Permissioned Blockchain Adoption in Supply Chains

Wenjun Wang

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Abstract

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We aim to identify factors that are critical in determining whether or not blockchain adoption arises in various market structures, and give guidance for addressing the challenges of blockchain implementation.

In Chapter 2, we construct an economic framework for understanding the incentives of the firms in a supply chain to form a blockchain consortium. We find that blockchain reduces information asymmetry for consumers, thereby enhancing consumer welfare. Consumer welfare gains can be sufficiently large that blockchain adoption is socially beneficial; nonetheless, we find that blockchain adoption does not arise in equilibrium. This situation arises because blockchain adoption costs are borne by manufacturers, and manufacturers cannot extract consumer gains through prices due to the competitive nature of the manufacturing sector. We offer a system of transfers to generate blockchain adoption in equilibrium when it is socially beneficial.

In Chapter 3, we investigate a variation of the model described in Chapter 2. Our analysis incorporates the blockchain's ability to trace shipments and generate cost savings for the manufacturers who join the blockchain. Although the blockchain enables early recalls of defective goods with higher probability, and thus, reduces expected unit costs for all the manufacturers on the blockchain, such gains are still competed away and blockchain adoption does not arise in equilibrium. This result strengthens our earlier findings on the incentive misalignment in a perfectly competitive setting. The associated welfare implications of this model are similar to those in Chapter 2.

In Chapter 4, we study a setting in which the consumer prices are determined exogenously. With this setting of sticky price, there exists a certain level of competition but it is not perfect. As a result, the manufacturer gains from blockchain adoption may be strictly positive, in contrast to two results in Chapters 2 and 3 where the gains are always competed away. We find that blockchain unequivocally benefits consumers but has an ambiguous effect upon the welfare of manufacturers. There exist conditions under which, although the blockchain improves global welfare, blockchain adoption does not arise in equilibrium. We refer to such a scenario as an adoption failure, and again a system of transfers is proposed to resolve that failure.

In Chapter 5, we examine whether blockchain adoption arises in equilibrium for a supply chain in which a single risk-averse manufacturer sells directly to consumers; thus, the manufacturer possesses market power. In this setting, we find that blockchain adoption always enhances manufacturer welfare when the adoption cost is zero. While two results in Chapters 2 and 3 demonstrate that blockchain adoption does not arise when the manufacturing sector is perfectly competitive, our findings clarify that the failure of blockchain adoption is not generic across all market structures. Rather, blockchain adoption arises in equilibrium for supply chains when the manufacturer possesses market power and when the adoption cost is sufficiently small.

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To my family.

Chapter 1: Introduction

Blockchains have emerged as an effective mechanism for information sharing that can ensure both data privacy and immutability. Furthermore, efficient implementations of multiparty secure computation and zero-knowledge proofs make it possible to compute provably verified aggregate measures of the immutable raw data, enabling the participants in a blockchain to make more informed decisions. For these reasons, blockchains can improve the efficiency of any industry with many players, with some of them holding private information. And yet, in practice, blockchains have not been widely adopted. Our goal is to identify factors that are critical in determining whether or not blockchain adoption arises in various market structures, and provide insight into the welfare implications of blockchain adoption.

1.1 Permissionless and Permissioned Blockchains

A blockchain is a decentralized shared trusted ledger. Cryptography and distributed consensus mechanisms ensure the integrity of this ledger, enabling all participants to agree on an immutable, unique version of the ground truth. Blockchains come in two varieties: permissionless and permissioned. Permissionless blockchains achieve consensus based on decentralized protocols, and, in theory, can have an unlimited number of participants or nodes. These protocols typically make only modest assumptions regarding the trustworthiness of nodes and do not require nodes to reveal identities beyond a pseudonymous identifier. Consequently, permissionless networks presume no level of trust in its participants as the identities are unknown, and these participants may manage multiple identities and freely acquire new ones and discard old ones. In contrast, permissioned blockchains restrict the set of nodes that can update the blockchain, restrict participation of nodes even as users, and, in some cases, provide full transparency regarding node identities.

A permissionless blockchain is one in which no special permission is required to add information to the blockchain. This does not imply that a participant may add information without satisfying some additional conditions, but rather that the additional conditions cannot depend on a participant's identity. For example, some permissionless blockchains, such as Bitcoin, specify a computational puzzle and allow any node that solves the puzzle to add information to the blockchain. Such a specification generates some security guarantees even when nodes are not trustworthy (see Nakamoto 2008), but also generates inefficiencies such as the possibility of prolonged disagreement regarding the content of the information stored on the blockchain (see Biais et al. 2019 and Hinzen et al. 2021). Since the information on the blockchain is ambiguous as long as this disagreement continues, permissionless blockchains are especially problematic in business settings because an extended ambiguity regarding the state of the system can be costly. Moreover, since business settings typically involve repeated interactions among a set of economically-motivated entities, some level of trust is an appropriate assumption.

The literature examining economic analysis of blockchains focuses largely on permissionless blockchains, and especially on Bitcoin. Some prominent papers that study Bitcoin include Yermack (2015), Biais et al. (2019), Easley et al. (2019), Foley et al. (2019), Griffin and Shams (2020), Capponi et al. (2021), Hinzen et al. (2021), Huberman et al. (2021), and Lehar and Parlour (2021). Beyond Bitcoin, some important research topics in permissionless blockchains include the design and analysis of consensus protocols, and smart contracts. As an example of the former, Proof-of-Stake (PoS) is a consensus protocol that is employed by a plurality of recently launched permissionless blockchains (see Irresberger et al. 2021) and is examined in many recent papers including Fanti et al. (2019), John et al. (2021), Rosu and Saleh (2021), and Saleh (2021). A smart contract refers to code that is uploaded to a blockchain. The code generally includes methods that can then be executed by users of the associated blockchain. Prominent papers examining the economic implications of smart contracts in permissionless blockchains include Tinn (2018), Cong and He (2019) and Cong et al. (2021). One particularly important application of smart contracts is an asset offering known as Initial Coin Offerings (ICOs). A typical ICO is conducted via a firm uploading code to a blockchain that issues a class of coins in exchange for capital. The terms of the issuance are generally specified directly in the smart contract code, which also guarantees the recipients particular rights. These mechanisms are further studied in Malinova and Park (2018), Catalini and Gans (2019), Lyandres et al. (2019), Howell et al. (2020), Li and Mann (2020), Chod and Lyandres (2021), Gan et al. (2021a), and Gan et al. (2021b).

Relative to permissionless blockchains, permissioned blockchains enjoy several key advantages, which make them better suited for managing supply chains. Permissioned blockchains tend to be more scalable and efficient. Moreover, for entities with shared interests (such as a consortium of companies in a particular industry), permissioned blockchains can be customized, and the governance structure can be designed by the participants. Most importantly, supply chains need known identities of participants in order to provide transparency and traceability. As a result, rather than permitting unrestricted entry and exit, supply chain blockchain solutions should be permissioned and confined to known entities. For these reasons, we focus on permissioned blockchains. We assume that only manufacturers can add information to the blockchain, and moreover all manufacturing-related transactions are stored on the blockchain, which makes the manufacturing process fully transparent to all participants on the blockchain. We emphasize though that restricting the set of nodes that can add information to the blockchain does not imply restricting the set of nodes that have access to information on the blockchain. Accordingly, we allow all consumers to read from the blockchain.

There is a growing body of research analyzing the economic implications of permissioned blockchains. Cao et al. (2019) and Cao et al. (2020) study permissioned blockchains in the context of auditing. Narang et al. (2019) demonstrate that permissioned blockchains facilitate online business-to-business collaboration under certain conditions. Pun et al. (2018) find that blockchains can be used to combat counterfeiting. Similar to this thesis, several other papers study blockchains within the specific context of supply chain management, which will be introduced in more detail in Section 1.2.3.

1.2 Blockchain Technology in Supply Chain Management

As we discuss in Section 1.1, permissioned blockchains are better suited for managing supply chains. The promising characteristics of blockchain technology have drew industry practitioners to use it for supply chain management (see Carson et al. 2018). Around the globe, a rising number of blockchain-based solutions are being created to optimize supply chain operations (see, e.g., IBM Food Trust, Provenance, SkuChain, BlockVerify, Vechain, Factom, Ripe.io, and OwlTing). The emerging technology has the potential to significantly enhance the traceability, speed, and coordination of supply chains. Among the many benefits of utilizing blockchain technology in supply chains, two of the most recognized are transparency and traceability.

1.2.1 Transparency

One of the key reasons for using blockchain into supply chain management is to increase information transparency. In comparison to established technologies for information sharing, such as enterprise resource planning (ERP) and electronic data interchange (EDI), blockchain offers novel benefits. For example, older technologies often restrict information sharing to one-on-one interactions with immediate suppliers and customers. This is because legacy concerns make integrating conventional information sharing systems (e.g., ERP) across numerous enterprises very costly and time-consuming, and the cost of integration might climb significantly as the number of firms increases (see Gaur and Gaiha 2020). However, blockchain has the potential to drastically reduce the expenses associated with adding new participants and merging multiple enterprises. This means that blockchain may facilitate information sharing inside a multi-firm supply chain network. Firms may learn how many other firms operate in the same tier of the supply chain, their identities, and their interactions with the supply chain's upstream and downstream layers, by joining the blockchain.

The new age of information sharing caused by blockchain has the potential to fundamentally alter how firms compete and collaborate within a particular industry (see Provenance 2015). Due to the fact that a supply chain network may comprise multiple tiers, each of which may contain multiple firms, when a supply chain network becomes visible, firms may unavoidably know the identities of their prospective rivals within the same tier. In reality, it is not uncommon for a supplier to sell via a number of manufacturers who compete in the end market. Additionally, when the supplier's capacity is depleted, the manufacturers must compete for the provider's limited capacity, resulting in a rationing game in which competing firms purchase more than their actual needs in order to be assigned additional capacity by the supplier (see Cachon and Lariviere 1999, Hall and Liu 2010, Cho and Tang 2014, and Cui and Zhang 2018). As a consequence, manufacturers face competition on both the demand and supply sides. How will making the supply chain network transparent impact competing firms in this scenario? On the one hand, insight into the supply chain network provides each manufacturer with more information, which may help them make better operational choices and generate advantages. On the other hand, knowledge about competing manufacturers might exacerbate competitiveness, which would be detrimental to firms. To summarize, blockchain poses a dilemma for firms. On the one side, it has the potential to dramatically increase supply chain information sharing, which has been extensively studied in the literature. On the other side, blockchain delivers this benefit by establishing a shared platform that enables rivals to have network visibility. Thus, it is critical to determine whether and under what circumstances each firm in a supply chain network would have the incentive to join a blockchain consortium, which is also one of the biggest challenges in implementing blockchain (see Gaur and Gaiha 2020).

Both vertical and horizontal information sharing have been investigated in many types of supply chains without blockchain in the information sharing literature. In vertical information sharing, previous research has examined firms' incentives to share private demand information (see, e.g., Lee et al. 2000, Raghunathan 2001, Simchi-Levi and Zhao 2003, Gaur et al. 2005, Li et al. 2014, and Chen et al. 2016), inventory information (see, e.g., Roy et al. 2019), and new product reliability information (see, e.g., Bakshi et al. 2015). Apart from vertical information sharing, a number of publications, mainly in the economics literature, examine whether firms in oligopoly marketplaces have an incentive to communicate their private information horizontally (see, e.g., Novshek and

Sonnenschein 1982, Clarke 1983, Vives 1984, Gal-Or 1985, Li 1985, Shapiro 1986, Kirby 1988, and Raith 1996). The context for this stream of work involves multiple firms competing in a market under a variety of conditions, including the competition mode (Cournot or Bertrand), the kind of goods (substitutes or complements), and the content of information to disclose (demand signal or production cost). They argue that firms have an (resp., no) incentive to communicate their private demand signals in a Cournot competition with complementary (resp., substitutable) goods or in a Bertrand competition with substitutable (resp., complimentary) goods (see Clarke 1983, Vives 1984, and Gal-Or 1985). This conclusion does not hold true, however, when firms share information about their private production costs (see Li 1985, Shapiro 1986, and Raith 1996). Additionally, Natarajan et al. (2012) analyze ex-ante and ex-post information sharing in a Cournot game and demonstrate that not all agreements to share private demand signals before observation can sustain ex-post. Our work is distinct from this stream of work because in a blockchain network, firms risk losing control over private information, which may be shared both vertically and horizontally simultaneously.

1.2.2 Traceability

Due to the fact that suppliers' efforts to enhance quality are often unobservable and non-contractable, moral hazard concerns and quality-related issues may occur. Recalls of products owing to quality issues are common in a variety of industries, including agri-food, pharmaceuticals, and consumer electronics. The inability to track the supply chain is frequent in industries such as agri-food and pharmaceuticals, posing difficulties in supply chain quality control. However, blockchain offers great potential to assist firms in overcoming the issues associated with a lack of traceability in supply chains (see Nikolakis et al. 2018 and Chen et al. 2020). Along with smart sensors, blockchain can enable traceability of the whole supply chain, including who, when, where, and how a given product was handled throughout the production and distribution processes. So far, a number of leading firms have used blockchain platforms. For instance, after IBM's unveiling of the world's first fully integrated, enterprise-grade production blockchain platform in late 2016, Walmart piloted the use of blockchain to trace pork supply chains in China (see Nash 2016). Walmart subsequently stated that some of its direct suppliers would be required to join its food-tracking blockchain (see Nash 2018). Alibaba and JD.com, two of China's largest e-commerce companies, have also embraced blockchain to combat counterfeit products (see Xiao 2017). In the United States, a Food Trust group was established in 2017 by 10 prominent retailers (e.g., Walmart) and food companies (e.g., Nestlé), with the goal of improving global food safety via the use of blockchain to track food and ingredients globally (see Aitken 2017).

Blockchain-enabled traceability can enhance supply chain operations in a variety of ways. To begin, the manufacturing process may include a large number of entities sequentially throughout a supply chain, and it may be impossible to determine which step of production resulted in the failure of the end product. In this situation, blockchain may be utilized to identify which tier(s) of the chain were responsible for the end product's failure. This sort of traceability is referred to as process traceability. Second, a firm may acquire the same goods from multiple sources without being able to pinpoint its origin. Once a defect occurs, the firm would be required to recall all products from the market, even if certain suppliers are non-defective. In this circumstance, blockchain can assist the firm in figuring out the supplier(s) responsible for the failure and distinguish between defective and non-defective products, allowing for an accurate recall of defective products. This sort of traceability is referred to as product traceability.

The quality management literature has explored supply chains without blockchain and studied ways to effectively increase suppliers' quality from a number of angles. Baiman et al. (2004) explore an assembly supply chain in which the buyer assembles an end product using components received from a variety of symmetric suppliers. They contrast the two contracts that demand individual testing with the group warranty contract, where all suppliers will be penalized without individual testing if the end product fails. Li (2012) conducts research on the optimal group warranty contract in various contexts including and without information asymmetry. Both articles emphasize the value of a group warranty contract in an assembly supply chain. Mu et al. (2016) investigate two quality-testing strategies for preventing purposeful adulteration by milk farmers. They examine how to strike a compromise between the high expenses of individual testing and the free-riding issue among farmers in mixed testing. While Dong et al. (2016) concentrate on quality management in a dyadic supply chain, they also evaluate the option of outsourcing part of the work to an independent contract manufacturer, which may be considered a special case of a serial supply chain. They investigate the buyer's choice for an inspection-based approach versus an external failure-based approach while taking the limited liability into account.

There is also a small body of work studying supply chain traceability without blockchain (mostly, enabled by RFID). Piramuthu et al. (2013) investigate the allocation of liability among the various players in the perishable supply chain based on traceability accuracy. Saak (2016) asserts that perfect traceability may not be optimal in a supply chain comprised of a single buyer and several suppliers. Yao and Zhu (2020) construct a game theoretical model to study the importance of traceability in combatting product label misconduct. Our work is different from this stream of work in that we focus on a new technology: blockchain, whose privacy-preserving transparency fosters confidence and trust in transaction data, which are critical for contamination containment and rapid recovery.

1.2.3 Operational Impacts

Apart from the literature mentioned in Sections 1.2.1 and 1.2.2, our work that studies information sharing (transparency) and quality management (traceability) in supply chains with blockchain and the associated economic implications is more closely related to the growing body of literature that investigates the operational impacts of blockchain in supply chain management. Babich and Hilary (2020) discuss key strengths and weaknesses of blockchain in a supply chain context, and identify a number of promising directions for further research. Blaettchen et al. (2020) conduct a simulation analysis to quantify the costs and benefits of blockchain being applied to food supply chains. Chod et al. (2019) demonstrate that signaling a firm's quality to lenders through inventory transactions is more efficient than signaling through loan requests and that blockchain technology enables verification of inventory transactions. Their results establish an important benefit of blockchain adoption, which enables a firm to secure favorable financing terms at lower signaling costs. Cui et al. (2020a) examine how blockchain impacts competing manufacturers purchasing from a supplier with limited capacity. In their analysis, blockchain enables manufacturers to observe each other's presence, thereby mitigating a manufacturer's over-order (under-order) incentive when supplier capacity is small (large). Moreover, they find that blockchain improves the total supply chain profit if, and only if, supplier capacity is sufficiently large. Cui et al. (2020b) study the implications of using blockchains to enable traceability in serial and parallel supply chains. They find that blockchain enhances profits for buyers and suppliers in a serial supply chain but that supplier profits may decline in a parallel supply chain. Dong et al. (2021) investigate a three-tier food supply chain with a combination of serial and parallel components, in which multiple parallel suppliers in the upstream make all-or-nothing efforts to reduce contamination risks, and a single supplier in the downstream makes purchase pricing decisions. They demonstrate that blockchain adoption does not necessarily benefit retailers.

1.3 A Three-Layer Supply Chain Setting

Throughout the thesis, we focus on a three-layer supply chain setting that consists of three types of agents: vendor(s), manufacturer(s) and consumers. Each chapter considers different variants and those specific details are introduced in each chapter. In the setting, vendor refers strictly to the initial supplier, and consumer refers strictly to the final user. In contrast, our usage of manufacturer simplifies all intermediate participants in the supply chain into one layer. This modeling choice allows us to parsimoniously examine how the intermediate layer of the manufacturer(s) interacts with layers above (i.e., the vendor(s)) and below (i.e., the consumers).

The vendor(s) supply the manufacturer(s) with goods, which are then used to fill consumer orders. Each manufacturer possesses a type that corresponds to the manufacturing process of her finished goods, and that type is unknown to consumers. Consumers are heterogeneous in their preferences for manufacturing processes (except for Chapter 4) and each has a unit demand. Each consumer seeks to fill her unit demand by selecting the manufacturer that maximizes her utility.

A consumer's utility increases in the value she ascribes to a manufacturer's good and decreases in the price of that good. The value that a consumer ascribes to a manufacturer's good depends on whether the given manufacturer employs a process that the consumer values. As in practice, these manufacturing processes are only imperfectly known by consumers, so the consumers act on incomplete information. We assume that consumers make a purchase decision that maximizes their utility. A consumer may select a good from a manufacturer or an outside option, which reflects the opportunity cost.

Heterogeneity in consumer preference of manufacturing processes is empirically well-established (see, e.g., Yiridoe et al. 2005 and Moser et al. 2011). For example, some consumers prioritize an environmentally-friendly manufacturing process whereas other consumers prioritize one that limits the usage of inorganic chemicals. Importantly, these characteristics of the manufacturing process are not easily observed; consequently, the consumer decisions will be sub-optimal relative to the case when consumers possess additional information. This phenomenon—consumers being poorly informed about attributes not easily observable (see, e.g., Poole and Baron 1996), yet valuing such attributes (see, e.g., Burton et al. 2001)—has been documented extensively in practice. Furthermore, it is common practice for manufacturers to specialize and cater to these heterogeneous consumer preferences. We capture these features in our models by assuming multiple manufacturer types over which consumers have preferences.

We assume that blockchain adoption improves the information environment for all agents. The improved information environment enables consumers to make more informed decisions because the blockchain tracks all important aspects of the manufacturing process, revealing these aspects not only to other manufacturers but also to consumers. In the absence of blockchain, each consumer has imperfect information regarding manufacturing processes and the manufacturer's quality so that she does not necessarily purchase from the manufacturer that provides her the highest utility ex post. In contrast, if a manufacturer joins the blockchain, then each consumer learns that manufacturer's type exactly from the information stored on the blockchain.

Developments in the blockchain industry support our assumption that blockchain improves the consumer information environment. For example, the IBM Food Trust blockchain initiative allows "essential... information that benefits consumers [to] be shared" with them. The information shared includes "product origin and quality" and "information about the nutritional properties of products and the potential presence of any allergens or questionable substances." [1] Consumers receive access to information by "scan[ning] a QR barcode on [the relevant item] with their phone."[2]

In Chapters 3, 4 and 5, we model the traceability of blockchain as enabling early and selective recalls of defective products. To clarify the roles of the vendor(s) and manufacturer(s) within those analyses, we briefly discuss here the example of a supply chain for fruit distribution.[3] Fruit is sourced at a farm and transported through various intermediaries to food stores where it finally reaches consumers. Within our framework, the farm corresponds to the vendor, and all supply chain participants other than the farm and consumers are simplified to the one manufacturer layer. A consumer values the fruit according to qualities that depend largely on the manufacturing process. We capture this using the notion of a type for the manufacturer(s) (distinguishing different sequences of steps before the fruit reaches the consumer) as well as for consumers (capturing preferences for different types of manufacturers). Nonetheless, there exists a concern relevant to both manufacturers and consumers that is not in the control of the manufacturer. In particular, fruits can become contaminated by dangerous microbes, which can cause illness in consumers, thus requiring a recall. This case corresponds to a (tail) scenario in which the fruit poses immediate health risks due to contamination rather than aspects of the subsequent processing affecting the perceived quality. We refer to fruit that poses such an immediate health risk as defective. Defects of this nature typically depend on farm practices which can lead to systematic contamination of entire batches of fruit.[4] Accordingly, we envision a vendor's type as governing the likelihood of the

[1] Source: https://www.carrefour.com/sites/default/files/2019-12/carrefour_press_release_81018_eng.pdf

[2] Source: https://www.reuters.com/article/us-carrefour-blockchain-idUSKCN1T42A5

[3] IBM Blockchain has partnered with various participants to develop blockchain-based food supply chains more generally. See https://www.ibm.com/blockchain/solutions/food-trust for further details.

[4] See https://tinyurl.com/y8ouwvap for context regarding microbial contamination of fruit due to farm practices.

goods being defective. In our models, the goods supplied by a vendor are assumed to be defective with an exogenous probability that is publicly known (Chapters 3 and 4) or with a probability that is determined by her effort choice (Chapter 5). Defective goods should be recalled and the cost of this is borne by the vendor.

From the perspective of the consumer, manufacturer type is conceptually unrelated to goods being defective. In the fruit distribution example, the goods are defective if consumption would lead to immediate illness. In contrast, a consumer ascribes a low value to a manufacturer if its manufacturing process does not prioritize the concerns of the consumer. For example, if the manufacturer does not prioritize environmental standards in processing whereas the consumer is of the type that highly values such standards, then the consumer ascribes a low value to that manufacturer's goods.

1.4 Main Contributions of This Thesis

This thesis contributes to the literature examining the economic implications of permissioned blockchains in a business setting. The thesis aims to identify factors that are critical in determining whether or not blockchain adoption arises in various market structures, and give guidance for addressing the challenges of blockchain implementation.

In Chapter 2, we construct an economic framework for understanding the incentives of the firms in a supply chain to form a blockchain consortium. We find that, in a perfectly competitive setting, blockchain adoption does not arise in equilibrium even when that outcome enhances the total welfare of all the participants. This is because the participants who stand to gain from blockchain adoption are distinct from those who control whether or not the blockchain is adopted. We resolve this misalignment of incentives by proposing a system of transfers so that adoption arises as an equilibrium outcome whenever it is sufficiently welfare enhancing. One distinguishing feature of our work is that it incorporates consumers into the analysis, allowing information stored on the blockchain to be shared with consumers and examining the associated welfare implications. This work is detailed in Iyengar et al. (2021b).

In Chapter 3, we investigate a variation of the model described in Chapter 2, in which we slightly simplify the vendor layer by removing the vertical differentiation and introduce an early recall mechanism. Our analysis incorporates the blockchain's ability to trace shipments and generate cost savings for the manufacturers who join the blockchain: if a manufacturer on the blockchain detects a defect, this is quickly shared with all the other manufacturers who are part of the blockchain, enabling an early recall of the defective product. This model addresses both the transparency and traceability of blockchain, whereas the model in Chapter 2 focuses on the transparency of blockchain. Although the blockchain enables early recalls of defective goods with higher probability, and thus, reduces expected unit costs for all the manufacturers on the blockchain, such gains are still competed away and transferred to consumers in the form of lower prices. Consequently, blockchain adoption does not arise in equilibrium. This result strengthens our earlier findings on the incentive misalignment in a perfectly competitive setting. The associated welfare implications and conclusions of this model are similar to those explored in Chapter 2.

Compared with the models in Chapters 2 and 3, our model in Chapter 4 further simplifies the vendor layer, assumes vertically differentiated manufacturers, allows more possibilities for the realization of manufacturers' types, and has a finer characterization of the early recall mechanism. In addition, one major difference is that the consumer prices are determined exogenously in this model. We study this price stickiness because in some circumstances, firms downstream need time to react to a change in price upstream, and hence, the manufacturers do not modify their consumer prices in a period of time. With the setting of sticky price, there exists a certain level of competition but it is not perfect. As a consequence, the manufacturer gains from blockchain adoption may be strictly positive, in contrast to two results in Chapters 2 and 3 where the gains are always competed away. We find that blockchain unequivocally benefits consumers but has an ambiguous effect upon the welfare of manufacturers. We demonstrate that there exist conditions under which, although the blockchain improves global welfare, blockchain adoption does not arise in equilibrium. We refer to such a scenario as an adoption failure, and again a system of transfers is proposed to resolve that failure.

In Chapter 5, we examine whether blockchain adoption arises in equilibrium for a supply chain in which a single risk-averse manufacturer sells directly to consumers; thus, the manufacturer possesses market power. Similar to Chapters 3 and 4, we model both the transparency and traceability of blockchain. In this setting, we demonstrate that blockchain adoption always strictly improves manufacturer welfare if blockchain adoption involves no fixed cost of implementation. When the manufacturer bears the cost of blockchain adoption for the supply chain as a whole, we find that blockchain adoption arises in equilibrium if the adoption cost is sufficiently small. Our work highlights the importance of manufacturer market power in generating blockchain adoption. In particular, while two results in Chapters 2 and 3 demonstrate that blockchain adoption does not arise when the manufacturing sector is perfectly competitive, our findings clarify that the failure of blockchain adoption is not generic across all market structures. Rather, blockchain adoption arises in equilibrium for supply chains when the manufacturer possesses market power and when the adoption cost is sufficiently small. This work is detailed in Iyengar et al. (2021a).

Chapter 2: Blockchain Adoption in a Supply Chain with Perfect Competition

2.1 Introduction

This chapter examines the three-layer supply chain setting described in Section 1.3 in the context of perfect competition among manufacturers and focuses on the transparency of blockchain. Our study aims to determine whether blockchain adoption is socially beneficial and whether such adoption arises in equilibrium in this setting. Within our analysis, there are two vertically differentiated vendors who supply manufacturers with goods, which are then used to fill consumer orders. The manufacturers are vertically differentiated into high and low quality manufacturers. In addition, the manufacturers are horizontally differentiated according to a type that corresponds to the manufacturing process of their finished goods. This two dimensional manufacturer type is unknown to consumers. Consumers have unit demand and are heterogeneous in their preferences over the manufacturing processes but strictly prefer the higher quality manufacturers.

We model each manufacturer as making two decisions. Manufacturers first simultaneously decide whether or not to join the blockchain; and then they simultaneously set prices for their goods.[1] A key finding in our analysis is that manufacturers do not gain from blockchain adoption due to perfect competition. To understand this point, it is important to recognize that blockchain adoption costs are one-time fixed costs that subsequently become sunk costs. Moreover, blockchain implementation differs from other capital expenditures in that blockchain adoption needs to be coordinated across competitors for any cost savings to materialize. This coordination, however, ensures competition in subsequent pricing decisions, and a standard Bertrand price competition ensues, with manufacturers competing away the gains.

[1]We model the pricing decision as subsequent to the blockchain adoption without loss of generality because, in practice, prices can be set at any point in time, but any pricing decision prior to the blockchain adoption decision is not relevant to our analysis regarding the economic implications of blockchain adoption.

Our first result, Proposition 2.3.1, establishes that consumers unambiguously benefit from blockchain adoption. This result arises both because blockchain adoption leads to an increase in utility for any consumer who purchases from a manufacturer and because blockchain adoption leads to an increase in the number of consumers purchasing from manufacturers. Blockchain adoption increases the utility for any consumer who purchases from a manufacturer because blockchain improves consumer information regarding manufacturer types and thereby enables consumers to make more informed decisions. Additionally, blockchain enables consumers to distinguish between high and low quality manufacturers, leading each low quality manufacturer to reduce her price to remain competitive. That price reduction, in turn, adds to the increase in utility for consumers purchasing from a manufacturer. Moreover, the increase in utility for each consumer purchasing from a manufacturer leads some consumers who would have otherwise selected the outside option to purchase from a manufacturer instead. Note that this increase in consumer demand represents an increase in consumer welfare because a consumer switches to purchasing from a manufacturer over the outside option only if doing so improves her utility, which implies that such switches must be welfare enhancing.

Although consumers benefit from blockchain adoption, manufacturers do not. In particular, Proposition 2.3.3 establishes that manufacturer welfare decreases with blockchain adoption. Crucially, blockchain adoption has implementation costs, and those costs are borne by manufacturers. Manufacturers cannot offset these adoption costs by increasing their profit margins because they are engaged in price competition with each other. Thus, manufacturer welfare necessarily declines when blockchain is adopted.

Our main result, Proposition 2.3.8, establishes that blockchain adoption can enhance total welfare, even by an arbitrarily large amount, and yet fail to arise in equilibrium. This possibility, termed an adoption failure, occurs because of a misalignment between those who gain from blockchain adoption and those who decide whether or not to adopt the blockchain. As discussed, each consumer unambiguously benefits from the blockchain, but the adoption costs fall fully on the manufacturers, and competition among manufacturers precludes manufacturers from extracting consumer gains. Accordingly, while adoption may be globally beneficial, such adoption does not arise in equilibrium because those required to implement the adoption (i.e., manufacturers) prefer not to adopt.

We construct a system of transfers, formalized within Proposition 2.3.9, to resolve adoption failures. This system charges consumers for access to the blockchain's information and then transfers the proceeds to manufacturers. The transfers must be large enough so that manufacturers are incentivized to adopt the blockchain, and yet small enough so that some consumers are willing to incur this additional fee. In practice, this system of transfers can be implemented easily, for example, by charging consumers for access to a web application that stores blockchain information relevant for consumer decision-making.

The remainder of Chapter 2 is organized as follows. Section 2.2 formally states our economic model of supply chains, with and without blockchain. Section 2.3 explores the welfare implications of blockchains, demonstrates the existence of adoption failures and puts forth a system of transfers to resolve those adoption failures.

2.2 Model

We begin in Section 2.2.1 with a description of the basic model without a blockchain, explaining how manufacturers set prices, how vendors determine whether to accept manufacturer orders and how consumers make purchase decisions. In Section 2.2.2 we describe the model in the presence of a blockchain, where manufacturers, in addition to setting prices, decide whether or not to join the blockchain, and consumers make their purchase decisions after observing the decisions of the manufacturers.

2.2.1 The Basic Model

We consider a supply chain consisting of two vendors, finitely many manufacturers, and a unit measure of consumers. Vendors are vertically differentiated, whereas manufacturers are both vertically and horizontally differentiated. Consumers are heterogeneous in their preferences across the horizontal differentiation among manufacturers. Consumers order products from manufacturers who supply them from vendors.

Let $V := \{h, l\}$, $h > l \geq 0$, denote the set of vendors. The type h (resp. l) vendor is the high (resp. low) type vendor. As we discuss later, the type h vendor has a higher cost of production as compared to the type l vendor, and each consumer values a good coming from the type h vendor more than the good coming from the type l vendor. Moreover, the manufacturers are vertically differentiated along $\{h, l\}$, in that manufacturers of the vertically differentiated type h (resp. l) procure from the type h (resp. type l) vendor.

Let $M := \{1, 2, \ldots, 4m\}$ be the set of manufacturers with $m \geq 2$.[2] Each manufacturer i has a two-dimensional type (q_i, ξ_i), where $q_i \in \{A, B\}$ is related to horizontal differentiation, whereas $\xi_i \in \{h, l\}$ corresponds to vertical differentiation. To that end, A and B are categorical variables, whereas, as discussed previously, $0 \leq l < h$ so that manufacturers with $\xi_i = h$ (resp. $\xi = l$) are of the high (resp. low) type. Letting M_i denote the set of type $i \in \{A, B\} \times \{h, l\}$, we assume that $|M_i| = m$ for all i, and that all of this is common knowledge. We assume that the type of a specific manufacturer is unknown to consumers, reflecting situations in which consumers are relatively poorly informed (see, e.g., Poole and Baron 1996). However, all our results apply equally, whether or not manufacturers know each others' types.

Each consumer $k \in [0, 1]$ also has type $t_k \in \{A, B\}$ with equal probability, and she prefers goods from a manufacturer of her own horizontally differentiated type (i.e, she prefers $q_i = t_k$ to $q_i \neq t_k$). Moreover, all consumers prefer manufacturers of the high type (i.e., $\xi_i = h$) to manufacturers of the low type (i.e., $\xi_i = l$). More formally, we assume that the random utility of consumer k from consuming a good by manufacturer i is given as follows:

$$V_{ik} = \underbrace{\eta_{ik}}_{\text{Horizontal Differentiation}} + \underbrace{\xi_i}_{\text{Vertical Differentiation}}, \tag{2.1}$$

where η_{ik} denotes the utility accrued due to horizontal differentiation across manufacturers and

[2]Supply chains generally involve many participants, including several manufacturers of the same type, which is reflected in our requiring $m \geq 2$.

$\xi_i \in \{h, l\}$ corresponds to the manufacturer's vertically differentiated type. η_{ik} is given explicitly as

$$\eta_{ik} = H \cdot \mathcal{I}(q_i = t_k) + L \cdot \mathcal{I}(q_i \neq t_k), \tag{2.2}$$

where $0 \leq L < H$.

We assume that consumers have access to signals that reflect the type of the manufacturers. Specifically, consumer $k \in [0, 1]$ receives a set of random signals $\{(\tilde{q}_{ik}, \tilde{\xi}_{ik})\}_{i \in M}$ with $\tilde{q}_{ik} \in \{A, B\}$ and $\tilde{\xi}_{ik} \in \{h, l\}$. We assume $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i) = \alpha = \mathbb{P}(\tilde{\xi}_{ik} = \xi_i \mid \xi_i)$ where $\alpha \in [\frac{1}{2}, 1)$ so that a consumer's signal does not fully reveal a particular manufacturer's type. Since all the main economic insights are present in the model with $\alpha = \frac{1}{2}$ and the exposition is significantly simpler, we restrict ourselves to $\alpha = \frac{1}{2}$ within the main text and Appendix A. Our main results are generalized in Appendix B. We discuss how blockchain affects consumer signals in Section 2.2.2.

We also assume that each consumer may forgo purchasing from a manufacturer and instead avail herself of an outside option. For consumer $k \in [0, 1]$, the utility from the outside option is ϕ_k, which is a random draw from a continuous distribution G that is supported on $[0, \infty)$. This outside option captures the opportunity cost of purchasing from a manufacturer, and could reflect, for example, the utility from an alternative good.

We model a finite horizon economy. At the outset, consumers learn their own types t_k, their utility from the outside option ϕ_k, and receive signals from each manufacturer. Then, all manufacturers act simultaneously, setting a consumer price P_i, $i \in M$, placing an order with the vendor of the manufacturer's vertically differentiated type ξ_i, and offering the vendor a price Ψ_i, $i \in M$. The manufacturer's order to the vendor is based upon anticipated demand, which is perfectly anticipated in equilibrium since our model involves no aggregate risk. Subsequent to the manufacturer's actions, consumers decide from which manufacturer, if any, to order the product, or take outside option instead. Finally, each vendor decides whether to accept, reject or partially fulfill each received order. The vendor then produces all goods that she agreed to produce and sends them to manufacturers who pass them along to consumers.

More formally, vendor h selects a fulfillment level, $\sigma_i \geq 0$, for each order from a manufacturer i with $\xi_i = h$ by solving the following optimization problem:

$$\max_{\{\sigma_i : \xi_i = h\}} \quad \underbrace{\sum_{i:\xi_i = h} (\Psi_i \cdot \sigma_i)}_{\text{Revenue}} - \underbrace{c_h\Big(\sum_{i:\xi_i = h} \sigma_i\Big)}_{\text{Cost}} \tag{2.3}$$

$$\text{s.t.} \quad 0 \leq \sigma_i \leq s_i \text{ for all } i : \xi_i = h$$

where $\Psi_i \geq 0$ denotes the price offered by manufacturer i, $s_i \geq 0$ denotes the anticipated demand of manufacturer i and $c_h(x)$ denotes the cost of vendor h producing x units. The constraint reflects that the fulfillment level σ_i is at most the consumer demand s_i.

Similarly, vendor l faces the following decision problem:

$$\max_{\{\sigma_i : \xi_i = l\}} \quad \underbrace{\sum_{i:\xi_i = l} (\Psi_i \cdot \sigma_i)}_{\text{Revenue}} - \underbrace{c_l\Big(\sum_{i:\xi_i = l} \sigma_i\Big)}_{\text{Cost}} \tag{2.4}$$

$$\text{s.t.} \quad 0 \leq \sigma_i \leq s_i \text{ for all } i : \xi_i = l$$

where $c_l(x)$ denotes the cost of vendor l producing x units.

We assume that c_h is linear, whereas c_l is strictly increasing and convex. This assumption enables us to study strategic price adjustments due to blockchain adoption while also maintaining tractability. In particular, our assumption implies that pricing adjustments come from only the low types, thereby simplifying our analysis. We impose further regularity to enable both high and low type manufacturers to co-exist in equilibrium.[3]

Manufacturer $i \in M$ anticipates both vendor and consumer behavior and sets her consumer

[3]We assume $c_l'(0) < c_h'(0)$ so that the cost of the low type manufacturer is sufficiently low to enable competition with high type manufacturers in equilibrium. We also require that $c_l'\Big(G(\frac{H+L+h+l}{2} - c_h'(0))\Big) > c_h'(0)$ to ensure that the cost of the low type manufacturer increase sufficiently fast so that high type manufacturers can compete with low type manufacturers in equilibrium. Finally, we assume $c_l(0) = c_h(0) = 0$ so that not fulfilling an order involves no cost.

price P_i and vendor price Ψ_i by solving the following problem which maximizes her net profit:

$$\max_{P_i, \Psi_i \geq 0} \quad \underbrace{P_i \cdot s_i}_{\text{Revenue}} - \underbrace{\Psi_i \cdot s_i}_{\text{Cost}} \qquad \text{s.t.} \quad \sigma_i = s_i \tag{2.5}$$

We assume that each manufacturer faces an arbitrarily large cost for not fulfilling any consumer order and therefore needs to ensure that all the anticipated demand s_i is fulfilled, i.e. $\sigma_i = s_i$. Note that both σ_i and s_i are endogenously determined, and both are affected by the prices selected by manufacturers. In particular, (2.3) and (2.4) show that σ_i depends upon Ψ_i, whereas s_i is a function of P_i as we discuss subsequently.

Consumers make purchase decisions after observing manufacturer prices and signals. Consumer k's utility, u_{ik}, for purchasing from manufacturer i is given by:

$$u_{ik} = \underbrace{\mathbb{E}[V_{ik} \mid \mathcal{F}_k]}_{\text{Expected Good Value}} - \underbrace{P_i}_{\text{Cost}} \tag{2.6}$$

where $\mathcal{F}_k$ denotes the information set of consumer $k \in [0, 1]$ that includes her own type, the value of her outside option, a type signal from each manufacturer, and the price posted by each manufacturer. Furthermore, as discussed, while consumer k does not know the type of any given manufacturer, it is public knowledge that there are exactly m manufacturers of each type, and consumers assume each manufacturer is equally likely to be of each type ex ante.

Consumer $k \in [0, 1]$ will purchase the good from one of the manufacturers if her utility from doing so (weakly) exceeds her outside option, i.e., if $\max_{i \in M} u_{ik} \geq \phi_k$. Let $i(k)$ be a manufacturer that offers her maximum utility, i.e., $i(k) \in \arg\max_{i \in M} u_{ik}$. Then, consumer k purchases the good from manufacturer $i(k)$ if her utility from doing so (weakly) exceeds ϕ_k; otherwise she chooses the outside option. Denoting this choice by $m(k)$ we see that $m(k)$ is either $i(k)$ or $\emptyset$. In this notation, the endogenous consumer demand for manufacturer $i \in M$, s_i, is:

$$s_i = \mu(\{k : m(k) = i\}) \tag{2.7}$$

with $\mu(S)$ denoting the measure of a set $S \subseteq [0,1]$ of consumers. Note that s_i depends on P_i because $m(k)$ depends on u_{ik} which depends on P_i.

2.2.2 Model with Blockchain

We enhance the basic model with a single blockchain. Any manufacturer i can join the blockchain by paying a cost $\chi_i > 0$. We let a_i be a binary decision variable that is set to 1 if manufacturer i joins the blockchain, and zero otherwise. We further assume that these decisions are publicly observable.

The presence of the blockchain changes the information environment of the consumer: for each manufacturer i on the blockchain, the signal observed by any consumer $k \in [0,1]$ reveals the manufacturer's type exactly. In other words, $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i, a_i = 1) = 1 = \mathbb{P}(\tilde{\xi}_{ik} = \xi_i \mid \xi_i, a_i = 1)$. This modeling choice, which is the major difference from the basic model, reflects the fact that the blockchain stores all relevant aspects of the manufacturing process and provides such information not only to manufacturers but also to consumers. Consistent with our basic model, we also assume that $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i, a_i = 0) = \alpha = \mathbb{P}(\tilde{\xi}_{ik} = \xi_i \mid \xi_i, a_i = 0)$.

We continue to model the economy with a finite horizon. As in the basic model, consumers learn their types and their utilities for the outside option at the outset. Then, before signals are generated, manufacturers simultaneously make blockchain adoption decisions. We specify the blockchain adoption decisions as before the signals are generated because, as discussed, the blockchain affects the distribution of signals. Subsequent to blockchain adoption decisions, each consumer observes signals from each manufacturer. Then, as in the basic model, manufacturers simultaneously set consumer prices and place vendor orders. Thereafter, consumers decide from which manufacturer, if any, to order the product; not choosing any manufacturer is thought of as choosing the outside option instead. Finally, each vendor decides whether to accept, reject or partially fulfill each received manufacturer order. Finally, each vendor produces all goods that she agreed to produce and sends them to manufacturers who pass them along to consumers.

As in the basic model, the high type vendor solves (2.3) and the low type vendor solves (2.4). In contrast, the manufacturer problem varies between the model with blockchain and the model without blockchain. In particular, when deciding whether to adopt blockchain, manufacturer $i \in M$ solves the following problem to maximize profit:

$$\max_{a_i \in \{0,1\}} \quad \Pi(a_i, a_{-i}) - \chi_i a_i, \tag{2.8}$$

where, similar to (2.5), the expected profit, $\Pi(a_i, a_{-i})$, is given by:

$$\begin{aligned} \Pi(a_i, a_{-i}) := \max_{P_i, \Psi_i \geq 0} \quad & P_i \cdot s_i - \Psi_i \cdot s_i \\ s.t. \quad & \sigma_i = s_i \end{aligned} \tag{2.9}$$

where $s_i \geq 0$ denotes the endogenously determined consumer demand for manufacturer i. Note that s_i depends upon (a_i, a_{-i}) because consumer demand depends on the signals about each manufacturer, and, as previously discussed, the signal accuracy depends on the adoption decisions of manufacturers.

As in the basic model, consumers observe manufacturer prices and signals and then make purchase decisions on that basis. In the presence of the blockchain, consumers observe whether or not a manufacturer joins the blockchain so that $\mathcal{F}_k$ also includes all adoption decisions; that observation, by itself, refines the consumer's information environment. Separately, consumers have accurate information about the types of the manufacturers who join the blockchain. These changes in the consumer information environment affect consumer decisions which, in turn, affect the endogenous demand observed by each manufacturer. Thus, the consumer problem specification is identical as in the basic model although u_{ik} now depends on whether or not manufacturer i joins the blockchain.

2.3 Economic Analysis

This section is devoted to an analysis of the total welfare of all the participants in the supply chain. Three scenarios, listed below, are of interest:

- **Non-Adoption:** No manufacturer joins the blockchain, i.e., $a_i = 0$ for all $i \in M$.
- **Full Adoption:** Every manufacturer joins the blockchain, i.e., $a_i = 1$ for all $i \in M$.
- **Partial Adoption:** Every high type manufacturer joins the blockchain, and no low type manufacturer joins the blockchain. That is, $a_i = 1$ for all $i : \xi_i = h$ and $a_i = 0$ for all $i : \xi_i = l$.

The welfare in the non-adoption setting can be thought of as a benchmark against which the welfare in the other two cases can be measured because the non-adoption setting is identical to our basic model without blockchain. This analysis is done in Section 2.3.1, which demonstrates that full and partial blockchain adoption always benefit consumers but never benefit the manufacturers. Our analysis also establishes that full and partial blockchain adoption have ambiguous effects on global welfare. In Section 2.3.2 we examine whether blockchain adoption arises in equilibrium, and find that neither full nor partial blockchain adoption arises in equilibrium. Consequently, there exist conditions under which full blockchain adoption does not arise in equilibrium even though such adoption enhances global welfare. We refer to such a situation as an adoption failure. In Section 2.3.2 we propose a system of transfers to resolve such failures.

2.3.1 Welfare Implications

The welfare of the vendor W_V, the welfare of the manufacturers W_M, the welfare of the consumers W_C, and the total welfare W are defined as follows:

$$W_V = \sum_{j:j\in\{h,l\}} \Big(\sum_{i:\xi_i=j} (\Psi_i \cdot \sigma_i) - c_j (\sum_{i:\xi_i=j} \sigma_i) \Big) \tag{2.10}$$

$$W_M = \sum_{i:i\in M} \max_{a_i\in\{0,1\}} \Pi(a_i, a_{-i}) - \chi_i a_i \tag{2.11}$$

$$W_C = \int_0^1 \mathbb{E}[\max\{\max_{i\in M} u_{ik},\ \phi_k\}]\ dk \tag{2.12}$$

$$W = W_V + W_M + W_C \tag{2.13}$$

Recall that in all of the three scenarios we examine the blockchain adoption decisions are fixed, but we determine all other quantities endogenously as solutions of the appropriate subgame perfect equilibrium. More explicitly, our welfare analysis requires determining consumer prices, $\{P_i\}_{i\in M}$, vendor prices, $\{\Psi_i\}_{i\in M}$, vendor fulfillment quantities per manufacturer $\{\sigma_i\}_{i\in M}$ and consumer demand per manufacturer, $\{s_i\}_{i\in M}$; all these quantities are determined endogenously, assuming the manufacturers, vendors and consumers act optimally in the subgame arising from the (fixed) adoption decisions of the manufacturers. Since all quantities vary across the three cases, we hereafter adopt the superscripts F, P and N to reference objects from the full, partial and non-adoption cases respectively.

Consumer Welfare

Proposition 2.3.1 (Blockchain Consumer Welfare Implications I)**.**
Both partial and full blockchain adoption always improve consumer welfare relative to the base model of non-adoption. Moreover, full adoption provides higher consumer welfare than partial adoption, i.e., $W_C^F \geq W_C^P > W_C^N$.

Proposition 2.3.1 establishes that consumer welfare unambiguously improves with blockchain adoption. This result arises due to two effects. In particular, the blockchain both enhances utility for each consumer who purchases from a manufacturer and also increases the overall measure of consumers purchasing from a manufacturer. We offer the following supplementary proposition to formalize that point:

Proposition 2.3.2 (Blockchain Consumer Welfare Implications II)**.**
Let $\mathcal{M} := \{k \in [0,1] : m(k) \in M\}$ denote the set of consumers that purchase from a manufacturer. We define $s := \mu(\mathcal{M})$ and $u := \left(\int_{k \in \mathcal{M}} u_{i(k)k} \, dk \right) / s$ so that s represents the total consumer demand and u denotes the utility per consumer among consumers who purchase from a manufacturer (i.e., k such that $k \in \mathcal{M}$). Then, the following results hold:

- *Expected Utility Per Consumer*
  The blockchain improves the utility per consumer who purchases from a manufacturer that uses blockchain, i.e., $u^F \geq u^P > u^N$.

- *Overall Consumer Demand*
  Some consumers switch from the outside option to purchasing from a manufacturer. In particular, consumer demand increases with blockchain adoption, i.e., $s^F \geq s^P \geq s^N$.

This first effect arises because the blockchain ensures the accuracy of the signal that each consumer receives from a given manufacturer. In turn, each consumer can find a manufacturer of her own horizontal type and can also distinguish among vertically differentiated manufacturers. The fact that consumers can distinguish among vertically differentiated manufacturers leads low quality manufacturers to reduce prices to remain competitive with high quality manufacturers. The first effect thus benefits consumers both because consumer decision-making improves make more informed decisions and because low quality manufacturers lower prices.

The second effect arises because the increase in utility for each consumer purchasing from a manufacturer leads some consumers, who would have otherwise selected their outside option, to optimally purchase from a manufacturer. The resulting increase in consumer demand reflects an increase in utility for each consumer switching away from the outside option as a consumer makes such a switch only if the consumer receives a higher utility by purchasing from a manufacturer.

Manufacturer Welfare

Proposition 2.3.3 (Blockchain Manufacturer Welfare Implications)**.**
Both full and partial blockchain adoption reduce manufacturer welfare relative to non-adoption, i.e., $W_M^F < W_M^P < W_M^N$.

Proposition 2.3.3 establishes that blockchain adoption always reduces manufacturer welfare. This is because manufacturers cannot extract consumer gains from blockchain adoption by raising prices, since competition in the manufacturing sector precludes manufacturers from such price increases. In addition, manufacturers pay a cost to implement blockchain, leaving them unambiguously worse off from blockchain adoption.

To understand this result, it is important to realize that the adoption cost becomes a sunk cost after blockchain adoption, whereas prices can be changed at any time, including after a blockchain adoption decision has been made. We capture this parsimoniously by modeling that the pricing decisions are made after the blockchain adoption decisions. For a manufacturer, undercutting a competitor's price always generates higher profits so long as the unit price exceeds the unit cost. As a consequence, manufacturers cannot raise prices beyond unit costs to internalize gains from blockchain adoption. Proposition 2.3.3 formalizes the fact that manufacturers are left with only the cost of adoption and no offsetting profits after the blockchain adoption decisions.

Vendor Welfare

Proposition 2.3.4 (Blockchain Vendor Welfare Implications I)**.**
Both full and partial blockchain adoption reduce vendor welfare relative to non-adoption, i.e., $W_V^F, W_V^P < W_V^N$.

Proposition 2.3.4 establishes that vendor welfare unambiguously decreases with blockchain adoption. This result arises because blockchain adoption reveals the low type manufacturers as being of low quality. In turn, low quality manufacturers respond by reducing prices to remain competitive with high quality manufacturers. The reduced prices, in turn, are passed through to the low quality vendor, leading to a reduction in profits for the low quality vendor. We formalize this point with the following supplementary proposition:

Proposition 2.3.5 (Blockchain Vendor Welfare Implications II)**.**
The price offered by low type manufacturers to the low type vendor increases with blockchain adoption, i.e., for all $i : \xi_i = l, \Psi_i^F, \Psi_i^P < \Psi_i^N$.

<u>**Global Welfare**</u>

Proposition 2.3.6 (Blockchain Welfare Implications)**.**
Blockchain adoption has ambiguous effects on global welfare. Let $\Delta := H - L$. *Then the following results hold.*

- <u>*Full Blockchain Adoption Could Enhance Global Welfare*</u>

  For Δ *sufficiently large, global welfare under full blockchain adoption exceeds global welfare without blockchain, i.e. there exists* $\underline{\Delta}$ *such that for all* $\Delta > \underline{\Delta} : W^F > W^N$.

- <u>*Partial Blockchain Adoption Could Enhance Global Welfare*</u>

  For Δ *sufficiently large, global welfare under partial blockchain adoption exceeds global welfare without blockchain, i.e., there exists* $\underline{\Delta}$ *such that for all* $\Delta > \underline{\Delta} : W^P > W^N$.

- <u>*Full Blockchain Adoption Could Reduce Global Welfare*</u>

  For $\sum_{i:i \in M} \chi_i$ *sufficiently large, global welfare under full blockchain adoption is lower than global welfare without blockchain, i.e., there exists* $\underline{\chi}$ *such that for all* $\sum_{i:i \in M} \chi_i > \underline{\chi} : W^F < W^N$.

- <u>*Partial Blockchain Adoption Could Reduce Global Welfare*</u>

  For $\sum_{i:\xi_i = h} \chi_i$ *sufficiently large, global welfare under partial blockchain adoption is lower than global welfare without blockchain, i.e., there exists* $\underline{\chi}$ *such that for all* $\sum_{i:\xi_i = h} \chi_i > \underline{\chi} : W^P < W^N$.

Proposition 2.3.6 establishes that blockchain adoption has ambiguous effects on global welfare. Recall that Proposition 2.3.1 establishes that consumer welfare increases with blockchain adoption, whereas Propositions 2.3.3 and 2.3.4 highlight that manufacturer and vendor welfare decreases with blockchain adoption. Proposition 2.3.6 highlights that neither consumer welfare gains nor manufacturer and vendor welfare losses necessarily dominates so that blockchain adoption has ambiguous welfare effects.

The possibility for blockchain adoption to enhance global welfare arises because blockchain improves every consumer's information accuracy, and thereby, leads each consumer to select a manufacturer of her type with higher probability. The welfare gain from selecting a manufacturer of her type increases in $\Delta := H - L > 0$. Proposition 2.3.6 establishes that a sufficiently high Δ ensures that consumer welfare increases sufficiently when blockchain is adopted to make blockchain adoption globally welfare enhancing.

Proposition 2.3.6 also highlights that blockchain adoption becomes globally welfare decreasing for all sufficiently high adoption costs. This result is straightforward since manufacturer welfare losses can be made arbitrarily large by setting adoption costs to be arbitrarily large.

2.3.2 Adoption Failures

Proposition 2.3.7 (Full and Partial Blockchain Adoption Non-Existence)**.**
There does not exist an equilibrium with full or partial blockchain adoption.

Proposition 2.3.7 follows from Propositions 2.3.1 and 2.3.3. In particular, manufacturer utility decreases when blockchain is adopted so that manufacturers optimally decide not to adopt blockchain in equilibrium. This result arises both because the benefits of blockchain accrue primarily to consumers (Proposition 2.3.1) and because the competitive nature of the manufacturing sector precludes manufacturers from extracting sufficient consumer welfare gains to offset blockchain adoption costs (Proposition 2.3.3).

Recall that an adoption failure refers to a case in which blockchain adoption does not arise in equilibrium even though such adoption enhances global welfare. Our next result, Proposition

2.3.8, follows from Propositions 2.3.6 and 2.3.7, establishing that adoption failures arise when Δ is sufficiently large:

Proposition 2.3.8 (Adoption Failures)**.**
An adoption failure arises for sufficiently large $\Delta := H - L$.

Intuitively, Proposition 2.3.8 arises from a misalignment between control over the adoption decision and the distribution of welfare gains generated from that decision. Recall that Proposition 2.3.1 establishes that consumers unambiguously benefit from blockchain adoption, and that benefit increases with Δ; moreover, this welfare enhancement is available to consumers for free. On the other hand, each manufacturer faces an adoption cost and no offsetting gain so that even a small adoption cost drives the manufacturer not to join the blockchain. This non-adoption occurs even if Δ is sufficiently large so that it necessarily enhances global welfare.

We propose a system of transfers that resolves the aforementioned adoption failures by transferring some surplus welfare from consumers to manufacturers. This system of transfers consists of charging consumers a fee κ to access the information on the blockchain, and using these fees to pay manufacturer i an amount $\tau_i \geq 0$. We require that the system of transfers $(\kappa, \{\tau_i\}_{i\in M})$ must be self-financing, in the sense that the payments to manufacturers are fully funded from consumer payments, without any external financing, i.e.

$$\kappa \cdot \mu_\kappa = \sum_{i:i\in M} \tau_i \tag{2.14}$$

where $\mu_\kappa \in [0, 1]$ denoting the measure of consumers that pay κ for access to the information on the blockchain.

We modify manufacturer i's problem to include the transfer payments, $\{\tau_i\}_{i\in M}$, as follows:

$$\max_{a_i\in\{0,1\}} \Pi(a_i, a_{-i}) - \chi_i a_i + \tau_i a_i \tag{2.15}$$

And, we modify consumer k's problem to include the blockchain access cost κ as follows:

$$\max_{b_k \in \{0,1\}} \mathbb{E}^{b_k}[\max\{\max_{i \in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \tag{2.16}$$

In this modified setting, consumer k selects her information environment $b_k \in \{0, 1\}$ prior to observing signals from manufacturers: $b_k = 1$ represents the consumer opting to gain access to the information on the blockchain, while $b_k = 0$ represents the consumer opting to forgo access to the information on the blockchain.

The consumer's information set $\mathcal{G}_k$ when selecting $b_k \in \{0, 1\}$ includes her outside option value, her own type, and manufacturer prices and adoption decisions. Note that $\mathcal{G}_k$ does not include the manufacturer signals since the decision $b_k \in \{0, 1\}$ determines the distribution of the manufacturer signals. In particular, by paying a fee κ, consumer k has the ability to receive more accurate type signals from any manufacturer who joins the blockchain. Formally, consumer k receives random signals $\{(\tilde{q}_{ik}, \tilde{\xi}_{ik})\}_{i \in M}$ such that $\mathbb{P}^{b_k}(\tilde{q}_{ik} = q_i \mid q_i, a_i) = \alpha + (1 - \alpha)a_i b_k = \mathbb{P}^{b_k}(\tilde{\xi}_{ik} = \xi_i \mid \xi_i, a_i)$.

In terms of the b_k variables, the measure μ_κ of the set of consumers who pay the access fee κ is given by $\mu_\kappa = \mu(\{k : b_k = 1\})$. Recall that the system of transfers $(\kappa, \{\tau_i\}_{i \in M})$ must be self-financing, i.e., (2.14) holds. We say that a system of transfers overcomes adoption failures if it induces an equilibrium in which all manufacturers adopt blockchain (i.e., $a_i = 1$ for all $i \in M$).

Proposition 2.3.9 (Resolving Adoption Failures)**.**
For sufficiently large Δ, there exists a system of transfers that overcomes adoption failures.

Proposition 2.3.9 establishes that our proposed transfers resolve adoption failures. The transfers must be set sufficiently high in order to offset manufacturers' adoption costs, but also sufficiently low to preserve some welfare gains for some consumers. The incremental welfare for manufacturers generates adoption because this welfare gain makes adopting blockchain incentive-compatible for manufacturers. Simultaneously, the preservation of a welfare gain for particular consumers ensures that those consumers are willing to pay for the blockchain information environment, which, in turn, finances the transfers to manufacturers.

The proportion of consumers purchasing from a manufacturer is less than that in the full adoption case discussed in Section 2.3.1. This is because consumers receive access to the information on the blockchain for free in the previous setting whereas consumers must pay a fee for that access in the revised setting. Since consumers have access to an outside option, the fee causes some consumers that would have purchased from a manufacturer in the previous case to select the outside option when a fee is charged. However, as Δ increases, it becomes optimal for all consumers to pay the fee.

Our work provides a concrete path to further blockchain adoption. In industries in which consumers would significantly benefit from information stored on the blockchain, consumers could be given access to some such information in return for financing blockchain adoption in that industry. Such financing payments correspond to transfer payments from consumers to manufacturers and thus, per Proposition 2.3.9, would facilitate blockchain adoption. The system we propose could be implemented in a straightforward manner through a paid web application, established by a consortium of all manufacturers. Any consumer would receive access to relevant information from the manufacturing process only if the consumer pays a fee. As discussed, the fee would be set sufficiently high that the sum of all fees offset blockchain adoption costs but not so high that too few consumers prefer to pay the fee. The proceeds of the fee revenue would be shared among all manufacturers, making blockchain adoption incentive-compatible.

Chapter 3: Impact of Early Recall on Blockchain Adoption

3.1 Introduction

In this chapter, we consider a variation of the model described in Chapter 2, in which we simplify the vendor layer by removing the vertical differentiation and introduce an early recall mechanism. This model addresses both the transparency and traceability of blockchain. We examine whether blockchain adoption arises in equilibrium in this setting. In our model, there is a single vendor who supplies the manufacturers with goods, which are then used to fill consumer orders.[1] Each manufacturer possesses a horizontally differentiated type that corresponds to the manufacturing process of her finished goods, and that manufacturer type is unknown to consumers. Consumers are heterogeneous in their preferences for manufacturing processes and each possesses unit demand.

