Essays on Information Economics

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ABSTRACT

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In this doctoral dissertation, I broadly study the impact of information on economies from both a theoretical and an empirical perspective. Specifically, I study how strategic agents in a heterogeneous interacting network make decisions under incomplete information and how their actions are affected by the parameters that define the incompleteness of the information, with an emphasis on the social value of information. I then estimate the impact of information disclosure on the stock market by studying the specific example of the annual CCAR and DFAST bank stress tests conducted by the Federal Reserve. This dissertation consists of two chapters.

In the first chapter, I study a game of heterogeneous strategic interactions under incomplete information. I characterize the equilibrium actions and compare them to the benchmark constrained-efficient allocation. I parameterize the available information in terms of pairwise information commonality and accuracy and study how changing the said commonality and accuracy affects the social welfare. I also study how the structure of interactions between players affects the social value of information. I find that the extent of the inefficiency of the economy dictates the social value of information. I provide a complete characterization of the comparative statics of the social welfare with respect to commonality and accuracy for completely efficient economies. I find that when interactions are heterogenous, it is possible for social welfare to be non-monotonic with respect to information commonality, a behavior unseen in economies with homogeneous interactions. For inefficient economies, I provide sufficient conditions under which the social welfare exhibits monotonic behavior.

In the second chapter, I study the predictability of the results of the annual Comprehensive Capital Analysis and Review (CCAR) and Dodd-Frank Act Stress Test (DFAST) conducted by the Federal Reserve. I find that these results are highly predictable on year-to-year basis. I also find a high degree of predictability within the adverse scenario and the severely adverse scenario results within a given year. I find that that these predictable trends hold over time, from 2012 to 2020. I also try to ascertain the impact of the announcement of these results on the stock market and find no statistically significant effect. Lastly, I study the fixed effect impact of the disclosure events on the stock and options market. I find that while there are individual instances of significant impact, there is no significant impact across the years. I discuss potential implications of these patterns for the further development and application of stress testing.

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Introduction

It is by now conventional wisdom that public communications and announcements are among the key instruments in policymakers' toolbox. They are used by Central Banks around the world as an instrument for the conduct of monetary policy. The most notable example is ECB President Mario Draghi's famous "whatever it takes" speech in 2012 (Randow and Speciale [50]), which resulted in a reduction in the bond yield spreads and borrowing costs of distressed economies in the eurozone (Wanke [54]). More recently, in 2019, the Federal Reserve Chairman Jerome Powell stated that the Federal Reserve is "conducting a public review of [its] monetary policy strategy, tools, and *communications*." (emphasis added) and that "we are looking at how we might improve the communication of our policy framework." (Powell [38]).

The importance of such public announcements, however, is not limited to central bankers or even policymakers. Many firms understand that their earnings announcements can have a significant effect on their stock prices. In fact, as documented by Johnson et al. [40], firms typically manage public expectations before their earnings announcements. For example, in Q2 2016, the investor-relations department at Citigroup referred Wall Street analysts to "comments made by Chief Executive Michael Corbat at a June 2nd investor conference . . . [that] the bank's second quarter profits were likely to be 'roughly flat' compared with the first quarter when Citigroup earned \$1.10 a share." (Gryta et al. [32]). On July 15th 2016, Citigroup's shares rose by 2% despite a 14% decline in year-over-year earnings because reported earnings per share was \$1.24, greater than the market forecast of \$1.10 (Johnson et al. [40]).

These examples highlight the importance of public announcements for economic outcomes. The fact that such announcements are public is in fact central to their ability to shape expectations and impact market outcomes. Public announcements affect the market by not only providing new information but also common information to the market.

In this thesis, I explore the role of public information in two contexts – one theoretical and one empirical. First, I study how public information affects social welfare in the presence of heterogeneous coordination motives. Second, I study the annual DFAST and CCAR bank stress tests conducted by the Federal Reserve and assess the impact of the disclosure of their results on the stock market.

