TEACHING MATHEMATICS FOR SOCIAL JUSTICE:

HOW STUDENTS IN AN ALL-GIRLS INDEPENDENT SCHOOL SETTING

USE MATHEMATICS TO READ AND WRITE THE WORLD

by

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Date 22 May 2019

Submitted in partial fulfillment of the
Requirements for the Degree of Doctor of Education in
Teachers College, Columbia University

2019

ABSTRACT

TEACHING MATHEMATICS FOR SOCIAL JUSTICE: HOW STUDENTS IN AN ALL-GIRLS INDEPENDENT SCHOOL SETTING USE MATHEMATICS TO READ AND WRITE THE WORLD

Lucretia Glover

Teaching mathematics for social justice or critical mathematical literacy is said to have the potential of providing all students with equal access to mathematics education. The researcher used a case study approach to investigate the factors that affect female students' development of sociopolitical consciousness and social agency through reading and writing the world with mathematics (RWWM). In conducting a 3-week study in an all-girls high school in New York, NY, students (N = 5) completed three mathematics lessons that addressed issues relating to racial profiling, education versus income earnings, and HIV/AIDS in Canada. This study contributes new insight into female students' learning outcomes and dispositions. In doing so, this study contributes to the research relating to students' development of sociopolitical consciousness and sense of agency as students "read and write the world" with mathematics.

The results indicated that although some students had some previous knowledge of social justice issues, the incorporation of social justice educated them about the most pressing issues of today, thereby creating an increased awareness. Although the majority

of the participants revealed that they developed a motivation to learn mathematics through a sociopolitical lens, some participants expressed negative feelings as a result of a social justice awareness. When investigating how students develop sociopolitical consciousness through reading the world with mathematics, participants reported using data as evidence of the severity of current social justice issues, relating mathematics to the issues in the real world, and an overall effect of developing a strong connection with the social justice issues. In participating in this study, participants noted the following positive aspects that encouraged them to use mathematics to write the world: having hard proof or evidence on the existence of social injustice, making mathematics more understandable and interesting, and developing an understanding of the real purpose of statistics. As for what prevents participants from developing social agency, students indicated that the lack of teacher guidance on how to take actions added to their not being clear about how to “write the world” with mathematics.

DEDICATION

To my siblings, Alicia, Ludetia, and Julius, when I needed you to make personal sacrifices for me to continue my educational journey, you rose to the occasion.

And

To my nephews and godchildren, never let your challenges in life keep you from walking into your destiny.

ACKNOWLEDGMENTS

It is the insight and contributions of many, that has added to my wealth of knowledge in the field of mathematics education and research. Given that education is ever evolving, the instructional tools and resources provided to me have held to lay a solid foundation to build upon continuously. It is through my years of experiences as a mathematics instructor and my tenure as a full-time graduate student of mathematics education that I been fortunate enough to learn from the experiences of others in their learning and doing of mathematics.

I do not believe that I am the same mathematics educator and research that I was 20 years ago when I enter the education field nor the same as I was four years ago when I began my doctoral program. I am incredibly grateful to the faculty members in New York University Steinhardt's Department of Teaching and Learning for further developing my teaching skills in a global education system. In transitioning to the Mathematics Department at Teachers College, Columbia University, I was challenged to engage in both the theory and practice of how students learn mathematics. Both my educational and professional experiences have built upon my expertise in the field of teaching and learning in mathematics and mathematics education.

As such, I am indebted to the faculty and staff members within the Mathematics, Science, and Technology Department at Teachers College, Columbia University. First, to my sponsor and Department Chair, Professor Erica Walker, your patience has been beyond all understanding. During my doctoral journey, you encouraged me to take my

time and embrace the experience. Although at times, the role of a doctoral student was challenging, you continued to provide the appropriate amount of support that was needed for me to continue. I cannot thank you enough for everything that you have done. I learned so much from you, and I am honored to have you both as my advisor and sponsor. Second, to Professor Stuart Weinberg, the Director of Student Teachers, your leadership and mentorship during my tenure as a graduate student and supervisor of student teachers have allowed me to be of greater service to preservice mathematics teachers. In working alongside you, I was able to help preservice teachers to improve, analyze as well as rethink their practice around classroom management, classroom instruction, classroom effectiveness, and pedagogical practices. To Professor Felicia Mensah, you constantly reminded me that not only do we learn from trial and error, but we also learn from others. Thank you for providing me with a lens to look deeper into critical issues in STEM education that affect students' learning outcomes. In addition to helping me to find my 'critical voice', you also placed the structures in place for me to develop a greater understanding of qualitative research. To Professor Peter Garrity, I can only hope that I will be able to be as impactful as a mathematics educator as you have over the years. In having participated in your courses in mathematics teaching and learning, I learned that a teacher's mathematical and pedagogical knowledge should not be confined but shared. For the greatest gift of being a teacher is enjoying what I do and sharing the love of teaching with others. To Professor Nicholas Wasserman, it was a pleasure watching you teach real analysis and the use of proof and deductive reasoning in mathematics education. It is because of you that I am able to understand how to better

bridge my years of experiences as a mathematics teacher in a K-12 setting and post-secondary instructor.

To my friends that I met during my matriculation at Teachers College, I have enjoyed collaborating with you on projects, learning more about how our interest align as well as how can we best support each other through our tenure on campus. Your collegiate support and friendship added to my ability to work more effectively as a team member, friend, collaborative research in the field of education. Finally, to my family, I thank you for your much-needed support and understanding. As I matriculated through my doctoral process, you have assisted me by taking care of family-related responsibilities. It is because of your sacrifices that I was able to move forward with accomplishing my educational and career goals. As I look forward to closing this chapter in my life, I do so with the understanding that I could not have done so without your support.

L.G.

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Chapter I

INTRODUCTION

The Need for the Study

According to Wannocott (2006), a mathematics curriculum grounded in critical mathematics or teaching mathematics for social justice (TMSJ) provides students with opportunities to discuss issues in the mathematics classroom relating to "relations of power, resources, inequities, and disparate opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences" (Gutstein, 2006, p. 26; see also Wonnacott, 2011, p. 2). More importantly, when teachers are able to help students find the connection between their lived experiences as it relates to issues pertaining to social justice explored in the classroom, researchers and practitioners in the field believe that students "feel increasingly challenged and obliged to respond to the challenge" (Freire, 1970/2003, p. 81; see also Wonnacott, 2011, p. 2). Therefore, the more teachers engage in conversations around TMSJ (Gutstein, 2003; Gutstein & Peterson, 2005), the more students are presented with opportunities to gain greater insight into how their experiences are nested within society's assumptions, inequities, limits, and possibilities.

With mathematics being a gatekeeper to career and economic advancement (Cobb, 2004; Moses, Kamii, Swap, & Howard, 1989; Wonnacott, 2011), a social justice

curriculum in a mathematics classroom allows the teaching of mathematics to be more than just conversations around the manipulation of numbers. By exposing students to opportunities to utilize mathematics in order to increase and strengthen understanding of the sociopolitical context of their lives through its use, students are able to "strengthen their conceptual understanding and procedural proficiencies in mathematics" (Gutstein, 2012, p. 299). Therefore, as facilitators of learning, Bigelow (1990) asserted that whether teachers want to or not, they are political agents who assist in shaping students' understanding of the larger society. However, in moving towards creating classrooms that support such structures, Romberg (1992) proclaimed that the

> challenge educators now face is creating a curriculum filled with responsible social and political issues that will help students understand the complexity of such problems, help them develop and understand the role of mathematics in their resolutions, and allow them, at the same time, to develop mathematical power. (p. 435; noted in Gutstein, 2003, p. 44)

According to NCTM (2000), students exhibit mathematical power when

> students confidently engage in complex mathematical tasks...draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress...are flexible and resourceful problem solvers...work productively and reflectively...communicate their ideas and results effectively...value mathematics and engage actively in learning it. (p. 3)

Despite the recent shift in research on mathematics and mathematics teaching and learning that highlights social justice (Appelbaum & Davila 2007; Gutstein, 2006; Nolan, 2009), only limited research has investigated the pedagogical practices that incorporate a social justice curriculum in K-12 classrooms (Bigelow, Christensen, Karp, Miner, & Peterson, 1994; Bigelow, Harvey, Karp, & Miller 2001; Gutstein, 2003; Sylvester, 1994; Wonnacott, 2011). According to Gutstein (2016), the amount of research on enacting

critical mathematics or teaching and learning mathematics for social justice (TMSJ) in Kindergarten through Grade 12 is insufficient. Although Gutstein acknowledged theories exist around critical mathematics, he argued that its implementation into the K-12 curriculum "has not been well worked out in practice and developed over time" (p. 455).

When creating an educational platform that used social justice pedagogy embedded with equity when teaching mathematics, Gutstein (2003) found evidence that "students began to read the world with mathematics, to develop mathematical power, and to change their orientation toward mathematics" (p. 45). In formulating a definition for reading the world with mathematics, Gutstein adapted from Freire's (Freire & Macedo, 1987) definition of reading the world (i.e., learning to read text). As such, Gutstein's (2003) working definition of "read[ing] the world with mathematics" translates as students being able to

> use mathematics to understand relations of power, resource inequities, and disparate opportunities, between different social groups and to understand explicit discrimination based on race, class, gender, language and other differences. Further, it means to dissect and deconstruct media and other forms of representation. It means to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them. (p. 45)

When expounding on his research on reading the world with mathematics to include writing the world, Gutstein (2016) used practitioner research to assist high school students with "develop[ing] a deep understanding of mathematics through using it to study their social reality and create (and extend) their own analyses of contradictions in society and their lives prepare them to change reality as they saw fit" (p. 455). In writing the world with mathematics, Gutstein (2006) interpreted this as "using mathematics to change the world" or seeing oneself as capable of making change through a means of

developing social agency (p. 27). When analyzing how students read and write the world with mathematics (RWWM), Gutstein (2016) asserted that reading and writing the world is an interdependent, nonlinear event that can become "dialectically interwoven as people participate in their daily life and reflect on their actions" (p. 456). Therefore,

> merely understanding social reality, however, does not liberate people, though it is both a precondition for and effect of, consciously transforming the world. According to Freire, reading the world needs writing the world; therefore, allowing someone to change their current reality. In the process, people develop social and individual agency, whether or not they engage with mathematics. (p. 456; see also Gutstein, 2006a)

Within the research that currently exists on TMSJ, Gutstein (2006) and Wonnacott (2011) indicated that much of it had examined the effects of critical mathematics pedagogy on marginalized groups of students who are grouped in relatively homogeneous settings. Bryd (2016) and Civitillo, Juang, Badra, and Schachner (2019) echoed the concerns of Gutstein (2006) and Wonnacott (2011). According to Bryd, the research relating to culturally relevant teaching and pedagogy that examines the educational "outcomes for diverse classrooms, for other students of color, and for White students" (p. 3) is limited. From an international perspective, Civitillo et al. acknowledged that most of the research that focuses on critical relevant teaching, cultural diversity beliefs, and self-reflection has been done through the perspectives of the preservice teacher.

Although the literature around TMSJ is growing, Wonnacott was one of the first researchers to utilize practitioners' research to examine the effects of TMSJ on affluent students in a heterogeneous setting. In having students "write the world with mathematics," Wonnacott explored the use of social justice issues in mathematics to promote social agency in affluent, middle school students. Despite Wonnacott's attempt

to extend TMSJ to students who are less affected by social justice issues than perhaps marginalized subgroups, there is limited research that has examined the effects of TMSJ in a single-sex setting.

In examining the discrepancy of learning and doing mathematics as it relates to gender inequalities, recent research has suggested that women are less likely nationally to persist in advanced mathematics courses through college. Mata, Monterio, and Peixoto (2012) shared similar results to Etsey and Snetzler (1998) of a meta-analysis of 96 studies (n = 30,490) examining the effect of gender on students' attitudes to mathematics. The results revealed that females hold more negative attitudes toward mathematics. Mata et al.'s (2012) study on Grade 5 to 12 students revealed that motivation-related variables were the primary predictor of why female students over time developed a negative attitude towards mathematics. When investigating the attitudes that senior high school students demonstrated toward mathematics as well as any sex differences in these attitudes among male and female students in Accra, Nigeria, Asante's (2012) findings indicated a significant difference between male and female students. According to Asante, the factors contributing to male-female differences are the school environment, teacher attitudes and beliefs, teaching styles, and parental attitudes.

As female students transition into college to obtain postsecondary degrees, research findings indicated that students' "mathematics confidence is a strong domain-specific indicator of STEM major choices" (Moakler & Kim, 2014, p. 139). In accordance with Moakler and Kim (2014), "both academic and mathematics confidence evolve from learned experiences and develop an outcome expectation to choose a STEM major" (p. 139). Given these conditions, it becomes difficult for all "students to use

mathematics to critically analyze their world to ultimately promote a democratic society in which all get an opportunity to participate fully" (Bartell, 2013, p. 130).

Linking current research on how both females and the dominant subgroup learn to read and write the world with mathematics (RWWM) may lead to interesting implications on how teachers can develop a greater understanding of how to employ learning and TMSJ to assist with achieving equity in mathematics education for all students. In further examining how students, and specifically female students, in an affluent setting, develop mathematical power through their use of RWWW can perhaps lead to finding some components that change the disposition of female students towards mathematics.

Statement of Purpose and Research Questions

The purpose of this qualitative study was to understand what high school students in an all-girls independent school in New York think about social justice issues, and how it may affect their learning of statistical concepts. More specifically, this study took a case study approach to investigate the factors that affect female students' development of sociopolitical consciousness and social agency through reading and writing the world with mathematics (RWWM).

The following research questions were considered in this study:

1. How does the incorporation of social justice issues into a high school statistics curriculum at an all-girls independent school affect female students?

2. How, if at all, do students in an all-girls high school statistics course develop sociopolitical consciousness through "reading the world with mathematics"[1]?
3. What factors contribute to or prevent students in an all-girls high school statistics course from "using mathematics to write the world"[2]?

Procedures

In identifying participants for this research study, the researcher used criterion sampling to identify and recruit potential participants. As a criterion for participating in this study, participants needed to be high school seniors currently taking a high school statistics course at an all-girls independent school located in a northeastern city. In meeting the prequalification of taking a statistics course at the site of interest, participants needed to have taken mathematics coursework in Algebra I, Geometry, Algebra II, and Precalculus with a score of 70 or higher. With only having one high school statistics course currently in session for the academic year, the mathematics teacher was first identified and recruited by the researcher. After the researcher did an initial visit to the classroom to introduce the study, five out of the eight total students in the high school statistics course agreed to participate.

In conducting this study, the researcher observed participants' interactions with a TMSJ curriculum over a unit of study comprising three lesson units that pertained to one

[1] RWWM means using mathematics to understand relation of power, resource inequities, and disparate opportunities among different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation, as well as to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them (Gutstein, 2003c, p. 45).

[2] This means to use mathematics to change the world or to see oneself capable of making change through a means of developing social agency (Gutstein, 2006, p. 27).

or more specific mathematical tasks. Over a period of 3 weeks, the researcher analyzed students' interactions with three lesson units that centered around social justice issues relating to racial profiling, education versus income earnings, and HIV/AIDS in Canada. During such time, the researcher observed the participants' interactions with the lessons within the units of study per each 70 minutes class section. Before starting the lesson units, the mathematics teacher and the researcher met to discuss and revise the lessons to ensure alignment with the current mathematics curriculum. Given that lesson units were adapted to meet the high school mathematics curriculum, all students enrolled in the course completed all classroom activities. Non-participants were not, however, required to participate in the research study. As such, required coursework to be completed by non-participating students was not shared with the researcher.

During the study, students were asked to interpret and make deductions from statistical data, charts/graphs, and text that addressed social justice issues. Surveys, coursework, interviews, and post-lesson reflections served as the primary data source for this research study and therefore aided in the process of triangulation. Field notes and classroom observations served as secondary data sources that helped to provided clarity around findings. In looking for patterns and relationships within the collection of data, data were transcribed and coded. In examining the data, the researcher looked for common patterns and themes within the given data set. With assistance from the qualitative data analysis software NVivo, the researcher used Braun and Clark's (2006) modified thematic analysis steps to analyze the data. This six-step process was completed four times for all primary data sources.

Addressing the Research Questions

To address the research questions in this study, the researcher administered questionnaires, conducted classroom observations, collected students' work and written reflections, and conducted interviews. During the observational period, the researcher took field notes in accordance with Creswell's (2007) observational protocol. After having first debriefed students about the purpose of the study and the expectations for the unit of study, an initial questionnaire was administered using the online platform Qualtrics at the start of class. The web-based questionnaire focused on questions about each interviewee's personal and family background, understanding of mathematics/ statistics, attitude towards or interest in mathematics/statistics, and learning and teaching mathematics for social justice.

Fieldnotes were taken daily and recorded participants' comments, classroom participation, and involvement within each lesson. Students' written and oral work was "used to determine students' level of engagement, and for the development of classic mathematical knowledge, critical knowledge and social agency" (Wonnacott, 2011, p. 33). Also, students' oral and written feedback served to examine their sense-making of mathematical representations in relation to "sociopolitical reality" and their ability to read the world with mathematics.

An assessment rubric adapted from *Middle Years Programme (MYP) Mathematics Guide* (2009) developed by the International Baccalaureate (IB) (see Appendix L) Organization was used to assess students' knowledge and understanding of mathematics concepts, communication in mathematics, and ability to reflect on the mathematics used. In promoting "the development of students who are knowledgeable,

inquirers, communicators, and reflective learners" (p. 2), the IB Organization partners with schools, government and international organizations. Currently, the *MYP Mathematics Guide* uses a scale ranging from 0-8 to assess students on the following four objectives as stated below:

- **Criterion A.** ***Knowledge and understanding promotes learning mathematics*** with understanding, allowing students to interpret results, make conjectures and use mathematical reasoning when solving problems in school and in real-life situations.

- **Criterion B.** ***Investigating patterns support inquiry-based learning.*** Through the use of investigations, teachers challenge students to experience mathematical discovery, recognize patterns and structures, describe these as relationships or general rules, and explain their reasoning using mathematical justifications and proofs.

- **Criterion C.** ***Communication in mathematics*** encourages students to use the language of mathematics and its different forms of representation, to communicate their findings and reasoning effectively, both orally and in writing.

- **Criterion D.** ***Reflection in mathematics*** provides an opportunity for students to reflect upon their processes and evaluate the significance of their findings in connection to real-life contexts. Reflection allows students to become aware of their strengths and the challenges they face as learners. (MYP, 2008, p. 2)

At the end of Unit 1, a face-to-face interview with each participant was conducted separately in an office within the school. Each interview took place at the close of Lesson 1 and lasted approximately 15 minutes. Interviews were used to clarify previous comments stated by participants on their free write activity in Lesson 2 in Unit 1 or on their post-lesson reflection that was given at the end of the unit. In addition to having students complete three lessons, students were to write a post-lesson reflection on how they were, if at all, affected by the lesson and if the lesson left them feeling inclined to take action. At the end of the study, an exit survey was used to see if, at all, (a) students'

perception of the use of mathematics had changed; (b) students were affected by doing the activities, or (c) students felt they were placed in a position in which they could use mathematics to take action or write the world.

Chapter II

REVIEW OF THE LITERATURE

At the start of the 21st century, school mathematics led with the charge of transitioning from a dualistic model "to a singular focus on a significant common core of mathematics for all students" (National Research Council [NRC], 1989, p. 11). To combat the potential challenges associated with preparing all students throughout their mathematics matriculation, the NRC proposed that a more student-centered approach replace the previous authoritarian model of teaching mathematics. With this intention, the teaching of mathematics would transition from a "transmission of knowledge" to teachers creating opportunities for "stimulation of learning." In recent years, mathematics researchers and mathematics practitioners have shifted their focus to examine the sociopolitical turn that is currently taking place in mathematics classrooms.

The primary purpose of this chapter is to highlight the landscape in mathematics education, explaining possible issues and outcomes of embedding social justice pedagogy and teaching into mathematics courses. Given that the literature on female students' learning of teaching mathematics and statistics for social justice is scant, this literature review gives prominence to some research around learning outcomes and disposition of female students towards learning and doing mathematics. This chapter begins by providing a review of the pedagogy and teaching practices associated with teaching

mathematics for social justice and its historical ties to critical mathematics, critical race theory, and culturally relevant pedagogy and teaching. Moreover, this chapter includes research on how students develop sociopolitical consciousness and social agency through writing and reading the world with mathematics (Gutstein, 2005). Following this discussion, the researcher discusses factors that impact female students' learning and dispositions towards mathematics, which might impede their ability to learn through a sociopolitical framework.

The Sociopolitical Turn in the Research on Mathematics Education

Mathematics education over the last two centuries points to how research on mathematics education has been molded by internal forces within the educational research arena (Kilpatrick, 1992). However, that is not to say that extraneous forces have not played a significant role in mathematics teaching and learning in conjunction with what areas of research in mathematics education take precedence in the field. Given that mathematics education is a relatively new discipline, Kilpatrick stated that mathematics is traditionally associated with developmental psychology and mathematics as a discipline. In confirming Kilpatrick's statement, Cobb (2004) identified the origins of mathematics education and mathematics research stemming from not only developmental psychology but cognitive development as well. Be that as it may, historically mathematics education and the research in mathematics education has struggled in the area in trying to establish its own identity (Kilpatrick,1992). As a result of an emergence of definitions, Wittmann (1998) considered mathematics education to be a field of study which investigates "the teaching of mathematics at all levels, including its premises, goals and [the] societal

environment" (p. 87). As a result, mathematicians and mathematics educators have shifted their attention to examine the "how and what mathematics is, or might be, taught and learned in school" (Belbase, 2010, p. 5; see also Kilpatrick, 1992). For that reason, it becomes even more critical to examine the way students formulate mathematical arguments to justifying their mathematical work.

During the 1980s, research in mathematics and mathematics education transitioned from examining how individuals acquire knowledge to now looking at the theoretical framework for deciphering the social origins of consciousness and knowledge (Lerman, 2004). Thus, researchers in the field of mathematics and mathematics education have shifted to look deeply at how students acquire and utilize their knowledge to make meaning of mathematics through lived experiences. In examining literature that appeared in 1988, Lerman referenced several textbooks (e.g., Bishop, 1988a; Lave, 1988; Walkerdine, 1988) which were significant in the wake of the social turn in mathematics education research. For example, in his book *Cognition in Practice*, Lave (1988) used studies to explore the mathematical practices of grocery shoppers and dieters to challenge cognitivism and transfer theory in relation to learning mathematics. Lave's findings suggested that in the process of learning and employing learning strategies and decision-making procedures, grocery shoppers and dieters can be considered "as situated within, and as products of, those social situations" (p. 9). Therefore, during the course of acquiring learning strategies and decision-making procedures, their experiences should be considered as "part of who one is 'becoming' in that practice" (p. 9). As a result of such interlocking factors relating to the community, Lerman (2000) argued that this gave rise to a new theoretical shift in academia whose sociocultural theories origins are rooted in

anthropology, sociology, and cultural psychology. More recent literature refers to this shift in theoretical perspectives in mathematics education as a sociopolitical turn (Gutiérrez, 2013).

In the article *The Sociopolitical Turn in Mathematics*, Gutiérrez (2013) argued for the need for and importance of why educators and practitioners alike should take a sociopolitical turn. Gutiérrez adopted the position that a sociopolitical turn in theoretical perspectives allows one to "see knowledge, power, and identity as interwoven and arising from (constituted within) social discourse" (p. 40). Gutiérrez believed that the purpose is "not just to better understand mathematics education in all of its social forms, but to transform mathematics education in ways that privilege more socially just practices" (p. 40). Although Gutiérrez did not believe that taking on issues relating to identity and power can be easily incorporated into the educational setting to change large-scale policy initiatives or serve as prescriptions for improving classroom practices, adopting a sociopolitical stance allows for "uncovering the taken-for-granted rules and ways of operating that privilege some individuals and exclude others" (p. 40). In further examining the challenges of theorizing from the social turn, Lerman (2000) stated that a major challenge involved accounting for individual cognition and differences in learning and doing mathematics as it relates to a product of social activity.

In the *History of Mathematics Teaching and Learning: Achievements, Problems, Prospects,* Karp and Furinghetti (2016) alluded to how the manifestation of mathematics education in retrospect to the influences of society is subject to the same social factors that determine the specific characteristics of education in general. In looking at the historical features of mathematics education, Karp and Furinghetti emphasized that since

segregation exists in education, mathematics education will follow suit. In providing more context around this statement, Karp and Furinghetti explained how the state and society process human experiences as having secondary importance to mathematics education. As with the labor market, the belief system of the dominant society and objectives pushed by a political system or the state has significantly shaped education; Karp and Furinghetti referenced that mathematics education is subject to these social influences in a similar manner.

For example, even after the Supreme Court ruling of *Brown v. Board of Education* (1954) ending *de jure* school segregation, historical events indicated that tracking continued to segregate school classrooms. According to Tate and Rousseau (2002), schools in the United States have typically operated under a tracking system that separates students according to curricular or ability levels. This, then, has led to systemic issues in which African American and poor youth were deprived of equal educational opportunities. Tate and Rousseau argued that because many middle and high schools incorporate some form of tracking, "the quality of teaching varies cross tracks" (p. 276). Tate and Rousseau also stated that although schools in the United States may offer two different geometry courses with the same course title, students in these courses are enrolled or arranged by academic ability. For instance, despite the same course title and the same number of course credits, the mathematical concepts as well as to the degree in which they are covered may vary. Tate and Rousseau referred to this as a hidden form of tracking or segregation that is more difficult to identify.

Moreover, with mathematics being a gatekeeper to career and economic advancement (Cobb, 2004; Moses, Kamii, Swap, & Howard, 1989), the idea of

segregation within mathematics education becomes all too real. In advancing the idea of mathematics for all (Allexsaht-Snider & Hart, 2001), the idea around equity for all students is more challenging. Depending upon the person asked, the term *equity* itself may carry with its varied definitions that present an array of possible outcomes for students' academic success. With equity in schools being one of the most challenging and problematic issues to achieve in mathematics education (Bartell, 2013; Forgasz & Rivera, 2012; NCTM, 2000), social justice issues revolving around poverty, wages, homelessness, exploitation, and discrimination continue to emerge in the forefront of the classroom. Despite the efforts of policymakers and educators to address shortcomings in mathematics education by raising mathematics standards and encouraging more mathematical thinking among students (Sherin, 2002), researchers have suggested that true equality in mathematics education, particularly for those who are in marginalized educational settings, begins with adjusting the mathematics curriculum to make it more relevant to students' lives (D'Ambrosio, 1997; Gutiérrez, 2002; Gutstein, 2003, 2005; Moses & Cobb, 2001; Moses, Kamii, Swap, & Howard, 1989). In doing so, mathematics researchers and practitioners have adopted curricula infused with culturally relevant teaching, critical mathematics, and teaching mathematics for social justice features.

Culturally Relevant Teaching and Pedagogy in Mathematics Education

Ladson-Billings' (1992c) working definition of culturally relevant or responsive teaching in mathematics education has been translated as being a pedagogy that resists oppression. According to Ladson-Billings (1995a), culturally relevant or responsive teaching resides in the following three propositions: "(a) students must experience

academic success; (b) students must develop and/or maintain cultural competence; and (c) students must develop a critical consciousness through which they challenge the status quo of the current social order" (p. 160). In 1995, Ladson-Billings added to the definition by stating that it is also "theoretical model that not only addresses students' achievement but also helps students to accept and affirm their cultural identity while developing critical perspectives that challenge inequities that schools (and other institutions) perpetuate" (p. 469). Be that as it may, her most current definition (2009) is "a pedagogy that empowers students intellectually, socially, emotionally, and politically by using cultural referents to impart knowledge, skills, and attitudes" (p. 20).

According to Ladson-Billings (2009), a student's culture is taken into consideration during culturally relevant teaching in order to preserve a student's culture while "transcend[ing] the negative effects of the dominant culture" (p. 19). Given that African and African American students deal with the dilemma of navigating through the demands of their academic studies while also demonstrating cultural competence, Ladson-Billings (1995) argued that "culturally relevant pedagogy must provide a way for students to maintain their cultural integrity while succeeding academically" (p. 476).

Although it may be challenging for teachers who possess different cultural backgrounds than their students (Murrell 1997; see also Gutstein, 2003), Ladson-Billings (1995) further explained teachers who can recognize social inequities and their causes must go beyond just encouraging students academically. As a means of encouraging students to exhibit cultural competence, Ladson-Billings indicated that teachers must assist their students in being able to identify, understand, and be critical of current global social inequities. Given that prospective teachers lack understanding of social inequities

and continually reject information about social inequities, teacher educators (Grant, 1989; Haberman, 1991; King & Ladson-Billings, 1990; Zeichner, 1992) have suggested that colleges must begin to recruit particular types of students into their educational teaching programs (Ladson-Billings, 1995, 1999; Sleeter, 1985, 2000). Giroux and Simon (1989) suggest that

> teachers who meet the cultural critique criteria must be engaged in a critical pedagogy which is: a deliberate attempt to influence how and what knowledge and identities are produced within and among particular sets of social relations. It can be understood as a practice through which people are incited to acquire a particular "moral character." As both a political and practical activity, it attempts to influence the occurrence and qualities of experiences. (p. 239)

In arguing a different approach to improving the quality of education for African American students, Ladson-Billings (2000) suggested that it is the practices of teacher preparation programs that have failed teachers. In arguing this position, Ladson-Billings stated that colleges and universities are not successful at coaching preservice teachers on how to teach African American students effectively. While looking at the uniqueness in the culture that students bring with them into the classroom, Ladson-Billings investigated how preservice teachers can provide instruction more effectively to African American students, focusing primarily on the social and cultural experiences and historical events in the history of African Americans. In emphasizing the need for preservice teachers to be equipped to manage the diverse issues that arise in the classroom, Ladson-Billings discussed how she worked with her preservice teachers in "understanding the ways oppression has worked against many groups of people based on their race, culture, class, gender, disability and sexual orientation" (p. 208). As did Carter (2003) and Franklin and Moss (1988), Ladson-Billings referenced the role of slavery and racism played in shaping the social and cultural experiences of African Americans. As such, Ladson-Billings

alluded to how the U.S. history of racial hierarchy (King, 1994) has assisted in creating the obstacles that teachers experience in the classroom. In combating these barriers, Ladson-Billings (2000) argued that "a more systematic comprehensive approach is needed" (p. 208). To assist preservice teachers in better meeting the needs of their diverse population, Ladson-Billings offered a more comprehensive approach which included the use autobiography, restructured field experiences, situated pedagogies, and returning to the classroom of experts.

Autobiography

Grounded in the research of Gomez and Tabachnick (1992), Ladson-Billings (2000) argued that as preservice teachers conduct their teaching practicum, they should complete personal/cultural autobiography (Hollins, 1990). In having preservice teachers write autobiographies, Jackson (1992) and Ladson-Billings (2000) believed that this would assist preservice teachers with reflecting on their experiences of working in a diverse classroom setting. In expanding this argument, Howard (2003) argued for institutions to go beyond recommending critical reflecting into conversations around issues of equity and social justice. According to Howard, institutions must take part in a critical reflection about the culture and race of teachers and their students. Given this, a critical reflection needs to be completed before an educational system incorporates issues of equity and social justice into its curricula and practices. As classrooms in the United States transition from being predominantly homogeneous—specifically, mostly White, female, and middle class—to being predominantly heterogeneous in that classrooms begin to include people of color as well as people from various socioeconomic backgrounds, Howard argued for teacher educators to provide new teachers with the

appropriate tools to best assist them in teaching a diverse student population. In short, "teacher educators must be able to help preservice teachers critically analyze important issues such as race, ethnicity, and culture, and recognize how these important concepts shape the learning experience for many students" (p. 195).

Restructured Field Experiences

In getting to know the "whole student" and the characteristics that amplify the differences among students, Ladson-Billings (2000) discussed how some institutional programs had reconstructed their field experiences programs to provide preservice teachers with practicum experiences in either a diverse classroom or diverse community that offers an immersion (e.g., see Mahan, 1982; Noordhoff & Kleingfeld, 1991) into THE day-to-day lives of their future students. Ladson-Billings argued that as new teachers take employment in urban school settings highly populated with African American students, teachers may find their previous training in White middle-class communities prove to be a mismatch to their current teaching placement. In furthering the argument for restricted field experiences, Ladson-Billings (2000) noted that diverse placements should be "accompany[ied] with the requisite understanding of African-American cultural and cultural practices" (p. 209).

Situated Pedagogies

Aligning with culturally specific pedagogies, Ladson-Billings (2000) "ask[ed] educators to think more carefully about the relationship of teacher preparation to the communities in which they are located and the school populations that their graduates are likely to serve" (p. 210). For example, urban schools are generally characterized as being

schools that are high needs and perform poorly and also have difficulty hiring staff (Lipman, 2011). In this case, rather than a recently hired teacher entering the educational workplace with an open mind, school labeling has the potential of creating a tainted outlook that can lead to preconceived judgments that further the injustice is proliferating in U.S. schools. Although there are advantages in making school districts' standardized assessments and performance results readily available on the internet, this further aids in identifying the educational gaps and lack of economic resources in disadvantaged or more marginalized areas. Might having access to this readily available data actually create additional biases for teachers entering the workplace?

Returning to the Classrooms of Experts

Regarding returning to classrooms of experts, Ladson-Billings (2000) referenced how teachers can employ pedagogical and programmatic practices such as academic achievement, cultural competence, and sociopolitical critique to assist with creating a classroom that fosters the academic achievement of African American students. As a result of implementing such strategies, teachers are able to both establish a classroom with established routines as well as drive high standards with expectations of students learning a rigorous and challenging curriculum. While regularly communicating these expectations to students, Ladson-Billings argued that some teachers implement constructivist approach (see also Fennema, Carpenter, Franke, & Carey, 1992) by including students' prior knowledge as a part of building blocks for structuring their newly learned knowledge. Moreover, Ladson-Billings discussed that these teachers employed sociopolitical critique as a means of assisting students with comprehending how social practices and structures that currently exist replicate inequities. In referencing

literature (Boggs, 1974), Ladson-Billings stated that "students must be challenged to ask questions about was that whole groups of people are systematically excluded from social benefits" (p. 201). Finally, Ladson-Billings (2000) shared that culturally relevant teachers foster forms of cultural competence into the classroom by helping, in this case, African American students understand the "aspects of their culture that facilitate their ability to communicate and relate to other members of their cultural group" (p. 210; see also Gay & Baber, 1987).

Culturally Relevant or Responsive Teaching in Practice

In a review of the literature explaining how students interact with a culturally relevant or responsive curriculum, Sealey-Ruiz (2007) conducted a study "to examine the significance of a culturally relevant curriculum to a group of Black female adult learners in a freshman composition course at a liberal arts college" (p. 45). According to Sealey-Ruiz, the study "probed how these women experienced a curriculum that was centered on their life experiences and sociohistorical backgrounds" (p. 45). Using a culturally relevant curriculum to look at the experiences of adult learners in relation to learning through a sociopolitical lens, Sealey-Ruiz conducted a qualitative study to answer the following research questions: (a) "How do Black female adults respond to a curriculum centered on their cultural ways of knowing?" and (b) "What happens when these women are instructed using a culturally relevant curriculum?" (p. 45).

For 13 weeks during a college semester, 15 black female adult learners attending a liberal arts college in Harlem, NY participated in journal writing, written assignments, note taking, class discussions, and teacher conferences in their freshman composition

course. According to Sealey-Ruiz, the study aimed at probing how the participants responded to a curriculum that was centralized around students' lived experiences and sociohistorical backgrounds. Although the results of this study supported that using a culturally relevant curriculum with Black female adult learners can strengthen their overall learning experience, the results of this study indicated that three significant themes emerged from the data. First, participants in the study showed a need for language validation with respect to the language spoken within their community. Second, by using a culturally relevant curriculum, participants were able to foster a positive self-identity and group identity. Third, Black females in the study were able to use the curriculum as a "weapon against oppression" (Asante, 1998, p. 68, as cited in Sealey-Ruiz, 2007, p. 56). As a result, participants reported that they were able to transition from feeling powerless to being empowered to affirm goals. Based on the findings presented in her research study, Sealey-Ruiz concluded in her argument that "using a CRC [culturally relevant curriculum] is the first step in creating a classroom that includes all voices" (p. 58).

