Assessing Pre-Service Elementary School Teachers' Use of Mathematics' Discipline-specific Language

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Abstract

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Existing studies have defined and assessed (a) disciplinary literacy, (b) mathematical literacy, and (c) general academic language. However, there is a need to define and assess teachers' use of mathematics' discipline-specific language (MDL), particularly elementary school teachers' use of MDL. The purpose of this study was twofold. First, I defined MDL, developed an instrument to measure it, and tested it with a specific population. Second, after validating the instrument, I used it to analyze the characteristics of teachers' use of MDL. This analysis was based on data collected from a sample of pre-service elementary teachers who were enrolled in the same special education graduate program.

The iterative development of the instrument resulted in an MDL scale (MDLS) that included 20 items rated on a 4-point Likert-like scale distributed between three distinct MDL categories: technical language, symbolic language, and visual language. Validity of the MDLS was confirmed using the set of 211 video recordings and corresponding lesson plans of mathematics lessons taught by pre-service special education elementary teachers. This research led to a modified MDL framework that indicated five distinct factors: technical language precision/explicitness, technical language fluency, symbolic language, visual language precision, and visual language fluency. Results highlighted varying levels of MDL use among elementary school teachers, with symbolic language use being particularly limited.

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List of Abbreviations

CCSS-M	Common Core State Standards for Mathematics
CCSS-MP	Common Core State Standards for Mathematical Practice
CFA	Confirmatory Factor Analysis
EdTPA	Educative Teacher Performance Assessment
EFA	Exploratory Factor Analysis
MDL	Mathematics' discipline-specific language
MDLS	Mathematics' discipline-specific language scale
MQI	Mathematical Quality of Instruction Coding Tool
SL	Symbolic language
SME	Subject matter expert
TL	Technical language
TLF	Technical language fluency
TLPE	Technical language precision/explicitness
VL	Visual language
VLF	Visual language fluency
VLP	Visual language precision

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Dedication

To my son, Santino: My parents taught me that with hard work, I can achieve anything I set my mind to. I want to pass this lesson on to you—dream big and chase those dreams with all your heart! I love you and I am amazed by you.

Chapter 1: Introduction

1.1 Need for the Study

Mathematical literacy can play an important role in solving problems encountered in today's rapidly evolving world. It is defined as "an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen" (Organisation for Economic Co-operation and Development [OECD], 2009, p. 14). The Common Core State Standards for Mathematical Practice (CCSS-MP) emphasize the importance of developing mathematical literacy and its fundamental components, one of which is discipline-specific mathematics language (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Discipline-specific mathematics language plays a critical role in helping individuals grasp abstract concepts, solve problems, and engage with mathematical ideas at deeper levels (Fang, 2012).

Discipline-specific language is defined as "the decontextualized oral and written language used within a specific discipline or profession with specialized vocabulary, syntax, and discourse patterns" (Zhang, 2014, p. 40). Mathematics' discipline-specific language (MDL) is different from everyday language (Machaba, 2017; Rothery, 1984). In other words, terms used in the mathematics classroom differ from those used outside a school setting. For instance, 'quotient' generally refers to the result of a division operation and is primarily used within mathematics. Other examples of MDL include vocabularies, symbols, and notations. Shanahan and Shanahan (2012) argued that a strong foundation of discipline-specific language is necessary to participate in the work of mathematics as a discipline.

MDL is used to encode information (Martin, 1989) and allows members of the discipline to communicate in discipline-specific ways (Bernstein, 1996). Strong pedagogy demands that teachers develop a thorough awareness of how MDL functions in different situations (Ball et al., 2005; Fang, 2014; Raiker, 2002; Thompson et al., 2008). Teachers require a knowledge of both discipline language and discipline content to serve the language demands of mathematics as a discipline (Siffrinn & Lew, 2018), and this knowledge must be developed through solid initial preparation (Ball et al., 2005). Teachers' understanding of MDL must then be translated to its use in the classroom. Furthermore, teachers must use precise MDL to enhance learners' conceptual understanding and procedural fluency (Machaba, 2017).

To develop students' MDL, elementary school teachers must be proficient in using their own MDL. This is crucial because teachers' use of correct MDL enables them to communicate mathematical concepts clearly and effectively. Such communication is essential for enhancing students' conceptual understanding of mathematics as a discipline (Ball et al., 2005), which will prepare them to succeed in subsequent grades. In addition, a lack of teachers' appropriate use of MDL can hinder student learning and become a source of misconceptions carried over to later mathematics education (Di Domenico, 2014; Köse, 2008). It is therefore vital that attention be given to MDL beginning at the elementary level (Siffrinn & Lew, 2018) since high-quality early mathematics instruction serves as a sound foundation for later learning in mathematics (Darling-Hammond & Bransford, 2005).

A population of teachers who should be studied in relation to their use of MDL at the elementary level is special education teachers. Many special education teachers have limited training in mathematics as a discipline. However, it is particularly important for special education teachers to use correct MDL in their instruction. Students with disabilities need their teachers to use precise MDL to ensure clarity of terminology, concepts, and procedures. According to Powell et al. (2019), teachers' explicit instruction in precise mathematics terminology improves clarity for students with disabilities, particularly for those who often struggle with abstract concepts. Special education teachers' use of effective MDL can help students with disabilities overcome learning barriers and succeed in mathematics (Powell et al., 2019).

Studies have examined teachers' knowledge of disciplinary literacy in mathematics (Colwell & Gregory, 2016; Di Domenico, 2014; Fang & Schleppegrell, 2010; Lenski & Thieman, 2013; Spires et al., 2018), but these studies have primarily been conducted at the secondary level. Research at the elementary level, focusing solely on discipline-specific language, has been limited to content areas other than mathematics (Cisco, 2016; Feez & Quinn, 2017; Lenski & Thieman, 2013; Ruzycki, 2015). It remains unclear what teachers' use of MDL looks like in the lower grades (kindergarten through fifth grade). Existing studies have defined and assessed (a) disciplinary literacy as a whole (Spires et al., 2018); (b) mathematical literacy (Jablonka, 2003; OECD, 2004); and (c) general academic language (Fang & Schleppegrell, 2010) at the secondary level.

While much of the existing research has focused on teachers' knowledge of MDL, less attention has been given to teachers' use of MDL, particularly at the elementary level. To the best of my knowledge, there have not been any studies conducted to define and assess elementary school teachers' use of MDL when teaching. Research on the use of MDL by special education teachers is also lacking. Since MDL is a major component of disciplinary literacy

(Fang, 2012; McConachie & Petrosky, 2009), connections that exist in literature can be used to lay the foundation for defining teachers' use of MDL at the elementary school level. A clear and specific framework for teachers' use of MDL is essential for understanding how MDL is used at the elementary school level. Development and validation of an instrument to assess the use of MDL can contribute to this theoretical understanding by collecting evidence and data from a particular group of participants, which can then inform the framework. Siffrinn and Lew (2018) argued that it is essential for elementary school teachers to be proficient in using MDL to make mathematical concepts more accessible to their students. Even if teachers are proficient in MDL, they may choose not to use it in their lessons. A quantitative assessment instrument, with implications for teacher education and professional development programs, can streamline the conversation about the expectations of elementary school teachers' use of MDL in teaching.

1.2 Purpose of the Study

The purpose of this study was twofold. Based on systemic analysis of literature, I defined MDL, developed an instrument for that definition, and tested that for a specific population. Then, after validating the instrument, I used it to analyze the characteristics of teachers' use of MDL based on data collected from a sample of pre-service elementary teachers enrolled in the same special education graduate program. The instrument for assessment of MDL is referred to as the Mathematics' Discipline-specific Language Scale (MDLS). The MDLS was validated using video recordings and corresponding lesson plans of mathematics lessons taught by these pre-service teachers. This study was guided by the following research questions:

RQ1. What are constructs of elementary school teachers' use of mathematics' discipline-specific language (MDL)?

RQ2. What are the characteristics of elementary school special education teachers' use of MDL?

1.3 Procedures of the Study

1.3.1 Part 1. Development and Validation of the MDLS

The process described in Part 1 addressed the first research question: What are constructs of elementary school teachers' use of MDL? In other words, how can we assess elementary school teachers' use of MDL?

Development of MDLS items was guided by practices defined by Hathcoat et al. (2016) and Boateng et al. (2018). First, specific items had to describe directly observed teachers' use of MDL in video recordings and corresponding lesson plans. Second, the following practices were used: (a) making items simple and specific, (b) ensuring that items are unidimensional and easy to read, and (c) generating twice as many items as needed.

To generate the preliminary statements for MDLS items, I collected and analyzed qualitative descriptors of teachers' use of MDL found in the Educative Teacher Performance Assessment (edTPA), the Mathematical Quality of Instruction Coding Tool (MQI), and the Common Core State Standards for Mathematical Practice (CCSS-MP). These statements were revised into performance-based items, then each was classified into one of the three categories of MDL: technical language, symbolic language, and visual language (Fang, 2012). Initially, a 4-point Likert-like scale (never, rarely, often, always) was used to measure frequency of occurrence for each MDLS item. An iterative process of item revisions at this stage aimed to ensure that each item was observable and independent of others (Initial Version of MDLS).

To test MDLS validity, I evaluated both (a) content validity and (b) construct validity of the scale. To test reliability, I evaluated (a) inter-rater reliability and (b) internal consistency of the scale. The sequence of stages in the MDLS development and validation process is shown in Figure 1.

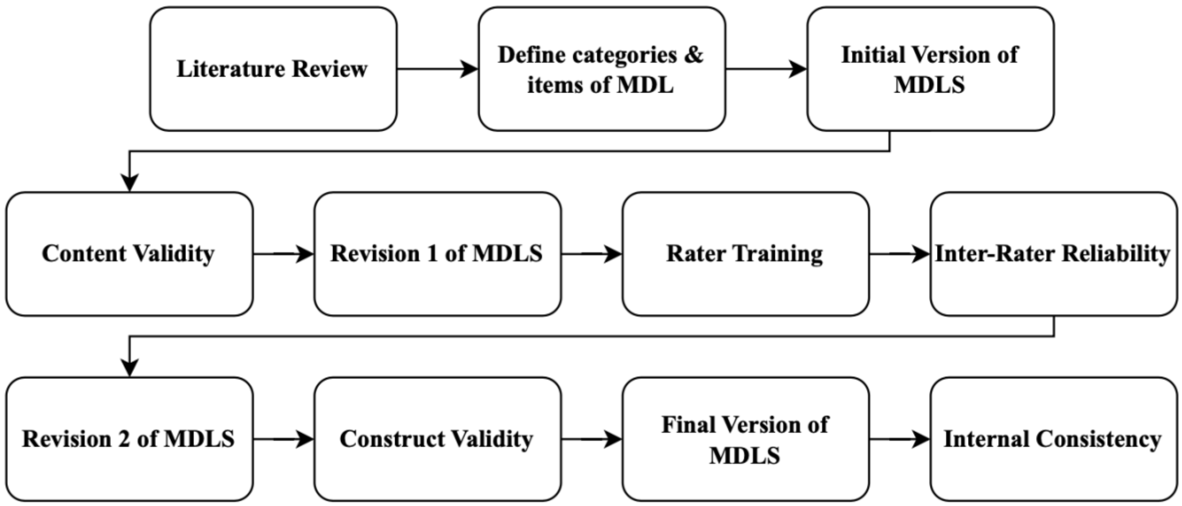


Figure 1. *Process of Developing and Validating the MDLS*

To evaluate the content validity, the developed scale along with specific tasks to complete were sent to two mathematical literacy experts. Both experts were scholars who conduct research in the area of disciplinary literacy in mathematics and have extensive experience helping teachers improve their mathematical literacy skills. Based on their feedback, the wording of several items was revised for clarity (Revision 1 of MDLS). Next, the inter-rater reliability of Revision 1 of MDLS was tested on 30 video recordings and corresponding lesson plans developed by pre-service elementary teachers enrolled in a special education graduate program. This data set was collected from 2012 to 2018 in grades K-5 (kindergarten through fifth grade) mathematics classrooms in urban public schools in the Northeast United States.

To test inter-rater reliability, I first trained two raters to use the MDLS with six videos and corresponding lesson plans selected from a data set of 247. Each rater then independently assessed a set of 30 video recordings and corresponding lesson plans in an initial round of scoring. Pearson correlation analysis was used on this set of scores followed by a debriefing with the raters. Based on the analysis and raters' comments, additional revisions were made to wording of items and the structure of Likert-like scale. This version of MDLS (Revision 2) was then used by the same two raters to score independently a new set of 30 video recordings and corresponding lesson plans. Inter-rater reliability was tested again using Pearson correlations and confirmed on this set of scores. The remaining data, 181 video recordings and corresponding lesson plans, were divided equally between the two raters to complete scoring using Revision 2 of MDLS.

Construct validity was then tested by comparing two models. The first model used a Confirmatory Factor Analysis (CFA) in SPSS AMOS 29 with three factors based on the theoretical framework. The second model used an Exploratory Factor Analysis (EFA) in SPSS 29 on the same dataset. Results of the EFA were then analyzed using a CFA in SPSS AMOS 29. Based on parameters for the goodness-of-fit (Bentler, 1990; Fabrigar, 1999; Kline, 2005), the better model was chosen as the Final Version of MDLS. The Final Version of MDLS was then tested for internal consistency using Cronbach's alpha.

1.3.2. Part 2. Exploring Elementary School Teachers' Use of MDL

The second part of the study used a descriptive design to analyze teachers' use of MDL based on data collected from a sample of pre-service elementary teachers enrolled in the same special education graduate program. This addressed the second research question: What are the characteristics of elementary school special education teachers' use of MDL?

Quantitative Analysis of Teachers' Use of MDL. Descriptive statistics in SPSS 29 were used to report the means, standard deviations, and ranges of each construct of MDL. A one-way ANOVA with post-hoc analysis was conducted to see if there was a statistically significant difference in the means of the constructs of MDL. A correlation analysis was then used to explore independence of the constructs and associations between them.

Qualitative Analysis of Teachers' Use of MDL. A content analysis was used to illustrate pre-service teachers' use of MDL at different levels. I used disproportionate stratified random sampling to select three lessons and corresponding videos to represent different levels of teachers' use of MDL (cases A-C). Based on conditions set by Hayes et al. (2015), the illustrative cases included descriptions, quotes, and key points of teachers' use of MDL from video recordings and lesson plans.

1.4 Concluding Remarks

In answering the research questions, I have (a) defined constructs of elementary school teachers' use of MDL, and (b) determined characteristics of elementary school special education teachers' use of MDL, based on data collected from a sample of pre-service elementary teachers enrolled in the same special education graduate program.

The theoretical contribution of this study lies in its development of the clear and specific constructs of MDL when used by elementary school teachers. This research led to a modified MDL framework that provides a more nuanced understanding of the MDL categories proposed in the literature. By offering a more detailed framework, this research advances the literature on disciplinary literacy and elementary teachers' use of MDL.

Theoretical frameworks need to be tested in practice (Grant & Osanloo, 2014), which is why I developed an instrument to assess teachers' use of MDL. Defining constructs of MDL bridged the gap between theoretical definitions and practical measurements of MDL in the field. The analysis of data collected from the MDLS provided a better understanding of how teachers used MDL at the elementary level. The illustrative case studies demonstrate how the MDLS can be used at different levels and may serve as a guide for its future use.

The MDLS functions as a practical tool for assessing teachers' use of MDL, enabling the identification of areas of strength and highlighting areas that may require further development. The MDLS could be helpful in research and evaluation to measure the MDL of pre- and in-service elementary school teachers. Furthermore, the MDLS has practical implications for teacher education and professional development with focus on development of all categories of MDL. It could be used to assess gaps and deficiencies in the MDL of pre- and in-service elementary school teachers to support their MDL development through focused programs.

Chapter 2: Literature Review

Dewey (1938) and Schwab (1978) argued the importance of literacy in elementary and secondary educational disciplines for years. The term *literacy* refers to reading, thinking, speaking, and content knowledge (McConachie & Petrosky, 2009). The focus on the role of literacy across various disciplines, however, only gained popularity within the past 30 years, following the early work of Ball et al. (1990). More and more teacher leaders adopted the focus of literacy across various disciplines in their schools. Curriculum writers and policymakers focused on ensuring that students learned how to read, understand, and explain information that came from various disciplines (Altieri, 2011). Since academic subject matter becomes more complex and differentiated by discipline, literacy definitions must be anchored to the specifics of individual disciplines (McConachie & Petrosky, 2009).

Mathematics' discipline-specific language (MDL) is a major component of both disciplinary literacy and mathematical literacy (Fang, 2012; McConachie & Petrosky, 2009). Since it is still unclear what teachers' use of MDL looks like at the elementary school level (Siffrinn & Lew, 2018), an analysis of literature on disciplinary literacy theory and mathematical literacy provides background information on what is currently known. This review was used to lay the foundation for defining elementary school teachers' use of MDL.

2.1 What Is Disciplinary Literacy?

The term *disciplinary literacy* was proposed by the Institute of Learning at the University of Pittsburgh in 2002 (McConachie & Petrosky, 2009). Disciplinary literacy was originally thought of as developing expertise in content, rhetorical processes, genres, methods, inquiries, and discipline (Ball et al., 1990). Shanahan and Shanahan (2014) adopted a more straightforward definition: “Disciplinary literacy refers to the idea that we should teach the specialized ways of reading, understanding, and thinking used in each academic discipline” (p. 636). Their definition suggested that mathematicians and scholars of other subject matters read their disciplinary texts rather differently. Shanahan and Shanahan proposed that disciplinary literacy should be introduced as early as elementary school.

McConachie and Petrosky (2009) suggested that literacy is rooted in disciplinary content and involves reading, speaking, writing, and thinking habits, such as reasoning and investigating, as practiced in each of the core disciplines. They provided a foundation for Shanahan and Shanahan’s (2014) work on disciplinary literacy by defining it as “the use of reading, reasoning, investigating, speaking, and writing required to learn and form complex content knowledge appropriate to a particular discipline” (p. 70). This definition emphasized (a) the knowledge of those who create and communicate within each discipline, (b) participation in the academic work of each discipline, and (c) the requirement of habitual thinking that many novices cannot do well yet. Additionally, McConachie and Petrosky (2009) identified the following factors of disciplinary literacy:

1. to read, write, talk, and reason as a junior member of a particular discipline;
2. to understand what counts as a good question, evidence, problem, or solution in a particular discipline;
3. to craft arguments in ways that members of disciplines do—conducting proofs in mathematics, for example;
4. to master the core ideas and concepts of a particular discipline and adopt the habits of thinking. (p. 132)

Fang (2012) defined disciplinary literacy as “the ability to engage in social, semiotic, and cognitive practices consistent with those of content experts” (p. 19).

Researchers proposed various viewpoints that challenged educators to think more critically about disciplinary literacy theory (McConachie & Petrosky, 2009; Moje, 2007; Shanahan & Shanahan, 2008). Moje (2007) found there are four different types of processes to developing disciplinary literacy: cognitive literacy processes, epistemological processes of the disciplines, linguistic processes of the disciplines, and linguistic and discursive navigation across cultural boundaries. Shanahan and Shanahan’s (2008) research focused specifically on reading and posed practical ways to utilize disciplinary literacy. Their work suggested that strong reading skills developed in the early years do not necessarily meet discipline-specific language requirements in later years. Therefore, specialized literacy instruction must increase as students matriculate through school. Shanahan and Shanahan proposed three progressive levels of literacy:

1. Basic literacy: decoding and knowledge of words that frequently occur in texts and are required for all tasks.
2. Intermediate literacy: generic comprehension strategies, common word meanings, and basic fluency required for many reading tasks.
3. Disciplinary literacy: specialized in specific subject matters.

McConachie and Petrosky’s (2009) disciplinary literacy framework concentrated on school systems as a whole by providing principles that allow all students the opportunity to participate in discipline-specific learning in the core subject areas. The disciplinary literacy principles of McConachie and Petrosky’s framework were as follows:

1. Knowledge and thinking must go hand in hand.

2. Learning is an apprenticeship.

3. Teachers mentor students as apprentices.

4. Classroom culture socializes intelligence.

5. Instruction and assessment drive each other. (p. 20)

In this framework, teachers were expected to provide all students the opportunity to participate in discipline-specific learning in the core subjects. However, this required the teachers themselves to possess the skills and knowledge to make sense of discipline-specific learning. Only then can teachers successfully transfer those skills and knowledge to their students. Once this occurs, students can apply their own skills and knowledge to schoolwork and, eventually, to the workplace (Moje, 2015).

Disciplinary literacy theory has been studied using two approaches: investigating functional linguistics (Christie & Derewianka, 2010; Fang, 2012; Fang & Schleppegrell, 2010; Fulda, 2009) and observing expert practices (Bullock, 1994; O'Halloran, 2008; Shanahan et al., 2011; Tall, 1991). An approach to studying disciplinary literacies involves analyzing texts to uncover the deeper meanings embedded within them. Individuals learn the preferred language functions used in that particular discipline, leading them to apply the same principles to their work. This approach, called "functional language analysis," is grounded in Systemic Functional Linguistics (Christie & Derewianka, 2010; Fang & Schleppegrell, 2010). The studies that investigated the functional linguistics approach focused on written text rather than oral language (Christie & Derewianka, 2010; Fang & Schleppegrell, 2010; Fulda, 2009). The *observing expert practices* approach to studying disciplinary literacy theory often involved comparing how experts and novices engaged with texts within a specific discipline (O'Halloran, 2008; Shanahan et al., 2011). Studies that used these approaches and specifically focused on mathematics (Fang, 2012; O'Halloran, 2008; Shanahan et al., 2011; Tall, 1991) are discussed further in the following section.

2.2 Connecting Disciplinary Literacy to MDL

Studies that focus on disciplinary literacy in mathematics observe experts either reading or communicating in their discipline (O'Halloran, 2008; Shanahan et al., 2011; Tall, 1991). According to Gee (2014), the interactions of experts communicating in their discipline are grounded in discourse theory. Gee's (1989, 2012) discourse theory incorporated disciplinary literacy theory by supporting classroom instruction that focused on what Shanahan and Shanahan (2011) described as thinking in ways similar to mathematicians. Gee's theory focused on everything from "isolated language to studying language in use shaped by the values of society and cultural context, including occupations" (Hillman, 2014, p. 398). Gee (2014) stressed that language is used for saying, doing, and being: When speaking, we inform; when acting, we do; and when we are something, we are engaged in the act of being. To understand what a writer or speaker is trying to say, we must grasp the complete meaning of their message. To summarize, Gee (2012) suggested that:

> Discourses are ways of behaving, interacting, valuing, thinking, believing, speaking, and often reading and writing, that are accepted as instantiations of particular identities by specific groups.... They are socially situated identities. They are, thus, always and everywhere social products of social histories. (p. 3)

Shanahan et al. (2011) observed experts reading mathematical texts aloud and found that they were very conscious of carefully reading each word and symbol. These mathematicians were also attending to possible errors since they understood how the smallest word can make the most significant difference. The experts agreed that mathematics is the most precise of all the disciplines; this differs from a novice's view of the discipline, who may assume that mathematics' brevity allows it to be a "quick read." Experts in the field understand how mathematics tends to be very condensed, only including the absolutely necessary information thus limiting the possibility of misinterpretation (O'Halloran, 2008; Shanahan et al., 2011).

Mathematicians view concepts as being deeply connected, malleable abstractions that can be applied to real-world phenomena, while novices view them in terms of examples specific to particular situations, lacking flexibility (Tall, 1991). Studies investigating functional linguistics have also found that mathematics is concerned with "definitions, axioms, theorems, and problems" (Fulda, 2009, p. 1437) and that the language of mathematics is abstract (Bullock, 1994).

