Essays on Financial Economics

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Abstract

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This dissertation consists of two chapters. In the first chapter, I revisit the role of Central Banks, the principal entity responsible for economic and financial stability. I consider a model of financial crises where two main attributes are liquidity dry-ups and information scarcity. Direct policies for market rejuvenation may not be able to target a sufficient mass of socially optimal surpluses, because the public authority lacks information regarding the quality of legacy assets and future projects. By contrast, informed agents such as asset managers who starve for liquidity can potentially finance socially favored projects and signal strong balance sheets, but they can also cherry pick, which can exacerbate the adverse selection in public markets where uninformed agents transact. Hence, I show that the public authority trades off the benefit of higher financing from liquidity provision to informed agents for the cost of a public market contraction. Moreover, I show that the optimal policy should involve a liquidity provision to informed agents as well as price support by direct asset-purchase. These results are in line with policies such as PPIP and LSAP that were implemented during the 2008 crisis. Finally, the paper proposes a proactive planner-of-last-resort role for the central bank as opposed to a naive lender of last resort suggested by Bagehot's dictum.

In the second chapter, I provide a first analysis of the concept of traveling times (hitting times or first passage times) for time-series of assets return. Traveling time is defined as the first passage time from one state to another and can be calculated from the knowledge of transition

probabilities. I analyze traveling times for important econometric time series and I provide a set of important results. First, I provide new pricing equations of a class of fixed-income assets, which their payoff would default to zero when particular states are triggered (similar to a risky bond). Second, I show that barrier like option prices can reveal transition probabilities of the underlying asset's return. Moreover, the paper discusses the estimation of the traveling times from historical data where I identify a considerable variation of traveling times across different assets such as known factor returns.

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Dedicated to my beloved mother, father and grandparents

Chapter 1: Planner of Last Resort

1.1 Introduction

Financial crises[1], as evidenced by the 2008 turmoil, are characterized by a collection of features of which scarce liquidity and substantial adverse selection are the most salient. First of all, liquidity dries up so that financing and investment cannot take place. Moreover, the profitable transactions can freeze when liquidity is scarce. Second, adverse selection restrains investment in socially optimal projects, because the information is insufficient to identify profitable projects. In response, the government (which includes central banks and other regulators in this paper) can use a variety of tools to reduce the contagion and restore markets. Among many different programs, the two most important policy interventions are liquidity provision and asset price support through direct asset purchases or guarantees.

An ongoing discussion in regulatory bodies focuses on the possible reforms for central banks to redesign the set of tools and legal frameworks to confront the next crisis. For example, in a recent panel of central banks (CB) chairs[2], Ben Bernanke suggested granting power to the CB to lend to a broader set of institutions, much like the CBs in the UK and Europe. Moreover, Bernanke suggested equipping the Federal Reserve a broader ex-ante legal authority to cope with future unforeseen circumstances. This change would give the CBs the flexibility to utilize existing tools in combination with the known ones to confront the crisis in a timely manner. Moreover, Mervyn King advocates for replacing the Lender of Last Resort (LOLR) with a "pawnbroker for all seasons", meaning the CB would be prepared to lend to any financial intermediary against proper assets and haircuts.

[1]I am grateful to professor Tano Santos for advice on this paper. I am also thankful to professors Patrick Bolton, Paolo Siconolfi and Olivier Darmouni for constructive comments.

[2]Annual Reviews, 2008 Financial Crisis

Although all these proposals are helpful to some extent, the government can take an active role of using all of the possible capabilities and tools together to achieve the most efficient outcome for society. Indeed, common practice is to use a variety of policies together, because different programs can have mutual first-order effects, and using one tool extensively may even exacerbate problems and destabilize the economy. Therefore, the government can act as the planner and carefully implement different programs together to gain the most out of the possible interactions of different tools in a timely manner. Hence, one can suggest a broader term for the role of the CB, and I propose the "planner of last resort" role for CBs, manifested in a simple model of market rejuvenation in which the CB can use a mixture of policies to utilize a complete set of tools in addition to the expertise of agents in the economy.

The phrase "planner of last resort" captures the main idea that the government can use a mixture of policies to attain the efficient social outcome. Prior literature has focused on the traditional role of an LOLR being the primary role of the CB. Few papers have rethought this role. For example, [1] introduced the term market maker of last resort, and [2] suggested the term dealer of last resort. The CB's role as that of a planner and an agent who actively contracts with different agents captures both previous ideas and adds more to it.

I also provide insights regarding the political and public concern that the public funds should not support too-big-to-fail or involved financial institutions that are generally a cause of the turmoil. The support for troubled institutions can generate a future moral hazard, and the funds can be used to exploit the disturbed markets. However, the distressed institutions can, in practice and if given the right incentives, leverage their critical position to benefit society. The reason is that the involved institutions usually have access to critical information and expertise. The public authority can, in general, utilize new mechanisms and contracts to incentivize different groups of agents to the benefit of the society.

Several important questions arise in this context: What is the optimal mixture of policies to restore markets to functioning? Can the traditional fire-fighting tools exacerbate the crisis? Should public aid be provided to a financial sector that has access to superior information and may itself be a cause of the distress? Should cheap liquidity be provided freely as suggested by Bagehot's dictum, and if so, to whom and how much? This paper provides a first analysis of how different policies for market rejuvenation interact and would provide important trade-offs that the policy maker should be aware of when examining different options.

A number of relevant and important policies arose during the 2008 crisis. These programs took many forms, such as direct asset-purchase and asset or debt guarantees. Another highly relevant policy is the Public-Private Investment Program (PPIP), whereby asset managers and the government pooled funds to purchase mortgage-backed securities to help market recovery and price discovery, while protecting tax payers money. The PPIP program captures the basic idea that the government should, under certain circumstances, rely on informed agents to restore market functioning.

This paper attempts to address the question of the optimal policy in a crisis during which market failure exists because of adverse selection. Additionally, some agents in the economy have superior information that can be utilized by the government. The optimal policy would be a mix of asset-purchase and liquidity provision. Specifically, this paper presents important trade-offs in a stylized model for providing liquidity to informed agents and shows how the government should rely on informed investors' superior knowledge of the assets' quality in the economy to rejuvenate the markets. Ideally, the government wants to finance all socially optimal projects in the presence of adverse selection, which would render direct interventions such as asset purchases and price support very costly. In this situation, the other arm to assist the government is through the expertise of informed agents. First, informed agents can finance projects that public markets cannot, because public markets may not be able to pay the right price. Moreover, informed agents are willing to reach the highest-yielding projects, which can correlate with the socially favored surplus. More importantly, informed trading can signal healthy balance sheets to the uninformed investors, and thus the capital can bypass the curse of adverse selection.

There is also a dark side to superior information, which is the ability of informed investors to cream-skim and leave the public markets with low average quality and even a market freeze in the extreme case. Therefore, the paper shows why a naive liquidity provider can exacerbate the crisis by providing fuel for the fire, and therefore, Bagehot's dictum would fail. Consequently, by providing liquidity in any form, the CB would face a trade-off. The public authority trades off the benefit of higher financing from liquidity provision to informed agents for the cost of a public market contraction. The public market distortion can be very costly. For example, one can consider a convex cost function of the size of the public market. However, in this paper, the cost is endogenous and is determined by the size of the government's asset-purchase program. The asset-purchase program would be costly because the government has to take a loss by removing a bulk of bad projects from the market. Moreover, I show that the government can improve social welfare by contracting with the known informed agents in order to align the incentives and prevent too much distortion.

I consider an economy with two types of investors, informed and uninformed. Informed investors (such as hedge funds and investment banks) have access to superior information technology and can identify and cherry-pick the best assets in the market. Moreover, the uninformed investors (mutual funds, pension funds, and commercial banks) can only assess the average quality of a pool of assets and would not make profits in expectation. Liquidity can be provided to either of the two types of agents; however, the informed one has a natural demand for liquidity, because it can reach the highest yielding projects. In an extension, uninformed agents can borrow cheap funds and lend to informed agents, which would magnify the results, because informed investors can lever up more. However, in this paper, I assume no link exists between the two types of investors, due to informational issues in a crisis. Moreover, I assume uninformed investors always have enough cash to invest. Thus, if lending is secure and costly, only informed agents would borrow. However, the liquidity provision to informed agents is costly from the social point of view, because the informed investors can cream-skim the best assets from the public markets. When informed investors cream-skim a lot, the average quality of the assets in the public market deteriorates so that new investment does not take place and a market freeze can happen. Moreover, a pool of low quality assets will increase the cost of other interventions by the government. On the other hand, the benefit of the informed agents for society is the ability to finance firms, which the public market cannot finance otherwise. This paper first presents this trade-off of liquidity provision to informed agents. When a crisis occurs, the government can intervene by providing liquidity to the intermediaries as well as price support to rejuvenate the public market. The government's objective is to maximize the total new surplus minus the cost of providing funds, and the government would trade off the cost and benefit of liquidity allocation to the informed.

There are some considerations in this paper. In this paper, I show novel trade-offs and externalities between different government programs, and I provide a rationale for why the government should rely on the informed agents as well as using other tools. There can be a liquidity provision to the uninformed agents and also mechanism designs from the government to disentangle the informed and uninformed and to incentivize the informed to shift financing to new projects outside the public market capacity. If uninformed investors have enough cash and do not lend to informed agents, any positive interest rate for collateralized lending would only attract the informed.

If the uninformed do not have enough cash, the government can provide collateralized liquidity to all agents indiscriminately, and therefore, the interest rate has to be zero so that uninformed agents can borrow. The liquidity provision policy interacts with the possible public-private partnership program. For example, if the government is providing cheap liquidity, a partnership program may not be feasible, because informed agents would get higher average returns from borrowing and investing for themselves. Thus, in this case, the government may prefer to shut down the free liquidity provision and engage in a partnership program. The government also may prefer to find a way to provide liquidity to uninformed agents or purchase a large amount of assets, thus acting as a public investor.

On the other hand, if a partnership is feasible, the government always prefers to implement it because it would not impose an additional social cost. Moreover, if the government can identify informed agents by lending costly liquidity, a partnership program is preferred if the benefit of the informed financing outside the public market is not greater than the benefit of removing a measure of low quality assets by the signaling mechanism in a partnership. If the government can identify the informed agents, it can control the amount of the liquidity provision and contract with informed agents to the benefit of society. If contracting is not feasible, and if the cost of the informed cream-skimming is high, the government can implement other basic policies to incentivize the informed. I consider one example in which the government distributes stakes of its asset-purchase program to offload costly losses. Finally, I leave the equilibrium occupational choice and ex-ante equilibrium project quality choice for future research.

Here, I summarize the main insights:

1. Public-private partnership trade-off: The government partnership with the informed agents will induce signaling for healthy assets at the expense of losing high-quality assets from the public market. The government thus trades off the social value of information for the distortion in the public markets.

2. Liquidity provision trade off: The government trades off the benefit of financing capacity outside of the public markets for the cost of rent-seeking when providing liquidity to the informed agents.

3. Failure of an LOLR dictum: Too much liquidity provision can lead to a market freeze even if it is risk free for the CB.

The paper is organized at follows. Section 1.3 outlines the model. Section 1.4 presents the benchmark cases and basic results. The equilibrium notion and the rejuvenation policy are presented in section 1.5, and section 2.7 concludes. Additional proofs and cases are in the appendix.

Related Literature. This paper is mainly related to the literature on liquidity provision and optimal market rejuvenation policies. The seminal work of [3] is the starting point for market failure and adverse selection. [4], [5] and [6] provide a dynamic model for endogenous liquidity and adverse selection. [7] provide an analysis of liquidity balance in the public and private markets, in which the uninformed agents can lend to informed agents who can cream-skim the best assets out of the public market. The price of assets in the public market is determined by the available cash in the hands of investors (which is also called the cash-in-the-market price which is first introduced in [8]).

The paper also relates to the literature on the role of CBs and public liquidity for market interventions. [9] emphasize the role of public liquidity to overcome aggregate shocks (see also [10] and [11]). [1] investigate the optimal timing of public liquidity and propose a market maker of last resort role for the government, which should facilitate the transfer of assets from short-run to long-run investors. Moreover, [2] suggests the term "dealer of last resort" as the Federal Reserve traded freely against a wide bid-ask spread during the 2008 crisis. This paper contributes to the research on central banking by suggesting the the CB should act as a planner and provide an optimal state-contingent mixed of policies.

This paper is also related to the literature on the social value of information. [12] is among the first papers to provide a formal analysis of distributive incentives of information acquisition, even when no social value is attached to that information. In this context, this paper is closely related to [13], who show how informed investors can cream-skim best assets and thus depress the public market price by lowering the quality of the pool in the public market. However, [13] mainly focus on cream-skimming as a non-linear effect to explain the growing size of the financial sector, whereas my model does not rely on nonlinearity. Rather, in this paper, a positive aspect of informed agents is that they can finance projects that the public market cannot afford to finance, as well as providing a social value by signaling healthy assets. Thus, there is a social value of the information in my model that can offset its distortionary effects.

This paper contributes to the literature on efficient market interventions. In the context of bank bailouts, [14] consider an efficient bailout design that maintains bankers' incentives while minimizing costs. [15] find optimal policies for recapitalization when banks suffer from debt overhang. [16] suggest a cap on the amount of short-term debt when a strategic complementarity exists between banks' leverage choices. [17] considers a model of LOLR where liquidity shocks cannot be distinguished from solvency shocks and they introduce the concept of "gambling for resurrection" which corresponds to a situation where an insolvent bank invests in a negative NPV project to hopefully benefit from upside. [18] provides a solution to hedge the risk of too big to fail institutions by considering an insurance mechanism from a public agent with special resolution authorities.

Two other closely related papers are [19] and [20]. Both explore optimal rejuvenation policies in the presence of adverse selection. The two papers are featured with an endogenous outside option when the government implements a mechanism. [20] consider the fungibility of legacy assets and new project to show debt contracts are optimal. By contrast, [19] considers legacy assets that can be sold separately and show the government optimally uses asset-purchase and equity injection to restore markets. My paper uses the stylized model in [19] to explore how the government policies interact. I do not explore the mechanism-design approach, because I consider a full non-pledgeability of new projects' revenue. Thus, I focus on the asset-purchase policy and liquidity provision.

1.2 Realism

In this section, I overview the relevant policies implemented around the world in 2008. In the US, the Federal Reserve and Treasury has provided immense support for the markets through different programs.

Massive amounts of liquidity were provided in many different forms. The Term Asset-Backed Securities Loan Facility (TALF) was a program created to increase the issuance of asset-backed securities (ABS) by providing loans to holders of ABS securities, which in turn would stimulate the flow of credit to households and businesses. Through Primary Dealer Credit Facility (PDCF), the Federal Reserve provided overnight collateralized loans to primary dealers to overcome tensions in the short-term financing market. The Fed also widened access to the discount window. However, due to the stigma of borrowing from the discount window, the Fed established the Term Auction Facility (TAF) program to support increasing demand for term funding, by offering collateralized loans. The TAF loans were available to sound depository institutions and were allocated using auctions in which the participating institutions placed bids for the amount and the interest rate of the loans.

Central banks directly purchased assets or provided guarantees under programs such as Large Scale Asset Purchase (LSAP), Troubled Asset Relief Program (TARP), Temporary Liquidity Guarantee Program (TLGP), and Capital Purchase Program (CPP) to provide price support and pump fresh capital into distressed institutions. The Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facility (AMLF) was designed to provide liquidity and price support for Asset-Backed Commercial Paper (ABCP) that strained Money Market Mutual Funds (MMMF) were forced to sell. The general goal of AMLP was to support ABCP and money markets by facilitating the flow of liquidity to those markets. The AMLF loans were issued to purchase ABCP and used the same ABCP as the collateral for the loan and thus the funds from AMLF were targeted to make a market only for ABCPs. The Money Market Investor Funding Facility (MMIFF) was introduced as a complement to AMLF to encourage investments longer than overnight and increase the availability of liquidity to MMMFs. Moreover, during 2008-2009, the Fed purchased near $1.25 trillion of agency assets under Agency Mortgage-Backed Securities Purchase Program to provide price support for longer-term assets and improve the market conditions. Moreover, under TARP, the government purchased near $420 billion of troubled assets and stocks of distressed financial institutions.[3]

As mentioned previously, another policy during the 2008 turmoil was PPIP, the purpose of which was to help market recovery and price discovery for the RMBS and CMBS markets and at the same time providing tax-payers with attractive returns. The target assets were AAA-rated securities issued before 2009, and the funds had an eight-year term. When the program ended (September 2013), the Treasury recouped $3.8 billion in net profit for the initial investment of $18.6 billion, corresponding to a 20% net return over three years of the funds' life. The Treasury committed liquidity in a mixture of equity and debt to funds managed by asset managers who also should have contributed nearly 25% in equity. The PPIP program captures the basic idea that the government should, under certain circumstances, rely on informed agents to restore market

[3]For a story about a fund managed by two wives of Wall Street mangers see "The Real Housewives of Wall Street" in Rollingstone.

functioning.

One of the prevalent policies in 2008, in most of the countries, was supporting the price of impaired assets of financial institutions by establishing vehicles in the forms of a fund to purchase and manage the impaired assets (under names such as bad banks, asset management companies and programs such as National Asset Management Agency (NAMA) in Ireland and SAREP in Spain). These asset management funds enjoyed benefits from the states (such as partial insurance) and took the form of a partnership or a new company. More complex asset relief solutions included partial ownership in the target bank for whom the assets were guaranteed.

In this paper, price support and guarantees are essentially the same tool. However, the precise role of guarantees can be different. As [21] points out: "The creation of the guarantee fund nipped the problem of liquidity risk in the bud, rather than trying to address liquidity problems after they arose.". Therefore it is crucial to credibly signal confidence so that private capital remains in the system to fund profitable investments. Providing guarantees is a way to engage the private capital. [22] notes the importance of guarantees in her legal piece: "Just-in-time guarantees keep private capital in the system, providing policymakers the time that they need to develop a viable plan to address deficiencies.". Please refer to the appendix for a discussion of guarantee schemes.

All Together, these programs suggest three main schemes in this paper: free liquidity provision, asset price support, and public-private partnerships. The goal of the paper is to provide a stylized model of market rejuvenation involving these programs.

1.3 The Model

The model builds on the stylized model presented in the first section of [19]. Agents, timing, and markets are as follows:

1.3.1 Agents, Preferences and Projects

Three types of agents exist, all of whom are risk neutral and don't discount the future: banks (or firms), informed investors and uninformed agents. The government is the policymaker.

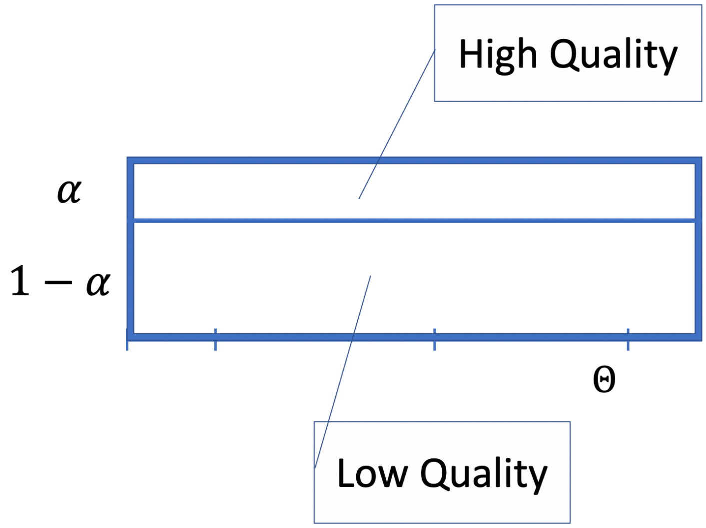

Figure 1.1: An overview of the assets in the economy.