In practice, a blockchain enables seamless tracking of goods so that any discovered defect is immediately known to all affected parties. We assume that a manufacturer detects a defective good with an exogenous probability. Then, consistent with practice, we assume that a defect detected by any manufacturer on the blockchain is immediately communicated to all the manufacturers on the blockchain, facilitating an early recall of defective goods. In the absence of blockchain, any detected defect would eventually become known to all manufacturers but there would be a delay in tracking the specific goods that need to be recalled and also in communicating the necessary information. Although the defective goods are replaced by the vendor in all cases, late recall of defective goods by a manufacturer results in a loss of reputation and goodwill, which is costly. A manufacturer can avoid this cost by joining the blockchain whenever that option is available.

We model each manufacturer as making two decisions. Manufacturers first simultaneously decide whether or not to join the blockchain; and then they simultaneously set prices for their

[1]Our assumption of a single vendor is purely for tractability, and does not affect the key economic insights.

goods.[2] A key finding in our analysis is that manufacturers do not gain from blockchain adoption because the benefits from doing so are passed onto consumers in the form of lower prices. To understand this point, it is important to recognize that blockchain adoption costs are one-time fixed costs that subsequently become sunk costs. Moreover, blockchain implementation differs from other capital expenditures in that blockchain adoption needs to be coordinated across competitors for any cost savings to materialize. This coordination, however, ensures competition in subsequent pricing decisions, and a standard Bertrand price competition ensues, with manufacturers competing away the cost savings through reduced prices.

Proposition 3.3.1 establishes that consumers unambiguously benefit from blockchain adoption. More precisely, while early recalls facilitate cost savings for manufacturers, those cost savings are passed onto consumers in the form of lower manufacturer prices due to price competition among manufacturers. Additionally, the improved information environment enables a consumer to more easily identify a manufacturer that employs a manufacturing process that she prefers, leading her to make a purchase that yields her more utility than in the absence of blockchain. As both these effects benefit consumers, blockchain adoption unambiguously enhances consumer welfare.

Proposition 3.3.3 establishes that manufacturer welfare decreases with blockchain adoption. Crucially, blockchain adoption also has costs, which are fully borne by the manufacturers. However, we shall show that the cost savings from blockchain adoption are competed away by the price competition between manufacturers, so that the cost savings do not offset blockchain adoption costs. Thus, manufacturer welfare necessarily declines when blockchain is adopted.

Our main result, Proposition 3.3.6, establishes that blockchain adoption can enhance total welfare, even by an arbitrarily large amount, and yet fail to arise in equilibrium. This possibility, termed an adoption failure, occurs because of a misalignment between those who gain from blockchain adoption and those who decide whether or not to adopt the blockchain. Each consumer unambiguously benefits from the blockchain, but the adoption costs fall fully on the manufacturers.

[2] We model the pricing decision as subsequent to the blockchain adoption without loss of generality because, in practice, prices can be set at any point in time, but any pricing decision prior to the blockchain adoption decision is not relevant to our analysis regarding the economic implications of blockchain adoption.

Accordingly, while adoption may be globally beneficial, such adoption does not arise in equilibrium because those required to implement the adoption (i.e., manufacturers) prefer not to adopt.

We offer a simple system of transfers, formalized within Proposition 3.3.7, to resolve adoption failures. This system charges consumers for access to the blockchain's information and then transfers the proceeds to manufacturers. The transfers must be large enough so that manufacturers are incentivized to adopt the blockchain, and yet small enough so that some consumers are willing to incur this additional fee. In practice, this system of transfers can be implemented easily, for example, by charging consumers for access to a web application that stores blockchain information relevant for consumer decision-making.

The remainder of Chapter 3 is organized as follows. Section 3.2 formally states our economic model of supply chains, with and without blockchain. Section 3.3 explores the welfare implications of blockchains, demonstrates the existence of adoption failures and puts forth a system of transfers to resolve those adoption failures.

3.2 Model

We begin in Section 3.2.1 with a description of the basic model without a blockchain and explain how manufacturers set prices and how consumers make purchasing decisions. In Section 3.2.2 we describe the model in the presence of a blockchain, where manufacturers, in addition to setting prices, decide whether or not to join the blockchain, and consumers make their purchase decisions after observing the decisions of the manufacturers.

3.2.1 The Basic Model

We consider a supply chain consisting of one vendor, finitely many manufacturers, and a unit measure of consumers. A single product, supplied by the vendor, reaches the consumers via manufacturers.

Let $M := \{1, 2, \ldots, 2m\}$ be the set of manufacturers with $m \geq 2$.[3] Each manufacturer i possesses a type q_i that is either A or B. Letting M_A and M_B denote the set of type A and type B manufacturers respectively, we assume that $|M_A| = |M_B| = m$, and that all of this is common knowledge. We assume that the type of a specific manufacturer is unknown to consumers, reflecting situations in which consumers are relatively poorly informed (see, e.g., Poole and Baron 1996). However, all our results apply equally, whether or not manufacturers know each others' types.

Each consumer $k \in [0, 1]$ also has type t_k which is equally likely to be A or B, and she prefers goods from a manufacturer of her own type. More precisely, each consumer receives utility H if she consumes a good from a manufacturer of her type whereas she receives utility L if she consumes a good of a manufacturer of the other type with $0 \leq L < H$. The aforementioned assumption reflects the heterogeneous preferences of consumers in practice (see, e.g., Yiridoe et al. 2005 and Moser et al. 2011). To help consumers make purchase decisions, we assume that consumers have access to signals that reflect the type of the manufacturers. Specifically, consumer $k \in [0, 1]$ receives a set of random signals $\{\tilde{q}_{ik} \in \{A, B\}\}_{i \in M}$. We assume $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i) = \alpha \in [\frac{1}{2}, 1)$ so that a consumer's signal does not fully reveal a particular manufacturer's type. Since all the main economic insights are present in the model with $\alpha = \frac{1}{2}$ and the exposition is significantly simpler, we restrict ourselves to $\alpha = \frac{1}{2}$ within the main text and Appendix C. All the results are generalized to $\alpha \in [\frac{1}{2}, 1)$ in Appendix D. We discuss how blockchain affects consumer signals in Section 3.2.2.

We also assume that each consumer may forgo purchasing from a manufacturer and instead avail herself of an outside option. For consumer $k \in [0, 1]$, the utility from the outside option is ϕ_k, which is a random draw from a continuous distribution G that is supported on $[0, \infty)$. This outside option captures the opportunity cost of purchasing from a manufacturer, and could reflect, for example, the utility from an alternative good.

We model a finite horizon economy with three periods, $t = 0, 1, 2$. At the beginning of period 0, consumers learn their own types, t_k, and their utility from the outside option, ϕ_k. Moreover, all manufacturers observe the unit price Ψ charged by the vendor for the good. In the middle of

[3] Supply chains generally involve many participants, including several manufacturers of the same type, which is reflected in our requiring $m \geq 2$.

period 0, manufacturers simultaneously set their own prices for consumers. At the end of period 0, consumers observe a signal from each manufacturer and then decide from which manufacturer, if any, to order the product; not choosing any manufacturer is thought of as choosing the outside option instead. These decisions determine the set of demands per manufacturer, given by $(s_1, s_2, ..., s_{2m})$, with s_i denoting the measure of consumers purchasing from manufacturer i. The demand set is observed by each manufacturer, who, in turn, places an order of the appropriate size with the vendor. The vendor produces the goods and sends them to manufacturers who pass them along to the consumers. This completes period 0.

The entire batch produced by the vendor is assumed to be defective with some exogenous probability, $p \in (0, 1)$. In period 1, if a manufacturer detects a defect, then that manufacturer requires the vendor to replace her goods, and pay all associated direct costs. Thus, manufacturers who discover a defect in period 1 do not incur any additional cost. We assume that defects are detected in a probabilistic fashion: if the vendor's batch is defective, then each manufacturer, independently of other manufacturers, receives news of this defect in period 1 with probability $\rho \in (0, 1)$.

In period 2, all outstanding defective goods are recalled. Manufacturers failing to recall defective goods in period 1 incur an additional cost that captures, among other things, a loss of reputation and goodwill.

Thus, the vendor's pay-off is:

$$(\underbrace{\Psi}_{Price} - \underbrace{(\gamma_0 + \gamma_1 p)}_{Expected\ Unit\ Cost}) \cdot \underbrace{\sum_{i=1}^{2m} s_i}_{Sales}, \tag{3.1}$$

where $\Psi \geq 0$ denotes the vendor's goods price, $\gamma_0 > 0$ denotes the vendor's unit cost of production and $\gamma_1 > 0$ denotes the vendor's unit cost of replacing an initially defective good. We emphasize that in our model the vendor pays to replace a defective good irrespective of the timing of the recall. As we discuss subsequently, the blockchain's primary role in our analysis is that it enables early recalls. The last two points collectively imply that the introduction of blockchain does not affect vendor unit costs so that vendor pricing provides limited insight into the implications of the introduction of blockchain. Accordingly, we hereafter assume that vendor pricing is competitive (i.e., $\Psi = \gamma_0 + \gamma_1 p$) to enable parsimonious analysis of the vendor while focusing our model around the manufacturer-consumer interaction.[4]

Manufacturer $i \in M$ sets her price P_i by solving the following problem that maximizes her net profit:

$$\max_{P_i \geq 0} \quad \mathbb{E}\big[\underbrace{P_i \cdot s_i}_{Revenue} - \underbrace{\Psi \cdot s_i}_{Cost\ of\ Goods} - \underbrace{c \cdot s_i \cdot Y_i}_{Late\ Recall\ Costs}\big], \tag{3.2}$$

where $s_i \geq 0$ denotes the (endogenously determined) consumer demand for manufacturer i, and thus, also the order placed at the vendor to fulfill that demand. Observe that the manufacturer's objective function consists of two parts: the profit from selling s_i units, at a per-unit profit of $(P_i - \Psi)$; and the loss due to not detecting a defective batch in period 1. This latter cost is assumed to be linear, with a per-unit cost of $c > 0$. Letting $Y_i \in \{0, 1\}$ denote the event that the vendor has a defective batch and manufacturer i does not detect the defect in period 1, we see that the number of units incurring the late detection cost is simply $Z_i = s_i \cdot Y_i$, so that the total cost from late recalls of defective goods equals $c \cdot s_i \cdot Y_i$.

Recall that the consumer base size, s_i, is determined endogenously. In particular, consumers observe manufacturer prices and signals and then make purchase decisions on that basis. Recall also that consumer $k \in [0, 1]$ receives utility H if she purchases from a manufacturer of her type (i.e., $t_k = q_i$), but utility L otherwise. Consumer k has imperfect information regarding manufacturer i's type so that her utility, u_{ik}, for purchasing from manufacturer i is:

$$u_{ik} = \underbrace{V_{ik}}_{Expected\ Good\ Value} - \underbrace{P_i}_{Cost} \tag{3.3}$$

with V_{ik} denoting the expected value for consumer k for manufacturer i's good. Note that V_{ik} is

[4]Our main results apply more generally for an exogenous vendor price $\Psi \geq \gamma_0 + \gamma_1 p$.

simply:

$$V_{ik} \equiv \underbrace{H \cdot \mathbb{P}(t_k = q_i \mid \mathcal{F}_k)}_{i\ and\ k\ are\ of\ same\ type} + \underbrace{L \cdot \mathbb{P}(t_k \neq q_i \mid \mathcal{F}_k),}_{i\ and\ k\ are\ of\ different\ types} \tag{3.4}$$

where $\mathcal{F}_k$ denotes the information set of consumer $k \in [0, 1]$ that includes her own type, the value of her outside option, a type signal from each manufacturer, and the price posted by each manufacturer. Furthermore, as discussed, while consumer k does not know the type of any given manufacturer, it is public knowledge that there are exactly m manufacturers of each type, and consumers assume each manufacturer is equally likely to be of either type ex ante.

Consumer $k \in [0, 1]$ will purchase the good if her utility from doing so (weakly) exceeds her outside option, i.e., if $\max_{i \in M} u_{ik} \geq \phi_k$. Let $i(k)$ be a manufacturer that offers her maximum utility, i.e., $i(k) \in \arg\max_{i \in M} u_{ik}$, with ties broken uniformly at random. Then, consumer k purchases the good from manufacturer $i(k)$ if her utility from doing so (weakly) exceeds ϕ_k; otherwise she chooses the outside option. Denoting this choice by $m(k)$ we see that $m(k)$ is either $i(k)$ or $\emptyset$. In this notation, the endogenous consumer demand for manufacturer $i \in M$, s_i, is:

$$s_i = \mu(\{k : m(k) = i\}) \tag{3.5}$$

with $\mu(S)$ denoting the measure of a set $S \subseteq [0, 1]$ of consumers.

3.2.2 Model with Blockchain

We enhance the basic model with a single blockchain. Any manufacturer i can join the blockchain by paying a cost $\chi_i > 0$. We let a_i be a binary decision variable that is set to 1 if manufacturer i joins the blockchain, and zero otherwise. We further assume that these decisions are publicly observable.

The presence of the blockchain changes the information environment of the consumer: for each manufacturer i on the blockchain, the signal observed by any consumer $k \in [0, 1]$ reveals the manufacturer's type exactly. In other words, $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i, a_i = 1) = 1$. This modeling choice, which is the first major difference from the basic model, reflects the fact that the blockchain stores all relevant aspects of the manufacturing process and provides such information not only to manufacturers but also to consumers. Consistent with our basic model, we also assume that $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i, a_i = 0) = \alpha \in [\frac{1}{2}, 1)$.

We continue to model the economy with three periods, $t = 0, 1, 2$. As in the basic model, consumers learn their types and their utility for the outside option at the earliest point of period 0, and manufacturers observe the price charged by the vendor for the good.[5] Subsequently, before any further action in period 0 but still in the beginning of the period, manufacturers simultaneously make blockchain adoption decisions. All blockchain adoption decisions are public knowledge. Then, as in the basic model, manufacturers simultaneously set prices in the middle of the period. At the end of period 0, consumers observe a signal from each manufacturer and then decide from which manufacturer, if any, to order the product; not choosing any manufacturer is thought of as choosing the outside option instead. We emphasize that the distributions from which signals are drawn are different depending on whether or not the manufacturer joins the blockchain. These decisions determine the demand observed by each manufacturer, who, in turn, places an order of the appropriate size with the vendor. The vendor produces the goods and sends them to manufacturers who pass them along to the consumers. This completes period 0.

As before, the entire batch produced by the vendor can be defective with some exogenous probability, $p \in (0, 1)$. In period 1, if a manufacturer detects a defect, then that manufacturer requires the vendor to replace the defective good, and pay all associated direct costs. This leads us to the second key difference from the basic model: if this manufacturer participates in the blockchain, this defect becomes known to all the manufacturers participating in the blockchain in period 1. That is, for each manufacturer on the blockchain, the *early* detection probability increases to $\rho_n > \rho$, where $n > 1$ denotes the number of manufacturers participating in the blockchain, and $\{\rho_n\}_{n=1}^{2m} \subseteq [0, 1]$ denotes a strictly increasing sequence of probabilities that reflect the increasing gains from information sharing via blockchain. This information sharing is only possible because

[5]The timing of the vendor's goods price being observable does not affect our results because it can be anticipated perfectly as it is a function of model parameters (i.e., $\Psi = \gamma_0 + \gamma_1 p$ with γ_0, γ_1 and p each being parameters and public knowledge).

the blockchain maintains an immutable record of the antecedents of each item and enables efficient tracking of the defect to the entire network. To maintain consistency with our basic model, we impose $\rho_1 := \rho$.

In period 2, all outstanding defective goods are recalled. Manufacturers failing to recall defective goods in period 1 incur an additional cost that captures, among other things, the loss of reputation and goodwill. We emphasize that this additional cost from a late recall does not reflect the replacement cost of the good as the replacement cost is always fully borne by the vendor irrespective of the timing of the recall. All pay-outs realize at the end of the period 2.

At the beginning of period 0, manufacturer $i \in M$ solves the following problem to maximize profit:

$$\max_{a_i \in \{0,1\}} \quad \Pi(a_i, a_{-i}) - \chi_i a_i, \tag{3.6}$$

where, similar to (3.2), the expected profit, $\Pi(a_i, a_{-i})$, is given by:

$$\Pi(a_i, a_{-i}) := \max_{P_i \geq 0} \quad \mathbb{E}\big[P_i \cdot s_i - \Psi \cdot s_i - c \cdot s_i \cdot Y_i\big], \tag{3.7}$$

where $s_i \geq 0$ denotes the endogenously determined consumer demand for manufacturer i that depends on observed manufacturer prices and consumer signals, which depends on the adoption decisions of manufacturers.

As before, consumers observe manufacturer prices and signals and then make purchase decisions on that basis. In the presence of the blockchain, consumers observe whether or not a manufacturer joins the blockchain so that $\mathcal{F}_k$ also includes all adoption decisions; that observation, by itself, refines the consumer's information environment. Separately, consumers have accurate information about the types of the manufacturers who join the blockchain. These changes in the consumer information environment affect consumer decisions which, in turn, affect the endogenous demand observed by each manufacturer. Thus, the consumer problem specification is identical as in the basic model although u_{ik} now depends on whether or not manufacturer i joins the blockchain.

3.3 Economic Analysis

This section is devoted to an analysis of the total welfare of all the participants in the supply chain. Three scenarios, listed below, are of interest:

- **Non-Adoption:** No manufacturer joins the blockchain, i.e., $a_i = 0$ for all $i \in M$.
- **Full Adoption:** Every manufacturer joins the blockchain, i.e., $a_i = 1$ for all $i \in M$.
- **Partial Adoption:** Every type A manufacturer joins the blockchain, and no type B manufacturer joins the blockchain. That is, $a_i = 1$ for all $i \in M_A$ and $a_i = 0$ for all $i \in M_B$.[6]

The welfare in the non-adoption setting can be thought of as a benchmark against which the welfare in the other two cases can be measured because the non-adoption setting is identical to our basic model without blockchain. This analysis is done in Section 3.3.1, which demonstrates that full and partial blockchain adoption always benefit consumers but never benefit the manufacturers. Our analysis also establishes that full and partial blockchain adoption have ambiguous effects on global welfare. In Section 3.3.2 we examine whether blockchain adoption arises in equilibrium, and find that neither full nor partial blockchain adoption arises in equilibrium under any conditions. Consequently, there exist conditions under which full blockchain adoption does not arise in equilibrium even though such adoption enhances global welfare. We refer to such a situation as an adoption failure. In Section 3.3.2 we propose a system of transfers to resolve such failures.

[6]Without loss of generality, this case reflects all symmetric pure strategies that correspond to partial adoption: symmetry between the two types of manufacturers in the model implies that the case in which all type B manufacturers join the blockchain and none of the type A manufacturers does is equivalent.

3.3.1 Welfare Implications

The welfare of the vendor W_V, the welfare of the manufacturers W_M, the welfare of the consumers W_C, and the total welfare W are defined as follows:

$$W_V = (\Psi - (\gamma_0 + \gamma_1 p)) \cdot \sum_{i:i\in M} s_i \tag{3.8}$$

$$W_M = \sum_{i:i\in M} \max_{a_i} \big(\max_{P_i\geq 0} \mathbb{E}\big[P_i \cdot s_i - \Psi \cdot s_i - c \cdot s_i \cdot Y_i\big]\big) - \chi_i a_i \tag{3.9}$$

$$W_C = \int_0^1 \mathbb{E}[\max\{\max_{i\in M} u_{ik},\ \phi_k\}]\ dk \tag{3.10}$$

$$W = W_V + W_M + W_C \tag{3.11}$$

Recall that we assume vendor pricing is competitive (i.e., $\Psi = \gamma_0 + \gamma_1 p$). Accordingly, $W_V = 0$, which implies:

$$W = W_M + W_C \tag{3.12}$$

Intuitively, competitive vendor pricing implies that the vendor welfare is unaffected by whether or not manufacturers join the blockchain, and is zero in all three scenarios. Thus, examining consumer and manufacturer welfare is sufficient to analyze global welfare. Recall that the three scenarios we examine cover all reasonable pure strategies for the manufacturers. In each scenario, the blockchain adoption decisions are fixed, but the welfare analysis requires determining prices, $\{P_i\}_{i\in M}$, and market shares, $\{s_i\}_{i\in M}$; these prices and market shares are determined endogenously, assuming the manufacturers and consumers act optimally in the subgame arising from the (fixed) adoption decisions of the manufacturers. Since prices, market shares and various other quantities vary across the three cases, we hereafter adopt the superscripts F, P and N to reference objects from the full, partial and non-adoption cases respectively.

Consumer Welfare

Proposition 3.3.1 (Blockchain Consumer Welfare Implications I)**.**
Both partial and full blockchain adoption always improve consumer welfare relative to the base model of non-adoption. Moreover, full adoption provides higher consumer welfare than partial adoption, i.e., $W_C^F > W_C^P > W_C^N$.

Proposition 3.3.1 establishes that consumer welfare unambiguously improves with blockchain adoption. This result arises due to three factors. First, the blockchain ensures the accuracy of the signal that each consumer receives from a given manufacturer, thereby ensuring that each consumer, who does not select the outside option, selects a manufacturer of her own type. Second, the blockchain enables early recalls of defective goods with higher probability, which generates cost savings for manufacturers; these cost savings then result in reduced prices, which further enhance consumer welfare. Finally, the improved signal accuracy and reduced good prices leads some consumers, who would have otherwise selected their outside option, to optimally purchase from a manufacturer using blockchain, resulting in an increase in the proportion of consumers purchasing from such manufacturers. The referenced increase in consumer demand for manufacturers using blockchain reflects an increase in utility for each consumer switching away from the outside option as such a switch is optimal only if the consumer receives a higher utility by purchasing from a manufacturer. In fact, this discussion establishes not only that blockchain adoption improves consumer welfare relative to non-adoption but also that full adoption improves consumer welfare relative to partial adoption. We provide the following supplementary proposition that formalizes each of the three factors:

Proposition 3.3.2 (Blockchain Consumer Welfare Implications II)**.**
Recall that $i(k)$ *denotes consumer* k*'s preferred manufacturer. Then, let* $V_k := \mathbb{E}[V_{i(k)k}]$ *denote the expected good value for consumer* k*, where the expectation is with respect to all possible signal realizations. Consequently, the following results hold:*

- *Factor 1: Expected Good Value*

  The blockchain improves the quality of consumer information, enabling a given consumer to select a manufacturer of her own type with higher probability. In turn, each consumer attains a higher expected good value, i.e., for all $k \in [0, 1]$*:* $V_k^F \geq V_k^P > V_k^N$.

- *Factor 2: Manufacturer Prices*

  The blockchain reduces the expected cost of production for manufacturers. These reduced costs lead to lower manufacturer prices, i.e., for all $i \in M$*:* $P_i^F < P_i^P \leq P_i^N$.

- *Factor 3: Consumer Demand*

  Due to the previous two factors, some consumers switch from the outside option to purchasing from a manufacturer. In particular, consumer demand increases with blockchain adoption, i.e., for all $i \in M$*:* $s_i^F \geq s_i^P \geq s_i^N$.

Manufacturer Welfare

Proposition 3.3.3 (Blockchain Manufacturer Welfare Implications)**.**
Both full and partial blockchain adoption reduce manufacturer welfare relative to non-adoption, i.e., $W_M^F < W_M^P < W_M^N$.

Proposition 3.3.3 establishes that blockchain adoption always reduces manufacturer welfare. This is because manufacturers compete away the gains from blockchain adoption, transferring them to consumers in the form of lower prices. In addition, manufacturers pay a fee to join the blockchain, leaving them unambiguously worse off from blockchain adoption.

To understand this result, it is important to realize that the adoption cost becomes a sunk cost after blockchain adoption, whereas prices can be changed at any time, including after a blockchain adoption decision has been made. We capture this parsimoniously by modeling a pricing decision immediately after blockchain adoption decisions have been made. As previously discussed, blockchain adoption reduces expected unit costs for the manufacturers who join the blockchain; however, per Proposition 3.3.2, the subsequent strategic pricing also leads to a reduction in manufacturer prices. In fact, since undercutting a competitor's price always generates higher profits so long as the unit price exceeds the unit cost, the ability to set prices leads manufacturers to fully compete away all the gains from blockchain adoption, transferring them to consumers. Proposition 3.3.3 formalizes the fact that manufacturers are left with only the cost of adoption and no offsetting revenues.

Global Welfare

Proposition 3.3.4 (Blockchain Welfare Implications)**.**
Blockchain adoption has ambiguous effects on global welfare. Let $\Delta := H - L$*. Then the following results hold.*

- *Full Blockchain Adoption Could Enhance Global Welfare*

  For Δ *sufficiently large, global welfare under full blockchain adoption exceeds global welfare without blockchain, i.e. there exists* $\underline{\Delta}$ *such that for all* $\Delta > \underline{\Delta} : W^F > W^N$.

- *Partial Blockchain Adoption Could Enhance Global Welfare*

  For Δ *sufficiently large, global welfare under partial blockchain adoption exceeds global welfare without blockchain, i.e., there exists* $\underline{\Delta}$ *such that for all* $\Delta > \underline{\Delta} : W^P > W^N$.

- *Full Blockchain Adoption Could Reduce Global Welfare*

  For $\sum_{i:i\in M} \chi_i$ *sufficiently large, global welfare under full blockchain adoption is lower than global welfare without blockchain, i.e., there exists* $\underline{\chi}$ *such that for all* $\sum_{i:i\in M} \chi_i > \underline{\chi} : W^F < W^N$.

- *Partial Blockchain Adoption Could Reduce Global Welfare*

  For $\sum_{i:i\in M} \chi_i$ *sufficiently large, global welfare under partial blockchain adoption is lower than global welfare without blockchain, i.e., there exists* $\underline{\chi}$ *such that for all* $\sum_{i:i\in M} \chi_i > \underline{\chi} : W^P < W^N$.

By combining both consumer and manufacturer welfare, Proposition 3.3.4 establishes that blockchain adoption has ambiguous effects on global welfare. Recall that Proposition 3.3.1 establishes that consumer welfare increases with blockchain adoption, whereas Proposition 3.3.3 highlights that manufacturer welfare decreases with blockchain adoption. Proposition 3.3.4 highlights that neither consumer welfare gains nor manufacturer welfare losses necessarily dominates so that blockchain adoption has ambiguous effects.

The possibility for blockchain adoption to enhance global welfare arises because blockchain improves every consumer's information accuracy, and thereby, leads each consumer to select a manufacturer of her type with higher probability. The welfare gain from selecting a manufacturer of her type increases in Δ. Proposition 3.3.4 establishes that a sufficiently high Δ ensures that consumer welfare increases sufficiently when blockchain is adopted to make blockchain adoption globally welfare enhancing.

Proposition 3.3.4 also highlights that blockchain adoption becomes globally welfare decreasing for all sufficiently high adoption costs. This result is straightforward since manufacturer welfare losses can be made arbitrarily large by setting adoption costs to be arbitrarily large.

3.3.2 Adoption Failures

We start with Proposition 3.3.5, which establishes that neither full nor partial blockchain adoption arises in equilibrium.

Proposition 3.3.5 (Full and Partial Blockchain Adoption Non-Existence)**.**
There does not exist an equilibrium with full or partial blockchain adoption.

This result follows because manufacturers anticipate the (previously discussed) price competition arising from joining the blockchain and recognize that the benefits from adoption by way of reduced prices will be passed along to the consumers while each of them bears the cost of adoption. Consequently, neither type A nor type B manufacturers adopt blockchain in any equilibrium.

We note that blockchain adoption differs from a traditional capital expenditure in that the potential benefits arise only if the adoption decisions are coordinated across competing participants:

this stems from the observation that blockchain is a technology for information sharing and such information sharing generates cost savings for a manufacturer only when it is coordinated across other manufacturers. However, multiple participants of the same type on the blockchain triggers a price competition, which results in lower prices for the consumers, but leaves the manufacturers joining the blockchain with only the sunk adoption cost and no offsetting revenues. And, therefore, neither type of manufacturer adopts blockchain in any equilibrium.[7]

Proposition 3.3.6 (Adoption Failures)**.**
An adoption failure arises for sufficiently large $\Delta := H - L$.

Proposition 3.3.6 follows from Propositions 3.3.4 and 3.3.5, establishing that adoption failures arise when Δ is sufficiently large. Recall that an adoption failure refers to a case in which full blockchain adoption does not arise in equilibrium even though such adoption enhances global welfare. Intuitively, Proposition 3.3.6 arises from a misalignment between control over the adoption decision and the distribution of welfare gains generated from that decision. Recall that Proposition 3.3.1 establishes that consumers unambiguously benefit from blockchain adoption, and that benefit increases with Δ; moreover, this welfare enhancement is available to consumers for free. On the other hand, each manufacturer faces an adoption cost and no offsetting gain due to price competition so that even a small adoption cost drives the manufacturer not to join the blockchain. This non-adoption occurs even if Δ is sufficiently large so that it necessarily enhances global welfare.

We propose a system of transfers that resolves the aforementioned adoption failures by transferring some surplus welfare from consumers to manufacturers. This system of transfers consists of charging consumers a fee κ to access the information on the blockchain, and using these fees to pay manufacturer i an amount $\tau_i \geq 0$. We require that the system of transfers $(\kappa, \{\tau_i\}_{i \in M})$ must be self-financing, in the sense that the payments to manufacturers are fully funded from consumer

[7] A similar feature of blockchain adoption is discussed in Cao et al. (2019) when competing auditors adopt a federated blockchain.

payments, without any external financing, i.e.

$$\kappa \cdot \mu_\kappa = \sum_{i:i\in M} \tau_i \tag{3.13}$$

where $\mu_\kappa \in [0,1]$ denoting the measure of consumers that pay κ for access to the information on the blockchain.

We modify manufacturer i's problem to include the transfer payments, $\{\tau_i\}_{i\in M}$, as follows:

$$\max_{a_i} \left(\max_{P_i\geq 0} \mathbb{E}\big[P_i \cdot s_i - \Psi \cdot s_i - c \cdot s_i \cdot Y_i\big]\right) - \chi_i a_i + \tau_i a_i \tag{3.14}$$

And, we modify consumer k's problem to include the blockchain access cost κ as follows:

$$\max_{b_k} \mathbb{E}^{b_k}[\ \max\{\max_{i\in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \tag{3.15}$$

In this modified setting, consumer k selects her information environment $b_k \in \{0,1\}$ prior to observing signals from manufacturers: $b_k = 1$ represents the consumer opting to gain access to the information on the blockchain, while $b_k = 0$ represents the consumer opting to forgo access to the information on the blockchain.

The consumer's information set $\mathcal{G}_k$ when selecting $b_k \in \{0,1\}$ includes her outside option value, her own type, and manufacturer prices and adoption decisions. Note that $\mathcal{G}_k$ does not include the manufacturer signals since the decision $b_k \in \{0,1\}$ determines the distribution of the manufacturer signals. In particular, by paying a fee κ, consumer k has the ability to receive more accurate type signals from any manufacturer who joins the blockchain. Formally, consumer k receives random signals $\{\tilde{q}_{ik}\}_{i\in M}$ such that $\mathbb{P}^{b_k}(\tilde{q}_{ik} = q_i \mid q_i, a_i) = \alpha + (1-\alpha)a_i b_k$.

In terms of the b_k variables, the measure μ_κ of the set of consumers who pay the access fee κ is given by $\mu_\kappa = \mu(\{k : b_k = 1\})$. Recall that the system of transfers $(\kappa, \{\tau_i\}_{i\in M})$ must be self-financing, i.e. (3.13) holds. We say that a system of transfers overcomes adoption failures if it induces an equilibrium in which all manufacturers adopt blockchain (i.e., $a_i = 1$ for all $i \in M$).

Proposition 3.3.7 (Resolving Adoption Failures)**.**

For sufficiently large Δ, there exists a system of transfers that overcomes adoption failures.

Proposition 3.3.7 establishes that our proposed transfers resolve adoption failures. The transfers must be set sufficiently high in order to offset manufacturers' adoption costs, but also sufficiently low to preserve some welfare gains for some consumers. The incremental welfare for manufacturers generates adoption because this welfare gain makes adopting blockchain incentive-compatible for manufacturers. Simultaneously, the preservation of a welfare gain for particular consumers ensures that those consumers are willing to pay for the blockchain information environment, which, in turn, finances the transfers to manufacturers.

The proportion of consumers purchasing from a manufacturer is less than that in the full adoption case discussed in Section 3.3.1. This is because consumers receive access to the information on the blockchain for free in the previous setting whereas consumers must pay a fee for that access in the revised setting. Since consumers have access to an outside option, the fee causes some consumers that would have purchased from a manufacturer in the previous case to select the outside option when a fee is charged. However, as Δ increases, it becomes optimal for all consumers to pay the fee.

Our work provides a concrete path to further blockchain adoption. In industries in which consumers would significantly benefit from information stored on the blockchain, consumers could be given access to some such information in return for financing blockchain adoption in that industry. Such financing payments correspond to transfer payments from consumers to manufacturers and thus, per Proposition 3.3.7, would facilitate blockchain adoption. The system we propose could be implemented in a straightforward manner through a paid web application, established by a consortium of all manufacturers. Any consumer would receive access to relevant information from the manufacturing process only if the consumer pays a fee. As discussed, the fee would be set sufficiently high that the sum of all fees offset blockchain adoption costs but not so high that too few consumers prefer to pay the fee. The proceeds of the fee revenue would be shared among all manufacturers, making blockchain adoption incentive-compatible.

Chapter 4: Blockchain Adoption in a Supply Chain with Sticky Price

4.1 Introduction

This chapter investigates the three-layer supply chain setting described in Section 1.3 in the context of sticky price and addresses both the transparency and traceability of blockchain. The goal of our study is to understand blockchain welfare implications and adoption or lack thereof in such an industry setting. Within our analysis, there exists a single vendor who supplies goods to manufacturers, which are then used to fill consumer orders. [1] The manufacturers are vertically differentiated into high and low quality manufacturers. Different from the models in the other chapters, consumers do not have the outside option and are homogeneous in their preferences: they strictly prefer the higher quality manufacturers. In addition, the consumer prices are determined exogenously and are all normalized to 1. We study this price stickiness because in some circumstances, firms downstream need time to react to a change in price upstream, and hence, the manufacturers do not modify their consumer prices in a period of time.

We model each manufacturer as making one key decision. Specifically, manufacturers simultaneously decide whether to adopt the blockchain. We assume that blockchain adoption improves the information environment for all agents. This improved information environment provides risk management benefits for manufacturers but also enables consumers to make more informed decisions. Blockchain adoption unambiguously improves welfare for consumers because the improved information environment enables consumers to improve decision-making. Blockchain adoption, however, also has costs, and these costs are fully borne by those implementing the system (i.e., manufacturers) so that blockchain adoption has an ambiguous effect upon manufacturer welfare. Our main result highlights that global welfare adoption might be arbitrarily large due to large

[1] Our single vendor assumption is purely for tractability, and does not affect our key economic insights.

consumer gains but that blockchain adoption may yet fail to arise because manufacturers prefer not to adopt.

In practice, a blockchain enables seamless tracking of goods so that any discovered defect is immediately known to all affected parties. We assume that a manufacturer detects a defective good with an exogenous probability. Then, consistent with practice, we assume, if a defect was detected by a manufacturer on the blockchain, it will be immediately known to all other manufacturers on the blockchain. We assume that such information transmission facilitates an early recall of defective goods, and this early recall, in turn, provides a cost saving to each manufacturer. The expected magnitude of this cost saving depends on a variety of factors including the likelihood of a defect, the detection probability and whether or not other manufacturers adopt the blockchain.

Nonetheless, blockchain adoption also has potential drawbacks for manufacturers. Most obviously, adoption entails a direct cost associated with implementation. Our model accounts for this cost by specifying an exogenous expense for any manufacturer adopting blockchain. When this cost is sufficiently large, a manufacturer will prefer not to adopt. The inverse implication, that sufficiently low direct costs induce adoption, does not hold in general. This finding arises because the improved consumer information environment may drive consumers away from low quality manufacturers so that low quality manufacturers may decide against adoption even if they can do so without a direct cost. We demonstrate that whether arbitrarily small adoption costs generate blockchain adoption depends upon consumer beliefs.

Our main result, Proposition 4.3.8, establishes that blockchain adoption may enhance global welfare, and yet fail to arise in equilibrium. We deem such an occurrence as an adoption failure. These adoption failures arise due to a misalignment between those who gain from blockchain adoption and those who decide whether to adopt blockchain and incur the cost. While each consumer unambiguously benefits from blockchain adoption, the adoption costs fall fully upon manufacturers. Accordingly, while adoption may be globally beneficial, such adoption does not arise in equilibrium because those required to implement the adoption (i.e., manufacturers) prefer not to adopt.

We offer a simple system of transfers, formalized within Proposition 4.3.9, to resolve the aforementioned adoption failures. This system charges consumers for access to the blockchain's information and then transfers the proceeds to manufacturers. The transfers must be large enough so that manufacturers are incentivized to adopt the blockchain, and yet small enough so that consumers are willing to incur this additional fee. Our analysis provides straightforward guidance for blockchain adoption in industries that would benefit from such adoption. We propose that consumers should partially finance blockchain adoption. This financing can be implemented, for example, by charging consumers for access to a web application that stores blockchain information relevant for consumer decision-making.

The remainder of Chapter 4 is organized as follows. Section 4.2 formally states our economic model of supply chain, with and without blockchain. Section 4.3 examines blockchain's welfare implications, demonstrates the existence of adoption failures and puts forth a system of transfers to resolve those adoption failures.

4.2 Model

We begin in Section 4.2.1 with a description of the basic model without a blockchain and explain how consumers make decisions. In Section 4.2.2 we describe the model in the presence of a blockchain, where manufacturers first decide whether or not to join the blockchain, and consumers make their purchase decisions after observing the decisions of the manufacturers.

4.2.1 The Basic Model

We consider a supply chain consisting of one vendor, $m \geq 2$ manufacturers, and a unit measure of identical consumers. A single product, supplied by the vendor, reaches the consumers via manufacturers. Each manufacturer i has a privately known type q_i that is equally likely to be H or L, with $0 < L < H$.[2] We abuse notation and let L and H also be the utility derived by a consumer

[2]In other words, all consumers and all manufacturers know the positive real numbers L and H, and all of them also know that, ex ante, each manufacturer is equally likely to be of either type; however, the realized type q_i of manufacturer i is known only to that manufacturer.

from purchasing the product from a manufacturer of type L and H respectively.

To help consumers make purchase decisions, we assume that consumers have access to signals that reflect the quality of the manufacturers. Specifically, consumer $k \in [0, 1]$ receives a set of random signals $\{\tilde{q}_{ik} \in \{H, L\}\}_{i=1}^{m}$. We assume $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i) = \alpha$ with $\alpha \in [\frac{1}{2}, 1)$.

We model a finite horizon economy with three periods, $t = 0, 1, 2$. In period 0, first, consumers observe a signal from each manufacturer and then decide from which manufacturer to order the product. These decisions determine the demand vector $(s_1, s_2, \ldots, s_m)$, with $\sum_i s_i = 1$, observed by each manufacturer, who, in turn, places an order of the appropriate size with the vendor. The vendor produces the goods and sends them to manufacturers who pass them along to the consumers. This completes period 0.

We assume that the entire batch produced by the vendor can be defective with some exogenous probability, $p \in (0, 1)$. In period 1, consumers may detect a defect, and report the defect to the manufacturer from whom they purchased the good. The manufacturer then asks the vendor to replace the batch, and pay all associated direct costs. Thus, manufacturers who discover a defect in period 1 do not incur any additional cost. We assume that defects are detected by consumers only in a probabilistic fashion: if the vendor's batch is defective, manufacturer i receives news of this defect from her consumer base with probability $1 - e^{-\lambda s_i}$ with $\lambda > 0$.[3].

In period 2, defects become public knowledge. Manufacturers failing to recall defective goods in period 1 incur an additional cost that captures, among other things, a loss of reputation and goodwill.

Note that in the basic model the vendor and the manufacturers have no decisions to make; the only agents who make decisions in the basic model are the continuum of consumers, and they do so based on observed signals. Specifically, consumer $k \in [0, 1]$ receives utility q_i from purchasing from manufacturer i, but she has imperfect information regarding q_i so she solves

$$\max_i \mathbb{E}_k[q_i], \tag{4.1}$$

[3]Note that $\mathbb{P}$(Defect not discovered from $\alpha + \beta$ consumers | Defect) = $\mathbb{P}$(Defect not discovered from α consumers | Defect) $\mathbb{P}$(Defect not discovered from β consumers | Defect)

where the expectation $\mathbb{E}_k[\cdot]$ is conditional upon her private information, i.e., the manufacturer quality signals, $\{\tilde{q}_{ik}\}_{i=1}^m$, that she observes. In the case that the consumer is indifferent among a set of manufacturers, she selects randomly and uniformly from that set.

4.2.2 Model with Blockchain

We enhance the basic model with a single blockchain. Manufacturer i can join the blockchain by paying a cost $\chi_i \geq 0$. We assume that this decision is publicly observable. Joining the blockchain changes the information environment of the consumer: for each manufacturer i on the blockchain, the signal observed by consumer $k \in [0, 1]$ satisfies $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i) = \alpha + \delta$ with $\alpha \in [\frac{1}{2}, 1)$ and $\delta \in (0, 1 - \alpha]$. That is, each consumer receives a more accurate signal regarding q_i for every manufacturer i on the blockchain. Thus, joining the blockchain can change the demand seen by a particular manufacturer, even though the total demand is always 1. Additionally, joining the blockchain may help a manufacturer detect defects earlier, as will be explained later. A manufacturer chooses to join this blockchain only if doing so results in a greater profit than not joining the blockchain.

We focus on the manufacturers' adoption decisions. We continue to model the economy with three periods, $t = 0, 1, 2$. At the beginning of period 0, manufacturers simultaneously make blockchain adoption decisions. All blockchain adoption decisions are public knowledge. Then, consumers receive signals and decide with which manufacturer to place an order. As before, the vendor produces the goods and sends them to manufacturers who pass them along to the consumers. Thus, the outcome is an adoption vector $(a_1, a_2, \ldots, a_m)$ and a demand vector $(s_1, s_2, \ldots, s_m)$, with each $a_i \in \{0, 1\}$ and $\sum_i s_i = 1$. This completes period 0.

As before, the entire batch produced by the vendor can be defective with some exogenous probability, $p \in (0, 1)$. In period 1, consumers may detect a defect, and report the defect to the manufacturer from whom they purchased the good. The manufacturer then asks the vendor to replace the defective good, and pay all associated direct costs. The key difference from the basic model is this: if this manufacturer participates in the blockchain, this defect becomes known to all the manufacturers participating in the blockchain. Thus, for each manufacturer on the blockchain, the early detection probability increases to $1 - \prod_{i \in I} e^{-\lambda s_i} = 1 - e^{-\lambda \sum_{i \in I} s_i}$, where I denotes the set of manufacturers participating in the blockchain. This is only possible because the blockchain maintains an immutable record of the antecedents of each item.

In period 2, defects become public knowledge. Manufacturers failing to recall defective goods in period 1 incur an additional cost that captures, among other things, the loss of reputation and goodwill. All pay-outs realize at the end of the period.

Let $M := \{1, 2, \ldots, m\}$ denote the set of manufacturers with $m \geq 2$. At the beginning of period 0, manufacturer $i \in M$ solves the following problem to maximize profit:

$$\max_{a_i} \quad \mathbb{E}_i[s_i(a_i, a_{-i})] - \mathbb{E}_i\big[c\big(s_i(a_i, a_{-i})Y_i\big)\big] - \chi_i a_i \tag{4.2}$$

where $a_i \in \{0, 1\}$ denotes the blockchain adoption decision with a cost $\chi_i a_i$, and $s_i \equiv s_i(a_i, a_{-i}) \geq 0$ denotes the endogenously determined consumer demand for manufacturer i, and thus, also the order placed at the vendor to fulfill that demand. Manufacturer i earns a profit which we normalize to 1 for each unit sold; however, she faces a cumulative non-decreasing convex cost $c(Z_i)$ on all defective units Z_i not detected in period 1. We assume that manufacturer profit in the absence of blockchain increases in revenues.[4] The variable $Y_i \in \{0, 1\}$ denotes the event that the vendor has a defective batch, and manufacturer i does not detect the defect in period 1. Recall that defect detection depends on the order sizes $s = \{s_i : i = 1, \ldots, m\}$, and the blockchain adoption decisions of all the manufacturers. Thus, $Z_i = s_i Y_i(a_i, a_{-i})$.

As before, each consumer solves Problem 4.1. The blockchain's existence affects the consumer problem by improving the consumer's information environment. In the presence of the blockchain, the consumer observes whether each manufacturer adopts the blockchain. That observation, by itself, refines the consumer's information environment. Separately, if a manufacturer adopts the blockchain, each consumer's signal for that manufacturer becomes more accurate thereby further

[4]Formally, we impose $\forall x : \frac{d}{dx}[x - pe^{-\lambda x} c(x)] \geq 0 \leftrightarrow c'(x) \leq \frac{e^{\lambda x}}{p} + \lambda c(x)$ and $c(0) = 0$.

enhancing the consumer's information regarding each manufacturer.

4.3 Economic Analysis

Our analysis involves three components. Section 4.3.1 examines the welfare implications of blockchain by comparing welfare in a setting without blockchain to that in a setting with adoption of blockchain by all manufacturers, hereafter referred to as full blockchain adoption. We demonstrate that full blockchain adoption always benefits consumers but has ambiguous effects for manufacturers. Our analysis also establishes that full blockchain adoption has ambiguous effects on global welfare. Section 4.3.2 then studies whether full blockchain adoption arises in equilibrium. We provide conditions under which such an equilibrium arises. Our analysis highlights that the equilibrium arising depends upon consumer beliefs. Section 4.3.3 establishes that there exist conditions under which blockchain adoption does not arise in equilibrium even though such adoption would enhance global welfare. We refer to such a situation as an adoption failure. Section 4.3.3 proposes a system of transfers to resolve such a failure.

4.3.1 Welfare Implications

It is nontrivial to determine whether full blockchain adoption improves welfare. Blockchain adoption benefits consumers but its effect on the manufacturing sector is ambiguous. The impact of full blockchain adoption on global welfare is also ambiguous.

The welfare of the manufacturers W_M, the welfare of the consumers W_C and the total welfare W are defined as follows:

$$W_M = \sum_{i=1}^{m} \mathbb{E}[\max_{a_i} \mathbb{E}_i[s_i(a_i, a_{-i})] - \mathbb{E}_i\left[c\left(s_i(a_i, a_{-i})Y_i\right)\right] - \chi_i a_i] \tag{4.3}$$

$$W_C = \int_0^1 \mathbb{E}[\max_i \mathbb{E}_k[q_i]]\ dk \tag{4.4}$$

$$W = W_M + W_C \tag{4.5}$$

Proposition 4.3.1 (Blockchain Consumer Welfare Implications)**.**
Blockchain adoption always improves consumer welfare.

Proposition 4.3.1 establishes that consumer welfare unambiguously improves with blockchain adoption. This result arises because the blockchain improves the accuracy of the signal that each consumer receives from a given manufacturer. The increased signal accuracy improves the probability that the consumer's observed signal corresponds to the true manufacturer quality. If a consumer observes a manufacturer with a high (low) signal then that manufacturer's quality is more likely to be high (low) when blockchain is adopted as compared to when blockchain is not adopted. Since the consumer selects a manufacturer from whom she receives a high signal whenever possible, the improved signal accuracy when the blockchain is adopted unequivocally improves consumer welfare.

To better understand this result, we consider the special case of $m = 2$. In this setting, the two manufacturers have the same quality with probability 0.5 and different qualities with probability 0.5. When the two manufacturers possess identical quality, the blockchain's existence does not affect consumer welfare. However, when the two manufacturers are different, blockchain adoption increases the likelihood that each consumer selects the manufacturer with high quality, and therefore, the consumer welfare increases.

Proposition 4.3.2 (Blockchain Manufacturer Welfare Implications I)**.**
Blockchain adoption may reduce manufacturer welfare. Even for arbitrarily small adoption costs, when blockchain is fully revealing (*i.e.,* $\delta = 1 - \alpha$) *and signals are otherwise uninformative* (*i.e.,* $\alpha = \frac{1}{2}$)*, blockchain adoption reduces manufacturer welfare for all sufficiently small* λ*.*

Blockchain adoption could decrease the welfare of the manufacturing sector. Proposition 4.3.2 formalizes that assertion. This result follows from the fact that the blockchain's benefits to manufacturers become arbitrarily small when risk-detection is sufficiently opaque (i.e., $\lambda \to 0^+$). Then, even a small adoption cost implies blockchain reduces welfare for the manufacturing sector.

To further clarify the relevant channel for this result, we consider a special case where early detection of a defect never occurs (i.e., $\lambda = 0$), the blockchain is fully revealing (i.e., $\alpha + \delta = 1$), and signals are uninformative in the absence of blockchain (i.e., $\alpha = \frac{1}{2}$). In this setting, blockchain adoption provides no benefit for manufacturers because no manufacturer ever discovers a defect and thus early recalls never occur irrespective of whether blockchain is adopted. Nonetheless, blockchain adoption requires an adoption cost. Thus, blockchain adoption increases expected costs to the manufacturing sector, and reduces the welfare of the manufacturing sector.

Proposition 4.3.3 (Blockchain Manufacturer Welfare Implications II)**.**
Blockchain adoption improves manufacturer welfare when λ is sufficiently large, and adoption costs are sufficiently low. For fixed adoption costs, blockchain adoption improves the welfare of the manufacturing sector most for an intermediate value of λ.

While blockchain adoption may reduce manufacturer welfare even with arbitrarily small adoption costs, blockchain adoption can, nonetheless, enhance manufacturer welfare with non-zero adoption costs for some values of λ. Proposition 4.3.3 establishes that assertion. This result is a consequence of the fact that the blockchain ensures any early defect detection is shared across all manufacturers, thereby, saving the entire sector from the cost of not recalling defective goods early. For a sufficiently high λ, this cost saving becomes large enough to outweigh blockchain adoption costs and thereby improves manufacturer welfare.

Proposition 4.3.3 also highlights that the benefits of blockchain adoption to the manufacturing sector are maximized at an intermediate value of λ. This is because at low values of λ no manufacturer detects a defect early with high probability so that information sharing has little value; in contrast, at high values of λ, each manufacturer individually detects a defect early with high probability so that the blockchain does not provide any additional benefit. We demonstrate that manufacturer welfare achieves a maximum for a well-defined intermediate value of λ, and for sufficiently small adoption costs, the maximum welfare gain is strictly positive.

Proposition 4.3.4 (Blockchain Welfare Implications)**.**
Blockchain adoption has ambiguous effects upon global welfare. Global welfare, under blockchain adoption, increases in the dispersion of product quality (i.e., $H - L$), and strictly exceeds the

global welfare without blockchain for sufficiently high product quality dispersion. Moreover, global welfare, under blockchain adoption, diverges with product quality dispersion. Global welfare, under blockchain adoption, decreases in adoption costs (i.e., $\sum_{i=1}^{m} \chi_i$*) and is strictly smaller than global welfare without blockchain for sufficiently high adoption costs.*

Incorporating both consumer and manufacturer welfare, Proposition 4.3.4 establishes that blockchain adoption has ambiguous effects upon global welfare. Proposition 4.3.1 establishes that consumer welfare increases with blockchain adoption, and Proposition 4.3.3 highlights that manufacturer welfare increases with blockchain adoption under certain conditions. Thus, it immediately follows that global blockchain adoption increases global welfare under certain conditions. Proposition 4.3.2 shows that manufacturer welfare decreases under certain conditions; Proposition 4.3.4 establishes that this welfare decrease can dominate the welfare increase to consumers, thereby rendering blockchain adoption welfare decreasing.

Proposition 4.3.4 also demonstrates that welfare benefits of blockchain adoption is a function of the product quality dispersion (i.e., $H - L$). As product quality dispersion increases so does the global welfare from blockchain adoption. This result follows because the blockchain improves every consumer's information accuracy, and thereby, leads each consumer to select a higher quality product with higher probability. The welfare gain from selecting the higher quality product, in turn, increases in product quality dispersion. Proposition 4.3.4 establishes that a sufficiently high product quality dispersion ensures that consumer welfare increases sufficiently when blockchain is adopted to make blockchain adoption globally welfare enhancing.

Finally, Proposition 4.3.4 highlights that blockchain adoption becomes globally welfare decreasing for all sufficiently high adoption costs (i.e., $\sum_{i=1}^{m} \chi_i$). This result is straight-forward since manufacturer welfare losses can be made arbitrarily large by setting adoption costs to be arbitrarily large whereas consumer welfare gains are bounded by the product quality dispersion.

4.3.2 Blockchain Adoption in Equilibrium

We next turn to examining if full blockchain adoption arises in equilibrium. We find that such adoption arises under certain conditions but that those conditions depend heavily upon consumer beliefs. Economic fundamentals do not uniquely determine consumer beliefs, and different beliefs that are consistent with an equilibrium lead to qualitatively different conditions for full blockchain adoption in equilibrium. More concretely, while certain consumer beliefs enable a full adoption equilibrium to arise for arbitrarily small adoption costs (i.e. $\forall i : \chi_i \to 0^+$), other consumer beliefs cannot generate a full adoption equilibrium even with zero adoption costs (i.e. $\forall i : \chi_i = 0$). In the following, we clarify our equilibrium concept and provide our equilibrium analysis.

Equilibrium Concept

We analyze the setting of Section 4.2 as an extensive form game involving both manufacturers and consumers. First, nature simultaneously assigns a type $q_i \in \{L, H\}$ to each manufacturer $i \in \{1, ..., m\}$. Then, all manufacturers simultaneously decide whether to adopt the blockchain. Subsequently, each consumer k receives signals $\{\tilde{q}_{ik}\}_{i=1}^m$ and observes all of manufacturers' adoption decisions, $\{a_i\}_{i=1}^m$. From this information, and without knowing true manufacturer types, each consumer simultaneously selects a manufacturer from which to make a purchase. A consumer receives the pay-off q_i if she purchases from manufacturer i. Manufacturer i receives pay-off $s_i(a_i, a_{-i}) - c(s_i(a_i, a_{-i})Y_i) - \chi_i a_i$. All agents maximize expected pay-offs, i.e., each consumer solves Problem 4.1, and each manufacturer solves Problem 4.2.

The strategy space for each manufacturer corresponds to the set of functions mapping manufacturer types, $q_i \in \{L, H\}$, into blockchain adoption decisions, $a_i \in \{0, 1\}$. This strategy space reflects that each manufacturer makes her adoption decision with knowledge of her own type but without the knowledge of the types of other manufacturers. Thus, the manufacturer strategy space is given by $G = \{g_1, g_2, g_3, g_4\}$ where $g_1(L) = 1 - g_1(H) = 1, g_2(L) = 1 - g_2(H) = 0, g_3(L) = g_3(H) = 1, g_4(L) = g_4(H) = 0$.

The strategy space for each consumer corresponds to the set of functions mapping all combinations of manufacturer adoption decisions, $\{a_i\}_{i=1}^m$, and manufacturer quality signals, $\{\tilde{q}_{ik}\}_{i=1}^m$, to the set of manufacturers. This strategy space reflects that each consumer selects a manufacturer after observing signals of manufacturer quality and whether or not each manufacturer adopts the blockchain.

We focus upon conditions that generate full blockchain adoption in equilibrium. For a full blockchain adoption equilibrium to arise, each manufacturer must earn a higher expected pay-off from adopting the blockchain as compared to not adopting the blockchain. A manufacturer's expected pay-off depends on anticipated consumer actions which, in turn, depend on consumer beliefs regarding manufacturer types. We employ Bayes' Law whenever feasible to determine consumer beliefs, but Bayes' Law cannot always be applied. In particular, since a consumer never observes a manufacturer not adopting blockchain in equilibrium, Bayes' Law cannot be applied to determine consumer beliefs when examining the out-of-equilibrium deviation of a manufacturer not adopting blockchain. To specify sensible consumer beliefs in such cases, we employ sequential equilibrium as our equilibrium concept. Sequential equilibrium refines perfect Bayesian equilibrium by restricting beliefs to those "consistent" with equilibrium behavior.[5]

Equilibrium Analysis

While sequential equilibrium restricts consumer beliefs, it does not necessarily identify those beliefs uniquely. As such, we consider two different reasonable consumer beliefs. We demonstrate that each of those consumer beliefs arise in a sequential equilibrium but that those beliefs require qualitatively different conditions to generate a full adoption equilibrium. We interpret our results as highlighting that blockchain adoption depends on consumer beliefs in a manner detached from economic fundamentals. We consider the following two beliefs:

Definition 4.3.1 (Consumer Belief 1)**.**

Consumer Belief 1, hereafter referred to as CB1, refers to the belief that each consumer believes

[5] We direct the interested reader to Kreps and Wilson (1982) for a full description of sequential equilibrium.

that manufacturer adoption decisions are independent of manufacturer quality (i.e., $\mathbb{P}(q_i = L \mid a_i = 0, \tilde{q}_{ik}) = \mathbb{P}(q_i = L \mid \tilde{q}_{ik})$).

Definition 4.3.2 (Consumer Belief 2)**.**
Consumer Belief 2, hereafter referred to as CB2, refers to the belief that each consumer believes any manufacturer not adopting is of low quality (i.e., $\mathbb{P}(q_i = L \mid a_i = 0, \tilde{q}_{ik}) = 1$).

These definitions specify beliefs only for cases in which a manufacturer does not adopt the blockchain. In all other cases, we determine beliefs via Bayes' Law.

Proposition 4.3.5 (Full Adoption Equilibrium CB1)**.**
Full adoption of blockchain is a sequential equilibrium when consumer beliefs are given by CB1 provided $x - pe^{-\lambda}c(x)$ is increasing and

$$\max_{1\leq i\leq m} \chi_i \leq \mathbb{E}\left[Z_{\tilde{\alpha}}^{L} - pe^{-\lambda}c(Z_{\tilde{\alpha}}^{L})\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right].$$

Proposition 4.3.6 (Full Adoption Equilibrium CB2)**.**
Full adoption of blockchain is a sequential equilibrium when consumer beliefs are given by CB2 provided $x - pe^{-\lambda}c(x)$ is increasing and

$$\max_{1\leq i\leq m} \chi_i \leq \mathbb{E}\left[Z_{\tilde{\alpha}}^{L} - pe^{-\lambda}c(Z_{\tilde{\alpha}}^{L})\right] - \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z_{\tilde{\alpha}}^{CB}}{m} - pe^{-\frac{Z_{\tilde{\alpha}}^{CB}}{m}\lambda}c(\frac{Z_{\tilde{\alpha}}^{CB}}{m})\right].$$

Both CB1 and CB2 arise as beliefs within a full adoption sequential equilibrium. Proposition 4.3.5 provides conditions for a full adoption sequential equilibrium when consumer beliefs follow CB1. Proposition 4.3.6 provides conditions for a full adoption sequential equilibrium when consumer beliefs follow CB2.

Corollary 4.3.7 (Full Adoption Equilibria)**.**

Assume $x - pe^{-\lambda}c(x)$ *is increasing on* $[0, 1]$. *Then,*

(I) Under CB1, a full blockchain adoption equilibrium may not arise even if adoption costs are zero. Formally, $\forall i : \chi_i = 0$ *is not sufficient for a full blockchain adoption equilibrium under consumer beliefs CB1.*

(II) Under CB2, a full blockchain adoption equilibrium arises for sufficiently small adoption costs. Formally, there exists $\underline{\chi} > 0$ *such that if* $\max_{1 \leq i \leq m} \chi_i < \underline{\chi}$ *then a full blockchain adoption equilibrium arises under consumer beliefs CB2.*

Corollary 4.3.7 highlights that the conditions for full blockchain adoption differ qualitatively depending upon consumer beliefs. More precisely, sufficiently small adoption costs generate a full adoption equilibrium under consumer beliefs CB2, but even zero adoption costs may be insufficient to generate a full adoption equilibrium under consumer beliefs CB1.

Our results, thus, highlight the importance of consumer beliefs in generating blockchain adoption. We emphasize that economic fundamentals do not determine consumer beliefs, so consumer beliefs must come from elsewhere. Practically speaking, our results suggest that encouraging consumers towards particular beliefs (e.g., CB2) would help generate blockchain adoption in equilibrium.

4.3.3 Adoption Failures

This section examines if full blockchain adoption arises in equilibrium when such adoption improves global welfare. We find that adoption may not arise in equilibrium even when it significantly enhances global welfare. We refer to such a situation as an adoption failure. We propose a system of transfers to resolve this failure.

Proposition 4.3.8 (Adoption Failures)**.**

An adoption failure arises for sufficiently large product quality dispersion and sufficiently high adoption costs.