Mathematics has a language of its own that is functional for constructing knowledge and reasoning in the subject (Fang, 2012). It includes unique content such as diagrams, numbers, and symbolic expressions. Gafoor and Sarabi (2015) classified this language in the following categories:

1. Object and concept symbols (e.g., numbers);
2. Operation and process symbols (e.g., arithmetic operations);
3. Relation symbols (e.g., greater than, less than, perpendicular, etc.);
4. Auxiliary symbols (e.g., parentheses, brackets, etc.);
5. Mathematical vocabulary and lexicon (e.g., variables, symbolic expressions, etc.), which includes verbal expression and can be broadly classified under:
    a. discipline-specific terms;
    b. common words that have a different meaning in mathematics. (p. 5)

Fang (2012) used functional language analysis to study the different ways language is used to present knowledge in the core academic disciplines, including mathematics. He did so by analyzing various texts to identify specific language used at the secondary level. For mathematics, he suggested that different language patterns can be categorized as technical, symbolic, and visual.

The definition of MDL used in the present study is based on Fang's (2012) work. I adopted technical language, symbolic language, and visual language as the categories of MDL. Fang defined technical language as discipline-specific grammatical features, structure, and vocabulary; symbolic language as mathematical symbols used to describe relationships between mathematical objects; and visual language as the use of visual representations, such as diagrams and charts, to convey meaning and support understanding in academic disciplines. My research is also grounded in McConachie and Petrosky's (2009) disciplinary literacy framework, which emphasized expectations of teachers—an aspect often overlooked by others in the field.

2.3 Connecting Mathematical Literacy to MDL

According to the Programme for International Student Assessment (OECD, 2003), also known as the PISA exam:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. (p. 24)

Mathematical literacy is about an individual's capacity to *use* and *apply* mathematical knowledge and cannot be theorized solely in mathematical knowledge (Jablonka, 2003). Mathematically literate individuals should know how to do mathematics and how to *speak* the language of mathematics. This is not learned by simply memorizing definitions but, rather, by applying the language of mathematics to their learning experiences (Hill et al., 2008). This can happen as early as the elementary grades since students will have described and identified numerous mathematical terms by the end of the second grade.

Studies in secondary education have been undertaken to assess teachers' knowledge of mathematical literacy, focusing on MDL (Di Domenico, 2014; Raiker, 2002; Tsamir et al., 2015). These studies used discourse analysis of classroom observations (Raiker, 2002) or analysis of self-reported survey responses (Di Domenico, 2014; Spires et al., 2018; Tsamir et al., 2015). Spires et al. (2018) concluded that secondary mathematics education's disciplinary literacy is a 'multidimensional construct' with three related factors: source literacy, expressive literacy, and analytic literacy. Source literacy refers to obtaining compositional credibility as a reader and writer, while expressive literacy generates literacy devices using voice and style. Analytic literacy refers to solution-oriented thinking, quantitative and technical terminology, and visual representations such as graphs and models. Within the construct of analytic literacy, the following disciplinary literacy items were observed: graph analysis, data interpretation, creating mathematical models, analysis of quantitative and technical terminology, and aiming for convergence upon a solution. The study concluded that the construct of analytic literacy was engaged with the most in secondary education (Spires et al., 2018). These studies have focused on teachers' knowledge rather than their use of mathematical literacy, particularly MDL.

2.4 Mathematics Education Perspective

Mathematics education literature suggests that language plays a key role in both teaching and student learning processes (Morgan et al., 2014; Pimm & Keyes, 1994). Language serves not only as a method of communication but also as a major component in understanding concepts. However, there has been variation in how the language of mathematics, or mathematical language, is defined. According to Pimm and Keyes (1994), "the language of mathematics" could be interpreted in several ways:

1. the spoken language of the mathematics classroom (including both teacher and student talk);
2. the use of particular words for mathematical ends (often referred to as the *mathematics register*);
3. the language of texts (conventional word problems or textbooks as a whole, including graphic material and other modes of representation);
4. the language of written symbolic forms. (p. 159)

Mathematical language is sometimes understood in a straightforward way as either terminology or symbols. In contrast, it can also extend beyond vocabulary and symbols to include diagrams, graphs, gestures, and other modes of communication (Morgan et al., 2014).

An important aspect of mathematical language is the use of symbols, which is a central focus of the field of semiotics. According to Presmeg et al. (2016), semiotics is the study of signs and symbols and how they are used to create and communicate meaning. Pimm and Keyes' (1994) social semiotics framework views language as a system of signs shaped by social contexts and interactions. This framework emphasizes the role of language in constructing meaning within the mathematics classroom, where both spoken and written forms of communication are used to convey mathematical ideas.

Furthermore, the "social turn" in mathematics education stressed the impact the social environment has on students' mathematical experiences (Lerman, 2000; Valero, 2004). This includes how students interact with each other, how mathematics is taught, and what the cultural attitudes toward mathematics are. Since these theoretical shifts, there has been growing recognition of the importance of language-rich activities in mathematics classrooms, often framed as 'conversation,' 'discussion,' or 'discourse.' Therefore, educators should be aware of the language demands of mathematics to support students' engagement with mathematical discourse and potentially enhance learning outcomes within the classroom (Morgan et al., 2014).

Due to the semiotic nature of mathematics requiring effective communication, there is a strong connection between linguistic competencies and mathematical development (Sfard, 2008). The exploration of language in mathematics education can contribute to a deeper understanding of how students learn concepts. It also helps teachers understand how students communicate those concepts and make sense of mathematics (İncebacak & Ersoy, 2021).

2.5 Teachers' Use of MDL

In the field of mathematics education, Halliday (1978) defined a specific set of meanings and language structures used in mathematics as the mathematics register. The mathematics register includes vocabulary, grammatical structures, precise expression, symbols, formality, and impersonality that is unique to mathematical communication (Halliday, 1978; Lee, 2006). Pimm (1987) expanded on Halliday's (1978) definition, emphasizing that the mathematics register is also about the ways of thinking and meaning that accompany words and expressions that are used. This concept is closely related to discipline-specific language, as defined by Zhang (2014), which refers to the specialized vocabulary, syntax, and discourse patterns used within a specific field, such as mathematics. However, the mathematics register refers to a narrower, more specific function or form of language within the discipline. Since both the mathematics register and MDL share key elements such as specialized vocabulary, precise expression, and formal structures to convey mathematical concepts, literature on teachers' use of the mathematics register can provide insights into teachers' use of MDL, an area that is under-researched in the field of disciplinary literacy.

Pimm and Keynes (1994) studied the relationship between language and mathematics in educational settings. Their work focused on improving communication through spoken language and the mathematics register of written language. Pimm and Keynes's (1994) findings emphasized the critical role that teachers' mathematical language has on influencing students' understanding of mathematics. Conditions must be met in teaching, many of which depend on both the students' and the teachers' language used in the classroom (Abele, 1998). Students often develop an understanding of the conventional mathematics register through the manner in which teachers present mathematical concepts. When teachers are explicit in using language to convey mathematics, they are guiding students to better articulate their thinking (Lee, 2006). Abele (1998) argued that teacher education and professional development programs should make it a goal to ensure that teachers develop a diverse set of strategies for leading classroom discussions. Teachers' decisions on the use of mathematical language are crucial for students' success. Moreover, the choice of language used in the context of problems, activities, and questions is central to whether the content is accessible to the learner (Clark, 1998).

2.6 Criteria Used to Assess MDL

Precision is a significant feature that distinguishes mathematical language from ordinary language (Di Domenico, 2014; Jamison, 2000; Tapson, 2000; Tsamir, 2015). Empirical and theoretical studies have shown that in most cases teachers used imprecise MDL (Jamison, 2000; Tapson, 2000), specifically at the secondary level (Di Domenico, 2014; Tsamir, 2015). For instance, Jamison (2000) and Tapson (2000) analyzed teachers' use of language as a tool for teaching mathematical concepts. They found that teachers often (a) used multiple different words to describe the same mathematical idea and (b) used one mathematical term to explain multiple notions (Jamison, 2000; Tapson, 2000). Both situations could result in students' confusion and teachers’ use of MDL becoming unnecessarily complicated. These issues with imprecision highlight the need for precision as a criterion for assessing MDL, as it ensures accuracy, clarity, and consistency in communicating mathematical concepts.

A second key criterion is explicitness. Lee (2006) explored how use of explicit language in teaching mathematics can enhance students’ ability to communicate their understanding. He suggested that teachers’ use of explicit language helps clarify students’ thinking. Powell et al. (2019) emphasized that explicit instruction in mathematics can significantly enhance clarity for students with disabilities, who may otherwise struggle with abstract concepts.

Finally, fluency is another important criterion for assessing MDL. Fluency in language involves understanding both individual terms and their relationships. It is closely related to the vocabulary and *conceptual density* of the language. Conceptually dense language consists of words or phrases that combine two or more concepts to form a new concept. Irujo (2007) found that vocabulary and semantics, ‘conceptually dense’ words or phrases, and multiple meanings (or terms for the same concept) caused significant difficulties for students when taught mathematics. Regarding vocabulary and semantics, words with math meanings different from their everyday meanings caused the most difficulty. Additionally, understanding a concept is more challenging when the concept is made up of the relationship between two words.

Assessments, such as the Educative Teacher Performance Assessment (edTPA) and the Mathematical Quality of Instruction Coding Tool (MQI), partially measure these criteria as part of overall instructional quality in the classroom. The edTPA evaluates pre-service teachers’ imprecision in language, and the MQI assesses pre- and in-service teachers’ explicitness of mathematical terminology and technical language fluency (Hill, 2010). These scoring rubrics with leveled progressions capture an overview of instructional quality and how well teachers support language demands for students. However, these instruments assess language as just one category within a much broader evaluation, rather than addressing the specific criteria of MDL in detail (Hill, 2010; Stanford Center for Assessment, Learning, & Equity, 2016). Each criterion is represented by a single measure, which limits the depth of assessment. Furthermore, these tools evaluate the teacher's ability to support students' language development rather than specifically addressing the quality of teachers' use of MDL.

Validation studies have been conducted on the edTPA to confirm the content validity, job relevance, and construct validity of the assessment (Stanford Center for Assessment, Learning, & Equity, 2019). The edTPA was validated for the population of pre-service teachers. The MQI was validated for the population of teachers from second to sixth grade classrooms (Boston et al., 2015).

According to the edTPA, precision is defined as being accurate with definitions and symbols in labeling, measurement, and numerical answers (Stanford Center for Assessment, Learning, & Equity, 2016). In the MQI, precision is assessed based on the following indicators: (a) teachers' instruction is free of major errors in spoken and written work; (b) errors that occur are captured and corrected within the teaching segment; (c) teachers solve problems correctly; and (d) teachers define terms correctly. Major mathematical errors or serious mathematical oversights are assessed using the following indicators: "forgetting a key condition in a definition or step in a problem, equating two non-identical mathematical terms, and neglecting to discuss key aspects of a problem" (Hill, 2010, n.p.).

According to the MQI (Hill, 2010), explicitness in mathematical terminology refers to whether teachers accurately use technical terms. This includes being clear, direct, and detailed. Fluency, or flexibility, was assessed by observing the density of mathematical language during periods of teacher talk. Density in this context refers to "lexical density," which means the proportion of discipline-specific words compared to the total number of words (Johansson, 2008). The MQI rubric categorizes the density of teachers' mathematical language as either high or low. Examples of the MQI progressions (from high to low) are as follows:

1. Teacher uses mathematical language as a vehicle for conveying content.
2. Teacher uses non-mathematical terms to describe mathematical ideas and procedures.
3. Teacher does not demonstrate fluency in mathematical language.
4. Teacher talk includes grammatical errors, unclear wording, or incorrect words.

2.7 Concluding Remarks

Several studies that focused on MDL in secondary education have assessed teachers' knowledge of disciplinary literacy and mathematical literacy (Di Domenico, 2014; Raiker, 2002; Tsamir et al., 2015). Raiker (2002) used discourse analysis of classroom observations. Tsamir et al. (2015) and Di Domenico (2014) analyzed self-reported survey responses. However, there are no studies that focus on elementary school teachers' use of MDL. Strong pedagogy demands that teachers develop a thorough awareness of how MDL functions in different situations (Ball et al., 2005; Fang, 2014; Raiker, 2002; Thompson et al., 2008), meaning the ways in which MDL serves particular purposes across different learning environments or contexts. In addition, teachers must use MDL to further enhance students' conceptual understanding and procedural fluency at the lowest grades (Machaba, 2017).

Disciplinary literacy and mathematical literacy frameworks can be used to accurately define, or operationalize, teachers' use of MDL at the elementary school level. However, there is a need to develop a framework specific to this context, which is part of the purpose of this study. In addition, the developed framework needs to be tested in practice. To address this, the present study defined MDL through a systemic analysis of the literature, then developed a research instrument aligned with the theoretical framework presented in the following section. The instrument was then tested with and validated for a specific population of pre-service special education elementary school teachers.

2.8 Theoretical Framework of the Study

To develop a theoretical framework for defining mathematics' discipline-specific language (MDL), I first outlined connections between existing frameworks on disciplinary literacy and studies on MDL. McConachie and Petrosky's (2009) disciplinary literacy framework defined standards necessary for students to develop discipline-specific literacy in the core subject areas. This framework suggested the following components of literacy: reading, reasoning, investigating, speaking, and writing. Gee (2012) suggested that language incorporates behaving, interacting, thinking, reading, speaking, and writing. Therefore, McConachie and Petrosky's (2009) speaking and writing components of disciplinary literacy can be considered part of language. In this study, McConachie and Petrosky's framework was adapted to focus solely on the language component of disciplinary literacy for the discipline of mathematics, a field with its own language that is functional for constructing knowledge and reasoning in the subject (Fang, 2012).

Further, Fang (2012) suggested that different language patterns of mathematics can be categorized as technical, symbolic, and visual. Fang defined technical language as discipline-specific grammatical features, structure, and vocabulary. Symbolic language is represented by mathematical symbols that are used to describe relationships between mathematical objects. Based on topics covered in elementary school mathematics curriculum, the following symbols are learned in grades K-5 (kindergarten through fifth grade): basic operations signs, the plus sign ($+$), the minus sign ($-$), the multiplication signs ($\times$, $\cdot$, or $*$), the division signs ($\div$ or $/$), the relation signs ($=$, $>$, $<$, $\geq$, and $\leq$), the fraction notation ($\frac{\square}{\square}$ or $\square / \square$), the place value signs (decimal point . and comma ,), units of measurement signs (feet ′ and inches ″), grouping symbols (parentheses, brackets, braces), and money signs ($ and ¢). Visual language at this level is represented by number lines, number paths, array and area models, strip diagrams, schematic diagrams, drawings, tables, graphs, other forms of pictorial representations, and/or concrete objects and manipulatives. These three categories of MDL were used to classify the patterns of MDL use in this study.

To assess the quality of teachers' use of MDL, I adapted criteria from the Educative Teacher Performance Assessment (edTPA) and the Mathematical Quality of Instruction Coding Tool (MQI) —precision, explicitness, and fluency—which I embedded in the MDL scale (MDLS) items that defined the three language categories. This was done with a specific focus on elementary school teachers' use of MDL. Here is one example, in which I integrated the assessment criterion of *precision* from the Common Core State Standards for Mathematical Practice (CCSS-MP) into the different categories of MDL:

> Precision is being accurate with definitions and symbols in labeling, measurement, and numerical answers (Stanford Center for Assessment, Learning, & Equity, 2019).
>
> - Assessing precision of technical language: Calculating accurately and expressing numeric answers appropriately for the context of the problem.

- Assessing precision of symbolic language: Using the equal sign consistently and appropriately.
- Assessing precision of visual language: Correctly labeling the axes of a graph.

My theoretical framework for the development of MDLS integrated two frameworks by assigning all three criteria of assessment to each category of MDL (Figure 2).

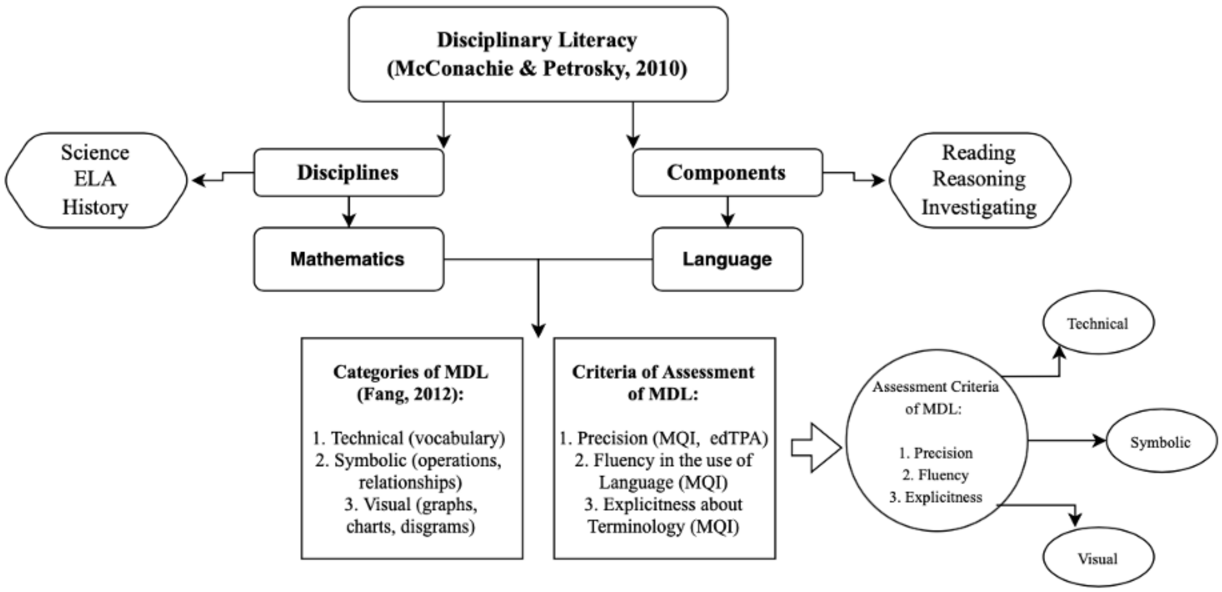


Figure 2. *Deriving My Framework from Existing Theories*

Chapter 3: Development and Validation of the Mathematics' Discipline-specific Language Scale

This chapter addresses the first research question, RQ1: What are constructs of elementary school teachers' use of mathematics' discipline-specific language (MDL)?

3.1 Development of Initial Version of the MDLS

The proposed theoretical framework integrated assessment criteria from existing instruments into a framework that defined categories of MDL, which created the foundation for the Mathematics' Discipline-Specific Language Scale (MDLS). Specifically, the MDLS was divided into three sections, according to Fang's (2012) three categories of MDL: technical language (TL), symbolic language (SL), and visual language (VL). I identified the assessment criteria—precision, explicitness, and fluency—through a review of the literature and the following major assessment documents: the Common Core State Standards for Mathematical Practice (CCSS-MP; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), the Educative Teacher Performance Assessment (edTPA; Stanford Center for Assessment, Learning, & Equity, 2019), and the Mathematical Quality of Instruction Coding Tool (MQI; Hill, 2010).

To develop observable items for each category of MDL using the assessment criteria, I first identified keywords (e.g., mathematical language, academic language, mathematics' disciplinary literacy, and elementary school mathematics language). These keywords guided the extraction of relevant phrases from the CCSS-MP, the EdTPA, and the MQI. The collected phrases were then analyzed in relation to teachers' use of MDL to generate preliminary statements for the MDLS items. These statements were classified into the three assessment criteria based on their definitions and classifications in the assessment documents. Table 1 shows the classification of this content analysis (Krippendorff, 2018).

Table 1

Content Analysis of Assessment Documents

Source	Preliminary Statements	Assessment Criteria of MDL
CCSS-MP (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010)	• Carefully formulate explanations • Use clear definitions when discussing reasoning • Express numerical answers appropriately for the problem context • Accurate calculations • Label quantities appropriately • Use appropriate symbols • State the meaning of the symbols used • Use the equal sign consistently and appropriately • Correctly specify units of measure • Label axes to clarify the correspondence with quantities in a problem	Precision
edTPA (Stanford Center for Assessment, Learning, & Equity, 2019)	• Being precise and accurate with definitions and symbols in labeling, measurement, and numerical answers • Correctly labeling the axes of a graph • Specifying units of measure during calculations • Calculating accurately and expressing numeric answers with appropriate precision for context of problem	
	• Organizing words or symbols together into phrases, clauses, sentences or visual representations • Organize language to convey meaning, e.g., formulas, long or elaborate noun phrases, conditional sentences	Fluency
MQI (Hill, 2010)	• Use of mathematical terms to describe mathematical ideas • Use of mathematical terms to describe procedures • Careful use of mathematical terms (reworded for clarity) • Lots of mathematical language used (reworded for clarity) • Use of mathematical language as a vehicle for conveying content • Density of mathematical language is high	
	• Use of explicitness about terminology • Accurate use of technical terms	Explicitness

Reference to the use of the equal sign emerged as a common theme during the content analysis conducted to generate preliminary statements for the MDLS. As a result, I decided to include a specific item related to the equal sign. Powell and Fuchs (2010) found teaching students to view the equal sign relationally and as a symbol of equivalence, instead of as a symbol indicating the result of a calculation, improves understanding of mathematical relationships and operations which are fundamental at the elementary level. Precisely using the equal sign, and emphasizing the relational meaning of the equal sign, can facilitate the transfer of skills to other mathematical concepts, improving students' overall mathematics proficiency (Powell & Fuchs, 2010).

Next, the preliminary statements were analyzed and non-performance-based statements were revised into performance-based statements. Then, each performance-based statement was classified into one of the three categories of MDL (Table 2). I combined some preliminary statements to reduce redundancy among statements from different documents. An iterative revision process occurred to ensure that each performance-based statement was observable. Visual language statements, which were already performance-based, were added based on a recommendation about the use of visual representations from What Works Clearinghouse (2012). Statements were further analyzed, and some were rephrased for clarity or to fit the needs of the MDLS. Table 3 shows additions and revisions at this stage.

I developed the initial version of MDLS by combining some of the statements and writing as items in Tables 2 and 3. I then classified each item into a category of MDL. Development of MDLS items was guided by practices defined by Hathcoat et al. (2016) and Boateng et al. (2018). First, category-specific items had to describe directly observed teachers'

Table 2

Revision and Classification of Preliminary Statements

Preliminary Statements	Performance-Based Statement	Category of MDL		
Precision		**Technical**	**Symbolic**	**Visual**
-Carefully formulate explanations -Use clear definitions when discussing reasoning	-The teacher states clear explanations and definitions.	✓		
Express numerical answers appropriately for the problem context	-The teacher states numerical answers appropriately for the problem context.	✓		
	-The teacher writes numerical answers appropriately for the problem context.		✓	
Accurate calculations	-The teacher performs accurate calculations.			✓
Label quantities appropriately	-The teacher labels quantities appropriately.			✓
Use appropriate symbols	-The teacher writes appropriate symbols.		✓	
State the meaning of the symbols used	-The teacher correctly states the meaning of the symbols used.		✓	
Using the equal sign consistently and appropriately	-The teacher uses the equal sign appropriately.		✓	
Correctly specify units of measure	-The teacher correctly states units of measure.	✓		
Label axes to clarify the correspondence with quantities in a problem	-The teacher correctly labels axes. -The teacher correctly clarifies how axes correspond with quantities in a problem.			✓

Table 2 (continued)

Preliminary Statements	Performance-Based Statement	Category of MDL		
Explicitness		**Technical**	**Symbolic**	**Visual**
Accurate use of technical terms	-The teacher accurately uses technical terms.	✓		
	-The teacher accurately uses symbols.		✓	
Fluency				
-Use of mathematical terms to describe mathematical ideas -Careful use of mathematical terms	-The teacher carefully uses mathematical terms to describe mathematical ideas.	✓		
	-The teacher correctly uses symbols to describe mathematical ideas.		✓	
Use of mathematical terms to describe procedures	-The teacher uses mathematical terms to describe procedures.	✓		
Lots of mathematical language used	-The teacher uses MDL with a high level of density.	✓		
	-The teacher uses mathematical symbols often.		✓	
	-The teacher uses mathematical language as a vehicle for conveying content.	✓		

Table 3

Additional Visual Language Statements Aligned with Assessment Criteria of MDL

Additional Visual Language Statements	Assessment Criteria of MDL
○ The teacher correctly uses a visual to represent and solve the problem. ○ The teacher translates abstract symbols and numbers into meaningful visual representations. ○ The teacher correctly expresses mathematical ideas using visual representations. ○ The teacher correctly converts visual representations into symbols. ○ The teacher correctly converts visually represented information into mathematical notation. ○ The teacher selects visual representations that are appropriate for students and the problems they are solving. ○ The teacher selects visual representations that focus on the relationship between quantities.	Precision
○ The teacher illustrates how material introduced in different parts of the lesson is connected. ○ The teacher explicitly links visual representations with the standard symbolic representations used in mathematics. ○ The teacher connects visual representations with standard notation used in mathematics.	Explicitness
○ The teacher only includes necessary details in visual representations. ○ The teacher uses different visual representations for different types of problems.	Fluency

use of MDL in video recordings and corresponding lesson plans. Second, the following practices were used: (a) making items simple and specific, (b) ensuring that items are unidimensional and easy to read, and (c) generating twice as many items as needed. I decided to include nine items per category with a total of 27 items in the initial version of MDLS (Table 4), following the recommendation by Tabachnick and Fidell (2007) to be over-inclusive when generating the initial item pool. A 4-point Likert-like scale (never, rarely, often, always) was used in the initial version of MDLS to measure frequency of occurrence for each MDLS item.

