What other prior knowledge might students need to answer this question?
b) Given this question, what might she be able to infer about students’ understanding of finding the equation of a line perpendicular to a given line that passes through a given point?
c) Provide an example question/prompt that you believe would assess students’ achievement of the learning objective. Be specific about the technology you would choose and why you would choose it. If you would not use technology, explain why not.

9 Mr. Kanumba would like to use Delta Math to review solving linear equations and inequalities in one variable, but has never done so before.
   a) To the best of your knowledge, explain to Mr. Kanumba the steps he needs to take to create the Delta Math quiz.
   b) To the best of your knowledge, explain what types of questions/prompts and feedback Mr. Kanumba can design on Delta Math.
   c) In your opinion, what are the benefits and drawbacks of using Delta Math when the topic is on solving linear equations and inequalities in one variable? Explain your response.

10 Julian was completing an online GeoGebra formative assessment for his Trigonometry class. After incorrectly responding to the question, he received the following automated feedback (outlined in red):

   a) Explain how this feedback does or does not help Julian understand his mistake or misconception? When Julian selected “Hint”, he received the diagram below:

   b) Explain how this feedback does or does not help Julian understand his mistake or misconception?
   c) Explain what adjustments would you make (if any) to this feedback?

11 Ms. Fischer and her students are working on graphing quadratic functions, and understanding how the coefficients in standard form transform the graph of a parabola (i.e., how different values of a, h, and c in \( ax^2 + bx + c \) transform the graph).
   a) Describe a technology enhanced formative assessment activity Ms. Fischer could engage students in?
   b) What resources and materials will Ms. Fischer and her students need to complete this activity?
   c) Provide an example of a question or prompt you might give students. Explain what you intend to measure about students’ understanding with this question/prompt (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate)?

12 Mr. Flint has the learning objective that:

   Students will be able to explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

   Using this learning objective Mr. Flint designed the following question in their Google Classroom:
**Expert Reviewer 1’s Comments on the First Draft of the Rubric**

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_This rubric is used to evaluate responses on the TEFA Literacy Test for Secondary Mathematics._

**Directions:**

- Each question on the test corresponds to most, but not all of the elements within each Component. Each question has "Applicable Rubrics" noted on the test.
- Literacy Levels for each element will be determined by simple majority.
  - For example, if a preservice teacher is determined to be "Partially Proficient" 4 times and "Proficient" 6 times out of 10 ICT applicable questions, they will be considered "Proficient" in ICT.
  - In the case of a "tie", the preservice teacher will be considered at the lower level. For example, if they are "Partially Proficient" 5 times and "Novice" 5 times out of 10 MLF applicable questions, they will be considered "Novice" in MLF.

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**Commented [A]:** not clear what this means

**Commented [A]:** I’m mostly concerned about the mapping from rubric to constructs – if each rubric has only one skill and the only score point is the proficiency level, then I am assuming that the sum of scores is the applicable mapping and perhaps my thought about a possible matrix solution is over-thinking it. The rules mentioned here lead me to believe that there is only one dimension on each rubric as in a sense each rubric is just one construct with one scale. However, the proposal that the simple majority wins might not be a good separator in every circumstance. e.g. if it’s a 5-4 decision out of nine items, because there could be weighting issues on each item in a cluster for a specific context in which there are multiple rubrics that would sway a judge to award a higher or lower overall score based on what else is happening to the scores.

You can make a case for just collecting data to find out what the eventual rules should be, or you could computationally model the whole thing and determine probabilities and then make some educated guesses.
### Component #1: Technology Integration

<table>
<thead>
<tr>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
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</thead>
<tbody>
<tr>
<td><strong>Identifying Classroom Technologies (ICT)</strong></td>
<td>Demonstrated knowledge of a variety of computer-based technologies. The knowledge includes that of a variety of mathematics specific technologies and the types of mathematics they can support.</td>
<td>Demonstrated knowledge of computer-based technologies. This knowledge includes that of some mathematics specific technologies and the types of mathematics they can support.</td>
<td>Some evidence of knowledge of computer-based technologies. This knowledge is limited to general computer-based technologies and does not include mathematics specific technologies.</td>
</tr>
<tr>
<td><strong>Operating Classroom Technologies (OCT)</strong></td>
<td>For a number of different mathematics specific technologies, the teacher is able to explain all of the following: - the necessary material/resources for operating the technology - how students interact with the technology</td>
<td>For more than one mathematics specific technology, the teacher is able to explain all of the following: - the necessary material/resources for operating the technology - how teachers interact with the technology - how students interact with the technology</td>
<td>For at least one mathematics specific technology, the teacher is able to explain the necessary materials/resources for operating the technology and at least one of the following: - how teachers interact with the technology - how students interact with the technology</td>
</tr>
<tr>
<td><strong>Technology and Instructional Goals (DIG)</strong></td>
<td>The teacher may or may not be able to explain how to make modifications to pre-designed TEFAs (when applicable).</td>
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### Component #2: Formative Assessment (FA)

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<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
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<tr>
<td><strong>Existing Student Understanding (ESU)</strong></td>
<td>The teacher consistently selects and designs questions/prompts that elicit information on student understanding. Questions/prompts consistently provide students with a variety of ways to demonstrate their thinking.</td>
<td>The teacher consistently selects and designs questions/prompts that elicit information on student understanding. Questions/prompts provide students with a variety of ways to demonstrate their thinking.</td>
<td>The teacher sometimes selects or designs questions/prompts that elicit information on student understanding. Some questions/prompts focus on memorization and computational procedures.</td>
</tr>
<tr>
<td><strong>Feedback Process (FP)</strong></td>
<td>The teacher plans for TEFAs that provide more than “correct” or “incorrect” feedback, and plans for targeted and supplemental feedback. The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, and provide clear alternatives. The teacher makes clear plans for students to be active participants in the feedback process.</td>
<td>The teacher plans for TEFAs that provide more than “correct” or “incorrect” feedback, and plans for targeted and supplemental feedback. The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, and provide clear alternatives. The teacher makes clear plans for students to be active participants in the feedback process.</td>
<td>The teacher plans for TEFAs that provide more than “correct” or “incorrect” feedback, but does not plan for providing more targeted or supplemental feedback. The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, but do not provide alternatives. The teacher might say they encourage students to be active participants, but do not make clear plans.</td>
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<tr>
<td><strong>Moving Learning Forward (MLF)</strong></td>
<td>The teacher consistently analyzes students’ data to determine what students know in relation to students’ attainment of the learning objective(s), the teacher consistently makes clear plans for instructional adjustments based on students’ feedback.</td>
<td>The teacher consistently analyzes students’ data to determine what students know in relation to students’ attainment of the learning objective(s), the teacher consistently makes clear plans for instructional adjustments based on students’ feedback.</td>
<td>The teacher attempts to analyze students’ data to determine what students know, it is not always related to students’ attainment of the learning objective(s) (e.g., focus on computation or presentation rather than mathematics). The teacher demonstrates that they want students to feedback to make adjustments to instruction, but do not make clear plans.</td>
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**Expert Reviewer 2**

**Expert Reviewer 2’s Comments on the Review Directions**

2) Please provide a review of the test with regard to the following. You are encouraged to make tracked changes on the document and provide general comments here:
<table>
<thead>
<tr>
<th>Topic</th>
<th>General Comments</th>
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<tbody>
<tr>
<td><strong>Clarity:</strong> Are the questions clear? Do you understand what they are asking? If not, how can they be improved?</td>
<td>In general the questions are expressed well and the meaning is clear. I have highlighted a few places where more clarity or different wording would be helpful but overall the questions are well written.</td>
</tr>
<tr>
<td><strong>Content:</strong> Do the questions address common skills and knowledge that would be associated with TEFA Literacy for Secondary Mathematic? If not, how can they be improved?</td>
<td>How comprehensively the responses will allow the researcher to fully explore the topic will, to some extent, depend on how detailed the responses are. Questions to explore the relevant areas of skills and knowledge are included in the test and probe into the respondents’ thinking but the quality of the answers is important. The researcher might therefore consider how full answers can be encouraged when administering the test.</td>
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<tr>
<td><strong>Eliciting:</strong> Do you think the questions will elicit responses that are adequate and measurable? If not, how can they be improved?</td>
<td>The questions should generate some interesting data. If answered in the detail requested, there should be a substantial amount of data that can be usefully analysed.</td>
</tr>
<tr>
<td><strong>Additional comments:</strong> This could be about any redundancies, questions you might eliminate, ambiguities that need to be addressed, potential leading questions, syntax, etc.</td>
<td>I have added some comments to the text where I had questions or suggestions. Some of these may be due being an ‘outsider’ and not having a full understanding of the aims of the study. In order to assess the appropriateness of the test questions it would have been useful to see the research questions and reference the test questions to these. With this information it would have been easier to assess whether the test questions were comprehensive enough and would generate the data needed to answer the research questions.</td>
</tr>
</tbody>
</table>

3) Please provide a review of the rubric with regard to the following. You are encouraged to make tracked changes on the document and provide general comments here:
**Directions & Use:** Do the scoring directions make sense? If not, how can they be improved?