Proposition 4.3.8 establishes that adoption failures arise when product quality dispersion (i.e., $H - L$) and adoption costs (i.e., $\max_i \chi_i$) are both large. This result follows from Propositions 4.3.4, 4.3.5 and 4.3.6. Proposition 4.3.4 demonstrates that global welfare unambiguously increases in product quality dispersion whereas both Propositions 4.3.5 and 4.3.6 establish that full blockchain adoption requires low adoption costs.

Intuitively, Proposition 4.3.8 arises from a misalignment between control over the adoption decision and distribution of welfare generated from that decision. Recall that Proposition 4.3.1 establishes that consumers unambiguously benefit from blockchain adoption, and that benefit increases with product quality dispersion; moreover, this welfare enhancement is available to consumers for free. However, each manufacturer faces an adoption cost but none benefit from product quality dispersion, so a sufficiently large adoption cost drives manufacturers to not adopt blockchain. This non-adoption occurs even if product quality dispersion is sufficiently large so that enhanced consumer welfare from such adoption dwarfs manufacturer adoption costs, and also enhances global welfare.

We propose a system of transfers to resolve adoption failures. Our system of transfers corrects the aforementioned misalignment by transferring surplus welfare from consumers to manufacturers. Our preceding analysis assumes that each consumer benefits from blockchain adoption without paying any cost. Subsequently, we relax that assumption, requiring that consumer k gains access to information on the blockchain only if she pays $\kappa \geq 0$ for that access.[6] We redistribute these consumer payments to manufacturers such that manufacturer i receives transfer $t_i \geq 0$ if manufacturer i adopts blockchain. We require $\kappa = \sum_{i=1}^{m} t_i$ so that manufacturer transfers are fully funded from consumer payments, and no external financing is necessary.

We modify manufacturer i's problem to include the transfer payments as follows:

$$\max_{a_i} \mathbb{E}_i[s_i(a_i, a_{-i})] - \mathbb{E}_i\left[c\left(s_i(a_i, a_{-i})Y_i\right)\right] - \chi_i a_i + t_i a_i \tag{4.6}$$

[6]We do not to allow κ to vary by consumer because such an assumption raises both practical and technical concerns. From a practical perspective, such an assumption would require price discrimination. From a technical perspective, the total value of payments would not be well-defined in general.

And, we modify consumer k's problem to include a blockchain access cost as follows:

$$\max_{b_k} \mathbb{E}^{b_k}[\max_i \mathbb{E}_k[q_i] - \kappa b_k] \tag{4.7}$$

In this modified setting, consumer k selects her information environment, $b_k \in \{0, 1\}$, prior to selecting a manufacturer from which to purchase: $b_k = 1$ represents the consumer opting to gain access to the blockchain's information, while $b_k = 0$ represents the consumer opting to forgo access to the blockchain's information. To gain access to the blockchain's information, the consumer must pay cost κ. In return, the consumer receives more accurate signals from any manufacturer that adopts the blockchain. Formally, consumer k receives random signals $\{\tilde{q}_{ki}\}_{i=1}^m$ such that $\mathbb{P}^{b_k}(\tilde{q}_{ik} = q_i \mid q_i) = \alpha + \delta a_i b_k$.

Proposition 4.3.9 (Resolving Adoption Failures)**.**
Suppose manufacturer adoption costs are symmetric (*i.e.,* $\forall i, j : \chi_i = \chi_j$) *and blockchain enables strictly more accurate manufacturer quality signals* (*i.e.,* $\delta > 0$). *Then, for sufficiently large product quality dispersion, a system of transfers overcomes an adoption failure. The transfers are given by* $\kappa = \delta \cdot (1 - 0.5^{m-1}) \cdot (H - L)$ *and* $\forall i : t_i = \frac{\delta \cdot (1-0.5^{m-1}) \cdot (H-L)}{m}$.

Proposition 4.3.9 establishes that our proposed transfers resolve adoption failures. The transfers are set sufficiently large to provide manufacturers enough incremental welfare to offset manufacturer costs but also sufficiently small to preserve some welfare gains for consumers. The incremental welfare for manufacturers generates adoption because this welfare gain makes adopting blockchain incentive-compatible for manufacturers. Simultaneously, the preservation of a welfare gain for each consumer ensures that each consumer is willing to pay for the blockchain information environment which in turn finances the transfers to manufacturers.

Our work provides a concrete path to further blockchain adoption. In industries in which consumers would significantly benefit from information stored on the blockchain, consumers could be given access to some such information in return for financing blockchain adoption in that industry. Such a system could be implemented in a straightforward manner by a paid web application.

Chapter 5: Blockchain Adoption in a Supply Chain with Market Power

5.1 Introduction

This chapter examines the three-layer supply chain setting described in Section 1.3 in the context of monopoly and addresses both the transparency and traceability of blockchain. We examine whether blockchain adoption arises in equilibrium for a supply chain in which a single risk-averse monopolist manufacturer sells directly to consumers, and therefore, possesses market power. [1] The manufacturer purchases goods (or raw materials) from vendors and sells to consumers. The manufacturer possesses a type that is unknown to consumers. Each consumer also possesses a type that reflects her preference over the type of the manufacturer. While consumers do not know the manufacturer type, each consumer nonetheless receives an imperfect signal of the manufacturer's type. We assume that the manufacturer has the ability to set prices for both the consumers and the vendors. In this setting, we demonstrate that blockchain adoption always strictly improves manufacturer welfare if blockchain adoption involves no fixed cost of implementation. When the manufacturer bears the cost of blockchain adoption for the supply chain as a whole, we find that blockchain adoption arises in equilibrium if the adoption cost is sufficiently small. [2]

Our economic analysis consists of four stages, two corresponding to the consumer-manufacturer interaction, and two corresponding to the manufacturer-vendor interaction. In the first stage, the manufacturer determines whether or not to adopt blockchain, and also sets a consumer price. In the second stage, each consumer reacts to the consumer price, deciding whether to purchase the manufacturer's good or else select an outside option. The outside option reflects the consumer's

[1] Our results generalize when there are finitely many manufacturers so long as each manufacturer sells a good that is sufficiently differentiated from the goods sold by any other manufacturer so that each manufacturer maintains pricing power.

[2] Since the manufacturer possesses market power in our setting, she can shift costs to other supply chain participants endogenously. Thus, the specific assumption regarding who bears the adoption cost is not crucial; our formal analysis assumes the adoption cost is borne by the manufacturer.

opportunity cost. In the third stage, the manufacturer selects a set of vendors from which to fill consumer orders and simultaneously selects a price to offer each selected vendor. In the final stage, each vendor who received an offer from the manufacturer reacts by either accepting or rejecting the offer. Each such vendor also simultaneously selects an effort level, that determines the likelihood that vendor's batch of goods is defective. A defective batch must be recalled and all associated revenues forfeited.

The manufacturer receives all vendor goods subsequent to the fourth subgame. At this point the manufacturer may find some of the goods to be defective. If the manufacturer can trace each such detected defect to the respective defective batch, then the manufacturer recalls only the defective batches. In that case, the manufacturer refunds sales to the associated consumers and also requires that vendors who produced those defective batches refund their sales to the manufacturer. In contrast, if the manufacturer cannot trace detected defects and nonetheless discovers some defects, then the manufacturer recalls all goods because she cannot determine the specific batches that are defective. In this second case, all sales from vendors to the manufacturer and all sales from the manufacturer to consumers are refunded. Any goods that are not recalled are released to consumers.

Our main finding—that blockchain adoption enhances manufacturer welfare—is supplemented with several additional results that explain why the blockchain is helpful. The blockchain enters our analysis in two ways. First, the blockchain enables the manufacturer to trace a defective good to its batch. Accordingly, in the absence of blockchain, a single defective batch from a vendor causes a recall of all goods, whereas, in the presence of blockchain, it causes a recall of only that vendor's batch. Second, the blockchain improves the accuracy of consumer signals. This latter primitive reflects the fact that a blockchain can store various relevant details of the manufacturing process, leading to a reduction in consumer information asymmetry regarding the manufacturer's type. Indeed, this is one of the critical factors in explaining why blockchain adoption increases manufacturer welfare. A second critical factor is the manufacturer's aversion to risk, and how blockchain adoption enables the manufacturer to "avoid" the risk of recalling defective goods: with the blockchain, the manufacturer has the ability to trace any defective good to the vendor who supplied it, which enables the manufacturer to fully diversify away individual vendor recall risks; although the manufacturer still faces the same expected number of recalled goods, the distribution becomes degenerate so that she faces no recall risk. This reduction in risk then enhances the manufacturer's welfare because she is a risk-averse agent. Most of the results in this chapter are driven by both factors described above.

Elaborating on the theme of risk, it is important to note that without the blockchain, it is optimal for a risk-averse manufacturer to specialize and procure goods from a single vendor; purchasing from multiple vendors in the absence of blockchain strictly increases the number of recalled goods since the manufacturer must recall all goods when any vendor produces a defective batch. In contrast, when blockchain is adopted, the manufacturer recalls a batch of goods only if that batch is defective; thus, increasing the number of vendors from which the manufacturer purchases does not alter the expected number of recalled goods, but it does allow the manufacturer to fully diversify away the risk that each vendor produces a defective batch. More formally, in the presence of blockchain, the manufacturer orders an infinitesimal amount from each vendor; and, therefore, faces a deterministic number of recalled goods but no recall risk. Since the manufacturer is risk averse, this risk reduction enhances the manufacturer's welfare. Moreover, there is a further, indirect positive effect on manufacturer welfare: the risk reduction reduces the marginal cost of the manufacturer, which increases her equilibrium sales volume. Accordingly, when blockchain is adopted, the manufacturer optimally expands her sales volume (by adjusting her consumer price), which leads to an increase in her expected profit and her overall welfare.

We now briefly discuss how the reduction in consumer information asymmetry enhances manufacturer welfare. When the manufacturer adopts the blockchain, each consumer's signal is more likely to properly identify the manufacturer's type. Consequently, if a consumer receives a signal that aligns her type with the manufacturer, then she values the manufacturer's good more so than in the absence of blockchain because the signal is more informative than in the absence of blockchain. In turn, the manufacturer reacts by raising the price charged to the consumer, thereby extracting some of the increase in consumer surplus. We demonstrate that this consumer price increase leads to a higher expected profit and increased manufacturer welfare.

We also examine the effect of blockchain adoption upon vendor and consumer welfare. We find that blockchain adoption has an ambiguous effect upon vendor welfare but unambiguously increases consumer welfare. With regard to the former, blockchain adoption has an ambiguous effect upon the price paid to vendors by the manufacturer and thus also an ambiguous effect upon vendor welfare. With regard to the latter, blockchain adoption unambiguously increases consumer welfare because it improves the consumer information environment, thereby enabling consumers to improve decision-making.

This work is most closely related to Chapters 2 and 3, which also focus on the blockchain adoption decision. This work differs from those two chapters though in that we consider a setting in which the manufacturer possesses market power, whereas Chapters 2 and 3 study a setting in which the manufacturing sector is perfectly competitive.

5.2 Model

We consider a static model consisting of infinitely many vendors, one manufacturer, and a unit measure of consumers. At the outset of our model, the manufacturer selects $a \in \{0, 1\}$ where $a = 1$ denotes the manufacturer adopting blockchain and $a = 0$ denotes the manufacturer not adopting blockchain. Thereafter, the manufacturer sets prices for consumers, and consumers make purchasing decisions. The manufacturer fulfills consumer demand by supplying from vendors.

We put forth our model primitives in Section 5.2.1. We then specify equilibrium conditions for the subgames arising from blockchain non-adoption (i.e., $a = 0$) and blockchain adoption (i.e., $a = 1$) in Sections 5.2.2 and 5.2.3 respectively. We specify those subgames separately to emphasize how blockchain's presence affects the economic environment. Section 5.2.4 provides model solutions.

5.2.1 Model Primitives

Manufacturer

There exists a single manufacturer who possesses a type q which is equally likely to be A or B. The manufacturer type impacts consumer utility, but it is unknown to consumers. Our assumptions reflect situations in which consumers are poorly informed regarding the specific type of a manufacturer (see, e.g., Poole and Baron 1996).

We assume that the manufacturer is risk averse and values her profits according to the following utility function:

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma} \tag{5.1}$$

where $\gamma \in (0,1)$ denotes the level of risk aversion of the manufacturer. Note that $U(x)$ is a Constant Relative Risk Aversion (CRRA) utility function and therefore satisfies the standard regularity conditions (i.e., $U'(x) \geq 0$, $U''(x) \leq 0$ and $U(0) = 0$). Moreover, $\lim_{\gamma \to 0^+} U(x) = x$ so that the manufacturers becomes risk-neutral as $\gamma \to 0^+$.

Consumers

There exists a unit mass of consumers $k \in [0,1]$. Each consumer k has a two-dimensional type $(v_k, t_k) \in [0,H] \times \{A,B\}$ where $0 < H \leq 1$. We assume that v_k is drawn uniformly from the interval $[0,H]$, and t_k is A or B with equal probability. Moreover, the utility V_k of a consumer of type (v_k, t_k) from consuming a good from the manufacturer is given by:

$$V_k = v_k \cdot \Big(\mathcal{I}\left(q = t_k\right) - \mathcal{I}\left(q \neq t_k\right)\Big) \tag{5.2}$$

where $\mathcal{I}(\cdot)$ is the indicator function that takes the value 1 if the argument is true and the value 0 otherwise. Note that consumer (v_k, t_k) earns positive utility $V_k \geq 0$ if her type matches that of the manufacturer (i.e., $q = t_k$) and negative utility otherwise. Our model thus reflects the heterogeneous preferences of consumers in practice (see, e.g., Yiridoe et al. 2005 and Moser et al. 2011).

Although consumers do not know the manufacturer's type, each consumer has access to a signal which provides information regarding the type of the manufacturer. Specifically, consumer $k \in [0, 1]$ receives a random signal $\tilde{q}_k \in \{A, B\}$. In the absence of blockchain, this signal is generated according to the following probability law:

$$\mathbb{P}(\tilde{q}_k = \tilde{q} \mid q) = \begin{cases} \alpha & \text{if } \tilde{q} = q \\ 1 - \alpha & \text{if } \tilde{q} \neq q \end{cases} \tag{5.3}$$

with $\alpha \in [\frac{1}{2}, 1)$ so that a signal does not fully reveal the manufacturer's type but nonetheless allows each consumer to possess some imperfect information regarding the manufacturer's type.

The presence of the blockchain changes the information environment of the consumer: if the manufacturer adopts the blockchain, then the signal observed by any consumer $k \in [0, 1]$ reveals the manufacturer's type with a higher probability. More precisely, when blockchain is adopted, consumer k's signal, $\tilde{q}_k \in \{A, B\}$ is generated according to the following probability law:

$$\mathbb{P}(\tilde{q}_k = \tilde{q} \mid q) = \begin{cases} \alpha + \delta & \text{if } \tilde{q} = q \\ 1 - \alpha - \delta & \text{if } \tilde{q} \neq q \end{cases} \tag{5.4}$$

with $\delta \in (0, 1 - \alpha]$ representing the extent to which blockchain improves the accuracy of consumer signals. This modeling choice reflects the fact that the blockchain stores all relevant aspects of the manufacturing process and provides such information not only to manufacturers but also to consumers.

We allow that each consumer may forgo purchasing from the manufacturer and instead avail herself of an outside option which provides value zero. This outside option captures the opportunity cost of purchasing from the manufacturer, and could reflect, for example, the utility from another manufacturer that we do not explicitly model.

Vendors

There exist countably infinite homogeneous vendors indexed by $j \in \mathbb{N}_+ = \{1, 2, \ldots\}$. Vendors receive orders from the manufacturer to fulfill consumer demand. In turn, each vendor decides whether to accept the order and a level of effort to exert if the order is accepted. Effort is costly for the vendor, but it determines the probability that the vendor's batch is defective. Defective batches must be recalled, and neither the vendor nor the manufacturer earn revenues from recalled batches.

More formally, the batch produced by vendor j is assumed to be defective with probability $\rho(e_j) \in [0, 1]$, where $e_j \in [0, 1]$ denotes vendor j's effort choice. We let $\rho(e_j) = 1 - \sqrt{e_j}$, implying $\rho(e_j) \in [0, 1]$ for all e_j and $\rho' < 0$ so that a higher vendor effort induces a lower defect probability. Exerting effort e_j results in a per unit cost $\frac{e_j}{2}$ to vendor j.

We assume that the manufacturer cannot expediently trace a defective unit to its batch in the absence of blockchain. As a consequence, when the manufacturer does not adopt blockchain, she recalls all vendor batches if she discovers any defects. This action represents precautionary measure taken in practice. Moreover, it leads us to the second key difference from when blockchain is adopted: if the manufacturer adopts the blockchain, every defective unit can be traced to the appropriate vendor(s); in turn, the manufacturer recalls only the defective vendor batches when blockchain is adopted. Our assumptions are consistent with practice as blockchain has been shown to produce practically relevant reductions in tracing times.[3]

Timeline

Our model evolves over three periods indexed by $t = 0, 1, 2$. At the beginning of period 0, consumers and the manufacturer learn their own types, and the manufacturer decides whether to adopt the blockchain immediately thereafter. In the middle of period 0, the manufacturer sets her price for consumers. At the end of period 0, each consumer observes a signal from the manufacturer and then decides whether to purchase from the manufacturer or choose the outside option instead. These decisions determine the demand of the manufacturer, $s \in [0, 1]$, where s represents the

[3]See, e.g., `https://www.hyperledger.org/learn/publications/walmart-case-study` which demonstrates a reduction of tracing times from 7 days to 2.2 seconds.

measure of consumers purchasing from the manufacturer and $1 - s$ represents the measure of consumers selecting the outside option.

At the beginning of period 1, the manufacturer observes the consumer demand and places orders with vendors to fulfill that demand. More precisely, the manufacturer selects a subset of vendors, equally divides the demand over the chosen subset, and offers each of the chosen vendors the same price. Since the vendors are homogeneous, choosing a subset of vendors is equivalent to choosing the number of vendors n, and we can, without loss of generality, assume that the set of vendors is $\{1, \dots, n\}$. Thereafter, each vendor that receives an offer chooses whether to accept the offer, and then selects an effort level. Each vendor that receives and accepts the offer produces the goods and sends them to the manufacturer. This completes period 1.

We assume that defects, if present, are detected at the beginning of period 2. Recall though that defective units cannot be traced expediently in the absence of blockchain. As a consequence, all batches are recalled if there exists any defect in the absence of blockchain, whereas only defective batches are recalled if blockchain were adopted. Then, at the end of period 2, all units that are not recalled are released to consumers, and pay-offs realize. Any consumer not having her order filled due to a recall receives a full refund and incurs no utility. Moreover, vendors and the manufacturer earn revenue only on goods that were not recalled.

5.2.2 Subgame Arising from Blockchain Non-Adoption (i.e., $a = 0$)

Since vendors are homogeneous, we restrict ourselves to examining equilibria in which vendors act symmetrically. Accordingly, if the manufacturer makes vendor j an offer and vendor j accepts and exerts effort e_j then vendor j's per unit expected profit is given by:

$$\mathcal{V}(\Psi, n, e_j, e_{-j}) := \underbrace{(1 - \rho(e_{-j}))^{n-1} \cdot (1 - \rho(e_j)) \cdot \Psi}_{\text{Per Unit Expected Revenue}} - \underbrace{\frac{e_j}{2}}_{\text{Per Unit Cost}} \tag{5.5}$$

where $\Psi \geq 0$ denotes the price offered by the manufacturer, $n \in \overline{\mathbb{N}}_+$ denotes the number of vendors that receive an offer from the manufacturer, and $e_{-j} \in [0, 1]$ denotes the symmetric effort choice of all other vendors. Note that $(1-\rho(e_{-j}))^{n-1}\cdot(1-\rho(e_j))$ corresponds to the probability that vendor j's batch is not recalled. This follows because, in the absence of blockchain, the manufacturer cannot trace defects to the offending vendor and thus recalls all batches when any batch is defective.

We assume that a vendor receives zero pay-off if she rejects the manufacturer's offer. Since $\mathcal{V}(\Psi, n, 0, e_{-j}) = 0$ for all $e_{-j} \in [0,1]$, we can, without loss of generality, assume that each vendor receiving an offer from the manufacturer accepts that offer. Thus, the effort choice for each vendor $e_j^*(\Psi, n) = e^*(\Psi, n) \in [0,1]$ is given as the solution for the following fixed-point problem:

$$e^*(\Psi, n) := \underset{e_j\in[0,1]}{\arg\max}\, \mathcal{V}(\Psi, n, e_j, e^*(\Psi, n)). \tag{5.6}$$

The manufacturer anticipates vendor effort choices. Therefore, independence of vendor defects implies that the manufacturer's expected utility is given by:

$$\mathcal{M}(P, s, \Psi, n) = \underbrace{\Big(1-\rho(e^*(\Psi, n))\Big)^n}_{\text{No Defect Probability}} \times \underbrace{U\Big((P-\Psi)\cdot s\Big)}_{\text{Utility If No Defect}} \tag{5.7}$$

where $s \geq 0$ denotes consumer demand for the manufacturer and $U(x)$ is given by (5.1).

Recall that the manufacturer first sets the price P she charges consumers. Then, observing consumer demand, she selects the number of vendors n to transact with, and the price she offers those vendors. Therefore, the number of vendors $n^*(P)$ and the price $\Psi^*(P)$ she offers these vendors are given by:

$$(\Psi^*(P), n^*(P)) := \underset{\Psi\geq 0, n\in\overline{\mathbb{N}}_+}{\arg\max}\ \mathcal{M}(P, s, \Psi, n) = \underset{\Psi\geq 0, n\in\overline{\mathbb{N}}_+}{\arg\max}\ \mathcal{M}(P, 1, \Psi, n) \tag{5.8}$$

where her choices are optimal conditional on all relevant and available information and thus depend upon the price she previously set for the consumer, $P \geq 0$. The equality in (5.8) follows from the fact that for CRRA functions U we have that $\underset{\Psi\geq 0, n\in\overline{\mathbb{N}}_+}{\arg\max}\ \mathcal{M}(P, 1, \Psi, n) = \underset{\Psi\geq 0, n\in\overline{\mathbb{N}}_+}{\arg\max}\ \mathcal{M}(P, s, \Psi, n)$ holds for all $s > 0$. Thus, it follows that optimal $(\Psi^*(P), n^*(P))$ are independent of the consumer demand s.

The consumer base size s is determined endogenously. Specifically, consumers make purchase decisions based on the manufacturer prices and observed signals. Recall that consumer $k \in [0, 1]$ receives utility V_k if she purchases from the manufacturer but her knowledge of V_k is imperfect and depends upon a signal, $\tilde{q}_k$ along with her own type, (v_k, t_k). Consequently, her expected pay-off for purchasing from the manufacturer is:

$$C(P, v_k, t_k, \tilde{q}_k) = \underbrace{\Big(1 - \rho(e^*(P))\Big)^{n^*(P)}}_{\text{No Defect Probability}} \times \underbrace{\mathbb{E}[V_k - P \mid v_k, t_k, \tilde{q}_k]}_{\text{Conditional Expected Utility If No Defect}} \tag{5.9}$$

where the expectation is taken over the manufacturer's type q and

$$e^*(P) := e^*(\Psi^*(P), n^*(P)). \tag{5.10}$$

Recall that the consumer possesses an outside option which we normalize to zero. Consequently, consumer k purchases from the manufacturer if and only if $C(P, v_k, t_k, \tilde{q}_k) > 0$, and thus the equilibrium consumer demand for the manufacturer, $s^*(P)$, is given by:

$$s^*(P) := \mu(\{k : C(P, v_k, t_k, \tilde{q}_k) > 0\}) \tag{5.11}$$

with $\mu(S)$ denoting the measure of a set $S \subseteq [0, 1]$ of consumers.

Finally, the manufacturer anticipates all future actions and therefore sets her price for consumers, P^*, according to:

$$P^* = \underset{P \geq 0}{\arg\max}\ \mathcal{M}(P) \tag{5.12}$$

where

$$\mathcal{M}(P) := \mathcal{M}(P, s^*(P), \Psi^*(P), n^*(P)). \tag{5.13}$$

Note that (5.6), (5.8), (5.11) and (5.12) provide equilibrium conditions for optimal behavior conditional on arbitrary behavior beforehand. More formally, the solution to these equations correspond to a subgame perfect Nash equilibrium in which agents behave optimally in a subgame even if that subgame is not on the equilibrium path of play. To ease the discussion of our results in Section 5.3, we denote all actions on the equilibrium path of play when blockchain is not adopted as follows:

$$e^* := e^*(P^*), \qquad \Psi^* := \Psi^*(P^*), \qquad n^* := n^*(P^*), \qquad s^* := s^*(P^*). \tag{5.14}$$

5.2.3 Subgame Arising from Blockchain Adoption (i.e., $a = 1$)

Since the blockchain enables the manufacturer to trace defects to the offending vendor, a generic vendor's objective function is not given by (5.5) when adoption occurs. Rather, vendor j's per unit expected profit is given by:

$$\mathcal{V}^{\mathcal{B}}(\Psi, n, e_j, e_{-j}) = (1 - \rho(e_j)) \cdot \Psi - \frac{e_j}{2} \tag{5.15}$$

where $\rho(e_j)$ corresponds to both the probability that vendor j's batch is defective and the probability that vendor j's batch is recalled. Recall that blockchain enables the manufacturer to trace defects to the offending vendor so that vendor j's batch is recalled if and only if it is defective. In turn, the symmetric vendor effort choice, $e^{**}(\Psi, n) \in [0, 1]$, is given by:

$$e^{**}(\Psi, n) := \underset{e_j \in [0,1]}{\arg\max}\, \mathcal{V}^{\mathcal{B}}(\Psi, n, e_j, e^{**}(\Psi, n)). \tag{5.16}$$

Note that we use the superscript ** to denote choices in the subgame arising from blockchain being adopted. This distinguishes such choices from those in the subgame arising from blockchain not being adopted. Recall, from Section 5.2.2, that we designated the latter with a superscript *.

Blockchain adoption entails an implementation cost, $\chi > 0$. Accordingly, when blockchain is adopted, the manufacturer's expected utility is given by:

$$\mathcal{M}^{\mathcal{B}}(P, s, \Psi, n) = \mathbb{E}[U\Big((P - \Psi) \cdot \frac{s}{n} \cdot N_s\Big)] - \chi \tag{5.17}$$

with N_s denoting the number of batches that were not defective. Note the random variable N_s that defines the demand when $a = 1$ satisfies $N_s \sim \text{Binomial}(n, 1 - \rho(e^{**}(\Psi)))$ with $e^{**}(\Psi) := e^{**}(\Psi, 1) = e^{**}(\Psi, n)$, where the equality follows from (5.16), and the definition of $\mathcal{V}^{\mathcal{B}}$ in (5.15).

In turn, the number of vendors $n^{**}(P)$ who receive an offer, and the offered price $\Psi^{**}(P)$ are given by:

$$(\Psi^{**}(P), n^{**}(P)) := \underset{\Psi \geq 0, n \in \overline{\mathbb{N}}_+}{\arg\max} \mathcal{M}^{\mathcal{B}}(P, s, \Psi, n) = \underset{\Psi \geq 0, n \in \overline{\mathbb{N}}_+}{\arg\max} \mathcal{M}^{\mathcal{B}}(P, 1, \Psi, n) \tag{5.18}$$

where the last equality follows for all s because the manufacturer utility U is CRRA.

Recall that blockchain adoption affects consumer demand by improving the accuracy of consumer signals (see (5.4)). Accordingly, consumer k's expected utility of purchasing from the manufacturer is:

$$C^{\mathcal{B}}(P, v_k, t_k, \tilde{q}_k) = \Big(1 - \rho(e^{**}(P))\Big) \times \mathbb{E}[V_k - P \mid v_k, t_k, \tilde{q}_k] \tag{5.19}$$

where the expectation is taken over the manufacturer's type q and

$$e^{**}(P) := e^{**}(\Psi^{**}(P), n^{**}(P)). \tag{5.20}$$

Consumer k purchases from a manufacturer if and only if $C^{\mathcal{B}}(P, v_k, t_k, \tilde{q}_k) > 0$. Consequently, consumer demand for the manufacturer, $s^{**}(P)$, is given by:

$$s^{**}(P) = \mu(\{k : C^{\mathcal{B}}(P, v_k, t_k, \tilde{q}_k) > 0\}). \tag{5.21}$$

The manufacturer anticipates all future actions when setting her price for consumers and deciding whether to adopt blockchain. Consequently, the consumer price when blockchain is adopted, P^{**}, is given explicitly by:

$$P^{**} = \arg\max_{P \geq 0} \mathcal{M}^{\mathcal{B}}(P) \tag{5.22}$$

where

$$\mathcal{M}^{\mathcal{B}}(P) := \mathcal{M}^{\mathcal{B}}(P, s^{**}(P), \Psi^{**}(P), n^{**}(P)). \tag{5.23}$$

Note that (5.16), (5.18), (5.21) and (5.22) provide equilibrium conditions for optimal behavior conditional on arbitrary behavior beforehand. More formally, the solution to these equations correspond to a subgame perfect Nash equilibrium in which agents behave optimally in a subgame even if that subgame is not on the equilibrium path of play. To ease the discussion of our results in Section 5.3, we denote all actions on the equilibrium path of play if blockchain is adopted as follows:

$$e^{**} := e^{**}(P^{**}), \qquad \Psi^{**} := \Psi^{**}(P^{**}), \qquad n^{**} := n^{**}(P^{**}), \qquad s^{**} := s^{**}(P^{**}). \tag{5.24}$$

5.2.4 Model Solution

When blockchain is not adopted, the subgame perfect Nash equilibrium is given explicitly as follows:

Proposition 5.2.1 (Solution when Blockchain is not Adopted)**.**

I. Each vendor's effort choice is given as follows:

$$e^{*}(\Psi, n) = \begin{cases} \Psi^2 & \textit{if } \Psi < 1 \textit{ and } n = 1 \\ 0 & \textit{if } \Psi < 1 \textit{ and } n > 1 \\ 1 & \textit{if } \Psi \geq 1 \textit{ and } n = 1 \\ 1 \textit{ or } 0 & \textit{if } \Psi \geq 1 \textit{ and } n = 2 \\ 1 \textit{ or } \Psi^{-\frac{2}{n-2}} \textit{ or } 0 & \textit{if } \Psi \geq 1 \textit{ and } n > 2 \end{cases}$$

II. The number of vendors from which the manufacturer buys, and the price that she offers those vendors, are given as follows:

- $\Psi^*(P) = \begin{cases} \frac{P}{2-\gamma} & \textit{if } P < 2-\gamma \\ 1 & \textit{if } P \geq 2-\gamma \end{cases}$
- $n^*(P) = 1$

III. Consumer demand for the manufacturer is given by:

$$s^*(P) = \begin{cases} \frac{1}{2}\left(1 - \frac{P}{(2\alpha-1)H}\right) & \textit{if } 1 - \frac{P}{(2\alpha-1)H} \geq 0 \\ 0 & \textit{if } 1 - \frac{P}{(2\alpha-1)H} < 0 \end{cases}$$

IV. The manufacturer price for consumers is given as follows:

$P^* = \frac{2-\gamma}{3-2\gamma}(2\alpha - 1)H$

In turn, Corollary 5.2.2 provides all solutions on the path of play if blockchain is not adopted:

Corollary 5.2.2 (Equilibrium Actions in the Absence of Blockchain)**.**

I. Each vendor's effort choice is given by $e^* = \left(\frac{(2\alpha-1)H}{3-2\gamma}\right)^2$.

II. The number of vendors from which the manufacturer buys, and the price that she offers those vendors, are given by $\Psi^* = \frac{(2\alpha-1)H}{3-2\gamma}$ *and* $n^* = 1$.

III. Consumer demand for the manufacturer is given by $s^* = \frac{1-\gamma}{6-4\gamma}$.

IV. The manufacturer price for consumers is given by $P^* = \frac{2-\gamma}{3-2\gamma}(2\alpha - 1)H$.

Equilibrium solutions differ when blockchain is adopted. In particular, the subgame perfect Nash equilibrium, when blockchain is adopted, is given as follows:

Proposition 5.2.3 (Solution when Blockchain is Adopted)**.**

I. Each vendor's effort choice is given as follows:

$$e^{**}(\Psi, n) = \begin{cases} \Psi^2 & \text{if } \Psi < 1 \\ 1 & \text{if } \Psi \geq 1 \end{cases}$$

II. The number of vendors from which the manufacturer buys and the price that she offers those vendors are given as follows:

- $\Psi^{**}(P) = \begin{cases} \frac{P}{2} & \text{if } P < 2 \\ 1 & \text{if } P \geq 2 \end{cases}$
- $n^{**}(P) = \infty$

III. Consumer demand for the manufacturer is given by:

$$s^{**}(P) = \begin{cases} \frac{1}{2}\left(1 - \frac{P}{(2(\alpha+\delta)-1)H}\right) & \text{if } 1 - \frac{P}{(2(\alpha+\delta)-1)H} \geq 0 \\ 0 & \text{if } 1 - \frac{P}{(2(\alpha+\delta)-1)H} < 0 \end{cases}$$

IV. The manufacturer price for consumers is given as follows:

$P^{**} = \frac{2}{3}(2(\alpha + \delta) - 1)H$

As a consequence, Corollary 5.2.4 provides solutions on the path of play when blockchain is adopted:

Corollary 5.2.4 (Equilibrium Actions when Blockchain is Adopted)**.**

I. Each vendor's effort choice is given by $e^{**} = \left(\frac{(2(\alpha+\delta)-1)H}{3}\right)^2$.

II. *The number of vendors from which the manufacturer buys, and the price that she offers those vendors, are given by* $\Psi^{**} = \frac{(2(\alpha+\delta)-1)H}{3}$ *and* $n^{**} = \infty$.

III. *Consumer demand for the manufacturer is given by* $s^{**} = \frac{1}{6}$.

IV. *The manufacturer price for consumers is given by* $P^{**} = \frac{2}{3}(2(\alpha + \delta) - 1)H$.

The manufacturer anticipates outcomes arising from her adoption decision so that she implements blockchain if and only if it enhances her overall utility:

$$a = 1 \Longleftrightarrow \mathcal{M}^{\mathcal{B}}(P^{**}) \geq \mathcal{M}(P^{*}). \tag{5.25}$$

Let Π^{**} denote the profit of the manufacturer (excluding the adoption cost) when she adopts blockchain and Π^{*} denote the profit of the manufacturer when she does not adopt blockchain. Then, the latter condition is equivalent to requiring that the incremental utility $\Omega := \mathbb{E}[U(\Pi^{**})] - \mathbb{E}[U(\Pi^{*})]$ associated with manufacturer profits exceeds the manufacturer's adoption cost, χ. More formally, the following equivalence holds:

$$\mathcal{M}^{\mathcal{B}}(P^{**}) \geq \mathcal{M}(P^{*}) \Longleftrightarrow \Omega \geq \chi \tag{5.26}$$

and thus we have the following result:

Proposition 5.2.5 (Endogenous Adoption Decision)**.**
In equilibrium, the manufacturer's adoption decision is given as follows:

$$a = \mathcal{I}\Big(\Omega \geq \chi\Big),$$

where the incremental utility $\Omega := \mathbb{E}[U(\Pi^{**})] - \mathbb{E}[U(\Pi^{*})]$ *is given explicitly by:*

$$\Omega = \frac{[(2(\alpha+\delta)-1)H]^{2-2\gamma}}{(1-\gamma)54^{1-\gamma}} - \frac{(1-\gamma)^{1-2\gamma}}{2^{1-\gamma}(3-2\gamma)^{3-2\gamma}}[(2\alpha-1)H]^{2-\gamma}.$$

5.3 Blockchain Adoption and Manufacturer Welfare

Our first result establishes that blockchain adoption always increases manufacturer welfare when adoption costs are zero. As a consequence, blockchain adoption is generic for sufficiently small adoption costs:

Proposition 5.3.1 (Blockchain Adoption in Equilibrium)**.**
The manufacturer's incremental utility from blockchain adoption due to a change in profits, $\Omega := \mathbb{E}[U(\Pi^{**})] - \mathbb{E}[U(\Pi^{*})]$*, is always strictly positive* (*i.e.,* $\Omega > 0$)*. Consequently, blockchain adoption arises in equilibrium for sufficiently small adoption costs, i.e.,* $a = 1$ *when* $0 < \chi \leq \Omega$*.*

We emphasize that the finding of Proposition 5.3.1, $\Omega > 0$, does not apply to arbitrary market settings. More precisely, two results in Chapters 2 and 3 demonstrate that $\Omega = 0$ when the manufacturing sector is competitive, and thus, blockchain adoption does not arise in equilibrium in that setting. To better understand how blockchain adoption arises in our setting, define

$$\Phi := U(\mathbb{E}[\Pi^{**}]) - U(\mathbb{E}[\Pi^{*}]). \tag{5.27}$$

Recall that Π^{*} denotes the manufacturer profit in the absence of blockchain and Π^{**} denotes the manufacturer profit (excluding the adoption cost) in the presence of blockchain. Thus, Φ corresponds to the incremental utility gain due to endogenous changes in the manufacturer's <u>expected</u> profit. In Section 5.3.2, we establish that $\Phi > 0$. Let

$$\Sigma := \Omega - \Phi. \tag{5.28}$$

Then, by definition, $\Phi + \Sigma = \Omega$, and Σ corresponds to incremental utility gain due to the reduction in the risk of the manufacturer's random profit. In Section 5.3.1 we establish that $\Sigma > 0$. Thus, establishing that the incremental gain Ω results from a strictly positive increase in the manufacturer's expected profit and a strictly positive reduction in the manufacturer's risk. We also discuss the underlying economic channels that generate Σ and Φ.

5.3.1 Manufacturer Profit Risk Reduction Σ

The information on the blockchain results in the manufacturer endogenously diversifying across all vendors, and thereby, driving the variance of the profit to zero. Since the manufacturer is risk averse, the aforementioned reduced risk increases her utility and thereby enables blockchain adoption for sufficiently small adoption costs. We formalize this insight in the following result:

Proposition 5.3.2 (Manufacturer Diversification Benefits)**.**
Let $\Phi := U(\mathbb{E}[\Pi^{**}]) - U(\mathbb{E}[\Pi^{*}])$ *and* $\Sigma := \Omega - \Phi$*. Then the following results hold:*

1. *Blockchain Endogenously Generates Vendor Diversification*

   $n^{**} = \infty > n^{*} = 1.$

2. *Blockchain Endogenously Generates Reduction In Profit Variance*

   $Var[\Pi^{**}] = 0 < Var[\Pi^{*}].$

3. *Diversification Benefits Are Always Strictly Positive But Vanish Without Risk Aversion*

   $\Sigma > 0$ *but* $\lim_{\gamma \to 0^{+}} \Sigma = 0.$

Proposition 5.3.2.1 establishes that the presence of the blockchain results in the manufacturer changing from purchasing from a single vendor (i.e., $n^{*} = 1$) to purchasing from infinitely many vendors (i.e., $n^{**} = \infty$). This effect arises endogenously because the blockchain enables the manufacturer to trace a defective item to the responsible vendor, thereby, allowing a targeted recall of only defective items. Without blockchain, the manufacturer cannot trace defective items and thus recalls all items whenever a single defect is detected. As a consequence, without blockchain, the manufacturer endogenously specializes to a single vendor ($n^{*} = 1$) in order to reduce the recall probability. In contrast, with blockchain, only defective items are recalled even when the manufacturer purchases from multiple vendors. Thus, since purchasing from many vendors does not increase recall probabilities but does reduce risk, the manufacturer optimally diversifies away all recall risks by splitting her order over infinitely many vendors (i.e., $n^{**} = \infty$).

Since consumers are infinitesimal, the manufacturer faces no aggregate risk from consumer demand. Rather, the manufacturer's profit variance arises entirely from the risk of vendor defects, which implies a distribution for the number of recalled goods, and thus, a distribution for profit. Since, the manufacturer splits her order over infinitely many vendors when blockchain is adopted, the distribution of defective goods received from vendors becomes degenerate, and thus, so too does the distribution of the number of recalled goods. In turn, in the presence of blockchain, the manufacturer profit also becomes degenerate, and she therefore incurs no profit variance (i.e., $Var[\Pi^{**}] = 0$). In contrast, without blockchain, the manufacturer orders from a single vendor, and she is, therefore, exposed to the risk that her chosen vendor sends her a defective batch (i.e., $Var[\Pi^{*}] > 0$). Proposition 5.3.2.2 formalizes this.

The manufacturer always gains from the discussed risk reduction (i.e., $\Sigma > 0$) because the manufacturer is risk averse (i.e., $\gamma > 0$). In fact, the manufacturer's gain due to risk reduction vanishes without risk aversion (i.e., $\lim_{\gamma \to 0^+} \Sigma = 0$), as demonstrated by Proposition 5.3.2.3.

5.3.2 Manufacturer Expected Profit Gains Φ

Blockchain adoption not only reduces the manufacturer's risk but also increases the manufacturer's expected profit. The increase in expected profits arises endogenously due to the manufacturer's risk aversion (i.e., $\gamma > 0$) and the blockchain's ability to ameliorate consumer information asymmetry (i.e., $\delta > 0$). We clarify these findings with our next result:

Proposition 5.3.3 (Expected Profit Gains)**.**
Let $\Phi := U(\mathbb{E}[\Pi^{**}]) - U(\mathbb{E}[\Pi^{*}])$. *Then, the following results hold:*

1. *Blockchain Endogenously Enhances Expected Profits*

   $\mathbb{E}[\Pi^{**}] > \mathbb{E}[\Pi^{*}]$ *and thus* $\Phi > 0$.

2. *Expected Profit Gains Vanish Without Risk Aversion And Without Information Asymmetry Reduction*

   $\lim_{\gamma,\delta\to 0^+} \frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]} = 1$ *and thus* $\lim_{\gamma,\delta\to 0^+} \Phi = 0.$

3. *Expected Profit Gains Do Not Vanish With Risk Aversion But Without Information Asymmetry Reduction*

   For fixed $\gamma > 0 : \lim_{\delta\to 0^+} \frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]} > 1$ *and thus* $\lim_{\delta\to 0^+} \Phi > 0.$

4. *Expected Profit Gains Do Not Vanish Without Risk Aversion But With Information Asymmetry Reduction*

   For fixed $\delta > 0 : \lim_{\gamma\to 0^+} \frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]} > 1$ *and thus* $\lim_{\gamma\to 0^+} \Phi > 0.$

Proposition 5.3.3.1 establishes that the manufacturer's expected profit always increases with blockchain adoption so that $\Phi > 0$. This increase in expected profits under blockchain adoption arises due to two factors: the manufacturer's risk aversion (i.e., $\gamma > 0$), and the blockchain reducing information asymmetry (i.e., $\delta > 0$). Propositions 5.3.3.2 - 5.3.3.4 formalize that point, establishing that the two factors are jointly necessary and separately sufficient for blockchain adoption to generate increased expected profit for the manufacturer.

The following result further elaborates on how the manufacturer's risk aversion $\gamma > 0$, and the information asymmetry reduction $\delta > 0$ interact to generate increased expected profits from blockchain adoption.

Proposition 5.3.4 (Determinants of Increased Expected Profits)**.**
The manufacturer's expected profit in the presence of blockchain is given as follows:

$$\mathbb{E}[\Pi^{**}] = f_{\Pi}(\gamma) \cdot g_{\Pi}(\alpha, \delta) \cdot \mathbb{E}[\Pi^{*}]$$

where $f_{\Pi}(\gamma)$ *and* $g_{\Pi}(\alpha, \delta)$ *separate the effect of manufacturer risk aversion,* $\gamma > 0$*, from that of the information asymmetry reduction,* $\delta > 0$*. More explicitly:*

$$f_{\Pi}(\gamma) = \frac{1}{27}\frac{(3-2\gamma)^3}{(1-\gamma)^2}, \qquad g_{\Pi}(\alpha, \delta) = (1 + \frac{2\delta}{2\alpha - 1})^2.$$

Moreover, the following results hold:

1. *Manufacturer Risk Aversion Puts Upward Pressure On Expected Profits*

   $f_\Pi(\gamma) > 1$ *and* $\frac{df_\Pi}{d\gamma} > 0$.

2. *Information Asymmetry Reduction Puts Upward Pressure On Expected Profits*

   $g_\Pi(\alpha, \delta) > 1$ *and* $\frac{\partial g_\Pi}{\partial \delta} > 0$.

Proposition 5.3.4 establishes that the proportional change in the manufacturer's expected profits with and without blockchain, $\frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]}$, decomposes into two multiplicative terms: $f_\Pi(\gamma)$ and $g_\Pi(\alpha, \delta)$. Those two multiplicative terms separate the effects of the manufacturer's risk aversion γ and the information asymmetry reduction δ. Note that $f_\Pi(\gamma) \cdot g_\Pi(\alpha, \delta) > 1$ is equivalent to $\mathbb{E}[\Pi^{**}] > \mathbb{E}[\Pi^{*}]$ so that $f_\Pi(\gamma) \cdot g_\Pi(\alpha, \delta) > 1$ implies that blockchain adoption increases expected profits. Next, we consider the study the effects of the manufacturer's risk aversion γ and the information asymmetry reduction δ separately. Since $f_\Pi(0) = g_\Pi(\alpha, 0) = 1$, $\mathbb{E}[\Pi^{**}] = f_\Pi(\gamma) \cdot \mathbb{E}[\Pi^{*}]$ when there is no information asymmetry reduction $\delta = 0$, and $\mathbb{E}[\Pi^{**}] = g_\Pi(\alpha, \delta) \cdot \mathbb{E}[\Pi^{*}]$ when the manufacturer is risk neutral i.e., $\gamma = 0$. Consequently, in examining each factor separately, we say that the manufacturer's risk aversion puts upward (downward) pressure on expected profits in the presence of blockchain if $f_\Pi(\gamma) > 1$ ($f_\Pi(\gamma) < 1$). Similarly, we say that the reduction in information asymmetry puts upward (downward) pressure on expected profits in the presence of blockchain if $g(\alpha, \delta) > 1$ ($g(\alpha, \delta) < 1$). In that context, Proposition 5.3.4 establishes that both the manufacturer's risk aversion, γ, and the reduction in information asymmetry, δ, put upward pressure on expected profits in the presence of blockchain. Additionally, this upward pressure augments as manufacturer risk aversion increases (i.e., $\frac{df_\Pi}{d\gamma} > 0$) and as information asymmetry reduction increases (i.e., $\frac{\partial g_\Pi}{\partial \delta} > 0$).

While the manufacturer's risk aversion and the reduction of information asymmetry both lead to higher expected profits for the manufacturer when blockchain is adopted, these channels operate in qualitatively different ways. We clarify that point with the following result:

Proposition 5.3.5 (Consumer Demand and Consumer Prices)**.**

Consumer demand in the presence of blockchain is given as follows:

$$s^{**} = f_s(\gamma) \cdot s^*, \quad \textit{with } f_s(\gamma) = \frac{3-2\gamma}{3-3\gamma}.$$

Additionally, the consumer price in the presence of blockchain is given as follows:

$$P^{**} = f_P(\gamma) \cdot g_P(\alpha, \delta) \cdot P^*$$

where $f_P(\gamma)$ and $g_P(\alpha, \delta)$ separate the effect of manufacturer risk aversion, $\gamma > 0$, from that of the information asymmetry reduction, $\delta > 0$. More explicitly:

$$f_P(\gamma) = \frac{6-4\gamma}{6-3\gamma}, \qquad g_P(\alpha, \delta) = 1 + \frac{2\delta}{2\alpha - 1}.$$

Moreover, the following results hold:

1. *Blockchain Adoption Increases Demand But Has An Ambiguous Effect On Prices*

   For all $\gamma, \delta : s^{**} > s^*$ but there exist $\gamma, \delta : P^{**} < P^*$ and there exist $\gamma, \delta : P^{**} > P^*$.

2. *Risk Aversion Pressures Demand Upward But Prices Downward*

   $f_s(\gamma) > 1$ and $\frac{df_s}{d\gamma} > 0$, but $f_P(\gamma) < 1$ and $\frac{df_P}{d\gamma} < 0$.

3. *Information Asymmetry Reduction Pressures Prices Upward*

   For all $\alpha : g_P(\alpha, \delta) > 1$ and $\frac{\partial g_P}{\partial \delta} > 0$, but $\frac{s^{**}}{s^*}$ is independent of δ.

Proposition 5.3.5.1 establishes that blockchain adoption increases consumer demand (i.e., $s^{**} > s^*$) but has an ambiguous effect on the consumer price (i.e., there exist $\gamma, \delta : P^{**} < P^*$ yet there exist $\gamma, \delta : P^{**} > P^*$). Propositions 5.3.5.2 and 5.3.5.3 clarify these findings, explaining how the manufacturer's risk aversion and the information asymmetry reduction generate the results. In particular, the manufacturer's risk aversion puts upward pressure on consumer demand when blockchain is adopted (i.e., $f_s(\gamma) > 1$) while information asymmetry reduction does not affect consumer demand when blockchain is adopted (i.e., $\frac{s^{**}}{s^*}$ does not depend upon δ); thus, as noted, blockchain adoption unambiguously increases consumer demand. In contrast, manufacturer risk aversion and information asymmetry reduction have opposing effects on the consumer price, which explains the aforementioned ambiguous effect of blockchain adoption on the consumer price.

The impact of the risk aversion effect is mediated by the risk associated with a recall and, when the manufacturer is exposed to such risk, she compensates by selling to fewer consumers, which reduces her overall profit risk. Recall that, as discussed in Section 5.3.1, the manufacturer faces a recall risk only in the absence of blockchain because the blockchain enables the manufacturer to achieve zero profit variance. Consequently, by removing the recall risk, blockchain adoption leads the manufacturer to increase her sales volume, which equals consumer demand in equilibrium. Since the manufacturer has market power, the manufacturer's consumer price determines consumer demand. In turn, a higher consumer demand necessarily entails a lower consumer price. Thus, as per Proposition 5.3.5.2, risk aversion puts upward pressure on consumer demand but downward pressure on the consumer price.

The reduction in information asymmetry increases demand because it increases each consumer's utility from purchasing the manufacturer's good. The manufacturer reacts to this increased consumer utility by raising the consumer price to extract some of the consumer's increased utility. Note that, since the increased consumer utility results in an upward shift in the consumer demand curve, an increase in the consumer price need not accompany a decrease in consumer demand. More formally, from Proposition 5.2.3, the endogenous consumer demand curve in the presence of blockchain is given as follows:

$$s^{**}(P) = \frac{1}{2}\Big(1 - \frac{P}{(2(\alpha + \delta) - 1)H}\Big). \tag{5.29}$$

Note that a reduction in information asymmetry shifts the entire consumer demand curve upward (i.e., for all $P : \frac{\partial s^{**}}{\partial \delta} > 0$). Thus, it is possible for blockchain adoption to generate a higher consumer price without decreasing consumer demand. In fact, when blockchain is adopted, our results highlight that the partial effect of the information asymmetry reduction upon the manufacturer is that the manufacturer increases her consumer price to the exact level of consumer demand without blockchain (i.e., $\frac{s^{**}}{s^{*}}$ does not depend upon δ, but $g_P(\alpha, \delta) > 1$).

The described effects for the manufacturer's risk aversion and the reduction in information asymmetry become more pronounced if the manufacturer is more risk averse or if the reduction in information asymmetry is larger respectively. More formally, Proposition 5.3.5.2 demonstrates that higher risk aversion levels generate higher upward pressure on consumer demand when blockchain is adopted (i.e., $\frac{df_s}{d\gamma} > 0$) and higher downward pressure on the consumer price when blockchain is adopted (i.e., $\frac{df_P}{d\gamma} < 0$). Additionally, Proposition 5.3.5.3 establishes that a higher information asymmetry reduction leads to higher upward pressure on the consumer price when blockchain is adopted (i.e., $\frac{\partial g_P}{\partial \delta} > 0$).

5.4 Vendor and Consumer Welfare

Both vendors and consumers are affected by blockchain adoption. We discuss the welfare implications for each in Sections 5.4.1 and 5.4.2 respectively.

5.4.1 Vendor Welfare

We find that blockchain adoption has an ambiguous effect upon vendor welfare. More formally, we define W_V^* and W_V^{**} as follows:

$$W_V^* = \mathcal{V}(\Psi^*, n^*, e^*, e^*) \cdot s^*, \qquad W_V^{**} = \mathcal{V}^{\mathcal{B}}(\Psi^{**}, n^{**}, e^{**}, e^{**}) \cdot s^{**}. \tag{5.30}$$

Recall that $\mathcal{V}$ corresponds to per unit vendor profit in the presence of blockchain, whereas $\mathcal{V}^{\mathcal{B}}$ corresponds to per unit vendor profit in the presence of blockchain. Then, since overall vendor sales equals consumer demand in equilibrium, W_V^* therefore denotes vendor welfare in the absence of blockchain and W_V^{**} denotes vendor welfare when blockchain is adopted. We relate W_V^* and W_V^{**} via the following result:

Proposition 5.4.1 (Vendor Welfare)**.**

Vendor welfare W_V depends on the price Ψ paid to vendors by manufacturers and consumer demand s as follows:

$$W_V^* = \frac{(\Psi^*)^2 \cdot s^*}{2}, \qquad W_V^{**} = \frac{(\Psi^{**})^2 \cdot s^{**}}{2}.$$

Consequently, vendor welfare in the presence of blockchain is given as follows:

$$W_V^{**} = \left(\frac{\Psi^{**}}{\Psi^*}\right)^2 \cdot \frac{s^{**}}{s^*} \cdot W_V^*$$

and blockchain adoption has an ambiguous effect upon vendor welfare (i.e., there exist $\gamma, \delta : W_V^{**} < W_V^*$ and there exist $\gamma, \delta : W_V^{**} > W_V^*$).

Proposition 5.4.1 establishes that vendor welfare depends positively upon both the price Ψ paid to vendors and consumer demand s. As we discuss subsequently, the effect of blockchain adoption upon the vendor price is ambiguous because blockchain adoption can increase or decrease the vendor price; in turn, blockchain adoption has an ambiguous effect upon vendor welfare despite the fact that blockchain adoption unambiguously increases consumer demand (see Proposition 5.3.5).

The effect of blockchain adoption upon the vendor price is summarized by the following result:

Proposition 5.4.2 (Vendor Price)**.**

The vendor price in the presence of blockchain is given as follows:

$$\Psi^{**} = f_\Psi(\gamma) \cdot g_\Psi(\alpha, \delta) \cdot \Psi^*$$

where $f_\Psi(\gamma)$ and $g_\Psi(\alpha, \delta)$ separate the effect of manufacturer risk aversion, $\gamma > 0$, from that of the information asymmetry reduction, $\delta > 0$. More explicitly:

$$f_\Psi(\gamma) = \frac{3 - 2\gamma}{3}, \qquad g_\Psi(\alpha, \delta) = 1 + \frac{2\delta}{2\alpha - 1}.$$

Moreover, the following results hold:

1. *Blockchain Adoption Has An Ambiguous Effect On Vendor Price*

   There exist $\gamma, \delta : \Psi^{**} < \Psi^{*}$ *and there exist* $\gamma, \delta : \Psi^{**} > \Psi^{*}$.

2. *Risk Aversion Pressures Vendor Price Downward*

   $f_{\Psi}(\gamma) < 1$ *and* $\frac{df_{\Psi}}{d\gamma} < 0$.

3. *Information Asymmetry Reduction Pressures Vendor Price Upward*

   $g_{\Psi}(\alpha, \delta) > 1$ *and* $\frac{\partial g_{\Psi}}{\partial \delta} > 0$.

Proposition 5.4.2.1 highlights that blockchain adoption has an ambiguous effect on the vendor price. This ambiguity arises because the manufacturer's risk aversion and information asymmetry reduction have opposing affects. In particular, Proposition 5.4.2.2 demonstrates that the former exerts downward pressure (i.e., $f_{\Psi}(\gamma) < 1$), whereas Proposition 5.4.2.3 demonstrates that the latter exerts upward pressure (i.e., $g_{\Psi}(\alpha, \delta) > 1$). Moreover, the referenced effects amplify as risk aversion and information asymmetry reduction increase respectively (i.e., $\frac{df_{\Psi}}{d\gamma} < 0$ and $\frac{\partial g_{\Psi}}{\partial \delta} > 0$).

The following result clarifies the impact of the manufacturer's risk aversion and the information asymmetry reduction on the vendor price:

Proposition 5.4.3 (Vendor Price and Consumer Price)**.**
The vendor price and consumer price are related as follows:

$$\Psi^{*} = h(\gamma) \cdot P^{*}, \qquad \Psi^{**} = h(0) \cdot P^{**}$$

whereas the vendor effort level is given as follows:

$$e^{*} = (\Psi^{*})^2 = (h(\gamma) \cdot P^{*})^2, \qquad e^{**} = (\Psi^{**})^2 = (h(0) \cdot P^{**})^2$$

where

$$h(\gamma) = \frac{1}{2 - \gamma}.$$

Note that $\frac{dh}{d\gamma} > 0$ *so that* $0 < h(0) < h(\gamma) \leq 1$.

Proposition 5.4.3 establishes that the manufacturer passes through a positive fraction of the consumer price to the vendor (i.e., $\frac{\Psi^*}{P^*} = h(\gamma) \in [0,1]$ and $\frac{\Psi^{**}}{P^{**}} = h(0) \in [0,1]$). This relationship arises because the manufacturer's forfeited revenue due to a defect is proportional to the consumer price and thus an increase in the consumer price leads the manufacturer to increase her vendor price in order to increase vendor effort and reduce the probability of forfeiting the associated revenue. Proposition 5.4.3 formalizes this, and shows that the vendor price increases in the consumer price and that optimal vendor effort also increases in the vendor price.

Note that the relative fraction which the manufacturer passes through to the vendor with and without blockchain is given by $\frac{h(0)}{h(\gamma)}$. Then, Proposition 5.3.5, together with Proposition 5.4.3, implies that the vendor price in the presence of blockchain can be written as follows:

$$\Psi^{**} = \frac{h(0)}{h(\gamma)} \cdot \frac{P^{**}}{P^*} \cdot \Psi^* = \underbrace{\frac{h(0)}{h(\gamma)} \cdot f_P(\gamma)}_{\text{Effect of } \gamma} \cdot \underbrace{g_P(\alpha, \delta)}_{\text{Effect of } \delta} \cdot \Psi^*. \tag{5.31}$$

Thus, (5.31) shows that the impact of information asymmetry reduction on the vendor price due to blockchain adoption is mediated by its impact on consumer price. In particular, since information asymmetry reduction pressures the consumer price upward when blockchain is adopted (i.e. , $g_P(\alpha, \delta) > 1$), it also pressures the vendor price upward when blockchain is adopted.

The impact of the manufacturer's risk aversion on the vendor price arises for two reasons. First, manufacturer risk aversion puts downward pressure on the consumer price when blockchain is adopted (i.e., $f_P(\gamma) < 1$), and this effect is passed through to the vendor price. Second, manufacturer risk aversion reduces the relative fraction that the manufacturer passes through to the vendor from $h(\gamma)$ to $h(0)$ when the blockchain is adopted because the manufacturer does not face recall risk when blockchain is adopted, and thus, has a weaker incentive to induce higher vendor effort as compared to the case without blockchain. Since these effects both pressure the vendor price downward, the cumulative effect is that manufacturer risk aversion puts downward pressure on the vendor price when blockchain is adopted as per Proposition 5.4.2.2.

5.4.2 Consumer Welfare

We find that blockchain adoption unambiguously improves consumer welfare. More formally, we define W_C^* and W_C^{**} as follows:

$$W_C^* = \int_{k:C_k^*>0} C_k^* \, d\mu, \qquad W_C^{**} = \int_{k:C_k^{**}>0} C_k^{**} \, d\mu \tag{5.32}$$

where

$$C_k^* := C(P^*, v_k, t_k, \tilde{q}_k), \qquad C_k^{**} := C^{\mathcal{B}}(P^{**}, v_k, t_k, \tilde{q}_k) \tag{5.33}$$

so that C_k^* denotes the equilibrium utility of consumer k if she purchases from the manufacturer in the absence of blockchain and C_k^{**} denotes the equilibrium utility of consumer k if she purchases from the manufacturer when blockchain is adopted. In turn, since μ denotes the measure over consumers and since consumers have access to an outside option that gives utility zero, W_C^* therefore denotes consumer welfare in the absence of blockchain and W_C^{**} denotes consumer welfare when blockchain is adopted. We relate W_C^* and W_C^{**} via the following result:

Proposition 5.4.4 (Consumer Welfare)**.**

Consumer welfare, W_C, in the presence of blockchain is given as follows:

$$W_C^{**} = f_{W_C}(\gamma) \cdot g_{W_C}(\alpha, \delta) \cdot W_C^*$$

where $f_{W_C}(\gamma)$ and $g_{W_C}(\alpha, \delta)$ separate the effect of manufacturer risk aversion, $\gamma > 0$, from that of the information asymmetry reduction, $\delta > 0$. More explicitly:

$$f_{W_C}(\gamma) = \frac{(3-2\gamma)^3}{27(1-\gamma)^2}, \qquad g_{W_C}(\alpha, \delta) = (1 + \frac{2\delta}{2\alpha - 1})^2.$$

Moreover, the following results hold:

1. *Blockchain Adoption Increases Consumer Welfare*

   $W_C^{**} > W_C^*$.