Byrd (2016) found three significant limitations in teaching through a lens of culturally relevant or responsive pedagogy. The first limitation addresses the lack of quality empirical studies that referenced the effects of employing culturally relevant teaching (Sleeter, 2012). Second, rather than taking into account students' perspectives, most of the research in this area took the approach of examining the perspective of the researcher or teachers (Howard, 2001). The primary aim of culturally relevant teaching helps African American students in identifying with African and African American culture that then assists in their development of a "relevant black personality" (Ladson-Billings, 2009, p. 19). Be that as it may, other researchers have employed culturally

relevant teaching and pedagogy with other racial groups as well (Au & Jordan, 1981; Byrd, 2016; Gutstein, 2003; Gutstein, Lipman, Hernandez, et al., 1997; Mohatt & Erickson, 1981). According to Byrd, the third limitation of studies addressing culturally relevant teaching in predominantly Black classrooms in similar settings (Morrison et al., 2008) is a limitation in itself.

To debunk the three limitations mentioned above, Byrd (2016) recruited 315 sixth to twelfth grade students from diverse racial backgrounds (62% female, 25% White, 25% Latino, 25% African American, and 25% Asian) to gauge their perspectives on the advantages of a culturally relevant pedagogy and positive socialization between school and race. In using a quantitative methodology approach to explore how "culturally relevant teaching and school racial socialization are associated with the academic and attitudinal outcomes of diverse middle and high school students" (p. 1), Byrd asked participants to complete surveys to gain additional information about their experiences. Participants were also asked questions pertaining to culturally relevant teaching, cultural socialization, and their experiences with learning about other cultures and racism. In addition to measuring participants' perceptions of culturally relevant teaching, Byrd's study aided in distinguishing between general constructivist and culture-based practices.

After conducting a linear regression, Byrd's (2016) findings supported her original hypothesis: "Perceptions of more constructivist teaching practice, cultural engagement, cultural socialization, promotion of cultural competence, support for positive interaction and critical consciousness socialization would be associated with better academic outcomes and more positive racial attitudes" (p. 3). Although she found that culturally relevant teaching correlated only weakly or moderately with the majority

of outcomes, as did racial socialization, the conclusion was that diverse teaching practices do lead to better outcomes, thereby upholding the literature on "student-centered, authentic instruction" (p. 6, see also Newmann et al., 1996; Saye & Social Studies Inquiry Research Collaborative, 2013). The findings in this study suggested that even with benefits of incorporating race and culture into the classroom, the noted limitations in this study cannot be ignored. Rather than just considering culturally relevant teaching as "good teaching" (Ladson-Billings, 1995a), the pros and cons of culturally relevant teaching must be considered before its implementation.

General Overview of Teaching Mathematics for Social Justice

With an increase in research in mathematics education (e.g., Freire,1970/2003; Gonzalez, 2009; Ladson-Billings, 1995) that looks more deeply at how to incorporate the social issues that impact mathematics teaching and learning, there is an increase in literature around critical mathematics or teaching for social justice (e.g., Gonzalez, 2009, Gutstein, 2003). As such, an examination of mathematics education from a sociopolitical framework is gaining popularity among researchers and practitioners and has been propelled to the forefront. This then has given rise to critical mathematics or teaching mathematics for social justice (TMSJ). Mathematics research developed from the birth of a sociopolitical turn is grounded in theoretical perspectives relating to critical race theory, critical mathematics/teaching mathematics for social justice, and culturally relevant pedagogy. Teaching for mathematics or critical mathematics is grounded in the work of culturally relevant teaching and pedagogy.

Teaching mathematics for social justice consists of students learning significant competencies in mathematics such as mathematical power (Moses & Cobb, 2001). According to Gutstein (2007), TMSJ provides students with an opportunity to ideally change their perception towards mathematics. Gutstein (2007) further argued that as students transition from viewing mathematics as a series of algorithms, students will begin to "create meaning and make sense of human and social experiences" (p. 4). Lastly, Gutstein believed that through acquired mathematical knowledge and application of learned mathematics skills, students can develop sociopolitical consciousness and a sense of social agency. Through such development, students can begin to see themselves as capable of effecting change against social injustice.

The research on teaching mathematics for social justice indicates that there is no uniform definition (Gonzalez, 2009). That being the case, the phrase *teaching mathematics for social justice* (TMSJ) can be used interchangeably with critical mathematics (Frankenstein, 2001), social justice mathematics, and criticalmathematical literacy (Freire & Macedo, 1987; Stinson & Wagner, 2012). Gutstein (2003) viewed teaching and learning of mathematics for social justice as a way of providing students with an educational platform to use mathematics to explain various phenomena that take place in their lives or the larger world around them. Similarly, Gutiérrez (2007) interrupted critical mathematics as "mathematics that squarely acknowledges the positioning of students as members of a society rife with issues of power and domination...[and] takes students' cultural identities and builds mathematics around them in ways that address social and political issues, especially highlighting the perspectives of marginalized groups" (p. 40).

In decoding teaching mathematics for social justice, the term *social justice* in recent literature has suggested that "contemporary notions of social justice coexist with [when an] expression of human rights, fairness, and equity is exercised" (Nelson & Creagh, 2013, p. 102). When conducting research that examined the Australian context of social justice in education and the issues relating to infusing social justice in education, Sturman (1997) aligned his viewpoint of social justice to both individual life chances and experiences of different societal groups. Sturman noted that one key aspect of theorizing over social justice is present in the recent debates on equity and equality. He further explained that given that social justice is not clearly defined, it translates into meaning "equal opportunities" or "equity."

Frankenstein (2001) explained that in order to examine data from a critical lens, a curriculum should introduce students to four goals of a critical mathematical literacy curriculum. First, in strengthening students' development and understanding of mathematics, Frankenstein indicated that students could minimize and disrupt the dichotomy between the instruction of mathematics and learning it. In understanding mathematics, Frankenstein believed that students should consider their cultural interactions in conjunction with how they develop. To deepen students' understanding of even the simplest of mathematical topics, Frankenstein suggested that the process of teachers asking students thoughtful questions can assist students with a better understanding of mathematics. Second, Frankenstein argued the need for "students to learn how mathematics skills and concepts can be used to understand the institutional structures of society" (p. 58). As students use numerical description and calculation to support their understanding and interpretation of raw data further, students are more

inclined to understand the politics of mathematics through reading the world. Third, with students developing an understanding the politics of mathematical knowledge, Frankenstein stated that they would begin to understand the presence of the political struggles and decisions that need to be made, around which researchers collect data. Fourth, the charge should be for students and teachers to "exchange our ways of thinking with each other and look together better ways of approaching the decodification of an object" (p. 62), thus creating greater possibilities around understanding the politics of knowledge.

According to Gonzalez (2009), the following four components guided her research in TMSJ and work with teachers. The first component is structured around giving students access to high-quality mathematics. Gonzales argued that "mathematics is needed to be a full participant in society" (p. 24). The second component of TMSJ "involves building upon the experiences of students from marginalized communities, while exploring issues of social justice through mathematics" (p. 25). The third component entails the "use of mathematics as a critical tool for understanding social life; one's position in society; and issues of power, agency, and oppression…[that is,]… critical mathematics" (p. 25). Finally, the fourth component of TMSJ "is the use of mathematics to radically reconfigure society so that it might be more just" (p. 25).

Reading and Writing the World With Mathematics

In earlier literature, critical mathematics and teaching mathematics for social justice are referred to as criticalmathematical literacy (Freire, 1970; Freire & Macedo, 1987). In this case, Freire (1970) and Freire and Macedo (1987) considered

criticalmathematical literacy and critical adult education to be the underlying context that is embedded in one's ability to read the world. During an interview, Freire (1985) was quoted as saying that "the act of reading cannot be explained as merely reading words since every act of reading words implies a previous reading of the world and subsequent rereading of the world" (p. 18). Freire argued that reading the world or "reading" reality goes beyond individuals developing the ability to read the world. According to Freire, people must also be able to write or rewrite their current reading reality. It is through the act of transformation that Freire believes taking a conscious practical action is at the core of literacy.

Using the guiding principle of reading the word, as outlined by Freire (1970) and later in research conducted by Freire and Macedo (1987), Gutstein modified Freire's working definition of reading the world or learning to read text (Freire & Macedo, 1987) and applied it to teaching and learning mathematics from a sociopolitical framework. As a result, Gutstein (2003c) defined reading the world with mathematics, as being able

> to use mathematics to understand the relation of power, resource inequities, and disparate opportunities, between different social groups and to understand explicit discrimination based on race, class, gender, language and other differences. Further, it means to dissect and deconstruct media and other forms of representation. It means to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them. (p. 45)

When creating an educational platform to promote a more just society through the use of social justice pedagogy embedded with the concept of equity work in mathematics teaching, Gutstein (2003) found evidence that suggested that "students began to read the world with mathematics, to develop mathematical power, and to change their orientation

toward mathematics" (p. 45). In defining mathematical power, Gutstein relied on NCTM's Principles and Standards (2000) working definition:

> Students confidently engage in complex mathematical tasks...draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress...are flexible and resourceful problem solvers...work productively and reflectively...communicate their ideas and results effectively...value mathematics and engage actively in learning it. (p. 3)

In adapting Freire's reading of the *word* to fit within the context of mathematics teaching and learning of social justice, Gutstein (2003) translated the reading of the *word* to imply that it is the act of "developing mathematical literacy and mathematical power" (p. 45). Gutstein's (2006) interpretation of writing the world with mathematics can best be translated as "using mathematics to change the world" (p. 27). In doing so, it positions a person to utilize mathematics to act against as well as bring attention to social injustices. It is during the process of writing the world that Gutstein believes students develop and employ social and individual agency. When analyzing how students read and write the world with mathematics (RWWM), Gutstein (2016) asserted that reading and writing the world can be considered as interdependent, nonlinear events that can become dialectically interwoven when people participate in their daily life and reflect on their actions (see Figure 1). Therefore,

> merely understanding social reality, however, does not liberate people, though it is both a precondition for and effect of, consciously transforming the world. Reading the world needs writing the world, for Freire, meant to change reality. In the process, people develop social and individual agency, whether or not they engage with mathematics. (p. 456; see also Gutstein, 2006)

In developing his theoretical framework for RWWM, Gutstein explained it through the learning and doing of mathematics that students develop a heighten

sociopolitical awareness. As a result, students are therefore able to utilize acquired mathematical skills to shape or change society. Therefore, it is through this dispositional shift that Gutstein believed the process that students begin to identify some of the limitations and usefulness of mathematics in being able to read and write the world.

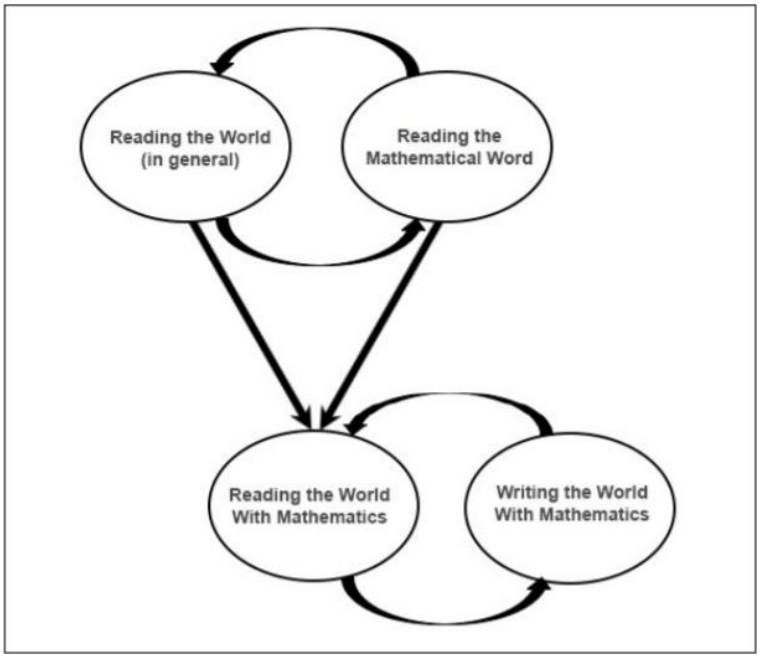

Figure 1. Relationship between reading the world and RWWM

Source. Retrieved from E. Gutstein. (2016). "Our issues, our people—Math as our weapon": Critical mathematics in a Chicago neighborhood high school. *Journal for Research in Mathematics Education*, *47*(5), 454-504. (p. 457)

In communicating the seriousness of facilitating instructions that embed RWWW, specifically for students in marginalized areas, Gutstein (2016) asserted that teaching with the intent to liberate could not fall short of teaching the mathematical or course content. According to Gutstein, the essence of learning and doing mathematics provides students with access to future career paths, economic survival for themselves and their families, reading and writing the world, and maximizing their human potential. For that

reason, teachers must ensure that their intentions of teaching from a social justice framework offers a balance of both content and sociopolitical awareness. In other words, helping students to RWWM cannot come at the expense of further widening an educational gap, especially for students who may already display a weaker mathematical understanding.

When asked by columnist Airaksinen (Campusreform.org, 2018) about the need to prepare and train mathematics teachers and other teachers in other disciplines about how to teach mathematics for social justice, Gutstein (2018) was quoted as arguing "that K-12 students need to be prepared through their mathematics education to investigate and critique injustice (such as racism and language discrimination), and to challenge, in words and actions, oppressive structures and acts" (n.p.). Gutstein added to his argument the need for teaching critical mathematics in that "critical mathematics is not an option for mathematics teachers, but rather a responsibility to our future" (n.p.). When asked to compare the traditional style of teaching mathematics to teaching critical mathematics, Gutstein indicated that traditional mathematics "is not sufficient and does not address many issues necessary to critical mathematics" (n.p.).

Given Gutstein's (2018) recent statement, mathematics educators and researchers alike would consider it important to provide the necessary buy-in for mathematics educators to get on board with teaching from a sociopolitical lens. However, does the need to prepare and train teachers to dismiss or outweigh Gutiérrez's (2013) argument that attending to identity and power issues in society within the classroom can be both straightforward and complicated to do? Addressing or even discussing issues of identity and power within the educational setting is difficult as well. With educators having to

attend to the demands of a standardized curriculum and high-stakes testing, Gutiérrez (2013) stated that there is little opportunity to "reflect upon how such students are constructing themselves and being constructed with respect to mathematics" (p. 37). Having brought attention to the level of difficulty in teaching from a sociopolitical lens, educators employing RWWM then ask what some of the challenges are of incorporating teaching mathematics for social justice.

As noted in his process of outlining the theory of RWWM, Gutstein (2016) identified two limitations existing within the practice of TMSJ or critical mathematics. In referencing the first limitation, Gutstein alluded to how the current pacing of the mathematics curriculum and standardized testing may play a factor in teachers not being able to fully adopt a social justice curriculum or expend social justice projects in their mathematics classroom. Other than his current research (Gutstein, 2016) that incorporates a year-long "enactment" of mathematics for racial and social justice for his Grade 12 mathematics class, Gutstein stated that the current research of this type is rare in the K-16 mathematics setting. In elaborating on this statement, Gutstein reported that no research has focused on assisting students with learning and using mathematics to understand and change their world. The second limitation addressed by Gutstein centers around student versus teacher choice. Although not in all cases (e.g., Turner, 2003/2009)), Gutstein pointed out that teachers' understanding of the world is typically limited. Instead of allowing students the autonomy to choose which topics they would like to explore, teachers are bounded by their understanding.

Participants in Bartell's (2013) study identified four challenges teachers encounter in learning to TMSJ. First, similar to the challenges identified by Gutstein (2016),

participants stated that as a teacher, they found it challenging to develop knowledge of social issues that can be readily incorporated into their mathematics classroom. Another challenge revolved around acquiring knowledge of the pedagogy needed to form a sociopolitical lens. As such, participants struggled with knowing what to teach. The third challenge pertained to teachers developing knowledge of their students' experiences and interests. The fourth challenge reported dealt with the implications of discussing and revealing one's personal life. Given the time constraints and limited understanding of teachers knowing how to foster safe classrooms that invite students culturally and lived experiences into the classroom environment, this problem is more common than usual. The challenge of "negotiating" both mathematics and social justice lends itself to teachers having to ask themselves difficult questions that lead to critical reflection. As teachers encourage and teach students how to read and write using mathematics, the noted challenges indicated that educators first need to approach this negotiation or incorporation by educating themselves not only on the pedagogical requirements for teaching from an RWWM or TMSJ framework, but also on their knowledge of their students' needs, social justice issues, and the flexibility of the curriculum and participants.

Applications of TMSJ in the Classroom

According to Gutstein (2003), "a pedagogy for social justice has three main goals: helping students develop sociopolitical consciousness, a sense of agency, and positive social and cultural identities" (p. 40). In building on this idea of social agency, Gutstein conducted a 2-year study to examine teaching and learning mathematics for social justice

28 urban middle-school Latino students. The participants were members of a public school that sets with a Mexican/Mexican American community. The student makeup in the elementary school consisted of 99% Latino, 98% from low-income families, and about 50% immigrants.

In lieu of helping students through the process of developing sociopolitical consciousness (e.g., *conscientizacao* an idea adopted from Freire [1992]), students can utilize mathematics as a mathematical tool to combat injustice, therefore shifting *conscientizacao* from being just a practice that is thought about to being a necessity in the act of creating change. As a deliberate attempt to use mathematics to help students "develop sociopolitical consciousness, a sense of agency and a positive social and cultural identities" (p. 66), Gutstein (2003) hoped to address the following research questions: "a) uncover and concretize components of teaching and learning mathematics for social justice and b) to understand the relationship of a Standards-based curriculum to that process" (p. 37). Seeking to fill in the gap of literature related to social justice pedagogy in the K-12 setting, Gutstein presented the students in his class with 17 real-world projects from Mathematics in Context (MiC) (National Center for Research in Mathematical Sciences Education and Freudenthal Institute, 1997-1998). Given that the projects were centered around the students' lives and lived experiences, Gutstein framed the projects around helping his students at Diego Rivera School to develop sociopolitical consciousness. It is also through this framework that Gutstein sought to "provide evidence for students' learning and changed disposition" (p. 42) towards mathematics.

As an effort to "use mathematics to understand—and potentially act on—their sociopolitical context (i.e., to read the world to develop mathematical power and in the

process to transform their attitude towards mathematics)" (p. 44), Gutstein incorporated the three teaching goals and objectives presented in Table 1 below.

Table 1

Teaching Goals and Objectives

Goals of Teaching for Social Justice		Specific Mathematics-Related Objectives
Develop Sociopolitical Consciousness		Read the World Using Mathematics
Develop Sense of Agency	⟷	Develop Mathematical Power
Develop Positive Social/Cultural Identities		Change Dispositions Toward Mathematics

Source: Retrieved from Gutstein, E. (2003). Teaching and learning mathematics for social justice in an urban, Latino school. *Journal for Research in Mathematics Education,* 37-73. (Gutstein, 2003, p. 43)

Gutstein's study found that through the process of his students' development of reading the world using mathematics, the majority of students developed mathematical power as well as changed their disposition towards mathematics. In having students discuss issues around social injustice, Gutstein (2003) stated that "there is nothing unusual about wanting students to be socially aware" (p. 40). Through the development of social awareness, students were propelled to ask important questions that were relevant to their lives, which then shaped the way they interpreted the forces and institutions that affected their world. According to Gutstein, a critical component of teaching for social justice lies in teachers "helping students understand, formulate, and address questions, and develop analyses of their society" (p. 40). Although the findings in this study indicated that their work presented varying levels of sophistication, students were able to use the social justice lessons to further develop viable arguments for solutions. Even though the results also indicated that not all students in the study developed an

appreciation for doing mathematics, 25 out of 28 students were reported to have shifted their attitudes towards mathematics.

Buenrostro (2016) drew from Gutstein's (2016) work on gaining students' perspectives on learning mathematics for social justice in a K-12 classroom setting. Buenrostro's work differed from most TMSJ studies in that it examined sociopolitical issues that directly affected the communities of the 13 participants in the study. In doing so, Buenrostro used a qualitative interview and narrative methodology approach to address the following research questions:

> 1) How did students experience attending a school focused on social justice? What aspects of this experience were most salient in students' narratives? 2) What meaning did students make of their experiences in the M4SJ class? What aspects of the class and their experience held salience for them? This includes reflections on the class learning with respect to experiences after high school including but not limited to post-secondary experiences, career goals and general life events. (pp. 6-7)

The participants in the study consisted of 12 Latin@ and 1 African American student. In highlighting the study's findings, this research noted two major findings. The first acknowledged that using mathematics to approach social justice issues makes the learning of mathematics more relatable and interesting. According to Buenrostro, the students' responses, in this case, served "as a counter-narrative to the notion that young people of color do not care about their education" (p. 216). Another major finding in the study pointed to low-income students of color being motivated to learn and use mathematics to discuss issues of social justice. According to the participants in the study, "students described the class as involving their hearts and their minds implicating a more committed level of engagement that from their perspective was largely facilitated through their interdependent ways of working together and the curricular context" (p. 218).

In contrast to Gutstein's (2003, 2006) and Buenrostro's (2016) work around TMSJ in marginalized school settings over an extended period time, Wonnacott (2011) employed a qualitative study to look at the effects of TMSJ on affluent students in a heterogeneous setting. As the study sought to investigate how affluent students used mathematics to promote social agency, the study adopted the following three research questions:

> 1) What does teaching mathematics for social justice (i.e., teaching strategies, resources, frequency, teacher-student interaction) look like in an affluent middle school classroom? 2) How does the incorporation of social justice issues in mathematics affect affluent, middle school students? 3) What factors affect the development of social agency in affluent, middle school students? (p. 31)

To conduct the study, Wonnacott took on the role of the practitioner in her year-long study and recruited 8 Caucasian, 9 South Asian, 1 Korean, and 1 Chinese student as participants from her Grade 10 mathematics course. The participants were between the ages of 4 to 16.

Classroom observation of students doing work in the class, student video reflections, students' work, and interviews for this study were collected during a unit on Analytic Geometry and Quadratic Relations. During the unit, Wonnacott had the students complete three mathematical activities involving social justice issues which investigated the relationship between income and education and HIV/AIDS in Canada. In coding the data, Wonnacott used Rogers's (1998) levels of engagement (e.g., cognitive, affective, existential, empowerment, and action) as her framework. Student work was assessed using an MYP Assessment Rubric (2008).

In the case of using TMSJ to students in an affluent, heterogenous setting, the study found that some students' cognitive and affective domains were affected. In other

instances, some students participating in the social justice activities presented in the mathematics course throughout the year felt the exercise empowered them to act. As such, “through critical reflection, learners [were able to] affirm, modify or dramatically transform their perspectives, worldviews or paradigms” (Rogers, 1998, p. 212).

According to Rogers (1998), students’ reaction to learning about global futures, aids in understanding how students thought about their experiences led them to have an “awakening of the heart” or “awakening of the soul” which led them to act. Lastly, the study’s findings suggested that students’ development of social agency was reported as linked to students’ age, personal connection to the topic, perception of responsibility, and amount of teacher direction.

Girls’ Learning Outcomes and Disposition

In maximizing students’ success and achieving equity in a mathematics classroom, the National Council of Teachers of Mathematics (NCTM) recommendation communicates that it requires mathematics teachers to work collectively with other stakeholders in an educational setting. While working to narrow existing learning gaps and establishing “high expectations and worthwhile opportunities for all students” (NCTM, 2000, p. 12), NCTM (2014) proclaimed that “educators at all levels must work to achieve equity with respect to student learning outcomes” (n.p.). In taking this position, NCTM recommended that “educators must have the knowledge, skills, and disposition necessary to support effective, equitable mathematics teaching and learning” (n.p.).

The NCTM's (2014) recommendation for all stakeholders in achieving access and equity in the mathematics classroom requires that (a) students' academic progress be frequently monitored such that, if needed, teachers and other stakeholders can provide accommodations; (b) students be provided with access to a rigorous curriculum and teachers be "skilled and effective" at providing "differentiate instruction as needed"; and (c) when appropriate, students be offered "remediation or additional challenges" (n.p.).

In attending to why educators must address equity in mathematics classrooms, Stiff and Harvey (1988) referred to secondary mathematics classroom as being "one of the most segregated places in American society" (p. 152; noted in Walker, 2012, p. 15). Similarly, Foote and Bartell (2011) argued that "mathematics education generates selection exclusion and segregation of students along the lines of gender, race, language, and socioeconomic status" (p. 4). Coupled with the fact that a student's school locality may contribute to the difference in student achievement, Asante (2012) asserted that the difference in male and female academic success is due to a school environment, teacher attitudes and beliefs, teaching styles, and parental attitudes. In supporting this argument, Allexsaht-Snider (2001) argued that the structural aspects of school districts (e.g., curricular and evaluative resources, financial, human, and leadership) also influence students' learning. These contributing factors add to students' experiences as well as further assist with creating a level of difficulty that hinders students' successful matriculation in mathematics courses (Bartell & Foote, 2011, p. 46) as well as their disposition toward mathematics. As such, girls' attitudes towards mathematics affects their mathematics learning outcomes, course takings in both high school and college, and future career choices.

Given that the present study aimed to investigate the factors that affect female students' development of sociopolitical consciousness and social agency through RWWM in a single-sex setting, it was necessary to investigate the learning outcomes and disposition of female students towards learning and doing mathematics. Given the limited TMSJ research that has specifically investigated girls, this section of the literature review begins with discussing students' learning of statistics, female course enrollment in mathematics and other STEM-related courses or fields. Also, given the claim that culturally relevant teaching is "a pedagogy that empowers students' intellectually, socially, emotionally, and politically by using cultural referents to impart knowledge, skills, and attitudes" (Ladson-Billings, 1994, p. 382), the research on some existing factors that various researchers have identified as contributing to female students' dispositions towards mathematics is important to understand.

Students' Learning of Statistics

When reflecting on how students learn statistics, earlier research in the field suggests that educators and researchers should first and foremost consider how students learn (Garfield, 1995). In the process of understanding how students transmit knowledge in mathematics and science, education communities allude to the constructivist approach as being the most recognized theory into how students acquire knowledge (Von Glasersfeld, 1987). According to Bada and Olusegun (2015), the constructivist approach to learn is as a result of students making connections between their prior knowledge to new found knowledge together. Bada and Olusegun refer to this action as mental construction. Due to being grounded in the field of psychology, a constructivist theory approach "suggest[s] that humans construct knowledge and meaning from their

experience" (Bada & Olusegun, 2015, p. 66). According to Jonassen (1994), it is through a constructivist approach that students try to make sense of how the world, as well as the things in it, operate. It is through this curiosity, that students "become engaged by applying their existing knowledge and real-world experience, learning to hypothesize, testing their theories, and ultimately drawing conclusions from their findings" (Bada & Olusegun, 2015, p. 66).

In students learning statistics through a constructivist approach, Garfield (1995) identified the following seven principles that teachers of statistics should consider:

1. Students learn by constructing knowledge.
2. Students learn by active involvement in learning activities.
3. Students learn to do well only what they practice doing.
4. Teachers should not underestimate the difficulty students have in understanding basic concepts of probability and statistics.
5. Teachers often overestimate how well their students understand basic concepts.
6. Learning is enhanced by having students become aware of and confront their misconceptions.
7. Calculators and computer should be used to help students visualize and explore data, not just to follow algorithms to predetermined ends. (Garfield, 1995, pp. 30 – 31)

When looking at the role of teachers play, researchers Schoenfeld (1992/2016) and Resnick (1988) look relationship teachers apply specific cognitive and cultural practice within the classroom. It is through the statistical enculturation

that statistics teachers act as a mentor and mediator to students during their development of statistical thinking. According to Ben-Zvi and Arcavi (2001), students must be able to use specific thinking tools alongside the communicative and collaborative process for statistical enculturation to occur. In students learning and exhibiting the language of mathematics, students seek out solutions, explore patterns as well as formulate conjectures (Schoenfeld, 2016). Schoenfeld state that students who are mathematically powerful are quantitatively literate. In that they are a) able to work with large amounts of quantitative data, b) make balanced judgments; c) use proportional reasoning; and d) flexible thinkers who are analytical.

In seeing oneself as a statistician, Wild and Pfannkuck (1999) propose that participants operate within four dimensions. First, statisticians must function within investigative cycles that will allow them to devise a plan to examine a real problem by devising a plan, collection data, analyzing information, and formulating a conclusion. Second, statisticians will employ different thinking modes which involve problem-solving strategies and statistical thinking modalities to search for evidence that helps support one's judgment. Third, statisticians should incorporate interrogative cycles which aim at one being able to question, critique, judge, as well as accept and disregard information. Fourth, statisticians should possess a disposition that allows one to challenge preconceptions.

STEM Course Taking

When looking at the portrayal of women in high-status occupations, particularly in fields identifying with mathematics, physics, and science, Eccles (1994) contended that, regardless of ongoing endeavors aimed at encouraging women to select STEM-related majors or pursue it as a profession, there remains a deficiency. For example, the National Science Board's Science & Engineering Indicators (2016) report indicated that despite women accounting for half of the college-educated workforce, women in 2013 accounted for only 29% of the science and engineering workforce (p. 3-83). According to this report, women in conjunction with some racial and ethnic minority groups (i.e., Blacks, Hispanics, American Indians, or Alaska Natives) in science and engineering in the United States have historically low representation in the field. In line with this argument, the Organization for Economic Co-operation and Development (OCED, 2013) indicated that women's participation in STEM-related training and degree attainment in STEM-related fields showed a continued decline for women in the United States and OECD countries (National Science Board, 2014; OECD, 2013).

Eccles, Vida, and Barber (2004), Nagy et al. (2008), and Watt (2005) asserted that when girls opt out of choosing advanced mathematics and science courses in high school, this contributes to the decrease in women's participation and career development in STEM. Although the research (Hill, Corbett, & St. Rose, 2010; National Science Foundation [NSF], 2012; see also Robnett, 2014) has pointed to women advancing in predominantly male-dominated fields in recent years, the results indicated that a gender imbalance in STEM-related fields remains. According to Robnett, gender imbalance in STEM includes but is not limited to "gendered socialization practices, stereotype threat,

work-family conflict, and gender bias (i.e., negative attitudes and treatment directed at girls and women in STEM)" (p. 20).

When investigating such gendered bias in research, Robnett referenced a qualitative study conducted by Leaper and Brown (2008). According to the literature, Leaper and Brown's study on adolescents found that the majority of female participants in their study reported experiencing gender bias in STEM at least one time. Robnett also noted that additional qualitative studies (Carlone & Johnson, 2007; Herzig, 2004) focusing on both undergraduate and graduate students have noted "that some women who pursue STEM degrees face barriers that can be directly traced to their gender and other background characteristics that interact with gender" (p. 21).

In another study conducted by Heilbronner (2013) that examined how women are represented in the STEM subjects, the researcher noted that women reported possessing lower self-efficacy in the subjects than men. As a result, only a few women chose to major in STEM subjects at the undergraduate level. The respondents in this study cited interest as the primary determinant of occupational choice for the two genders. A report by the United Nations Educational Scientific and Cultural Organization (UNESCO, 2017) indicated that girls were more likely to lose interest than boys in science-based studies as their age increased. As a result, girls started to drop out of STEM studies as early as the secondary level, while boys started to lose interest in their advanced levels. According to the report, girls and boys between the ages of 10 and 11 were typically equally represented in STEM, with boys accounting for 75% and girls 72%. As early as 18 years of age, female and male representation in STEM fell to 19% for female students and 33% for male students.

When factoring in race/ethnicity, Farinde and Lewis's (2012) study examining the representation of African American female students in STEM courses revealed that compared to males, females accounted for 9.7% of the undergraduate population who pursued STEM majors, compared to 25.5% males. When the researchers factored in race/ethnicity, they found that White students account/ed for 16% of STEM majors, compared to 14.9% African American. A related study conducted by Goodrum et al. (2011) to examine students' enrollment in STEM courses in Australia revealed that the percentage of learners pursuing science courses in Year 12 went down from 94.1% to 51.42% in 2010. In the same light, Sarkar, Tytler, and Palmer (2014) noticed that Year 12 enrollments in advanced mathematics decreased by 22% between 2000 and 2012 and by 34% between 1995 and 2012. The trend affected female students the most. Additional research provided by the Office of Science and Technology Policy (2016) showed that although women and minority groups made up 70% of the total number of students enrolled in U.S. colleges, only 45% of this population pursued STEM degrees.

Factor 1. Teacher and student interactions and anxiety. According to a study by Christensen, Knezek, and Tyler-Wood (2015), male students tended to have positive dispositions towards STEM while girls' dispositions tend/ed to be negative. Boaler (1997) analyzed the issue of girls' disposition towards mathematics using various approaches, such as teacher-student interactions, teachers' beliefs of girls' mathematical ability, student-student engagements, and learners' engagement with instruction from educators and various activities. The researcher noted that girls' participation in STEM subjects like mathematics was influenced by their negative attitudes towards the subjects, stereotypical views that mathematics is a male domain, inadequate teaching methods,

socialization, reduced exposure to mathematical activities after school, as well as the lack of proper knowledge about the importance of mathematics in career choices.

Anxiety, as a negative scale within attitudes towards mathematics, was one of the key variables (Baloglu & Kocak, 2006; Dowker, Bennett, & Smith, 2012; Jain & Dowson, 2009), which had a particularly significant impact on mathematics performance and achievement (Miller & Bichsel, 2004). Mathematical anxiety is defined as students' psychological reactions, including feelings of nervousness, stress, and helplessness when dealing with mathematics (Fennema & Sherman, 1976; OECD, 2013b). Lang (1968) suggested that mathematical anxiety could affect individuals from three independent aspects: physiological reactions, psychological reactions, and behavior.

Generally, females show/ed relatively higher anxiety. However, some studies (e.g., Baloglu & Kocak, 2006; Gierl & Bisanz, 1995) found different results. After conducting a study to examine the development of mathematics anxiety experienced by elementary schools students, Gierl and Bisanz (1995) found evidence that supported the existence of two different forms of anxiety (mathematics test anxiety and mathematics problem-solving anxiety) among students in Grades 3 and 6. Although the findings indicated no gender differences in these types of anxiety were identified, the results revealed that younger students presented a more negative disposition towards mathematics than older students. Vale (2008) and Thomson (2014) found that women were more anxious about mathematics than men. Wu, Barth, Amin, Malcarne, and Menon (2012) found that anxiety had a detrimental influence on children's mathematics achievement, despite their situational and social experience of doing mathematics.

Beilock and Willingham (2015) report/ed that anxiety associated with learning and doing mathematics was not limited to a particular region of the world. That is, students experience mathematics anxiety both in the United States and abroad. When reviewing a case study by Jones (2001), Beilock and Willingham (2015) found that "an estimated 25% of 4-year college students and up to 80% of the community college students suffer from a moderate to a high degree of math anxiety in the US" (p. 32). According to Beilock and Willingham, research (Maleoney & Bellockthat, 2012) has suggested students "pick up on from parents, teachers or peers that math is, indeed, worthy of anxiety" (p. 31). Beilock and Willingham also found research (Maloney, Gunderson, Ramirez, Levine & Sian, 2014) that suggested a link between students' mathematics performance and teachers' performance. Other research reviewed by Beilock and Willingham revealed that mathematics anxiety can be experienced by individuals outside of academia. For example, additional studies reviewed by Beilock and Willingham found that mathematics anxiety was associated with nurses' incorrect calculation of medication (McMullan, Jones, & Lea, 2012).

When looking more closely at teacher-student interactions with mathematics, the research revealed that female students developed anxiety around doing and learning mathematics as early as elementary school. A study conducted by Beilock et al. (2010) noted that when female elementary school teachers were anxious about teaching mathematics, their anxiety always had a negative consequence on the performance of female students in the subject. Beilock's study determined that many elementary school teachers were females and their anxieties always had a positive correlation with girls' scores in mathematics and their perceptions on who was naturally good in the subject.