Yes, the scoring directions are clear. I am assuming that if the test question is not answered this would be counted as evidence of ‘Novice’. Most of the ‘Novice’ descriptors commence with “little or no evidence” which makes this clear but it would be helpful to make sure that the “no evidence” scenario is included in all the ‘Novice’ descriptors.

**Clarity:** Is the wording of the rubric clear and concise? Do you understand what is being evaluated? If not, how can it be improved?

Yes, the wording of the rubric is clear but the quantity of description gives the reader a lot of information to digest and process about one category. It would be worth trying to shorten the wording in places. In several places there are two or more criteria within the description (sometimes as separate paragraphs) and it is not always immediately clear whether all of these need to be evidenced or only one. It would be worth considering whether you need all these criteria. Perhaps you could reduce some of these down to less criteria that still capture the most important indicators of the level.

Where there are key words in the descriptions that mark the difference between levels, perhaps these could be typed in **bold**?

**Content:** Does the content of the rubric align to the components/elements being measured? If not, how can it be improved?

Yes, the different concepts seem clear in the rubric.

**Progression:** Does the progression of levels make sense? Are there too big or too small of differences between one level and the next?

The progression level generally make sense. The changes in wording across the table indicate a progression and are understandable. There is however always a difficulty of what the measures or levels mean when using word descriptions since words like “some” or “most” can be interpreted differently. I am not suggesting you change these but it does mean there will inevitably be unequal differences between the levels, depending on how these words are interpreted.
**Additional comments:** This could be about any redundancies, elements you might eliminate/add, ambiguities that need to be addressed, syntax, etc.

I do have a concern about how the rubric is used with the test questions when the rubric specifies multiple instances and the test question only requires one. For example, Q1 asks for “a computer-based” technology but to be scored at “advanced” level there needs to be evidence of “a variety of computer-based technologies”. The same happens in some other test questions. This seems to be a design fault that could skew the results away from the advanced level.

4) **Alignment:** Please comment on the alignment between the test and the rubric. Are there any instances of misalignment? Unclear connections? Improvements to be made?

The point above is relevant since it is about this connection as well as the scoring.

5) **Additional Comments:** Please provide any additional comments that may help in the development of the test and rubric below.

I can see that a lot of careful thought has gone into the construction of the test and rubric. The test questions are interesting and are crafted to probe into the thinking of the respondents. I would suggest the researcher considers some minor revisions and then undertakes a pilot to examine the functionality in practice. Although this takes time, it often uncovers other issues and leads to better research instruments – and therefore better data.

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**Expert Reviewer 2’s Comments on the First Draft of the Test**

**TEFA Literacy Test for Secondary Mathematics**

In all scenarios, please assume that the teacher and students are working in a secondary mathematics classroom that is aligned to the Core Common Standards for Mathematics (2010).

Applicable Rubrics are identified by their acronyms; for example, "Identifying Classroom Technologies" is "ICT".

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<thead>
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<th>Scenario</th>
<th>Applicable Rubrics</th>
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   a) Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions.  
   b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.  
   c) Given the question/prompt that you designed in part (b), what might you be able to infer about students’ learning? (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?) | ICT, TIG, ESU, SL, MP |
| 2   | Students in Ms. Glow’s Geometry class have recently learned about the AA and SSS conjectures for similar triangles, and have begun to use two-column proofs. Following one of the lessons, Ms. Glow created a formative assessment on GeoGebra ©. One of the problems asked: | MLF, FP, SL, MP, ESU |

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*Commented [A1]: “fixed” could mean different things. Is this a learning outcome or a pre-requisite for some other activity?

*Commented [A2]: The following problem is not written in question form so “asked” seems inappropriate. Perhaps just “state”?
a) Sam provided the following answer:

The triangles are not similar because they do not have matching angles

What can you determine about Sam’s understanding of similar triangles?

b) GeoGebra® provided the following feedback to Sam (in green):

In your opinion, is this feedback useful for Sam? Explain.

c) Provide an example of how you might expand upon or change this GeoGebra® problem to help make students’ understanding of similar triangles more visible. Be specific about any technological changes.

3 Ms. Knope is working with students on the properties of exponents, both whole number and rational, and decided to use a Khan Academy® quiz to formatively assess students. One of the questions asks:

OCT

a) What mathematical concepts is this question assessing? What mathematical proficiencies (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive disposition) is this question prompting? Explain your responses.

b) The computer only offers students correct/incorrect feedback. What additional feedback (if any) would you provide students if they made a mistake? Explain your response.

c) To the best of your knowledge, explain to Ms. Knope how they can access a report on students’ performance on this Khan Academy® quiz.
4. Valeria turned in her homework on linear and exponential growth on Desmos ©. The first question asked her to analyze the table and determine if they demonstrated linear or exponential growth.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
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<tr>
<td>2</td>
<td>6</td>
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<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>48</td>
</tr>
</tbody>
</table>

- a) What can you determine about Valeria’s understanding of this topic?
- b) To the best of your knowledge, how else could you use Desmos © to formatively assess students’ understanding about the difference between linear and exponential growth? Be as detailed as possible.
- c) Based on your response to part (b), what do you anticipate you will be able to determine about a student’s understanding?

5. Ms. Zhang is teaching a Mathematical Modeling class and is introducing her students to finding the curve of best fit using spreadsheet technology (e.g., Excel, Google Sheets).

- a) What do you think Ms. Zhang will need to teach students about the spreadsheet technology before they begin the project?
- b) After initial instruction, students begin working in pairs to create their first curve of best fit for a given data set.
- c) How should Ms. Zhang collect formative assessment data on students’ learning process while they are working?
- d) Given your response in (b), what mathematical proficiencies (conceptional understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) will the formative assessment measure?

6. Back in Ms. Zhang’s class, students have finished their first drafts of their modeling projects on finding the curve of best fit. They now have to give a short, digital presentation of their findings.

- a) What platform/method would you use to give the digital presentation? Explain your choice.
- b) How would you encourage students to give peer-feedback during the presentations? Be specific.
- c) Given the most common mistake on students’ projects was confusing the correlation coefficient (R) with the coefficient of determination (R²), what feedback might you give students? Describe how you would provide this feedback.

7. Mr. Kanumba’s Statistics class is working on the learning objective:

Students will be able to use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

- a) Identify a computer-based technology that Mr. Kanumba could use to help students achieve this objective.
- b) Using the technology you identified in part (a), provide an example of a question/promp you would incorporate in a formative assessment.
- c) Given the question/promp that you designed in (b), what might you be able to infer about students’ learning? (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?)

8. Students in Ms. Swanson’s class have demonstrated that they know:

1) how to find the equation of a line parallel to a given line that passes through a given point.
2) that perpendicular lines must intersect at a 90 degree angle.

The learning objective for today’s lesson is:

Students will be able to find the equation of a line perpendicular to a given line that passes through a given point.

Ms. Swanson asked students the following question on Desmos ©:
213

| 9 | Mr. Kanumba would like to use Delta Math to review solving linear equations and inequalities in one variable, but has never done so before.  
|   | a) To the best of your knowledge, explain to Mr. Kanumba the steps he needs to take to create the Delta Math quiz.  
|   | b) To the best of your knowledge, explain what types of questions/prompts and feedback Mr. Kanumba can design on Delta Math.  
|   | c) In your opinion, what are the benefits and drawbacks of using Delta Math when the topic is on solving linear equations and inequalities in one variable? Explain your response. |

| 10 | Julian was completing an online GeoGebra ® formative assessment for his Trigonometry class. After incorrectly responding to the question, he received the following automated feedback (outlined in red):  
|    | a) Explain how this feedback does or does not help Julian understand his mistake or misconception? When Julian selected “Hint”, he received the diagram below: |

| 11 | Ms. Fischer and her students are working on graphing quadratic functions, and understanding how the coefficients in standard form transform the graph of a parabola (i.e., how different values of \( a \), \( b \), and \( c \) in \( ax^2 + bx + c \) transform the graph).  
|    | a) Describe a technology enhanced formative assessment activity Ms. Fisher could engage students in?  
|    | b) What resources and materials will Ms. Fisher and her students need to complete this activity?  
|    | c) Provide an example of a question or prompt you might give students. Explain what you intend to measure about students’ understanding with this question/prompt (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem-solving skills will they be able to demonstrate?) |

| 12 | Mr. Flint has the learning objective that:  
|    | Students will be able to explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.  
|    | Using this learning objective Mr. Flint designed the following question in their Google Classroom: |
Expert Reviewer 2’s Comments on the First Draft of the Rubric

