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Abstract

This paper studies how job-security provisions affect labor market fluidity and, through it, aggregate productivity and wages. We develop a search-and-matching model with multi-worker firms in which redundant workers can be dismissed only gradually, slowing the replacement of unproductive matches. The novel component is the presence of unproductive employees who are protected and receive equal treatment in wage setting, generating labor hoarding within firms. Quantitative analysis shows that easing dismissal restrictions, i.e., increasing the probability that unproductive matches can be dissolved, raises labor productivity, output, and wages, while the unemployment rate responds only modestly because higher separations are largely offset by stronger vacancy creation and faster job finding.

JEL classification: E32, J20, J64.

Keywords: labor market fluidity, dismissal restrictions, labor hoarding, search and matching, multi-worker firms.

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1 Introduction

A growing literature links labor market fluidity—the pace of worker reallocation across employers—to aggregate productivity and economic performance (Davis and Haltiwanger, 2014; Decker et al., 2020; OECD, 2025).[1] At the same time, fluidity is shaped by labor market institutions, including employment protection that limits firms' ability to separate from redundant workers.[2] This raises a central policy question: how do job-security provisions that slow worker reallocation affect aggregate productivity, and at what cost in terms of unemployment? The question is especially salient in periods of rapid technological and structural change—when the efficient reallocation of workers across firms and tasks becomes critical—and it has gained renewed policy attention in recent discussions of advanced economies.

This paper explores the link between job-security provision and productivity by developing a novel theory of employment protection. The standard model of employment protection, such as Hopenhayn and Rogerson (1993), assumes a firing *tax*, which is imposed on firms wishing to reduce the optimal level of employment. In other words, employment protection in the literature has been modeled as a cost of labor adjustment. In contrast, our model explicitly describes *those who are protected*. Specifically, each firm employs a mass of employees and some of them randomly turn into the *unproductive* type. Employment protection is captured by two features: (i) firms can dismiss redundant (unproductive) workers only gradually, and (ii) firms must treat productive and unproductive employees equally in wage setting. Because unproductive employees contribute little to production yet must be paid the same wage, firms would like to replace them; dismissal restrictions slow this replacement, generating labor hoarding and an employment-composition channel from job security to aggregate productivity.

Multi-worker firms are essential for this mechanism. In a standard search-and-matching model with endogenous job destruction and one worker per job, an unprofitable match would dissolve immediately (Mortensen and Pissarides, 1994). With a mass of employees, however, firms can pool productive and unproductive labor and remain solvent, al-

[1] Using linked employer–employee data across advanced economies, OECD (2025) documents that job-to-job mobility that reallocates workers toward higher-wage and higher-productivity firms accounts for a sizeable fraction of aggregate wage and productivity growth.

[2] Cross-country evidence suggests that stringent dismissal regulations reduce efficiency-enhancing job reallocation by dampening vacancy creation and job mobility, with negative consequences for productivity growth (Bassanini, Nunziata, and Venn, 2009; Andrews and Cingano, 2014; Bartelsman, Haltiwanger, and Scarpetta, 2013).

lowing low-productivity matches to persist when dismissals are difficult. Building on multi-worker search-and-matching frameworks (Smith, 1999; Cahuc et al., 2008; Kudoh et al., 2019), we examine how dismissal frictions interact with vacancy creation, wages, and aggregate outcomes.

While the key policy instrument in the traditional model of employment protection is the firing tax rate, our policy instrument is the *firing probability* which measures the difficulty of dismissal in an economy. If this probability is zero, then no firing is allowed so that the firm must wait until unproductive employees voluntarily leave the firm, which occurs at the same rate as other productive employees. If the firing probability is one, then the firm can dismiss all unproductive employees within a period. The reciprocal of the firing probability is the average duration of employment for an unproductive employee.

We present two complementary versions of the model. The first delivers analytical results under a simplified wage-setting protocol.[3] We analytically establish that an increase in the firing probability increases the steady-state labor productivity. The mechanism is straightforward. When dismissal is difficult, unproductive matches persist, the within-firm composition of employment deteriorates, and average labor productivity declines. The second model incorporates intra-firm wage bargaining (Cahuc et al., 2008; Kudoh et al., 2019) and is calibrated to match key moments of the Japanese labor market. Quantitatively, easing dismissal restrictions increases output, labor productivity, and wages, while the unemployment rate responds only modestly because higher separations are largely offset by stronger vacancy creation and faster job finding. These findings highlight an employment-composition channel through which dismissal restrictions can depress aggregate productivity even in a stationary environment by slowing the replacement of unproductive matches.

This paper relates to several strands of literature. First, it contributes to research on labor market fluidity, reallocation, and aggregate performance (Davis and Haltiwanger, 2014; Decker et al., 2020, OECD, 2025). Where this literature documents that job-to-job mobility toward better firms is an important component of wage and productivity growth, we provide a structural mechanism that maps a specific institution—dismissal restrictions—into slower replacement of low-quality matches and weaker aggregate performance. Second, it complements work on employment protection and reallocation by identifying a distinct composition-based mechanism that operates even in a stationary

[3]Specifically, we follow Cooper et al. (2006) to assume that the wage rate is determined by firm's take-it-or-leave-it offer, under which the equilibrium wage rate equals the (constant) flow value of nonwork.

environment. In particular, while existing studies emphasize firing taxes or adjustment costs and their implications for reallocation and growth (Hopenhayn and Rogerson, 1993; Mukoyama and Osotimehin, 2019), we model job security as a restriction on the speed at which unproductive matches can be dissolved, combined with equal treatment in wage setting, which generates labor hoarding and deteriorates within-firm employment composition. Finally, our mechanism is conceptually related to the "zombie" phenomenon emphasized in the corporate restructuring literature: just as zombie firms can depress productivity by congesting reallocation, dismissal restrictions can sustain low-productivity matches—"zombie matches"—and slow the economy's reallocation of labor toward productive uses (Hoshi, 2006; Caballero, Hoshi, and Kashyap, 2008).

The rest of the paper is organized as follows. Section 2 documents cross-country evidence linking job tenure—a proxy for labor market fluidity—to productivity and wage growth. Section 3 presents the baseline model and analytical results. Section 4 develops the quantitative model and discusses calibration. Section 5 reports quantitative results and robustness. Section 6 concludes.

2 Cross-country Evidence: Job Tenure, Productivity, and Wage Growth

This section documents stylized facts that motivate the analysis and provide empirical benchmarks for the quantitative model. We focus on labor market fluidity and its relationship with aggregate productivity and wage growth.

Measuring labor market fluidity in a harmonized way across countries is challenging because internationally comparable measures of worker flows—especially employer-to-employer (job-to-job) transitions—are limited. For this reason, we use average job tenure as an informative and widely available proxy for labor market fluidity. The rationale is straightforward: in more fluid labor markets, worker mobility is more active and average tenure is shorter; conversely, longer tenure is indicative of more persistent matches and a lower pace of worker reallocation.

Figure 1 plots average job tenure against labor productivity across countries. A clear negative relationship emerges: economies with longer job tenure tend to exhibit lower labor productivity. The correlation coefficient is about -0.45. This pattern suggests that a rigid labor market, as reflected in long job tenure, may be associated with weaker aggregate productivity performance.