The researcher discovered that the higher the level of mathematics anxiety in these teachers, the higher the likelihood of girls endorsing the perception that boys were good at the subject, and the lower their performances were in mathematics.

In a similar study by Ramirez et al. (2018), the researchers observed the same relationship between teacher anxiety and girls' performance in mathematics, namely that increased levels of mathematics anxiety in teachers resulted in decreased performance in mathematics for students. According to Ramirez et al., this "relationship is partially mediated by the students' perception that their teacher believes not everyone can be good at mathematics" (p. 1).

In highlighting that mathematics anxieties extend long past primary grades, Escalera-Chávez et al. (2016) conducted a study that examined the anxiety levels of high school students in learning and doing mathematics. Their research showed that high school students have anxiety about mathematics and that the female level of anxiety was always higher than that of boys. In other studies (Radišić, Videnović & Baucal, 2015; García-Santillán et al., 2016), the researchers noted that factors such as an individual's interest and performance in mathematics, ability to grasp mathematics concepts easily, and the environment within the school and the classroom were found to contribute to low anxiety levels in students. Moreover, their study determined that indiscipline in class, strict rules in school, and the socioeconomic status of a school contributed to higher levels of anxiety. The researcher noted that if a student was studying in a school of higher socioeconomic status than her own, the student would feel anxious towards mathematics due to higher expectations on students' achievement in that school.

Mathematics anxiety also was a social factor. In conducting a field study, Maloney, Ramirez, Gunderson, Levine, and Beilock (2015) examined if parents' mathematics anxiety impacted their students' academic performance and anxiety levels in mathematics. Of the 868 student participants, Maloney et al. were able to collect data pertaining to parents' math anxiety and frequency of homework help from 529 children. Based on the results, when parents' mathematics anxiety was high, the mathematics anxiety of children increased. In addition, when parents with high levels of anxiety provided homework help to their child, there was a negative impact on their child's learning of mathematics. Thus, the frequent interaction of students and parents with the latter helping with homework provided opportunities for parents who have mathematics anxiety to express their own dislikes or discomfort with learning and doing mathematics. According to Maloney et al., these intersections could result in demotivation, which would likely reduce the amount of effort children put into mathematics.

However, this did not only hold true for parents. Interactions with early elementary school teachers was also a factor (Beilock & Maloney, 2015). Over the course of an academic school year, Beilock, Gunderson, Ramirez, and Levine (2010) assessed the anxiety levels and dispositions towards mathematics of 17 first- and second-grade female teachers and their students. The findings indicated that when teachers' mathematics anxiety increased, it negatively impacted students' performance. At the beginning of the study, the results indicated no significant relation between students' mathematics performance and teachers' levels of mathematics anxiety. However, at the closing of the study, the findings indicated that when female elementary teachers showed higher signs of mathematics anxiety, their female students acquired less mathematical

knowledge. Although the data found no significant relationship between female teachers and boys, the findings suggested that female elementary school teachers endorsed the negative stereotypes that believed boys were better at doing mathematics than girls.

The reported high levels of mathematics anxiety and mathematics avoidance among learners made it essential to come up with practical approaches to remediation. The need to adopt individual approaches to addressing mathematics anxiety is always advocated for in the literature. Strategies such as relaxation, where an individual is taught to learn how to manage muscle tension, can help reduce anxiety. Cue-controlled relaxation has also been found to reduce mathematics anxiety and lead to improved performance in mathematics. Moreover, mindfulness training is beneficial to learners as it helps them reduce physiological stress that is usually brought about by mathematics tasks (Chiesa, Calati, & Serretti, 2011; Conrad & Roth, 2007; Maloney et al., 2014). Other techniques suggested by researchers to reduce mathematics anxiety include reappraisal, bibliotherapy, and expressive writing (Maloney et al., 2014).

Additional strategies that can help to alleviate mathematics anxiety include cognitive behavioral approaches (Gregor, 2005; Hembree,1988), which have been found to be effective, especially in reducing trait mathematic anxiety. These techniques are designed to address individual thoughts, attitudes, and beliefs that bring about trait mathematics anxiety. The need to address maladaptive beliefs, such as certain myths about mathematics and worry expressed towards mathematical tasks, can be helpful in dealing with negative dispositions towards mathematics. Educators should identify and challenge students' negative thoughts and beliefs about mathematics and encourage learners to embrace only positive feelings and behaviors towards math. The tendency of

individuals to procrastinate doing mathematics due to fear should also be eliminated by encouraging learners to regularly expose themselves to the same tasks that they consider challenging to help them deal with anxiety. In supporting this theory, researchers Buckley et al. (2016) found that cognitive behavioral approaches can be effectively used in alleviating learners' anxiety towards mathematics, especially if such techniques are used in conjunction with relaxation.

Factor 2. Gender-related bias. Reis and Park (2001) conducted a study that examined gender differences among high-achieving students. During a review of literature, Reis and Park noted that teachers' actions in the classroom pointed to stereotyped behaviors that affect girls' perceptions as well as performance, both in life and academia. In finding literature to support this claim, Reis and Park noted a study by Kissane (1986) which found that teachers overwhelmingly called on male students who scored higher on the SAT over females who performed well on quantitative reasoning. Reis and Park noted that such behaviors contribute to females' shortcomings in schools. Given that society's beliefs and attitudes view mathematics and related fields as a male domain where males dominate the mathematics classroom and workplace, both teachers and parents view females as being less capable of doing mathematics (Forgasz, Leder, Mittelberg, Tan, & Murimo, 2015). After conducting an international comparison of gendered beliefs about mathematics and technology, Tan, Forgasz, Leder, and McLeod (2012) found evidence that the nine countries (e.g., Canada, China, Egypt, India, Israel, Singapore, UAE, UK, and Australia) represented in their study consistently, to some varying degree, had traditional gender-stereotyped views of males and females doing mathematics. Despite all nine countries recognizing mathematics as significant course

needed for both male and female students, the results of the study indicated that these countries viewed mathematics, to some degree, as a male-dominated field as well.

According to Kane and Mertz (2012), gender equality and various sociocultural factors greatly determine girls' and boys' performance in mathematics. This research confirmed that women who are well educated and earn good money are likely to influence their children's performance positively, especially in mathematics, than those with less education and earning income. These results are consistent with previous research that found that the socioeconomic status of the home environment has a considerable influence on the performance of children. Kane and Mertz noted that eliminating gender inequalities in earnings might help to enhance girls' learning outcomes in challenging subjects like mathematics and other sciences. Other cultural aspects like beliefs about women's and girls' role in the workplace can also affect their participation in STEM fields. It is argued that:

> "gender-related stereotype threat", which can be transmitted by parents, teachers and the wider society from the youngest ages, affects not only a girl's attitude towards mathematics and science and performance in these subjects, but also a woman's interest and performance in STEM in their studies and careers. (Salmon, 2015, p. 22)

In a study that examined gender-based mathematics performance, DeRosa (2011) conducted a qualitative study to determine variables that might be connected with the success or lack of success of girls enrolled in mathematics. In recruiting both third-grade girls and their mothers from an affluent school district, the researchers used both parents' and students' responses to mathematics-related questions to look for any existing correlations. When looking at female attitudinal factors and their potential relationship between female students' perceptions and mathematics performances, DeRosa's study

revealed 40 findings. In summarizing these findings, the study found that mothers from affluent school communities with high-performing considered their stay-at-home status and strong involvement as affecting their daughter's academic success. As the participants' mothers believed, greater performance and understanding of learned mathematics skills positively aided their daughters' future life choices. Therefore, their daughters could get into elite colleges and secure desirable, lucrative jobs later.

In adding to the research that examined parental beliefs in addition to students' self-perception of ability, Bleeker and Jacobs (2004) took a longitudinal follow-up approach to examine the relationship between mothers' earlier gender stereotypes and perceptions concerning their children's later mathematics-science achievement and career choices. Using Jacobs and Eccles' (1992) 143 sixth-grade participants from predominantly White middle- and working-class, suburban communities in Michigan, the findings produced new evidence of the ongoing significance of parents' beliefs and their impact on how children perceived themselves and made career choices. After some 12 years later, the research produced four major findings. First, as predicted, "early perceptions of one's child may be shaped by general stereotypic beliefs that, in turn, are related to children's later self-beliefs and performance" (p. 107). Second, "adolescents' self-perceptions of math ability during the 10th grade mediate[d] the relation between mothers' perception and adolescents' math[ematics]-science career self-efficacy" (p.107). Third, "the interaction of an adolescent's gender and a mother's prediction of her child's ability to succeed in a math[ematics] career was a direct indicator of whether a young adult chooses a career in physical science-computing, as opposed to nonscience or life science-business career" (p. 107). Fourth, "the current findings indicate that gender

differences in attitudes are present in early adolescence and continue throughout middle and high school" (p. 107). The results of this study confirmed that girls' self-perceptions of being able to do mathematics were lower than boys' perceptions.

Another study by Wang and Degol (2017) found that factors such as cognitive ability, career choices or interests, work values, gender stereotypes and biases, and ability beliefs were the primary determinants of women's underrepresentation in the STEM fields with intense mathematics learning. Also, the researchers in this study noted that ability differences and strengths, an individual's career choice, and lifestyle preferences were biologically related because they stemmed from the hormonal differences between men and women. Moreover, it was noted that sociocultural factors, like one's social beliefs and society's perceptions on the differences in men and women's abilities, as well as the cultural issues that force females to pursue feminine careers, were also likely to influence women's career choices.

In a study conducted by Piatek-Jimenez, Cribbs, and Gill (2018) to examine the stereotypical views on female and male learners' qualities, the researchers noted that society stereotypically attributed certain personality traits to girls while others were also attached to boys. The extent of these beliefs was determined by gender and parental education and whether students were planning to pursue a STEM-based course. Thus, the researchers highlighted how gender stereotypes might be contributing to the low enrollment and the low performance of female students in STEM courses. Sax, Kanny, Riggers-Piehl, Whang, and Paulson (2015) examined Mathematics Self-Concept (MCS) based on how it varies across the various STEM subjects among university students. The research revealed there was a gender gap in MSC in almost all the STEM majors.

However, it was determined that the importance of MSC in determining women's selection of STEM majors has been reducing over time. The researchers noted that though women's low self-concept of mathematics influenced their STEM choices, its effects in determining women's representation in these courses has become weaker over time.

In combating gender-related biases, some researchers have suggested that single-gender mathematics classroom produce positive learning outcomes for females in mathematics. For instance, James (2009) investigated the differences in learning styles of boys and girls. When examining these differences, James indicated that male and female students have different learning approaches stemming from their preferences, motivations, and interests. As such, James argued that boys and girls process and store information in different sections of the brain. In accounting for the difference between male and female students' performance in STEM-related subjects, James' findings suggested that cognitive gender differences affect/ed how students process information. Regarding these findings, James argued that because of these dissimilarities, female and male students should not be enrolled in the same classrooms.

In further advancing the argument for separate mathematics classrooms for boys and girls, Else-Quest and Peterca's (2015) quasi-experimental study revealed that although single-sex schools were associated with low performance in reading and mathematics among boys, girls' achievement in reading, mathematics, science, and writing were significantly higher. Studies also indicated that when girls were placed in single-sex classrooms, they perform/ed better in mathematics. According to Bowe, Desjardins, Covington Clarkson, and Lawrenz (2017), female learners' academic

performances in single-sex mathematics classrooms increased because such environments have been found to mitigate the effects of math stereotypes for both the students and their teachers. Research on girls' and boys' performance in mathematics in mixed schools showed that the males often outperformed the females in mathematics due to the latter's lack of confidence on the subject, personal interests, and teaching methods (Kyei, Apam, & Nokoe, 2011).

Other researchers (e.g. Bofah & Hannula, 2016) have indicated that girls attending single-sex schools tend to have an increased level of self-confidence in mathematics than those attending mixed-gender or coeducational schools. It has also been noticed that girls in coed schools tend to have lower self-confidence in math than their counterparts in other types of schools. Although these researchers found a difference in girls' performance in mathematics between those attending coed and single-sex schools, other studies (Doris, O'Neill, & Sweetman, 2013; Gilson, 2002; Norton & Rennie, 1998) found that there is no significant difference and call for further extensive research on the issue.

Factor 3: Mindset. Even though studies have shown that women to major in STEM subjects in college are well qualified, many of them drop these majors not long after they start college (Seymour & Hewitt, 1997). Low enrollment of women in STEM fields and majors is common even among gifted women. For instance, Benbow (1998) and Benbow and Minor (1986) found that among a sample of youths gifted in both languages and mathematics, only 20% of the girls indicated that they planned to pursue a career in a science- or mathematics-related field, compared to 40% of boys. According to Dweck (2007), the confusion that comes with having to learn new concepts also

contributes to low performance in mathematics among female students. The researcher noted that even among females with higher IQs, confusion could lead to reduced performance. For instance, for girls with higher IQs, performance in mathematics was reduced whenever they faced difficulties that challenged their abilities.

In contrast, males labeled as having a higher IQ were reported as being less confused than females and could perform better when presented with challenges in mathematics. This research supported Dweck's (1986) previous findings that indicated that in situations where girls, more particularly bright girls, felt overly challenged, they avoided the risk of failure. Rather than approaching the challenge as an opportunity to learn, students generally provided the correct answer without explaining how they derived their answer. Dweck referred to this process of guaranteeing success without the consequence of risking being wrong as maladaptive motivational patterns.

Although girls and women take fewer mathematics than their male peers, Boaler's (2002) observation of girls and boys indicated that unlike boys who are more concerned with getting the answer right, girls desire to know why their solutions to mathematics problems work. In changing students' perceptions about being able to do and learn mathematics, Dweck (2014) stressed the importance of getting students to change their mindset from fixed to growth (e.g., when students understand that their abilities can be developed over time.

Dweck (2015) called for "teachers who understand the growth mindset do everything in their power to unlock that" (p. 2). In shifting students' fixed mindset, Dweck argued that teachers must go beyond just looking at a growth mindset as being about students applying effort. Although students' applied effort is a vital key to their

achievement, Dweck stated that students' effort could not stand alone. In shifting from a fixed to a growth mindset, students must be willing to approach mathematics problems from varied approaches. When facing difficult problems, they must also be willing to seek assistance and feedback from others. In addition to students having a growth mindset in mathematics, Dweck indicated that parents must understand the implications of endorsing a growth mindset, for if parents respond to students' mathematical errors in a negative manner, there is the potential of creating a reversal effect. In helping students foster a growth mindset, all stakeholders must understand (a) that everyone will not shift from having a fixed to a growth mindset, and (b) what triggers the fixed-mindset reaction in students during challenging situations.

In further elaborating the need to allow students to use growth mindset work through their mathematical errors, Boaler (2015) emphasized the need for teachers to encourage students to explore their misconceptions about the mathematics topic further. In helping students make sense of what they are learning in mathematics, Boaler suggested the use of concrete manipulatives starting in the earlier years of students' mathematics journey to improve students' learning in mathematics. In dispelling the myth that people who can do math are the smartest or cleverest people, Boaler stressed the need for people to understand how pervasive and damaging this idea can be on students' learning outcomes. Therefore, in mediating the problem, teachers should encourage students to examine the various pathways to approaching and solving a mathematical problem. In shifting students' mindset from fixed to growth, Boaler suggested the following intervention strategies for parents and teachers:

> paying attention to the mathematics questions and task that students work on, the ways teachers and parents encourage or grade students, the forms of grouping

> used in classrooms, the ways mistakes are dealt with, the norms developed in classrooms, the math messages we can give to students, and the strategies they learn to approach mathematics. (p. xiii)

Motivation can be defined as the attitudinal indicator that directs a person's certain behaviors (Elliot & Covington, 2001). The concept of Mathematical Motivation is generated from the broader psychological conception and narrows down to the scale within the students' attitudes towards a certain subject—mathematics.

Students' disposition towards mathematics is most often due to their childhood experiences (Sparrow & Hurst, 2010). From the early years, children are exposed to messages on mathematics via toys, social media, and words from their parents and teachers (Adelson & McCoach, 2011). Students with strong mathematical motivation show a positive attitude towards mathematics and are more open to learning the subject (Erdem & Genç, 2013). It was also identified that girls' motivation was higher than boys, and students' motivation and attitudes towards mathematics were improved from lower to higher grades. However, several studies identified that boys showed greater interest and enjoyment of mathematics learning (Thomson, 2014; Vale, 2008; Watt, Eccles, & Durik, 2006).

Factor 4: Sociocultural factors. Academic self-perceptions and career choices of children are affected by how their parents perceive their academic abilities, have specific academic achievement expectations, and provide support (Ahmed, Minnaert, van der Werf, & Kuyper, 2010; Bowen, Hopson, Rose, & Glennie, 2012; Ferry, Fouad, & Smith, 2000; Frome & Eccles, 1998). A longitudinal study conducted by Wilkins and Ma (2003) suggested that students' mathematical outcomes and mathematics disposition were influenced by the amount of support they received from their parents. Using data from

the Longitudinal Study of American Youth (LSAY; Miller, Kimmel, Hoffer, & Nelson, 2000), Wilkins and Ma analyzed data from 3,116 seventh-grade students in schools in the United States. To look at the development of student outcomes related to mathematics and science course takings over a 6-year period, Wilkins and Ma used a three-level hierarchical linear model to investigate the change in students' mathematics disposition and beliefs. The results of the study indicated that although students in the seventh grade have an average disposition towards mathematics, their disposition shifted negatively during their middle and high school at a rate of 0.06 points per year. According to Wilkins and Ma, this decrease in disposition can be linked to peer influence, teacher and parent push, and media exposure. Although Wilkins and Ma's findings indicated that parents and teachers play/ed distinctively different roles in the lives of their children, more support provided by teachers and parents can promote science and mathematics outcomes of students in comparable ways.

Burris (2003) used a quantitative, quasi-experimental design to examine the achievement effects and later advanced mathematics course takings as a result of heterogeneous grouping in a suburban school in New York. In addition to replacing the middle school mathematics curriculum for eighth grade with a more advanced algebra-based curriculum, mathematics course was detracked. This allowed students from various economic and racial backgrounds as well as mixed academic ability to take advanced mathematics together. Using a quantitative cohort design, the study sought to answer two research questions. The first "inquire[d] where or not all students, including initial low achievers, benefit from an accelerated algebra course in eighth grade" (p. 8), while the second focused on "what has been an important and contentious aspect of the tracking

debate: whether the achievement of initial high achievers is diminished by inclusion of all learners in heterogeneously grouped math[ematics] classes" (p. 9). The results of the study were positive. A descriptive statistical analysis showed that all students taking higher-level mathematics in a heterogeneously group middle school setting resulted in more students taking Advanced Placement Calculus in high school. Students previously on a lower track matriculated to more advanced mathematics classes. As a result, the number of students dropping out of mathematics decreased prior to graduation. The data revealed that this pattern was the same across the board for minority students, low-socioeconomic status, and students of varied performance levels in mathematics.

Aside from women being minority in STEM careers, race is also another factor. Based on research, there are only a few African American or Latino women (Koebler, 2011) in STEM careers. One reason attributed to this is that only a few African American or Latino women graduate from college. On a positive note, recent research (Haeverlo, 2010) has shown that female students' race/ethnicity does not greatly affect their confidence or interest in mathematics and science. However, Delpit (2012) and Ladson-Billings (2009) indicated that teachers who hold stereotyped beliefs about what Black students can achieve in school can lead to lower teacher expectations for those students. Pringle, Brkich, Adams, West-Olatunii, and Archer-Banks (2012) found such beliefs can cause some urban teachers to discourage African American girls in particular from aspiring to or entering the mathematics and science fields as early as elementary school. Other studies (Brand, Glasson, & Green, 2006; Pringle, Lyons, & Booker, 2010) indicated that some teachers not only direct derogatory statements based on stereotypes at their Black students but also demonstrate differential treatment.

According to Walker (2012), "it is certainly true that urban youth and Blacks and Latino/as are not the only one missing from positive discourses about mathematics" and that "women's positioning with regard to mathematics is also problematic" (p. 9). In her book *Building Mathematics Learning Communities: Improving Outcomes in Urban High Schools*, Walker looked at the effects that popular media and culture in the United States have played in creating this false narrative that only a select few "can do" mathematics. Rather than seeing themselves as mathematicians or doers of mathematics, Blacks, Latino/as, and women, in general, are often overlooked. Even with Blacks, Latino/as, and women considering themselves to be great mathematicians, Walker argued that the perception of doing mathematics is negatively tainted in that doers of mathematics are often considered to be a little odd or weird or to have significant mental problems (p. 8). Coupled with the fact that the media construct images that represent White, male, or those of Asian descent as being "overachievers," the evidence of limited representation and the mental impact forced on Blacks, Latino/as, and women in the mathematics field goes without saying.

Given that educational reform discussions on student learning in mathematics education take a "top down, deficit-orientation preceptive" more specifically in urban settings, Walker (2012) argued that such reform movements do not do enough to consider students' strengths and interests appropriately. As such, Walker offered three key principles to help students and teachers in urban school communities with increasing students' outcomes in mathematics. First, members of the teaching and learning community should know that "urban high school students want to be engaged in mathematics and have developed communities that support that engagement" (p. 2).

Walker added that despite common portraits of urban high school students in the media being disinterested in mathematics, the data from Lowell High School indicated that students in this subgroup saw themselves as learners and doers of mathematics. Second, "teachers and administrators can learn from how students develop mathematics communities and use this knowledge to build collaborative communities that support mathematics achievement and engagement" (p. 2). Third, teachers and administrators must be made aware that they can "inadvertently create obstacles that thwart the mathematics potential of these students" (p. 2). In addition to district-, school-, and classroom-level decisions, Walker argued that external policies contribute to and impede students' mathematical outcomes. Although these three fundamental principles are not specifically addressed for female learners and doers of mathematics specifically, females in urban high school communities are also members of an underrepresented group (e.g., Black, Latino/as, and women) who stands to benefit from the three principles above.

Theoretical Framework

In developing the theoretical framework for this study, the researcher drew from three conceptual perspectives around TSMJ (e.g., critical mathematics or Criticalmathematics literacy). Given that the approach of this qualitative study toward the mathematical research was through a sociopolitical lens, the researcher looked at theories relating to critical race theory, culturally relevant/responsive pedagogy and teaching, and reading and writing the world with mathematics (RWWM). Moreover, because the researcher also looked at the effects of teaching and learning mathematics for social

justice, this study also extended to Rogers's (1998) levels of engagement (e.g., cognitive, affective, existential, empowerment, and action).

Critical Theory/CRT

Critical theory was developed from the Frankfurt School, according to Stinson et al. (2007). Although the definition of critical theory has been often revised through the years, critical theory is perceived as having emerged in part from the struggle for liberation (Wellmer, 2014). Therefore, in the fight to liberate all human beings, critical theory exists to ensure that all people may experience the same opportunities for self-development (Horkheimer, 1972). Marx and Engels (1970) argued that "power" and "ideology" are the two critical analytical components of critical theory (Brookfield, 2014). Stinson (2009) cited Bottomore who, in 2001, suggested that "critical theory maintains sociopolitical critiques on social practices and ideology that mask 'systematically distorted accounts of reality which attempt to conceal and legitimate asymmetrical power relations'" (p. 506). In conducting an analysis of how conflicts and social interests are manifested by systems of domination (Bottomore, 2001), Stinson (2009) reported that critical theorists believe that such research "will bring about an awakening of consciousness and awareness of social injustices, motivating self-empowerment and social transformation" (p. 506). Stinson also referred to Freire (1970/2000b), whose work on liberating, empowering, and transformed individuals come from becoming aware of the multiple contradictions that exist on multiple levels of society (e.g., political and economic) and to fight against oppression. One powerful means of fighting oppression and ensuring the liberation of members of society is through education and the curricula that structure learning in schools. The development

of critical pedagogy in all subjects (and specifically mathematics for this study) is the first step toward social justice.

Historically in the United States, African Americans, American Indians, and Latin@s have been and continue to be viewed as inferior or deviant (Gutiérrez, 2013). Only recently have mathematics researchers and mathematics educators began to discuss how racism unfolds in mathematics education, but educational scholars outside the field have long explored how racism manifests itself at the four levels: individual, institutional, societal, and global/epistemology (Scheurich & Young, 1997). Drawing from feminist, poststructuralist, legal, liberal, and even Marxist theories and methodologies, "critical race theory provides a different theoretical analysis of how discourse of race and racism operate within the U.S. social structures" (Stinson, 2009, p. 505).

The lived experiences of both students and teachers in American law schools during the mid-1970s gave birth to critical race theory (CRT) (Martinez, 2014). In challenging the "constrictive role that racial ideology plays in the composition and culture of American institutions" (Gotanda & Peller, 1995, as cited in Martinez, 2014, p. 18), law students and teachers spotlighted how racial ideology and power are represented in the American legal system (Gotanda & Peller,1995). The primary aim of CRT was to transform existing legal doctrines in a way that illuminated the interrelationships of race, racism, and power in the law (Gotanda & Peller, 1995). Although CRT was born out of frustration with critical legal studies, marginalized or oppressed people of color have resonated with the principles of CRT. Dixon and Rousseau (2006) suggested that CRT recognizes that racism is indeed pervasive in America, and to even begin the job of ending racism, society must analyze itself through

a contextual/historical lens; reject claims of meritocracy and neutrality; understand how racism actually constructs dis/advantage among different groups; and embrace the experiential knowledge of people of color. To this end, the five principles of CRT are: "(1) the notion that racism is ordinary and not aberrational [in American society]; (2) the idea of an interest convergence; (3) the social construction of race; (4) the idea of storytelling and counter-storytelling; and (5) the notion that whites have actually been recipients of civil rights legislation" (Stinson, 2009, as cited in Hartlep, 2010, p. 3).

Culturally Relevant Teaching and Pedagogy

Culture is defined as a "group's individual and collective ways of believing, thinking and knowing, which include shared experiences, skills, consciousness, values, forms of expression, behavior and social institution" (Tillman, 2002, p. 4). Given that the experiences, research needs, and worldviews of researchers and scholars vary, culture is often defined, described, and conceptualized differently by those who undertake to study it (Tillman, 2002). For example, in conducting culturally sensitive research on African American communities, the question remains of who has more knowledge to discuss and interpret Black culture. Tillman's work around culturally sensitive research on African Americans outlined a framework highlighting the following five components: "culturally congruent research methods, culturally specific knowledge, cultural resistance to theoretical dominance, and culturally informed theory and practice" (p. 6).

Ladson-Billings (2000) defined cultural competence more as one's efforts to be effective within the culture in which one was born. As well, the individual in that culture develops and incorporates the knowledge, attitudes, and beliefs that allow that individual to participate in the culture and also transform oneself (Leonard, Brooks, Barnes-

Johnson, & Berry, 2010). At the school level, culturally relevant pedagogy or culturally responsive teaching encourages teachers to implement cultural practices as official knowledge in some form (Ladson-Billings, 1995a). This then allows teachers to demonstrate cultural competence while also encouraging students to interact with the course material from their own cultural perspectives (Scheriff & Spector, 2011).

Ladson-Billings is credited for the popularity and development of culturally relevant pedagogy and its application in education (Gay, 2010). In defining the theoretical model of culturally relevant pedagogy, Ladson-Billings (1995) not only acknowledged that it speaks to students' achievement but, at the same time, it supports students in embracing and affirming their cultural identity as they also sharpen critical perspectives that empower them to challenge the injustices around them (including in schools and other institutions). She noted that culturally relevant pedagogy is intended to support students' academic success, cultural competence, and development of critical consciousness. Meaningful culturally relevant pedagogy should engage cultural practices that already exist in learning communities; this serves then to motivate and encourage student discourse and build student-teacher relationships.

According to researchers (Leonard, 2009; Martin, 2007, 2009), mathematics education is a social construction that has become gendered and racialized. For example, some teachers and parents believe that boys are better at doing mathematics than girls. As a result of this belief, teachers call on boys in mathematics class more than girls. In other instances, some teachers think that Black students have less academic ability as their peers. In this case, teachers lower their expectations of their Black students (Delpit, 2012; Ladson-Billings, 2009). These stereotypical beliefs cause students to question whether

society sees them as even capable of succeeding in mathematics (Stiff & Harvey, 1988). Culturally responsive mathematics pedagogy has the potential to construct students' positive mathematics identities as they challenge, critique, and seek to change the injustices that can lead to negative identities (Leonard et al., 2010). Martin (2000) argued that students' mathematical identity develops when students reflect on their "(a) ability to do mathematics, (b) the significance of mathematical knowledge, (c) the opportunities and barriers to enter mathematics fields, and (d) the motivation and persistence needed to obtain mathematics knowledge" (p. 19).

In adopting critical pedagogy curriculum into the mathematics classroom, teachers and students are able to work with theories of pedagogy and practices of critical pedagogy while also using mathematics as a tool to analyze and examine social injustices critically (Stinson, Bidwell, Powell, & Thurman, 2008). As such, critical mathematics pedagogy or teaching mathematics for social justice is viewed as liberatory education (Du Bois, 1935; Perry, 2003; Provenzo, 2002) "that encourage[s] both teachers and students to develop an understanding of the interconnecting relationship among ideology, power, and culture" (Leistyna & Woodrum, 1996, as cited in Stinson et al., 2008, p. 3). Critical mathematics pedagogy consists of two groups of pedagogical goals (Gutstein, 2006): a pedagogical goal and a mathematical goal. Although Gutstein explained that the pedagogical goal focuses on social justice while the mathematical goal focuses on mathematics, he argued that both foci are vital to examine the dynamics and relationships that are dialectical. Gutstein (2006) noted that:

> The three social justice pedagogical goals are (1) reading the world with mathematics, (2) writing the world with mathematics, and (3) developing positive cultural and social identities. The three mathematics pedagogical goals (1) reading

> the mathematical word, (2) succeeding academically in the traditional sense, and (3) changing one's orientation to mathematics. (p. 24)

As a result of this theoretical framework, the present study sought to fill the gap in the literature that examines the experiences of non-minority students as they interact with mathematics curriculum developed around theories relating to culturally relevant teaching and pedagogy, critical theory, and CRT.

Chapter III

METHODOLOGY

The primary purpose of this qualitative case study was to understand what high school students in an all-girls independent school in New York think about social justice issues, and how it may have affected their learning of statistical concepts. Furthermore, this study examined what, if any, factors may affect female students' development of sociopolitical consciousness and social agency by using mathematics to read and write the world. This study further aimed at answering the following research questions:

1. How does the incorporation of social justice issues into a high school statistics curriculum at an all-girls independent school affect female students?
2. How, if at all, do students in an all-girls high school statistics course develop sociopolitical consciousness through "reading the world with mathematics"[1]?
3. What factors contribute to or prevent students in an all-girls high school statistics course from "using mathematics to write the world"[2]?

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[1] RWWM means using mathematics to understand relation of power, resource inequities, and disparate opportunities among different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation, as well as to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them (Gutstein, 2003c, p. 45).

[2] This means to use mathematics to change the world or to see oneself capable of making change through a means of developing social agency (Gutstein, 2006, p. 27).

In this chapter, a description of the qualitative methods used to investigate how the participants thought about social justice issues is presented. Further information regarding the participants and the setting, methods for data collection, data analysis, and limitations are provided in this chapter. A summary of the research methodology concludes this chapter.

Research Design

The study investigated how students think about embedding social justice issues into their current mathematics curriculum through a case study approach using qualitative research methods. Given that this research study sought to examine a bounded phenomenon through in-depth data collection methods involving multiple data sources, the case study approach helped with "understand[ing] the behavioral conditions through the actor's perspective" (Zainal, 2007, p. 1). In using a case study approach, this study lent itself to adopting a qualitative design (Creswell, 2002) to address the aforementioned research questions.

With the qualitative case study approach, this study could examine in-depth the possible effects of incorporating social justice issues into a high school statistics curriculum at an all-girls independent school. As such, the procedures taken in this case study led to examining if the participants' experiences encouraged or discouraged them from taking action to change the world. Additionally, the study sought to determine if the female students had an increased understanding of the relation of power, resource inequities, and disparate opportunities among different social groups.

For the purpose of this study, the researcher adopted and operated from Yin's (2009) working definition of a case study. "A case study is an empirical inquiry that: (a) investigates a contemporary phenomenon (the 'case') in depth and within its real-world context, especially when (b) the boundaries between phenomenon and context may not be clearly evident" (p. 18).

Merriam (1998) added that a case study is an exploration and investigation of a particular phenomenon such as the following: "a program, event, person, strategy, institution, or group" (p. 9, as cited in Wang, 2015). A qualitative case study then requires the setting to be a bounded system where the limits between the case being examined and the contextual case conditions, be they temporal or spatial, are often unclear and obscure (Yin, 2013, p. 6, as cited in Wang, 2015, p. 351). In this case, the five female students learning about social justice issues through mathematics inside a statistics classroom constitute a bounded phenomenon.

Wang (2015) explained how case studies allow the researchers to answer their "how and why" (p. 351) questions within a bounded system. In this case, the current study explored how the integration of the concepts of mathematics and social justice issues affected the social consciousness of the five female students inside a classroom. Given the context and setting of the study, the case study approach was deemed to be the most appropriate. A phenomenological study was not chosen because the focus was not on the lived experiences of the students, but rather on how and why the integration affected their overall social consciousness. Further, the researcher eliminated the choice of using a grounded theory approach early on as the researcher's goal was not to develop a theory upon study completion, but rather to identify how and why the incorporation of

mathematics can help or hinder the students in fully understanding and realizing the importance of current social justice issues.

In investigating the use of social justice issues in mathematics to promote social agency in female high school students, this research study replicated elements from Wonnacott's (2011) research. Although this study aligned itself with elements of previous studies (Gutstein, 2006, 2007, 2016; Wonnacott, 2011) that utilized site-based research to explore and understand learning and teaching mathematics for social justice, this study differed in that it did not conduct the study from the practitioner's point of view. Although the researcher is employed at the participating site, she served as a principal investigator who collected information by means of surveying, observing, and interviewing.

The Setting

This section serves to describe the school and relies on data obtained from the school's website. Northwestern Academy (a pseudonym) is an independent school for girls located in the New York metropolitan area. Within its K-12 population of over 500 female students, students are distributed across three divisions which include the lower school (K-4), middle school (5-8), and upper school (9-12). As recently as 2016, the school's records indicated that little over 14% of the student body population are persons of color. Currently, 88% of the school's teaching staff hold advanced degrees. The student-faculty ratio is 6:1. Students who attend Northwestern Academy come from the five boroughs of New York City (e.g., Manhattan, Brooklyn, the Bronx, Queens, and Staten Island) as well as from New Jersey, Connecticut, and Westchester County.

The tuition at Northwestern Academy for its 2017-2018 academic year was over $40,000 for students in K-12. Of the students who attend Northwestern Academy, the school reported that its need-based financial aid program provided scholarships to approximately 20% of the student body; that is, students are provided grants ranging from partial to almost full tuition.

To create a more diverse and inclusive community at Northwestern Academy, the school has taken on diversity and inclusivity initiatives developed around employing research-based practices for educating girls. In furthering the school's efforts in educational equity and inclusion, Northwestern Academy partners with an outside organization that provides training around dismantling patterns of racism and injustice in schools and communities. To assist White educators at Northwestern Academy with the historical framework, interpersonal skills, and strategies to develop curriculum that are needed to teach diverse students, Northwestern Academy also partners with TFL (a pseudonym). Once a year, Northwestern Academy sends faculty and students to attend the annual conference for independent schools that aims at providing a safe space for both educators and students of color and allies to participate in leadership and professional development and networking opportunities.

At Northwestern Academy diversity and inclusivity work take place as soon as students are enrolled in the lower school. For example, in the lower school's social studies class, questions are embedded in the curriculum to challenge students to think about stereotypes and their individual identities as well as to celebrate cultural diversity. When students enter middle school, the English/Language Arts and Foreign Language curriculum opens students up to hearing works by various writers from diverse cultures

and experiences. As students enter upper school and assume leadership roles, they are encouraged to take a stance on matters they find relevant or interesting.