Upon the conclusion of these revisions, the initial version of MDLS was ready to send to experts to review for content validity. Table 4 shows revisions made to the MDLS during the instrument validation process, as well as the final validated instrument.

Table 4

Iteration Analysis of the MDLS

Label	Initial Version	Revision 1	Revision 2	Final Version
TL1 Precision	The teacher states/writes numerical answers that are relevant to the problem's context.	The teacher states/writes numerical answers that are relevant to the problem's context.	The teacher states/writes numerical answers that are relevant to the problem's context.	The teacher states/writes numerical answers that are relevant to the problem's context.
TL2 Precision	The teacher uses discipline-specific words to provide clear definitions.	The teacher uses discipline-specific words to provide clear definitions.	The teacher uses discipline-specific words to provide clear definitions.	The teacher uses discipline-specific words to provide clear definitions.
TL3 Precision	The teacher correctly states/specifies units of measurements (when applicable).	The teacher correctly states/specifies units of measurements (when applicable).	*The teacher states/specifies units of measurements, or puts meaning to the numerical value.*	The teacher states/specifies units of measurements, or puts meaning to the numerical value.
TL4 Explicitness	The teacher correctly uses at least two different terms to describe the same mathematical idea (when applicable).	*The teacher correctly uses at least two different terms to describe the same mathematical concept (when applicable).*	*The teacher uses at least two different terms to describe the same mathematical concept.*	*Item was removed*
TL5 Explicitness	The teacher discusses the meaning of used discipline-specific words.	The teacher discusses the meaning of used discipline-specific words.	The teacher discusses the meaning of used discipline-specific words.	The teacher discusses the meaning of used discipline-specific words.

Table 4 (continued)

Label	Initial Version	Revision 1	Revision 2	Final Version
TL6 Explicitness	The teacher compares and contrasts everyday language with technical language.	*The teacher explains everyday words that represent mathematical concepts.*	The teacher explains everyday words that represent mathematical concepts.	The teacher explains everyday words that represent mathematical concepts.
TL7 Fluency	The teacher provides correct explanations (when applicable).	The teacher provides correct explanations (when applicable).	*The teacher provides correct explanations.*	The teacher provides correct explanations.
TL8 Fluency	The teacher correctly uses terms to describe mathematical ideas.	*The teacher correctly uses mathematical terms to describe concepts.*	*The teacher uses mathematical terms to describe concepts.*	The teacher uses mathematical terms to describe concepts.
TL9 Fluency	The teacher uses mathematical language as a vehicle for conveying content.	The teacher uses mathematical language as a vehicle for conveying content.	The teacher uses mathematical language as a vehicle for conveying content.	The teacher uses mathematical language as a vehicle for conveying content.
SL10 Precision	The teacher uses symbols correctly.	The teacher uses symbols correctly.	*The teacher uses symbols correctly (when applicable).*	The teacher uses symbols correctly (when applicable).
SL11 Precision	The teacher correctly uses the equal sign.	The teacher correctly uses the equal sign.	The teacher correctly uses the equal sign.	*Item was removed*
SL12 Precision	The teacher uses mathematical notation to provide clear definitions.	The teacher uses mathematical notation to provide clear definitions.	The teacher uses mathematical notation to provide clear definitions.	The teacher uses mathematical notation to provide clear definitions.
SL13 Explicitness	The teacher discusses the meaning of the symbols used.	The teacher discusses the meaning of the symbols used.	*The teacher discusses the meaning of mathematical symbols.*	The teacher discusses the meaning of mathematical symbols.

Table 4 (continued)

Label	Initial Version	Revision 1	Revision 2	Final Version
SL14 Explicitness	The teacher makes connections between symbols and mathematical ideas.	*The teacher makes connections between symbols and mathematical concepts.*	The teacher makes connections between symbols and mathematical concepts.	The teacher makes connections between symbols and mathematical concepts.
SL15 Explicitness	The teacher supports the meaning of the equal sign as relational rather than operational.	The teacher supports the meaning of the equal sign as relational rather than operational.	*The teacher supports the meaning of the equal sign as relational rather than operational (when applicable).*	The teacher supports the meaning of the equal sign as relational rather than operational (when applicable).
SL16 Fluency	The teacher uses mathematical symbols when providing explanations.	The teacher uses mathematical symbols when providing explanations.	The teacher uses mathematical symbols when providing explanations.	The teacher uses mathematical symbols when providing explanations.
SL17 Fluency	The teacher correctly uses symbols to describe mathematical ideas.	*The teacher correctly uses symbols to describe mathematical concepts.*	*The teacher uses symbols to describe mathematical concepts.*	*Item was removed*
SL18 Fluency	The teacher uses simple, concise language to connect symbols to their meaning.	The teacher uses simple, concise language to connect symbols to their meaning.	The teacher uses simple, concise language to connect symbols to their meaning.	The teacher uses simple, concise language to connect symbols to their meaning.
VL19 Precision	The teacher selects visual representations that are appropriate for the structure of the problem.	The teacher selects visual representations that are appropriate for the structure of the problem.	The teacher selects visual representations that are appropriate for the structure of the problem.	*Item was removed*
VL20 Precision	The teacher correctly labels elements of a visual representation.	The teacher correctly labels elements of a visual representation.	*The teacher labels elements of a visual representation.*	The teacher labels elements of a visual representation.

Table 4 (continued)

Label	Initial Version	Revision 1	Revision 2	Final Version
VL21 Precision	The teacher correctly converts visually represented information into mathematical notation.	The teacher correctly converts visually represented information into mathematical notation.	*The teacher converts visually represented information into mathematical notation.*	The teacher converts visually represented information into mathematical notation.
VL22 Explicitness	The teacher states what the visuals represent.	The teacher states what the visuals represent.	The teacher states what the visuals represent.	*Item was removed*
VL23 Explicitness	The teacher links visual representations with quantities in the problem.	The teacher links visual representations with quantities in the problem.	The teacher links visual representations with quantities in the problem.	*Item was removed*
VL24 Explicitness	The teacher includes only necessary details in visual representations.	The teacher includes only necessary details in visual representations.	*The teacher includes only necessary information in visual representations.*	*Item was removed*
VL25 Fluency	The teacher correctly uses visual representations to describe mathematical ideas.	*The teacher correctly uses visual representations to describe concepts.*	*The teacher uses visual representations to describe concepts.*	The teacher uses visual representations to describe concepts.
VL26 Fluency	The teacher uses discipline-specific words when describing visual representations.	The teacher uses discipline-specific words when describing visual representations.	The teacher uses discipline-specific words when describing visual representations.	The teacher uses discipline-specific words when describing visual representations.
VL27 Fluency	The teacher uses different visual representations for different types of problems.	The teacher uses different visual representations for different types of problems.	*The teacher uses different visual representations for the same problem.*	The teacher uses different visual representations for the same problem.

Note. Revised items in each iteration of the MDLS are represented in Italic.

3.2 Testing Validity and Reliability of the MDLS

Testing validity involved “accumulating evidence to provide a sound scientific basis for the proposed score interpretations” (AERA, 1999, p. 9). Reliability refers to the consistency found in repeated measurements of the same phenomenon. The more consistent the results from repeated measurements, the higher the reliability (Carmines & Zeller, 1979). I first evaluated content validity of the MDLS (section 3.2.1), defined as “the extent to which the items on a test are fairly representative of the entire domain the test seeks to measure” (Salkind, 2010, p. 239). Next, I assessed inter-rater reliability (section 3.2.3), which is the degree of agreement among raters who independently rate various features (Hallgren, 2012). Then, I evaluated construct validity (section 3.2.4), or the extent to which an instrument measures the domain or construct that it is meant to measure (Cook & Campbell, 1979). Finally, I tested the internal consistency of the final MDLS (section 3.2.5), which reflects the extent to which items within an instrument measure various aspects of the same construct (Revelle, 1979).

3.2.1 Content Validity

After developing the initial version of MDLS, the instrument was sent to two subject matter experts (SMEs) in the field of mathematical literacy for teachers. The first SME (expert 1) received a Doctor of Philosophy in Curriculum Literacies from a public university in Australia. She is now a literacy specialist for elementary schools, with research expertise in the area of disciplinary literacy in the mathematics classroom. The second SME (expert 2) earned a Doctor of Philosophy in Literacy and Language Education from a private university in midwestern United States. He is a Professor of Education in the School of Teaching and Learning at a large public university in southeastern United States. His research focuses on disciplinary literacy in mathematics as well as reading practices of mathematicians. The SMEs were provided the initial version of MDLS and asked to complete the following tasks:

1. Review the categories of elementary school teachers' use of MDL and provide feedback on whether these categories are appropriate in representing MDL.
2. Review the items for each category and determine whether they belong in/contribute to the corresponding categories.
3. Evaluate the wording of each item to check for ambiguity, measurability, and precision.
4. Determine whether there is a sufficient number and range of items to verify internal consistency.

When the SMEs completed a review of the MDLS, I met with each SME to receive feedback on the initial version of the scale. SME 1 suggested making an adjustment to the instructions of the MDLS so that teachers' use of MDL can still be assessed in student-centered classrooms. Therefore, guidelines to "consider these items to be true if the teacher provides students with the opportunity to…." were added to the MDLS instructions. SME 1 also recommended replacing the term *visual representation* with *mathematical model* to better align the MDLS with elementary school materials. However, I rejected this suggestion because of the definition of visual language in the theoretical framework. SME 2 recommended using the term *concept* instead of *idea* and made suggestions for the wording of TL6. Based on SME 2's feedback, the term *idea* in TL4, TL8, SL14, SL17, and VL25 was changed to *concept* and TL6 was reworded for clarity. Table 5 summarizes all changes with direct quotes from SMEs explaining why they were suggested. Once revisions were completed, Revision 1 of MDLS was sent back to the SMEs and received their approval (see Revision 1 in Table 4).

Table 5

Changes Made to Initial MDLS Based on Feedback from SMEs

Label	Before	After	Explanation
	No specific guidelines existed at first	Added to instructions: *Can also consider these items to be true if the teacher provides students with the opportunity to…*	Expert 1 raised the concern about a classroom that follows the workshop model, where the teacher is the facilitator and the students do most of the talking. *"Since the accompanying lesson plans will be paired with videos for the assessment, if MDL is evident in the lesson plan, then it can be checked off on the Likert-like scale. This will also cover classrooms that are not as teacher-centered as others."*
TL4, TL8 SL14, SL17 VL25	Term in items: Idea	Term in items: Concept	Expert 2 suggested using the term *concept* instead of *idea* for these items. *"Correctly making connections between mathematical concepts, or providing students with the opportunity to see the relationships between mathematical concepts, is crucial for elementary school mathematics lessons. For example, when adding, finding the total, or the sum (a total in an addition problem is called a sum), the total can be anything - it is imprecise. In making these items about the mathematical relationship instead, it can help move people from imprecise to more precise language."*
TL6	Term in items: compares and contrasts everyday language with technical language	Term in items: explains everyday words that represent mathematical concepts	Based on Expert 2's suggestion, reworded for clarity.

3.2.2 Data Set

This study used an existing data set. The data set included 247 lesson plans and videos collected as part of the pre-service elementary school special education teachers' regular coursework, accumulated over eight consecutive semesters at a public university in northeastern United States. Each pre-service elementary school teacher was assigned to develop, teach, and videotape one mathematics lesson during the graduate pedagogy course on teaching mathematics and science with technology to students with learning disabilities.

3.2.3 Inter-Rater Reliability

To test inter-rater reliability, two independent raters were selected based on their expertise in mathematical content knowledge. Both raters have a Bachelor of Science degree in Mathematics and Computer Science from a four-year liberal arts college in New York City. Rater 1 is pursuing a Doctor of Philosophy in Mathematics. Rater 2 is a software engineer.

To prepare raters for scoring, I conducted a raters' training meeting on the MDLS. To prepare for this meeting, I created a list of examples and identified six videos and corresponding lesson plans from the data set of 247. The training started with an explanation of the instrument, followed by scoring three sets of videos and corresponding lesson plans together. The raters then scored independently from each other three additional sets of videos and corresponding lesson plans. Independent scoring was followed by a discussion of any disagreements between the scores to ensure the raters' complete understanding of the MDLS.

After the training, each rater independently scored 30 sets of videos and corresponding lesson plans. Pearson's correlation analysis was used to determine the degree of agreement between them. The Pearson *r*-values for Revision 1 of MDLS ranged from .63 to .86. All items with *r*-values below .7 were discussed with the raters during the debriefing that followed, and the wording of these MDLS items was revised based on feedback from the raters. Revision 2 in Table 4 showed the revised items, while Table 6 includes explanations as to why these changes were necessary. In addition, following the raters' recommendation, I changed the rating scale from measuring frequency of occurrences (e.g., never, rarely, often, always) to assessing the quality of MDL use (e.g., not evident, incorrect, somewhat correct, correct).

Table 6

Changes Made to Revision 1 MDLS Based on Analysis of Debriefing

Label	Change Made	Explanation
TL3, TL4, TL7, TL8, SL10, SL15, SL17, VL20, VL21, VL25	Removed the word "correctly" from items, or added/removed "(when applicable)" as needed.	Adjusted due to change in rating scale
TL3	Added "or puts meaning to the numerical value"	For clarity
SL13	Replaced "the symbols used" with "mathematical symbols"	
VL24	Replaced "details in visual representations" with "information in visual representations"	
VL27	Replaced "different types of problems" with "the same problem"	

Revision 2 of MDLS was then independently used by the same two raters to score a new set of 30 video recordings and corresponding lesson plans. Inter-rater reliability was tested again using Pearson correlations on this set of scores. For the scoring with Revision 2 of MDLS, the Pearson *r*-values ranged from .71 to .89, confirming high inter-rater reliability (Asuero et al., 2006). The remaining 181 video recordings and corresponding lesson plans were divided between the raters to complete scoring using Revision 2 of MDLS.

3.2.4 Construct Validity

To assess construct validity, I used 211 scores that were collected using the same version of MDLS (Revision 2) to compare two models. First, I employed a confirmatory factor analysis (CFA) using AMOS 29 to determine the extent to which the data supported the three-factor model hypothesized by the theoretical framework. The second model was developed using a two-step approach. Initially, an exploratory factor analysis (EFA) using SPSS 29 was conducted to explore the underlying structure of the data and identify potential factors. Based on the results of the EFA, I hypothesized the factor structure of the MDLS. This structure was then tested using a CFA in AMOS 29. I compared the two models to select the best one, based on parameters for the goodness-of-fit (Bentler, 1990; Fabrigar, 1999; Kline, 2005).

Evaluating the Theoretical Model via CFA. The requirements of sample sizes for conducting a CFA are inconsistent across studies (MacCallum et al., 1999; Nunally, 1978). I utilized recommendations by Nunally (1978) that a sample size of at least 200 participants, with a ratio of the sample size to the number of variables of at least 10 is needed for conducting a CFA. Based on this recommendation, the number of lessons and videos (211) was sufficient to conduct a CFA to evaluate the structure of the MDLS suggested by the theoretical framework.

The CFA was conducted in SPSS AMOS 29. To create the model, I used three factors representing the three categories of MDL: technical, symbolic, and visual language. Each factor included nine items according to the Final Version of MDLS. The analysis resulted in a significant chi-squared $X^2(321) = 934.9$ ($p < .001$). A significant chi-squared statistic suggests that the model does not fit the data well. However, if all other fit indices are acceptable, it is likely that the result is influenced by the large sample size, as chi-squared tests are sensitive to sample size (Fields, 2018). The comparative fit index (CFI = .791) and the Tucker-Lewis index

(TLI = .772) were both below the acceptable value of .9 (Bentler, 1990). In addition, the standardized root mean square residual (SRMR = .088) and the root mean square error for approximation (RMSEA = .095) were higher than the acceptable value of .08 (Bentler, 1990). To improve the fit of the model, five items (TL2, TL4, SL11, SL17 and VL26) that had very high modification indices were removed. The resulting three-factor model is shown in Figure 3.

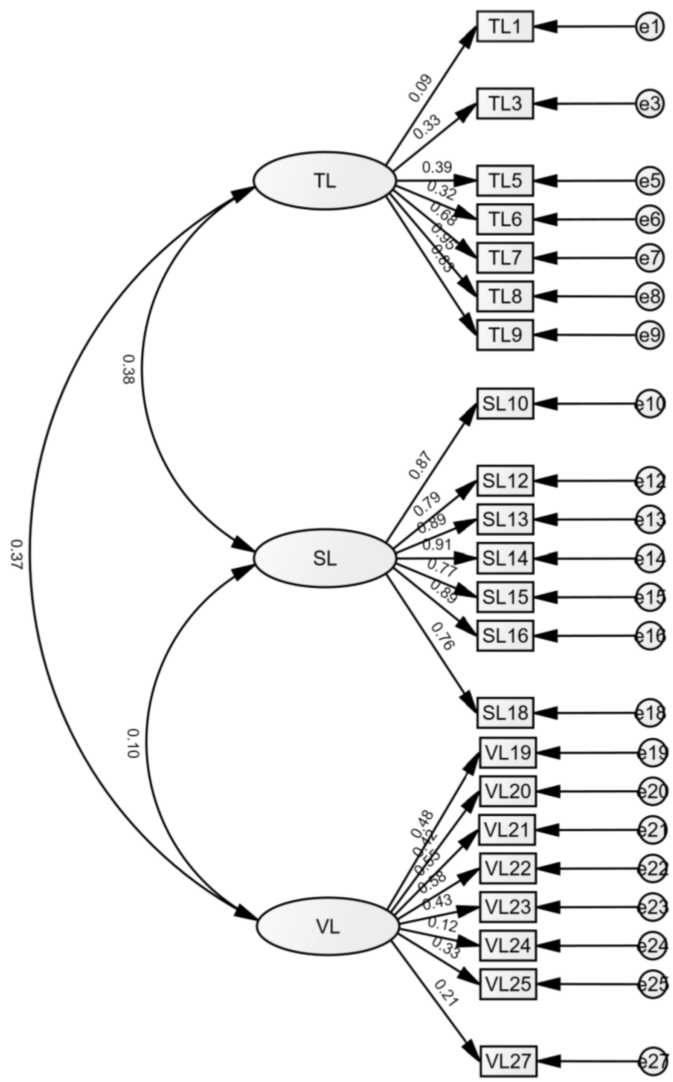


Figure 3. *AMOS Path Diagram for Three-Factor Model of MDLS*

The ratio X^2/df = 2.121 < 3.0 indicated a good fit (Kline, 2005). The comparative fit index (CFI = .886) and the Tucker-Lewis Index (TLI = .872) were both slightly below the accepted value of .9 (Bentler, 1990). Both the standardized root mean square residual (SRMR = .0792 < .08) and the root mean square error for approximation (RMSEA = .073 < .08) were acceptable. However, a root mean square error for approximation value closer to .05 is preferred for a good fit (Fabrigar, 1999). The model resulted in a significant chi-squared $X^2(206) = 437.0$ ($p < .001$). The three-factor model showed a decent fit, but it did not meet all parameters for a reasonable fit.

Data-Driven Model Development with EFA and CFA. To develop a data-driven model of the MDLS, I conducted an EFA on the collected MDLS data using SPSS 29. All items that loaded onto more than one factor were removed (TL4, SL11, VL19, VL22, VL23, and VL24). The resulting model consists of five factors (Table 7). The procedure for factor extraction in SPSS included the following options: principal component analysis as the extraction method, varimax rotation, using a scree plot to determine the number of factors to retain, and a threshold for suppressed values set at .42 for factor loadings to be considered significant.

Table 7

Final Rotated Component Matrix

Item	Component					Assessment Criteria
	1	2	3	4	5	
TL1			.551			Precision
TL2			.743			Precision
TL3			.589			Precision
TL5			.594			Explicitness
TL6			.578			Explicitness
TL7		.794				Fluency
TL8		.870				Fluency

Table 7 (continued)

Item	Component					Assessment Criteria
TL9		.813				Fluency
SL10	.867					Precision
SL12	.775					Precision
SL13	.877					Explicitness
SL14	.899					Explicitness
SL15	.781					Explicitness
SL16	.923					Fluency
SL17	.913					Fluency
SL18	.812					Fluency
VL20					.759	Precision
VL21					.830	Precision
VL25				.688		Fluency
VL26				.656		Fluency
VL27				.718		Fluency

The EFA led to two separate factors for technical language—one that focuses on precision and explicitness (TLPE) and one that focuses on fluency (TLF). Similarly, two separate factors were extracted for visual language, one that focuses on precision (VLP) and one that focuses on fluency (VLF). Symbolic language (SL) remained a single factor. Therefore, the model suggested by the EFA includes three categories of MDL, with two categories each represented by two factors according to different assessment criteria.

To determine whether the five-factor model is a better fit for the data than the three-factor model, I conducted a CFA in SPSS AMOS 29 to evaluate the five-factor model. The analysis resulted in a significant chi-squared $X^2(179) = 438.1$ ($p < .001$). The comparative fit index (CFI

= .895) and the Tucker-Lewis index (TLI = .876) were both slightly below the accepted value of .9 (Bentler, 1990). The standardized root mean square residual (SRMR = .047 < .08) indicated a very good model fit, but the root mean square error for approximation (RMSEA = .083) was slightly higher than the acceptable value of .08 (Bentler, 1990). To improve the model, I removed item SL17 due to its high covariance with SL16 (60.228, while covariances for all other items were < 20.0). Removing items with high covariance can be justified as long as they do not substantially contribute to construct validity or reliability (Kock, 2016; Shi et al., 2019). The final five-factor model is shown in Figure 4.

