Exploring the Role of Cross-Cultural Professional Development on Educational Acculturation Strategies of Kazakhstani Mathematics Faculty

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Abstract

Exploring Acculturation Strategies of Mathematics Faculty in Two Kazakhstan Universities: A Case Study

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This study addresses the limited attention given to cultural factors when evaluating the influence of professional development (PD) programs rooted in one mathematics educational tradition on participants from another educational tradition. The study examines the effect of a cross-cultural PD program led by a team of United States and Kazakhstani mathematics education experts on six Kazakhstani mathematics faculty perceptions and teaching practices. Data were collected through surveys, interviews, and lesson recordings. Findings indicate that cross-cultural PD primarily influenced perceptions and teaching practices that required minimal effort, such as the integration of technology tools, while culturally rooted aspects, such as teaching approaches, were not affected. The study proposes a theoretical framework that provides guidance for future research on the effectiveness of cross-cultural PD as an acculturation process.

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Acknowledgments

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Chapter 1: Introduction

1.1 Need for the Study

Mathematics, often viewed as a universal subject, varies significantly in how it is taught across cultures due to distinct educational traditions shaped by historical and cultural contexts. Beyond its academic purpose, mathematics contributes to individuals' reasoning and cognitive development (Wang & Liu, 2002). The differences in social and cultural contexts could contribute to the differences in mathematics educational traditions, which shapes teaching practices (Bishop, 1988, 2001; D'Ambrosio, 1994, 2000; Schmidt, et al., 1996). In this study, the mathematics educational tradition of a group refers to the shared dominant values and expectations present in mathematics education, which are shaped by the cultural, historical, and environmental contexts of a particular society. Numerous studies have documented substantial evidence of variations in mathematics educational traditions across different cultural contexts (Leung, et al., 2006). However, a significant gap exists in the literature regarding the distinct mathematics educational traditions of Central Asian countries.

In Central Asia, Islamic cultural influences since the ninth century (Leung, 2008) and, later, Union of Soviet Socialist Republics (USSR) pedagogical models have shaped mathematics teaching approaches (Kaiser & Blomeke, 2013; OECD, 2017). The substantial influences of social, cultural, and historical factors on mathematics educational traditions in Central Asian countries suggests it is imperative to examine their educational traditions independently of other Asian countries (Bishop,1998). Furthermore, in recent decades, many Central Asian countries have pursued distinct paths of educational reforms, underscoring the need to consider the educational traditions of each country individually. This study focuses on the Republic of Kazakhstan.

Following its independence in 1991, Kazakhstan reformed its social and educational systems, shifting their goals away from Sovietized structures towards more flexible and autonomous ones. (Lang, 2019). In its efforts to align with international educational standards, Kazakhstan has focused on increasing financial support to education programs, technology integration in schools, and developing programs to improve teacher quality (Ma & Meng, 2011).

As an important milestone of increasing autonomy, in 1993 the government legally authorized the establishment of private educational institutions. This policy granted higher education institutions varying degrees of independence (Chu, 2009). Currently, the Kazakhstan Ministry of Education and Science controls curriculum and standards, but private universities have greater flexibility in areas such as partnerships and professional development (PD) compared to public universities (OECD, 2017). To further align with international standards, the Kazakhstani government facilitates partnerships between local educational institutions and foreign experts, including those from the United States, to provide faculty PD programs (OECD, 2017). These experts often focus on introducing reformed teaching practices that emphasize student-centered learning, technology integration, and real-world applications (Drijvers, 2015; Sawad, et.al, 2002). However, the stark differences between Kazakhstani traditional education norms and these reformed approaches may create challenges for faculty striving to balance reform goals with preserving valuable educational heritage (Chu, 2009). The institutional differences may also influence the nature and extent of changes in faculty teaching practices resulting from PD (Desimone, 2009).

Limited research has explored how educators established educational traditions shape effectiveness of PD grounded in different educational traditions. (Hereafter, this type of PD will be referred to as cross-cultural PD). A gap exists in understanding how participants' educational traditions affect their engagement with PD developed elsewhere. This study addresses this gap by exploring the influence of a cross-cultural PD program led by a team of a U.S. and a Kazakhstani mathematics education expert. The PD promoted reformed teaching approaches, particularly technology integration and STEM applications in teaching mathematics, and the study explored the cultural and contextual factors moderating the influence of this PD on faculty perceptions, instructional practices, and acculturation strategies. To further understand the influence of PD grounded in one educational tradition on the participants from another educational tradition, the study proposes a theoretical framework connecting cross-cultural PD effectiveness and the acculturation process.

1.2 Theoretical Framework

To understand the effect of PD features on the outcome of faculty participating in the PD, this study adapted Desimone's (2009) Framework for Studying the Effects of PD on Teachers. The framework identifies five critical components of effective PD: content focus (what the participants are expected to learn from the PD), active learning (opportunities for participants to actively engage), coherence (relevance of the PD to participants' teaching situation), duration (days and hours of the PD), and collective participation (whether participants are participating PD with colleagues from the same institution). These features are suggested for assessing PD's impact on participants' perceptions and instructional practices. However, while this framework incorporates contextual factors such as institutional norms and teaching environments, it does not account for the potential influence of educational traditions of participants and PD providers.

In order to explore the influence of cross-cultural PD on perceptions and teaching practices of Kazakhstani mathematics faculty, this study also adapted Berry's (2003) Framework of Acculturation. This framework suggests that acculturation outcomes can be observed as behavior changes and acculturation stress at the individual level, and it identifies four acculturation strategies (Berry, 2003; Berry & Sam, 2016). The strategies range from fully embracing the new culture (assimilation), maintaining both cultures (integration), rejecting the new culture (separation), or disconnecting from both cultures (marginalization). Thus, faculty perceptions and behaviors may reflect varying degrees of retention of heritage educational traditions and adoption of new educational traditions. This study modified Berry's definitions of acculturation strategies to fit the context of education. In addition, due to its irrelevance to the education context, the marginalization strategy was removed from this study framework.

This study positions differences in educational traditions as a significant factor influencing the effectiveness of cross-cultural PD. It proposes a theoretical framework to guide the exploration of the effectiveness of cross-cultural PD by connecting Desimone's (2009) Framework for Studying the Effects of PD on teachers and a modified Berry's Framework of Acculturation (Berry, 2003; Berry & Sam, 2016). The theoretical framework recognizes that educators' perceptions and practices are shaped not only by PD features but also by their decisions in balancing heritage and new educational traditions.

1.3 Purpose of the Study

The purpose of this study was to examine the influence of a cross-cultural PD program, that focused on reformed teaching and technology integration, on acculturation strategies of Kazakhstani mathematics faculty teaching undergraduate discrete mathematics courses to students preparing to become mathematics teachers.

This study followed a case study methodology and was guided by the following research questions:

1. How were the Kazakhstani mathematics faculty perceptions of teaching and learning mathematics influenced by their experiences in the cross-cultural PD?
2. How were the Kazakhstani mathematics faculty intended and enacted teaching practices influenced by their experiences in the cross-cultural PD?
3. Which educational acculturation strategies did the Kazakhstani mathematics faculty demonstrate in their teaching practices after participating in the cross-cultural PD?
4. Which PD features did the Kazakhstani mathematics faculty identify as reasons for (a) changes or lack of changes in their perceptions of teaching and learning mathematics, and (b) implementing or rejecting new educational traditions?
5. Did institutional contexts influence the Kazakhstani mathematics faculty perceptions and educational acculturation strategies?

1.4 Procedures

This study was based on a year-long program funded by the American Councils for International Education as part of the Central Asia University Partnerships Program (UniCEN). The project ran from 2020 to 2021 to build partnership between a private university in the United

States and the two Kazakhstani universities. The participants in this study were six mathematics faculty who taught English-based discrete mathematics courses for future mathematics teachers. U1 is a privately funded institution in the Southern region of Kazakhstan, and U2 is a state-funded university in the country's Northern region. Both universities offer programs in three languages: Kazakh, Russian, and English. The experts' team included one mathematics education expert from a private university in the Northeastern USA and one mathematics education expert from U1.

Due to the COVID-19 pandemic, the study was conducted virtually. During the Fall 2020 semester, the six mathematics faculty participated in three synchronous online PD sessions led by a team of experts. PD materials included a custom discrete mathematics textbook developed collaboratively by the team and an online GeoGebra supplement book created by the U.S. expert. The custom textbook introduced mathematical concepts using a formal abstract approach supported by non-mathematical examples and connections to secondary school topics. The GeoGebra book featured dynamic simulations, virtual labs, and problems illustrating discrete mathematics applications in STEM to promote student engagement in explorations and investigations. In Spring 2021, faculty taught discrete mathematics courses in which they were expected to implement both the project materials and the new teaching approaches. Those courses were also taught online due to COVID-19. Monthly online meetings with the experts' team were held to provide participants with opportunities for sharing their experiences and to support the implementation process.

The study spanned 29 weeks in 2020–2021, with data collection timelines varying due to differences in the spring semester schedules of the two universities and each faculty teaching schedule.

1.4.1 Data Collection

Faculty were surveyed before and after the three PD sessions in the fall of 2020, using the **Faculty Efficacy and Attitudes Toward STEM Survey (F-STEM)**, which was adapted from the Teacher-STEM Survey developed by the Friday Institute for Educational Innovation (2012). Additionally, faculty were interviewed via Zoom for 30 – 40 minutes after the three PD sessions and after the teaching of the course. The **Faculty Semi-Structured Individual Interview (F-SII) protocol** was modified from the Contextualize to Learn (C2L) interview protocol developed by the Wisconsin Center for Education Research (n.d.). Finally, each faculty submitted video recordings of three lessons of their choice taught during the Spring 2021 semester.

1.4.2 Data Analysis

The F-STEM surveys were scored following the guide provided by the Friday Institute for Educational Innovation (2012). The average scores of each subscale were calculated for each faculty. The scores were then used to examine faculty perceptions.

Faculty lesson video recordings were scored following the guidelines included with **Reform Teaching Observation Protocol (RTOP),** developed by the Arizona Collaborative for Excellence in the Preparation of Teachers (Sawada et al., 2002). The scores of each lesson were then used to identify the level of enacted reformed teaching. **Field Notes** that focused on lesson settings, teaching practices, use of instructional tools, and faculty-student interactions were taken while watching the videos. Field notes were analyzed by identifying the key events, such as practices incorporating PD materials and pedagogies, and identifying the patterns in instruction and faculty-student interaction. The field notes were used to explain the RTOP scores.

The video recordings of individual interviews were transcribed, and the transcripts were analyzed using thematic analysis. Initial codes were created based on the study's theoretical framework. These codes focused on PD features (content focus, active learning, coherence, duration, collective participation) and educational traditions (teaching methodology, assessment practice, organizational structure, culture norms). New codes were developed during the coding process as necessary. Through this analysis, the study identified (a) PD features that faculty found influential, (b) faculty perceptions of educational traditions, and (c) possible connections between PD features and faculty perceptions of teaching and learning mathematics.

1.4.3 Addressing Research Questions

In order to answer **RQ1** and identify changes (or lack of changes) in faculty perceptions of teaching and learning after the PD program, differences in the scores of pre- and post-F-STEM surveys were calculated. The themes that emerged from the analysis of the faculty interviews were used to explain changes (if any) in F-STEM surveys.

In order to answer **RQ2,** the intended and enacted teaching practices were evaluated based on the analysis of both faculty interviews and teaching practice observed in lessons. Then the intended and enacted teaching practices were compared.

In order to answer **RQ3,** faculty educational acculturation strategies were identified based on RTOP scores, analysis of field notes, and faculty interviews. RTOP scores reflected the degree of reformed teaching observed in the lesson videos. Analysis of field notes revealed whether evidence of technology use and STEM applications was present. Themes that emerged from the interviews captured perceptions toward, and intended and self-reported enacted implementation of new educational traditions (reformed teaching, technology integration, and connecting mathematics teaching and learning to STEM disciplines).

In order to answer **RQ4a**, themes that emerged from the analysis of faculty interviews were analyzed to identify influential PD features that corresponded with the perceptions changes identified in RQ1 (if any). For **RQ4b**, these themes were analyzed to identify influential PD features that corresponded with the faculty intended and enacted teaching practices identified in RQ2.

In order to answer **RQ5**, faculty perceptions of teaching and learning mathematics, instructional practices, acculturation strategies, and influential PD features were compared between the two participating universities.

Chapter 2: Literature Review

Culture represents a shared system of knowledge, values, beliefs, and practices that connects individuals within a society, enabling interactions with their environment and among themselves (D‘Ambrosio, 2000; White, 1959, as cited in Bishop, 1988). It forms shared values, knowledge and behaviors through which people engage with their environment, communicate, and resolve communal challenges (Bishop, 1988).

Mathematics is inherently tied to practical needs and the social environment as it is defined as “a way of knowing, being, and doing developed in response to a need to deal with quantities, space, and relationships and to solve problems” (NCTM, 2024, p. 2). D‘Ambrosio (1994) critiques conventional historiography for neglecting "the factors influencing the consumption of what we might call academic mathematics produced in an alien cultural environment" (p. 29). Anthropological and cross-cultural studies have demonstrated that mathematics is not universal or neutral; instead, it varies significantly among cultures, reflecting the unique historical, environmental, and cultural conditions of each society (Bishop, 1988; D ‘Ambrosio, 1994). Like language, religious beliefs, or rituals, mathematics is an inherently cultural phenomenon, universally generated yet culturally specific. It functions as an essential conceptual and practical tool connected to tool-making and problem-solving activities (Bishop, 1988).

Symbols, practices, and mathematical products hold value because individuals and institutions attribute meaning to them within particular cultural contexts (Bishop, 2001). Since cultures do not share the same values in mathematics, each culture possesses distinct values concerning and expectations toward mathematics education (Bishop, 2001; Gerdes, 1998). Those values may change as culture changes. D‘Ambrosio (1994, 2000) further emphasizes that mathematics education must account for cultural dynamics—balancing acceptance of change (modernity) with the protective influences of cultural ethos. Therefore, members of the same cultural group are likely to share certain core values towards mathematics and mathematics education. In this study, *mathematics educational traditions* of a group refer to the shared traditional values and expectations of mathematics education, shaped by the cultural, historical, and environmental contexts of a certain society.

This chapter provides an overview of the literature on variations in mathematics educational traditions across cultural groups and countries and highlights the need for focused research on Central Asian educational traditions, particularly within the context of the Kazakhstan educational reform. The review identifies key gaps in the existing literature and introduces the theoretical framework guiding this study.

2.1 Mathematics, Education, and Culture

Boylan (2016) proposed viewing mathematics as a social and cultural practice, emphasizing its historical and cultural significance as well as its role as a collective human achievement. Bishop (1988) conceptualized mathematics as a cultural product, arguing that all mathematical knowledge emerges from cultural practices such as counting, measuring, designing, playing, explaining, and locating. Mathematics represents a legacy built by previous generations, positioning individuals as active contributors to the ongoing development of shared human knowledge and cultural heritage (Boylan, 2016). D‘Ambrosio (2001) further developed this idea through his theory of "ethnomathematics," which acknowledges the mathematical practices embedded within diverse cultural contexts and challenges the notion of mathematics as a culturally neutral discipline.

Because of the profound connections between mathematics and the sociocultural contexts in which it develops, mathematicians and mathematics educators from different cultures hold very distinct views regarding the nature of mathematics, which lead to different views of how mathematics should be taught (Bishop, 1988). Mathematics extends beyond academic purposes, contributing to individuals' holistic development and preparing learners to engage with reality (D 'Ambrosio, 2000, 2001; Wang & Liu, 2002). Consequently, people from diverse cultural contexts value different qualities in mathematics and mathematics education, leading to varied beliefs about effective teaching practices. As cultural contexts change, these values in mathematics and mathematics education evolve accordingly. The way that culture shapes the teaching approaches of mathematics is similar to its influence on the teaching of language and history (Schmidt et. al, 1996). Differences in social and cultural contexts can therefore predict variations in mathematics educational traditions across different cultures.

2.1.1 Differences in Educational Traditions

Mathematics education does not exist in isolation from the broader cultural context in which it operates (Bishop, 1988; D'Ambrosio, 2001). Many lectures were found about the differences in mathematics education, and they highlighted the potential cultural influences on what people from different cultures value in their mathematics teaching approaches. Furthermore, these beliefs and practices are not static but evolve in response to social and cultural changes.

Andrews and Hatch (2000) compared conceptions and teaching of mathematics between English and Hungarian middle school teachers. Their study identified within-country patterns and highlighted significant differences between the two nations' philosophies of mathematics education, demonstrating how these philosophies shaped teachers' perceptions of mathematics and pedagogy, and consequently their teaching practices. These national beliefs, rooted in cultural, historical, and philosophical contexts, played a critical role in determining how mathematics was perceived and taught in these countries. Schmidt et al. (1996) examined curriculum and pedagogy through mixed-method case studies in France, Japan, Norway, Spain, Switzerland, and the United States. Their findings identified shared characteristics across these nations but highlighted that each country maintained distinct and consistent features in curriculum implementation, content, and teaching practices. They observed that variations in lesson characteristics were more significant across countries than within any single country. The content of mathematics did not disrupt the consistent interaction between curriculum and pedagogy within individual countries. Instead, differences in teaching practices were influenced by teachers' training, experiences, and educational beliefs shaped by national ideologies.

Extended study on Chinese mathematics education also reveals the influences of Confucian culture in its mathematics educational tradition. Confucian culture emphasizes moral cultivation, social harmony, and respect for knowledge and authority, and Chinese mathematics education tends to value hard work, perseverance, teacher exemplification, and extrinsic motivation. The teaching approach in China often promotes centralized teaching approaches, prioritizes fundamental mathematics content and essential procedures or skills, and views memorization, practice, and structured learning as necessary and advantageous components of the learning process (Huang and Asghar, 2018; Kaiser & Blomeke, 2013; Leung, 2001, 2006; Leung, et al., 2006; Wang & Liu, 2002). Those educational traditions largely differ from countries that are not influenced by the same culture. Kaiser et al. (2006) found that mathematics education in France, Germany, and England values individual learning opportunities and personal development. London teachers in Leung's (2006) study valued student-centered teaching approaches in mathematics education, emphasizing practical applications and shaping mathematics content to individual aptitude and personal interest.

The development of current mathematics educational traditions in the United States is deeply connected to broader historical and societal trends of liberty and democracy. The United States was founded on principles of self-governance and citizen participation, and its culture has historically emphasized individualism, self-reliance, and personal agency (Tocqueville, 1835). As Dewey (1916, p. 166) emphasized, "No thought, no idea, can possibly be conveyed as an idea from one person to another… only by wrestling with the conditions of the problem firsthand, seeking and finding his [her] own way out, does he [she] think" (Dewey, 1916, p. 166). In recent decades, dynamic, student-centered approaches that emphasize student leadership, active participation, and inquiry-based learning have been strongly promoted in the United States. Reformed teaching, which positions students as active knowledge constructors who create and develop their own understanding and knowledge through experience and reflection rather than passively receiving information, reflects these ideas (Sawada et al., 2002; Windschitl, 2002). This approach signals a paradigm shift from traditional teacher-centered approaches to activity-based learning environments that provide multiple opportunities for students to build their own knowledge in inquiry-based, problem-solving activities that are connected to meaningful contexts and often involve student collaboration (Sawada et al., 2002; Windschitl, 2002). In the United States, this teaching approach resonates with broader cultural values of independence, innovation, and democratic participation.

Educational traditions can evolve as the culture changes and can synthesize multiple cultural influences. For example, Leung's (2006) study found that because Hong Kong was influenced by both Chinese and British culture, its mathematics educational traditions also draw from both of them. Teachers' beliefs and teaching practices reflect a blend of both educational traditions. In addition, Japanese mathematics education also evolved as its society and culture was influenced by Europe and the United States in the Meiji era (Kaiser et al., 2006). Japanese mathematics education managed to maintain its distinctive characteristics even as it imported educational traditions, often adapting them to fit local contexts.

Consequently, Japanese mathematics educational traditions preserve Japanese characteristics while sharing similarities with the imported educational traditions. These two cases show that mathematics educational traditions can be influenced by imported cultures, though these changes typically emerge gradually, and there is not necessarily a complete shift from one to another (Kaiser et al., 2006; Leung, 2006). Mathematics educational traditions can maintain distinctive cultural characteristics of the heritage culture while incorporating imported beliefs and approaches, creating an integrated tradition reflecting multiple cultural foundations.

Comparative studies reveal that differences in mathematics educational traditions between individual countries are pronounced. Significant differences in teaching approaches across countries are often deeply rooted in national historical, social, and cultural contexts. The literature strongly indicates that individuals' beliefs and teaching practices are associated with the educational traditions of their group. However, individuals' teaching practices do not always mirror their beliefs or the mainstream values of their educational traditions (Leung, et al., 2006). This gap between belief and practice makes it important to explore whether cultural differences influence educators' perceptions and practices in the teaching of mathematics.

2.1.2 Central Asian Educational Traditions

Central Asian countries have been influenced by Islamic culture since the ninth century, which has significantly shaped their cultural and educational traditions (Niyozov, 2008). However, the subsequent influence of the Union of Soviet Socialist Republics (USSR) diminished the role of religion in education through the implementation of a strongly centralized education model (Achilov, 2012; Niyozov, 2008). Under this Soviet paradigm, standardized curricula, textbooks, and administrative structures were mandated across all educational institutions, with centralized decision-making processes that severely limited institutional autonomy (Kuraev, 2016; Yakavets, 2014). This pervasive centralization contributed substantially to forming distinctive educational traditions characterized by teacher-centered and lecture-based teaching approaches, structured exercises, strict adherence to rules, and teacher control throughout the whole country, including Central Asian republics (Niyozov, 2008; OECD, 2007; International Commission on Mathematical Instruction, 1966). The system rigidly reinforced state philosophy rather than typical civic education. By mandating centralized national curriculum across the USSR's diverse ethnic populations, the system effectively subordinated cultural heritage. Despite the existence of a long liberal local tradition in pedagogical philosophy, there was uniform application of pedagogical practices across the USSR (Yakavets, 2014).

Education in the USSR was highly specialized to meet the labor demands of the centrally planned economy. School curricula, textbooks, and teaching materials were meticulously developed to prepare students for roles that would serve the needs of the state. Within this framework, mathematics occupied a privileged position as the cornerstone of technical and military-technical education, with clear expectations that mathematical study should connect to productive labor (Karp, 2014, 2021; Yakavets, 2014). Unlike other academic disciplines, mathematics education remained relatively sheltered from the ideological pressures and political campaigns that significantly influenced other subjects. This exceptional status enabled Soviet mathematics education to maintain its focus on academic content and intellectual development rather than ideological conformity. Mathematics classes were deliberately structured to teach the subject matter specifically and substantively (Karp, 2014, 2021).

Soviet mathematics teaching approaches prioritized strong fundamental knowledge and deep theoretical understanding, with courses characterized by systematic, proof-based exposition and rigorous problem-solving practices (Karp, 2014, 2021). The students were required to work independently, but pedagogical approaches operated under strict control (Karp, 2014; Yakavets, 2014). Teachers were required to exercise clear authority in their classrooms, carefully supervising all student activities and work (Karp, 2014, 2021; Yakavets, 2014). The centralization and teacher-centered culture in Soviet education continued to influence post-Soviet countries' mathematics educational traditions (Karp, 2021), leaving a legacy of centralized and rigorous control by authorities, procedural fluency through repetitive practice, teacher-centered teaching approaches, and rigorous theoretical foundations (Yakavets, 2014). These features stand in contrast to educational traditions in countries like the United States, which value individualization in mathematics education and employ teaching approaches such as reformed teaching.