Participants

The average adjusted gross income reported for 2012 for the population surrounding Northwestern Academy was $512,000, of which the average salary reported was $218,413. The demographics reported in the statistics course indicated that 50% of the students stated they were White, 25% Hispanic or Latino, and 25% other; no students reported they were Black, Asian, American Indian or Alaska Native, or Native Hawaiian or Pacific Islander. Of the eight female students enrolled in a year-long, high school statistics course at Northwestern Academy, five of these students agreed to participate in the study: Carla, Jennifer, Kate, Ruby, and Susan (all pseudonyms). On the demographic questionnaires, Jennifer, Kate, and Ruby reported that their race/ethnicity was White, while Susan and Carla indicated they were Hispanic or Latina. Although all five participants reported in their initial questionnaire that they were 17 years of age and seniors at Northwestern Academy, Ruby reported during her face-to-face interview that she had a birthday during the study and turned 18.

Through a process of criterion sampling, the participants were selected from a group of students currently enrolled in a Grade 12 statistics course. Based on the school's guidelines, students could enroll in the high school statistics course if they had completed the prerequisite high school mathematics coursework (e.g., Algebra II, Geometry, and Precalculus) with a score of 70 or higher. According to Mr. Tyler, the mathematics teacher, the grouping of students in the statistics course at Northwestern Academy was

heterogeneous. Depending on a student's interest, previous coursework completed, and educational track (e.g., honors or general education), a statistics course at Northwestern Academy may consist of both Grade 11 and 12 students.

The high school's master schedule of courses and records were used to identify the current high school teacher scheduled to teach statistics. In soliciting the instructor of the statistics course, the researcher conducted a face-to-face meeting to determine what, if any, background or previous teaching experience the teacher had in teaching around social justice. The teacher identified was a White, male teacher with 2 years of teaching experience. He indicated that his educational background consisted of a Bachelor of Arts degree in mathematics, with a minor in ethics studies and a Master's degree in mathematics education.

In implementing the lesson plans, the statistics teacher worked alongside the researcher to adapt lesson plans. Based on classroom observations, reading of students' coursework, and interactions with students, the statistics instructor was asked to provide general information about each student's understanding of the mathematical concepts being taught as well as the participants' previous performance in statistics.

Materials/Instruments

This study was based on three statistical lesson units framed around teaching mathematics for social justice (TMSJ). The lesson units embedded in the study were adapted from Gutstein (2006), Gutstein and Peterson (2005), and Wonnacott (2011). In addition to questionnaires, post-lesson reflections, and personal interviews, these instruments assisted with examining how the female learners thought about social justice

issues. In addition, these instruments aided in determining if or how participants' experiences with a TMSJ curriculum embedded into a high school statistics course may affect their ability to want to effect change in the world.

In obtaining each participant's consent, the researcher met with the class prior to the start of the study to provide an overview of the study. During that time, the researcher provided the class with a Participants' Informed Consent and Participants' Rights Form (Appendix A) as well as a Parent Permission Form (Appendix B). During the meeting, students were provided with ample time to ask questions about the procedures and were informed of their right to decline participation. In checking to see if they comprehended the study and the requirements, students were randomly called on to summarize their understanding of the study to the class. With the study taking place during normal school hours, the teacher and the researcher also informed the participants that although the class would participate in the lessons taught by the teacher, students had the option of not participating in the study. Therefore, any information obtained by the teacher of students who did not wish to participate would not be shared with the researcher.

All observations and individual interviews were audiotaped with the participants' written consent. Prior to being asked to partake in each activity or interview, participants were required to read and give their consent by signing the consent agreement to be audiotaped once parents agreed to their participation in the study. Any participant wishing not to be recorded during a whole group observation had the option of not participating in the discussion or having their responses deleted from the audio. If anyone during an individual interview wished to decline the audio recording but consented to the interview, the student was informed that her interview would be transcribed.

In starting the study, the researcher asked the class to participate in an initial questionnaire (see Appendix C) administered using the online platform Qualtrics at the start of class on November 30, 2017. To obtain information about participants' demographics and current volunteer work, the researcher adapted questions in this section of the survey from Holodick-Reed (2013). Additional questions pertaining to the understanding of mathematics/statistics, attitudes towards or interest in mathematics/ statistics, and understanding of learning and teaching mathematics for social justice were adapted from Wannocott's (2011) study and tested to examine the effects of teaching mathematics for social justice to an affluent population.

TMSJ Lesson Units

Over the course of 3 weeks, the researcher observed five students' interactions with a TMSJ curriculum comprised of three lesson units around social justice issues relating to racial profiling, education versus income earnings, and HIV/AIDS in Canada. In embedding a social justice lens into the current statistics curriculum, the researched adapted several lessons pertaining to one or more specific mathematical tasks to cover 3 weeks of material. Before starting the lesson units, the mathematics teacher and the researcher met to discuss and revise the lessons to ensure alignment with the current mathematics curriculum. The lessons in those units are described as follows:

Unit 1. In completing Unit 1, students were asked to participate in three different mathematics activities which required them to use mathematics to analyze racial profiling data against actual data. Using the mathematics lessons *Driving While Black/Driving While Brown (DWB/DWB)—A Mathematical Investigation of Racial Profiling: It is Racism?* adapted from *Rethinking Mathematics Teaching Social Justice by the Numbers*

(Gutstein & Peterson, 2005) and *Reading and Writing the World with Mathematics: Toward a Pedagogy for Social Justice* (Gutstein, 2006), students were asked to participate in a probability simulation. The activity asked students to use the law of large numbers to determine if mathematics can be used as a tool to investigate whether African American and Latinos/as in Chicago, Illinois were perhaps racially profiled by police officers, as previously claimed by African Americans and Latinos/as.

The statistics teacher used students' results from the simulation to open up a class discussion around racial profiling and racism. Within Lesson 1 were three mathematics activities (see Appendices D, E, and F) and a post-lesson reflection (see Appendix G) embedded at the end of the lesson. In developing students' understanding of randomness, experiment, sample size, simulation, experimental and theoretical probability, and the law of large numbers (Gutstein, 2012), students were asked to flip a coin 100 times and record the outcome. The second activity asked students to participate in a simulation involving randomly pulling different colored chips from a paper bag to find the racial breakdown of Chicago in 2010. The third activity involved having students investigate driving while Black or Brown in Chicago from 1987-1997 by calculating the results of simulating 100 discretionary stops (Gutstein, 2006; Gutstein & Peterson, 2005).

Unit 2. Unit 2, *Schooling Versus Income Earnings* (see Appendix H), was adapted from Wonnacott's (2011) study. The goals of the lesson were to determine whether: (a) a relationship existed between the number of years a person spends in school and their income, and (b) there is a difference between male and female earning power. The data for this lesson were retrieved from the 2010 Current Population Report taken from the U.S. Department of Commerce, Bureau of Labor Statistics.

In participating in this lesson, students were asked to examine the data on median annual income and level of education in 2010 to write a hypothesis. Next, students were asked to present both the male and female data in a scatter plot and determine the line of best fit and *r*-value. From this information, students were asked to determine if a relationship existed between the number of years a person spent in school and his or her income. As related to earning power, participants were asked to use the line of best fit to examine the difference in male and female earning power.

Unit 3. In seeing how students were affected by social justice issues relating to individuals living outside of the United States, participants were asked to take part in a third lesson which investigated the HIV/AIDS cases in Canada (see Appendix I). This lesson plan was developed by Michelle Munk and adapted from the Wonnacott (2011) study. After reading a brief background of the number of HIV/AIDS cases reported by the Public Health Agency of Canada (PHAC) between 1996 to 2016, participants were instructed to use technology (e.g., graphing calculator or online tools) to construct a scatter plot with quadratic features. When looking at the data over time, participants were asked to use the data presented to make a statistical forecast to explain when the HIV/AIDS epidemic in Canada would conclude.

Given that these lesson units were adapted to meet the high school mathematics curriculum, all students enrolled in the course were required to complete all classroom activities. They were not, however, required to participate in the research study. As such, required coursework completed by non-participating students was not shared with the researcher.

In determining if students (a) possessed the level of mathematical understanding needed to address statistical concepts raised in the TMSJ unit study, or (b) could "draw on [their] knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress" (NCTM, 2000, p. 3, see also Gutstein, 2003, p. 46), the teacher during the course of the study taught concepts around basic probability of large numbers, linear regression, correlation coefficient, and quadratic modeling.

Follow-up Materials/Instruments

To gauge students' initial experience with the social justice curriculum that had been embedded in their statistics classroom, the researcher constructed a face-to-face interview (see Appendix J). According to Creswell (2013), Denzin and Lincoln (2013a, 2013b), and Marshall and Rossman (2015), one of the significant benefits of employing an interview is that researchers are presented with an opportunity to capture a person's perspective of an event or experience through in-depth interviews (see also Bloomberg & Volpe, 2016). In this study, the design of the questions aimed at gaining additional information about students' background as well as their learning style. Moreover, the survey question design sought to gather information about participants' overall experiences with doing and learning statistics and mathematics before the start of the study. Item numbers 13-19 were placed on the post-lesson survey to gauge students' experience and determine if they were affected by engaging in the first TMSJ lesson on racial profiling. More specifically, Task 14 sought to examine how participants may have used mathematics or statistics in the process of developing sociopolitical consciousness

to investigate racial profiling or driving while Black or Brown. To investigate if participants had developed a sense of social agency through writing the world with mathematics, Task 17 solicited information on whether or not participants felt compelled to want to take action after they had participated in Unit 1.

Given that the initial online survey gathered general information about the participants and how they understood teaching and learning mathematics for social justice prior to the mathematics teachings, an exit survey (see Appendix K) was conducted to have students reflect on their initial responses. Other than being easy to administer, an online survey was unobtrusive (Fink, 2013; Fowler, 2014). The exit survey was structured so that open-ended questions could tap into personal experiences (Bloomberg & Volpe, 2016). In doing so, the survey aided in investigating if participants' perception of teaching and learning mathematics for social justice had changed. With the open-ended structured questions, the researcher hoped to shed light on determining if students' attitudes towards mathematics were altered.

Table 2

Learning and Teaching Mathematics for Social Justice Post-Lesson Survey

Item Number	Question
13	When doing activities in Unit 1 that investigated racial profiling or driving while Black or Brown (Gutstein & Peterson, 2013; Gutstein, 2006), did you find yourself using those same steps or approaches as in Question 12 to complete problems? If so, explain.
14	On your free write activity for Unit 1, you stated that the following mathematics or statistics concepts ***(list concepts)*** may or may not be useful in determining if African American and Latinos/as are stopped, searched, harassed and arrested because they "fit" a racial profile in Chicago (Gutstein & Peterson, 2013; Gutstein, 2006). Can you provide more detail about the use of mathematics or statistics you believe would answer this question?

15	Tell me about what immediate emotions developed as a result of participating in the activity on racial profiling.
16	In your post-lesson reflection, you shared that you (were/were not) affected by doing this activity. Can you explain a little more about why this might be the case?
17	Based on your participation in the three activities in lesson one on racial profiling, did you feel compelled to want to take action? Why or why not?
18	In using mathematics, did it change your initial response to how mathematics can be used to investigate racial profiling? Please explain why or why not.
19	If at all, how did participating in lesson one differ from the previous statistics lesson taught by your teacher? Explain.

Furthermore, the exit survey was intended to gather information on whether participants initially believed that the purpose of teaching and learning mathematics for social justice had been altered by partaking in TMSJ lessons. In determining if this was, in fact, the case, participants were presented with their original statements as asked to revisit the questions. At such time, participants were instructed to indicated if there were no changes or provided additional information to explain their change in thinking.

Data Collection, Processing, and Analysis

Confidentiality

In ensuring the participants' confidentiality, the name of the independent school and its location were not disclosed. For the purpose of this study, the school and the participants were assigned pseudonyms. The record of participants and assigned pseudonyms were kept confidential and protected by an online password site. All observations, interviews, and participants' work samples were obtained with the participants' written consent. These documents along with consent forms were scanned or

retrieved by digital form and were password-protected. In maintaining the ethics of this study, the data were linked to each participant and assigned pseudonyms during their interviews as well as placed on written communications and forms. After the teacher graded the work for his purposes, the data were shared with the researcher. The researcher then removed the participants' names by voiding them out and assigning pseudonyms for maintaining confidentiality during the study. In ensuring the anonymity of the participants, the pseudonyms were not shared with the teacher of the course nor the participants. In reference to the staff or faculty members in this study, the following pseudonyms were also established: Mr. Tyler, the mathematics teacher; Dr. Goodwin, the principal; and Dr. Parker, the head of school.

As a means of storing raw data, the researcher collected and stored the raw data on a password-protected computer. As a requirement for obtaining access to the documents, a password or viewing rights are needed to view data. Although the raw data were shared with an outside agency to analyze the data, the participants' anonymity was maintained. To ensure that ethical standards were in place, the consent forms, instrument papers, and coded and data materials were stored in a locked file cabinet in the researcher's office. All digital formatted materials and audio recordings (e.g., coded or not coded) were stored on a password-protected computer.

Data Collection Method

Throughout the TMSJ unit of study, classes at Northwestern Academy operated on a 6-day rotation and lasted 70 minutes per section. Based on the class schedule, students met for 4 days out of a 6-day rotation. For the purpose of teaching three lesson units developed around teaching mathematics for social justice, the study started at

Northwestern Academy on November 30, 2017, and the teaching of the lesson units concluded on Friday, December 22, 2017. Approximately 2½ months after the first lesson, the researcher administered a follow-up survey on February 13 to see if participants' initial comments were altered by their experiences. In providing a timeline of the events that took place over the course of the study, an outline of the lesson activities administered, and their duration is presented in Table 3.

Survey. After administering the initial survey to gather students' general demographics and their understanding of teaching mathematics for social justice, the teacher taught the three lesson units. During three lessons, the researcher observed from the back of the room the participants' mathematical thought processes as well as their immediate reactions to participating in statistics lessons embedded with a social justice lens. To record the data, the researcher took field notes in a Google document and audio-recorded the instruction. The participants' and mathematics teacher's comments and mathematical calculations were recorded on the left of the field notes, and the researcher's reflective notes were transcribed on the right. Any information obtained from non-participants during the audio recordings was deleted. As noted above, participants' names were replaced with their assigned pseudonyms.

Table 3

Teaching Mathematics for Social Justice Outline of Activities

Activity	Unit/Lesson Number and Activity	Duration
Initial Survey: Background Information and TMSJ Focus	Online Survey	15 minutes
Introductory to Probability and Law of Large Number	Unit 1 Lesson1	2 days

Find Chicago's Racial Breakdown Simulation Including Post-Lesson Reflection	Unit 1 Lesson 2	2 days
Interview: Initial Thoughts About Reading and Writing the World with Mathematics on Lesson 1	Face to Face Interview	15 minutes
Investigating DWB/DWB	Unit 1 Lesson 3	2 days
Education versus Income	Unit 2	2 days
HIV/AIDS Canada	Unit 3	3 days
Exit Survey: Experience with TMSJ and Attitude Towards Mathematics	Online Survey	15 minutes

Post-lesson reflections. Immediately following each lesson, participants were asked to complete a post-lesson reflection. In allowing students to reflect on their learned experience from having participated in each lesson activity, the researcher presented participants with similar questions per each post-lesson reflection. For the purpose of students communicating their thoughts in written form, post-lesson reflections to an online Google document were sent to the mathematics teacher. Then near the end of the lesson, the teacher provided students with the link. In providing the participants' responses to the research, the mathematics teacher then shared participants' Google documents with the researcher for analysis and coding. The participants' post-lesson reflections were structured with open-ended questions that pertained to gathering information about whether students' experiences had led them to develop sociopolitical consciousness and social agency. One of the benefits of having participants type their responses was that it allowed the researcher to capture precisely what each participant

was alluding to. Rather than conducting a focus group, the written post-reflections provided less distraction and took approximately 10 minutes to complete.

Interviews. After Unit, participants were contacted via email to schedule a face-to-face interview to discuss if, in fact, the TMSJ lesson on racial profiling provided insight into society's assumptions, inequities, and limits. Based on the participants' availability, the researcher met with each participant for 15 minutes in an office space to ask open-ended questions. During the interviews, participants were informed of the interview format and how they could conclude the interview at any time. In explaining the interview process to the participants, the researcher noted that the first portion of the interview would provide additional information on personal demographics. The second part of the interview would provide insight into the experience students took from Unit 1 and whether they were affected by the activity. The last few questions would inform the researcher of whether Unit 1 led students wanted to take action.

With the participants' consent, their feedback from the survey was audio-recorded and later transcribed using the online platform Temi. As such, all transcriptions were time-stamped and stored in a digital Dropbox.

Exit survey. At the end of Unit 3, the researcher conducted an exit survey to investigate how or if participants were affected and whether the lesson resulted in students wanting to take action. To allow students time to reflect on their experience, the exit survey took place 2½ months after the initial questionnaire. To collect information about participants' experiences with the TMSJ curriculum embedded in a statistics course and their attitudes towards mathematics, the survey was constructed in a Google document format. This allowed each participant to be presented the questions along with

her initial feedback provided on the entrance questionnaire. In sharing the file with the participants, each participant was added as a collaborator to the specific document and given rights to edit the document. Upon completion, the participants' right to edit the Google document was removed. This allowed the researcher to remove and replace all identifying names with a pseudonym.

In organizing the data obtained for the study, scanned items (e.g., response course work, reflections, and free write activities) and transcriptions of observations and interviews were sorted in a table within a Google document. The table listed each participant and provided a cell to record if each participant data were collected for ag given task. When the data were collected, they were then linked to the document through the use of Google sharing. As a precaution, the data were also stored in a password-protected computer.

Data Analysis

The four sources above were analyzed through a qualitative thematic analysis which has the general aim of obtaining themes from the data. In this case, all four data sources were used as the main foundations of the study themes. Braun and Clarke (2006) described thematic analysis as "a method for identifying, analyzing, and then reporting" the most common patterns and themes from the data analyzed (p. 79). The researcher then repeated the cycle (see Figure 1) for all data sources such as the survey/free write activity, post-lesson reflection, interview, and exit survey in order to address the three research questions of the study adequately.

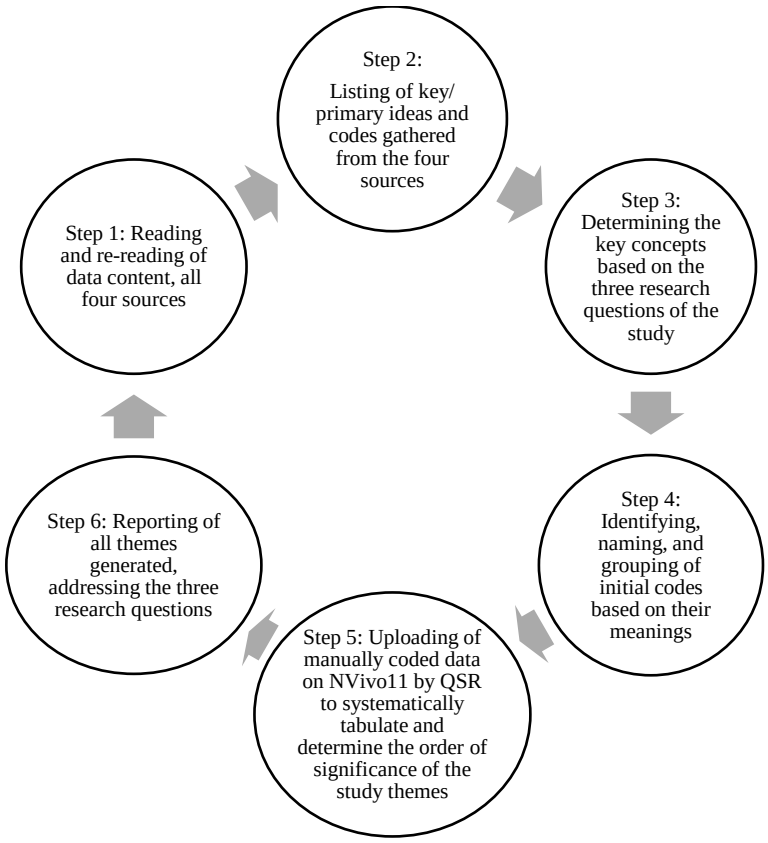


Figure 2. Modified thematic analysis steps for the current study as modeled by Braun and Clarke (2006)

Validity and Reliability

Researchers have the accountability to truthfully and fully interpret the participants' shared perceptions from the data and report them to the research study (Yin, 2014). To fully warrant the process of the trustworthiness of a qualitative study, Lincoln and Guba's (1985) four constructs of a good qualitative research study were followed. The constructs were credibility, transferability, dependability, and confirmability.

Credibility

The research study's credibility was attained by performing a member check with each of the five students' surveys, interviews, and gathered reflections (Merriam &

Tisdell, 2015). The member check was performed by asking the participants to thoroughly read, re-read, examine, and re-check all four data sources gathered and reported. Upon the presentation, the female students had the opportunity to correct any wrong information or interpretation and edit them as requested. Merriam and Tisdell (2015) explained the strategy was vital to confirm the soundness and legitimacy of the collected data. The member checks with the five students then validated the researcher's data collection, analysis, and interpretation.

Transferability

The second construct was the transferability of the research study wherein the data gathered from the interviews were presented and discussed in complete detail. Merriam and Tisdell (2015) emphasized the need for another strategy which was the provision of "rich and thick descriptions" of the data (p. 259). In this case, the researcher sought to report the shared perceptions and experiences of the participants from all sources based on the actual data and in verbatim form to ensure the current research was comprehensive enough to be applied to other contexts or settings.

Dependability

To ensure the dependability of the research results, the researcher ascertained that the questions asked to the participants focused only on the integration of mathematics and social justice issues. This was to limit the boundaries of the scope and concept of the study to the actual subject being explored. Further, the researcher ensured during the data collection that participants were comfortable enough and had no issues or problems with

any of the tasks or responsibilities connected with the research study. This was to guarantee that participants were able to provide sound and unbiased perceptions.

Confirmability

The conformability of the study was achieved through the completion of an audit trail for the whole research study process. Merriam and Tisdell (2015) shared an audit trail that provided a detailed account of the study's "methods, procedures, and decision points" (p. 259). In the current research, the researcher recorded and presented the data collection and analysis steps to access them at any point easily and when needed. Additionally, this was done to provide pieces of evidence that the study was performed without any bias or other probable influences that may have influenced the result(s) of the study.

Ethical Assurances

To follow and conform with the policies set in place by the University's Institutional Review Board (IRB), which are required when dealing with and involving human participants in research studies, the researcher submitted the procedures to the IRB. The research study processes and planned data methodology were conducted to seek approval and have the go-signal to contact and recruit participants for the study. This was also performed to guarantee the welfare and safety of the participants (the five female students).

With the approval of the IRB, the researcher started contacting the participants and reviewed and presented the purpose and other information about the study. Once the participants agreed to join the research process, an informed consent form containing the

role of the researcher, the scope of the participants' participation, probable risks when joining, protection of their identities, and an overview of the research study were discussed and presented to the participants. No study procedure and data collection were conducted until all five participants read and signed the informed consent, in accordance with strict IRB guidelines. To further protect the participants' well-being, their confidentiality and anonymity were also guaranteed. The researcher developed five code names for the five female participants to ensure their identities would not in any way be revealed or determined in the research study. Participants were also informed how data were to be collected, presented, and stored upon study completion. In terms of data filing and storage, all participant names were removed and replaced with pseudonyms such as Participants Kate, Jennifer, Ruby, Susan, and Carla.

Additionally, data were stored on a password-protected computer for which only the researcher had knowledge of the lock code; all printed and soft copies of any data related to the study were stored inside a locked filing cabinet after every use. The data will be safely stored and protected for 6-7 years, after which the data will be completely destroyed. Finally, all five participants volunteered for the study; no one was forced or coerced to participate. No incentives or rewards were presented, and the researcher solely hoped for the students' cooperation and a sense of social consciousness upon seeing and learning of the purpose of the study.

Summary

The third chapter discussed the methodology of the research study. A single-case study was chosen to report the four data sources collected from the five female students

who belonged in a bounded system or inside a single classroom. The four sources such as the introduction survey and free write activity, post-lesson reflection, interview, and exit survey were analyzed using Braun and Clarke's (2006) thematic analysis. All sources were then coded and analyzed with the aim of addressing the three research questions of the study. Data were collected with the assurance that all IRB procedures were strictly honored. Further, limitations were minimized as the researcher worked to delineate the effects on the actual results of the study and Lincoln and Guba's (1985) criteria of a good qualitative research study was followed. In the next chapter, the results of the thematic analysis of the four data sources in response to the three research questions are presented and thoroughly discussed.

Chapter IV

RESULTS

The purpose of this qualitative study was to understand what high school students in an all-girls independent school in New York think about social justice issues, and how it may have affected their learning of statistical concepts. More specifically, the researcher used a case study approach to investigate the factors that affect female students' development of sociopolitical consciousness and social agency through reading and writing the world with mathematics (RWWM). The research questions asked:

1. How does the incorporation of social justice issues into a high school statistics curriculum at an all-girls independent school affect female students?
2. How, if at all, do students in an all-girls high school statistics course develop sociopolitical consciousness through "reading the world with mathematics"[1]?
3. What factors contribute to or prevent students in an all-girls high school statistics course from using "mathematics to write the world"[2]?

[1] RWWM means using mathematics to understand relation of power, resource inequities, and disparate opportunities among different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation, as well as to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them (Gutstein, 2003c, p. 45).

[2] This means to use mathematics to change the world or to see oneself capable of making change through a means of developing social agency (Gutstein, 2006, p. 27).

This chapter opens with the profile portraits of five female students. The information included highlights participants' background and initial thoughts on the purpose of mathematics and statistics as well as seeks to gather information about each participant's initial definition of understanding of teaching mathematics for social justice. In providing a portrait of the classroom discussion and activities in each lesson unit, the researcher included some of the mathematical computation and responses from the classroom discussion and worksheets. Following this section is a data analysis of the four data sources (e.g., initial survey/free writes, lesson reflection, face-to-face interview, and exit survey) interwoven in Unit 1—Racial Profiling, Unit 2—Education versus Income, and Unit 3—HIV/AIDS in Canada. The presentation of findings highlights a breakdown of themes and discussion of the results, followed by a summary of the chapter.

Portraits of Participants

Case 1: Carla

General background. Carla, a student with high interest and participation in the performing arts and art-related work, lives near Northwestern Academy. Her father has a Master's degree and her mother is an immigrant from a Latin American country with some graduate or professional schooling. Carla stated during her interview that up until her hometown became a little dangerous, she and her family would often visit where her mother grew up. On her initial questionnaire, Carla referenced that she has a 3.6-4.0 grade point average and an A- in statistics. Her extracurricular or sports-related activities involve crew, participating in the school plays and musicals, and the knitting club. Carla currently serves as the Co-Ambassador of an affinity group and Co-Editor of the school's

paper. In the last 5 years, Carla has volunteered at church-related functions, at Christmas donation sponsored events, and in a program geared towards helping prisoners integrate back into civilian life.

When asked about her least favorite class, Carla articulated during her face-to-face interview that she did not like science. Carla attributed her current dislike for learning science to the way teachers at her school structure the course. During her interview, Carla stated: “I hate the sort of pedagogy learning that we are doing. It’s like all students lead the learning. I like the small school environment with a small student-to-teacher ratio.” She further elaborated that she did not understand why students in the science class did not take advantage of the small class sizes. When asked to discussed mathematics, Carla indicated that sometimes learning mathematics can be fun. However, she explained that mathematics can become difficult when she has to break down challenging mathematics problems into smaller parts.

Initial thoughts on the purpose of mathematics and statistics. When asking about her initial thoughts about the purpose of learning mathematics, Carla communicated in her initial survey: “I think that the purpose of learning math is learning to problem solve. Most people choose to not become mathematicians after high school, but it helps develop skills in learning to take on challenging tasks.” To expand on if Carla thought people should learn mathematics, Carla responded:

> I don’t think mathematics is the most necessary school subject, and I don’t understand why it needs to be taken for all four years of high school, while you can choose not to take other subjects. However, I suppose it is important to have a basic understanding of math to learn logical thinking and understand the math that reoccurs in everyday life.

Carla was then asked what she thought the purpose of teaching statistics as a mathematics course in high school should be. In answering this question, Carla stated, “I think that the purpose of learning statistics in high school is to be able to see when the stats shown on the news or Internet are accurate so that you can draw true conclusions about the world.”

Initial definition and understanding of TMSJ. Although Carla stated during her initial survey that she discussed issues of social justice in her History, French, and English/Language Arts class, Carla was also asked to comment on what she thought the effect would be if social justice issues were incorporated into a statistics or mathematics classroom. Carla believed students would develop an interest to learn statistics or mathematics through social justice because “it would give it relevance in the real world, as opposed to completely random problems”; therefore, it would be more interesting to learn. In articulating her working definition of teaching mathematics for social justice, Carla stated that “teaching mathematics for social justice could mean learning how to calculate numbers that would give perspective on the world and the issues that many people face.”

Case 2: Jennifer

General background. Jennifer, a native of an Eastern European country, stated that her native culture and Jewish background definitely had an impact on her transition into an all-girls independent school. Without knowing if her family would return to their native homeland, Jennifer’s family first enrolled her into an Orthodox Jewish school. Upon entering a public middle school in the United States, Jennifer pursued her interest in coding. Now during her senior year, Jennifer believed that she is no longer interested in STEM-related activities. She claimed that ever since she started Northwestern

Academy, her academic performance has “gone downhill.” Jennifer believed that her previous educational experience outside of Northwestern Academy was filled with her learning through drills. According to Jennifer, this has led her to not understand the meaning behind learning.

As for her approach to learning concepts in statistics, Jennifer indicated that she met with her statistics teacher regularly outside of class. Jennifer currently has a C average in statistics and a grade point average of 3.1-3.5 on a 4.0 grade point scale. When struggling with a mathematics concept, Jennifer stated that she felt as if she needed one-on-one attention from the teacher. When working in large groups in mathematics class, Jennifer felt nervous and afraid to ask questions. Jennifer attributed her actions of failing to ask questions to being lazy as well as concerned with the group’s dynamics rather than her own learning.

When discussing the educational background of parents, Jennifer indicated that her father holds a Bachelor’s degree and her mother obtained a doctoral degree. She further explained that although she has no prejudice, her parents’ upbringing in other countries may at times have caused them to say things that she felt were not politically correct. Jennifer further explained that depending upon the situation, her parents might say things that appear to be slightly racist. Although Jennifer stated that her parents did not mean the things they said, she attributed her parents’ actions to their background and what people have informed them as the norm.

As for Jennifer’s involvement in school, her extracurricular activities include theatre production and community service. Within the last 5 years, Jennifer has participated in over 100 hours of community service events held or sponsored by

Northwestern Academy as well as programs associated with her previous middle school that relate to Jewish holidays. Jennifer reported that she is a member of the Gay-Straight Alliance.

Initial thoughts on the purpose of mathematics and statistics. Jennifer stated that she thought the purpose of learning mathematics or statistics was "to understand our world from a quantitative perspective, to train our brain to think a certain way." She added to her initial thoughts on her initial survey that people "should learn it in order to train their brain and learn how to perceive things in a quantitative way." In doing so, Jennifer believed it will "help students learn how to assess problems and data within our world and challenge information rather than simply receive it and trust it."

Initial definition and understanding of TMSJ. Jennifer's previous experience with learning about social justice in a K-12 setting extended to her school and is a frequent event. In sharing how social justice issues are explored at Northwestern Academy, Jennifer wrote that social justice issues are "explored in class by relating topic matters to relevant events that currently happen in our society (because it associates emotion and applicability with the material we are learning in class)." Jennifer is currently interested in social justice issues related to Black culture and stigma as well as mental health and stigma. In sharing her reason for this interest, Jennifer wrote, "I like to be informed about history before I make assumptions about certain groups of people." She also added that she was currently reading *Hip Hop Wars: What We Talk About When We Talk About Hip Hop—and Why It Matters* (Rose, 2008).

When asking Jennifer to expand on her knowledge of learning for social justice to formulate a working definition of teaching mathematics for social justice, Jennifer wrote

in her initial survey: “Learning about statistics such as incarceration rates, because when you learn about these statistics it can be very eye-opening, and you feel truly informed about issues that people ‘talk about’.” She further explained that the effects of incorporating social justice issues into a mathematics or statistics curriculum would render a situation in which “students would be more educated and genuinely interested in the course because it has real-world applicability.”

Case 3: Kate

General background. Kate indicated that she had attended Northwestern Academy since Kindergarten and lives in an apartment with her mother near the school. During her face-to-face interview, Kate stated that she has a 13-year-old brother who attends a college preparatory school for boys in the New York area. When asked about her parents’ educational background, she reported that her father had some college background and her mother had a Bachelor’s degree. Kate was involved in numerous clubs and sports at school. She also wrote for the school newspaper, participated in the animal rights club, and worked in a journalism/media internship. Within the last 5 years, Kate has volunteered for over 50 hours at nonprofit daycares and in soup kitchens.

As for her academic performance in statistics, Kate indicated that she currently has an average of an A- in the course. Kate indicated that although she was fascinated with statistics, the large numbers were hard to visualize. Therefore, in learning mathematics concepts, Kate reported that she needed hands-on activities to avoid zoning out in class. In an interview, Kate indicated that her approach to learning concepts in

statistics was through trial and error. In answering a statistics problem, Kate stated that she usually attempted the mathematics problem three or four times before figuring it out. Kate acknowledged that learning statistics was different than learning precalculus. During her interview, she stated that she struggled with memorizing the formulas used in precalculus. However, in doing statistics, she felt that the teacher was asking her to approach mathematics problems through the use of common sense.

Initial thoughts on the purpose of mathematics and statistics. To gauge Kate's initial thoughts on the purpose of learning mathematics and why people should learn mathematics, Kate voiced in her initial survey:

> There are many careers that mathematics and statistics are applicable to, so they can help you from a career standpoint. Additionally, it is helpful to learn statistics for numerous everyday tasks such as figuring out how much of something you need to purchase, understanding your chances of getting into colleges and schools from an acceptance rate standpoint, and identifying when statistics in articles, newspapers or ads are inflated or incorrect. People should learn mathematics to figure out if they have a passion for it and want to go into a career involving mathematics. Additionally, I think it is an important life skill to learn because it can help you in a variety of circumstances.

In comparing her initial response about the purpose and the need for learning mathematics to statistics, Kate indicated that "statistics are a part of the world." As such, statistics is

> found in articles, newspapers, ads and on television networks and news channels. It is important to learn that statistics can be skewed and not always accurate, and it is equally important to learn how to identify when that is the case. Additionally, there are many careers that utilize statistics, so it is important for students to explore it in order to learn whether or not they have a passion for it.

Initial definition and understanding of TMSJ. Regarding the incorporation of social justice awareness or education for the whole school, Kate stated in her initial survey "a few times each school year we have 'fixed days' dedicated to discussing social

justice issues. Additionally, in some English classes, we often compare the social issues in the books we read to modern day social issues." When asked what some social justice related issues that she was interested in studying were and why, Kate responded: "A social justice related issue that I am interested in studying is women's rights because as a woman, I feel it is important to educate myself on these issues. However, I also think it's important that I learn about social justice issues not completely related to myself such as racism and LGBTQ rights."

When looking at whether Kate can use her prior knowledge and experience to form a working definition of teaching mathematics for social justice, she responded: "I think that teaching mathematics for social justice mostly involves statistics and encompasses learning how to read, write as well as interpret statistics regarding social issues." However, she added, "If social justice issues were to be incorporated into a statistics or mathematics curriculum, many more students would be educated about these issues subsequently giving them the opportunity to be passionate about them and be an advocate for them."