Banks and Legacy Assets. A measure $1-\mu$ of solvent banks owns $1-\alpha$ units of legacy assets, which will pay a sure amount equal to θ in the future. The value of θ is distributed uniformly on $[\gamma, 1]$ and is known only to the banks and the informed investors. Each solvent bank also has an additional α unit of a high-quality legacy asset that pays off 1 at maturity. Additionally, a measure μ of banks exists with a legacy asset with valuation equal to 0 (corresponding to an atom on $\theta = 0$). These banks correspond to the insolvent banks that should ideally not participate in any market. Following the assumptions in [19], only the owner of the legacy asset can enjoy its payoff, and it cannot be divided or shared. This assumption means banks only rely on selling legacy assets as a source of funding. In figure 1.1, an overview of the assets in the economy is presented.

New Project. A new project with a positive surplus is available to all banks except the insolvent banks. This project can correspond to the value that can be generated after the crisis and can be interpreted as a growth option or a new surplus. The new project needs fund I and would payoff $I + S$ at maturity with S being the net present value. Moreover, I only consider the extreme case in which none of the revenue $I + S$ is pledgeable. Thus, the only source of funding for banks is through asset sales to the public markets, the government, or the informed investors. However, the informed investors correctly anticipate the new surplus when negotiating the price of the legacy assets. The insolvent banks also have access to a new project that produces a negative surplus $S = -I$ but delivers a small positive private benefit to the bank. [4]

Note that if there is no cost of deficit, the government should ideally remove the insolvent banks from any financing market but can purchase their asset at a positive price. On the contrary, if running deficits incurs a cost, the government should not provide financial aid to the insolvent banks from any financial aid.

Informed Investors. Informed investors have total cash of M^I and stand ready to purchase assets at discounts. The informed agents can identify the types of assets at no cost and have a limited capacity to purchase and hold assets. Therefore, I assume a measure η for the informed, and each can purchase at most one unit of assets. They can borrow from the government, but they cannot borrow from other agents. As will become apparent, there is not a perfect competition in the informed market, and informed agents make positive rents. However, informed agents are free to choose to trade in public or private markets, wherever bring them more returns.

Uninformed Agents. Uninformed agents cannot identify types, and they would break even in expectation when they purchase assets. Moreover, cash in the hands of uninformed agents is sufficient, but they do not lend any excess cash.

The Public-Private Partnership. I consider a partnership program involving informed agents to purchase high quality assets from the banks. Informed agents are willing to participate in a partnership as long as it provides them with sufficient return. Note that a slightly higher price than the public market would attract the solvent banks to sell their high-quality assets to the partnership. When the program is implemented, public investors can distinguish participating banks from the insolvent banks and other banks who do not participate. The benefit of this program is that the solvent banks can signal their strength, but it can also impose a distortion in the public markets. To make the analysis simple, I only consider the case when the government's partnership program is extensive, meaning either no bank or all the banks in the public market would sell their high-

[4] Note the implicit correlation between the value of legacy assets for the insolvent banks and the negative future surplus is by assumption for simplicity but one can consider a given correlation for all the banks and solve the model accordingly.

quality assets to the partnership. Moreover, I assume a constant price for the partnership to not allow information revelation from the possibility of different prices.

The government. The government consists of all the relevant public authorities such as the Treasury, CB, and regulatory body. I assume that the government knows the informed agents, which means these agents would self-select into the relevant government' partnership program, which I explain below.

The government provides liquidity only to informed agents, and a government program consists of these elements: the government offers liquidity m^I with interest rate 0 to the informed as a liquidity provision, posts price p^g as an asset-purchase program, and provides funds to implement the public-private partnership program. Let $1^p = 1$ indicate the case in which a partnership program is implemented. Then, the government chooses

$$G = (1^p, m^I, p^g)$$

to maximize the social welfare function

$$F = S \times v - I \times v' - B(M^g, D),$$

where v is the total mass of projects with positive surplus financed, v' is the mass of projects implemented with negative surplus, D is the deficit, M^g is the total funds that the government provides and B is the cost for the government. The cost for the government is a function of the deficit and the total amount of funds provided. For simplicity, I assume $B = \lambda \times D$ to show the main trade-offs. However, one can consider a convex form for function B. The case $\lambda = 0$ means the deficit is not costly and maximizing total net surplus would be the government's objective.

The government's objective should be interpreted with some caution. In case of a partnership program, the government may prefer not to implement an asset-purchase because in practice, the low-quality firms that are excluded from the public markets may find it indiscriminate to not participate in any asset-purchase program by the government. It can be the case in reality, because

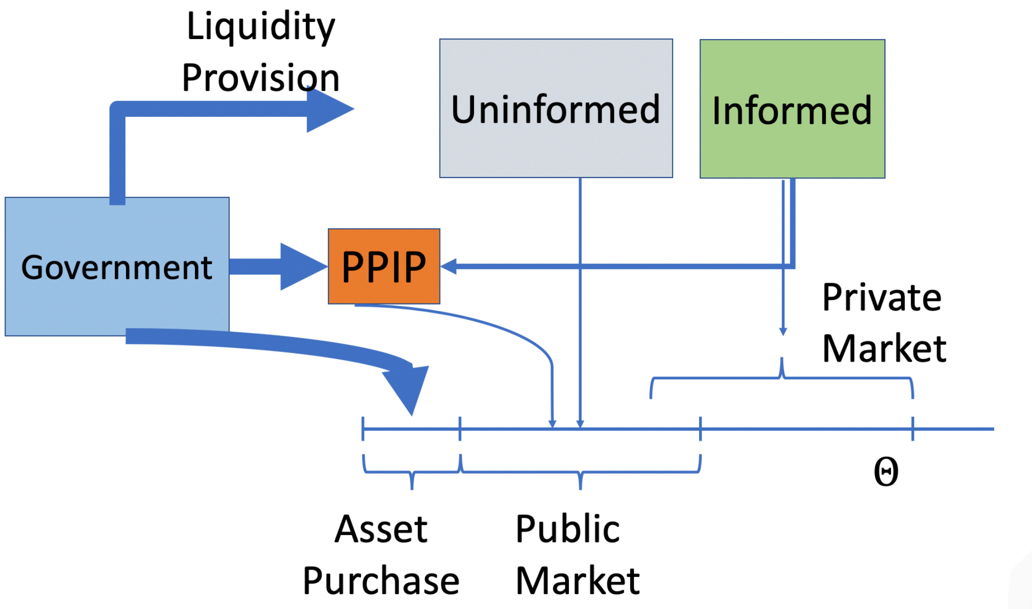

Figure 1.2: An overview of the economy.

the role of the public authority is to ensure fairness. However, the government can provide price support within the range of assets that are anticipated to correlate with a positive surplus. In this paper, I assume a partnership is extensive, and the government can exclude the very low types from the market when implementing a partnership. A snapshot of the components of the economy is presented in Figure 1.2.

Timing and Information. The timing is as follows: At stage 1, banks learn the quality of their legacy assets. At stage 2, the government's partnership program purchases assets, and uninformed agents, informed agents and the government's asset-purchase program make offers. At stage 3, the banks sell or keep their assets. At stage 4, payoffs are realized, and agents consume. The public investors can distinguish solvent from insolvent banks if the partnership program is implemented. Moreover, the government does not have more information than the public investors, but as in [19], the government would take the lower tail of assets when offering the same price as the price in the public market.

1.3.2 Markets

In this section, I outline the price-formation mechanism in the informed and uninformed markets. The market structure mainly follows the same dual market of [13], in which the private market is an OTC market and the price is determined by Nash bargaining based on the agent's outside options. This is a natural choice as firms always have the option of selling in a public market and informed agents can make private deals by offering better prices.

Public Market. In the public market, uninformed agents offer a price that makes them break even.

Denote the price for one unit of an asset in the public market by p^m, and if the public market is frozen, set $p^m = 0$. Also, let p^p denote the price of one unit of a high-quality asset in a partnership program and note we always have $p^m \leq p^p$. Also note that insolvent banks would always sell at any positive price.

Consider the case with no informed agent and no partnership program. In this case, a type θ will sell and do the project if both of these conditions hold: $I \leq p^m$ and $\alpha + (1-\alpha)\theta \leq p^m + S$, where the latter is equivalent to

$$\theta \leq \frac{p^m + S - \alpha}{1-\alpha}.$$

The reason is that the firm should have enough funds to implement the project, and furthermore, the new surplus added to the selling price should be above the outside option of keeping the asset to consume.

In case of a partnership program, the firm can sell the high-quality asset at a higher price, and the low-quality asset can be sold to the public. I assume selling one portion of the asset would be insufficient to finance the new project. In summary, in the case of a partnership program in place, the firm θ will sell and implement the project if these conditions hold:

$$\alpha p^p < I,\ (1-\alpha)p^m < I,$$

$$I \leq \alpha p^p + (1-\alpha)p^m,$$

and

$$\alpha + (1-\alpha)\theta \leq \alpha p^p + (1-\alpha)p^m + S,$$

where the latter is equivalent to

$$\theta \leq \frac{\alpha p^p + (1-\alpha)p^m + S - \alpha}{1-\alpha}.$$

Note the price $p^m < I$ cannot be sustained, because only the types with a true valuation less than p^m would sell, and then the average price in the public market would be less than p^m (unless only the lowest types participate); thus, the market unravels.

Now, in the absence of a partnership program, all of the insolvent banks would sell and if $I \leq p^m$, the types $\alpha + (1-\alpha)\theta \leq p^m + S$ would also sell. Thus, the average price is equal to (assuming no distortion by informed trading as I illustrate the informed market later)

$$p^{av} = \mu \times 0 + \frac{(1-\mu)(\alpha + (1-\alpha)\gamma + p^m + S)}{2}.$$

And if a partnership program is in place and $I \leq p^m$, the insolvent banks and high-quality assets are excluded from the public market, and the average price for one unit of asset is equal to (assuming no distortion by informed trading except the partnership program)

$$p^{av} = \frac{1}{2}\left(\gamma + \frac{\alpha p^p + (1-\alpha)p^m + S - \alpha}{1-\alpha}\right)$$

$$= \frac{(1-\alpha)\gamma + \alpha p^p + (1-\alpha)p^m + S - \alpha}{2(1-\alpha)}.$$

Note we should have $p^m \leq p^{av}$ so that the public market can work. Without a partnership program and adding the constraint $I \leq p^m$, one can observe that if

$$\frac{(1-\mu)(\alpha + (1-\alpha)\gamma + S)}{1+\mu} < I,$$

the public markets will not be working in the absence of the government's asset-purchase.

In the case of a partnership program and with the constraint $I \leq \alpha p^p + (1-\alpha)p^m$, if

$$(1-\alpha)\gamma + 2\alpha p^p + S - \alpha < I,$$

the public markets will not be working in the absence of the government's asset-purchase.

One can observe a trade-off for the government's partnership program: by engaging in a partnership, the government removes a load of low-quality assets from the public market while introducing low-quality assets of the banks that participate in the public market. For example, if $\gamma = 0$, $\mu = 0$, and

$$I - \alpha < S < I + \alpha - 2\alpha p^p,$$

introducing a partnership can cause a working market to freeze. Moreover, if $\gamma = 0$ and μ is high enough so that

$$I + \alpha - 2\alpha p^p < S < I(1 + \frac{2\mu}{1-\mu}) - \alpha,$$

introducing a partnership can rejuvenate a frozen market. This trade-off for a partnership program is one of the main messages of this paper. However, entry into a partnership program is endogenous because informed agents can wait and make deals on private OTC markets.

Private Market. Next, I show how the price of a deal between a bank and the informed investors is formed. Denote by $p^i(\theta)$ the price of the deal between an informed agent and the bank θ. Banks are always willing to sell to the informed investors as opposed to the public market because they can get better terms. Moreover, assume informed agents split rents with bargaining when they make deals with the banks. Let k be the bargaining coefficient for the informed so that they acquire a proportion $1-k$ of the surplus. The case $k = 0$ represents the case in which an informed agent will acquire the total surplus. Parameter k is the same for all informed agents and represents the return in the informed market compared with the public markets. I assume each informed agent would match to a bank, and all informed agents would make offers at the same time. Thus, the banks cannot sell their high-quality asset and the rest of their assets to different informed agents.

However, when a partnership is in place, the separation of the asset sale would become feasible. These considerations would not complicate the analysis, and the main insights would remain the same.

Now, I describe the price formation and the rate of return for informed agents. First, consider the case of no partnership and let $I \leq p^m$. Consider the types

$$\frac{p^m + S - \alpha}{1 - \alpha} < \theta,$$

and note these types would not sell in the public market. Now, an informed agent can at least offer $\alpha + (1 - \alpha)\theta - S$ for the asset, and type θ would have the incentive to sell and implement the project. Thus, a surplus S could be negotiated between the informed agent and the firm; therefore, the bargaining price would be $p^i(\theta) = (\alpha + (1 - \alpha)\theta - S) + kS$. The net profit for the informed agent when dealing with type θ is $(\alpha + (1 - \alpha)\theta) - p^i(\theta)$; therefore, the informed agent's return for type θ would be

$$r^i(\theta) = \frac{(1 - k)S}{\alpha + (1 - \alpha)\theta - (1 - k)S}.$$

Now, consider the types

$$\theta < \frac{p^m + S - \alpha}{1 - \alpha},$$

and note these types can sell in the public market, implement the project and make a payoff equal to $p^m + S$. For these types, an informed agent can offer a price higher than p^m, and the bargaining price for the informed would be $p^i(\theta) = p^m + k(\alpha + (1 - \alpha)\theta - p^m)$ and the return would be equal to

$$r^i(\theta) = \frac{(1 - k)(\alpha + (1 - \alpha)\theta - p^m)}{p^m + k(\alpha + (1 - \alpha)\theta - p^m)},$$

as long as this net return is positive.

Finally, one can easily observe that if the public market is not working the rate of return for informed agents is equivalent to the case in which $p^m = I$.

Now, consider the case in which a partnership is in place, and let $I \leq \alpha p^p + (1 - \alpha) p^m$. Consider the types

$$\frac{\alpha p^p + (1-\alpha)p^m + S - \alpha}{1-\alpha} < \theta,$$

and note these types would not sell in the public market. Now, the analysis is unchanged from the previous case, and the informed will make a return equal to

$$r^i(\theta) = \frac{(1-k)S}{\alpha + (1-\alpha)\theta - (1-k)S}.$$

Now, consider the types

$$\theta < \frac{\alpha p^p + (1-\alpha)p^m + S - \alpha}{1-\alpha},$$

and note these types can sell in the public market, do the project, and receive a payoff equal to $\alpha p^p + (1-\alpha)p^m + S$. For these types, an informed investor can offer a price higher than $\alpha p^p + (1-\alpha)p^m$, and the banks would always find foregoing partnership and public markets to be beneficial. The informed return would then be

$$r^i(\theta) = \frac{(1-k)(\alpha + (1-\alpha)\theta - \alpha p^p - (1-\alpha)p^m)}{\alpha p^p + (1-\alpha)p^m + k(\alpha + (1-\alpha)\theta - \alpha p^p - (1-\alpha)p^m)},$$

as long as this return is positive.

Figure 1.3 shows the return on the informed capital curve for $I = p^m = 0.3$, $S = 0.2$, $\alpha = 0$ and $k = 0, 0.5$. The intuition of the non-monotonic curve in Figure 1.3 is as follows. The surplus of the project is fixed for all types, and thus, the higher the outside option of the firm, the lower the return of a potential deal. However, the return for informed agents is increasing for the middle values of θ because they are participating in the public markets and higher types are foregoing more surplus. The informed return is zero for low values of θ because they should be offered at least I, which leaves no profit for an informed agent. Furthermore, when a partnership is in place for a positive value of α, the same intuition holds. Moreover, when a partnership is in place, the peak of the informed return curve would realize for a higher type because the outside option of firms in the public market would be higher.

Now, I find the types that would sell to informed agents. I illustrate this for the case in which $\alpha = 0$. For other cases, derivations would be similar. Informed agents exhaust their cash on the highest-yielding deals until they run out of capacity, because I assumed each informed agent has a capacity for one unit of the asset. The return on the marginal types who receive deals from an informed agent should be equal, and solving for

$$\frac{(1-k)(\theta - p^m)}{p^m + k(\theta - p^m)} = \frac{(1-k)S}{\theta - (1-k)S} = x,$$

one can observe that informed agents spend all their cash on the interval

$$B(p^m, x) = \left[\frac{p^m(1-k)(1+x)}{1-k-kx}, S(1-k)\frac{1+x}{x}\right].$$

The value of x is the highest for $\theta = p^m$ and is lowest for $\theta = 1$ because lower values of x would not be feasible in the equilibrium given the limited capacity of the informed. The upper bound and the lower bound for x is given by

$$x \in \left[\frac{(1-k)S}{1-(1-k)S}, \frac{S(1-k)}{p^m + kS}\right].$$

The measure of the interval $B(p^m, x)$ cannot exceed η, the capacity constraint of the informed.

One can observe the ramification of the informed from Figure 1.3. The informed optimal choice corresponds to an interval in the middle values of θ. The private market consists of high values of θ, whereas the public market corresponds to lower values of θ. Therefore, the informed purchase would increase the surplus in the private market while shrinking the size of the public market. This insight of dual role of the information for the benefit of the society is new. Moreover, the benefit of the information is greater when the informed trading can signal healthy banks.

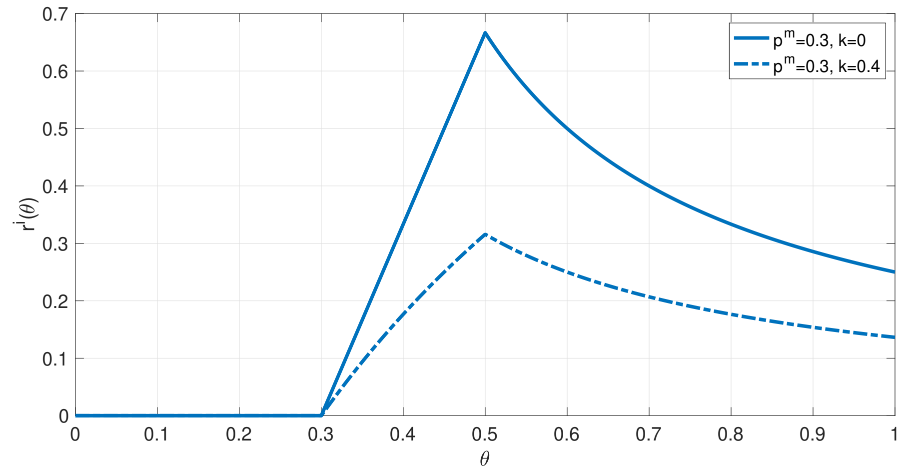

Figure 1.3: Informed rate of return vs θ.

1.4 Benchmarks

In this section, I provide several benchmarks that would signify the main trade-offs. First, I start with the efficient outcome where the government knows all the types.

1.4.1 Efficient Allocation

In the absence of friction, the government excludes all the insolvent banks and finances I for all of the projects with full repayments. The social surplus would be:

$$F^0 = S \times (1 - \mu).$$

However, consider the case in which the frictions remain in place, but the government knows the types. If the deficit is not costly, the government will optimally spend an amount equal to I for each solvent bank to create all the surplus in the economy as before. However, when the deficit is costly, the government excludes all the insolvent banks and finances as many projects as possible. I assumed the legacy asset cannot as a collateral, and thus, the only tool for the government is to purchase the legacy asset of each type, and I assume a full bargaining for the firms. Therefore, the government purchases the legacy assets at price I for types $\theta \in [\theta_1, 1]$, where for type θ_1, the marginal cost of the deficit is equal to the marginal benefit of generating surplus S. The value of θ_1 is given by

$$\alpha + (1-\alpha)\theta_1 = \max\{I - \frac{S}{\lambda}, \gamma\},$$

and the social surplus is

$$F^1 = S(1-\mu)\frac{1-\theta_1}{1-\gamma} - \lambda \times D < F^0.$$

The government objective is to achieve a social welfare level close to F^1, given the limited information and the limited set of tools assumed. Note that if the government posts the price I for all the types, as I show later, the deficit would be higher because now more low-quality types would sell. In addition, the very high types would not sell to the government, and thus, the surplus would decrease and the welfare level would be lower than F^1. The main goal of this paper is to show novel trade-offs and the interaction of different programs, and although maximizing social welfare is an objective, obtaining the first best is not the main focus of the analysis in this paper.