2. *Risk Aversion Pressures Consumer Welfare Upward*

   $f_{W_C}(\gamma) > 1$ *and* $\frac{df_{W_C}}{d\gamma} > 0$.

3. *Information Asymmetry Reduction Pressures Consumer Welfare Upward*

   $g_{W_C}(\alpha, \delta) > 1$ *and* $\frac{\partial g_{W_C}}{\partial \delta} > 0$.

Proposition 5.4.4 establishes that consumer welfare unambiguously increases with blockchain adoption because of both the manufacturer's risk aversion, γ, and the information asymmetry reduction, δ. In particular, as per Propositions 5.4.4.2 and 5.4.4.3, both the manufacturer's risk aversion and the information asymmetry reduction put upward pressure on consumer welfare when blockchain is adopted (i.e., $f_{W_C}(\gamma) > 1$ and $g_{W_C}(\alpha, \delta) > 1$). Moreover, those effects augment with the manufacturer's risk aversion and the information asymmetry reduction respectively (i.e., $\frac{df_{W_C}}{d\gamma} > 0$ and $\frac{\partial g_{W_C}}{\partial \delta}$).

Consumer welfare increases due to the manufacturer's risk aversion when blockchain is adopted because, as discussed in Proposition 5.3.5.2, blockchain adoption leads a risk-averse manufacturer to lower her consumer price. The lower consumer price, in turn, enhances consumer welfare.

The information asymmetry reduction enhances consumer welfare when blockchain is adopted because it improves endogenous consumer decision-making. More precisely, when blockchain is adopted, consumer signals regarding the manufacturer's type become more accurate. In turn, consumer decision-making improves and thus consumer welfare increases.

Conclusion

This thesis demonstrates that the market structure is crucial in determining whether or not blockchain adoption arises:

(a). In the setting with perfect competition discussed in Chapters 2 and 3, blockchain adoption does not arise in equilibrium due to a misalignment of the benefits and costs of blockchain adoption. We show how adoption failures can be overcome by charging the consumers a fee that serves as a transfer payment to the manufacturer.

(b). When markets are not perfectly competitive, as in Chapter 4, blockchain has an ambiguous effect upon manufacturer welfare. We demonstrate that there exist conditions under which, although the blockchain improves global welfare, blockchain adoption does not arise in equilibrium, resulting in an adoption failure.

(c). In the monopolistic setting discussed in Chapter 5, blockchain adoption arises in equilibrium when the adoption cost is sufficiently small. In particular, market power enables a manufacturer with market power to adjust prices, and thereby, extract some of gains that are accrued to other supply chain participants, and thus makes blockchain adoption incentive compatible for sufficiently small adoption costs. This result contrasts with that of Chapters 2 and 3.

Although we explored two most common types of market structures, namely perfect competition and monopoly, other market organizations have been left for the future work. One example is on oligopoly. The subtleties of this new context will have some influence, but based upon the findings we know for perfect competition and monopoly, it is fair to conjecture that the result for oligopoly would lay somewhere in between. In an oligopolistic manufacturing sector, each manufacturer possesses a certain market power but none of them dominates the entire market; as a consequence, the manufacturer welfare in this case would be between zero and the monopolistic manufacturer welfare. Allowing multiple products in the supply chain could be another direction that is worth considering. This will expand our models and make the entire setting closer to the real world. Despite the complexity introduced by multiple products, we expect that the market structure is still the dominant factor in determining whether or not blockchain adoption arises. In summary, future work would concern deeper analysis of other market structures and incorporating new features to make the existing models richer.

References

Aitken R (2017) IBM forges Blockchain collaboration with Nestlé & Walmart in global food safety. Forbes (August 22) URL `https://www.forbes.com/sites/rogeraitken/2017/08/22/ibm-forges-blockchain-collaboration-with-nestle-walmart-for-global-food-safety/`.

Babich V, Hilary G (2020) Om forum—distributed ledgers and operations: What operations management researchers should know about blockchain technology. Manufacturing & Service Operations Management 22(2):223–240.

Baiman S, Netessine S, Kunreuther H (2004) Procurement in supply chains when the end-product exhibits the "weakest link" property. Working Paper .

Bakshi N, Kim SH, Savva N (2015) Signaling new product reliability with after-sales service contracts. Management Science 61(8):1812–1829.

Biais B, Bisiere C, Bouvard M, Casamatta C (2019) The blockchain folk theorem. Review of Financial Studies 32(5):1662–1715.

Blaettchen P, Jagmohan A, Ratakonda K, Franceschini M (2020) A information network for food safety: A simulation approach. Working Paper .

Burton M, Rigby D, Young T, James S (2001) Consumer attitudes to genetically modified organisms in food in the UK. European Review of Agricultural Economics 28(4):479–498.

Cachon GP, Lariviere MA (1999) Capacity choice and allocation: Strategic behavior and supply chain performance. Management Science 45(8):1091–1108.

Cao S, Cong LW, Han M, Hou Q, Yang B (2020) Blockchain architecture for auditing automation and trust building in public markets. Computer 53(7):20–28.

Cao S, Cong LW, Yang B (2019) Financial reporting and blockchains: Audit pricing, misstatements, and regulation. Working Paper .

Capponi A, Olafsson S, Alsabah H (2021) Proof-of-work cryptocurrencies: Does mining technology undermine decentralization? Working Paper .

Carson B, Romanelli G, Walsh P, Zhumaev A (2018) Blockchain beyond the hype: What is the strategic business value? Digital McKinsey (June 19) URL `https://www.mckinsey.com/business-functions/mckinsey-digital/our-insights/blockchain-beyond-the-hype-what-is-the-strategic-business-value`.

Catalini C, Gans J (2019) Initial coin offerings and the value of crypto tokens. NBER Working Paper .

Chen F, Lai G, Xiao W (2016) Provision of incentives for information acquisition: Forecast-based contracts vs. menus of linear contracts. Management Science 62(7):1899–1914.

Chen Y, Dai T, Korpeoglu CG, Körpeoğlu E, Sahin O, Tang CS, Xiao S (2020) Innovative online platforms: Research opportunities. Manufacturing & Service Operations Management 22(3):430–445.

Cho SH, Tang CS (2014) Capacity allocation under retail competition: Uniform and competitive allocations. Operations Research 62(1):72–80.

Chod J, Lyandres E (2021) A theory of icos: Diversification, agency, and information asymmetry. Management Science Forthcoming.

Chod J, Trichakis N, Tsoukalas G, Aspegren H, Weber M (2019) Blockchain and the value of operational transparency for supply chain finance. Management Science Forthcoming.

Clarke RN (1983) Collusion and the incentives for information sharing. Bell Journal of Economics 383–394.

Cong LW, He Z (2019) Blockchain disruption and smart contracts. Review of Financial Studies 32(5):1754–1797.

Cong LW, Li Y, Wang N (2021) Tokenomics: Dynamic adoption and valuation. Review of Financial Studies 34(3):1105–1155.

Cui TH, Zhang Y (2018) Cognitive hierarchy in capacity allocation games. Management Science 64(3):1250–1270.

Cui Y, Gaur V, Liu J (2020a) Blockchain collaboration with competing firms in a shared supply chain: Benefits and challenges. Working Paper .

Cui Y, Hu M, Liu J (2020b) Values of traceability in supply chains. Working Paper .

Dixon H (1987) Approximate Bertrand equilibria in a replicated industry. Review of Economic Studies 54:47–62.

Dong L, Jiang P, Xu F (2021) Blockchain adoption for traceability in food supply chain networks. Working Paper .

Dong Y, Xu K, Xu Y, Wan X (2016) Quality management in multi-level supply chains with outsourced manufacturing. Production and Operations Management 25(2):290–305.

Easley D, O'Hara M, Basu S (2019) From mining to markets: The evolution of bitcoin transaction fees. Journal of Financial Economics 134(1):91–109.

Fanti G, Kogan L, Viswanath P (2019) Economics of proof-of-stake payment systems. Working Paper .

Foley S, Karlsen JR, Putnins TJ (2019) Sex, drugs, and bitcoin: How much illegal activity is financed through cryptocurrencies? Review of Financial Studies 32(5):1798–1853.

Gal-Or E (1985) Information sharing in oligopoly. Econometrica 329–343.

Gan J, Tsoukalas G, Netessine S (2021a) Inventory, speculators, and initial coin offerings. Management Science 67:914 – 931.

Gan J, Tsoukalas G, Netessine S (2021b) To infinity and beyond: Financing platforms with uncapped crypto tokens. Working Paper .

Gaur V, Gaiha A (2020) Building a transparent supply chain. Harvard Business Review URL https://hbr.org/2020/05/building-a-transparent-supply-chain.

Gaur V, Giloni A, Seshadri S (2005) Information sharing in a supply chain under arma demand. Management Science 51(6):961–969.

Griffin JM, Shams A (2020) Is bitcoin really un-tethered? Journal of Finance Forthcoming.

Hall NG, Liu Z (2010) Capacity allocation and scheduling in supply chains. Operations Research 58(6):1711–1725.

Hinzen FJ, John K, Saleh F (2021) Bitcoin's limited adoption problem. Working Paper .

Howell ST, Niessner M, Yermack D (2020) Initial coin offerings: Financing growth with cryptocurrency token sales. Review of Financial Studies 33(9):3925–3974.

Huberman G, Leshno JD, Moallemi C (2021) An economic analysis of the bitcoin payment system. Review of Economic Studies Forthcoming.

Irresberger F, John K, Saleh F (2021) The public blockchain ecosystem: An empirical analysis. NYU Stern Working Paper .

Iyengar G, Saleh F, Sethuraman J, Wang W (2021a) Blockchain adoption in a supply chain with market power. Working Paper .

Iyengar G, Saleh F, Sethuraman J, Wang W (2021b) Economics of permissioned blockchain adoption. Working Paper .

John K, Rivera T, Saleh F (2021) Economic implications of scaling blockchains: Why the consensus protocol matters. Working Paper .

Kirby AJ (1988) Trade associations as information exchange mechanisms. RAND Journal of Economics 138–146.

Kreps DM, Wilson R (1982) Sequential equilibria. Econometrica 50(4):863–894.

Lee HL, So KC, Tang CS (2000) The value of information sharing in a two-level supply chain. Management Science 46(5):626–643.

Lehar A, Parlour C (2021) Miner collusion and the bitcoin protocol. Working Paper .

Li J, Mann W (2020) Digital tokens and platform building. Working Paper .

Li L (1985) Cournot oligopoly with information sharing. RAND Journal of Economics 521–536.

Li L (2012) Capacity and quality contract design in supply chain. Ph.D. thesis, Northwestern University.

Li Z, Gilbert SM, Lai G (2014) Supplier encroachment under asymmetric information. Management Science 60(2):449–462.

Lyandres E, Palazzo B, Rabetti D (2019) Do tokens behave like securities? an anatomy of initial coin offerings. Working Paper .

Malinova K, Park A (2018) Tokenomics: When tokens beat equity. Working Paper .

Moser R, Raffaelli R, Thilmany DD (2011) Consumer preferences for fruit and vegetables with credence-based attributes: A review. International Food and Agribusiness Management Review 22, URL www.ifama.org.

Mu L, Dawande M, Geng X, Mookerjee V (2016) Milking the quality test: Improving the milk supply chain under competing collection intermediaries. Management Science 62(5):1259–1277.

Nakamoto S (2008) Bitcoin: A peer-to-peer electronic cash system. https://bitcoin.org/bitcoin.pdf .

Narang S, Byali M, Dayama P, Pandit V, Narahari Y (2019) Design of trusted b2b market platforms using permissioned blockchains and game theory. 2019 IEEE International Conference on Blockchain and Cryptocurrency (ICBC), 385–393.

Nash KS (2016) Wal-Mart readies Blockchain pilot for tracking U.S. produce, China pork. Wall Street Journal (December 16) URL https://blogs.wsj.com/cio/2016/12/16/wal-mart-readies-blockchain-pilot-for-tracking-u-s-produce-china-pork/.

Nash KS (2018) Walmart requires lettuce, spinach suppliers to join Blockchain. Wall Street Journal (September 24) URL https://blogs.wsj.com/cio/2018/09/24/walmart-requires-lettuce-spinach-suppliers-to-join-blockchain/.

Natarajan K, Kostamis D, Merserean A (2012) Strategic demand information sharing between competitors. Working Paper .

Nikolakis W, John L, Krishnan H (2018) How blockchain can shape sustainable global value chains: An evidence, verifiability, and enforceability (EVE) framework. Sustainability 10(11):3926.

Novshek W, Sonnenschein H (1982) Fulfilled expectations cournot duopoly with information acquisition and release. Bell Journal of Economics 214–218.

Piramuthu S, Farahani P, Grunow M (2013) RFID-generated traceability for contaminated product recall in perishable food supply networks. European Journal of Operational Research 225(2):253–262.

Poole N, Baron L (1996) Consumer awareness of citrus fruit attributes. British Food Journal 1(98):23–28.

Provenance (2015) Blockchain: The solution for transparency in product supply chains. https://www.provenance.org/whitepaper .

Pun H, Swaminathan JM, Hou P (2018) Blockchain adoption for combating deceptive counterfeits. Working Paper .

Raghunathan S (2001) Information sharing in a supply chain: A note on its value when demand is nonstationary. Management Science 47(4):605–610.

Raith M (1996) A general model of information sharing in oligopoly. Journal of Economic Theory 71(1):260–288.

Rosu I, Saleh F (2021) Evolution of shares in a proof-of-stake cryptocurrency. Management Science 67:661–672.

Roy A, Gilbert SM, Lai G (2019) The implications of visibility on the use of strategic inventory in a supply chain. Management Science 65(4):1752–1767.

Saak AE (2016) Traceability and reputation in supply chains. International Journal of Production Economics 177:149–162.

Saleh F (2021) Blockchain without waste: Proof-of-stake. Review of Financial Studies 34:1156–1190.

Shapiro C (1986) Exchange of cost information in oligopoly. Review of Economic Studies 53(3):433–446.

Simchi-Levi D, Zhao Y (2003) The value of information sharing in a two-stage supply chain with production capacity constraints. Naval Research Logistics 50(8):888–916.

Tinn K (2018) 'Smart' contracts and external financing. Working Paper .

Vives X (1984) Duopoly information equilibrium: Cournot and Bertrand. Journal of Economic Theory 34(1):71–94.

Xiao E (2017) Alibaba, JD tackle China's fake goods problem with Blockchain. Techinasia (October 16) URL `https://www.techinasia.com/alibaba-jd-ecommerce-giants-fight-fake-goods-blockchain`.

Yao S, Zhu K (2020) Combating product label misconduct: The role of traceability and market inspection. European Journal of Operational Research 282(2):559–568.

Yermack D (2015) Is bitcoin a real currency? an economic appraisal. Handbook of Digital Currency 31–43.

Yiridoe EK, Bonti-Ankomah S, Martin RC (2005) Comparison of consumer perceptions and preference toward organic versus conventionally produced foods: A review and update of the literature. Renewable Agriculture and Food Systems 20(4):193–205.

Appendix A: Chapter 2 Proofs for $\alpha = \frac{1}{2}$

A.1 Main Proofs

Appendix A.1.2 provides proofs of all propositions stated in the body of Chapter 2. These proofs rely on model solutions of prices, market shares and expected consumer utilities. We derive the endogenous values of all such quantities in Proposition A.1.1, which is stated and proved in Appendix A.1.1. Since c_h is linear and strictly increasing, we assume $c_h(x) = \gamma x$ with $\gamma > 0$ in the appendices. All proofs in this appendix assume $\alpha = \frac{1}{2}$. All the results are extended to $\alpha \in [\frac{1}{2}, 1)$ in Appendix B.

A.1.1 Model Solutions

Recall that we assume the following regularity conditions on the cost functions c_h and c_l of the high and low type vendors:

Assumption A.1.1. We assume that the cost function $c_h(x) = \gamma x$ of the high type vendor and the cost function $c_l(x)$ of the low type vendor satisfy the following regularity conditions:

1. $c_l(0) = c_h(0) = 0$.

2. c_l is convex with a strictly increasing derivative c_l'.

3. $c_l'(0) < \gamma < c_l'\Big(G(\frac{H+L+h+l}{2} - \gamma)\Big)$.

Proposition A.1.1. *Define* $M_h := \{i : \xi_i = h\}$, $M_l := \{i : \xi_i = l\}$, $s_h := \sum_{i:i\in M_h} s_i$, *and* $s_l := \sum_{i:i\in M_l} s_i$. *The equilibrium prices, market shares, and expected consumer utilities under full blockchain adoption, partial blockchain adoption, and blockchain non-adoption are as follows.*

1. ***Full Blockchain Adoption:*** $P_i^F = \Psi_i^F = \Psi_l^F := \max\{\gamma - (h-l), 0\}$ *for all* $i \in M_l$, $P_i^F = \Psi_i^F = \Psi_h^F := \gamma$ *for all* $i \in M_h$, $s_l^F = (c_l')^{-1}(\max\{\gamma-(h-l),0\})$, $s_h^F = G(H+h-\gamma)-s_l^F$, *and* $u_{i(k)k} = H + h - \gamma$ *for all* $k \in [0,1]$.

2. ***Partial Blockchain Adoption:*** $P_i^P = \Psi_i^P = \Psi_l^P := \max\{\gamma - (h-l) - \frac{H-L}{2}, 0\}$ *for all* $i \in M_l$, $P_i^P = \Psi_i^P = \Psi_h^P := \gamma$ *for all* $i \in M_h$, $s_l^P = (c_l')^{-1}(\max\{\gamma - (h-l) - \frac{H-L}{2}, 0\})$, $s_h^P = G(H+h-\gamma) - s_l^P$, *and* $u_{i(k)k} = H + h - \gamma$ *for all* $k \in [0,1]$.

3. ***Blockchain Non-Adoption:*** $P_i^N = \Psi_i^N = \Psi_l^N = \Psi_h^N := \gamma$ *for all* $i \in M$, $s_l^N = (c_l')^{-1}(\gamma)$, $s_h^N = G(\frac{H+L+h+l}{2} - \gamma) - s_l^N$, *and* $u_{i(k)k} = \frac{H+L+h+l}{2} - \gamma$ *for all* $k \in [0,1]$.

Proof. For manufacturer i, once adoption decision a_i is made, adoption cost $\chi_i a_i$ becomes a sunk cost and she is now in the subgame of choosing her consumer price P_i and vendor price Ψ_i. In the following, we look for a symmetric equilibrium where the vendor prices of the manufacturers of the same vertical type are the same, i.e., $\Psi_i = \Psi_h$ for all $i \in M_h$ and $\Psi_i = \Psi_l$ for all $i \in M_l$.

- **Full Blockchain Adoption:** Given the vendor price Ψ_h^F, vendor h faces the following optimization problem:

$$\max_{\{\sigma_i : i \in M_h\}} \Psi_h^F \cdot \sum_{i: i \in M_h} \sigma_i - \gamma \cdot \sum_{i: i \in M_h} \sigma_i$$
$$\text{s.t. } 0 \le \sigma_i \le s_i^F \text{ for all } i : i \in M_h$$

  Therefore, the optimal fulfillment level $\sigma_i^* = s_i^F \cdot \mathcal{I}(\Psi_h^F \ge \gamma)$ for all $i : i \in M_h$. Since each type h manufacturer faces an arbitrarily large cost for not fulfilling any consumer order and thus wants to set Ψ_h^F to the lowest possible price such that $\sigma_i^* = s_i^F$, it follows that $\Psi_h^F = \gamma$. Given the vendor price Ψ_l^F, vendor l faces the following decision problem:

$$\max_{\{\sigma_i : i \in M_l\}} \Psi_l^F \cdot \sum_{i: i \in M_l} \sigma_i - c_l(\sum_{i: i \in M_l} \sigma_i)$$
$$\text{s.t. } 0 \le \sigma_i \le s_i^F \text{ for all } i : i \in M_l$$

Define $\sigma_l := \sum_{i:i\in M_l} \sigma_i$. Then, the objective $\Psi_l^F \sigma_l - c_l(\sigma_l)$ is strictly concave in σ_l, and the first order condition tells us that its unconstrained maximum is attained at $\sigma_l^* = (c_l')^{-1}(\Psi_l^F)$. So long as $\Psi_l^F \geq c_l'(s_l^F)$, the constrained maximum can be attained at $\sigma_l^* = s_l^F$. Thus, because c_l' is strictly increasing, the lowest possible price such that $\sigma_l^* = s_l^F$ satisfies $\Psi_l^F = c_l'(s_l^F)$.

Under full blockchain adoption, consumers know each manufacturer's type and will select a manufacturer of her own type. For consumer k whose type is t_k, we have that

$$\mathbb{E}[V_{ik} \mid \mathcal{F}_k] = \begin{cases} H+h & \text{if } i \in M_{t_k h} \\ H+l & \text{if } i \in M_{t_k l} \\ L+h & \text{if } i \in M_{\bar{t}_k h} \\ L+l & \text{if } i \in M_{\bar{t}_k l} \end{cases},$$

where $\bar{t}_k \in \{A, B\}$ is the other type that is different from t_k. Type Ah (resp. type Bh) manufacturers are competing for the market consisting of type A (resp. type B) consumers; type Al (resp. type Bl) manufacturers are competing for the market consisting of type A (resp. type B) consumers. Thus, we have four Bertrand games for type Ah, type Al, type Bh, and type Bl manufacturers respectively. Due to price competition, manufacturer i will set her consumer price P_i to her cost (i.e., her vendor price) so that the other manufacturers of the same type have no chance to undercut to obtain the entire market. Accordingly, $P_i^F = \Psi_h^F$ for all $i \in M_h$ and $P_i^F = \Psi_l^F$ for all $i \in M_l$. Define $P_h^F := \Psi_h^F$ and $P_l^F := \Psi_l^F$. Furthermore, type Al (resp. type Bl) manufacturers are willing to lower their consumer prices to compete with type Ah (resp. type Bh) manufacturers so that a type A (resp. type B) consumer is indifferent between a type Al (resp. type Bl) manufacturer and a type Ah (resp. type Bh) manufacturer. This gives us $H + l - P_l^F = H + h - P_h^F$ and thus $P_l^F = P_h^F - (h - l)$. It is possible that even if a type Al (resp. type Bl) manufacturer sets her consumer price to zero, the expected utility she provides to a type A (resp. type B) consumer is still lower than that provided by a type Ah (resp. type Bh) manufacturer. Taking this situation into consideration, we have that

$P_l^F = \max\{P_h^F - (h-l), 0\}$.

The final piece is the market share. From the above analysis, we know that consumer k will always select a type h or type l manufacturer of her own type, and the corresponding expected utility $u_{i(k)k} = H + h - P_h^F$. She will then compare $u_{i(k)k}$ to her outside option ϕ_k. Consequently, we have that $s_h^F + s_l^F = \mathbb{P}(u_{i(k)k} \geq \phi_k) = G(H + h - P_h^F)$.

Combining all the above, we obtain the following system of equations:

$$\begin{aligned}
\Psi_h^F &= \gamma, \\
\Psi_l^F &= c_l'(s_l^F), \\
P_h^F &= \Psi_h^F, \\
P_l^F &= \Psi_l^F, \\
P_l^F &= \max\{P_h^F - (h-l), 0\}, \\
s_h^F + s_l^F &= G(H + h - P_h^F).
\end{aligned}$$

The solutions are

$$\begin{aligned}
\Psi_h^F &= \gamma, \\
\Psi_l^F &= \max\{\gamma - (h-l), 0\}, \\
s_h^F &= G(H + h - \gamma) - s_l^F, \\
s_l^F &= (c_l')^{-1}(\max\{\gamma - (h-l), 0\}).
\end{aligned}$$

The conditions in Assumption A.1.1 ensure a positive s_h^F as $c_l'\Big(G(H+h-\gamma)\Big) \geq c_l'\Big(G(\frac{H+L+h+l}{2} - \gamma)\Big) > \gamma > \max\{\gamma - (h-l), 0\}$. It can be verified that the above solutions constitute a separating equilibrium. In equilibrium, type Ah (resp. type Bh, type Al, type Bl) manufacturers split the market share $\frac{s_h^F}{2}$ (resp. $\frac{s_h^F}{2}, \frac{s_l^F}{2}, \frac{s_l^F}{2}$) evenly.

- **Partial Blockchain Adoption:** Following the analysis we did for full blockchain adoption, we have that $\Psi_h^P = \gamma$ and $\Psi_l^P = (c_l')^{-1}(s_l^P)$. Under partial blockchain adoption, consumers know which manufacturers are of type Ah and of type Bh since every type h manufacturer joins the blockchain. Consequently, the consumers know that the remaining manufacturers are all of type l, but they cannot distinguish the horizontal type (i.e., type A or type B) among these manufacturers. Therefore, for consumer k whose type is t_k, we have that

$$\mathbb{E}[V_{ik} \mid \mathcal{F}_k] = \begin{cases} H + h & \text{if } i \in M_{t_k h} \\ L + h & \text{if } i \in M_{\bar{t}_k h} \\ \frac{H+L}{2} + l & \text{if } i \in M_l \end{cases},$$

where $\bar{t}_k \in \{A, B\}$ is the other type that is different from t_k. Similarly, we have three Bertrand games here: type Ah (resp. type Bh) manufacturers are competing for the market consisting of type A (resp. type B) consumers, and all type l manufacturers are competing for the entire market. The equilibrium consumer price for manufacturer i in the subgame is equal to her cost (i.e., her vendor price), so $P_i^P = \Psi_h^P$ for all $i \in M_h$ and $P_i^P = \Psi_l^P$ for all $i \in M_l$. Define $P_h^P := \Psi_h^P$ and $P_l^P := \Psi_l^P$. Furthermore, type l manufacturers are willing to lower their consumer prices to compete with type h manufacturers so that a type A (resp. type B) consumer is indifferent between a type l manufacturer and a type Ah (resp. type Bh) manufacturer. This gives us $\frac{H+L}{2} + l - P_l^P = H + h - P_h^P$. Performing the adjustment we did for P_l^F in full blockchain adoption, we have that $P_l^P = \max\{P_h^P - (h - l) - \frac{H-L}{2}, 0\}$.

The final piece is the market share. From the above analysis, we know that consumer k will always select a type h manufacturer of her own type or a type l manufacturer, and the corresponding expected utility $u_{i(k)k} = H + h - P_h^P$. She will then compare $u_{i(k)k}$ to her outside option ϕ_k. Consequently, we have that $s_h^P + s_l^P = \mathbb{P}(u_{i(k)k} \geq \phi_k) = G(H + h - P_h^P)$.

Combining all the above, we obtain the following system of equations:

$$\begin{aligned}
\Psi_h^P &= \gamma, \\
\Psi_l^P &= c_l'(s_l^P), \\
P_h^P &= \Psi_h^P, \\
P_l^P &= \Psi_l^P, \\
P_l^P &= \max\{P_h^P - (h-l) - \frac{H-L}{2}, 0\}, \\
s_h^P + s_l^P &= G(H + h - P_h^P).
\end{aligned}$$

The solutions are

$$\begin{aligned}
\Psi_h^P &= \gamma, \\
\Psi_l^P &= \max\{\gamma - (h-l) - \frac{H-L}{2}, 0\}, \\
s_h^P &= G(H + h - \gamma) - s_l^P, \\
s_l^P &= (c_l')^{-1}(\max\{\gamma - (h-l) - \frac{H-L}{2}, 0\}).
\end{aligned}$$

The conditions in Assumption A.1.1 guarantee a positive s_h^P as $c_l'\Big(G(H + h - \gamma)\Big) \geq c_l'\Big(G(\frac{H+L+h+l}{2} - \gamma)\Big) > \gamma > \max\{\gamma - (h-l) - \frac{H-L}{2}, 0\}$. It can be verified that the above solutions constitute a separating equilibrium. In equilibrium, type Ah (resp. type Bh, type l) manufacturers split the market share $\frac{s_h^P}{2}$ (resp. $\frac{s_h^P}{2}, s_l^P$) evenly.

- **Blockchain Non-Adoption:** Following the analysis we did for full blockchain adoption, we have that $\Psi_h^N = \gamma$ and $\Psi_l^N = (c_l')^{-1}(s_l^N)$. Since $\alpha = \frac{1}{2}$, type signals are uninformative; therefore, $\mathbb{P}(q_i = t_k \mid \mathcal{F}_k) = \mathbb{P}(\xi_i = h \mid \mathcal{F}_k) = \frac{1}{2}$ for all $i \in M$ and $k \in [0,1]$, i.e. the manufacturers are indistinguishable from the consumers' perspective. Consequently, consumers will randomly choose from the set of manufacturers with the lowest price. Due to price competition, all manufacturers will charge exactly the same price, and this price will equal the cost, i.e., $P_i^N = P_h^N := \Psi_h^N$ for all $i \in M_h$, $P_i^N = P_l^N := \Psi_l^N$ for all $i \in M_l$, and $P_h^N = P_l^N$. A separating equilibrium with $P_h^N \neq P_l^N$ is impossible here because a type l manufacturer can mimick the type h manufacturer at no cost in the absence of blockchain, which results in a profitable deviation. Note that $u_{ik} = \frac{H+L}{2} + \frac{h+l}{2} - P_i^N = \frac{H+L+h+l}{2} - P_h^N$ for all $i \in M$ and $k \in [0,1]$. Thus, the market share $s_h^N + s_l^N = \mathbb{P}(u_{i(k)k} \geq \phi_k) = G(\frac{H+L+h+l}{2} - P_h^N)$.

Combining all the above, we obtain the following system of equations:

$$
\begin{aligned}
\Psi_h^N &= \gamma, \\
\Psi_l^N &= c_l'(s_l^N), \\
P_h^N &= \Psi_h^N, \\
P_l^N &= \Psi_l^N, \\
P_h^N &= P_l^N, \\
s_h^N + s_l^N &= G(\frac{H+L+h+l}{2} - P_h^N).
\end{aligned}
$$

The solutions are

$$
\begin{aligned}
\Psi_h^N &= \gamma, \\
\Psi_l^N &= \gamma, \\
s_h^N &= G(\frac{H+L+h+l}{2} - \gamma) - s_l^N, \\
s_l^N &= (c_l')^{-1}(\gamma).
\end{aligned}
$$

The conditions in Assumption A.1.1 ensure a positive s_h^N. It can be verified that the above solutions constitute a pooling equilibrium. In equilibrium, type h (resp. type l) manufacturers split the market share s_h^N (resp. s_l^N) evenly.

□

A.1.2 Proofs of Propositions Stated in Chapter 2

Proof of Proposition 2.3.1. Recall that G denotes the cumulative distribution function of the outside option ϕ_k and is supported on $[0, +\infty)$. Let $p(v)$ denote the corresponding probability density function, which is also defined on $[0, +\infty)$. Define an auxiliary function $f_1(x) := xG(x) + \int_x^{+\infty} vp(v)dv$. Since $f_1'(x) = G(x) > 0$ for all $x > 0$, $f_1(x)$ is strictly increasing in x on $[0, +\infty)$. By (2.12) and Proposition A.1.1, we have that

$$\begin{aligned}
W_C^F &= \int_0^1 \mathbb{E}[\max\{\max_{i \in M} u_{ik}, \phi_k\}] \, dk \\
&= \mathbb{E}[\max\{H + h - \gamma, \phi_k\}] \\
&= \mathbb{E}[H + h - \gamma \mid H + h - \gamma \geq \phi_k]\mathbb{P}(H + h - \gamma \geq \phi_k) + \mathbb{E}[\phi_k \mid H + h - \gamma < \phi_k]\mathbb{P}(H + h - \gamma < \phi_k) \\
&= (H + h - \gamma)G(H + h - \gamma) + \int_{H+h-\gamma}^{+\infty} vp(v) \, dv \\
&= f_1(H + h - \gamma).
\end{aligned}$$

Similarly,

$$\begin{aligned}
W_C^P &= \int_0^1 \mathbb{E}[\max\{\max_{i \in M} u_{ik}, \phi_k\}] \, dk \\
&= \mathbb{E}[\max\{H + h - \gamma, \phi_k\}] \\
&= f_1(H + h - \gamma)
\end{aligned}$$

and

$$\begin{aligned}
W_C^N &= \int_0^1 \mathbb{E}[\max\{\max_{i \in M} u_{ik}, \phi_k\}] \, dk \\
&= \mathbb{E}[\max\{\frac{H + L + h + l}{2} - \gamma, \phi_k\}] \\
&= f_1(\frac{H + L + h + l}{2} - \gamma).
\end{aligned}$$

Note that $H > L$ and $h > l$. It readily follows that $W_C^F \geq W_C^P > W_C^N$ since $f_1(x)$ is strictly increasing. □

Proof of Proposition 2.3.2. Proposition 2.3.2 directly follows from Proposition A.1.1.

- **Expected Utility Per Consumer:** By Proposition A.1.1 and the definition of u, we get that

$$u^F = H + h - \gamma,$$
$$u^P = H + h - \gamma,$$
$$u^N = \frac{H + L + h + l}{2} - \gamma.$$

  Since $H > L$ and $h > l$, $u^F \geq u^P > u^N$ follows.

- **Consumer Demand:** By Proposition A.1.1, $s^F = s_h^F + s_l^F = G(H + h - \gamma)$, $s^P = s_h^P + s_l^P = G(H + h - \gamma)$, and $s^N = s_h^N + s_l^N = G(\frac{H+L+h+l}{2} - \gamma)$. Since G is increasing, we have that $s^F \geq s^P \geq s^N$.

□

Proof of Proposition 2.3.3. By Proposition A.1.1, each manufacturer has zero expected profit in the subgame because of price competition, regardless of the level of blockchain adoption. It follows that $W_M = 0 - \sum_{i:i \in M} \chi_i a_i$, so $W_M^F = -\sum_{i:i \in M} \chi_i$, $W_M^P = -\sum_{i:i \in M_h} \chi_i$, and $W_M^N = 0$. We can see that $W_M^F < W_M^P < W_M^N$. □

Proof of Proposition 2.3.4. Define an auxiliary function $f_2(x) := c_l'(x) \cdot x - c_l(x)$. Then, $f_2(0) = 0$ and $f_2'(x) = c_l''(x) \cdot x > 0$ for all $x > 0$, so $f_2(x)$ is strictly increasing in x on $[0, +\infty)$. By (2.10) and Proposition A.1.1, we have that

$$\begin{aligned} W_V^F &= (\Psi_h^F \cdot s_h^F - \gamma \cdot s_h^F) + (\Psi_l^F \cdot s_l^F - c_l(s_l^F)) \\ &= 0 + (c_l'(s_l^F) \cdot s_l^F - c_l(s_l^F)) \\ &= f_2(s_l^F). \end{aligned}$$

Similarly, $W_V^P = f_2(s_l^P)$ and $W_V^N = f_2(s_l^N)$. Since $(c_l')^{-1}$ is strictly increasing, by Proposition A.1.1, $s_l^F = (c_l')^{-1}(\max\{\gamma - (h-l), 0\}) < (c_l')^{-1}(\gamma) = s_l^N$ and $s_l^P = (c_l')^{-1}(\max\{\gamma - (h-l) - \frac{H-L}{2}, 0\}) < (c_l')^{-1}(\gamma) = s_l^N$ hold. Therefore, $W_V^F = f_2(s_l^F) < f_2(s_l^N) = W_V^N$ and $W_V^P = f_2(s_l^P) < f_2(s_l^N) = W_V^N$. □

Proof of Proposition 2.3.5. By Proposition A.1.1, we know that $\Psi_l^F = c_l'(s_l^F)$, $\Psi_l^P = c_l'(s_l^P)$, and $\Psi_l^N = c_l'(s_l^N)$. Since c_l' is strictly increasing and we have shown in the proof of Proposition 2.3.4 that $s_l^F < s_l^N$ and $s_l^P < s_l^N$, it immediately follows that $\Psi_l^F < \Psi_l^N$ and $\Psi_l^P < \Psi_l^N$. □

Proof of Proposition 2.3.6. By (2.13), Proposition 2.3.1, Proposition 2.3.3, and Proposition 2.3.4, we have that

$$
\begin{aligned}
W^F &= W_C^F + W_V^F + W_M^F \\
&= f_1(H + h - \gamma) + f_2(s_l^F) - \sum_{i:i\in M} \chi_i \\
&= f_1(\Delta + L + h - \gamma) + f_2(s_l^F) - \sum_{i:i\in M} \chi_i, \\
W^P &= W_C^P + W_V^P + W_M^P \\
&= f_1(H + h - \gamma) + f_2(s_l^P) - \sum_{i:i\in M_h} \chi_i \\
&= f_1(\Delta + L + h - \gamma) + f_2(s_l^P) - \sum_{i:i\in M_h} \chi_i, \\
W^N &= W_C^N + W_V^N + W_M^N \\
&= f_1(\frac{H + L + h + l}{2} - \gamma) + f_2(s_l^N) + 0 \\
&= f_1(\frac{\Delta}{2} + L + \frac{h + l}{2} - \gamma) + f_2(s_l^N).
\end{aligned}
$$

View L as a fixed parameter and allow Δ to vary. Since $f_1(x)$ is strictly increasing, W^F, W^P and W^N all strictly increase in Δ. W^F decreases in $\sum_{i:i\in M} \chi_i$ and is strictly smaller than W^N for sufficiently large $\sum_{i:i\in M} \chi_i$. The same argument applies to W^P.

Note that

$$W_F - W_N = \Big(f_1(\Delta + L + h - \gamma) - f_1(\frac{\Delta}{2} + L + \frac{h+l}{2} - \gamma)\Big) + \Big(f_2(s_l^F) - f_2(s_l^N) - \sum_{i:i\in M} \chi_i\Big).$$

By Proposition A.1.1, $s_l^N = (c_l')^{-1}(\gamma)$. Thus, the second term of $W_F - W_N$, $f_2(s_l^F) - f_2(s_l^N) - \sum_{i:i\in M}\chi_i \geq f_2(0) - f_2(s_l^N) - \sum_{i:i\in M}\chi_i = -f_2((c_l')^{-1}(\gamma)) - \sum_{i:i\in M}\chi_i$. Fixing parameter γ and adoption costs $\chi_i : i \in M$,

$$\begin{aligned}\frac{\partial(W^F - W^N)}{\partial\Delta} &= G(\Delta + L + h - \gamma) - \frac{1}{2}G(\frac{\Delta}{2} + L + \frac{h+l}{2} - \gamma)\\ &\geq G(\Delta + L + h - \gamma) - \frac{1}{2}.\end{aligned}$$

Because $\lim_{\Delta\to+\infty} G(\Delta + L + h - \gamma) = 1$, $\frac{\partial(W^F - W^N)}{\partial\Delta} \geq \frac{1}{4}$ when Δ is large enough. This implies that $\lim_{\Delta\to+\infty} W^F - W^N = +\infty$ no matter how large $f_2((c_l')^{-1}(\gamma))$ and $\sum_{i:i\in M}\chi_i$ are, meaning that W^F strictly exceeds W^N for sufficiently large Δ. Applying a similar argument, we can also show that W^P strictly exceeds W^N for sufficiently large Δ.

$W^F - W^N$ (resp. $W^P - W^N$) is negative for sufficiently high adoption costs and positive for sufficiently large Δ, so blockchain adoption has ambiguous effects on global welfare. □

Proof of Proposition 2.3.7. By Proposition A.1.1, a type h manufacturer i (i.e., $i \in M_h$) adopts the blockchain and receives zero expected profit in the subgame due to price competition, i.e., $\Pi(1, a_{-i}) = 0$, under blockchain adoption (either full or partial). Hence, her expected profit for adopting the blockchain is $0 - \chi_i = -\chi_i$. However, if she chooses not to adopt the blockchain, then her expected profit is always non-negative, which is better than $-\chi_i$. Thus, with blockchain adoption (either full or partial), manufacturer i has incentive to deviate so there does not exist an equilibrium. □

Proof of Proposition 2.3.8. Proposition 2.3.6 tells us that full blockchain adoption is welfare-enhancing for sufficiently large Δ. However, by Proposition 2.3.7, there does not exist an equilibrium with full blockchain adoption. Hence, an adoption failure arises by definition. □

Proof of Proposition 2.3.9. Recall that G denotes the cumulative distribution function of the outside option ϕ_k of consumer k. We consider the system of transfers where $\tau_i = \chi_i$ for all $i \in M$, and $\kappa = \frac{\sum_{i:i\in M}\chi_i}{G(\phi)}$, where ϕ is to be determined. We will first show that for all sufficiently large Δ, there exists ϕ such that $b_k = 1$ if and only if $\phi_k \le \phi$ and thus $\mu_\kappa = \mu(\{k : b_k = 1\}) = \mathbb{P}(\phi_k \le \phi) = G(\phi)$. Since $\kappa \cdot \mu_\kappa = \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} \cdot G(\phi) = \sum_{i:i\in M}\chi_i = \sum_{i:i\in M}\tau_i$, it follows that the system of transfers specified above is self-financing. Next, we will show that $\{a_i = 1, P_i = \Psi_i = \gamma\}_{i\in M_h} \cup \{a_i = 1, P_i = \Psi_i = \max\{\gamma - (h-l), 0\}\}_{i\in M_l}$ can be sustained as an equilibrium under such a system of transfers. Thereby, full blockchain adoption arises in equilibrium and the adoption failure is resolved.

Suppose $a_i = 1$ for all $i \in M$, $P_i = \Psi_i = \Psi_h := \gamma$ for all $i \in M_h$, and $P_i = \Psi_i = \Psi_l := \max\{\gamma - (h-l), 0\}$ for all $i \in M_l$. Consumers are now in the subgame of making the b_k decision. The generic consumer k solves the following optimization problem:

$$\max_{b_k\in\{0,1\}} g(b_k, \phi_k; \phi) := \mathbb{E}^{b_k}[\max\{\max_{i\in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k],$$

where consumer k's information set is given by $\mathcal{G}_k = \sigma(t_k, \phi_k, \{a_i, P_i\}_{i\in M})$. When $b_k = 1$, i.e. consumer k has access to the information on the blockchain, the result in Proposition A.1.1 regarding the full adoption case implies that $\max_{i\in M} u_{ik} = H + h - \gamma = \Delta + L + h - \gamma$. Therefore,

$$\begin{aligned} g(1, \phi_k; \phi) &= \mathbb{E}^1[\max\{\max_{i\in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \\ &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)}. \end{aligned}$$

When $b_k = 0$, consumer k opts to forgo access to the information on the blockchain. Here, the case is a bit different from the non-adoption case: since $P_i = \gamma$ for all $i \in M_h$, $P_i = \max\{\gamma - (h-l), 0\}$ for all $i \in M_l$, and $\gamma \ne \max\{\gamma - (h-l), 0\}$, the consumers are able to distinguish the vertically differentiated type (i.e., type h or type l) among the manufacturers, but they cannot distinguish the horizontally differentiated type (i.e., type A or type B) among the manufacturers. Accordingly,

$\max_{i \in M} u_{ik} = \frac{H+L}{2} + h - \gamma = \frac{\Delta}{2} + L + h - \gamma$. Therefore,

$$g(0, \phi_k; \phi) = \mathbb{E}^0[\max\{\max_{i \in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k]$$
$$= \max\{\frac{\Delta}{2} + L + h - \gamma,\ \phi_k\}.$$

Consumer k chooses to gain access to the information on the blockchain, i.e. $b_k = 1$, if and only if

$$h(\phi_k; \phi) := g(1, \phi_k; \phi) - g(0, \phi_k; \phi) \geq 0.$$

By Lemma A.2.1(b) in Appendix A.2, it follows that there exists $\phi^* \in [0, +\infty)$ such that $h(\phi^*; \phi^*) = 0$ and $h(\phi_k; \phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. Thus, $b_k = 1$ if and only if $\phi_k \leq \phi^*$ is an optimal decision rule for all consumers. Since measure μ_κ of the consumers who pay the fee κ to gain access to the information on the blockchain satisfies $\mu_\kappa = \mu(\{k : b_k = 1\}) = \mathbb{P}(\phi_k \leq \phi^*) = G(\phi^*)$, it follows that the payment collected from the consumers is $\kappa \cdot \mu_\kappa = \frac{\sum_{i:i \in M} \chi_i}{G(\phi^*)} \cdot G(\phi^*) = \sum_{i:i \in M} \chi_i = \sum_{i:i \in M} \tau_i$, indicating that the system of transfers is self-financing.

Note that $\phi^* \leq \Delta + L + h - \gamma$ according to the proof of Lemma A.2.1(b). When $b_k = 1$, we have that $\phi_k \leq \phi^* \leq \Delta + L + h - \gamma = \max_{i \in M} u_{ik}$ holds and thus all the consumers that choose $b_k = 1$ will purchase from a manufacturer rather than taking the outside option. Similarly, when $b_k = 0$, we have that $\phi_k > \phi^* > \frac{\Delta}{2} + L + h - \gamma = \max_{i \in M} u_{ik}$ and thus all the consumers that choose $b_k = 0$ will take the outside option. Following the analysis we did for full blockchain adoption in Proposition A.1.1, we can obtain the following system of equations to derive the market shares s_h and s_l under the system of transfers:

$$\begin{aligned}\Psi_h &= \gamma,\\ \Psi_l &= c_l'(s_l),\\ P_h &= \Psi_h,\\ P_l &= \Psi_l,\\ P_l &= \max\{P_h - (h-l), 0\},\\ s_h + s_l &= G(\phi^*).\end{aligned}$$

Next, we take one step back to the stage where manufacturers make their adoption and pricing decisions. Fix i and suppose $a_j = 1$ for all $j \in M \setminus \{i\}$, $P_j = \Psi_j = \Psi_h$ for all $j \in M_h \setminus \{i\}$, and $P_j = \Psi_j = \Psi_l$ for all $j \in M_l \setminus \{i\}$. For manufacturer i, if she adopts the blockchain, then she obviously has no incentive to increase her vendor price $\Psi_i = \Psi_{\xi_i}$; she also has no incentive to lower $\Psi_i = \Psi_{\xi_i}$ because that results in under-fulfilling her consumer order. Additionally, she has no incentive to change her consumer price from $P_i = \Psi_{\xi_i}$ because of price competition. Her expected profit is zero in this case since $P_i = \Psi_i$ and the transfer $\tau_i = \chi_i$ just covers her adoption cost χ_i. If she does not adopt the blockchain, the consumers still know her type as she is the only manufacturer that is not on the blockchain. She makes her pricing decisions P_i and Ψ_i as if she joins the blockchain. Consequently, her consumer price P_i remains equal to her vendor price Ψ_i in this case. This implies that her expected profit from not adopting the blockchain is zero as well, so she has no incentive to deviate from adopting the blockchain. To conclude, manufacturer i has no incentive to deviate from the adoption decision $a_i = 1$ and the pricing decisions $P_i = \Psi_i = \Psi_{\xi_i}$. This completes the proof. □

A.2 Supplementary Lemma and Example

Lemma A.2.1. *The function $h(\phi_k; \phi) := g(1, \phi_k; \phi) - g(0, \phi_k; \phi)$ has the following properties.*

(a). $h(\phi_k;\phi)$ *is continuous and weakly decreasing in* ϕ_k *on* $[0,+\infty)$.

(b). For all sufficiently large Δ, *there exists* $\phi^* \in [0,+\infty)$ *such that* $h(\phi^*;\phi^*) = 0$ *and* $h(\phi_k;\phi^*) \geq 0$ *if and only if* $\phi_k \leq \phi^*$. *Therefore,* $b_k = 1$ *if and only if* $\phi_k \leq \phi^*$. *Moreover,* ϕ^* *is not necessarily unique with a general cumulative distribution function* G.

Proof. By definition,

$$\begin{aligned} h(\phi_k;\phi) &= g(1,\phi_k;\phi) - g(0,\phi_k;\phi) \\ &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L + h - \gamma,\ \phi_k\}. \end{aligned}$$

Each term of it is continuous in ϕ_k on $[0,+\infty)$, so $h(\phi_k;\phi)$ as a function of ϕ_k is continuous on $[0,+\infty)$. Depending on how large Δ is, there are three cases, each of which we subsequently examine separately: $\frac{\Delta}{2}+L+h-\gamma > 0$, $\frac{\Delta}{2}+L+h-\gamma \leq 0$ but $\Delta+L+h-\gamma > 0$, and $\Delta+L+h-\gamma \leq 0$. Consider the following three cases:

(1). $\frac{\Delta}{2} + L + h - \gamma > 0$

In this case, we can partition $[0,+\infty)$ into three intervals $[0, \frac{\Delta}{2} + L + h - \gamma]$, $(\frac{\Delta}{2} + L + h - \gamma, \Delta + L + h - \gamma]$, and $(\Delta + L + h - \gamma, +\infty)$. $h(\phi_k;\phi)$ as a function of ϕ_k is constant on the first and last intervals, and linearly strictly decreasing on the middle interval. Moreover, $h(\phi_k;\phi)$ is continuous so that $h(\phi_k;\phi)$ is continuous and weakly decreasing in general.

When $\phi_k \in [0, \frac{\Delta}{2} + L + h - \gamma]$, $h(\phi_k;\phi)$ is given by:

$$\begin{aligned} h(\phi_k;\phi) &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L + h - \gamma,\ \phi_k\} \\ &= (\Delta + L + h - \gamma) - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} - (\frac{\Delta}{2} + L + h - \gamma) \\ &= \frac{\Delta}{2} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)}, \end{aligned}$$

which does not depend upon ϕ_k.

When $\phi_k \in (\frac{\Delta}{2} + L + h - \gamma, \Delta + L + h - \gamma]$, $h(\phi_k; \phi)$ is given by:

$$\begin{aligned} h(\phi_k; \phi) &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L + h - \gamma,\ \phi_k\} \\ &= (\Delta + L + h - \gamma) - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \phi_k, \end{aligned}$$

which is linear and strictly decreasing in ϕ_k.

When $\phi_k \in (\Delta + L + h - \gamma, +\infty)$, $h(\phi_k; \phi)$ is given by:

$$\begin{aligned} h(\phi_k; \phi) &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L + h - \gamma,\ \phi_k\} \\ &= \phi_k - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \phi_k \\ &= -\frac{\sum_{i:i\in M} \chi_i}{G(\phi)}, \end{aligned}$$

which does not depend upon ϕ_k.

(2). $\frac{\Delta}{2} + L + h - \gamma \leq 0$ but $\Delta + L + h - \gamma > 0$

In this case, we can partition $[0, +\infty)$ into two intervals $[0, \Delta + L + h - \gamma]$ and $(\Delta + L + h - \gamma, +\infty)$. By a similar argument, we can prove that $h(\phi_k; \phi)$ is continuous and weakly decreasing on $[0, +\infty)$. More specifically, it is strictly decreasing on $[0, \Delta + L + h - \gamma]$ and flat on $(\Delta + L + h - \gamma, +\infty)$.

(3). $\Delta + L + h - \gamma \leq 0$

In this case, $h(\phi_k; \phi) = -\frac{\sum_{i:i\in M} \chi_i}{G(\phi)}$ is a constant function on $[0, +\infty)$ so that it is trivially continuous and weakly decreasing.

In conclusion, across all three cases, $h(\phi_k; \phi)$ is continuous and weakly decreasing in ϕ_k on $[0, +\infty)$. This completes the proof of (a).

When $\frac{\Delta}{2} > G^{-1}(0.5) - L - h + \gamma$, $\frac{\Delta}{2} + L + h - \gamma > G^{-1}(0.5) > 0$ holds true. It follows from Case (1) in the first property that $h(\phi_k; \phi)$ with such Δ is continuous and weakly decreasing in ϕ_k on $[0, +\infty)$. More specifically, it is strictly decreasing on $(\frac{\Delta}{2} + L + h - \gamma, \Delta + L + h - \gamma]$ and flat on $[0, \frac{\Delta}{2} + L + h - \gamma]$ and $(\Delta + L + h - \gamma, +\infty)$. Thus, if we can find a ϕ^* such that $\phi^* \in (\frac{\Delta}{2} + L + h - \gamma, \Delta + L + h - \gamma]$ and $h(\phi^*; \phi^*) = 0$, then $h(\phi_k; \phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$.

Note that

$$\begin{aligned} l(\phi) &:= h(\phi; \phi) \\ &= g(1, \phi; \phi) - g(0, \phi; \phi) \\ &= \max\{\Delta + L + h - \gamma,\ \phi\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L + h - \gamma,\ \phi\} \end{aligned}$$

is continuous in ϕ on $(0, +\infty)$ and

$$l(\Delta + L + h - \gamma) = -\frac{\sum_{i:i\in M} \chi_i}{G(\Delta + L + h - \gamma)} < 0.$$

When $\frac{\Delta}{2} > \max\{2 \sum_{i:i\in M} \chi_i, G^{-1}(0.5) - L - h + \gamma\}$,

$$\begin{aligned} l(\frac{\Delta}{2} + L + h - \gamma) &= \frac{\Delta}{2} - \frac{\sum_{i:i\in M} \chi_i}{G(\frac{\Delta}{2} + L + h - \gamma)} \\ &> \frac{\Delta}{2} - \frac{\sum_{i:i\in M} \chi_i}{G(G^{-1}(0.5))} \\ &= \frac{\Delta}{2} - 2 \sum_{i:i\in M} \chi_i \\ &> 0. \end{aligned}$$

This implies that when $\frac{\Delta}{2} > \max\{2 \sum_{i:i\in M} \chi_i, G^{-1}(0.5) - L - h + \gamma\}$, by intermediate value theorem, there exists a solution $\phi^* \in (\frac{\Delta}{2} + L + h - \gamma, \Delta + L + h - \gamma]$ such that $h(\phi^*; \phi^*) = l(\phi^*) = 0$. With this ϕ^*, we have that $h(\phi_k; \phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. ϕ^* is not necessarily unique with a general cumulative distribution function G; Example A.2.1 below is such an example. This completes the proof of (b). □

Example A.2.1. Recall that we want to find a ϕ^* such that $\phi^* \in (\frac{\Delta}{2} + L + h - \gamma, \Delta + L + h - \gamma] =: I$ and

$$l(\phi^*) = \max\{\Delta + L + h - \gamma,\ \phi^*\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi^*)} - \max\{\frac{\Delta}{2} + L + h - \gamma,\ \phi^*\} = 0.$$

It is equivalent to solving $y_1(\phi) = y_2(\phi)$ for $\phi \in I$, or finding the intersection points of two curves $y_1(\phi)$ and $y_2(\phi)$ on I, where

$$\begin{aligned} y_1(\phi) &= \max\{\Delta + L + h - \gamma,\ \phi\} - \max\{\frac{\Delta}{2} + L + h - \gamma,\ \phi\}, \\ y_2(\phi) &= \frac{\sum_{i:i\in M} \chi_i}{G(\phi)}. \end{aligned}$$

Set $\Delta = \frac{13}{3}, L = h = \frac{1}{2}, \gamma = 1, \sum_{i:i\in M} \chi_i = 1$, and

$$G(\phi) = \begin{cases} \frac{\phi}{4} & \phi \in [0, 3] \\ \phi - \frac{9}{4} & \phi \in (3, \frac{13}{4}] \\ 1 & \phi \in (\frac{13}{4}, +\infty) \end{cases}$$

being piecewise linear. Then, $I = (\frac{13}{6}, \frac{13}{3}]$ and

$$\begin{aligned} y_1(\phi) &= \max\{\frac{13}{3},\ \phi\} - \max\{\frac{13}{6},\ \phi\}, \\ y_2(\phi) &= \frac{1}{G(\phi)}. \end{aligned}$$

There are two solutions $\phi^* = 3$ and $\phi^* = \frac{10}{3}$ satisfying $\phi^* \in I$ and $l(\phi^*) = 0$.

In this example, $y_2(\phi)$ is decreasing but piecewise convex with a kink. It is possible to further twist the shape of $y_2(\phi)$ while keeping it decreasing to generate more intersection points with $y_1(\phi)$ over I.

Appendix B: Chapter 2 Proofs for $\alpha \in [\frac{1}{2}, 1)$

This appendix generalizes our results to the case of $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i) = \alpha \in [\frac{1}{2}, 1)$ and $\mathbb{P}(\tilde{\xi}_{ik} = \xi_i \mid \xi_i) = \frac{1}{2}$. Appendix A included complete proofs for $\alpha = \frac{1}{2}$. The extension to $\alpha \in (\frac{1}{2}, 1)$ involves additional mathematical complexity that is developed here. We begin with several supporting lemmas in Appendix B.1, and use these results to re-derive all the propositions in Appendix B.2.

B.1 Supplementary Lemmas

When $\alpha \in (\frac{1}{2}, 1)$, the signals for the horizontal type of each manufacturer are partially informative. Therefore, the proportion of consumers that select a given manufacturer depends upon the distribution of manufacturer signals. Lemmas B.1.1 - B.1.4 provide intermediate results that enable us to explicitly compute the proportions. These results are necessary for several proofs in Appendix B.2. Additionally, to determine the equilibrium with a system of transfers, it is necessary to determine the incremental utility that consumers achieve from full information regarding manufacturer types relative to partial information. Lemma B.1.5 provides an intermediate result for that purpose, which we rely upon in the proof of Proposition 2.3.9 in Appendix B.2.

Recall that it is common knowledge that there are exactly $2m$ manufacturers of each horizontal type (i.e., type A or type B). Let r_i, $i = 1, \ldots, 4m$, denote the true horizontal type of manufacturer i. We assume that every consumer k has prior distribution that is uniform over the set of possible types

$$E = \{(r_1, \ldots, r_{4m}) \mid r_v \in \{A, B\}, \forall v \in M, |\{v : r_v = A\}| = 2m\}.$$

Lemma B.1.1. *Suppose none of the manufacturers adopts the blockchain. Fix a consumer k. Suppose the signal from manufacturer i is the same as her type, i.e. $\tilde{q}_{ik} = t_k$, and the signal from manufacturer j is not the same, i.e. $\tilde{q}_{jk} \neq t_k$. Then, for any signal $\tilde{r}_{vk} \in \{A, B\}, \forall v \in M \setminus \{i, j\}$, we have*

$$\mathbb{P}\Big(q_i = t_k \;\Big|\; t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{v:v\in M\setminus\{i,j\}} \{\tilde{q}_{vk} = \tilde{r}_{vk}\}\Big)$$
$$\geq \mathbb{P}\Big(q_j = t_k \;\Big|\; t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{v:v\in M\setminus\{i,j\}} \{\tilde{q}_{vk} = \tilde{r}_{vk}\}\Big).$$

Moreover, the inequality is strict when $\alpha \in (\frac{1}{2}, 1)$.

Proof. Define the following five auxiliary sets

$$\begin{aligned}
E_{ij} &= \{(r_1, \ldots, r_{4m}) \mid (r_1, \ldots, r_{4m}) \in E,\; r_i = r_j = t_k\}, \\
E_{i\bar{j}} &= \{(r_1, \ldots, r_{4m}) \mid (r_1, \ldots, r_{4m}) \in E,\; r_i = t_k, r_j \neq t_k\}, \\
E_{\bar{i}j} &= \{(r_1, \ldots, r_{4m}) \mid (r_1, \ldots, r_{4m}) \in E,\; r_i \neq t_k, r_j = t_k\}, \\
E_i &= E_{ij} \cup E_{i\bar{j}}, \\
E_j &= E_{ij} \cup E_{\bar{i}j}.
\end{aligned}$$

Denote $\tilde{r}_{ik}$ and $\tilde{r}_{jk}$ as the realizations of $\tilde{q}_{ik}$ and $\tilde{q}_{jk}$, respectively. Then, $\tilde{r}_{ik} = t_k$ and $\tilde{r}_{jk} \neq t_k$.