Case 4: Ruby

General background. Ruby came from a family in which both parents had obtained a doctoral degree. In her face-to-face interview, Ruby stated that she has lived in the metropolitan part of New York all her life and resides 11 blocks from Northwestern Academy. Ruby believed that because she was shy, her parents wanted her to attend a smaller school for female students. Therefore, her parents enrolled her in Northwestern Academy in Kindergarten. Ruby's current statistics grade was an A, and her grade point

average was a 3.6-4.0 on a 4.0 grade point scale. She reported that her strength in statistics was writing responses to problem sets. Ruby's extracurricular activities included co-heading Northwestern Academy's art and literary magazine. She was also a member of the Space Club. Over a span of 3 years, Ruby has completed over 75 hours of volunteer work at her temple of worship. During her free time, Ruby stated that she liked to read and write creatively as well as play with her pet chinchilla.

During a face-to-face interview, Ruby indicated that her parents enrolled her in Northwestern Academy because of the smaller school setting. With Ruby being shy, her family wanted to a school that she would feel comfortable in an all-women environment. Ruby considered herself to be both an introverted and extroverted type of person with excellent listening skills and witty personality. According to Ruby, she learned best through reading text and watching videos. For example, Ruby's favorite class was are language arts classes pertaining to literally monsters or AP literature. In explaining why she enjoyed her AP Literature class, Ruby indicated that the structure of the course allowed for really good conversations about how to analyze literature.

In learning and doing statistics, Ruby explained during her interview that she enjoyed working on her problem sets with her friends after school. When describing how her learning style differed in statistics from other subjects, Ruby reported that learning mathematics concepts required "a different kind of mindset." Rather than "looking for charter growth," Ruby believed that mathematics course required students to manipulate numbers into formulas. Ruby articulated that she viewed her strength in mathematics and statistics as being able to communicate her understanding of concepts through writing. As

for areas that she felt she needed more support, Ruby indicated that she wanted support with understanding certain formulas in different patterns. She also stated that she wanted to know about why and when certain formulas for a particular situation work/ed. In finding clarity about some of her mathematics questions, Ruby's response indicated that she used her friends as a resource from which to draw.

Initial thoughts on the purpose of mathematics and statistics. Ruby noted on her initial survey that "math is a skill we need to navigate the modern world." When asking her to elaborate on the purpose of learning mathematics or statistics, Ruby wrote: "Even if we only ever count out change at a register, or compare deals in a store, we need to understand numbers to make the best decisions. As for statistics, they're everywhere in the news, and if we do not understand them, we cannot understand how they might be manipulated, which could well lead to us getting manipulated in turn." In vocalizing why people should learn mathematics, Ruby stated:

> I think they should because without math many basic life skills will be impossible to complete. For example, there will be taxes to pay, and eventually maybe credit card balances to manage. Money to keep track of, in other words and money is crucial in this day and age. And if not money, there are many jobs which require at least a basic understanding of math, from doctor to architect.

As related to what the purpose of teaching statistics as a mathematics course in high school should be, Ruby stated, "I think it should mainly give us enough knowledge of statistics to recognize how manipulative they can be. If we're 'literate' in statistics, we can tell when someone is using a piece of the truth to lie to us. Also, statistics may open doors to greater understandings [and] revelations about our world at large."

Initial definition and understanding of TMSJ. Prior to participating in the study, Ruby indicated that she discussed social justice-related issues at Northwestern Academy in her Language Arts/English and History course. When asked to describe how social justice issues were embedded in the class discussions, Ruby stated:

> We discuss social justice mainly, I think, in history classes and certain English courses. In history we look at social justice issues and how people fought for their rights, the revolutions they lead, and how power dynamics have or haven't changed. In English just a few days ago we were discussing the social justice issues raised by *A Raisin in the Sun* [*e.g., A text written by* Hansberry (1984)], and have since moved on to examining gender in *As You Like It* [e.g., an article about Shakespeare's play].

In sharing whether social justice issues were discussed school-wide at Northwestern, Ruby stated, "We also have a day devoted to diversity and understanding how that relates to us all."

When asked what some social justice-related issues students were interested in studying, Ruby wrote in her initial survey, "the death penalty could be an interesting moral debate." Given that students were presented with a chance to learn about social justice issues at Northwestern Academy, Ruby believed it provided them with an opportunity to possibly "look at access to education" in hopes of "understanding why certain groups are consistently relegated to lower-income jobs and the like." Ruby further explained that "part of learning statistics should probably be learning to use them to make a point." For example, Ruby stated that maybe statistics could be used in the case of climate change as a means of providing evidence for any deniers.

When asked to provide a working definition of teaching mathematics for social justice, Ruby articulated in her initial survey that she thought "teaching mathematics for

social justice refers to using numbers and statistics to notice trends relating to various demographics in a given society." Ruby then provided the following example to support her working definition of the possible use of TMSJ, in that the "incarceration rates of Black men in America, say, would be a revealing set of numbers." If perhaps social justice issues were incorporated into a statistics or mathematics curriculum, Ruby noted that the effects "would depend heavily on the interests of the students involved." In that matter, Ruby stated, "I suppose best case scenario is the information the students learn galvanized them to the extent that they feel motivated to take action to correct some injustices."

Case 5: Susan

General background. Susan, according to the initial questionnaire, currently had an A- average in statistics. Both her mother and father had a high school diploma or equivalent. She was involved in social justice activities that bring awareness to women's health. Currently, she served on a member of Latinx Affinity Group, played badminton, peer tutored, and wrote for the school newspaper. Outside of school, she worked for a clothing store and tutored for a non-profit organization. For the last 6 years, Susan has been a part of an academic program called the DBK Fellowship (a pseudonym). Over the previous 5 years, Susan worked over 120 volunteer hours at a food pantry located in New York and organized books at the library. Susan indicated that in addition to her school requiring her to do volunteer work, she enjoys helping others.

Initial thoughts on the purpose of mathematics and statistics. When asked to provide her initial thoughts of the purpose of mathematics and statistics, Susan wrote: "I

think the purpose of learning mathematics is to give individuals basic skills (addition, percentages, etc.) for life. I think it also allows for critical thinking, which is also essential." In elaborating on her idea, Susan then explained what she thought the purpose of teaching statistics as a mathematics course in high school should be. According to Susan, discussed how statistics could be used as a tool to assist with creative thinking. On her initial survey, Susan written statement read as:

> I think statistics is different from most of my previous classes because it leaves room for more creative thinking. In my previous math classes, there is always a certain method to solve a problem. But, statistics requires more critical thinking and improvisation.

In defending her stance on why she thought people should learn mathematics, Susan wrote: "They should learn mathematics because mathematics can serve as a way to analyze data and make predictions. If you understand numbers (and how to identify trends), you can use it as evidence for your arguments."

Initial definition and understanding of TMSJ. Prior to participating in the unit activities, Susan was also asked to provide her definition and understanding of TMSJ. On her initial survey, Susan defined TMSJ as "a way to analyze data revolving around social issues that will help students realize the authenticity and severity of such issues." When asked what she thought the effects would be if social justice issue were to be incorporated into a statistics or mathematics curriculum, Susan provided the following response:

> There are people in the world that do not acknowledge and refuse to believe certain social justice issues. However, by incorporating it into the mathematics curriculum, there would be concrete evidence and room for discussions among peers.

Summary of Units 1-3 Classroom Discussion and Mathematical Computation

Unit 1: Racial Profiling

The purpose of Lesson 2 in Unit 1 was to investigate racial profiling or Driving While Black or Driving While Brown (DWB/DWB). As outlined in Part 2 of the activity (see Appendix E), each group was given a small bag with colored counting chips to match the racial proportions in Chicago, Illinois. Without opening their bags, participants were asked to determine how many chips of each color they thought were in their bag. Blue counting chips represented African American, green counting chips represented White, red counting chips Latinos/as, and yellow counting chips represented Asians/ Pacific Islanders/Native Americans.

Without opening their bag, Ruby indicated that her group estimated 25 total counting chips. In providing her reasoning, Ruby stated:

> Given this, we estimated the number of tiles per color using the percentages from the class data. For example, $\frac{133}{401}$ green tiles are around 33%. Thirty-three percent of 25 is closest to 8, so we estimated that there would be eight green tiles in the bag. As a class, we thought there would be eight green tiles, eight blue tiles, seven red, and two yellow.

Next, Ruby was asked to revisit her cumulative table (see Figure 3) to examine how close her group's first row was to their final answer. In doing so, Ruby's group was asked if their previously mentioned comment would remain the same or change after only 10 picks. Ruby provided the following response:

> As we picked more times, outlining numbers were "evened out" by all the data. As such, our percentages became more accurate. In our first two rows, we saw a pattern of distribution which matched the final answer. However, our answer would not be as accurate if we did not complete more tests. The more we picked, the less coincidences or random could affect our results. This is evidenced by the fact that our cumulative total was more representative of the actual number of tiles in the bag than an individual test of ten.

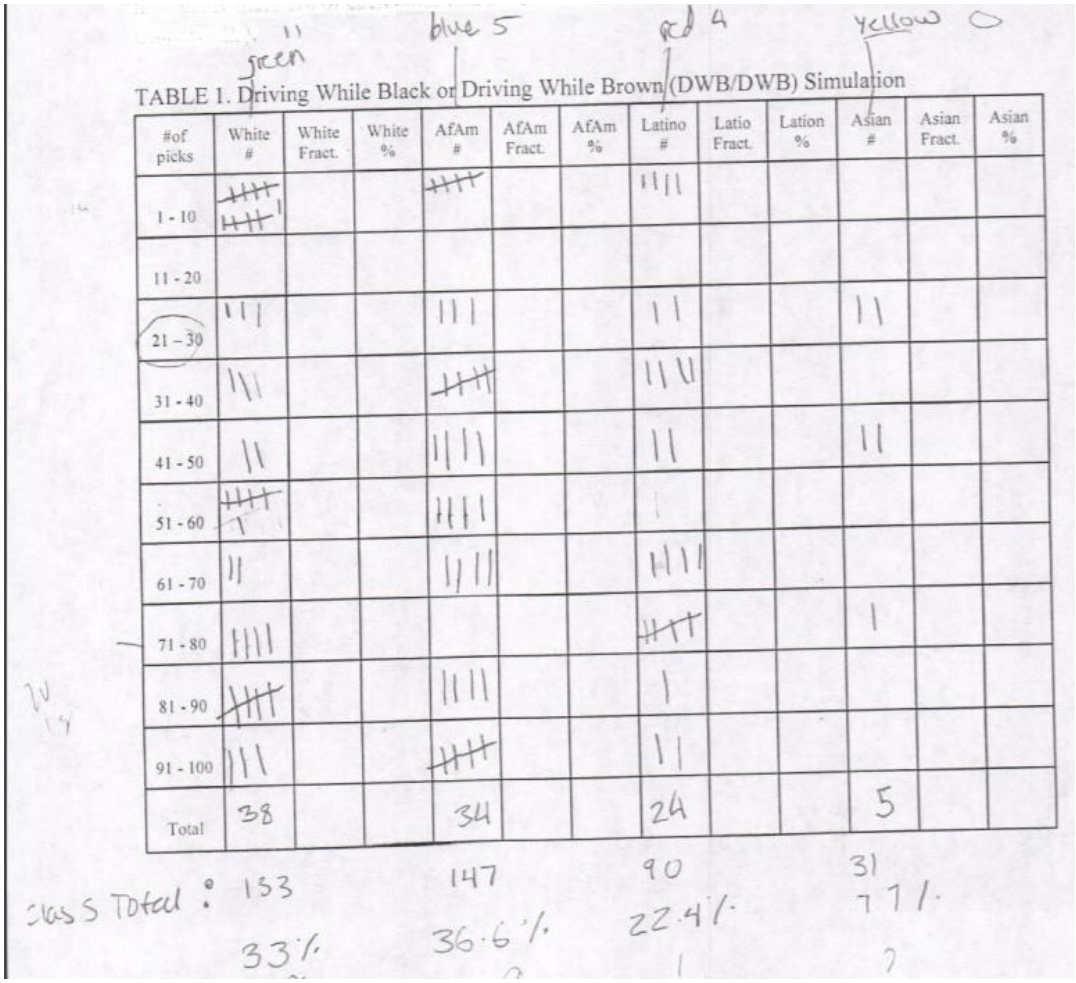


Figure 3. Actual teaching mathematics for social justice—Unit 1, Lesson 2, Driving While Black or Driving While Brown (DWB/DWB) simulation table: Results from Ruby's group

When expanding on this concept, Kate's group was asked to reflect and make a comparison between 10 picks and 100 picks. Kate replied:

> I think that after a hundred picks versus ten picks there would not be much of a difference in percentages at all. This is because all proportionate, so even though a hundred picks are a bigger sample size, the actual percentage was not changed much. However, there are benefits to having a larger sample size, and therefore the information that we gather after a hundred picks is more accurate than the information that we would gather after ten picks.

Participants were then asked to use mathematics to explain what would happen if their group were to pick the colored tiles 1,000,000 times. Ruby's, Kate's, and Susan's groups discussed how completing more trials increases the chances of accuracy. Ruby responded:

> Increasing the number of times, we picked tiles increases the accuracy of our percentages. If we picked a million times, I would assume our guess as to the actual breakdown and number of tiles in the bag would be extremely close to the reality though there is not, of course, a guarantee.

Susan added to the conversation, by stating: "If we picked a million times, the experimental probability would approach the theoretical probability. You will probably get more accurate results." As for Kate, her group stated: "If we picked a million times, the likelihood the data we receive from that will be more accurate and even close to correct is much greater. Increasing sample size usually results in more accurate data; however, it is not a guarantee."

In continuing the discussion on racial profiling, the participants also completed a third activity (see Appendix F). To determine if racial profiling occurred, students were asked to simulate the data from traffic stops in Chicago, Illinois. Using sample data from police reports from 1987-1997, students were asked to set their own simulation of the situation using counting chips to represents Latino/a divers. In an area of about 1,000,000 motorists with approximately 28,000 Latinos/as, the participants in the class indicated that $\frac{28000}{1{,}000{,}000}$ or 2.8% of the motorist were Latino/a. With 3,100 out of 14,750 of the traffic stops involving Latino/as, members of the class indicated that their mathematics calculation indicated that 21% of the discretionary traffic stops involved Latino/as.

After conducting the simulation, the participants were asked to compare the discretionary stop data to the class' simulation data. To determine if the simulation data supported the claim of racial profiling of Latino/as, the class combined their data (see Table 4) from their discretionary stop simulation. The participants were then asked to consider the simulation results and compare them to the discretional stop data. After making a comparison between theoretical probability and experimental probability, participants were asked if their simulation data supported the claim of racial profiling in Chicago, Illinois.

Table 4

Whole Class Data From Discretionary Stops Simulation

Group Number	**Red Chips**	**Other Chips**	**Total**
Group 1	3	97	100
Group 2	3	97	100
Group 3	2	97	100
Group 4	1	97	100
Total	9	391	400

In computing the relative error, the class first calculated the theoretical probability equaling $\frac{1}{36}$ or 2.77%. Next, the class calculated the experimental probability by dividing the number of total red chips (N = 9) by the total number of simulation trials (N = 400) completed by class. The experimental probability equaled $\frac{9}{400}$ or 2.25%. The class then subtracted the theoretical probability 2.77% minus the experimental probability 2.25% to account for a difference of .52%. Using the percent formula $\frac{Theretical\ Probability - Experimental\ Probability}{Theretical\ Probability}$, the class calculated the relative error as $\frac{.52}{2.77} = .187 = 18.7\%$.

Unit 2: Income Versus Education

The goal of Unit 2 was for participants to determine whether there was a (a) relationship between the number of years a person spent in school and their income, and (b) difference in male and female earning power. Students were asked to take a few minutes to write a hypothesis of the data presented in Figure 4 (e.g., Median Annual Income, by Level of Education, 2010). Although the several students provided a response to the question, the teacher asked the students to narrow the response down two to items the participants leaned toward. For example, Jennifer stated, "The more education you have, the higher the income you get with occasional expectations." Carla indicated that "The more years spent in school, the higher income and your debt," and Kate responded, "The correlation depends on the degree of study and the amount of time spent at *x* school." As such, the class concluded that more education obtained by a person leads to a higher income.

	Elementary/Secondary			College				
Sex & Year	Less than 9th grade	9th to 12 grade, no completion[1]	High school completion (includes equivalency)[2]	Some college, no degree[3]	Associate degree[4]	Bachelor degree[5]	Master degree[6]	Professional degree[6]
Men 2010	24,000	29,000	40,000	46,000	50,000	64,000	81,000	115,000
Female 2010	18,000	21,000	30,000	33,000	38,000	47,000	59,000	77,000

Note: Year-round, full-time workers 25 years and older. (—) = not available.

Figure 4. Median annual income, by level of education, 2010

Next, the teacher asked the students to discuss among themselves about how they would use the data presented in Figure 4 to graph a scatter plot. Kate then suggested to the class that should color code the data (see Figure 5). Prior to teacher using Desmos, an

online graphing feature, to construct the scatter plot, the teacher asked the participants to determine what value each level of schooling would represent on the graph.

While transferring the scatter plot onto a coordinate grid in their mathematics packet, Jennifer, Carla and Kate showed signs of difficulty. Jennifer struggled with what graph to use as well as saying that she was scared to make a mistake. After Jennifer graphed the data for males, she asked: “Does anyone see how ridiculous it is that women make less than men?” She then referenced that there was no justification for the difference in pay for men and women. Shortly later, she asked the teacher if their current school had also taken part of the problem of unequal pay of female and male teachers. As for Carla, she needed a little more time to grapple with the data. She was the last person to take out her computer to plot the information into her computer.

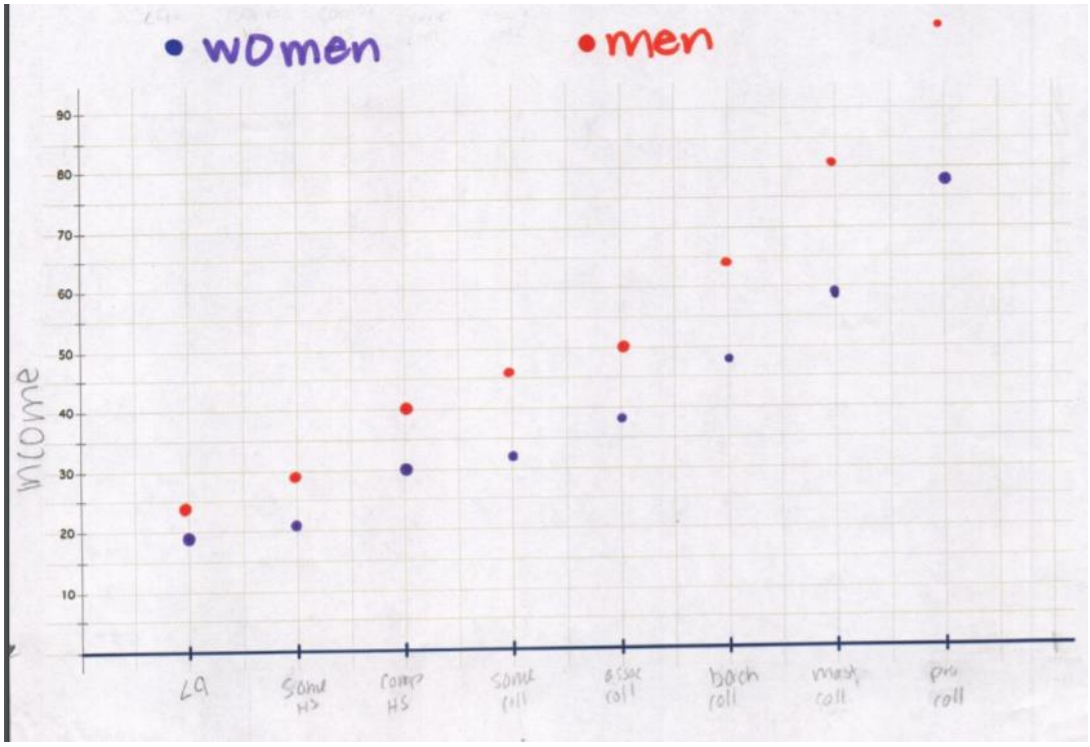


Figure 5. Kate’s graph of the median annual income, by level of education, 2010

Although Ruby was able to use Desmos to explain to the class how to construct a line of best fit for both males and females, she was unable to transfer the computer-generated graph onto her worksheet. Rather than drawing the line of regression, Kate, Ruby, and Susan connected the points in the scatter plot as if it were a line graph. During the exercise, both Kate and Carla continued to have difficulty with completing the activity.

After several conversations with peers as well as asking the teacher for assistance, Carla and Kate stopped working on the assignment. Their worksheet packet for Unit 2 showed that they failed to complete the remaining activities. Therefore, without providing enough evidence to assess the students' communication in mathematics, the ability to use mathematics to reflect, and knowledge and understanding on the MYP Assessment Rubric (2008), both Carla and Kate scored a "0" for each category (see Table 5, Abbreviated MYP Assessment Results from Unit 2).

When asked to present the equation of the line of best fit for both male and females as well as the *r*-value for both equations, Jennifer did not provide a response. However, when asked to explain what the slope of the line of best fit represented, Jennifer indicated that the slope of the line represented "a positive correlation between education and income." In supporting her answer about whether a relationship existed between the number of years a person spends in school and their income, Jennifer responded, "Yes. The more education, the more income you make." In elaborating whether the equations of the line of best fit indicated a difference in male and female earning power, Jennifer indicated that men made more than women. When the teacher asked what might contribute to the difference in pay, Jennifer told the class that she

believed that gender inequality equated for the difference in salary between men and women.

As the lesson progressed, Ruby explained that her group calculated the *r*-value at 0.8959. In explaining what her group's *r*-value represented, Ruby stated, "It tells you how concentrated the points are, and how strong the correlation there is." In explaining if a relationship existed between the numbers of years a person spent in school and income, Ruby indicated that the longer a person spent in school, the higher their income would be. She also stated that the data indicated that for every additional degree, a person earned more. Susan concluded the conversation by adding, "The slope of the line of best fit represents the amount income increase by per category of education. The y-intercept represents the starting income for all." Susan then went on to explain that there existed a relationship between the number of years a person spent in school and income. Susan stated, "Yes, there is. The correlation is high. The longer you spend in school, the higher your income will be."

After completing the exercises in Unit 2, the researcher used an MYP Assessment Rubric (2008) (see Appendix L) to assess the participants' communication in mathematics, ability to use mathematics to reflect, and knowledge and understanding. The participants' abbreviated results are indicated in Table 5.

Unit 3: HIV/AIDS in Canada

The last activity used quadratic modeling to examine the number of HIV/AIDS cases in Canada. For the first part of this activity, participants were asked used the data reported to the Public Health Agency of Canada from 1996-2014 to construct a scatter

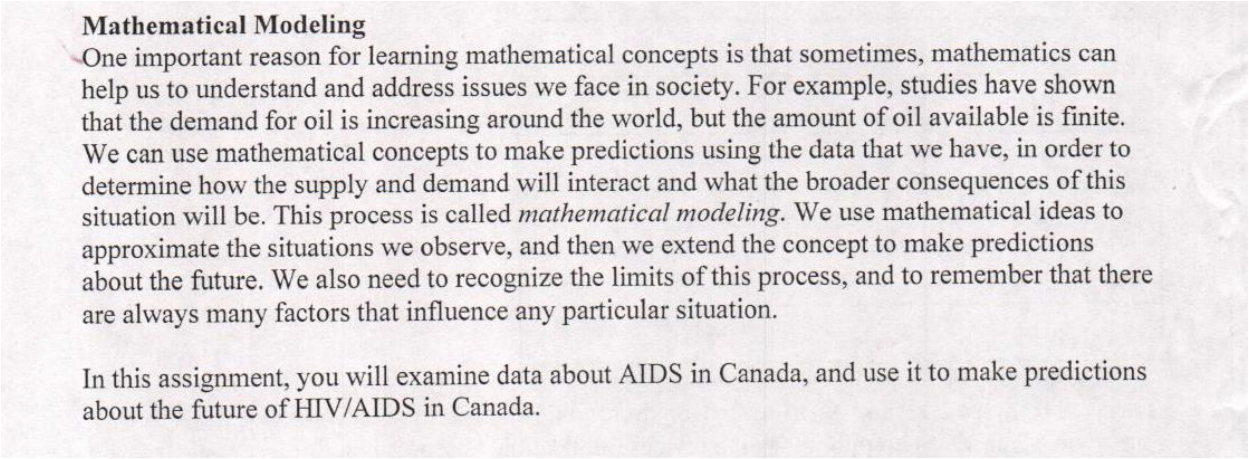
Table 5

Abbreviated MYP Assessment Results From Unit 2

Participant	***Descriptor***	***Possible Score***	***Achievement Level***	***Description***
Case 1. Carla	Knowledge and Understanding in Criterion A	0-8	0	Carla did not provide enough evidence to give the student a grade on Knowledge and Understanding.
	Communication in Mathematics Criterion C	0-6	0	Carla did not provide enough evidence to give the student a grade on Communication in Mathematics.
	Reflection in Mathematics Criterion D	0-6	0	Carla did not reach the standard, in that the student did not make an attempt to explain whether his or her results make sense in the context of the problem. Therefore, there is not enough evidence to give the student a grade on Reflection in mathematics.
Case 2. Jennifer	Knowledge and Understanding in Criterion A	0-8	1-2	Jennifer attempted to make deductions when solving a simple problem in familiar contexts.
	Communication in Mathematics Criterion C	0-6	0	There is not enough evidence to give Jennifer a grade on Communication in Mathematics.
	Reflection in Mathematics Criterion D	0-6	1-2	Jennifer attempted to explain whether his or her results make sense in the context of the problem. The student attempted to describe the importance of his or her findings in connection with real life.
Case 3. Kate	Knowledge and Understanding in Criterion A	0-8	0	There is not enough evidence to give Kate a grade on Knowledge and Understanding in mathematics.
	Communication in Mathematics Criterion C	0-6	0	There is not enough evidence to give Kate a grade on Communication in mathematics.
	Reflection in Mathematics Criterion D	0-6	0	There is not enough evidence to give Kate a grade on Reflection in mathematics.
Case 4. Ruby	Knowledge and Understanding in Criterion A	0-8	3-4	Ruby sometimes made appropriate deductions when solving simple and more complex problems in familiar contexts.
	Communication in Mathematics Criterion C	0-6	3-4	Ruby showed sufficient use of mathematical language and forms of mathematical representation. The lines of reasoning are clear though not always logical or completed. She moved between different forms of representation with some success.
	Reflection in Mathematics Criterion D	0-6	1-2	Ruby attempts to explain whether his or her results make sense in the context of the problem. In addition, Ruby attempted to describe the importance of his or her findings in connection with real life.
Case 5. Susan	Knowledge and Understanding in Criterion A	0-8	5-6	When solving a problem, Susan included some of the information to show most or all of the information to show that she understood the problem. Generally, Susan made appropriate deductions when solving challenging problems as well as used mathematics vocabulary and symbols correctly.
	Communication in Mathematics Criterion C	0-6	5-6	Susan showed good use of mathematical language and forms of mathematical representation. Her lines of reasoning are concise, logical and complete. She showed solutions using numbers and symbols as well as moves effectively between different forms of representation. At times. Susan showed solutions in more than one form.
	Reflection in Mathematics Criterion D	0-6	5-6	Susan was able to critically explain whether her results made sense. In explaining her responses, she provided an explanation of the importance of her findings in connection to real life.

Note: Mathematics descriptors are adopted from MYP Assessment Rubric for Mathematics (2008)

plot. Figure 6 contains the explanation provided by the instructor for to how mathematics assists with understanding as well as becoming more aware of societal issues. Questions on quadratic equation, trends, and other inquiries about the data were posted as well.



Mathematical Modeling

One important reason for learning mathematical concepts is that sometimes, mathematics can help us to understand and address issues we face in society. For example, studies have shown that the demand for oil is increasing around the world, but the amount of oil available is finite. We can use mathematical concepts to make predictions using the data that we have, in order to determine how the supply and demand will interact and what the broader consequences of this situation will be. This process is called *mathematical modeling*. We use mathematical ideas to approximate the situations we observe, and then we extend the concept to make predictions about the future. We also need to recognize the limits of this process, and to remember that there are always many factors that influence any particular situation.

In this assignment, you will examine data about AIDS in Canada, and use it to make predictions about the future of HIV/AIDS in Canada.

Figure 6. Sample explanation of mathematical modeling in an activity

The second part of the lesson asked students to convert a quadratic equation from standard form to vertex form. In assisting the students, the teacher provided the students with the vertex form and standard form equations on the whiteboard. After immediately starting the exercise, Jennifer indicated that she was having difficulty completing the directions. Based on her feedback to the teacher, Jennifer struggled with converting from a standard form of a quadratic equation to vertex form. Given that Jennifer, as well as some of the other students, did not understand the problem, the teacher asked if they had completed their homework. The students then indicated that they did not complete their homework assignment. To help with decreasing some of the confusion, the teacher then allowed time for the students to complete their incomplete homework assignment during the instructional period.

During an observation, the researcher noticed that Jennifer used her instructional time to sketch out drawings on her paper. She was also observed looking around the room at other students' work posted on the walls in the classroom. At times, Jennifer read over the problems on the homework worksheet. Approximately 7 minutes after restarting her work, the teacher checked in on Jennifer's status. When approaching Jennifer's desk, the teacher restated the directions. Jennifer then asked for assistance on identifying where the formulas were on the board.

Although students have provided with oral directions on the steps taken to construct a scatter plot using the data reported by the Public Health Agency of Canada on HIV/AIDs cases between 1996-2014, Jennifer continued to appear distracted. At times, she took her pencil and practiced moving between each of her figures. After rejoining the lesson, a classmate helped Jennifer to find the value of *a* in the mathematical equation.

When the teacher asked the participants to inspect their scatter plot and provide a reasonable approximation of *h*, Ruby raised her hand to respond. Ruby then stated that *h* in the vertex form of a quadratic equation represented the year in which reported cases reached a maximum. By inspection of her scatter plot, Ruby told the class that she estimated that 7 years was a reasonable approximation of *h*. In explaining the role of *k*, Ruby stated that *k* represented the maximum number of reported cases. Carla agreed with Ruby and stated that the value of *k* represented 2400 recorded cases of HIV/AIDS in Canada. Using a calculator to assist with her mathematical calculation (see below), Ruby then calculated the *h* value in the vertex of the parabola. Ruby identified the location (7.312, 2.446.77) as her vertex of the graph. The teacher then wrote the steps to find

vertex form of the standard form equation $y = -2.526x^2 + 36.942x + 2311.696$ as follows:

Write the standard form equation into the vertex form of the equation.

$Standard\ Form: y = ax^2 + bx + c$ when the vertex equals $(\frac{-b}{2a}, f(\frac{-b}{2a}))$.

Step 1: $y = -2.526x^2 + 36.942x + 2311.696$

$the\ x\ value = \frac{-b}{2a} = \frac{-36.942}{2(-2.525)}$ = 7.312

$y = a(x - 7.312)^2 + 2446.77$

Vertex = $(\frac{-b}{2a}, f(\frac{-b}{2a}))$ = (7.312, 24446.77)

Step 2: Let (x, y) = (12, 2391.316) find the value of *a*.

$y = a(x - 7.312)^2 + 2446.77$

$2391.316 = a(12 - 7.312)^2 + 2446.77$

$-2446.77 \qquad\qquad -\ 2446.77$

$-55.454 = a(4.688)^2$

$a = -2.523$

Therefore,

$y = -2523(x - 7.312)^2 + 2446.77\ with\ a\ vertex\ of\ (7.312, 2446.77)$

Susan concluded the problem with stating that the vertex (h, k) of the graph was at (7.312, 2446.77). When asking what year the value 7.312 represented in the data set, the class stated that maximum number of HIV/AIDS cases in Canada must have occurred during 2002.

To follow up the discussion, the teacher then asked the participants to conduct a data analysis of the data. After looking at the trends observed in the data, Ruby stated, “I

have no idea what is going on in Canada." Based on Ruby's response, the teacher asked the class to revisit the data once more. Seeing that the students were struggling with the analysis, the teacher stated, "Perhaps a quadratic equation was not the best type of graph to use with this data. Perhaps a cubic or quintic root graph would be best."

Prior to transitioning to the next questions, Ruby expressed her frustration with being asked questions relating to a country for which she knew nothing of their current status. Ruby stated, "I have no idea what is going on in Canada. The decline in the number of HIV/AIDS cases could be related to changes in policy or medical care." She then added, "Perhaps someone really homophobic came into the office, and they did not report all the cases."

Jennifer joined the conversation and stated that perhaps the decline in HIV/AIDS cases was related to a conspiracy theory led by Canada's government and that perhaps the data were mishandled or finagled to show a decline in HIV/AIDS cases. When discussing when the data suggested HIV/AIDS cases would end, Jennifer stated that she thought that would be around 2024. Ruby, on the other hand, disagreed and stated that by the class's calculation, the data reflected there would be no new HIV/AIDS cases starting in the beginning of 2034. Using the vertex equation, the class performed the following computations to determine when there would be no more AIDS cases in Canada:

Step 1.

$$0 = -2.526(x - 7.312)^2 + 2446.77$$

$$2446.77 = -2.526(x - 2446.77)$$

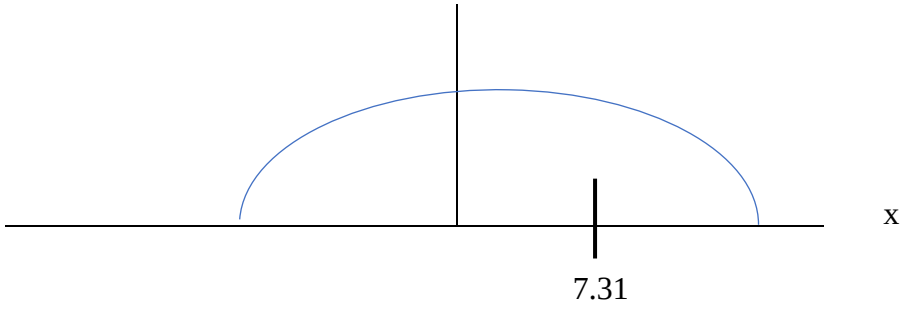
$$968.634 = (x - 7.312)^2$$

$$\sqrt{968.634} = \sqrt{(x - 7.312)^2}$$

$$x = 38.435$$

Step 2. The quadratic regression equation

$$y = -2.526(x - 7.312)^2 + 2446.77$$



Step 3. Determining the year there would be no more HIV/AIDS cases

Solve for x.

$$-2.526\,(x - 7.312)^2 = -2446.77$$

$$\frac{-2.526(x - 7.312)^2}{-2.526} = \frac{-2446.77}{-2.526}$$

$$\sqrt{(x - 7.312)^2} = \sqrt{968.634}$$

$$x - 7.312 = \pm\sqrt{968.634}$$

$$x = 7.312 \pm \sqrt{968.634}$$

$$x = 7.312 \pm 31.23$$

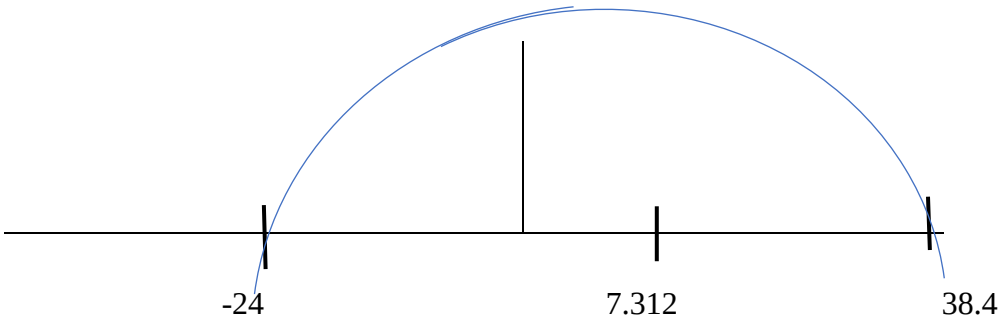


Table 6

Abbreviated MYP Assessment Results From Unit 3

Participant	Descriptor	Possible Score	Achievement Level	Description
Case 1. Carla	Knowledge and Understanding in Criterion A	0-8	1-2	When solving a problem, Carla included some of the information to show that they understand at least part of the problem. Carla used correct use of mathematics vocabulary and symbols is limited and sometimes incorrect.
	Communication in Mathematics Criterion C	0-6	1-2	Carla showed basic use of mathematical language and/or forms mathematical representation. The lines of reasoning are difficult to follow. Carla answered part of the problem, but the solutions may be unclear or hard to follow. She sometimes showed solutions in more than one form and may make connections between the representations.
	Reflection in Mathematics Criterion D	0-6	0	There is not enough evidence to give Carla a grade on Reflection in Mathematics.
Case 2. Jennifer	Knowledge and Understanding in Criterion A	0-8	1-2	Jennifer attempted to make deductions when solving a simple problem in familiar contexts.
	Communication in Mathematics Criterion C	0-6	1-2	Jennifer showed basic use of mathematical language and/or forms of mathematical representation. The lines of reasoning are difficult to follow.
	Reflection in Mathematics Criterion D	0-6	1-2	Jennifer attempted to explain whether his or her results make sense in the context of the problem. She also attempted to describe the importance of his or her findings in connection with real life.
Case 3. Kate	Knowledge and Understanding in Criterion A	0-8	-	No results
	Communication in Mathematics Criterion C	0-6	-	No results
	Reflection in Mathematics Criterion D	0-6	-	No results
Case 4. Ruby	Knowledge and Understanding in Criterion A	0-8	5-6	Ruby generally made appropriate deductions when solving challenging problems in a variety of familiar contexts.
	Communication in Mathematics Criterion C	0-6	5-6	Ruby showed good use of mathematical language and forms of mathematical representation. The lines of reasoning are concise, logical and complete. Ruby moved effectively between different forms of representation.
	Reflection in Mathematics Criterion D	0-6	3-4	Ruby correctly but briefly explained whether his or her results make sense in the context of the problem and describes the importance of his or her findings in connection to real life. She also attempted to justify the degree of accuracy of his or her results where appropriate.
Case 5. Susan	Knowledge and Understanding in Criterion A	0-8	5-6	Susan generally made appropriate deduction when solving challenging problems in a variety of familiar context.
	Communication in Mathematics Criterion C	0-6	3-4	Susan showed sufficient use of mathematical language and forms of mathematical representation. The lines of reasoning clear though not always logical or complete. The student moves between different forms of representation with some success.
	Reflection in Mathematics Criterion D	0-6	3-4	Susan correctly but briefly explains whether his or her results make sense in the context of the problem and described the importance of her findings in connection to real life. Susan also attempted to justify the degree of accuracy of her results when appropriate.