1.4.2 No Intervention

Consider the case with no government intervention. In this scenario, all of the insolvent banks would sell and if p^m is the price for one unit of legacy asset and $I \leq p^m$, the types $\alpha + (1-\alpha)\theta \leq p^m + S$ would also sell.

Thus, if no informed trading occurs, the average price is equal to

$$p^{av} = \mu \times 0 + \frac{(1-\mu)(\alpha + (1-\alpha)\gamma + p^m + S)}{2}.$$

We should have $p^m \leq p^{av}$ so the public market can work. Then, if

$$\frac{(1-\mu)(\alpha + (1-\alpha)\gamma + S)}{1+\mu} < I,$$

the public market would not be working. And if

$$I < \frac{(1-\mu)(\alpha + (1-\alpha)\gamma + S)}{1+\mu},$$

the public market can coexist with the private market. However, if too many informed deals occur, the average price p^{av} can shrink to the level where $p^{av} < I$ and the public market would unravel. The following proposition summarizes the no-intervention outcome:

Proposition 1 (No Intervention) *Suppose the government does not intervene. If*

$$\frac{(1-\mu)(\alpha + (1-\alpha)\gamma + S)}{1+\mu} < I,$$

the public market breaks down and the informed agents spend their cash on the interval $B(I, x^*)$ for a value of x^*, which makes the size of the interval $B(I, x^*)$ equal to η. And if

$$I < \frac{(1-\mu)(\alpha + (1-\alpha)\gamma + S)}{1+\mu},$$

the public and private markets can coexist. However, in this case, if informed cash is high enough so that $M^* < M^I$, the public market breaks down because of the presence of the informed and the informed spend their cash on the interval $B(I, x^*)$.

The last part of proposition 1 is a new insight that we see later in more detail when the government provides liquidity. The main finding is that if the informed cream-skim a lot, the average quality in the public markets will not be high enough to keep the public market vital. Thus, the cream-skimming mechanism can itself cause market failure. This finding is not the case in [13], because this paper includes an investment level that should be fulfilled to keep the transactions profitable.

This is the dark side of superior information, because a considerable amount of surplus can vanish due to low-quality assets in the public market. This social cost can be much more severe than the benefit of the informed financing projects outside the capacity of the public market. In addition, the failure of the public markets due to the excess liquidity in the hands of the informed is a sudden phenomenon. Precisely, for $M^I = M^* - \epsilon$, a small increase in M^I would cause a sudden market freeze. This finding is also new: even a small increase in the liquidity can cause a sudden and severe drop in the level of the output. The message for policymakers would be to regulate excess liquidity to foster investments rather than allowing a siphoning to the informed private markets.

Next, I consider the case with no informed agent.

1.4.3 Benchmark with No Informed Investor

In this section, I analyze the case with no informed private market but with a partnership program possibly in place; thus, a partnership is the only way the informed can participate. First, consider the case with no partnership, as given in [19]. Consider the situation in which the public market is not working as described above. The government can help markets restore functioning, by posting a price p^g to buy a fraction b of low-type assets. In equilibrium, it should be that $p^m = p^g$. Otherwise, if $p^g < p^m$, the low-type firms would not sell to the government and would sell to public investors at a higher price. Thus, in equilibrium, the government posts the same price as the public, low-type firms would sell to the government, and other types would participate in the public pool. One can easily see that only the case $I \leq p^g = p^m$ is feasible. If $I \leq p^m$, all the types

$$\theta < \frac{p^m + S - \alpha}{1 - \alpha},$$

would be willing to sell their asset at price p^m. For the public investors to break even, the government has to take up all the types

$$\alpha + (1 - \alpha)\theta < p^m - S,$$

(including the insolvent banks) so that public investors break even. Note we should have $\alpha + (1 - \alpha)\gamma < p^m - S$, and if the latter is not true, the prices goes up so that the average quality becomes the same as the market price. The optimal amount for p^m, which we denote by p^*, is obtained by solving the below program:

$$\max F(p^m) = \frac{S(1-\mu)}{1-\gamma}\left(\frac{p^m + S - \alpha}{1-\alpha} - \gamma\right)$$
$$-\frac{\lambda(1-\mu)}{2(1-\gamma)(1-\alpha)}\left((p^m - \alpha - (1-\alpha)\gamma)^2 - S^2\right) - \lambda\mu p^m - \mu I,$$

where we assume $I \leq p^m$. Additionally, for the government's participation, we should have $0 < F(p^*)$. We can then easily show the following proposition:

Proposition 2 (No Informed Investor and No Partnership) *Suppose the informed are cashless so they do not participate, and there no partnership program is in place. If public markets are not working, that is,*

$$\frac{(1-\mu)(\alpha + (1-\alpha)\gamma + S)}{1+\mu} < I,$$

the optimal value for p^m is

$$p^* = \frac{S}{\lambda} - \frac{\mu(1-\gamma)(1-\alpha)}{1-\mu} + \alpha + (1-\alpha)\gamma,$$

as long as $I < p^*$. If $p^* < I$, the optimal value for p^m is $p^m = I$ as long as $0 < F(I)$. If $F(I) < 0$, the government does not intervene and market failure is preferable to intervention.

Intuitively, the optimal price is increasing S because the government would benefit more from financing future surplus through direct asset-purchase. Moreover, the optimal price is increasing in γ and α because there would be a lower loss for the government from purchasing low-quality assets.

One can also observe that the optimal price is decreasing in the mass of insolvent banks μ and the deficit coefficient λ. The intuition is that if there is a low measure of insolvent banks, the government can tolerate a higher deficit by posting a high price, and the same is true when the cost of the deficit is low.

However, if the deficit is very costly or there is a high measure of insolvent banks, the government would benefit from implementing a market stimulation. The main cost for the government is the surplus lost when the insolvent banks divert funds to an inefficient use.

Now, consider the case in which no informed transaction is feasible except a partnership. If $I \leq \alpha p^p + (1-\alpha)p^m$, all the types

$$\theta \leq \frac{\alpha p^p + (1-\alpha)p^m + S - \alpha}{1-\alpha},$$

would be willing to sell their assets. For the public investors to break even, the government has to take up all the types

$$\theta < \frac{(1-\alpha)p^m - \alpha p^p - S + \alpha}{1-\alpha},$$

so that public investors break-even. The optimal amount for p^m denoted by p^* is obtained by solving the following equation:

$$\max F(p^m) = \frac{S(1-\mu)}{1-\gamma}\left(\frac{\alpha p^p + (1-\alpha)p^m + S - \alpha}{1-\alpha} - \gamma\right)$$
$$-\frac{\lambda(1-\mu)}{2(1-\gamma)(1-\alpha)}\left(((1-\alpha)p^m - (1-\alpha)\gamma)^2 - (S - \alpha + \alpha p^p)^2\right),$$

where I assume $I \leq \alpha p^p + (1-\alpha)p^m$. In addition, for the government's participation, we should have $0 < F(p^*)$. Note that, in practice, the government would also have a stake in the partnership; however, I do not consider the stake in this paper. The following proposition can then be easily shown:

Proposition 3 (No Informed Investor with Partnership) *Suppose the informed are cashless and do not participate, but a partnership program is in place. If public markets are not working, that is,*

$$\frac{(1-\mu)(\alpha + (1-\alpha)\gamma + S)}{1+\mu} < I,$$

the optimal value for p^m is

$$p^* = \frac{S}{\lambda(1-\alpha)} + \gamma,$$

as long as $I' < p^*$*, where*

$$I' = \frac{I - \alpha p^p}{1 - \alpha}.$$

If $p^* < I'$*, the optimal value for* p^m *is* $p^m = I'$*, as long as* $0 < F(I')$*. If* $F(I') < 0$*, the government does not intervene and a market failure is preferable to intervention.*

One can observe that the optimal price is increasing in α. The intuition is that for a higher α, the government's asset-purchase consists of fewer low-quality assets. Therefore, the government can be provide a higher price support. Moreover, for a higher γ, the optimal price would increase as well and the intuition is similar. In addition, the optimal price is increasing in the social surplus S, because the government is willing to take up more deficit to create more surplus. Furthermore, the price would be decreasing in λ because using the deficit to support the future surplus would be costly.

Another important message is that implementing the partnership program involves a tradeoff. By implementing the partnership program, the insolvent banks are excluded from participating in the public market, but the government has to purchase a higher load of low-quality legacy assets. Intuitively, if there is a high mass of insolvent banks, a partnership is beneficial for the government, but if this mass is low and the cost of the deficit is high, a partnership would not be profitable.

Next, I consider the case in which the government provides liquidity to the informed.

1.4.4 Liquidity Provision for the Informed and No Partnership

In this section, I consider liquidity provision for the informed investors and demonstrate how liquidity provision would interact with the government asset-purchase programs. The implicit assumption is that the government can credibly secure the funds it provides to the informed, and therefore, there is no need for collateral (or the informed can produce credible collateral). This assumption can hold in reality when the government increases the level of cash in the economy and then liquidity flows to the informed section. I consider direct liquidity provision to the informed, because this scheme was among the recent proposals for rethinking the role of the central bank.

The government faces a tradeoff for allocating liquidity to the informed agents: more liquidity means more projects get financed that the public market cannot finance otherwise, but this allocation would increase the cost for the government's asset-purchase, because some of the high-quality assets are taken out of the public market pool by the informed. The liquidity allocation problem is equivalent to first finding the optimal amount x that informed investors should purchase for a given p^m and then finding the optimal p^m. Note only $p^m + S < 1$ is interesting, which means a possible private market would exist to begin with. Thus, if p^m is the price in the public market and x corresponds to the minimum return of the informed, the informed would purchase the types

$$B(p^m, x) = \left[\frac{p^m(1-k)(1+x)}{1-k-kx}, S(1-k)\frac{1+x}{x}\right].$$

The government purchases types

$$\theta < 2p^m - \frac{p^m(1-k)(1+x)}{1-k-kx} = \frac{2-(1+k)(1+x)}{1-k-kx}$$

at price p^m and allocates liquidity m^I to the informed so that they acquire $B(p^m, x)$. The reason is that the average quality of the types remaining in the public market should be equal to p^m; otherwise, the government cannot acquire the lower tails of the projects in the market.

Public investors would break even by purchasing types

$$\theta \in \left[\frac{2-(1+k)(1+x)}{1-k-kx}, \frac{p^m(1-k)(1+x)}{1-k-kx}\right]$$

at price p^m. Define x_1, x_2 as the lower and upper bound for x as described before:

$$x \in \left[\frac{(1-k)S}{1-(1-k)S}, \frac{S(1-k)}{p^m + kS}\right] = \left[x_1, x_2\right].$$

Note

$$x_2 < \frac{1-k}{k}.$$

Therefore,

$$x < \frac{1-k}{k},$$

which means

$$0 < 1 - k - kx.$$

Note the measure of the interval $B(p^m, x)$ should be less than the capacity constraint η. Now, for a given p^m, the government chooses x to maximize

$$\max F_1(x) = \frac{S(1-\mu)}{1-\gamma}\left(\frac{S(1-k)(1+x)}{x} - \gamma\right)$$

$$-\frac{\lambda(1-\mu)}{2(1-\gamma)}\left((p^m-\gamma)^2 - (\frac{p^m x}{1-k-kx})^2\right) - \lambda\mu p^m - \mu I,$$

such that the capacity constraint η is not violated. Subsequently, the government chooses p^m to maximize

$$\max F_2(p^m) = \frac{S(1-\mu)}{1-\gamma}\left(\frac{S(1-k)(1+x)}{x} - \gamma\right)$$

$$-\frac{\lambda(1-\mu)}{2(1-\gamma)}\left((p^m-\gamma)^2 - (\frac{p^m x}{1-k-kx})^2\right) - \lambda\mu p^m - \mu I,$$

Let's investigate the optimal amount of liquidity provision by the government which I denote by x^*.

One can easily observe the following(??):

$$0 < F_1''(x),$$

because,

$$F_1'(x) = \frac{-S^2(1-k)}{x^2} + \lambda\frac{(p^m)^2(1-k)x}{(1-k-kx)^3}$$

is increasing in x given the fact that $0 < 1 - k - kx$.

Moreover, the signs of $F_1'(x_1)$ and $F_1'(x_2)$ are independent of k, where

$$F_1'(x_1) = \frac{(1-(1-k)S)^2}{1-k}(-1+\lambda\frac{(p^m)^2 S}{(1-S)^3}),$$

and

$$F_1'(x_2) = \frac{(p^m+kS)^2}{1-k}(-1+\lambda\frac{S}{p^m}).$$

Note $0 < F_1'(x_2)$, if and only if

$$\frac{p^m}{S} < \lambda.$$

Moreover, $0 < F'(x_1)$ if and only if

$$\frac{(1-S)^3}{S(p^m)^2} < \lambda.$$

Denote

$$\lambda_1^* = \frac{p^m}{S},$$

and

$$\lambda_2^* = \frac{(1-S)^3}{S(p^m)^2}.$$

Note that under the condition $p^m + S < 1$, we would have

$$\lambda_1^* < \lambda_2^*.$$

Proposition 4 (Zero-Interest Liquidity Provision for the Informed) *For $I \leq p^m$, the optimal liquidity provision is to provide the informed the maximum feasible liquidity if λ is low enough (suffices to have $\lambda < \lambda_1^*$) or provide no liquidity if λ is high enough (suffices to have $\lambda_2^* < \lambda$).*

The intuition regarding the liquidity provision to the informed is as follows. The government trades-off the benefit of liquidity provision to the informed for the cost of distortion in the public market. The benefit of the informed for the is to indirectly creating a surplus that the public market cannot finance, because the high-quality types only transact in private markets. However, the informed can reduce the quality of the pool of assets in the public market, which would ultimately translate to a higher cost of deficit. If the cost of the deficit is high enough, the government fore-going the positive surplus in the private markets would be optimal. Moreover, if the deficit is not costly, the government provides the maximum feasible liquidity to the informed while providing a higher price support for the public market. In addition, if the cost function for deficit has a convex form, the optimal liquidity provision would happen in an interior point rather than the extreme sides of possible scenarios.

If the number of informed intermediaries is finite, the government can contract with them to provide liquidity, only if they do not increase the intervention costs. If contracting is not feasible, the government can bundle the liquidity with equity in the asset-purchase program to share the cost of cream-skimming (see appendix for an example). We have the following proposition:

Proposition 5 (Contracting with Informed) *Consider the case with a finite number of informed intermediaries. The government can maximize social welfare by contracting with informed intermediaries to shift their purchases to outside of the public market to minimize the cost for the government's intervention.*

To understand the above proposition, consider the government price $p^g = I$. The government can write a contract with the informed to shift their purchases to the interval $[I + S, 1]$; therefore, the social welfare would be

$$F^1 = S(1 - \mu) - \lambda \times D - \mu \times I < F^0.$$

Combining propositions 2 and 4, if λ is low enough, the price in the public market would be high and the informed would receive maximum liquidity. However, if λ is high enough, the government should keep its intervention at a minimum and provide no liquidity to the informed. I consider the optimal price choice by the government in the appendix.

1.5 Equilibrium and Results

Here, I first define an equilibrium notion when a partnership, liquidity provision, and asset-purchase can be implemented. Then, I illustrate the government's optimal choice and discuss the practical policy recommendations.

1.5.1 Equilibrium Notion

The main question is how the government can use all the tools and capabilities of the agents to most benefit society. One main consideration is how the price of a partnership program is determined. Moreover, the informed can always refuse to participate in a partnership and wait to make deals privately. Therefore, another important concern is whether the informed have the right incentive to participate in a partnership. The final aspect would be the possible interaction of a partnership and liquidity-provision program.

The price p^p can be set via Nash bargaining as in the private markets. However, I set $p^p = p^m$ for a more tractable analysis withought losing the generality of the main message. The case $p^p = p^m$, corresponds to the case in which the informed can make the highest possible return out of a partnership, and this return would be higher than the return of direct informed dealing in the private market. Therefore, the informed would always have the incentive to participate in a partnership.

The government offers liquidity m^I to the informed with the interest rate 0. Moreover, the government posts price $p^g = p^m$ as an asset-purchase program and provides funds to implement the public-private partnership program with the price $p^p = p^m$. Let 1^p indicate whether the partnership program is implemented or not.

Then, the government chooses

$$G = (1^p, m^I, p^g)$$

to maximize social welfare function

$$F = S \times v - I \times v' - B(M^g, D),$$

where ν is the total mass of projects with positive surplus financed, ν' is the mass of projects implemented with negative surplus, D is the deficit, M^g is the total funds that the government provide, and B is the cost for the government. The cost for the government is a function of the deficit and the total amount of funds provided. Here, I consider a strictly convex cost for the deficit to highlight interior solutions. Assume $B = \lambda(D)$ is strictly convex and $\lambda(0) = 0$.

I define the equilibrium notion as follows:

Definition 1 (Equilibrium Notion) *An equilibrium is a choice of policy $G = (1^p, m^I, p^g)$, where the government maximizes the social welfare function, and if $1^p = 1$, the informed agents' expected rate of return from entering in a partnership should be greater than or equal to the expected return of making deals privately.*

In this equilibrium notion, I maintain all other market structures in place. I next describe the government's optimal choice of action.

1.5.2 The Government's Optimal Choice

In this section, I summarize the government's optimization and the choice for the optimal policy. Then, the government either sets $1^p = 0$, $p^m = p^*$ and provides m^I, or the government implements a partnership policy $1^p = 1$, sets $p^m = p^*$ and provides m^I.

We immediately have the following theorem:

Theorem 1 (Indiscriminate Liquidity-Provision Equilibrium) *In the equilibrium, if μ is high enough and the deficit cost is low enough, the government would implement a partnership and provide liquidity for private markets. The government trades off the benefit of a partnership with the lost surplus in the private markets due to the informed participation and limited capacity.*

This is intuitive: if the partnership requires a high mass of the informed investors, there would be a surplus loss in the private markets as well a higher cost of asset-purchase compared with the benefit of removing a mass of insolvent banks.

In the figure 1.4, I present the optimal policy for different combination of parameters μ and λ.

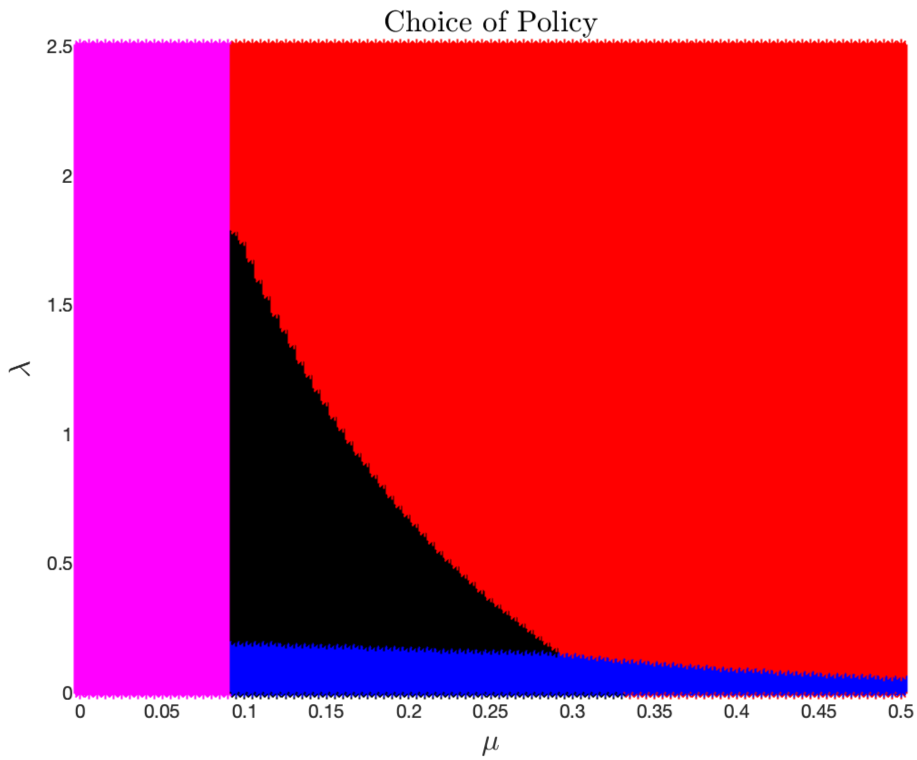

Figure 1.4: Optimal policy: Blue: PPIP, black: liquidity provision, red: only asset-purchase, cyan: public market is working.