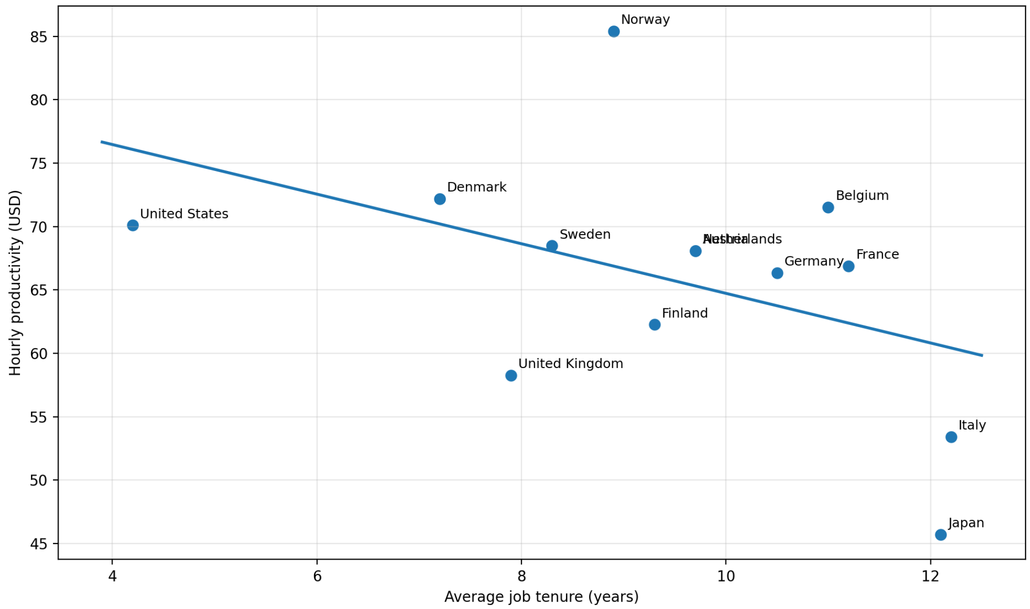

Figure 1: Job Tenures and Labor Productivity

We turn to examine the relationship between labor market fluidity and wage growth. Figure 2 shows a similarly negative relationship between job tenure and wage growth. Economies with longer tenure tend to experience slower wage growth (the correlation coefficient is about -0.64).

These relationships are descriptive and do not identify causal effects, but they provide a useful empirical motivation for studying how institutions that impede worker reallocation can affect aggregate outcomes.

In addition, we construct a complementary proxy that captures fluidity through the unemployment margin. Specifically, we define an unemployment-duration-based mobility indicator as the ratio of short-term unemployment to long-term unemployment:

$$\text{Ease of labor mobility} \equiv \frac{U^{<1y}}{U^{\geq 1y}},$$

where $U^{<1y}$ denotes the number of unemployed with duration less than one year and $U^{\geq 1y}$ denotes the number of unemployed with duration one year or more. This indicator is interpreted as a summary measure of the persistence of unemployment and the extent of labor market mismatch: a lower prevalence of long-term unemployment suggests smoother reallocation through unemployment.

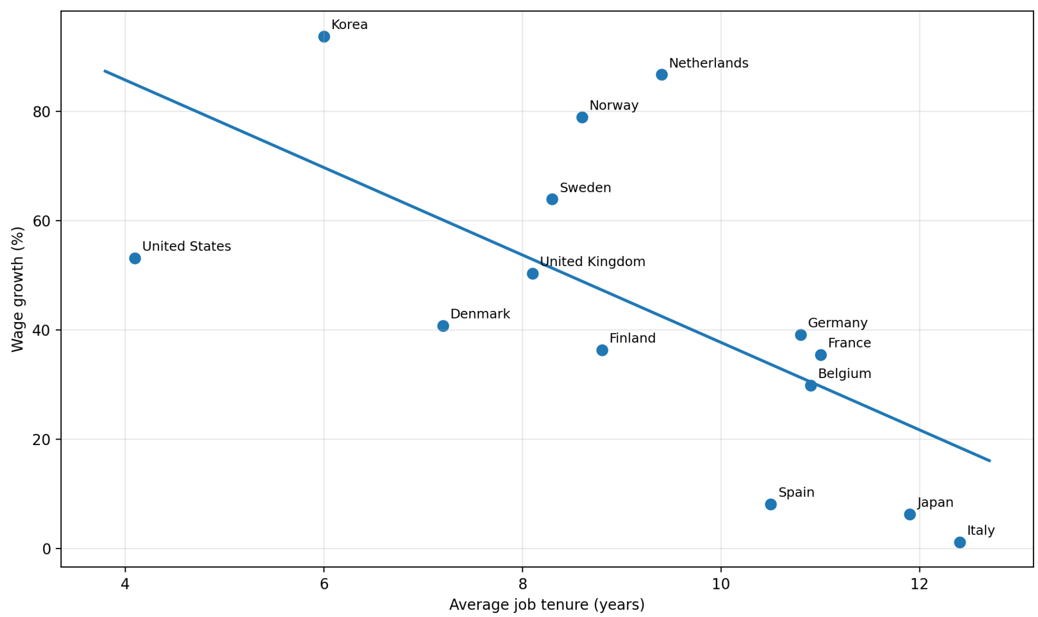

Figure 2: Job Tenures and Wage Growth

Figures 3 and 4 relate the unemployment-duration-based mobility indicator to labor productivity and wage growth, respectively. The relationship between the indicator and labor productivity is positive but modest in magnitude: the correlation is about 0.16. The indicator is strongly positively related to wage growth and the correlation is about 0.67. These cross-country patterns are consistent with the idea that more fluid labor markets are associated with stronger aggregate performance.

3 Employment Protection and Productivity

To highlight the novelty of our model of employment protection, this section presents a stylized search-matching model of the labor market in which, once employed, even workers with zero productivity can only be fired gradually. This sharply contrasts with the conventional model of firing restrictions, in which firms can dismiss redundant workers *instantly* by paying a firing cost. As a result, there are productive workers and unproductive workers *within each firm*. For brevity of exposition, we follow Cooper et al. (2007) to assume that the wage rate is determined by a take-it-or-leave-it offer by firms. A richer model of wage bargaining is presented and explored in Section 4.

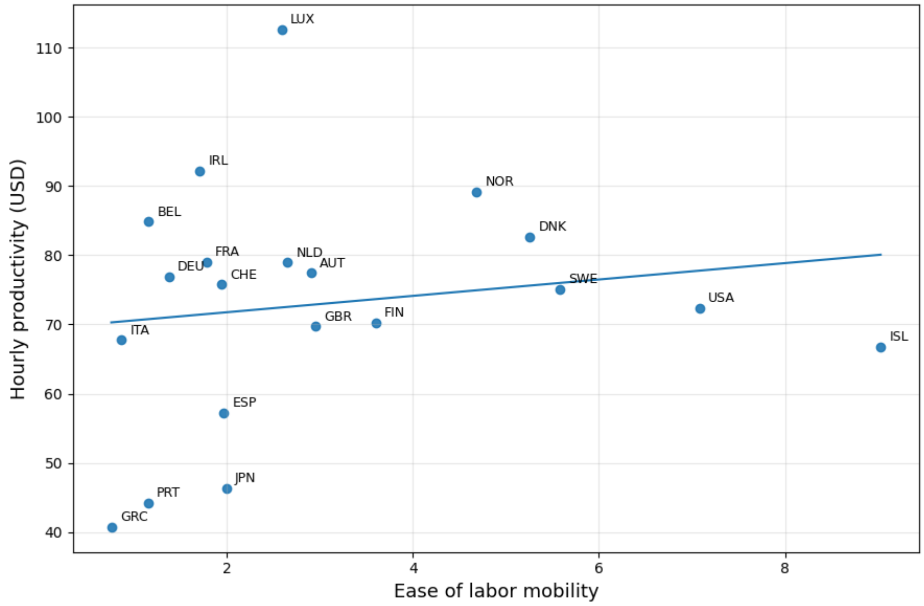

Figure 3: Ease of Labor Mobility and Labor Productivity

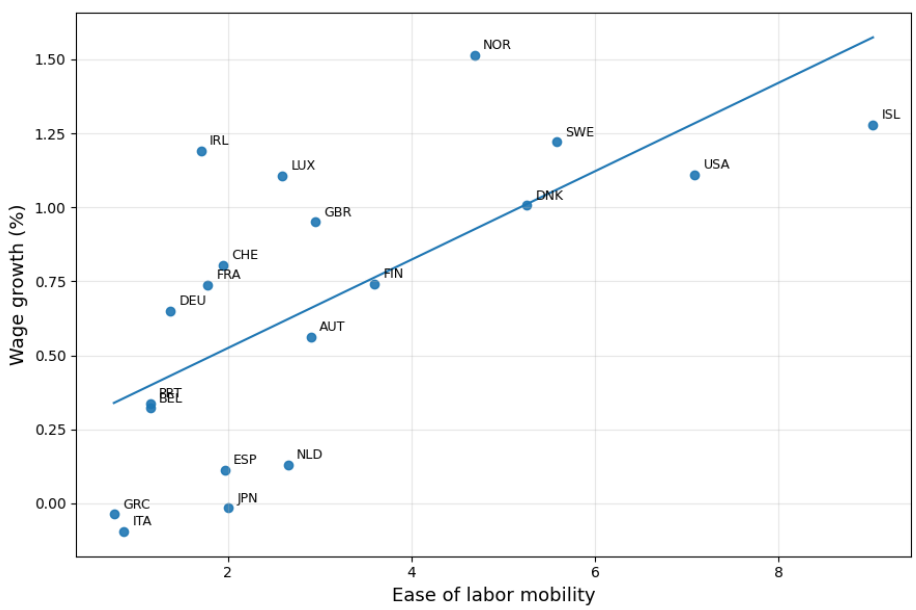

Figure 4: Ease of Labor Mobility and Wage Growth

3.1 Production

The novelty of our model is worker heterogeneity *within* a firm. While individuals are ex-ante homogeneous, once employed, some of them will eventually become *unproductive*. We intend to capture scenarios in which the skills required of employees evolve over time. For example, the skills demanded of middle managers may differ fundamentally from those required of newly hired employees. Another plausible scenario involves ongoing technological advancement, such as in information technology, which gradually alters the skill requirements year by year. We assume that a certain proportion of workers fail to adapt to these changes, thereby becoming unproductive.