Note that

$$
\begin{aligned}
&\mathbb{P}\Big(q_i = t_k \,\Big|\, t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{v:v\in M\setminus\{i,j\}} \{\tilde{q}_{vk} = \tilde{r}_{vk}\}\Big) \\
&\quad = \frac{\mathbb{P}(q_i = t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid t_k)}{\mathbb{P}(\bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid t_k)} \\
&\quad = \frac{\sum_{r_i = t_k, r_v\in\{A,B\},\forall v\in M\setminus\{i\}} \mathbb{P}(\bigcap_{v=1}^{4m}\{q_v = r_v\}, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid t_k)}{\sum_{r_v\in\{A,B\},\forall v\in M} \mathbb{P}(\bigcap_{v=1}^{4m}\{q_v = r_v\}, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid t_k)} \\
&\quad = \frac{\sum_{r_i = t_k, r_v\in\{A,B\},\forall v\in M\setminus\{i\}} \mathbb{P}(\bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid \bigcap_{v=1}^{4m}\{q_v = r_v\}, t_k)\mathbb{P}(\bigcap_{v=1}^{4m}\{q_v = r_v\} \mid t_k)}{\sum_{r_v\in\{A,B\},\forall v\in M} \mathbb{P}(\bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid \bigcap_{v=1}^{4m}\{q_v = r_v\}, t_k)\mathbb{P}(\bigcap_{v=1}^{4m}\{q_v = r_v\} \mid t_k)} \\
&\quad = \frac{\sum_{(r_1,\ldots,r_{4m})\in E_i} \mathbb{P}(\bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid \bigcap_{v=1}^{4m}\{q_v = r_v\}, t_k)\mathbb{P}(\bigcap_{v=1}^{4m}\{q_v = r_v\} \mid t_k)}{\sum_{(r_1,\ldots,r_{4m})\in E} \mathbb{P}(\bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid \bigcap_{v=1}^{4m}\{q_v = r_v\}, t_k)\mathbb{P}(\bigcap_{v=1}^{4m}\{q_v = r_v\} \mid t_k)} \\
&\quad = \frac{\sum_{(r_1,\ldots,r_{4m})\in E_i} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v)}{\sum_{(r_1,\ldots,r_{4m})\in E} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v)} \\
&\quad = \frac{\sum_{(r_1,\ldots,r_{4m})\in E_i} \prod_{v=1}^{4m} \big(\alpha \mathbf{1}_{\{\tilde{r}_{vk} = r_v\}} + (1-\alpha)\mathbf{1}_{\{\tilde{r}_{vk} \neq r_v\}}\big)}{\sum_{(r_1,\ldots,r_{4m})\in E} \prod_{v=1}^{4m} \big(\alpha \mathbf{1}_{\{\tilde{r}_{vk} = r_v\}} + (1-\alpha)\mathbf{1}_{\{\tilde{r}_{vk} \neq r_v\}}\big)}.
\end{aligned}
$$

Since the denominators of $\mathbb{P}(q_i = t_k \mid t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{v:v\in M\setminus\{i,j\}}\{\tilde{q}_{vk} = \tilde{r}_{vk}\})$ and $\mathbb{P}(q_j = t_k \mid t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{v:v\in M\setminus\{i,j\}}\{\tilde{q}_{vk} = \tilde{r}_{vk}\})$ are the same, it suffices to compare their numerators $\sum_{(r_1,\ldots,r_{4m})\in E_i} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v)$ and $\sum_{(r_1,\ldots,r_{4m})\in E_j} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v)$.

Let $\bar{t}_k \in \{A, B\}$ denote the other type that is different from t_k. There is a one-to-one mapping between two sets $E_{i\bar{j}}$ and $E_{\bar{i}j}$:

$$
\begin{aligned}
(r_1, \ldots, r_{4m}) &= (r_1, \ldots, r_{i-1}, t_k, r_{i+1}, \ldots, r_{j-1}, \bar{t}_k, r_{j+1}, \ldots, r_{4m}) \\
&\mapsto (r_1, \ldots, r_{i-1}, \bar{t}_k, r_{i+1}, \ldots, r_{j-1}, t_k, r_{j+1}, \ldots, r_{4m}) \\
&= (r'_1, \ldots, r'_{4m}).
\end{aligned}
$$

Because $\tilde{r}_{ik} = t_k$ and $\tilde{r}_{jk} \neq t_k$, it is easy to see that for any $(r_1, \ldots, r_{4m}) \in E_{i\bar{j}}$,

$$\prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) = \alpha^2 \prod_{v:v\in M\setminus\{i,j\}} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v).$$

Similarly, for any $(r'_1, \ldots, r'_{4m}) \in E_{\bar{i}j}$,

$$\prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r'_v) = (1-\alpha)^2 \prod_{v:v\in M\setminus\{i,j\}} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r'_v).$$

If $(r'_1, \ldots, r'_{4m})$ corresponds to $(r_1, \ldots, r_{4m})$, then

$$\alpha^2 \prod_{v:v\in M\setminus\{i,j\}} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) \geq (1-\alpha)^2 \prod_{v:v\in M\setminus\{i,j\}} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r'_v)$$

since $\alpha \geq \frac{1}{2}$ and $r_v = r'_v$ for all $v \in M \setminus \{i, j\}$. Note that the inequality is strict when $\alpha > \frac{1}{2}$. Combining all these results, we obtain that

$$\begin{aligned}
&\sum_{(r_1,\ldots,r_{4m})\in E_i} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) \\
&= \sum_{(r_1,\ldots,r_{4m})\in E_{ij}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) + \sum_{(r_1,\ldots,r_{4m})\in E_{i\bar{j}}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) \\
&= \sum_{(r_1,\ldots,r_{4m})\in E_{ij}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) + \sum_{(r_1,\ldots,r_{4m})\in E_{i\bar{j}}} \alpha^2 \prod_{v:v\in M\setminus\{i,j\}} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) \\
&\geq \sum_{(r_1,\ldots,r_{4m})\in E_{ij}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) + \sum_{(r'_1,\ldots,r'_{4m})\in E_{\bar{i}j}} (1-\alpha)^2 \prod_{v:v\in M\setminus\{i,j\}} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r'_v) \\
&= \sum_{(r_1,\ldots,r_{4m})\in E_{ij}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) + \sum_{(r'_1,\ldots,r'_{4m})\in E_{\bar{i}j}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r'_v) \\
&= \sum_{(r_1,\ldots,r_{4m})\in E_j} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v).
\end{aligned}$$

Note that the inequality in the fourth line is strict if $\alpha \in (\frac{1}{2}, 1)$ so that the statement of the lemma holds with a strict inequality in that case. This concludes the proof. □

Consider the case where none of the manufacturers adopts the blockchain. Consider a generic consumer k. Recall that $i(k)$ denotes consumer k's preferred manufacturer, and the expected utility $u_{i(k)k}$ that she receives from purchasing from manufacturer $i(k)$ is given by

$$\begin{aligned} u_{i(k)k} &= \mathbb{E}[V_{i(k)k} \mid \mathcal{F}_k] - P^N_{i(k)} \\ &= (H - L)\mathbb{P}\Big(q_{i(k)} = t_k \;\Big|\; t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\}\Big) + L + \frac{h+l}{2} - \gamma. \end{aligned}$$

We call a signal $\tilde{q}_{ik}$ from manufacturer i favorable if the realization $\tilde{r}_{ik} = t_k$. Next, we show that the number of favorable signals $|\{v : \tilde{r}_{vk} = t_k\}|$ is a sufficient statistic for computing $\mathbb{P}(q_{i(k)} = t_k \mid t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\})$.

Lemma B.1.2. *Consider the case where none of the manufacturers adopts the blockchain. Then, the number of favorable signals $|\{v : \tilde{r}_{vk} = t_k\}|$ is a sufficient statistic for the conditional probability $\mathbb{P}(q_{i(k)} = t_k \mid t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\})$, i.e.*

$$\mathbb{P}\Big(q_{i(k)} = t_k \;\Big|\; t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\}\Big) = \theta\big(|\{v : \tilde{r}_{vk} = t_k\}|\big)$$

for some function $\theta(\cdot) : \{0, \dots, 4m\} \to [0,1]$. Moreover, $0 < \theta_z < 1$ for $z = 0, 1, \dots, 4m$, where $\theta_z := \theta(z)$.

Proof. Let $\{\tilde{r}_{vk}\}_{v \in M}$ denote the realizations of the signals from the manufacturers. Let $z := |\{v : \tilde{r}_{vk} = t_k\}|$ denote the number of favorable signals received by consumer k.

First consider the case where $z > 0$. In this case, Lemma B.1.1 implies that $\tilde{r}_{i(k)k} = t_k$. Since the prior is uniform over the set E, the posterior probability

$$\mathbb{P}\Big(q_{i(k)} = t_k \;\Big|\; t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\}\Big) = \frac{\sum_{(r_1,\dots,r_{4m})\in E_{i(k)}} \mathbb{P}(\bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid \bigcap_{v=1}^{4m}\{q_v = r_v\}, t_k)}{\sum_{(r_1,\dots,r_{4m})\in E} \mathbb{P}(\bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\} \mid \bigcap_{v=1}^{4m}\{q_v = r_v\}, t_k)}$$
$$= \frac{\sum_{(r_1,\dots,r_{4m})\in E_{i(k)}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v)}{\sum_{(r_1,\dots,r_{4m})\in E} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v)},$$

where $E_{i(k)} = \{(r_1, \dots, r_{4m}) \mid (r_1, \dots, r_{4m}) \in E, r_{i(k)} = t_k\}$. First consider the numerator. We know that there are exactly $2m$ manufacturers with type $r_i = t_k$, $2m$ manufacturers with type $r_i \neq t_k$, and $r_{i(k)} = t_k$. The summation in the numerator can be equivalently computed by summing over the possible values for the number of favorable signals $x \leq \min\{z, 2m\}$ that come from manufacturers with type $r_i = t_k$. We can define

$$D_x = \{(r_1, \dots, r_{4m}) \mid |\{v : r_v = \tilde{r}_{vk} = t_k\}| = x)\}$$

as the set of all possible types such that there are x favorable signals coming from manufacturers with type $r_i = t_k$. Then, the summation can be rewritten as

$$\sum_{(r_1,\dots,r_{4m})\in E_{i(k)}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) = \sum_{x=\max\{1,z-2m\}}^{\min\{z,2m\}} \sum_{(r_1,\dots,r_{4m})\in E_{i(k)}\cap D_x} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v).$$

Recall that $|\{v : \tilde{r}_{vk} = t_k\}| = z$ and note that

$$E_{i(k)} \cap D_x = \{(r_1, \dots, r_{4m}) \mid r_{i(k)} = t_k, |\{v : r_v = t_k\}| = 2m, |\{v : r_v = \tilde{r}_{vk} = t_k\}| = x\}.$$

For any $(r_1, \ldots, r_{4m}) \in E_{i(k)} \cap D_x$, we have

$$\begin{aligned}
|\{v : r_v = \tilde{r}_{vk} = t_k\}| &= x, \\
|\{v : r_v \neq t_k, \tilde{r}_{vk} = t_k\}| &= z - x, \\
|\{v : r_v = t_k, \tilde{r}_{vk} \neq t_k\}| &= 2m - x, \\
|\{v : r_v \neq t_k, \tilde{r}_{vk} \neq t_k\}| &= 2m - z + x,
\end{aligned}$$

and thus $\prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) = \alpha^{2m+2x-z}(1-\alpha)^{2m-2x+z}$. Moreover, it is not hard to get that $|E_{i(k)} \cap D_x| = \binom{z-1}{x-1}\binom{4m-z}{2m-x}$. Thus, the numerator

$$\begin{aligned}
\eta_1(z) := & \sum_{(r_1,\ldots,r_{4m}) \in E_{i(k)}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) \\
= & \sum_{x=\max\{1,z-2m\}}^{\min\{z,2m\}} \sum_{(r_1,\ldots,r_{4m}) \in E_{i(k)} \cap D_x} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) \\
= & \sum_{x=\max\{1,z-2m\}}^{\min\{z,2m\}} \sum_{(r_1,\ldots,r_{4m}) \in E_{i(k)} \cap D_x} \alpha^{2m+2x-z}(1-\alpha)^{2m-2x+z} \\
= & \sum_{x=\max\{1,z-2m\}}^{\min\{z,2m\}} |E_{i(k)} \cap D_x| \cdot \alpha^{2m+2x-z}(1-\alpha)^{2m-2x+z} \\
= & \sum_{x=\max\{1,z-2m\}}^{\min\{z,2m\}} \binom{z-1}{x-1}\binom{4m-z}{2m-x} \alpha^{2m+2x-z}(1-\alpha)^{2m-2x+z}
\end{aligned}$$

is only a function of z.

Next, note that the denominator

$$\begin{aligned}
& \sum_{(r_1,\ldots,r_{4m}) \in E} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) \\
& \quad = \sum_{(r_1,\ldots,r_{4m}) \in E_{i(k)}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) + \sum_{(r_1,\ldots,r_{4m}) \in E \setminus E_{i(k)}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v).
\end{aligned}$$

We already have an expression for the first term. So, consider the second term. Since $(r_1, \ldots, r_{4m}) \in$

$E \setminus E_{i(k)}$, we have that $r_{i(k)} \neq t_k$. Using an analysis similar to the one above, we get that

$$\begin{aligned}
&\sum_{(r_1,\ldots,r_{4m})\in E\setminus E_{i(k)}} \prod_{v=1}^{4m} \mathbb{P}(\tilde{q}_{vk} = \tilde{r}_{vk} \mid q_v = r_v) \\
&= \sum_{x=\max\{0,z-2m\}}^{\min\{z-1,2m\}} \binom{z-1}{x}\binom{4m-z}{2m-x} \alpha^{2m+2x-z}(1-\alpha)^{2m-2x+z} =: \eta_2(z).
\end{aligned}$$

Thus, it follows that

$$\mathbb{P}\Big(q_{i(k)} = t_k \;\Big|\; t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\}\Big) = \frac{\eta_1(z)}{\eta_1(z)+\eta_2(z)} =: \theta(z).$$

This establishes that z is a sufficient statistic. Moreover, since $\eta_1(z)$ (resp. $\eta_2(z)$) contains at least one term and $\alpha \in [\frac{1}{2}, 1)$, $\eta_1(z)$ (resp. $\eta_2(z)$) is positive. Therefore, $0 < \theta_z < 1$ for all $z > 0$.

When $z = 0$, none of the signals consumer k observes is equal to t_k. Consumer k will randomly select a manufacturer from M. Since there are $2m$ type A manufacturers and $2m$ type B manufacturers, it is easy to argue that $\theta_0 := \theta(0) = \frac{1}{2} \in (0, 1)$. □

Lemma B.1.3. *Define w_z as the proportion of the consumers who observe z favorable signals, for $z = 0, 1, \ldots, 4m$. Then,*

$$w_z = \sum_{x+y=z, 0\le x,y\le 2m} \binom{2m}{x}\binom{2m}{y} \alpha^{2m+x-y}(1-\alpha)^{2m-x+y}$$

and $\sum_{z=0}^{4m} w_z = 1$, so $\{w_z\}_{z=0}^{4m}$ constitutes a probability mass function.

Proof. Define w_z^A as the measure of the type A consumers who observe z favorable signals, for $z = 0, 1, \ldots, 4m$. Then,

$$\begin{aligned}
w_z^A &= \frac{1}{2} \sum_{x+y=z, 0\le x,y\le 2m} \binom{2m}{x} \alpha^x (1-\alpha)^{2m-x} \binom{2m}{y} \alpha^{2m-y}(1-\alpha)^y \\
&= \frac{1}{2} \sum_{x+y=z, 0\le x,y\le 2m} \binom{2m}{x}\binom{2m}{y} \alpha^{2m+x-y}(1-\alpha)^{2m-x+y},
\end{aligned}$$

where x (resp. y) denotes the number of type A (resp. type B) manufacturers that emit type A signals. The term $\binom{2m}{x}\alpha^x(1-\alpha)^{2m-x}\binom{2m}{y}\alpha^{2m-y}(1-\alpha)^y$ represents the proportion of the type A consumers who observe x favorable signals from type A manufacturers and y favorable signals from type B manufacturers. Similarly, we can get w_z^B and actually $w_z^B = w_z^A$. Hence,

$$\begin{aligned} w_z &= w_z^A + w_z^B \\ &= \sum_{x+y=z, 0\le x,y\le 2m} \binom{2m}{x}\binom{2m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y}. \end{aligned}$$

Given the expression for w_z, we have that

$$\begin{aligned} \sum_{z=0}^{4m} w_z &= \sum_{z=0}^{4m} \sum_{x+y=z, 0\le x,y\le 2m} \binom{2m}{x}\binom{2m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y} \\ &= \left(\sum_{x=0}^{2m}\binom{2m}{x}\alpha^x(1-\alpha)^{2m-x}\right)\left(\sum_{y=0}^{2m}\binom{2m}{y}\alpha^{2m-y}(1-\alpha)^y\right) \\ &= 1. \end{aligned}$$

□

Lemma B.1.4. *Define π_z as the proportion of the consumers who observe z favorable signals that are all emitted from type l manufacturers, for $z = 1, 2, \ldots, 4m$. Then,*

$$\pi_z = \sum_{x+y=z, 0\le x,y\le m} \binom{m}{x}\binom{m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y}$$

for $z = 1, 2, \ldots, 2m$ and $\pi_z = 0$ for $z > 2m$. If we define $\pi_0 := \alpha^{2m}(1-\alpha)^{2m}$, then $\sum_{z=0}^{2m}\pi_z = \alpha^m(1-\alpha)^m$.

Proof. When $z > 2m$, it is impossible that all z favorable signals come from type l manufacturers, as there are $2m$ type l manufacturers in total. Thus, $\pi_z = 0$ for $z > 2m$. When $1 \le z \le 2m$, define π_z^A as the measure of the type A consumers who observe z favorable signals that are all emitted from type l manufacturers. Consider a type A consumer who observes x favorable signals from type A manufacturers and y favorable signals from type B manufacturers. We know from the proof of Lemma B.1.3 that the measure of such consumers equals $\frac{1}{2}\binom{2m}{x}\binom{2m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y}$. Next, we derive the probability that all these $x+y$ favorable signals come from type l manufacturers; denote this probability by $p^A_{x,y}$. When $x > m$ or $y > m$, it is easy to see that $p^A_{x,y} = 0$ since there are m type Al manufacturers and m type Bl manufacturers in total. When $0 \le x, y \le m$, $p^A_{x,y} = \binom{2m-x}{m-x}\cdot\binom{2m-y}{m-y}/\binom{2m}{m}^2$ by simple combinatorics. Combining all the above, we obtain that

$$\begin{aligned}
\pi^A_z &= \frac{1}{2}\sum_{x+y=z,0\le x,y\le 2m}\binom{2m}{x}\binom{2m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y}\cdot p^A_{x,y} \\
&= \frac{1}{2}\sum_{x+y=z,0\le x,y\le m}\binom{2m}{x}\binom{2m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y}\cdot\frac{\binom{2m-x}{m-x}\binom{2m-y}{m-y}}{\binom{2m}{m}^2} \\
&= \frac{1}{2}\sum_{x+y=z,0\le x,y\le m}\binom{m}{x}\binom{m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y}.
\end{aligned}$$

Similarly, we can get π^B_z and actually $\pi^B_z = \pi^A_z$. Hence,

$$\begin{aligned}
\pi_z &= \pi^A_z + \pi^B_z \\
&= \sum_{x+y=z,0\le x,y\le m}\binom{m}{x}\binom{m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y}.
\end{aligned}$$

Given the expression for π_z, we have that

$$\begin{aligned}
\sum_{z=0}^{2m}\pi_z = \pi_0 + \sum_{z=1}^{2m}\pi_z & \\
&= \alpha^{2m}(1-\alpha)^{2m} + \sum_{z=1}^{2m}\sum_{x+y=z,0\le x,y\le m}\binom{m}{x}\binom{m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y} \\
&= \sum_{z=0}^{2m}\sum_{x+y=z,0\le x,y\le m}\binom{m}{x}\binom{m}{y}\alpha^{2m+x-y}(1-\alpha)^{2m-x+y} \\
&= \left(\sum_{x=0}^{m}\binom{m}{x}\alpha^{x}(1-\alpha)^{2m-x}\right)\left(\sum_{y=0}^{m}\binom{m}{y}\alpha^{2m-y}(1-\alpha)^{y}\right) \\
&= \alpha^m(1-\alpha)^m.
\end{aligned}$$

□

Lemma B.1.5. *The function $h(\phi_k;\phi) := g(1,\phi_k;\phi) - g(0,\phi_k;\phi)$ has the following properties.*

(a). $h(\phi_k;\phi)$ is continuous, piecewise linear and weakly decreasing in ϕ_k on $[0,+\infty)$.

(b). For all sufficiently large Δ, there exists $\phi^* \in [0,+\infty)$ such that $h(\phi^*;\phi^*) = 0$ and $h(\phi_k;\phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. Therefore, $b_k = 1$ if and only if $\phi_k \leq \phi^*$.

Proof. By definition,

$$\begin{aligned} h(\phi_k;\phi) &= g(1,\phi_k;\phi) - g(0,\phi_k;\phi) \\ &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} - \sum_{z=0}^{2m}\bar{w}_z \cdot \max\{\tilde{\theta}_z\Delta + L + h - \gamma,\ \phi_k\}. \end{aligned}$$

Each term of it is continuous in ϕ_k on $[0,+\infty)$, so $h(\phi_k;\phi)$ as a function of ϕ_k is continuous on $[0,+\infty)$. By Lemma B.1.2, $0 < \min_{0\leq z\leq 2m}\tilde{\theta}_z < 1$. Depending on how large Δ is, there are three cases, each of which we subsequently examine separately: $\min_{0\leq z\leq 2m}\tilde{\theta}_z \cdot \Delta + L + h - \gamma > 0$, $\min_{0\leq z\leq 2m}\tilde{\theta}_z \cdot \Delta + L + h - \gamma \leq 0$ but $\Delta + L + h - \gamma > 0$, and $\Delta + L + h - \gamma \leq 0$. Consider the following three cases:

(1). $\min_{0\leq z\leq 2m}\tilde{\theta}_z \cdot \Delta + L + h - \gamma > 0$

In this case, we can partition $[0,+\infty)$ into three intervals $[0, \min_{0\leq z\leq 2m}\tilde{\theta}_z \cdot \Delta + L + h - \gamma]$, $(\min_{0\leq z\leq 2m}\tilde{\theta}_z \cdot \Delta + L + h - \gamma, \Delta + L + h - \gamma]$, and $(\Delta + L + h - \gamma, +\infty)$. $h(\phi_k;\phi)$ as a function of ϕ_k is constant on the first and last intervals, and piecewise linearly strictly decreasing on the middle interval. Moreover, $h(\phi_k;\phi)$ is continuous so that $h(\phi_k;\phi)$ is continuous, piecewise linear and weakly decreasing in general.

When $\phi_k \in [0, \min_{0 \le z \le 2m} \tilde{\theta}_z \cdot \Delta + L + h - \gamma]$, $h(\phi_k; \phi)$ is given by:

$$
\begin{aligned}
h(\phi_k; \phi) &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i \in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} \bar{w}_z \cdot \max\{\tilde{\theta}_z \Delta + L + h - \gamma,\ \phi_k\} \\
&= (\Delta + L + h - \gamma) - \frac{\sum_{i:i \in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} \bar{w}_z (\tilde{\theta}_z \Delta + L + h - \gamma) \\
&= \left(1 - \sum_{z=0}^{2m} \bar{w}_z \tilde{\theta}_z\right) \Delta - \frac{\sum_{i:i \in M} \chi_i}{G(\phi)},
\end{aligned}
$$

which does not depend upon ϕ_k.

When $\phi_k \in (\min_{0 \le z \le 2m} \tilde{\theta}_z \cdot \Delta + L + h - \gamma, \Delta + L + h - \gamma]$, $h(\phi_k; \phi)$ is given by:

$$
\begin{aligned}
h(\phi_k; \phi) \quad &= \quad \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i \in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} \bar{w}_z \cdot \max\{\tilde{\theta}_z \Delta + L + h - \gamma,\ \phi_k\} \\
&= \quad (\Delta + L + h - \gamma) - \frac{\sum_{i:i \in M} \chi_i}{G(\phi)} - \sum_{z:\tilde{\theta}_z \Delta + L + h - \gamma > \phi_k} \bar{w}_z (\tilde{\theta}_z \Delta + L + h - \gamma) \\
&\qquad - \sum_{z:\tilde{\theta}_z \Delta + L + h - \gamma \le \phi_k} \bar{w}_z \phi_k.
\end{aligned}
$$

Note that $\sum_{z:\tilde{\theta}_z \Delta + L + h - \gamma \le \phi_k} \bar{w}_z > 0$ when $\phi_k \in (\min_{0 \le z \le 2m} \tilde{\theta}_z \cdot \Delta + L + h - \gamma, \Delta + L + h - \gamma]$, so $h(\phi_k; \phi)$ is piecewise linear and strictly decreasing in ϕ_k on this interval.

When $\phi_k \in (\Delta + L + h - \gamma, +\infty)$, $h(\phi_k; \phi)$ is given by:

$$
\begin{aligned}
h(\phi_k; \phi) &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i \in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} \bar{w}_z \cdot \max\{\tilde{\theta}_z \Delta + L + h - \gamma,\ \phi_k\} \\
&= \phi_k - \frac{\sum_{i:i \in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} \bar{w}_z \phi_k \\
&= -\frac{\sum_{i:i \in M} \chi_i}{G(\phi)},
\end{aligned}
$$

which does not depend upon ϕ_k.

(2). $\min_{0\le z\le 2m} \tilde{\theta}_z \cdot \Delta + L + h - \gamma \le 0$ but $\Delta + L + h - \gamma > 0$

In this case, we can partition $[0, +\infty)$ into two intervals $[0, \Delta + L + h - \gamma]$ and $(\Delta + L + h - \gamma, +\infty)$. By a similar argument, we can prove that $h(\phi_k;\phi)$ is continuous, piecewise linear and weakly decreasing on $[0, +\infty)$. More specifically, it is strictly decreasing on $[0, \Delta + L + h - \gamma]$ and flat on $(\Delta + L + h - \gamma, +\infty)$.

(3). $\Delta + L + h - \gamma \le 0$

In this case, $h(\phi_k;\phi) = -\frac{\sum_{i:i\in M}\chi_i}{G(\phi)}$ is a constant function on $[0, +\infty)$ so that it is trivially continuous, piecewise linear and weakly decreasing.

In conclusion, across all three cases, $h(\phi_k;\phi)$ is continuous, piecewise linear and weakly decreasing in ϕ_k on $[0, +\infty)$. This completes the proof of (a).

When $\Delta > \frac{G^{-1}(0.5) - L - h + \gamma}{\min_{0\le z\le 2m}\tilde{\theta}_z}$, $\min_{0\le z\le 2m} \tilde{\theta}_z \cdot \Delta + L + h - \gamma > G^{-1}(0.5) > 0$ holds true. It follows from Case (1) in Property (a) that $h(\phi_k;\phi)$ with such Δ is continuous, piecewise linear and weakly decreasing in ϕ_k on $[0, +\infty)$. More specifically, it is strictly decreasing on $(\min_{0\le z\le 2m} \tilde{\theta}_z \cdot \Delta + L + h - \gamma, \Delta + L + h - \gamma]$ and flat on $[0, \min_{0\le z\le 2m} \tilde{\theta}_z \cdot \Delta + L + h - \gamma]$ and $(\Delta + L + h - \gamma, +\infty)$. Thus, if we can find a ϕ^* such that $\phi^* \in (\min_{0\le z\le 2m} \tilde{\theta}_z \cdot \Delta + L + h - \gamma, \Delta + L + h - \gamma]$ and $h(\phi^*;\phi^*) = 0$, then $h(\phi_k;\phi^*) \ge 0$ if and only if $\phi_k \le \phi^*$.

Note that

$$
\begin{aligned}
l(\phi) &:= h(\phi;\phi) \\
&= g(1, \phi;\phi) - g(0, \phi;\phi) \\
&= \max\{\Delta + L + h - \gamma,\ \phi\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} - \sum_{z=0}^{2m} \bar{w}_z \cdot \max\{\tilde{\theta}_z\Delta + L + h - \gamma,\ \phi\}
\end{aligned}
$$

is continuous in ϕ on $(0, +\infty)$ and

$$
l(\Delta + L + h - \gamma) = -\frac{\sum_{i:i\in M}\chi_i}{G(\Delta + L + h - \gamma)} < 0.
$$

By Lemma B.1.2, $0 < \max_{0\le z\le 2m} \tilde{\theta}_z < 1$. When $\Delta > \max\{\frac{2\sum_{i:i\in M}\chi_i}{1-\max_{0\le z\le 2m}\tilde{\theta}_z}, \frac{G^{-1}(0.5)-L-h+\gamma}{\min_{0\le z\le 2m}\tilde{\theta}_z}\}$,

$$
\begin{aligned}
l(\min_{0\le z\le 2m}\tilde{\theta}_z\cdot\Delta+L+h-\gamma) &= \left(1-\sum_{z=0}^{2m}\bar{w}_z\tilde{\theta}_z\right)\Delta-\frac{\sum_{i:i\in M}\chi_i}{G(\min_{0\le z\le 2m}\tilde{\theta}_z\cdot\Delta+L+h-\gamma)}\\
&\ge \left(1-\max_{0\le z\le 2m}\tilde{\theta}_z\cdot\sum_{z=0}^{2m}\bar{w}_z\right)\Delta-\frac{\sum_{i:i\in M}\chi_i}{G(G^{-1}(0.5))}\\
&= \left(1-\max_{0\le z\le 2m}\tilde{\theta}_z\right)\Delta-2\sum_{i:i\in M}\chi_i\\
&> 0.
\end{aligned}
$$

This implies that when $\Delta > \max\{\frac{2\sum_{i:i\in M}\chi_i}{1-\max_{0\le z\le 2m}\tilde{\theta}_z}, \frac{G^{-1}(0.5)-L-h+\gamma}{\min_{0\le z\le 2m}\tilde{\theta}_z}\}$, by intermediate value theorem, there exists a solution $\phi^* \in (\min_{0\le z\le 2m}\tilde{\theta}_z\cdot\Delta+L+h-\gamma, \Delta+L+h-\gamma]$ such that $h(\phi^*;\phi^*) = l(\phi^*) = 0$. With this ϕ^*, we have that $h(\phi_k;\phi^*) \ge 0$ if and only if $\phi_k \le \phi^*$. This completes the proof of (b). □

B.2 Main Proofs

Appendix B.2 provides proofs of propositions stated in the body of Chapter 2 for $\mathbb{P}(\tilde{q}_{ik} = q_i \mid q_i) = \alpha \in [\frac{1}{2}, 1)$ and $\mathbb{P}(\tilde{\xi}_{ik} = \xi_i \mid \xi_i) = \frac{1}{2}$. For simplicity, we focus upon the cases of full and non-adoption of blockchain. Our proofs rely on the lemmas provided within Appendix B.1. Moreover, these proofs rely on model solutions of prices, market shares and expected consumer utilities. We derive the endogenous values of all such quantities in Proposition B.2.1, which is stated and proved at the beginning of this section. Since c_h is linear and strictly increasing, we assume $c_h(x) = \gamma x$ with $\gamma > 0$ in the appendix.

We impose the following regularity conditions on the cost functions c_h and c_l of the high and low type vendors:

Assumption B.2.1. We assume that the cost function $c_h(x) = \gamma x$ of the high type vendor and the cost function $c_l(x)$ of the low type vendor satisfy the following regularity conditions:

1. $c_l(0) = c_h(0) = 0$.

2. c_l is convex with a strictly increasing derivative c_l'.

3. $c_l'(\alpha^m(1-\alpha)^m) < \gamma < c_l'\Big(G\big(\frac{H+L+h+l}{2} - \gamma\big) - \alpha^m(1-\alpha)^m\Big)$.

Proposition B.2.1. *Define $M_h := \{i : \xi_i = h\}$, $M_l := \{i : \xi_i = l\}$, $s_h := \sum_{i:i\in M_h} s_i$, and $s_l := \sum_{i:i\in M_l} s_i$. The equilibrium prices, market shares, and expected consumer utilities under full blockchain adoption and blockchain non-adoption are as follows.*

1. ***Full Blockchain Adoption:*** *$P_i^F = \Psi_i^F = \Psi_l^F := \max\{\gamma - (h-l), 0\}$ for all $i \in M_l$, $P_i^F = \Psi_i^F = \Psi_h^F := \gamma$ for all $i \in M_h$, $s_l^F = (c_l')^{-1}(\max\{\gamma-(h-l),0\})$, $s_h^F = G(H+h-\gamma) - s_l^F$, and $u_{i(k)k} = H + h - \gamma$ for all $k \in [0,1]$.*

2. ***Blockchain Non-Adoption:*** *$P_i^N = \Psi_i^N = \Psi_l^N = \Psi_h^N := \gamma$ for all $i \in M$, $s_l^N = (c_l')^{-1}(\gamma)$, $s_h^N = \sum_{z=0}^{4m} w_z G(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma) - s_l^N$, and $u_{i(k)k} = \theta_z(H-L) + L + \frac{h+l}{2} - \gamma$ w.p. w_z for $z = 0, 1, \ldots, 4m$ for all $k \in [0,1]$.*

Variables z, θ_z and w_z are all defined in Appendix B.1.

Proof. For manufacturer i, once adoption decision a_i is made, adoption cost $\chi_i a_i$ becomes a sunk cost and she is now in the subgame of choosing her consumer price P_i and vendor price Ψ_i. In the following, we look for a symmetric equilibrium where the vendor prices of the manufacturers of the same vertical type are the same, i.e., $\Psi_i = \Psi_h$ for all $i \in M_h$ and $\Psi_i = \Psi_l$ for all $i \in M_l$.

- **Full Blockchain Adoption:** Given the vendor price Ψ_h^F, vendor h faces the following optimization problem:

$$\max_{\{\sigma_i : i\in M_h\}} \Psi_h^F \cdot \sum_{i:i\in M_h} \sigma_i - \gamma \cdot \sum_{i:i\in M_h} \sigma_i$$
$$\text{s.t. } 0 \le \sigma_i \le s_i^F \text{ for all } i : i \in M_h$$

  Therefore, the optimal fulfillment level $\sigma_i^* = s_i^F \cdot \mathcal{I}(\Psi_h^F \ge \gamma)$ for all $i : i \in M_h$. Since each type h manufacturer faces an arbitrarily large cost for not fulfilling any consumer order and thus wants to set Ψ_h^F to the lowest possible price such that $\sigma_i^* = s_i^F$, it follows that $\Psi_h^F = \gamma$. Given the vendor price Ψ_l^F, vendor l faces the following decision problem:

$$\max_{\{\sigma_i : i \in M_l\}} \ \Psi_l^F \cdot \sum_{i: i \in M_l} \sigma_i - c_l \Big(\sum_{i: i \in M_l} \sigma_i \Big)$$

$$\text{s.t.} \ \ 0 \le \sigma_i \le s_i^F \text{ for all } i : i \in M_l$$

Define $\sigma_l := \sum_{i:i \in M_l} \sigma_i$. Then, the objective $\Psi_l^F \sigma_l - c_l(\sigma_l)$ is strictly concave in σ_l, and the first order condition tells us that its unconstrained maximum is attained at $\sigma_l^* = (c_l')^{-1}(\Psi_l^F)$. So long as $\Psi_l^F \ge c_l'(s_l^F)$, the constrained maximum can be attained at $\sigma_l^* = s_l^F$. Thus, because c_l' is strictly increasing, the lowest possible price such that $\sigma_l^* = s_l^F$ satisfies $\Psi_l^F = c_l'(s_l^F)$.

Under full blockchain adoption, consumers know each manufacturer's type and will select a manufacturer of her own type. For consumer k whose type is t_k, we have that

$$\mathbb{E}[V_{ik} \mid \mathcal{F}_k] = \begin{cases} H + h & \text{if } i \in M_{t_k h} \\ H + l & \text{if } i \in M_{t_k l} \\ L + h & \text{if } i \in M_{\bar{t}_k h} \\ L + l & \text{if } i \in M_{\bar{t}_k l} \end{cases},$$

where $\bar{t}_k \in \{A, B\}$ is the other type that is different from t_k. Type Ah (resp. type Bh) manufacturers are competing for the market consisting of type A (resp. type B) consumers; type Al (resp. type Bl) manufacturers are competing for the market consisting of type A (resp. type B) consumers. Thus, we have four Bertrand games for type Ah, type Al, type Bh, and type Bl manufacturers respectively. Due to price competition, manufacturer i will set her consumer price P_i to her cost (i.e., her vendor price) so that the other manufacturers of the same type have no chance to undercut to obtain the entire market. Accordingly, $P_i^F = \Psi_h^F$ for all $i \in M_h$ and $P_i^F = \Psi_l^F$ for all $i \in M_l$. Define $P_h^F := \Psi_h^F$ and $P_l^F := \Psi_l^F$. Furthermore, type Al (resp. type Bl) manufacturers are willing to lower their consumer prices to compete with type Ah (resp. type Bh) manufacturers so that a type A (resp. type B) consumer is indifferent between a type Al (resp. type Bl) manufacturer and a type Ah (resp. type Bh) manufacturer. This gives us $H + l - P_l^F = H + h - P_h^F$ and thus $P_l^F = P_h^F - (h - l)$. It is possible that even if a type Al (resp. type Bl) manufacturer sets her consumer price to zero, the expected utility she provides to a type A (resp. type B) consumer is still lower than that provided by a type Ah (resp. type Bh) manufacturer. Taking this situation into consideration, we have that $P_l^F = \max\{P_h^F - (h - l), 0\}$.

The final piece is the market share. From the above analysis, we know that consumer k will always select a type h or type l manufacturer of her own type, and the corresponding expected utility $u_{i(k)k} = H + h - P_h^F$. She will then compare $u_{i(k)k}$ to her outside option ϕ_k. Consequently, we have that $s_h^F + s_l^F = \mathbb{P}(u_{i(k)k} \geq \phi_k) = G(H + h - P_h^F)$.

Combining all the above, we obtain the following system of equations:

$$
\begin{aligned}
\Psi_h^F &= \gamma, \\
\Psi_l^F &= c_l'(s_l^F), \\
P_h^F &= \Psi_h^F, \\
P_l^F &= \Psi_l^F, \\
P_l^F &= \max\{P_h^F - (h - l), 0\}, \\
s_h^F + s_l^F &= G(H + h - P_h^F).
\end{aligned}
$$

The solutions are

$$
\begin{aligned}
\Psi_h^F &= \gamma, \\
\Psi_l^F &= \max\{\gamma - (h - l), 0\}, \\
s_h^F &= G(H + h - \gamma) - s_l^F, \\
s_l^F &= (c_l')^{-1}(\max\{\gamma - (h - l), 0\}).
\end{aligned}
$$

The conditions in Assumption B.2.1 ensure a positive s_h^F as $c_l'\Big(G(H+h-\gamma)\Big) \geq c_l'\Big(G(\frac{H+L+h+l}{2} - \gamma)\Big) > c_l'\Big(G(\frac{H+L+h+l}{2} - \gamma) - \alpha^m(1-\alpha)^m\Big) > \gamma > \max\{\gamma - (h-l), 0\}$. It can be verified that the above solutions constitute a separating equilibrium. In equilibrium, type Ah (resp. type Bh, type Al, type Bl) manufacturers split the market share $\frac{s_h^F}{2}$ (resp. $\frac{s_h^F}{2}, \frac{s_l^F}{2}, \frac{s_l^F}{2}$) evenly.

- **Blockchain Non-Adoption:** Following the analysis we did for full blockchain adoption, we have that $\Psi_h^N = \gamma$ and $\Psi_l^N = (c_l')^{-1}(s_l^N)$. Consider the competition among the manufacturers of the same type (type Ah, type Bh, type Al, or type Bl). Suppose there exists a symmetric Nash equilibrium. Then, the price competition implies that the equilibrium consumer price for manufacturer i in the subgame must equal her cost (i.e., her vendor price), i.e., $P_i^N = P_h^N := \Psi_h^N$ for all $i \in M_h$ and $P_i^N = P_l^N := \Psi_l^N$ for all $i \in M_l$. Again, we consider the pooling equilibrium in which $P_h^N = P_l^N$; a separating equilibrium with $P_h^N \neq P_l^N$ is impossible here because a type l manufacturer can mimick the type h manufacturer at no cost in the absence of blockchain, which results in a profitable deviation. For $\alpha = \frac{1}{2}$, such a pooling equilibrium exists in the subgame, and the equilibrium prices are given by $P_i^N = P_h^N = P_l^N = \Psi_i^N = \Psi_l^N = \Psi_h^N = \gamma$ for all $i \in M$.

  However, for $\alpha \in (\frac{1}{2}, 1)$, no such equilibrium exists in the subgame because each manufacturer has a unilateral profitable deviation where she raises her consumer price by a sufficiently small amount. In particular, suppose manufacturer i raises her price by $\delta > 0$. Then, if δ is sufficiently small, consumer k still prefers manufacturer i to all other manufacturers if and only if $\tilde{q}_{ik} = t_k$ and for all $j \neq i : \tilde{q}_{jk} \neq t_k$. Note that when δ is too large, no consumer would purchase from manufacturer i; in fact, a crude bound is $\delta \leq H - L$. Moreover, Lemma B.1.3 establishes that the measure of consumers that prefer manufacturer i to all other manufacturers for a sufficiently small price increase is bounded above by $\frac{w_1}{4m}$. Even if all of these consumers forgo the outside option, the incremental profit from manufacturer i raising her price above the expected cost is bounded above by $\varepsilon_m = \frac{w_1}{4m} \cdot (H-L)$. Since $\lim_{m\to\infty} w_1 = 0$, it follows that $\lim_{m\to\infty} \varepsilon_m = 0$. Thus, the manufacturer's incremental profit vanishes as $m \to \infty$.

Indeed, the non-existence of pure strategy equilibria has been recognized as a major conceptual difficulty in price-setting models (see Dixon 1987), and one alternative that has been explored is the notion of approximate equilibria. For example, Dixon (Dixon 1987, Definition 3, page 47) considers the notion of an ε-equilibrium, defined as one in which each agent is within ε of her best payoff, given the actions of the other agents. (In practice there are some small but positive costs associated with a price adjustment that can justify an ε-equilibrium.) For the price setting problem faced by the manufacturers in our model, we use this definition of ε-equilibrium. It is clear that our solution is an ε-equilibrium in the subgame for all $\varepsilon \leq \varepsilon_m$ with $\lim_{m\to\infty} \varepsilon_m = 0$. In economic terms, a manufacturer's gain from unilaterally raising her price is negligible, particularly in markets with many competitors. Thus, for $\alpha \in (\frac{1}{2}, 1)$, $P_i^N = P_h^N = P_l^N = \Psi_i^N = \Psi_l^N = \Psi_h^N = \gamma$ for all $i \in M$ are the prices corresponding to an ε-equilibrium in the subgame for $\varepsilon = \frac{w_1(H-L)}{4m}$, and this approximate notion of equilibrium becomes exact in the limit (i.e., as $m \to \infty$).

Next, we derive the mathematical expression for the market share $s_h^N + s_l^N$. Consider a generic consumer k. By definition, consumer k's information set

$$\mathcal{F}_k = \sigma\left(t_k, \phi_k, \{a_v, P_v^N, \tilde{q}_{vk}, \tilde{\xi}_{vk}\}_{v\in M}\right).$$

Note that ϕ_k is independent of t_k and $\{a_v, P_v^N, q_v, \tilde{q}_{vk}, \xi_v, \tilde{\xi}_{vk}\}_{v\in M}$ and has no impact on $\{V_{vk}, u_{vk}\}_{v\in M}$, so we can safely drop it from $\mathcal{F}_k$ in the following discussion. Since the adoption decisions and the equilibrium consumer prices under blockchain non-adoption are the same for all manufacturers, the adoption decisions $\bigcap_{v=1}^{4m}\{a_v = 0\}$ and the consumer prices $\bigcap_{v=1}^{4m}\{P_v^N = \gamma\}$ do not provide extra information for consumer k to distinguish between manufacturers. Therefore, we do not include $\{a_v, P_v^N\}_{v\in M}$ into $\mathcal{F}_k$ in the following discussion.

To sum up, $\mathcal{F}_k$ is now simplified to $\sigma\left(t_k, \{\tilde{q}_{vk}, \tilde{\xi}_{vk}\}_{v\in M}\right)$. It follows that

$$\begin{aligned}
\mathbb{E}[V_{ik} \mid \mathcal{F}_k] &= \mathbb{E}[\eta_{ik} \mid \mathcal{F}_k] + \mathbb{E}[\xi_i \mid \mathcal{F}_k] \\
&= (H-L)\mathbb{P}(q_i = t_k \mid t_k, \{\tilde{q}_{vk}, \tilde{\xi}_{vk}\}_{v\in M}) + L + \mathbb{E}[\xi_i \mid t_k, \{\tilde{q}_{vk}, \tilde{\xi}_{vk}\}_{v\in M}] \\
&= (H-L)\mathbb{P}(q_i = t_k \mid t_k, \{\tilde{q}_{vk}\}_{v\in M}) + L + \mathbb{E}[\xi_i \mid \{\tilde{\xi}_{vk}\}_{v\in M}],
\end{aligned}$$

where the third equality holds because $\{q_v, \tilde{q}_{vk}\}_{v\in M}$ is independent of $\{\xi_v, \tilde{\xi}_{vk}\}_{v\in M}$. Since $\mathbb{P}(\tilde{\xi}_{ik} = \xi_i \mid \xi_i) = \frac{1}{2}$ for all $i \in M$, the vertically differentiated type signals $\{\tilde{\xi}_{vk}\}_{v\in M}$ are uninformative. Therefore, conditional on $\{\tilde{\xi}_{vk}\}_{v\in M}$, ξ_i equals h and l with equal probability, and we get that $\mathbb{E}[\xi_i \mid \{\tilde{\xi}_{vk}\}_{v\in M}] = \frac{h+l}{2}$. Note that $P_i^N = \gamma$ for all $i \in M$ and

$$\mathbb{E}[V_{ik} \mid \mathcal{F}_k] = (H-L)\mathbb{P}(q_i = t_k \mid t_k, \{\tilde{q}_{vk}\}_{v\in M}) + L + \frac{h+l}{2}$$

after simplification, so consumer k only relies on the horizontally differentiated type signals she received to select among the manufacturers. Treat $\{\tilde{q}_{vk}\}_{v\in M}$ as random variables and suppose that the signals consumer k observes are $\{\tilde{r}_{vk}\}_{v\in M}$, which are the realizations of $\{\tilde{q}_{vk}\}_{v\in M}$. For any realization t_k, $\mathbb{P}(q_i = t_k \mid t_k, \{\tilde{q}_{vk}\}_{v\in M})$ can be regarded as a function mapping $(\tilde{r}_{1k}, \ldots, \tilde{r}_{4m,k})$ to a number between $[0,1]$, namely a probability. Evaluating $\mathbb{P}(q_i = t_k \mid t_k, \{\tilde{q}_{vk}\}_{v\in M})$ at t_k and $\{\tilde{r}_{vk}\}_{v\in M}$, we get the conditional probability $\mathbb{P}(q_i = t_k \mid t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\})$.

From Lemma B.1.2, it follows that

$$\begin{aligned}
u_{i(k)k} &= \mathbb{E}[V_{i(k)k} \mid \mathcal{F}_k] - P_{i(k)}^N \\
&= (H-L)\mathbb{P}(q_{i(k)} = t_k \mid t_k, \{\tilde{q}_{vk}\}_{v\in M}) + L + \frac{h+l}{2} - \gamma \\
&= \theta_Z(H-L) + L + \frac{h+l}{2} - \gamma,
\end{aligned}$$

where $Z := |\{v : \tilde{q}_{vk} = t_k\}|$ is a random variable representing the number of favorable horizontally differentiated type signals received by consumer k. Lemma B.1.3 gives the mathematical expression for w_z, the proportion of the consumers who observe z favorable signals. Consequently, $\mathbb{P}(Z = z) = w_z$ for $z = 0, 1, \ldots, 4m$. Of the w_z fraction of consumers who observe z favorable signals, a $1 - G(\theta_z(H - L) + L + \frac{h+l}{2} - \gamma)$ will take the outside option because $\phi_k > u_{i(k)k} = \theta_z(H - L) + L + \frac{h+l}{2} - \gamma$, and the remaining will select a manufacturer from the ones that emit favorable signals to order the product with. Thus, the market share under blockchain non-adoption $s_h^N + s_l^N = \sum_{z=0}^{4m} w_z G(\theta_z(H - L) + L + \frac{h+l}{2} - \gamma)$.

Combining all the above, we obtain the following system of equations:

$$\begin{aligned}
\Psi_h^N &= \gamma, \\
\Psi_l^N &= c_l'(s_l^N), \\
P_h^N &= \Psi_h^N, \\
P_l^N &= \Psi_l^N, \\
P_h^N &= P_l^N, \\
s_h^N + s_l^N &= \sum_{z=0}^{4m} w_z G\big(\theta_z(H - L) + L + \frac{h + l}{2} - P_h^N\big).
\end{aligned}$$

The solutions are

$$\begin{aligned}
\Psi_h^N &= \gamma, \\
\Psi_l^N &= \gamma, \\
s_h^N &= \sum_{z=0}^{4m} w_z G\big(\theta_z(H - L) + L + \frac{h + l}{2} - \gamma\big) - s_l^N, \\
s_l^N &= (c_l')^{-1}(\gamma).
\end{aligned}$$

Note that some consumers receive favorable signals from only type l manufacturers, and therefore, select a type l manufacturer if they possess a sufficiently low outside option. As a consequence, our solution must satisfy the following condition:

$$s_l^N \geq \sum_{z=1}^{2m} \pi_z G\big(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma\big).$$

By symmetry, the same argument applies for type h manufacturers and thus our solution must also satisfy the following condition:

$$s_h^N \geq \sum_{z=1}^{2m} \pi_z G\big(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma\big).$$

Both these conditions hold due to Assumption B.2.1 and Lemmas B.1.2 - B.1.4, thereby completing the proof. More explicitly, the first condition holds because:

$$s_l^N = (c_l')^{-1}(\gamma) > \alpha^m(1-\alpha)^m \geq \sum_{z=1}^{2m} \pi_z G\big(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma\big)$$

where the first inequality relies on Assumption B.2.1 and the second inequality follows, via Lemma B.1.4, from $\alpha^m(1-\alpha)^m = \sum_{z=0}^{2m} \pi_z \geq \sum_{z=1}^{2m} \pi_z G\big(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma\big)$.

To see why the second condition holds, note that

$$\begin{aligned} s_h^N &= \sum_{z=0}^{4m} w_z G\big(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma\big) - (c_l')^{-1}(\gamma) \\ &\geq \sum_{z=0}^{4m} w_z G\big(\frac{H+L+h+l}{2} - \gamma\big) - (c_l')^{-1}(\gamma) \\ &= G\big(\frac{H+L+h+l}{2} - \gamma\big) - (c_l')^{-1}(\gamma), \end{aligned}$$

by Lemma B.1.2 and Lemma B.1.3, and because $\sum_{z=1}^{2m} \pi_z G\big(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma\big) \leq \sum_{z=0}^{2m} \pi_z = \alpha^m(1-\alpha)^m$ by Lemma B.1.4. Thus, a sufficient condition for the above to hold true is $c_l'\Big(G\big(\frac{H+L+h+l}{2} - \gamma\big) - \alpha^m(1-\alpha)^m\Big) > \gamma$, which is imposed by Assumption B.2.1.

□

Proof of Proposition 2.3.1. Recall that G denotes the cumulative distribution function of the outside option ϕ_k and is supported on $[0, +\infty)$. Let $p(v)$ denote the corresponding probability density function, which is also defined on $[0, +\infty)$. Define an auxiliary function $f_1(x) := xG(x) + \int_x^{+\infty} vp(v)dv$. Since $f_1'(x) = G(x) > 0$ for all $x > 0$, $f_1(x)$ is strictly increasing in x on $[0, +\infty)$. By (2.12) and Proposition B.2.1, we have that

$$\begin{aligned}
W_C^F &= \int_0^1 \mathbb{E}[\max\{\max_{i\in M} u_{ik}, \phi_k\}]\ dk \\
&= \mathbb{E}[\max\{H + h - \gamma, \phi_k\}] \\
&= \mathbb{E}[H + h - \gamma \mid H + h - \gamma \geq \phi_k]\mathbb{P}(H + h - \gamma \geq \phi_k) + \mathbb{E}[\phi_k \mid H + h - \gamma < \phi_k]\mathbb{P}(H + h - \gamma < \phi_k) \\
&= (H + h - \gamma)G(H + h - \gamma) + \int_{H+h-\gamma}^{+\infty} vp(v)\ dv \\
&= f_1(H + h - \gamma).
\end{aligned}$$

Similarly,

$$\begin{aligned}
W_C^N &= \int_0^1 \mathbb{E}[\max\{\max_{i\in M} u_{ik}, \phi_k\}]\ dk \\
&= \sum_{z=0}^{4m} w_z \mathbb{E}[\max\{\theta_z(H - L) + L + \frac{h + l}{2} - \gamma, \phi_k\}] \\
&= \sum_{z=0}^{4m} w_z f_1(\theta_z(H - L) + L + \frac{h + l}{2} - \gamma).
\end{aligned}$$

By $H > L$, $h > l$, and Lemma B.1.2, we get that $H > \theta_z(H - L) + L$ and $h > \frac{h+l}{2}$. It readily follows that $W_C^F > W_C^N$ since $f_1(x)$ is strictly increasing and $\sum_{z=0}^{4m} w_z = 1$. □

Proof of Proposition 2.3.2. Proposition 2.3.2 directly follows from Proposition B.2.1, Lemma B.1.2, and Lemma B.1.3.

- **Expected Utility Per Consumer:** By Proposition B.2.1, Lemma B.1.2, and the definition of

u, since $H > L$ and $h > l$, we get that

$$
\begin{aligned}
u^F &= H + h - \gamma, \\
u^N &= \frac{\sum_{z=0}^{4m} w_z G(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma) \cdot \left(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma\right)}{\sum_{z=0}^{4m} w_z G(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma)} \\
&< \frac{\sum_{z=0}^{4m} w_z G(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma) \cdot (H + h - \gamma)}{\sum_{z=0}^{4m} w_z G(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma)} \\
&= H + h - \gamma \\
&= u^F.
\end{aligned}
$$

- **Consumer Demand:** By Proposition B.2.1, $s^F = s_h^F + s_l^F = G(H+h-\gamma)$ and $s^N = s_h^N + s_l^N = \sum_{z=0}^{4m} w_z G(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma)$. Since $\theta_z < 1$ by Lemma B.1.2, $\sum_{z=0}^{4m} w_z = 1$ by Lemma B.1.3, and G is increasing, we have that

$$
\begin{aligned}
s^N &= \sum_{z=0}^{4m} w_z G(\theta_z(H-L) + L + \frac{h+l}{2} - \gamma) \\
&\leq \sum_{z=0}^{4m} w_z G(H + h - \gamma) \\
&= G(H + h - \gamma) \\
&= s^F.
\end{aligned}
$$

□

Proof of Proposition 2.3.3. By Proposition B.2.1, each manufacturer has zero expected profit in the subgame because of price competition, regardless of the level of blockchain adoption. It follows that $W_M = 0 - \sum_{i:i\in M} \chi_i a_i$, so $W_M^F = -\sum_{i:i\in M} \chi_i$ and $W_M^N = 0$. We can see that $W_M^F < W_M^N$. □

Proof of Proposition 2.3.4. Define an auxiliary function $f_2(x) := c_l'(x) \cdot x - c_l(x)$. Then, $f_2(0) = 0$ and $f_2'(x) = c_l''(x) \cdot x > 0$ for all $x > 0$, so $f_2(x)$ is strictly increasing in x on $[0, +\infty)$. By (2.10) and Proposition B.2.1, we have that

$$
\begin{aligned}
W_V^F &= (\Psi_h^F \cdot s_h^F - \gamma \cdot s_h^F) + (\Psi_l^F \cdot s_l^F - c_l(s_l^F)) \\
&= 0 + (c_l'(s_l^F) \cdot s_l^F - c_l(s_l^F)) \\
&= f_2(s_l^F).
\end{aligned}
$$

Similarly, $W_V^N = f_2(s_l^N)$. Since $(c_l')^{-1}$ is strictly increasing, by Proposition B.2.1, $s_l^F = (c_l')^{-1}(\max\{\gamma - (h-l), 0\}) < (c_l')^{-1}(\gamma) = s_l^N$ holds. Therefore, $W_V^F = f_2(s_l^F) < f_2(s_l^N) = W_V^N$. □

Proof of Proposition 2.3.5. By Proposition B.2.1, we know that $\Psi_l^F = c_l'(s_l^F)$ and $\Psi_l^N = c_l'(s_l^N)$. Since c_l' is strictly increasing and we have shown in the proof of Proposition 2.3.4 that $s_l^F < s_l^N$, it immediately follows that $\Psi_l^F < \Psi_l^N$. □

Proof of Proposition 2.3.6. By (2.13), Proposition 2.3.1, Proposition 2.3.3, and Proposition 2.3.4, we have that

$$
\begin{aligned}
W^F &= W_C^F + W_V^F + W_M^F \\
&= f_1(H + h - \gamma) + f_2(s_l^F) - \sum_{i:i\in M} \chi_i \\
&= f_1(\Delta + L + h - \gamma) + f_2(s_l^F) - \sum_{i:i\in M} \chi_i, \\
W^N &= W_C^N + W_V^N + W_M^N \\
&= \sum_{z=0}^{4m} w_z f_1(\theta_z(H - L) + L + \frac{h+l}{2} - \gamma) + f_2(s_l^N) + 0 \\
&= \sum_{z=0}^{4m} w_z f_1(\theta_z \Delta + L + \frac{h+l}{2} - \gamma) + f_2(s_l^N).
\end{aligned}
$$

View L as a fixed parameter and allow Δ to vary. Since $f_1(x)$ is strictly increasing, W^F and W^N all strictly increase in Δ. W^F decreases in $\sum_{i:i\in M} \chi_i$ and is strictly smaller than W^N for sufficiently large $\sum_{i:i\in M} \chi_i$.

Note that

$$W_F - W_N = \Big(f_1(\Delta + L + h - \gamma) - \sum_{z=0}^{4m} w_z f_1(\theta_z \Delta + L + \frac{h+l}{2} - \gamma)\Big) + \Big(f_2(s_l^F) - f_2(s_l^N) - \sum_{i:i \in M} \chi_i\Big).$$

By Proposition B.2.1, $s_l^N = (c_l')^{-1}(\gamma)$. Thus, the second term of $W_F - W_N$, $f_2(s_l^F) - f_2(s_l^N) - \sum_{i:i\in M} \chi_i \geq f_2(0) - f_2(s_l^N) - \sum_{i:i\in M} \chi_i = -f_2((c_l')^{-1}(\gamma)) - \sum_{i:i\in M} \chi_i$. Fixing parameter γ and adoption costs $\chi_i : i \in M$,

$$\begin{aligned}
\frac{\partial(W^F - W^N)}{\partial \Delta} &= G(\Delta + L + h - \gamma) - \sum_{z=0}^{4m} w_z \theta_z G(\theta_z \Delta + L + \frac{h+l}{2} - \gamma) \\
&\geq G(\Delta + L + h - \gamma) - \sum_{z=0}^{4m} w_z \theta_z \cdot 1 \\
&\geq G(\Delta + L + h - \gamma) - \left(\max_{0 \leq z \leq 4m} \theta_z\right) \sum_{z=0}^{4m} w_z \\
&= G(\Delta + L + h - \gamma) - \max_{0 \leq z \leq 4m} \theta_z.
\end{aligned}$$

By Lemma B.1.2, $0 < \max_{0\leq z \leq 4m} \theta_z < 1$. Because $\max_{0\leq z \leq 4m} \theta_z < 1$ and $\lim_{\Delta \to +\infty} G(\Delta + L + h - \gamma) = 1$, $\frac{\partial(W^F - W^N)}{\partial \Delta} \geq \frac{1}{2}(1 - \max_{0\leq z \leq 4m} \theta_z) > 0$ when Δ is large enough. This implies that $\lim_{\Delta\to+\infty} W^F - W^N = +\infty$ no matter how large $f_2((c_l')^{-1}(\gamma))$ and $\sum_{i:i\in M} \chi_i$ are, meaning that W^F strictly exceeds W^N for sufficiently large Δ.

$W^F - W^N$ is negative for sufficiently high adoption costs and positive for sufficiently large Δ, so blockchain adoption has ambiguous effects on global welfare. □

Proof of Proposition 2.3.7. By Proposition B.2.1, a type h manufacturer i (i.e., $i \in M_h$) adopts the blockchain and receives zero expected profit in the subgame due to price competition, i.e., $\Pi(1, a_{-i}) = 0$, under full blockchain adoption. Hence, her expected profit for adopting the blockchain is $0 - \chi_i = -\chi_i$. However, if she chooses not to adopt the blockchain, then her expected profit is always non-negative, which is better than $-\chi_i$. Thus, with full blockchain adoption, manufacturer i has incentive to deviate so there does not exist an equilibrium. □

Proof of Proposition 2.3.8. Proposition 2.3.6 tells us that full blockchain adoption is welfare-enhancing for sufficiently large Δ. However, by Proposition 2.3.7, there does not exist an equilibrium with full blockchain adoption. Hence, an adoption failure arises by definition. □

Proof of Proposition 2.3.9. Recall that G denotes the cumulative distribution function of the outside option ϕ_k of consumer k. We consider the system of transfers where $\tau_i = \chi_i$ for all $i \in M$, and $\kappa = \frac{\sum_{i:i\in M}\chi_i}{G(\phi)}$, where ϕ is to be determined. We will first show that for all sufficiently large Δ, there exists ϕ such that $b_k = 1$ if and only if $\phi_k \leq \phi$ and thus $\mu_\kappa = \mu(\{k : b_k = 1\}) = \mathbb{P}(\phi_k \leq \phi) = G(\phi)$. Since $\kappa \cdot \mu_\kappa = \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} \cdot G(\phi) = \sum_{i:i\in M}\chi_i = \sum_{i:i\in M}\tau_i$, it follows that the system of transfers specified above is self-financing. Next, we will show that $\{a_i = 1, P_i = \Psi_i = \gamma\}_{i\in M_h} \cup \{a_i = 1, P_i = \Psi_i = \max\{\gamma - (h-l), 0\}\}_{i\in M_l}$ can be sustained as an equilibrium under such a system of transfers. Thereby, full blockchain adoption arises in equilibrium and the adoption failure is resolved.