Note: Mathematics descriptors are adopted from MYP Assessment Rubric for Mathematics (2008)

Data Analysis

All four sources of data were analyzed using qualitative thematic analysis. The method focused on determining the most common and significant patterns from the data collected from the five participants. According to Braun and Clarke (2006), thematic analysis is used when searching for and identifying the most essential and meaningful responses of the participants in response to the research questions of the study. In the current research, data were analyzed in line with the three research questions such as (A) how the incorporation of social justice issues into a high school statistics curriculum at an all-girls independent school affect five female students; (B) how the students in an all-girls high school statistics course develop sociopolitical consciousness through reading the world with mathematics; and (C) the factors that contribute to or prevent students in an all-girls high school statistics course from using mathematics to write the world. The next section contains additional findings from the analysis. Only the major themes that occur the most are presented and discussed. Themes with fewer references or other most essential perceptions and experiences were considered as minor themes but are presented only in the tables. Subthemes are also included to better explain and detail the major and minor themes of the study.

Results

Research Question 1. How does the incorporation of social justice issues into a high school statistics curriculum at an all-girls independent school affect female students?

The first research question discussed how the incorporation of social justice issues into a high school statistics curriculum at an all-girls independent school affected five female students. Four sources were analyzed to address the first research question. From the qualitative thematic analysis of the data, five major themes were established. In the opening activity, participants reported the incorporation of social justice educated them about the most pressing issues of today. Although the post-lesson reflection indicated the majority of the participants' previous knowledge of some of the issues, it was found they also experienced an increased awareness. Strong reactions were also discovered from the interviews as the participants expressed negative feelings with the social injustice awareness. Although some participants indicated that some of the activities produced limited to no effect, the majority of the participants revealed from the exit survey that they developed the motivation to learn mathematics through the social justice issues.

Theme 1: Incorporation of social justice educated them about the most pressing issues of today. The opening survey and free-write activity introduced the participants' initial thoughts on the incorporation of mathematics and social justice issues. The analysis indicated the participants' positive reaction to being educated on the most pressing issues of today. They believed the incorporation provided concrete evidence on racial profiling given the real-world applicability of the concepts;, therefore leading one to take action in the form of spreading awareness about social justice issues or advocating for change.

For example, four of the five participants shared on their opening survey and free-write activity that they developed a newfound knowledge of and interest in the most pressing issues of today. Through the real-world applicability and presence of concrete

evidence, participants revealed the incorporation increased their awareness as well as trust on the data presented discussing the issues on racial profiling, inequality in wage earnings, and trends of AIDS cases.

During her one-on-one interview with the researcher, Jennifer referenced about being educated of the most pressing issues of today:

> Um, well, similar to like what I've been saying before, it's educated me. It's made me more aware if those aren't synonymous. I guess they are pretty synonymous, but they're different in the way that we can be aware. But when you get educated, it's more like prepping you to do something to stop it. Like you're not going to be that one. You're going to be one less person who's uneducated and is discriminatory of people around you. And that's how it works. You banish something by slowly and gradually eliminating. Like a parasite from something. Like I considered racist a parasite. And you have to. And I would say that education is a part of the remedy. It's not everything, but it's definitely a part of it.

Subtheme 1A: Having real-world applicability. The first subtheme encompassed the feature of the integration as a positive practice given its real-world applicability. Jennifer described the integration of the two concepts as an "eye-opener" where students were able to use mathematical concepts to learn about the actual issues of which people should be knowledgeable. For Jennifer, the real-world applicability of the issues encouraged the students to explore and learn more about these issues as these may also affect them in the future; she said:

> Learning about statistics such as incarceration rates, because when you learn about these statistics it can be very eye-opening and you feel truly informed about issues that people "talk about." Students would be more educated and genuinely interested in the course because it has real-world applicability.

Meanwhile, Kate had the same perception as Jennifer. For Kate, the incorporation was an opportunity for the students to learn the concepts of mathematics and, at the same time, develop the passion for learning about the social justice issues and be more

proactive to address them. Kate said, “If social justice issues were to be incorporated into a statistics or mathematics curriculum, many more students would be educated about these issues subsequently giving them the opportunity to be passionate about them and be an advocate for them.”

Subtheme 1B: Having concrete evidence on racial profiling. Another subtheme that followed was the use of mathematics to provide concrete evidence that these social justice issues actually exist. In particular, the majority of the participants discussed the racial profiling experienced today. Ruby believed the mathematical concepts provide evidence, confirming the presence of these issues through statistics and probability:

> Yes, mathematics would be useful in the above scenario, because it would move their claims out of the realm of feeling (they feel or think they are being stopped, searched, harassed, and arrested because they “fit” a racial profile) and into the world of hard facts. Their claims become hard evidence if there is proof that they are, in fact, being stopped, searched, etc. more than other ethnicities, and that proof can be gained through statistics/probability.

Further, Susan was already previously aware of the presence of these issues. However, she shared that the integration allowed her to confirm the news and reports on the issues by looking at the actual numbers and trends:

> I know that there are currently statistics (unfortunately, I don’t know the specifics) that show that people of color are more often stopped by cops than white individuals. Thus, it is a matter of collecting data (numbers) and seeing the trends present within the data collected.

Theme 2: Having prior knowledge on social justice issues. Three of the five participants reported that although they had previous knowledge about social justice issues, the integration of social justice concepts provided the opportunity for them to increase their knowledge and awareness of the importance of the issues. In their post-lesson reflection notes, Carla, Ruby, Susan, and Jennifer all reported they had previous

knowledge and understanding of the different social justice issues discussed. However, they emphasized the integration of concepts allowed them to develop an increased awareness of and interest in completing the three TMSJ Units. Similar to the findings from the initial survey and introductory free-write activity, Carla, Ruby, and Susan gained heightened awareness through interacting with the TMSJ lessons.

Both Ruby and Susan shared another subtheme that emerged from the data. Ruby reported an improved awareness upon completing Unit 1 on racial profiling, saying: "I think I'm more aware now than I was about the extent of the issue." Correspondingly, Susan shared the facts she learned about the big disparity in the incomes of males and females as well as those with varying educational attainments, saying:

> While I did know gender pay gap was a thing, I never really saw actual digits to show this. It was also interesting to see the drastic difference in income in someone that finished high school versus someone that didn't. Even that diploma can have a drastic impact.

In a follow-up interview about her previous response to the gender pay gap and education versus income lesson in Unit 2, Susan responded:

> I knew that income had to do with schooling. Like growing up, my parents always told me to get an education if you really want to make money. But I really didn't actually realize the drastic differences between certain levels of education. For example, after looking at the data, I did not know that not having a having a high school diploma or not having one could determine the difference of if you live in poverty. Like, the difference in income is drastic. The data made me more aware of the differences in income for different degrees levels and gender pay gaps. Because going to an all-girl school you are constantly reminded of it, but you never actually see the digits to kind of like see that. However, having it in front of you makes you look at reality.

She further elaborated her views on gender inequality and the pay gap by adding to the conversation:

> I think the gender pay gap is ridiculous. If two people are doing the same job, they should get the same amount of money. No matter the demographics, they are doing the same job. I think women for a long time were expected to say at home and take care of the kids. So, when they started to introduce themselves to the work field, I think their employers doubted their skills and thought that women are used to being at home. They have not lived the life males have lived for the past decades, employers don't think women's skills won't be the same as males. I think it is like the repetition of stay at home mom taking care of the kids doing domestic choirs led to women being underestimated.

Theme 3: Strong feelings associated with social justice awareness. An analysis of the interview transcripts revealed that the majority of the participants expressed strong reactions on how the incorporation of teaching mathematics for social justice affected their social consciousness. As evidence of influences of the incorporation of mathematics and statistics, four of the five participants expressed their negative feelings for learning about social injustices present in society today.

Jennifer, Ruby, Kate, and Carla all reported the negative feelings they developed upon their social injustice awareness. These participants felt angry, upset, and frustrated at learning more about the racial profiling experienced by many. Aside from the negative feelings, participants shared they were also able to reflect more on the said issue even after the actual classes.

Jennifer and Ruby shared that they felt angry as they believed they should have taken the initiative to learn more about these issues earlier. Initially, Jennifer shared that she was mad at herself for not knowing about the presence of concrete racism issues in society. She then found the activity to be enlightening and added how education is the first step to the change the society has been waiting for:

> Um, obviously I am mad. Like I don't know, these things myself. I don't know why I expect myself to know it. But at the same time, I'm like, oh my gosh, you are so racist for not even knowing that this happens. Um, and like I have to tell myself separately like it doesn't make me racist that I didn't know it. It is if I

> didn't accepted that would be racist. But in terms of doing this activity, um, it's been enlightening. If that's the correct word, and obviously I don't exactly know what to do to stop it. Like, I know that one person loved can stop racism in America, but I think that education is definitely the first step.

Similarly, Ruby shared felt angry after completing Unit 1 on racial profiling. She stated how she should have done something about these issues earlier and not allow the injustices to grow bigger:

> Um, I know angry is a strong word. I think maybe photos feeling, but just sort of one surprise that I suppose it's such a thing I've been allowed to continue then maybe I shouldn't be. And Yeah. Um, I think I was struck by the injustice that appeared to be present.

During her interview, Kate stated she felt upset upon learning about the racial profiling issues in Unit 1. She then added that the lessons allowed her to reflect further as well: "Yeah, I think…I think definitely brings emotion into mathematics, which we don't usually deal with. It's not just numbers. Um, and I think it's upsetting, but at the same time it makes you reflect."

Subtheme 3A: Feeling upset due to racial profiling. The first subtheme that followed was being upset because of racial profiling. Kate stated she felt upset upon learning about the racial profiling issues. She then added that the lessons allowed her to reflect further as well: "Yeah, I think…I think definitely brings emotion into mathematics, which we don't usually deal with. It's not just numbers. Um, and I think it's upsetting, but at the same time it makes you reflect."

Subtheme 3B: Feeling frustrated upon confirmation of racial profiling issues. The second subtheme was the feeling of frustration as shared by Carla. The participant indicated during her face-to-face interview that the integration of racial profiling

provided substantial evidence these issues exist, and the confirmation made her feel frustrated as a part of the society that promotes such racism and discrimination:

> Yeah, I mean I don't think I personally had let, I don't think it had a big effect on me just cause it was stuff like I already know and like with the racial profiling stuff, um, so like it's already been something that affected me, but it was definitely frustrating to have that confirmed.

Theme 4: Motivation to learn mathematics through social justice issues. Three of the five participants noted in their exit survey that they believed the integration led to the development and increase of motivation to learn mathematics in the light of the social justice issues. As such, the effects of integrating social justice issues into the statistics curriculum led three of the five participants to feel empowered to learn mathematics.

The participants shared how the lessons within the units of study allowed them to develop social consciousness as they were more motivated to learn more about the issues even after the lessons were completed. Jennifer admitted finding the integration and the lessons to be interesting. The incorporation made mathematics more exciting to learn and explore: "I found the three social justice lessons to be really interesting, and it made me feel more motivated to learn mathematics/statistics." Meanwhile, Kate added, "Now, I think that teaching mathematics for social justice means conducting statistic activities or exercises that encompass and are related to social justice issues." Finally, Susan expressed the integration as a helpful way of discussing both the mathematics and social justice concepts:

> Before, I never thought about combining my interests in social justice and mathematics. However, now I see that it is possible. I think mathematics/statistics can be a very engaging way to teach people about social issues by showing data and the trends within the data.

Theme 5: Social justice lessons had limited to no effect. For Carla, Unit 1 had a limited effect on her social consciousness. Nevertheless, the participant still highlighted that she developed an interest in the other lessons due to the real-world application of the lessons. She stated: "I was not particularly affected by the social justice activities in Unit One. However, it did make math more interesting because we considered a real-world application." When asked to reflect on Unit 3, Ruby stated that the topic of AIDS did not really affect her or the people closest to her. In addition, the lessons did not provide specific data on how the issues could be resolved. However, the participant still believed that by opening the table for discussion of social justice issues, society can experience a positive shift in the long run:

> AIDS affects neither me nor my family personally, but this data was an interesting look into a topic that I know affects thousands the world over. There didn't seem to me to be obvious ways this data reflected social justice issues or revealed ways I could help solve this problem. I suppose just a willingness to talk about AIDS, and acknowledge the stigma that surrounds it, might be a big step from where we were as a society a few decades ago.

On the other hand, Carla and Kate had a different outlook on how the lesson activities affected them. For Carla, the activities left her wanting to spread awareness in hopes of advocating for change. During the free-write opening activity after Unit 1 on racial profiling, Carla stated:

> If social justice issues were to be incorporated into a statistics or mathematics curriculum, many more students would be educated about these issues subsequently giving them the opportunity to be passionate about them and be an advocate for them.

Thus, having concrete evidence on racial profiling after completing Unit 1, Kate said:

> Mathematicians can then make note of this disproportion and use these numbers to spread awareness and make a change. Using math in the debate on institutional racism shows hard proof that racial profiling exists and thus needs to stop.

Research Question 2. How, if at all, do students in an all-girls high school statistics course develop sociopolitical consciousness through "reading the world with mathematics"?

The second research question explored how the students in an all-girls high school statistics course developed sociopolitical consciousness through reading the world with mathematics. Four other major themes were established focusing on using data as evidence on the severity of current social justice issues, relating mathematics to the issues in the real world, and an overall effect of developing a strong connection with the social justice issues. Participants believed upon an integration of social justice issues in mathematics, they have since developed their sociopolitical consciousness.

Theme 6: Using data as evidence on the severity of current social justice issues. An analysis of Unit 1 on social justice issues pertaining to racial profiling revealed that the majority of the participants reported the numerical and statistical evidence allowed them to discover and learn more about the current social justice issues.

For Carla, the fact that mathematics can provide evidence that a particular social issue such as racial profiling exists is a big factor. Through the presentation of percentages, statistics, and disproportions of the different cases, students became aware of the actual profiling that occurs against African Americans and Latinos:

> Mathematics can be a useful tool in answering questions about racial profiling because can give numerical proof on the existence of racial profiling. Mathematicians can find the percentage of African American and Latino drivers that have been pulled over [wrongfully] and compare it to statistics on the white population. Mathematicians can then make a note of this disproportion and use these numbers to spread awareness and make a change.

Jennifer described mathematics as a "useful tool" that can be used to address the disproportions in social justice issues and cases. An example shared was the number of

times people of color are stopped by the police without any actual violation performed. This participant believed probability could help in presenting and determining the disparities in the current statistics concerning racial profiling, saying:

> I think mathematics can be a useful tool in helping us answer these questions because we can use statistics related to how many more or less people of color are stopped randomly on the street or on the road, and how many more or less times the police actually catch someone on the street or the road doing something illegal. Probability can also help us calculate certain things, such as "how likely is it for a person of color to get pulled over as opposed to a white person getting pulled over," and we'd use current statistics to calculate these probabilities.

Susan emphasized how concrete data would make certain individuals believe social justice issues. By having concrete evidence, people would develop an interest on said issues and allow room for discussion leading to awareness of and enlightenment on the social injustices happening in the country today: "There are people in the world that do not acknowledge and refuse to believe certain social justice issues. However, by incorporating it into the mathematics curriculum, there would be concrete evidence and room for discussions among peers."

Finally, Ruby believed mathematics was a valuable tool for exploring and learning about the world. She gave several daily actions and practices that encompassed the significance of mathematics in the lives of the people:

> Math is a skill we need to navigate the modern world. Even if we only ever count out change at a register, or compare deals in a store, we need to understand numbers to make the best decisions. As for statistics, they're everywhere in the news, and if we don't understand them we can't understand how they might be manipulated, which could well lead to us getting manipulated in turn.

Theme 7: Relating mathematics to the issues in the real world. From a thematic analysis, the post-reflections revealed how participants were able to relate mathematics to the issues in the real world upon completing the three units of study. In particular, they

used percentages, statistics, and patterns of inequality on racial profiling in Unit 1 and education versus income earning in Unit 2.

For example, Carla shared how the first lesson on racial profiling in Unit 1 provided her with tools to understand better the patterns of inequality experienced by many in her community, saying: “Lessons in Unit One helped me to use mathematics to examine racial profiling in the broader social world. It also gave me the tools to understand how to mathematically find patterns of inequality in my own community.” Kate added that the integration of mathematics and the social justice issues provided answers to her previous doubts and questions about the experiences of many in society today. The lesson activities also allowed her to reflect on and determine how she can help to improve the current situation of many today:

> Mathematics allowed me to learn about some of statistics regarding social justice issues through hands-on activities and questions asking me to reflect on said activities. I feel like I learned the most about racial profiling, as the activity with the chips was very straightforward and the questions afterward allowed me to further explore similar statistics. The activities we completed in Unit Two educated on me on important social justice issues through allowing me to obtain statistics, and then relating to them to these issues. Because I learned more about these issues, I feel that I want to take action and help, and I will also from now on be more aware of social justice issues such as racial profiling.

In addition, Ruby believed that mathematics could play an essential role in changing the situations and victims of racial profiling. With the statistics and trends over the years, more people can be enlightened of the injustices and unfair treatment received by people of color, as she said:

> We used mathematics to examine how our totally random statistics compared with the actual statistics, which helped us to draw conclusions about the role race plays in traffic stops. We had no information on the cops themselves, or what racial groups they belonged to, so power derived from race being implemented here is not necessarily a conclusion I think we could draw. However, because our

> statistics are so disparate from reality, we could claim that something unjust is occurring with regards to race in these discretionary stops.

Susan shared how discrimination or racial profiling was highlighted in their lesson. Through numbers and percentages, she found the actual inequalities and disparities in the number of population and the number of times the police stop the people of color, as compared to their White counterparts:

> I used mathematics in lesson under to understand discrimination based on race. Once we got the population of a specific location and the amount of people that identified with a certain race in that group, we were able to see how people of color can make up a small percentage of the population, yet still be stopped by cops an absurd amount of times in comparison to their white peers.

Correspondingly, aside from racial profiling, Susan seemed to be very much affected by learning about the skill and gender pay gap even with the same level of educational attainment, as discussed in Unit 2. Mathematics allowed for the participant to distinguish the differences, especially in the different levels of education and impact of schooling on the earnings of individuals:

> I learned that one's income is highly correlated to their years of schooling. It was also interesting to see the gender pay gap, despite people have the same education. I think using mathematics helped me fully process the differences in income between the various degree of education. While I knew about it, it is more shocking and helpful to see the actual numbers.

An analysis of the face-to-face interviews uncovered the same major theme as the reflections shared by the participants. The analysis showed how the students started to relate mathematics to the issues found and experienced in the real world. Although two of the five participants were already aware of the issues, the lessons allowed them to explore and reflect on them further, leading to higher social consciousness.

Carla now believed mathematics can be an essential tool for knowing more about the real world. Given technological changes and advances, mathematics has become increasingly valuable in the daily dealings and practices of individuals:

> Yeah, I mean, I think mathematics like definitely can be used and should be used in the real world and I think that the way that the world is changing, people are using more logical and mathematical ways to approach things and debate too.

Meanwhile, Jennifer shared she was already familiar with the issues discussed. However, mathematics allowed her to explore the issues further and integrate daily practices into the concepts of mathematics:

> No, like, I don't want to say it's like, oh my God, like I already know why it's relevant, but like it makes a lot of sense that we can use statistics to investigate racial profiling because essentially statistics is proof. It's evidence of things. Unless the statistics are altered by the police. I would have zero clue whether that happens or not. Um, but I'd want to be like a smart Alec, but like it makes sense that like we were using statistics to explore racial profiling because I wouldn't use say Algebra or I wouldn't use calculus. I would use statistics.

Theme 8: Developing a strong connection with social justice issues. The eighth major theme that emerged was the development of a strong connection with social justice issues. In particular, two participants already expressed their desire to take action on the gender wage gap issue. This is because as women, they felt a strong connection and empathy with those who are experiencing the injustices first-hand. Jennifer explained how the gender wage gap urged her to take concrete actions in order to see better outcomes in the future: "The lesson about the wage gap motivated me to want to take action because I myself am a woman and therefore may experience this injustice first-hand." Additionally, Kate shared that all the lessons were significant to her. However, Kate shared in her exit survey that the gender wage gap resounded with her and was the most meaningful:

> All of the lessons resonated with me, however the one about the wage gap between men and women who had the same levels of education resonated with me the most. I think this is because I am a woman who works hard in school and plans to keep doing so through undergraduate and graduate school. Seeing that despite the time, money and effort being put into my education men at my same level of education will statistically probably make more than me and be more successful is upsetting and discouraging. Because the lesson about the wage gap between men and women resonated the most with me, it's also the lesson that made me feel motivated to want to take action.

As for Ruby, the lesson on HIV/AIDS in Unit 3 resonated more with her. In her post-lesson reflection on HIV/AIDS cases in Canada, Ruby stated:

> It may have helped me make a stronger connection between AIDS and social justice issues by bringing my attention to the fact that this syndrome affects so many people, of all different socioeconomic and ethnic backgrounds, sexual orientations, age groups, et cetera.

Subtheme 8A: Realizing the need to educate oneself further on the issues. By contrast, Susan developed a strong connection with Unit 1. In realizing the need to educate oneself further on the issues, Susan wrote:

> I think that it was interesting to use the data from Chicago because Chicago is part of the USA; thus we are inclined to feel a connection since the city is part of our country. It makes us aware that this issue is going on near us. For me, learning about one specific scenario sparks more interest, and I would like to explore the statistics of other places in the United States.

Theme 9: Using statistics as valid evidence to confirm arguments. Another major theme was the use of statistics as valid evidence to confirm the participants' arguments. During a post-lesson reflection after Unit 2, Jennifer stated:

> Relating to my answer in Question Four, using mathematics in Unit Two introduced mathematics to me as a way of confirming evidence and basing accurate arguments (related to social justice), i.e., being able to rely on "fact" more than "feeling."

During the exit survey, two other participants shared how the lessons broadened her understanding of mathematics as well as social justice issues. Ruby believed statistics need further credibility when interpreted, saying:

> I think these lessons have broadened my understanding of the uses of statistics. Mostly I saw them as having a negative connotation before, as they can be so easily manipulated and have been used to mislead, but now I understand that they're simply a tool. Statistics can help people see the truth of certain situations or in certain patterns of behavior, and prove that certain problems are widespread, and can thereby be used for good.

Susan added the value of mathematics when discussing the social justice issue. Again, the participant highlighted how the numbers served as evidence that these issues exist, and concrete actions must be done to address them:

> I still agree with my initial response. I also think that teaching mathematics for social justice allows us to look at data and make predictions about upcoming years based on trends. There are people in the world that do not acknowledge and refuse to believe certain social justice issues. However, by incorporating it into the mathematics curriculum, there would be concrete evidence and room for discussions among peers.

During her face-to-face interview, Kate argued on the need to have stronger statistical evidence to confirm the issues. In discussing how mathematics might be used as a tool to confirm an argument, Kate said:

> I'm sure that there are ways that mathematics can be a useful tool in helping us answer this question, however, I'm not sure how. Maybe it's possible that you can look at statistics regarding similar situations and observe and decipher it in order to interpret the likeliness of the situation. However, I'm not sure that you can determine that a situation definitely a hundred percent happened through the use of statistics, as statistics are usually used in order to make an educated guess or relay information.

Similarly, Ruby indicated that statistics can be used as valid evidence to confirm arguments. As the researcher and Ruby discussed this issue during a one-on-one interview after Unit 1, Ruby stated:

> I think the more statistics we have, the better, the stronger the case becomes. So, statistics regarding, uh, the general population of the neighborhood or the racial makeup of the neighborhood, um, uh, even the race of the police officers involved, um, how frequent the stops were in general? I think I was aware of racial profiling in the scenario being a problem. Um, I think the school has done a pretty decent job of bringing that sort of thing to our attention. Um, but I would say it was striking when we compare it to are completely random data to the data provided in reality because that showed there was a really stark difference and our results that. Yeah.
>
> Um, I think so. Maybe just changed my view of how statistics can be used to in general. I tend to think of them not as untruths but as being used to manipulate and a lot of cases, especially in the news, uh, we spent some of the first portions of the year looking at statistics that had been, you know, the graphs have been changed or the, uh, the scales were wildly different. That kind of proved to me that a lot of statistics that are fed to us or misleading, um, but this was a much more sort of positive spin on how we can highlight injustice in our world and how we can prove that it's happening.

Research Question 3. What factors contribute to or prevent students in an all-girls high school statistics course from "using mathematics to write the world"?

The third research question explored the factors that contributed to or prevented students in an all-girls high school statistics course from using mathematics to write the world. For the five participants, they found the positive aspects that encouraged them to use mathematics: having hard proof or evidence for the existence of social injustice, having a desire to make change through the use of evidence, making mathematics more understandable and interesting, and developing an understanding of the real purpose of statistics. All four participants shared their positive feelings and perceptions on completing the integrated lessons, providing confirmation of the positive influences of the integration when applied accordingly.

Theme 10: Having hard proof or evidence on the existence of social injustice.

The tenth major theme of the study was the finding that having a hard proof or evidence on the existence of social injustice played a big role in the students' willingness to use

mathematics to write the world. In particular, mathematics presents tools and data to answer current issues on racial profiling. For four of the five participants, having concrete evidence that these social issues exist and are happening was an essential factor in encouraging them to use "mathematics to write the world."

For Susan, it was vital to use mathematics and prove the reports on the issues through statistical data and numbers, saying:

> They should learn mathematics because mathematics can serve as a way to analyze data and make predictions. If you understand numbers (and how to identify trends), you can use it as evidence for your arguments.

Subtheme 10A: Presence of tools and data to answer current issues on racial profiling. One subtheme under this major theme was the presence of tools and data to answer the issues of racial profiling. Carla shared how mathematics provided evidence to those who were questioning the existence and truthfulness of social issues. Further, the participant indicated the numbers can be used to open discussion in order to help solve the issues going forward:

> Using math in the debate on institutional racism shows hard proof that racial profiling exists and thus needs to stop.… Mathematics can be a useful tool in answering questions about racial profiling because can give numerical proof on the existence of racial profiling. Mathematicians can find the percentage of African American and Latino drivers that have been pulled over and compare it to statistics on the white population. Mathematicians can then make a note of this disproportion and use these numbers to spread awareness and make change. Using math in the debate on institutional racism shows hard proof that racial profiling exists and thus needs to stop.

Additionally, Jennifer shared the practical uses of mathematics in the daily lives of the people, saying:

> Probability can also help us calculate certain things, such as "how likely is it for a person of color to get pulled over as opposed to a White person getting pulled over," and we'd use current statistics to calculate these probabilities.

Finally, Ruby agreed mathematics was effective in understanding and hopefully addressing the issues in the future. Mathematics provided evidence and answers to the previous assumptions and feelings that she had before completing the lessons. In responding to Unit 1's opening activity investigating racial profiling or Driving While Black or Driving While Brown (DWB/DWB), Ruby noted mathematics could show "undeniable proof" to "strengthen" the value of the issues.

Theme 11: Having a desire to make change or take action through the use of evidence. The post-lesson reflections that followed had the same finding as the first source analyzed in Research Question 3. Again, the reflection established the positive feature of the integration as having solid and concrete evidence to prove the existence of the said issues. The key difference was the result of having the desire to make a change.

For Carla, although the lessons in Unit 1 on racial profiling had a limited effect, it was a positive change that she now saw how statistics can motivate the society to act and fight against the issues:

> After participating, I don't really see myself "using mathematics to change the world" because I am not mathematically inclined, but I do see myself understanding how statistics can be used as a mirror on society and can be helpful in social movements.

Further, Jennifer agreed having evidence could encourage more people to open their eyes and do something about the looming issues:

> I could, because I could prove my arguments and therefore persuade more people to take action because I can prove that there is something to take action for (and perhaps I can create anger in others because anger can start a revolution for the better).

Kate added how mathematics allowed her to learn about these vital issues. For her, education was the first step towards positive changes. Hopefully, more and more people can be educated, and change can be achieved in the near future:

> In a sense, yes. Using these statistics to learn about these important issues is the first step to making change, as I feel that having education on a topic is what first allows you to identify it as a problem. It's after you, for yourself, identify that something is a problem that makes you want to take action and address that problem. Therefore, if we become educated on certain problems through mathematics, it's mathematics that will inspire people to make change.

Although Kate acknowledged that it is through education that a person is able to identify problems of social injustice, she identified one of the challenges associated with taking action or writing the world. During her face-to-face interview, Kate shared that although social injustices involving discrimination upset her and made her want to take action, the activities in the study did not teach her how to take action. When discussing if the lesson in Unit 1 on racial compelled her to want to take action, Kate stated:

> Yes. I think anytime or at that I feel that I am educated on a subject that has to do with discrimination or you know, something that I disagree with, I always want to take action on it. Just because it's always upsetting to me. But the one thing I don't think I took away from the activities was learning how to take action, but more so being education, which is obviously the first step.

Susan was happy as she was able to fulfill her mission or goal of doing something for the society, saying:

> Yes. I have always wanted to do something related to public policy when I'm older**.** I feel like having statistics to back up my data while I propose something would be most beneficial and label me as reliable, not someone just fueled by emotions and opinions.

When discussing the issue of education versus income, Susan further elaborated on her desire to use data to make change. In a face-to-face interview, Susan stated:

> I just think that being previously aware of gender pay gaps and like the impact that schooling has on income, I don't think I was really wasn't that affected. I just think that it's kind of regurgitated what it means to me. However, I did think that having the actual data held to deepen my understanding. I guess. It motivated me to want to make a change even more. Since I was seeing actual numbers in front of me, this is like true. So, there should be something done about it. Like knowing that government officials have these numbers but are not doing anything about it affects me. Like, it just makes me frustrated.

Although Carla indicated during a face-to-face interview that she previously created a C-SPAN like video to spread awareness about similar social justice issues, she did not think that she would take a mathematical approach to taking action. When reflecting on the exercises in Unit 1, Carla stated:

> I think like there are people who take action through finding these statistics and through making that clear and I think that's not how I would have thought about approaching it. I think I would've let the statistics affect me, but I would rather let mathematicians do that and maybe spread awareness and do things like that.

Similarly, when asked if she felt compelled to want to take action after Unit 2 on income earnings and education levels, Ruby responded:

> It seems quixotic to think I, a high-schooler, could change the wage gap. More likely I'll someday advocate for equality on a personal level, which might ripple outwards to change the world.

Theme 12: Making mathematics more understandable and interesting. The majority of the interviewed students believed in the effectiveness of the integration as they found mathematics to be more understandable and interesting after the incorporation of concepts. Participants shared how they enjoyed this kind of lesson plan as compared to the traditional set-up.

In providing background to how the unit lessons assisted with making mathematics more understandable and interesting, Carla described the integration as

"fun" and made solving the problems more interesting. The participant found the problems to be more sensible and relevant as well:

> Yes, I did. That was actually pretty fun as like a problem. I think having that sort of problem kind of made statistics even more interesting too because it was like [inaudible], it's like related to something and not a random math problem.

Meanwhile, Jennifer echoed how the integration was more sensible and meaningful, saying:

> No, it made more sense when we were doing. Like what we do besides the study is very abstract and like I don't even know what it means. But what we've been doing this study, it makes a lot more sense. To a very large extent. I can't express that more.

Lastly, Kate believed it was helpful to be educated about said social justice issues and integrating them with the mathematics concepts was an effective practice:

> I think I learned a little bit more about, um, racial profiling. So, I think that's helpful to be educated about issues like that. Um, especially since we're forced to take mathematics in school and we're not always forced to learn about those other things. So, I think it was helpful having someone tell you that and like the pattern before to tell you that.

Theme 13: Developing an understanding of the real purpose of statistics. From the exit survey, the majority of the participants reported they have since developed an understanding of the real purpose of statistics. Kate and Susan were able to improve their knowledge and awareness of the issues when compared to the usual or traditional concepts. As Kate shared:

> I now believe that there is a much broader range of reasons as to why students should learn statistics. Learning about social justice issues through statistics is one of those reasons, as I think it's hands-on, easy to understand and informative. As such, learning about social justice issues through statistics has made statistics slightly easier to grasp, as social justice issues tie statistics into real-world scenarios and problems.

Similarly, in her exit survey, Susan agreed on the importance of the integration and the focus of educating students on the issues. As for making mathematics more understandable and interesting, Susan wrote:

> Yes. I think showing people, including students and government officials, this data is important. For students, this will give them the incentive to continue their education because they can see their possible income and maybe rethink their choices and future life. Government officials must be bombarded with this data because I think it is unfair that the lack of even a high school diploma can send someone into poverty.

Chapter Summary

The chapter presented a discussion of the thematic analyses of the four data sources. The purpose of this qualitative study was to understand what high school students in an all-girls independent school in New York thought about social justice issues and how it may have affected their learning of statistical concepts. In particular, the researcher used a case study approach to investigate the factors that affected female students' development of sociopolitical consciousness and social agency through reading and writing the world with mathematics (RWWM). A total of 13 major themes were generated from the thematic analysis, all addressing the three research questions of the study. The next chapter contains a more in-depth discussion of the findings and themes in relation to the literature. Further, the recommendations, implications, and conclusions are discussed as well.