The mass of total employees, L, satisfies $L = \ell + n$, where ℓ is the mass of productive employee and n is the mass of unproductive ones. We treat ℓ and n as continuous to ensure differentiability. Production technology we consider is

$$y = A\ell^{\alpha}k^{1-\alpha}, \tag{1}$$

where $0 < \alpha < 1$, $A > 0$, and k denotes capital used in production. Production function (1) postulates that unproductive employees do not contribute to production. While extreme, this assumption guarantees that firms have strong incentive to dismiss those employees. With employment protection, firms are forced to keep those workers. As a result, the labor productivity of the firm is given by

$$\frac{A\ell^{\alpha}k^{1-\alpha}}{\ell + n}. \tag{2}$$

It is not surprising that the labor productivity decreases with n.

3.2 Worker Flows

There is a unit measure of ex-ante homogeneous labor force. The labor market is with search-matching frictions. The mass of matches in period t is determined by the Cobb-Douglas matching function $m_0 U_t^{\xi} V_t^{1-\xi}$, where U_t is the mass of job seekers, V_t is the mass of aggregate job vacancies, and $m_0 > 0$ and $0 < \xi < 1$ are parameters. A vacancy is matched to a worker during a period with probability $q_t = m_0 U_t^{\xi} V_t^{1-\xi} / V_t = m_0 \theta_t^{-\xi} \equiv q(\theta_t)$, where $\theta_t \equiv V_t / U_t$. Similarly, the probability that a worker is matched with a vacancy, or the job finding rate, is given by $m_0 U_t^{\xi} V_t^{1-\xi} / U_t = m_0 \theta_t^{1-\xi} = \theta_t q(\theta_t)$.

Time is discrete. Each period starts with ℓ_t productive employees and n_t unproductive employees. After the final consumption good is produced, a fraction λ_F of unproductive employees can be dismissed, where $\lambda_F \in (0,1)$ is exogenous. We interpret λ_F as a policy parameter governing the ease of dismissal: a lower [redacted]F corresponds to tighter firing restrictions, while $\lambda_F = 1$ implies that all unproductive employees can be dismissed at the end of the period. When [redacted]$_F = 0$, direct dismissal is impossible, so firms must retain all unproductive employees unless the match is dissolved through a common exogenous separation margin, which occurs with probability $\lambda_Q \in (0,1)$. The parameter λ_Q applies to all ongoing matches, regardless of whether the worker is productive or unproductive, and captures quits and other separations not directly governed by dismissal regulation. At the end of each period, productive employees are hit by a negative productivity shock with probability $0 < \phi < 1$. Those who are hit by this shock become unproductive employees in the next period.

We normalize the mass of firms to unity. Thus, $V_t = v_t$, where v_t is the mass of each firm's vacancies. Similarly, ℓ_t and n_t also mean the aggregate mass of productive workers and unproductive workers, respectively. Worker flows are summarized by the following transition equations

$$\ell_{t+1} = \left(1 - \lambda_Q\right)(1-\phi)\,\ell_t + q(\theta_t)v_t, \tag{3}$$

$$n_{t+1} = (1-\lambda_F)\left(1-\lambda_Q\right) n_t + \left(1-\lambda_Q\right)\phi\ell_t, \tag{4}$$

$$U_{t+1} = \left[1 - \theta_t q(\theta_t)\right] U_t + \lambda_Q \ell_t + \left[\lambda_F + (1-\lambda_F)\,\lambda_Q\right] n_t, \tag{5}$$

and $1 = U_t + \ell_t + n_t$.

3.3 Firms

Firms are homogeneous and competitive in the product market. Each firm produces $A\ell^\alpha k^{1-\alpha}$ units of the final consumption good, pays wages and vacancy costs, and acquires investment good to build capital. With the vacancy-filling rate, $q(\theta)$, the mass of productive employees in the next period is given by $(1-\lambda_Q)(1-\phi)\ell + q(\theta)v$. We assume that each firm possesses a technology that converts one unit of the final consumption good into a unit of investment good. Then, the stock of capital used in the next period satisfies $(1-\delta)k + i$, where δ is the rate of capital depreciation and i is the level of investment good.

Let $S = (A, \ell, n, k, U)$ be the set of state variables. The value of a firm $J(S)$ satisfies the following Bellman equation:

$$J(S) = \max_{v,i}\left\{A\ell^\alpha k^{1-\alpha} - w(S)(\ell+n) - cv - i + \beta J(S_{+1})\right\}, \tag{6}$$

and the maximization is subject to $\ell_{+1} = (1-\lambda_Q)(1-\phi)\ell + q(\theta)v$ and $k_{+1} = (1-\delta)\,k + i$, where $\beta = 1/(1+r)$ is the discount factor and $c > 0$ is the vacancy cost. The wage rate $w(S)$ generally depends on the set of state variables. It is important to note that the firm pays the wage rate w to *all* employees, productive and unproductive ones. In other words, the firm is unable to set a distinct wage rate for unproductive workers. This assumption is crucial for our theory of employment protection because otherwise, the firm will set the wage rate of zero for unproductive employees.

The first-order conditions with respect to v and i imply

$$\beta J_\ell\left(S_{+1}\right) = \frac{c}{q(\theta)}, \tag{7}$$

$$\beta J_k\left(S_{+1}\right) = 1. \tag{8}$$

The envelope conditions yield

$$J_\ell\left(S\right) = \alpha A \ell^{\alpha-1} k^{1-\alpha} - w(S) - w_\ell(S)(\ell+n) + \left(1-\lambda_Q\right)\left(1-\phi\right)\beta J_\ell\left(S_{+1}\right), \tag{9}$$

$$J_k\left(S\right) = \left(1-\alpha\right) A \ell^{\alpha} k^{-\alpha} - w_k(h;S)(\ell+n) + \left(1-\delta\right)\beta J_k\left(S_{+1}\right). \tag{10}$$

3.4 Workers

The value of being a productive employee, $J^E(S)$, satisfies

$$J^E\left(S\right) = w\left(S\right) + \lambda_Q \beta J^U\left(S_{+1}\right) + \left(1-\lambda_Q\right)\left[\left(1-\phi\right)\beta J^E\left(S_{+1}\right) + \phi\beta J^N\left(S_{+1}\right)\right], \tag{11}$$

where $J^N\left(S\right)$ and $J^U(S)$ are the values of being an unproductive employee and a job-seeker, respectively. Once employed, the worker engages in production at the wage rate $w(S)$. After production, the match separates for exogenous reasons with probability λ_Q. If separation does not occur, the worker remains employed but may transit to an unproductive state with probability ϕ.