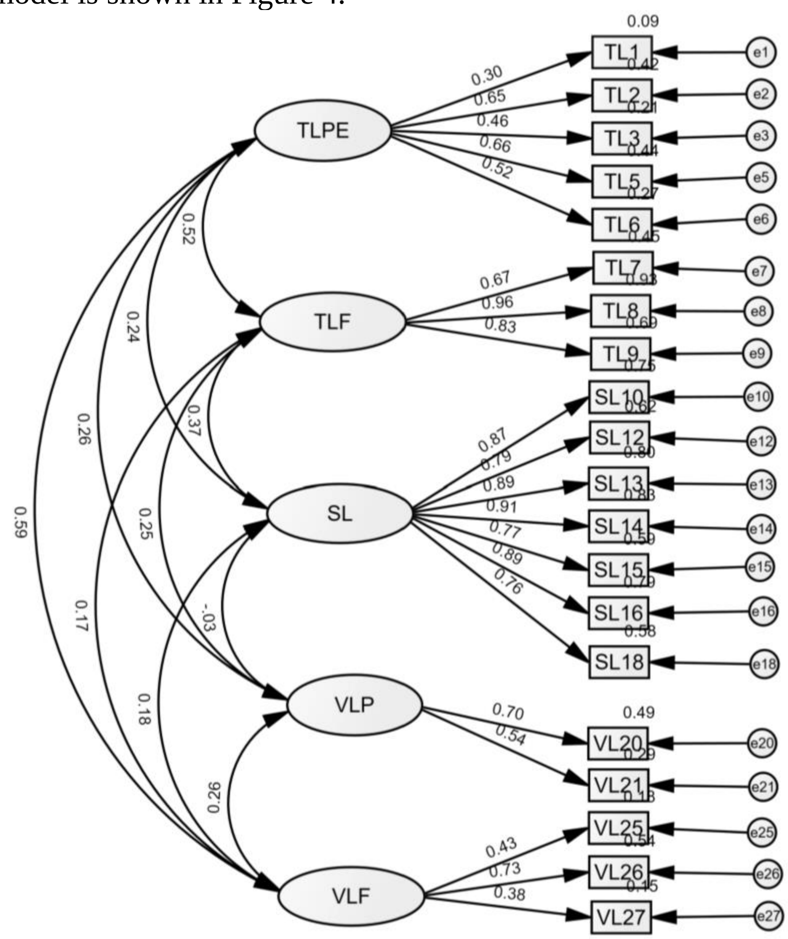


Figure 4. *AMOS Path Diagram for Final Five-Factor Model of MDLS*

The ratio X^2/df = 1.959 < 3.0 improved. The comparative fit index (CFI = .924 > .9) and the Tucker-Lewis Index (TLI = .910 > .9) both indicated a good fit (Bentler, 1990). Similarly, the standardized root mean square residual (SRMR = .0592 < .08) and the root mean square error for approximation (RMSEA = .068 < .08) were smaller and indicated a good model fit. The model resulted in a lower but still a significant chi-squared $X^2(160) = 313.5$ ($p < .001$). However, all other indices indicated a good model fit. Therefore, the five-factor model provided a better fit than the three-factor model and was accepted as the final structure of the MDLS. Thus, the study confirmed the construct validity of MDLS. With 20 items in the final version of MDLS, the factor ratio (lessons to items, in this study) was larger than 10, meeting requirements for factor analysis (Nunally, 1978).

3.2.5 Internal Consistency and Final Version of MDLS

I used Cronbach's alpha method to measure internal consistency of the final MDLS. Cronbach's alpha calculates the average of all possible pairwise correlations between items, providing a generalizable estimate of the internal consistency of the scale (Cronbach, 1951). George and Mallery (2003) provided rules for acceptable values of Cronbach's alpha: > .9 is excellent, > .8 is good, > .7 is acceptable, > .6 is questionable, > .5 is poor, and < .5 is unacceptable. Cronbach (1951) suggested applying the formula separately to each subscale in the assessment instrument. The value of alpha for symbolic language subscale (α = .943) is excellent, for the technical language subscale (α = .778) is good, and just a little below acceptable for visual language subscale (α = .514), with very good overall value of 0.872 indicating acceptable internal consistency of the MDLS. Therefore, the final version of the MDLS consists of 20 items on a 4-point Likert-like scale, including five precision/explicitness items and three fluency items in the technical language category, seven items in the symbolic language category, and two precision items and three fluency items in the visual language category (Table 8).

Table 8

Final Version of MDLS

Label	Assessment Criteria	Item
TL1	Precision	The teacher states/writes numerical answers that are relevant to the problem's context.
TL2	Precision	The teacher uses discipline-specific words to provide clear definitions.
TL3	Precision	The teacher states/specifies units of measurements, or puts meaning to the numerical value.
TL5	Explicitness	The teacher discusses the meaning of used discipline-specific words.
TL6	Explicitness	The teacher explains everyday words that represent mathematical concepts.
TL7	Fluency	The teacher provides correct explanations.
TL8	Fluency	The teacher uses mathematical terms to describe concepts.
TL9	Fluency	The teacher uses mathematical language as a vehicle for conveying content.
SL10	Precision	The teacher uses symbols correctly (when applicable).
SL12	Precision	The teacher uses mathematical notation to provide clear definitions.
SL13	Explicitness	The teacher discusses the meaning of mathematical symbols.
SL14	Explicitness	The teacher makes connections between symbols and mathematical concepts.
SL15	Explicitness	The teacher supports the meaning of the equal sign as relational rather than operational (when applicable).
SL16	Fluency	The teacher uses mathematical symbols when providing explanations.
SL18	Fluency	The teacher uses simple, concise language to connect symbols to their meaning.
VL20	Precision	The teacher labels elements of a visual representation.
VL21	Precision	The teacher converts visually represented information into mathematical notation.
VL25	Fluency	The teacher uses visual representations to describe concepts.
VL26	Fluency	The teacher uses discipline-specific words when describing visual representations.
VL27	Fluency	The teacher uses different visual representations for the same problem.

3.3 Results and Concluding Remarks

The process described in this chapter addressed the first research question, RQ1: What are constructs of elementary school teachers' use of MDL? In the process of MDLS development, I defined MDL for elementary school teachers as three constructs with five distinct factors: technical language precision/explicitness, technical language fluency, symbolic language, visual language precision, and visual language fluency. Validation of the MDLS for this particular population led to modification of the theoretical framework that initially hypothesized three distinct categories to a five-factor model of MDL specifying that technical language and visual language categories are each made up of two distinct factors, while symbolic language remains a single factor (Figure 5).

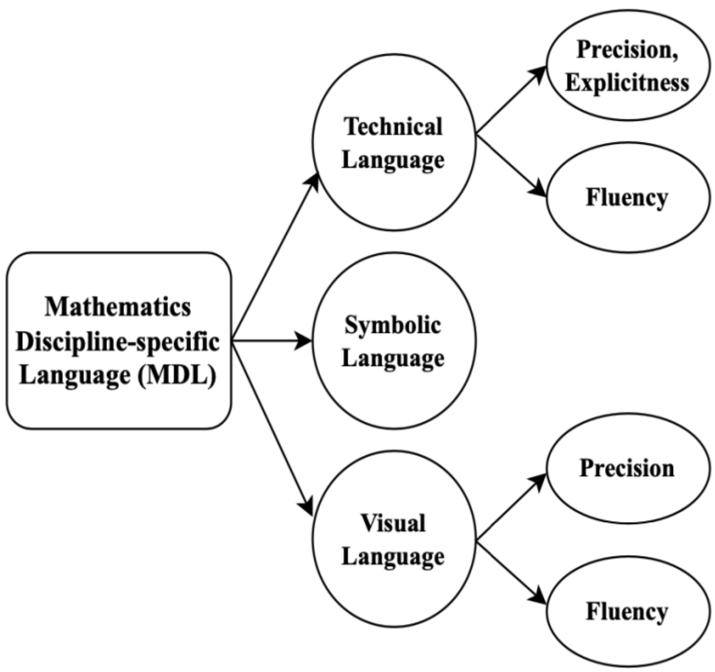


Figure 5. *Modification of My Theoretical Framework*

Therefore, this research enhances the clarity and measurability of elementary school teachers' MDL use. This modified framework not only extended the literature on disciplinary literacy and elementary school teachers' use of MDL, but also resulted in the development of a validated instrument, the MDLS, that can serve as a practical tool for assessing teachers' use of MDL for the purpose of identifying specific areas of strength and targeting areas in need of improvement.

Chapter 4: Exploring Elementary School Teachers' Use of Mathematics' Discipline-specific Language

This chapter describes the research design, participants, and data analysis methods that address RQ2: What are the characteristics of elementary school special education teachers' use of mathematics' discipline-specific language (MDL)?

4.1 Research Design

This part of the study followed a quantitative descriptive design to describe systematically the MDL characteristics of pre-service special education elementary school teachers' use of MDL based on the scores of video recordings and corresponding lesson plans of mathematics lessons taught by them. The quantitative analysis was followed by a qualitative case study to illustrate examples of MDL use at different levels using the Mathematics' Discipline-specific Language Scale (MDLS).

4.2 Participants and Case Selection

The data used in the study were collected from the participants, referred to as pre-service teachers because they were enrolled in an initial certification program for special education. However, all of them had completed initial certification in another area prior to enrollment in a graduate program: 43% of participants already held a license in general childhood education, 15% in early childhood, 3% in adolescence English, 1% in adolescence Mathematics, and 38% in adolescence Social Studies. Although only 1% had a background in mathematics, all participants enrolled in mathematics courses required for initial certification and for their graduate program. The majority of participants (67%) were between the ages of 23 and 26 years old, 7% between the ages of 18 and 22, 15% between the ages of 27 and 32, and 11% were 33 years of age or older. Males comprised 5% of the sample and females comprised 95%.

4.3 Data Analysis Methods

First, I used descriptive statistics in SPSS 29 to report the means, standard deviations (SD), and ranges of each factor of MDL. Next, a one-way ANOVA with post-hoc analysis was used to determine whether there was a statistically significant difference between the means of the factors of MDL: technical language precision/explicitness (TLPE), technical language fluency (TLF), symbolic language (SL), visual language precision (VLP), and visual language fluency (VLF). Then, Pearson r correlation analysis was used to determine whether there were any relationships between the factors.

A content analysis was then used to provide examples of the application of the MDLS at various levels. To find the MDL score of a lesson, I calculated the average of the scores from the 20 MDLS items. To determine the level of MDL use, these scores were rounded to the nearest integer. For the purpose of the analysis, I used disproportionate stratified sampling. Stratified sampling involved dividing the MDLS scores into three subgroups, then randomly selecting a lesson and corresponding video among each of the three subgroups to form the final sample. The sampling was disproportionate to ensure different levels of MDL use were represented.

The analysis of the three lessons and corresponding videos represented each of the three cases of MDL use at distinct levels. The video recordings for each case were uploaded to a transcript generator. Once transcripts of the closed captions of each video recording were generated, the transcripts were read while the video recordings were simultaneously watched to correct any errors. Per suggestions from Hayes et al. (2015), I found evidence (e.g., descriptions, quotes, key points, etc.) from each case's video transcript and corresponding lesson plan to explain the MDLS score. For each lesson, I used the collected evidence to make observations and to describe features of the lesson related to MDLS items.

4.3.1 Quantitative Analysis of Teachers' Use of MDL

In this section, I present results of the analysis completed to address RQ2: What are the characteristics of elementary school special education teachers' use of MDL? First, I discuss the quantitative analysis. Next, I provide an illustration of three levels of MDL use demonstrated by these teachers while teaching mathematics in elementary classrooms. I further discuss these results connecting them to the existing literature.

4.4 Results

The descriptive statistics of each factor of MDL and the percentages of lessons at a particular MDL level are presented in Table 9.

Table 9

Descriptive Statistics of Elementary School Teachers' Use of MDL (N = 211)

MDL	Mean	SD	Min	Max	Level 0	Level 1	Level 2	Level 3
Technical Language (TL)	2.37	.741	1.0	3.0	0.0%	4.7%	44.5%	50.7%
TL Precision/Explicitness	2.42	.586	1.4	3.0	0.0%	1.4%	56.8%	41.7%
TL Fluency	2.28	.939	0.0	3.0	6.2%	9.0%	35.5%	49.3%
Symbolic Language (SL)	1.53	1.196	0.0	3.0	26.5%	17.5%	33.6%	22.3%
Visual Language (VL)	2.39	.608	1.4	3.0	0.0%	1.0%	61.1%	37.9%
VL Precision	2.35	.636	0.0	3.0	0.5%	1.4%	43.1%	55.0%
VL Fluency	2.42	.587	1.0	3.0	0.0%	1.4%	53.6%	45.0%
Overall	2.08	.988	1.0	2.95	0.0%	14.2%	63.0%	22.7%

A one-way ANOVA was performed to compare the MDLS scores for the factors of MDL: technical language precision/explicitness, technical language fluency, symbolic language, visual language precision, and visual language fluency. The one-way ANOVA indicated that there was a statistically significant difference in mean MDLS scores between the five factors ($F(4, 4215) = 214.551$, $p < .001$). Therefore, Tukey HSD (Honestly Significant Difference) post-hoc tests were conducted to examine pairwise differences (Table 10).

Table 10

Results of Tukey HSD Post-hoc Tests

Factor of MDL		Mean Difference	Standard Error	Significance P-values	95% Confidence Interval	
					Lower Bound	Upper Bound
TLPE	TLF	.131*	.045	.032	.01	.25
	SL	.886*	.036	<.001	.79	.98
	VLP	.064	.052	.726	-.08	.21
	VLF	-.003	.045	1.000	-.13	.12
TLF	TLPE	-.131*	.045	.032	-.25	-.01
	SL	.755*	.043	<.001	.64	.87
	VLP	-0.66	.057	.767	-.22	.09
	VLF	-.134	.051	.061	-.27	.00
SL	TLPE	-.886*	.036	<.001	-.98	-.79
	TLF	-.755*	.043	<.001	-.87	-.64
	VLP	-.821*	.050	<.001	-.96	-.69
	VLF	-.889*	.043	<.001	-1.01	-.77
VLP	TLPE	-.064	.052	.726	-.21	.08
	TLF	.066	.057	.767	-.09	.22
	SL	.821*	.050	<.001	.69	.96
	VLF	-.068	.057	.751	-.22	.09

Table 10 (continued)

Factor of MDL		Mean Difference	Standard Error	Significance P-values	Confidence Interval	
					Lower Bound	Upper Bound
VLF	TLPE	.003	.045	1.000	-.12	.13
	TLF	.134	.051	.061	.00	.27
	SL	.889*	.043	<.001	.77	1.01
	VLP	.068	.057	.751	-.09	.22

Note. *The mean difference is significant at the 0.05 level.

Based on the post-hoc analysis, symbolic language scores were statistically significantly lower than both technical language and visual language scores, and technical language fluency scores were statistically significantly lower than technical language precision/explicitness scores. All other pairwise comparisons were not statistically significant. Further, Pearson r correlation analysis was performed on TL, SL, and VL (Table 11).

Table 11

Correlations between TL, SL, and VL

	TLPE	TLF	SL	VLP	VLF
TLPE	1				
TLF	.240**	1			
SL	.086**	.082*	1		
VLP	.007	.093	.366**	1	
VLF	.233**	.079*	.158**	.064	1

Note. **Correlation is significant at the 0.01 level (2-tailed).

*Correlation is significant at the 0.05 level (2-tailed).

Pearson r analysis indicated no correlations between VLP and both TL factors. According to Evans (1996), correlations < 0.20 are interpreted as very weak, 0.20-0.39 as weak, 0.40-0.59 as moderate, 0.60-0.79 as strong, and > 0.80 as very strong. Statistically significant weak positive correlations (Evans, 1996) were found between SL and VLP ($r = .366$), TLPE and TLF ($r = .240$), and TLPE and VLF ($r = .233$). All other correlations, while statistically significant, were very weak ($r \leq .158$). Weak and very weak correlations suggest relative independence of these factors.

4.4.1 Analysis of MDL Cases in Elementary Classrooms

Analysis of elementary school teachers' use of MDL indicated that there were no lessons at Level 0 (no evidence of MDL). Therefore, this section presents the analysis of only three cases of MDL use at distinct levels: Level 1 (use of incorrect MDL), Level 2 (use of somewhat correct MDL), and Level 3 (use of correct MDL).

1. Case of Teacher A: Evidence of MDL Use at Level 1

The Case of Teacher A was developed using two data sources: the lesson plan and the video recording of the lesson that was taught. According to the lesson plan, Teacher A taught in a third grade integrated co-teaching (ICT) classroom. An ICT classroom contains both students with and without disabilities and is co-taught by a general education teacher and a special education teacher. 10 students were pulled from the larger class to participate in this lesson, which lasted for 55 minutes. The topic of the lesson was The Order of Operations. The learning objectives stated in the lesson plan were:

1. *Students will be able to use order of operations to solve an expression.*
2. *Students will have a better understanding of order of operations.*

Teacher A planned to address the following two Common Core State Standards for Mathematics (CCSSM):

> 3.OA.B.5: *Apply properties of operations as strategies to multiply and divide.*
>
> 3.OA.C.7: *Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.*

The lesson plan detailed the beginning of Teacher A's lesson, which was not caught on video. Teacher A planned to start the lesson with a warm-up problem, circulating the classroom and choosing two students' answers to share out. After that, Teacher A planned to write the two answers on the interactive whiteboard, then ask students to discuss what they noticed and what they wondered with their partners. Teacher A intended on using this discussion to transition to the mini-lesson on how to use the order of operations to evaluate an expression. She planned to provide each student with a handout about the order of operations (shown in Figure 6). Teacher A planned to use the PEMDAS handout to model solving three problems in front of the class.

Anchor Chart

P	Parenthesis
E	Exponents
MD	Multiplication & Division
AS	Addition & Subtraction

Purple Elephants May Destroy A School!

Figure 6. *PEMDAS Handout from Teacher A's Lesson*

Next, Teacher A planned for the students to work alone to solve practice problems. When students were working independently, Teacher A planned to circulate the room and assess students when they were ready to transition from individual work to group work. After all students finished this practice, her plan was to place all students into homogeneous groups to create equations for other students to solve.

Since students would be working on their own to complete the remaining tasks in the lesson, Teacher A planned to give each student their own laptop. Next, Teacher A planned to instruct students on how to join an online lesson she developed using Nearpod interactive multimedia platform (https://nearpod.com/). Teacher A wrote that she would remind students of PEMDAS, then have them play a game where they would match the numerical expressions to the correct answers. As students worked, Teacher A planned to circulate the room and address their questions. The rest of the lesson plan was implemented as planned; therefore, I only describe what I observed in the video recording. When most students were done, Teacher A informed the class that they would conclude the lesson with a five-question quiz. The video ended with Teacher A telling students that they would review the matching game and quiz questions as a class, but this was not included in the lesson plan or the video. The next section provides detailed analysis of Teacher A's use of technical language (TL), symbolic language (SL), and visual language (VL) as demonstrated in the lesson plan and video of teaching.

Analysis of Technical Language. In this lesson, Teacher A demonstrated inconsistent use of technical language (M = 1.65, SD = 0.92), specifically for precision/explicitness, M_{pe} = 2.2, SD = 0.44 and for fluency, M_f = 0.67, SD = 0.58. For most parts of the lesson, Teacher A used incorrect technical language (TL), while in some cases she did not use TL at all, and in other cases, she used somewhat correct TL. This is shown by the analysis of both sources.

In the following example, a discipline-specific word, *term*, could have been used but was not (item TL8): "*Which one's* [term is] *first? You multiply first so…"* (Teacher A, video transcript). There were other instances throughout the lesson when Teacher A used discipline-specific words, but did not discuss their meaning with the students (item TL5). For example:

> *So, guys, order of operations, what we just learned, right? Yes, so I wanted to remind you guys of your PEMDAS. Right? You have it in front of you so you do not have to worry, but just in case remember whenever we do any of these problems "purple elephants may destroy a school"* [a mnemonic that Teacher A's class used to refer to PEMDAS or the order of operations], *right? Our PEMDAS. And remember to work from left to right.* (Teacher A, video transcript).

Here (as well as throughout the lesson plan and the video recording), Teacher A used the acronym PEMDAS to represent order of operations instead of using appropriate mathematical language. PEMDAS is not a mathematics term; it is an acronym that is offered to students to memorize a sequence of operations which could lead to misconceptions. Learning the order of operations involves understanding how to evaluate mathematical expressions using properties of operations rather than thoughtless imitation (Ameis, 1992). Methods such as developing the 'hierarchy of operators' triangle (Ameis, 2011) and identifying the structure of the terms and factors in an expression first (Taff, 2017) while using precise mathematical language would be more appropriate for students to develop their conceptual thinking and understanding.

In addition, Teacher A introduced the topic of this lesson incorrectly and inappropriately for this grade level in her lesson plan. Based on curriculum standards, third-grade students have not yet learned about exponents. The CCSSM for third grade only includes order of operations with four basic operations: addition, subtraction, multiplication, and division. Teacher A also selected two CCSSM that were not aligned to the topic of her lesson: 3.OA.B.5 and 3.OA.C.7:

> 3.OA.B.5: *Apply properties of operations as strategies to multiply and divide.*
>
> 3.OA.C.7: *Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations.* (Teacher A, lesson plan).

“Properties of operations” in the above standards is not referring to the order of operations. “Properties of operations” is referring to the four number properties: commutative property, associative property, distributive property and identity property.

In other instances of using mathematical language in this lesson, Teacher A did not provide correct explanations (item TL7). To illustrate, Teacher A wrote the following in her lesson plan:

> *Just like in today’s warm up, we sometimes have to solve expressions that have more than one operation. We may even come across expressions that have three or more operations! In order to solve these expressions, we need to know the correct order to solve these expressions in. To help us, we are going to learn an acronym that will direct us in which one to solve first.*

In this example, mathematical language is evident in Teacher A saying, “solve expressions that have more than one operation,” as well as using terms such as *expression* and *operations*. However, usage of the verb “solve” is incorrect because expressions do not have a solution to find; they can only be evaluated by substituting values and simplifying. Teacher A used “solve” instead of “evaluate” throughout the entire lesson, including in the lesson’s learning objectives.

Analysis of Symbolic Language. Teacher A did not use explicitly symbolic language (SL) for most of this lesson (M = 0.29, SD = 0.49). When she did use SL, she used incorrect SL. This is shown by the analysis of both sources.

Teacher A’s lesson plan stated: “*The example 4 + 5 x 3 will be written on the Smart Board for them to solve in their notebooks.”* Throughout the entire lesson, however, Teacher A did not use simple, concise language to explain the symbols used and what they mean in relation to the order of operations (items SL13, SL18). For instance, Teacher A introduced the NearPod activity by saying, *“Here’s what we’re gonna do, guys. When you guys are ready, there is a matching game on your screen. Take your time. Match the expression to the answer”* (Teacher

A, video transcript). Symbols and their meanings were not explicitly explained, and students went right into solving problems. One of the students did not know how to begin. In Teacher A's response, symbolic language was evident in her use of *times* and *plus*:

Teacher A: *What do you do first? Multiplication or addition?*

Student: *Multiplication?*

Teacher A: *Yeah, so what's ten times one? Yep.*

Student: *Plus three.*

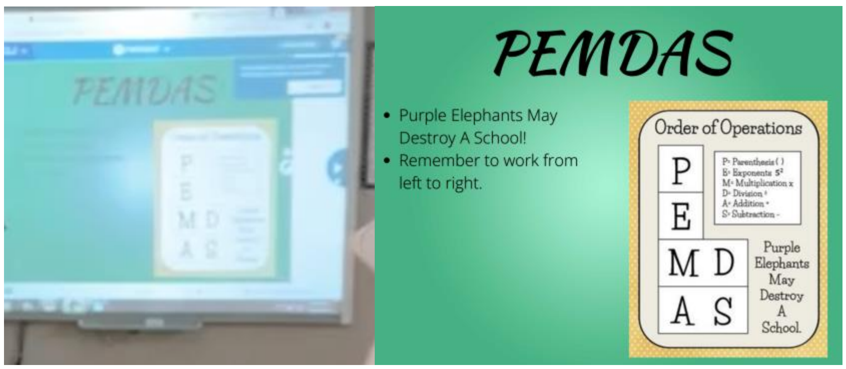
Teacher A: *There you go now.* (Teacher A, video transcript).

Yet, based on the video, there were many other symbolic terms affiliated with the order of operations that Teacher A did not mention to her class (e.g., divided by, parentheses, etc.).

Furthermore, Teacher A did not make any explicit connections between symbols of the four operations and names of these operations (item SL14). The topic of order of operations discusses properties and hierarchical relationships between mathematics operations, which are represented by symbols—for example, +, -, x, /, as well as ( ). If Teacher A connected the symbols of operations with the names of operations, and parentheses with the concept of grouping, that would be an appropriate use of SL.