TEFA Literacy for Secondary Mathematics
Rubric

Directions:
- Each question on the test corresponds to most, but not all of the elements within each Component. Each question has "Applicable Rubrics" noted on the test.
- Literacy Levels for each element will be determined by simple majority.
  - For example, if a preservice teacher is determined to be "Partially Proficient" 4 times and "Proficient" 6 times out of 10 ICT applicable questions, they will be considered "Proficient" in ICT.
  - In the case of a "tie", the preservice teacher will be considered at the lower level. For example, if they are "Partially Proficient" 5 times and "Novice" 5 times out of 10 MLF applicable questions, they will be considered "Novice" in MLF.
Component #1: Technology Integration

<table>
<thead>
<tr>
<th>Component: Identification of ICT (ICT)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demonstrated knowledge of a variety of computer-based technologies. The knowledge includes that of a variety of mathematics specific technologies and the types of mathematics they can support.</td>
<td>Demonstrated knowledge of a variety of computer-based technologies. The knowledge includes that of a variety of mathematics specific technologies and the types of mathematics they can support.</td>
<td>Some evidence of knowledge of computer-based technologies. The knowledge is limited to general computer-based technologies and does NOT include mathematics specific technologies.</td>
<td>Little to no evidence of knowledge of computer-based technologies.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operating Classroom Technologies (OCT)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>For a number of different mathematics specific technologies, the teacher is able to explain all of the following: - the necessary material/resources for operating the technology - how students interact with the technology. The teacher is also able to explain how to make modifications to pre-designed TEFAs (when applicable).</td>
<td>For more than one mathematics specific technology, the teacher is able to explain all of the following: - the necessary material/resources for operating the technology - how teachers interact with the technology. The teacher may or may not be able to explain how to make modification to pre-designed TEFAs (when applicable).</td>
<td>For at least one mathematics specific technology, the teacher is able to explain the necessary material/resources for operating the technology and at least one of the following: - how teachers interact with the technology - how students interact with the technology. The teacher cannot explain how to make modification to pre-designed TEFAs (when applicable).</td>
<td>Little to no evidence of how to operate computer-based technologies, both non-mathematics and mathematics specific.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Technology and Instructional Goal (TIG)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher is able to analyze, select, and design TEFAs that align to the purposes of formative assessment and gives mathematical learning objectives.</td>
<td>The teacher is able to analyze, select, and design TEFAs that align to the purposes of formative assessment and gives mathematical learning objectives.</td>
<td>The teacher is able to design TEFAs that align to the purpose offormative assessment and gives mathematical objectives, but often struggle to design their own.</td>
<td>Little to no demonstrated understanding of how to align the TEFAs to the instructional goal of formative assessment or mathematics.</td>
<td></td>
</tr>
</tbody>
</table>

Component #2: Formative Assessment (FA)

<table>
<thead>
<tr>
<th>Component: Exiling Student Understanding (EU)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher consistently selects questions/promptsthat elicit information on student understanding. Questions/prompts consistently provide students with a variety of ways to demonstrate their thinking.</td>
<td>The teacher often selects or designs questions/prompts that elicit information on student understanding. Questions/prompts provide students with a variety of ways to demonstrate their thinking; few questions/prompts focus on memorization and computational procedures.</td>
<td>The teacher sometimes selects or designs questions/prompts that elicit information on student understanding. Some questions/prompts focus on memorization and computational procedures.</td>
<td>The teacher consistently selects questions/prompts that elicit little to no information on student understanding. Questions/prompts focus on memorization and computational procedures.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Feedback Process (FP)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher consistently plans for TEFAs that provide more than “correct” or “incorrect” feedback and plans for targeted and supplemental feedback. The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, and use clear alternatives. The teacher makes clear plans for students to be active participants in the feedback process.</td>
<td>The teacher consistently plans for TEFAs that provide more than “correct” or “incorrect” feedback, and plans for targeted or supplemental feedback. The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, alternatives are vague or missing. The teacher makes clear plans for students to be active participants in the feedback process.</td>
<td>The teacher plans for TEFAs that provide more than “correct” or “incorrect” feedback, but does not plan for providing more targeted or supplemental feedback. The teacher demonstrates that they do not consider “correct” or “incorrect” feedback adequate and appropriate, but do not provide alternatives. The teacher might say they encourage students to be active participants, but do not make clear plans.</td>
<td>The teacher plans for TEFAs as that primarily provide “correct” or “incorrect” feedback. The teacher consistently demonstrates that they consider “correct” or “incorrect” feedback adequate and appropriate. The teacher does not plan for or encourage students to be active participants in the feedback process.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Moving Learning Forward (MLF)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher consistently analyzes students’ data to determine what students know in relation to the learning objectives. The teacher consistently makes clear plans for instructional adjustments based on students’ feedback.</td>
<td>The teacher demonstrates that they can analyze students’ data to determine what students know in relation to the learning objectives, but may not do so consistently. The teacher demonstrates that they can plan for instructional adjustments based on students’ feedback.</td>
<td>The teacher attempts to analyze students’ data to determine what students know, it is not always related to students’ attainment of the learning objective(s), but may not do so consistently. The teacher demonstrates that they want to use student-feedback to make adjustments to instruction, but do not make clear plans.</td>
<td>The teacher struggles to analyze students’ data to determine what students know. Teacher demonstrates that they do not plan to use student-feedback to make adjustments to instruction.</td>
<td></td>
</tr>
</tbody>
</table>

Comments:
- Commented (A1): If the last question only asks for one, how can this level be evidenced?
- Commented (A2): Both? Or either?
- Commented (A3): Does the teacher have to evidence this and the two points above to achieve this level? Perhaps this extra criterion could be added to the two above instead of having a whole new paragraph?
- Commented (A4): This doesn’t really help me with assessing the evidence. Perhaps it could be left out.
- Commented (A5): Do you really need to say this? If it is not required could you just leave it out?
- Commented (A6): It may be better to leave out the word “often.” They either struggle or they do not.
- Commented (A7): These look the same.
<table>
<thead>
<tr>
<th>Component #3: Mathematics Pedagogy (MP)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Student Learning (SL)</strong></td>
</tr>
<tr>
<td>Advanced</td>
</tr>
<tr>
<td>The teacher is consistently selecting, analyzing or designing mathematical tasks on TEFAs that align with students' prior knowledge and typical learning processes.</td>
</tr>
<tr>
<td>Mathematics content on the TEFA consistently prompts students to make connections, conjecture, and reason mathematically.</td>
</tr>
<tr>
<td>Proficient</td>
</tr>
<tr>
<td>The teacher is often selecting, analyzing or designing mathematical tasks on TEFAs that align with students' prior knowledge and typical learning processes.</td>
</tr>
<tr>
<td>Mathematics content on the TEFA often prompts students to make connections and reason mathematically.</td>
</tr>
<tr>
<td>Partially Proficient</td>
</tr>
<tr>
<td>The teacher is sometimes selecting, analyzing or designing mathematical tasks on TEFA that align with students' prior knowledge and typical learning processes.</td>
</tr>
<tr>
<td>Mathematics content on the TEFA sometimes prompts students to make connections or reason mathematically.</td>
</tr>
<tr>
<td>Novice</td>
</tr>
<tr>
<td>Little to no evidence that the teacher is selecting, analyzing or designing mathematical tasks on TEFA that align with students' prior knowledge and typical learning processes.</td>
</tr>
<tr>
<td>Mathematics content on the TEFA does not prompt students to make connections or reason mathematically.</td>
</tr>
</tbody>
</table>

**Comments**

A11: Are these two sides of the same coin? Do you need both?

A12: As before, perhaps this can be shortened and combined into one statement?
Appendix J: External Rater Evaluation and Review

External Raters’ Evaluation of Mock Responses, Review of the Rubric and TEFA Literacy Test for Secondary Mathematics

External Rater A

NOTE: External Reviewer A did not leave commentary on the Rubric.

External Rater A’s Evaluation of Mock Responses: Overall Literacy Levels

Component #1: Mathematics (M)

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematical Proficiency (MP)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>(weak in prod. Disposition)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Student Learning (SL)</strong></td>
<td></td>
<td>X</td>
<td>pt. 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9b → pt. 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>6c → pt. 2</td>
<td></td>
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</tbody>
</table>

Component #2: Technology

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Align to Instructional Goals (AIG)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>X Lean here b/c 7a/11a and lack of GeoGebra understanding</td>
<td></td>
</tr>
<tr>
<td><strong>Technology Knowledge (TK)</strong></td>
<td></td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Only fluent in Desmos/Google sheets &amp; slides</td>
<td></td>
<td></td>
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</tbody>
</table>

Component #3: Formative Assessment

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Eliciting Student Understanding (ESU)</strong></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1b, 7b, 8c, 11 pure proced. or computation</td>
<td></td>
</tr>
<tr>
<td>Feedback Process (FP)</td>
<td>X</td>
<td>Variety of mtds. Teacher has plans</td>
<td>(50%)</td>
<td></td>
</tr>
<tr>
<td>----------------------</td>
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<td>-------</td>
<td>---</td>
</tr>
<tr>
<td>Moving Learning Forward (MLF)</td>
<td></td>
<td></td>
<td>X</td>
<td>Relies only on stud. feedback rather than teacher.</td>
</tr>
</tbody>
</table>

**External Rater A’s Comments on the Review Directions**

Thank you for agreeing to review the *TEFA Literacy Test for Secondary Mathematics* and corresponding *Rubric*. Your feedback will be invaluable to the development of a valid tool to be used in my dissertation study on TEFA Literacy for Secondary Mathematics.