Chapter V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

With mathematics being a gatekeeper to career and economic advancement (Cobb, 2004; Moses, Kamii, Swap, & Howard, 1989; Wonnacott, 2011), it has been argued that there is a need to provide students with opportunities to discuss social justice issues relating to "power, resources, inequities, and disparate opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language, and other difference" (Gutstein, 2006, p. 26). As teachers help students to make connections between lived experiences relating to social justice issues and mathematics, Gutstein argued that these connections have the potential to deepen students' understanding of the sociopolitical context through the lens of their lived experience. Therefore, as students "read and write the world with mathematics," they increase their conceptual understanding and procedural proficiencies in mathematics.

The purpose of this qualitative case study was to explore the possible effects of embedding social justice issues into a high school statistics course at an all-girls independent school located in a northeastern U.S. metropolitan city. Furthermore, this study investigated factors that may affect female students' development of sociopolitical consciousness and social agency as they read and write the world with mathematics. Out

of eight enrolled students, five students agreed to participate in the study. In agreeing to take part in the study, the participants took part in three TMSJ lesson units relating to racial profiling, education versus income earnings, and HIV/AIDS in Canada. Over the course of 3 weeks, participants took part in several 70-minute lessons involving completing classroom discussion, simulations, mathematical calculation, and graphical representation about social justice issues.

In determining how students were affected by the embedding of a social justice curriculum into a high school statistics course as well as how they used mathematics to "read" and "write" the world, the researcher used an initial survey, face-to-face interview, pre- and post- reflections, as well as an exit survey as primary data sources. Field notes and observations where used as secondary sources. As a method for identifying, analyzing, and then reporting the common themes found in the primary sources, the researcher took a qualitative thematic analysis (Braun & Clarke, 2006) approach.

In addition to asking participants to complete class activities and discussions around teaching mathematics for social justice, this study utilized the Middle Years Programme (MYP) International Baccalaureate (2008) rubric to analyze student work. Using MYP as an informal assessment tool assisted the researcher in gaining greater insight into students' knowledge and understanding of mathematics concepts as they interpreted results, made conjectures, and used mathematical reasoning to solve problems. In looking at how the participants communicated in mathematics, the MYP rubric was used to encourage them to communicate their findings orally and in writing. The MYP rubric also provided an opportunity for participants to reflect the processes and evaluate the significance of their findings.

The literature helped to provide additional context around the sociopolitical turn in mathematics education as well as the girls' learning outcomes and disposition towards mathematics. In examining how female students read and write the world with mathematics, the theoretical framework drew from three conceptual perspectives (e.g., culturally relevant/responsive pedagogy and teaching, critical race theory, and reading and writing the world with mathematics) around TMSJ. These collective ideas helped to add clarity and context to emerging themes.

Conclusions

Using a case study methodology, this study sought to answer three research questions. As such, answers to these questions are discussed in the following sections:

1. How does the incorporation of social justice issues into a high school statistics curriculum at an all-girls independent school affect female students?
2. How, if at all, do students in an all-girls high school statistics course develop sociopolitical consciousness through "reading the world with mathematics"?
3. What factors contribute to or prevent students in an all-girls high school statistics course from "using mathematics to write the world"?

Research Question 1—The Effects

The first research question sought to investigate the effects of embedding a social justice curriculum into a statistics course at an all-girls, independent high school. An analysis of the primary data sources revealed variations in experiences as learners, thus suggesting that a variation in response to learning occurred (Rogers, 1998). As

participants interacted with the three unit lessons, five major themes emerged from the data:

1. Incorporation of social justice educated them about the most pressing issues of today;
2. Having prior knowledge of social justice issues;
3. Strong feelings associated with social justice awareness;
4. Motivation to learn mathematics through social justice issues; and
5. Social justice lessons had limited to no effect.

The results of the study indicated that four of the five female students were presented with an opportunity to acquire newfound knowledge, allowing them to be educated on the most pressing issues today. As Jennifer explained, the real-world applicability of the issues discussed in the lesson encouraged her to learn the issues and how they may affect her in the future. Although Carla, Jessica, Ruby, and Susan entered the study with prior knowledge of the social justice issues, the integration of a social justice curriculum into the statistical course presented Susan with the tools to be able to verify or debunk media sources on related issues. As for Kate, the investigations of social justice issues through the lens of mathematics helped her develop a passion for learning about social justice issues as well as become more proactive in addressing social justice issues.

These results suggested that students who enter the classroom with prior knowledge of social justice issues may be able to use their knowledge to make meaning of mathematics through their lived experiences. In previous literature, Ladson-Billings (2000) argued that when some teachers implemented a constructivist approach, prior

knowledge was used as part of the building blocks for structuring students' newly learned knowledge. As presented in the literature of social constructivism and cognitive constructivism (Liu & Chen, 2010), prior knowledge can play a vital role in how individuals construct their knowledge in an active way. The findings in this study are consistent with the research (Saunders, 1992) suggesting that the participants in this study responded to their sensory experiences through building "in their minds, schemas or cognitive structures which constitute the meaning and understanding of their world" (p. 136). As participants made sense of the phenomenon they encountered during the study, the literature suggested that it was at this time that they made meaning (Resnick, 1983). According to Resnick and Saunders, meaning cannot be transferred by the teacher to the students. It is, therefore, developed by the cognitive apparatus of the learner through a student's process of sensory interaction with the world.

In having students participate in a statistics curriculum infused with social justice issues around racial profiling, income versus education, and HIV/AIDS in Canada, the teacher and researcher adjusted the statistics curriculum to make it more relevant to students' lives (D'Ambrosio, 1997; Gutiérrez, 2002; Gutstein, 2003, 2005; Moses & Cobb, 2001; Moses et al., 1989). In providing a possible explanation for adopting a curriculum that shares features of culturally relevant teaching, critical mathematics, and teaching mathematics for social justice, the researcher and teacher took a sociopolitical turn, as argued by Gutiérrez (2013), namely that taking a theoretical perspective would allow participants to "see knowledge, power, and identity as interwoven and arising from (constituted within) social discourse" (p. 40). The literature on the "real-world" applicability of mathematics concepts suggested that participants transferred their school

learned mathematics to real-world situations (Boaler, 1993). According to Boaler, past and current research has suggested that students' academic outcomes vary when they are presented with "abstract" and "in context" calculations that offer the same mathematical demand or level rigor.

In having students engaged in units of study involving racial profiling, income versus education, and HIV/AIDS in Canada, the participants reported having an increased awareness about social justice issues that exist. When asked to complete a post-lesson reflection after the three activities included in Unit 1, both Kate and Ruby indicated that the activities involving the claims of arrest associated with driving while Black or Brown left them with a sense of awareness about social justice issues. Kate indicated that partaking in the activities educated her on social justice issues. Ruby also stated that by having participated in the class activities, she was more aware now than she was about the extent of the issue.

As a result of participating in these activities, Kate and Ruby had what Rogers (1998) referred to as an "awakening of the mind." According to Rogers, this refers to the "cognitive or intellectual dimensions of learning, including knowledge acquisition, ways of thinking, and overall perspective [is] taken" (p. 206). It is through an awakening of the mind that Rogers stated one can "question, critique, or dramatically change their entire world view or perspective" (p. 206). Given that an "awakening of the mind" may stimulate many reactions, Rogers also noted that students might associate what they are feeling as intellectual overwhelming, pessimism, paralysis, or even incapacity. As a result of trying to balance or manage feelings associated with the cognitive demands of learning or thinking, Rogers stated that students, in turn, take action.

Although there is a risk of exposing some students to a present-day issue that affect civilization for the first time, Slaughter (1989) believed that it is a necessary part for students to be able to see the "interconnectedness and systematicity that characterizes the global system" (Rogers, 1998, p. 206). It is through this process that Slaughter felt students will be able to comprehend and manage such issues that presently affect their current or future state.

For example, after participating in the activities in Unit 1, Jennifer stated that in the context of driving, she learned that statistics could prove that racial profiling was in "fact real." Similarly, Ruby reported that she was angry that the application statistics and mathematics assisted in establishing proof or concrete evidence that an issue such as racial profiling real exists. According to Jennifer, although the activity provided a powerful visualization, she exited the activities feeling angrier about the injustice than when entering the activity.

Susan indicated that although she entered the study having prior knowledge of racial profiling as a prominent issue in society by looking at the news and readings, her participation in the study helped to confirm her beliefs about racial profiling. As a result of her participation, Susan believed that the effect of incorporating social justice issues into her statistics class provided her with the mathematical tools and mathematical knowledge needed to support her claims. Carla, on the other hand, believed that she was not particularly affected by social justice activities. Given that Carla felt the lesson made mathematics more interesting, she shared how this allowed the class to participate in activities designed around a real-world application.

The literature described the negative feelings or anger associated with learning and teaching for social justice issues, as discussed by Carla, Ruby, and Jennifer, which resembled an affective effect suggesting that an emotion influenced their perspective on the lesson. Susan's responses in the post-lesson reflection and interview indicated that the activities also had an affective effect on her in that they helped to merge her interest in social justice and mathematics. According to Masters, Barden, and Ford (1979), affective states "have an indirect effect on behavior and learning- an impact mediated by the actions of others whose behavior towards an individual is influenced by that individual's apparent affective state" (p. 381). When considering how positive (happy), neutral, and negative (sad) affective states impact the way children can master learning, Masters et al. noted that unlike a positive affective state which enhances children's learning, a negative affective state can interfere with or hinder children's learning process. When an affective state is coupled with an active or passive tempo, students' rate of learning may increase or decrease when solving problems. For example, a student displaying a positive affective state and an active tempo would more likely have an increased rate of learning and make fewer errors in the work. However, the findings in this study suggested that the participants' anger (e.g., negative affective state) motivated them to use mathematics to "read the world" and, in some instances, "write the world with mathematics."

When exploring students' responses to learning about global futures or "the study of alternative futures of human and planetary life" (Rogers, 1998, p. 203), Rogers claimed that students might take on cognitive, affective, or soulful aspects that cause learners to take possible action. According to Rogers (1994/1998), the term *soul* is

defined as the "essence of the human being, the core values one holds, the theistic or secular meaning for existence and the sense of life purpose" (p. 209).

Wonnacott's (2011) study of the use of social justice issues in mathematics to promote social agency in affluent, middle-school students supported this notion of outcomes ranging from positive to negative to no effects. Wonnacott found that the incorporation of social justice issues led to students' cognitive and affective domains being affected. For example, the "effects included students: developing an awareness of the social justice issues, applying their knowledge of classical mathematics and changing their perception of mathematics" (p. 58). The cognitive effects ranged from students demonstrating evidence of applying mathematical concepts, to having prior knowledge, or reporting that their perception of mathematics changed while participating in the study.

In the case of an affective effect, some participants in Wonnacott's study also reported negative feelings associated with social justice issues. For instance, a participant reported feeling a sense of helplessness and anger. Similar to this study, others reported no development of feelings after partaking in three cycles (e.g., involving lessons around gender, income, and education; resource distribution in affluent and impoverished areas in the Greater Toronto Area; and modeling with linear and quadratic equations and the HIV/AIDS epidemic in Canada) of practitioner research. Lastly, the findings presented in Wonnacott's study suggest/ed that students' development of social agency was linked to factors revolving around students' age, personal connection to the topic, amount of teacher direction, and students' perception of responsibility. Although the manner in which the data that this researcher collected from the participants confirmed Wonnacott's study, the present findings also enlarge upon that work.

Research Question 2—"Reading the World With Mathematics"

The second research question examined how, if at all, did participants develop sociopolitical consciousness through reading the world with mathematics. Gutstein (2003) defined 'reading the world with mathematics' as being able

> to use mathematics to understand the relation of power, resource inequities, and disparate opportunities, between different social groups and to understand explicit discrimination based on race, class, gender, language and other differences. Further, it means to dissect and deconstruct media and other forms of representation. It means to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them. (p. 45)

As the participants interacted with the three unit lessons, four additional major themes emerged from the data:

6. Using data as evidence on the severity of current social justice issues;
7. Relating mathematics to the issues in the real world;
8. Developing a strong connection with social justice issues; and
9. Using statistics as valid evidence to confirm arguments.

As a result of completing Unit 1 activities, Carla reported that mathematics could serve as a tool to answer questions relating to racial profiling. For Carla, mathematics can be used to provide solid proof that racial profiling of African Americans and Latinos is an issue as well as serve as an essential tool in learning more about the real world. In the case of Jennifer, mathematics can be used as a tool to assist with answering questions around unwarranted stop-and-frisk practices or traffic stops. Although Jennifer stated that she had prior knowledge of social justice issues, she believed that statistics can be used to explore these issues further. By having a strong connection to social justice issues relating to the gender wage gap, Jennifer felt that her first-hand experience urged her to

take concrete actions. As a means of improving better outcomes for women in the future, Jennifer believed that using statistics can be used to confirm arguments. Ruby also agreed that mathematics could be used as a tool, and in her case to explore and learn about the world. As for Susan, her statements emphasized how the incorporation of social justice issues into the curriculum can provide people with concrete data and proof that would make a certain individual believe specific social justice issues. Being affected by the activity involving education and gender, Susan believed that mathematics could be used to distinguish the difference in levels of education and the impact of school on individual earnings. Although during a face-to-face interview, Kate indicated she was unsure of how mathematics could be used as a tool and argued for the need to have stronger statistical evidence to confirm issues. Upon completing the exercise on the wage gap between men and women, Kate reported that Unit 2 resonated the most with her. As a result, she felt motivated to want to take action.

The findings in this study showed a direct relationship with the research on a curriculum designed around TMSJ, "reading the world with mathematics," and critical mathematical literacy. For instance, Frankenstein (2001) argued that one of four goals of critical mathematical literacy curriculum should be for "students to learn how mathematics skills and concepts can be used to understand the institutional structures of society" (p. 58). According to Frankenstein, as students used numerical description and calculation to support their understanding and interpretation of raw data further, they were more inclined to understand the politics of mathematics through reading the world. In alignment with these findings, Gonzalez's (2009) research in TMSJ indicated that in addition to building upon students' experiences, mathematics should be used as a vital

tool for understanding how society works, what one's place in society is, and how do issues of power and agency as well as oppression impact society. According to Gonzalez, "mathematics is needed to be a full participant in society" (p. 24).

As students interact with a TSMJ curriculum, Gutstein (2003) argued that such a curriculum should help "students develop sociopolitical consciousness, a sense of agency, and positive social and cultural identities" (p. 40). According to Gutstein, TMSJ positions students to use mathematics to explain various phenomena that take place in one's immediate life and broader social world. Through students' development of sociopolitical consciousness (e.g., *conscientizacao*), Gutstein argued, students could use mathematics as a tool to combat injustice. As students make sense of human social experience, Gutstein (2007) claimed that students will develop a critical compression of such experiences. Therefore, using mathematics as an analytical tool assists with changing their perception of mathematics.

For example, in a 2-year study that used a qualitative, practitioner-research methodology to investigate TMSJ in a Latino classroom, Gutstein (2003) learned that students began to read the world with mathematics as a way to increase mathematical power but also change their disposition or orientation toward the subject. Though a series of real-world projects, middle school students in an urban school setting began to connect mathematical ideas with an understanding of society over time. As presented in tests, quizzes, projects, and classwork, the findings indicated that 27 out of 28 students developed aspects of mathematical power that ranged in levels of sophistication. According to Gutstein, over time students were able to create mathematical generalizations and construct their solutions to non-routine mathematics problems. As

students began to use mathematics to develop sociopolitical awareness and question the validity of sources and facts, their disposition towards mathematics altered over time as well. According to Gutstein, although not all students reported having a love for mathematics, the majority of the students in his study understood that mathematics could be used as a tool to both solve problems and better understand issues relating to inequality within the context of solving societal issues.

Research Question 3—"Writing the World With Mathematics"

The third research question examined what factors contributed to or prevented students in an all-girls high school statistics course from using "mathematics to write the world." As participants interacted with the three unit lessons, four additional major themes emerged from the data:

10. Having hard proof or evidence on the existence of social justice issues;
11. Having a desire to make change or take action through the use of evidence;
12. Making mathematics more understandable and interesting; and
13. Developing an understanding of the real purpose of statistics.

Four out of five participants (e.g., Carla, Jennifer, Ruby, and Susan) indicated that having hard proof or evidence of the existence of social injustice encouraged them to use mathematics to write the world. Although Carla noted that she did not really see herself as being mathematically inclined or able to use mathematics to alter her current reality or change the world, she could see herself understanding how statistics could be helpful in social movements. Having participated in the three units of study, Carla found the integration of social justice activities into a statistics course provided problems that were

more sensible and relevant than random mathematics problems so much so that she indicated it made mathematics fun and interesting.

Jennifer and Kate echoed that the interrogation played a role in their learning outcome. For example, Jennifer believed that the activities presented in the study made a lot of sense. As for Jennifer, she believed having evidence allowed her to prove arguments as well as helped her to persuade more people to take action. If she is able to create anger in others, Jennifer believed that "anger can start a revolution for the better." Kate stressed how the method of integrating social justice issues into the curriculum was effective in helping her learn a little bit more about issues like racial profiling. Lastly, both Kate and Susan noted they were able to improve their knowledge and awareness of issues when compared to the more traditional structure of teaching.

Although some participants noted a desire to take action through the use of evidence it is important to note that in certain cases some students did not see themselves qualified to make social changes. Given her role as a high school student, Ruby believed that mathematicians should take on the job of spreading awareness. Both Carla and Ruby's response indicated that perhaps they do not identify as mathematicians capable of changing current reality.

The findings in this study add to the conversation around culturally relevant teaching and pedagogy in mathematics education, writing the world with mathematics, and growth mindset. In having students participate in activities involving social justice issues in a statistics course, the findings suggested that the teacher assisted the students in exhibiting cultural competence. As Carla began to participate in activities that allowed her to identify, understand, and offer critical perspectives of today's social inequities, she

reported a positive change in her disposition toward mathematics. Although neither Carla nor the other participants referenced their experience as transitioning from a fixed mindset to a growth mindset, the results presented in the MYP rubric and participants' responses suggested that many participants appeared to show a willingness to approach statistical problems from varied approaches. Given that the research (Walker, 2012) suggested that "women's position with regard to mathematics is also problematic" (p. 9), the findings in this study helped with seeing whether the female students saw themselves as doers of mathematics. In addition, it provided an avenue in which the researcher could see whether the female students' interactions with a social justice curriculum shifted their disposition towards mathematics.

Regarding writing the world with mathematics, Freire (1985) indicated that people must be able to write and rewrite their current reality. In having students use mathematics against inequities or to bring attention to specific social justice issues, Gutstein (2016) believed that students develop and employ social and individual agency. Not only do students develop a heightened sociopolitical awareness and need to shape or change the world, but through this disposition shift, that Gutstein believed that students learn the limitations and usefulness of reading and writing the world with mathematics.

When looking at participants' responses on whether or not the structure of the unit lessons and the teacher's presentation of the content materials made mathematics more understandable and interesting, the participants' responses helped to shed more light on female students' learning outcomes and disposition towards mathematics. For instance, two of the recommendations made by the NCTM (2014) required teachers to be skillful and effective at providing differentiated instruction as needed. In addition, the NCTM

indicated that when appropriate, teachers should offer students' remediation or additional challenges. Although there were various reasons for the girls' attitudes towards mathematics, many participants in this study indicated that the incorporation of social justice issues made learning mathematics fun and interesting. In other cases, some participants indicated that the lesson format (e.g., nontraditional versus traditional) helped to increase their understanding. Given that Helbronner (2013) reported that women possess lower self-efficacy in STEM-related subject than men, the findings in this study indicated that the social justice lesson embedded in a mathematics curriculum aided students in increasing interest in mathematics. Therefore, it is possible that the incorporation of social justice issues in other STEM-related courses may produce similar results.

The act of participants being able to 'write the world with mathematics' through teachers making mathematics more understandable and interesting points to the implications of how academic engagement is driven by students' interest. In defining academic engagement, Newmann, Wehlage, and Lamborn (1992) translate the term as being actively involved, having a commitment, as well as attending to attention. Rather than students withdrawing from the social justice lessons, students' motivation and engagement with the lesson draws them into wanting to learn more about how to take action against social justice issues.

Methodological Assumptions, Limitations, and Delimitations

This study employed a qualitative case study approach to analyze multiple data sources (e.g., introduction survey and free-writing activity, post-lesson reflection,

interviews, and exit survey). The limited or small number of recruited participants may have also limited the generalizability of the study. Although the researcher assumed the students answered the interviews, surveys, and reflections honestly and truthfully, the data collected from the students may have been restricted due to the possibility that students may have been afraid their teachers or the school administrators may identify them. Their perceptions may have been limited, and their actual feelings and experiences may not have been shared completely.

To minimize the effects of the limitations above and as seen in Chapter IV, the researcher collected, analyzed, and presented the data in an organized manner, coding and reporting them based on the themes recorded per research question. Meanwhile, the small number of participants allowed the researcher to analyze all data carefully and completely, ensuring that the most significant and meaningful responses were coded and reported in Chapter IV. The final limitation was also moderated as the researcher continually assured the participants that their teachers and school stakeholders would not in any way identify them. By doing so, the students would not feel intimidated or scared to share even their negative or not-so-affirmative perceptions of the lesson integration.

Recommendations

In completing this study, the researcher offers recommendations based on the literature reviewed, findings, analysis, and conclusions presented in this study. The recommendations that follow offer support to (a) current mathematics teachers, (b) students, (c) school administrators and mathematical coaches, and (d) further research in the area.

Recommendations for Mathematics Teachers

Prior to adopting a social justice curriculum into a mathematics course, mathematics teachers should:

- Develop a platform in which students can communicate their interest in the social justice topics they would like the teacher to consider as part of the class discussion. Although the lessons in this study were either adapted from previous studies, including students in the conversation about their learning aids them in taking ownership of their learning.
- Ensure that lessons are age-appropriate as well as challenging to students in a way that will draw them into the lesson. With many of the lessons reviewed by the researcher geared towards middle school students, teachers wanting to teach mathematics through a sociopolitical lens may have to consider developing new lessons or modifying previously created lessons to meet their content objective and grade level.
- Critically reflect on the impact the lessons will have on the students and the classroom dynamics. Before teaching mathematics for social justice, set expectations in place that will cultivate a learning environment to invite and support difficult conversations and different perspectives. This then allows each participant to feel she is a valued member of the classroom.
- Critically reflect on the teacher's role in fostering students' learning of mathematics that engages students in conversations around examining social justice issues. Given that some of the lessons may ask students to recognize, understand, and critique current social inequities, teachers will need to

understand that students may want to know if their teacher is an advocate, a silent voice, or an intruder.

- Design lessons that provide avenues and activities for students, more specifically disengaged or struggling learners, to see them as successful learners and doers of mathematics.
- Consider widening the scope of the audience. In teaching culturally relevant lessons or mathematics for social justice, considering reaching an audience outside of the scope of being marginalized or fitting a particular racial make-up. The issues of global society are forever changing and affect people from a variety of backgrounds, socioeconomic statuses, cultures, and genders. Therefore, the conversations around critical issues or social injustice should be widened to include varied racial demographics, genders, learning levels, and socioeconomic statuses.
- Provide a support system (e.g., counselor, peer teacher, department chair, community group, parents, or administrators) for students to communicate their personal emotions or feelings developed as a result of participating in teaching and learning mathematics for social justice.
- Create lessons that allow students time to grapple with the material, their emotions, and the mathematics content.
- Understand that in teaching students to read the world with mathematics, students may desire that teachers already have support structures in place to assist them with taking action (e.g., writing the world with mathematics)

against certain injustices. As the results in this study showed, students desired assistance with making a change.

Recommendations for Students

Students participating in learning mathematics for social justice should:

- Understand that learning about social justice issues may increase awareness of sociopolitical issues that can affect them differently and at times create a sense of awareness that is uncomfortable. Therefore, students should be aware of the support system the school and teacher have put in place.
- Consider speaking with the teacher before engaging in the lesson about what topics will be covered and to what depth.
- Understand and respect that all students' views and prior experiences about a particular issue may not be shared by any classmate or teacher.
- Conduct an ongoing assessment of how each lesson may or may not affect the student personally. Given that some lessons may ignite emotions or feelings that move students towards taking action, it is essential to know that ***not*** having these feelings are valued as well.
- Take sufficient time not only to grapple with the social justice issue, but also to acquire the mathematical knowledge needed to understand the objective(s) of each lesson fully.

Recommendations for School Administrators and Mathematics Coaches

School administrators and mathematics coaches alike wishing to incorporate social justice activities into mathematics curriculum should consider that:

- A degree of discomfort is often associated and experienced by both teachers and students when discussing issues of race, social class, disabilities, and sexual orientation.
- In addition to asking faculty and staff to reflect on the role 'White privilege' plays in creating culturally responsive classrooms, educational intuitions must also discuss the role 'teacher privilege' plays out in fostering unequal opportunities and levels of discomfort for some students.
- In addition to coaching around what is culturally responsive teaching, school administrators and mathematics coaches should provide additional resources and coach around how to design, implement, and assess mathematics lessons that discuss social justice issues.
- As school administrators ask mathematics teachers to incorporate culturally responsive lessons into the mathematics classroom, they should do so only after allowing teachers to express their level of comfort and understanding of teaching sociopolitical topics within their current school setting.
- In asking both teachers and students to work with a curriculum designed to elaborate and increase awareness of social inequities using mathematics, school administrators and staff members must create a supportive environment that assists with confronting bias and diverse viewpoints.
- Both students and teachers will need help with partnering with community partners in their efforts to write the world with mathematics.

Recommendations for Future Research

The researcher recommends that additional studies be conducted to gain a more comprehensive understanding of the role students' gender, diversity (e.g., regarding racial and gender diversity as well as mathematical knowledge) within the classroom, and mindset play in how students are able to "read and write the world with mathematics." In addition, such studies should also consider teachers' beliefs, teaching experiences, racial and gender backgrounds in incorporating culturally relevant lessons into a mathematics classroom.

Considering this, the following research interests should be considered:

- Given that the current research suggested that males display a higher level of self-efficacy than their female peers as well as receive less biased feedback from parents and teachers in learning and doing mathematics, the mathematics disposition of male students in a heterogeneous setting may yield different learning outcomes. Therefore, a similar study using the same criteria in students reading and writing the world with mathematics should be undertaken with male high school students in an independent school setting.
- Although this research touched briefly on girls' mindset, further studies should be conducted to determine how, if at all, does having a growth or fixed mindset impacts students' development of sociopolitical consciousness or desire to write the world. In considering students' mindset upon entering and exiting the study, a comparison and analysis should be done to assess mindset differences between male and female students in their learning of sociopolitical concepts in a mathematics classroom.

- In capturing the range in differences of mathematical knowledge that male and female students display while participating in TMSJ lessons, the researcher recommends that a quantitative study be completed to determine the level of significance. As such, researchers may want to look more closely at how students develop and display mathematical knowledge throughout all phases of their learning of social justice issues in a mathematics setting.
- A longitudinal study on the female population used in this study or similar studies should be undertaken to examine the long-term effects of participating in a study on TMSJ. Such a study would lend itself to seeing if and at what point students, who previously indicated that they needed help knowing how to write the world with mathematics, have since developed the knowledge and tools to do so. In addition, how, if at all, did students transition from wanting to take action to actually engaging in social and political issues in their communities to fight against social injustice?
- Given that much of the research existing around culturally relevant teaching and pedagogy has focused on preservice teachers, the research in the field should explore the impact of TMSJ with teachers who are more experienced. To that end, researchers should include teachers from diverse racial and gender backgrounds.
- Given the literature indicating that teachers' beliefs and mathematical knowledge impact their students' learning of mathematics, a cross-sectional study should be conducted to investigate the possible relationship between teachers' beliefs about TMSJ and students' ability to use the information

obtained during TMSJ lessons to “read and write the world with mathematics.” In doing so, the research could compare two sample populations at a given point in time to see if a correlation exists.

- Given two different groups who either indicated being affected or not affected by participating in the social justice lessons in this study, a cohort study or prospective observation study could be done at a later time.
- Given that this study was similar to Wonnacott’s study using affluent students and Gutstein’s study using Latino students, researchers may want to conduct a meta-analysis. In combining the findings of all three studies, researchers can look more deeply into the common effects of students interacting with social justice lessons.
- In capturing how the teacher in this study might have been affected by embedding a social justice lesson into a statistics curriculum as well as teaching it to an all-girl population, a narrative inquiry should be conducted. In that way, it will help with capturing the experience of the teacher through means of storytelling.

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Appendix A

Participants' Informed Consent and Participants' Right Form

INTRODUCTION

You are being invited to participate in this research study called "Teaching Mathematics for Social Justice: How Students in an All Girls Independent School Setting Use Mathematics to Read and Write the World." You may qualify to take part in this research study because are between the ages of fifteen and eighteen and are in a statistics class. You will be recruited during normal school hours. If you opt out of participating in the activities, you will be asked to proceed with your normal classroom routine independent of the classroom instructor. Daily routines may include but are not limited to written assignments, journal reflections, individual research requirements. If you are presently participating in another study, you can be part of this study. Approximately fifteen people will participate in this study and it will take six weeks of your time to complete.

WHY IS THIS STUDY BEING DONE? This study is being done to gain an understanding of what high school students think about social justice issues, and how it might affect their learning of statistical concepts in a mathematics classroom.

WHAT WILL I BE ASKED TO DO IF I AGREE TO TAKE PART IN THIS STUDY?

If you decide to participate, you will participate in an online questionnaire. The electronic questionnaire can be completed in the privacy of your home. The questionnaire will last approximately twenty minutes. During the questionnaire, you will be asked questions regarding your personal and family background, understanding of mathematics/ statistics, attitudes towards or interest in mathematics/statistics, and learning and teaching mathematics for social justice. You will be asked to provide a pseudonym or false name in order to keep your identity confidential.

You will then be asked to participate in three mathematics lessons over the course of six weeks that will allow you to discuss your experience with the mathematics activities related to social justice and the teaching and learning of statistics during a regular 70-minute course. All classroom activities will be observed and audio-recorded by the Lucretia Glover, the Principal Investigator. After the recorded classroom observations have been transcribe, the original recording will be deleted. If you do not wish to be audio-recorded, you will still be able to participate. Towards the end of each lesson, you will be asked to write a fifteen-minute post reflection regarding what you learned and how you were affected by the activity. In keeping your identity confidential, the pseudonym or false name will be coded and recorded.

If you opt out of participating in these activities, you will be asked to proceed with your normal classroom routine independent of the classroom instructor. Daily routines may include but are not limited to written assignments, journal reflections, individual research requirements.

Finally, you may then be asked to participate in a focus group ran by Lucretia Glover, the Principal Investigator where you will discuss their experiences over the course of the six-weeks mathematics/statistics lessons. This will also be audio-recorded, and the tapes destroyed after they are transcribed by a professional transcriptionist. Everyone will be asked not to discuss what is being spoken about outside of the group, but it is impossible to guarantee complete confidentiality. This portion of study will take place in classroom for about approximately thirty minutes during lunch time.

WHAT POSSIBLE RISKS OR DISCOMFORTS CAN I EXPECT FROM TAKING PART IN THIS STUDY?

This is a minimal risk study, which means the harms or discomforts that you may experience are not greater than you would ordinarily encounter in daily life while taking routine physical or psychological examinations or tests. However, there are some risks to consider. You might feel embarrassed to discuss problems that you experienced with learning mathematics or statistics concepts. However, you do not have to answer any questions or divulge anything you don't want to talk about. You can stop participating in the study at any time without penalty. You might feel concerned that things you say might get back to your principal. The principal investigator is taking precautions to keep your information confidential and prevent anyone from discovering or guessing your identity, such as using a pseudonym instead of your name and keeping all information on a password protected computer and locked in a file drawer.

WHAT POSSIBLE BENEFITS CAN I EXPECT FROM TAKING PART IN THIS STUDY?

There is no direct benefit to you for participating in this study. However, the principal investigator is a mathematics teacher and will be honored if your child were to participate in this study.

WILL I BE PAID FOR BEING IN THIS STUDY?

Your child will not be paid to participate, but they will receive a small token of appreciation (e.g. a five-dollar gift card). There are no costs to you for taking part in this study.

WHEN IS THE STUDY OVER? CAN I LEAVE THE STUDY BEFORE IT ENDS?

The study is over when you have completed the online questionnaire, classroom observation and post reflection, and focus group. However, your child can leave the study at any time even if they haven't finished. They will still receive a five-dollar gift card.

PROTECTION OF YOUR CONFIDENTIALITY

The investigator will keep all written materials locked in a desk drawer in a locked office. Any electronic or digital information (including audio recordings) will be stored on a computer that is password protected. What is on the audio-recording will be written down and the audio-recording will then be destroyed. There will be no record matching your child's real name with their pseudonym. Research data concerning children will be kept for five years.

HOW WILL THE RESULTS BE USED?

The results of this study will be published in journals and presented at academic conferences. Your name or any identifying information about you will not be published. This study is being conducted as part of the dissertation of the principal investigator.

CONSENT FOR AUDIO RECORDING

Audio recording is part of this research study. You can choose whether to give permission to be recorded. If you decide that you don't wish to be recorded, you will still be able to participate in this study.
Pick one option below:

_____I give my consent to be recorded

_____
Signature

_____I **do not** consent to be recorded

_____
Signature

WHO MAY VIEW MY PARTICIPATION IN THIS STUDY

Pick one option below:

___I consent to allow written, audio taped materials viewed at an educational setting or at a conference outside of Teachers College

_____
Signature

___I **do not** consent to allow written, audio taped materials viewed outside of Teachers College Columbia University

_____
Signature

OPTIONAL CONSENT FOR FUTURE CONTACT

The investigator may wish to contact you in the future. Please initial the appropriate statements to indicate whether or not you give permission for future contact.

I give permission to be contacted in the future for research purposes:

Yes _____ No_____
Initial Initial

I give permission to be contacted in the future for information relating to this study:

Yes _____ No_____
Initial Initial

PARTICIPANT'S RIGHTS

- I have read and discussed the informed consent with the researcher. I have had ample opportunity to ask questions about the purposes, procedures, risks and benefits regarding this research study.
- I understand that my participation is voluntary. I may refuse to participate or withdraw participation at any time without penalty to future student status or grades; services that I would otherwise receive.
- The researcher may withdraw me from the research at his or her professional discretion.
- If, during the course of the study, significant new information that has been developed becomes available which may relate to my willingness to continue my participation, the investigator will provide this information to me.
- Any information derived from the research study that personally identifies me will not be voluntarily released or disclosed without my separate consent, except as specifically required by law.
- I should receive a copy of the Informed Consent document.

My signature means that I agree to participate in this study

Print Name: _____ **Date**: _____

Signature: _____

Appendix B

Parent Permission Form

INTRODUCTION

Your child is being invited to participate in this research study called "Teaching Mathematics for Social Justice: How Students in an All Girls Independent School Setting Use Mathematics to Read and Write the World." Your child may qualify to take part in this research study because they are between the ages of fifteen and eighteen and are in a statistics class. Students will be recruited during normal school hours. Students who opt out of participating in the activities will be asked to proceed with their normal classroom routine independent of the classroom instructor. Daily routines may include but are not limited to written assignments, journal reflections, individual research requirements. If they are presently participating in another study, they can be part of this study. Approximately fifteen children will participate in this study and it will take place over the course of six weeks to complete.

WHY IS THIS STUDY BEING DONE? This study is being done to gain an understanding what high school students think about social justice issues, and how it might affect their learning of statistical concepts in a mathematics classroom.

WHAT WILL MY CHILD BE ASKED TO DO IF I AGREE THAT MY CHILD CAN TAKE PART IN THIS STUDY?

If you decide to allow your child to take part in this study, your child will participate in an online questionnaire. The electronic questionnaire can be completed in the privacy of your home. The questionnaire will last approximately twenty minutes and will focus on questions regarding the interviewee's personal and family background, understanding of mathematics/statistics, attitudes towards or interest in mathematics/statistics, and learning and teaching mathematics for social justice. Your child will be asked to provide a pseudonym or false name in order to keep their identity confidential.