The value of an unproductive employee satisfies

$$J^N\left(S\right) = w\left(S\right) + \lambda_F \beta J^U\left(S_{+1}\right) + \left(1-\lambda_F\right)\left[\lambda_Q \beta J^U\left(S_{+1}\right) + (1-\lambda_Q)\beta J^N\left(S_{+1}\right)\right]. \tag{12}$$

A key assumption here is that *all workers are treated equally*, and as a result, unproductive workers receive the same wage rate $w(S)$ as productive ones do.[4] As is evident from the

[4] This should be viewed as part of the dismissal restriction in our model economy. To see why, suppose instead that the firm can set the wage rate flexibly for each employee when firing is prohibited. With wage flexibility, the firm then sets wages for unproductive workers so low that they are willing to leave the firm. Thus, wage flexibility for each employee neutralizes the firing restriction. The equal-treatment assumption is therefore a key device through which firms cannot undo dismissal restrictions through wage cuts or demotions.

production function (1), these workers contribute nothing to output or profit. From the firm's perspective, it would be optimal to dismiss all unproductive workers immediately; however, due to firing restrictions, their employment status is partially protected. As a result, there is a mass of employees who are unwilling to quit the current job whereas the firm wishes to dismiss them.

Whether an unproductive worker is dismissed in a period is determined randomly. Specifically, an unproductive employee will be fired after production with probability λ_F. Its reciprocal, $1/\lambda_F$, represents the average duration of employment for unproductive workers, which we interpret as an index of the strength of firing restrictions. If direct dismissal does not occur, the match may still separate through the common exogenous separation margin λ_Q, which applies to all matches. Notice that our model captures an important asymmetry between workers and firms: unproductive workers are unwilling to quit the current job at the going wage rate whereas the firm wishes to dismiss them.

We assume that whether an employee is productive or not is purely firm-specific. Thus, an unproductive employee can be productive in a different firm. As a result, all job seekers are homogeneous. The value of a job seeker is therefore standard:

$$J^U(S) = z + \theta q(\theta)\beta J^E(S_{+1}) + (1-\theta q(\theta))\beta J^U(S_{+1}), \tag{13}$$

where z is the flow value of nonwork.

3.5 Some Analytical Results

For brevity of exposition, we follow Cooper et al. (2007) to assume that the wage rate is determined such that the firm makes a take-it-or-leave-it offer to the worker, under which the worker is indifferent between employment and unemployment. Because the firm has no incentive to keep any unproductive worker, the firm's offer must make *productive workers* just indifferent between employment and unemployment, or

$$J^E(S_t) = J^U(S_t). \tag{14}$$

An important caveat is that the firm rather has an incentive to set wages to make sure $J^N(S_t) < J^U(S_t)$, under which all unproductive employees self-select to leave the firm voluntarily. To focus on our scenario in which the firm unwillingly hoard labor, we look for an equilibrium in which $J^N(S_t) \geq J^U(S_t)$.

Proposition 1 *The firm's take-it-or-leave-it wage offer satisfies $w = z$.*

Proof. In Appendix A. ■

With this wage rate, $w_\ell(S) = w_k(S) = 0$ holds in (9) and (10). Substitute these equations into (7) and (8) to obtain the job-creation condition and the optimal capital-labor ratio,

$$\left[1 + r - \left(1 - \lambda_Q\right)\left(1 - \phi\right)\right] \frac{c}{q(\theta)} = \alpha A \left(k/\ell\right)^{1-\alpha} - z, \tag{15}$$

$$k/\ell = \left(\frac{1-\alpha}{r+\delta} A\right)^{\frac{1}{\alpha}}, \tag{16}$$

which jointly determine the unique steady-state θ. We can then establish the following.

Proposition 2 *Suppose $\phi > 0$. A weaker firing restriction induces a higher steady-state labor productivity.*

Proof. In Appendix B. ■

An increase in λ_F implies that firing restrictions are relaxed. This reduces the proportion of unproductive employees within each firm. Observe that labor productivity is independent of the common exogenous separation rate λ_Q and the dismissal rate λ_F if $\phi = 0$.

4 A Quantitative Model

The previous section has presented a model in which some employees become unproductive. We have used the model to analytically show that the labor productivity increases with λ_F. An important limitation of the previous model is that the wage rate does not directly respond to a change in the firm's output because of the take-it-or-leave-it bargaining protocol. To consider a richer wage bargaining protocol, this section extends the previous model with an alternative production technology capturing the idea that unproductive workers cause productivity losses. We continue to assume the same transition equations (3)–(5) and the worker values (11)–(13).

4.1 Firms

The mass of total employees, L, satisfies $L = \ell + n$, where, as in the previous section, ℓ is the mass of productive employees and n is the mass of unproductive ones. Production technology we consider in this section is

$$y = A\left(1+n\right)^{-\psi}\left(\ell + n\right)^{\alpha} k^{1-\alpha} = \tilde{A} L^{\alpha} k^{1-\alpha}, \tag{17}$$

where $\psi > 0$, $0 < \alpha < 1$, $A > 0$, and k denotes capital used in production. Total factor productivity $\tilde{A}$ satisfies $\tilde{A} = A(1+n)^{-\psi}$. This implies that as n increases, the firm's productivity declines, creating the firm's incentive to dismiss those workers. As in the previous section, employment protection leads to output losses. As $n \to 0$, we have a standard production function $y = A\ell^{\alpha}k^{1-\alpha}$.

Transition equations (3) and (4) imply that the mass of total employees L_t evolves according to

$$\begin{aligned} L_{t+1} = \ell_{t+1} + n_{t+1} &= \left(1-\lambda_Q\right)(1-\phi)\,\ell_t + q(\theta_t)v_t + (1-\lambda_F)\left(1-\lambda_Q\right)n_t + \left(1-\lambda_Q\right)\phi\ell_t \\ &= \left(1-\lambda_Q\right)L_t + q(\theta_t)v_t - \lambda_F\left(1-\lambda_Q\right)n_t. \end{aligned} \tag{18}$$

Let $S = (\tilde{A}, L, k, U)$ be the set of state variables. The value of a firm $J(S)$ satisfies the following Bellman equation:

$$J(S) = \max_{v,i}\left\{\tilde{A}L^{\alpha}k^{1-\alpha} - w(S)L - cv - i + \beta J(S_{+1})\right\}, \tag{19}$$

where $\beta \equiv 1/(1+r)$ is the discount factor, $\tilde{A} = A(1+n)^{-\psi}$, $L = \ell + n$, and the maximization is subject to $L_{+1} = \left(1-\lambda_Q\right)L + q(\theta)v - \lambda_F\left(1-\lambda_Q\right)n$ and $k_{+1} = (1-\delta)k + i$. The first-order conditions with respect to v and i imply

$$\beta J_L(S_{+1}) = \frac{c}{q(\theta)}, \tag{20}$$

$$\beta J_k(S_{+1}) = 1. \tag{21}$$

The envelope conditions yield

$$J_L(S) = \alpha\tilde{A}L^{\alpha-1}k^{1-\alpha} - w(S) - w_L(S)L + \left(1-\lambda_Q\right)\beta J_L(S_{+1}), \tag{22}$$

$$J_k(S) = (1-\alpha)\tilde{A}L^{\alpha}k^{-\alpha} - w_k(S)L + (1-\delta)\beta J_k(S_{+1}). \tag{23}$$

Observe that (20) and (22) jointly imply

$$J_L(S) = \alpha\tilde{A}L^{\alpha-1}k^{1-\alpha} - w(S) - w_L(S)L + \left(1-\lambda_Q\right)\frac{c}{q(\theta)}, \tag{24}$$

which is the marginal contribution to the value of the firm of an increase in the number of employees.

4.2 Wage Determination

Contract is determined by the *representative marginal worker* and the firm. All other workers accept the contract. The wage rate is determined by

$$\eta J_L(S) = (1-\eta)\left[J^E(S) - J^U(S)\right], \tag{25}$$

where η is the worker bargaining power.