**Step 2:** Please grade the MOCK TEST using the Rubric. There are directions on the Rubric to assist you in the process, if you are unsure about anything, please use “Tracked Changes” in Word to make a comment about what you are unsure of and why.

**Step 3:** Please provide a review of the test. You are encouraged to make “Tracked Changes” on the test, or you may just comment below:

<table>
<thead>
<tr>
<th>Topic</th>
<th>General Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clarity: Are the questions clear? Do you understand what they are asking? If not, how can they be improved?</td>
<td>See 2c “technological changes” * See 3a “terms” 5c 7c See: Delta Math? #9, #12</td>
</tr>
<tr>
<td>Content: Do the questions address common skills and knowledge that would be associated with TEFA Literacy for Secondary Mathematic? If not, how can they be improved?</td>
<td>I think this is where the Delta Math comment might be applicable and the comment on the pedagogical terms used…</td>
</tr>
<tr>
<td>Eliciting: Do you think the questions will elicit responses that are adequate and measurable? If not, how can they be improved?</td>
<td>Not sure I found the rubric difficult to use as an “overall” measure (Oops … ignore this …)</td>
</tr>
</tbody>
</table>
**Step 4:** Please provide a review of the rubric with regard to the following. You are encouraged to make tracked changes on the document and provide general comments here:

<table>
<thead>
<tr>
<th>Topic</th>
<th>General Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Directions &amp; Use:</strong> Do the scoring directions make sense? If not, how can they be improved?</td>
<td>I found the rubric hard to use as an “overall” measure &amp; if not for the cheat sheet of applicable questions would have been lost.</td>
</tr>
<tr>
<td><strong>Clarity:</strong> Is the wording of the rubric clear and concise? Do you understand what is being evaluated? If not, how can it be improved?</td>
<td>I personally worked my way through each section, repeatedly going back over the test, so I did all the MP questions, then started over and did the SL, etc. – a long process. The wording is good &amp; I understand the process.</td>
</tr>
<tr>
<td><strong>Content:</strong> Does the content of the rubric align to the components/elements being measured? If not, how can it be improved?</td>
<td>Content also good …</td>
</tr>
<tr>
<td><strong>Progression:</strong> Does the progression of levels make sense? Are there too big or too small of differences between one level and the next?</td>
<td>I get that this is qualitative data, but I would love numbers (sorry) and somehow average to get an overall score, b/c the teacher was good in some parts &amp; meg in others.</td>
</tr>
<tr>
<td><strong>Additional comments:</strong> This could be about any redundancies, elements you might eliminate/add, ambiguities that need to be addressed, syntax, etc.</td>
<td></td>
</tr>
</tbody>
</table>

**Alignment:** Please comment on the alignment between the test and the rubric. Are there any instances of misalignment? Unclear connections? Improvements to be made?

* I get that you needed to test for validity & thus needed to see if we’d come up with same scores on certain aspects, but it felt contrived advanced in MP but novice in MLF?

Lastly, I do need to eliminate at least 1 question, possibly 2 to ensure the test does not take more than 60 minutes for participants to take. Are there any questions you would eliminate? And why?
I would eliminate either scenario 5 or 6

Get rid of Zhang 5/6 - 6c was very confusing & too many stats questions … (Love #10 with the trig …)

**External Rater A’s Comments on the Second Draft of the Test (Containing Mock Responses)**

---

**Scenario 1:**
Mr. O’Connor, an Algebra I teacher, is working with students to graph linear functions in slope-intercept and point-slope form. The learning objective is that: Students will be able to graph the function on the x-y-axis, identify the x- and y-intercepts, and calculate the slope.

| AI/TK |  
a) Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions.  
They could use Desmos  
| MP/ESU |  
b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.  
Given the graph of the linear function (I would use Desmos to provide the graph), find the equation of the line in  
a) slope-intercept form:  
b) point-slope form:  
c) What do you notice about these two equations?  
| SL |  
c) Given the question/prompt that you designed in part (b), what prior knowledge would students need in order to answer this question?  
They would have to have knowledge about the coordinate plane, linear graphs, how to identify/plot points, how to find/slope slopes. They would also need to know the difference between slope-intercept and point-slope form. |

---

**Scenario 2:**
Students in Ms. Glow’s Geometry class have recently learned about the AA and SSS conjectures for similar triangles, and have begun to use two-column proofs. Following one of the lessons, Ms. Glow created a formative assessment on GeoGebra ©. One of the problems was:

![Geogebra Diagram](image_url)  

| MP/SL/MLF* |  
Sam provided the following answer:  
**The triangles are not similar because they do not have matching angles**  
a) What can you infer about Sam’s understanding of similar triangles? Do you think Sam has any misconceptions about this concept? If so, what?  
Based on Sam’s answer, it does seem like he knows that the triangles are not similar and it is because of the angle measures. He does not use the AA conjecture or a formal proof, which may mean he does not understand what is meant by “justify your response”. Since I do not know more of the structure of the class, I cannot determine if this is because of a misconception or a lack of teaching.  
| MP |  
GeoGebra © provided the following feedback to Sam (in green):  
**Snippet from GeoGebra feedback**
b) In your opinion, is this feedback useful for helping Sam to better understand similar triangles? Explain.
This feedback only provides the student with a partially correct answer and, considering the class is working on two-column proofs, this feedback is only partially helpful. It does not prompt the student to think about what they did wrong, or even to ask them a question to make them try to come up with the correct answer.

AXG
TK
ESU
c) Provide an example of how you might expand upon or change this GeoGebra problem to help make students’ understanding of similar triangles more visible. Be specific about any technological changes.
I think I would change the directions of the problem to ask for a proof that uses the conjectures students have learned, instead of just saying “justify.” I think I might also include the side lengths of the triangles so they can use those to further prove that they are not proportional.

Scenario 3:
Ms. Knopp is working with students on the properties of exponents, both whole number and rational, and decided to use a Khan Academy quiz to formatively assess students. One of the questions asks:

MP
a) What mathematical concepts is this question assessing? What mathematical proficiencies (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive disposition) is this question prompting?
This is assessing procedural fluency since the student must look at the steps (procedures) that Neil is taking and decide if they are correct or not.

FP
b) The computer provides students with correct/incorrect feedback. What additional feedback (if any) would you provide

Cached (103): I assume that people know what these terms mean -- I had to look them up...
TK  

a) What can you infer about Valeria’s understanding of this topic?

b) To the best of your knowledge, how else could you use Desmos to formatively assess students’ understanding about the difference between linear and exponential growth? Be as detailed as possible.

Not a lot – She may know that it is exponential, but she may also just have guessed since there are really only two options.

Esu

Ms. Zhang is teaching a Mathematical Modeling class and is introducing her students to finding the curve of best fit using spreadsheet technology (e.g., Excel, Google Sheets).

SL  

a) Assuming this is the first time students have used spreadsheet technology in her class, what do you think Ms. Zhang will need to teach students about the spreadsheet technology before they begin the project?

If this is the first time, and I am going to assume they are using google sheets, they will need to know how to log on, how to label and input values, how to generate graphs (scatter plots) from these values, how to use sheets to generate a curve of best fit, how to use sheets to determine other things like the R^2 and residuals (if that is how far Ms. Zhang is going with the topic).

b) After initial instruction, students begin working in pairs to create their first curves of best fit for a given data set.

Provide an example of a question/prompt you would incorporate as students are working to formatively assess students’ understanding and progress. Include how you would pose this question/prompt (verbally, on paper, with technology, etc.).
<table>
<thead>
<tr>
<th>SCENARIO 6:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Back in Ms. Zhang’s class, students have finished their first drafts of their modeling projects on finding the curve of best fit. They now have to give a short, digital presentation of their findings.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ARG TK MLF</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) What platform/method would you use to give the digital presentations? Explain how your choice might help with formative assessment.</td>
</tr>
</tbody>
</table>

Since I decided in the last question to use Google Sheets, I would use Google Slides for the presentations. These can be shared with anyone, so I can review the presentations and other students could comment on them both of which would help with formative assessment.