Suppose $a_i = 1$ for all $i \in M$, $P_i = \Psi_i = \Psi_h := \gamma$ for all $i \in M_h$, and $P_i = \Psi_i = \Psi_l := \max\{\gamma - (h-l), 0\}$ for all $i \in M_l$. Consumers are now in the subgame of making the b_k decision. The generic consumer k solves the following optimization problem:

$$\max_{b_k\in\{0,1\}} g(b_k, \phi_k; \phi) := \mathbb{E}^{b_k}[\max\{\max_{i\in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k],$$

where consumer k's information set is given by $\mathcal{G}_k = \sigma(t_k, \phi_k, \{a_i, P_i\}_{i\in M})$. When $b_k = 1$, i.e. consumer k has access to the information on the blockchain, the result in Proposition B.2.1 regarding the full adoption case implies that $\max_{i\in M} u_{ik} = H + h - \gamma = \Delta + L + h - \gamma$. Therefore,

$$\begin{aligned} g(1, \phi_k; \phi) &= \mathbb{E}^1[\max\{\max_{i\in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \\ &= \max\{\Delta + L + h - \gamma,\ \phi_k\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)}. \end{aligned}$$

When $b_k = 0$, consumer k opts to forgo access to the information on the blockchain. Here, the case is a bit different from the non-adoption case: since $P_i = \gamma$ for all $i \in M_h$, $P_i = \max\{\gamma - (h-l), 0\}$ for all $i \in M_l$, and $\gamma \neq \max\{\gamma - (h-l), 0\}$, the consumers are able to distinguish the vertically differentiated type (i.e., type h or type l) among the manufacturers, but they cannot distinguish the horizontally differentiated type (i.e., type A or type B) among the manufacturers. Recall the definition of θ_z in Lemma B.1.2:

$$\theta_z := \mathbb{P}\Big(q_{i(k)} = t_k \;\Big|\; t_k, \bigcap_{v=1}^{4m}\{\tilde{q}_{vk} = \tilde{r}_{vk}\}\Big),$$

where $z = |\{v : \tilde{r}_{vk} = t_k\}|$. Note that θ_z and w_z are defined for consumers selecting among $4m$ manufacturers. Similarly, we can define $\tilde{\theta}_z$ and $\tilde{w}_z$ for a consumer selecting among $2m$ type h (resp. type l) manufacturers. Accordingly, $\max_{i\in M_h} u_{ik} = \tilde{\theta}_{Z_h}(H-L) + L + h - \gamma = \tilde{\theta}_{Z_h}\Delta + L + h - \gamma$ and $\max_{i\in M_l} u_{ik} = \tilde{\theta}_{Z_l}(H-L) + L + h - \gamma = \tilde{\theta}_{Z_l}\Delta + L + h - \gamma$, where $Z_h := |\{v : \tilde{q}_{vk} = t_k, v \in M_h\}|$ and $Z_l := |\{v : \tilde{q}_{vk} = t_k, v \in M_l\}|$ are two i.i.d. random variables and $Z_h = z$ w.p. $\tilde{w}_z$ for $z = 0, 1, \ldots, 2m$. Consumer k finally compares $\max_{i\in M_h} u_{ik}$ and $\max_{i\in M_l} u_{ik}$ to determine $\max_{i\in M} u_{ik}$. Therefore,

$$\begin{aligned} g(0,\phi_k;\phi) &= \mathbb{E}^0[\max\{\max_{i\in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \\ &= \mathbb{E}^0[\max\{\max\{\tilde{\theta}_{Z_h}, \tilde{\theta}_{Z_l}\}\cdot\Delta + L + h - \gamma,\ \phi_k\} \mid \mathcal{G}_k] \\ &= \sum_{z=0}^{2m} \bar{w}_z \cdot \max\{\tilde{\theta}_z\Delta + L + h - \gamma,\ \phi_k\}, \end{aligned}$$

where $\bar{w}_z := \mathbb{P}(\max\{\tilde{\theta}_{Z_h}, \tilde{\theta}_{Z_l}\} = \tilde{\theta}_z)$ for $z = 0, 1, \ldots, 2m$. Note that we still have the properties $0 < \tilde{\theta}_z < 1$ for $z = 0, 1, \ldots, 2m$ and $\sum_{z=0}^{2m} \bar{w}_z = 1$ by the variants of Lemma B.1.2 and Lemma B.1.3 (i.e., replacing m with $\frac{m}{2}$ in those proofs). Consumer k chooses to gain access to the information on the blockchain, i.e. $b_k = 1$, if and only if

$$h(\phi_k;\phi) := g(1,\phi_k;\phi) - g(0,\phi_k;\phi) \geq 0.$$

By Lemma B.1.5(b), it follows that there exists $\phi^* \in [0, +\infty)$ such that $h(\phi^*;\phi^*) = 0$ and $h(\phi_k;\phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. Thus, $b_k = 1$ if and only if $\phi_k \leq \phi^*$ is an optimal decision rule for all consumers. Since measure μ_κ of the consumers who pay the fee κ to gain access to the information on the blockchain satisfies $\mu_\kappa = \mu(\{k : b_k = 1\}) = \mathbb{P}(\phi_k \leq \phi^*) = G(\phi^*)$, it follows that the payment collected from the consumers is $\kappa \cdot \mu_\kappa = \frac{\sum_{i:i\in M} \chi_i}{G(\phi^*)} \cdot G(\phi^*) = \sum_{i:i\in M} \chi_i = \sum_{i:i\in M} \tau_i$, indicating that the system of transfers is self-financing.

Next, we take one step back to the stage where manufacturers make their adoption and pricing decisions. Fix i and suppose $a_j = 1$ for all $j \in M \setminus \{i\}$, $P_j = \Psi_j = \Psi_h$ for all $j \in M_h \setminus \{i\}$, and $P_j = \Psi_j = \Psi_l$ for all $j \in M_l \setminus \{i\}$. For manufacturer i, if she adopts the blockchain, then she obviously has no incentive to increase her vendor price $\Psi_i = \Psi_{\xi_i}$; she also has no incentive to lower $\Psi_i = \Psi_{\xi_i}$ because that results in under-fulfilling her consumer order. Additionally, she has no incentive to change her consumer price from $P_i = \Psi_{\xi_i}$ because of price competition. Her expected profit is zero in this case since $P_i = \Psi_i$ and the transfer $\tau_i = \chi_i$ just covers her adoption cost χ_i. If she does not adopt the blockchain, the consumers still know her type as she is the only manufacturer that is not on the blockchain. She makes her pricing decisions P_i and Ψ_i as if she joins the blockchain. Consequently, her consumer price P_i remains equal to her vendor price Ψ_i in this case. This implies that her expected profit from not adopting the blockchain is zero as well, so she has no incentive to deviate from adopting the blockchain. To conclude, manufacturer i has no incentive to deviate from the adoption decision $a_i = 1$ and the pricing decisions $P_i = \Psi_i = \Psi_{\xi_i}$. This completes the proof. □

Appendix C: Chapter 3 Proofs for $\alpha = \frac{1}{2}$

C.1 Main Proofs

Appendix C.1.2 provides proofs of all propositions stated in the body of Chapter 3. These proofs rely on model solutions of expected good values, prices and market shares. We derive the endogenous values of all such quantities in Proposition C.1.1, which is stated and proved in Appendix C.1.1. All proofs in this appendix assume $\alpha = \frac{1}{2}$. All the results are extended to $\alpha \in [\frac{1}{2}, 1)$ in Appendix D.

C.1.1 Model Solutions

Proposition C.1.1. *The equilibrium expected good values, prices, and market shares under full blockchain adoption, partial blockchain adoption, and blockchain non-adoption are as follows.*

1. ***Full Blockchain Adoption:*** $V_k^F = H$ *for all* $k \in [0,1]$, $P_i^F = P^F := \Psi + cp(1-\rho_{2m})$ *for all* $i \in M$, $s_i^F = \frac{1}{2m}G(H - P^F)$ *for all* $i \in M$, *and* $u_{i(k)k} = H - P^F$ *for all* $k \in [0,1]$.

2. ***Partial Blockchain Adoption:*** $V_k^P = H$ *for all* $k \in [0,1]$, $P_i^P = P_A^P := \Psi + cp(1-\rho_m)$ *for all* $i \in M_A$, $P_i^P = P_B^P := \Psi + cp(1-\rho_1)$ *for all* $i \in M_B$, $s_i^P = \frac{1}{2m}G(H - P_i^P)$ *for all* $i \in M$, *and* $u_{i(k)k} = H - P_{t_k}^P$ *for all* $k \in [0,1]$.

3. ***Blockchain Non-Adoption:*** $V_k^N = \frac{H+L}{2}$ *for all* $k \in [0,1]$, $P_i^N = P^N := \Psi + cp(1-\rho_1)$ *for all* $i \in M$, $s_i^N = \frac{1}{2m}G(\frac{H+L}{2} - P^N)$ *for all* $i \in M$, *and* $u_{i(k)k} = \frac{H+L}{2} - P^N$ *for all* $k \in [0,1]$.

Proof. For manufacturer i, once adoption decision a_i is made, adoption cost $\chi_i a_i$ becomes a sunk cost and she is now in the subgame of choosing her price P_i.

- **Full Blockchain Adoption:** Consumers know each manufacturer's type and will select a manufacturer of her own type, so $V_k^F = H$ for all $k \in [0,1]$. Type A (resp. type B) manufacturers are competing for the market consisting of type A (resp. type B) consumers; we have two Bertrand games for type A and type B manufacturers respectively. Due to price competition, manufacturer i will set P_i to her expected cost so that the other manufacturers of the same type have no chance to undercut to obtain the entire market. Accordingly, $P_i^F = P_A^F := \Psi + cp(1-\rho_{2m})$ for all $i \in M_A$ and $P_i^F = P_B^F := \Psi + cp(1-\rho_{2m})$ for all $i \in M_B$; type A (resp. type B) manufacturers split the market consisting of type A (resp. type B) consumers evenly, $s_i^F = \frac{1}{2m}\mathbb{P}(H - P^F \geq \phi_k) = \frac{1}{2m}G(H - P^F)$ for all $i \in M$. By definition, $\max_{i \in M} u_{ik} = u_{i(k)k} = H - P^F$ for all $k \in [0,1]$.

- **Partial Blockchain Adoption:** Since all type A manufacturers adopt the blockchain and the blockchain is fully revealing, consumers know that a manufacturer with $a_i = 1$ is of type A. As it is a common knowledge that there are exactly m manufacturers of each type, the remaining m manufacturers with $a_i = 0$ must be of type B. Accordingly, consumers know each manufacturer's type based on manufacturers' adoption decisions; $a_i = 1$ implies that manufacturer i is of type A, and $a_i = 0$ implies that manufacturer i is of type B. Hence, $V_k^P = H$ for all $k \in [0,1]$. Again, we have price competitions and the equilibrium price for manufacturer i in the subgame is equal to her expected cost. It follows that $P_i^P = P_A^P := \Psi + cp(1-\rho_m)$ for all $i \in M_A$ and $P_i^P = P_B^P := \Psi + cp(1-\rho_1)$ for all $i \in M_B$; type A (resp. type B) manufacturers split the market consisting of type A (resp. type B) consumers evenly, $s_i^P = s_A^P := \frac{1}{2m}G(H - P_A^P)$ for all $i \in M_A$ and $s_i^P = s_B^P := \frac{1}{2m}G(H - P_B^P)$ for all $i \in M_B$. In order for type A (resp. type B) consumers to stick to type A (resp. type B) manufacturers, an additional condition to be added is $H - P_B^P \geq L - P_A^P$, which is equivalent to $H - L \geq cp(\rho_m - \rho_1)$. By definition, $\max_{i \in M} u_{ik} = u_{i(k)k} = H - P_A^P$ for all type A consumers and $\max_{i \in M} u_{ik} = u_{i(k)k} = H - P_B^P$ for all type B consumers.

- **Blockchain Non-Adoption:** Since $\alpha = \frac{1}{2}$, type signals are uninformative; therefore, $\mathbb{P}(q_i = t_k \mid \mathcal{F}_k) = \frac{1}{2}$ for all $i \in M$ and $k \in [0,1]$, i.e. the manufacturers are indistinguishable from the consumers' perspective. It follows that $V_{ik} = V_k^N = \frac{H+L}{2}$ for all $i \in M$ and $k \in [0,1]$, and consequently, consumers will randomly choose from the set of manufacturers with the lowest price. Hence, due to price competition, all manufacturers will charge exactly the same price, and this price will equal the expected cost, i.e., $P_i^N = P^N := \Psi + cp(1-\rho_1)$ for all $i \in M$. Thus, the market share $s_i^N = \frac{1}{2m}\mathbb{P}(\frac{H+L}{2} - P^N \geq \phi_k) = \frac{1}{2m}G(\frac{H+L}{2} - P^N)$ for all manufacturers $i \in M$, and $\max_{i \in M} u_{ik} = u_{i(k)k} = \frac{H+L}{2} - P^N$ for all consumers $k \in [0,1]$.

□

C.1.2 Proofs of Propositions Stated in Chapter 3

Proof of Proposition 3.3.1. Recall that G denotes the cumulative distribution function of the outside option ϕ_k and supported on $[0,+\infty)$. Let $p(v)$ denote the corresponding probability density function, which is also defined on $[0,+\infty)$. Define an auxiliary function $f(x) := xG(x) + \int_x^{+\infty} vp(v)dv$. Since $f'(x) = G(x) > 0$ for all $x > 0$, $f(x)$ is strictly increasing in x on $[0,+\infty)$. By (3.10) and Proposition C.1.1, we have that

$$
\begin{aligned}
W_C^F &= \int_0^1 \mathbb{E}[\max\{\max_{i\in M} u_{ik}, \phi_k\}]\, dk \\
&= \mathbb{E}[\max\{H - P^F, \phi_k\}] \\
&= \mathbb{E}[H - P^F \mid H - P^F \geq \phi_k]\mathbb{P}(H - P^F \geq \phi_k) + \mathbb{E}[\phi_k \mid H - P^F < \phi_k]\mathbb{P}(H - P^F < \phi_k) \\
&= (H - P^F)G(H - P^F) + \int_{H-P^F}^{+\infty} vp(v)dv \\
&= f(H - P^F).
\end{aligned}
$$

Similarly,

$$\begin{aligned} W_C^P &= \int_0^1 \mathbb{E}[\max\{\max_{i \in M} u_{ik}, \phi_k\}] \ dk \\ &= \frac{1}{2}\mathbb{E}[\max\{H - P_A^P, \phi_k\}] + \frac{1}{2}\mathbb{E}[\max\{H - P_B^P, \phi_k\}] \\ &= \frac{1}{2} f(H - P_A^P) + \frac{1}{2} f(H - P_B^P) \end{aligned}$$

and

$$\begin{aligned} W_C^N &= \int_0^1 \mathbb{E}[\max\{\max_{i \in M} u_{ik}, \phi_k\}] \ dk \\ &= \mathbb{E}[\max\{\frac{H + L}{2} - P^N, \phi_k\}] \\ &= f(\frac{H + L}{2} - P^N). \end{aligned}$$

Proposition C.1.1 tells us $P^F < P_A^P < P_B^P = P^N$, so $H - P^F > H - P_A^P > H - P_B^P > \frac{H+L}{2} - P^N$. It follows that $W_C^F > W_C^P > W_C^N$ since $f(x)$ is strictly increasing. □

Proof of Proposition 3.3.2. Proposition 3.3.2 directly follows from Proposition C.1.1.

- **Manufacturer Prices:** By Proposition C.1.1, $P^F < P_A^P < P^N$ and $P^F < P_B^P = P^N$, so $P_i^F < P_i^P \leq P_i^N$ for all $i \in M$.

- **Expected Good Value:** By Proposition C.1.1, $V_k^F \geq V_k^P > V_k^N$ for all $k \in [0, 1]$ as $H = H > \frac{H+L}{2}$.

- **Consumer Demand:** Since $P_i^P \leq P_i^N$ and G is increasing, $s_i^N = \frac{1}{2m} G(\frac{H+L}{2} - P^N) = \frac{1}{2m} G(\frac{H+L}{2} - P_i^N) \leq \frac{1}{2m} G(H - P_i^P) = s_i^P$. Since $P_i^F < P_i^P$ and G is increasing, $s_i^P = \frac{1}{2m} G(H - P_i^P) \leq \frac{1}{2m} G(H - P_i^F) = \frac{1}{2m} G(H - P^F) = s_i^F$. Hence, $s_i^F \geq s_i^P \geq s_i^N$ for all $i \in M$. If G is strictly increasing, then the strict inequalities hold, i.e. $s_i^F > s_i^P > s_i^N$ for all $i \in M$.

□

Proof of Proposition 3.3.3. By Proposition C.1.1, each manufacturer has zero expected profit in the subgame of choosing price because of price competition, regardless of the level of blockchain adoption. It follows that $W_M = 0 - \sum_{i:i\in M} \chi_i a_i$, so $W_M^F = -\sum_{i:i\in M} \chi_i, W_M^P = -\sum_{i:i\in M_A} \chi_i$, and $W_M^N = 0$. We can see that $W_M^F < W_M^P < W_M^N$. □

Proof of Proposition 3.3.4. By (3.12), Proposition 3.3.1, and Proposition 3.3.3, we have that

$$\begin{aligned}
W^F &= W_C^F + W_M^F \\
&= f(H - P^F) - \sum_{i:i\in M} \chi_i \\
&= f(\Delta + L - P^F) - \sum_{i:i\in M} \chi_i, \\
W^P &= W_C^P + W_M^P \\
&= \frac{1}{2} f(H - P_A^P) + \frac{1}{2} f(H - P_B^P) - \sum_{i:i\in M_A} \chi_i \\
&= \frac{1}{2} f(\Delta + L - P_A^P) + \frac{1}{2} f(\Delta + L - P_B^P) - \sum_{i:i\in M_A} \chi_i, \\
W^N &= W_C^N + W_M^N \\
&= f(\frac{H + L}{2} - P^N) + 0 \\
&= f(\frac{\Delta}{2} + L - P^N).
\end{aligned}$$

View L as a fixed parameter and allow Δ to vary. Since $f(x)$ is strictly increasing, W^F, W^P and W^N all increase in Δ. W^F decreases in $\sum_{i:i\in M} \chi_i$ and is strictly smaller than W^N for sufficiently large $\sum_{i:i\in M} \chi_i$. The same argument applies to W^P.

Fixing adoption costs $\chi_i, \forall i \in M$,

$$\begin{aligned}
\frac{\partial(W^F - W^N)}{\partial \Delta} &= G(\Delta + L - P^F) - \frac{1}{2} G(\frac{\Delta}{2} + L - P^N) \\
&\geq G(\Delta + L - P^F) - \frac{1}{2}.
\end{aligned}$$

Because $\lim_{\Delta\to+\infty} G(\Delta + L - P^F) = 1$, $\frac{\partial(W^F - W^N)}{\partial \Delta} \geq \frac{1}{4}$ when Δ is large enough. This implies that

$\lim_{\Delta\to+\infty} W^F - W^N = +\infty$ no matter how large $\sum_{i:i\in M} \chi_i$ is, meaning that W^F strictly exceeds W^N for sufficiently large Δ. Applying a similar argument, we can also show that W^P strictly exceeds W^N for sufficiently large Δ.

$W^F - W^N$ (resp. $W^P - W^N$) is negative for sufficiently high adoption costs and positive for sufficiently large Δ, so blockchain adoption has ambiguous effects on global welfare. □

Proof of Proposition 3.3.5. By Proposition C.1.1, a type A manufacturer i (i.e. $i \in M_A$) adopts the blockchain and receives zero expected profit in the subgame of choosing price due to price competition, i.e. $\Pi(1, a_{-i}) = 0$, under blockchain adoption (either full or partial). Hence, her expected profit for adopting the blockchain is $0 - \chi_i = -\chi_i$. However, if she chooses not to adopt the blockchain, then her expected profit is always non-negative, which is better than $-\chi_i$. Thus, with blockchain adoption (either full or partial), manufacturer i has incentive to deviate so there does not exist an equilibrium. □

Proof of Proposition 3.3.6. Proposition 3.3.4 tells us that full blockchain adoption is welfare-enhancing for sufficiently large Δ. However, by Proposition 3.3.5, there does not exist an equilibrium with full blockchain adoption. Hence, an adoption failure arises by definition. □

Proof of Proposition 3.3.7. Recall that G denotes the cumulative distribution function of the outside option ϕ_k of consumer k. We consider the system of transfers where $\tau_i = \chi_i$ for all $i \in M$, and $\kappa = \frac{\sum_{i:i\in M} \chi_i}{G(\phi)}$, where ϕ is to be determined. We will first show that for all sufficiently large Δ, there exists ϕ such that $b_k = 1$ if and only if $\phi_k \le \phi$ and thus $\mu_\kappa = \mu(\{k : b_k = 1\}) = \mathbb{P}(\phi_k \le \phi) = G(\phi)$. Since $\kappa \cdot \mu_\kappa = \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} \cdot G(\phi) = \sum_{i:i\in M} \chi_i = \sum_{i:i\in M} \tau_i$, it follows that the system of transfers specified above is self-financing. Next, we will show that $\{a_i = 1, P_i = \Psi + cp(1-\rho_{2m})\}_{i\in M}$ can be sustained as an equilibrium under such a system of transfers. Thereby, full blockchain adoption arises in equilibrium and the adoption failure is resolved.

Suppose $a_i = 1$ and $P_i = P^F := \Psi + cp(1 - \rho_{2m})$ for all $i \in M$. Consumers are now in the subgame of making the b_k decision. The generic consumer k solves the following optimization problem:

$$\max_{b_k} g(b_k, \phi_k; \phi) := \mathbb{E}^{b_k}[\max\{\max_{i \in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k],$$

where consumer k's information set is given by $\mathcal{G}_k = \sigma(t_k, \phi_k, \{a_l, P_l\}_{l \in M})$. When $b_k = 1$, i.e. consumer k has access to the information on the blockchain, the result in Proposition C.1.1 regarding the full adoption case implies that $\max_{i \in M} u_{ik} = H - P^F = \Delta + L - P^F$. Therefore,

$$\begin{aligned} g(1, \phi_k; \phi) &= \mathbb{E}^1[\max\{\max_{i \in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \\ &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i \in M} \chi_i}{G(\phi)}. \end{aligned}$$

When $b_k = 0$, consumer k opts to forgo access to the information on the blockchain. The result in Proposition C.1.1 regarding the non-adoption case implies that $\max_{i \in M} u_{ik} = \frac{H+L}{2} - P^F = \frac{\Delta}{2} + L - P^F$. Therefore,

$$\begin{aligned} g(0, \phi_k; \phi) &= \mathbb{E}^0[\max\{\max_{i \in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \\ &= \max\{\frac{\Delta}{2} + L - P^F,\ \phi_k\}. \end{aligned}$$

Consumer k chooses to gain access to the information on the blockchain, i.e. $b_k = 1$, if and only if

$$h(\phi_k; \phi) := g(1, \phi_k; \phi) - g(0, \phi_k; \phi) \geq 0.$$

By Lemma C.2.1(b) in Appendix C.2, it follows that there exists $\phi^* \in [0, +\infty)$ such that $h(\phi^*; \phi^*) = 0$ and $h(\phi_k; \phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. Thus, $b_k = 1$ if and only if $\phi_k \leq \phi^*$ is an optimal decision rule for all consumers. Since measure μ_κ of the consumers who pay the fee κ to gain access to the information on the blockchain satisfies $\mu_\kappa = \mu(\{k : b_k = 1\}) = \mathbb{P}(\phi_k \leq \phi^*) = G(\phi^*)$, it follows that the payment collected from the consumers is $\kappa \cdot \mu_\kappa = \frac{\sum_{i:i \in M} \chi_i}{G(\phi^*)} \cdot G(\phi^*) = \sum_{i:i \in M} \chi_i = \sum_{i:i \in M} \tau_i$, indicating that the system of transfers is self-financing.

Next, we take one step back to the stage where manufacturers make their adoption and pricing decisions. For manufacturer i, if she adopts the blockchain, then she has no incentive to change her price from P^F because of price competition. Her expected profit is zero in this case since her price $P_i = P^F$ is equal to her expected cost and the transfer $\tau_i = \chi_i$ just covers her adoption cost χ_i. If she does not adopt the blockchain, she no longer has access to the information on the blockchain, and her expected cost goes up to $\Psi + cp(1 - \rho_1)$. Accordingly, in order to obtain non-negative profit, her price $P_i \geq \Psi + cp(1 - \rho_1) > \Psi + cp(1 - \rho_{2m}) = P^F$. Since $V_{ik} \leq H = V_{jk}$ for all j such that $j \neq i$ and $q_j = t_k$, it follows that $u_{ik} = V_{ik} - P_i < V_{jk} - P^F = u_{jk}$ for all j such that $j \neq i$ and $q_j = t_k$. Consequently, no consumer will select manufacturer i. This implies that her expected profit from not adopting the blockchain is zero as well, so she has no incentive to deviate from adopting the blockchain. To conclude, manufacturer i has no incentive to deviate from the adoption decision $a_i = 1$ and the pricing decision $P_i = \Psi + cp(1 - \rho_{2m})$. This completes the proof. □

C.2 Supplementary Lemma and Example

Lemma C.2.1. *The function $h(\phi_k; \phi) := g(1, \phi_k; \phi) - g(0, \phi_k; \phi)$ has the following properties.*

(a). $h(\phi_k; \phi)$ is continuous and weakly decreasing in ϕ_k on $[0, +\infty)$.

(b). For all sufficiently large Δ, there exists $\phi^* \in [0, +\infty)$ such that $h(\phi^*; \phi^*) = 0$ and $h(\phi_k; \phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. Therefore, $b_k = 1$ if and only if $\phi_k \leq \phi^*$. Moreover, ϕ^* is not necessarily unique with a general cumulative distribution function G.

Proof. By definition,

$$\begin{aligned} h(\phi_k; \phi) &= g(1, \phi_k; \phi) - g(0, \phi_k; \phi) \\ &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i: i \in M} \chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L - P^F,\ \phi_k\}. \end{aligned}$$

Each term of it is continuous in ϕ_k on $[0, +\infty)$, so $h(\phi_k; \phi)$ as a function of ϕ_k is continuous on $[0, +\infty)$. Depending on how large Δ is, there are three cases, each of which we subsequently examine separately: $\frac{\Delta}{2} + L - P^F > 0$, $\frac{\Delta}{2} + L - P^F \leq 0$ but $\Delta + L - P^F > 0$, and $\Delta + L - P^F \leq 0$. Consider the following three cases:

(1). $\frac{\Delta}{2} + L - P^F > 0$

In this case, we can partition $[0, +\infty)$ into three intervals $[0, \frac{\Delta}{2} + L - P^F]$, $(\frac{\Delta}{2} + L - P^F, \Delta + L - P^F]$, and $(\Delta + L - P^F, +\infty)$. $h(\phi_k; \phi)$ as a function of ϕ_k is constant on the first and last intervals, and linearly strictly decreasing on the middle interval. Moreover, $h(\phi_k; \phi)$ is continuous so that $h(\phi_k; \phi)$ is continuous and weakly decreasing in general.

When $\phi_k \in [0, \frac{\Delta}{2} + L - P^F]$, $h(\phi_k; \phi)$ is given by:

$$\begin{aligned} h(\phi_k; \phi) &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L - P^F,\ \phi_k\} \\ &= (\Delta + L - P^F) - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - (\frac{\Delta}{2} + L - P^F) \\ &= \frac{\Delta}{2} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)}, \end{aligned}$$

which does not depend upon ϕ_k.

When $\phi_k \in (\frac{\Delta}{2} + L - P^F, \Delta + L - P^F]$, $h(\phi_k; \phi)$ is given by:

$$\begin{aligned} h(\phi_k; \phi) &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L - P^F,\ \phi_k\} \\ &= (\Delta + L - P^F) - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \phi_k, \end{aligned}$$

which is linear and strictly decreasing in ϕ_k.

When $\phi_k \in (\Delta + L - P^F, +\infty)$, $h(\phi_k; \phi)$ is given by:

$$\begin{aligned} h(\phi_k; \phi) &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L - P^F,\ \phi_k\} \\ &= \phi_k - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \phi_k \\ &= -\frac{\sum_{i:i\in M} \chi_i}{G(\phi)}, \end{aligned}$$

which does not depend upon ϕ_k.

(2). $\frac{\Delta}{2} + L - P^F \leq 0$ but $\Delta + L - P^F > 0$

In this case, we can partition $[0, +\infty)$ into two intervals $[0, \Delta + L - P^F]$ and $(\Delta + L - P^F, +\infty)$. By a similar argument, we can prove that $h(\phi_k; \phi)$ is continuous and weakly decreasing on $[0, +\infty)$. More specifically, it is strictly decreasing on $[0, \Delta + L - P^F]$ and flat on $(\Delta + L - P^F, +\infty)$.

(3). $\Delta + L - P^F \leq 0$

In this case, $h(\phi_k; \phi) = -\frac{\sum_{i:i\in M} \chi_i}{G(\phi)}$ is a constant function on $[0, +\infty)$ so that it is trivially continuous and weakly decreasing.

In conclusion, across all three cases, $h(\phi_k; \phi)$ is continuous and weakly decreasing in ϕ_k on $[0, +\infty)$. This completes the proof of (a).

When $\frac{\Delta}{2} > G^{-1}(0.5) - L + P^F$, $\frac{\Delta}{2} + L - P^F > G^{-1}(0.5) > 0$ holds true. It follows from Case (1) in the first property that $h(\phi_k; \phi)$ with such Δ is continuous and weakly decreasing in ϕ_k on $[0, +\infty)$. More specifically, it is strictly decreasing on $(\frac{\Delta}{2} + L - P^F, \Delta + L - P^F]$ and flat on $[0, \frac{\Delta}{2} + L - P^F]$ and $(\Delta + L - P^F, +\infty)$. Thus, if we can find a ϕ^* such that $\phi^* \in (\frac{\Delta}{2} + L - P^F, \Delta + L - P^F]$ and $h(\phi^*; \phi^*) = 0$, then $h(\phi_k; \phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$.

Note that

$$\begin{aligned} l(\phi) &:= h(\phi; \phi) \\ &= g(1, \phi; \phi) - g(0, \phi; \phi) \\ &= \max\{\Delta + L - P^F,\ \phi\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \max\{\frac{\Delta}{2} + L - P^F,\ \phi\} \end{aligned}$$

is continuous in ϕ on $(0, +\infty)$ and

$$l(\Delta + L - P^F) = -\frac{\sum_{i:i\in M} \chi_i}{G(\Delta + L - P^F)} < 0.$$

When $\frac{\Delta}{2} > \max\{2\sum_{i:i\in M}\chi_i, G^{-1}(0.5) - L + P^F\}$,

$$\begin{aligned} l(\frac{\Delta}{2} + L - P^F) &= \frac{\Delta}{2} - \frac{\sum_{i:i\in M}\chi_i}{G(\frac{\Delta}{2} + L - P^F)} \\ &> \frac{\Delta}{2} - \frac{\sum_{i:i\in M}\chi_i}{G(G^{-1}(0.5))} \\ &= \frac{\Delta}{2} - 2\sum_{i:i\in M}\chi_i \\ &> 0. \end{aligned}$$

This implies that when $\frac{\Delta}{2} > \max\{2\sum_{i:i\in M}\chi_i, G^{-1}(0.5) - L + P^F\}$, by intermediate value theorem, there exists a solution $\phi^* \in (\frac{\Delta}{2} + L - P^F, \Delta + L - P^F]$ such that $h(\phi^*;\phi^*) = l(\phi^*) = 0$. With this ϕ^*, we have that $h(\phi_k;\phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. ϕ^* is not necessarily unique with a general cumulative distribution function G; Example C.2.1 below is such an example. This completes the proof of (b). □

Example C.2.1. Recall that we want to find a ϕ^* such that $\phi^* \in (\frac{\Delta}{2} + L - P^F, \Delta + L - P^F] =: I$ and

$$l(\phi^*) = \max\{\Delta + L - P^F,\ \phi^*\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi^*)} - \max\{\frac{\Delta}{2} + L - P^F,\ \phi^*\} = 0.$$

It is equivalent to solving $y_1(\phi) = y_2(\phi)$ for $\phi \in I$, or finding the intersection points of two curves $y_1(\phi)$ and $y_2(\phi)$ on I, where

$$\begin{aligned} y_1(\phi) &= \max\{\Delta + L - P^F,\ \phi\} - \max\{\frac{\Delta}{2} + L - P^F,\ \phi\}, \\ y_2(\phi) &= \frac{\sum_{i:i\in M}\chi_i}{G(\phi)}. \end{aligned}$$

Set $\Delta = \frac{13}{3}$, $L = P^F = 0$, $\sum_{i:i \in M} \chi_i = 1$, and

$$G(\phi) = \begin{cases} \frac{\phi}{4} & \phi \in [0, 3] \\ \phi - \frac{9}{4} & \phi \in (3, \frac{13}{4}] \\ 1 & \phi \in (\frac{13}{4}, +\infty) \end{cases}$$

being piecewise linear. Then, $I = (\frac{13}{6}, \frac{13}{3}]$ and

$$y_1(\phi) = \max\{\frac{13}{3},\ \phi\} - \max\{\frac{13}{6},\ \phi\},$$
$$y_2(\phi) = \frac{1}{G(\phi)}.$$

There are two solutions $\phi^* = 3$ and $\phi^* = \frac{10}{3}$ satisfying $\phi^* \in I$ and $l(\phi^*) = 0$.

In this example, $y_2(\phi)$ is decreasing but piecewise convex with a kink. It is possible to further twist the shape of $y_2(\phi)$ while keeping it decreasing to generate more intersection points with $y_1(\phi)$ over I.

Appendix D: Chapter 3 Proofs for $\alpha \in [\frac{1}{2}, 1)$

This Appendix includes proofs of all the results stated in Chapter 3 for $\alpha \in [\frac{1}{2}, 1)$. Appendix C included complete proofs for $\alpha = \frac{1}{2}$. The extension to $\alpha \in (\frac{1}{2}, 1)$ involves additional mathematical complexity that is developed here. We begin with several supporting lemmas in Appendix D.1, and use these results to re-derive all the propositions in Appendix D.2.

D.1 Supplementary Lemmas

When $\alpha \in (\frac{1}{2}, 1)$, signals are partially informative. Therefore, the proportion of consumers that select a given manufacturer depends upon the distribution of manufacturer signals. Lemmas D.1.1 - D.1.3 provide intermediate results that enable us to explicitly compute the proportions for each manufacturer. These results are necessary for several proofs in Appendix D.2. Additionally, to determine the equilibrium with a system of transfers, it is necessary to determine the incremental utility that consumers achieve from full information regarding manufacturer types relative to partial information. Lemma D.1.4 provides an intermediate result for that purpose, which we rely upon in the proof of Proposition 3.3.7 in Appendix D.2.

Recall that it is common knowledge that there are exactly m manufacturers of each type. Let r_i, $i = 1, \ldots, 2m$, denote the true type of manufacturer i. We assume that every consumer k has prior distribution that is uniform over the set of possible types

$$E = \{(r_1, \ldots, r_{2m}) \mid r_l \in \{A, B\}, \forall l \in M, |\{l : r_l = A\}| = m\}.$$

Lemma D.1.1. *Suppose none of the manufacturers adopts the blockchain. Fix a consumer k. Suppose the signal from manufacturer i is the same as her type, i.e. $\tilde{q}_{ik} = t_k$, and the signal from manufacturer j is not the same, i.e. $\tilde{q}_{jk} \neq t_k$. Then, for any signal $\tilde{r}_{lk} \in \{A, B\}, \forall l \in M \setminus \{i, j\}$,* *we have*

$$\mathbb{P}\Big(q_i = t_k \;\Big|\; t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{l: l \in M \setminus \{i,j\}} \{\tilde{q}_{lk} = \tilde{r}_{lk}\}\Big)$$
$$\geq \mathbb{P}\Big(q_j = t_k \;\Big|\; t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{l: l \in M \setminus \{i,j\}} \{\tilde{q}_{lk} = \tilde{r}_{lk}\}\Big).$$

Moreover, the inequality is strict when $\alpha \in (\frac{1}{2}, 1)$.

Proof. Define the following five auxiliary sets

$$\begin{aligned}
E_{ij} &= \{(r_1, \ldots, r_{2m}) \mid (r_1, \ldots, r_{2m}) \in E,\ r_i = r_j = t_k\}, \\
E_{i\bar{j}} &= \{(r_1, \ldots, r_{2m}) \mid (r_1, \ldots, r_{2m}) \in E,\ r_i = t_k, r_j \neq t_k\}, \\
E_{\bar{i}j} &= \{(r_1, \ldots, r_{2m}) \mid (r_1, \ldots, r_{2m}) \in E,\ r_i \neq t_k, r_j = t_k\}, \\
E_i &= E_{ij} \cup E_{i\bar{j}}, \\
E_j &= E_{ij} \cup E_{\bar{i}j}.
\end{aligned}$$

Denote $\tilde{r}_{ik}$ and $\tilde{r}_{jk}$ as the realizations of $\tilde{q}_{ik}$ and $\tilde{q}_{jk}$, respectively. Then, $\tilde{r}_{ik} = t_k$ and $\tilde{r}_{jk} \neq t_k$.

Note that

$$
\begin{aligned}
&\mathbb{P}\Big(q_i = t_k \,\Big|\, t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{l:l\in M\setminus\{i,j\}} \{\tilde{q}_{lk} = \tilde{r}_{lk}\}\Big) \\
&\quad = \frac{\mathbb{P}(q_i = t_k, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid t_k)}{\mathbb{P}(\bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid t_k)} \\
&\quad = \frac{\sum_{r_i = t_k, r_l\in\{A,B\},\forall l\in M\setminus\{i\}} \mathbb{P}(\bigcap_{l=1}^{2m}\{q_l = r_l\}, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid t_k)}{\sum_{r_l\in\{A,B\},\forall l\in M} \mathbb{P}(\bigcap_{l=1}^{2m}\{q_l = r_l\}, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid t_k)} \\
&\quad = \frac{\sum_{r_i = t_k, r_l\in\{A,B\},\forall l\in M\setminus\{i\}} \mathbb{P}(\bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid \bigcap_{l=1}^{2m}\{q_l = r_l\}, t_k)\mathbb{P}(\bigcap_{l=1}^{2m}\{q_l = r_l\} \mid t_k)}{\sum_{r_l\in\{A,B\},\forall l\in M} \mathbb{P}(\bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid \bigcap_{l=1}^{2m}\{q_l = r_l\}, t_k)\mathbb{P}(\bigcap_{l=1}^{2m}\{q_l = r_l\} \mid t_k)} \\
&\quad = \frac{\sum_{(r_1,\dots,r_{2m})\in E_i} \mathbb{P}(\bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid \bigcap_{l=1}^{2m}\{q_l = r_l\}, t_k)\mathbb{P}(\bigcap_{l=1}^{2m}\{q_l = r_l\} \mid t_k)}{\sum_{(r_1,\dots,r_{2m})\in E} \mathbb{P}(\bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid \bigcap_{l=1}^{2m}\{q_l = r_l\}, t_k)\mathbb{P}(\bigcap_{l=1}^{2m}\{q_l = r_l\} \mid t_k)} \\
&\quad = \frac{\sum_{(r_1,\dots,r_{2m})\in E_i} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l)}{\sum_{(r_1,\dots,r_{2m})\in E} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l)} \\
&\quad = \frac{\sum_{(r_1,\dots,r_{2m})\in E_i} \prod_{l=1}^{2m} \left(\alpha \mathbf{1}_{\{\tilde{r}_{lk} = r_l\}} + (1-\alpha)\mathbf{1}_{\{\tilde{r}_{lk} \neq r_l\}}\right)}{\sum_{(r_1,\dots,r_{2m})\in E} \prod_{l=1}^{2m} \left(\alpha \mathbf{1}_{\{\tilde{r}_{lk} = r_l\}} + (1-\alpha)\mathbf{1}_{\{\tilde{r}_{lk} \neq r_l\}}\right)}.
\end{aligned}
$$

Since the denominators of $\mathbb{P}(q_i = t_k \mid t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{l:l\in M\setminus\{i,j\}}\{\tilde{q}_{lk} = \tilde{r}_{lk}\})$ and $\mathbb{P}(q_j = t_k \mid t_k, \tilde{q}_{ik} = t_k, \tilde{q}_{jk} \neq t_k, \bigcap_{l:l\in M\setminus\{i,j\}}\{\tilde{q}_{lk} = \tilde{r}_{lk}\})$ are the same, it suffices to compare their numerators $\sum_{(r_1,\dots,r_{2m})\in E_i} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l)$ and $\sum_{(r_1,\dots,r_{2m})\in E_j} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l)$.

Let $\bar{t}_k$ denote the type that is different from t_k. There is a one-to-one mapping between two sets $E_{i\bar{j}}$ and $E_{\bar{i}j}$:

$$
\begin{aligned}
(r_1, \dots, r_{2m}) &= (r_1, \dots, r_{i-1}, t_k, r_{i+1}, \dots, r_{j-1}, \bar{t}_k, r_{j+1}, \dots, r_{2m}) \\
&\mapsto (r_1, \dots, r_{i-1}, \bar{t}_k, r_{i+1}, \dots, r_{j-1}, t_k, r_{j+1}, \dots, r_{2m}) \\
&= (r'_1, \dots, r'_{2m}).
\end{aligned}
$$

Because $\tilde{r}_{ik} = t_k$ and $\tilde{r}_{jk} \neq t_k$, it is easy to see that for any $(r_1, \dots, r_{2m}) \in E_{i\bar{j}}$,

$$
\prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) = \alpha^2 \prod_{l:l\in M\setminus\{i,j\}} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l).
$$

Similarly, for any $(r'_1, \ldots, r'_{2m}) \in E_{\bar{i}j}$,

$$\prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r'_l) = (1-\alpha)^2 \prod_{l:l \in M \setminus \{i,j\}} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r'_l).$$

If $(r'_1, \ldots, r'_{2m})$ corresponds to $(r_1, \ldots, r_{2m})$, then

$$\alpha^2 \prod_{l:l \in M \setminus \{i,j\}} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) \geq (1-\alpha)^2 \prod_{l:l \in M \setminus \{i,j\}} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r'_l)$$

since $\alpha \geq \frac{1}{2}$ and $r_l = r'_l$ for all $l \in M \setminus \{i, j\}$. Note that the inequality is strict when $\alpha > \frac{1}{2}$. Combining all these results, we obtain that

$$\begin{aligned}
&\sum_{(r_1,\ldots,r_{2m}) \in E_i} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) \\
&= \sum_{(r_1,\ldots,r_{2m}) \in E_{ij}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) + \sum_{(r_1,\ldots,r_{2m}) \in E_{i\bar{j}}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) \\
&= \sum_{(r_1,\ldots,r_{2m}) \in E_{ij}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) + \sum_{(r_1,\ldots,r_{2m}) \in E_{i\bar{j}}} \alpha^2 \prod_{l:l \in M \setminus \{i,j\}} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) \\
&\geq \sum_{(r_1,\ldots,r_{2m}) \in E_{ij}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) + \sum_{(r'_1,\ldots,r'_{2m}) \in E_{\bar{i}j}} (1-\alpha)^2 \prod_{l:l \in M \setminus \{i,j\}} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r'_l) \\
&= \sum_{(r_1,\ldots,r_{2m}) \in E_{ij}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) + \sum_{(r'_1,\ldots,r'_{2m}) \in E_{\bar{i}j}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r'_l) \\
&= \sum_{(r_1,\ldots,r_{2m}) \in E_j} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l).
\end{aligned}$$

Note that the inequality in the fourth line is strict if $\alpha \in (\frac{1}{2}, 1)$ so that the statement of the lemma holds with a strict inequality in that case. This concludes the proof. □

Consider the case where none of the manufacturers adopts the blockchain. Consider a generic consumer k. Recall that $i(k)$ denotes consumer k's preferred manufacturer, and the utility $u_{i(k)k}$

that she receives from purchasing from manufacturer $i(k)$ is given by

$$\begin{aligned} u_{i(k)k} &= V_{i(k)k} - P^N_{i(k)} \\ &= (H-L)\mathbb{P}\Big(q_{i(k)} = t_k \,\Big|\, t_k, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\}\Big) + L - P^N_{i(k)}. \end{aligned}$$

We call a signal $\tilde{q}_{ik}$ from manufacturer i favorable if the realization $\tilde{r}_{ik} = t_k$. Next, we show that the number of favorable signals $|\{l : \tilde{r}_{lk} = t_k\}|$ is a sufficient statistic for computing $\mathbb{P}(q_{i(k)} = t_k \mid t_k, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\})$.

Lemma D.1.2. *Consider the case where none of the manufacturers adopts the blockchain. Then, the number of favorable signals $|\{l : \tilde{r}_{lk} = t_k\}|$ is a sufficient statistic for the conditional probability $\mathbb{P}(q_{i(k)} = t_k \mid t_k, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\})$, i.e.*

$$\mathbb{P}\Big(q_{i(k)} = t_k \,\Big|\, t_k, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\}\Big) = \theta\big(\,|\{l : \tilde{r}_{lk} = t_k\}|\,\big)$$

for some function $\theta(\cdot) : \{0, \dots, 2m\} \to [0,1]$. Moreover, $0 < \theta_z < 1$ for $z = 0, 1, \dots, 2m$, where $\theta_z := \theta(z)$.

Proof. Let $\{\tilde{r}_{lk}\}_{l \in M}$ denote the realizations of the signals from the manufacturers. Let $z := |\{l : \tilde{r}_{lk} = t_k\}|$ denote the number of favorable signals received by consumer k.

First consider the case where $z > 0$. In this case, Lemma D.1.1 implies that $\tilde{r}_{i(k)k} = t_k$. Since the prior is uniform over the set E, the posterior probability

$$\begin{aligned} \mathbb{P}\Big(q_{i(k)} = t_k \,\Big|\, t_k, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\}\Big) &= \frac{\sum_{(r_1,\dots,r_{2m}) \in E_{i(k)}} \mathbb{P}(\bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid \bigcap_{l=1}^{2m}\{q_l = r_l\}, t_k)}{\sum_{(r_1,\dots,r_{2m}) \in E} \mathbb{P}(\bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\} \mid \bigcap_{l=1}^{2m}\{q_l = r_l\}, t_k)} \\ &= \frac{\sum_{(r_1,\dots,r_{2m}) \in E_{i(k)}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l)}{\sum_{(r_1,\dots,r_{2m}) \in E} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l)}, \end{aligned}$$

where $E_{i(k)} = \{(r_1, \dots, r_{2m}) \mid (r_1, \dots, r_{2m}) \in E, r_{i(k)} = t_k\}$. First consider the numerator. We know that there are exactly m manufacturers with type $r_i = t_k$, m manufacturers with type $r_i \neq t_k$, and $r_{i(k)} = t_k$. The summation in the numerator can be equivalently computed by summing over the possible values for the number of favorable signals $x \leq \min\{z, m\}$ that come from manufacturers with type $r_i = t_k$. We can define

$$D_x = \{(r_1, \ldots, r_{2m}) \mid |\{l : r_l = \tilde{r}_{lk} = t_k\}| = x)\}$$

as the set of all possible types such that there are x favorable signals coming from manufacturers with type $r_i = t_k$. Then, the summation can be rewritten as

$$\sum_{(r_1,\ldots,r_{2m})\in E_{i(k)}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) = \sum_{x=\max\{1,z-m\}}^{\min\{z,m\}} \sum_{(r_1,\ldots,r_{2m})\in E_{i(k)}\cap D_x} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l).$$

Recall that $|\{l : \tilde{r}_{lk} = t_k\}| = z$ and note that

$$E_{i(k)} \cap D_x = \{(r_1, \ldots, r_{2m}) \mid r_{i(k)} = t_k, |\{l : r_l = t_k\}| = m, |\{l : r_l = \tilde{r}_{lk} = t_k\}| = x\}.$$

For any $(r_1, \ldots, r_{2m}) \in E_{i(k)} \cap D_x$, we have

$$\begin{aligned} |\{l : r_l = \tilde{r}_{lk} = t_k\}| &= x, \\ |\{l : r_l \neq t_k, \tilde{r}_{lk} = t_k\}| &= z - x, \\ |\{l : r_l = t_k, \tilde{r}_{lk} \neq t_k\}| &= m - x, \\ |\{l : r_l \neq t_k, \tilde{r}_{lk} \neq t_k\}| &= m - z + x, \end{aligned}$$

and thus $\prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) = \alpha^{m+2x-z}(1-\alpha)^{m-2x+z}$. Moreover, it is not hard to get that

$|E_{i(k)} \cap D_x| = \binom{z-1}{x-1}\binom{2m-z}{m-x}$. Thus, the numerator

$$
\begin{aligned}
\eta_1(z) &:= \sum_{(r_1,\ldots,r_{2m}) \in E_{i(k)}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) \\
&= \sum_{x=\max\{1,z-m\}}^{\min\{z,m\}} \sum_{(r_1,\ldots,r_{2m}) \in E_{i(k)} \cap D_x} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) \\
&= \sum_{x=\max\{1,z-m\}}^{\min\{z,m\}} \sum_{(r_1,\ldots,r_{2m}) \in E_{i(k)} \cap D_x} \alpha^{m+2x-z}(1-\alpha)^{m-2x+z} \\
&= \sum_{x=\max\{1,z-m\}}^{\min\{z,m\}} |E_{i(k)} \cap D_x| \cdot \alpha^{m+2x-z}(1-\alpha)^{m-2x+z} \\
&= \sum_{x=\max\{1,z-m\}}^{\min\{z,m\}} \binom{z-1}{x-1}\binom{2m-z}{m-x}\alpha^{m+2x-z}(1-\alpha)^{m-2x+z}
\end{aligned}
$$

is only a function of z.

Next, note that the denominator

$$
\begin{aligned}
&\sum_{(r_1,\ldots,r_{2m}) \in E} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) \\
&\quad = \sum_{(r_1,\ldots,r_{2m}) \in E_{i(k)}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) + \sum_{(r_1,\ldots,r_{2m}) \in E \setminus E_{i(k)}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l).
\end{aligned}
$$

We already have an expression for the first term. So, consider the second term. Since $(r_1, \ldots, r_{2m}) \in E \setminus E_{i(k)}$, we have that $r_{i(k)} \neq t_k$. Using an analysis similar to the one above, we get that

$$
\begin{aligned}
&\sum_{(r_1,\ldots,r_{2m}) \in E \setminus E_{i(k)}} \prod_{l=1}^{2m} \mathbb{P}(\tilde{q}_{lk} = \tilde{r}_{lk} \mid q_l = r_l) \\
&\quad = \sum_{x=\max\{0,z-m\}}^{\min\{z-1,m\}} \binom{z-1}{x}\binom{2m-z}{m-x}\alpha^{m+2x-z}(1-\alpha)^{m-2x+z} =: \eta_2(z).
\end{aligned}
$$

Thus, it follows that

$$\mathbb{P}\Big(q_{i(k)} = t_k \,\Big|\, t_k, \bigcap_{l=1}^{2m} \{\tilde{q}_{lk} = \tilde{r}_{lk}\}\Big) = \frac{\eta_1(z)}{\eta_1(z) + \eta_2(z)} =: \theta(z).$$

This establishes that z is a sufficient statistic. Moreover, since $\eta_1(z)$ (resp. $\eta_2(z)$) contains at least one term and $\alpha \in [\frac{1}{2}, 1)$, $\eta_1(z)$ (resp. $\eta_2(z)$) is positive. Therefore, $0 < \theta_z < 1$ for all $z > 0$.

When $z = 0$, none of the signals consumer k observes is equal to t_k. Consumer k will randomly select a manufacturer from M. Since there are m type A manufacturers and m type B manufacturers, it is easy to argue that $\theta_0 := \theta(0) = \frac{1}{2} \in (0, 1)$.

□

Lemma D.1.3. *Define w_z as the proportion of the consumers who observe z favorable signals, for $z = 0, 1, \ldots, 2m$. Then,*

$$w_z = \sum_{x+y=z, 0 \le x, y \le m} \binom{m}{x}\binom{m}{y} \alpha^{m+x-y} (1-\alpha)^{m-x+y}$$

and $\sum_{z=0}^{2m} w_z = 1$, so $\{w_z\}_{z=0}^{2m}$ constitutes a probability mass function.

Proof. Define w_z^A as the measure of the type A consumers who observe z favorable signals, for $z = 0, 1, \ldots, 2m$. Then,

$$\begin{aligned} w_z^A &= \frac{1}{2} \sum_{x+y=z, 0 \le x, y \le m} \binom{m}{x} \alpha^x (1-\alpha)^{m-x} \binom{m}{y} \alpha^{m-y} (1-\alpha)^y \\ &= \frac{1}{2} \sum_{x+y=z, 0 \le x, y \le m} \binom{m}{x}\binom{m}{y} \alpha^{m+x-y} (1-\alpha)^{m-x+y}, \end{aligned}$$

where x (resp. y) denotes the number of type A (resp. type B) manufacturers that emit type A signals. The term $\binom{m}{x}\alpha^x(1-\alpha)^{m-x}\binom{m}{y}\alpha^{m-y}(1-\alpha)^y$ represents the measure of the type A consumers who observe x favorable signals from type A manufacturers and y favorable signals from type B

manufacturers. Similarly, we can get w_z^B and actually $w_z^B = w_z^A$. Hence,

$$\begin{aligned} w_z &= w_z^A + w_z^B \\ &= \sum_{x+y=z, 0\le x,y\le m} \binom{m}{x}\binom{m}{y} \alpha^{m+x-y}(1-\alpha)^{m-x+y}. \end{aligned}$$

Given the expression for w_z, we have that

$$\begin{aligned} \sum_{z=0}^{2m} w_z &= \sum_{z=0}^{2m} \sum_{x+y=z, 0\le x,y\le m} \binom{m}{x}\binom{m}{y} \alpha^{m+x-y}(1-\alpha)^{m-x+y} \\ &= \left(\sum_{x=0}^{m} \binom{m}{x} \alpha^{x}(1-\alpha)^{m-x}\right)\left(\sum_{y=0}^{m} \binom{m}{y} \alpha^{m-y}(1-\alpha)^{y}\right) \\ &= 1. \end{aligned}$$

□

Lemma D.1.4. *The function $h(\phi_k;\phi) := g(1,\phi_k;\phi) - g(0,\phi_k;\phi)$ has the following properties.*

(a). $h(\phi_k;\phi)$ is continuous, piecewise linear and weakly decreasing in ϕ_k on $[0,+\infty)$.

(b). For all sufficiently large Δ, there exists $\phi^* \in [0,+\infty)$ such that $h(\phi^*;\phi^*) = 0$ and $h(\phi_k;\phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. Therefore, $b_k = 1$ if and only if $\phi_k \leq \phi^*$.

Proof. By definition,

$$\begin{aligned} h(\phi_k;\phi) &= g(1,\phi_k;\phi) - g(0,\phi_k;\phi) \\ &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} - \sum_{z=0}^{2m} w_z \cdot \max\{\theta_z\Delta + L - P^F,\ \phi_k\}. \end{aligned}$$

Each term of it is continuous in ϕ_k on $[0,+\infty)$, so $h(\phi_k;\phi)$ as a function of ϕ_k is continuous on $[0,+\infty)$. By Lemma D.1.2, $0 < \min_{0\le z\le 2m} \theta_z < 1$. Depending on how large Δ is, there are three cases, each of which we subsequently examine separately: $\min_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F > 0$,

$\min_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F \le 0$ but $\Delta + L - P^F > 0$, and $\Delta + L - P^F \le 0$. Consider the following three cases:

(1). $\min_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F > 0$

In this case, we can partition $[0, +\infty)$ into three intervals $[0, \min_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F]$, $(\min_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F, \Delta + L - P^F]$, and $(\Delta + L - P^F, +\infty)$. $h(\phi_k; \phi)$ as a function of ϕ_k is constant on the first and last intervals, and piecewise linearly strictly decreasing on the middle interval. Moreover, $h(\phi_k; \phi)$ is continuous so that $h(\phi_k; \phi)$ is continuous, piecewise linear and weakly decreasing in general.

When $\phi_k \in [0, \min_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F]$, $h(\phi_k; \phi)$ is given by:

$$\begin{aligned} h(\phi_k; \phi) &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} w_z \cdot \max\{\theta_z \Delta + L - P^F,\ \phi_k\} \\ &= (\Delta + L - P^F) - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} w_z (\theta_z \Delta + L - P^F) \\ &= \left(1 - \sum_{z=0}^{2m} w_z \theta_z\right) \Delta - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)}, \end{aligned}$$

which does not depend upon ϕ_k.

When $\phi_k \in (\min_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F, \Delta + L - P^F]$, $h(\phi_k; \phi)$ is given by:

$$\begin{aligned} h(\phi_k; \phi) &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} w_z \cdot \max\{\theta_z \Delta + L - P^F,\ \phi_k\} \\ &= (\Delta + L - P^F) - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \sum_{z:\theta_z\Delta + L - P^F > \phi_k} w_z (\theta_z \Delta + L - P^F) - \sum_{z:\theta_z\Delta + L - P^F \le \phi_k} w_z \phi_k. \end{aligned}$$

Note that $\sum_{z:\theta_z\Delta+L-P^F\le\phi_k} w_z > 0$ when $\phi_k \in (\min_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F, \Delta + L - P^F]$, so $h(\phi_k; \phi)$ is piecewise linear and strictly decreasing in ϕ_k on this interval.

When $\phi_k \in (\Delta + L - P^F, +\infty)$, $h(\phi_k; \phi)$ is given by:

$$\begin{aligned} h(\phi_k; \phi) &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} w_z \cdot \max\{\theta_z \Delta + L - P^F,\ \phi_k\} \\ &= \phi_k - \frac{\sum_{i:i\in M} \chi_i}{G(\phi)} - \sum_{z=0}^{2m} w_z \phi_k \\ &= -\frac{\sum_{i:i\in M} \chi_i}{G(\phi)}, \end{aligned}$$

which does not depend upon ϕ_k. Note that the second line follows from the first because Lemma D.1.2 implies $\theta_z < 1$, and the third line follows from the second because Lemma D.1.3 implies $\sum_{z=0}^{2m} w_z = 1$.

(2). $\min\limits_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F \le 0$ but $\Delta + L - P^F > 0$

In this case, we can partition $[0, +\infty)$ into two intervals $[0, \Delta+L-P^F]$ and $(\Delta+L-P^F, +\infty)$. By a similar argument, we can prove that $h(\phi_k; \phi)$ is continuous, piecewise linear and weakly decreasing on $[0, +\infty)$. More specifically, it is strictly decreasing on $[0, \Delta + L - P^F]$ and flat on $(\Delta + L - P^F, +\infty)$.

(3). $\Delta + L - P^F \le 0$

In this case, $h(\phi_k; \phi) = -\frac{\sum_{i:i\in M} \chi_i}{G(\phi)}$ is a constant function on $[0, +\infty)$ so that it is trivially continuous, piecewise linear and weakly decreasing.

In conclusion, across all three cases, $h(\phi_k; \phi)$ is continuous, piecewise linear and weakly decreasing in ϕ_k on $[0, +\infty)$. This completes the proof of (a).

When $\Delta > \frac{G^{-1}(0.5) - L + P^F}{\min\limits_{0\le z\le 2m} \theta_z}$, $\min\limits_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F > G^{-1}(0.5) > 0$ holds true. It follows from Case (1) in Property (a) that $h(\phi_k; \phi)$ with such Δ is continuous, piecewise linear and weakly decreasing in ϕ_k on $[0, +\infty)$. More specifically, it is strictly decreasing on $(\min\limits_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F, \Delta + L - P^F]$ and flat on $[0, \min\limits_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F]$ and $(\Delta + L - P^F, +\infty)$. Thus, if we can find a ϕ^* such that $\phi^* \in (\min\limits_{0\le z\le 2m} \theta_z \cdot \Delta + L - P^F, \Delta + L - P^F]$ and $h(\phi^*; \phi^*) = 0$, then $h(\phi_k; \phi^*) \ge 0$ if and only if $\phi_k \le \phi^*$.