In chapter one, I present a theoretical model to study the social value of information in the presence of heterogeneous interactions among strategic agents under incomplete information. When agents coordinate under incomplete information, they are subject to information frictions that prevent them from coordinating efficiently. There are two kinds of information frictions that could potentially arise – incompleteness frictions and heterogeneity frictions. Incompleteness frictions arise due to the fact that players do not know the true state of the economy accurately. Heterogeneity frictions arise when agents have different sources of (incomplete) information. Note that in an economy that is subject to incompleteness frictions but not heterogeneity frictions, agents can still arrive at a consensus about the state of the world, albeit an inaccurate one. Adding heterogeneity frictions further weakens the agents' ability to coordinate. In such a scenario, any public information will be used by agents to form higher-order expectations, leading to imperfect coordination. As such, notions of commonality and accuracy of information become important in determining the equilibrium outcomes in the economy. The commonality of information addresses the ability of agents to arrive at a consensus and the accuracy of the information addresses the discrepancy between the achieved consensus and the true state of the economy. I lay out these notions explicitly in chapter one.

I explore the questions of coordination and social value of information using a framework in which a finite group of decision makers interact with one another in the presence of strategic interactions that are potentially asymmetric both in degree and nature. Specifically, I focus on the class of standard quadratic-payoff games, while allowing for an arbitrary pattern of strategic complementarities and/or substitutabilities between pairs of agents. I explicitly characterize the equilibrium actions and compare them to the benchmark of the constrained-efficient allocation. The constrained-efficient allocation may differ from the equilbrium actions because the social planner accounts for the direct payoff externalities between players that players do not account for in equilibrium.

The presence of information frictions within the economy leads to social welfare losses, which depend on the structure and nature of interactions between the players. The social value of information depends on the extent of the inefficiency of the equilibrium. Indeed, in chapter one, I provide a complete characterization of the behavior of the social welfare in efficient economies with respect to the underlying commonality and accuracy of the information structure. When all players' interactions are complementary, increasing information commonality necessarily increases the social welfare. However, when there are substitutive interactions within the economy, because even-numbered higher-order interactions are complementary, it can potentially result in a net complementary effect. Indeed, I find that when payoff interactions between agents are heterogeneous, social welfare can potentially be non-monotonic in the commonality in agents' information sets. This happens only when payoff interactions are heterogeneous. I conclude by studying the social value of information in inefficient economies and provide sufficient conditions for monotonic behavior of social welfare with respect to information commonality and accuracy.

In chapter two, I try to quantify the impact of the announcement of the annual stress tests, conducted by the Fed, on the stock market. In the aftermath of the 2008 financial crisis, the Federal Reserve started conducting regulatory stress tests on big banks to determine their capital levels as part of the Dodd-Frank Wall Street Reform and Consumer Protection Act. It started in 2009 with the Supervisory Capital Assessment Program (SCAP). As part of the SCAP, the loan portfolios of qualifying banks were subjected to a hypothetical stress scenario and the resulting losses and loss rates were made public. Over the years, the SCAP evolved into the Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Frank Act Stress Test (DFAST). As part of the DFAST, the Fed released loss values and rates of loan portfolios for qualifying banks under two separate scenarios – the adverse scenario and the severely adverse scenario. I study the predictability of these reported loss values and loss rates. I find that both losses and loss rates are highly predictable within the two scenarios reported, both across banks and across loan categories. This is very surprising because the defined scenarios are roughly 250-dimensional objects, yet the predictability between the severely adverse and the adverse results effectively reduces the stress severity to one dimension and suggests a missed opportunity to diversify the types of stresses tested. I also evaluate the predictability of these reported losses and loss rates across time. Once again, I find that losses reported the previous year are very highly predictive of the losses reported in the current year across all banks and loan categories. This calls into question the usefulness and the informativeness of these results. In order to study whether the stress tests provide new information to the stock market, I perform a regression to see if there is a tangible correlation between the announced loss rates and the unexpected return of the respective bank on the day after the announcement. I find that there is statistically significant correlation between the two. I also repeat this exercise with the unexpected loss rate (computed from the two previous years' worth data) and the unexpected return for the bank on the day after the announcement. Once again, I find no statistically significant correlation between the two. This is consistent with the finding above that the announced stress test results are very predictable.