<table>
<thead>
<tr>
<th>FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) How would you encourage students to give peer-feedback during the presentations? Be specific about the methods and resources you would use.</td>
</tr>
</tbody>
</table>

Like I said in the last questions, in Google Sheets students could comment on each other’s work, so I would have them do that. I would make sure to explain to them that they need to be commenting on the mathematics and not just the look or neatness of the presentation.

<table>
<thead>
<tr>
<th>SL TK FF</th>
</tr>
</thead>
<tbody>
<tr>
<td>c) Given the most common mistake on students’ projects was confusing the correlation coefficient (R) with the coefficient of determination (R^2), what feedback might you give students? Describe how you would use the platform/method you chose in part (a), if possible, to provide this feedback. (If not possible, describe an alternative way to provide feedback.)</td>
</tr>
</tbody>
</table>

This mistake may be because of the similarity of the names and symbols for the terms and I think I would identify the mistake and then ask students to compare and contrast the two terms. On Google Sheets, I could leave a comment on their work.

---

<table>
<thead>
<tr>
<th>SCENARIO 7:</th>
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</thead>
<tbody>
<tr>
<td>Mr. Kanumba’s Statistics class is working on the learning objective: Students will be able to use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ARG TK</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Identify a computer-based technology that Mr. Kanumba could use to formatively assess students on this objective.</td>
</tr>
</tbody>
</table>

Possibly Desmos, but I am not really sure since I don’t have much experience teaching statistics.

<table>
<thead>
<tr>
<th>MP ESU</th>
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</thead>
<tbody>
<tr>
<td>b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.</td>
</tr>
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</table>

Again, I am not really sure if I can use Desmos for statistics, but if I can, I would ask something like: Given the two histograms (and have two histograms displayed) identify the a) Mean b) Median c) Interquartile range d) Standard deviation What is the same in these two data sets? What is different? How do you know?

<table>
<thead>
<tr>
<th>MP</th>
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<tbody>
<tr>
<td>c) Given the question/prompt that you designed in (b), what might you be able to infer about students’ learning? (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?)</td>
</tr>
</tbody>
</table>

I would be able to determine their procedural fluency in finding the mean, median, interquartile range, and sd of a data set. I would also be able to see some reasoning and conceptual understanding in their ability to compare the two data sets.

---

<table>
<thead>
<tr>
<th>SCENARIO 8:</th>
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</thead>
<tbody>
<tr>
<td>Students in Ms. Swanson’s class have demonstrated that they know:</td>
</tr>
<tr>
<td>(1) how to find the equation of a line parallel to a given line that passes through a given point and</td>
</tr>
<tr>
<td>(2) that perpendicular lines must intersect at a 90 degree angle.</td>
</tr>
</tbody>
</table>

The learning objective for today’s lesson is:
Students will be able to find the equation of a line perpendicular to a given line that passes through a given point.

Ms. Swanson asked students the following question on Demo C:

What is the equation of the line perpendicular to y=3x+7 that travels through the point (1,5)?

SL

a) What other prior knowledge might students need to answer this question?

What is means for lines to be parallel and perpendicular, the relationship between the slopes of parallel and perpendicular lines, slope intercept and point-slope forms for lines, basic arithmetic

MP

b) Given this question, what might she be able to infer about students' understanding of finding the equation of a line perpendicular to a given line that passes through a given point?

Not much -- this is multiple choice, so we cannot know if the student guessed right or actually knew the right answer.

MP

c) Provide an example question/prompt that you believe would assess students' achievement of the learning objective. Be specific about the technology you would choose and why you would choose it. If you would not use technology, explain why not.

I would want to do a few different questions on paper to really understand their learning. For one, I would ask students to find the equation of the line perpendicular to (equation) that travels through a point (point) and give them space to show their work instead of just a multiple choice. I think it would just be easier for students to show their work on a problem like this on paper rather than having to type it out on a computer.

Scenario 9:

Mr. Kanumba would like to use Delta Math to formatively assess students understanding of solving linear equations and inequalities in one variable, but has never done so before.

TK

a) To the best of your knowledge, explain to Mr. Kanumba the steps he needs to take to create the Delta Math quiz.

First he needs to create an account, next he needs to search for the topic, and then Delta Math will automatically generate questions for him.

Sl

After your instructions, Mr. Kanumba begins using Delta Math and has students works on the following problem:

Carson has $38 to spend at a bicycle store for some new gear and biking outfits. Assume all prices listed include tax.

• He buys a new bicycle for $187.94.
• He buys a bicycle reflector for $8.45 each and a pair of bike gloves for $4.96.
• He plans to spend some or all of the money he has left to buy new biking outfits for $4.96 each.

What is the greatest number of outfits Carson can buy with the money that’s left over?

MP

c) In your opinion, what are the benefits and drawbacks of using Delta Math when the topic is on solving linear equations and inequalities in one variable? Explain your response.

I haven’t used Delta Math very much, but it seems like what students can input is just an answer, so I would not be able to see their work to identify any mistakes or misconceptions. I do think it gives automated feedback with the correct answer, so this would prevent me from trying to have them figure out the correct answer on their own.

Scenario 10:

Julian was completing an online GeoGebra formative assessment for his Trigonometry class. After incorrectly responding to the
question, he received the following automated feedback (outlined in red):

a) What might be Julian’s mistake or misconception? Explain how this feedback does or does not help Julian understand his mistake or misconception?

It’s hard to tell what his mistake is since this is multiple choice, but by selecting (b) it shows he subtracted 360 from 510 correctly and selected the one that had 150 in it. The feedback is pretty helpful, it is directing him to think about where the angle lies while not giving away the answer.

When Julian selected “Hint”, he received the diagram below:

b) Explain how this feedback does or does not help Julian understand his mistake or misconception?

This helps Julian to visualize what the hint was saying while still not completely giving away the answer. It may help him remember which way we measure angles.

c) Explain what adjustments would you make (if any) to this feedback?

I think I would change the “510 lies in quadrant 2, and cosine of the angle should be negative” to “510 lies in quadrant 2, and cosine of the angle should be negative” to “510 lies in quadrant 2, and cosine of the angle should be negative.”

2, what does this mean about the value of cosine?” Doing this makes the student have to think more instead of just giving the answer away.

Scenario 11:

Ms. Fisher and her students are working on graphing quadratic functions, and understanding how the coefficients in standard form transform the graph of a parabola (i.e., how different values of $a$, $b$, and $c$ in $ax^2 + bx + c$ transform the graph).

a) Describe a technology enhanced formative assessment activity Ms. Fisher could engage students in?

I would use a Desmos Quiz.

b) Provide an example of a question or prompt you might give students. Explain what you intend to measure about students’ understanding with this question/prompt (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate?)

I would give students the graph of a quadratic and ask them to figure out the equation in standard form. This would assess their procedural fluency in identifying the values in a graph that correspond to the values in the equation. It would also assess their conceptual understanding about this relationship.

Scenario 12:

Mr. First has the learning objective that:

Students will be able to rewrite expressions involving radicals and rational exponents using the properties of exponents.

a) What prior knowledge must students have to complete this learning objective?

Knowledge of fractions, order of operations, radicals, rational numbers, exponents, properties of exponents, and how to simplify exponents.
External Rater B:

External Rater B’s Evaluation of Mock Responses: Overall Literacy Levels

Component #1: Mathematics (M)

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Proficiency (MP)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student Learning (SL)</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
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</tbody>
</table>

Component #2: Technology

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Align to Instructional Goals (AIG)</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology Knowledge (TK)</td>
<td></td>
<td>X</td>
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</table>

Component #3: Formative Assessment

<table>
<thead>
<tr>
<th></th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eliciting Student Understanding</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>
External Rater B’s Comments on the Review Directions

Thank you for agreeing to review the *TEFA Literacy Test for Secondary Mathematics* and corresponding *Rubric*. Your feedback will be invaluable to the development of a valid tool to be used in my dissertation study on TEFA Literacy for Secondary Mathematics.

**Step 2:** Please grade the MOCK TEST using the Rubric. There are directions on the Rubric to assist you in the process, if you are unsure about anything, please use “Tracked Changes” in Word to make a comment about what you are unsure of and why.