Lastly, Teacher A did not use mathematical notation to provide clear definitions (item SL12). For instance, the board with Nearpod Slide 1 displayed PEMDAS with it defined as "Purple Elephants May Destroy A School" to help students remember the acronym (Figure 7). Symbols were included on Nearpod Slide 1, but Teacher A did not reference them or use symbolic language to explain the order of operations to the class. At that time, Teacher A said *"PEMDAS, it's in front of us, it's in our brains, it's glued there, we're good?"* without further explanation (Teacher A, video transcript). When PEMDAS was elaborated on throughout the lesson, it was referred to as "Purple Elephants May Destroy A School." The following statement from Teacher A's lesson plan showed that "Purple Elephants May Destroy A School" was used intentionally as an explanation:

> *The paper monitor (student) will hand everyone their anchor sheet with the acronym PEMDAS on it. This will be taped into their math notebooks. The teacher will explain that PEMDAS stands for Purple Elephants May Destroy A School.*



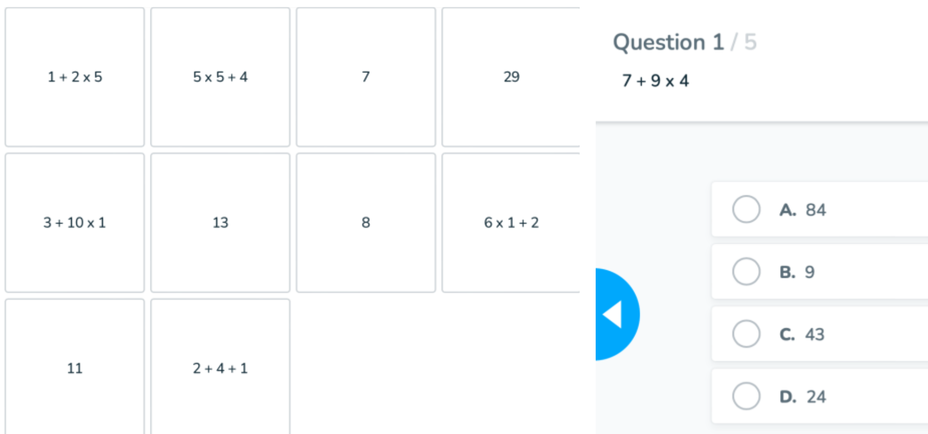
(a) (b)

Figure 7. *Images from Video Recording (a) and Nearpod Slide 1 (b)*

Analysis of Visual Language. Similar to her use of TL, Teacher A demonstrated inconsistent use of visual language (VL) in this lesson, with instances of both not evident VL and somewhat correct VL (M = 1.4, SD = 0.55). Specifically, for precision, $M_p = 2.0$, SD = 0.0 and for fluency, $M_f = 1.0$, SD = 0.0. This is shown by the analysis of both sources.

Teacher A provided students with the PEMDAS handout (Figure 6) to refer to when working on problems. The PEMDAS handout included a table, which is a visual representation as defined by the theoretical framework. The table stated what the acronym PEMDAS represented (parenthesis, exponents, multiplication, division, addition, and subtraction). However, in the lesson plan and video recording Teacher A stated that the acronym PEMDAS represented *"purple elephants may destroy a school"* (a mnemonic that her class used to remember PEMDAS) instead of using discipline-specific words to describe the order of operations (item VL26).

There were instances in the lesson plan and video where visual representations were not used to describe how to solve problems using the order of operations (item VL25). As seen in Figure 8, for example, students were given expressions only in symbolic form.



(a) (b)

Figure 8. *Nearpod Slide 2 (a) and Example from Five-Question Quiz (b)*

Note. The instructions for the matching game in Nearpod stated, *"Match the problem with its answer."*

In conclusion, Teacher A used inconsistent technical language and visual language, while her use of symbolic language was not evident or incorrect in all instances, contributing to the low level of MDL use (M = 1.0, SD = 0.91), as illustrated in this case.

2. Case of Teacher B: Evidence of MDL Use at Level 2

The Case of Teacher B was developed using two data sources: the lesson plan (including a handout packet that was provided to students) and the video recording of the lesson that was taught. According to the lesson plan, Teacher B taught in a second-grade general education classroom. There were 23 students in the class and the lesson lasted for 90 minutes. The topic of the lesson was Skip Counting by Fives to solve multiplication word problems. The learning objective stated in the lesson plan was:

> *Students will solve multiplication word problems and equations by grouping objects into fives and skip counting by fives, write an equation, and explain their strategy.*

Teacher B planned to address the following two CCSSM:

> 2.OA.C.3: *Determine whether a group of objects (up to twenty) has an odd or even number of members, e.g., by pairing objects or counting them by twos; write an equation to express an even number as a sum of two equal addends.*

> 2.OA.C.4: *Use addition to find the total number of objects arranged in rectangular arrays with up to five rows and up to five columns; write an equation to express the total as a sum of equal addends.*

The video recording of the lesson taught by Teacher B included the whole lesson; therefore, the lesson description is based mostly on the video. The description also includes any discrepancies between the planned and enacted lessons. According to the video, the lesson started with a group discussion on the rug in front of the board between Teacher B and the students. Teacher B began by asking students about the types of multiplication word problems they have worked on and what different methods they have learned to solve them. This led to Teacher B briefly reviewing skip counting by twos.

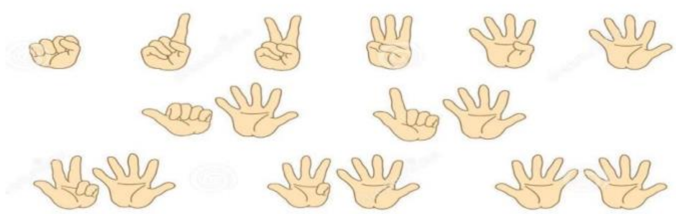
Teacher B continued the discussion by explaining to the class how to skip count by fives to solve a multiplication word problem involving the five times table. She had the students use their fingers to help them learn how to skip count by fives. Together, they worked on solving a word problem on the board. Teacher B called a student to the board to complete the blanks in the multiplication sentence of the problem. She concluded this portion of the lesson by providing directions for the stations activity that came next. The students moved from the rug to their desks that were previously set up for stations and arranged for working in groups of four (with one group of three). Learning materials were already at every group, including handout packets with directions and corresponding worksheets for each station. The Math Station Rules were written in the students' handout packet as follows:

1. *You have eight minutes at each station.*
2. *Follow the directions on your worksheet and complete it with your partner.*
3. *When the timer goes off, you and your group move to the next station.*
4. *When you finish Station 6, come to Station 1 at the Smart Board.*
5. *Have fun!*

At this point, only one station labeled Station 1, where students were using Smart Board, was recorded on video. The game on the Smart Board had students match a multiplication sentence to the correct answer, match a multiplication sentence to the correct number of grouped objects (2 x 5 would be matched to two groups of five displayed objects), and answer related questions. Based on the lesson plan, the following activities were happening at other stations:

- Station 2: Students used dotted grid graph paper to solve two multiplication word problems and then wrote an explanation of how they found their answers.
- Station 3: Students used mental math to solve eight multiplication problems, including filling in missing numbers in a sequence, completing multiplication sentences, and writing a multiplication sentence for a provided number of grouped objects (five groups of five = ? x ?).
- Station 4: Students completed multiplication sentences by rolling a die, multiplying the number it landed on by five, recording the answer, and repeating until time was called.
- Station 5: Students matched multiplication sentences to the correct number of grouped objects using flash cards.

- Station 6: Students labeled the hands image (Figure 9a) after skip counting by fives on each finger: one finger up is five, four fingers up is 20, and so on. Students at this station went on to skip count by fives using a hundreds chart (10×10 square containing the numbers 1-100; Figure 9b).



(a)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

(b)

Figure 9. *Images from Station 6 Handout—Hands Image (a) and Hundreds Chart (b)*

Based on the lesson plan and video, students had 8 minutes to complete the tasks at each station. The groups of students rotated from station to station until they completed all six.

The video showed the lesson ending with students returning to the rug for a brief discussion about the different ways they solved multiplication word problems throughout the lesson. Teacher B placed emphasis on the skip counting by five method and asked students to use their fingers to do so one last time to conclude the lesson. Both the lesson plan and video of teaching were used in the analysis of Teacher B’s use of technical, symbolic, and visual language.

Analysis of Technical Language. Teacher B mostly used somewhat correct technical language (TL) in this lesson ($M = 2.0$, $SD = 0.53$). Specifically, for precision/explicitness, $M_{pe} = 2.2$, $SD = 0.44$, and for fluency, $M_f = 1.67$, $SD = 0.58$, as shown by the analysis of both sources.

Although Teacher B used some mathematical terms in the lesson plan and video such as *count, equal,* and *multiplication* to describe counting strategies such as skip counting by twos and fives, she left out terms such as *addend, even number, equation,* and *odd number* (item TL8). Teacher B used language and contexts familiar to students to introduce the concept of skip counting. To help students fully understand TL, however, Teacher B could have linked familiar language and contexts directly to TL in her explanations (Maton, 2013). Teacher B's explanations were inconsistent throughout the lesson. While there were instances of correct explanations, there were also instances when Teacher B did not provide explanations, as shown in the example below (item TL7):

> Teacher B: *So what did you and your partner discuss? I want you to say what your partner said. What did your partner say?*
>
> Student 5: *My partner said it was easier to count by fives.*
>
> Teacher B: *Excellent, did anybody explain why it's easier to count by fives?*
>
> Student 1: *I know it's easier to count by fives, it's because when you have a two it's two, four, six…*
>
> Teacher B: *Okay, so you think it's easier to count by fives, so it saves us time. Excellent.* (Teacher B, video transcript)

Teacher B did not elaborate on the meaning of skip counting as repeated addition and she did not explain why skip counting saves time, e.g., counts numbers in groups, multiplication strategy.

There were instances when Teacher B put meaning to numerical values in this lesson (item TL3). When reading aloud the problem below (Figure 10), Teacher B added context to clarify the problem for students, *"...Each bunch has five flowers. How many flowers does she have?"* (Teacher B, video transcript). Students counted the total number of flowers and included *flowers* in their solution sentence. Teacher B also had students count on their fingers throughout this example and the rest of the lesson, which put meaning to numerical values to some extent.

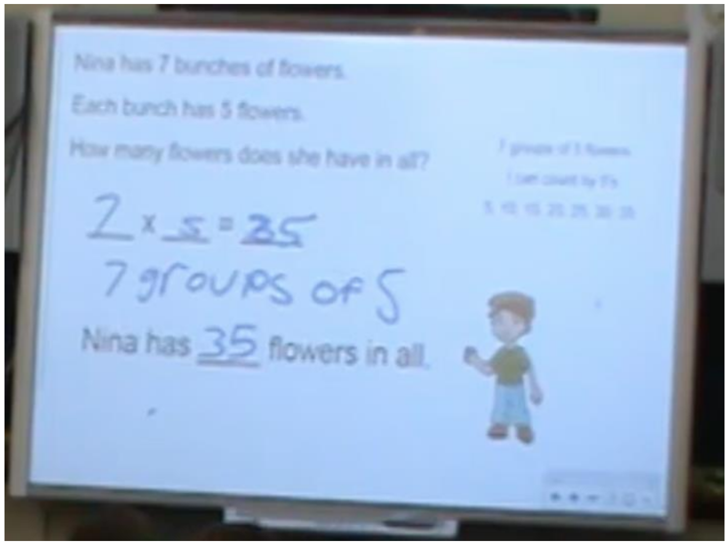


Figure 10. *Image from Video Recording—Flowers Problem*

Analysis of Symbolic Language. Teacher B used mostly somewhat correct symbolic language (SL) and sometimes correct SL in this lesson (M = 2.29, SD = 0.49), as shown by the analysis of both sources.

According to both the lesson plan and video, Teacher B used the multiplication sign and the equal sign correctly (items SL10, SL16). Figure 11 is one example of this.

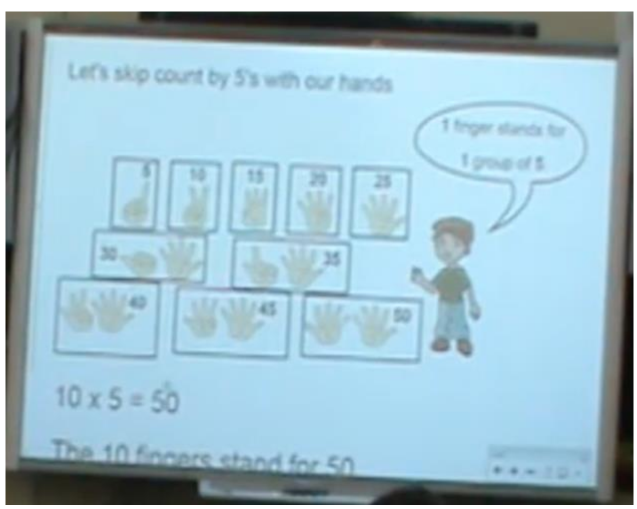
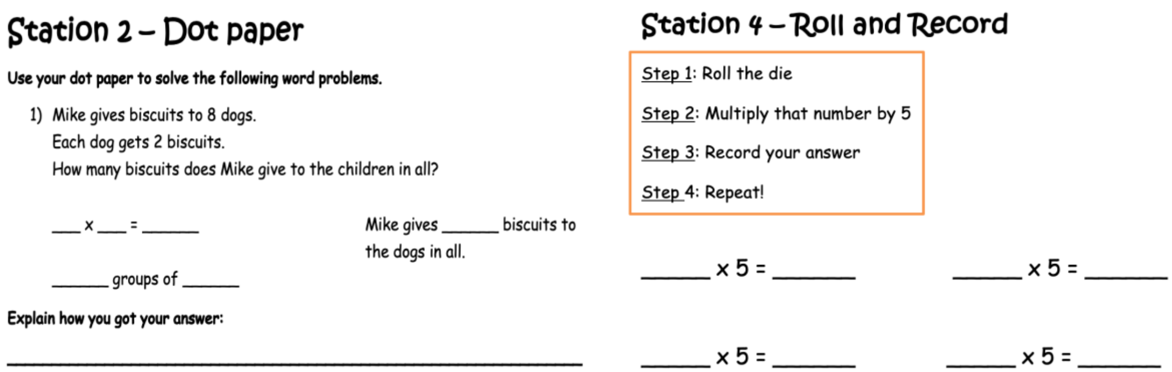


Figure 11. *Image from Video Recording—Skip Counting by Fives*

Teacher B incorporated the meaning of mathematical symbols in many instances, such as having students complete number sentences (Figure 12a) and write their own equations (Figure 12b).


Station 2 – Dot paper

Use your dot paper to solve the following word problems.

1) Mike gives biscuits to 8 dogs.
Each dog gets 2 biscuits.
How many biscuits does Mike give to the children in all?

___ x ___ = ______

Mike gives ______ biscuits to the dogs in all.

______ groups of ______

Explain how you got your answer:

____

Station 4 – Roll and Record

Step 1: Roll the die

Step 2: Multiply that number by 5

Step 3: Record your answer

Step 4: Repeat!

______ x 5 = _______ ______ x 5 = _______

______ x 5 = _______ ______ x 5 = _______

(a) (b)

Figure 12. *Images from Station 2 Handout (a) and Station 4 Handout (b)*

However, when Teacher B explained problems to students in some instances, she did not explicitly discuss the meaning of mathematical symbols (item SL13). For example, the multiplication symbol was repeatedly used in Station 4. The steps (Figure 12b) and Teacher B's instructions in the video both included the term *multiply*:

> *Station 4, Roll and Record, so you're rolling the die you're multiplying whatever number you roll by five, so if I roll a three, I multiply it by five. Then you record your answer… Then you repeat, you keep going.* (Teacher B, video transcript)

Yet, the term *multiply* was not defined in the lesson plan or in the video recording. According to the CCSSM, this is the first lesson for gaining foundations for multiplication. The lesson's learning objective states, *"Students will solve multiplication word problems and equations by grouping objects into fives and skip counting by fives, write an equation, and explain their strategy"* (Teacher B, lesson plan). Introducing definitions upfront provides students with a solid foundation for learning a new concept and solving relevant problems (Edwards & Ward, 2008).

Based on the video recording, Teacher B made connections between symbols and mathematical concepts (item SL14). Teacher B used the terms *equation*, *groups of*, and *multiplication sentence* when referring to SL, but did not emphasize that skip counting using fingers was only a strategy used to solve multiplication problems. Teacher B did not make clear distinctions between skip counting (e.g., 5, 10, 15, 20…) and multiplication (e.g., 5 x 2 = 10) during the lesson.

Analysis of Visual Language. Teacher B used somewhat correct visual language (VL) throughout the lesson (M = 2.0, SD = 0.0). Specifically, for precision, M_p = 2.0, SD = 0.0, and for fluency, M_f = 2.0, SD = 0.0, as shown by the analysis of both sources.

According to both the lesson plan and the video, Teacher B used different visual representations for the same problem in some instances (item VL27). Visuals in this case consisted of the hands image (Figure 9a, Figure 11), the hundreds chart (Figure 9b), and the dice. However, Teacher B did not use different visual representations for a multiplication problem involving using equal groups to multiply (Figure 13).

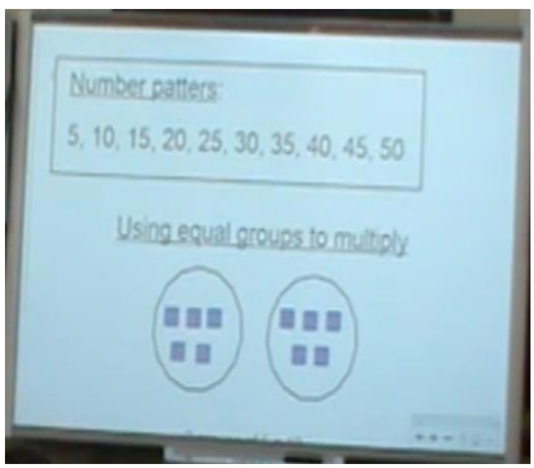


Figure 13. *Image from Video Recording—Using Equal Groups to Multiply*

Teacher B used visual representations to review skip counting by fives during the stations activity (item VL25). When Teacher B planned to model the following problem and two others at the board, however, she did not use visual representation to describe how to solve: "*Megan* *buys 2 books for $4 each. She pays with a 10-dollar bill. How much money does she have left?"* (Teacher B, lesson plan).

In conclusion, the average score of Teacher B's use of MDL was 2.1 with a standard deviation of 0.45. As illustrated in this case, Teacher B's use of symbolic language was slightly more correct than her use of technical language and visual language which were both somewhat correct throughout her lesson.

3. Case of Teacher C: Evidence of MDL Use at Level 3

The Case of Teacher C was developed using two data sources: the lesson plan and the video recording of the lesson that was taught. According to the lesson plan, Teacher C taught in a second grade ICT classroom. Five students were pulled from the larger class to participate in this lesson, which lasted for 50 minutes. The topic of the lesson was Place Value. The learning objectives stated in the lesson plan were:

1. *Using "Expanded Form-Number to 1,000" interactive online game, students will write three-digit numbers in expanded form by answering seven out of the ten questions correctly.*

2. *Using "Expanded Form-Number to 1,000," students will read three-digit numbers in expanded form by orally answering two out of three questions presented by the teacher during independent work.*

Teacher C planned to address the following CCSSM:

> CCSS 2.NBT.3: *Read and write numbers to 1000 using base-ten numerals, number names, and expanded form*.

The lesson description was based on the video of the taught lesson, which matched the lesson plan. There were no discrepancies between the planned and enacted lessons. Based on the video recording, Teacher C began the lesson by playing a video on the Smart Board titled, "Place Value Math Song: Ones, Tens, Hundreds," which reviewed the place value of each digit in a three-digit number using base-ten blocks. Teacher C then displayed 125 in standard form and asked the students to demonstrate different ways to represent the three-digit number on dry erase boards. As students were working, Teacher C asked guiding questions to help them recall their prior knowledge of place value.

Next, Teacher C introduced “expanded form” using a definition, an example (100 + 20 + 5), and a slinky to show students the concept of expanding. Teacher C then explained how to create a place value chart and base-ten blocks using the Smart Board’s virtual tools. They converted three numbers from standard form to expanded form together; each student had the chance to use the Smart Board’s virtual tools.

In the activity to follow, students practiced reading and writing three-digit numbers in expanded form by playing an interactive online game titled, “Expanded Form-Number to 1,000.” The students worked independently on iPads during this activity. While the students played the game, they answered inquiry-based questions on a corresponding handout. Once this activity was finished, Teacher C led a whole-class discussion by projecting the game on the Smart Board and having each student come up to share an answer. Teacher C also had students share aloud their responses to the questions on the corresponding handout. Teacher C concluded the lesson by asking students to define expanded form of a three-digit number and to expand the number 103.

The video recording of the lesson taught by Teacher C includes the whole lesson; therefore, analysis of Teacher C’s use of technical, symbolic, and visual language was based on the video.

Analysis of Technical Language. Teacher C mostly used correct technical language (TL), with very few instances of somewhat correct TL, in this lesson ($M = 2.88$, $SD = 0.35$). Specifically, for precision/explicitness, $M_{pe} = 2.8$, $SD = 0.45$ and for fluency, $M_f = 3.0$, $SD = 0.0$. This is shown by the analysis of the video.

Based on the video, Teacher C discussed the meaning of used discipline-specific words (item TL5). In the following example, a discipline-specific term, *expanded form,* was explained by Teacher C: *"So if we have 125 and it's this small, we're going to expand it, okay? And expanded form is one way of pulling a number apart and writing it as the sum of the values of each digit…"* (Teacher C, video transcript). She also provided correct explanations (item TL7). For instance, in the video, Teacher C showed an example of a number in standard form and asked, "*How do we expand—stretch them out? By showing their what?*" Two students responded with *"their numbers"* and *"their values."* Teacher C elaborated on the students' responses to provide a clear and correct explanation: *"Their values, very good. Their sum—the sum of their values."* She also linked their responses directly to the formal definition of *expanded form* that she provided earlier in the lesson.

Teacher C utilized mathematical terms to describe concepts (item TL8). In this instance, she used many mathematical terms related to the topic of place value, such as *expanded form, adding, sums, ones, tens,* and *hundreds* to expand 125:

> Teacher C: *To make this number in expanded form, what can I do? We need our plus signs because we're adding the sums of each value. So how many ones did we say we had? …*
>
> Student 1: *Five.*
>
> Teacher C: *Very good. Five ones. Do you see how Teacher C lines it up with the chart? How many tens did we have? Student 3, do you know?*
>
> Student 3: *Um two.*
>
> Teacher C: *Two tens, but ten times two equals tw-*
>
> Student 3: *Twenty!*
>
> Teacher C: *Very good and how many hundreds did we have?*
>
> Student 3: *Oh -oh! One hundred.*

Teacher C: *So, we have to put the number 100 and boys and girls, we just expanded our number.*

Analysis of Symbolic Language. In the same manner that Teacher C used technical language, Teacher C mostly used correct symbolic language (SL), with very few instances of somewhat correct SL, in this lesson (M = 2.86, SD = 0.38). This is shown by the analysis of the video.

Teacher C consistently used simple, concise language to connect symbols to their meaning during the lesson (item SL18). For example, Teacher C explained how to take a number in standard form and use addition (the plus sign) to expand it in a clear, organized way:

Teacher C: W*hat's the first thing we have to do when we're writing our number in expanded form? … What are those signs called?*

Student: *The plus signs.*

Teacher C: *So we'll put our two plus signs. Very good…* (Teacher C, video transcript).