Proposition 3 *In any steady state, the wage rate satisfies*

$$w\left(L\right)=\frac{\alpha\tilde{A}k^{1-\alpha}}{\alpha+\frac{1}{\eta}-1}L^{\alpha-1}+\left(1-\eta\right)z+\theta\eta c$$
$$+\left(1-\eta\right)\left(1-\lambda_{Q}\right)\phi\beta\frac{\frac{\alpha\tilde{A}k^{1-\alpha}}{\alpha+\frac{1}{\eta}-1}L^{\alpha-1}-\eta z+\left[\frac{-\eta}{1-\eta}\right]\theta\eta c+\left(1-\lambda_{Q}\right)\phi\eta\frac{c}{q(\theta)}}{1-\left[\left(1-\lambda_{F}\right)-\phi\left(1-\eta\right)\right]\left(1-\lambda_{Q}\right)\beta}. \tag{26}$$

Proof. In Appendix C. ■

*Similarly, in any steady state, the job-creation condition and the optimal capital imply

$$\frac{\left(1+r\right)c}{q(\theta)}=\frac{1}{\alpha\eta+1-\eta}\alpha\tilde{A}L^{\alpha-1}k^{1-\alpha}-w\left(L\right)+\left(1-\lambda_{Q}\right)\frac{c}{q(\theta)}, \tag{27}$$

$$r+\delta=\left(1-\alpha\right)\tilde{A}L^{\alpha}k^{-\alpha}-\left(1-\alpha\right)\frac{\alpha\tilde{A}k^{-\alpha}}{\alpha+\frac{1}{\eta}-1}L^{\alpha}. \tag{28}$$

4.3 Steady-State Equilibrium Conditions

Note that $\beta=1/(1+r)$. Job-creation and capital demand:

$$\frac{\left(1+r\right)c}{q(\theta)}=\frac{1}{\alpha\eta+1-\eta}\alpha\tilde{A}L^{\alpha-1}k^{1-\alpha}-w\left(L\right)+\left(1-\lambda_{Q}\right)\frac{c}{q(\theta)}, \tag{29}$$

$$r+\delta=\left(1-\alpha\right)\tilde{A}L^{\alpha}k^{-\alpha}-\left(1-\alpha\right)\frac{\alpha\tilde{A}k^{-\alpha}}{\alpha+\frac{1}{\eta}-1}L^{\alpha}. \tag{30}$$

The wage rate:

$$w\left(L\right)=\frac{\alpha\tilde{A}k^{1-\alpha}}{\alpha+\frac{1}{\eta}-1}L^{\alpha-1}+\left(1-\eta\right)z+\theta\eta c$$
$$+\left(1-\eta\right)\left(1-\lambda_{Q}\right)\phi\beta\frac{\frac{\alpha\tilde{A}k^{1-\alpha}}{\alpha+\frac{1}{\eta}-1}L^{\alpha-1}-\eta z+\left[\frac{-\eta}{1-\eta}\right]\theta\eta c+\left(1-\lambda_{Q}\right)\phi\eta\frac{c}{q(\theta)}}{1-\left[\left(1-\lambda_{F}\right)-\phi\left(1-\eta\right)\right]\left(1-\lambda_{Q}\right)\beta}. \tag{31}$$

Worker flows

$$1=U+\ell+n, \tag{32}$$

$$\ell=\left(1-\lambda_{Q}\right)\left(1-\phi\right)\ell+\theta q(\theta)U, \tag{33}$$

$$n=\left(1-\lambda_{F}\right)\left(1-\lambda_{Q}\right)n+\left(1-\lambda_{Q}\right)\phi\ell, \tag{34}$$

$$U=\left[1-\theta q(\theta)\right]U+\lambda_{Q}\ell+\left[\lambda_{F}+\left(1-\lambda_{F}\right)\lambda_{Q}\right]n, \tag{35}$$

$$\theta=\frac{v}{U} \tag{36}$$

Observe that one of the above is redundant (because of $1 = U + \ell + n$). Thus,

$$\left[1 - \left(1 - \lambda_Q\right)(1 - \phi)\right] \ell = \theta q(\theta) U,$$
$$\theta q(\theta) U = \lambda_Q \ell + \left[\lambda_F + \left(1 - \lambda_F\right) \lambda_Q\right] \left(1 - U - \ell\right),$$

from which

$$U = \frac{\lambda_F + \left(1 - \lambda_F\right) \lambda_Q}{\frac{1 - \left(1 - \lambda_Q\right)(1 - \phi) + \left(1 - \lambda_Q\right) \lambda_F}{1 - \left(1 - \lambda_Q\right)(1 - \phi)} \theta q(\theta) + \lambda_F + \left(1 - \lambda_F\right) \lambda_Q}.$$

Output:

$$y = A\left(1 + n\right)^{-\psi} L^{\alpha} k^{1-\alpha} \tag{37}$$

The aggregate labor productivity is

$$\frac{y}{L} = \frac{A\left(1 + n\right)^{-\psi} L^{\alpha} k^{1-\alpha}}{L} = A\left(1 + n\right)^{-\psi} L^{\alpha-1} k^{1-\alpha} \tag{38}$$

5 Quantitative Analysis

This section quantifies the implications of dismissal restrictions for aggregate productivity and labor market outcomes using the model developed in Section 4. We first calibrate the model to key features of the Japanese labor market. We then use the calibrated model to assess how changes in the incidence of unproductive workers and in the ease of dismissing unproductive workers affect productivity, wages, and unemployment. Finally, we examine the robustness of the results under alternative calibrations and more extreme labor-hoarding scenarios.

5.1 Calibration

We calibrate the benchmark economy to key labor-market facts for Japan, following the strategy in Kudoh et al. (2019). We choose the model period to be one quarter and set the steady-state subjective discount rate to $r = 0.01$, so that the discount factor is $\beta = 1/(1 + r) = 0.99$. This implies an annual real interest rate of roughly four percent.

In the production function, we set the labor share parameter to $\alpha = 2/3$. Following Braun et al. (2006), the quarterly depreciation rate is set to $\delta = 0.028$. We normalize steady-state total factor productivity to $A = 1$. The parameter ψ, which governs the output loss associated with unproductive (hoarded) labor in our production technology, is set to $\psi = 1$ in the benchmark; we consider $\psi = 2$ in the sensitivity analysis.

The matching function takes the Cobb–Douglas form $m(U, V) = m_0 U_t^{\xi} V_t^{1-\xi}$, where m_0 is matching efficiency and ξ is the elasticity of matches with respect to job seekers. Following Lin and Miyamoto (2014), we set $\xi = 0.6$, which lies within the plausible range surveyed by Petrongolo and Pissarides (2001). We choose m_0 to match the job-finding probability and labor market tightness. Using the worker-flow estimates in Miyamoto (2025), the monthly job-finding probability is about 13 percent, the monthly separation probability is about 0.4 percent, and average labor market tightness is $\theta = 0.95$. As in Kudoh et al. (2019), we convert monthly flow probabilities to quarterly frequency by multiplying them by three. Given $\xi = 0.6$, the implied quarterly job-finding rate satisfies $f = m_0 \theta^{1-\xi}$, so matching efficiency is $m_0 = f/\theta^{1-\xi} = 0.398$.

Separations occur through two margins in the model. The first is a common exogenous separation margin, governed by λ_Q, which applies to all matches and captures quits and other separations not directly governed by dismissal regulation. The second is the dismissal margin, governed by λ_F, which applies only to unproductive workers. We discipline these parameters so that the model reproduces (i) the overall separation rate and (ii) the empirical importance of non-dismissal separations relative to dismissals. In the data, a large fraction of separations reflects quits and other worker-side exits rather than dismissals. To pin down λ_Q, we assume that 65 percent of total separations are accounted for by this non-dismissal margin. Although this calibration is motivated by evidence on voluntary separations, the model interpretation of λ_Q is broader: it captures all separations outside the dismissal margin. This decomposition strategy follows Silva and Toledo (2009) and Kuo and Miyamoto (2019), who split separations into exogenous and endogenous components.

We set the probability that an employed worker becomes unproductive, ϕ, by targeting the share of labor-hoarding workers in the data. The Recruit Works Institute estimates the number of hoarded workers by comparing "required employment"—implied by firms' performance and related indicators—with actual employment. While the labor-hoarding share varies over time, we target 5.2 percent, the average computed at five-year intervals over 1995–2015. Given nontrivial variation in the underlying estimates, we subsequently conduct sensitivity analysis with respect to ϕ.

There is no consensus in the literature regarding the worker's bargaining power in wage determination. We, therefore, focus on the symmetric case, $\eta = 1/2$. We set the unemployment benefit z to satisfy $z = 0.6W$, consistent with Japan's replacement ratio reported in Nickell (1997). Finally, we determine the remaining parameter(s)—in particular the vacancy posting cost c—by solving the steady-state equilibrium conditions of the model. Table 1 reports the calibrated parameter values.