**Step 3:** Please provide a review of the *test*. You are encouraged to make “Tracked Changes” on the test, or you may just comment below:

<table>
<thead>
<tr>
<th>Topic</th>
<th>General Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Clarity:</strong> Are the questions clear? Do you understand what they are asking? If not, how can they be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Content:</strong> Do the questions address common skills and knowledge that would be associated with TEFA Literacy for Secondary Mathematics? If not, how can they be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Eliciting:</strong> Do you think the questions will elicit responses that are adequate and measurable? If not, how can they be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Additional comments:</strong> This could be about any redundancies, questions you might eliminate, ambiguities that need to be addressed, potential leading questions, syntax, etc.</td>
<td></td>
</tr>
</tbody>
</table>
Step 4: Please provide a review of the rubric with regard to the following. You are encouraged to make tracked changes on the document and provide general comments here:

<table>
<thead>
<tr>
<th>Topic</th>
<th>General Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Directions &amp; Use:</strong> Do the scoring directions make sense? If not, how can they be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Clarity:</strong> Is the wording of the rubric clear and concise? Do you understand what is being evaluated? If not, how can it be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Content:</strong> Does the content of the rubric align to the components/elements being measured? If not, how can it be improved?</td>
<td></td>
</tr>
<tr>
<td><strong>Progression:</strong> Does the progression of levels make sense? Are there too big or too small of differences between one level and the next?</td>
<td></td>
</tr>
<tr>
<td><strong>Additional comments:</strong> This could be about any redundancies, elements you might eliminate/add, ambiguities that need to be addressed, syntax, etc.</td>
<td>I think the only one I had issues comprehending how to use was TK. I felt that the criteria made it too restrictive to really give a meaningful rating. I preface this with I may have misunderstood what was written.</td>
</tr>
</tbody>
</table>

Alignment: Please comment on the alignment between the test and the rubric. Are there any instances of misalignment? Unclear connections? Improvements to be made?

Lastly, I do need to eliminate at least 1 question, possibly 2 to ensure the test does not take more than 60 minutes for participants to take. Are there any questions you would eliminate? And why?

I think Scenario 8. I did not see the impact of technology on this question. Rather, I do not see how technology would make a difference in the delivery of the question that was posed, “What is the equation of the line…” Even with part c that focuses on the use of technology, I think it needs to ties into the use more, and why it really facilitates formatively assessing the student.

External Rater B’s Comments on the Second Draft of the Test (Containing Mock Responses)
Technology Enhanced Formative Assessment (TEFA)

Literacy Test for Secondary Mathematics

Thank you for participating in the TEFA Literacy Test for Secondary Mathematics. The goal of this test is to measure your knowledge and skills in TEFA for the secondary mathematics classroom. Please answer each question to the best of your ability.

In all scenarios, please assume that the teacher and students are working in a secondary mathematics classroom that is aligned to the Common Core State Standards for Mathematics (2010).

---

Scenario 1:
Mr. O’Connor, an Algebra 1 teacher, is working with students to graph linear functions in slope-intercept and point-slope form. The learning objective is that: Students will be able to graph the function on the x-y-axis, identify the x- and y-intercepts, and calculate the slope.

**AOG TK**
a) Identify a computer-based technology that Mr. O’Connor and his students could use to graph these functions.

*They could use Desmos.*

**MP ESU**
b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.

*Given the graph of the linear function (I would use Desmos to provide the graph), find the equation of the line in:
- slope-intercept form:
- point-slope form:
- What do you notice about these two equations?*

**SL**
c) Given the question/prompt that you designed in part (b), what prior knowledge would students need in order to answer this question?

*They would have to have knowledge about the coordinate plane, linear graphs, how to identify/plot points, how to find/plot slopes. They would also need to know the difference between slope-intercept and point-slope form.*

---

Scenario 2:
Students in Ms. Glow’s Geometry class have recently learned about the AA and SSS conjectures for similar triangles, and have begun to use two-column proofs. Following one of the lessons, Ms. Glow created a formative assessment on GeoGebra ©. One of the problems was:

*The triangles are not similar because they do not have matching angles*

a) What can you infer about Sam’s understanding of similar triangles? Do you think Sam has any misconceptions about this concept? If so, what?

*Based on Sam’s answer, it does seem like he knows that the triangles are not similar and it is because of the angle measures. He does not use the AAA conjecture or a formal proof, which may mean he does not understand what is meant by “justify your response”. Since I do not know more of the structure of the class, I cannot determine if this is because of a misconception or a lack of teaching.*

**MF GeoGebra ©** provided the following feedback to Sam (in green):

---


*Commented (A2): Proficient. I think this question is more of a motivating question more so than a formative assessment.*

*Commented (A3): Advanced. Student is asked to provide their observations.*

*Commented (A4): Advanced.*

*Commented (A5): Advanced Adv. Teacher excels in this category. Teacher is able to pinpoint where the student went wrong and analyzes the quality of the student’s learning on this assignment.*

*Commented (A7): Advanced. It seems like the teacher appropriately assessed the student’s response.*

*Commented (A8): I am not sure this applies here totally. I am having trouble on seeing how to rate using this category.*
b) In your opinion, is this feedback useful for helping Sam to better understand similar triangles? Explain.

This feedback only provides the student with a partially correct answer and, considering the class is working on two-column proofs, this feedback is only partially helpful. It does not prompt the student to think about what they did wrong, we usually try to ask them a question to make them try to come up with the correct answer.

ARU

C) Provide an example of how you might expand upon or change this GeoGebra problem to help make students understand of similar triangles more visible. Be specific about any technological changes.

I think I would change the directions of the problem to ask for a proof that uses the conjectures students have learned instead of just saying "justify." I think I might also include the side lengths of the triangles so they can use those to further prove that they are not proportional.

Scenario 3:

Mr. Knope is working with students on the properties of exponents, both whole number and rational, and decided to use a Khan Academy quiz to formatively assess students. One of the questions asks:

\[ \frac{s^{-6}}{s^{-2}} = \frac{s^{1}}{s^{3}} \]

Did Neil make a mistake? If so, in which step?

Choose 1 answer:

\[ \begin{align*}
1 & \quad \text{Neil did not make a mistake.} \\
2 & \quad \text{Neil made a mistake in Step 1.} \\
3 & \quad \text{Neil made a mistake in Step 2.} \\
4 & \quad \text{Neil made a mistake in Step 3.}
\end{align*} \]

MP

a) What mathematical concepts in this question assessing? What mathematical proficiencies (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and/or productive dispositions) in this question prompting?

This is assessing procedural fluency since the student must look at the steps (procedures) that Neil is taking and decide if they are correct or not.

FR

b) The computer provides students with correct/incorrect feedback. What additional feedback (if any) would you provide...
Students if they made a mistake? Explain your response.

In this problem, Neil did not make a mistake, but if Neil had made a mistake I would provide additional feedback on asking them to correct the mistake and evaluate their work as well. This would help students to think more deeply about the kinds of mistakes that can be made in problems like these and, hopefully, not make them.

c) To the best of your knowledge, explain to Mr. Knox how they can access a report on students' performance on this Khan Academy quiz.

I am not sure — I have not used Khan Academy like this before.

Scenario 4:

Valeria turned in her homework on linear and exponential growth on Desmos ©. The first question asked her to analyze the table and determine if they demonstrated linear or exponential growth.

a) What can you infer about Valeria’s understanding of this topic?

Not a lot — She may know that it is exponential, but she may also just have guessed since there are really only two options.

b) To the best of your knowledge, how else could you use Desmos © to formatively assess students’ understanding about the difference between linear and exponential growth? Be as detailed as possible.

You could change the prompt of this question to ask the student to explain why they know. Desmos also allows graphing (point and lines) which means students could be asked to generate their own linear and exponential graphs and say what is different. Similarly, they could compare and contrast the already made graphs of linear and exponential, using math vocabulary to explain distinct features.

c) Based on your response to part (b), what do you anticipate you will be able to determine about a student’s understanding?

If they are asked to explain how they know it is linear or exponential, I will be able to determine if they have more of a conceptual understanding about the differences. If they generate their own graphs, I will be able to determine if they are able to go between algebraic and graphic representations. Compare and contrast also gives me more information about their conceptual understanding.

Scenario 5:

Ms. Zhang is teaching a Mathematical Modeling class and is introducing her students to finding the curve of best fit using spreadsheet technology (e.g., Excel, Google Sheets).

a) Assuming this is the first time students have used spreadsheet technology in her class, what do you think Ms. Zhang will need to teach students about the spreadsheet technology before they begin the project?

If this is the first time, and I am going to assume they are using google sheets, they will need to know how to log on, how to label and input values, how to generate graphs (scatter plots) from these values, how to use sheets to generate a curve of best fit, how to use sheets to determine other things like the R² and residuals (if that is how far Ms. Zhang is going with the topic).

b) Provide an example of a question/prompt you would incorporate as students are working to formatively assess students’ understanding and progress. Include how you would pose this question/prompt (verbally, on paper, with technology, etc.).
Scenario 6:
Back in Ms. Zhang’s class, students have finished their first drafts of their modeling projects on finding the curve of best fit. They now have to give a short, digital presentation of their findings.

AN
a) What platform/method would you use to give the digital presentations? Explain how your choice might help with formative assessment.

ML
Since I decided in the last question to use Google Sheets, I would use Google Slides for the presentations. These can be shared with anyone, so I can review the presentations and other students could comment on them both of which would help with formative assessment.