Bishop (1998) underscored the profound impact of social, cultural, and historical factors on mathematics education in Central Asia, emphasizing that findings from studies in other Asian countries do not apply to this region. After independence from the USSR, Central Asian nations have pursued distinct reform trajectories shaped by their individual cultural foundations and national goals. Following the dissolution of the USSR, Central Asian nations began re-establishing their connections to Islamic identity (Achilov, 2012), while also striving to revive and promote their national cultures, traditions, heritage, and local languages (Achilov, 2012; Niyozov, 2008). These reforms have led to significant transformations in social and educational objectives. The diversity in cultural heritage and reform pathways across Central Asian countries has likely contributed to the development of unique educational traditions within each nation. Therefore, it is imperative to examine the mathematics educational traditions of Central Asian countries independently.

2.2 Higher Education System and Educational Reform in Kazakhstan

In October 1997, the President of Kazakhstan presented the "Strategy for Development of the Republic of Kazakhstan until 2030," which identified educational reform as a crucial step in national development (President of the Republic of Kazakhstan, n.d.). At that time the Kazakhstani higher education system was deeply rooted in the educational traditions of the USSR (Lang, 2019). During the Soviet era, the education system prioritized the needs of the country over individual and institutional objectives. Under this model, standardized curricula, textbooks, and administrative structures were imposed across all institutions; centralized decision-making processes left little room for institutional autonomy; and the superintendents, who possessed ultimate authority within their institutions, had to ensure compliance with communal directives of the country (Kuraev, 2016; Wang & Bai 2021). After establishing its independence in 1991, to establish a national higher education system distinct from the Sovietized and Russified structures, Kazakhstan introduced a series of significant societal reforms. The reforms included the transformation of the country's political, economic, and educational systems to be more flexible and autonomous (Lang, 2019; Ma & Meng, 2011). The Ministry of Education and Science of the Republic of Kazakhstan (MESRK) released the State Program for Educational Development (SPED) 2011-2020, which outlined educational reform objectives and strategies (2010). In SPED, government officials acknowledged that the Kazakhstani education system required modification at all levels to address the requirements of the global socio-economic landscape and meet its ambition to become part of "the club of 50 most competitive countries in the world" (MESRK, 2010, p. 5). The rebuilding of Kazakhstani national culture involves integrating systems and practices from other countries in the national educational reform effort (Chu, 2009; Lang, 2019; Ma & Meng, 2011). The reform does not aim to transform the Kazakhstani education system to mirror those of other countries, but rather to learn from and incorporate innovative ideas and practices within the specific context of Kazakhstan. The educational reform specifically focuses on increasing the autonomy and internationalization of the institutions.

2.2.1 Increase of Autonomy

Higher education reform is a critical component of these transitions, and the Kazakhstani government is carrying it out by changing the curricula, pedagogy, institutional structure, and laws (Hartley et al., 2016). In order to increase institutional autonomy, the Kazakhstani higher education institutions moved away from the rigid control of the MESRK (Chu, 2009; Hartley et al., 2016; Yakavets et al, 2023). Following the introduction of the Higher Education Law in 1993, which authorized the creation of private institutions, the number of private higher education institutions rapidly expanded. By 1997, public institutions accounted for only 43.2% of all higher education institutions in Kazakhstan, while private institutions made up 56.8% (Wang & Bai 2021). By 2006, out of a total of 177 higher education institutions, 109 were private (OECD, 2007). The establishment of private higher education institutions has provided young adults with increased access to higher education (Chu, 2009; Nadirova, 2018; OECD, 2007). In 2010, the MESRK increased academic freedom by allowing higher education institutions to gradually gain more control over their elective curriculum. By 2020, higher education institutions were expected to have control over 70% of the curriculum for undergraduate programs, 80% for master's programs, and up to 95% for doctoral programs (MERSK, 2010).

At the time of this study, the MESRK oversaw and regulated activities in both public and private higher educational institutions. Its responsibilities included defining and executing educational policy, drafting regulations for educational funding, preparing educational standards and curricula, organizing and implementing assessment systems, and setting requirements for teacher education, among other tasks (OECD, 2017). In contrast with public higher education institutions, private higher education institutions have greater autonomy in appointing administrative teams, managing funding, establishing partnerships with local and global institutions, organizing staff PD, and organizing exchanges for faculty and students (OECD, 2007). With fewer government restrictions on financial operations, private higher education institutions have more flexibility in allocating resources to meet institutional needs, including adopting different tools and materials for delivering courses (OECD, 2017). However, a significant challenge to the decentralization efforts was the lack of a comprehensive legal structure combined with inadequate staffing, especially the lack of competent educational administrators at both individual and institutional levels (Yakavets, 2014). Higher education institutions' leaders have expressed a desire for clearer guidance from the government, reflecting a paradoxical dependence on centralized decision-making despite their autonomy (Li & Ashirbekov, 2014).

Although the Kazakhstani government has made significant efforts to increase institutional autonomy, rigid control mechanisms create barriers to achieving full operational independence of both public and private higher educational institutions (OECD, 2007; OECD, 2017; Yakavets, 2014). While the curriculum and standards for the same course in private and public universities are not expected to vary significantly, differences in the levels of operational autonomy between these institutions can lead to variations in other aspects of instruction, such as faculty learning of new skill sets (Desimone, 2009).

2.2.2 Internationalization

Internationalization has been a key focus of Kazakhstani higher education reform, and Kazakhstan has actively pursued this direction since the 1990s by aligning its educational system with European higher education's structure and standards (MERSK, 2010; Wang & Bai, 2021). These efforts include but are not limited to transitioning from the Soviet model to the three-tier degree model (bachelor's, master's, and doctoral), developing the higher education curriculum to include relevant international dimensions, increasing financial support to education programs, popularizing the use of the Internet and technology in schools, and developing programs to improve the quality of teachers (Ma & Meng, 2011; OECD, 2007). This effort gained significant momentum in 2010 when Kazakhstan became a full member of the Bologna Process, advancing the nation's commitment to integrating its higher education system into the global landscape by adopting European structures and international standards (Lang, 2019; MERSK, 2010; OECD, 2017). Higher education curricula in Kazakhstan are anticipated to adopt more student-centered approaches, focusing on achieving desired learning outcomes rather than adhering strictly to inputs like study hours. Flexibility in course structure is encouraged as long as the intended outcomes are achieved (MERSK, 2010; OECD, 2017).

Language reform has been a pivotal aspect of Kazakhstan's reform of higher education. The shift from bilingual education in Kazakh and Russian to a trilingual system, incorporating English, reflects the government's 2007 proposal to prepare citizens for the global labor market and economic integration (Sarmurzin et al., 2024; Kucherbayeva & Smagulova, 2023; Lang, 2019; Mehisto et al., 2023; OECD, 2017). This initiative aims to not only position Kazakhstan within the global economy but also to facilitate access to advanced academic knowledge, as English dominates high-impact journals and global research outputs. The adoption of English supports the national goal of improving education quality and aligning with international standards (Sarmurzin et al., 2024; Lang, 2019; OECD, 2017). Higher education institutions have built partnerships with leading global universities and fostered collaborations with English-speaking institutions, further promoting internationalization (Lang, 2019; Mehisto et al., 2023; Sarmurzin et al., 2024). Kazakhstan envisions transitioning to full English-medium instruction in higher education institutions in the near future (Lang, 2019; OECD, 2017). Achieving this vision requires developing English-based curricula and preparing educators to align with these reforms.

However, the implementation of English education in Kazakhstan presents significant challenges. The policy has faced criticism due to concerns about its potential impact on the Kazakh language and culture. Opponents worry that the introduction of English, alongside the already widespread use of Russian, may marginalize the Kazakh language and diminish its use (Kucherbayeva & Smagulova, 2023; Sarmurzin et al., 2024). As a relatively new addition to the education system, the implementation of English-based instruction faces obstacles due to a shortage of fluent English speakers. Both students' and teachers' lack of English proficiency has prevented the effective implementation of trilingual education so far (Sarmurzin et al., 2024). Furthermore, Kazakhstan lacks an English-speaking environment that could naturally support language acquisition. As a multi-ethnic country, it is a home to a diverse range of spoken languages, but English is not widely used or prevalent. Additionally, Kazakhstan is geographically surrounded by non-English-speaking countries, further limiting its population's exposure to English (Li & Ashirbekov, 2014; Sarmurzin et al., 2024). To address these barriers, the government has established English-based teacher training programs. More importantly, there is a critical need to enhance English proficiency among educators at all levels and provide specialized training for content-area instruction in English. However, language acquisition is inherently time-intensive, and teaching subject matter in English requires specialized pedagogical preparation. Given the relatively recent integration of English into the education system, insufficient language proficiency among both teachers and students may significantly hinder the effectiveness of English-based content instruction.

Kazakhstan also aims to enhance internationalization by integrating advanced technologies with education and expanding the use of technology-enabled learning (OECD, 2017). To support teachers, the government developed centralized digital repositories that include instructional guides and materials to facilitate high-quality teaching. For students, the expansion in the use of technology aims to ensure that when they encounter academic challenges, they will have access to remedial resources and additional support. However, reaching these ambitious goals may be constrained by infrastructure challenges, which must be addressed to realize the full potential of these reforms (OECD, 2017).

The 2007 and 2017 OECD reports on Kazakhstan's education system indicate that the implementation of these reforms has been challenging and slow. Despite Kazakhstan's acceptance to the Bologna Process, substantial changes have not yet materialized, largely due to the continued centralization of the education system. This persistence of centralized planning emphasizes control and monitoring as the primary mechanisms for quality assurance, limiting the system's ability to adopt more decentralized and flexible practices. As a result, course structure and content remain relatively rigid, impeding the shift toward a more dynamic and student-focused education system (OECD, 2017).

The student mathematics learning experiences provided by the current university faculty are deeply embedded in the educational traditions that emphasize teacher-centered instruction, where student engagement is limited to drills and exercises, lacking opportunities for exploration and active participation (OECD, 2017). Consequently, many faculty continue teaching mathematics in ways that reflect these traditions, often excluding student-centered and inquiry-based approaches. On both governmental and institutional levels, efforts to enhance autonomy and internationalization in the Kazakhstani education system are ongoing but require greater momentum. Furthermore, future research should explore on an individual-level what factors are contributing to the continued use of traditional teaching approaches among faculty, despite top-down educational reform efforts.

2.2.3 Professional Development for Educators in Kazakhstan

To support these transitions toward greater educational autonomy and internationalization, educators in Kazakhstan must acquire updated skills to effectively support both themselves and their students (Chu, 2009; Lang, 2019; Ma & Meng, 2011; Tampayeva, 2015). Aligning with the government's goal of developing educators and students to meet international standards (Duman, 2024), there exists a need for university faculty to participate in international collaborative opportunities, learn about different teaching approaches, acquire and/or develop new instructional materials, improve English for instruction, learn to incorporate technology into instruction, etc. (Pardała et al. 2015; Toybazarova & Nazarova, 2018). An example of the government's efforts to address these goals is the Strategy for Academic Mobility program that provides funding for educators and students to study abroad with the requirement that they return to work in Kazakhstan for at least five years. The flagship initiative is the Bolashak International Scholarship, established in 1993, which initially supported both undergraduate and graduate education. Currently, the Bolashak program priorities have shifted toward supporting graduate students in specialized fields aligned with the Kazakhstan State Industrial and Innovative Development Program. The program aims to enable individuals to learn innovative practices abroad and contribute to the country's development upon their return (OECD, 2007, 2017). Another aspect of internationalization involves the government's endeavor to align the Kazakhstani education system with European standards by encouraging educational institutions to host experts and specialists from countries leading in specific innovative fields (OECD, 2007, 2017). A central feature of the Kazakhstani educational reform is the emphasis on student-centered approaches that foster intrinsic motivation, critical thinking, and engagement in meaning-oriented, application-oriented, self-managed, and cooperative learning (Yakavets et al., 2023). These pedagogical priorities align closely with the concept of reformed teaching which is valued in U.S. mathematics educational traditions (Leung, 2001; Sawad et al., 2002). Experts from foreign countries like the United States were invited to develop PD programs for Kazakhstani educators focused on innovative pedagogical approaches such as reformed teaching.

Another feature of Kazakhstani educational reform is a strong emphasis on teaching mathematics with technology and connecting mathematics to other STEM disciplines (Drijvers, 2015). PD is the most commonly used approach for educators to gain and develop new knowledge and skills (Desimone, 2009). Although the effectiveness of PD varies across contexts and implementations, evidence shows that well-designed PD can influence faculty attitudes, beliefs, and perceptions across various dimensions of education (Edwards et al., 2015; Tuan et al., 2017). These factors, in turn, shape faculty practices in the classroom, which may significantly affect student learning and achievement over time (Postareff et al., 2007, 2008; Stes et al., 2010). The influx of experts from other countries to provide PD to Kazakhstani educators has led to the need for studies to analyze effectiveness of such programs, especially taking into account differences in educational traditions of invited experts and Kazakhstani educators.

2.3 Educational Challenges During the COVID-19 Pandemic

The abrupt shift from in-person to online teaching during the COVID-19 pandemic presented unprecedented challenges for education worldwide. During that time, educators did not have time to carefully plan online learning experiences, and the lessons often lacked the deliberate design and preparation time that characterizes effective online education (Rapanta et al., 2020). In this transition, educators faced challenges as they rapidly adapted pedagogical methods to online formats, particularly in the areas of hands-on, laboratory-based, or performance-oriented teaching that traditionally rely on physical presence (Johnson et al., 2020). The transition also influenced teaching effectiveness and learning outcomes, as maintaining student engagement became challenging, fostering interactive learning experiences proved to be more difficult, and sustaining student motivation in remote contexts presented significant difficulty (Adedoyin & Soykan, 2020; Rapanta et al., 2020; Xia et al., 2022). Additionally, these transitions in learning environments added stress and anxiety for both educators and students, compounded by the broader societal impacts of the pandemic. Accessibility to the internet, technology, and other digital tools complicated the online teaching and learning situation (Rapanta et al., 2020). Teaching with technology and online instruction represents a fairly novel experience, particularly for Kazakhstani educators, as integration of technology in education is a goal in progress. The educators likely had little training and were not prepared in online teaching. These impacts of COVID-19 may have added another challenge to Kazakhstan's transition towards an innovative educational system and likely affected the participants in the cross-cultural PD in this study.

2.4 Summary

Differences in educational cultures lead to variations in mathematics educational traditions. Therefore, participants in cross-cultural PD programs may encounter significant differences between their own heritage educational traditions and the new educational traditions brought by the experts leading the PD programs. These differences become particularly relevant in the context of Kazakhstan, where cross-cultural PD programs are a critical part of ongoing higher educational reform. While variations in teaching practices are influenced by diversity in mathematics educational traditions, the role of educational traditions is often overlooked or underemphasized in cross-cultural educational research, particularly when examining the educators' adaptation to new pedagogy. During the reform process, educators in Kazakhstan face the complex challenge of balancing elements of the country's educational system, accomplishing educational reform goals, and building a unique national educational culture that maintains the valuable heritage of Kazakhstan (Chu, 2009; Duman, 2024; Liang, 2001). There is the essential need to understand how the cross-cultural PD programs affect their participants to ensure sustainable progress, but there has been little research conducted regarding cross-cultural PD or faculty training in Kazakhstan.

To improve the effectiveness of such PD programs, researchers and practitioners need to understand how differences in education traditions influence perceptions and teaching practices of PD participants. However, cultural influences on the effectiveness of PD have been understudied in mathematics education, particularly the influence of PDs designed based on educational traditions from one culture for participants from a different culture. Furthermore, the research on the cultural perspective of PD programs often concentrates on pedagogy that embraces students' culture, rather than how the PD participants experience acculturation when acquiring new knowledge and skills from the experts of different educational cultures. There is a need to study the effectiveness of cross-cultural PD programs, examining not only the significant features of the PD itself but also investigating the role that difference in educational traditions plays in shaping participants' experiences and outcomes.

2.5 Theoretical Framework

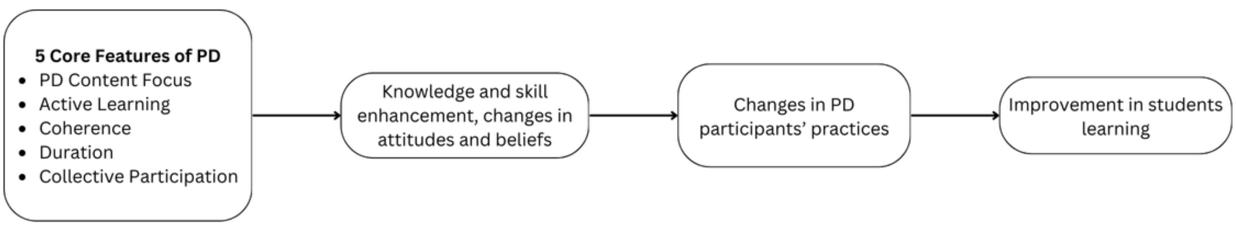
This study investigated how cross-cultural PD that promotes reformed teaching, enhancing mathematics teaching with technology and strengthening connections with STEM disciplines, influenced the perceptions and teaching practices of higher education faculty in Kazakhstan. Understanding this complex intersection of educational cultures requires a theoretical framework that accounts for both PD effectiveness and educational acculturation processes. To explore the factors that could moderate the PD's influence I adapted Desimone's (2009) Framework for the Studying the Effects of PD on Teachers and Berry's Acculturation Strategies framework (Berry & Sam, 2016). The following sections detail each framework and explain their synthesis into the study's theoretical framework.

2.5.1 Framework for Studying the Effects of PD

The Framework for Studying the Effects of PD on Teachers (Desimone, 2009) "represents interactive, non-recursive relationships between the critical features of PD, teacher knowledge and beliefs, classroom practice, and student outcomes" (p. 184). In this study, I focus on examining how PD influences participants' perceptions and teaching practices. The framework suggests that PD has a sequence of effects starting with the PD experience and then followed by knowledge and skill enhancement, changes in attitudes and beliefs, implementation of newly acquired approaches, and, finally, improvements in teaching practices and student achievements. This framework identifies five core features of effective PD: content focus, active learning, coherence, duration, and collective participation. It also suggests that these features should be measured to assess the PD's impact, and three variables - changes in participants' perceptions, changes in practices, and changes in students' achievement - should be the measures that reflect the influence of the PD (Desimone, 2009). This study focused on one course of influence (Figure 1).

Figure 1

Framework for Studying the Effects of PD



Note: Re-created based on the graphic of the framework from "Improving Impact Studies of Teachers' Professional Development: Toward Better Conceptualizations and Measures," by L. Desimone, 2009, *Educational Researcher, 38*(3).

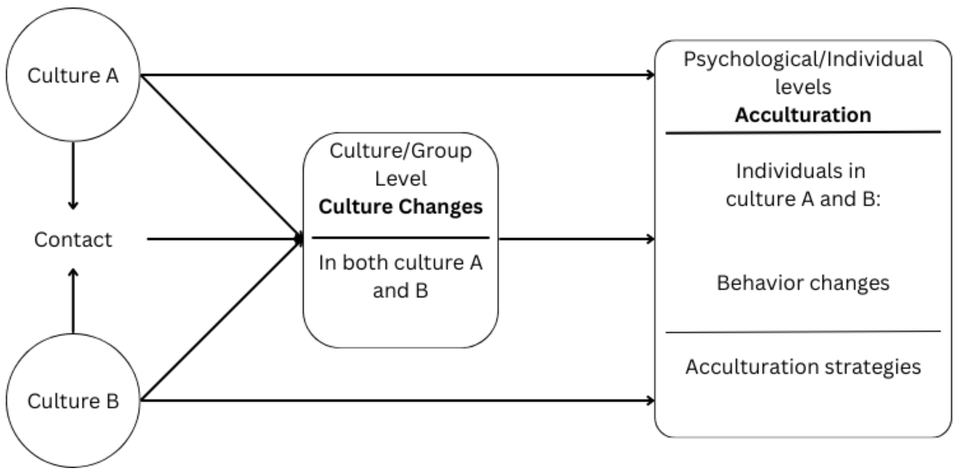
While the PD framework takes into account teaching and learning contexts, which could include teacher and student knowledge and attitude, and the environmental factors such as class, school, and district level, it does not account for the potential influence of the education culture backgrounds of participants and PD providers. Therefore, in this study I consider education culture separately from the context. When different education cultures intersect, the reactions of participants can be complex. Understanding the potential cultural influences of the new education traditions on Kazakhstani faculty in PD is a central focus of this study.

2.5.2 Framework of Acculturation

Acculturation research emerged as scholars sought to understand the complex processes that occur when individuals and groups from different cultural backgrounds come into sustained contact. In fields such as psychology, sociology, and anthropology, frameworks like Berry's framework of acculturation were developed to systematically examine how people navigate cultural encounters in contexts such as immigration. Berry's (2003) Framework of Acculturation provides guidance in examining the process of effects that occur when two groups from different cultures come into continuous first-hand contact, which can manifest impacts at both cultural/group and psychological/individual levels. Rather than representing separate phenomena, cultural/group and psychological/individual acculturation constitute interconnected processes. Each level simultaneously constrains and enables the other through multiple pathways operating across varied temporal scales and life domains. Cultural/group acculturation refers to changes in social structures and institutions, while psychological/individual acculturation involves changes in individual behaviors, identity, values, and attitudes (Berry & Sam, 2016). Individual behaviors are influenced by acculturation at the moment of contact, and the vast differences among individuals (even in the same group or society) under the same culture contact experiences lead to the distinction of social and individual levels. Hence it is important to explore individuals' acculturation experiences. In the long term, the acculturation effects are defined as three types of adaptation (adaptation refers to both possible positive and negative outcomes) (Berry, 2017). The acculturation process extends beyond the behavior changes within the immediate period of cultural contact. Long-term adaptations may span years or even generations following initial contact experiences (Berry, 2017; Berry & Sam, 2016). This study only focuses on acculturation at the psychological/individual levels (Figure 2).

Figure 2

Framework for Conceptualizing Acculturation Components and Relationships

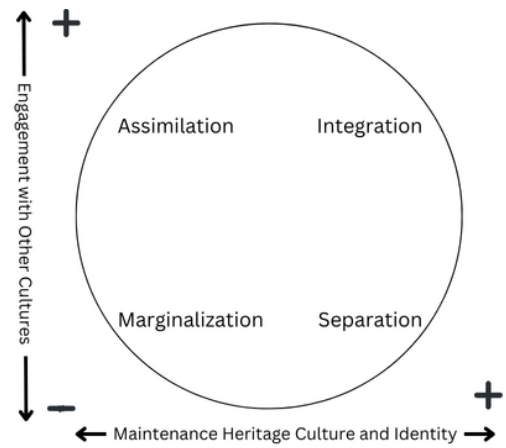


Note: Re-created based on the graphic from "*The Cambridge Handbook of Acculturation Psychology*" by J. Berry and D. Sam, 2016, Cambridge University Press.

In cultural contact situations, individuals typically display changes that occur without significant difficulty (Berry, 2003). The acculturation strategies indicate dimensions that should be considered when measuring individuals' responses during cultural contacts as well as potential outcomes and their meanings. Because there are often gaps between individuals' perceptions and actions, it is suggested that acculturation strategies should be assessed through both intended and enacted behaviors (Berry, 2003). The changes can be measured based on two fundamental dimensions: individuals' preference for maintaining their heritage culture and their engagement with other cultures (Berry & Sam, 2016). When these two dimensions are considered simultaneously, they generate four distinct acculturation strategies (Figure 3).