In another instance, Teacher C demonstrated expanding 125 (dialogue previously shown in the TL section) and 132 (Figure 14). She connected symbols to their meanings by explaining how

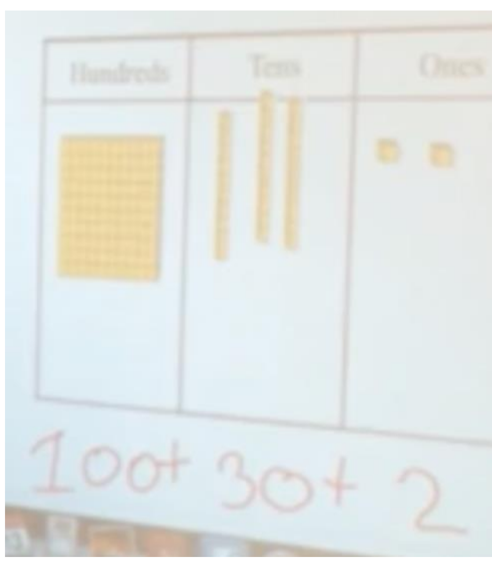


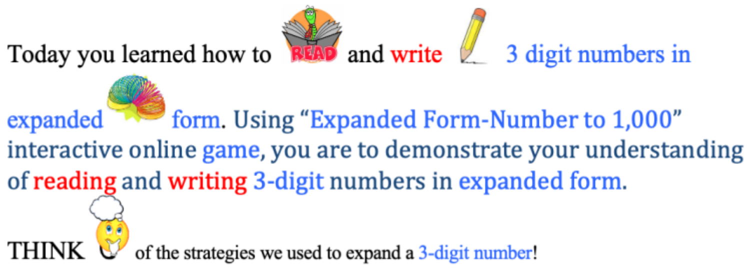
Figure 14. *Image from Video Recording—Expanding 132*

the plus signs in 100 + 20 + 5 and 100 + 30 + 2 represent the process of addition, which resulted in finding the sum of the ones, tens, and hundreds places. Similar examples occurred throughout the lesson.

Also shown in the above example, Teacher C made connections between symbols and mathematical concepts by having students *add* the ones, tens, and hundreds places throughout the video (item SL14). In addition, Teacher C used symbols, such as the plus sign and the equal sign, correctly when providing explanations (items SL10, SL16). Lastly, throughout the lesson, Teacher C supported the meaning of the equal sign as relational rather than operational using virtual tools to help students visualize the equal sign as a comparison of two equivalent values (item SL15).

Analysis of Visual Language. Teacher C consistently used correct visual language (VL) throughout this lesson ($M = 3.0$, $SD = 0.0$). Specifically, for precision, $M_p = 3.0$, $SD = 0.0$, and for fluency, $M_f = 3.0$, $SD = 0.0$, as shown by the analysis of both sources.

Teacher C used visual representations to describe place value during this lesson (item VL25). For example, a slinky was used as a visual to aid students in understanding the concept of expanding. Teacher C expanded a slinky in the front of the room and stated, "*Expanded form is when we take a number like a slinky and we expand it*" (Teacher C, video transcript). She then provided students with a formal definition of expanded form. The students' corresponding handout included an image of a slinky as a reminder of this explanation (Figure 15).


Today you learned how to READ and write 3 digit numbers in expanded form. Using "Expanded Form-Number to 1,000" interactive online game, you are to demonstrate your understanding of reading and writing 3-digit numbers in expanded form.

THINK of the strategies we used to expand a 3-digit number!

Figure 15. *Image from Handout—Slinky*

Teacher C also used virtual tools on the Smart Board to describe place value when writing three-digit numbers in expanded form (Figure 16). A place value chart was used to show the separation of the ones, tens, and hundreds places of the three-digit number, and then base-ten blocks were used to model the size of place values in relation to each other. Throughout the entire lesson, Teacher C used discipline-specific words when describing visuals (item VL26).

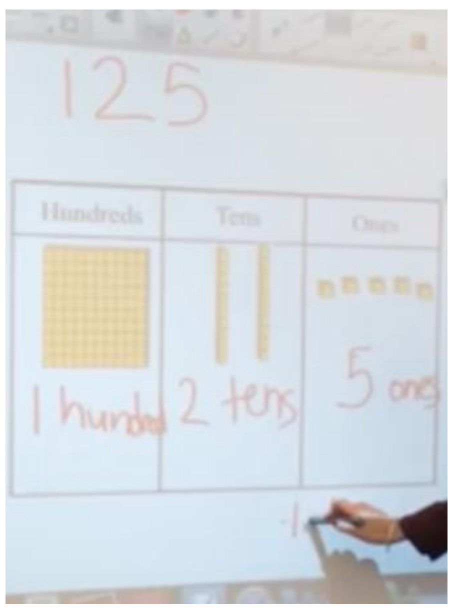


Figure 16. *Image from Video Recording—Place Value Chart and Base-Ten Blocks*

Although it is not shown in Figure 16, Teacher C finished each example by writing the corresponding number sentence on the board. Teacher C converted visually represented information into mathematical notation throughout the lesson (item VL20; VL21). Further evidence from the video to support this is included below:

> Teacher C: *Student 6, why don't you come up and show me how to use the manipulatives to move them across the charts? Okay, so for 222, how many ones do we need to put? How many cubes?*
>
> Student 6: *One, two.*
>
> Teacher C: *Very good. And how many tens do we have?*
>
> Student 6: *We need two tens.*
>
> Teacher C: *Very good. One, two, great, and how many hundreds? Okay. Now how can we write this number in expanded form?… Student 3, can you read this number in expanded form for me? Right here. Very good. Read the sum, what are we adding?*
>
> Student 3: *Um 200, and twenty, and two.*

In conclusion, Teacher C used correct technical language, symbolic language, and visual language throughout the lesson (overall M = 2.9, SD = 0.31) with just a few instances of using somewhat correct TL and SL.

4.5 Discussion and Conclusion

This chapter presented a quantitative analysis of pre-service elementary school teachers' use of MDL as well as a content analysis of video transcripts and corresponding lesson plans that illustrated MDL use at Level 1, Level 2, and Level 3.

The quantitative analysis indicated that teachers' use of technical, symbolic, and visual language varies, demonstrating the utility of the MDL scale in distinguishing between different aspects of teachers' use of MDL. The results of analysis also indicated a weak positive correlation between teachers' use of different aspects of MDL, suggesting that teacher education and professional development programs should pay attention to developing each aspect of MDL independently. Additionally, the quantitative analysis showed that this sample of pre-service special education elementary teachers' use of symbolic language was particularly poor. These results highlight symbolic language as an area of highest need in MDL for teacher preparation programs to focus on. An important consideration when interpreting these results is that the study assessed teachers' use of MDL, not their knowledge of MDL.

The illustrative case studies displayed teachers' MDL use at different levels (Table 12).

Table 12

Descriptive Statistics of Teachers' Use of MDL in Three Cases

Teacher	TLPE		TLF		SL		VLP		VLF		Overall	
	M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
A	2.2	0.44	0.67	0.58	0.29	0.49	2.0	0.0	1.0	0.0	1.0	0.91
B	2.2	0.44	1.67	0.58	2.29	0.49	2.0	0.0	2.0	0.0	2.1	0.45
C	2.8	0.45	3.0	0.0	2.86	0.38	3.0	0.0	3.0	0.0	2.9	0.31

Teacher A's precision and explicitness in technical language (TLPE), as well as her precision and fluency in visual language (VLP and VLF), were inconsistent throughout the lesson. There was no evidence of symbolic language (SL), and fluency in technical language (TLF) was either not evident or incorrect, which is characteristic of MDL use at Level 1. Teacher B demonstrated somewhat correct precision and explicitness in technical language, fluency in technical language, precision and fluency in visual language, and use of symbolic language throughout the lesson, which is characteristic of MDL use at Level 2. Teacher C's use of symbolic language, precision and explicitness in technical language, fluency in technical language, and precision and fluency in visual language were correct throughout the lesson, which is characteristic of MDL use at Level 3. She also frequently made connections between the five components of MDL, which is also characteristic of MDL use at Level 3.

Several factors could have contributed to the variations in teachers' use of MDL across the three cases. For instance, some teachers may not use MDL due to limited knowledge or confidence in using it. Even teachers familiar with MDL might choose not to incorporate it into their lessons, preferring to use everyday language instead. This preference may stem from a belief that everyday language is more accessible for students. Additionally, some teachers may have concerns about whether students, particularly those in special education, would fully benefit from MDL.

It is worth noting that all participants in this study were preparing for special education certification. Given this, it is important to consider whether there might be underlying reasons influencing their use—or lack thereof—of MDL in the classroom. Powell et al. (2019) found that students with disabilities require precise MDL and explicit instruction to ensure clarity of terminology and concepts. The teachers in this study may not have fully recognized how using MDL can help students with disabilities overcome learning barriers in mathematics. Moreover, a lack of training in the use of MDL may hinder teachers' ability to implement it effectively.

Chapter 5: Summary, Conclusions, and Recommendations

The purpose of this study was to (a) define mathematics' discipline-specific language (MDL) and confirm that by developing an instrument to assess elementary school teachers' use of MDL, and (b) analyze the characteristics of teachers' use of MDL based on data collected from a sample of pre-service elementary teachers enrolled in the same special education graduate program. Therefore, this dissertation study was guided by the following research questions: RQ1: What are constructs of elementary school teachers' use of MDL? and RQ2: What are the characteristics of elementary school special education teachers' use of MDL?

5.1 Conclusions

RQ1: What are constructs of elementary school teachers' use of MDL?

This study developed and validated a quantitative instrument for external assessment of MDL of elementary school teachers. To the best of my knowledge, this is the first instrument that defines aspects of teachers' use of MDL at the elementary school level.

The development of the Mathematics' Discipline-specific Language Scale (MDLS) led to the identification of three categories of teachers' use of MDL: technical language, symbolic language, and visual language. Initially, the theoretical framework for this study defined MDL by incorporating three assessment criteria—precision, explicitness, and fluency—into the scale items that defined its three categories. However, the data-driven process of MDLS validation for this particular population led to a modification of the theoretical framework, and the categories of MDL were further specified into five distinct factors: technical language precision/ explicitness, technical language fluency, symbolic language, visual language precision, and visual language fluency. While both technical language and visual language were each divided into two factors, symbolic language remained as a single factor. This revision provides a clear framework to define and measure MDL use, helping address a gap in the literature on the theoretical understanding of the structure of MDL used by elementary teachers.

RQ2: What are the characteristics of elementary school special education teachers' use of MDL?

Most teacher-participants demonstrated incorrect or somewhat correct use of technical and visual language components. However, teachers' use of symbolic language was even more limited, with 26.5% of participants not using symbolic language at all and only 22.3% of them using it correctly.

For the study participants, symbolic language scores were statistically significantly lower than both technical language and visual language scores, and technical language fluency scores were statistically significantly lower than technical language precision/explicitness scores. The results also indicated that pairwise correlations between all factors of MDL were weak or very weak, which suggests that development of each factor might need to be done independently.

The analysis of three lessons and corresponding videos illustrated different levels of MDL use in three cases. The purpose of these cases was to (a) provide guidance and support for practitioners on how to use MDLS, and (b) connect observations and features of the lessons to MDLS items.

5.2 Study Significance

The theoretical contribution of this study lies in its development of the clear and specific aspects of MDL used by elementary school teachers. This study first defined MDL for elementary school teachers, then developed and validated a five-factor model of MDL that explains the structure of the language. By offering a more detailed framework, this research advances the literature on disciplinary literacy and elementary teachers' use of MDL. A clear and specific framework for MDL in elementary schools is essential for understanding how teachers use MDL. The development and validation of an assessment instrument like the MDLS contributes to this understanding by defining clear performance-based indicators of MDL, which in turn informs and refines the theoretical framework. In addition, the framework for MDL helps to solidify the role that MDL plays in broader disciplinary literacy frameworks.

This study also addressed a gap in the literature on elementary teachers' and special education teachers' use of MDL. This contribution is important, as most existing research has focused on teachers' knowledge of MDL rather than their actual use of it, particularly at the elementary level. Since MDL is a key component of disciplinary literacy (Fang, 2012; McConachie & Petrosky, 2009), this work draws on existing literature to create a foundation for defining and understanding how special education teachers use MDL at the elementary level.

The five-factor model introduced in this study serves as a starting point for further research on teachers' use of MDL. Researchers can use the MDLS as a valid and reliable instrument for studying teachers' use of MDL. The MDLS can be validated for general education elementary school teachers, and in different settings, including varying geographic and socioeconomic contexts, as well as among teachers with different levels of teaching experience. Researchers can also examine the impact teachers' use of MDL might have on student outcomes in mathematics. They can conduct longitudinal studies to track whether there is a correlation between teachers' use of MDL and student achievement in mathematics.

The MDLS has various practical applications for teachers, teacher educators, school administrators, program developers, and policymakers.

For teachers, it can serve as a self-assessment of their use of MDL in the classroom. By reflecting on their MDLS scores, teachers can identify which MDL aspects they use most and areas where improvement is needed, adjusting lesson plans and refining instruction to ensure MDL is used precisely, explicitly, and fluently. Teachers can also collaborate with colleagues by using the MDLS as a peer assessment and sharing MDLS scores and MDL strategies, both of which foster a professional learning community focused on the use of correct MDL. While professional development could certainly be beneficial, it may also be that teachers require additional support in finding effective ways to incorporate MDL into their classrooms.

In teacher preparation programs, the MDLS can be integrated into training to provide formative feedback for both pre-service and in-service teachers. The MDLS can also function as a pre- and post-assessment to evaluate the program's impact on teachers' use of MDL. Teacher educators can use the MDLS to assess pre-service and novice teachers' use of MDL and guide them in using correct MDL from the start of their careers. Furthermore, MDLS data can help teacher educators design mathematics education courses and develop scaffolded support to strengthen pre-service and in-service teachers' use of MDL as needed.

School administrators can use the MDLS to assess teachers' strengths and weaknesses in using MDL, then make data-driven decisions on where to invest in training and resources. MDLS data can identify schoolwide trends in teachers' use of MDL, inform targeted professional development for teachers needing extra support, and help create a shared vision on expectations for MDL use across grade levels. Additionally, administrators can use this data to strategically delegate MDL instruction to teachers with a strong background in mathematics, especially in elementary schools where some teachers may lack this expertise.

Program developers can use the MDLS to guide the development of curricula and instructional materials that support teachers' use of correct MDL. By analyzing MDLS data, program developers can identify areas where MDL is often not evident or incorrect and create programs to address these gaps. They can also collaborate with teacher educators and administrators to ensure that the programs they create align with the broader goals for MDL, establishing a shared vision for its use across schools.

Policymakers can use MDLS data to develop policies that provide the resources, training, and guidance teachers need to use MDL effectively. By sharing MDLS data and MDL resources across schools and districts, policymakers can align expectations for MDL use on a larger scale.

5.3 Limitations and Future Research

My theoretical framework was validated for elementary school teachers who were pre-service special education teachers. The framework needs to be validated for elementary school teachers or special education teachers in general.

This study raises new research questions on (a) why technical language and visual language have two separate factors; (b) why factors are different for technical language and visual language (precision/explicitness vs. precision); and (c) why symbolic language remains a single factor. Future studies could also explore the variables influencing teachers' use of MDL and examine the impact of teachers' use of MDL on students' mathematical learning.

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Appendix A: Mathematics' Discipline-specific Language Scale

Table A1

Mathematics' Discipline-specific Language Scale

Scoring Guide:

3 - Correct (indicator holds true for all teacher's comments)

2 - Somewhat Correct/Evident (indicator holds true for more than half of teacher's comments; inconsistency/instances of correct and not evident within same item)

1 - Incorrect (indicator does not hold true in the classroom; instances of somewhat correct and not evident within same item)

0 - Not Evident

Assessment Criteria	Can also consider these items to be true if the teacher provides students with the opportunity to… **ITEM**	Level (0 - 3)	Evidence and Notes
Technical Language			
Precision	1. The teacher states/writes numerical answers that are relevant to the problem's context.		
	2. The teacher uses discipline-specific words to provide clear definitions.		
	3. The teacher states/specifies units of measurements, or puts meaning to the numerical value.		
Explicitness	4. The teacher discusses the meaning of used discipline-specific words.		
	5. The teacher explains everyday words that represent mathematical concepts.		
Fluency	6. The teacher provides correct explanations.		
	7. The teacher uses mathematical terms to describe concepts.		
	8. The teacher uses mathematical language as a vehicle for conveying content.		

Assessment Criteria	ITEM	Level (0 - 3)	Evidence and Notes
Symbolic Language			
Precision	9. The teacher uses symbols correctly (when applicable).		
	10. The teacher uses mathematical notation to provide clear definitions.		
Explicitness	11. The teacher discusses the meaning of mathematical symbols.		
	12. The teacher makes connections between symbols and mathematical concepts.		
	13. The teacher supports the meaning of the equal sign as relational rather than operational (when applicable).		
Fluency	14. The teacher uses mathematical symbols when providing explanations.		
	15. The teacher uses simple, concise language to connect symbols to their meaning.		
Visual Language			
Precision	16. The teacher labels elements of a visual representation.		
	17. The teacher converts visually represented information into mathematical notation.		
Fluency	18. The teacher uses visual representations to describe concepts.		
	19. The teacher uses discipline-specific words when describing visual representations.		
	20. The teacher uses different visual representations for the same problem.		

Mean overall score: ____

Appendix B: Catalog of Mathematics' Discipline-specific Language at the Elementary School Level

Technical language:

Technical language (TL) in elementary school mathematics curriculum is primarily represented by specialized vocabulary related to mathematical concepts. Since TL may vary by grade level, it is recommended to refer to academic word lists for mathematics that align with the standards and grade level of the lessons. For instance, in the Case of Teacher B, the following second-grade Common Core State Standards vocabulary word list (Partners for Learning, 2019) was utilized:

2-dimensional
3-dimensional
a.m.
add/addition
addend
Additive Identity Property of 0
analog clock
analyze
angle
array
Associative Property of Addition
attribute
bar graph
base ten
category
cent
centimeter (cm)
circle
column
Commutative Property of Addition
compare
compose
count
cube
data
decompose
determine
difference
different/difference
digit
digital clock
dime
dollar
equal shares
equal/equivalent
equation
estimate/estimation
even number
expanded form
face
fourths
greater than
half circle
half/halves
hexagon
horizontal
hundreds
inch
length
less than
line plot
meter (m)
minute
money
nickel
number line
numeral
odd number
ones
operation
p.m.
partition
pentagon
picture graph
place value
quadrilateral
quarter
rectangle
row
ruler
scale
set
skip count
solve
square

standard units of measure	thirds	unknown number
subtract/subtraction	time	value
sum	triangle	whole
symbol	twice	whole number
tens	unit	yard

Symbolic language:

Symbolic language (SL) refers to mathematical symbols that are used to describe relationships between mathematical objects (Fang, 2012). In the elementary school mathematics curriculum, SL is represented by basic operations signs, the plus sign (+), the minus sign (−), the multiplication signs (×, ·, or ∗), the division signs (÷ or /), the relation signs (=, >, <, ≥, and ≤), the fraction notation ($\frac{\square}{\square}$ or $\square/\square$), the place value signs (decimal point . and comma ,), units of measurement signs (feet ′ and inches ″), grouping symbols (parentheses, brackets, braces), and money signs ($ and ¢).

Visual language:

Visual language in elementary school mathematics curriculum is represented by number lines, number paths, array and area models, strip diagrams, schematic diagrams, drawings, tables, graphs, other forms of pictorial representations, and/or concrete objects/manipulatives.

Appendix C: Teacher A

Lesson Plan

Student Name:	**Lesson Topic:**	
Student 143	Order of Operations	
Subject Area:	**Grade Level:**	**Number of students:**
Mathematics	3rd grade	10
Type of classroom:	**Time Allotment (continuous class only):**	
ICT	55 minutes	

Description of Lesson (include grouping of students):

Students will be learning order of operations through a variety of platforms. They will working in homogeneous groups with each other and do in class activities. They will also participate in a Nearpod lesson with two assessment activities included.

State Curriculum Standards met in this lesson:

3.OA.B.5: Understand properties of multiplication and the relationship between multiplication and division. Apply properties of operations as strategies to multiply and divide.

3.OA.C.7: Multiply and divide within 100. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations. By the end if Grade 3, know from memory all products of two one-digit numbers.

Instructional Objective(s):

Students will be able to use order of operations to solve an expression.

Students will have a better understanding of order of operations.

Materials, Resources and Technology

1. Teacher and TA checklist notes
2. Smartboard
3. Nearpod Activity https://share.nearpod.com/xAIPCiunkV
4. Laptop Cart
5. Google slides
6. Go Math workbook pages 431-432
7. Teaching Order of Operations resource https://teacherthrive.com/2014/08/teaching-order-of-operations.html

Teacher Preparation:

1. Print order of operations anchor chart for each student.
2. Reserve laptop cart.
3. Have digital copy of anchor chart to project.

Active Participation

Students will come into the classroom and begin a warm up activity. The example "4 + 5 x 3" will be written on the Smartboard for them to solve in their notebooks. The teacher and TA will circulate around the room and take notes on how the students are solving the problems. The teacher will take two different answers and write them on the Smartboard. (Example 4 + 5 = 9 and 9 x 3 = 27 or 5 x 3 = 15 and 15 + 4 = 19) The students will be asked their notices and wonders about the two different ways of solving the problem. They will be told to turn and talk to their partner about them. The teacher will explain, "Just like in today's warm up, we can sometimes have to solve expressions that have more than one operation. We may even come across expressions that have three or more operations! In order to solve these expressions, we need to know the correct order to solve these expressions in. To help us, we are going to learn an acronym that will direct us in which one to solve first." The paper monitor (student) will hand everyone their anchor sheet with the acronym PEMDAS on it. This will be taped into their math notebooks. The teacher will explain that PEMDAS stands for "Purple Elephants May Destroy A School." The teacher will have a version of the anchor chart projected on the Smartboard so that they can go over it together. The students will not focus on exponents because in third grade, students do not work with exponents. The teacher will make sure students know that MD and AS are next to each other because when the expression only involves those operations, they can just multiply and divide from left to right or add and subtract from left to right. The teacher will now begin to model. Megan buys 2 books for $4 each. She pays with a 10-dollar bill. How much money does she have left? She says, "after reading this situation I know 2 books for 4 dollars each would be 2 X 4 and if I am paying with a 10-dollar bill I have to subtract how much the books cost from the 10 dollars I give them. It will look like: 10 - (2 X 4). Looking at our acronym, I see that multiplication comes before subtraction. Therefore, I must do 2 X 4 first." The teacher will put parentheses around (2 X 4). 5 X 3 = 15. "Then, I have to subtract 10 - 8 which equals 2 dollars." The teacher will model two more problems.

Next, the students will try problems on their own. The teacher and TA will circulate to see if the students have mastered the problems. If the majority of the class has mastered the skill, they will move on to group work. Students will be grouped homogeneously. The students who need more help will work together to create equations for each other and solve them. The other students will use google slides on their laptops to create problems for the class, as a review at a later date, to show they have mastered the skill. The students will now all participate in the Nearpod activity as a class. Each student will receive a laptop and enter the code for their Nearpod activity. They will first review PEMDAS. They will participate in a matching activity that asks them to match the equation and the solution. They will have a few minutes to ask the teachers any questions before moving on to the final quiz on Nearpod. The quiz consists of 5 questions and will serve as their summative assessment for the end of the period.

National Education Technology Standards for Students (NETS•S) met in this lesson:
1. Empowered Learner 1c. Students use technology to seek feedback that informs and improves their practice and demonstrate their learning in a variety of ways.
Differentiation Strategies, Including Adaptations for Special Learners:
Students who need enrichment in the lesson will be able to refer back to their anchor chart as needed. These students will also be able to work in homogeneous pairs to create questions for each other to solve and to help each other better understand the topic.
Assessment/Evaluation:
Students will be assessed based on the notes from both the teacher and TA during the lesson. A formative assessment will be given as a matching game in the Nearpod activity. There will also be a summative quiz at the end of the lesson on Nearpod. The students will be assigned homework to bring back with them the next day and receive a quiz at the end of the week on the skills they have learned. The students who are struggling will have time to meet with the teacher one on one for further feedback and assistance at a later date.