PF
b) How would you encourage students to give peer feedback during the presentations? Be specific about the methods and resources you would use.

Like I said in the last questions, in Google Sheets students could comment on each other’s work, so I would have them do that. I would make sure to explain to them that they need to be commenting on the mathematics and not just the look or neatness of the presentation.

SL
c) Given the most common mistake on students’ projects was confusing the correlation coefficient (R) with the coefficient of determination (R^2), what feedback might you give students? Describe how you could use the platform/method you choose in part (a), if possible, to provide this feedback. (If not possible, describe an alternative way to provide feedback.)

This mistake may be because of the similarity of the names and symbols for the terms and I think I would identify the mistake and then ask students to compare and contrast the two terms. On Google Sheets, I could leave a comment on their work.

Scenario 7:
Mr. Kanumba’s Statistics class is working on the learning objective:
Students will be able to use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

AG
a) Identify a computer-based technology that Mr. Kanumba could use to formatively assess students on this objective.

Possibly Desmos, but I am not really sure since I don’t have much experience teaching statistics.

MP
b) Using the technology you identified in part (a), provide an example of a question/prompt you would incorporate in a formative assessment.

Again, I am not really sure if I can use Desmos for statistics, but if I can, I would ask something like: Given the two histograms (and have two histograms displayed) identify the
a) Mean
b) Median
c) Interquartile range
d) Standard deviation

What is the same in these two data sets? What is different? How do you know?

Scenario 8:
Students in Ms. Swanson’s class have demonstrated that they know:
(1) how to find the equation of a line parallel to a given line that passes through a given point and (2) that perpendicular lines must intersect at a 90 degree angle.
The learning objective for today’s lesson is:
Students will be able to find the equation of a line perpendicular to a given line that passes through a given point.

Ms. Swanson asked students the following question on Desmos C:

What is the equation of the line perpendicular to y = 2x + 3 that passes through the point (1,5)?
- y = x + 2
- y = 2x + 2
- y = -x - 4
- y = -2x + 3

b) Given this question, what might she be able to infer about students’ understanding of finding the equation of a line perpendicular to a given line that passes through a given point?

Not much – this is multiple choice, so we cannot know if the student guessed right or actually knew the right answer.

c) Provide an example question/prompt that you believe would assess students’ achievement of the learning objective. Be specific about the technology you would choose and why you would choose it. If you would not use technology, explain why not.

I would want to do a few different questions on paper to really understand their learning. For one, I would ask students to find the equation of the line perpendicular to [equation] that travels through a point [point] and give them space to show their work instead of just a multiple choice. I think it would just be easier for students to show their work on a problem like this on paper rather than having to type it out on a computer.

Scenario 9:
Mr. Kanumba would like to use Delta Math to formatively assess students understanding of solving linear equations and inequalities in one variable, but has never done so before.

a) To the best of your knowledge, explain to Mr. Kanumba the steps he needs to take to create the Delta Math quiz.

First he needs to create an account, next he needs to search for the topic, and then Delta Math will automatically generate questions for him.

b) Mr. Kanumba notices students are struggling to begin, how might you help students? Provide as many details as possible.

I think I would help students by having a class discussion and asking them identify what each of the numbers in the problem represents, how this relates to what we have been doing with inequalities, and then try to get them to set up their own equations.

c) In your opinion, what are the benefits and drawbacks of using Delta Math when the topic is on solving linear equations and inequalities in one variable? Explain your response.

I haven’t used Delta Math very much, but it seems like what students can input is just an answer, so I wouldn’t be able to see their work to identify any mistakes or misconceptions. I do think it gives automated feedback with the correct answer, so this would prevent me from trying to have them figure out the correct answer on their own.

Scenario 10:
Julian was completing an online Geogebra © formative assessment for his Trigonometry class. After incorrectly responding to the
question, he received the following automated feedback (outlined in red):

SL: a. What might be Julian’s mistake or misconception? Explain how this feedback does or does not help Julian understand his mistake or misconception?

FP: It’s hard to tell what his mistake is since this is multiple choice, but by selecting (b) it shows he subtracted 360 from 510 correctly and selected the one that had 150 in it.

The feedback is pretty helpful, it is directing him to think about where the angle lies while not giving away the answer.

SL: When Julian selected “Hint”, he received the diagram below:

b. Explain how this feedback does or does not help Julian understand his mistake or misconception?

FP: This helps Julian to visualize what the hint was saying while still not completely giving away the answer. It may help him remember which way we measure angles.

SL: c. Explain what adjustments would you make (if any) to this feedback?

FP: I think I would change the '510 lies in quadrant 2, and cosine of the angle should be negative’ to '510 lies in quadrant

2, what does this mean about the value of cosine?' Doing this makes the student have to think more instead of just giving the answer away.

Scenario 1:
Ms. Fischer and her students are working on graphing quadratic functions, and understanding how the coefficients in standard form transform the graph of a parabola (i.e., how different values of a, b, c in ax^2 + bx + c transform the graph).

MP: a. Describe a technology enhanced formative assessment activity Ms. Fisher could engage students in?

AR: I would use a Desmos Quiz

MP: b. Provide an example of a question or prompt you might give students. Explain what you intend to measure about students’ understanding with this question/prompt (e.g., what knowledge, understanding, procedural fluency, reasoning, and/or problem solving skills will they be able to demonstrate)?

IK: I would give students the graph of a quadratic and ask them to figure out what the equation would be in standard form. This would assess their procedural fluency in identifying the values in a graph that correspond to the values in the equation. It would also assess their conceptual understanding of the relationship.

SL: c. Given your response in part (b), what mistake might students make? Why might they make these mistakes?

IK: Students will most likely struggle with understanding the a and c values as you have to do more algebraic manipulation after you identify the roots in the graph.

Scenario 2:
Mr. Flint has the learning objective that:
Students will be able to rewrite expressions involving radicals and rational exponents using the properties of exponents.

SL: a. What prior knowledge must students have to complete this learning objective?

FP: Knowledge of fractions, order of operations, radicals, rational numbers, exponents, properties of exponents, and how to simplify exponents.
External Rater B’s Comments on the Second Draft of the Rubric

**Component #1: Mathematics**

<table>
<thead>
<tr>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student Learning (SL)</td>
<td>The teacher demonstrates that they are effectively using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs. The teacher is able to identify all common mistakes and misconceptions; they demonstrate that they are thinking deeply about why these mistakes and misconceptions happen.</td>
<td>The teacher demonstrates that they are using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs, but this is not always done effectively. The teacher is able to identify most common mistakes and misconceptions. They demonstrate basic understanding as to why these mistakes and misconceptions happen.</td>
<td>There is some evidence that the teacher is using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs. The teacher is able to identify some common mistakes, but struggles to identify common misconceptions. They struggle to demonstrate why these mistakes and misconceptions happen.</td>
<td>Little to no evidence that the teacher is using students’ prior knowledge to analyze, select, or design questions/prompts on TEFAs. Little to no evidence that the teacher is able to identify mistakes or misconceptions in student responses to TEFAs questions.</td>
</tr>
</tbody>
</table>

| Mathematical Proficiency (MP) | The teacher is able to analyze, select, and design questions/prompts on TEFAs that measure students’ mathematical proficiency. The questions/prompts address: - procedural fluency - conceptual understanding - strategic competence - adaptive reasoning - productive disposition | The teacher is able to analyze, select, or design questions/prompts on TEFAs that measure most of students’ mathematical proficiency. The questions/prompts address: - procedural fluency - conceptual understanding - and at least 1 more strand | Some evidence that the teacher is able to analyze, select, or design questions/prompts on TEFAs that measure some of students’ mathematical proficiency. The questions/prompts address: - procedural fluency - conceptual understanding - and at least 1 more strand | Little to no evidence that the teacher is able to analyze, select, or design questions/prompts on TEFAs that measure the 3 Strands of Mathematical Proficiency | 1b, 2a, 3a, 4a, 5c, 6c, 8a, 9b, 10a, b, 11c, 12a, b |
### Component #2: Technology