Note that

$$\begin{aligned}
l(\phi) &:= h(\phi;\phi) \\
&= g(1,\phi;\phi) - g(0,\phi;\phi) \\
&= \max\{\Delta + L - P^F,\ \phi\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} - \sum_{z=0}^{2m} w_z \cdot \max\{\theta_z\Delta + L - P^F,\ \phi\}
\end{aligned}$$

is continuous in ϕ on $(0,+\infty)$ and

$$l(\Delta + L - P^F) = -\frac{\sum_{i:i\in M}\chi_i}{G(\Delta + L - P^F)} < 0.$$

By Lemma D.1.2, $0 < \max\limits_{0\le z\le 2m}\theta_z < 1$. When $\Delta > \max\{\frac{2\sum_{i:i\in M}\chi_i}{1-\max\limits_{0\le z\le 2m}\theta_z}, \frac{G^{-1}(0.5)-L+P^F}{\min\limits_{0\le z\le 2m}\theta_z}\}$,

$$\begin{aligned}
l(\min_{0\le z\le 2m}\theta_z \cdot \Delta + L - P^F) &= \left(1 - \sum_{z=0}^{2m} w_z\theta_z\right)\Delta - \frac{\sum_{i:i\in M}\chi_i}{G(\min\limits_{0\le z\le 2m}\theta_z \cdot \Delta + L - P^F)} \\
&\ge \left(1 - \max_{0\le z\le 2m}\theta_z \cdot \sum_{z=0}^{2m} w_z\right)\Delta - \frac{\sum_{i:i\in M}\chi_i}{G(G^{-1}(0.5))} \\
&= \left(1 - \max_{0\le z\le 2m}\theta_z\right)\Delta - 2\sum_{i:i\in M}\chi_i \\
&> 0.
\end{aligned}$$

This implies that when $\Delta > \max\{\frac{2\sum_{i:i\in M}\chi_i}{1-\max\limits_{0\le z\le 2m}\theta_z}, \frac{G^{-1}(0.5)-L+P^F}{\min\limits_{0\le z\le 2m}\theta_z}\}$, by intermediate value theorem, there exists a solution $\phi^* \in (\min\limits_{0\le z\le 2m}\theta_z \cdot \Delta + L - P^F, \Delta + L - P^F]$ such that $h(\phi^*;\phi^*) = l(\phi^*) = 0$. With this ϕ^*, we have that $h(\phi_k;\phi^*) \ge 0$ if and only if $\phi_k \le \phi^*$. This completes the proof of (b). □

D.2 Main Proofs

Appendix D.2 provides proofs of all propositions stated in the body of Chapter 3 for $\alpha \in [\frac{1}{2}, 1)$. These proofs rely on the lemmas provided within Appendix D.1. Moreover, these proofs rely on model solutions of expected good values, prices and market shares. We derive the endogenous values of all such quantities in Proposition D.2.1, which is stated and proved at the beginning of this section.

Proposition D.2.1. *The equilibrium expected good values, prices, and market shares under full blockchain adoption, partial blockchain adoption, and blockchain non-adoption are as follows.*

1. ***Full Blockchain Adoption:*** $V_k^F = H$ *for all* $k \in [0,1]$, $P_i^F = P^F := \Psi + cp(1-\rho_{2m})$ *for all* $i \in M$, $s_i^F = \frac{1}{2m}G(H - P^F)$ *for all* $i \in M$, *and* $u_{i(k)k} = H - P^F$ *for all* $k \in [0,1]$.

2. ***Partial Blockchain Adoption:*** $V_k^P = H$ *for all* $k \in [0,1]$, $P_i^P = P_A^P := \Psi + cp(1-\rho_m)$ *for all* $i \in M_A$, $P_i^P = P_B^P := \Psi + cp(1-\rho_1)$ *for all* $i \in M_B$, $s_i^P = \frac{1}{2m}G(H - P_i^P)$ *for all* $i \in M$, *and* $u_{i(k)k} = H - P_{t_k}^P$ *for all* $k \in [0,1]$.

3. ***Blockchain Non-Adoption:*** $V_k^N = \left(\sum_{z=0}^{2m} w_z\theta_z\right)(H-L) + L$ *for all* $k \in [0,1]$, $P_i^N = P^N := \Psi + cp(1-\rho_1)$ *for all* $i \in M$, $s_i^N = \frac{1}{2m}\sum_{z=0}^{2m} w_z G(\theta_z(H-L) + L - P^N)$ *for all* $i \in M$, *and* $u_{i(k)k} = \theta_z(H-L) + L - P^N$ *w.p.* w_z *for* $z = 0, 1, \ldots, 2m$ *for all* $k \in [0,1]$.

Variables z, θ_z *and* w_z *are all defined in Appendix D.1.*

Proof. For manufacturer i, once adoption decision a_i is made, adoption cost $\chi_i a_i$ becomes a sunk cost and she is now in the subgame of choosing her price P_i.

- **Full Blockchain Adoption:** Consumers know each manufacturer's type and will select a manufacturer of her own type, so $V_k^F = H$ for all $k \in [0,1]$. Type A (resp. type B) manufacturers are competing for the market consisting of type A (resp. type B) consumers; we have two Bertrand games for type A and type B manufacturers respectively. Due to price competition, manufacturer i will set P_i to her expected cost so that the other manufacturers of the same type have no chance to undercut to obtain the entire market. Accordingly, $P_i^F = P_A^F := \Psi + cp(1-\rho_{2m})$ for all $i \in M_A$ and $P_i^F = P_B^F := \Psi + cp(1-\rho_{2m})$ for all $i \in M_B$; type A (resp. type B) manufacturers split the market consisting of type A (resp. type B) consumers evenly, $s_i^F = \frac{1}{2m}\mathbb{P}(H - P^F \geq \phi_k) = \frac{1}{2m}G(H - P^F)$ for all $i \in M$. By definition, $\max_{i \in M} u_{ik} = u_{i(k)k} = H - P^F$ for all $k \in [0,1]$.

- **Partial Blockchain Adoption:** Since all type A manufacturers adopt the blockchain and the blockchain is fully revealing, consumers know that a manufacturer with $a_i = 1$ is of type A. As it is a common knowledge that there are exactly m manufacturers of each type, the remaining m manufacturers with $a_i = 0$ must be of type B. Accordingly, consumers know each manufacturer's type based on manufacturers' adoption decisions; $a_i = 1$ implies that manufacturer i is of type A, and $a_i = 0$ implies that manufacturer i is of type B. Hence, $V_k^P = H$ for all $k \in [0, 1]$. Again, we have price competitions and the equilibrium price for manufacturer i in the subgame is equal to her expected cost. It follows that $P_i^P = P_A^P := \Psi + cp(1 - \rho_m)$ for all $i \in M_A$ and $P_i^P = P_B^P := \Psi + cp(1 - \rho_1)$ for all $i \in M_B$; type A (resp. type B) manufacturers split the market consisting of type A (resp. type B) consumers evenly, $s_i^P = s_A^P := \frac{1}{2m} G(H - P_A^P)$ for all $i \in M_A$ and $s_i^P = s_B^P := \frac{1}{2m} G(H - P_B^P)$ for all $i \in M_B$. In order for type A (resp. type B) consumers to stick to type A (resp. type B) manufacturers, an additional condition to be added is $H - P_B^P \geq L - P_A^P$, which is equivalent to $H - L \geq cp(\rho_m - \rho_1)$. By definition, $\max_{i \in M} u_{ik} = u_{i(k)k} = H - P_A^P$ for all type A consumers and $\max_{i \in M} u_{ik} = u_{i(k)k} = H - P_B^P$ for all type B consumers.

- **Blockchain Non-Adoption:** Consider the competition among the manufacturers of the same type (either type A or type B). Suppose there exists a symmetric Nash equilibrium. Then, the price competition implies that the equilibrium price for manufacturer i in the subgame must equal her expected cost. For $\alpha = \frac{1}{2}$, such an equilibrium exists in the subgame, and the equilibrium prices are given by $P_i^N = P^N := \Psi + cp(1 - \rho_1)$ for all $i \in M$.

  However, for $\alpha \in (\frac{1}{2}, 1)$, no such equilibrium exists in the subgame because each manufacturer has a unilateral profitable deviation where she raises her price by a sufficiently small amount. In particular, suppose i raises her price by $\delta > 0$. Then, if δ is sufficiently small, consumer k prefers manufacturer i to all other manufacturers if and only if $\tilde{q}_{ik} = t_k$ and for all $j \neq i : \tilde{q}_{jk} \neq t_k$. Note that when δ is too large, no consumer would purchase from manufacturer i; in fact, a crude bound is $\delta \leq H - L$. Moreover, Lemma D.1.3 establishes that the measure of consumers that prefer manufacturer i to all other manufacturers for a sufficiently small price increase is bounded above by $\frac{w_1}{2m}$. Even if all of these consumers forgo the outside option, the incremental profit from manufacturer i raising her price above the expected cost is bounded above by $\varepsilon_m = \frac{w_1}{2m} \cdot (H - L)$. Since $\lim_{m \to \infty} w_1 = 0$, it follows that $\lim_{m \to \infty} \varepsilon_m = 0$. Thus, the manufacturer's incremental profit vanishes as $m \to \infty$.

Indeed, the non-existence of pure strategy equilibria has been recognized as a major conceptual difficulty in price-setting models (see Dixon 1987), and one alternative that has been explored is the notion of approximate equilibria. For example, Dixon (Dixon 1987, Definition 3, page 47) considers the notion of an ε-equilibrium, defined as one in which each agent is within ε of her best payoff, given the actions of the other agents. (In practice there are some small but positive costs associated with a price adjustment that can justify an ε-equilibrium.) For the price setting problem faced by the manufacturers in our model, we use this definition of ε-equilibrium.

It is clear that our solution is an ε-equilibrium in the subgame for all $\varepsilon \leq \varepsilon_m$ with $\lim_{m \to \infty} \varepsilon_m = 0$. In economic terms, a manufacturer's gain from unilaterally raising her price is negligible, particularly in markets with many competitors. Thus, for $\alpha \in (\frac{1}{2}, 1)$, $P_i^N = P^N = \Psi + cp(1 - \rho_1)$ for all $i \in M$ are the prices corresponding to an ε-equilibrium in the subgame for $\varepsilon = \frac{w_1(H-L)}{2m}$, and this approximate notion of equilibrium becomes exact in the limit (i.e., as $m \to \infty$).

Since $P_i^N = P^N$ are the same for all $i \in M$, $s_i^N = s_A^N$ for all $i \in M_A$ and $s_i^N = s_B^N$ for all $i \in M_B$ by the symmetry of the manufacturers of the same type from the consumers' perspective. Since half of the consumers are of type A and half of them are of type B, by symmetry, if there are an amount of type A (resp. type B) consumers selecting a type A manufacturer, then there are the same amount of type B (resp. type A) consumers selecting a type B manufacturer. Thus, $s_A^N = s_B^N$. In the following, we will derive the mathematical expression for s_i^N.

Consider a generic consumer k. By definition, consumer k's information set

$$\mathcal{F}_k = \sigma\left(t_k, \phi_k, \{a_l, P_l^N, \tilde{q}_{lk}\}_{l\in M}\right).$$

Note that ϕ_k is independent of t_k and $\{a_l, P_l^N, q_l, \tilde{q}_{lk}\}_{l\in M}$ and has no impact on $\{V_{lk}, u_{lk}\}_{l\in M}$, so we can safely drop it from $\mathcal{F}_k$ in the following discussion. Since the adoption decisions and the equilibrium prices under blockchain non-adoption are the same for all manufacturers, the adoption decisions $\bigcap_{l=1}^{2m}\{a_l = 0\}$ and the prices $\bigcap_{l=1}^{2m}\{P_l^N = \Psi + cp(1-\rho_1)\}$ do not provide extra information for consumer k to distinguish between manufacturers. Therefore, we do not include $\{a_l, P_l^N\}_{l\in M}$ into $\mathcal{F}_k$ in the following discussion. To sum up, $\mathcal{F}_k$ is now simplified to $\sigma\left(t_k, \{\tilde{q}_{lk}\}_{l\in M}\right)$. Treat $\{\tilde{q}_{lk}\}_{l\in M}$ as random variables and suppose that the signals consumer k observes are $\{\tilde{r}_{lk}\}_{l\in M}$, which are the realizations of $\{\tilde{q}_{lk}\}_{l\in M}$. For any realization t_k, $\mathbb{P}(q_i = t_k \mid \mathcal{F}_k)$ can be regarded as a function mapping $(\tilde{r}_{1k}, \ldots, \tilde{r}_{2m,k})$ to a number between $[0, 1]$, namely a probability. Evaluating $\mathbb{P}(q_i = t_k \mid \mathcal{F}_k)$ at t_k and $\{\tilde{r}_{lk}\}_{l\in M}$, we get the conditional probability $\mathbb{P}(q_i = t_k \mid t_k, \bigcap_{l=1}^{2m}\{\tilde{q}_{lk} = \tilde{r}_{lk}\})$.

From Lemma D.1.2, it follows that

$$V_{i(k)k} = (H-L)\mathbb{P}(q_{i(k)} = t_k \mid \mathcal{F}_k) + L = \theta_Z(H-L) + L$$

and

$$u_{i(k)k} = V_{i(k)k} - P_{i(k)}^N = \theta_Z(H-L) + L - P^N,$$

where $Z := |\{l : \tilde{q}_{lk} = t_k\}|$ is a random variable representing the number of favorable signals received by consumer k. Lemma D.1.3 gives the mathematical expression for w_z, the proportion of the consumers who observe z favorable signals. Consequently, $\mathbb{P}(Z = z) = w_z$ for $z = 0, 1, \ldots, 2m$, and $V_k^N = \mathbb{E}[V_{i(k)k}] = \left(\sum_{z=0}^{2m} w_z\theta_z\right)(H-L) + L$.

Of the w_z fraction of consumers who observe z favorable signals, a $1 - G(\theta_z(H-L) +$

$L - P^N$) will take the outside option because $\phi_k > u_{i(k)k} = \theta_z(H - L) + L - P^N$, and the remaining will randomly select a manufacturer from the ones that emit favorable signals to order the product with. Hence, the total sales under blockchain non-adoption $\sum_{i=1}^{2m} s_i^N = \sum_{z=0}^{2m} w_z G(\theta_z(H-L)+L-P^N)$. Since we have already established that the $2m$ manufacturers split the entire market evenly, therefore, $s_i^N = \frac{1}{2m}\sum_{z=0}^{2m} w_z G(\theta_z(H - L) + L - P^N)$.

□

Proof of Proposition 3.3.1. Recall that G denotes the cumulative distribution function of the outside option ϕ_k and supported on $[0, +\infty)$. Let $p(v)$ denote the corresponding probability density function, which is also defined on $[0, +\infty)$. Define an auxiliary function $f(x) := xG(x) + \int_x^{+\infty} vp(v)dv$. Since $f'(x) = G(x) > 0$ for all $x > 0$, $f(x)$ is strictly increasing in x on $[0, +\infty)$. By (3.10) and Proposition D.2.1, we have that

$$
\begin{aligned}
W_C^F &= \int_0^1 \mathbb{E}[\max\{\max_{i\in M} u_{ik}, \phi_k\}]\ dk \\
&= \mathbb{E}[\max\{H - P^F, \phi_k\}] \\
&= \mathbb{E}[H - P^F \mid H - P^F \geq \phi_k]\mathbb{P}(H - P^F \geq \phi_k) + \mathbb{E}[\phi_k \mid H - P^F < \phi_k]\mathbb{P}(H - P^F < \phi_k) \\
&= (H - P^F)G(H - P^F) + \int_{H-P^F}^{+\infty} vp(v)dv \\
&= f(H - P^F).
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
W_C^P &= \int_0^1 \mathbb{E}[\max\{\max_{i\in M} u_{ik}, \phi_k\}]\ dk \\
&= \frac{1}{2}\mathbb{E}[\max\{H - P_A^P, \phi_k\}] + \frac{1}{2}\mathbb{E}[\max\{H - P_B^P, \phi_k\}] \\
&= \frac{1}{2}f(H - P_A^P) + \frac{1}{2}f(H - P_B^P)
\end{aligned}
$$

and

$$
\begin{aligned}
W_C^N &= \int_0^1 \mathbb{E}[\max\{\max_{i \in M} u_{ik}, \phi_k\}]\ dk \\
&= \sum_{z=0}^{2m} w_z \mathbb{E}[\max\{\theta_z(H-L)+L-P^N, \phi_k\}] \\
&= \sum_{z=0}^{2m} w_z f(\theta_z(H-L)+L-P^N).
\end{aligned}
$$

Lemma D.1.2 and Proposition D.2.1 tell us $H > \theta_z(H-L)+L$ and $P^F < P_A^P < P_B^P = P^N$, so $H - P^F > H - P_A^P > H - P_B^P > \theta_z(H-L)+L-P^N$. It follows that $W_C^F > W_C^P > W_C^N$ since $f(x)$ is strictly increasing. □

Proof of Proposition 3.3.2.

- **Manufacturer Prices:** By Proposition D.2.1, $P^F < P_A^P < P^N$ and $P^F < P_B^P = P^N$, so $P_i^F < P_i^P \leq P_i^N$ for all $i \in M$.

- **Expected Good Value:** By Lemmas D.1.2 and D.1.3, $\sum_{z=0}^{2m} w_z\theta_z < \sum_{z=0}^{2m} w_z = 1$, so $H = H > \left(\sum_{z=0}^{2m} w_z\theta_z\right)(H-L)+L$ and $V_k^F \geq V_k^P > V_k^N$ for all $k \in [0,1]$.

- **Consumer Demand:** Since $P_i^P \leq P_i^N$ and G is increasing, by Lemmas D.1.2 and D.1.3,

$$
\begin{aligned}
s_i^N &= \frac{1}{2m}\sum_{z=0}^{2m} w_z G(\theta_z(H-L)+L-P^N) \\
&= \frac{1}{2m}\sum_{z=0}^{2m} w_z G(\theta_z(H-L)+L-P_i^N) \\
&\leq \frac{1}{2m}\sum_{z=0}^{2m} w_z G(H-P_i^P) \\
&= \frac{1}{2m} G(H-P_i^P)\sum_{z=0}^{2m} w_z \\
&= \frac{1}{2m} G(H-P_i^P) \\
&= s_i^P.
\end{aligned}
$$

Since $P_i^F < P_i^P$ and G is increasing, $s_i^P = \frac{1}{2m}G(H-P_i^P) \leq \frac{1}{2m}G(H-P_i^F) = \frac{1}{2m}G(H-P^F) = s_i^F$. Hence, $s_i^F \geq s_i^P \geq s_i^N$ for all $i \in M$. If G is strictly increasing, then the strict inequalities hold, i.e. $s_i^F > s_i^P > s_i^N$ for all $i \in M$.

□

Proof of Proposition 3.3.3. By Proposition D.2.1, each manufacturer has zero expected profit in the subgame of choosing price because of price competition, regardless of the level of blockchain adoption. It follows that $W_M = 0 - \sum_{i:i\in M} \chi_i a_i$, so $W_M^F = -\sum_{i:i\in M} \chi_i$, $W_M^P = -\sum_{i:i\in M_A} \chi_i$, and $W_M^N = 0$. We can see that $W_M^F < W_M^P < W_M^N$. □

Proof of Proposition 3.3.4. By (3.12), Proposition 3.3.1, and Proposition 3.3.3, we have that

$$\begin{aligned}
W^F &= W_C^F + W_M^F \\
&= f(H - P^F) - \sum_{i:i\in M} \chi_i \\
&= f(\Delta + L - P^F) - \sum_{i:i\in M} \chi_i, \\
W^P &= W_C^P + W_M^P \\
&= \frac{1}{2} f(H - P_A^P) + \frac{1}{2} f(H - P_B^P) - \sum_{i:i\in M_A} \chi_i \\
&= \frac{1}{2} f(\Delta + L - P_A^P) + \frac{1}{2} f(\Delta + L - P_B^P) - \sum_{i:i\in M_A} \chi_i, \\
W^N &= W_C^N + W_M^N \\
&= \sum_{z=0}^{2m} w_z f(\theta_z(H - L) + L - P^N) + 0 \\
&= \sum_{z=0}^{2m} w_z f(\theta_z \Delta + L - P^N).
\end{aligned}$$

View L as a fixed parameter and allow Δ to vary. Since $f(x)$ is strictly increasing, W^F, W^P and W^N all increase in Δ. W^F decreases in $\sum_{i:i\in M} \chi_i$ and is strictly smaller than W^N for sufficiently large $\sum_{i:i\in M} \chi_i$. The same argument applies to W^P.

Fixing adoption costs $\chi_i, \forall i \in M$,

$$\begin{aligned}\frac{\partial(W^F - W^N)}{\partial\Delta} &= G(\Delta + L - P^F) - \sum_{z=0}^{2m} w_z\theta_z G(\theta_z\Delta + L - P^N) \\ &\geq G(\Delta + L - P^F) - \sum_{z=0}^{2m} w_z\theta_z \cdot 1 \\ &\geq G(\Delta + L - P^F) - \left(\max_{0\leq z\leq 2m} \theta_z\right)\sum_{z=0}^{2m} w_z \\ &= G(\Delta + L - P^F) - \max_{0\leq z\leq 2m} \theta_z.\end{aligned}$$

By Lemma D.1.2, $0 < \max_{0\leq z\leq 2m} \theta_z < 1$. Because $\max_{0\leq z\leq 2m} \theta_z < 1$ and $\lim_{\Delta\to+\infty} G(\Delta + L - P^F) = 1$, $\frac{\partial(W^F-W^N)}{\partial\Delta} \geq \frac{1}{2}(1 - \max_{0\leq z\leq 2m} \theta_z) > 0$ when Δ is large enough. This implies that $\lim_{\Delta\to+\infty} W^F - W^N = +\infty$ no matter how large $\sum_{i:i\in M} \chi_i$ is, meaning that W^F strictly exceeds W^N for sufficiently large Δ. Applying a similar argument, we can also show that W^P strictly exceeds W^N for sufficiently large Δ.

$W^F - W^N$ (resp. $W^P - W^N$) is negative for sufficiently high adoption costs and positive for sufficiently large Δ, so blockchain adoption has ambiguous effects on global welfare. □

Proof of Proposition 3.3.5. By Proposition D.2.1, a type A manufacturer i (i.e. $i \in M_A$) adopts the blockchain and receives zero expected profit in the subgame of choosing price due to price competition, i.e. $\Pi(1, a_{-i}) = 0$, under blockchain adoption (either full or partial). Hence, her expected profit for adopting the blockchain is $0 - \chi_i = -\chi_i$. However, if she chooses not to adopt the blockchain, then her expected profit is always non-negative, which is better than $-\chi_i$. Thus, with blockchain adoption (either full or partial), manufacturer i has incentive to deviate so there does not exist an equilibrium. □

Proof of Proposition 3.3.6. Proposition 3.3.4 tells us that full blockchain adoption is welfare-enhancing for sufficiently large Δ. However, by Proposition 3.3.5, there does not exist an equilibrium with full blockchain adoption. Hence, an adoption failure arises by definition. □

Proof of Proposition 3.3.7. Recall that G denotes the cumulative distribution function of the outside option ϕ_k of consumer k. We consider the system of transfers where $\tau_i = \chi_i$ for all $i \in M$, and $\kappa = \frac{\sum_{i:i\in M}\chi_i}{G(\phi)}$, where ϕ is to be determined. We will first show that for all sufficiently large Δ, there exists ϕ such that $b_k = 1$ if and only if $\phi_k \le \phi$ and thus $\mu_\kappa = \mu(\{k : b_k = 1\}) = \mathbb{P}(\phi_k \le \phi) = G(\phi)$. Since $\kappa \cdot \mu_\kappa = \frac{\sum_{i:i\in M}\chi_i}{G(\phi)} \cdot G(\phi) = \sum_{i:i\in M}\chi_i = \sum_{i:i\in M}\tau_i$, it follows that the system of transfers specified above is self-financing. Next, we will show that $\{a_i = 1, P_i = \Psi + cp(1-\rho_{2m})\}_{i\in M}$ can be sustained as an equilibrium under such a system of transfers. Thereby, full blockchain adoption arises in equilibrium and the adoption failure is resolved.

Suppose $a_i = 1$ and $P_i = P^F := \Psi + cp(1-\rho_{2m})$ for all $i \in M$. Consumers are now in the subgame of making the b_k decision. A generic consumer k solves the following optimization problem:

$$\max_{b_k} g(b_k, \phi_k; \phi) := \mathbb{E}^{b_k}[\max\{\max_{i\in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k],$$

where consumer k's information set is given by $\mathcal{G}_k = \sigma(t_k, \phi_k, \{a_l, P_l\}_{l\in M})$. When $b_k = 1$, i.e. consumer k has access to the information on the blockchain, the result in Proposition D.2.1 regarding the full adoption case implies that $\max_{i\in M} u_{ik} = H - P^F = \Delta + L - P^F$. Therefore,

$$\begin{aligned} g(1, \phi_k; \phi) &= \mathbb{E}^1[\max\{\max_{i\in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \\ &= \max\{\Delta + L - P^F,\ \phi_k\} - \frac{\sum_{i:i\in M}\chi_i}{G(\phi)}. \end{aligned}$$

When $b_k = 0$, consumer k opts to forgo access to the information on the blockchain. The result in Proposition D.2.1 regarding the non-adoption case implies that $\max_{i\in M} u_{ik} = \theta_Z(H - L) + L - P^F = \theta_Z\Delta + L - P^F$ where $Z := |\{l : \tilde{q}_{lk} = t_k\}|$ is a random variable representing the number of favorable signals received by consumer k and $Z = z$ w.p. w_z for $z = 0, 1, \ldots, 2m$. Therefore,

$$\begin{aligned} g(0, \phi_k; \phi) &= \mathbb{E}^0[\max\{\max_{i \in M} u_{ik},\ \phi_k\} - \kappa b_k \mid \mathcal{G}_k] \\ &= \mathbb{E}^0[\max\{\theta_Z \Delta + L - P^F,\ \phi_k\} \mid \mathcal{G}_k] \\ &= \sum_{z=0}^{2m} w_z \cdot \max\{\theta_z \Delta + L - P^F,\ \phi_k\}. \end{aligned}$$

Consumer k chooses to gain access to the information on the blockchain, i.e. $b_k = 1$, if and only if

$$h(\phi_k; \phi) := g(1, \phi_k; \phi) - g(0, \phi_k; \phi) \geq 0.$$

By Lemma D.1.4(b), it follows that there exists $\phi^* \in [0, +\infty)$ such that $h(\phi^*; \phi^*) = 0$ and $h(\phi_k; \phi^*) \geq 0$ if and only if $\phi_k \leq \phi^*$. Thus, $b_k = 1$ if and only if $\phi_k \leq \phi^*$ is an optimal decision rule for all consumers. Since measure μ_κ of the consumers who pay the fee κ to gain access to the information on the blockchain satisfies $\mu_\kappa = \mu(\{k : b_k = 1\}) = \mathbb{P}(\phi_k \leq \phi^*) = G(\phi^*)$, it follows that the payment collected from the consumers is $\kappa \cdot \mu_\kappa = \frac{\sum_{i:i \in M} \chi_i}{G(\phi^*)} \cdot G(\phi^*) = \sum_{i:i \in M} \chi_i = \sum_{i:i \in M} \tau_i$, indicating that the system of transfers is self-financing.

Next, we take one step back to the stage where manufacturers make their adoption and pricing decisions. For manufacturer i, if she adopts the blockchain, then she has no incentive to change her price from P^F because of price competition. Her expected profit is zero in this case since her price $P_i = P^F$ is equal to her expected cost and the transfer $\tau_i = \chi_i$ just covers her adoption cost χ_i. If she does not adopt the blockchain, she no longer has access to the information on the blockchain, and her expected cost goes up to $\Psi + cp(1 - \rho_1)$. Accordingly, in order to obtain non-negative profit, her price $P_i \geq \Psi + cp(1 - \rho_1) > \Psi + cp(1 - \rho_{2m}) = P^F$. Since $V_{ik} \leq H = V_{jk}$ for all j such that $j \neq i$ and $q_j = t_k$, it follows that $u_{ik} = V_{ik} - P_i < V_{jk} - P^F = u_{jk}$ for all j such that $j \neq i$ and $q_j = t_k$. Consequently, no consumer will select manufacturer i. This implies that her expected profit from not adopting the blockchain is zero as well, so she has no incentive to deviate from adopting the blockchain. To conclude, manufacturer i has no incentive to deviate from the adoption decision $a_i = 1$ and the pricing decision $P_i = \Psi + cp(1 - \rho_{2m})$. This completes the proof. $\square$

Appendix E: Chapter 4 Proofs

Proof of Proposition 4.3.1. Define $\tilde{\alpha} = \alpha + \delta$. Note that $\alpha \in [\frac{1}{2}, 1)$, $\delta \in (0, 1-\alpha]$ and $\tilde{\alpha} \in (\frac{1}{2}, 1]$. Consider a generic consumer. Let $\{\tilde{q}_i : i = 1, \ldots, m\}$ be the set of quality signals she receives from the m manufacturers.

Under full adoption, $\mathbb{P}(q_i = H, \tilde{q}_i = H) = \mathbb{P}(q_i = L, \tilde{q}_i = L) = \frac{1}{2}\tilde{\alpha}$ and $\mathbb{P}(q_i = H, \tilde{q}_i = L) = \mathbb{P}(q_i = L, \tilde{q}_i = H) = \frac{1}{2}(1-\tilde{\alpha})$. Therefore, $\mathbb{P}(\tilde{q}_i = H) = \mathbb{P}(\tilde{q}_i = L) = \frac{1}{2}$ and $\mathbb{P}(q_i = H \mid \tilde{q}_i = H) = 1 - \mathbb{P}(q_i = L \mid \tilde{q}_i = H) = \mathbb{P}(q_i = L \mid \tilde{q}_i = L) = 1 - \mathbb{P}(q_i = H \mid \tilde{q}_i = L) = \tilde{\alpha}$. Since $\tilde{\alpha} > \frac{1}{2} > 1 - \tilde{\alpha}$, it follows that the consumer will always place an order with a manufacturer with $\tilde{q}_i = H$, if such a manufacturer exists. Hence, we need to consider the following two cases:

1. $\tilde{q}_i = H$ for some $i \in \{1, \ldots, m\}$: This case occurs with probability $1 - 0.5^m$, and in this case, the consumer places an order with any manufacturer i with $\tilde{q}_i = H$, and receives the expected quality $\mathbb{E}[q_i \mid \tilde{q}_i = H] = \tilde{\alpha}(H - L) + L$.

2. $\tilde{q}_i = L$ for all $i \in \{1, \ldots, m\}$: This case occurs with probability 0.5^m, and in this case, the consumer places an order with any manufacturer, and receives the expected quality $\mathbb{E}[q_i \mid \tilde{q}_i = L] = (1-\tilde{\alpha})(H - L) + L$.

Since all the consumers are identical, the total consumer welfare W_C^{FA} is equal to the expected utility of a generic consumer. Thus, the consumer welfare

$$\begin{aligned} W_C^{FA} &= \int_0^1 \mathbb{E}[\max_i \mathbb{E}_k[q_i]]\mathrm{d}k \\ &= [\tilde{\alpha}(1-0.5^m) + (1-\tilde{\alpha})0.5^m]\,(H-L) + L. \end{aligned}$$

The no blockchain or non-adoption of blockchain corresponds to $\delta = 0$. In this case, the consumer welfare W_C^{NA} is given by

$$W_C^{NA} = [\alpha(1 - 0.5^m) + (1 - \alpha)0.5^m]\,(H - L) + L.$$

Since $g(x) = x(1 - 0.5^m) + (1 - x)0.5^m$ is increasing in terms of x over $[0, 1]$ and $\delta > 0$, we conclude that blockchain always improves consumer welfare. □

Proof of Proposition 4.3.2. Fix an arbitrary manufacturer i. Suppose there is no blockchain or none of the manufacturers adopts the blockchain. Fix the types of all manufacturers, and fix a generic consumer, and let $\{\tilde{q}_i : i = 1, \ldots, m\}$ denote the quality signals that this consumer receives from the m manufacturers. Define

$$\beta_i = \mathbf{1}_{\{\tilde{q}_i = H\}},$$
$$\theta_i = \mathbb{E}[\beta_i \mid q_i] = \mathbb{P}(\tilde{q}_i = H \mid q_i) = \alpha\mathbf{1}_{\{q_i=H\}} + (1 - \alpha)\mathbf{1}_{\{q_i=L\}},$$

where $\mathbf{1}_A \in \{0, 1\}$ denotes the indicator of the event A.

Let Z_α^L (resp. Z_α^H) denote the market share manufacturer i can get when $q_i = L$ (resp. $q_i = H$). Thus, the expected welfare of manufacturer i is given by $\frac{1}{2}\mathbb{E}\left[Z_\alpha^L - pe^{-Z_\alpha^L\lambda}c(Z_\alpha^L)\right] + \frac{1}{2}\mathbb{E}\left[Z_\alpha^H - pe^{-Z_\alpha^H\lambda}c(Z_\alpha^H)\right]$. Next, we compute an expression for Z_α^L and Z_α^H.

First, suppose $q_i = L$. Then we have the following two cases.

1. $\tilde{q}_i = L$: This case occurs with probability α, and in this case, manufacturer i is able to attract market share, if and only if, $\tilde{q}_j = L$, for all $j \neq i$, and in this case, manufacturer i shares the unit demand with all other manufacturers. Thus, the market share $Z_\alpha^L(L)$ is given by

$$Z_\alpha^L(L) = \frac{1}{m}\left[\prod_{j\neq i}\mathbb{P}(\tilde{q}_j = L \mid q_j)\right] = \frac{1}{m}\prod_{j\neq i}(1 - \theta_j).$$

Using the fact that the type of each manufacturer is independently drawn, and $\mathbb{E}_{q_j}[1 - \theta_j] = 0.5$, it follows that $\mathbb{E}_{q_{-i}}[Z_\alpha^L(L)] = \frac{0.5^{m-1}}{m}$.

2. $\tilde{q}_i = H$: This case occurs with probability $1-\alpha$, and in this case, manufacturer i shares the unit demand with all other manufacturers j with $\tilde{q}_j = H$. Thus, the market share $Z_\alpha^L(H)$ is given by

$$
\begin{aligned}
Z_\alpha^L(H) &= \sum_{\tilde{q}_{-i}\in\{L,H\}^{m-1}} \frac{\prod_{j\neq i}\mathbb{P}(\tilde{q}_j = L \mid q_j)^{\mathbf{1}_{\{\tilde{q}_j=L\}}}\mathbb{P}(\tilde{q}_j = H \mid q_j)^{\mathbf{1}_{\{\tilde{q}_j=H\}}}}{1+\sum_{j\neq i}\mathbf{1}_{\{\tilde{q}_j=H\}}} \\
&= \sum_{\beta_{-i}\in\{0,1\}^{m-1}} \frac{\prod_{j\neq i}(1-\theta_j)^{1-\beta_j}\theta_j^{\beta_j}}{1+\sum_{j\neq i}\beta_j}.
\end{aligned}
$$

Using the fact the types of all manufacturers are drawn independently, $\mathbb{E}_{q_j}[(1-\theta_j)^{1-\beta_j}\theta_j^{\beta_j}] = 0.5$, and there are $\binom{m-1}{k}$ sequences with exactly k signals $\tilde{q}_j$ equal to H, it follows that $\mathbb{E}_{q_{-i}}[Z_\alpha^L(H)] = 0.5^m \sum_{k=0}^{m-1}\binom{m-1}{k}\frac{1}{k+1}$.

Since we consider a continuum of infinitesimal consumers, the probability immediately translates to fractions, i.e. α fraction of the consumers receives the signal $\tilde{q}_i = L$ and $1-\alpha$ fraction of the consumers receive the signal $\tilde{q}_i = H$. Thus, it follows that the market share

$$
\begin{aligned}
Z_\alpha^L &= \alpha Z_\alpha^L(L) + (1-\alpha) Z_\alpha^L(H) \\
&= \alpha\left(\frac{1}{m}\prod_{j\neq i}(1-\theta_j)\right) + (1-\alpha)\left(\sum_{\beta_{-i}\in\{0,1\}^{m-1}} \frac{\prod_{j\neq i}\theta_j^{\beta_j}(1-\theta_j)^{1-\beta_j}}{1+\sum_{j\neq i}\beta_j}\right).
\end{aligned}
$$

Using a similar analysis, it follows that the market share Z_α^H when $q_i = H$ is given by

$$
Z_\alpha^H = (1-\alpha)\left(\frac{1}{m}\prod_{j\neq i}(1-\theta_j)\right) + \alpha\left(\sum_{\beta_{-i}\in\{0,1\}^{m-1}} \frac{\prod_{j\neq i}\theta_j^{\beta_j}(1-\theta_j)^{1-\beta_j}}{1+\sum_{j\neq i}\beta_j}\right).
$$

Thus, the manufacturer welfare is given by

$$\begin{aligned}
W_{M,\alpha}^{NA} &= m\left(\frac{1}{2}\mathbb{E}\left[Z_\alpha^L - pe^{-Z_\alpha^L\lambda}c(Z_\alpha^L)\right] + \frac{1}{2}\mathbb{E}\left[Z_\alpha^H - pe^{-Z_\alpha^H\lambda}c(Z_\alpha^H)\right]\right)\\
&= m\left(0.5^m\sum_{k=0}^{m-1}\binom{m-1}{k}\frac{1}{k+1} + 0.5^m\frac{1}{m} - \mathbb{E}\left[0.5pe^{-Z_\alpha^L\lambda}c(Z_\alpha^L) + 0.5pe^{-Z_\alpha^H\lambda}c(Z_\alpha^H)\right]\right)\\
&= 0.5^m\sum_{k=0}^{m}\binom{m}{k} - 0.5mp\mathbb{E}\left[e^{-Z_\alpha^L\lambda}c(Z_\alpha^L) + e^{-Z_\alpha^H\lambda}c(Z_\alpha^H)\right]\\
&= 1 - 0.5mp\mathbb{E}\left[e^{-Z_\alpha^L\lambda}c(Z_\alpha^L) + e^{-Z_\alpha^H\lambda}c(Z_\alpha^H)\right]
\end{aligned}$$

where the last equality follows from the fact that the first term is the sum of all terms in the binomial series with $p = q = \frac{1}{2}$.

If $\alpha = \frac{1}{2}$, then $Z_{\frac{1}{2}}^L = Z_{\frac{1}{2}}^H = \frac{1}{m}$. Therefore, the manufacturer welfare is given by

$$W_{M,\frac{1}{2}}^{NA} = 1 - p\ e^{-\frac{\lambda}{m}}\ m\ c\left(\frac{1}{m}\right).$$

Under full adoption, the expected market shares are given by $Z_{\tilde{\alpha}}^L$ and $Z_{\tilde{\alpha}}^H$, where $\tilde{\alpha} = \alpha + \delta$, and the manufacturer welfare is given by

$$\begin{aligned}
W_{M,\tilde{\alpha}}^{FA} &= m\left(\frac{1}{2}\mathbb{E}\left[Z_{\tilde{\alpha}}^L - pe^{-\lambda}c(Z_{\tilde{\alpha}}^L)\right] + \frac{1}{2}\mathbb{E}\left[Z_{\tilde{\alpha}}^H - pe^{-\lambda}c(Z_{\tilde{\alpha}}^H)\right]\right) - \sum_{i=1}^{m}\chi_i\\
&= m\left(0.5^m\sum_{k=0}^{m-1}\binom{m-1}{k}\frac{1}{k+1} + 0.5^m\frac{1}{m} - \mathbb{E}\left[0.5pe^{-\lambda}c(Z_{\tilde{\alpha}}^L) + 0.5pe^{-\lambda}c(Z_{\tilde{\alpha}}^H)\right]\right) - \sum_{i=1}^{m}\chi_i\\
&= 1 - 0.5mpe^{-\lambda}\mathbb{E}\left[c(Z_{\tilde{\alpha}}^L) + c(Z_{\tilde{\alpha}}^H)\right] - \sum_{i=1}^{m}\chi_i
\end{aligned}$$

When $\tilde{\alpha} = 1$, i.e. the blockchain is fully-revealing,

$$Z_1^L = \begin{cases} \frac{1}{m} & \text{w.p. } 0.5^{m-1},\\ 0 & \text{w.p. } 1 - 0.5^{m-1}, \end{cases}$$

and $Z_1^H = \frac{1}{k+1}$, with probability $0.5^{m-1}\binom{m-1}{k}$, for $k = 0, 1, ..., m-1$. Therefore,

$$\begin{aligned}
W_{M,1}^{FA} &= 1 - 0.5mpe^{-\lambda}\left[0.5^{m-1}c\Big(\frac{1}{m}\Big) + 0.5^{m-1}\sum_{k=0}^{m-1}\binom{m-1}{k}\cdot c\Big(\frac{1}{k+1}\Big)\right] - \sum_{i=1}^{m}\chi_i \\
&= 1 - 0.5^m pe^{-\lambda}\left[m\cdot c\Big(\frac{1}{m}\Big) + \sum_{k=0}^{m-1}\binom{m-1}{k}m\cdot c\Big(\frac{1}{k+1}\Big)\right] - \sum_{i=1}^{m}\chi_i \\
&= 1 - 0.5^m pe^{-\lambda}\left[m\cdot c\Big(\frac{1}{m}\Big) + \sum_{k=1}^{m}\binom{m}{k}k\cdot c\Big(\frac{1}{k}\Big)\right] - \sum_{i=1}^{m}\chi_i.
\end{aligned}$$

The difference $\Delta(\lambda)$ between the manufacturer welfare under full adoption and no blockchain is given by

$$\begin{aligned}
\Delta(\lambda) &= W_{M,1}^{FA} - W_{M,\frac{1}{2}}^{NA} \\
&= \left(1 - 0.5^m pe^{-\lambda}\left[m\cdot c\Big(\frac{1}{m}\Big) + \sum_{k=1}^{m}\binom{m}{k}k\cdot c(\frac{1}{k})\right] - \sum_{i=1}^{m}\chi_i\right) - \left(1 - pe^{-\frac{\lambda}{m}}\cdot m\cdot c\Big(\frac{1}{m}\Big)\right) \\
&= 0.5^m p\left[(e^{-\frac{\lambda}{m}} - e^{-\lambda})mc\Big(\frac{1}{m}\Big) + \sum_{k=1}^{m}\binom{m}{k}\left(e^{-\frac{\lambda}{m}}mc\Big(\frac{1}{m}\Big) - e^{-\lambda}kc\Big(\frac{1}{k}\Big)\right)\right] - \sum_{i=1}^{m}\chi_i.
\end{aligned}$$

Jensen's inequality applied to the convex cost function c, we get $kc(\frac{1}{k}) = \sum_{i=1}^{k} c(\frac{1}{k}) + \sum_{i=k+1}^{m} c(0) \geq mc(\frac{1}{m})$. Hence,

$$\begin{aligned}
\Delta(0) &= 0.5^m p\sum_{k=1}^{m}\binom{m}{k}\left(mc\Big(\frac{1}{m}\Big) - kc\Big(\frac{1}{k}\Big)\right) - \sum_{i=1}^{m}\chi_i \\
&\leq 0.5^m p\sum_{k=1}^{m}\binom{m}{k}\left(mc\Big(\frac{1}{m}\Big) - mc\Big(\frac{1}{m}\Big)\right) - \sum_{i=1}^{m}\chi_i \\
&< 0.
\end{aligned}$$

Since $\Delta(\lambda)$ is a continuous function, $\Delta(\lambda)$ is negative for λ that is sufficiently small. □

Proof of Proposition 4.3.3. By the result of Proposition 4.3.2, under full adoption, the manufacturer welfare is given by

$$W_{M,\tilde{\alpha}}^{FA} = 1 - 0.5mpe^{-\lambda}\mathbb{E}\left[c(Z_{\tilde{\alpha}}^{L}) + c(Z_{\tilde{\alpha}}^{H})\right] - \sum_{i=1}^{m}\chi_i;$$

under non-adoption or no blockchain case, the manufacturer welfare is given by

$$W_{M,\alpha}^{NA} = 1 - 0.5mp\mathbb{E}\left[e^{-Z_{\alpha}^{L}\lambda}c(Z_{\alpha}^{L}) + e^{-Z_{\alpha}^{H}\lambda}c(Z_{\alpha}^{H})\right].$$

Hence, the difference in the manufacturer welfare under full adoption and no blockchain is given by

$$\begin{aligned}
\Delta(\lambda) &= W_{M,\tilde{\alpha}}^{FA} - W_{M,\alpha}^{NA} \\
&= \left(1 - 0.5mpe^{-\lambda}\mathbb{E}\left[c(Z_{\alpha}^{L}) + c(Z_{\alpha}^{H})\right]\right) - \left(1 - 0.5mp\mathbb{E}\left[e^{-Z_{\tilde{\alpha}}^{L}\lambda}c(Z_{\tilde{\alpha}}^{L}) + e^{-Z_{\tilde{\alpha}}^{H}\lambda}c(Z_{\tilde{\alpha}}^{H})\right]\right) - \sum_{i=1}^{m}\chi_i \\
&= 0.5mp\left(\mathbb{E}\left[e^{-Z_{\alpha}^{L}\lambda}c(Z_{\alpha}^{L}) + e^{-Z_{\alpha}^{H}\lambda}c(Z_{\alpha}^{H})\right] - e^{-\lambda}\mathbb{E}\left[c(Z_{\tilde{\alpha}}^{L}) + c(Z_{\tilde{\alpha}}^{H})\right]\right) - \sum_{i=1}^{m}\chi_i.
\end{aligned}$$

Since $\alpha < 1$, there exist $z_{\alpha,\min}$ and $z_{\alpha,\max}$ such that $0 < z_{\alpha,\min} \leq Z_{\alpha}^{L}, Z_{\alpha}^{H} \leq z_{\alpha,\max} < 1$. Also, we have $0 \leq Z_{\tilde{\alpha}}^{L}, Z_{\tilde{\alpha}}^{H} \leq 1$. Suppose $c(z_{\alpha,\min}) > 0$. Since $1 - z_{\alpha,\max} > 0$ and the reputation cost function c is increasing, we have that for all sufficiently large λ, the first term in the expression for $\Delta(\lambda)$

$$\begin{aligned}
&0.5mp\left(\mathbb{E}\left[e^{-Z_{\alpha}^{L}\lambda}c(Z_{\alpha}^{L}) + e^{-Z_{\alpha}^{H}\lambda}c(Z_{\alpha}^{H})\right] - e^{-\lambda}\mathbb{E}\left[c(Z_{\tilde{\alpha}}^{L}) + c(Z_{\tilde{\alpha}}^{H})\right]\right) \\
&\geq mp\left(e^{-z_{\alpha,\max}\lambda}c(z_{\alpha,\min}) - e^{-\lambda}c(1)\right) \\
&= mpe^{-\lambda}\left(e^{(1-z_{\alpha,\max})\lambda}c(z_{\alpha,\min}) - c(1)\right) \\
&> 0.
\end{aligned}$$

Thus, it follows that there exists a λ_0 such that $\Delta(\lambda_0) > 0$, blockchain improves manufacturer welfare, when the total adoption cost $\sum_{i=1}^{m}\chi_i$ is sufficiently small.

Note that $\lim_{\lambda \to +\infty} \Delta(\lambda) = -\sum_{i=1}^{m} \chi_i < 0$ and $\Delta(\lambda)$ is continuous on $[0, +\infty)$. This implies that $\max_{\lambda \geq 0} \Delta(\lambda) \geq \Delta(\lambda_0) > 0$, is attained by some intermediate value of λ. $\square$

Proof of Proposition 4.3.4. From the expressions for consumer welfare in Proposition 4.3.1 and manufacturer welfare in Proposition 4.3.2, it follows that, under full adoption, the total welfare

$$\begin{aligned} W_{\tilde{\alpha}}^{FA} &= \left(\left[\tilde{\alpha}(1-0.5^m) + (1-\tilde{\alpha})0.5^m\right](H-L) + L \right) \\ &\quad + \left(1 - 0.5mpe^{-\lambda}\mathbb{E}\left[c(Z_{\tilde{\alpha}}^L) + c(Z_{\tilde{\alpha}}^H)\right] - \sum_{i=1}^{m} \chi_i \right); \end{aligned}$$

under non-adoption or no blockchain case, the total welfare

$$\begin{aligned} W_{\alpha}^{NA} &= \left(\left[\alpha(1-0.5^m) + (1-\alpha)0.5^m\right](H-L) + L \right) \\ &\quad + \left(1 - 0.5mp\mathbb{E}\left[e^{-Z_{\alpha}^L\lambda}c(Z_{\alpha}^L) + e^{-Z_{\alpha}^H\lambda}c(Z_{\alpha}^H)\right]\right). \end{aligned}$$

Both $W_{\tilde{\alpha}}$ and W_{α} are increasing and linear in the product quality dispersion $H - L$, so they diverge with the product quality dispersion.

The change in welfare

$$\begin{aligned} \Delta_W &= W_{\tilde{\alpha}}^{FA} - W_{\alpha}^{NA} \\ &= \left(1 - 0.5mpe^{-\lambda}\mathbb{E}\left[c(Z_{\alpha}^L) + c(Z_{\alpha}^H)\right]\right) - \left(1 - 0.5mp\mathbb{E}\left[e^{-Z_{\tilde{\alpha}}^L\lambda}c(Z_{\tilde{\alpha}}^L) + e^{-Z_{\tilde{\alpha}}^H\lambda}c(Z_{\tilde{\alpha}}^H)\right]\right) \\ &\quad +(\tilde{\alpha} - \alpha)(1 - 0.5^{m-1})(H - L) - \sum_{i=1}^{m} \chi_i \\ &= 0.5mp\left(\mathbb{E}\left[e^{-Z_{\alpha}^L\lambda}c(Z_{\alpha}^L) + e^{-Z_{\alpha}^H\lambda}c(Z_{\alpha}^H)\right] - e^{-\lambda}\mathbb{E}\left[c(Z_{\tilde{\alpha}}^L) + c(Z_{\tilde{\alpha}}^H)\right]\right) \\ &\quad +\delta(1 - 0.5^{m-1})(H - L) - \sum_{i=1}^{m} \chi_i. \end{aligned}$$

When $H - L$ is sufficiently large, $\Delta_W > 0$, which indicates that the global welfare under blockchain adoption strictly exceeds the global welfare without blockchain for sufficiently high product quality dispersion; when $\sum_{i=1}^{m} \chi_i$ is sufficiently large, $\Delta_W < 0$, which indicates that the global welfare under blockchain adoption is strictly smaller than the global welfare without blockchain for sufficiently high adoption costs. □

Proof of Proposition 4.3.5. Fix an arbitrary manufacturer i. Suppose $q_i = L$ and all other manufacturers play the adoption strategy g_3. First, suppose manufacturer i adopts the blockchain, i.e. $a_i = 1$. By the result of Proposition 4.3.2, the expected payoff of manufacturer i is

$$\mathbb{E}_{q_{-i}}\left[s_i(1, g_3(q_{-i})) - pe^{-\lambda\left[s_i(1,g_3(q_{-i}))+\sum_{k\neq i} g_3(q_k)s_k\right]}c(s_i(1, g_3(q_{-i})))\right] - \chi_i = \mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right] - \chi_i.$$

Next, suppose manufacturer i does not adopt the blockchain, i.e. $a_i = 0$. Let $Z^{CB}_{\tilde{\alpha}}$ denote the market share manufacturer i can get in this case. Fix the types of all manufacturers, and fix a generic consumer, and let $\{\tilde{q}_i : i = 1, \ldots, m\}$ denote the quality signals that this consumer receives from the m manufacturers. Then we have the following two cases.

1. $\tilde{q}_i = L$: This case occurs with probability α. Since all other manufacturers adopt the blockchain, the posterior quality for all manufacturer $j \neq i$ is given by

$$\mathbb{E}[q_j \mid a_j = 1, \tilde{q}_j = L] = (1 - \tilde{\alpha})(H - L) + L,$$
$$\mathbb{E}[q_j \mid a_j = 1, \tilde{q}_j = H] = \tilde{\alpha}(H - L) + L.$$

   Since manufacturer i's expected quality $\mathbb{E}[q_i \mid a_i = 0, \tilde{q}_i = L] = (1 - \alpha)(H - L) + L$ satisfies

$$\mathbb{E}[q_j \mid a_j = 1, \tilde{q}_j = L] < \mathbb{E}[q_i \mid a_i = 0, \tilde{q}_i = L] < \mathbb{E}[q_j \mid a_j = 1, \tilde{q}_j = H],$$

   it follows that manufacturer i receives market share, if and only if, $\tilde{q}_j = L$ for all $j \neq i$, and in this case, she receives the entire unit demand. Thus, the market share $Z^{CB}_{\tilde{\alpha}}(L) = \prod_{j\neq i} \mathbb{P}(\tilde{q}_j = L \mid q_j) = \prod_{j\neq i}(1 - \theta_j)$.

2. $\tilde{q}_i = H$: This case occurs with probability $1 - \alpha$. Since all other manufacturers adopt the

blockchain, an argument similar to the case when $\tilde{q}_i = L$, establishes that

$$\mathbb{E}[q_j \mid a_j = 1, \tilde{q}_j = L] < \mathbb{E}[q_i \mid a_i = 0, \tilde{q}_i = L] = \alpha(H - L) + L < \mathbb{E}[q_j \mid a_j = 1, \tilde{q}_j = H].$$

Thus, manufacturer i receives market share, if and only if, $\tilde{q}_j = L$ for all $j \neq i$, and in this case, she receives the entire unit demand. Thus, the expected market share $Z_{\tilde{\alpha}}^{CB}(H) = \prod_{j \neq i} \mathbb{P}(\tilde{q}_j = L \mid q_j) = \prod_{j \neq i}(1 - \theta_j)$.

Since α fraction of the consumers receive the signal $\tilde{q}_i = L$, and $1 - \alpha$ fraction of the consumers receive the signal $\tilde{q}_i = H$, it follows that

$$\begin{aligned} Z_{\tilde{\alpha}}^{CB} &= \alpha Z_{\tilde{\alpha}}^{CB}(L) + (1 - \alpha) Z_{\tilde{\alpha}}^{CB}(H) \\ &= \prod_{j \neq i}(1 - \theta_j), \end{aligned} \tag{E.1}$$

where $\theta_j = \tilde{\alpha}$ and $1 - \tilde{\alpha}$ with equal probability, the expected payoff of manufacturer i is

$$\mathbb{E}_{q_{-i}}\left[s_i(0, g_3(q_{-i})) - pe^{-\lambda s_i(0, g_3(q_{-i}))} c(s_i(0, g_3(q_{-i})))\right] = \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda} c(Z_{\tilde{\alpha}}^{CB})\right].$$

In order for $(f_1 = g_3, \ldots, f_m = g_3)$ to be a Bayesian Nash equilibrium, manufacturer i should prefer $a_i = g_3(L) = 1$ to $a_i = 0$, i.e. we must have

$$\begin{aligned} &\mathbb{E}_{q_{-i}}\left[s_i(1, g_3(q_{-i})) - pe^{-\lambda\left[s_i(1, g_3(q_{-i})) + \sum_{k \neq i} g_3(q_k) s_k\right]} c(s_i(1, g_3(q_{-i})))\right] - \chi_i \\ &= \mathbb{E}\left[Z_{\tilde{\alpha}}^{L} - pe^{-\lambda} c(Z_{\tilde{\alpha}}^{L})\right] - \chi_i \\ &\geq \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda} c(Z_{\tilde{\alpha}}^{CB})\right] \\ &= \mathbb{E}_{q_{-i}}\left[s_i(0, g_3(q_{-i})) - pe^{-\lambda s_i(0, g_3(q_{-i}))} c(s_i(0, g_3(q_{-i})))\right]. \end{aligned}$$

This inequality holds provided

$$\chi_i \leq \mathbb{E}\left[Z_{\tilde{\alpha}}^{L} - pe^{-\lambda} c(Z_{\tilde{\alpha}}^{L})\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda} c(Z_{\tilde{\alpha}}^{CB})\right].$$

Next, suppose $q_i = H$ and all other manufacturers play strategy g_3. From an analysis similar to the one used above, it follows that in order for $(f_1 = g_3, ..., f_m = g_3)$ to be a Bayesian Nash equilibrium, manufacturer i should prefer $a_i = g_3(H) = 1$ to $a_i = 0$, i.e. we must have

$$\begin{aligned}
&\mathbb{E}_{q_{-i}}\left[s_i(1, g_3(q_{-i})) - pe^{-\lambda\left[s_i(1,g_3(q_{-i}))+\sum_{k\neq i} g_3(q_k)s_k\right]}c(s_i(1, g_3(q_{-i})))\right] - \chi_i \\
&\quad = \mathbb{E}\left[Z_{\tilde{\alpha}}^H - pe^{-\lambda}c(Z_{\tilde{\alpha}}^H)\right] - \chi_i \\
&\quad \geq \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right] \\
&\quad = \mathbb{E}_{q_{-i}}\left[s_i(0, g_3(q_{-i})) - pe^{-\lambda s_i(0,g_3(q_{-i}))}c(s_i(0, g_3(q_{-i})))\right].
\end{aligned}$$

This inequality holds provided

$$\chi_i \leq \mathbb{E}\left[Z_{\tilde{\alpha}}^H - pe^{-\lambda}c(Z_{\tilde{\alpha}}^H)\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right].$$

Since $\tilde{\alpha} > 0.5$, $Z_{\tilde{\alpha}}^H \geq Z_{\tilde{\alpha}}^L$ holds for each ω. Suppose $x - pe^{-\lambda}c(x)$ is increasing for $x \in [0, 1]$, it follows that

$$\begin{aligned}
&\left(\mathbb{E}\left[Z_{\tilde{\alpha}}^H - pe^{-\lambda}c(Z_{\tilde{\alpha}}^H)\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right]\right) \\
&\qquad - \left(\mathbb{E}\left[Z_{\tilde{\alpha}}^L - pe^{-\lambda}c(Z_{\tilde{\alpha}}^L)\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right]\right) \\
&\qquad\qquad = \mathbb{E}\left[(Z_{\tilde{\alpha}}^H - pe^{-\lambda}c(Z_{\tilde{\alpha}}^H)) - (Z_{\tilde{\alpha}}^L - pe^{-\lambda}c(Z_{\tilde{\alpha}}^L))\right] \geq 0.
\end{aligned}$$

Thus, $(f_1 = g_3, ..., f_m = g_3)$ is a Bayesian Nash equilibrium provided that

$$\begin{aligned}
\max_{1\leq i\leq m} \chi_i \leq \min\Big\{&\mathbb{E}\left[Z_{\tilde{\alpha}}^L - pe^{-\lambda}c(Z_{\tilde{\alpha}}^L)\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right], \\
&\mathbb{E}\left[Z_{\tilde{\alpha}}^H - pe^{-\lambda}c(Z_{\tilde{\alpha}}^H)\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right]\Big\}, \\
= \mathbb{E}&\left[Z_{\tilde{\alpha}}^L - pe^{-\lambda}c(Z_{\tilde{\alpha}}^L)\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right],
\end{aligned}$$

where the tighter upper bound is from the low-quality situation.

In the rest of this proof, we show that the Bayesian Nash equilibrium above is actually a sequential equilibrium. We denote the set of manufacturers by M and the set of consumers by C. The strategy space for manufacturer i is $\sigma_i^M(t) \in \{0, 1\}$ denote the action of manufacturer i when her type is t.

The information set A_k^C for a generic consumer k is given by

$$A_k^C = \left\{\left(a_1^m = (a_1, \ldots, a_m), \tilde{q}_1^m = (\tilde{q}_1, \ldots, \tilde{q}_m)\right)\middle| a_i \in \{0, 1\}, \tilde{q}_i \in \{L, H\}, i = 1, ..., m\right\}. \quad \text{(E.2)}$$

The strategy σ_k^C for consumer k is a mapping from A_k^C to $\{1, \ldots, m\}$, where $\sigma_k^C(a_1^m, \tilde{q}_1^m) \in \{1, \ldots, m\}$ denotes the manufacturer from whom the consumer purchases.

In terms of this new notation, the Bayesian Nash equilibrium obtained above, can be written as follows. For all manufacturers $i \in \{1, \ldots, m\}$,

$$\sigma_i^M(H) = \sigma_i^M(L) = 1.$$

Define

$$I_1 = \{i | a_i = 1, \tilde{q}_i = H\},$$
$$I_2 = \{i | a_i = 0, \tilde{q}_i = H\},$$
$$I_3 = \{i | a_i = 0, \tilde{q}_i = L\},$$
$$I_4 = \{i | a_i = 1, \tilde{q}_i = L\},$$

and $j_0 = \min\{j : I_j \neq \emptyset\}$. Since $\mathbb{E}[q_i \mid a_i = 1, \tilde{q}_i = H] > \mathbb{E}[q_i \mid a_i = 0, \tilde{q}_i = H] > \mathbb{E}[q_i \mid a_i = 0, \tilde{q}_i = L] > \mathbb{E}[q_i \mid a_i = 1, \tilde{q}_i = L]$, the consumer strategy $\sigma_k^C(a_1^m, \tilde{q}_1^m)$ is to uniformly choose a manufacturer $i_0 \in I_{j_0}$, and place an order with manufacturer i_0.