1.6 Conclusions

In this paper, I provided a first analysis of how the government policies for market rejuvenation interact. The government trades off relying on the information of informed agents with the cost of public market contraction. This tradeoff is manifested in the two partnership and liquidity allocation policies interacted with the asset-purchase program. Informed agents can finance projects outside the capacity of the public markets, and the informed trading can signal strong balance sheets. This benefit of information is in contrast to the cost of cream-skimming, in which informed agents' trading can exacerbate the adverse selection in public markets and cause further contractions. Thus, informed agents, who can be a cause of the turmoil, can also assist the government in allocating liquidity efficiently, because their expertise can be utilized to the benefit of society. These results can be extended in a few directions for future work. First, liquidity can be provided to uninformed agents, who can lend this liquidity to informed agents besides the direct government lending. Moreover, if an atom-less mass of informed agents is present, the government can design mechanisms to disentangle informed and uninformed agents to incentivize informed agents to shift the financing to new projects outside the public market capacity.

Appendix

1.7 Conditional Liquidity Provision for the Informed

Now, we explore new policy directions and we only consider zero-interest liquidity for presenting the main insights. The government can use other mechanisms to improve the social welfare function. For example, the government can condition liquidity provision by obligating the informed investors to hold equity in the government's program. Additionally, the government can condition repayments on indices of the public market performance. In this way, some of the losses are offset by the profits of informed investors and the government can induce the informed to reduce the externality for the government.

Consider a new policy whereby the government offers (m^I, p, α), where α is the stake in the government's program that investors should hold in addition to paying back liquidity m^I. Under this policy, for a public market price of p^m, the government loss would be

$$L(\theta') = \frac{1}{1-\gamma}((p^m - \gamma)^2 - (p^m - \theta')^2),$$

where the government is purchasing types $\theta < 2p^m - \theta'$ and the informed are taking a subset of the types $[\theta', 1]$. Thus, if the informed take asset θ' that could be traded on the public market, the marginal loss for the informed would be

$$-\alpha L'(\theta') = \alpha \frac{2(\theta' - p^m)}{1-\gamma},$$

and the total marginal return would be

$$r^i(\theta', \alpha) = r^i(\theta')(1 - \frac{2\alpha(\theta' - p^m)}{p^i(\theta')(1-\gamma)}),$$

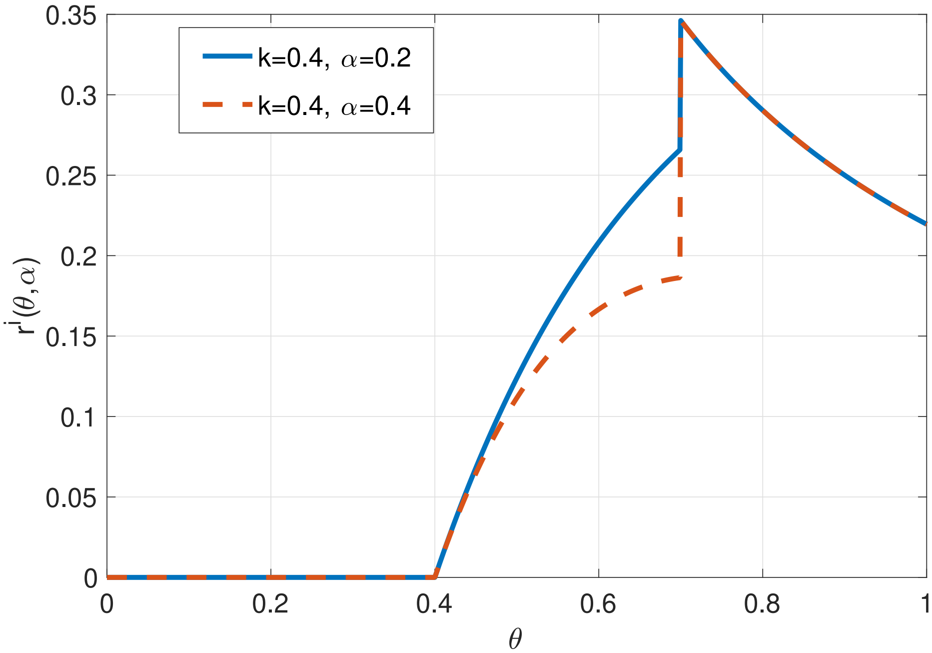

Figure 1.5: Informed rate of return vs θ.

where substituting $p^i(\theta) = p^m + k(\theta - p^m)$ would be equal to

$$r^i(\theta', \alpha) = r^i(\theta')(1 - \frac{2\alpha(\theta' - p^m)}{(p^m + k(\theta' - p^m))(1 - \gamma)}).$$

Figure 1.5 shows the return on the informed capital curve for $p^m = 0.4$, $S = 0.3$, and $\gamma = 0$.

Next, we consider the government's choice of optimal α. Note that increasing α is beneficial only for the government, because it would induce the informed investors to shift their deals to types that would not have been on the public market. This statement holds as long as the informed investors' profit is higher than their equity in the government's program. Thus, if α is sufficiently high, the informed would spend all their capital on the interval $[p^m, 1]$. The minimum amount of α necessary for the informed to spend all their capital on the interval $[p^m, 1]$ would be

$$\frac{(1-k)S}{p+kS}(1 - \frac{cS}{p+kS}) = \frac{(1-k)S}{p^m + \mu + S - (1-k)S},$$

where

$$c = \frac{2\alpha}{1-\gamma}.$$

Then,

$$\alpha^* = \frac{(1-\gamma)(p^m + kS)\mu}{2S(p^m + kS + \mu)},$$

which is increasing in μ, k, and p^m. Increasing α beyond α^* acts only as a transfer of more losses to the informed.

Proposition 6 (Conditional Zero-Interest Liquidity Provision for the Informed) *For every $I < p^m$, providing the informed an equity α in the government's program is optimal.*

However, the above policy suggestion is not practical but can be relevant to future crises. Consider what the government can do to optimize the outcome. If the number of informed intermediaries is finite, the government can provide liquidity and write a steep contract with the informed involving a punishment for cream-skimming and thus preventing them from distorting the public markets.

1.7.1 The government's price choice with liquidity provision

We assume μ, the total capacity for the informed, is small enough, which means $x^*(p^m)$ would be such that

$$|B(p^m, x^*)| = \frac{S(1+x^*)}{x^*} - p^m(1+x^*) = \mu.$$

Now, we can determine the optimal p^m. For that, the government chooses p^m to maximize

$$F(x) = S(\frac{S(1+x^*)}{x^*} - \gamma) - \frac{\lambda}{2}((p^m - \gamma)^2 - (p^m x^*)^2).$$

Because

$$\frac{S(1+x^*)}{x^*} - p^m(1+x^*) = \mu,$$

the government should choose p^m to maximize

$$F(x) = S(\mu + p^m(1 + x^*) - \gamma) - \frac{\lambda}{2}((p^m - \gamma)^2 - (p^m x^*)^2).$$

It can be the case that the government benefits from increasing price.

1.8 Guarantee Programs

There were many programs launched as a response to the 2008 crisis. One of the most prevalent tools was implementing guarantee programs to reduce the financing burden for the firms. Guarantees were introduced in more than 19 countries. We will provide a glimpse of these guarantee programs and discuss their performance and implications.

Japan had one of the largest guarantee programs (near 300 billion us dollars). [23] explores the interaction of guarantee program and relationship lending and finds that there was a reduction in firm performance for firms who borrowed from a main bank. Moreover, they find that main banks substituted non-guaranteed loans with guaranteed ones but in overall the firms received more financing.

Judge (2018) proposes a guarantor of last resort (or emergency guarantor authority, EGA), which helps to stop the financial panic and keep the private capital remaining in the system. The EGA would also provide the time and option for further policy intervention and addressing the information and capital gaps. Judge (2018) also raises important issues in implementing an EGA in an environment where government and agents can have different information about healthy institutions and frictions in liquidity allocation. On the other hand, she correctly mentions that EGA needs a careful assessment of liquidity and credit risk and a precise judgment when it faces a pool of good and bad firms.

In the U.S., the public authority implemented a number of guarantee programs. The FDIC extended deposit insurance to include deposits more than $100,000. Treasury issued guarantees for money market mutual funds to not allow these to funds break the buck. The MMF guarantee included all the eligible funds and was in the form of an \$50b Exchange Stabilization Fund (ESF).

Moreover, FDIC has added a debt guarantee program (DGP) to boost inter-bank lending and ultimately improve bank lending to businesses. FDIC also established a transaction account guarantee program (TAGP) to prevent runs on small banks. The two programs TAGP and DGP are under the temporary liquidity guarantee program (TLGP). TLGP was drafted in a way that it only relies on its own fees.

The DGP program issued guarantees for unsecured debts issued between Oct 14 2008, and June 30 2009, and was expected to mature before June 30 2012. In 2009, the authorities extended the termination date to Dec 31 2012. The main goal of the program was to revive the inter-bank lending. However, a guarantee debt can cause over lending and ultimately a misuse of funds.

The DGP did not include short-term senior unsecured debt with maturity less than 30 days. The fees for guarantees were increasing with debt maturity and were around 75 basis points. Participating in DGP was with another cost as the DGP imposed the restriction of max out the guaranteed debt capacity first and disclosing non-guaranteed debt.

These programs were in general effective to engage private lender in uncertain markets. The size of DGP became as large as 346b at some point. While there was a risk of adverse selection and defaults, TGLP generated more revenues than its losses. The DGP gathered 10.4b revenue out of fees and 153m in losses. Moreover, TAGP collected 1.2b in revenue while incurring 2.1b in losses.

There are several important questions in this context. How establishing a 50b ESF fund can prevent runs from MMFs that have a size of more than 2t. How was the debt guarantee program designed in term of the entry and exit decision for the government and the coverage terms (duration, seniority, fees, etc)? What is the cost and benefit of a guarantee program as opposed to traditional LOLR?

Chapter 2: Traveling Times of Asset Return

2.1 Introduction

Features[1] and moments of asset returns can be helpful for examining markets, predicting future quantities and gauging macroeconomic fluctuations. Although the univariate distribution of returns are studied substantially, the bivariate distribution of returns can reveal useful additional information. If a process is i.i.d., different draws of the process at different points in time are independent, and knowledge of the underlying probability distribution is enough to pin down all moments. However, many financial processes are dynamic in nature and have time interdependencies. Therefore, knowledge of mean, variance or higher return moments at each point in time is not enough to fully pin down the process. One needs to know the joint distribution of the process for different points in time.

In the paper, I introduce the fundamental and yet less explored concept of expected traveling time (ETT henceforth) along with transition probabilities (TPs henceforth) for asset returns and discuss their estimation, computation and application. ETT is intuitively the expected time for the first visit to a given state, starting from another given state. The concept of traveling time is well known in Markov chain applications (under the names hitting time or passage time) but is largely missing from the finance literature with few exceptions. Given a Markov chain, the chain behaviour can be fully characterized by knowledge of the transition probabilities. Therefore, if a process can be approximated by a Markov chain, a first step to characterize the process would be to quantify and estimate its transition probabilities. Afterwards, other features could be fully calculated from the quantified transition probabilities, such as traveling times. Intuitively, if a process is moving swiftly between states, then the traveling times are low and if a process is persistent, the traveling

[1] I am grateful to professor John Donaldson for advice on this paper and providing the data. I am thankful to professors Rajnish Mehra, Harry Mamayaski, Michael Johannes, Paul Tetlock and Tano Santos for constructive comments.

times would be high. As a result, changes in transition probabilities or traveling times can signal important market expectations.

Consider a given underlying asset X and its return time series R_t. Similarly, one can also consider the price time series P_t or any other time-related series that can be modeled as stationary over a finite time interval. One can partition the range of values for R_t into different regions and identify the regions as different states. Starting from a state S_1 and making random draws, the process can end up in another state S_2. The first passage time it takes to travel from S_1 to S_2 is a random variable with values in the set of positive integers. Define the expected traveling time (ETT) as the average of the random times it takes for the first passage from S_1 to S_2. For example, one can calculate the TP and ETT from a high return state defined by $R_t > R_H$, to a low return state $R_t < R_L$ or to a mid-range return $R_L < R_t < R_H$.

What is the importance of ETT? ETTs and TPs capture fundamental moments on the joint distribution of the process, for two different points in time, as opposed to the mean and standard deviation are univariate moments. For a given process x_t, ETT is capturing information about the joint distribution of x_t and x_{t+i} which is information beyond autocorrelations.

Having an estimate of the TPs and ETTs would result in a finer understanding of the return process. Once the TPs and ETTs are quantified for a process (with proper statistical significance), a number of questions follow. Is the process different from a random walk or AR1? Can one implement a trading strategy to benefit from the transition of the process between different states? What drives the changes in time series and cross-section of TPs and ETTs for different assets? Can the changes in TPs or ETTs signal market expectations or forecast future returns? How can we price assets for which the payoffs are linked to the first passage to particular states?

In the paper, I first review the concept of expected traveling time (ETT) and provide useful results for calculating the ETT. Then, I exploit my analysis of ETT to provide pricing equations and insights for an economy which is governed by a stationary Markov chain. I then provide results regarding the ETT for important processes such as an autoregressive of order one (AR1), moving average (MA) and random walk (RW). I then discuss the estimation of TPs and ETTs using historical data and option prices. Finally, I estimate ETTs using the historical data for a number of assets to show the variation of ETT across assets and across time.

More specifically, I provide the following results:

1. The paper characterizes the TPs and ETTs for important time-series such as AR1, MA1 and RW. Moreover, I provide a procedure for estimating the ETTs from historical data and apply it to a production asset pricing model. In addition, I show that option prices, such as Knock-out and barrier type options, can reveal the transition probabilities for an asset's return process.

2. The paper provides closed-form solutions for particular fixed-income assets for which payoffs would terminate when a particular state is realized. Examples are risky debt, insurance contracts and long forward-looking bonds. These results are new to Finance, Economics and Markov chain literature.

An outline of the paper is as follows. First, I provide a basic example in trading to motivate the concept of traveling time. In Section 8, I formally introduce the concepts TP and ETT. In Section 2.3, I provide the application of my analysis to an economy governed by an arbitrary stationary Markov chain. Section 2.4 provides an analysis of ETTs for important time-series in econometrics. Section 2.5 discusses estimation methods of TPs and ETT. In Section 2.6, I estimate ETTs for a production economy and equity factor returns while Section 2.7 concludes the paper.

2.1.1 Motivations

To motivate the concept of expected travelling time, I start with an example from trading and then I provide a numerical example to compare with its autocorrelation.

There is an interpretation of ETT when the process under study is regarded as a price and not a return process. Prices are usually non-stationary, although a detrended price or a price series in a short time interval can be regarded as stationary. For example, the residual from a trading strategy or a high-frequency price in a short time frame can be regarded as stationary.

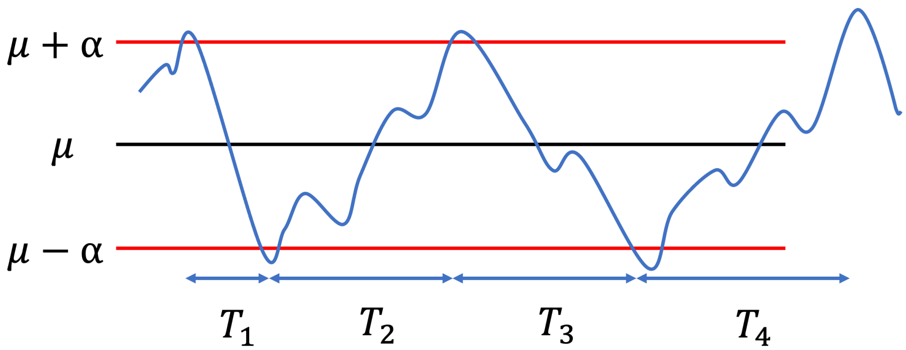

Figure 2.1: An example in trading, T_is are the times to travel from high to low and from low to high.

In these cases, the success of a trading strategy is linked to the concept of traveling times. To see this, assume a trading strategy that buys when the price hits (or exceeds) $\mu - \alpha\sigma$ and sells at $\mu + \alpha\sigma$ and again short sells at $\mu + \alpha\sigma$ and so on. The profits for this trading strategy depends on how much time it takes to travel from a low to a high state or vice versa, as is depicted in Figure 2.1. If the transaction cost per trade is c, the cost of waiting per period is b (for example, the marginal cost of technology maintenance or the borrowing cost per period), and there are n trades in a sample of length T, then the total expected profit per trade would be

$$y(\alpha) = \frac{(2\alpha\sigma - c)n - bT}{n},$$

or

$$y(\alpha) = (2\alpha\sigma - c) - bETT(\alpha),$$

where the expected traveling time (ETT) is defined as $\frac{T}{n}$ (I will formally explain this in the section 2.5).

Taking a derivative of $y(\alpha)$ w.r.t. α would yield

$$2\sigma - bETT'(\alpha^*) = 0,$$

or

$$ETT'(\alpha^*) = \frac{2\sigma}{b}.$$

Therefore, the trader can optimally choose α^*, which is decreasing in the waiting cost b if and only if the $ETT(\alpha)$ curve is convex with respect to α. Later in section 2.4, I will provide conditions for special cases of expected traveling times curves to be concave or convex.

Now, consider the following example. Denote ρ as the autocorrelation of a given time series. Consider two two-state Markov Chains defined by transitions probabilities p_{12} (from state one to state two) and p_{21} (from state two to state one) as:

- Series 1: $p_{12} = 0.99$ and $p_{21} = 0.11$.

- Series 2: $p_{12} = p_{21} = 0.3$.

As in [24], one can show that for the two aforementioned series (see also Example 2) the autocorrelation is

$$\rho = 1 - p_{12} - p_{21},$$

and,

$$ETT(0) = \frac{1}{1-\rho}\left(\frac{p_{21}}{p_{12}} + \frac{p_{12}}{p_{21}}\right).$$

Therefore for the above series:

- Series 1:

$$\rho = -0.1, \quad ETT(0) = 8.28.$$

- Series 2:

$$\rho = \ 0.4, \quad ETT(0) = 3.33.$$

Finance research has been mostly silent on which series is more persistent: a series can have high traveling times yet a negative autocorrelation. How do factor returns, test assets and stocks behave when comparing ρ and ETT? I will touch on this important question in the Section 2.6.

2.1.2 Related Literature

In this section, I connect the concepts in this paper to different parts of the literature. The paper is related to strands of literature in asset pricing, mathematical finance, operation research and macroeconomics [2].

The basic mathematical results about Markov chains can be found in books such as [27] and [28]. I will also use results from [29] which explores identities regarding hitting times of regular Markov chain. Markov chain models have applications to macroeconomics and asset pricing. Such applications are often under the Markov switching model; see [30] and [31] for finance related applications, [32] for an econometrics analysis and [33] for an introduction. [34] and [35] use similar measures related to hitting times in a two-state MC to study inflation persistence and mean-reversion.