Figure 3

Four Acculturation Strategies for Small Groups and Their Individual Members



Note: Re-created based on the graphic from "*The Oxford Handbook of Acculturation and Health*" by J. Berry, 2017, Oxford University Press.

2.4.3 Theoretical Framework for the Study

Connecting the Framework for Studying the Effects of PD on Teachers with the modified Acculturation Framework led to the development of the theoretical framework of this study: the Cross-Cultural PD Educational Acculturation. In this study I focus on individual acculturation experiences during a year-long PD program to provide insights into how the cultural context affected faculty perceptual and behavior changes. While acculturation is well defined and observable outside of educational contexts, I needed a theoretical guide to identify established measurement parameters and possible change patterns within an educational context. Therefore, I adapted the acculturation strategy framework to guide my examination of faculty educational acculturation during their contact with a new educational tradition. This study adapted Berry's definitions of acculturation strategies to the educational context (Table 1). However, the marginalization strategy used in the original framework is not applicable in an educational context. Educators must employ some teaching approaches that align with either traditional teacher-centered approaches or more contemporary student-centered approaches, or some combination of the two. Therefore, *marginalization* was removed from this study framework.

Table 1

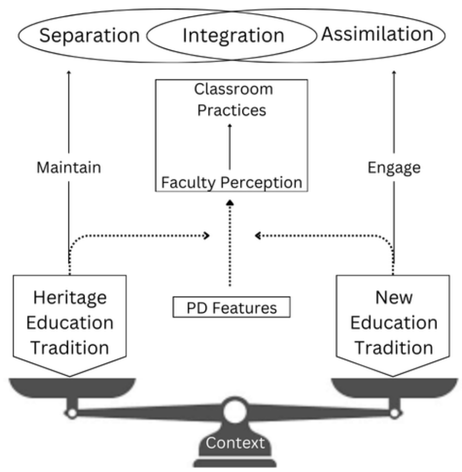
Definitions of Acculturation Strategies

Strategies	Original Definition	Definition for Education Culture
Assimilation	Individuals do not wish to maintain their cultural identity and seek daily interaction with other cultures	Instructors do not wish to maintain their traditional pedagogical practices and seek daily implementation of pedagogical practices from another educational culture
Separation	Individuals place a value on holding on to their original culture and at the same time wish to avoid interaction with others	Instructors place value on their traditional pedagogical practices and at the same time wish to avoid implementation of new pedagogical practices from another education culture
Integration	Individuals have an interest in both maintaining their original culture and having daily interactions with other groups; there is some degree of cultural integrity maintained, while at the same time the individual seeks to participate as an integral part of the larger society	Instructors in some degree maintain their traditional pedagogical practices and at the same time seek to implement some pedagogical practices from another educational culture
Marginalization	Individuals have little possibility of, or interest in, cultural maintenance, and little interest in having relations with others	This strategy is not applicable in the context of education

Perceptions and behaviors of educators participating in the PD are shaped not only by the PD features but also by their degree of desire to balance heritage and new educational traditions which can be observed by measuring faculty education acculturation strategies. Therefore, education culture was added as another factor that can influence the outcomes of PD (Figure 4).

Figure 4

The Cross-Cultural PD Educational Acculturation Model



While the faculty go through the acculturation processes, we consider that both their perceptions and practices of their teaching might be affected. The differences in cultural context could contribute to the differences in perceptions of mathematics and mathematics pedagogy, which could further lead to differences in teaching (Schmidt, et.al, 1996). Faculty balance between the two educational traditions can be observed through their perceptions and approaches to teaching practices. In this study the contextual factors include the institutional contexts such as power dynamics and support within the university. Since Kazakhstan's public and private higher education institutions provide different contexts, the contextual factors of the schools could moderate the PD influence on faculty educational acculturation strategies.

Chapter 3: Methodology

This chapter describes the methodology employed in this study to collect and analyze data. The following sections will restate the research purpose and questions as well as discuss the study context and data resources, the demographics of participants, the development and implication of data collection instruments, and the procedures of data analysis.

3.1 Research Design and Research Questions

The purpose of this study was to examine the acculturation strategies of Kazakhstani mathematics faculty who participated in a cross-cultural PD program that focused on reformed teaching and technology integration. Those faculty were all teachers of discrete mathematics courses to students preparing to become mathematics teachers. In order to identify acculturation strategies, I needed to have in-depth analysis of faculty perceptions and actions regarding the incorporation and application of educational traditions that were new to them. Thus, this study used a case study methodology to collect a variety of data from the participants. I collected both qualitative and quantitative data for analysis in order to address the following research questions:

1. How were the Kazakhstani mathematics faculty perceptions of teaching and learning mathematics influenced by their experiences in the cross-cultural PD?
2. How were the Kazakhstani mathematics faculty intended and enacted teaching practices influenced by their experiences in the cross-cultural PD?
3. Which educational acculturation strategies did the Kazakhstani mathematics faculty demonstrate in their teaching practices after participating in the cross-cultural PD?

4. Which PD features did the Kazakhstani mathematics faculty identify as reasons for (a) changes or lack of changes in their perceptions of teaching and learning mathematics, (b) implementing or rejecting new educational traditions?
5. Did institutional contexts influence the Kazakhstani mathematics faculty perceptions and educational acculturation strategies?

3.2 Description of Professional Development Program

This study is based on a year-long PD program funded by the American Councils for International Education completed in 2020-2021 as part of the Central Asia University Partnerships Program (UniCEN) to build a partnership between one private university in the Northeastern USA and the two Kazakhstani universities. The UniCEN grant program requires active collaboration from both university partners. Applications must demonstrate joint initiative and commitment from both the U.S. and Kazakhstani institutions. This UniCEN project was designed for Kazakhstani mathematics faculty teaching English-based undergraduate discrete mathematics courses for pre-service mathematics teachers. Its purpose was to promote inquiry-based instruction, the integration of mathematics-specific instructional technology and STEM applications, and the enabling of students to connect concepts in discrete mathematics to the topics in the secondary school mathematics curriculum. The PD program and its materials were developed through a direct partnership between two mathematics education experts – one from the U.S. university and another from one of the two Kazakhstani universities.

Due to the COVID-19 pandemic, synchronous PD sessions, synchronous course teaching, and all data collection procedures were held online. In the first part of the project conducted during the Fall 2020 semester, six mathematics faculty from two Kazakhstani universities, one public and one private, participated in the three PD sessions. These sessions were held on consecutive Saturdays, led by the two experts. In the 2021 Spring semester, all faculty who participated in the PD taught a discrete mathematics course at their universities.

3.1.1 Description of PD Materials

The PD materials included a custom discrete mathematics textbook (Lyublinskaya & Bagisbayev, 2000) and a GeoGebra book supplement (Lyublinskaya, n.d.). The custom textbook was developed collaboratively by the two experts and includes six major sections: (a) mathematics logic, (b) set theory, (c) relations and functions, (d) mathematical induction and combinatorics, (e) number system, and (7) complex polynomial functions and the fundamental theorem of algebra. The textbook introduces each concept through three key features. First, formal mathematical statements – definitions and theorems – establish a rigorous foundation. Next, readily understandable examples connect abstract ideas to STEM applications and real-life experiences. Finally, familiar examples link the concept to the secondary school curriculum, reinforcing continuity and relevance. An example of these features used for explanation of the pigeonhole principle in the chapter on combinatorics is shown in Figure 5.

Figure 5

Features of The Custom Textbook for the Pigeonhole Principle

4.2.9. Pigeonhole Principle

Theorem. If m pigeons are put into n pigeonholes, then at least one pigeonhole must contain more than one pigeon if $m > n$, and at least one pigeonhole must be empty if $m < n$.

Example.

Hair-counting. More than 1,500,000 people live in Almaty, Kazakhstan. It is known that the number of hairs on a human head is less than 1,000,000. Therefore, at least two people in Almaty have exactly the same number of hairs on their heads.

Exercise.

The Birthday Problem. Show that if the number of students at a school is 370 then there exist at least two students who share the same birthday, and if number of students is 360 then there exist days when no birthdays are celebrated at the school.

Generalized Pigeonhole Principle.

If m pigeons are put into n pigeonholes, then there is at least one pigeonhole containing at least $\left\lceil \frac{m}{n} \right\rceil$ pigeons and there is at least one pigeonhole containing at most $\left\lfloor \frac{m}{n} \right\rfloor$ pigeons, where $\lceil\ \rceil$ and $\lfloor\ \rfloor$ are, respectively, the "ceiling" and "floor" functions, i.e. $\lceil x \rceil$ and $\lfloor x \rfloor$ are the closest to x integers such that $\lfloor x \rfloor \le x \le \lceil x \rceil$, for example, $\lceil \pi \rceil = 4, \lfloor \pi \rfloor = 3, \lceil 2 \rceil = \lfloor 2 \rfloor = 2$.

Example.

Among 1500 students at the university there are at least $\left\lceil \frac{1500}{366} \right\rceil = 5$ students with the same birthday and there is at least one day of the year when the number of birthdays is at most $\left\lfloor \frac{1500}{366} \right\rfloor = 4$.

Note: Reprinted with permission from "*Introduction to Discrete Mathematics: Course Materials for Pre-Service Mathematics Teachers"* by K. Bagisbayev and I. Lyublinskaya, 2020, Unpublished manuscript. © Authors

Here the theorem presents a formal introduction of the principle. The first example about hair counting presents an “outside of mathematics” context, and the last example demonstrates connection of this principle to the Kazakhstan high school mathematics curriculum.

The GeoGebra book was created by the U.S. expert to provide a supplement for the course with opportunities for students to actively engage with the course topics through demonstrations, explorations, exercises, and applications. The book included at least one interactive activity for each topic covered in the custom textbook as well as an appendix that covered graph theory and directed trees and graphs, topics not included in the textbook. The GeoGebra book comprised a total of 75 activities. Each activity included instructions for students, allowing for independent work. An example of activity is presented in Figure 6.

Figure 6

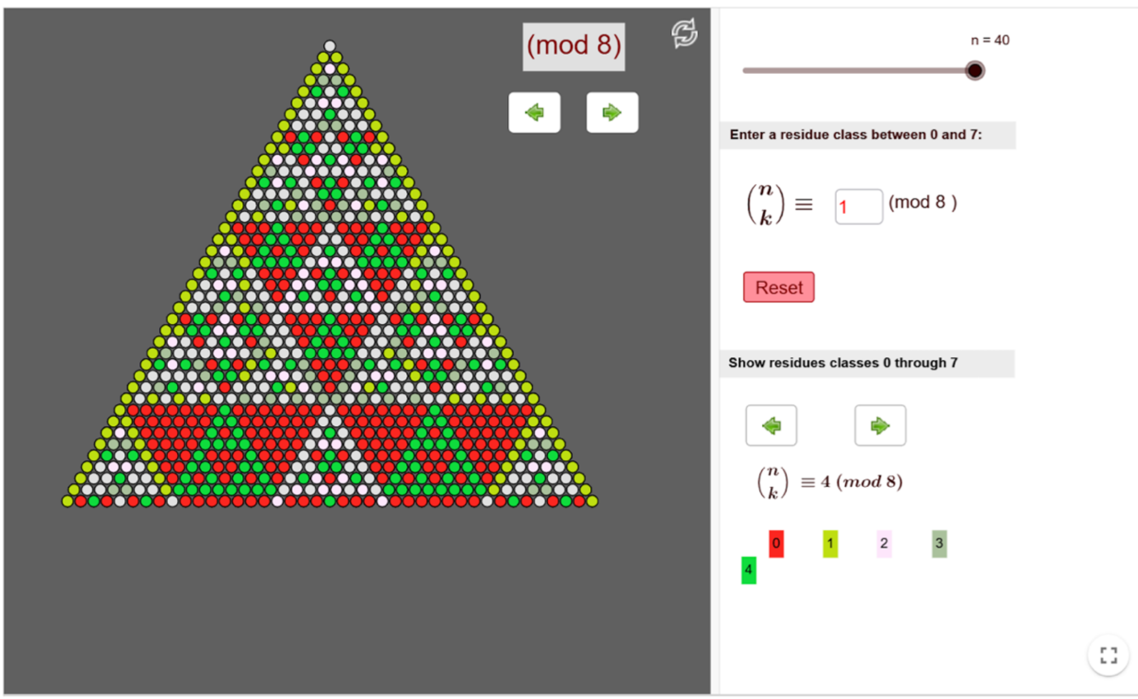
Exploration of Patterns in Pascal's Triangle Modulo m

Instructions

This applet looks at patterns in the Pascal triangle based on the divisibility of the binomial coefficients. Explore the patterns of divisibility of the binomial coefficients by any number *m*. Look for patterns in the distribution of the binomial coefficients based on the possible remainders 0, 1, 2, ..., *m*-1 (the residue class).

- Use the slider to "build" Pascal's triangle, $n \in [1, 40]$
- Change *m* by using the left and right green arrows in the sketch
- To display one residue class, enter a number between 0 and *m*-1 in the input box
- To display simultaneously several consecutive residue classes, click on the right green arrow.

Use the reset icon in the sketch to return back to initial sketch. Use the red Reset button to reset residue classes.



Note: Reprinted from "*Discrete Mathematics for Teachers: Technology Investigations*" by I. Lyublinskaya, n.d., GeoGebra, http://bit.ly/KazakhstanDiscreteMath

3.2.2 PD Content and Structure

The year-long, cross-cultural PD program included two parts. The first part was held in the Fall of 2020 and involved three three-hour-long sessions held on consecutive Saturdays, the agendas of each session are presented in Table 2. During the first session, participants compared the methodological traditions in teaching mathematics in different countries and discussed their benefits and challenges. They were then introduced to the course philosophy and features presented in the custom textbook and went on to review content-based curriculum materials. The second session focused on the role of technology in the course with emphasis on research-based foundations for effective use of technology. Participants were introduced to GeoGebra and reviewed supplementary GeoGebra book activities. The last session included discussion of STEM applications and technology investigations in discrete mathematics, and there was a brief workshop on using GeoGebra for development of online materials for students. Faculty teams also had time to plan for technology integration. After each session participants were expected to spend a half-day in completing independent tasks related to the focus of each session; during the sessions there was a dedicated time for the participants to share these tasks.

Table 2

Overview of Professional Development Program

PD Sessions	Session Content	Independent tasks
Day 1, November 15 Philosophy and Overview of the course "Discrete Mathematics for Teachers"	• Discrete mathematics course philosophy • Review of content-based curricular materials • Methodological traditions in teaching mathematics in the U.S. and other countries • Illustrations of the teaching approach	• Develop additional non-mathematics examples for each course chapter • Develop weekly plans for the discrete mathematics lectures and seminars

PD Sessions	Session Content	Independent tasks

Day 2, November 22
Technology Investigations for the Course
	• Role of technology in the discrete mathematics course
	• Review of supplementary GeoGebra materials
	• Introduction to GeoGebra workshop
	• Foundations for effective use of technology (research base)
		• Develop discrete mathematics test questions
		• Complete the GeoGebra basic skills tutorial
		• Select a discrete mathematics activity from the GeoGebra supplement book
		• Analyze and prepare presentation:
		  • Demonstrate activity features
		  • Explain how the activity will be used in teaching

Day 3, November 29 STEM Applications of Discrete Mathematics Projects	• Exploration of STEM applications and technology integration • Faculty presentations of GeoGebra activities • GeoGebra workshop on advanced features	• Classify discrete mathematics test items using the provided classification guidelines • Develop a scoring guide for the discrete mathematics test

Overall pedagogical focus of the PD was on reformed teaching, which emphasizes active student participation and engagement (Sawada et al., 2002). In the context of mathematics teaching, this approach focuses on fostering deeper understanding by meaningfully connecting abstract mathematical concepts to students' prior knowledge. Reformed teaching prioritizes inquiry-based learning experiences where students discover mathematical concepts through guided exploration rather than direct lectures alone, reflecting contemporary understanding of effective mathematics pedagogy that connects theoretical concepts with practical applications.

The PD also emphasized integrating mathematics-specific technology such as GeoGebra, enabling students to actively explore mathematical concepts, apply their mathematical knowledge in authentic scenarios, and engage in problem-solving processes. This integration of technology aimed to support the core reformed teaching principles by facilitating student inquiry and discovery within structured learning environments.

3.2.3 Classroom Implementation

After the PD sessions were completed in 2020, faculty were expected to apply what they learned in the PD in their discrete mathematics courses during the Spring 2021 semester. Faculty followed the curriculum of their respective universities. Both universities used *Discrete Mathematics and its Applications* (7th Ed.) by Rosen, K.H. (2011) as the main textbook and offered the option of using the custom textbook as a supplementary resource. All lessons were delivered online at both universities due to the COVID-19 pandemic.

To foster ongoing support and peer collaboration throughout the implementation, five monthly online meetings were held with faculty participants. These meetings served three primary purposes: (a) to facilitate discussions about the implementation process, (b) to enable faculty to share their experiences, and (c) to provide additional support as needed. Additionally, each faculty taught a micro-lesson in one of these meetings, demonstrating a selected lesson to their colleagues. These demonstrations were accompanied by discussions and reflections on their classroom experiences, fostering a collaborative exchange of ideas.

3.2.4 Participants

The participants of this study consisted of six mathematics faculty from two Kazakhstani universities who participated in the UniCEN project. These faculty were all instructors in undergraduate discrete mathematics courses offered to pre-service secondary mathematics teachers during the Spring 2021 semester.

University 1 (U1) is a privately funded higher educational institution located in a suburban area in the Southern region of the country and embraces a trilingual educational model. English serves as the medium of instruction for 81% of its degree programs, while Kazakh accounts for 16%, and Russian for 3% of the curriculum. U1 boasts a comprehensive range of academic offerings, encompassing 28 undergraduate programs, 21 master's programs, and seven doctoral programs. The university has 428 faculty and over 8000 students, with 36% of the faculty holding Ph.D. degrees. For more than ten years, U1 has offered a selective program for future teachers in English. Students must have and intermediate level (B1) of English proficiency to be admitted to this program.

University 2 (U2) is a state-funded urban higher educational institution located in the Northern region of the country. It offers 58 bachelor's programs, 42 master's programs, and ten doctoral programs, with over 400 faculty and 7000 students. U2 provides teacher preparation programs in Kazakh, Russian, and English. While there are no specific English language proficiency requirements for admission to the English-based program, students are expected to take English in their first two to three semesters.

Since the study participants were teaching English-based discrete mathematics courses, they were expected to be fluent in English. Thus, data collection instruments were all developed in English. Demographic information and teaching assignments for the faculty are shown below in Table 3.

Table 3

Demographics and Teaching Assignments of Faculty-Participants

Faculty ID	Affiliation	Highest Degree	Teaching Experience	Teaching Assignment	Number of Students Taught
F1	U1	Master's	7 Years	Lecture	38
F2	U1	Master's	24 Years	Practice seminar	19
F3	U1	Master's	2 Years	Practice seminar	19
F4	U2	Master's	10 Years	Lecture + practice seminar	16
F5	U2	Master's	32 Years	Lecture + practice seminar	16
F6	U2	PhD	13 Years	Lecture + practice seminar	16

The two participating universities had different structures of faculty teaching assignments. In U1, F1 taught only lectures for all students. The students were divided into two groups for practice seminars. F2 and F3 each taught one of the practice seminars. Lectures were scheduled once per week for 50 minutes, and the practice sessions were scheduled twice per week for 50 minutes each.

In U2, each faculty taught both the lectures and the practice seminars for about a third of a semester to the same group of students. The lectures were scheduled twice per week for 50 minutes, and the practice seminars were scheduled once per week for 50 minutes.

3.3 Data Collection and Research Instruments

Data collection spanned 29 weeks in the academic year of 2020-2021. The timeline (in weeks) is presented in Table 4. Due to different academic calendars, the data collection timeline varied slightly between the two universities.

Table 4

Timeline of Data Collection

		PD Workshops				Winter Break	Implementation																				
Weeks in Study		1	2	3	4		9			12			15			18			21			24			27		29
F-STEM																											
F-SII																						U1	U1			U2	U2	
Lesson Videos	U1														F2	F1 F3			F1 F2 F3	F1	F2 F3	Semester End					
	U2												F5					F4 F6	F4 F6		F5	F5	F4	F6		Semester End	

I collected both quantitative and qualitative data. All participating faculty completed the efficacy and attitudes toward STEM survey. Faculty also participated in two semi-structured interviews and submitted recordings of three lessons they taught. In the following sections, I first introduce each data collection instrument employed in this study, followed by the description of data collection methods.

3.3.1 Faculty Efficacy and Attitudes Toward STEM Survey (F-STEM)

Faculty perceptions about teaching and learning mathematics were collected using the F-STEM survey. The F-STEM survey was modified from the T-STEM survey for mathematics teachers, which was developed and validated by the Friday Institute for Educational Innovation (2012). The original survey included 63 5-point Likert-scale questions in seven subscales as follows: (a) Mathematics Teaching Efficacy and Belief (MTEB); (b) Mathematics Teaching Outcome Expectancy (MTOE); (c) Student Technology Use (STU); (d) Mathematics Instruction (MI); (e) 21st Century Learning Attitudes; (f) Teacher Leadership Attitudes; and (g) STEM Career Awareness. Each survey item was valued on a scale of 1 (strongly disagree) to 5 (strongly agree), with 0 representing no response or not applicable. The validity of the original T-STEAM survey for mathematics teachers was tested based on a sample of 102 math teachers (Friday Institute for Educational Innovation, 2012). The F-STEM survey in this study only included the first four subscales that were relevant to mathematics teaching and learning. The Cronbach's alpha values for the adopted subscales shown in Table 5 confirmed the internal consistency of the survey.

Table 5

T-STEM Survey Internal Consistency

Construct	**Number of Items**	**Cronbach's Alpha (n=102)**
Mathematics Teaching Efficacy and Belief	11	.943
Mathematics Teaching Outcome Expectancy	9	.849
Student Technology Use	8	.869
Mathematics Instruction	14	.929

Note: Information from "*Teacher Efficacy and Beliefs toward STEM Survey*" by Friday Institute for Educational Innovation, 2012.

In addition, the word "teachers" was replaced by word "faculty" in both surveys. The present tense verbs were replaced with the future tense on questions in the post survey to assess faculty anticipated actions after participating in PD sessions, e.g. "During mathematics instructional meetings, how often **will** (changed from "do") your students use technology to support higher-order thinking?" The survey was distributed via Qualtrics online platform before (week 1) and after (week 4) of the first part of the PD program.

The pre and post scores of F-STEM surveys were compared and used to identify changes or stability in faculty perceptions before and after participating in the PD sessions. The patterns of change in perceptions helped assess faculty acculturation strategies.

3.3.2 Reformed Teaching Observation Protocol (RTOP)

The Reformed Teaching Observation Protocol (Sawada et al., 2002) was used to analyze faculty teaching practices in the collected videos and lesson plans. This protocol was developed by the Arizona Collaborative for Excellence in the Preparation of Teachers Program to assess the extent to which K-20 mathematics and science classrooms have shifted from traditional didactic practice and lecture to student-centered inquiry-oriented learning (Sawada et al., 2002). The protocol was used for both qualitative and quantitative data collection. The protocol measures 25 items on a 5-point Likert-scale and provides sections for field notes. The 25 items were divided into five subscales within three subsets: Lesson Design and Implementation (five items), Content (ten items): Propositional (five items) and Procedural (five items) Pedagogic Knowledge and Classroom Culture (ten items): Communicative Interactions (five items), and Student/Teacher Relationships (five items).