Table 1: Calibrated Parameter Values

Parameters	Description	Values
r	Real interest rate	0.01
α	Labor share parameter	2/3
m_0	Matching efficiency	0.3981
ξ	Matching elasticity w.r.t. job seekers	0.6
z	Unemployment flow value	1.0023
ψ	Productivity penalty from hoarding	1.0
δ	Capital depreciation rate	0.0094×3
A	Aggregate productivity	1.0
η	Worker bargaining power	0.5
λ_Q	Common exogenous separation probability	0.0084
λ_F	Dismissal probability of an unproductive worker	0.0876
ϕ	Probability a productive worker becomes unproductive	0.0053
c	Vacancy cost	0.6700

5.2 Results

Table 2 reports the steady-state values of key endogenous variables implied by the calibrated model. The benchmark economy sets $\psi = 1$, which governs the severity of productivity losses associated with unproductive (hoarded) workers.

The model reproduces the labor-market moments targeted in the calibration. In particular, labor market tightness is $\theta = 0.95$ and the share of unproductive (hoarded) workers is $n/(\ell + n) = 0.052$, both of which are explicitly calibration targets. Consistent with these targets, the benchmark steady state implies an unemployment rate of $u = 0.032$, productive employment $\ell = 0.918$, unproductive employment $n = 0.050$, output $y = 2.058$, labor productivity $y/L = 2.126$, and the wage rate is $w = 1.671$.

We next examine how the steady state responds to changes in (i) the incidence of unproductive workers and (ii) the ease with which firms can dismiss unproductive workers.

Table 2: Results

	$\psi = 1$			$\psi = 2$		
	Baseline ($\phi = .0053, \lambda_F = .0876$)	$\phi = .0047$	$\lambda_F = .105$	Baseline ($\phi = .0053, \lambda_F = .0876$)	$\phi = .0047$	$\lambda_F = .105$
θ	0.95	0.984	0.976	0.95	1.017	1.002
u	0.032	0.030	0.032	0.032	0.029	0.032
ℓ	0.918	0.930	0.925	0.918	0.930	0.925
n	0.050	0.041	0.043	0.050	0.041	0.043
$n/(\ell + n)$	0.052	0.042	0.044	0.052	0.042	0.044
k	10.773	10.949	10.889	10.008	10.315	10.227
y	2.058	2.091	2.080	1.911	1.970	1.953
y/L	2.126	2.155	2.148	1.975	2.029	2.017
w	1.671	1.693	1.688	1.552	1.594	1.585

We begin by reducing the probability that a productive worker becomes unproductive by 20 percent, from $\phi = 0.0053$ to $\phi = 0.0047$. This change lowers unproductive employment from $n = 0.050$ to $n = 0.041$, reducing the hoarding share from 5.2 percent to 4.2 percent. The resulting improvement in employment composition increases output from 2.058 to 2.091 and raises labor productivity from 2.126 to 2.155. Labor market tightness rises from $\theta = 0.95$ to $\theta = 0.984$, while unemployment declines slightly from 3.2% to 3.0%. Wages also increase, from 1.671 to 1.693.

The underlying mechanism operates through the endogenous within-firm composition of employment. Productive workers become unproductive with probability ϕ, while unproductive workers exit the firm either through dismissals—governed by λ_F, which is lower under stronger firing restrictions—or through the common exogenous separation margin λ_Q, which captures quits and other separations not directly tied to dismissal regulation. In steady state, inflows into unproductive status are exactly offset by outflow-sthrough these two margins, pinning down the stock of unproductive workers n. Because unproductive workers reduce effective productivity through the production technology, a larger steady-state n translates into lower labor productivity.

This composition channel then feeds back to job creation. The job-creation condition links θ to the expected marginal surplus from a new hire. When ϕ is high and/or firing restrictions are strong (low λ_F), hiring becomes less attractive because a newly hired productive worker is more likely to turn into an unproductive worker who remains on the payroll for longer. This lowers the expected surplus, reduces vacancy posting, and depresses equilibrium labor market tightness. Conversely, when unproductive employment becomes less prevalent, he expected surplus rises. Because wages depend on labor market conditions and the marginal product of productive labor—while unproductive workers do not participate in the wage negotiation—reductions in hoarding raise productivity and strengthen the surplus associated with productive employment, leading to higher wages.

We next consider a relaxation of firing restrictions by increasing the probability that an unproductive worker is dismissed by 20 percent, from $\lambda_F = 0.0876$ to $\lambda_F = 0.105$. As in the previous exercise, increasing λ_F reduces the steady-state stock of unproductive workers and improves productivity. Unproductive workers falls to $n = 0.043$, and the hoarding share declines to 4.4 percent. Output increases to $y = 2.080$, labor productivity rises to 2.148, and labor market tightness increases to $\theta = 0.976$. At the same time, the unemployment rate remains essentially unchanged at 0.032. This reflects an offsetting force: while relaxing firing restrictions accelerates separations from the unproductive state—raising inflows into unemployment—it also strengthens job creation and job finding through higher tightness. As a result, changes in λ_F primarily operate through the composition and productivity margin, whereas their impact on the unemployment stock is muted by the simultaneous adjustment of unemployment inflows and outflows in equilibrium.

5.3 Robustness and Sensitivity Analysis

We now examine the robustness of the quantitative results. We begin by considering the role of the parameter ψ, which governs the severity of the productivity penalty associated with labor hoarding. In the benchmark specification, we set $\psi = 1$. Here, we consider a higher value, $\psi = 2$, and report the results in Table 2.

Because a larger ψ amplifies the productivity loss from hoarding, changes that reduce the stock of unproductive workers translate into larger improvements in output, productivity, and wages, as well as somewhat larger increases in labor market tightness. For example, when $\psi = 2$, a reduction in ϕ raises output y from 1.9115 to 1.9404 and increases labor market tightness θ from 0.95 to 0.983. Similarly, increasing λ_F raises output to 1.9339 and tightness to 0.978. Despite these amplified effects on output and labor market tightness, the unemployment response remains small, reflecting the same inflow–outflow offset emphasized above. Overall, variations in ψ affect the quantitative magnitude of the results but do not alter the qualitative conclusions. In what follows, we therefore return to the benchmark case with $\psi = 1$ and conduct further sensitivity analysis.

In the benchmark calibration, model parameters were chosen to match a labor-hoarding share of 5.2 percent, corresponding to the average share of unproductive workers over the period 1995–2015. However, the labor-hoarding share has been trending upward. According to estimates by the Recruit Works Institute (2015), it is projected to reach 8.2 percent by 2025. We therefore re-examine the quantitative implications of the model using this higher value as the calibration target.

To this end, we recalibrate the vacancy posting cost c, the flow value of unemployment z, the dismissal probability of unproductive workers λ_F, and the probability that an employed worker becomes unproductive ϕ so that the model matches the new target. The resulting parameter values are $c = 0.6422$, $z = 0.9622$, $\lambda_F = 0.056$, and $\phi = 0.0057$.

Under these recalibrated parameters, we conduct comparative statics exercises analogous to those in the benchmark analysis. Specifically, we consider a 20 percent increase in λ_F and a 20 percent reduction in ϕ relative to their benchmark values. Table 3 reports the steady-state outcomes and comparative statics results under the new calibration. For ease of comparison, the results are presented as percentage changes relative to the benchmark steady state rather than as levels.

Table 3: Sensitivity Analysis

	Benchmark			Target 8.2%			Extreme	
	Values	λ_F 20%up	ϕ 20% down	Values	λ_F 20%up	ϕ 20% down	Values	λ_F 100%up
θ	0.950	2.74	3.54	0.950	3.93	5.32	0.950	25.00
U	0.032	-0.26	-7.76	0.032	-0.31	-8.20	0.032	-0.11
V	0.030	2.46	-4.50	0.030	3.61	-3.32	0.030	24.86
ℓ	0.918	0.82	1.31	0.889	1.24	1.94	0.774	9.21
n	0.050	-14.73	-18.95	0.079	-13.73	-18.45	0.194	-36.84
$n/(\ell+n)$	0.052	-14.74	-19.16	0.082	-13.74	-18.67	0.200	-36.84
k	10.773	1.08	1.64	10.341	1.54	2.35	8.893	9.69
y	2.058	1.08	1.64	1.975	1.54	2.35	1.698	9.69
y/L	2.126	1.07	1.38	2.040	1.53	2.07	1.755	9.68
w	1.671	1.06	1.36	1.604	1.52	2.05	1.379	9.60

Note: Except for the columns labeled Values, all entries report percentage changes relative to the benchmark steady state, rather than levels.