<table>
<thead>
<tr>
<th>Align to Instructional Goals (AIG)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher is able to analyze, select, and design TEFAs that align to the purposes of formative assessment and given mathematical learning objectives.</td>
<td>The teacher is able to analyze and select TEFAs that align to the purposes of formative assessment and given mathematical learning objectives, but they struggle to design their own.</td>
<td>The teacher struggles to select or analyze TEFAs that align to the instructional goal of formative assessment or mathematics. They cannot design their own.</td>
<td>Little to no demonstrated understanding of how to align the TEFAs to the instructional goal of formative assessment or mathematics.</td>
<td>1a. 2c. 4b. 5c. 6a. 7a. 8a. 9c. 11a.</td>
<td></td>
</tr>
<tr>
<td><strong>Technology Knowledge (TK)</strong></td>
<td>The teacher can identify 4-5 different technologies that are appropriate for formative assessments of 4-5 different topics in mathematics. The teacher can provide detailed explanations on how to use these different technologies from both the student and teacher perspective.</td>
<td>The teacher can identify 4-5 different technologies that are appropriate for formative assessments of 4-5 different topics in mathematics. The teacher can explain basic use of these different technologies from both the student and teacher perspective, but does not provide detailed or advanced directions.</td>
<td>The teacher can identify 2-3 different technologies that are appropriate for formative assessments of 2-3 different topics in mathematics. The teacher attempts to use these different technologies from both the student and teacher perspective.</td>
<td>Little to no evidence of knowledge of computer-based technologies or how to operate them.</td>
<td>3a. 3c. 3e. 4b. 5a. 6a.* 6a.* 7a. 8c. 9a.* 11a.</td>
</tr>
</tbody>
</table>

### Component #3: Formative Assessment

<table>
<thead>
<tr>
<th>Eliciting Student Understanding (ESU)</th>
<th>Advanced</th>
<th>Proficient</th>
<th>Partially Proficient</th>
<th>Novice</th>
<th>Applicable Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that provide students with a variety of ways to demonstrate their understanding. i.e.: There is strong emphasis on explanation, justification, and problem solving. No questions utilize memorization and computational procedures.</td>
<td>The teacher demonstrates that they are analyzing, selecting, or designing questions/prompts on TEFAs that provide students with more than 1 way to demonstrate their understanding. i.e.: There is more emphasis on explanation and justification and almost no questions utilize memorization and computational procedures.</td>
<td>There is some evidence that the teacher is analyzing, selecting, or designing questions/prompts on TEFAs that provide students with a way to demonstrate their understanding. i.e.: There is some emphasis on explanation and justification, although many questions still utilize memorization and computational procedures.</td>
<td>Little to no evidence that the teacher is analyzing, selecting, or designing questions/prompts on TEFAs that elicit student understanding. i.e.: Questions/prompts focus on memorization and computational procedures.</td>
<td>1b. 2c. 4b. 5b. 7c. 8c. 9c. 11b.</td>
<td></td>
</tr>
<tr>
<td>Feedback Process (FP)</td>
<td>The teacher is able to analyze, select, or design TEFAs that provide targeted feedback and plans for supplemental feedback. The teacher plans for students to be active participants in the feedback process in a variety of ways (e.g. discussions, guided commenting, peer use of rubrics/success criteria, etc.).</td>
<td>The teacher is able to analyze, select, or design TEFAs that provide targeted feedback, but does not plan for supplemental feedback. The teacher plans for students to be active participants in the feedback process primarily through class/pair discussions.</td>
<td>The teacher is able to analyze, select, or design TEFAs that provide more than “correct” or “incorrect” feedback, although this feedback may not be targeted. The teacher might state they encourage students to be active participants, but do not make clear plans for how to do this.</td>
<td>The teacher selects or designs TEFAs that primarily provide “correct” or “incorrect” feedback. The teacher does not plan for students to be active participants in the feedback process.</td>
<td>2b. 2a.* 2b.* 6a.* 9a.* 10a,b,c</td>
</tr>
<tr>
<td>Moving Learning Forward (MLF)</td>
<td>The teacher demonstrates that they are able to analyze student data to determine what students know and use this data to make appropriate adjustments to instruction.</td>
<td>The teacher demonstrates that they are able to analyze students’ data to determine what students know and use this data to make adjustments to instruction. These adjustments may not always be aligned to the student data.</td>
<td>The teacher may struggle to appropriately analyze students’ data to determine what students know (i.e., Not focused on attainment of the learning objective(s), but on semantics or presentation), but they use this data to make adjustments to instruction.</td>
<td>Teacher demonstrates that they do not analyze student feedback, nor do they plan to use student feedback to make adjustments to instruction.</td>
<td>2a. 2a.* 9b. 1c.</td>
</tr>
</tbody>
</table>
Appendix K: Comparison of Evaluations of Heather’s Responses

Comparison of Evaluations of Heather’s Responses to the TEFA Literacy Test for Secondary Mathematics

<table>
<thead>
<tr>
<th>Total Number of Evaluations (preliminary and overall)</th>
<th>64</th>
<th>Percentage of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Exact Matches (EM)</td>
<td>42</td>
<td>65.6%</td>
</tr>
<tr>
<td>Total +/-1 Matches (1M)</td>
<td>16</td>
<td>25%</td>
</tr>
<tr>
<td>Sum of EM + 1M</td>
<td>58</td>
<td>90.6%</td>
</tr>
</tbody>
</table>

Evaluations

<table>
<thead>
<tr>
<th>Scenario, Question</th>
<th>My Evaluation</th>
<th>External Rater B’s Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mathematical Proficiency</td>
</tr>
<tr>
<td>1b</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>2a</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>3a</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
<tr>
<td>4a</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>4c</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>5c</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>6c</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>7b</td>
<td>Advanced</td>
<td>Proficient</td>
</tr>
<tr>
<td>8c*</td>
<td>NA</td>
<td>N/A</td>
</tr>
<tr>
<td>10b</td>
<td>Proficient</td>
<td>Advanced</td>
</tr>
<tr>
<td>Overall</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>Student Learning</td>
<td>1c</td>
<td>Advanced</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>------</td>
<td>----------</td>
</tr>
<tr>
<td>2a</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>5a</td>
<td>Proficient</td>
<td>Partially proficient</td>
</tr>
<tr>
<td>7a</td>
<td>Proficient</td>
<td>Proficient</td>
</tr>
<tr>
<td>8b</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
<tr>
<td>9a</td>
<td>Proficient</td>
<td>Advanced</td>
</tr>
<tr>
<td>10c</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
<tr>
<td>11a</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
<tr>
<td>Overall</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Alignment to Instructional Goals</th>
<th>1a</th>
<th>Advanced</th>
<th>Proficient</th>
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</thead>
<tbody>
<tr>
<td>4b</td>
<td>Advanced</td>
<td>Advanced</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5b</td>
<td>Proficient</td>
<td>Advanced</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6a</td>
<td>Novice</td>
<td>Novice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7c*</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8c*</td>
<td>Novice</td>
<td>N/A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10a</td>
<td>Advanced</td>
<td>Advanced</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11c*</td>
<td>Advanced</td>
<td>N/A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td>Proficient</td>
<td>Proficient</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th>Technology Knowledge</th>
<th>1a</th>
<th>Partially Proficient</th>
<th>Novice/Partially proficient</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2c</td>
<td>Advanced</td>
<td>Advanced</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3c</td>
<td>Partially Proficient</td>
<td>Proficient</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>4b</strong></td>
<td><strong>5a</strong></td>
<td><strong>5b</strong></td>
<td><strong>6a</strong></td>
</tr>
<tr>
<td>----</td>
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<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td></td>
<td>Proficient</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Novice</td>
</tr>
<tr>
<td></td>
<td>Partially proficient</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Novice</td>
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### Elicitation of Student Understanding

<table>
<thead>
<tr>
<th></th>
<th><strong>1b</strong></th>
<th><strong>2c</strong></th>
<th><strong>4b</strong></th>
<th><strong>5b</strong></th>
<th><strong>6b</strong></th>
<th><strong>7b</strong></th>
<th><strong>7c</strong></th>
<th><strong>8c</strong>*</th>
<th><strong>10a</strong>*</th>
<th><strong>10b</strong></th>
<th><strong>Overall</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Advanced</td>
<td>Advanced</td>
<td>Advanced</td>
<td>Advanced</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
<tr>
<td></td>
<td>Proficient/Advanced</td>
<td>Advanced</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Advanced</td>
<td>Novice</td>
<td>Proficient</td>
<td>Advanced</td>
<td>Advanced</td>
</tr>
</tbody>
</table>

### Feedback Process

<p>| <strong>2b</strong> | Advanced | Advanced |</p>
<table>
<thead>
<tr>
<th></th>
<th>3b</th>
<th>Proficient</th>
<th>Partially proficient/proficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>5b</td>
<td>Partially Proficient</td>
<td>Proficient</td>
<td></td>
</tr>
<tr>
<td>8b*</td>
<td>Proficient</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>8c*</td>
<td>NA</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>9a</td>
<td>Advanced</td>
<td>Advanced</td>
<td></td>
</tr>
<tr>
<td>9b</td>
<td>Partially Proficient to Proficient</td>
<td>Advanced</td>
<td></td>
</tr>
<tr>
<td>9c</td>
<td>Advanced</td>
<td>Advanced</td>
<td></td>
</tr>
<tr>
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<td>Proficient</td>
<td>Advanced</td>
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</tr>
<tr>
<td>11c</td>
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<tr>
<td>Overall</td>
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<td>Advanced</td>
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</tbody>
</table>