The inter-rater reliability of the RTOP was confirmed using linear regression. The R2 values ranged from 0.670 to 0.954 for the scale as a whole and for the five subscales. The study also confirmed face validity, construct validity, and predictive validity using standard methods (Sawad et al., 2002). The construct validity findings indicate that RTOP scores accurately reflect the student-centered and inquiry-oriented features of a lesson, making it a suitable tool for this study (Sawada & Piburn, 2000).

Based on the degree to which the items were characteristic of the lesson, on a scale of 0 (not observed) to 4 (very descriptive) the total score of all items ranged from 0 (traditional, lecture-based) to 100 (reformed, student-driven) points. According to Treagust et al. (1996), the range of 0–25 represents teacher-centered teaching approaches, 26–50 represents transitional teaching approaches, 51–75 represents student-centered teaching approaches, and 76–100 represents highly reformed teaching approaches. The lessons with higher scores reflect a greater degree of reformed teaching, which means that the faculty engaged more with the trending mathematics teaching approaches that are valued by the educational traditions in the United States. The RTOP scores serve as quantitative indicators of faculty engagement with reformed teaching, providing insights into one aspect of their engagement of the new educational tradition to determine their acculturation strategies.

The field notes were recorded for each submitted lesson alongside that lesson's scores and following the RTOP guidelines. The field notes documented the key events occurring during the submitted lesson videos, especially if faculty demonstrated evidence of applying what they learned in PD and from resources provided during the PD in the classroom. The teaching events were organized in tables with timestamps from the videos. The field notes included classroom settings, key teaching events, and faculty-student interactions. I also recorded faculty or students' statements verbatim as evidence. An example of the field notes is presented in Appendix B. These notes provided additional qualitative data about the teaching practices observed.

To ensure accurate application of RTOP, I trained myself using the official *RTOP Training Guide* (Sawada & Piburn, 2000) and the online training program to ensure that I used the protocol appropriately.

The alignment of RTOP with the objectives of this research allows for robust evidence collection to assess the type of teaching approaches enacted by faculty and to contribute to the process of identifying their educational acculturation strategies. Lower RTOP scores (below 26) indicate traditional teacher-centered teaching approaches that largely maintain heritage teaching traditions, aligning with separation acculturation strategies. Mid-range scores (25-50) demonstrate transitional teaching that integrates elements from both traditional and reformed approaches, reflecting integration strategies. Higher scores (above 50) indicate predominantly student-centered, inquiry-based teaching approaches, suggesting assimilation toward the new educational practices (Berry, 2003; Sawada et al., 2002).

However, faculty educational acculturation strategies must be evaluated holistically. The level of engagement with new teaching practices does not necessarily fully represent faculty acculturation; we should also consider their attempts, successes, and failed engagements with technology and STEM applications as well as expressed willingness to implement different aspects of new educational traditions. Since the PD sessions focused on multiple dimensions – reformed teaching approaches, mathematics-specific technology integration, and STEM applications – acculturation should be assessed across all these domains rather than through

RTOP scores alone. Table 6 provides categories for identifying acculturation strategies based on multiple criteria.

Table 6

Acculturation Strategy Identification

Criteria	Separation	Integration	Assimilation
	No engagement & Full maintenance → Full engagement & No maintenance		
Teaching Practices	RTOP ≤ 25	25 < RTOP ≤ 50	RTOP > 50
Perceptions of Teaching Approaches	Teacher-centered	Transitional	Student-centered Inquiry-based
Implementation of Mathematics-Specific Technology	Technology is not used for mathematics teaching or learning	Technology use for demonstrations (faculty) and drills (students)	Student use of technology for investigations and explorations
Implementation of STEM Applications	No connections	Superficial connections	Meaningful connections

Note: "Engagement" refers to engagement with new educational traditions, while "maintenance" refers to the maintenance of heritage educational traditions.

3.3.3 Faculty Semi-Structured Individual Interviews (F-SII)

Individual interviews with faculty were used to explore the influence of PD on faculty perceptions and teaching practices of mathematics. The F-SII protocol was modified from Contextualize to Learn (C2L) faculty interview protocols by the Wisconsin Center for Education Research (n.d.). The F-SII protocols included seven open-ended questions concerning faculty attitudes and perceptions of PD experiences, materials, and their course implementation.

The aim of the C2L interview protocol is to gather insights into the manner in which faculty interpreted their PD experiences, focusing on implementation contextualization in instructing college students with mathematics (Wisconsin Center for Education Research, n.d.). In order to gather insights into the manner in which faculty interpreted the influence of the cross-cultural PD on their teaching of the discrete mathematics course, C2L faculty interview protocols were modified (Table 7). Considering the fact that English was not the native language for all participating faculty, language in the questions was simplified, with long questions divided into multiple short questions. Each faculty was interviewed for 30 to 40 minutes via Zoom before (week 4) and after teaching the course (weeks 24-28).

Table 7

Changes to First Interview Protocol Questions

C2L Questions	F-SII Questions
1. To begin, could you tell me about your role at the college? What was your professional experience before teaching at [institution name]? Can you tell me a bit about your experiences teaching math?	1. For the record, please state your name and name of your university. What was your professional experience before teaching at [University name]? Can you tell me a bit about your experiences teaching mathematics?
2. After having participated in the professional development activities (give college-specific examples) on math contextualization, how do you make sense of that experience? What would you say are the "good" and "bad" about it? Do you think there is anything that can be done differently to make it better?	2. After having participated in the professional development workshop, how do you make sense of that experience?
	3. Specifically, a. In your opinion, what was the most valuable experience? b. In your opinion, what was the least valuable experience?
6. Do you see yourself using any of the contextualization techniques or approaches you learned as a result of the workshop in your future practice teaching math?	4. How do you see using the PD-provided materials, techniques, or approaches in your future practice teaching discrete mathematics?
3. Thinking about contextualization of math, what did that mean to you prior to the workshop? What does it mean to you now? Have there been differences in how you define and think of math contextualization as a result of the workshop?	5. Thinking about technology investigations and STEM applications for the discrete mathematics course, what did that mean to you prior to the workshop? What does it mean to you now? Do you think differently about teaching the discrete mathematics course as a result of the workshop?

C2L Questions	F-SII Questions
4. Could you describe your experiences, if any, working with other faculty during or as a result of the professional development activities? 5. What are your expectations for a community of practice around teaching math to allow you to interact with other faculty that teach math or in the technical areas?	Excluded
7. Could you describe any areas where you may need assistance or resources, in the future, in developing knowledge and skills, or making changes to your own practices around teaching math?	6. Could you describe any areas where you may need assistance or additional resources in developing knowledge and skills, or making changes to your own practices of teaching discrete mathematics?
8. Is there anything else you would like to share with me related to what we talked about, or any other aspects of your experiences with professional development in teaching math?	7. Is there anything else you would like to share with me related to what we talked about or to any other aspects of your experiences with the professional development workshop?

Following the first round of interviews, written versions of the same questions were sent to two faculty who encountered difficulties with verbal English during their interview session. Their written responses provided additional data for analysis. In order to improve comprehension during verbal communication with the participants, questions for the second round of interviews were further shortened, rephrased, and translated using Google Translate. These translations were provided to the participants during the interview via the Zoom chat.

The F-SII were used to provide details on faculty perceptions, their intended teaching practices, and their reported enacted teaching practices as well as to support identification of effective PD features. Faculty perceptions and their intended and reported enacted teaching practices were used to assist the identification of their educational acculturation strategies.

3.4 Data Analysis

This section describes the data analysis methods. The first section describes quantitative data analysis methods used with F-STEM surveys and RTOP scores; the second section describes qualitative data analysis methods used with F-SII and RTOP field notes. Finally, the last section explains how results of the data analysis were used to address research questions.

3.4.1 Quantitative Analysis

F-STEM surveys were analyzed following the guide provided by the Friday Institute for Educational Innovation (2012). Reverse scoring was applied to negative questions. Google Sheets were used to calculate descriptive statistics for each subscale for individual faculty and for each university as an aggregate. The differences in the scores on pre and post surveys were also calculated. The results were then visually represented.

Videos of taught lessons were scored using the RTOP. Each recorded lesson received a total score ranging from 0 to 100 points and was categorized as one of the teaching approaches on the total score.

3.4.2 Qualitative Analysis

The semi-structured individual interviews were recorded and transcribed using Zoom's automatic transcription feature. To ensure data integrity, accuracy, and completeness, I reviewed the Zoom-generated transcripts, correcting any discrepancies and verified every statement against the corresponding interview recordings. The verified transcripts were then organized in an Excel spreadsheet and labeled with the faculty ID and timestamp for easy reference and analysis (Appendix A).

A codebook was created, including initial codes and categories developed based on the study’s theoretical framework (Table 8). The five PD features identified as influential by Desimone (2009) were included in the codebook as codes. The code definitions also followed Desimone's definition of each feature (Desimone, 2009; Kang et al., 2013). Faculty intention and implementation of content goals, mathematical beliefs, and pedagogical orientations can be revealed through their responses to four fundamental questions: What are students expected to learn? Who delivers the instruction? How is instruction organized? What have students learned? (Schmidt et al., 1996). Based on Schmidt et al.'s (1996) framework, the codebook also included codes that defined teaching methodologies, curriculum design, assessment practices, organizational structures, and cultural norms observed in the videos.

An expert holding a PhD in mathematics education with experience in qualitative research was invited to participate in assessing the validity and reliability of the codebook. The validity of the codebook was checked through the following steps:1) I met with the expert to review the codebook, explained the study background and purposes, and provided explanations for each code and its definition; 2) the expert reviewed the codebook and asked questions for clarification; 3) the expert provided suggestions on the language of the code definitions and examples, and the codebook was revised to improve the mutual understanding of the codes; 4) the expert and I coded the same interview transcript independently using the codebook and we met to discuss and make changes to the codebook as necessary; and 5) we repeated steps 3 and 4 with different transcripts until we reached consensus on the codebook. Intercoder reliability was calculated by dividing the number of coding agreements by the total number of coding decisions (agreements plus disagreements). The process was repeated with another excerpt until the intercoder reliability calculated via percent agreement reached 82% under the PD features category and 89% under the educational tradition category, which is considered acceptable (Tracy, 2020).

Table 8

Codebook of This Study

Codes	Definitions	Example
PD Features		
Content Focus	Faculty perceptions of the PD materials (e.g. custom textbook, GeoGebra tutorials and book, and presentations)	I think the most valuable thing [about the PD] was about the GeoGebra.
Learning Opportunity	Faculty perceptions about their learning opportunities during PD	Maybe if we can do more practice, I think it will be better.
Coherence	Faculty perceptions about the relevance of PD to their teaching	I think it is also my field. So, I'm very happy to be in this workshop.
Duration	Faculty perceptions about the length of the PD program	It is not enough time.
Collective Participation	Faculty perceptions about their experience of PD with work peers	I asked some things that we had … in our workshop from them [peers].
Educational Traditions		
Teaching Methodology	Teaching strategies and techniques to deliver mathematics content and facilitate students' learning	I used GeoGebra as a demonstration, for demonstration examples of theories.
Teaching Objectives	The knowledge and skills faculty expect students to learn from the lesson	They have to memorize that formula, and they have to calculate it by hand.
Assessment Practice	The methods faculty use to assess students' learning	I had to check their solutions by hand, but if we had some technologies which help to do that for us, it would be very delightful.
Organizational Structure	Classroom management. How lessons are structured, including how student work is organized	I never think that each student can also use their own computer in the same lecture.
Cultural Norms	Unwritten rules that guide faculty-student relationships and behaviors in the classroom	Generally, in lectures, we will just … teach by presentation.

The finalized transcripts were analyzed using the thematic analysis method. First, I read through each transcript thoroughly and assigned codes to statements that aligned with predetermined code definitions. After all interview transcripts were coded, statements with the same code were compiled in a single table for the first and second interviews, and analytical notes were created for each statement to identify key insights. Using these notes, I developed initial categories by identifying common patterns and relationships among related codes. These categories were subsequently reviewed, reorganized, and merged until they adequately captured the concepts within the dataset, forming initial themes. The initial themes from both interviews had many similarities, so I further reorganized, merged, and refined them until they represented the complete dataset. Finally, these refined themes of both interviews were reviewed to ensure that they clearly presented the meanings emerging from the data and that each theme maintained a specific focus on a distinct aspect of the data. These themes were then named in one or two sentences by identifying the essential concepts that characterized each theme.

The analysis of RTOP field notes focused on capturing patterns of lesson design, teaching practices, and faculty-student interactions for each submitted lesson. While rewatching each lesson video, I took field notes and added memos alongside the corresponding events (Appendix B). I then revisited the field notes and memos to form a comprehensive summary for each lesson, identifying the key teaching events, and identifying the patterns in teaching practices and student-faculty interactions. The resulting summaries identified patterns across all three videos for each faculty.

3.4.3 Addressing Research Questions

In order to answer RQ1: "How were the Kazakhstani mathematics faculty perceptions of teaching and learning mathematics influenced by their experiences in the cross-cultural PD?" I compared the results of pre and post F-STEM surveys to identify changes or lack of changes in faculty perceptions. Then, I used the themes that emerged from the analysis of the faculty interviews to provide explanations of the changes in faculty perceptions after the PD program.

In order to answer RQ2: "How were the Kazakhstani mathematics faculty intended and enacted teaching practices influenced by their experiences in the cross-cultural PD?" I identified the faculty intended and enacted teaching practices from the themes that emerged from the analysis of the interviews and from the analysis of the lesson videos. I then compared and contrasted the intended and enacted practices for each faculty.

In order to answer RQ3: "Which educational acculturation strategies did the Kazakhstani mathematics faculty demonstrate in their teaching practices after participating in the cross-cultural PD?" I triangulated multiple data sources to identify faculty educational acculturation strategies following the definition established in 3.3.2 Reformed Teaching Operation Protocol section of this chapter (Table 6). This analysis integrated RTOP scores to identify levels of exhibited reformed teaching approaches, field notes to identify evidence of technology use and STEM applications in teaching practices, and themes that emerged from interviews to identify faculty perceptions and intended practices.

In order to answer RQ4 (a): "Which PD features did the Kazakhstani mathematics faculty identify as reasons for changes or lack of changes in their perceptions of teaching and learning mathematics?" I determined the PD features that faculty identified as influential on their perceptions from the themes that emerged from the analysis of the faculty interviews. Then, the faculty perception changes (answers to RQ1) were compared with those identified PD features to indicate which PD features might be associated with the changes in faculty perceptions. To answer RQ4. (b): "Which PD features did the Kazakhstani mathematics faculty identify as reasons for implementing or rejecting new educational traditions?" I determined the PD features that faculty identified as influential for their teaching practices of the discrete mathematics course from the themes that emerged from the faculty interviews. I then triangulated faculty intended and enacted teaching practices (answer to RQ 2) with the identified PD features to indicate what PD features might be associated with faculty implementing or rejecting implementation of the PD materials and new teaching approaches.

In order to answer RQ5, "Did institutional contexts influence the Kazakhstani mathematics faculty perceptions and educational acculturation strategies?" I reorganized the results from the previous four research questions, including faculty perceptions, teaching practices, acculturation strategies, identified influential PD features by universities, and compared the data from the two universities to identify similarities and differences.

Chapter 4: Results

This chapter presents the findings from the analysis of faculty responses to the pre and post F-STEM surveys, faculty individual interviews, and lesson videos of the discrete mathematics course. The results are organized in three sections that correspond to the data collection methods.

The "Analysis of Faculty STEM (F-STEM) Surveys" section presents the scores on subscales of F-STEM surveys, comparing pre and post results to identify changes or lack of changes in each subscale by the whole group, by universities, and by individuals. The "Analysis of Lesson Videos" section presents the RTOP scores and analysis of field notes for each faculty lesson video to evaluate faculty enacted teaching practices. The "Analysis of Faculty Individual Semi-Structured Interviews (F-ISS)" section presents themes which emerged from F-SII interviews. The results of the analysis reported in this chapter will be combined in chapter 5 to answer research questions.

4.1 Analysis of Faculty STEM (F-STEM) Surveys

The F- STEM survey has four subscales measuring faculty perceptions: Mathematics Teaching Efficacy and Beliefs (MTEB), Mathematics Teaching Outcome Expectancy (MTOE), Student Technology Use (STU), and Mathematics Instruction (MI). The data were analyzed on each subscale separately.

4.1.1 Cumulative Descriptive Statistics for Pre and Post Survey Results

The cumulative descriptive statistics for the pre and post survey scores in each subscale are shown in Table 9 and Figure 7.

Table 9

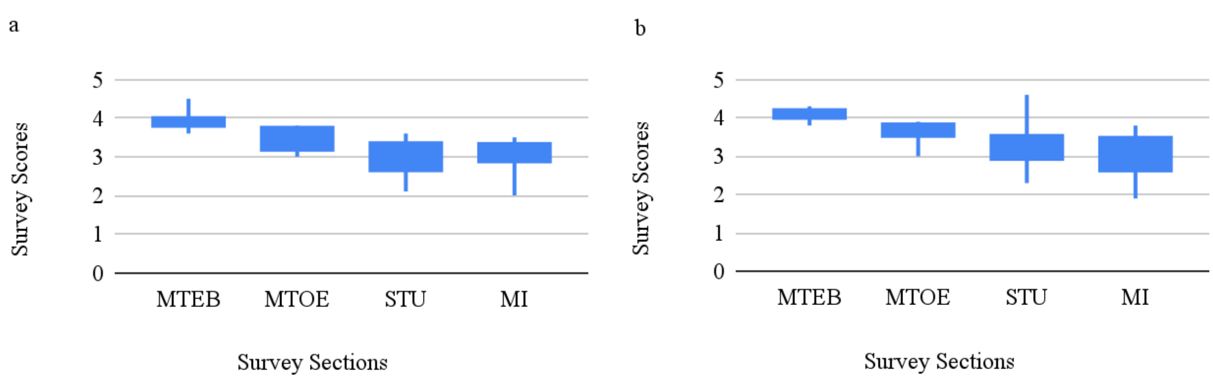
Descriptive Statistics of F-STEM Surveys

Pre	Mean	SD	Min	Q1	Median	Q3	Max
MTEB	3.9	0.31	3.6	3.8	3.9	4	4.5
MTOE	3.5	0.34	3.0	3.18	3.5	3.75	3.8
STU	2.9	0.56	2.1	2.65	2.9	3.35	3.6
MI	3.0	0.54	2.0	2.88	3.1	3.33	3.5
Post	**Mean**	**SD**	**Min**	**Q1**	**Median**	**Q3**	**Max**
MTEB	4.1	0.18	3.8	4	4.1	4.2	4.3
MTOE	3.6	0.33	3.0	3.53	3.6	3.83	3.9
STU	3.3	0.78	2.3	2.93	3.2	3.53	4.6
MI	3.1	0.73	1.9	2.63	3.4	3.48	3.8

Figure 7 presents the five-number summary of cumulative scores on the pre survey and post survey.

Figure 7

Pre and Post F-STEM Survey Scores for All Faculty



Note: Figure 7a presents pre survey data; figure 7b presents post survey data.

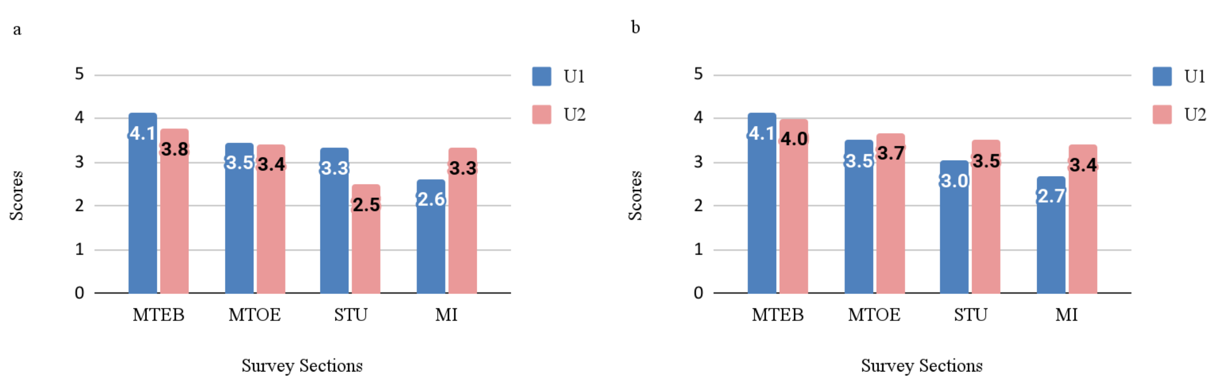
Based on Table 9 and Figure 7 we can see that the four survey subscales (MTEB, MTOE, STU, and MI) increased in their mean and median scores from the pre to post survey, and this suggests overall faculty perceptions had positive shifts. Based on Table 9, the changes of scores on the MTEB (0.2 points, 5.1%) and MTOE (0.1 points, 2.9%) demonstrated an improvement in mean scores, with a notable decrease in variability and a narrowing of the score range. Faculty perception on mathematics teaching efficacy and confidence increased, and the differences in the scores of individual faculty were smaller. The STU scores showed the largest increase in the mean score (0.4 points, 13.8%), though there was an increase in variability (0.56 to 0.78) making responses to this subscale greatly diverse among individuals. The MI mean scores exhibited a more modest increase (0.1 points, 3.3%) but a larger variability (0.54 to 0.73) demonstrating a variety of responses on the individual level.

4.1.2 Comparison of Pre and Post Survey Results by Universities

In order to examine whether there were differences between universities, the scores of the survey were grouped by university (Figure 8).

Figure 8

F-STEM Survey Score Comparison Between U1 and U2



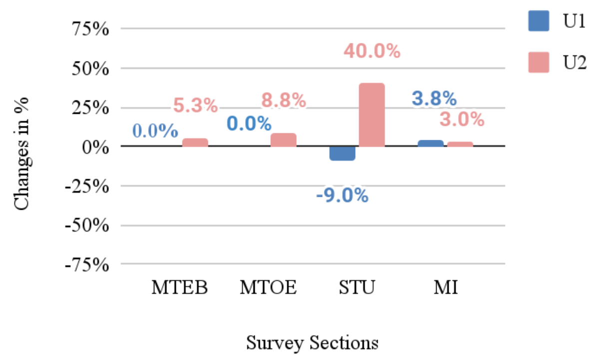
Note: Figure 8a presents pre survey data; figure 8b presents post survey data.

The pre-PD comparison between the two universities reveals differences in the baseline. Faculty from U1 scored higher on three of the four F-STEM subscales (MTEB, MTOE, and

STU) when compared to U2, and MI was the only exception (Figure 8a). The post survey scores demonstrated that after PD sessions, faculty from U2 exceeded U1 on MTOE and STU, closed the gap in MTEB, and remained constant in its difference in MI (Figure 8b). The percentage changes in pre and post surveys of each subscale by university are shown in Figure 9.

Figure 9

Percent Changes in Pre and Post F-STEM Survey Score Comparison Between U1 and U2



Note: Mathematics Teaching Efficacy and Beliefs (MTEB), Mathematics Teaching Outcome Expectancy (MTOE), Student Technology Use (STU), and Mathematics Instruction (MI).

For U1, the MTEB and MTOE scores remained unchanged, while STU scores decreased by 9%. However, U1 had a slight rise to 3.8% in MI scores. In contrast, U2 exhibited a consistent rise across all subscales: MTEB and MTOE scores increased by 5.3% and 8.8%, respectively, while STU scores increased by 40% and MI increased by 3.0%. Overall, U2 faculty showed larger growth across all subscales compared to U1 faculty (Figure 9), which may indicate that the PD had more positive influence on U2 faculty perceptions than on U1 faculty.