Your child will then be asked to participate in three mathematics lessons over the course of six-weeks that will allow them to discuss their experience with the mathematics activities related to social justice and the teaching and learning of statistics during a regular 70-minute course. All classroom activities will be observed and audio-recorded by the Lucretia Glover, the Principal Investigator. After the recorded classroom observations have been transcribed, the original recording will be deleted. If you do not wish your child to be audio-recorded, your child will still be able to participate. Towards the end of each lesson, your child will be asked to write a fifteen-minute post reflection regarding what they learned and how they were affected by the activity. In keeping your child's identify confidential, the pseudonym or false name will be coded and recorded. Students who opt out of participating in these activities will be asked to proceed with their normal classroom routine independent of the classroom instructor. Daily routines may include but are not limited to written assignments, journal reflections, individual research requirements.

Finally, your child may then be asked to participate in a focus group run by Lucretia Glover, the Principal Investigator where children will discuss their experiences over the course of the six-weeks mathematics/statistics lessons. This will also be audio-recorded, and the tapes destroyed after they are transcribed by a professional transcriptionist. Everyone will be asked not to discuss what is being spoken about outside of the group, but it is impossible to guarantee complete confidentiality. This portion of study will take place in classroom for about approximately thirty minutes during lunch time.

WHAT POSSIBLE RISKS OR DISCOMFORTS CAN MY CHILD EXPECT FROM TAKING PART IN THIS STUDY?

This is a minimal risk study, which means the harms or discomforts that your child may experience are not greater than your child would ordinarily encounter in daily life while taking routine physical or psychological examinations or tests. Your child might feel embarrassed to discuss problems learning math. However, your child does not have to answer any questions or divulge anything they don't want to talk about. Your child can stop participating in the study at any time without penalty. The principal investigator is taking precautions to keep your child's information confidential and prevent anyone from discovering what they say or their identity, such as using a pseudonym instead of their name and keeping all information on a password protected computer and locked in a file drawer.

WHAT POSSIBLE BENEFITS CAN MY CHILD EXPECT FROM TAKING PART IN THIS STUDY?

There is no direct benefit to your child for participating in this study. However, the principal investigator is a mathematics teacher and will be honored if your child were to participate in this study.

WILL MY CHILD BE PAID FOR BEING IN THIS STUDY?

Your child will not be paid to participate, but they will receive a small token of appreciation (e.g. a five-dollar gift card). There are no costs to you for your child's taking part in this study.

WHEN IS THE STUDY OVER? CAN MY CHILD LEAVE THE STUDY BEFORE IT ENDS?

The study is over when your child has completed the online questionnaire, classroom observation and post reflection, and focus group. However, your child can leave the study at any time even if they haven't finished. They will still receive a five-dollar gift card.

PROTECTION OF YOUR CHILD'S CONFIDENTIALITY

The investigator will keep all written materials locked in a desk drawer in a locked office. Any electronic or digital information (including audio recordings) will be stored on a computer that is password protected. What is on the audio-recording will be written down and the audio-recording will then be destroyed. There will be no record matching your child's real name with their pseudonym. Research data concerning children will be kept for five years.

HOW WILL THE RESULTS BE USED?

The results of this study will be published in journals and presented at academic conferences. Your child's name or any identifying information about your child will not be published. This study is being conducted as part of the dissertation of the principal investigator.

CONSENT FOR AUDIO RECORDING

Audio recording is part of this research study. You can choose whether to give permission for your child to be recorded. If you decide that you don't wish your child be recorded, they will still be able to participate in this research study.

Pick one option:

_____I give my consent for my child to be recorded _____

Signature

_____I **do not** consent for my child to be recorded _____

Signature

WHO MAY VIEW MY CHILD'S PARTICIPATION IN THIS STUDY

Pick one option:

____I consent to allow my child's written, audio taped materials viewed at an educational setting or at a conference outside of Teachers College._____

Signature

_____I **do not** consent to allow my child's written, audio taped materials viewed outside of Teachers College Columbia University _____

Signature

OPTIONAL CONSENT FOR FUTURE CONTACT

The investigator may wish to contact you in the future. Please initial the appropriate statements to indicate whether or not you give permission for future contact.

I give permission to be contacted in the future for research purposes:

Yes _____ No_____

Initial Initial

I give permission to be contacted in the future for information relating to this study:

Yes _____ No_____

Initial Initial

PARTICIPANT'S RIGHTS

- I have read and discussed the informed consent with the investigator. I have had ample opportunity to ask questions about the purposes, procedures, risks and benefits regarding this research study.
- I understand that my child's participation is voluntary. I may refuse to allow my child to participate or withdraw participation at any time without penalty to future student status or grades; services that my child would otherwise receive. I understand that my child may refuse to participate without penalty.
- The investigator may withdraw my child from the research at her professional discretion.
- If, during the course of the study, significant new information that has been developed becomes available which may relate to my willingness to allow my child to continue participation, the investigator will provide this information to me.
- Any information derived from the research study that personally identifies my child will not be voluntarily released or disclosed without my separate consent, except as specifically required by law.
- I should receive a copy of the Informed Consent document.

My signature means that I agree to allow my child to participate in this study.

Child's name: _____

Print Parent or guardian's name: _____

Parent or guardian's signature: _____

Date: _____

Appendix C

Protocol for Initial Questionnaire

Adapted from Wannocott (2011) and Holodick-Reed (2013)

Time of Interview: _____ Date: _____
Location:
_____
Interviewer: _____ Interviewee (Pseudonym): _____

Consent Form Signed: (*Select One*) Yes or No

General Information
You have been invited to participate in this online survey that explores issues related to social justice and the teaching and learning of statistics. You may qualify to take part in this research study because we are interested in understanding what high school students think about these issues, and how it might affect their learning of statistical concepts. To assist with our note-taking and maintaining the authenticity in your response, we would like you to participate in a web-based questionnaire. For your information, only the researcher and professional transcriptionist on the project are privy to the electronic responses. Any information obtained during the initial interview is confidential; therefore, you are asked to provide a pseudonym or false name. Participation in this research study is voluntary. At any time during our interview or at any time after our interview you feel uncomfortable, you have the right to withdraw from parts of the interview, the entire interview or from the entire study.

This web-based questionnaire is planned to last for approximately 15 - 20 minutes. During the online questionnaire, you will be asked questions relating to your background information, understanding of mathematics/statistics, attitude towards or interest in mathematics/statistics, and learning and teaching mathematics for social justice.

Background Information
1. First Name

2. What is your current age?

3. Ethnicity origin (or Race): Please specify your ethnicity.
o White o Black or African American
o American Indian or Alaska Native o Hispanic or Latino
o Asian o Native Hawaiian or Pacific Islander
o Other

4. How many years have you attended your current school?
o 0 - 3 years o 4 - 6 years o 7 - 9 years o 10 or more years

5. If you previously attend a different middle or different high school, indicate the name of the school.

6. What is your anticipated graduation year?
o 2018 o 2019

7. Other than statistics, what other high school mathematics courses have you taken? Select all that apply.
▢ Algebra 1A and Algebra 1B ▢ Algebra I
▢ Algebra 2 ▢ Honors Algebra 2
▢ Geometry ▢ Honors Geometry
▢ Precalculus ▢ Honors Precalculus
▢ Calculus ▢ AP Calculus AB
▢ Adv. Problem Solving & Mathematical Modeling

8. To the best of your knowledge, what is your current grade in your statistics course?
o A o A-
o B+ o B o B-
o C+ o C (7) o C-
o D+ o D o D- o F

9. What is your most recent cumulative GPA?
o Below a 2.5 o 2.6 - 3.0 o 3.1 - 3.5 o 3.6 - 4.0 o 4.1 or higher

10. Parents' Educational Background: Highest Level of Completion

	Father or Guardian 1	Mother or Guardian 2
	Check the highest level	Check the highest level
Elementary School	o	o
Middle/Jr. High School	o	o
High School Diploma or Equivalent	o	o
Business or trade school	o	o
Some college	o	o
Associate degree	o	o
Bachelor degree	o	o
Some graduate or professional schooling	o	o
Master degree	o	o

Doctoral degree	o	o

11. Briefly, describe your involvement in extracurricular or sports related activities in high school.

12. Briefly, describe your involvement in your community or other areas.

13. Have you participated in any volunteer work in the last five years? If so, can you briefly explain why you selected to volunteer?

14. If you answered yes to question 10, what was your role at the site location you volunteered at?

15. On average, how many volunteer hours have you completed?

Purpose of Mathematics

16. What do you think is the purpose of learning mathematics or statistics?

17. If you think people should learn mathematics, why should they learn it?

18. What do you think the purpose of teaching statistics as a mathematics course in high school should be?

Learning and Teaching Mathematics for Social Justice

19. To your best ability, define teaching mathematics for social justice.

20. What do you think the effects would be if social justice issue were to be incorporated into a statistics or mathematics curriculum?

21. Do you currently learn or discuss social justice related issues in other classes or within your school? If so, how were these topics explored or discussed during class?

22. What are some social justice related issues that you are interested in studying and why?

Appendix D

Unit 1—Lesson 1: Review of Basic Probability Ideas

Adapted from Teachers for Social Justice (2009) and *Reading and Writing with Mathematics: Toward a Pedagogy for Social Justice* (Gutstein, 2012)

To understand racial profiling, you will need to understand several concepts: *randomness, experiment, simulation, sample size, experimental and theoretical probability* and *the law of large numbers*. One way for students to begin discussing these ideas is to have you work in pairs to toss a coin 100 times (the experiment) and record results. Once done, we will combine the class data and have the whole class together examine how the combined data comes closer to a 50-50 split than do the individual pairs (the *law of large numbers*).

Introduction to Probability

We now turn from counting to probability. As we will see, this is not much of a turn. Many probability questions are really counting questions. First let's take a general look at what probability is and try to define it. Suppose we have a quarter, where one side is heads and the other side is tails. If we flip the coin over and over and over again, we expect that the proportion of flips that come up heads to be roughly one half. So, we say that the probability that a coin flip will turn up heads is one half.

This is probability: the proportion of times that we expect an outcome to occur if we perform the experiment over and over.

Notice that we can't say the word *exact* proportion of times we have success. We cannot predict with 100% accuracy the outcome. This is the difficulty in defining probability. Probability only exists because we are trying to measure an event that is not definite.

Our coin example is how we will in general approach most probability problems. We figure out how many equally likely outcomes there are, then count how many of these outcomes are what we call "success" or "favorable". Our probability is the ratio of the number of successful outcomes to the total number of possible outcomes.

If all outcomes are equally likely, then the probability of success is $\frac{number\ of\ successful\ outcomes}{number\ of\ possible\ outcomes}$.

Sometimes it will be necessary to distinguish between theoretical probability (we expect the coin to land heads up ½ of the time) vs. experimental probability (flip a coin a bunch of times, do you actually get half of the coin flips to land head up - not likely). Experimental probability is the ratio of the number of times an event occurs to the total number of times the activity is performed.

There are many things we cannot theoretically model so easily as a coin flip or a roll of a die. In these cases, we rely heavily on experimental probability. For example, we look at free throw percentage as the likelihood the basketball player will make their next free throw shot. Wall Street investors use experimental probability to make their bets on where the market will go based on their knowledge of previously studied data and what has happened in the past under similar circumstances.

Instructions for the Activity

In groups of two you will be performing repeated coin flips and recording data. There are two jobs: flipping the coin, recording the data. You will flip coins in sets of ten repeatedly. Every 20th flip you are to calculate the running total. Once you have 100 coin flips, compile your total in the final table. Then we will compare each group's data as well as compare individual group's data to the total class data.

1. What is the set of possible outcomes of a coin flip?
2. In any experiment, the set of all possible outcomes of the experiment is called the _____.

Coin Flips #1-10

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #11-20

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Sum of Total Coin Flips

Sum of Coin Flips #1-20

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #21-30

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #31-40

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #41-50

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Sum of Coin Flips #1-40

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #51-60

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #61-70

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #71-80

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #81-90

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Sum of Coin Flips # 1-60

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Sum of Coin Flips # 1-80

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Coin Flips #91-100

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Sum of Coin Flips # 1-100

Outcome	# of times occurred	Experimental probability
Heads		
Tails		

Outcome	**# of times occurred in each group (out of 100)**	**Total**	**Experimental Probability**
Heads			
Tails			

Whole Class' Data

Sum: _____

Appendix E

Unit 1—Lesson 2: Racial Profiling Activity

Adapted from *Rethinking Mathematics Teaching Social Justice by the Numbers* (Gutstein & Peterson, 2013) and *Reading and Writing the World with Mathematics: Toward a Pedagogy for Social Justice* (Gutstein, 2006)

Part 1. **Opening Activity**

The purpose of this project is to investigate racial profiling or Driving While Black or Driving While Brown (DWB/DWB). African Americans and Latinos/as having complained, filed suit, and organized against what they believe are racist police practices- being stopped, searched, harassed, and arrested because they "fit" a racial profile- they are African American (Black) or Latino/a (Brown). But it this true? How do we know? And can mathematics be a useful tool in helping us answer this question?

Whole Group Discussion Questions

1. Do you think the statement above about racial profiling is true?
2. Regardless of whether you answered yes or no, how do you reach your conclusion?
3. Do you think mathematics can be a useful tool in helping answer the question? If so, explain your reasoning behind your response.
4. Are there particular concepts in mathematics or statistics that you think might be helpful in addressing this question or questions like this one? If so, what mathematics or statistics concepts might be useful and why?

Part 2: Find Chicago's Racial Breakdown

As a group, each of you will be given a small bag with colored counting chips to match the racial proportions in Chicago, IL from the 2010 Census' data. Blue counting chips will represent African Americans, green counting chips will represent Whites, red counting chips Latinos/as and yellow counting chips will represent Asian/Pacific Islanders/Native Americans.

Procedures:

Without counting the total number of counting chips in your bag, pick one cube without looking. Then record its color and replace the counting chip. Record the results of each 10 picks in the chart provided (tally marks work well). You are to *conduct an experiment* (picking/replacing 100 times), *collect data* (recording each pick), and *analyze data* (determine from the simulation how many there are of each color and the total, and what is the racial/ethnic percentages of Chicago, IL).

TABLE 1. Driving While Black or Driving While Brown (DWB/DWB) Simulation

# of picks	White #	White Fract.	White %	AfAm #	AfAm Fract.	AfAm %	Latino #	Latino Fract.	Latino %	Asian #	Asian Fract.	Asian %
1 - 10												
11 - 20												
21 – 30												
31 - 40												
41 - 50												
51 - 60												
61 - 70												
71 - 80												
81 - 90												
91 - 100												
Total												

Part I: Follow Up Questions.

1. Without opening your bag, how many cubes of each color do you think are in it? Why? Discuss your reasoning in your group, in detail, and explain how you used mathematics.

2. Look at your cumulative table - what happened as you picked more times? How close was your first row to your final answer to question 1 above? That is if you had to answer question 1 after just 10 picks, would your answer be the same or different as after 100 picks, and why?

Part 2: Whole Group Exercise.

1. Work class to combine all of the data of all groups on the board to find the percentage for each race. What happened when we combined everyone's data together? Why?

2. What do you think would happen if you picked 1,000,000 times? Use mathematics to explain your reasoning.

Appendix F

Unit 1—Lesson 3: Investigating DWB/DWB Exploration Activity

Adapted from *Rethinking Mathematics Teaching Social Justice by the Numbers* (Gutstein & Peterson, 2013) and *Reading and Writing the World with Mathematics: Toward a Pedagogy for Social Justice* (Gutstein, 2006)

For this activity, you simulate the data from traffic stops to decide whether you think racial profiling occurred. Here are sample Illinois data based on police reports from 1987 - 1997. Racial profiling data for any locality can usually be found through web searches. In an area of about 1,000,000 motorists, approximately 28,000 were Latinos/as. Over a certain period of time, state police made 14,750 discretionary traffic stops (e.g., If a driver changes lane without signaling, or drives 1 - 5 mph over the speed limit, police may stop her or him but do not have to). Of these stops, 3,100 were of Latino/a drivers.

Use what you learned in Activity 2 to set up your own simulation of the situation using counting chips. If needed, add additional counting chips. Then, pick and replace, record the data, and calculate the results of simulating 100 "discretionary" stops.

Follow-up Questions

1. What percentage of the motorist in Activity 3 were Latino/a?
2. What percentage of the discretionary traffic stops involved Latino/a?
3. How did you set up the simulation for Activity #3 (how many "Latino/a" counting chips and how many total)? Why did you choose those numbers?
4. How many Latinos/as were picked out of 100 picks, and what percentage is that?
5. Consider your simulation results. Compare them to the discretionary stop data. Does your simulation data in Number 4 support the claim of racial profiling? Why or why not?

Whole Class Activity

Combine individual groups' results and analyze as a whole class.

Whole Class Discussion Question

1. Based on the data, do you think racial profiling is a problem, and if so, what do you think should be done about it?
2. What questions does this project raise in your mind? Is there anything you would want to know more about?
3. What, if anything, would you like to share out about this experience?

Appendix G

Post-Lesson Reflection for Unit 1 (Lessons 1-3)

The purpose of the post-lesson reflection is to collect students' feedback about the social justice activity. Prior to typing on this form, please make a copy of the Google Document for yourself. Then share a copy of the new document with your responses with your teacher.

1. What did you learn from participating in statistics activities which examined social justice issues around racial profiling or driving while Black or Brown?

2. Describe anyway, in which, you may have been affected by the social justice activities in Unit 1?

3. Were there any challenges that you came across in doing these activities? If so, provide a brief overview.

4. Explain how you, if at all, used mathematics in Lesson 1 to understand relation of power, resource inequities, or disparate opportunities, between different social groups and discrimination based on race, class, gender, language or other differences[1].

5. Explain how, if at all, the lessons in Unit 1 may have helped you to use mathematics to examine these various phenomena as described in Question 4 "both in one's immediate life and in the broader social world and to identify relationship and make connections between them," (Gutstein, 2003, p. 45).

6. After having participated in this activity, could you see yourself "using mathematics to change the world[2]" (Gutstein, 2006, p. 27). If so, how?

7. What questions do you still have?

---

[1] To read the world with mathematics.

[2] To write the world with mathematics.

Appendix H

Unit 2—Schooling Versus Income Earnings

Adapted from the dissertation entitled *Teaching Mathematics for Social Justice and its Effects on Affluent Students* (Wonnacott, 2011, pp. 109 - 110)

Goal of the Activity:

- Determine whether there is the relationship between the number of years you spend in school and your income.
- Determine whether there is a difference in male and female earning power.

Activity

In determining whether there is a relationship, use the data provided in the table below to complete steps 1-6.

1. Write a hypothesis.
2. Which columns will to construct a graph? Determine if you need to graph all of columns or can you omit some out?
3. Using the data provided above, construct one scatter plot for both male and female data.
4. Draw two lines of the line of best fit, one for males and another for females.
5. Determine the equation of the line of best fit for both males and females.
6. Determine the r-value for both lines.

Table A. Median Annual Income, by Level of Education, 2010

	Elementary/Secondary			College				
Sex & Year	**Less than 9th grade**	**9th to 12 grade, no completion[1]**	**High school completion (includes equivalency)[2]**	**Some college, no degree[3]**	**Associate degree[4]**	**Bachelor degree[5]**	**Master degree**	**Professional degree**
Men 2010	24,000	29,000	40,000	46,000	50,000	64,000	81,000	115,000
Female 2010	18,000	21,000	30,000	33,000	38,000	47,000	59,000	77,000

Note: Year-round, full-time workers 25 years and older. (—) = not available.

[1] Includes 1 to 3 years high school for 1990.

[2] Includes 4 years of high school for 1990, and equivalency certificates for the other years.

[3] Includes 1 to 3 years of college and associate degrees for 1990.

[4] Not reported separately for 1990. "or higher" for 2009.

[5] Includes 4 years of college for 1990.

Source: U.S. Dept. of Commerce, Bureau of the Census, Current Population Reports, Series P-60, "Money Income of Households, Families, and Persons in the United States," "Income, Poverty, and Valuation of Noncash Benefits," various years; and Series P-60, "Money Income in the United States," various years. From *Digest of Education Statistics 2005.* For 2009: nces.ed.gov. https://www.infoplease.com/median-annual-income-level-education-1990-2010

Follow-up Questions

1. What is the calculated *r*-value? What does the *r*-value tell you?

2. Explain what the slope of the line of best fit represents?

3. Explain what the y-intercept represents?

4. Using the information that you have found, is there a relationship between the number of years you spend in school and your income? Support your answer.

5. Using the equations of the line of best fit, is there a difference in male and female earning power? Support your answer. If there is a difference between earning power, what real-life factors might contribute to this?

6. What assumptions did you make? How would these assumptions affect the accuracy of your answers?

7. How could the findings from this assignment be applied to real-life? Why are these findings important?

8. Is there another model that would be a better fit for this data?

Appendix I

Unit 3: Teaching Mathematics for Social Justice
Quadratic Modeling: HIV/AIDS in Canada

Developed by Michelle Munk, an adaption from the dissertation entitled *Teaching Mathematics for Social Justice and its Effects on Affluent Students* (Wonnacott, 2011, pp. 111-113)

Background information

The Human Immunodeficiency Virus (HIV) is the virus that causes Acquired Immunodeficiency Syndrome (AIDS). HIV attacks the immune system, resulting in a chronic, progressive illness and leaving infected people vulnerable to opportunistic infections and cancers. The median time from infection to AIDS diagnosis now exceeds 10 years. AIDS is fatal. There is no cure, (Health Canada, Wetherly, p. 100, 2008).

There are over 15,000 people living with HIV/AIDS (PHAs) in Toronto. They represent different socio-economic and ethnic backgrounds, sexual orientations, age groups, and they belong to many different cultural communities within the city.

PHAs may face social and physical challenges that affect their health and well-being. These challenges take many forms: HIV-related stigma and discrimination, for example, tends to isolate PHAs from their communities, while periods of ill-health can make it difficult for them to maintain steady, full-time employment.

The needs of PHAs are increasingly complex. As people live longer with HIV/AIDS, the social and physical challenges they face can become increasingly complicated and difficult to overcome. Social isolation, due not just to the stigma attached to HIV/AIDS but to the gradual passing away of their peers, the side-effects of long-term medication use, and difficult-to-meet nutritional requirements — all of these things can have a negative impact on their well-being. Factors such as gender, sexual orientation, socio-economic status, and cultural background can further complicate the situation. (from the AIDS Committee of Toronto.)

Mathematical Modeling

One important reason for learning mathematical concepts is that sometimes, mathematics can help us to understand and address issues we face in society. For example, studies have shown that the demand for oil is increasing around the world, but the amount of oil available is finite. We can use mathematical concepts to make predictions using the data that we have, in order to determine how the supply and demand will interact and what the broader consequences of this situation will be. This process is called *mathematical modeling*. We use mathematical ideas to approximate the situations we observe, and then we extend the concept to make predictions about the future. We also need to recognize the limits of this process, and to remember that there are always many factors that influence any particular situation.

In this assignment, you will examine data about AIDS in Canada, and use it to make predictions about the future of HIV/AIDS in Canada.

Table 1: Number of HIV cases by year of test (all ages)

Year of Test	Number of Cases Reported to Public Health Agency of Canada (PHAC)
1996	2,729
1997	2,460
1998	2,290
1999	2,184
2000	2,092
2001	2,216
2002	2,460
2003	2,468
2004	2,520
2005	2,476
2006	2,537
2007	2,439
2008	2,620
2009	2,391
2010	2,330
2011	2,290
2012	2,081
2013	2,076
2014	2,044

Retrieved from the HIV and AIDS in Canada: Surveillance Report to December 31, 2014 (n.d.) from https://www.canada.ca/en/public-health/services/publications/diseases-conditions/hiv-aids-canada-surveillance-report-december-31-2014/page-10-data-tables.html#t14

Constructing A Scatter Plot

The table above indicates the number of HIV cases reported to Public Health Agency of Canada (PHAC) from 1996 to 2014. Use the data from the table above, construct a scatter plot. You may use a spreadsheet, statistical software programme, or a graphing calculator to assist you with graphing.

Questions

1. In general, what does *h* represent in the vertex form of a quadratic equation?
2. Specific to this dataset, what does *h* represent?
3. By inspection of your scatter plot, what is a reasonable approximation of *h*?
4. In general, what does *k* represent in the vertex form of a quadratic equation?
5. Specific to this dataset, what does *k* represent?
6. By inspection of your scatter plot, what is a reasonable approximation of *k*?
7. What is the difference in the parabola between a quadratic equation with a positive a value and a quadratic equation with a negative *a* value?
8. What happens if *a* is zero? Discuss in terms of both the equation and the resulting graph.
9. If a quadratic equation has a positive *a* value, will the vertex of the parabola be a maximum or a minimum? Why?
10. How does changing the value of *a* affect the shape of a parabola?
11. What is your best value for *a*?
12. What is your best value for *h*?
13. What is your best value for *k*?
14. Write your equation here in the form: $a(x - h)^2 + k$
15. How are *h* and *k* related?
16. What is the vertex of your graph? What does this mean in terms of this dataset?
17. Print your graph and include it with your assignment.

Analysis (5 Communication marks overall)

1. Describe the trend(s) you observe in the data.
2. Why do you think the data forms a parabola? What factors contribute to this shape?
3. According to your equation for a curve of good fit, how many AIDS cases would there have been in 2017? in 2025?
4. According to your equation, when will there be no more AIDS cases in Canada? Do you think that this is a realistic prediction? Why or why not?
5. Based on the data, can we conclude that the AIDS epidemic in Canada is coming to an end? Explain.
6. Do you think that a parabola is a good model for the situation of AIDS in Canada? Why or why not?
7. Add the data for 2015- 2023 to your graph. How does the data after 2015 compare to the trends you observed for 1996-2014? How does this new data affect your opinion of the accuracy of this mathematical model? Why do you think this later trend is occurring?
8. Write at least two questions you have based on this data (e.g. questions for further research).

Appendix J

Interview One—Follow-up of Lesson One

Participant: _____Date: _____ Time: _____

Previously, you took a questionnaire on (*date*) that asked you questions about your background, the purpose of mathematics, as well as on your preliminary thoughts on what is learning and teaching of mathematics for social justice. This survey aims at gathering additional information about your demographics as well about your feelings and experiences about teaching and learning mathematics for social justice. This survey will ask you to reflect on the three activities with lesson one. Additionally, this survey may ask you to clarify or add on to your written responses from your free write activity or post reflection responses that you shared with your teacher in lesson one.

Personal Background and Learning Style

1. Share with me a little about who you are and where you are from.
2. What do you like to do with your free time?
3. You stated that you have attended, *school's name* for approximately _____ years. Can you tell me about why your parents or you may have selected for you to attend *school's name*?
4. Did any of your parents or siblings previously attended or attend *school's name*?
5. Briefly describe your personality and learning style.
6. What is your favorite class and why?
7. What is your least favorite class and why?

The Learning and Doing of Statistics

8. Tell me about a time in which you enjoyed a mathematics experience in your statistics class.
9. Describe what in what ways do you like to learn statistics in your classroom.
10. Does your learning style differ for statistics than other subjects? Why do you believe this is the case?
11. What do believe your academic strengths and/or weakness in mathematics or statistics?
12. What steps or approaches do you take to learn concepts in statistics?

Learning and Teaching for Social Justice

13. When doing activities in lesson one that investigated racial profiling or driving while Black or Brown (Gutstein & Peterson, 2013; Gutstein, 2006), did you find yourself using those same steps or approaches to complete the problem? If so, explain.

14. On your free write activity for lesson one, you stated that the following mathematics or statistics concepts ***(list concepts)*** may or may not be useful in determining if African American and Latinos/as are stopped, searched, harassed and arrested because they "fit" a racial profile in Chicago (Gutstein & Peterson, 2013; Gutstein, 2006). Can you provide more detail about the use of mathematics or statistics you believe would answer this question?
15. Tell me about what immediate emotions develop as a result of participating in the activity on racial profiling?
16. In your post lesson reflection, you shared that you (were/were not) affected by doing this activity. Can you explain a little more about why this might be the case?
17. Based on your participation in the three activities in lesson one on racial profiling, did you feel complied to want to take action? Why or why not?
18. In using mathematics, did it help to change your initial response to how mathematics can be used to investigate racial profiling?
19. If at all, how did participating in lesson one differ than the previous statistics lesson taught by your teacher? Explain.

Appendix K

TMSJ: Online Exit Survey

Adapted from Wannocott (2011)

Name: _____ Date: _____

Directions: In responding to this survey, you will be asked to revisit some of your previous comments or statements made on your initial survey. Please read each question and/or statement made and determine if at all your response has changed.

Purpose of Mathematics

Questions	**Your Initial Response**	**As you look back at the question and your response, how, if at all has your response changed.**
1. What do you think is the purpose of learning mathematics or statistics?		
2. If you think people should learn mathematics, why should they learn it?		
3. What do you think the purpose of teaching statistics as a mathematics course in high school should be?		

Learning and Teaching Mathematics for Social Justice

Questions	**Your Initial Response**	**As you look back at the question and your response, how, if at all has your response changed.**
1. To your best ability, define teaching mathematics for social justice.		
2. What do you think the effects would be if social justice issue were to be incorporated into a statistics or mathematics curriculum?		

Additional Questions

Questions	**Your Response**
1. After participating in the three social justice lessons, how if at all has your attitude or disposition changed towards learning mathematics/statistics?	
2. Did any of the lessons resonate more with you? If so, which one and why?	
3. If at all, which of the three lessons did you feel motivated you to want to take action, and why?	

Appendix L

MYP Assessment Rubric for Mathematics Criteria

Adapted from Middle Years Programme (MYP) Mathematics Guide (2008)

Achievement Level **MYP**	**Communication in Mathematics** **Criterion C Descriptors**	**Decoder**
0	The student does not reach a standard described by any of the descriptors given below	• There is not enough evidence to give the student a grade on Communication in Mathematics.
1-2	The student shows basic use of mathematical language and/or forms of mathematical representation. The lines of reasoning are difficult to follow.	• The student can show solutions using numbers, symbols, and/or words, although some mistakes may be present. • The student answers parts of the problem, but the solution may be unclear or hard to follow. • The student can sometimes show solutions in more than one form and may make connections between the representations
3-4	The student shows sufficient use of mathematical language and forms of mathematical representation. The lines of reasoning are clear though not always logical or complete. The student moves between different forms of representation with some success.	• The student usually can show solutions using numbers, symbols, and/or words properly. • The student answers most parts of the problem in a way that is often clear, mostly complete, and fairly easy to follow. • The student can usually show solutions in more than one form while making connections between the representations
5-6	The student shows good use of mathematical language and forms of mathematical representation. The lines of reasoning are concise, logical and complete. The student moves effectively between different forms of representation.	• The student can show solutions using numbers, symbols, and/or words properly. • The student answers all parts of the problem in a clear and complete solution that is easy to follow. • The student can show solutions in more than one form while making connections between the representations.

Achievement Level MYP	Reflection in Mathematics Criterion D Descriptor	Decoder
0	The student does not reach a standard described by any of the descriptors given below.	● There is not enough evidence to give the student a grade on Reflection in mathematics.
1-2	The student attempts to explain whether his or her results make sense in the context of the problem. The student attempts to describe the importance of his or her findings in connection to real life.	● The student makes an effort to explain their answer. ● The student can see and connect their results to real life.
3-4	The student correctly but briefly explains whether his or her results make sense in the context of the problem and describes the importance of his or her findings in connection to real life. The student attempts to justify the degree of accuracy of his or her results where appropriate.	● The student can look back at their answer and tries to explain if their answer is reasonable. ● The student can see and connect their results to real life. ● The student attempts to justify or give some reasons why their level of accuracy is appropriate.
5-6	The student critically explains whether his or her results make sense in the context of the problem and provides a detailed explanation of the importance of his or her findings in connection to real life. The student justifies the degree of accuracy of his or her results where appropriate. The student suggests improvements to the method when necessary.	● The student can look back at their answer and explain if their answer is reasonable. ● The student can see and explain in detail how their results are important to real life. ● The student can justify or give reasons why their level of accuracy is appropriate. ● The student can suggest improvements to the way they solved the problem when necessary.

Achievement Level MYP	Knowledge and Understanding Criterion A Descriptor	Decoder
0	The student does not reach a standard described by any of the descriptors given below.	● There is not enough evidence to give the student a grade on Knowledge of Understanding.
1-2	The student attempts to make deductions when solving simple problems in familiar contexts.	● When solving a problem, students include some of the information to show that they understand at least part of the problem. ● The answer to the problem is not completely correct ● Correct use of math vocabulary and symbols is limited and sometimes incorrect. ● The problem given is easier and similar to those that have been seen in class.
3-4	The student sometimes makes appropriate deductions when solving simple and more complex problems in familiar contexts.	● When solving a problem, students include some of the information to show that they understand at least part of the problem. ● Sometimes answers are correct and other times only part of the answer is correct. ● Correct use of math vocabulary and symbols is limited. ● The problem given is more difficult but still similar to those that have been seen in class.
5-6	The student generally makes appropriate deductions when solving challenging problems in a variety of familiar contexts.	● When solving a problem, students include most or all of the information to show that they understand the problem. ● Usually the answer to the problem is correct. ● Math vocabulary and symbols are used correctly. ● The problem given is more difficult but still similar to those that have been seen in class.
7-8	The student consistently makes appropriate deductions when solving challenging problems in a variety of contexts including unfamiliar situations.	● When solving a problem, students include all needed information to show that they completely understand the problem. ● The answer to the problem is almost always correct. Math vocabulary and symbols are used correctly. ● The problem given is more difficult and is different than those seen before in class.

Appendix M

Definitions of Terms and Acronyms

- **Case Study** - "A case study is an empirical inquiry that: (a) investigates a contemporary phenomenon (the "case") in depth and within its real-world context, especially when (b) the boundaries between phenomenon and context may not be clearly evident" (Yin, 2009, p. 18).

- **Cultural Competence** - "The ability to function effectively in one's culture of origin" (Ladson-Billings, 2000, p. 210).

- **Culturally Relevant Pedagogy** - "Theoretical model that not only addresses student achievement but also helps students to accept and affirm their cultural identity while developing critical perspectives that challenge inequities that schools (and other institutions) perpetuate" (p. 469).

- **Culture** - A "group's individual and collective ways of believing, thinking and knowing, which include shared experiences, skills, consciousness, values, forms of expression, behavior and social institution" (Tillman, 2002, p. 4).

- **Global Futures** - The study of alternative futures of human and planetary life" (Rogers, 1998, p. 203).

- **Mathematical Anxiety** - Students' psychological reactions including feelings of nervousness, stress and helplessness when dealing with mathematics (Fennema & Sherman, 1976; OECD, 2013b)

- **Mathematical Education** - A field of study which investigates "the teaching of mathematics at all levels, including its premises, goals and [the] societal environment" (Wittmann, 1998, p. 87).

- **Mathematical Power** – "Mathematical power exists when students confidently engage in complex mathematical tasks...draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress...are flexible and resourceful problem solvers...work productively and reflectively...communicate their ideas and results effectively...value mathematics and engage actively in learning it." (NCTM, 2000, p. 3)

- **Motivation** - The attitudinal indicators that direct a person's certain behaviors (Elliot & Covington, 2001).

- **Reading the World with Mathematics –** "To use mathematics to understand relation of power, resource inequities, and disparate opportunities, between different social groups and to understand explicit discrimination based on race, class, gender, language and other differences. Further, it means to dissect and deconstruct media and other forms of representation. It means to use mathematics to examine these various phenomena both in one's immediate life and in the broader social world and to identify relationships and make connections between them." (Gutstein, 2003c, p. 45)

- **Writing the World with Mathematics –** "Using mathematics to change the world or to see oneself capable of making change through a means of developing social agency" (Gutstein, 2006, p. 27).

Acronyms

- International Baccalaureate - IB
- Middle Years Programme Mathematics guide - MYP
- National Council of Teachers for Mathematics – NCTM
- Read and Write the World with Mathematics - RWWM
- Teaching Mathematics for Social Justice – TMSJ
- Science, Technology, Engineering and Mathematics – STEM