This paper is slightly related to the literature in quantitative finance and operation research that studies hitting times. A number of papers characterize the optimal trading threshold in continuous-time for a mean-reverting process such as [36], [37], [38], [39], [40], [41] and [42]. For pairs trading (which is a long position with a short position in similar security), see [43] and [44]. [45] characterize an optimal buy low and sell high strategy for a GBM price process (see also [46] for an analysis of a buy and hold strategy), and [47] investigates market-making profits for mean-reverting price processes. The paper is also related to the literature that studies the statistical properties of asset returns including statistical arbitrage. [48] is the main paper on statistical arbitrage, which studies the performance of various strategies, and [49] studies statistical arbitrage for commodity spreads. The previous works in operation research are related to hitting times of continuous-time stochastic processes; however, I will be distinct as I study the hitting times and their estimation in discrete-time with different applications to trading.

The paper is related to works that document the time-series behavior of assets returns, such as mean-reversion. [24] provides a thorough study and comparison with their newly defined metric of Average Crossing Time or (ACT), which is related to hitting times, as a proxy for mean-reversion.

[2] For applications of hitting times in dynamical systems see [25] and [26].

For other empirical investigation of mean-reversion refer to [50], [51], [52], [53], [54], [55]). I will also compare the concept of traveling time to autocorrelation and document a few empirical observations for factor portfolios. [56] shows that momentum exists in well-diversified factor portfolios with surprisingly a negative autocorrelation. This means that there would be a difference between momentum and autocorrelation. In this paper, I also make a difference between the concept of ETT and autocorrelation. Therefore a natural question for future research would be if the concept of Momentum is linked to the concept of ETT.

I present theoretical results for the pricing of certain fixed-income securities, which is related to the literature on pricing of securities in an MC setting and the literature on recovery of certain parameters from the observed prices. [57] is the first paper to provide a recovery theorem. [58] provides recovery in a more generalized setting. [59] provide an analysis of bond pricing in an MC setting and also discuss the recovery of subjective probabilities from observed prices. [60] discuss recovery in the presence of long-term risks to disentangle investor's beliefs (see also [61]).

While mean hitting time, mean escape times and other measures are studied in mathematics, there are few papers that discuss perspicuous connections to financial concepts. [62] studies risk and return of stock prices by considering a Heston model and analyzing the escape times. [63] study the hitting times' distribution of stocks under different models and compare to the empirical density of hitting times. Moreover, [64] investigates the first passage time for different diffusion processes and studies an application in trading and provides a bubble model and its first passage time. In this paper, I have a more direct approach for estimating and using traveling times for financial concepts. I do not rely on any diffusion process but rather estimate the quantities from historical data, and I provide connections to pricing specific securities and the recovery of state prices.

There are two close papers to this work. [24] studies the average time that it takes to cross the mean as a measure of persistence and studies the behavior of this measure in a general equilibrium macro model. Moreover, [65] studies the forward-looking estimation of autocorrelation. In this paper, I adopt the same method introduced in [65] to infer forward-looking ETTs and TPs.

2.2 Expected Traveling Time (ETT)

In this section, I introduce the concept of expected traveling time and provide several results. While the main focus is the concept of traveling time, I will also introduce other related notions that are known in the Markov chain literature.

Consider an N state Markov chain (MC) with states $S_1, S_2, ..., S_N$ and let p_{ij} be the transition probability from state i to state j. Denote the transition matrix by P with the ijth entry equal to p_{ij}. Note that for $i = 1, 2, ...N$,

$$\sum_{j=1}^{N} p_{ij} = 1.$$

In matrix form

$$P\mathbf{1} = \mathbf{1},$$

where $\mathbf{1}$ is a column vector with all entries equal to 1. Therefore, $\mathbf{1}$ is the right eigenvector of P and one can show that it is the unique (up to a constant) eigenvector corresponding to the eigenvalue 1.

I only consider regular Markov chains which means chains with the positive probability of moving from any state to another after a finite number of steps. For a chain to be regular, it is equivalent to assume that P^n has only positive entries for some natural number n.

Let π_i be the stationary unconditional probability of state i and note that

$$\sum_{i=1}^{N} \pi_i = 1.$$

When the chain is in equilibrium, state i would occur with probability π_i and one can show that for regular MCs all the entries of π are strictly positive. Standard conditional probability arguments show that the probabilities π_i satisfy

$$\pi_i = \sum_{j=1}^{N} \pi_j p_{ji}\,.$$

In matrix form (superscript T corresponds to matrix or vector transpose):

$$\pi^T P = \pi^T.$$

Therefore, π is the left eigenvector of P corresponding to eigenvalue 1 and one can show that π is the unique vector (up to a constant) with this property.

Define the matrix W to be the $N \times N$ matrix with all rows equal to π^T. Then it is easy to show that

$$WP = PW = W,$$

and

$$W^2 = W.$$

Now, I define the expected traveling time as the expectation of the first hitting time for a particular state starting from another given state.

Definition 2 *Consider a MC with N states denoted by $1, 2, ..., N$. Suppose the current state is i and consider a random sequence of realizations of the chain and denote the first time that another state j is observed by the random variable T_{ij}. Define $t_{ij} = E\,T_{ij}$ as the expected traveling time (ETT) from state i to state j. Moreover, for $i = 1, 2, ..., N$, define $t_{ii} = 0$. Denote the ETT matrix by $T = [t_{ij}]$.*

It is useful to define the expected traveling time from a given state to itself.

Definition 3 *Define $m_{ii} = E\,T_{ii}$ as the expected traveling time (ETT) starting from state i and ending to state i at some future point.*

I now define a measure of traveling time that averages two separate ETTs using stationary probabilities π.

Definition 4 *For a stationary process X_t with unconditional mean μ and standard deviation σ, define $ETT(\alpha)$ as follows. Consider a Markov chain defined on 3 states $X < \mu - \alpha\sigma$, $\mu - \alpha\sigma <$*

$X < \mu + \alpha\sigma$ and $\mu + \alpha\sigma < X$ and number the states to be 1, 2, 3 *respectively. Define,*

$$ETT(\alpha) = \pi_1 t_{13} + \pi_3 t_{31}.$$

The autocorrelation of a general Markov chain depends an the numerical values assigned to the different states (except for a chain with only two states). I am interested to introduce measures that are defined for a chain itself without reference to the numerical values assigned to each state. For this purpose, I define different measures. For a given state i, define the expected cover time as the expected traveling times to all other states starting from state i. Moreover, the expected cover time for the chain is defined as the expected cover time for individual states where the expectation is done using stationary probabilities. I formalize these definitions.

Definition 5 *Define the expected cover time for state i as*

$$C_i = \sum_{j=1}^{N} \pi_j t_{ij} \ .$$

Moreover, the expected cover time for the chain is defined as:

$$ECT = \sum_{j=1}^{N} \pi_j C_j \ .$$

ECT can be interpreted as a measure of persistence of a chain that is defined using only the TPs and not the particular values assigned to different states. I normalize ECT so that for an i.i.d. chain the following measure becomes equal to zero.

Definition 6 *Define the magnitude of persistence of a given N state Markov chain as* $1 - (N - 1)/ECT$.

It is easy to prove that in a two state Markov chain the ECT is related directly to the autocorrelation of the chain denoted by ρ (in a two state chain the autocorrelation is only a function of transition probabilities P). For calculations please refer to Example 2 in Section 2.4.

Proposition 7 *In a two state Markov chain*

$$ECT = \frac{1}{1-\rho} = \frac{1}{p_{12} + p_{21}}.$$

Therefore, for a two state chain, if $\rho = 1$ then $ECT = \infty$ and if $\rho = 0$ then $ECT = 1$. There is a one-to-one correspondence between the auto-correlation of a two state chain and its ECT.

I am interested to find closed form expressions for ETT and ECT as a function of the transition matrix P. For calculating t_{ij}, consider the immediate next state after i and call it k. The time to travel from the state i to the state j would be one plus the ETT from state k to state j and taking an average with respect to the TPs yields the following result:

Proposition 8 *The following system of equations relates ETTs to TPs. For $i, j = 1, 2, ..., N$ and $i \neq j$:*

$$t_{ij} = \sum_{k=1}^{N} p_{ik}(1 + t_{kj}) = 1 + \sum_{k=1}^{N} p_{ik} t_{kj} \,,$$

where $t_{ii} = 0$.

I now demonstrate a closed form solution for ETTs using results from the Markov Chain literature. Define $W = \mathbf{1} \times \pi^T$ and denote I as the identity matrix. The fundamental matrix Z is defined as $Z = (I - P + W)^{-1}$. Note that the inverse of $I - P + W$ always exists (see [29]).

In the following proposition, I mention basic properties of the matrix Z.

The following results are known in the Markov chain literature (see [29]) and I restate them in this paper's notation.

Proposition 9 **(Known Result)** Let Z be the fundamental matrix for some Markov Chain with entries Z_{ij}. For $i, j = 1, 2, ..., N$:

$$t_{ij} = \frac{Z_{jj} - Z_{ij}}{\pi_j} \,.$$

There are closed form solutions for expected covering times which are given in the proposition below (see [29] for a proof of the below result and discussion of similar results).

Proposition 10 (Known Result) Denote $\lambda_1 = 1, \lambda_2, ..., \lambda_N$ as eigenvalues of the transition matrix P. Then the value of C_i does not depend on i. Thus, for every i,

$$ECT = C_i = tr(Z) - 1 = \sum_{j=2}^{N} \frac{1}{1-\lambda_j}.$$

Calculation of covering times require knowledge of all the entries of the transition matrix. The above Proposition shows that the ECT is invariant of state i and thus the ECT encapsulates information about the whole chain. Moreover, the above proposition proves provides an intuitive closed form expression based on the eigenvalues. If eigenvalues are close to 1, then the ECT would be high suggesting that it takes a long time for the process to cover all the states.

Here, I propose a lower bound for ECT which is a tight bound for $N = 2$ and helps to build intuition.

Proposition 11 *The ECT is bounded by*

$$ECT \geq \frac{(N-1)^2}{N - p_{11} - p_{22} - ... - p_{NN}}.$$

Proof. Note that the function $f(x) = 1/x$ is convex. Using the Jensen's inequality for positive real numbers $x_1, x_2, ..., x_n$, one can conclude the below inequality

$$(\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_n})(x_1 + x_2 + ... + x_n) \geq n^2.$$

Using the above inequality for

$$\frac{1}{1-\lambda_1}, \frac{1}{1-\lambda_2}, ..., \frac{1}{1-\lambda_N}$$

completes the proof. ■

Therefore, intuitively, if the sum of transition probabilities from all states to themselves is high, it would take a long time for the chain to cover all the states as the chain would be likely to continue to be in the previous state with a high probability.

2.3 An Economic Model

In this section, I introduce and analyze the properties of particular claims that are naturally related to hitting times in an economy governed by a Markov chain. Consider a fixed income market which is complete with a state variable which takes N different values corresponding to states $1, 2, ..., N$. I assume there is no arbitrage.

The closest paper for this section is [59] who examines properties of a long bond and the recovery of Arrow-Debreu prices. My analysis considers the same setting as in [59] but I consider a new set of securities related to hitting times and provide new results for pricing those assets, recovering Arrow-Debreu prices and studying the properties of stochastic discount factors.

Let A_{ij} be the Arrow-Debreu price of a security in state i that pays \$1 if the next state happens to be j and let matrix A be $A = [A_{ij}]$. The transition probability matrix is P with $p_{ij} > 0$. Then, $A_{ij} > 0$ and the sum of ith row of A is the price of a risk-less bond in the state i. The Arrow-Debreu prices are related to transition probabilities. For example, [59] show that in a basic setting A_{ij} can be thought of p_{ij} times the ratio of marginal utility of a representative investor

$$A_{ij} = \alpha \pi_{ij} \frac{u'(j)}{u'(i)}, \tag{2.1}$$

where α is the time discount rate.

My examination of traveling times is useful to analyze pricing of particular claims. Define a *Risky Debt* as follows: if the current state is i, consider an asset that pays a constant equal to \$1 and it pays 0 after and including the first time that another specific state j is realized and denote by D_{ij} as the price of this asset. Risky debt corresponds to risk-less claims unless particular default states triger zero payoff.

The following two propositions relate D_{ij} to entries of A.

Proposition 12 *For* $i, j = 1, 2, ..., N$ *and* $i \neq j$

$$D_{ij} = \sum_{k=1}^{N} A_{ik}(1 + D_{kj}) .$$

where, I define $D_{ii} = 0$*.*

Using the same approach as in Section 1, I present closed form expressions D assets. I consider the risk-neutral case for simplicity in which it is easy to note that $D_{ij} = t_{ij}$, where t_{ij} is the expected traveling time.

Let's b be a left eigenvector of A such that

$$b^T = b^T A$$

and

$$\mathbf{1}^T b = \mathbf{1}.$$

Define $W = \mathbf{1} \times b^T$ and define the matrix Φ as $\Phi = (I - A + W)^{-1}$.

Proposition 13 *Consider the risk-neutral case and with no discounting. For* $i, j = 1, 2, ..., N$*, the price of D assets are as follows*

$$D_{ij} = \frac{\Phi_{jj} - \Phi_{ij}}{b_j} .$$

Proof. Since $D_{ij} = t_{ij}$, the expression for D_{ij} follows the closed expression for t_{ij} as in Proposition 10. ■

Now, I state a recovery theorem which states that the knowledge of prices D_{ij} is enough to recover all Arrow-Debreu prices A_{ij}. This result does not mean a recovery of subjective probabilities and marginal utilities from observed prices (such as [57] and [58]) as I assumed a risk-neutral case with no discounting. The importance of this result is a new emphasize about risky debt: A full knowledge of all risky debt prices (for which only one state triggers default) would be enough to pin down the economy.

Proposition 14 *The knowledge of D_{ij} would recover all Arrow-Debreu prices A_{ij}.*

Proof. Since $D_{ij} = t_{ij}$, it is enough to show that the knowledge of all t_{ij} would recover all Arrow-Debreu prices A_{ij}. The latter follows from Theorem 4.4.12 of [28]. ■

2.4 ETT Analysis for Time-Series Models

In this section, I provide basic examples of ETT for different processes. I consider discrete time Markov chains and processes such as an AR1 process and random walk (RW).

Example 1 (i.i.d. process): Consider an i.i.d. process x_t with an arbitrary probability distribution. The transition probability from state S to state U is given by unconditional probability of state U which I denote by $P(U)$. The latter is true since at each point in time we have an independent draw. Moreover, the probability of traveling from state S to state U is given by the reciprocal of unconditional probability of state U. This can easily be proven by noting that if T is the expected time to travel to state U (from any state), then:

$$T = P(U)(1) + (1 - P(U))(1 + T) \Rightarrow T = \frac{1}{P(U)}.$$

The following proposition summarizes the i.i.d. case.

Proposition 15 *Consider a Markov chain with N states which resembles an i.i.d. process such that $p_{ij} = \pi_j$ and denote by μ and σ the mean and standard deviation. Then,*

$$t_{ij} = 1/\pi_j,$$

$$ECT = N - 1,$$

and

$$ETT(\alpha) = \frac{2}{1 - P(|X - \mu| < \alpha\sigma)}.$$

Returning to my example of trading, the $ETT(\alpha)$ curve can be concave or convex. Consider an i.i.d. process with C.D.F. $F(x)$. Then, ETT from any state to $X > \mu - \alpha\sigma$ would be

$$ETT_1(\alpha) = \frac{1}{1 - F(\mu - \alpha\sigma)}.$$

Thus,

$$ETT_1'(\alpha) = \frac{\sigma f(\mu - \alpha\sigma)}{1 - F(\mu - \alpha\sigma)}.$$

Therefore $ETT(\alpha)$ is convex in α if and only if the hazard function for the C.D.F. F is increasing.

Example 2 (Two states Markov chain): Consider a Markov Chain with two states and transition probabilities p_{12} and p_{21}. As shown in [24], the autocorrelation for such a chain would be $\rho = 1 - p_{12} - p_{21}$ and:

$$\pi_1 = \frac{1 - p_{22}}{2 - p_{11} - p_{22}},$$
$$\pi_2 = \frac{1 - p_{11}}{2 - p_{11} - p_{22}},$$

and

$$t_{12} = \frac{1}{p_{12}},$$
$$t_{21} = \frac{1}{p_{21}}.$$

Moreover, the ECT for this chain would be

$$ECT = \frac{1}{p_{12} + p_{21}} = \frac{1}{1 - \rho}.$$

[24] define the Average Crossing Time or ACT as

$$\pi_1 t_{12} + \pi_2 t_{21} = \frac{1}{2 - p_{11} - p_{22}} \left(\frac{p_{21}}{p_{12}} + \frac{p_{12}}{p_{21}} \right),$$

and examine its properties. The relation between ACT and ρ is not a one-to-one relation, as opposed to ECT and ρ. However, if the Markov chain with two states is symmetric so that $p_{12} = p_{21} = p$, then ECT and ACT would coincide up to a constant factor.

Example 3 (Three states Markov chain): Consider a Markov chain with three states. We can find the ETT by solving a system of equations as in Proposition 8. One can find a closed form expression for ETTs.

Proposition 16 *For a three states Markov chain with TP matrix P, the closed form expression for t_{12} is given by*

$$t_{12} = \frac{1 - p_{33} + p_{13}}{(1 - p_{11})(1 - p_{33}) - p_{13}p_{31}}.$$

Therefore, for a three states Markov chain, t_{12} is a function of TPs alone and one can observe that if $p_{13} = p_{23} = 0$, the expression for t_{12} in a 3-state MC would become the same as t_{12} in a 2-state MC.

To provide an example of ECT, let's consider a chain of three states with the transition probabilities given by the below matrix

$$P = \begin{pmatrix} \alpha & 1-\alpha & 0 \\ 0 & \beta & 1-\beta \\ 1-\gamma & 0 & \gamma \end{pmatrix}.$$

It is easy to find expressions for the stationary probabilities. One can show that

$$\pi_1 = \frac{(1-\gamma)(1-\beta)}{(1-\alpha)(1-\beta) + (1-\alpha)(1-\gamma) + (1-\beta)(1-\gamma)}.$$

If the eigenvalues of P are $1, x, y$, then by Proposition 10

$$ECT = \frac{1}{1-x} + \frac{1}{1-y} = \frac{2-x-y}{1-x-y+xy}.$$

The eigenvalues satisfy the characteristics equation

$$(\lambda - 1)(\lambda - x)(\lambda - y) = (\lambda - \alpha)(\lambda - \beta)(\lambda - \gamma) - (1 - \alpha)(1 - \beta)(1 - \gamma),$$

and thus,

$$1 + x + y = \alpha + \beta + \gamma,$$

$$xy + x + y = \alpha\beta + \alpha\gamma + \beta\gamma.$$

Therefore,

$$ECT = \frac{3 - (\alpha + \beta + \gamma)}{3 + \alpha\beta + \alpha\gamma + \beta\gamma - 2(\alpha + \beta + \gamma)}.$$

In the next example, I explore the important time-series process AR1.

Example 4 (AR1): Consider an AR1 process, $x_{t+1} = \rho x_t + \sigma \epsilon_{t+1}$, where the residual ϵ_t is i.i.d. standard normal and ρ is the autocorrelation parameter. Define $y_t = x_t/m$, where $m = (\sigma/\sqrt{1-\rho^2})$ such that y_t has unconditional variance equal to 1. I normalize the σ to 1 with no loss of generality.

Consider 3 states defined by whether the value $x_t = x$ is within these regions: $x < b, b < x < a, a < x$. For example, the first state corresponds to the region $x < b$ which will happen when the process is below a given threshold b.

I now calculate the transition probabilities between different states. For example, for calculating the transition probability from the state $x > a$ to the state $x < b$, denoted by $P(a, b)$, we should compute the conditional probability

$$P(a, b|\rho) = P(x_{t+1} < b|x_t > a) = P(\rho x_t + \epsilon_{t+1} < b|x_t > a)$$

$$= P(\rho m y + \epsilon_{t+1} < b|my > a),$$

which is the area under $y > a/m$ and $\epsilon < b - (\rho m)y$ (for a two dimensional standard normal distribution) divided by the probability of $y > a/m$.