These differences could suggest that institutional factors might be influencing the effect of the PD on faculty perceptions.

4.1.3 Results by Individual Faculty

Because the previous analysis in the whole group showed variance among individuals, the changes between pre and post survey were analyzed separately for each faculty (Table 10). The highest possible score is 5 and the lowest is 0.

Table 10

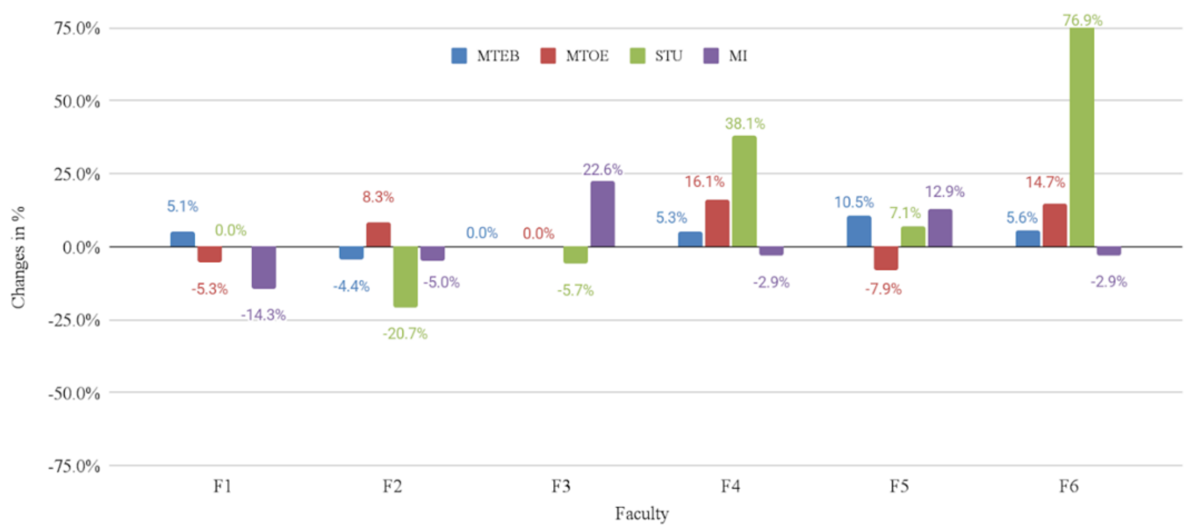
Results of Pre and Post F-STEM Surveys

		MTEB	MTOE	STU	MI
			U1		
F1	Pre	3.9	3.8	3.6	2.8
	Post	4.1	3.6	3.6	2.4
F2	Pre	4.5	3.6	2.9	2.0
	Post	4.3	3.9	2.3	1.9
F3	Pre	4.0	3.0	3.5	3.1
	Post	4.0	3.0	3.3	3.8
			U2		
F4	Pre	3.8	3.1	2.1	3.4
	Post	4.0	3.6	2.9	3.3
F5	Pre	3.8	3.8	2.8	3.1
	Post	4.2	3.5	3.0	3.5
F6	Pre	3.6	3.4	2.6	3.5
	Post	3.8	3.9	4.6	3.4

Percent changes in survey scores across the four subscales for the six faculty are presented in Figure 10.

Figure 10

Percent Changes in Pre and Post F-STEM Survey Score for All Faculty

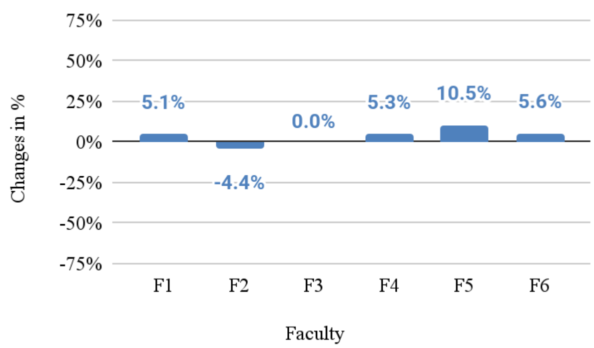


Note: Mathematics Teaching Efficacy and Beliefs (MTEB), Mathematics Teaching Outcome Expectancy (MTOE), Student Technology Use (STU), and Mathematics Instruction (MI).

All faculty demonstrated increased scores in at least one subscale after participating in the PD sessions. However, no consistent pattern was observed across all six participants, as changes in their scores varied. Some trends were identifiable within each university. Faculty from U1 (F1, F2, and F3) displayed mixed results, with each faculty showing increased scores in only one of the four subscales between pre and post surveys. In contrast, each faculty from U2 (F4, F5, and F6) showed increased scores in three of the four subscales. All three U2 faculty demonstrated growth in MTEB and STU. Detailed analysis of score changes for each faculty across all subscales is presented in Figures 11 - 14.

Figure 11

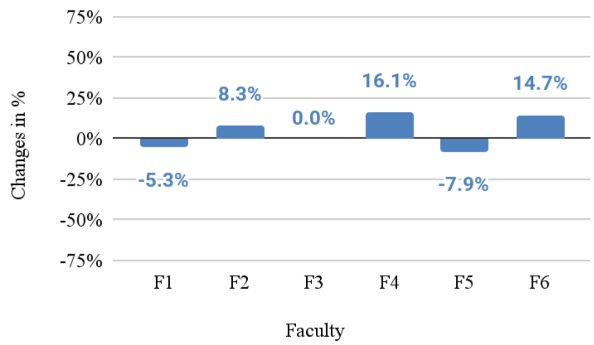
Percent Changes in Faculty Pre and Post Scores on Mathematics Teaching Efficacy and Beliefs



The MTEB subscale assessed faculty perceptions of their self-efficacy and confidence in teaching mathematics. F1's score increased by 5.1%, the score of F3 did not change, F4's score increased by 5.3%, F5 score showed the most noticeable growth at 10.5%, and F6's score improved by 5.6%, F2 was the only faculty whose score decreased (-4.4%) (Figure 11). The increases may suggest that these faculty developed greater confidence in their capabilities to effectively teach mathematical concepts, potentially indicating their comfort with their teaching approaches and content knowledge at the moment of taking the survey. The decrease of F2's score on MTEB may suggest that he developed more critical opinions of his mathematics teaching approaches or encountered some challenges.

Figure 12

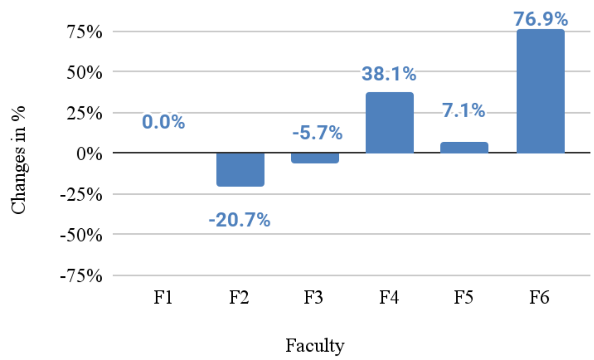
Percent Changes in Faculty Pre and Post Scores on Mathematics Teaching Outcome Expectancy



The MTOE subscale assessed faculty perception of the effectiveness of their teaching practices on the students' learning outcomes in mathematics. Three faculty demonstrated growth in their scores: F2 showed an 8.3% increase, F4 showed a 16.1% increase, and F6 showed a 14.7% increase (Figure 12). These increases may suggest these faculty developed more confidence that their teaching approaches could aid student mathematics achievement regardless of external challenges. Conversely, F1 and F5 exhibited decreases in their MTOE scores by 5.3% and 7.9% respectively. These negative changes may suggest these faculty have developed more reservations about the effect of their teaching practices that influence student learning. F3's unchanged score suggests stable beliefs about teaching outcome expectancy.

Figure 13

Percent Changes in Faculty Pre and Post Scores on Student Technology Use

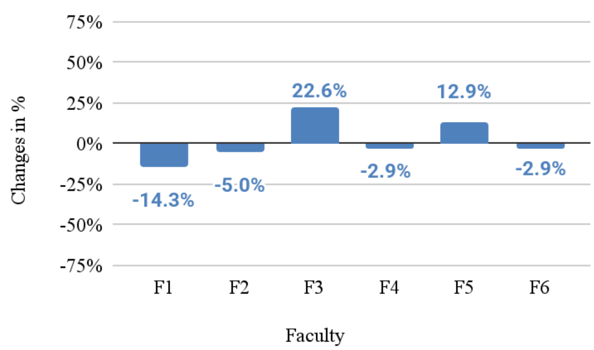


The STU subscale assessed faculty perceptions of how frequently they expect students to use technology in their lessons to learn mathematics concepts. The pre survey measured how often faculty expected students to use technology in their current mathematics lessons at the time of the survey, while the post survey focused on faculty anticipation of student technology use frequency in future lessons.

As presented in Figure 13, F4's score showed an increase of 38.1%, F5 increased by 7.1%, and F6 exhibited the most noticeable change with an increase of 76.9%. These positive changes may suggest these faculty developed stronger expectations and intentions to integrate technology into their lessons for students use. The F2 and F3 exhibited decreases in scores of 20.7% and 5.7% respectively, suggesting they may have developed lower expectations about students' technology use as part of their teaching approaches. F1's unchanged scores suggest F1 maintained consistent expectations. The negative changes and unchanged may suggest faculty anticipated challenges of implementation.

Figure 14

Percent Changes in Faculty Pre and Post Scores on Mathematics Instruction



The MI subscale measured faculty perceptions of how frequently they employed student-centered and inquiry-based mathematics teaching approaches in their practices. The pre survey assessed faculty perceptions of how frequently they employed student-centered and inquiry-based teaching practices in their mathematics lessons at the time of survey completion. The post survey examined faculty anticipation for their future implementation of these student-centered and inquiry-based teaching approaches.

As presented in Figure 14, only F3 and F5 showed increases in scores on this subscale, of 22.6% and 12.9% respectively. The changes may suggest these faculty have developed stronger intentions to implement student-centered and inquiry-based teaching approaches in their mathematics lessons. F1 exhibited the greatest decrease in score by 14.3%, F2 decreased by 5.0%, and both F4's and F6's scores decreased by 2.9%. The changes may suggest these faculty anticipated less implementation of student-centered and inquiry-based teaching approaches, which may also be related to anticipated challenges with implementation.

4.2 Analysis of Lesson Videos

In this section, the results of analysis of lesson videos, including the RTOP score and analysis of the field notes, are presented. Table 11 shows overall RTOP scores for the lesson videos submitted by each faculty.

Table 11

Overall RTOP Scorers and Teaching Classification of Faculty-Submitted Lesson Videos

Faculty	Lesson 1	Lesson 2	Lesson 3
F1 (lecture only)	22 (Teacher-centered)	22 (Teacher-centered)	21 (Teacher-centered)
F2 (practice only)	37 (Transitional)	37 (Transitional)	21 (Teacher-centered)
F3 (practice only)	18 (Teacher-centered)	18 (Teacher-centered)	25 (Teacher-centered)
F4	18 (lecture) (Teacher-centered)	25 (practice) (Teacher-centered)	17 (lecture) (Teacher-centered)
F5	16 (lecture) (Teacher-centered)	15 (lecture) (Teacher-centered)	20 (practice) (Teacher-centered)
F6 (lecture only)	19 (Teacher-centered)	16 (Teacher-centered)	19 (Teacher-centered)

The range of RTOP scores (15 to 25 points) for most of the submitted lessons videos indicated use of teacher-centered approaches. F2 had two practice seminars that used transitional teaching approaches, both scoring 37, while F3 and F4 each had one practice seminar that scored 25 points, approaching the transitional teaching approach threshold.

4.2.1 Analysis of Lessons of F1

F1 taught only lectures, with each lesson scoring 22, 22, and 21 on the RTOP scales. All lectures followed the same sequence. F1 started with a proof-oriented presentation of the theoretical material. The presentation was followed by several worked examples of problems that demonstrated the application of the introduced theorems and proofs. Students were then assigned similar problems to complete independently. In the three submitted lessons, F1 made minimal connections between discrete mathematics topics and students' real-life experiences or other disciplines. Such a connection was observed only once, during the second lesson. In that instance, and after students presented solutions for converting a sequence from explicit to recursive form, F1 remarked, "Explicit [form] is easy to find a value, but recursive [form] is better for use in programming."

F1's recorded lessons indicated that he mostly used Microsoft OneNote to present his lectures and to work through examples on an interactive whiteboard similarly to the way he would use a standard classroom whiteboard. RTOP subscale scores for F1's lessons are shown in Table 12.

Table 12

Subscale Scores of F1's Lesson Videos

Subsets	Subscales	Lesson 1	Lesson 2	Lesson 3
Lesson Design and Implementation		2	2	1
Content	Propositional Knowledge	14	16	14
	Procedural Knowledge	3	2	3
Classroom Culture	Communicative Interactions	1	1	1
	Student/Teacher Relationships	2	2	2

As seen in Table 12, out of the subscales, F1 only scored high on his Propositional Knowledge, which represents his deep understanding of the discipline. However, the Lesson Design and Implementation, and the Classroom Culture scores were low, and the total scores reflected a traditional teacher-centered approach. In addition, few attempts at implementing PD materials were observed in these submitted lessons.

4.2.2 Analysis of Lessons of F2

F2 taught practice seminars, in which the RTOP scores for the three lessons were 37, 37, and 21. Each seminar began with F2 recalling the key content from the lecture, followed by practice problems for students to solve. Lessons 1 and 2 featured student participation, with opportunities for students to work on problems independently and share their solutions. In contrast, lesson 3 was more teacher-centered, with F2 demonstrating and modeling problem-solving strategies.

In all three lessons, but especially in lesson 2, students used a shared screen to annotate solutions of practice problems, which occasionally led to student discussions about alternative solutions. For example, in lesson 2, when one student presented her solution to a sequence problem, F2 prompted the class to comment, resulting in disagreements and alternative approaches being shared. Such discussions were teacher-facilitated through prompts like "Do you agree?" and "Does anyone disagree?"

In lesson 3, F2 used GeoGebra to present practice problems, with students answering by annotating on the screen rather than interacting with the technology directly. F2 also integrated STEM application problems into lessons 2 and 3 and offered bonus points for participation. For example, students were assigned a task to investigate air pollution in Almaty city and to use recursively defined sequences to estimate future pollution levels. RTOP subscale scores for F2's lessons are shown in Table 13.

Table 13

Subscale Scores of F2's Lesson Videos

Subsets	Subscales	Lesson 1	Lesson 2	Lesson 3
Lesson Design and Implementation		3	4	2
Content	Propositional Knowledge	16	16	14
	Procedural Knowledge	4	4	1
Classroom Culture	Communicative Interactions	7	4	2
	Student/Teacher Relationships	7	9	2

According to RTOP criteria, F2 consistently demonstrated strong content knowledge as he scored high on the Propositional Knowledge subscale in all three lessons. Compared to lesson 3, lessons 1 and 2 scored higher in Classroom Culture due to a larger proportion of time used for student participation. Overall, lessons 1 and 2 fell within the range of transitional teaching, whereas lesson 3 remained in the traditional teacher-centered teaching range. In addition, within these submitted lessons, F2 implemented some of the PD materials and practices.

4.2.3 Analysis of Lessons of F3

F3 taught practice seminars, with RTOP scores of 18, 18, and 25. Lessons 1 and 2 followed a consistent structure in which F3 began by reviewing the key concepts from the lecture and followed by presenting worked examples. Students were then presented with a few similar problems for practice without time to work on the problems independently. F3 raised questions to lead the problem-solving process immediately after presenting the problems. When students shared and explained their solutions following F3's guidance, F3 often intervened, completing the solutions herself or providing answers without significant student input.

While the structure in lesson 3 remained consistent with lessons 1 and 2, there was an increased proportion of student talk and participation, particularly toward the end of the lesson when students verbally presented all solutions without F3's intervention. Students primarily used Kazakh to answer questions, explain their solutions, seek clarification, and engage in discussions with F3 and their classmates. F3 responded to students' questions in both English and Kazakh.

F3 utilized Microsoft Word as an alternative whiteboard during her lessons. No implementation of GeoGebra materials, or STEM applications were observed in the three submitted lessons. RTOP subscale scores for F3's lessons are presented in Table 14.

Table 14

Subscale Scores of F3's Lesson Videos

Subsets	Subscales	Lesson 1	Lesson 2	Lesson 3
Lesson Design and Implementation		4	2	2
Content	Propositional Knowledge	13	14	14
	Procedural Knowledge	0	0	2
Classroom Culture	Communicative Interactions	1	2	3
	Student/Teacher Relationships	0	0	4

According to RTOP subscales, F3 scored high on Propositional Knowledge across all three lessons, demonstrating a strong understanding of the discipline. Lesson 3 scored higher in the Classroom Culture subset due to the increased proportion of student participation and discussions. The RTOP scores of the three submitted lessons were categorized as teacher-centered, with lesson 3 showing a shift toward transitional teaching due to its increased student participation and discussion. No PD materials were observed in the submitted lessons, though some transitional teaching practices were observed in lesson 3.

4.2.4 Analysis of Lessons of F4

F4 taught both lectures and practice seminars, choosing to submit two lectures (lessons 1 and 3) and one seminar (lesson 2). His lectures scored 18 and 17, while his practice seminar scored 25 on the RTOP scale. His lectures started with content-focused presentations followed by demonstrations of worked examples and concluded with student practice. During F4's lectures students had fewer problems to solve independently, and F4 often wrote the problem solutions himself on the shared screen, rather than asking students to share their solutions.

During the practice seminar (lesson 2) F4 demonstrated several different ways of providing feedback to the students after they worked on the practice problems independently. For simple fact-based problems he first projected a problem on the screen to the entire class and immediately asked students to verbally share their answers. After students shared the answers, F4 explained the reasoning behind students' answers. If students didn't provide an answer right away or gave incorrect answers, then F4 would demonstrate his procedures for solving the problems, explaining the reasoning behind the solutions. For other problems that required graphing, F4 first demonstrated how to solve these problems using the Graph Online application. Then, students were assigned similar problems to solve independently using Graph Online and were asked to submit their answers in a group chat. F4 would then either share the students' solutions from the group chat or provide his solutions after each practice problem. The group chat provided a space for students to share their solutions and view others' solutions.

F4 incorporated real-world elements into students' practice problems in both lectures and practice seminars. For example, students were asked to use graphs to present their relationships with friends in lesson 2 and to find the shortest paths between real cities in Kazakhstan in lesson

3. F4 also used GeoGebra in lessons 1 and 3 to demonstrate concepts to students. For example, he introduced Dijkstra's algorithm by presenting relevant examples in the GeoGebra book to students.

F4 used GeoGebra materials and other mathematics-specific instructional technology to support subject matter demonstration and students' practice. Although the choice of tools was appropriate for the topics taught, technology merely replaced non-technological materials for presentation; students did not interact with technologies for inquiry. RTOP subscale scores for F4's lessons are shown in Table 15.

Table 15

Subscale Scores of F4's Lesson Videos

Subsets	Subscales	Lesson 1	Lesson 2	Lesson 3
Lesson Design and Implementation		2	3	1
Content	Propositional Knowledge	15	17	15
	Procedural Knowledge	0	3	0
Classroom Culture	Communicative Interactions	0	0	0
	Student/Teacher Relationships	1	2	1

F4 demonstrated high scores in the Propositional Knowledge subscale; however, his scores in Lesson Design and Implementation, Procedural Knowledge, and Classroom Culture were low. The total RTOP scores indicated that his lessons remained teacher centered. In lesson 2, F4 provided opportunities for students to engage in learning activities, and he also paid attention to students' prior knowledge and provided explanations to students, which improved the score on the subscale of Classroom Culture. According to RTOP scores, F4's lecture lessons were teacher-centered, and his practice seminar reflects a shift toward transitional teaching. F4 also implemented PD materials and practices.

4.2.5 Analysis of Lessons of F5

F5 submitted three lessons: two lectures and one practice seminar. The lectures (lessons 1 and 2) scored 16 and 15 respectively on the RTOP scale, while the practice seminar (lesson 3) scored 21. F5's lectures began with a presentation of theorems or definitions of concepts, followed by demonstrations of worked examples on a shared screen. Student participation was minimal, with few opportunities for students to answer questions or work on problems independently. GeoGebra books were used in both lectures for demonstration purposes only. For instance, in lesson 2, F5 presented a GeoGebra activity on permutations and combinations, demonstrating how the GeoGebra activity could function as a combinatorics calculator. However, students did not interact with GeoGebra themselves; F5 used it only to verify answers she had calculated manually for the problems.

In lesson 2, F5 misused mathematical terminology and provided false solutions multiple times, which lowered her score on the Propositional Knowledge subscale. For example, when F5 presented a permutation problem about arranging three fruits (an apple, a banana, and a pear) in different orders, F5 correctly listed all six possible permutation arrangements. Then, she stated both on her PowerPoint and verbally that there were "six combinations or permutations" for these three fruits when she was presenting permutations of the fruits rather than combinations.

The practice seminar (lesson 3) began with F5 sharing her screen to present problems to students, and she continued by asking students to provide oral responses. When students failed to answer immediately or gave incorrect solutions, F5 tended to interrupt, providing definitions, correct answers, and justifications herself. Students were rarely given opportunities to explain or justify their answers, as F5 often explained the reasoning immediately after receiving a correct response. This seminar incorporated real-world connections where F5 linked the concept of tree diagrams to relationships within a Kazakh family to demonstrate the mathematical concept to students. RTOP subscale scores for F5's lessons are shown in Table 16.

Table 16

Subscale Scores of F5's Lesson Videos

Subsets	Subscales	Lesson 1	Lesson 2	Lesson 3
Lesson Design and Implementation		2	2	2
Content	Propositional Knowledge	14	13	15
	Procedural Knowledge	0	0	0
Classroom Culture	Communicative Interactions	0	0	2
	Student/Teacher Relationships	0	0	2

As presented in Table 16, F5 scored high in Propositional Knowledge for all three submitted lessons. The score for this subscale was lower for lesson 2 due to F5's misuse of terminology and provision of incorrect solutions to worked problems. F5's scores in Lesson Design and Implementation, Procedural Knowledge, and Classroom Culture were consistently low. In the practice seminar (lesson 3), the scores in the Classroom Culture subset were higher due to the fact that students had slightly more opportunities to participate in problem-solving activities, though these opportunities remained limited. GeoGebra and STEM applications were used exclusively for teacher demonstrations. According to RTOP scores, F5's lessons were teacher-centered, although F5 attempted to implement PD materials.

4.2.6 Analysis of Lessons of F6

F6 submitted three lecture lessons, which scored 19, 16, and 19 on RTOP scales. All of the lectures followed a consistent structure. Each lesson began with F6 presenting and reading aloud the lesson objectives and the pedagogical tools to be used, followed by the presentation and reading of definitions or theorems and corresponding worked examples. Opportunities for active student participation were limited across the submitted lessons, particularly in lessons 2 and 3, where students were not provided with opportunities to work independently on problems, discuss, or share their work.

In lesson 1, after F6 presented the definitions of propositions, she shared a link to an online propositions calculator and displayed practice problems on the screen so that students could solve the problems independently using the calculator. After a brief waiting time, F6 asked students to share their screens and demonstrate how they used the calculator to find their solutions with no reflection or justification of the solutions by either the students or F6. In lesson 3, F6 presented step-by-step instructions for finding Euler and Hamilton paths using Kazakhstan cities and the road network of the country with Graph Online and the Electronic Educational Atlas on Geography. While students were introduced to these tools and instructed in their use, they did not have the opportunity to use the technology or solve the problems themselves. RTOP subscale scores for F6's lessons are shown in Table 17.