We first consider the scenario in which λ_F increases. The direction of change for all endogenous variables is the same as in the benchmark case. Focusing on magnitudes, however, the responses of most variables—except for unproductive employment and its share in total employment—are larger under the new calibration than under the benchmark calibration. A similar pattern emerges in the scenario in which ϕ declines. Taken together, these results indicate that when the labor-hoarding share is higher (8.2 percent), the qualitative comparative statics remain unchanged, while the quantitative responses of wages and productivity become more pronounced.

We also consider an extreme case in which unproductive workers account for 20 percent of total employment—approximately four times the benchmark level. We recalibrate the model to target this share, which implies a dismissal probability of $\lambda_F = 0.023$. Whereas the previous exercises examined a 20 percent increase in λ_F, we now analyze the effects of a 100 percent increase, corresponding to a substantial relaxation of dismissal restrictions. Table 3 also reports the resulting steady-state values.

The results show that a large increase in λ_F leads to sizable changes in the model's endogenous variables. In particular, unproductive employment declines by 37 percent, and the share of unproductive workers in total employment falls by a similar magnitude. By contrast, productive employment increases by 9 percent, resulting in a 9.7 percent increase in labor productivity. Notably, despite the large separations of unproductive workers, the unemployment rate remains virtually unchanged. This muted unemployment response reflects a simultaneous and substantial expansion in job creation, with vacancies increasing by 25 percent. In sum, even when dismissal restrictions are substantially relaxed, aggregate productivity improves markedly, while the impact on unemployment remains limited.

6 Conclusion

This paper studied the link between employment protection and aggregate productivity through the lens of labor market fluidity. Motivated by cross-country evidence that economies with longer job tenure tend to display weaker productivity performance and slower wage growth, we developed a search-and-matching framework with multi-worker firms in which redundant workers can be dismissed only gradually. The novel component of the model is a stock of unproductive employees who are protected and treated equally as productive employees in wage setting. This equal-treatment restriction prevents firms from adjusting wages for unproductive labor, generates labor hoarding, and creates an employment-composition channel through which dismissal restrictions affect aggregate outcomes.

The mechanism is straightforward. When dismissal is difficult, unproductive matches persist, the within-firm composition of employment deteriorates, and average labor productivity declines. In the quantitative model calibrated to the Japanese labor market, relaxing dismissal restrictions increases long-run productivity and output and raises wages, while the unemployment rate responds only modestly because higher separations are largely offset by stronger vacancy creation and faster job finding.

These findings highlight a channel through which employment protection can depress aggregate productivity even in the absence of aggregate shocks. By slowing the replacement of unproductive matches, dismissal restrictions lower the average productivity of employed labor and weaken wage growth. From a policy perspective, reforms that raise labor market fluidity—whether through dismissal procedures or related institutions that facilitate reallocation—can generate meaningful efficiency gains. At the same time, any assessment of labor market reform should account for transitional dynamics and distributional consequences, as well as potential complementarities with social insurance and active labor market policies.

Several directions are left for future work. First, extending the framework to allow for job-to-job transitions and on-the-job search would enable a closer mapping between fluidity, wage growth, and career dynamics emphasized in recent empirical work. Second, labor market fluidity may shape the effectiveness of macroeconomic policies. Recent work (e.g., Kuo and Miyamoto, 2023) suggests that fiscal multipliers can depend on labor market fluidity, yet the underlying mechanism remains incompletely understood. Extending the framework developed here to analyze how labor market institutions interact with the transmission of macroeconomic policy is a promising direction.

Finally, the relevance of labor market fluidity extends beyond productivity and wages. As economies face rapid technological and structural change—including AI- and robot-driven automation and reallocation pressures associated with decarbonization—both firms and workers must adapt by reallocating across firms, tasks, and occupations. A more fluid labor market can facilitate this adjustment by speeding the replacement of obsolete matches and enabling labor to move toward expanding activities (Kudoh and Miyamoto, 2025). Integrating the dismissal-restriction mechanism studied here with quantitative models of automation and structural transformation is therefore a promising avenue for future research.

Appendix

A Proof of Proposition 1

From (11) and (13), we obtain

$$\begin{aligned} J^E(S) - J^U(S) &= w - z + (1-\lambda_Q)(1-\phi)\beta\left[J^E(S_{+1}) - J^U(S_{+1})\right] \\ &\quad + (1-\lambda_Q)\phi\beta\left[J^N(S_{+1}) - J^U(S_{+1})\right] - \theta q(\theta)\beta\left[J^E(S_{+1}) - J^U(S_{+1})\right], \end{aligned}$$

from which the contract (14) implies

$$w = z - \left(1-\lambda_Q\right)\phi\beta\left[J^N(S_{+1}) - J^U(S_{+1})\right]. \quad (39)$$

Similarly, (12) and (13) imply

$$\begin{aligned} J^N(S) - J^U(S) &= w - z + (1-\lambda_F)\lambda_Q\beta\left[J^U(S_{+1}) - J^U(S_{+1})\right] \\ &\quad + (1-\lambda_F)(1-\lambda_Q)\beta\left[J^N(S_{+1}) - J^U(S_{+1})\right] - \theta q(\theta)\beta\left[J^E(S_{+1}) - J^U(S_{+1})\right], \end{aligned}$$

from which we obtain

$$J^N(S) - J^U(S) = w - z + (1-\lambda_F)(1-\lambda_Q)\beta\left[J^N(S_{+1}) - J^U(S_{+1})\right]. \quad (40)$$

Because the wage rate is the same for all workers, we evaluate (40) at the contract (39) to obtain

$$J^N(S) - J^U(S) = \chi\left[J^N(S_{+1}) - J^U(S_{+1})\right],$$

where $\chi = [(1-\lambda_F) - \phi](1-\lambda_Q)\beta$. Thus,

$$\begin{aligned} J^N(S_t) - J^U(S_t) &= \chi\left[J^N(S_{t+1}) - J^U(S_{t+1})\right] \\ &= \lim_{T\to\infty}\chi^T\left[J^N(S_{t+T}) - J^U(S_{t+T})\right] \\ &= 0 \end{aligned}$$

holds for all t. This implies that the contract satisfies

$$w = z.$$

As in Cooper et al. (2007), the wage rate is independent of output.

B Proof of Proposition 2

Given θ, worker flows imply that the steady-state levels of U, ℓ, and n are given by

$$U = \frac{\lambda_F + (1-\lambda_F)\lambda_Q}{\frac{1-\left(1-\lambda_Q\right)(1-\phi)+\left(1-\lambda_Q\right)\lambda_F}{1-\left(1-\lambda_Q\right)(1-\phi)}\theta q(\theta) + \lambda_F + (1-\lambda_F)\lambda_Q}, \tag{41}$$

$$\ell = \frac{\theta q(\theta)}{1-\left(1-\lambda_Q\right)(1-\phi) + \frac{1-(1-\lambda_F)\left(1-\lambda_Q\right)+\left(1-\lambda_Q\right)\phi}{1-(1-\lambda_F)\left(1-\lambda_Q\right)}\theta q(\theta)}, \tag{42}$$

$$n = \frac{\left(1-\lambda_Q\right)\phi}{1-(1-\lambda_F)\left(1-\lambda_Q\right)}\ell. \tag{43}$$

Substitute these results into (2) to obtain the steady-state labor productivity:

$$\begin{aligned} LP = \frac{A\ell^{\alpha}k^{1-\alpha}}{\ell+n} &= \frac{\left(\frac{1-\alpha}{r+\delta}\right)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}\ell}{\ell + \frac{\left(1-\lambda_Q\right)\phi}{1-(1-\lambda_F)\left(1-\lambda_Q\right)}\ell} \\ &= \frac{1-(1-\lambda_F)\left(1-\lambda_Q\right)}{1-(1-\lambda_F)\left(1-\lambda_Q\right)+\left(1-\lambda_Q\right)\phi}\left(\frac{1-\alpha}{r+\delta}\right)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}}, \end{aligned} \tag{44}$$

from which

$$\frac{\partial LP}{\partial \lambda_F} = \frac{\left(1-\lambda_Q\right)^2\phi}{\left[1-(1-\lambda_F)\left(1-\lambda_Q\right)+\left(1-\lambda_Q\right)\phi\right]^2}\left(\frac{1-\alpha}{r+\delta}\right)^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} > 0$$

for $\phi > 0$.