Note that one can write the above expression as

$$P(a, b|\rho) = P((1/m)(\rho x_t + \epsilon_{t+1}) < b/m|x_t/m > a/m)$$

$$= P(X < b/m|X' > a/m),$$

where $X = (1/m)(\rho x_t + \epsilon_{t+1})$ and $X' = x_t/m$ have variance 1 and are jointly normal with correlation ρ.

This is summarized in the proposition below.

Proposition 17 *Consider the AR1 process, $x_{t+1} = \rho x_t + \epsilon_{t+1}$, where the residual ϵ_t is i.i.d. standard normal and ρ is the autocorrelation parameter. Moreover, denote $F(x)$ and $f(x)$ to be the C.D.F. and p.d.f. of the standard normal distribution, respectively, and define $m = (1/\sqrt{1-\rho^2})$. The transition probability from the state $x > a$ to the state $x < b$ for the AR1 process, denoted by $P(a, b)$, is given by*

$$P(a, b|\rho) = \frac{\int_{a/m}^{\infty} F(b - \rho m x) f(x) dx}{1 - F(a/m)}. \tag{2.2}$$

I will not present a closed form for $P(a, b)$ except in one special case. One can show that (see [24]):

$$P(0, 0|\rho) = G(0, 0, \rho) = \frac{\pi/2 - tg^{-1}(\rho/\sqrt{1-\rho^2})}{\pi}.$$

This is related to the Average Crossing Time (ACT) from the mean introduced in [24]. After finding the transition probabilities for the 3 states, one can find the ETTs. I will turn into estimation of ETTs from historical data for an AR1 process in the next section. I will use historical frequency and compare it to numerically solving transition probabilities suggested by similar expressions as in 2.2.

Moreover, for an AR1 process defined as above, one can find the ECT for the case where we consider only two states $x > 0$ and $x < 0$.

Proposition 18 *Consider the AR1 process, $x_{t+1} = \rho x_t + \sigma\epsilon_{t+1}$, where the residual ϵ_t is i.i.d. standard normal and ρ is the autocorrelation parameter. Then, if we consider the derived Markov chain with two states $x_t > 0$ and $x_t < 0$, we would have*

$$ECT = \frac{\pi}{\pi - 2tg^{-1}(\rho/\sqrt{1-\rho^2})}.$$

Example 5 (MA1): Consider a moving average (MA1) process, $x_{t+1} = \epsilon_{t+1} + w\epsilon_t$, where ϵ_t is i.i.d. standard normal. Define $m'^2 = Var(x_t) = 1 + w^2$ and

$$\rho' = Corr(x_{t+1}, x_t) = Corr(\epsilon_{t+1} + w\epsilon_t, \epsilon_t + w\epsilon_{t-1}) = w/(1+w^2).$$

Note that

$$P(x_{t+1} < b | x_t > a) = \frac{P(\epsilon_{t+1} + w\epsilon_t < b, \epsilon_t + w\epsilon_{t-1} > a)}{P(\epsilon_t + w\epsilon_{t-1} > a)},$$
$$= P(x_{t+1}/m' < b/m' | x_t/m' > a/m').$$

The following proposition can be proved using standard probability arguments.

Proposition 19 *Consider the MA1 process, $x_{t+1} = \epsilon_{t+1} + w\epsilon_t$, where ϵ_t is i.i.d. standard normal. Moreover, denote $F(x)$ and $f(x)$ to be the C.D.F. and p.d.f. of the standard normal distribution, respectively. Define $m'^2 = Var(x_t) = 1 + w^2$ and*

$$\rho' = w/(1+w^2).$$

The transition probability from the state $x > a$ to the state $x < b$ for the MA1 process, denoted by $Q(a, b)$, is given by

$$Q(a, b|w) = \frac{\int_{-\infty}^{+\infty} F(b - wx)(1 - F(a/w - x/w)) f(x) dx}{1 - F(a/\sqrt{1+w^2})}. \tag{2.3}$$

Moreover,

$$Q(0,0|w) = G(0,0,\rho') = \frac{\pi/2 - tg^{-1}(\rho'/\sqrt{1-\rho'^2})}{\pi}. \tag{2.4}$$

I now present the ECT for an MA1 process for the case where we consider only two states $x > 0$ and $x < 0$.

Proposition 20 *Consider the MA1 process, $x_{t+1} = \epsilon_{t+1} + w\epsilon_t$, where ϵ_t is i.i.d. standard normal. Define $m'^2 = Var(x_t) = 1 + w^2$ and*

$$\rho' = w/(1+w^2).$$

Then, if we consider the Markov chain with two states $x_t > 0$ and $x_t < 0$, we would have

$$ECT = \frac{\pi}{\pi - 2tg^{-1}(\rho'/\sqrt{1-\rho'^2})}.$$

One can find a similar relationship for $AR(q)$, $MA(p)$ or other important processes such as GARCH and ARCH.

The next example considers a random walk process which is a special case of an AR1 process.

Example 6 (RW): Consider a RW process, $x_{t+1} = x_t + \sigma\epsilon_{t+1}$, where residual ϵ is i.i.d. standard normal. A RW is a special case of an AR1 process when the autocorrelation parameter is 1. Consider $\rho \to 1$ in an AR1 process, then $m \to \infty$ which means

$$P(a,b|1,\sigma) \to 0.$$

We can show the following proposition which is intuitive for a RW process and is generally known in probability theory but I will restate it in the context of this paper.

Proposition 21 *For a RW process, the ETTs would be infinite which is for $i \neq j$*

$$t_{ij} = \infty.$$

The above proposition mean that the Traveling times would be very high in a finite sample for a RW and eventually they would be infinite. I postulate that this can be used design tests to detect a random walk process.

2.5 Estimation of ETT

In this section, I estimate ETT for historical data for an AR1 process, few specific assets returns and an asset return derived from a production economy. I discuss how ETT can vary within assets.

Given a time series, how can we calculate p_{ij} and t_{ij} from historical data? The most straightforward way is to calculate the frequency of transition and the average traveling time using sample averages. For example, for calculating the ETT from state i to state j we observe all the traveling times from the state i to the state j that have occurred in the data and take a sample average to estimate the ETT from the state i to the state j. We can calculate the standard deviations from the sample averages as well and we should have enough data in order for the estimates to be meaningful. In exactly the same manner, one can estimate $ETT(\alpha)$ and ECT for a given process.

Precisely, I consider a time-series of length T and calculate the expected traveling time for the observed sample, which I call ETT_i for the ith sample. Next, I redo this process M times to record ETT_i for $i = 1, 2, ..., M$. I then estimate the ETT for the process as

$$\widehat{ETT} = \frac{ETT_1 + ETT_2 + ... + ETT_M}{M},$$

and the standard error of ETT as

$$\hat{\sigma}(ETT) = \frac{\hat{\sigma}(ETT_i)}{\sqrt{M}}.$$

2.5.1 AR1 and MA1

The traveling times for an AR1 process with parameters ρ and σ is estimated as follows. I simulate a long sample (10000 observations) for an AR1 process. I then observe how many times the process ends up in a particular state to estimate the stationary probability of that state. For estimating $ETT(\alpha)$, I observe the number of times for transition between the states $X > \alpha\sigma$ and $X < -\alpha\sigma$ and take a sample average. I then repeat the above procedure 1000 times and take an average of sample averages and the standard deviation of the estimates will be the standard deviation for the sample averages.

Figure 2.2 plots $ETT(\alpha)$ for different values of α for an AR1 process with parameters ρ and σ. The $ETT(\alpha)$ is increasing with correlation parameter ρ which is reasonable as the process becomes more persistence (in the sense of a high autocorrelation). Moreover, $ETT(\alpha)$ is increasing and convex with α which means it would take more time to travel from a high value state to a low value one.

Figure 2.4 plots the standard deviation band for $T = M = 10^3$ and $\rho = 0.6$ for the estimation of ETT suggesting that the band is tight enough for the finite sample. Moreover, Figure 2.3 plots $ETT(\alpha)$ for different AR1 processes for different values of ρ and α. One can again observe that $ETT(\alpha)$ is increasing in α. In addition, the slope of the $ETT(\alpha)$ curve is higher for a higher ρ suggesting that traveling times increase more, when increasing α marginally, when the series has more autocorrelation.

Next, in Figure 2.5, I plot the ETT for a MA1 process in which one can see the concavity of ETT. Now, let's compare the ETT for a MA1 process and an AR1 with the same autocorrelation. For this purpose, I choose $w = 1$ and $\rho = 0.5$ and I plot the ETT in Figure 2.6 in which one can see the variation of ETT for two simple processes with the same autocorrelation.

In conclusion, the plots show that for an AR1 process there is a monotonic relation between ETT and autocorrelation and there would not be new information when one considers ETT as AR1 is simple process in nature which is determined fully by the knowledge of the autocorrelation. Later in Section 2.6 I will show that this monotonic relation is not always the case.

2.5.2 Estimation of ETT Using Option Prices

In this section, I discuss how to calculate TP and ETT from option prices. Utilizing historical data may not give a suitable proxy of future expectations of market participants about the under-

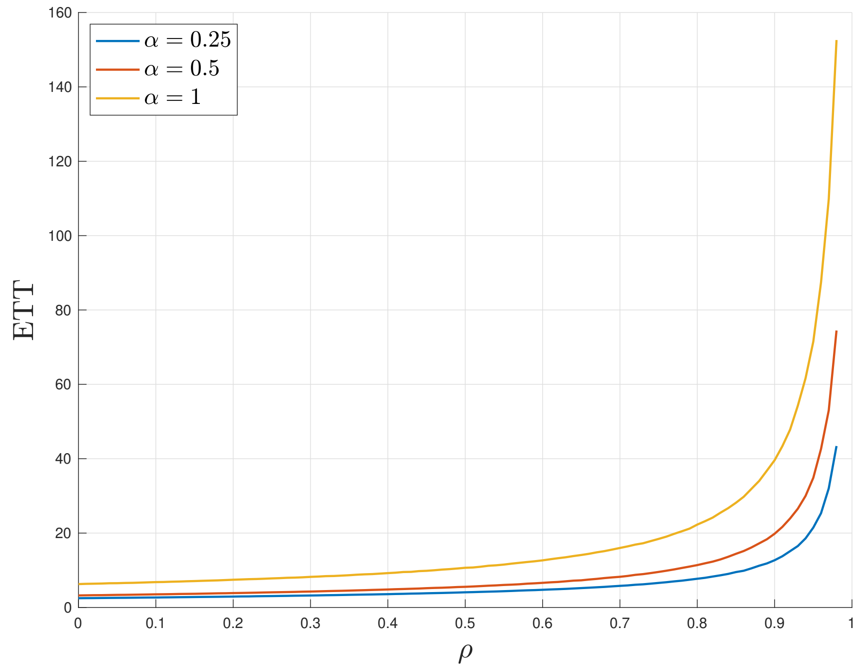

Figure 2.2: $ETT(\alpha)$ for an AR1 process.

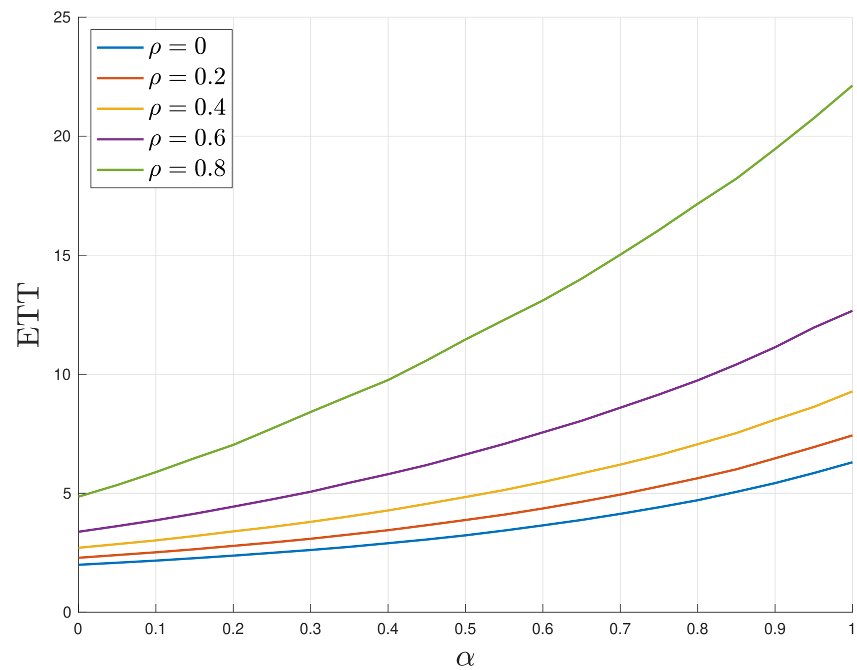

Figure 2.3: $ETT(\alpha)$ for an AR1 process.

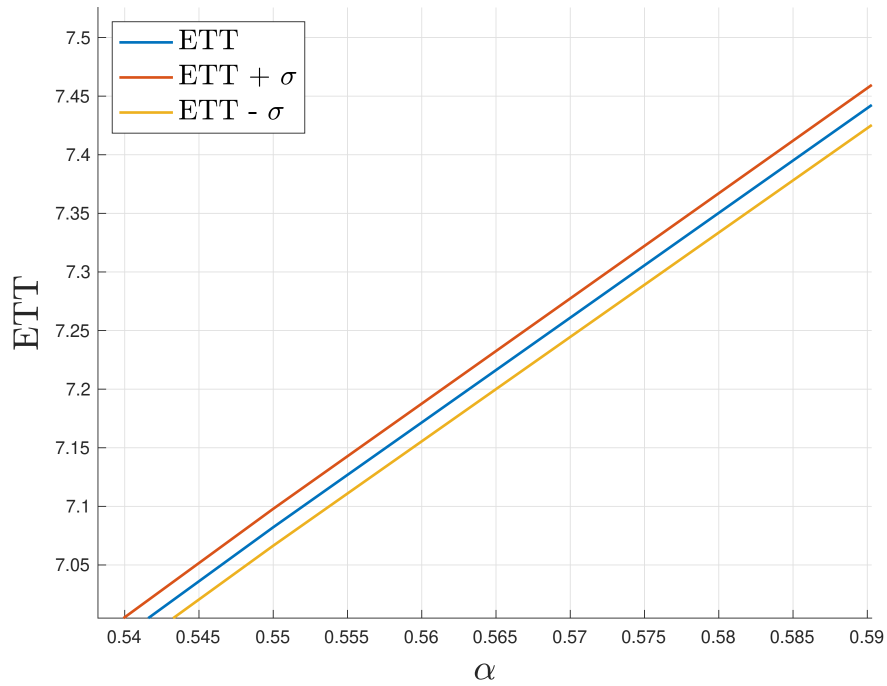

Figure 2.4: Standard deviation band for $E\hat{T}T(\alpha)$ for an AR1 process, here σ denotes the standard error of the estimation.

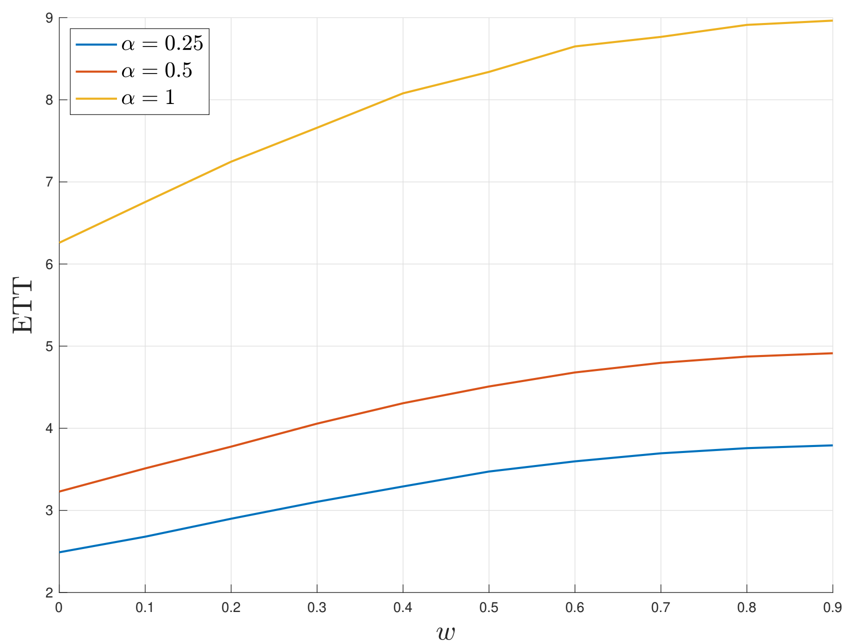

Figure 2.5: $ETT(\alpha)$ for a MA1 process.

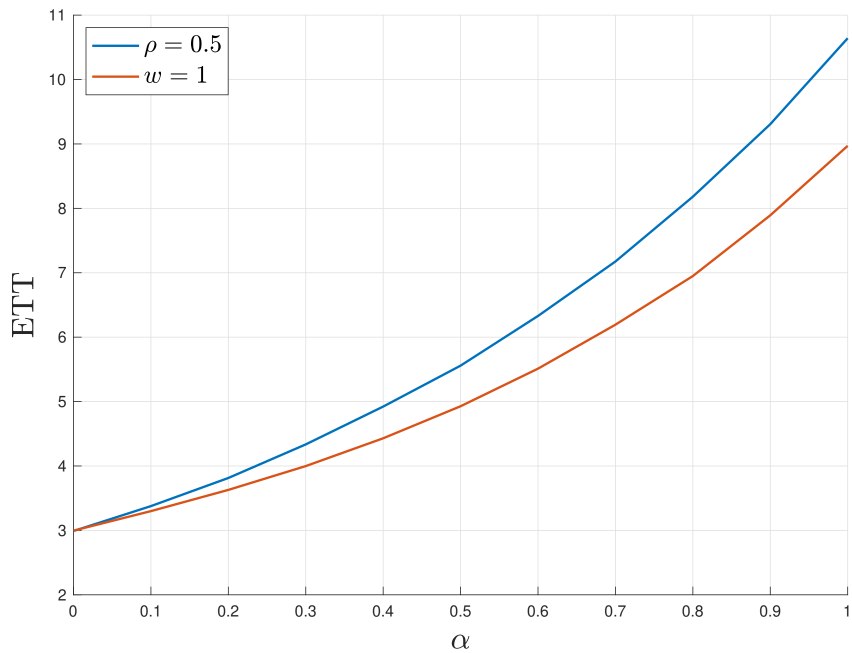

Figure 2.6: $ETT(\alpha)$ for MA1 and AR1 processes.

lying asset. However, option prices encapsulate future expectations of market participants about the underlying process. Therefore, the option prices can be used to estimate forward-looking moments. These forward looking estimates can be then used to infer signals for future behaviour of the asset or the economy. Calculating the TPs under the risk-neutral measure is done using basic option pricing techniques. For doing the calculation under the physical measure, I use the same strategy introduced in [66] for the market return as the underlying return series. I assume a particular specification for the stochastic discount factor (SDF) for a change of measure from risk-neutral to physical probabilities. Although this specification is derived from economic intuition, altering the assumption on the SDF would result on other computations of the forward looking moments. Therefore, the results should be considered as a proxy for the true moments under the physical measure. For calculating the TPs of Individual stocks under the physical measure, one needs a more elaborate pricing model. I then discuss if the theory can be used in practice.

To fix the notation, note that price at time t of any claim X_T at maturity T, with no interim payoffs or dividends, is given by:

$$Price_t(X_T) = E_t(X_T M_T) = E_t^*(X_T)/R_{fT},$$

where E^* is risk-neutral expectation, R_{fT} is risk-free rate for discounting constant payoffs at maturity T and M_T is the stochastic discount factor (SDF) for pricing claims at time T.