Table 17

Subscale Scores of F6's Lesson Videos

Subsets	Subscales	Lesson 1	Lesson 2	Lesson 3
Lesson Design and Implementation		1	1	2
Content	Propositional Knowledge	15	15	17
	Procedural Knowledge	0	0	0
Classroom Culture	Communicative Interactions	2	0	0
	Student/Teacher Relationships	1	0	0

F6 scored high in Propositional Knowledge across all three lessons, demonstrating her solid knowledge of mathematics content. The application of graph theory to real-life geography scenarios in lesson 3 resulted in an even higher score in the Propositional Knowledge subscale for that lesson. Students' use of the propositional calculator for practice and sharing their solutions with the class led to a relatively higher score in the Classroom Culture subset in lesson 1. F6's scores in Lesson Design and Implementation, Procedural Knowledge, and Classroom Culture were consistently low. The RTOP scores indicated that F6's lessons were teacher centered. F6 did not implement PD materials but incorporated mathematics-specific technology and STEM applications primarily for demonstrations and to aid students' practice exercises.

4.3 Analysis of Faculty Individual Semi-Structured Interviews (F-ISS)

The faculty individual interviews were analyzed using the thematic analysis method explained in Chapter 3. Comparing the themes from the first and second interviews indicated significant similarities between them. Consequently, I reorganized and refined these themes to establish the final themes. In the following sections, the final themes are presented in separate sections. The final themes are:

1. Faculty developed positive perceptions about using the supplementary GeoGebra book in teaching discrete mathematics after participating in the cross-cultural PD.
2. Faculty developed positive perceptions and new knowledge about using STEM applications in teaching discrete mathematics after participating in the cross-cultural PD.
3. Faculty found the custom textbook to be somewhat valuable as a resource for teaching and practice in their discrete mathematics courses.
4. Faculty found that collaborative participation in the PD enhanced their learning.
5. Faculty continued to favor teacher-centered teaching approaches and used the provided materials in a manner consistent with these views.

4.1.1 Theme 1: Faculty Developed Positive Perceptions About Using the Supplementary GeoGebra Book in Teaching Discrete Mathematics After Participating in the Cross-Cultural PD.

Faculty developed positive perceptions of using GeoGebra to teach discrete mathematics following the PD sessions. They felt prepared to use the GeoGebra book, finding it relevant and easy to use for teaching relevant topics.

In both the first and second interviews, all faculty identified GeoGebra as the most valuable component of the PD session. During the **first interview**, some faculty, particularly those who had not previously used technology in teaching, found that the PD introduced not only a new instructional tool but also the broader concept of integrating technology into mathematics education. F5 noted, “I think [PD] changed my opinion on teaching math or discrete math [using] technology such as GeoGebra. [Before PD], I didn't know what [technology] is useful for teaching, [what] is easy and interesting" (F5, 1st interview). GeoGebra was a new tool for teaching discrete mathematics to all faculty, although some had previously encountered it and dismissed its potential, perceiving it as limited in functionality or relevant only for teaching geometry or graph-related topics. F1 recalled, "[GeoGebra] … wasn't something new for me. … When I first saw it… GeoGebra was not so good" (F1, 1st interview).

Before teaching the course, five out of six faculty perceived that the PD sessions on GeoGebra helped them feel prepared to use it in teaching discrete mathematics. F1 stated, "I feel comfortable to [use GeoGebra]. Because… during the sessions… GeoGebra … was clear for me" (F1, 1st interview). Some faculty were even inspired to integrate GeoGebra into other courses they teach. F3 expressed this intention: "I think if I start to use [GeoGebra] in my [courses], and for other subjects, it will also be beneficial if I learn how to do everything [with GeoGebra]" (F3, 1st interview). For F6, who had prior experience with GeoGebra and other technology, the PD provided inspiration to develop new tools:

> The workshop helped me gain new skills… I know [how to] program GeoGebra and use it. … [What was] new for me in this workshop was the creation of interactive resources. … I will look at [resources on] how to create an electronic textbook [provided by the PD] and create our own interactive resource using the same methods. (F6, 1st interview)

She also planned to create her own GeoGebra activities for other courses she teaches after the PD sessions. In contrast, F5 felt less prepared to use GeoGebra, stating, "I like GeoGebra, and [it is a] very interesting program, but I don't understand what [it does] and how it works, etc." (F5, 1st interview). She explained that missing the second workshop due to illness limited her learning, yet she still identified GeoGebra as the most valuable takeaway from the PD and planned to use it in her discrete mathematics course.

During the **second interview**, after teaching the course, all faculty continued to cite the PD's focus on GeoGebra as its greatest benefit. F2 stated, "[When] I [first] learned about GeoGebra, it [was] used only in geometry for figures" (F2, 2nd interview). Likewise, F5 initially viewed GeoGebra as primarily applicable to geometry but acknowledged that her understanding expanded after the PD sessions: "I see that I use [GeoGebra] in different subjects, teaching different subjects" (F5, 2nd interview).

After teaching, all of the faculty described the PD-provided GeoGebra book as highly relevant and easy to use in their discrete mathematics courses. Faculty expressed enthusiasm about incorporating the GeoGebra book into their courses and confirmed its usefulness after teaching. Even F5, who felt underprepared in the first interview, used the GeoGebra book in her teaching: "I tried to use the GeoGebra book from PD. … For example, when we solve problems with combinatorics, I use the [activity] for counting the permutations in the GeoGebra book" (F5, 2nd interview). However, two faculty (F1, F6) also indicated that the GeoGebra book does not cover all concepts in the course. F1 shared, "The teaching [with] GeoGebra was great for especially … discrete mathematics. Approximately 30% of my lectures [used] GeoGebra. [GeoGebra book] was not usable for all topics" (F1, 2nd interview).

In summary, faculty identified the PD content on teaching discrete mathematics with GeoGebra as the most valuable component of their PD experience. After teaching, faculty confirmed their perception that GeoGebra was an effective instructional tool. The relevance and ease of use of the GeoGebra book supported faculty in teaching the topics covered in the book.

4.1.2 Theme 2: Faculty Developed Positive Perceptions and New Knowledge About Using STEM Applications in Teaching Discrete Mathematics After Participating in the Cross-Cultural PD.

Faculty expanded their knowledge and developed positive perceptions of teaching discrete mathematics with STEM applications after the PD.

Before participating in the PD (**first interview**), all faculty had some awareness of STEM applications in mathematics teaching, but their understanding was limited. F1 noted, “I have some ideas about STEM application, but … it wasn’t so clear before my participation [in] the PD” (F1, 1st interview). For faculty who had already incorporated STEM applications in their teaching, the PD reinforced their beliefs about the importance of connecting mathematics with other STEM disciplines and real-world examples. F5 explained, “I always tried to show the connection between mathematical concepts and some applications in other fields. In the process of our collaborative work, I became even more firmly established in my views” (F5, 1st interview).

All faculty recognized the value of integrating STEM applications in teaching discrete mathematics. F1 commented, “I think it will be better [for] students to understand [the discrete mathematics content] with [STEM] applications” (F1, 1st interview). Similarly, F2 shared, “I think the STEM application is very effective for students because students [can] search many ways to use multiple integrals in real-world problems” (F2, 1st interview). F4 reflected on how STEM applications could make discrete mathematics more engaging for students: “I have never seen … STEM application in teaching discrete mathematics … I think it would be very interesting for students [to see] how discrete mathematics [is] connected with science and with everyday life" (F4, 1st interview).

After teaching the course (**second interview**), all faculty maintained positive perceptions about implementing STEM applications in teaching mathematics. F5 highlighted how STEM applications changed her perception of discrete mathematics:

> I think such that discrete math is [a] very difficult subject, [it is] not [an] interesting subject in the beginning, but now I [have] change[d] my opinions about this subject … because I can choose or show [its] application in other subject[s] like computer science, chemistry, physics, etc. (F5, 2nd interview)

In summary, the PD sessions effectively expanded faculty knowledge and fostered favorable perceptions of teaching discrete mathematics with STEM applications.

4.1.3 Theme 3: Faculty Found the Custom Textbook to Be Somewhat Valuable as a Resource for Teaching and Practice in Their Discrete Mathematics Courses.

Faculty found the examples and problems in the custom textbook to be highly relevant and valuable for their demonstration and student practice in discrete mathematics courses. These views became stronger after teaching the course; however, two faculty were concerned about the insufficient number of examples and practice problems.

Before teaching (**first interview**), all faculty recognized the potential value of the textbook. For example, F2 stated: "[the custom textbook] is … a bit more clear for students" (F2, 1st interview), and F5 stated "I also liked the [custom] textbook … The main material in the textbook is presented compactly and strictly" (F5, 1st interview). F1 and F2 acknowledged that the textbook contained all the necessary course information. Nonetheless, concerns emerged regarding the small number of examples and problems available in the custom textbook. Both F2 and F6 specifically noted insufficient numbers of problems for student practice. F2 expressed the need for additional examples and questions, while F6 emphasized the need for more similar problems. F6 stated: "I saw that in the [custom] textbook there are no tasks of the same type for a group of students (for example, 20 students)" (F2, 1st interview).

After teaching the course (**second interview**), faculty unanimously affirmed the value of the examples and problems in the custom textbook for demonstration and student practice. All faculty stated they primarily utilized the custom textbook as a source for demonstration examples and practice problems. “I used examples from this book. … in all my lectures that topic was covered in this [custom] textbook," F4 said in the second interview. Beyond examples and problems, F3 mentioned that he occasionally integrated definitions and theoretical content from the textbook into his teaching practices. F5 and F6 found the contents of the custom textbook to be clear and reported successfully incorporating it into their lesson preparation:

> I use the [custom] textbook in preparation of theoretical material for the lesson, it is needed and necessary… I'm preparing a presentation in PowerPoint for my lesson using the theoretical material of this textbook. And used tasks from this textbook, … for students in practice lessons. I use it in topics like sets, graphs. (F6, 2nd interview)

The concern regarding insufficient examples and practice problems was reported during the second interview only by F6.

In summary, the faculty viewed the custom textbook as particularly valuable and helpful, primarily as a source of examples and problems in their teaching practices. All faculty found the custom textbook highly relevant to their course content and appreciated its ease of use. One of the two faculty who initially expressed concerns about the number of examples and problems in the custom textbook found it satisfactory after teaching with it, while the other faculty continued to express this concern after teaching the course.

4.1.4 Theme 4: Faculty Found That Collaborative Participation in the PD Enhanced Their Learning.

Faculty highly valued the opportunity to participate in the PD with their colleagues as it supported their learning during the PD.

Five out of six faculty found that participating in learning opportunities in the PD with their colleagues was helpful (**first interview**). U1 faculty noted that collaborative participation supported them in addressing issues related to the PD sessions and teaching the discrete mathematics course. For example, F2, while discussing his uncertainty about teaching with STEM applications, stated, “I will talk about [STEM application with] the team. Maybe we can find some solutions [for this issue]” (F2, 1st interview). F1 also emphasized the benefits of team-based planning for teaching the course: “We planned the course and how we will teach it together, which was helpful” (F1, 1st interview, paraphrased). As the least experienced faculty in U1, F3 found collaborative participation particularly valuable. She noted, “When [my colleagues and I] worked together, I might ask them about discrete topics and teaching questions” (F3, 1st interview).

Similarly, U2 faculty found that collaborative participation was beneficial in that it allowed them to learn from one another. Through communication within U2 and among the entire group, they were able to exchange ideas and teaching strategies. F4 mentioned, “There were some presentations in the last workshop … my colleagues, from the other university and from my university [were] showing some useful examples in GeoGebra” (F4, 1st interview). F5 expressed similar sentiments about collaborative learning opportunities during PD: “Talk[ing] about the experience of my colleagues, [about the new] method and techniques of teaching … I think that I increased my achievement in this project and my work of teaching" (F5, 1st interview).

After teaching their courses (**second interview**), four faculty continued to value the collaborative aspects of the PD. F3 highlighted the benefit of learning from her colleagues' teaching practices: "We discussed how other teachers also are teaching, their experiences also, their teaching methods … how they are taking feedback from students, preparing questions for them" (F3, 2nd interview). F5 stated: "It was very interesting and [helpful] for me [to] work with other participants [during] this project" (F5, 2nd interview).

In summary, faculty from both universities valued collaborative participation in the PD. The opportunity to participate in the same learning opportunities was beneficial as it allowed faculty to learn from and work with each other to address teaching challenges, share teaching strategies, and exchange ideas.

4.1.5 Theme 5: Faculty Continued to Favor Teacher-Centered Teaching Approaches and Used the Provided Materials in a Manner Consistent With These Views.

Faculty traditional views of instructors' responsibility for content delivery remained unchanged, with technology and STEM applications used primarily to support content presentation and student practice.

Before teaching the course (**first interview**), all faculty primarily viewed technology—particularly GeoGebra—as a tool to enhance their presentations and facilitate student learning through improved visualization. They intended to use the GeoGebra book for more effective content delivery. F2 stated, "We can make [the content] clear for students, … if we [visualize] some symbols [with GeoGebra]" (F2, 1st interview). Faculty described using GeoGebra to help students visualize abstract mathematical concepts. For instance, F4 explained, “I can use, for example, GeoGebra as a tool to show some mathematical ideas for students” (F4, 1st interview). The faculty did not intend to use GeoGebra explorations for students’ inquiry but only planned to use it for their demonstrations.

Similarly, all faculty intended to use STEM applications for content presentations and student practice. When F4 talked about her intention of implementing STEM applications, she considered presenting students with connections by showing them examples: “Combination of [mathematics and] these science areas, … we can use [technology] to show how these areas are connected with each other. (F4, 1st interview). F1, F2, and F3 explicitly expressed their uncertainty about how STEM applications influence their implementation. F1 considered using GeoGebra as his strategy for teaching with STEM applications. F2 expressed a lack of clarity, stating, “STEM education. I think it was a bit vague because we just talked[ed] about GeoGebra. [Thinking about teaching with STEM applications, I am not sure about] what kind of real-world problems we are going to give students” (F2, 1st interview). Similarly, F3 summarized her unpreparedness for integrating STEM applications: “We had [a] session according to STEM; however, I have to try to [study] about this one more deeply by myself … I think I don't have [enough] knowledge…about STEM” (F3, 1st interview).

All faculty intended to use the custom textbook in their courses as resources for preparation, examples, or practice problems. Two faculty expressed concerns about effectively implementing the custom textbook beyond using the examples and practice problems. F1 acknowledged that the textbook contained all the necessary course information. However, he wished there had been more explanation during the PD sessions of how the PD providers would use all sections of the custom textbook for teaching. Similarly, F2 valued the textbook but desired additional clarification during the PD sessions concerning effective teaching practices using it. He directly stated, "I don't know how to use [the custom textbook] more effectively" (F2, 1st interview).

After teaching the course (**second interview**), faculty reported that their enacted teaching practices with GeoGebra were largely aligned with their intention, which was to use GeoGebra primarily for concept demonstration and presentation. F1 highlighted its usefulness for visual presentations:

> That [GeoGebra] was great to show to students, especially online, … technological use for us to show them good graphics to understand them better used in set theories. In logical laws it was also very useful to see how it works. (F1, 2nd interview)

F3 and F4 also reported similar usage of GeoGebra: “While I was explaining the topic, I was showing [the students] the [content by] demonstrating … [GeoGebra] website [on the screen]” (F3, 2nd interview), and “I used GeoGebra as a demonstration, for demonstration examples of theories” (F4, 2nd interview). The students had no chance to explore the activities in the GeoGebra book. Four faculty (F2, F3, F4, F5) also reported using GeoGebra activities for student practice. F4 noted, “We used [GeoGebra] in lectures and also in the practices. Because there are a lot of problems which students can also solve … like a problem book” (F4, 2nd interview).

As two faculty mentioned in theme 1, not all topics had corresponding activities in the GeoGebra book, and that may have limited their implementation of the GeoGebra materials. F6, who found limited suitable activities in the GeoGebra book, opted to use other technologies in her course for student practice: “Students solved problems … [with] their propositional logic calculator. This calculator allows the students to check independently solved problems and find the tautologies in oppositional” (F6, 2nd interview).

All faculty reported that they incorporated STEM applications and real-life related problems in their presentations. For instance, F1 noted, “I tried to show examples, … in logical law … I showed the example of 0 1, binary number assignment [in computer science]” (F1, 2nd interview). F5 also emphasized using STEM applications to demonstrate real-world relevance: “I can choose or show [discrete mathematics] applications in other subject[s], for example, for computer science, … for chemistry, physics, etc.” (F5, 2nd interview). Additionally, three faculty (F2, F4, F6) assigned STEM-related problems for students to practice. For example, F2 provided STEM application problems as bonus assignments: For example, F2 intended to give students practice problems connecting discrete mathematics topics to real-world and STEM disciplines: “We use STEM applications for functions, for relations, for combinatorics, for graph theory… [to practice] after lessons. In sequence, [I] gave [students data] from a company [that] produces recursively. [Students use the data as] sources to define sequence” (F2, 2nd interview).

The three faculty who initially felt underprepared continued to express the need for additional support in teaching discrete mathematics with STEM applications. F3 emphasized that she needed more information to feel confident in teaching with STEM applications: “I know [STEM application means] connecting with other [disciplines] … I don't have many information on how to use it [STEM application] in my teaching.” (F3, 2nd interview). The uncertainty may have prevented faculty implementation.

After teaching, all faculty reported that the custom textbook served as a useful resource for examples and practice problems. Faculty primarily used the textbook as a problem set rather than as a recourse to implementing new teaching approaches. F4 noted a lack of understanding regarding the teaching approaches underlying the custom textbook, which made him feel limited in his ability to implement this textbook effectively in his teaching.

In summary, faculty recognized the benefits of using GeoGebra and STEM applications in their teaching (themes 1 and 2), but they primarily viewed these tools as means to enhance their presentations and facilitate student practice. The custom textbook was also used as a resource for examples and practice problems. Based on faculty reported intended and enacted teaching practices, no changes occurred in shifting from teacher-centered to student-centered methods. Faculty perceived their responsibility for teaching as delivering content effectively and providing students with practice opportunities. Faculty limited ability to create new GeoGebra activities, insufficient understanding of STEM applications, and incomplete comprehension of how to use the custom textbook hindered their implementation of the PD materials and their use of those materials to support new teaching approaches.

Chapter 5: Discussion and Conclusion

This chapter presents answers to the study research questions, the discussion of the study findings, and conclusions. The "Answering Research Questions" section addresses each research question based on the results of the analysis presented in Chapter 4. The section titled "Connecting Findings to Literature" examines the study findings in relation to existing literature on PD effects and acculturation processes. Theoretical contributions and practical implications will then be discussed, and the last section addresses the study's limitations and makes recommendations for future research.

5.1 Answering Research Questions

To answer RQ1 regarding faculty perceptions of teaching and learning, I identified the changes and lack of changes in faculty perceptions after they participated in the PD sessions. The data used in this discussion were the scores of the F-STEM survey subscale results, which reflected the changes, and the themes that emerged from faculty interviews, which explained them. To answer RQ2, I determined faculty enacted teaching practices based on the RTOP scores and the results of analysis of field notes of the submitted lesson videos. I then identified faculty intended teaching practices and their self-reported enacted practices based on themes that emerged from faculty interviews. Intended and enacted teaching practices discussion focused on faculty implementation of reformed teaching approaches and use of PD materials. The intended and enacted teaching practices were then compared. To answer RQ3, I identified faculty educational acculturation strategies based on the guide presented in Table 6 in Chapter 3, using faculty perceptions (RQ1), and their intended and enacted teaching practices (RQ2). To answer RQ4 (a), I identified influential PD features from faculty interview themes that faculty indicated and identified how the features corresponding with any changes in perception (RQ1). For RQ4 (b), I identified influential PD features based on the faculty interview themes that the faculty indicated and identified how the feature corresponding with their intended and enacted teaching practices (RQ2). Finally, all results were compared between U1 and U2 to address RQ5, identifying any differences or similarities between the institutions.

5.1.1 Answering RQ1: How were the Kazakhstani Mathematics Faculty Perceptions of Teaching and Learning Mathematics Influenced by Their Experiences in the Cross-Cultural PD?

In order to explain the influence of cross-cultural PD on faculty perceptions about teaching and learning mathematics, results of F-STEM survey analysis were compared to the results of interview analysis. According to the F-STEM scores, the most significant positive changes occurred in attitudes toward Student Technology Use (STU). However, the themes that emerged from the interview analysis indicated that technology was used primarily by the faculty for demonstrations, and students were only using technology for drills and problem-solving practice (Theme 5). Moreover, in both interviews all faculty expressed preference for their traditional teacher-centered approaches to teaching, while a few also expressed a lack of understanding about reformed teaching. The findings from the interviews are consistent with faculty skeptical attitudes about student-centered, inquiry-based teaching approaches as evident from the decrease in scores for the Mathematics Instruction (MI) subscale of F-STEM survey for four out of six faculty.

Overall, the mixed pattern of attitudes indicates faculty selectively embraced aspects of the new education tradition. They integrated compatible technologies but resisted fundamental changes challenging their established teacher-centered approaches.

5.1.2 Answering RQ2: How Were the Kazakhstani Mathematics Faculty Intended and Enacted Teaching Practices Influenced by Their Experiences in the Cross-Cultural PD?

Faculty teaching practices were examined through multiple data sources and comparison of their intended practices (based on interview analysis) with enacted practices (based on RTOP analysis and interviews analysis). The findings revealed that faculty predominantly intended and enacted teacher-centered teaching approaches, utilizing PD materials primarily as presentation resources for teachers or practice materials for students rather than as catalysts for teaching approach transformation. All faculty intended to implement GeoGebra and STEM applications in their teaching practices. However, not all faculty were observed enacting these implementations in the submitted lesson videos. Faculty self-reported teaching approaches confirmed that they implemented these materials in non-submitted lessons. An interesting pattern emerged where practice seminars consistently demonstrated more implementation of reformed teaching practices than lectures. In these seminars, students had more opportunities for active learning, such as independent problem-solving and whole-class discussion of solution strategies.

In summary, faculty teaching approaches remained predominantly teacher-centered in both intention and enactment. However, as a result of PD sessions, all faculty demonstrated high willingness and made attempts to implement PD materials even though not all submitted lessons showed evidence of PD material implementation. Faculty tended to incorporate more reformed teaching practices in practice seminars compared to lectures.

5.1.3 Answering RQ3: Which Educational Acculturation Strategies Did the Kazakhstani Mathematics Faculty Demonstrate in Their Teaching Practices After Participating in the Cross-Cultural PD?

To determine faculty educational acculturation strategies, results from RTOP analysis of submitted lesson videos were combined with the results of interview analysis. Each faculty enacted teaching approaches were identified using complete RTOP scores from all their submitted lessons, while perceptions, intended and self-reported enacted teaching practices were identified through interview analysis, and the implementation of PD materials was determined through field note analysis. Because acculturation is a process involving complex negotiation rather than simple adoption or rejection, nuanced interpretation is essential (Berry, 2017). For example, when faculty incorporated mathematics-specific technology like GeoGebra within their traditional teacher-centered teaching approaches, this demonstrated their willingness and attempts to engage with new educational traditions. Despite potentially low overall RTOP scores, such faculty demonstrated implementation of integration strategies rather than separation.