Next, we define the associated beliefs for the manufacturers and the consumers. The belief space B^C for any consumer k is the 2^m-dimensional simplex

$$B^C = \left\{ p \in \mathbb{R}^{2^m} \middle| p_{q_1^m} \geq 0, \sum_{q_1^m} p_{q_1^m} = 1 \right\}.$$

Since the consumers use prior beliefs when manufacturers do not adopt the blockchain, the posterior distribution

$$\mathbb{P}(q_i \mid \tilde{q}_i, a_i) = \begin{cases} \alpha \mathbf{1}_{\{q_i = \tilde{q}_i\}} + (1-\alpha)\mathbf{1}_{\{q_i \neq \tilde{q}_i\}}, \, a_i = 0, \\ \tilde{\alpha} \mathbf{1}_{\{q_i = \tilde{q}_i\}} + (1-\tilde{\alpha})\mathbf{1}_{\{q_i \neq \tilde{q}_i\}}, \, a_i = 1, \end{cases}$$

and, for $(a_1^m, \tilde{q}_1^m) \in A_k^C$, the belief $\mu_k^C(a_1^m, \tilde{q}_1^m) \in B^C$ is given by the posterior distribution

$$\mu_k^C(a_1^m, \tilde{q}_1^m)_{q_1^m} = \prod_{i=1}^{m} \mathbb{P}(q_i \mid \tilde{q}_i, a_i)$$

over the true types of the manufacturers given the information $(a_1^m, \tilde{q}_1^m)$. Note that the consumer strategy σ_k^C is Bayes' optimal with respect to this posterior belief μ_k^C.

We establish that σ is sequentially rational given μ. Let the manufacturer policy $\sigma_i^{M,n}(t)$ be Bernoulli$(1 - \frac{1}{n})$ for $t \in \{H, L\}$, and let strategy $\sigma_k^{C,n}$ for consumer k be $\sigma_k^{C,n} = \sigma_k^C$ for all $n \geq 1$. Then $\sigma^n \to \sigma$ as $n \to \infty$.

For each of the manufacturer i, trivially $\lim_{n\to\infty} \mu_i^{M,n} = \mu_i^M$. Since $\mathbb{P}(q_i = H) = \mathbb{P}(q_i = L) = \frac{1}{2}$, and the manufacturer policy is independent of her type, Bayes's Law implies that

$$\mu_k^{C,n}(a_1^m, \tilde{q}_1^m) = \mu_k^C(a_1^m, \tilde{q}_1^m)$$

for $(a_1^m, \tilde{q}_1^m) \in A_k^C$ and $n \geq 1$. Thus, we have that $\lim_{n\to\infty} \mu_k^{C,n} = \mu_k^C$. It follows that $\mu^n \to \mu$ as $n \to \infty$. This completes the proof. □

Proof of Proposition 4.3.6. Fix an arbitrary manufacturer i. Suppose $q_i = L$ and all other manufacturers play the adoption strategy g_3. First, suppose manufacturer i adopts the blockchain, i.e.

$a_i = 1$. From Proposition 4.3.2, the expected payoff of manufacturer i is

$$\mathbb{E}_{q_{-i}}\left[s_i(1, g_3(q_{-i})) - pe^{-\lambda\left[s_i(1,g_3(q_{-i}))+\sum_{k\neq i} g_3(q_k)s_k\right]}c(s_i(1, g_3(q_{-i})))\right] - \chi_i = \mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right] - \chi_i.$$

Next, suppose manufacturer i does not adopt the blockchain, i.e. $a_i = 0$. Since consumers believe that a manufacturer not adopting the blockchain is of low quality, all consumers believe that manufacturer i quality is L. For all the other manufacturers $j \neq i$ is $(1 - \tilde{\alpha})(H - L) + L$ if $\tilde{q}_j = L$, and $\tilde{\alpha}(H - L) + L$ if $\tilde{q}_j = H$. We have to consider the following two cases:

1. $\delta < 1 - \alpha$, i.e. $\tilde{\alpha} < 1$: In this case, the expected quality of manufacturer $j \neq i$ is greater than L regardless of the signal $\tilde{q}_i$, therefore, manufacturer i will never be considered by the consumers and the payoff of manufacturer i is 0.

2. $\delta = 1 - \alpha$, i.e. $\tilde{\alpha} = 1$: Here the blockchain is fully revealing, and $\tilde{q}_j = q_j$ with probability 1. Manufacturer i is able to attract market share, if and only if, $\tilde{q}_j = L$, for all $j \neq i$, and in this case, manufacturer i shares the unit demand with all other manufacturers. And, the market share for manufacturer i is $\frac{Z^{CB}_{\tilde{\alpha}}}{m}$ where $Z^{CB}_{\tilde{\alpha}}$ is defined in (E.1), and the expected payoff of manufacturer i is

$$\mathbb{E}_{q_{-i}}\left[s_i(0, g_3(q_{-i})) - pe^{-\lambda s_i(0,g_3(q_{-i}))}c(s_i(0, g_3(q_{-i})))\right] = \mathbb{E}\left[\frac{Z^{CB}_{\tilde{\alpha}}}{m} - pe^{-\frac{Z^{CB}_{\tilde{\alpha}}}{m}\lambda}c(\frac{Z^{CB}_{\tilde{\alpha}}}{m})\right].$$

In order for $(f_1 = g_3, ..., f_m = g_3)$ to be a Bayesian Nash equilibrium, manufacturer i should prefer $a_i = g_3(L) = 1$ to $a_i = 0$, i.e. we must have

$$\begin{aligned}
&\mathbb{E}_{q_{-i}}\left[s_i(1, g_3(q_{-i})) - pe^{-\lambda\left[s_i(1,g_3(q_{-i}))+\sum_{k\neq i} g_3(q_k)s_k\right]}c(s_i(1, g_3(q_{-i})))\right] - \chi_i \\
&= \mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right] - \chi_i, \\
&\geq \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z^{CB}_{\tilde{\alpha}}}{m} - pe^{-\frac{Z^{CB}_{\tilde{\alpha}}}{m}\lambda}c(\frac{Z^{CB}_{\tilde{\alpha}}}{m})\right], \\
&= \mathbb{E}_{q_{-i}}\left[s_i(0, g_3(q_{-i})) - pe^{-\lambda s_i(0,g_3(q_{-i}))}c(s_i(0, g_3(q_{-i})))\right].
\end{aligned}$$

This inequality holds provided

$$\chi_i \leq \mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right] - \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z^{CB}_{\tilde{\alpha}}}{m} - pe^{-\frac{Z^{CB}_{\tilde{\alpha}}}{m}\lambda}c(\frac{Z^{CB}_{\tilde{\alpha}}}{m})\right].$$

Suppose $q_i = H$ and all other manufacturers play the strategy g_3. From an analysis similar to the one used above, in order for $(f_1 = g_3, ..., f_m = g_3)$ to be a Bayesian Nash equilibrium, manufacturer i should prefer $a_i = g_3(H) = 1$ to $a_i = 0$, i.e. must have

$$\begin{aligned}
\mathbb{E}_{q_{-i}} & \left[s_i(1, g_3(q_{-i})) - pe^{-\lambda\left[s_i(1,g_3(q_{-i}))+\sum_{k\neq i} g_3(q_k)s_k\right]}c(s_i(1, g_3(q_{-i})))\right] - \chi_i \\
&= \mathbb{E}\left[Z^H_{\tilde{\alpha}} - pe^{-\lambda}c(Z^H_{\tilde{\alpha}})\right] - \chi_i, \\
&\geq \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z^{CB}_{\tilde{\alpha}}}{m} - pe^{-\frac{Z^{CB}_{\tilde{\alpha}}}{m}\lambda}c(\frac{Z^{CB}_{\tilde{\alpha}}}{m})\right], \\
&= \mathbb{E}_{q_{-i}}\left[s_i(0, g_3(q_{-i})) - pe^{-\lambda s_i(0,g_3(q_{-i}))}c(s_i(0, g_3(q_{-i})))\right].
\end{aligned}$$

This inequality holds provided

$$\chi_i \leq \mathbb{E}\left[Z^H_{\tilde{\alpha}} - pe^{-\lambda}c(Z^H_{\tilde{\alpha}})\right] - \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z^{CB}_{\tilde{\alpha}}}{m} - pe^{-\frac{Z^{CB}_{\tilde{\alpha}}}{m}\lambda}c(\frac{Z^{CB}_{\tilde{\alpha}}}{m})\right].$$

Suppose $x - pe^{-\lambda}c(x)$ is increasing for $x \in [0, 1]$. Since $\tilde{\alpha} > 0.5$, $Z^H_{\tilde{\alpha}} \geq Z^L_{\tilde{\alpha}}$, and it follows that $\mathbb{E}\left[Z^H_{\tilde{\alpha}} - pe^{-\lambda}c(Z^H_{\tilde{\alpha}})\right] \geq \mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right]$. Hence, $(f_1 = g_3, ..., f_m = g_3)$ is a Bayesian Nash equilibrium provided that

$$\begin{aligned}
\max_{1\leq i\leq m} \chi_i &\leq \min\left\{\mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right], \mathbb{E}\left[Z^H_{\tilde{\alpha}} - pe^{-\lambda}c(Z^H_{\tilde{\alpha}})\right]\right\} \\
&\qquad - \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z^{CB}_{\tilde{\alpha}}}{m} - pe^{-\frac{Z^{CB}_{\tilde{\alpha}}}{m}\lambda}c(\frac{Z^{CB}_{\tilde{\alpha}}}{m})\right] \\
&= \mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right] - \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z^{CB}_{\tilde{\alpha}}}{m} - pe^{-\frac{Z^{CB}_{\tilde{\alpha}}}{m}\lambda}c(\frac{Z^{CB}_{\tilde{\alpha}}}{m})\right],
\end{aligned}$$

where the tighter upper bound is from the low-quality situation.

Using the notation in Proposition 4.3.5, the Bayesian Nash equilibrium above can be written as follows. The manufacturer strategy $\sigma_i^M(t) = 1$ for all $t \in \{H, L\}$, the consumer information set A_k^C is given by (E.2), and each consumer randomizes over all manufacturers with the highest posterior quality. These two terms together with μ_i^M are identical to the corresponding terms in Proposition 4.3.5.

The consumer beliefs μ_k^C are different. Since the consumers assume that the consumers are of type L when they do not adopt the blockchain, the posterior distribution over types is given by

$$\mathbb{P}(q_i \mid \tilde{q}_i, a_i) = \begin{cases} \mathbf{1}_{\{q_i = L\}} & a_i = 0, \\ \tilde{\alpha}\mathbf{1}_{\{q_i = \tilde{q}_i\}} + (1 - \tilde{\alpha})\mathbf{1}_{\{q_i \neq \tilde{q}_i\}}, & a_i = 1. \end{cases}$$

The equilibrium can be supported as a sequential equilibrium by defining $\sigma_i^{M,n}(H) \sim \text{Bernoulli}(1 - \frac{1}{n^2})$, $\sigma_i^{M,n}(L) \sim \text{Bernoulli}(1 - \frac{1}{n})$, and $\sigma_k^{C,n} = \sigma_k^C$. Note that ratio of the $\mathbb{P}(a_i = 0 \mid q_i = L)/\mathbb{P}(a_i = 0 \mid q_i = H) = n \to \infty$ as $n \to \infty$. □

Proof of Corollary 4.3.7.

(I) Here the consumer beliefs CB1 hold. Suppose $\delta = 1 - \alpha$, i.e. $\tilde{\alpha} = 1$, and the blockchain is fully revealing. In this case,

$$Z_1^L = \begin{cases} \dfrac{1}{m} & \text{w.p. } 0.5^{m-1}, \\ 0 & \text{w.p. } 1 - 0.5^{m-1}, \end{cases}$$

and $Z_1^{CB} = mZ_1^L$. In this case, for sufficiently large λ or sufficiently small p,

$$\begin{aligned}
&\mathbb{E}\left[Z_{\tilde{\alpha}}^L - pe^{-\lambda}c(Z_{\tilde{\alpha}}^L)\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right] \\
&\quad = \mathbb{E}\left[Z_1^L - pe^{-\lambda}c(Z_1^L)\right] - \mathbb{E}\left[Z_1^{CB} - pe^{-Z_1^{CB}\lambda}c(Z_1^{CB})\right], \\
&\quad = 0.5^{m-1}\left(\frac{1}{m} - pe^{-\lambda}c(\frac{1}{m})\right) - 0.5^{m-1}\left(1 - pe^{-\lambda}c(1)\right), \\
&\quad = -0.5^{m-1}\frac{m-1}{m} + 0.5^{m-1}pe^{-\lambda}\left(c(1) - c(\frac{1}{m})\right), \\
&\quad \approx -0.5^{m-1}\frac{m-1}{m} \\
&\quad < 0.
\end{aligned}$$

Since the adoption costs $\chi_i \geq 0$, it follows from Proposition 4.3.5 that there is no full blockchain adoption equilibrium for this parameter setting.

(II) Here the consumer beliefs CB2 holds. We have to consider two cases:

1. $\delta = 1 - \alpha$, i.e. $\tilde{\alpha} = 1$ and the blockchain is fully revealing. In this case,

$$Z_1^L = \begin{cases} \frac{1}{m} & \text{w.p. } 0.5^{m-1}, \\ 0 & \text{w.p. } 1 - 0.5^{m-1}, \end{cases}$$

and $\frac{Z_1^{CB}}{m} = Z_1^L$. In this case, since $p, \lambda > 0$,

$$\begin{aligned}
&\mathbb{E}\left[Z_{\tilde{\alpha}}^L - pe^{-\lambda}c(Z_{\tilde{\alpha}}^L)\right] - \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z_{\tilde{\alpha}}^{CB}}{m} - pe^{-\frac{Z_{\tilde{\alpha}}^{CB}}{m}\lambda}c(\frac{Z_{\tilde{\alpha}}^{CB}}{m})\right] \\
&\quad = \mathbb{E}\left[Z_1^L - pe^{-\lambda}c(Z_1^L)\right] - \mathbb{E}\left[\frac{Z_1^{CB}}{m} - pe^{-\frac{Z_1^{CB}}{m}\lambda}c(\frac{Z_1^{CB}}{m})\right], \\
&\quad = \mathbb{E}\left[p\left(e^{-Z_1\lambda} - e^{-\lambda}\right)c(Z_1)\right] \\
&\quad = 0.5^{m-1}p\left(e^{-\frac{\lambda}{m}} - e^{-\lambda}\right)c(\frac{1}{m}) \\
&\quad > 0.
\end{aligned}$$

2. $\delta < 1 - \alpha$, i.e. $\tilde{\alpha} < 1$, and the blockchain is not fully revealing. There exist $z_{\tilde{\alpha},\min}$ and $z_{\tilde{\alpha},\max}$ such that $0 < z_{\tilde{\alpha},\min} \leq Z^L_{\tilde{\alpha}} \leq z_{\tilde{\alpha},\max} < 1$. Since $x - pe^{-\lambda}c(x)$ is increasing,

$$\begin{aligned}
&\mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right] - \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z^{CB}_{\tilde{\alpha}}}{m} - pe^{-\frac{Z^{CB}_{\tilde{\alpha}}}{m}\lambda}c\Big(\frac{Z^{CB}_{\tilde{\alpha}}}{m}\Big)\right] \\
&= \mathbb{E}\left[Z^L_1 - pe^{-\lambda}c(Z^L_1)\right], \\
&\geq z_{\tilde{\alpha},\min} - pe^{-\lambda}c(z_{\tilde{\alpha},\min}), \\
&> 0 - pe^{-\lambda}c(0), \\
&= 0.
\end{aligned}$$

Set $\underline{\chi} = \min\left\{0.5^{m-1}p\big(e^{-\frac{\lambda}{m}} - e^{-\lambda}\big)c\big(\frac{1}{m}\big), z_{\tilde{\alpha},\min} - pe^{-\lambda}c(z_{\tilde{\alpha},\min})\right\} > 0$. Then Proposition 4.3.6 implies that a full blockchain adoption equilibrium arises if $\max_{1\leq i\leq m}\chi_i < \underline{\chi}$.

□

Proof of Proposition 4.3.8. Under CB1, Proposition 4.3.4 and Proposition 4.3.5 imply that an adoption failure arises when

$$\begin{aligned}
\sum_{i=1}^{m}\chi_i < 0.5mp\Big(&\mathbb{E}\left[e^{-Z^L_\alpha\lambda}c(Z^L_\alpha) + e^{-Z^H_\alpha\lambda}c(Z^H_\alpha)\right] - e^{-\lambda}\mathbb{E}\left[c(Z^L_{\tilde{\alpha}}) + c(Z^H_{\tilde{\alpha}})\right]\Big) \\
&+ \delta(1 - 0.5^{m-1})(H - L),
\end{aligned}$$

and

$$\max_{1\leq i\leq m}\chi_i > \mathbb{E}\left[Z^L_{\tilde{\alpha}} - pe^{-\lambda}c(Z^L_{\tilde{\alpha}})\right] - \mathbb{E}\left[Z^{CB}_{\tilde{\alpha}} - pe^{-Z^{CB}_{\tilde{\alpha}}\lambda}c(Z^{CB}_{\tilde{\alpha}})\right].$$

Holding everything else constant, for sufficiently high adoption costs $\{\chi_i : i = 1, \ldots, m\}$, the second inequality holds. Fixing χ_i's such that the second inequality holds, for sufficiently large product quality dispersion $H - L$, the first inequality holds since $\delta > 0$ and $1 - 0.5^{m-1} > 0$.

Under CB2, Proposition 4.3.4 and Proposition 4.3.6 imply that an adoption failure arises when

$$\sum_{i=1}^{m} \chi_i < 0.5mp\left(\mathbb{E}\left[e^{-Z_{\alpha}^{L}\lambda}c(Z_{\alpha}^{L}) + e^{-Z_{\alpha}^{H}\lambda}c(Z_{\alpha}^{H})\right] - e^{-\lambda}\mathbb{E}\left[c(Z_{\tilde{\alpha}}^{L}) + c(Z_{\tilde{\alpha}}^{H})\right]\right)$$
$$+ \delta(1 - 0.5^{m-1})(H - L),$$

and

$$\max_{1\le i\le m} \chi_i > \mathbb{E}\left[Z_{\tilde{\alpha}}^{L} - pe^{-\lambda}c(Z_{\tilde{\alpha}}^{L})\right] - \mathbf{1}_{\{\delta=1-\alpha\}}\mathbb{E}\left[\frac{Z_{\tilde{\alpha}}^{CB}}{m} - pe^{-\frac{Z_{\tilde{\alpha}}^{CB}}{m}\lambda}c(\frac{Z_{\tilde{\alpha}}^{CB}}{m})\right].$$

The rest of the proof is similar to that for CB1. □

Proof of Proposition 4.3.9. Under CB1, Proposition 4.3.4 and Proposition 4.3.5 imply that an adoption failure arises when

$$\sum_{i=1}^{m} \chi_i < 0.5mp\left(\mathbb{E}\left[e^{-Z_{\alpha}^{L}\lambda}c(Z_{\alpha}^{L}) + e^{-Z_{\alpha}^{H}\lambda}c(Z_{\alpha}^{H})\right] - e^{-\lambda}\mathbb{E}\left[c(Z_{\tilde{\alpha}}^{L}) + c(Z_{\tilde{\alpha}}^{H})\right]\right)$$
$$+ \delta(1 - 0.5^{m-1})(H - L),$$

and

$$\max_{1\le i\le m} \chi_i > \mathbb{E}\left[Z_{\tilde{\alpha}}^{L} - pe^{-\lambda}c(Z_{\tilde{\alpha}}^{L})\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right].$$

The transfers $t_i = \frac{\delta(1-0.5^{m-1})(H-L)}{m} > 0$, reduce the manufacturer adoption costs to $\chi_i^{\text{transfer}} = \chi_i - t_i$. For sufficiently large product quality dispersion $H - L$,

$$\max_{1\le i\le m} \chi_i^{\text{transfer}} = \chi_1^{\text{transfer}} \le \mathbb{E}\left[Z_{\tilde{\alpha}}^{L} - pe^{-\lambda}c(Z_{\tilde{\alpha}}^{L})\right] - \mathbb{E}\left[Z_{\tilde{\alpha}}^{CB} - pe^{-Z_{\tilde{\alpha}}^{CB}\lambda}c(Z_{\tilde{\alpha}}^{CB})\right].$$

Therefore, from Proposition 4.3.5 we have that a full blockchain adoption equilibrium arises. From

Proposition 4.3.1, we have that the consumer welfare when they adopt the blockchain

$$\begin{aligned}
\mathbb{E}^1[\max_i \mathbb{E}_k[q_i]] - \kappa &= ([\tilde{\alpha}(1-0.5^m) + (1-\tilde{\alpha})0.5^m](H-L) + L) - \kappa \\
&= ([\alpha(1-0.5^m) + (1-\alpha)0.5^m](H-L) + L), \\
&= \mathbb{E}^0[\max_i \mathbb{E}_k[q_i]],
\end{aligned}$$

i.e. the welfare gain is 0, and paying κ for the access is weakly incentive compatible for the consumers. Thus, the system of transfers successfully overcomes the adoption failure. A similar proof can be applied to establish the argument under CB2. □

Appendix F: Chapter 5 Proofs

Proof of Proposition 5.2.1.

I. Given Ψ and n, defined in (5.5) and (5.6), the effort choice for each vendor, e^*, is a solution for the following fixed-point problem:

$$\begin{aligned} e^* &= \arg\max_{e_j \in [0,1]} \mathcal{V}(\Psi, n, e_j, e^*) \\ &= \arg\max_{e_j \in [0,1]} \left(1 - \rho(e^*)\right)^{n-1} \cdot \left(1 - \rho(e_j)\right) \cdot \Psi - \frac{e_j}{2} \\ &= \arg\max_{e_j \in [0,1]} (e^*)^{\frac{n-1}{2}} \cdot e_j^{\frac{1}{2}} \cdot \Psi - \frac{e_j}{2}. \end{aligned}$$

The objective is a quadratic function in terms of $e_j^{\frac{1}{2}}$ (i.e., $(e^*)^{\frac{n-1}{2}} \cdot \Psi \cdot x - \frac{x^2}{2}$) and its unconstrained maximum is attained at $e_j^{\frac{1}{2}} = (e^*)^{\frac{n-1}{2}} \cdot \Psi$. Therefore, the effort choice $e^* = \min\{(e^*)^{n-1} \cdot \Psi^2, 1\}$.

When $n = 1$, $e^* = \min\{\Psi^2, 1\}$. When $n = 2$, $e^* = \min\{e^* \cdot \Psi^2, 1\}$; if $\Psi < 1$, the only solution is $e^* = 0$, and if $\Psi \geq 1$, two solutions are $e^* = 1$ and $e^* = 0$. When $n > 2$, $e^* = \min\{(e^*)^{n-1} \cdot \Psi^2, 1\}$; if $\Psi < 1$, the only solution is $e^* = 0$, and if $\Psi \geq 1$, two solutions are $e^* = \Psi^{-\frac{2}{n-2}}$ and $e^* = 0$ coming from solving $e^* = (e^*)^{n-1} \cdot \Psi^2$ and the other solution is

$e^* = 1$. To conclude, we have that

$$e^*(\Psi, n) = \begin{cases} \Psi^2 & \text{if } \Psi < 1 \text{ and } n = 1 \\ 0 & \text{if } \Psi < 1 \text{ and } n > 1 \\ 1 & \text{if } \Psi \geq 1 \text{ and } n = 1 \\ 1 \text{ or } 0 & \text{if } \Psi \geq 1 \text{ and } n = 2 \\ 1 \text{ or } \Psi^{-\frac{2}{n-2}} \text{ or } 0 & \text{if } \Psi \geq 1 \text{ and } n > 2 \end{cases} \cdot$$

II. With P and s, defined in (5.7) and (5.8), the number of vendors n^* and the price Ψ^* the manufacturer offers the vendors are given by:

$$\begin{aligned}(\Psi^*, n^*) &= \underset{\Psi \geq 0, n \in \overline{\mathbb{N}}_+}{\arg\max}\ \mathcal{M}(P, s, \Psi, n) \\ &= \underset{\Psi \geq 0, n \in \overline{\mathbb{N}}_+}{\arg\max}\ \Big(1 - \rho(e^*(\Psi, n))\Big)^n \cdot \frac{\big((P - \Psi) \cdot s\big)^{1-\gamma}}{1 - \gamma} \\ &= \underset{\Psi \geq 0, n \in \overline{\mathbb{N}}_+}{\arg\max}\ (e^*(\Psi, n))^{\frac{n}{2}} \cdot \frac{(P - \Psi)^{1-\gamma}}{1 - \gamma}.\end{aligned}$$

The last equality indicates that Ψ^* and n^* do not depend upon the consumer demand s.

If the manufacturer chooses a $\Psi < 1$, then $e^*(\Psi, n) = 0$ for any $n > 1$. Thus, in this case, $n^* = 1$ and

$$\begin{aligned}\Psi_1^* &:= \underset{\Psi \in [0,1)}{\arg\max}\ \frac{\Psi \cdot (P - \Psi)^{1-\gamma}}{1 - \gamma} \\ &= \begin{cases} \frac{P}{2-\gamma} & \text{if } P < 2 - \gamma \\ 1^- & \text{if } P \geq 2 - \gamma \end{cases}.\end{aligned}$$

If the manufacturer chooses a $\Psi \geq 1$, then the value of $e^*(\Psi, n)$ is ambiguous by the result of part I. However, there always exists a n such that $e^*(\Psi, n) = 1$ attains the maximum. The manufacturer will select such a n as her n^*. Consequently,

$$\begin{aligned}\Psi_2^* &:= \arg\max_{\Psi \geq 1} \frac{(P-\Psi)^{1-\gamma}}{1-\gamma} \\ &= 1.\end{aligned}$$

Finally, the manufacturer chooses between Ψ_1^* and Ψ_2^* to maximize her expected utility. If $P \geq 2-\gamma$, then $\Psi^* = \Psi_1^* = \Psi_2^* = 1$. If $P < 1$, choosing Ψ_2^* results in a negative expected utility, so $\Psi^* = \Psi_1^*$. If $1 \leq P < 2-\gamma$, by the definition of Ψ_1^* and Ψ_2^*,

$$\frac{\Psi_1^* \cdot (P-\Psi_1^*)^{1-\gamma}}{1-\gamma} > \frac{1 \cdot (P-1)^{1-\gamma}}{1-\gamma} = \frac{(P-\Psi_2^*)^{1-\gamma}}{1-\gamma}.$$

Hence, the expected utility of choosing Ψ_1^* is greater than that of choosing Ψ_2^*, meaning $\Psi^* = \Psi_1^*$. To conclude, we have that

$$\Psi^*(P) = \begin{cases} \frac{P}{2-\gamma} & \text{if } P < 2-\gamma \\ 1 & \text{if } P \geq 2-\gamma \end{cases}$$

and

$$n^*(P) = \begin{cases} 1 & \text{if } P < 2-\gamma \\ \text{ambiguous} & \text{if } P \geq 2-\gamma \end{cases}.$$

In part IV, we will see that the equilibrium price for consumers, P^*, is less than $2-\gamma$, so we actually do not need to deal with the ambiguous case.

III. By Bayes' theorem, $\mathbb{P}(q = \tilde{q}_k \mid \tilde{q}_k) = \alpha$. It follows that

$$\begin{aligned}\mathbb{E}[V_k \mid v_k, t_k, \tilde{q}_k] &= \mathbb{E}[v_k \cdot \big(\mathcal{I}(q = t_k) - \mathcal{I}(q \neq t_k)\big) \mid v_k, t_k, \tilde{q}_k] \\ &= v_k \cdot \Big(2 \cdot \mathbb{P}(q = t_k \mid t_k, \tilde{q}_k) - 1\Big) \\ &= \begin{cases} v_k \cdot (2\alpha - 1) & \text{if } \tilde{q}_k = t_k \\ -v_k \cdot (2\alpha - 1) & \text{if } \tilde{q}_k \neq t_k \end{cases}.\end{aligned}$$

For P given by (5.9), consumer k's expected payoff for purchasing from the manufacturer is:

$$\begin{aligned}C(P, v_k, t_k, \tilde{q}_k) &= \Big(1 - \rho(e^*(P))\Big)^{n^*(P)} \cdot \mathbb{E}[V_k - P \mid v_k, t_k, \tilde{q}_k] \\ &= (e^*(P))^{\frac{n^*(P)}{2}} \cdot (\mathbb{E}[V_k \mid v_k, t_k, \tilde{q}_k] - P),\end{aligned}$$

where $e^*(P) = e^*(\Psi^*(P), n^*(P))$. Consumer k purchases from the manufacturer if and only if $C(P, v_k, t_k, \tilde{q}_k) > 0$, which is equivalent to $\mathbb{E}[V_k \mid v_k, t_k, \tilde{q}_k] > P$. Note that when $\tilde{q}_k \neq t_k$, $\mathbb{E}[V_k \mid v_k, t_k, \tilde{q}_k] = -v_k \cdot (2\alpha - 1) \leq 0 \leq P$ and thus consumer k will take her outside option. Therefore, consumer k purchases from the manufacturer if and only if $\tilde{q}_k = t_k$ and $\mathbb{E}[V_k \mid v_k, t_k, \tilde{q}_k] = v_k \cdot (2\alpha - 1) > P$.

By the model setting, half of the consumers are of type A and half of the consumers are of type B, and the manufacturer knows her type q. Thus, the consumer demand

$$\begin{aligned}s^*(P) &= \underbrace{\frac{1}{2} \cdot \mathbb{P}(\tilde{q}_k = A \mid q) \cdot \mathbb{P}(v_k \cdot (2\alpha - 1) > P)}_{\text{Type } A \text{ Consumers' Demand}} + \underbrace{\frac{1}{2} \cdot \mathbb{P}(\tilde{q}_k = B \mid q) \cdot \mathbb{P}(v_k \cdot (2\alpha - 1) > P)}_{\text{Type } B \text{ Consumers' Demand}} \\ &= \frac{1}{2} \cdot \mathbb{P}(v_k \cdot (2\alpha - 1) > P) \\ &= \frac{1}{2} \cdot \max\left\{1 - \frac{P}{(2\alpha - 1)H}, 0\right\}.\end{aligned}$$

IV. By (5.12), the manufacturer anticipates all future actions and sets her price for consumers, P^*, such that:

$$\begin{aligned}
P^* &= \arg\max_{P\geq 0} \mathcal{M}(P) \\
&= \arg\max_{P\geq 0} \mathcal{M}(P, s^*(P), \Psi^*(P), n^*(P)) \\
&= \arg\max_{P\geq 0} \Big(1-\rho(e^*(P))\Big)^{n^*(P)} \cdot \frac{\Big((P-\Psi^*(P))\cdot s^*(P)\Big)^{1-\gamma}}{1-\gamma} \\
&= \arg\max_{P\geq 0} (e^*(P))^{\frac{n^*(P)}{2}} \cdot \frac{\Big((P-\Psi^*(P))\cdot s^*(P)\Big)^{1-\gamma}}{1-\gamma},
\end{aligned}$$

where $e^*(P) = e^*(\Psi^*(P), n^*(P))$.

Define

$$P_1^* := \arg\max_{0\leq P<2-\gamma} (e^*(P))^{\frac{n^*(P)}{2}} \cdot \frac{\Big((P-\Psi^*(P))\cdot s^*(P)\Big)^{1-\gamma}}{1-\gamma}$$

and

$$P_2^* := \arg\max_{P\geq 2-\gamma} (e^*(P))^{\frac{n^*(P)}{2}} \cdot \frac{\Big((P-\Psi^*(P))\cdot s^*(P)\Big)^{1-\gamma}}{1-\gamma}.$$

If $P < 2-\gamma$, then $\Psi^*(P) = \frac{P}{2-\gamma}$, $n^*(P) = 1$, and $e^*(P) = \left(\frac{P}{2-\gamma}\right)^2$ by part I and part II. Hence,

$$\begin{aligned}
P_1^* &= \arg\max_{0\leq P<2-\gamma} \frac{P}{2-\gamma} \cdot \frac{\Big((P-\frac{P}{2-\gamma})\cdot\frac{1}{2}\cdot\max\Big\{1-\frac{P}{(2\alpha-1)H},0\Big\}\Big)^{1-\gamma}}{1-\gamma} \\
&= \arg\max_{0\leq P<(2\alpha-1)H} \frac{(1-\gamma)^{-\gamma}}{2^{1-\gamma}(2-\gamma)^{2-\gamma}} \cdot P^{2-\gamma} \cdot \left(1-\frac{P}{(2\alpha-1)H}\right)^{1-\gamma}.
\end{aligned}$$

Note that the second line follows from the first because $(2\alpha-1)H < 1 < 2-\gamma$ and $\max\Big\{1-\frac{P}{(2\alpha-1)H},0\Big\} = 0$ when $P \in [(2\alpha-1)H, 2-\gamma)$. The objective is a unimodal function and the corresponding first order condition tells us that its maximum is attained at $P = \frac{2-\gamma}{3-2\gamma}(2\alpha - 1)H$, which is in the interval $[0, (2\alpha - 1)H)$. Therefore, we can conclude that $P_1^* = \frac{2-\gamma}{3-2\gamma}(2\alpha - 1)H$ and the corresponding maximum is positive. If $P \geq 2 - \gamma$, then $\Psi^*(P) = 1$ and $(e^*(P))^{n^*(P)} = 1$ by part I and part II. Hence,

$$P_2^* = \arg\max_{P \geq 2-\gamma} \frac{\left((P-1) \cdot \frac{1}{2} \cdot \max\left\{1 - \frac{P}{(2\alpha-1)H}, 0\right\}\right)^{1-\gamma}}{1-\gamma}.$$

Note that $(2\alpha - 1)H < 1 < 2 - \gamma$ and $\max\left\{1 - \frac{P}{(2\alpha-1)H}, 0\right\} = 0$ when $P \geq 2 - \gamma$. Thus, the objective is always zero. In conclusion, the manufacturer chooses $P^* = P_1^* = \frac{2-\gamma}{3-2\gamma}(2\alpha - 1)H$ to maximize her expected utility.

□

Proof of Corollary 5.2.2. By Proposition 5.2.1, solutions for all equilibrium actions in the absence of blockchain are given by:

$$\begin{aligned}
e^* &= e^*(P^*) = \left(\frac{(2\alpha - 1)H}{3 - 2\gamma}\right)^2, \\
\Psi^* &= \Psi^*(P^*) = \frac{(2\alpha - 1)H}{3 - 2\gamma}, \\
n^* &= n^*(P^*) = 1, \\
s^* &= s^*(P^*) = \frac{1 - \gamma}{6 - 4\gamma}, \\
P^* &= \frac{2 - \gamma}{3 - 2\gamma}(2\alpha - 1)H.
\end{aligned}$$

□

Proof of Proposition 5.2.3.

I. When Ψ and n are given by (5.15) and (5.16), the vendor effort choice e^{**} solves:

$$\begin{aligned}
e^{**} &= \arg\max_{e_j \in [0,1]} \mathcal{V}^{\mathcal{B}}(\Psi, n, e_j, e^{**}) \\
&= \arg\max_{e_j \in [0,1]} (1 - \rho(e_j)) \cdot \Psi - \frac{e_j}{2} \\
&= \arg\max_{e_j \in [0,1]} e_j^{\frac{1}{2}} \cdot \Psi - \frac{e_j}{2}.
\end{aligned}$$

The objective is a quadratic function in terms of $e_j^{\frac{1}{2}}$ (i.e., $\Psi x - \frac{x^2}{2}$). The constrained maximum is attained at $e_j^{\frac{1}{2}} = \min\{\Psi, 1\}$, so the effort choice $e^{**} = \min\{\Psi^2, 1\}$. To sum up, we have that

$$e^{**}(\Psi, n) = \begin{cases} \Psi^2 & \text{if } \Psi < 1 \\ 1 & \text{if } \Psi \geq 1 \end{cases}.$$

II. For P and s given by (5.17) and (5.18), the number of vendors n^{**} to whom the manufacturer makes an offer and the offer price Ψ^{**} are given by:

$$\begin{aligned}
(\Psi^{**}, n^{**}) &= \arg\max_{\Psi \geq 0, n \in \overline{\mathbb{N}}_+} \mathcal{M}^{\mathcal{B}}(P, s, \Psi, n) \\
&= \arg\max_{\Psi \geq 0, n \in \overline{\mathbb{N}}_+} \mathbb{E}\Big[U\Big((P - \Psi) \cdot \frac{s}{n} \cdot N_s\Big)\Big] - \chi,
\end{aligned}$$

where $N_s \sim \text{Binomial}(n, 1 - \rho(e^{**}(\Psi)))$ with $e^{**}(\Psi) := e^{**}(\Psi, 1) = e^{**}(\Psi, n)$.

Define $X_n := (P - \Psi) \cdot \frac{s}{n} \cdot N_s$ and $c := \mathbb{E}[X_n] = (P - \Psi) \cdot s \cdot (e^{**}(\Psi))^{\frac{1}{2}}$. By strong law of large numbers, $X_\infty = c$. Since U is strictly concave on $\mathbb{R}_+$, for given Ψ and any finite n, $\mathbb{E}[U(X_n)] < U(\mathbb{E}[X_n]) = U(c) = \mathbb{E}[U(X_\infty)]$ holds by Jensen's inequality. Thus, $n = \infty$ is a dominant strategy and $n^{**} = \infty$, which is independent of Ψ. It follows that

$$\begin{aligned}\Psi^{**} &= \arg\max_{\Psi \geq 0} \mathbb{E}[U(X_\infty)] - \chi \\ &= \arg\max_{\Psi \geq 0} U\Big((P - \Psi) \cdot s \cdot (e^{**}(\Psi))^{\frac{1}{2}}\Big) - \chi \\ &= \arg\max_{\Psi \geq 0} (P - \Psi) \cdot (e^{**}(\Psi))^{\frac{1}{2}} \\ &= \arg\max_{\Psi \geq 0} (P - \Psi) \cdot \min\{\Psi, 1\} \\ &= \begin{cases} \frac{P}{2} & \text{if } P < 2 \\ 1 & \text{if } P \geq 2 \end{cases}.\end{aligned}$$

Note that the third line follows from the second because U is strictly increasing, and the fourth line holds true due to the result of part I. To sum up, we have that

$$\Psi^{**}(P) = \begin{cases} \frac{P}{2} & \text{if } P < 2 \\ 1 & \text{if } P \geq 2 \end{cases},$$

$$n^{**}(P) = \infty.$$

III. The argument to get $s^{**}(P)$ is very similar to what we did in Proposition 5.2.1 part III; the only difference here is that we need to replace α with $\alpha + \delta$ in the proof of Proposition 5.2.1 part III. Consequently, we get that $s^{**}(P) = \frac{1}{2} \cdot \max\left\{1 - \frac{P}{(2(\alpha+\delta)-1)H}, 0\right\}$.

IV. By (5.22) and the results of part I, part II and part III,

$$\begin{aligned}
P^{**} &:= \arg\max_{P\geq 0} \mathcal{M}^{\mathcal{B}}(P) \\
&= \arg\max_{P\geq 0} \mathcal{M}^{\mathcal{B}}(P, s^{**}(P), \Psi^{**}(P), n^{**}(P)) \\
&= \arg\max_{P\geq 0} \mathbb{E}[U\Big((P-\Psi^{**}(P))\cdot \frac{s^{**}(P)}{n^{**}(P)}\cdot N_s\Big)] - \chi \\
&= \arg\max_{P\geq 0} U\Big((P-\Psi^{**}(P))\cdot s^{**}(P)\cdot (e^{**}(P))^{\frac{1}{2}}\Big) - \chi \\
&= \arg\max_{P\geq 0} (P-\Psi^{**}(P))\cdot s^{**}(P)\cdot (e^{**}(P))^{\frac{1}{2}} \\
&= \arg\max_{P\geq 0} \Big(P - \min\{\frac{P}{2},1\}\Big)\cdot\frac{1}{2}\cdot\max\Big\{1-\frac{P}{(2(\alpha+\delta)-1)H},0\Big\}\cdot\min\{\frac{P}{2},1\},
\end{aligned}$$

where $N_s \sim \text{Binomial}(n^{**}(P), 1-\rho(e^{**}(P)))$ and $e^{**}(P) := e^{**}(\Psi^{**}(P)) = e^{**}(\Psi^{**}(P),1)$. Note that if $P \geq 2$, then $1-\frac{P}{(2(\alpha+\delta)-1)H} \leq 1-\frac{2}{(2(\alpha+\delta)-1)H} \leq 1-2 < 0$ because $(2(\alpha+\delta)-1)H \leq 1$. Consequently, in this case, $\max\Big\{1-\frac{P}{(2(\alpha+\delta)-1)H},0\Big\} = 0$ and the objective is always zero. It immediately follows that

$$\begin{aligned}
P^{**} &= \arg\max_{0\leq P<2} \Big(P - \min\{\frac{P}{2},1\}\Big)\cdot\frac{1}{2}\cdot\max\Big\{1-\frac{P}{(2(\alpha+\delta)-1)H},0\Big\}\cdot\min\{\frac{P}{2},1\} \\
&= \arg\max_{0\leq P<2} \frac{P}{2}\cdot\frac{1}{2}\cdot\Big(1-\frac{P}{(2(\alpha+\delta)-1)H}\Big)\cdot\frac{P}{2}.
\end{aligned}$$

The objective is a unimodal function and the corresponding first order condition tells us that its maximum is attained at $P = \frac{2}{3}(2(\alpha+\delta)-1)H$, which is in the interval $[0,2)$. Therefore, $P^{**} = \frac{2}{3}(2(\alpha+\delta)-1)H$.

□

Proof of Corollary 5.2.4. By Proposition 5.2.3, solutions for all equilibrium actions when blockchain is adopted are given by:

$$\begin{aligned}
e^{**} &= e^{**}(P^{**}) = \Big(\frac{(2(\alpha+\delta)-1)H}{3}\Big)^2,\\
\Psi^{**} &= \Psi^{**}(P^{**}) = \frac{(2(\alpha+\delta)-1)H}{3},\\
n^{**} &= n^{**}(P^{**}) = \infty,\\
s^{**} &= s^{**}(P^{**}) = \frac{1}{6},\\
P^{**} &= \frac{2}{3}(2(\alpha+\delta)-1)H.
\end{aligned}$$

□

Proof of Proposition 5.2.5. By Proposition 5.2.1 part IV,

$$\begin{aligned}
\mathcal{M}(P^*) &= \frac{(1-\gamma)^{-\gamma}}{2^{1-\gamma}(2-\gamma)^{2-\gamma}} \cdot P^{2-\gamma} \cdot \Big(1 - \frac{P}{(2\alpha-1)H}\Big)^{1-\gamma}\Big|_{P=P^*}\\
&= \frac{(1-\gamma)^{1-2\gamma}}{2^{1-\gamma}(3-2\gamma)^{3-2\gamma}}[(2\alpha-1)H]^{2-\gamma}.
\end{aligned}$$

By Proposition 5.2.3 part IV,

$$\begin{aligned}
\mathcal{M}^{\mathcal{B}}(P^{**}) &= U\Big(\frac{P^2}{8} \cdot \Big(1 - \frac{P}{(2(\alpha+\delta)-1)H}\Big)\Big) - \chi\Big|_{P=P^{**}}\\
&= \frac{[(2(\alpha+\delta)-1)H]^{2-2\gamma}}{(1-\gamma)54^{1-\gamma}} - \chi.
\end{aligned}$$

Define

$$\begin{aligned}
\Omega &:= \mathbb{E}[U(\Pi^{**})] - \mathbb{E}[U(\Pi^*)]\\
&= (\mathcal{M}^{\mathcal{B}}(P^{**}) + \chi) - \mathcal{M}(P^*)\\
&= \frac{[(2(\alpha+\delta)-1)H]^{2-2\gamma}}{(1-\gamma)54^{1-\gamma}} - \frac{(1-\gamma)^{1-2\gamma}}{2^{1-\gamma}(3-2\gamma)^{3-2\gamma}}[(2\alpha-1)H]^{2-\gamma}.
\end{aligned}$$

Then, the manufacturer adopts the blockchain, i.e., $a = 1$, if and only if $\Omega \geq \chi$. Hence, $a = \mathcal{I}\Big(\Omega \geq \chi\Big)$. □

Proof of Proposition 5.3.1. By Proposition 5.2.5, $\Omega = \frac{[(2(\alpha+\delta)-1)H]^{2-2\gamma}}{(1-\gamma)54^{1-\gamma}} - \frac{(1-\gamma)^{1-2\gamma}}{2^{1-\gamma}(3-2\gamma)^{3-2\gamma}}[(2\alpha-1)H]^{2-\gamma}$. In order to show that $\Omega > 0$, it suffices to show that the ratio of two terms is greater than 1, i.e.,

$$\frac{\frac{[(2(\alpha+\delta)-1)H]^{2-2\gamma}}{(1-\gamma)54^{1-\gamma}}}{\frac{(1-\gamma)^{1-2\gamma}}{2^{1-\gamma}(3-2\gamma)^{3-2\gamma}}[(2\alpha-1)H]^{2-\gamma}} = \frac{[(2(\alpha+\delta)-1)H]^{2-2\gamma}}{[(2\alpha-1)H]^{2-\gamma}} \cdot \frac{(3-2\gamma)^{3-2\gamma}}{27^{1-\gamma}(1-\gamma)^{2-2\gamma}} > 1.$$

Define an auxiliary function $l_1(\gamma) := \ln\left(\frac{(3-2\gamma)^{3-2\gamma}}{27^{1-\gamma}(1-\gamma)^{2-2\gamma}}\right) = (3-2\gamma)\ln(3-2\gamma) - (1-\gamma)\ln 27 - (2-2\gamma)\ln(1-\gamma)$. Then, $l_1'(\gamma) = -2\ln(3-2\gamma) + 2\ln(1-\gamma) + \ln 27 = 2\ln\left(\frac{1}{2} - \frac{1}{6-4\gamma}\right) + \ln 27$, which is decreasing in γ on $(0,1)$. Define $\gamma^* := \frac{21-3\sqrt{3}}{23}$. Then, $l_1'(\gamma) > 0$ when $\gamma < \gamma^*$ and $l_1'(\gamma) < 0$ when $\gamma > \gamma^*$. Hence, $l_1(\gamma)$ is strictly increasing on $(0,\gamma^*)$ and strictly decreasing on $(\gamma^*, 1)$. As $\lim_{\gamma\to 0^+} l_1(\gamma) = \lim_{\gamma\to 1^-} l_1(\gamma) = 0$, $l_1(\gamma)$ is thereby positive on $(0,1)$. Combining all the above, we have that

$$\frac{[(2(\alpha+\delta)-1)H]^{2-2\gamma}}{[(2\alpha-1)H]^{2-\gamma}} \cdot \frac{(3-2\gamma)^{3-2\gamma}}{27^{1-\gamma}(1-\gamma)^{2-2\gamma}} = \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^{2-2\gamma} \cdot \frac{1}{((2\alpha-1)H)^{\gamma}} \cdot e^{l_1(\gamma)} > 1 \cdot 1 \cdot e^0 = 1,$$

which completes the proof. □

Proof of Proposition 5.3.2. By Corollary 5.2.2,

$$\begin{aligned}\Pi^* &= (P^* - \Psi^*) \cdot s^* \cdot \prod_{j=1}^{n^*} I_j \\ &= \frac{(1-\gamma)^2}{2(3-2\gamma)^2}(2\alpha-1)H \cdot I_1,\end{aligned}$$

where $I_j \sim \text{Bernoulli}(1-\rho(e^*)) = \text{Bernoulli}\left(\frac{(2\alpha-1)H}{3-2\gamma}\right)$ are i.i.d. Bernoulli random variables. By

Corollary 5.2.4,

$$\begin{aligned}\Pi^{**} &= (P^{**} - \Psi^{**}) \cdot s^{**} \cdot \frac{N_s}{n^{**}} \\ &= \frac{[(2(\alpha+\delta)-1)H]^2}{54}\end{aligned}$$

by strong law of large numbers, where $N_s \sim \text{Binomial}(n^{**}, 1-\rho(e^{**})) = \text{Binomial}\left(n^{**}, \frac{(2(\alpha+\delta)-1)H}{3}\right)$. Given the above results, we are ready to prove Proposition 5.3.2.

1. It is easy to see that $n^{**} = \infty > 1 = n^*$.

2. Π^{**} is a constant whereas Π^* is a nondegenerate random variable, so $Var[\Pi^{**}] = 0 < Var[\Pi^*]$.

3. Note that Π^{**} is a constant and $\Omega = \mathbb{E}[U(\Pi^{**})] - \mathbb{E}[U(\Pi^*)] = U(\Pi^{**}) - \mathbb{E}[U(\Pi^*)]$, so $\Sigma = \Omega - \Phi = U(\mathbb{E}[\Pi^*]) - \mathbb{E}[U(\Pi^*)]$. Since U is strictly concave, $\Sigma > 0$ holds by Jensen's inequality. Moreover,

$$\begin{aligned}\Sigma &= U(\mathbb{E}[\Pi^*]) - \mathbb{E}[U(\Pi^*)] \\ &= \frac{(1-\gamma)^{1-2\gamma}}{2^{1-\gamma}(3-2\gamma)^{3-3\gamma}}[(2\alpha-1)H]^{2-2\gamma} - \frac{(1-\gamma)^{1-2\gamma}}{2^{1-\gamma}(3-2\gamma)^{3-2\gamma}}[(2\alpha-1)H]^{2-\gamma} \\ &= \frac{(1-\gamma)^{1-2\gamma}[(2\alpha-1)H]^{2-2\gamma}}{2^{1-\gamma}(3-2\gamma)^{3-3\gamma}}\left(1-\left(\frac{(2\alpha-1)H}{3-2\gamma}\right)^{\gamma}\right) \\ &\to 0\end{aligned}$$

as $\gamma \to 0^+$.

□

Proof of Proposition 5.3.3. By the proof of Proposition 5.3.2, we know that $\Pi^* = \frac{(1-\gamma)^2}{2(3-2\gamma)^2}(2\alpha - 1)H \cdot I_1$ and $\Pi^{**} = \frac{[2(\alpha+\delta)-1)H]^2}{54}$. Therefore, $\mathbb{E}[\Pi^*] = \frac{(1-\gamma)^2}{2(3-2\gamma)^3}[(2\alpha-1)H]^2$ and $\mathbb{E}[\Pi^{**}] = \frac{[(2(\alpha+\delta)-1)H]^2}{54}$.

1. Note that

$$\frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]} = \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^2 \cdot \frac{(3-2\gamma)^3}{27(1-\gamma)^2}$$
$$> \frac{(3-2\gamma)^3}{27(1-\gamma)^2}$$
$$=: e^{l_2(\gamma)}.$$

The auxiliary function $l_2(\gamma) = \ln\left(\frac{(3-2\gamma)^3}{27(1-\gamma)^2}\right) = 3\ln(3-2\gamma) - \ln 27 - 2\ln(1-\gamma)$. Then, $l_2'(\gamma) = \frac{2\gamma}{(1-\gamma)(3-2\gamma)} > 0$ and thus $l_2(\gamma)$ is strictly increasing in γ on $(0,1)$. As $\lim_{\gamma\to 0^+} l_2(\gamma) = 0$, $l_2(\gamma) > 0$ holds for all $\gamma \in (0,1)$. Consequently, $\frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]} > e^{l_2(\gamma)} > e^0 = 1$ and $\mathbb{E}[\Pi^{**}] > \mathbb{E}[\Pi^{*}]$. Since U is strictly increasing, $\Phi > 0$ follows.

2. We have that

$$\lim_{\gamma,\delta\to 0^+} \frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]} = \lim_{\gamma,\delta\to 0^+} \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^2 \cdot \frac{(3-2\gamma)^3}{27(1-\gamma)^2}$$
$$= \left(\frac{2\alpha-1}{2\alpha-1}\right)^2 \cdot \frac{3^3}{27\cdot 1^2}$$
$$= 1.$$

Thus,

$$\lim_{\gamma,\delta\to 0^+} \Phi = \lim_{\gamma,\delta\to 0^+} \left(\frac{U(\mathbb{E}[\Pi^{**}])}{U(\mathbb{E}[\Pi^{*}])} - 1\right) U(\mathbb{E}[\Pi^{*}])$$
$$= \lim_{\gamma,\delta\to 0^+} \left(\left(\frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]}\right)^{1-\gamma} - 1\right) U(\mathbb{E}[\Pi^{*}])$$
$$= \left(\lim_{\gamma,\delta\to 0^+} \left(\frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^{*}]}\right)^{1-\gamma} - 1\right) \cdot \lim_{\gamma,\delta\to 0^+} U(\mathbb{E}[\Pi^{*}])$$
$$= 0.$$

3. For fixed $\gamma > 0$,

$$\lim_{\delta\to 0^+} \frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^*]} = \lim_{\delta\to 0^+} \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^2 \cdot \frac{(3-2\gamma)^3}{27(1-\gamma)^2}$$
$$= \frac{(3-2\gamma)^3}{27(1-\gamma)^2}.$$

It has been shown in part 1 that $\frac{(3-2\gamma)^3}{27(1-\gamma)^2} > 1$ for all $\gamma \in (0,1)$. Thus, $\lim_{\delta\to 0^+} \frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^*]} > 1$.

4. For fixed $\delta > 0$,

$$\lim_{\gamma\to 0^+} \frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^*]} = \lim_{\gamma\to 0^+} \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^2 \cdot \frac{(3-2\gamma)^3}{27(1-\gamma)^2}$$
$$= \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^2$$
$$> 1.$$

□

Proof of Proposition 5.3.4. By the proof of Proposition 5.3.3, we know that $\frac{\mathbb{E}[\Pi^{**}]}{\mathbb{E}[\Pi^*]} = \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^2 \cdot \frac{(3-2\gamma)^3}{27(1-\gamma)^2} = f_\Pi(\gamma) \cdot g_\Pi(\alpha,\delta)$.

1. That $f_\Pi(\gamma) > 1$ has been proved in the proof of Proposition 5.3.3. Moreover, $\frac{df_\Pi}{d\gamma} = \frac{2\gamma(3-2\gamma)^2}{27(1-\gamma)^3} > 0$.

2. It is easy to see that $g_\Pi(\alpha,\delta) > 1$. Moreover, $\frac{\partial g_\Pi}{\partial\delta} = \frac{4}{2\alpha-1}(1 + \frac{2\delta}{2\alpha-1}) > 0$.

□

Proof of Proposition 5.3.5. By Corollaries 5.2.2 and 5.2.4,

$$\frac{s^{**}}{s^*} = \frac{3-2\gamma}{3-3\gamma} = f_s(\gamma),$$
$$\frac{P^{**}}{P^*} = \frac{2(\alpha+\delta)-1}{2\alpha-1} \cdot \frac{6-4\gamma}{6-3\gamma} = f_P(\gamma) \cdot g_P(\alpha,\delta).$$

1. For all $\gamma \in (0,1)$ and $\delta \in (0, 1-\alpha]$, $f_s(\gamma) > 1$ and thus $s^{**} > s^*$. When $\alpha = \frac{3}{4}, \gamma = \frac{1}{2}, \delta = \frac{1}{64}$, $P^{**} < P^*$. When $\alpha = \frac{3}{4}, \gamma = \frac{1}{2}, \delta = \frac{1}{4}$, $P^{**} > P^*$.

2. We have that $f_s(\gamma) > 1$ and $\frac{df_s}{d\gamma} = \frac{1}{3(1-\gamma)^2} > 0$, but $f_P(\gamma) < 1$ and $\frac{df_P}{d\gamma} = -\frac{2}{3(2-\gamma)^2} < 0$.

3. For all $\alpha \in [\frac{1}{2}, 1)$ and $\delta \in (0, 1-\alpha]$, it is easy to see that $g_P(\alpha, \delta) > 1$. Moreover, $\frac{\partial g_P}{\partial \delta} = \frac{2}{2\alpha - 1} > 0$.

□

Proof of Proposition 5.4.1. By (5.30) and Corollaries 5.2.2 and 5.2.4, we have that

$$\begin{aligned} W_V^* &= \left((1-\rho(e^*))^{n^*} \cdot \Psi^* - \frac{e^*}{2}\right) \cdot s^* \\ &= \frac{(\Psi^*)^2 \cdot s^*}{2} \\ &= \frac{1-\gamma}{4(3-2\gamma)^3}[(2\alpha - 1)H]^2 \end{aligned}$$

and

$$\begin{aligned} W_V^{**} &= \left((1-\rho(e^{**})) \cdot \Psi^{**} - \frac{e^{**}}{2}\right) \cdot s^{**} \\ &= \frac{(\Psi^{**})^2 \cdot s^{**}}{2} \\ &= \frac{[(2(\alpha+\delta)-1)H]^2}{108}. \end{aligned}$$

Thus, $\frac{W_V^{**}}{W_V^*} = \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^2 \cdot \frac{(3-2\gamma)^3}{27(1-\gamma)}$. When $\alpha = \frac{3}{4}, \gamma = \frac{1}{2}, \delta = \frac{1}{64}$, $W_V^{**} < W_V^*$. When $\alpha = \frac{3}{4}, \gamma = \frac{1}{2}, \delta = \frac{1}{4}$, $W_V^{**} > W_V^*$. □

Proof of Proposition 5.4.2. By Corollaries 5.2.2 and 5.2.4, $\frac{\Psi^{**}}{\Psi^*} = \frac{2(\alpha+\delta)-1}{2\alpha-1} \cdot \frac{3-2\gamma}{3} = f_\Psi(\gamma) \cdot g_\Psi(\alpha, \delta)$.

1. When $\alpha = \frac{3}{4}, \gamma = \frac{1}{2}, \delta = \frac{1}{64}$, $\Psi^{**} < \Psi^*$. When $\alpha = \frac{3}{4}, \gamma = \frac{1}{2}, \delta = \frac{1}{4}$, $\Psi^{**} > \Psi^*$.

2. It is easy to see that $f_\Psi(\gamma) < 1$. Moreover, $\frac{\partial f_\Psi}{\partial \gamma} = -\frac{2}{3} < 0$.

3. It is easy to see that $g_\Psi(\alpha, \delta) > 1$. Moreover, $\frac{\partial g_\Psi}{\partial \delta} = \frac{2}{2\alpha - 1} > 0$.

□

Proof of Proposition 5.4.3. By Corollaries 5.2.2 and 5.2.4, we have that

$$\begin{aligned}
\frac{\Psi^*}{P^*} &= \frac{1}{2-\gamma} = h(\gamma), \\
\frac{\Psi^{**}}{P^{**}} &= \frac{1}{2} = h(0), \\
\frac{e^*}{(P^*)^2} &= \left(\frac{\Psi^*}{P^*}\right)^2 = \frac{1}{(2-\gamma)^2} = (h(\gamma))^2, \\
\frac{e^{**}}{(P^{**})^2} &= \left(\frac{\Psi^{**}}{P^{**}}\right)^2 = \frac{1}{4} = (h(0))^2.
\end{aligned}$$

Moreover, $\frac{dh}{d\gamma} = \frac{1}{(2-\gamma)^2} > 0$. □

Proof of Proposition 5.4.4. By (5.32) and Corollaries 5.2.2 and 5.2.4, we have that

$$\begin{aligned}
W_C^* &= (1-\rho(e^*))^{n^*} \cdot \frac{1}{2} \cdot \int_0^1 \mathbb{E}[\left(v_k \cdot (2\alpha - 1) - P^*\right)\mathcal{I}\left(v_k \cdot (2\alpha-1) > P^*\right)]\, dk \\
&= \frac{\Psi^*}{2} \cdot \frac{\left((2\alpha-1)H - P^*\right)^2}{2(2\alpha-1)H} \\
&= \frac{(1-\gamma)^2}{4(3-2\gamma)^3}[(2\alpha-1)H]^2
\end{aligned}$$

and

$$\begin{aligned}
W_C^{**} &= (1-\rho(e^{**})) \cdot \frac{1}{2} \cdot \int_0^1 \mathbb{E}[\left(v_k \cdot (2(\alpha+\delta) - 1) - P^{**}\right)\mathcal{I}\left(v_k \cdot (2(\alpha+\delta)-1) > P^{**}\right)]\, dk \\
&= \frac{\Psi^{**}}{2} \cdot \frac{\left((2(\alpha+\delta)-1)H - P^{**}\right)^2}{2(2(\alpha+\delta)-1)H} \\
&= \frac{[(2(\alpha+\delta)-1)H]^2}{108}.
\end{aligned}$$

Thus, $\frac{W_C^{**}}{W_C^*} = \left(\frac{2(\alpha+\delta)-1}{2\alpha-1}\right)^2 \cdot \frac{(3-2\gamma)^3}{27(1-\gamma)^2} = f_{W_C}(\gamma) \cdot g_{W_C}(\alpha, \delta)$.

1. That $f_{W_C}(\gamma) > 1$ has been proved in the proof of Proposition 5.3.3. It is obvious that $g_{W_C}(\alpha, \delta) > 1$. Consequently, $\frac{W_C^{**}}{W_C^*} > 1$ and thus $W_C^{**} > W_C^*$.

2. $\frac{df_{w_C}}{d\gamma} = \frac{2\gamma(3-2\gamma)^2}{27(1-\gamma)^3} > 0.$

3. $\frac{\partial g_{w_C}}{\partial \delta} = \frac{4}{2\alpha-1}(1 + \frac{2\delta}{2\alpha-1}) > 0.$

$\square$