As an example of risk-neutral pricing, note that the risk-neutral expectation of return of any traded asset would be equal to the risk-free rate R_{fT}. This is because $X_t = Price_t(X_T)$ and

$$1 = Price_t(R_T) = E_t^*(R_T)/R_{fT} \Rightarrow R_{fT} = E_t^*(R_T).$$

For another example, note that any SDF should price any return R_T including R_{fT} and thus:

$$1 = E_t(R_{fT} M_T) \Rightarrow 1/R_{fT} = E_t(M_T).$$

First, I illustrate what the transition probabilities would be in the Black-Scholes continuous time setting. In the Black-Scholes setting, the underlying stock follows a geometric Brownian motion (GMB)

$$dS_t = \mu S_t + \sigma dB_t,$$

where μ is the drift parameter and σ is the diffusion parameter. Define the return R_{t+1} as the return from time t to time $t+1$, which is $R_{t+1} = S_{t+1}/S_t$. I will now show that R_t and R_{t+1} are independent. Note that

$$S_t = S_0 \exp\left((\mu - \frac{\sigma^2}{2})t + \sigma B_t\right),$$

therefore

$$R_{t+1} = \exp\left((\mu - \frac{\sigma^2}{2}) + \sigma(B_{t+1} - B_t)\right).$$

For a Brownian motion, $B_{t+1} - B_t$ would be independent of $B_t - B_{t-1}$ and thus the same would be true for R_t and R_{t+1}. Note that $B_{t+1} - B_t \sim N(0, \sigma^2)$. The following proposition is true.

Proposition 22 *For a GBM with return time-series R_t, the returns R_t and R_{t+1} would be independent and thus the TPs would be*

$$P(R_{t+1} < a|R_t) = P(R_{t+1} < a) = F(a/\sigma),$$

where F is the C.D.F. for a standard normal distribution.

Although, a GBM has independent returns, there might be some dependency in the actual data (that may not follow a GBM) that could be revealed from market prices. In the next section, I illustrate how to find certain features of an asset return under risk-neutral distribution from prices of options on the underlying asset.

Risk-Neutral Measure

In this section, I derive expressions for TPs under the risk-neutral measure from prices of options on the underlying asset. I first start by finding the probability distribution of future return R_T under the risk-neutral measure (which is also studied in [66]) and then I calculate the transition probabilities.

What would be the risk-neutral C.D.F. of the future return? I should find the probability that the future return is bigger than a given threshold K? In mathematical notation, if P^* indicates probability under the risk-neutral measure, what would be the value of $G^*(K) = P^*_t(R_T > K)$? The following is true

$$G^*(K) = P^*_t(R_T > K) = E^*_t(1_{\{R_T>K\}}) = E^*_t(1_{\{S_T>KS_t\}}) = E^*_t(1_{\{S_T>K'\}})$$

$$= R_{fT}Price_t(1_{S_T>K'}) = R_{fT}B(KS_t),$$

where $K' = KS_t$ and $B(K')$ is price of a binary option with strike K'. Hence, the risk-neutral probability $P^*_t(R_T > K)$ would be equal to the risk-free rate times the price of a binary option where a binary option is defined as a claim with payoff equal to 1 when $S_T > K'$ and payoff 0 otherwise. Note that so far, I have not assumed a specific process for the underlying asset. In the Black-Scholes environment, where the underlying follows a geometric Brownian motion, one can show that $B(K) = Price_t(1_{S_T>K}) = N(d_2)/R_{fT}$, where $N(d_2)$ is the 2nd term in the BlackScholes formula.

Therefore, if we can observe binary option prices for different values of K', we can fully reconstruct the risk-neutral probability distribution of future returns from binary option prices. Note the limiting cases

$$P_t^*(R_T > 0) = 1,$$

and

$$P_t^*(R_T > \infty) = 0.$$

Let's denote $B(K)$ as the price of a binary option with barrier K and $C(K)$ as the price of a European call option with strike K. A standard no-arbitrage argument shows that

$$B(K) = -\frac{dC(K)}{dK} = -C'(K).$$

Moreover, denote $f(k)$ be the p.d.f. of R_T under the risk-neutral probability. We can find $f(K)$ by taking a derivative of

$$F(K) = 1 - P_t^*(R_T > K) = 1 - R_{fT}B(KS_t)$$

with respect to K. Therefore,

$$f(K) = -R_{fT}S_tB'(KS_t) = R_{fT}S_tC''(KS_t).$$

The p.d.f. $f(K)$ can be interpreted as a constant times the price of Arrow-Debreu security $AD(K)$, defined as the price of a claim which pays a non-zero amount only if the underlying asset at the time of maturity ends up in the interval $[K, K + dK]$.

Having specified the risk-neutral probability distribution from option prices, let's calculate the risk-neutral transition probabilities, which is defined as the transition probability between states under the risk-neutral measure. For illustration, I only consider three states which are defined as $R_T > a$, $b < R_T < a$ and $R_T < b$, where here R_T is the return from time t to time T and I only analyse transition from $T = t + 1$ to $T = t + 2$, where current time is $t = 0$.

As an example, what is the transition probability from state $R_1 < b$ to state $R_2 > a$? Or in mathematical notion, given the time t information, what is the value of risk-neutral probability $P_t^*(R_2 > a|R_1 < b)$?

Note that the return from time 1 to time 2, which I call R_{12}, is simply R_2/R_1. For calculating the transition probabilities from R_1 to $R_{1,2}$, I will first calculate the joint probability $P_t^*(R_2 > a, R_1 > b)$. We can write:

$$P_t^*(R_2 > a, R_1 > b) =$$

$$= R_{f2} \times Price_t(1_{\{S_2 > aS_t, S_1 > bS_t\}}).$$

The term

$$P(a, b) = Price_t(1_{\{S_2 > a/S_t, S_1 > b/S_t\}})$$

is equal to the price of a type of barrier or knock-out options, a claim on S_2 given that the midway price is not below a threshold. If we can observe the price $P(a, b)$ from market, then we can calculate the risk-neutral transition probabilities and traveling times. However, we need to calculate the joint probability of $R_{1,2}$ and R_1 which is $Q(a, b) = P_t^*(R_2/R_1 > a, R_1 > b)$. We can approximate $Q(a, b)$ by taking a sum of the joint probability $P(x, y)$ for particular set of values of x, y. In addition, we can find $Q(a, b)$ by looking at the market price of particular claims (if they exist)

$$Q(a, b) = P_t^*(R_2/R_1 > a, R_1 > b) = P_t^*(R_2 > aR_1, R_1 > b)$$

$$= R_{f2} \times Price_t(1_{\{S_2 > aS_1, S_1 > bS_t\}}),$$

which is the market price of a forward looking barrier type option. Moreover, note that

$$B^f(a) = P_t^*(R_2/R_1 > a) = R_{f2} \times Price_t(1_{\{S_2 > aS_1\}}),$$

which is price of forward binary option.

One needs to find the data for such options (from public or OTC markets) or find a way to approximate those options from conventional calls and puts. I summarize the findings in this section in the following proposition.

Proposition 23 *If there is no arbitrage, prices of certain barrier options like claims would reveal the TPs for the return of the underlying asset. Precisely,*

$$P_t^*(R_2/R_1 > a | R_1 < b) = \frac{R_{f2} \times \left(Price_t(1_{\{S_2 > aS_1\}}) - Price_t(1_{\{S_2 > aS_1, S_1 > bS_t\}})\right)}{1 - R_{fT} B(bS_t)},$$

or

$$P_t^*(R_2/R_1 > a | R_1 < b) = \frac{B^f(a) - Q(a,b)}{1 - R_{fT}.B(bS_t)}.$$

In the next section, I present the above findings under the physical measure instead of a risk-neutral measure, when the underlying is the market portfolio.

Physical Measure

I explain how to calculate the transition probabilities under the physical measure by assuming a specific SDF. I first present the probability distribution of future return R_T under the physical measure (which is pointed out in Martin (2017) as well). Next, I will derive expressions for TPs under the physical measure.

I only consider the underlying to be the market portfolio for simplicity of calculations. I use the same method as in [66], in which it is assumed that there exists a log utility investor who invests in the market. Consequently, $M_T = 1/R_T$ is a discount factor and one can write

$$E_t(X_T) = E_t(M_T R_T X_T) = Price_t(R_T X_T).$$

If P denotes the probability under the physical measure, I first calculate the physical probability $G(K) = P_t(R_T > K)$ (which is also done in [66]):

$$P_t(R_T > K) = E_t(1_{\{R_T > K\}}) = E_t(1_{\{S_T > KS_t\}}) = E_t(1_{\{S_T > KS_t\}})$$

$$= Price_t(R_T 1_{S_T > KS_t}) = (1/S_t)Price_t(S_T 1_{S_T > KS_t}).$$

$$= (1/S_t)(C(KS_t) + (KS_t)B(KS_t)).$$

If $K' = KS_t$, the price of the claim $S_T 1_{S_T > K'}$ is equal to the price of a plain option plus price of a binary option. In the Black-Scholes world,

$$(1/S_t)Price_t(S_T 1_{S_T > K'}) = N(d_1),$$

where $N(d_1)$ is the first term in the BlackScholes formula.

Note the limiting cases:

$$P_t(R_T > 0) = (1/S_t)Price_t(S_T 1_{S_T > 0}) = (1/S_t)S_t = 1,$$

and

$$P_t(R_T > \infty) = (1/S_t)Price_t(S_T 1_{S_T > \infty}) = 0.$$

Now, the C.D.F. of R_T under the physical measure is:

$$F(K) = P_t(R_T < K) = 1 - (1/S_t)(C(KS_t) + (KS_t)B(KS_t)).$$

And the p.d.f. of R_T under the physical measure would be:

$$f(K) = F'(K) = -C'(KS_t) - KS_t B'(KS_t) - B(KS_t).$$

$$= -KS_tB'(KS_t).$$

We can compare the physical and risk-neutral probability distributions.

$$G^*(K) = R_{fT}B(KS_t),$$

$$G(K) = (1/S_t)(C(KS_t) + (KS_t)B(KS_t)).$$

Therefore,

$$G^* < R_{fT}G/K.$$

$$S_t(G(K/S_t) - G^*(K/S_t)) = C(K) + KB(K) - R_{fT}S_tB(K)$$

$$= Price_t((S_T - R_{fT}S_t)1_{S_T>K}) \geq Price_t((S_T - R_{fT}S_t)1_{S_T>0})$$

Since $Price_t((S_T - R_{fT}S_t)1_{S_T>0}) = 0$,

$$G^* \leq G.$$

We have proved the following proposition.

Proposition 24 *If $F^*(K) = 1 - G^*(K)$ and $F(K) = 1 - G(K)$ are the risk-neutral and the physical probability distribution of return under the SDF assumption in the paper, then no arbitrage implies*

$$F^*(K) \geq F(K).$$

Also recall

$$f^*(K) = -R_{fT}S_tB'(KS_t),$$

and

$$f(K) = -KS_tB'(KS_t).$$

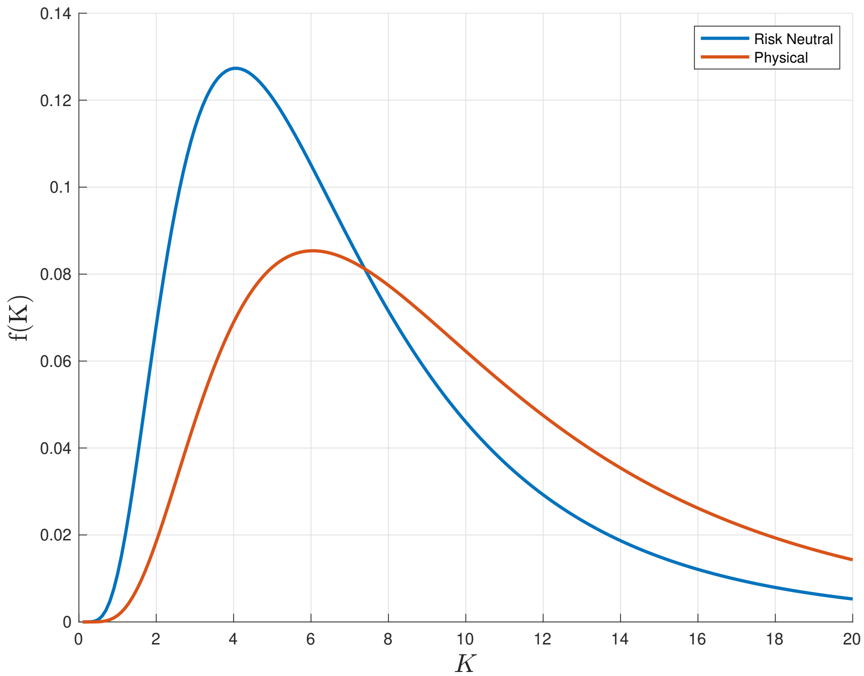

Figure 2.7: Arrow-Debreu prices in BlackScholes environment for $S_0 = 70$, $\sigma = 0.2$, $r = 0.02$, $T = 10$.

Thus,

$$f = Kf^*/R_{fT}.$$

Figure 2.7 plots the Arrow-Debreu prices for different values of K, assuming the BlackScholes environment (r is the continuously compounded risk-free rate). In addition, Figure 2.8 plots $F(K)$ for different values of K, assuming the BlackScholes environment.

Now, I calculate the transition probabilities under the physical measure. Consider three states as before which are defined as $R_T > a$, $b < R_T < a$ and $R_T < b$, where here R_T is the return from time t to time T and I only consider the transition from $T = t + 1$ to $T = t + 2$, where current time is $t = 0$.

As an example, what is the transition probability $P_t(R_2 > a|R_1 < b)$? Denote the return from time 1 to time 2 as $R_{12} = R_2/R_1$. For calculating the transition probabilities from R_1 to R_{12}, I will

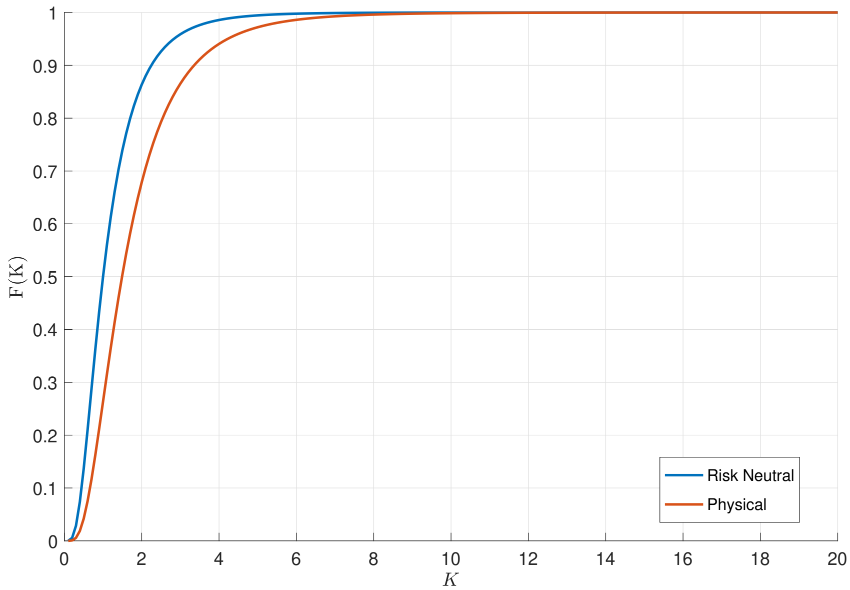

Figure 2.8: CDF of R_T in BlackScholes environment for $S_0 = 70$, $\sigma = 0.2$, $r = 0.02$, $T = 10$.

first calculate the joint probability $P_t(R_2 > a, R_1 > b)$. We can write:

$$P_t(R_2 > a, R1 > b) =$$

$$= Price_t(R_2 1_{\{S_2 > aS_t, S_1 > bS_t\}}),$$

$$= (1/S_t) Price_t(S_2 1_{\{S_2 > aS_t, S_1 > bS_t\}}).$$

It would be enough to calculate

$$P'(a, b) = Price_t(S_2 1_{\{S_2 > a/S_t, S_1 > b/S_t\}}),$$

which is type of a barrier or knock-out option.

I next calculate the joint probability of $R_{1,2}$ and R_1 which is $Q'(a, b) = P_t(R_2/R_1 > a, R_1 > b)$. We can find $Q'(a, b)$ by observing market price of particular claims (if they exists)

$$Q'(a, b) = P_t(R_2/R_1 > a, R_1 > b) = P_t(R_2 > aR_1, R_1 > b)$$

$$= Price_t(R_2 1_{\{S_2 > aS_1, S_1 > bS_t\}}),$$

$$= (1/S_t) Price_t(S_2 1_{\{S_2 > aS_1, S_1 > bS_t\}}),$$

which is the market price of a forward looking barrier type option. Moreover, note that

$$B'^f(a) = P_t(R_2/R_1 > a) = (1/S_t) Price_t(S_2 1_{\{S_2 > aS_1\}}),$$

which is price of forward binary option. The following proposition summarizes the physical probability case.

Proposition 25 *If there is no arbitrage, prices of certain barrier options like claims would reveal the TPs for the return of the underlying asset. If R_T denotes the return of the market portfolio and under the assumption that $1/R_T$ is a SDF,*

$$P_t(R_2/R_1 > a | R_1 < b) = \frac{(1/S_t) Price_t(S_2 1_{\{S_2 > aS_1\}}) - (1/S_t) Price_t(S_2 1_{\{S_2 > aS_1, S_1 > bS_t\}})}{1 - (1/S_t)(C(bS_t) + (bS_t) B(bS_t))},$$

or

$$P_t(R_2/R_1 > a | R_1 < b) = \frac{B'^f(a) - Q'(a,b)}{1 - (1/S_t)(C(bS_t) + (bS_t) B(bS_t))}.$$

In this chapter, I showed the new importance of Knock-Out options. Knock-Out options reveal information about the TPs of the underlying asset's return. This information is useful beyond the expected return, and expected volatility of the underlying asset as TPs are more fundamental than unconditional moments.

2.6 Application to Asset Data

Stock returns constitute several empirical properties. As [67] points out, stock prices have low autocorrelation, heavy tails (even conditional heavy tails after correcting for volatility clustering), an asymmetry between gains and losses, aggregational Gaussianity (which means that the distributions of return are not the same for different time horizons and the distribution of return looks more like normal as the time horizon increases) and slow decay of absolute return autocorrelation (long-range dependence). In this section, I provide a new perspective of how asset returns behave when considering the TPs and ETTs. These new measures can shed light on differences in asset returns beyond autocorrelation. In this section, I first show the ETT for some conventional assets such as Bond and Commodities. Next, I calculate the ETT for a few assets return derived from a simulation of production asset pricing model as introduced in [24]. Finally, I calculate the ETT for some well-known factor returns and discuss the possible importance of this measure to study asset prices.

2.6.1 Asset Data

I use data [3] from nearly 20 years from 1997-2017 to estimate the ETT for different asset returns. In Table 2.1, ETT1 is the expected traveling time from $+\sigma$ to $-\sigma$, ETT2 is from $+0.5\sigma$ to -0.5σ and ρ is the autocorrelation. We can see that overall there is a positive relation between autocorrelation and ETT. Gold returns exhibit a less negative autocorrelation than Bond and lower traveling times which is intuitive as a negative autocorrelation means a swifter moves between high and low states. Oil has a high positive autocorrelation and high traveling times.

Moreover, Figure 2.9 plots the ETT for different assets for a variety of bandwidth α. These assets are AGG (Barclays Aggregate Bond Fund), EEM (MSCI Emerging Markets Index ETF), SPY (SPDR SP 500 ETF), GLD (SPDR Gold Shares), USO (United States Oil Fund ETF), TIP (Barclays TIPS Bond Fund ETF), VTI (Vanguard Total Stock Market EFT), VNQ (Vanguard Real Estate EFT) and VIX (CBOE Volatility Index).

We can draw a number of observations. First, we can see that the whole spectrum of α is useful to distinguish between different series: while some series have similar ETTs for low values of α, they would differ when α is high enough which maybe because of some fat tails in the joint distribution of a given asset return between to consecutive dates (t and $t + 1$). Second, one can observe a crossing behavior for different assets such as SPY and VNQ: the ETT can be lower for

[3] Data is from [24].