Video Transcript

[00:01] Teacher: Okay, perfect. So guys we're gonna use the website. We're gonna play a lot of games on it. We're just gonna show you how to do this. Okay?

Student 1: Awesome. Cool, sounds good.

Teacher: You guys are very [inaudible] All right let's get computers.

Other Teacher: Oh wait the computer monitor is absent right? All right Student 2, take over.

Teacher: All right.

Student 2: Hello okay!

Other teacher: Oh you want to hit the - okay

Student 2: Can somebody help me give out the laptops?

[01:51] Other Teacher: Alright so teacher is going to show you what you're going to go on to when you get on your computer. I'm also going to write the website here so as soon as you log on and go on google chrome you're going to type in this website. Okay?

Student 2: No no right no three right? Okay.

[students chat]

Teacher: All right guys so here's what you're gonna do on your computers you're going to go to the internet you're going to put in www.nearpod.com.

Student 3: What's a Nearpod?

Teacher: When you have it, give me a thumbs up.

Student 3: Oh.

Student 4: Oh.

[02:50] Teacher: Perfect.

Student 4: Oh I have so many websites.

Other teacher: Just go to the website.

[students chat]

Teacher: Okay. Good. Good. [Teacher walks around] Good, good. Good, okay. So, guys where it says enter code. It should be right on top. Perfect, perfect. You are going to put in our lovely little code vkclj.

Student 5: Is it going to be capital or not?

[inaudible]

[05:30] Teacher: Okay. Beautiful. Don't worry about others, just put your name. [Continues to walk around and help students] Your first name. Perfect. Alright. We're all in. Put your thumbs up if you're in. So guys order of operations, what we just learned, right?

Student 1: Yup. [inaudible]

[06:50] Teacher: Yes, so I wanted to remind you guys of your PEMDAS. Right? You have it in front of you so you do not have to worry, but just in case remember whenever we do any of these problems "purple elements may destroy school," right? Our PEMDAS. And remember to work from left to right.

[inaudible]

Okay. Here's what we're gonna do guys when you guys are ready there is a matching game on your screen. Take your time. Match the expression to the answer.

Student 2: Okay.

Teacher: Nice. Go slowly, take your time. You have plenty of time.

Student 3: I press one of these?

Teacher: Yes, so if you press this expression, what's the answer to that?

[students chat]

Other teacher: Pick another one, good job guys. I love this thanks Teacher, this is awesome.

Teacher: Good job guys. What do you do first? Multiplication or addition?

Student 3: Multiplication?

Teacher: Yeah so what's 10 times 1? Yep.

Student: Plus 3.

Teacher: There you go now.

Other teacher: I love this you have to show me how to make this yeah this is amazing.

Teacher: Absolutely.

Other teacher: You guys are working too fast so just go left to right.

Teacher: Slow down guys, you have all the time you need.

Other teacher: I like that I mean it's writing it down, excellent.

Teacher: So, what's in that one?

[10:00] Other teacher: All by yourself. Teacher, Student 5 said she even didn't have to look at that, she remembered.

Teacher: Good!

Student 1: I finished.

Teacher: Yeah, there you go. Very good guys. Good job.

Student 2: I was working so fast. I love working fast.

Teacher: You did. You went through them really fast.

Other teacher: Slow and steady wins the race.

Teacher: That's true.

[students chat]

Student 3: So what's the next thing?

Other teacher: Yeah just give me two more things. Okay, good. Thanks. Nice guys.

Teacher: All right do we have any questions before we move on? We get this?

Student 4: No.

[12:20] Teacher: Because the next thing we're going to do - is going to be a little quiz and it's going to be timed.

Student 5: Oh no.

Teacher: I promise it's not scary. All right, no questions before we move on, right?

Student 1: No.

Student 2: No.

Teacher: PEMDAS, it’s in front of us, it’s in our brains, it’s glued there, we’re good?

Student 3: Mhm [nods head].

Student 4: Now we’re going to go to the quiz?

Teacher: All right.

Student 5: Two minutes?

Student 1: Yes, I love challenges.

Teacher: It’s okay. It’s two minutes per question. All right you’re good. You guys have plenty of time.

[inaudible]

Student 2: I can’t do this.

Teacher: What do you mean? Don’t use that word in my classroom.

Student 3: Use your sheet.

Student 4: Yes yes.

[13:47] Teacher: Which one’s first? You multiply first so--

Student 2: I don’t know.

[students chat]

[15:27] Teacher: What was that problem?

Student 1: There was a hard question.

Teacher: Do you remember what the question was?

Other teacher: Oh it was the last one?

Student 1: Yeah it was it was - it was like -

Other teacher: Do you remember? I think I might I think it was 1 plus 2 times 5 minus --

Teacher: Oh did you all have different questions? Yes all right then we're gonna go over them. Okay.

Student 3: Three two one let's go. Go.

Teacher: It might give you the same -- yes you guys are rushing through these. You're flying.

Other teacher: Guys don't just click answers, do it on your paper.

[17:30] Student 4: I have 55 seconds. Oh man.

[students chat]

Teacher: We're going to go over them together guys.

Appendix D: Teacher B

Lesson Plan

Student Name:	**Lesson Topic:**	
Student 106	Skip Counting By 5's To Solve Multiplication Word Problems	
Subject Area:	**Grade Level:**	**Number of students:**
Mathematics	Second	**23**
Type of classroom:	**Time Allotment:**	
General Ed	90 Minutes	

Description of Lesson (include grouping of students):
In this lesson, students will be reviewing their multiplication skills (using mental math, repeated addition, and skip counting by 2's). Students will engage in a whole group mini lesson, then students will go with their assigned groups and rotate from station to station, then return for a recap of lesson.

State Curriculum Standards met in this lesson:

Mathematics Grade 2
Operations and Algebraic Thinking 2.OA
Work with equal groups of objects to gain foundations for multiplication.
3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Instructional Objective(s):

Using their prior knowledge, students will build on their knowledge of multiplication strategies. Students will solve multiplication word problems and equations by grouping objects into 5's and skip counting by 5's, write an equation, and explain their strategy.

Materials, Resources and Technology
Math Textbook: Math in Focus, Singapore Math by Marshall Cavendish Education

Kheong, F. H., Ramakrishnan, C., Choo, M., & Marshall Cavendish International (Asia) Pte. Ltd. (2013). Chapter 6 Multiplication Tables of 2, 5, and 10. In *Math in Focus: Singapore math by Marshall Cavendish* (pp. 149A-188E). Singapore, Singapore: Marshall Cavendish Education.

Teacher Materials:
- *Laptop*
- *Smart Board*
- *Math Game using Smart Lab in Smart Notebook*

- *Student handout with direction sheet*
- *Timer*

Student Materials:

- *. Directions sheet*
- *Independent packet* – 1 page per station
- *Pencil*
- Dot paper

Station 1: SMART Board – game for students.
Station 2: Dot Paper – Dot paper in sheet protector, dry erase markers, erasers and pencils.
Station 3: Mental Math – pencils.
Station 4: Roll and Record – Two large die with numbers 1-6 and pencils.
Station 5: Flash Match – Two packs of flash cards, pencils.
Station 6: Skip by 5's – Highlighters and pencils.

Teacher Preparation:

1. Create SMART Board lesson to use during mini lesson using Smart Notebook.
2. Using Smart Notebook, use Smart Lab to create game using one of the simulations, to create an engaging task for the students, which includes sorting multiplication facts.
3. Create student packet including worksheets for each station.
4. Connect laptop to Smart Board to test game and activity before teaching lesson.
5. Move desks into groups of four. One group will have three students; the rest will have four students.

Active Participation

The lesson will begin with a group discussion on the rug between the teacher and students to activate their prior knowledge. The teacher will ask the students about the type of multiplication word problems they have been working on, and what different strategies they have learned to solve them (mental math, repeated addition, pictures, and skip counting by 2's for the two times table).

Next, the class will be taught a mini lesson on how to skip count by 5's to solve multiplication word problems involving the five times table. After practicing how to skip count and using our fingers to help skip count by 5's, together we will solve a word problem that involves students to skip count by 5's and complete the blanks in a multiplication sentence. When the word problem is solved, the students will review the rules of the stations and the directions for each station.

Then, students will return to their assigned groups with their materials and begin working. Students will follow the directions on the worksheet in their packet for that station. The group at station 1 (SMART Board) will play three games. The first game, students have to match the multiplication sentence to the correct solution. The second game, students have to match the phrase groups of to the multiplication sentence (for example: 2 groups of 5 would be displayed, students have to match the correct multiplication sentence 2 x 5 into the blank spot. The third game, each student picks a character to race. In order to continue around the track, the students have to answer each of the five questions correctly to receive a double boost. Station2, students will use their dot paper to help solve two multiplication word problems, and then have to explain how they got their answer. Station 3, students use the strategy of mental math to solve different multiplication problems including: missing numbers in a pattern, multiplication sentences, and writing out the meaning groups of multiplication sentence (5 groups of 5 = ? x ? = ?). Station 4, students will use two big dice (one per two students). They will roll the die and use the number it landed on (numbers 1-6) and multiply it by 5. Each time they roll the die they must record the number they rolled and the answer to the multiplication sentence. Station 5, students (in groups of two) will match their flash cards (multiplication sentence and answer, groups of and multiplication sentence. Each time they make a match they must record it on their worksheet. Station 6, students are skip counting by 5's. Like in the mini lesson, students will be labeling pictures of hands by counting by 5's (1 finger up is 5, 4 fingers up is 20, etc.). Next, students in this station will use a highlighter and skip count by 5's on the 100's chart. Each time they skip count by 5's they have to highlight their answer. Students will have eight minutes on the timer to complete the task at their station. When the timer goes off, students move to the next station (students at station 1 move to station 2, students at station 6 move to station 1 with their packets, dot paper, and pencil).

After, the class will return back to the rug to recap what we have learned. We will discuss the different ways to solve multiplication word problems, specifically concentrating on skip counting by 5's. As we wrap up we will use our fingers to skip by 5's up to 50.

National Education Technology Standards for Students (NETS•S) met in this lesson:

International Society for Technology in Education (ISTE)
2. Communication and collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.

a. Interact, collaborate, and publish with peers, experts, or others employing a variety of digital environments and media.

d. Contribute to project teams to produce original works or solve problems.

Adaptations for Special Learners:

For students with visual impairments, accommodations will have the text on their screens enlarged, and darkening or brightening of the screen, which will put less strain on their eyes.

For students with hearing impairments, auditory assistance will be provided. A teacher or paraprofessionals will real aloud the directions and questions to the student. Students may also use their headphones if and when needed.

For students with a speech or language impairment, words or photos will be incorporated into their exit slips. Accommodations of directions and questions being read a loud will also be provided.

For students with a learning disability, differentiated worksheets will be created with simpler equations and photos relating to the topic.

Assessment/Evaluation:

Formative:
Students will be assessed on an ongoing basis during whole group and small group instruction. After the stations packet is completed, students will receive their grade along with a graded rubric, which was provided to students prior to the lesson, and in student's possession throughout the time of the stations. Using a Post-it, students will express one of their strengths, and identify their next steps.

Summative:
Students will be assessed using a rubric (1-4) on their knowledge of skip counting by 5's to solve multiplication word problems and equations. Students will also be assessed on their use and completion of the SMART Board games as well as how they navigated through the games on the SMART Board.

Video Transcript

[00:01] Teacher: Okay. So, today - we've been learning a lot about multiplication. So, today we're going to - our I can statement is "I can skip count by 5 to solve multiplication word problems." So what do we think that means, skip counting by 5?

[inaudible]

Teacher: Awesome now is that a little bit easier than counting 1, 2…?

Student 1: Yeah.

Student 2: Yeah.

Teacher: Okay, great. Okay, so like before we were skip counting by 2's because this chapter, remember, it's the time tables for 2, 5, and 10. So we learned to skip count by 2's right? We mastered it. Now, we're moving on to skip counting by 5's and by the looks of it, I think we're all gonna be masters. Alright? So, let's take a look, so turn our bodies to the Smart Board. Awesome so just like Student 1 was saying the number of patterns 5, 10, 15, 20. Can we do it together?

[students count out loud]

Teacher: Let's do it all together, ready?

All students: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100.

Teacher: Awesome. We can keep going all day, but we're going to stop there for now. Let's do the magic 5. Face the Smart Board.

[students chat]

Teacher: Good so we're noticing a pattern we're noticing that they either end in 5s or they end in 0s. Excellent so just like we use equal groups to multiply so if we were using this - so if looking at this picture right here what would our number sentence be, what are we thinking?

[student comes up to board]

Teacher: Excellent we can also rephrase it: two groups of 5 equals 10. There's multiple ways we can say it. Okay. So, let's skip count by 5 with our hands. So, I noticed the other day for homework you had to do something where you had to label the fingers with the different numbers. Yes, no ... ? We all did it, I looked at it. All right so - stop, look, and listen.

All students: Okay.

[03:50] Teacher: So do we notice something about the fingers though? What do we know about the fingers? Student 2.

Student 2: I noticed that they both have different numbers like they have 1 and then they have 2 and they have 3. And they all have different numbers.

Teacher: They all have different numbers with the fingers, yes.

Student 2: And they all go up to 50.

Teacher: They all go up to 50, so, yes, that's a good observation. So, I'm noticing something else though too. Student 3, what do you notice?

Student 4: Um I notice that it goes by 5s. 5, 10...

All students: 15, 20, 25, 30, 35, 40, 45, 50.

Teacher: Awesome. So, do we notice something else though? So… so, our friends in the corner over here is telling us 1 finger stands for 1 group of 5. Do we notice that? So 1 finger, so 1 times 5 equals…

All students: 5.

[05:00] Teacher: If I'm holding up two fingers how many does that equal?

All students: 10

[teacher holds up 3, 4, 5, 6, 7, 8, 9, 10 fingers]

All students: 15, 20, 25, 30, 35, 40, 45, 50.

Teacher: Excellent so let's try that again, but I want everyone to use their fingers. So get your fingers out. Let me see your hands. So - stop, look, and listen.

All students: Okay.

Teacher: Let's see those two hands. Okay, ready? Let's do it all together.

[05:38] everyone: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50.

Teacher: 50. Excellent. So we have tools on our hands, right? All right, so let's do the magic 5. Face front. What are we gonna do? We're getting to it. Okay so if I was to put this in a multiplication sentence.

[student 4 raises hand]

Teacher: Um yes but that would be a little bit you know what if you're counting by 8 if you memorize things so like when I know - so Student 4 just made a good point so there are other ways that you can skip count or count so she said can you can you skip count by 8s so you can - stop look and listen.

All students: Okay.

Teacher: Some of us can do it, some of us might get a little trouble. But there are ways you can do it so let's take a look. Ah so she noticed that there's a problem there. What problem do we have here? Can you explain it to us?

[inaudible]

Teacher: Stop, look, and listen.

All students: Okay.

Teacher: She's trying to explain something to us because no 1 noticed this yet, so let's give her - let's give her our attention and make sure we're listening. So just like we said 10 fingers equals 50, so 10 ten times 5, right? I want you to turn and talk to your partner about how this is useful. How is this more useful than counting by twos? Mustard, it's your turn to go first 1 two three. Um Ketchup now it is your turn. All right three. Do the magic 5. Face the Smart Board.

[10:10] So when I asked you to turn and talk - when I asked you to turn and talk, I wanted you to try and see is it easier to skip count by 5s or is it easier to skip count by two? So what can you and your partner - stop look and listen.

All students: Okay.

Teacher: So what did you and your partner discuss? I want you to say what your partner said. What did your partner say?

Student 5: My partner said it was easier to count 5.

Teacher: Excellent, did anybody explain why it's easier to count by 5?

Student 1: I know it's easier to count by 5. It's because when you have a 2 it's 2, 4, 6.

Teacher: Okay, so you think it's easier to count by 5, so it saves us time. Excellent. Student 2 and then Student 3. Let's keep in mind that we're being respectful to our friends. We're facing front. So, Student 4 I need you to face front. Turn around Student 5 Smart Board.

Okay, so let's try a word problem and let's try skip counting by 5 see if it's a little bit easier. So, okay. Each bunch has 5 flowers. How many flowers does she have? Think about it, what does she - what are we looking for? Student 6, what are we looking for? Stop, look, and listen.

All students: Okay.

Teacher: We're listening. So he's doing 7 times 5. So how can- using skip counting, how can you figure the answer out? Stop, look, and listen.

All students: Okay.

Teacher: Fix your bodies. Criss cross applesauce. Facing front. Student 7, how can you skip count by 5s? Where did you stop? Can you show me on your hands?

[15:10] So Nina has 35 flowers. Alright, so 7 groups of 5 flowers. I can count by 5s so just like math did right he did 5 10 15 20 25 30 35 and he stopped here because he knew he had 7. Stop, look, and listen.

Okay so now we're going to move into our Stations. So remember we each have our packets. Let's do the magic 5 base front on your bottoms.

So Station 2, you're solving the word problem. You're writing your explanation and you're using your dot paper so you have your dot paper on your desk. Also the dry erase markers are in the bin with an eraser for you.

Station 3, you're using mental math. Using mental math you can use your fingers to count right figuring out the answers.

Station 4, "Roll and Record," so you're rolling the die you're multiplying whatever number you roll by 5 so if I roll a 3, I multiply it by 5. Then you record your answer so you're going to record it as many times as you need to. Then you repeat you keep going. There's 2 die on the table so groups of 2.

Station 5, flash match. So you have the flash cards on your table, right? So you're matching the multiplication sentence to the answer and then you're writing it down. There's two different groups. There's groups of and there's multiplication sentences. Counting by 5 so you have just like we did before here you have to label the hands with the numbers.

[17:34] Then here you have to skip count by 5 and highlight the number you stopped at, okay? And there's highlighters in your bin on the table. Okay make sure when you're going Station to Station you're bringing your materials with your packet, your dot paper, and your pencil.

Station 1, so group 1. Student 1 and Student 2 you're going to remain here on the carpet with me, okay? Yes we're going to go back to our table.

Group 5. Awesome. Group 3. Group 2. And Group 4. Okay. Everybody stop.

All students: Hands on top.

Teacher: Okay just like we've done in this week you're moving to the next number Station. So if you're at Station Four, you're moving the Station 5. Okay so Station 6, so Station 6, you're going to be coming here to the Smart Board for Station 1 okay. Okay? So Student 3 and Student 4 last you'll just have 1 side of yourself okay okay any questions before we continue?

Second Teacher: Pencils down boys and girls. Student 5 that's the second time.

[inaudible]

Teacher: You have to write the numbers on the hand and you have to highlight the different groups okay. Okay I'm going to set the timer for about maybe eight minutes to see how we do. Okay? Remember. Stop, look, and listen.

All students: Okay.

Teacher: Stop, look, and listen.

All students: Okay.

[20:30] Teacher: Remember you're working as a very soft whisper to be able to hear you all right.

[students chat]

[26:20] Everyone's working at their own Stations. We're now rotating Stations.

[30:00] [students chat and do group work]

Teacher: Okay so the bell has rang so what we're going to do is we'll go to lunch and we'll finish this when we get back.

[01:03:32] Teacher: Okay at this point in time take your worksheet and whatever materials you have your pencils Station six come to the rug. Okay okay. Everyone needs to respond. Please come to the carpet with all your belongings. Your pencils. Thank you very much and Station 5 please come to the carpet with your dot papers. Yes.

Stop, look, and listen. All right, so who can recap today what we did today? What was our "I can" statement? Who remembers? Student 7.

Student 7: Today in math, we were learning how to skip count by twos and now we are learning to skip count by 5s.

Teacher: Excellent so we learned how to skip count by 5s and each Station you have to do something with counting by 5s in different ways so who can remember one of the ways we skip counted by 5? Student 8.

[01:06:10] Student 8: Um we [draws dice on board].

Teacher: Okay so we use dice that was 1 of the Stations and whatever number you would multiplied by 5.

Student 9: We also used dot paper.

Teacher: Why is it important? I noticed a lot of us especially when we were coming up to the board to do the Smart Board game, I noticed a lot of us were counting on our fingers. Why are you doing that? Student 9.

Student 9: Because we have 5 fingers and we just have 5 5s like that.

Teacher: So can we do that all together? All right, excellent sit down. Thank you, boys. So let's use it everybody get your hands out let's do it 1 more time, ready?

Excellent so - stop, look, and listen. Now looking over today's math skip counting by 5 is it a little bit easier for us to solve a problem? We will learn to skip count by 10s. Yes, so - but right now we're just concentrating on 5s. Stop, look, and listen. At this point in time I want everyone to just sit on their papers. Sit on your papers. Just because it's a distraction.

Appendix E: Teacher C

Lesson Plan

Student Name:	**Lesson Title:**	
Student 028	Place Value	
Subject Area:	**Grade Level:**	**Number of students:**
Mathematics	2nd	5
Type of classroom:	**Time Allotment:**	
ICT	1 Period (50 minutes)	

Description of Lesson (include grouping of students):
In this lesson, students will practice reading and writing three-digit numbers in expanded form. They will engage in an interactive SMART Notebook activity as a group. Independently, on iPads, they will access the internet game "Expanded Form," to practice reading and writing numbers in expanded form.

State Curriculum Standards met in this lesson:

2.NBT.3. **Read and write numbers to 1000 using** base-ten numerals, number names, **and expanded form.**

Instructional Objective(s):

Using "Expanded Form-Number to 1,000" interactive online game, students will write 3-digit numbers in expanded form by answering 7 out of the 10 questions correctly.

Using "Expanded Form-Number to 1,000" students will read 3-digit numbers in expanded form by orally answering 2 out of 3 questions presented by teacher during independent work.

Materials, Resources and Technology

- Teacher's computer
- Smart Board
- LCD projector
- Smart Notebook
- Flash drive
- Video- "Place Value Math Song: Ones, Tens, Hundreds" https://www.youtube.com/watch?v=5W47G-h7myY
- 5 Dry erase boards with markers
- Interactive Game (website)- "Expanded Form Game" https://www.studyladder.com/games/activity/expanded-form-numbers-to-1000-28083
- 5 iPads

- Headphones
- Instruction Handout Activity Sheets
- Slinky
- Clipboard to record observations
- Pencils
- Houghton Mifflin Harcourt: Go Math!

Dixon, J. (2012). Numbers to 1,000. In *Go Math!* (Common Core ed.). Orlando, Florida: Houghton Mifflin Harcourt.

Teacher Preparation:

1. Schedule appointment for iPad rental from library.
2. Schedule a period to pull out five students for this lesson.
3. Print handout instructions and activity handout.
4. Turn on classroom computer.
5. Turn on SMART Board and insert flash drive.
6. Open SMART Notebook and display on SMART Board.
7. Download video “Place Value Math Song: Ones, Tens, Hundreds” from flash drive to computer.
8. Open virtual manipulatives on SMART Notebook.
9. Make sure iPads are charged and turned on.
10. Make sure iPads are connected to Wifi.
11. Bookmark “Expanded-Form Game” on all students’ iPads.
12. Make sure sound on iPad is turned on.

Active Participation

The teacher will begin the lesson playing a motivational video “Place Value Math Song: Ones, Tens, Hundreds” on the SMART Board. The video will review place value of each digit in a three-digit number using base-ten blocks. The teacher will ask students questions about place value based on the video. The students will have opportunities to demonstrate their understanding of place value.

The students will be presented with a three-digit number in standard form. They will be asked to demonstrate different ways to represent the number on their dry erase boards. The teacher will introduce the number in expanded form. Students will learn that “expanded form is one way of pulling a number apart and expressing it as a sum of the values of each digit” (“Expanded Form,” 2014). A slinky will be expanded as a visual aide to help students understand the concept of expanding. The teacher will model how to use SMART Notebooks virtual tools and pens to demonstrate expanded form. The students will learn how to utilize a place value chart and base-ten blocks on SMART Notebook, to identify the expanded form of a three-digit number. Each student will have an opportunity to participate in the interactive SMART Notebook class activity. The activity will present the students with a number in standard form, which they will have to convert to expanded form using the tools and strategies learned in the lesson.