RTOP score results revealed that faculty predominantly employed teacher-centered teaching approaches. One notable exception was F2's practice seminars, where RTOP scores demonstrated characteristics of transitional teaching approaches. This indicates that F2 taught these lessons with more reformed teaching practices though still retaining teacher-centered core elements. However, evidence from field notes and faculty-reported enacted teaching practices indicated that all faculty adopted some aspects of new educational traditions while maintaining their teacher-centered approaches. Interview data results revealed that all faculty developed favorable beliefs about the PD materials and were willing to implement mathematics-specific technology and STEM applications into their current teaching practice (themes 1, 2, 3). However, these implementations remained for teacher demonstrations or student drill practice rather than inquiry-based learning (theme 5).

In summary, all faculty demonstrated both willingness to incorporate and actual implementation of PD materials within their teacher-centered teaching practices. According to educational acculturation strategy definitions, this pattern represents an integration acculturation strategy, wherein faculty selectively adopted aspects of new educational traditions that could enhance their teaching while maintaining the teacher-centered approaches rooted in their heritage core pedagogical identity.

5.1.4 Answering RQ4.a: Which PD Features Did the Kazakhstani Mathematics Faculty Identify as Reasons for Changes or Lack of Changes in Their Perceptions of Teaching and Learning Mathematics?

The interview analysis results, connected with the F-STEM survey results, identified the PD features that faculty reported as influencing their perceptions of teaching and learning mathematics. Content Focus emerged as the most significant factor, with all faculty identifying GeoGebra-focused content as the most valuable component of the PD. Faculty developed positive perceptions of this tool regardless of their prior experience (theme 1), and this positive change in faculty perception toward using technology was reflected in growth on the STU subscales of the survey. Similarly, faculty perceived PD expanded their knowledge of STEM applications, and how to connect abstract mathematics to other disciplines and real-life situations (theme 2).

Additionally, the Coherence of PD sessions and materials, designed to be directly applicable to faculty courses and teaching situations, helped faculty recognize the potential benefits of teaching with these new resources (themes 1, 3). Collective Participation emerged as another beneficial feature, as faculty valued collaborating with colleagues during and after PD sessions to resolve questions and learn from peers, which was particularly valuable for less experienced faculty (theme 4). This peer support contributed significantly to positive perceptions of the overall PD experience.

However, faculty hold skeptical perception towards the reformed teaching and the philosophy embedded behind these materials. The decreases in faculty scores on the MI subscales on the F-STEM survey further reflect a lack of change in faculty perception of this aspect. Comparison of time allocation across different topics within the PD sessions revealed more time devoted to GeoGebra (spanning two days) but limited time allocated to the pedagogical foundations of PD (Day 1). This Duration feature likely contributed to both the prominence of GeoGebra in faculty recollections and the reduced impact of reformed teaching on faculty perceptions.

In summary, Content Focus, Coherence, and Collaborative Participation were defined by faculty as influential on their perceptions of the new educational tradition. The Duration factor was not mentioned by the faculty but may also have contributed to their lack of change of perceptions.

5.1.5 Answering RQ4.b: Which PD Features Did the Kazakhstani Mathematics Faculty Identify as Reasons for Implementing or Rejecting New Educational Traditions?

Interview analysis results were connected with faculty intended and enacted teaching practices to identify the PD features that influenced faculty implementation of new educational traditions. Faculty identified both the Content Focus of the PD and the Coherence of PD materials to their teaching contexts as primary factors supporting implementation. Specifically, faculty found that the GeoGebra book and the custom textbook provided direct examples and practice problems that they could adapt to their lessons, facilitating the easier integration of these materials into their teaching of the topics covered by the PD materials (themes 1, 3, 5). Two faculty in the first interview and one faculty in the second interview commented on their lack of understanding and knowledge regarding the pedagogical foundations of how to utilize the custom textbook for reformed teaching approaches. They expressed that insufficient Content Focus on this aspect limited their ability and prevented them from implementing new teaching practices (theme 5). The lack of reformed teaching practices was evident in both submitted lessons and reported teaching practices.

When materials were not prepared for direct adaptation, faculty identified the need for extended content focus on certain topics, such as creating new GeoGebra resources (theme 5). Differences among faculty intended, enacted, and reported teaching practices also revealed that faculty implemented the PD materials when the topics taught were covered and did not implement them when the topics were not covered by the materials.

In summary, Content Focus and Coherence of the PD sessions facilitated faculty implementation of PD materials and practices for teaching specific mathematics concepts. However, insufficient Content Focus on reformed teaching prevented the implementation of new teaching practices. Improvements in these areas may enable faculty to move beyond using provided resources and achieve greater implementation of new educational traditions.

5.1.6 Answering RQ5: Did Institutional Contexts Influence the Kazakhstani Mathematics Faculty Perceptions and Educational Acculturation Strategies?

The results of the F-STEM survey, RTOP analysis, and interview analysis for both U1 and U2 were compared, revealing distinct patterns in faculty perceptions and teaching practices in different universities.

U1 faculty displayed more varied responses to the F-STEM survey, showing no changes on the MTEB and MTOE subscales, a minor increase in the MI subscale, and even decreases on the STU subscale. Their high baseline scores on the pre-survey suggest they may have perceived less need to modify their teaching approaches. In contrast, U2 faculty demonstrated consistent increases across all survey subscales, particularly on the STU subscale. This pattern aligned with evidence that U2 faculty incorporated more mathematics instructional technologies in their submitted lesson videos.

The RTOP analysis revealed an interesting contrast between the institutions. U1 faculty achieved higher RTOP scores because their lessons featured more opportunities for student discussion and presentation of reasoning, despite implementing less technology. Meanwhile, U2 faculty incorporated more technology and STEM applications but maintained more teacher-centered instructional practices. The differences in lesson type may have contributed to these patterns, with U1 submitting proportionally more practice seminars and U2 submitting more lectures. In addition, the institutional context may also have contributed to the differences in teaching approaches. U1, operating as a private university, functioned with less stringent centralized control, which may have enabled faculty to implement more individualized pedagogical practices. In contrast, U2, as a public institution, exhibited greater uniformity and centralized characteristics in its institutional operations, which may have contributed to more teacher-centered practices in the classrooms. Collaborative teaching within each university appears to have reinforced consistent pedagogical approaches among faculty at the same institution.

Differences emerged between U1 and U2 faculty in both perception changes and teaching practices. The variations in faculty perception changes may be related to the different baseline perceptions between the two groups. Meanwhile, differences in teaching practices may correlate with both the types of lessons submitted (practice seminars vs. lectures) and the broader institutional context (private vs. public university).

5.2 Connecting Findings to Literature

This research aimed to examine the influence of a cross-cultural PD program, that focused on reformed teaching and technology integration, on acculturation strategies of Kazakhstani mathematics faculty teaching undergraduate discrete mathematics courses to students preparing to become mathematics teachers. The study employed a case study methodology. Both quantitative and qualitative data was collected, including pre and post surveys, videos of taught lessons, and semi-structured interviews. The study aimed to provide an understanding of the faculty experiences of the PD and its influence on their perceptions and teaching practices through the lens of the Cross-Cultural PD Educational Acculturation Model.

5.2.1 Influential PD Features on Faculty Perceptions and Teaching Practices

The faculty revealed in their interviews that Content Focus, Coherence, and Collaborative Participation were the PD features that positively influenced their perceptions and practices. These findings are consistent with studies indicating that when PD sessions are highly concentrated on a specific content, closely aligned with participants' teaching situations, and involve participation with colleagues, participants are likely to develop greater confidence and lead to implementation of the new curriculum (Edwards et al., 2015; Penuel et al., 2007).

In this study, integrating technology, GeoGebra, in discrete mathematics teaching was a focus of the PD session. Faculty perceived the GeoGebra aspect of the PD as most effective; they expressed feelings of preparedness and a serious intent to implement it. Evidence of implementation was observed in their teaching practices in submitted lessons and in their self-reported implementation. In contrast, there was less explicit emphasis placed on reformed teaching approaches during the PD. Some faculty reported limited understanding of reformed teaching; there were few increases in their perception and little observed practice in faculty submitted lessons and in their self-reported practices. This finding echoes Penuel et al.'s (2007) study which concluded that the focus of content predicts teacher participants' preparedness for implementation. Tuan et al. (2017) also found that PD programs' focus on specific applications can effectively change teachers' conceptions and instructional approaches.

In this study, there was a clear connection between the Coherence feature and the effectiveness of PD. Faculty perceived a strong alignment between PD sessions and materials with their discrete mathematics courses. They appreciated even before teaching their courses that PD materials were readily applicable in their classrooms and recognized their value in enhancing presentations and demonstrations. The implementation of these materials was evident through observed teaching practices and faculty self-reports. This finding corresponds with research by Penuel et al. (2007) and Garet et al. (2001), which demonstrated that teachers' perception of coherence between PD and their teaching context serves as a positive predictor of curriculum implementation.

Collaborative Participation also emerged as an effective feature of the PD program. Faculty reported that participating in PD with their colleagues provided valuable peer support. Through collaboration, faculty were able to ask questions, clarify concepts, learn, and share knowledge, which helped resolve problems and deepen their understanding of the PD content. This finding is consistent with Vangrieken et al.'s (2015) systematic review, which identified that participating in professional learning opportunities with colleagues fosters collective learning and knowledge sharing, leading to increased learning as well as improved teacher confidence and self-efficacy.

The lack of change in faculty perceptions and practices regarding student-centered and inquiry-based approaches may also relate to the duration (session hours or time span) of the PD. Postareff et al. (2007, 2008) compared faculty who participated in intra-cultural PD programs of varying lengths and suggested that transitioning from a teacher-focused to a student-focused approach is possible. Tuan et al. (2017) found significant improvement in both science and mathematics teachers' conceptions of inquiry-based teaching approaches during a 9-week PD program totaling 36 hours of training. Postareff et al. (2007, 2008) discovered that the transformation of faculty perceptions and practices from teacher-centered to student-centered is a gradual process that requires at least one year of specific pedagogical training (approximately 750 to 900 hours). Although this cross-cultural PD spanned a year and provided subject-specific materials and training that promoted reformed teaching, the shorter training hours (nine hours of PD sessions) may have contributed to the lack of change in faculty perceptions and practices regarding teaching approaches.

5.2.2 Influential Context Features on Faculty Perceptions and Practices

Context features are also influential on the effectiveness of the PD (Desimone, 2009). In this study, the PD program was adapted to an online format due to the impact of the COVID-19 pandemic. Despite Fishman et al. (2013) finding no significant differences between in-person and online PD sessions regarding influence on participants' instructional perceptions and practices, faculty may face considerable challenges when shifting from in-person teaching to online courses.

Trust and Whalen (2020) documented that most K-12 teachers lacked preparedness for online instruction when the pandemic emerged, largely due to inadequate training in effective online teaching approaches. Through interviews with online teaching experts, Rapanta et al. (2020) emphasized the necessity for training university faculty in specialized pedagogical content knowledge to equip them for teaching online. Limited experience and inadequate training in online teaching methods likely created significant challenges for instructors, contributing to their feelings of being overwhelmed (Rapanta et al., 2020; Trust & Whalen, 2020). The PD program was originally designed for in-person classrooms, and faculty were teaching their first online discrete mathematics course in the spring of 2021. Because of their preparedness and the stress of transforming to online teaching, faculty may have found it challenging to simultaneously implement the new materials and teaching approaches introduced during the PD program. For that reason, they may have chosen to maintain their traditional teaching approaches.

Moreover, faculty and students' lack of English proficiency likely contributed to challenges in implementing PD materials and new teaching approaches in the discrete mathematics course. While the program assumed all faculty possessed sufficient language skills for English instruction and full participation in PD activities, conversation during the interviews and observations of submitted lessons revealed varying levels of English proficiency among the six faculty. During the PD, collaborative participation allowed faculty to use their native language to communicate with each other and resolve issues in understanding PD sessions or materials. As English is a relatively recent addition to the instructional languages in the country and differs significantly from Kazakh and Russian, many Kazakhstani educators lack adequate training in English language skills (Sarmurzin et al., 2024). Faculty may have encountered difficulties in effectively conveying complex mathematical concepts and engaging students, particularly in a non-native language. In addition, during some observed lessons, students spoke languages other than English. This suggests that students may have also struggled with learning the course content in a foreign language. When both faculty and students have language barriers, it can create additional obstacles to the successful implementation of PD content.

In addition, research suggests the course structures contribute to the differences in faculty perception and teaching approaches between institutions (Prosser & Trigwell, 1997). At U1, one faculty taught lectures to all students, while the other two faculty taught only practice seminars. At U2, all faculty taught both lectures and practice seminars. Furthermore, class sizes varied considerably, with F1 teaching 38 students compared to his colleagues who each had 19 students, and U2 faculty each teaching 16 students. While I couldn't identify specific institutional factors due to the lack of consistent patterns in perceptions and teaching approaches between faculty at the two universities, contextual influences likely explain why practice seminars consistently exhibited more reformed features than lectures taught by the same faculty. Practice seminars are designed for students to practice solving problems using knowledge they acquired from lectures. Prosser and Trigwell's (1997) presented in their study, that instructors were likely to implement teaching approaches they felt were valued institutionally Naturally, seminars necessitate greater student involvement and encourage faculty to implement a more student-centered approach. Because faculty viewed technology and STEM applications primarily as presentation and drill materials, they were more likely to engage students in using these tools during practice seminars. In addition, they also claimed that instructors more readily adopted student-centered teaching approaches when they perceived control over content and their teaching practices in the classroom and had manageable class sizes. The higher student participation observed in U1 lessons may be attributed to differing levels of autonomy between the universities, despite similar class sizes for practice sessions at both institutions.

Participants in this study had a diverse range of teaching experience, which could potentially influence the impact of the PD sessions on their perceptions and practices. In a large-scale study on a federal educational innovation, Berman and McLaughlin (1978) revealed that more experienced teachers exhibited a lower likelihood of altering their perceptions, adopting novel strategies, or sustaining innovations following the intervention. Similarly, Thibaut et al. (2018) discovered an inverse correlation between teachers' length of teaching experience and their attitudes toward STEM-integrated, problem-based, and inquiry-based methods introduced during a PD program. However, this study found no discernible patterns related to faculty teaching experience. The faculty who had less teaching experience did not exhibit more transition to the new traditions either in perception or teaching approaches compared to the faculty who had long years of teaching.

5.2.3 Cultural Factors and Acculturation Strategies

Research indicates that curriculum development, pedagogical beliefs, and teaching practices are deeply embedded in national cultures (Kaiser et al., 2006; Kaiser & Blomeke, 2013; Leung, 2006; Leung, et al., 2006; Schmidt et al.,1996; Wang & Liu, 2002). Consequently, this study considers the cross-cultural PD experiences for participants from different educational traditions as an acculturation process. The transition from teacher-centered to student-centered approaches represents a fundamental transformation in beliefs that extends beyond methodological changes. These challenges are deeply embedded in cultural values and established teaching approaches, particularly in contexts with strong hierarchical educational traditions (Hallinger & Lu, 2011). The faculty demonstrated limited change in instructional practices and even declining perceptions regarding student-centered and inquiry-based teaching approaches. However, they did implement technology and STEM applications within their teacher-centered approaches. This pattern indicates complex negotiation between their heritage educational tradition and the new educational traditions, rather than a complete separation of the two.

When the study was conducted, the education system and educational traditions in Kazakhstan, despite reform efforts, were changing slowly, and the system remained centralized. The primary teaching approaches in classrooms continued to be teacher-centered (OECD, 2017). Although only one faculty attended higher education before the reform, it is likely that all faculty were educated with the traditional teacher-centered approach. The pre-F-STEM survey results revealed faculty high levels of confidence in their teaching approaches (Mathematics Teaching Efficacy and Belief (MTEB): 3.6 to 4.5, Mathematics Teaching Outcome Expectancy (MTOE): 3.0 to 3.8). These findings can be explained by that faculty draw on their experiences as students in the classroom regarding effective instructional activities when determining their own teaching approaches. They consider how they themselves learned during the same developmental stage as their current students, and what "unique set of values, codes of conduct, and epistemological views" of the subject matter they developed during their learning experiences within a specific cultural context (Oleson & Hora, 2014). When the faculty were educated under the Kazakhstan heritage mathematics educational traditions, their teachers' approaches were likely rooted in these traditions. They are satisfied with their current approaches precisely because these methods reflect how they themselves successfully learned mathematics.

The findings regarding the Kazakhstani faculty confidence in heritage teacher-centered approaches also parallel findings from some studies conducted in East Asia. Under a centralized system and the continuous influence of Confucian culture in education, which similarly emphasizes centralization and teacher authority in the classroom, educators typically maintain strong convictions about the effectiveness of teacher-centered approaches for student success (Ho, et al., 2001; Huang & Asghar, 2018). Burkhalter and Shegebayev (2012) suggested that the context for Kazakhstani educators to change their instructional beliefs and practices is complex, as fully adopting new teaching approaches may conflict with their established beliefs about education and teacher roles, potentially resulting in reduced engagement with new educational traditions. The persistence of a centralized education system and educational traditions in Kazakhstan (OECD, 2017; Li & Ashirbekov, 2014) may further reinforce faculty comfort with maintaining traditional classroom dynamics rather than pivoting to new teaching approaches. Faculty partial adaption of the new educational tradition in this study aligns with Hallinger and Lu's (2011) findings in the first three years of their study, in which they found significant challenges in shifting deeply ingrained beliefs about education and teacher roles during their implementation of problem-based learning at a university in Thailand. With longer contact time or training, faculty may demonstrate greater adaptation to new educational traditions, similar to the faculty at the Thai university.

Faculty educational beliefs and practices are deeply embedded within their sociocultural context, shaped by both personal experiences and broader social structures (Bishop, 1998; Pajares, 1992). Acculturation theory suggests that while individuals can undergo acculturation independently of their broader society, the extent and nature of societal transformation significantly influences individual adaptation processes (Berry, 2003). In the case of Kazakhstan, despite decades of independence and reform initiatives, many fundamental features of the Soviet educational system persist in contemporary practice (Lang, 2019; OECD, 2007, 2017) such as the maintenance of mathematics course structures, particularly the distinct separation between lectures and practice seminars. This structure embodies specific beliefs about the roles and responsibilities of faculty and students in the classroom. The persistence of Soviet-era educational traditions is not unique to Kazakhstan. A nation's education curricula and regulatory frameworks fundamentally reflect broader societal transformations and evolving perspectives on educational challenges. Russia's experience provides an informative parallel, where despite significant political and social transformations, the educational system maintains substantial continuity with Soviet-era practices, textbooks, etc. (Karp, 2021). The interconnection between national culture, curricula, and educational traditions (Kaiser et al., 2006; Schmidt et al., 1996) suggest that with an unchanged bigger, national picture, faculty as individuals are not likely to acculturate toward new education traditions. However, ongoing shift toward greater institutional autonomy and internationalization in Kazakhstan may create an environment more conducive to pedagogical innovation (Hartley et al., 2016). This transition could also provide increasing opportunities and motivation for faculty to engage with new educational traditions.

Moreover, Berry's study on immigrants in Canada found that the result of individual acculturation is relevant to the acculturation of the environment they are moving to and is most successful when individuals' preferred strategies align with those encouraged by the broader society (Berry, 2006). The complex process of acculturation suggests that individual changes influence the group/cultural level of acculturation, and acculturation at the group/cultural level also influences acculturation at the psychological/individual level (Berry, 2017; Berry & Sam, 2016). The changes made by individual Kazakhstani faculty are currently constrained by the remnants of centralization in its current educational system, curriculum, and typical teaching approaches.

With ongoing reforms in Kazakhstan and its educational system, the centralized and unified features will be reduced (as part of the reform goals), and the further adaptations to the new educational traditions among Kazakhstani faculty can be expected over time. As Berry's (2003) theory suggests, while immediate changes may be observable following initial cultural contact, acculturation is a gradual process. For example, Hallinger and Lu's (2011) study in Thailand reported high levels of rejection at the beginning of their research. However, after several years, participants in their study successfully adapted to the new teaching approaches developed from a very different educational tradition. The span of this study enables us to observe the influences of cultural contact at a personal level, such as perceptual and behavioral adjustments, but group/cultural level changes require longer time to become observable. As the current study only explored the acculturation of individuals, and the duration of the study was one year, the result may be insufficient to examine the development of new values or traditions as groups.

That the Kazakhstani faculty in this study adopted an integration acculturation strategy is consistent with research indicating that individuals in multicultural environments are more likely to choose integration as their preferred acculturation strategy (Fox et al., 2013; Phinney et al., 1992). Berry’s (2003) framework suggested that when acculturation experiences push people beyond their ability to cope, changes manifest as acculturative stress that disrupts normal daily functioning. Research on acculturation processes demonstrates that individuals who choose integration strategies typically adapt more successfully to new cultural contexts and maintain better psychological well-being. Berry and Sam (2010) attribute this success to these individuals' ability to utilize resources and skills from both their original and new cultural contexts, enhancing their capacity to navigate cultural transitions effectively. While these findings come from immigration studies and cannot be directly generalized to educational contexts, the underlying principle appears relevant to pedagogical adaptation. Faculty who successfully maintain valuable aspects of their heritage educational traditions while engaging with new educational traditions may benefit from expanded knowledge and skills in pedagogy. This integration approach could represent a promising pathway for Kazakhstani faculty navigating educational reform, allowing them to build upon existing pedagogical strengths while incorporating innovative teaching approaches.

5.3 Conclusion

PD features demonstrated influence on both faculty perceptions and teaching practices as the result of the PD. Faculty perceptions and implementation of PD materials showed significantly greater changes compared to their teaching approaches and pedagogical beliefs, where changes were more limited. Contextual features (class types, institutional structures, educational system) of Kazakhstani educational traditions contributed also to faculty changes. By the end of the PD program, faculty participants were employing the integration acculturation strategy, balancing implementation of the new instructional materials within their existing teacher-centered approaches.

The cross-cultural PD experience represents an ongoing acculturation process rather than a single learning event. The changes in faculty perceptions and teaching practices observed should be understood as a moment within a longer trajectory rather than a fixed outcome of intervention. Acculturation is a complex process occurring at both cultural and individual levels when cultures come into contact. The changes happening at the cultural/group level can also influence individual acculturation (Berry, 2003, 2017). Thus, under the process of educational reform, faculty negotiation between heritage and new educational traditions may be continuing now that the PD has concluded. As Kazakhstani educational reform continues its efforts to increase autonomy and internationalization, the cultural environment may evolve and create new educational conditions. Faculty acculturation strategies toward new educational traditions may continue to evolve, potentially leading to different perceptions and teaching practices over time.

This understanding has significant implications for how we conceptualize and evaluate cross-cultural PD. It suggests the need for sustained support that acknowledges the gradual nature of educational change when navigating between different educational traditions. Rather than expecting immediate transformation, PD initiatives should establish foundations for the ongoing acculturation process while respecting valuable aspects of established practices that faculty wish to preserve.

5.4 Theoretical Contributions

This study developed the Cross-Cultural PD Educational Acculturation Model by extending Berry's (2003) Acculturation Framework into educational contexts and integrating it with Desimone's (2009) Framework for Studying the Effects of Professional Development on Teachers. Rather than viewing PD as a simple skill-acquisition process, this framework reconceptualizes cross-cultural PDs as an acculturation process where educators navigate between different educational traditions that promote different and even conflicting teaching philosophies and approaches than their heritage educational traditions. When educators encounter PD that originates from educational traditions different from their own, the participation in the PD constitutes a cultural contact experience. This perspective extends beyond traditional views of PD as merely transmitting skills or knowledge, positioning it instead as a complex cultural negotiation where participants must reconcile new pedagogical approaches with deeply held educational values and practices. The learning and implementation process becomes an acculturation journey rather than simple adoption.