C Proof of Proposition 3

Substitute (25) into (20) to obtain

$$\beta\left[J^E(S_{+1}) - J^U(S_{+1})\right] = \frac{\eta}{1-\eta}\frac{c}{q(\theta)}. \tag{45}$$

Observe that

$$\begin{aligned} &J^E(S) - J^U(S) = w(S) - z + \Omega(\theta), \\ &J^N(S) - J^U(S) = w(S) - z + (1-\lambda_F)\beta\mathbb{E}(1-\lambda_Q)\left[J^N(S_{+1}) - J^U(S_{+1})\right] - \theta q(\theta)\frac{\eta}{1-\eta}\frac{c}{q(\theta)}, \end{aligned}$$

where

$$\Omega\left(\theta\right)=\left[\left(1-\lambda_{Q}\right)\left(1-\phi\right)-\theta q(\theta)\right]\frac{\eta}{1-\eta}\frac{c}{q(\theta)}+(1-\lambda_{Q})\beta\mathbb{E}\phi\left[J^{N}\left(S_{+1}\right)-J^{U}\left(S_{+1}\right)\right].$$

Thus, the wage offer satisfies

$$\eta\left[\alpha\tilde{A}L^{\alpha-1}k^{1-\alpha}-w\left(S\right)-w_{L}\left(S\right)L+\left(1-\lambda_{Q}\right)\frac{c}{q(\theta)}\right]=\left(1-\eta\right)\left[w\left(S\right)-z+\Omega\left(\theta\right)\right],$$

from which

$$w_{L}\left(L\right)L+\frac{1}{\eta}w\left(L\right)=\alpha\tilde{A}L^{\alpha-1}k^{1-\alpha}+\left(1-\lambda_{Q}\right)\frac{c}{q(\theta)}+\frac{1-\eta}{\eta}\left[z-\Omega\left(\theta\right)\right],$$

or

$$w_{L}\left(L\right)L+\frac{1}{\eta}w\left(L\right)=\alpha\tilde{A}L^{\alpha-1}k^{1-\alpha}+T, \tag{46}$$

where

$$T=\left(1-\lambda_{Q}\right)\frac{c}{q(\theta)}+\frac{1-\eta}{\eta}\left[z-\Omega\left(\theta\right)\right]$$

is the term independent of L. The differential equation (46) satisfies for all $L\geq 0$, along with the condition that $w\left(L\right)L\leq\tilde{A}L^{\alpha}k^{1-\alpha}$, which requires that the total wage payment does not exceed the firm's revenue. It is useful to observe that

$$\begin{aligned}\frac{\partial}{\partial L}\left[w(L)L^{\frac{1}{\eta}}\right]&=\left[w_{L}(L)L+\frac{1}{\eta}w(L)\right]L^{\frac{1}{\eta}-1}\\&=\left[\alpha\tilde{A}L^{\alpha-1}k^{1-\alpha}+T\right]L^{\frac{1}{\eta}-1}\\&=\alpha\tilde{A}L^{\alpha+\frac{1}{\eta}-2}k^{1-\alpha}+TL^{\frac{1}{\eta}-1}.\end{aligned}$$

Since () implies $w(L)L^{\frac{1}{\beta}}\leq AL^{\alpha+\frac{1}{\eta}-1}k^{1-\alpha}$, we have $\lim_{L\to 0}w(L)L^{\frac{1}{\eta}}=0$. Thus, it follows that

$$w(L)L^{\frac{1}{\eta}}=\int_{0}^{L}\left\{\alpha\tilde{A}j^{\alpha+\frac{1}{\eta}-2}k^{1-\alpha}+Tj^{\frac{1}{\eta}-1}\right\}dj=\frac{\alpha\tilde{A}k^{1-\alpha}}{\alpha+\frac{1}{\eta}-1}L^{\alpha+\frac{1}{\eta}-1}+\eta TL^{\frac{1}{\eta}}.$$

Thus,

$$\begin{aligned}w\left(L\right)=&\frac{\alpha\tilde{A}k^{1-\alpha}}{\alpha+\frac{1}{\eta}-1}L^{\alpha-1}+\left(1-\eta\right)z+\theta\eta c\\&+(1-\lambda_{Q})\phi\left\{\eta\frac{c}{q(\theta)}-\left(1-\eta\right)\beta\mathbb{E}\left[J^{N}\left(S_{+1}\right)-J^{U}\left(S_{+1}\right)\right]\right\}.\end{aligned} \tag{47}$$

This is the wage rate, which explicitly depends on $\mathbb{E}[J^N(S_{+1}) - J^U(S_{+1})]$. When $\phi = 0$, this wage equation reduces to the standard wage equation under multi-worker firms (Cahuc et al., 2008; Kudoh and Sasaki, 2011; Kudoh et al., 2019).

From (47), we obtain

$$w_L(L) = (\alpha - 1)\frac{\alpha \tilde{A} k^{1-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha-2},$$
$$w_k(L) = (1-\alpha)\frac{\alpha \tilde{A} k^{-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha-1}.$$

Thus,

$$J_L(S) = \alpha \tilde{A} L^{\alpha-1} k^{1-\alpha} + (1-\alpha)\frac{\alpha \tilde{A} k^{1-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha-1} - w(S) + \left(1 - \lambda_Q\right)\frac{c}{q(\theta)},$$
$$J_k(S) = (1-\alpha)\tilde{A} L^{\alpha} k^{-\alpha} - (1-\alpha)\frac{\alpha \tilde{A} k^{-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha} + (1-\delta).$$

Now consider the term $J^N(S_{+1}) - J^U(S_{+1})$. From the value of unproductive employee,

$$J^N(S) - J^U(S) = w(S) - z + (1-\lambda_F)\beta\mathbb{E}(1-\lambda_Q)\left[J^N(S_{+1}) - J^U(S_{+1})\right] - \theta q(\theta)\frac{\eta}{1-\eta}\frac{c}{q(\theta)},$$

We *evaluate this expression at the wage rate for productive employee*. We then obtain

$$J^N(S) - J^U(S) = \frac{\alpha \tilde{A} k^{1-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha-1} - \eta z + \left[\frac{-\eta}{1-\eta}\right]\theta\eta c + (1-\lambda_Q)\phi\eta\frac{c}{q(\theta)} + \left[(1-\lambda_F) - \phi(1-\eta)\right](1-\lambda_Q)\beta\mathbb{E}\left[J^N(S_{+1}) - J^U(S_{+1})\right],$$

from which, in any steady state,

$$J^N(S) - J^U(S) = \frac{\frac{\alpha \tilde{A} k^{1-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha-1} - \eta z + \left[\frac{-\eta}{1-\eta}\right]\theta\eta c + (1-\lambda_Q)\phi\eta\frac{c}{q(\theta)}}{1 - \left[(1-\lambda_F) - \phi(1-\eta)\right](1-\lambda_Q)\beta}.$$

Therefore, in any steady state, the wage rate satisfies

$$w(L) = \frac{\alpha \tilde{A} k^{1-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha-1} + (1-\eta) z + \theta\eta c + (1-\eta)(1-\lambda_Q)\phi\beta\frac{\frac{\alpha \tilde{A} k^{1-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha-1} - \eta z + \left[\frac{-\eta}{1-\eta}\right]\theta\eta c + (1-\lambda_Q)\phi\eta\frac{c}{q(\theta)}}{1 - \left[(1-\lambda_F) - \phi(1-\eta)\right](1-\lambda_Q)\beta}.$$

When $\phi = 0$, it reduces to

$$w(L) = \frac{\alpha A k^{1-\alpha}}{\alpha + \frac{1}{\eta} - 1} L^{\alpha - 1} + (1 - \eta) z + \theta \eta c,$$

where $\tilde{A} = A(1+n)^{-\psi} = A$ when $\phi = 0$.

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