Students will independently engage in the interactive online game “Expanded form- Number to 1,000.” “Expanded form- Number to 1,000” will help students practice reading and writing numbers in expanded form. They will follow steps on the technology instruction handout to access and play the game. In the game, students will be presented with ten 3-digit numbers in standard form and asked to identify the value of the missing number in expanded form. The students will use their pointer finger to type the missing value in the game's calculator. The students can earn points for each answer they complete correctly. The activity provides immediate feedback for the player when he or she correctly answers a question. If a player incorrectly answers a question, it is highlighted in red, to aware the player of his or her error. While the students play "Expanded form-Number to 1,000," they will answer provided questions on the activity handout.

After the students have completed "Expanded form-Number to 1,000" and activity questions, they will meet the teacher at the rug to share their findings. The teacher will project the internet activity on to the SMART Board. Each student will have the opportunity to come up to the board and share his or her answer. Students will also share their answers to the activity handout's inquiry-based questions.

National Education Technology Standards for Students (NETS•S) met in this lesson:

5b. Exhibit a positive attitude toward using technology collaboration, learning, and productivity.
5c. Demonstrate personal responsibility for lifelong learning.

Adaptations for Special Learners:

Students with learning disabilities will wear headphones to listen to the questions read aloud in the game. They will also have visuals on the instructional handout to guide them through the interactive activity. Instructions will be read aloud by the teacher to the students. The activity will scaffold students' learning by providing at least two-digit values in expanded form and have the student identify the third digit's value. Students will also have extra-time to complete the interactive activity. A paraprofessional and teacher will assist students in using the technology.

Students with physical impairments will have the opportunity to demonstrate their understanding by typing in the expanded form of a number on a digital calculator. Students will have opportunities to use virtual manipulatives to identify place value of digits. Paraprofessional will assist student in writing the answers from the game on to the activity sheet. Student's activity sheet will provide three possible choices in expanded form for the student to circle.

Assessment/Evaluation:

- The teacher will formally assess the students by scoring their activity handout. Students, who receive a score of 7 or more, have mastered the objective of reading and writing three-digit numbers in expanded form.
- The teacher will observe students while they are engaged in "Place Value- Numbers to 1,000." As an informal assessment, the teacher will record observations of each student during the activity. Teacher will also ask students inquiry-based questions during the interactive Smart Notebook activity. The questions may include: How would you read this number in expanded from? How do we write this number in expanded form? How many hundreds are there? How many tens are there? How many

ones are there? If we have two tens, how could we write the expanded form of this value?

- Students will receive feedback formally and informally throughout the lesson. The "Place Value-Numbers to 1,000" game positively reinforces students when their answer is correct. The teacher will also walk around and provide verbal reinforcement to students while they are engaged in the activity. During the SMART Notebook class activity, the teacher will positively reinforce students while they are answering questions. The questions may include: How can we expand a number? What is the difference between standard and expanded form? How many ones are there? How many cubes do we need to represent the place value of the given number? The teacher will give formal feedback to students by scoring their activity handout and providing future goals.

Dixon, J. (2012). Numbers to 1,000. *Go Math!* (Common Core ed.). Orlando, Florida: Houghton Mifflin Harcourt.

Expanded Form. (2014). Retrieved September 30, 2015, from http://www.icoachmath.com/math_dictionary/expanded_form.html

Expanded form - Numbers to 1000, Mathematics skills online, interactive activity lessons. (2013). Retrieved September 30, 2015, from https://www.studyladder.com/games/activity/expanded-form-numbers-to- 1000-28083

Mr. R. (2011, October 2). *Place value math song: Ones, tens, hundreds*. Retrieved September 30, 2015, from https://www.youtube.com/watch?v=5W47G-h7myY

Video Transcript

[00:01] Teacher: Hi boys and girls my name is Teacher as you remember. I've been working with you for a few weeks now and we've been doing place value this entire month. Right? And we've been working with different ways we could show the values of numbers. Well, I chose you guys to do this lesson with me because it's a really fun lesson and we're going to review place value and go over one of the ways that we can show that we know place value. Okay? Reading it and writing it. Okay? So now we're gonna watch a fun little video.

["Ones, tens, hundreds song" plays]

[02:32] Teacher: Did you guys like that video?

All students: Yes!

Teacher: Okay so I went over place value because today we are going to be able to read and write numbers in expanded form. So let's go over the video. What did they say, what value came first?

Student: Um the ones.

Teacher: The ones, very good. Student 1, what value came second?

Student 1: Ten.

Teacher: Very good. And Student 3, what value came third?

Student 3: Um the hundreds.

Teacher: Very good. And what represented the ones?

Student 3: Little cubes.

Teacher: The cubes, very good. And which picture represented the tens, what do we call that? Student 2.

Student 2: The hmm…

Teacher: What are these called?

Student: Sticks.

Teacher: Somebody want to help her out?

Student: Ten sticks.

Teacher: The ten sticks or the longs, right? And what's this big one called and what does it represent?

Student: Hundreds.

Teacher: The - okay, it represents the hundreds, but what do we call it?

Student: A big cube.

Student: Cube. [students chat]

Student: A big -?

Student: A hundreds block.

Teacher: A hundreds block. Very good and once we had 10 ones what did we have to do? Can we stay in the ones value or do we have to move on over?

Student: Move on over.

Teacher: Move on over to what value?

Student: The ten.

Teacher: To the tens very good and once we had the tens and we had ten tens what did we have to do?

Student: We had to move to the hundreds place.

Teacher: Okay, very good. So I'm going to show you a number -

[announcement interruption]

[05:00] Teacher: The number is 125 and what I want you to do… I would like you to turn and talk to your partner and represent the number 125 another way. What other way can we show the number 125? It doesn't matter whose markers it is, okay? I just grabbed one from the bin - we'll return them, okay? So, how many ways can we write or show the number?

[students chat]

Teacher: How many ways can we write or show the number 125? What are the different ways? Okay, turn and talk to your partner, maybe your partner knows another way. There's the three of you so you guys could turn and talk. Oh I like what we're doing.

[students chat]

Teacher: Remember we learned about place value all month. What were some of the ways that we could show a number?

Student: We could show a number by um -

Teacher: So tell your partner.

Student: You could show numbers by writing or drawing.

Teacher: Drawing. Very good, I love drawing and writing. How can we write the number? What are some of the ways we can write it?

Student: Um you can write it in hundreds, tens, and ones.

Teacher: Okay, so you can write how many hundreds in word form, how many hundreds there are, how many tens, and how many ones. Very good. Anyone else? Okay. So we said drawing, we said how many numbers are in the tens, how many are in the ones, and how many are in the hundreds, but what no one said what was in the TP: expanded form. Expanded form is when we take a number like a slinky and we expand it.

[07:15] Student: Woah.

Teacher: Wow, right? So if we have 125 and it's this small, we're going to expand it, okay? And expanded form is one way of pulling a number apart and writing it as the sum of the values of each digit so what I want you to do is, I want you to hold up your - your pictures - a lot of you went first for the drawing - very nice, great job, great, excellent, very good. You're on your way, Student 3? Very good.

Teacher: So I'm going to show you a way that we can do this on Smart Notebook activity. I want everyone to close their markers. You could put your - your dry erase board in front of you. You could leave your thing up or you could erase it, it's fine. Okay, we need to pay attention to Teacher. Student 2 look up. Student 3 check in. Okay. So, I'm gonna write. We're on Smart Notebook. Who has seen your teachers work with Smart Notebooks before? They've - they've used this - this sort of paper. You've never seen them at the Smart Board?

Student: No.

Teacher: No?

Student: They just like draw it out like this - they just put 100 -

Teacher: No, I'm saying in general in any subject they've used the Smart Board. I've seen them use it.

Student: Yeah but not for like -

Teacher: Not - well, let me show you something that's really cool. So, I'm gonna press the markers on and then I'm gonna pick up my marker and I'm gonna choose the red. And I'm gonna write my number -

Student: You have to hit the -

Teacher: I did. One, two, five, and these are called virtual manipulatives. Okay? Because we can clone them and make more of them. So what I'm going to do is I'm going - gonna represent my number using my base 10 blocks and I'm going to put it in my base 10 chart to help me - to help scaffold my thinking of how am I going to make this number in expanded form? So first I'm going to start with my ones. I'll take the little cube - I forgot to go back to the - oh - erase it because I made a mistake.

[10:00] Teacher: Okay so now I touch the arrow so that I could touch the cube and I'm gonna move it all the way to the ones. Look how cool that is. I'm going to go back. How many do I have to put here?

Students: Oh - oh - oh!

Student: Five.

Teacher: Very good. Three, four, five. How many tens am I gonna have to put?

Student: You're gonna have to put five.

Teacher: Ten fives?

Student: Two.

Teacher: Very good. So one, two. And how many hundred cubes am I gonna have to put?

Student: Um one.

Student: One.

Teacher: Do you see how we could keep taking more because I cloned it. So now what's your name again?

Student 1: Student 1.

Teacher: Student 1 said you could do it by saying how many ones there are, how many tens and how many hundreds, right? That's another way. So, what I'm going to do is I'm going to press the marker right in blue and how many ones do we have?

Student: We have five ones and two tens.

Teacher: Okay one second. Five ones.

Student: And two tens.

Teacher: And we have two tens.

Student: And we have one hundred.

Teacher: Very good. One hundred. That was another way. But now, to make this number in expanded form, what can I do? We need our plus signs because we're adding the sums of each value. So how many ones did we say we had?

Student: Two.

Teacher: Ones?

Student: Five.

Teacher: Very good. Five ones. Do you see how Teacher lines it up with the chart? How many tens did we have? Student 3, do you know?

Student 3: Um two.

Teacher: Two tens, but ten times two equals tw -

Student 3: Twenty!

Teacher: Very good and how many hundreds did we have?

Student 3: Oh -oh! One hundred.

[12:30] Teacher: So we have to put the number 100 and boys and girls we just expanded our number like a slinky and that's what you're gonna be -that's what you're going to do. So now I'm gonna give you a few numbers in standard form that means it's going to look like this and I want you to write it in expanded form and then I'm going to let you come up to the board and move the manipulatives into the chart and show the number in expanded form. Okay, everyone's going to have a chance to come up.

Student: Yay!

Student: Wait, do we make our own?

Teacher: No, I'm giving you a number. Okay? Ready? The first number is 132. I'll write the number for you in standard form on the paper as soon as I erase my work.

Student: Did it.

Teacher: Okay. Turn and talk to your partner and see if he has the same one.

[students chat]

Teacher: Did you write your number in expanded form? Very good. I'm gonna need Student 1 to come up. I love the way he's sitting. And check, do you see the arrow? Is the arrow blue? That means we could touch our first cube so what was our number, what did we say?

Student 1: 132.

Teacher: So - very good - so how many ones, Student 1, do we need?

Student 1: Two.

Teacher: Very good. And how many tens?

[15:00] Student 1: Three.

Teacher: Very good. And how many hundreds?

Student 1: One.

Teacher: Very good. Good job. Thank you, you could sit down. Student 4, can you come up and write the number and expanded form based on -Hold on. What do you have to do first? I'll press it for you because you can't reach. Okay, pick up your pen, pick a color. Very good. So now let's make - what's the first thing we have to do when we're writing our number in expanded form? What are the - what are those signs called?

Student: The plus signs.

Teacher: So we'll put our two plus signs. Very good, so how many ones are in the number 132? Very good, and how many tens do we have?

Student: 30.

Very good, and how many hundreds? I just raised-

Student: 1.

Teacher: Very good. Now, I'm going to write another number, and I want you to expand it. Ready? This one's a silly one. Ready? 222.

Student 1: What?

Student 2: It's the same number.

Teacher: It's not the same number - oh there's the same - So now you have to show me what cubes you need. I write in an expanded form. Okay, 222. You did it? Hi, so what's your name again?

Student 6: Student 6.

[17:25] Teacher: Student 6, why don't you come up and show me how to use the manipulatives to move them across the charts? Okay, so for 222 how many ones do we need to put? How many cubes?

Student 6: One, two.

Teacher: Very good. And how many tens do we have?

Student 6: We need 2 tens.

Teacher: Very good. One, two, great, and how many hundreds? Okay. Now how can we write this number in expanded form? Would you like to do it, Student 2? Okay sit down. So, now I'll press it for you because you can't reach. Press a color. Very good so now what signs do we need? What were the two signs? Good plus, that was the question I was asking you, 220 plus...

Student 3, can you read this number in expanded form for me? Right here. Very good. Read the sum, what are we adding?

Student 3: Um 200, and 20, and 2.

Teacher: So, 200 plus 20 plus 2 is the same as saying 200 and what?

Student 3: Twenty-two.

Teacher: Good, we're going to do one more and then we're gonna go to the iPads.

All students: Yay!

Teacher: Okay, so your number is 365. Do you know why I chose the number 365?

Student: Why?

Student: You're 65.

Teacher: I'm not 65 [laughs]. Oh no, now we're being silly. No one knows? 365.

Student: I wrote 55.

[19:55] Other teacher: 65.

Teacher: I'm gonna write it for you. 365.

Student: I did a plus to it.

Teacher: Very good.

Student: Me too.

Teacher: Turn to your partner and read your expanded form number. Okay, how would you read it in expanded form?

Student: 300 plus 65.

Teacher: Plus 60 plus five. Not plus 65 plus five. Very good. Student 3 would you like to come up to the board and do it? You're the only one that didn't come up yet. Come on. You could come do the expanded form once Student 3 goes up to the board, okay?

So Student 3 I'll press the arrow for you already because you can't reach. What do we grab first? You want to do the hundreds, okay. How many do we need? Let's look at our hundreds. Good. It's okay. Leave it like that, let them overlap.

Wow now how many tens do we need? Are we doing the right thing? I don't want to see any more markers in anyone's hands right now.

Good.

Good.

Good job.

You want to put it there.

Okay, don't worry about it. Look, you know you overlap this one. You have to be able to see it - right - I - just leave it like that. That's fine. Let's look at our ones. How many ones Student 3?

Student 3: 4, 5.

[22:48] Teacher: Good job. Just leave it like that, it's fine. Okay four. Five. Did yours look like that girls? They did a good job. Okay go ahead Student 1 show us how you expanded the number 365 and okay by the way girls you know why I chose 365? Go ahead Student 1, because there's 365 days in a year.

Student: It's a lot.

Student: Yes, it's very a lot.

Teacher: Good job. Student 6 how can - how would we read this number? Good job Student 1. 300 plus 60 plus five. Great 365. Okay so now we're gonna play the game that's the best part of all. Okay so this sheet is going to be at your desk. I set up a - sit down - I set up a station - Student 1 you don't have to clean that right now, pay attention up here.

Okay I set up the station with the iPads with the headphones with the activity sheet and with the technology guide. Everyone said at the beginning of class they all know how to use iPads so this should be easy then. Okay?

So here's the guide and today you learned how to read and write numbers and expanded form - three-digit numbers - you've done this for a while but we were just brushing up on it. Okay so what you're going to do is first you're going to look at the technology guide.

[25:00] Teacher: It's for you. It's taped on the side to the right of your activity sheet. Okay you know how to turn on your iPad you know how to swipe the screen from left to right because everyone says they have iPads at home or they've worked on iPads before.

And who knows how to find the safari? Great. When you tap the safari you- I have that you could click on the book icon in the top left corner but some of the iPads might already have the website up for you. Okay? Then you'll click expanded form game and then you'll press the sign that looks like this. It looks like a woman is speaking and she'll talk to you, you'll put your headphones on when you get up to this step and she'll talk to you and tell you what to do.

This - in this game you're going to fill in the missing number in the expanded form so remember we had a hundreds number sum, a ten sum, and a one sum? One of the sums are going to be missing and you're gonna type it in because you're - the game gives you - you'll press start, you'll press the listen to the question, and you'll use the digital calculator to enter like I said the missing number and then you'll write the answer to that missing number on your sheet. All of the questions that are in the game are on this sheet, so all you have to do is the number that you typed in, fill in that number here and then you're going to win coins.

[students gasp]

Teacher: Okay, if you get one wrong it's okay we all make mistakes go on and go to the next one - we're gonna review it. You mean a flaw. A flaw. There's ten of them, okay? It's easy. Piece of cake, right? You all know how to work your iPads. Yes. And then we'll go and then you'll answer these questions when you finish the game and then we're going to come back and go and we're going to play the game together, okay? So you can go to your - pick a station right now.

Okay you're walking, you don't need your smart - you can leave it on the floor.

Student: Okay.

Student: I need an iPad.

[27:30] Teacher: There's one right here, Student 4. Okay sit at one. Okay, very nice. I love how Student 4's sitting. Don't worry about that you could leave it flat, okay?

Student: Oh there is no password.

Teacher: No password. So what are you doing? Yeah, look. The technology's right here for you.

Student: Is this it? Yes it's there. You mean it starts?

Teacher: Here's the guide.

Student: You have to hit start?

Teacher: Look here, what does it say? Press start and then who do you have to press to hear the question? Good job. Where's your headphones? Put them on. Remember as you're answering the questions and winning coins you have to use your pencil to fill in the answer. Good job Student 2. You have to press the person. Press the green to hear it. Okay you're writing the answer as you do it. As you do it you're writing. Good job. As you fill in the missing number you're writing. Where's the pencil? There's pencils. So which one did you put in right here? Great, I love it. Okay. Okay, what do you press? Okay and then you have to press the next. Remember? It says it right here. Good job. If you have a question, raise your hands.

[30:15] Student: There's another one, do I go to the next page?

Teacher: Yes. Once you go to the - once you did three questions, you flip to the next page. Thank you for asking. Hi. Which one goes here? Good job, I love how everyone's working. Let's do the next page. Good job. Oh wow all of you are winning so many points - so many coins. I'm gonna have to bring more prizes for the amount of coins that you win next time.

Student: Do you have to put your name?

Teacher: Yes.

Student: Done.

[32:30] Teacher: You finished? Good job, so now, how do you exit? It says it right here. Good job, you're doing the right thing. And when it's done, flip over to answer these questions. Okay. You're done? You can take your headphones off. How do you exit? How do you exit? What does it say on the technology guide? Right here. Where do you - what do you press? You know if you said you know how to use the iPad. What do you press? Look at the picture, you know. Good job. Excellent, great. What is expanded form? What did we say expanded form was? When we take the number and ... ?

[students whisper amongst themselves]

Think about it. You’re done Student 4? How do you exit the game? Good job. Excellent. Girls, if you want you could take your work to the floor to answer. Student 6 uh why don’t you go with the answer to the floor? Student 5 too.

How did you show? How did you show?

[35:15] Teacher: All ten, right? Great. Now you can go over there and answer it. So what is expanded form? Go ahead. So what did we say expanded form was? You could give an example for that one. Yes. What was this called? Good job.

[helps more students]

[37:34] In a few minutes. we’re gonna go over the game. Look at all the points you’re getting. It’s okay.

[students whisper amongst themselves]

Teacher: Thanks. Why don’t you take this to the meeting area and we’re gonna go -we’re gonna play the game together. Teacher is gonna play with you. Okay so Teacher is doing it on the computer. Okay Student 3, okay thank you.

[points to cartoon character] Silly guy. Did everyone enjoy the game?

Student 1: Yeah.

Student 2: Yes.

Teacher: Did you like the game? Okay. So, I’m gonna press speaker.

Game: Press start when you are ready. You will earn spending points for every correct answer.

Teacher: I’m ready.

Game: The three-digit number is expanded as shown. What’s the value of the missing number [inaudible].

[39:55] Teacher: So what - Student 1 come up - what’s missing? Good job and what do we press? Yay.

[students clap]

Teacher: Student 4, come up. So it says the three digit number -

Game: The three digit number is expanded as shown. What’s the value of the missing number? 337. 300 plus 30 plus.

Teacher: Good job. Look at Student 4 she's the head of the teacher. Come up, Student 2. The three digit number.

Game: The three-digit number is expanded as shown. What's the value of the missing number? 586. plus 80 plus 6.

Teacher: Okay where's the five and the ... ? So what do we have to make that a five ... ? Good and we call that number five ... ?

Student: Hundred.

Teacher: And press - we forgot to press next. We can't go on. Look at that we've already got so many coins.

Game: The three-digit number is expanded as shown.

Teacher: Come on Student 5.

Game: What's the value of the missing number?

Teacher: Good job. Student 1, you want to come up again?

Game: The three digit number is expanded as shown. What's the value of the missing number? 898. 800 plus 90 plus.

Teacher: Go ahead, you're ahead over her, you don't even need her help.

Game: The three digit number is expanded as shown. What's the value of the missing number? 672. Plus 70 plus 2.

Teacher: Okay so what - what was missing?

Student: 600

Teacher: Great, what number are you typing in? Can you re - can you read the number in expanded form?

Student: 600 plus 70 plus 2.

[42:30] Teacher: Great, can you press next? Student 3. Student 3 it's your turn. You don't want her - you don't want her help? Okay so -

Game: What's the value of the missing number? 938 to 900 plus … plus 8.

Teacher: Good job. Can you read the number for me in expanded form?

Student: 3.

Teacher: 3. 30- what number did you type here?

Student: 30. What number is this?

Student: 900. 900 plus 30 plus 8.

Teacher: Good job. Okay three more left. Student 6, come up.

Game: The three digit number is expanded as shown. What's -

Teacher: Oh look at that she said - oh you got that one wrong. That one was -

Other teacher: She got it - she - okay -

Teacher: It's okay, maybe you hit the zero instead. It's okay. See what happens. You can hit a button. Go ahead Student 2 so which one's missing -

Game: The three-digit number is expanded as shown. What's the value of the missing number 771? … Plus 70 plus 1.

Teacher: Good, what do you have to press? Good would you read that, 771? In expanded form? Help her out Student 1. How would you have read 771 in expanded form?

Student: Uh 700, 70, 1. Plus... Plus 1.

Teacher: Good.

Student: Can I go?

Teacher: Okay, let Student 3 go one more time because he didn't get - he only went once. So what's missing Greg.

Student: Um.

Teacher: What's missing?

[44:51] Teacher: Good job. Excellent. Keep it up! Look, we got another coin. We got one - I just want to ask you one more question.

A few more. So what did we say expanded form was? What did we say expanded form is?

Student: Expanded form is to expand the word.

Teacher: Not the word.

Student: The number.

Teacher: The number. The number and how do we do that? How do we expand - stretch them out? By showing their what?

Student: Their numbers.

Student: Their values.

Teacher: Their values, very good. Their sum - the sum of their values. Girls pay attention. And how can we expand the number 103? This one's a tricky one.

All students: Oh! Oh!

Teacher: What goes in the hundreds? I don't think you - you can't do it on here, type it in. Very good sit down and we'll go to the tens.

All students: Oh! Oh!

Other teacher: Don't call out.

Teacher: Great. See that one was tricky, right? And Student 1 what has to go in the ones? And can you read the number in expanded form for me?

Student: 100 and -

Teacher: 100 plus. What do we have in the middle?

Student: Plus 0 plus 3.

Teacher: Very good and how did we do - how did we show expanded form in the game? What did we use? What was the tool that we used?

Student: The headphones.

Teacher: The headphones helped us and… ?

Student: And the iPad.

Teacher: And the iPad and what was in the game on the iPad? We wanted to help you out?

Student: Um the thing that we typed in that we were typing.

Teacher: What's that called?

[47:30] Student: It was called uh-

Teacher: With a c. It starts with a c. Also we use it in math. Well, you guys don't use it yet. Calculator. That was a calculator in the game. And the very last question. How did you... What tools did you use to help you expand it?

Student: We used the headphones on the iPad.

Teacher: Very good and what else to expand the number? We just said the tool.

Student: We um we used the calculator.

Teacher: The calculator in the game, very good and we also thought of our…

Student: Our..

Teacher: What were the things we used to help us?

Student: The cubes.

Teacher: The virtual manipulatives. Who liked playing this game?

Student: Me.

Student: Yeah.

Teacher: You liked it? Well, everyone did a great job.