The findings of this study contribute to both acculturation studies and mathematics education research by illuminating PD as a cultural adaptation processes and outcomes of PD as educational acculturation results. The results suggest that educational acculturation follows patterns similar to those observed in broader cultural adaptation research. As Berry (2003, 2017) documented in sociocultural contexts, educational acculturation is a gradual process where changes first appear in aspects requiring less cognitive or emotional investment. In the educational context, PD participants may also more readily adopt new instructional materials and tools while maintaining their heritage teaching approaches and beliefs about teaching and learning a subject – similar to how individuals in cross-cultural or multicultural settings might adjust external behaviors before core cultural values (Berry, 2003, 2017).

The acculturation process continues well beyond the formal PD period, as participants continually negotiate between maintaining heritage educational traditions and engaging with new ones (Berry, 2017; Berry et al., 2006). The outcomes measured at any particular point reflect a specific stage in this ongoing acculturation journey rather than final adoption or rejection. The study confirmed that educational acculturation is also a longitudinal process, with changes in PD participants occurring gradually, and that changes in the early stages of contact can be observed through tasks that require less effort to cope, behaviorally or mentally, such as development of positive perceptions toward the educational traditions (RQ1) and implementation of new instructional materials (RQ 2) (Berry, 2003). The changes in aspects that are deeply connected to cultural or societal background, such as pedagogy, may appear after multiple years of contact (Berry, 2017). As demonstrated by the immigrants or refugees in Berry's studies, the process of acculturation will continue after the firsthand contact, in this case after the end of the PD program, and participants' acculturation strategies may continuously change.

Educational acculturation strategies in this framework should be understood as descriptive outcomes of how individuals navigate between educational traditions rather than hierarchical goals or evaluative measurements. Integration, assimilation, and separation are not ranked in terms of desirability or success, but, rather, represent different responses of individual participants based on their unique circumstances, needs, and contexts. Each strategy reflects their agency in determining their balance of heritage and new educational traditions in ways that are meaningful and practical within their professional contexts (Sam & Berry, 2010).

Moreover, consistent with Berry's findings across diverse contexts, integration emerged as the predominant acculturation strategy among faculty participants in this study (RQ3). This strategy allows participants to maintain a certain level of heritage educational traditions while exploring and experimenting with the new educational traditions, which may reduce the possibility of encountering acculturation stress (Berry, 2017). This study suggests that participants in cross-cultural PD are likely to choose integration strategies based on the particular new educational. Because integrating new materials is an easy change to make, participants are unlikely to separate themselves fully from all PD materials, just as successful immigrant adaptation often involves selective integration rather than full assimilation. The changes in pedagogy may start with positive perceptions toward new teaching approaches, as the faculty had in this study. Thus, integration is the most likely acculturation strategy to be observed.

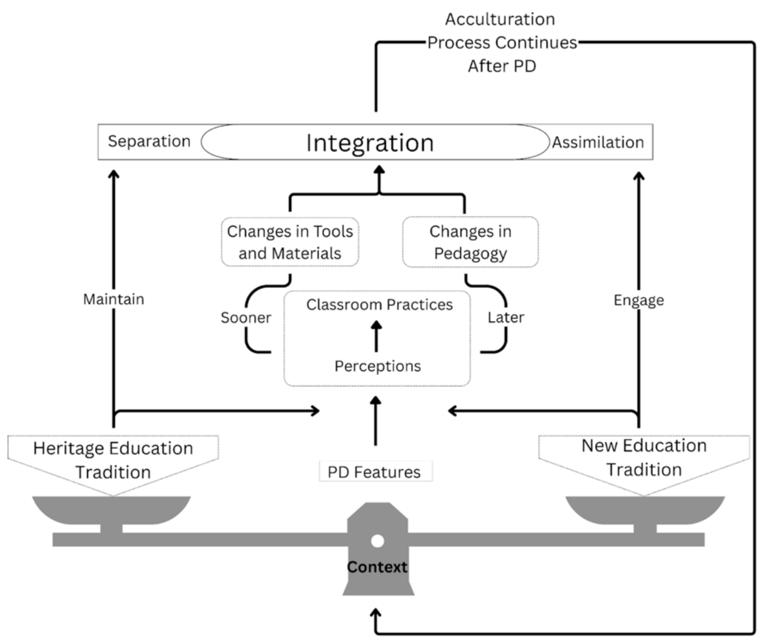
The specific context in which the acculturation takes place significantly influences how PD participants respond to new educational traditions (RQ 5). Just as immigrants' acculturation experiences vary based on host community receptivity, power dynamics, and institutional support (Berry, 2006; Berry et al., 2006; Schwartz et al., 2010), the effectiveness of cross-cultural PD can also be influenced by education system structures, institutional factors, leadership support, broader educational reforms initiatives, language, etc. While it is possible for individuals to make changes locally, this can be very difficult when the broader environment remains unchanged. When acculturation happens at the cultural or group level, it substantially contributes to the acculturation of individuals. When a cross-cultural PD program is aligned with broader societal goals, it generally contributes to participants' development of positive perceptions toward new educational traditions as well as greater willingness to use or attempt to use new materials and teaching approaches in their existing practices (Schwartz et al., 2010).

This study confirms that Desimone's (2009) core PD features remain influential in cross-cultural PD contexts (RQ 4). Content focus and coherence proved foundational to the cultural contact experience, enabling participants to understand and anticipate the potential value of new educational traditions within their specific contexts. The coherence of PD materials reduced difficulties in implementing changes and expanded possibilities for participants to adapt materials from the new educational traditions (Penuel et al., 2007). While active learning opportunities and sufficient duration provided essential support for participants to develop deeper conceptual understanding and practical skills needed for navigating between educational traditions, collaborative participation created peer support networks and internal learning communities that helped participants overcome acculturation barriers. These features functioned not merely as technical elements but as bridges, enabling participants to access new educational traditions while preserving sufficient space for their own cultural negotiations.

The Cross-Cultural PD Educational Acculturation Model has significant applications beyond this specific study. It provides a valuable theoretical lens for investigating the effectiveness of cross-cultural PD programs that are developed based on educational traditions different from those familiar to participants. The refined Cross-Cultural PD Educational Acculturation Model is adjusted based on the findings of this study as shown in Figure 15. By conceptualizing cross-cultural PD as an acculturation process rather than simply a knowledge transmission endeavor, researchers and PD designers gain new insights into how PD participants navigate tensions between heritage traditions and new educational traditions. This framework is particularly valuable for international educational collaborations and educational reform initiatives. It offers both analytical guidance for researchers studying PD effectiveness across cultural boundaries and practical guidance for designers seeking to create more culturally responsive PD experiences.

Figure 15

The Cross-Cultural PD Educational Acculturation Model



5.5 Practical Implications

The observed positive perceptions and high rate of integration of new instructional tools in this study suggest that the effectiveness of a cross-cultural PD program can be enhanced by providing coherent, ready-to-use instructional materials that participants can immediately implement in their classrooms. While the participants take time to familiarize and negotiate with values and philosophies of new educational traditions, the ready-to-use materials provide them with chances to actively engage with some aspects of new educational traditions in their classrooms. The appropriate resources may significantly increase the effectiveness of the PD and the likelihood of implementation during early acculturation phases. The limited recognition of student-centered approaches in this study indicates a need for PD sessions to explicitly address new pedagogies, particularly those unfamiliar to participants. PD content should clearly articulate that pedagogical change involves embracing different philosophical approaches to teaching and learning, not just implementing new tools or textbooks.

Furthermore, the observed differences in implementation levels between instructional tools and teaching approaches suggest that pedagogy requires significantly more time and effort to transform. In this study, the resistance to change may stem from deep roots in the Kazakhstani educational traditions, which have developed within the country's specific cultural and historical contexts. Consequently, pedagogy transformation is unlikely to manifest during one PD program. Although acculturation processes continue beyond the initial period of cultural contact, structured long-term projects with sustained participant support are suggested for improving the effectiveness of PD to support more difficult transformations like pedagogy, especially within a cross-cultural context. Besides offering specific workshops, PD providers should foster sustained support for participants, thereby allowing PD participants to have sufficient time to consider and experiment with new instructional tools, materials, and approaches. PD providers should incorporate structured follow-up components to address emergent implementation issues and reinforce learning. In this study, due to funding constraints and the restriction on the project timeline established by the funding agency, support sessions of the PD could not continue beyond the program's one-year duration. Funding can also be a contextual influencer of the effectiveness of a PD program.

Other contextual factors such as the mode of teaching and university course structures may have influenced outcomes of PD in this study. Thus, it is suggested that PD programs should be designed in a way that considers various contextual factors that could shape the effectiveness of the PD. More importantly, the study findings revealed how the educational system structures manifest in classroom practices, as indicated by differences in faculty teaching practices between lectures and practice seminars. Acculturation on a culture/group level significantly contributes to acculturation on the individual/psychological level. Countries and institutions pursuing educational reforms should therefore invest in structural and policy changes that align with reform objectives and provide educators with opportunities for acculturation. Without corresponding systemic changes, transformations in individual classrooms or among educators will likely remain limited.

While developing a cross-cultural PD, the providers should develop an understanding of the participants' heritage educational tradition, considering how balancing the educational traditions could improve the effectiveness of PD. The PD should create a culturally sensitive environment to support its participants. Before conducting a cross-cultural PD, it is suggested that PD designers should conduct thorough contextual analysis, adapting their approaches to align with existing institutional norms and constraints.

5.6 Limitations and Future Research

The primary limitation of this study lies in its small sample size. With only six participants, the study results cannot be generalized. To investigate the potential influences of contextual factors, future studies should expand the sample size, ensuring a more comprehensive representation of the target population. Furthermore, the small number of participants from each university limited the possibility of conducting an in-depth analysis of institutional factors that might have affected the PD outcomes. To gain a more thorough understanding of the differences between various types of institutions and the influences of institutional factors, future research would benefit from involving a larger sample of institutions and including more participants from each institution. This approach would enable researchers to identify and explore the complex interplay between institutional characteristics and the effectiveness of PD.

The PD program in this study lasted one year and included 3, 3-hour PD sessions and five 1-hour follow-up meetings. Considering the gradual nature of acculturation and change in pedagogy, along with ongoing shifts in society and educational systems, this study was limited in its ability to observe the potential long-term effects of cross-cultural PD on faculty. Future studies should extend the research period, if resources permit, to better examine the long-term effects of PD on participants' acculturation. Additionally, future research could lengthen the PD program by providing continued support after the initial training sessions. Moran and McMaster (2009) suggest that extensive follow-up after training may strengthen the development of faculty perceptions. Future research would benefit from assessing participants' perceptions during different stages of the extended follow-up period to determine the potential impact of sustained support and engagement.

Additionally, the COVID-19 pandemic necessitated change to the PD and the discrete mathematics course mode from in-person to online, which may have reduced the effectiveness of the PD and increased the complexity of implementation for the faculty. While future studies are unlikely to encounter the same conditions, it is suggested that researchers adjust the design of the PD to accommodate new situations if similar circumstances arise.

During the acculturation process, group-level phenomena can establish parameters for individual adaptation (Berry, 2003). Future research on acculturation in educational contexts should examine the interaction between group-level and individual-level acculturation processes. Future studies on the effectiveness of cross-cultural PD would benefit from expanding the exploration beyond individual participants to incorporate institutional-level analysis that examines historical trajectories and cultural dynamics in both originating and receiving groups. Additionally, the reciprocal nature of acculturation merits attention, particularly the exploration of whether PD providers also experience acculturation during cultural contact and, if so, what influences this has on their practices. This bidirectional focus would extend understanding of how power dynamics between educational "exporters" and "importers" shape mutual adaptation processes and program effectiveness (Berry, 2006).

For future research, the findings of this study suggest viewing PD program as educational acculturation experiences for both sides. This study not only provided PD participants with insights into different educational traditions but also offered PD providers an understanding of the participants' educational traditions and their influence on the adaptation of new educational traditions. Future PD providers are encouraged to take participants' educational traditions into account when designing PDs and to provide culturally sensitive workshops and materials to better suit the needs and situations of participants. It may ultimately enhance the effectiveness and sustainability of the cross-cultural PD experience.

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Appendix A: Example of Coded Interview Transcripts

Time	Who	Statements	PD Feature	Ed Traditions	Notes
0:00	I	Q1. How has this workshop affected your teaching discrete mathematics course?			
0:11	F	That was a great workshop, so it was helpful to make our lectures seminars better.	Coherence		Faculty think PD help them to rise course quality
		So, I learned new, new things for me was GeoGebra. The teaches of (with) GeoGebra, there was great for especially this, this discrete mathematics.	Content Focus		Faculty gain new knowledge and skill of teaching with GG Faculty think GG is great for teaching discrete
		And also, we used some other new (text)books for us, because we used only one (text)book we used for three years, yeah yes, we used one book and it was good to use extra books also.	Content Focus		PD textbook ais new for faculty in three years Faculty feel good to use a new textbook
0:58	I	Q2. What has been the most useful part of the workshop?			
1:05	F	I guess GeoGebra. But it will be better if we have to learn [it with] more time yeah.	Content Focus, Duration		GeoGebra was most helpful More time on GG would be better
1:22	I	Q3. What has been the least useful part of the workshop?			
1:28	F	Nothing, I guess. Everything was great.			
1:35	I	Q4. Did you use the project textbook in your teaching discrete mathematics?			
1:39	F	Yeah yeah. I use it to solve examples mostly. And some theoretical [explanations] but mostly I use it for solving examples.	Content Focus	Teaching Methodology	PD textbook is useful Faculty used PD textbook for problem/practices examples PD textbook used for theoretical explanations
1:56	I	How did you use the textbook, can you give an example?			

2:01	F	How I use it, you mean yeah? [I use the book] on lessons I give [examples and problems] from that book, some good examples, examples to solve to students. That's how I use it. And some homework assignments to students also from the book. And some theories I use it from the books, which was different from others' books, from my book that I use.	Coherence	Teaching Methodolog y	Faculty show students solving process with PD textbook examples Faculty give students examples from PD textbook as task and HW some theory in PD textbook is different than another book and faculty used
2:41	I	Did students work with the textbook during class?			
2:46	F	Yeah yeah, I said that before, they solved some examples from the book. And so, some exercises, assignment for homework.	Coherence	Teaching Methodolog y	Students used PD textbook as problem sets for exercises and HW
3:04	I	Q5. Did you use GeoGebra supplements in your teaching discrete mathematics?			
3:11	F	Yeah, approximately in 30% of my lectures I used GeoGebra. That was not usable for all topics, but used was usable in logical laws, sets and in relations of functions.	Coherence, Content Focus		Faculty used GG in 30% of their lessons GG is relevant to some discrete topics but not all GG cannot be used for some topics
3:34	I	Can you give an example?			
3:38	F	How I used it yeah? We used, as I said in set theory. That was great to show to students, especially in online, because the problem was online not face to face lectures, so that's why it needs more technical, technological use for us to show them good graphics to understand them better used in set theories. In logical laws also was very useful to see how its work.	Content Focus	Teaching Methodolog y, Organizatio nal Structure	Faculty use GG to present content to students online Faculty thin online is a reason why need technology in teaching GG provide good visual representation for students to understand the content better Faculty presentation leading lesson
4:32	I	Did student use GeoGebra applet that during class?			

4:39	F	Yeah, so I asked questions from them, that's only [time] they used [it]. But, in future, I think I will I will put into syllabus the GeoGebra as mandatory to learn them and make at least one topic to solve, to make one topic to move, to one topic exercises from GeoGebra, so they will make it. They will create new exercises.		Teaching Methodology, Learning objectives for students	Students use GG for practicing problems when faculty give tasks on GG Faculty think in future to add GG as mandatory skills to students to learn
5:19	I	Can you give an example, how students use GeoGebra during class?			
5:28	F	Oh yeah I am asking question with showing them GeoGebra, they are solving with GeoGebra.		Teaching Methodology	faculty show students how to use GG and ask students to solve with GG
5:40	I	So, students are solving questions on GeoGebra?			
5:44	F	Yeah yeah yeah. There we have a lot of examples, so before the final answer, finally us for showing answers, they give their own answer, and we are check it for.	Content Focus, Coherence	Teaching Methodology, Culture Norms	Faculty find GG examples can be used in their lesson Students use GG individually to find answer for questions and faculty show right answer with whole class to check with GG

Appendix B: Example of Field Notes for Lesson Recordings

CONTEXTUAL BACKGROUND AND ACTIVITIES	Online lecture lesson with 38 students (supposed to be). Faculty share screen. No students' cameras were turned on, faculty cameras were not turned on.	
Time	Field Notes	Analytic Notes
0:13	Faculty shared the screen with a blank note page and introduced students to today's class to continue on the topic mathematics induction. The goal of the class is to solve problems, and the faculty want to do practice with students.	
0:55	Faculty asked what is mathematics induction, how to define and how to prove with mathematical induction. Faculty continued to review the basic steps of mathematical induction. The basic steps and inductive steps were written on the shared screen. After that the faculty asked "any questions?" but leave no time before go into the next event	Did not give students time to think. Didn't ask students go give answer Faculty write definitions on board
2:29	Faculty choose the first example from HW, and write the question on board. [1+2+2^2..+2^n=2^n+1-1] Faculty asked "how can we solve this?" Give a couple seconds before calling on a student.	I can't tell whether the student volunteered.
4:05	S1: there is basic step faculty write "basic step" onboard and asked "which number you will choose, let's set this as P(n)" Faculty set P(n) before student answer S1: P(1) F: "Let's check. If it's 1, it will work" Then faculty write out the process on board, and test P(1) satisfied. "now inductive step. I assume this one is k is true..." Faculty continue writing on the beard the solving process. "Then we need to prove p(k+1) is also true" Faculty continues the proofing process on board."This is true, k+1. .. I think is this our example".	Student did not explain her answer, but faculty give the explanation Faculty asked the students to share their answers but F led the solving process and explained everything themselves.
6:50	F: "anyone else, how can we prove this one?" S : "P(k)? P(k+2) ia part of k+1? Part of P(k+1)" F: "Yes. You mean this part (writing on board) is part of this expression (drawing arrow)" Faculty explain the relation between expression P(k) and P(k+1) by writing on the board. F:"Any questions?" GIve thinking time	Cannot tell whether it is the same student Faculty cut off students before they say P(k+1) Faculty explanation was accurate but not what the student said
8:11	Go to the next example. F: " I guess you know what a geometric sequence is, or (writing "progression" on board)". Faculty explain and wiring on board the initial term and ratio. Give a geometric sequence. Faculty wrote a series of given sequences and asked: "how can we identify the formula, what is the formula?" After the waiting time no students answered, so faculty wrote out the series in form with initial terms and ratios on board, and then wrote out the initial term, and asked "how do we shorten the expression, what does this look like?" no student responde.	Faculty assumed students knew the geometric sequence. The question about formula is not clear what is wanted, and when no student answers the faculty didn't give further direction but gave the answer. Faculty then asks students to simplify the expression, and asks to connect with periouse examples.
13:02	F:"if you look at the previous example, it looks like this one" Faculty copy down the expression of the first example to hint students. F: "Any ideas?	Students give an answer that is not clear or explained.

	How can we rewrite this expression in short form". No student answered, then the faculty called on two students who gave no responde, and the third student said she didn't have the answer. And the fourth student: "q^n-1?" F: "let us see... you mean we can write this (ex2) one in this (ex1) form right?" Faculty write out 1+q+...=q^n)-1 "Let's check. Let q=3, n=3". Faculty write the checking process on board. "this one is not equal"	Faculty gave his interpretation of students' answers.
18:10	Faculty wrote another idea to solve this question on board. "This should give some ideas. … if you see this one, then you will see q^n)-1)/q-1=q^n-1)+q^n-2…+1 for the geometric sequence. As you see here (sum of sequence we can substitute by this (evolved expression)" Faculty copy down the original sum expression and write down how he transforms the expression to a simpler form.	Faculty provided a new way of thinking, but he completed the whole process by himself.
21:30	Then faculty go into the proof with an induction process. "basic step. I will get our basis, S1 … will be b1. … This is true. Let's do n=2… same as this one. There is no problem with the basis." After writing, the process of checking basic faculty continues. F: "we will assume that n=k is true....then we need to prove n=k+1 is also true." Faculty stopped after writing the equation of n=k-1 and asked students to work on their own "starting from this step, you will do it yourself, I will give you two minutes. Then I will ask you questions, or anyone which to solve. if you have any question you can ask, you have to prove this is true". (24:50 -26:50)	Faculty give students time to work on their own after all the main ideas and procedures are given.
26:50	F: "Can anyone tell us their solution? Any ideas?" Faculty called on three students, all of them didn't have an answer to this question. F:" you can answer in Russian or Kazakh" F: "Let's see how to solve this." Faculty writing out the proof step by step on board. Faculty gave some explanation of his steps (show the previous expression as references to rewrite the current one) and read out loud what he wrote.	Faculty allow students to speak Russian or Kazakh in class if they are more comfortable. Students are not active to answer faculty's questions.
33:08	A student said something but it's not clear. Faculty called the student, and she responde "no questions".	
33:17	Next example. F: " what kind of questions we can see with mathematics inductions. Inequalities." Faculty write down an inequality expressed on board and ask "when would it be true? I think it will be true all the time. We have to show it is true for all positive integers" (Sounds like a student giving an answer but it is not clear). Faculty started to write the proof:"Basis, the first integer is 1 ... then the next one is 2.... it is true. Then inductive step, our assumption will be n=k (write k<2^k). Then we have to show that it is true when n=k+1. Anyone want to solve this one, this example?" After waiting time "also if you have an idea", give more waiting time, still not respond. THen we will continue. Faculty write step by step on board and read out loud, "this is how we can prove inequalities".	Faculty did not give the student time to answer the question he gave. Students were not active to answer/solve given questions although sometimes the faculty waited.
39:45	F: "Let's see one more example. Again if you have any question you can ask" Faculty write the questions. Faculty give some review about factorial to explain why there is a conditional restriction for this problem. Then faculty write on board basis steps of the proof, and setted n=k and n=k+1 situation for inductive steps. F: "how we can prove?" One student answered without faculty calling: "I open k+1 factorial" F: "what is that?" S:"K+1) k!" Then faculty recorded n>=4, so in this case k is at least 5. Faculty gives explanations on why the left side is less without asking	Faculty explain the situation with prior knowledge Faculty explains the students' answers to them. Students do not give the final explanation

	students.	
45:27	Next example. Faculty write the question on board, 3|N^3)-n. "I will give you one minute" F: "you know what is divisible yes? (Someone said yes) I hope you know how to prove this one… I hope you know what this sign means". Give divisible examples. F:"Do you have any ideas? n?" A student answered without calling on: " First step we can just put 1, 1-1 =0, 0 divisible by 3." Faculty wrote down what the student said, and " this is true. Next step." Faculty didn't let the student continue, but himself set up the inductive step, and wrote all the steps to prove n=k is true and set the equation for n=k+1. A student continued the steps, and the faculty wrote down what he said (not sure if it is the same student. Student: "as we said before 3^3 is divisible by 3..." Faculty cut the students and say " or we can substitute by a1". Student: "Yes. The final expression is divisible by 3".	Faculty gives students questions to solve from the first step. Faculty asked if students knew prior knowledge and gave explanations and examples. One student contributed to the first step, but the faculty didn't let him complete the whole problem. Faculty cut off students' when they talked about their steps and explained the final answer for the student.