Asset	ETT1	ETT2	ρ
Bond (TIP)	7.00	3.56	-0.03
Gold (GLD)	6.50	3.17	-0.11
Oil (USO)	9.07	4.39	0.29

Table 2.1: ETT for different assets

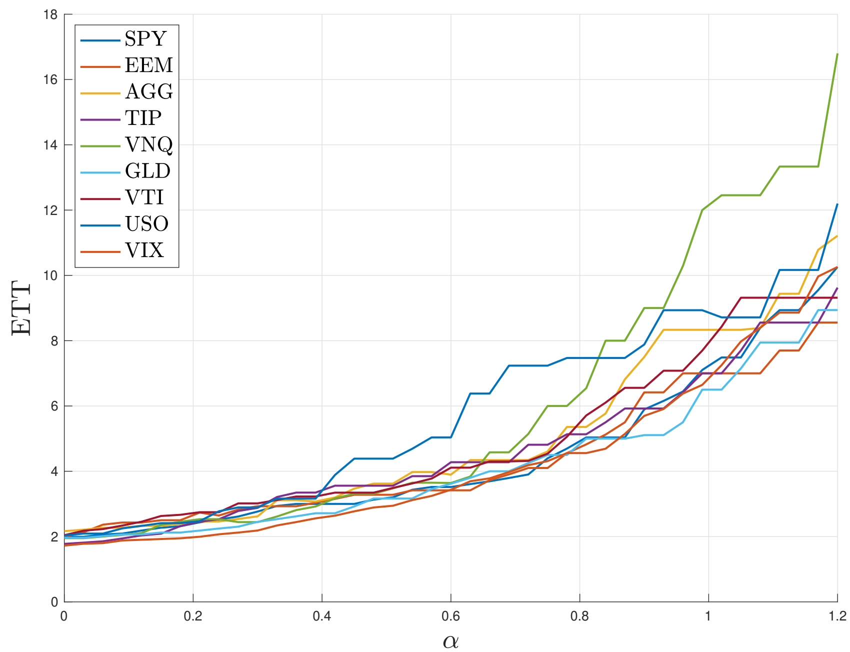

Figure 2.9: $ETT(\alpha)$ for different assets.

one asset compared to another one for low values of α and higher for other values of α which shows that the assets behave fundamentally different for regions close to their mean and far apart from their mean. The last observation is that the asset returns seem to be more distinguishable in ETT terms for high values of α with the caveat that for high values of α the finite sample standard error would also be higher.

2.6.2 Production Asset Pricing Model

Next, I calculate the ETT for a simulation of a production economy which is also studied in Section 3 of [24]. In the production economy, the representative agent decides on consumption, labor and investment: consider a representative agent neoclassical macroeconomic model that chooses consumption c_t, labour v_t and investment i_t to maximize utility function $\ln(c_t)$ according to the program

$$\max\ E\left(\sum_{t=0}^{\infty}\beta^t ln(c_t)\right),$$

subject to the constraints $c_t + i_t \leq k_t^{\alpha} e^{\lambda_t}$, $k_{t+1} = (1-\Omega)k_t + i_t$ and $\lambda_{t+1} = \rho\lambda_t + \epsilon_{t+1}$ where Ω is the depreciation rate, k_t is the capital stock, ϵ_t is i.i.d normal noise with mean zero and variance σ^2. Let's p_t^e, p_t^b, r_t^e and r_t^p denote the price of equity, price of risk-free bond, return on equity and the equity premium, respectively. There are closed form solutions for the prices and returns in this economy which I exploit for simulation of prices and returns processes.

I am interested in plotting the ETTs for different prices and returns in this economy to observe if the measure ETT is important in this economy.

Tables 2.2 and 2.3 show the ETT for $\alpha = 1$ and $\alpha = 0.5$. We can notice a few interesting patterns. For a given autocorrelation ρ, the two series with the highest ETT are the price and return for bond which indicates some sort of persistence in the bond return compared to equity and this is intuitive as bond returns changes slowly compared to risky assets. However, when ρ is high, then the highest ETT would be for the equity price series suggesting that the price is not stationary. Another interesting pattern is the fact that ETT is not monotonically increasing with autocorrelation for bond price and return as opposed to equity price and return. Therefore, ETT suggests that the return on bond and equity are inherently different. This can be an investigation for future research.

Moreover, Figure 2.10 plots the ETT for the equity return for different values of correlation parameter ρ in the productivity shock and α. One can again observe that the ETT for equity return is increasing with α and ρ suggesting that a high autocorrelation translates to a high ETT and vice versa.

At last, in Figure 2.11, I compare the ETT for the return of equity, bond and the equity premium for different values of α where $\rho = 0.4$. One can repeat the observation in Tables 2.2 and 2.3. The return on bond has higher ETTs suggesting the process is more persistent. It is interesting that only for high values of α, there is a substantial difference between the ETTs of the equity return and

ρ	0	0.2	0.4	0.6	0.8	0.95
p_t^e	4.6121	5.5818	6.9875	9.4069	15.475	43.952
p_t^b	4.6079	6.2628	8.6204	10.224	8.004	5.252
r_t^e	2.3864	2.6082	2.8901	3.2606	3.7761	4.3542
r_t^b	4.6041	6.2654	8.6035	10.233	7.9952	5.2627
r_t^p	2.6658	2.9549	3.3101	3.7398	4.2491	4.5699

Table 2.2: ETT for different series, $\alpha = 0.5$

ρ	0	0.2	0.4	0.6	0.8	0.95
p_t^e	8.8315	10.487	12.877	17.236	28.7	86.294
p_t^b	8.8324	11.529	15.539	18.495	16.535	10.856
r_t^e	4.4897	4.7452	5.1168	5.6885	6.6668	8.0673
r_t^b	8.8355	11.532	15.546	18.518	16.477	10.868
r_t^p	5.0915	5.5289	6.0915	6.8619	7.9054	8.7328

Table 2.3: ETT for different series, $\alpha = 1$

equity premium. Therefore, ETT can provide useful information to distinguish different properties of equity return and the equity premium.

2.6.3 Equity Factors Return

In this section, I provide an analysis of traveling times for equity factor returns. The factors are obtained based on sorts on different characteristics and are widely known in the literature. I show that there is a considerable variation of ETT across different factor returns, and I derive a few empirical irregularities from which I postulate conjectures for future research.

In the Figures 2.12 and 2.13, I calculate the ETT for various factor returns which includes 18 Size portfolios' return, 19 Book-to-Market portfolios and 10 Momentum portfolios[4]. There is a considerable variation of ETT. Note that ETT is invariant of scaling a series and thus, variation of ETTs can correspond to variation of autocorrelation and other moments of the joint distribution of returns between two consecutive days.

In Figure 2.14, I compare the ETT for the well-known Fama-French four-factor returns. There are some interesting observations. First, the ETT would be high for Momentum factor return. In addition, there is a similarity between the ETT of SMB and HML for low values of α while their

[4]Data is from Fama-French library.

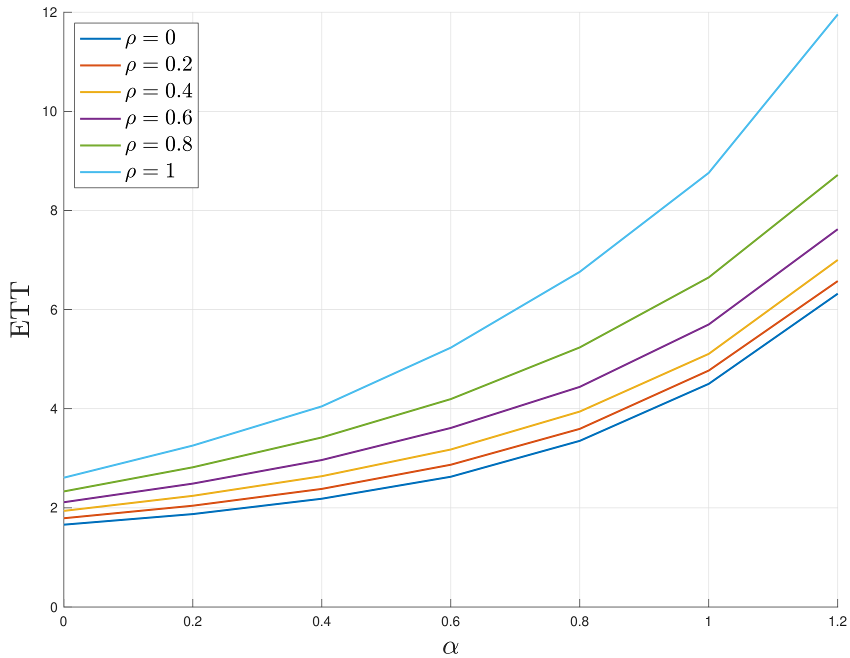

Figure 2.10: $ETT(\alpha)$ for equity return.

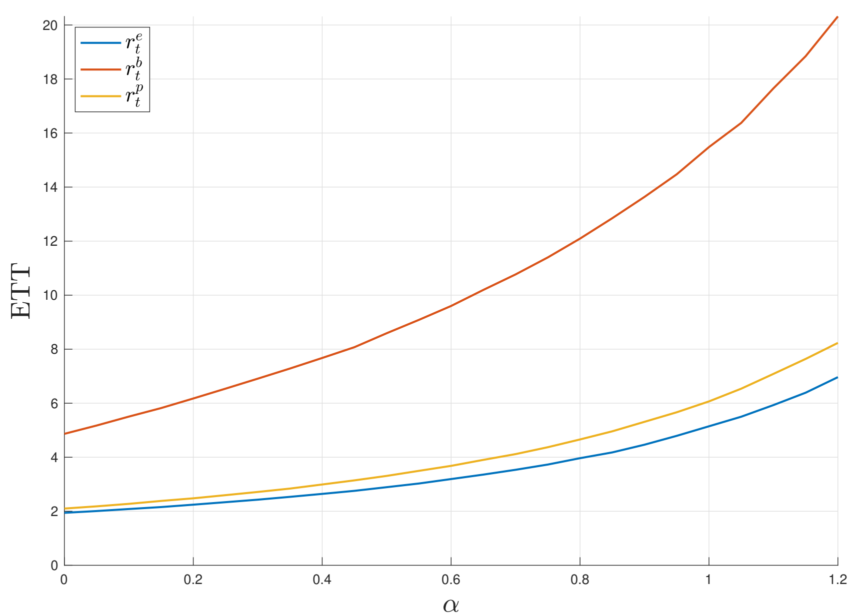

Figure 2.11: $ETT(\alpha)$ for different return series for $\rho = 0.4$.

ETT would be significantly different for high values of α, suggesting that the joint distribution of return for two consecutive dates would behave very differently for these factors only for high values of α. Moreover, the similarity of ETT for high values of α for the Size and Market factor returns suggest the same tail behaviour for high bandwidth α.

Finally, I plot autocorrelation vs $ETT(0)$ in the Figure 2.15 for 10 monthly Momentum portfolios returns. Surprisingly, I document a negative relation with autocorrelation, which indicates that for a fixed autocorrelation the value of ETT can vary considerably.

To conclude this chapter, I summarize the empirical findings and postulate conjectures for future research:

1. Although the ETT is invariant with a scale of the series (adding a scalar plus multiplying a scalar), the ETT varies considerably among different assets.

2. ETT can be non-monotonically related to autocorrelation.

3. ETT can even have a negative relationship to autocorrelation for certain classes of assets suggesting a considerable variation of ETT for a fixed value of autocorrelation.

4. Momentum factor returns have a high ETT suggesting that there can be a link to series exhibiting momentum in returns and high ETTs.

One can also ask for more in-depth empirical analysis of the concept of ETT and how it correlates with other moments or features of assets and factors, and I leave this direction for future research.

2.7 Conclusion

In this paper, I introduced the concept of traveling times for assets returns and provided several results. I provided pricing equations of risky debts in a Markov Chain economy, and I showed that barrier like option prices can reveal transition probabilities of the underlying asset's return.

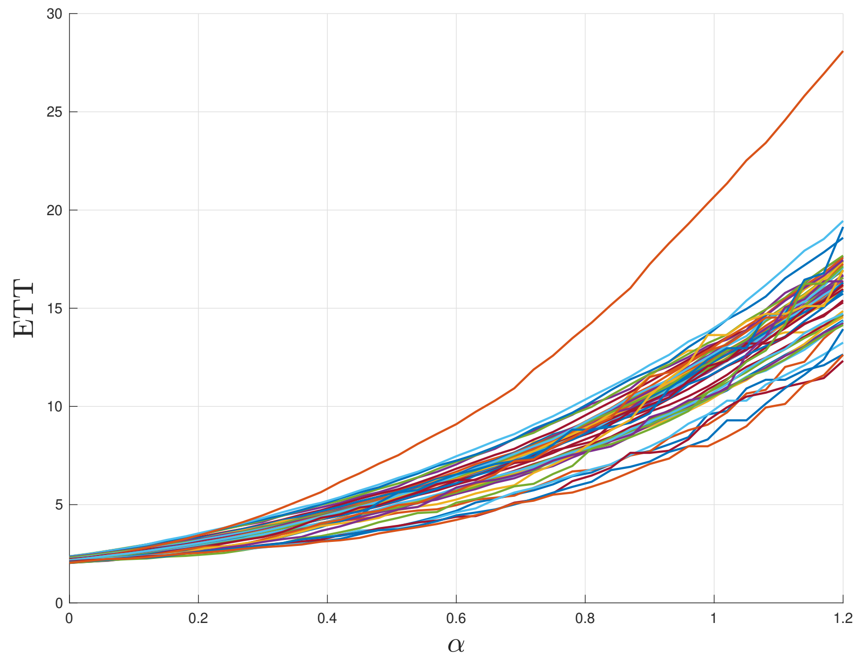

Figure 2.12: $ETT(\alpha)$ for different return series for $\rho = 0.4$.

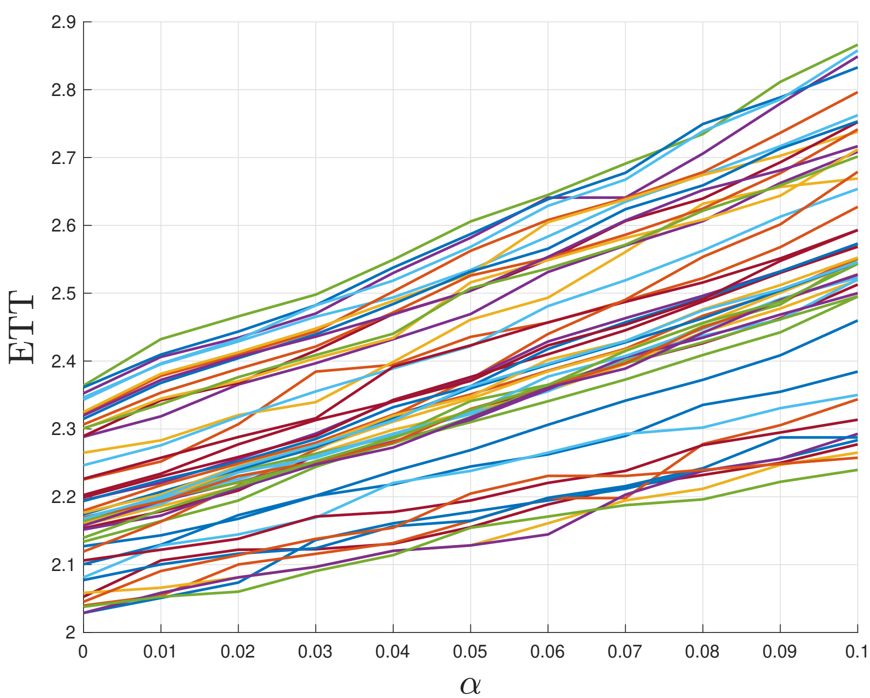

Figure 2.13: $ETT(\alpha)$ for factor return series for α close to zero.

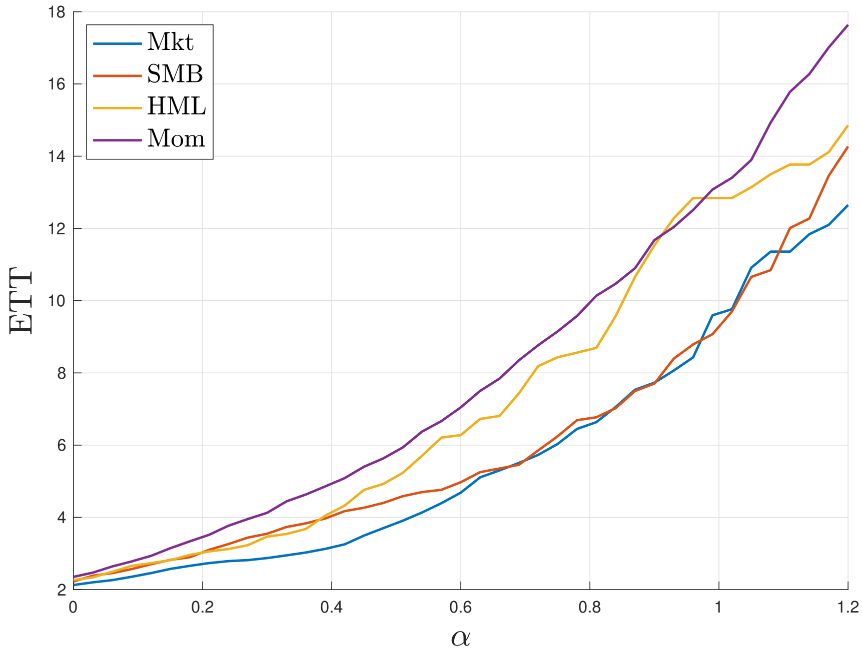

Figure 2.14: $ETT(\alpha)$ for different return series for $\rho = 0.4$.

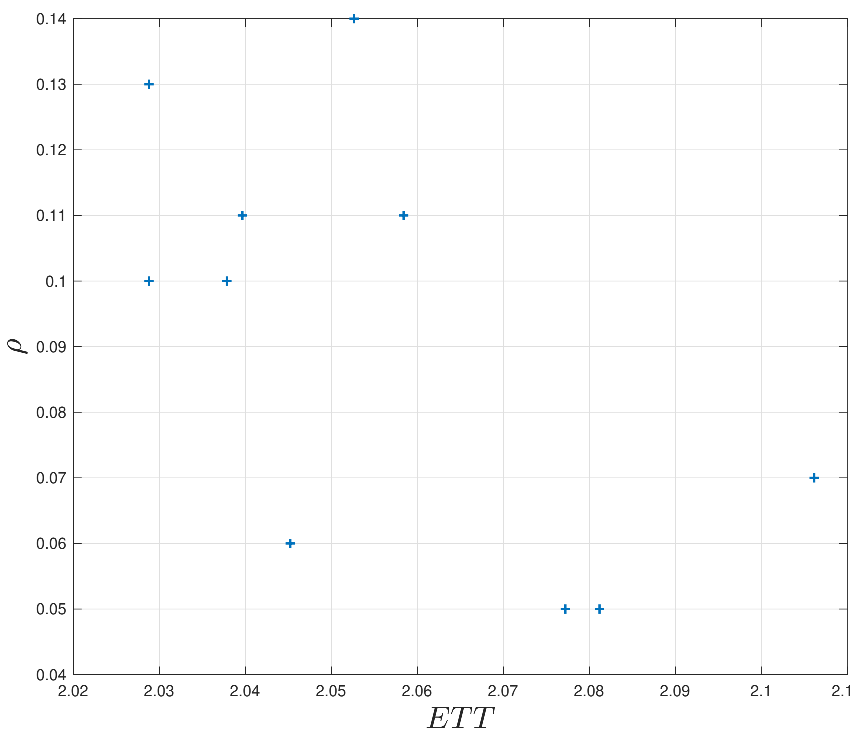

Figure 2.15: ρ and $ETT(0)$ for monthly momentum portfolio returns.

Moreover, I discussed the estimation of the traveling times from historical data. There is a considerable variation of traveling times across known factor returns which leaves interesting directions for future research.

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