Finally, I evaluate the fixed effect impact of the announcement itself as an event on the stock and option market. I find significant effects in some years, but no significant effects when data is pooled across years, suggesting that there is no overall effect of the announcement. The presence of statistically significant effects is somewhat surprising because the date of the announcement of the stress results is known well in advance. This suggests that the activity on the day of the announcement is driven mostly by speculation.

Chapter 1

Network Interactions and the Social Value of Information

1.1 Introduction

Many economic interactions feature coordination and miscoordination motives in the form strategic complementarities and substitutabilities. Consider, for example, the classical example of bank runs discussed in Diamond and Dybvig [27]: if a depositor believes that other depositors will likely run on the bank that they have commonly invested in, she has stronger incentives to run on the bank herself. Depositors' actions in such a scenario—whether to run or not—are strategic complements. Similarly, in a market where various producers compete a la Cournot, if a producer believes that its competitors would increase production, she has an incentive to decrease her own output. In this case, producers' actions are strategic substitutes.

As is evident from the above examples, an agent's optimal actions in an environment that exhibits such strategic interactions are highly dependent not only on her expectations of the economy's underlying fundamentals—say, the solvency of the bank in the first example or consumer demand in the second—but also on her expectations of the decisions of other agents' actions. As a result, any uncertainty about the fundamental and/or expectations of other agents can play a first-order role in determining economic outcomes.[1]

[1]This insight has a long history, going back to Keynes [43], who famously described investments in the stock market to a fictional beauty contest game in which participants are supposed to choose six prettiest pictures out of a hundred from a newspaper. Those who choose the most popular pictures will be given a reward. Explaining the reasoning behind the choice made by a rational participant, Keynes writes: "It is not a case of choosing those that, to the best of one's judgment,

In this chapter, I study how information frictions surrounding the state of the economy affect the social value of information, specifically in the presence of heterogeneous strategic interactions. I generalize the framework of Angeletos and Pavan (2007) [17] and characterize how heterogeneity in the nature and extent of strategic interactions shape the behavior of individual agents within the economy and determine the private and social values of information. My results indicate that benchmark homogeneous-interactions models such as Angeletos and Pavan (2007) [17] may mask key insights on how strategic complementarities and substitutabilities shape the social value of information.

I explore these questions using a framework in which a finite group of decision makers interact with one another in the presence of strategic interactions that are potentially asymmetric both in degree and nature. Specifically, I focus on the class of standard quadratic-payoff games, while allowing for an arbitrary pattern of complementarities and/or substitutabilities between pairs of agents. As is standard in the literature, I capture these asymmetric patterns of interactions by a directed, weighted network, which I refer to as the economy's *interaction network*. While reduced-form, this framework is general enough to nest various types of economic interactions, such as peer effects Ballester et al. [18], competition between firms that interact with one another in various, potentially overlapping markets Myatt and Wallace (2015) [47], and price-setting behavior in production networks La'O and Tahbaz-Salehi [45].

To study the private and social values of information, I endow the above-mentioned framework with incomplete information. More specifically, I assume that agents' payoffs are affected by an unknown state—what I refer to as the economy's *fundamental*—and that each agent receives a private signal that is potentially informative

are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees."

about both the fundamental and other agents' signals. Thus, an agent's signal not only helps her form an imperfect estimate for the economy's underlying state, but also informs her about the signals of all the other agents, which in turn allows her to form expectations about their equilibrium actions. To capture the dual role of signals in my framework, I parameterize the economy's information structure using two sets of parameters. A first set of parameters, which I refer to signals' *accuracies*, capture the correlation of each agent's signal with the fundamental. The second set of parameters, which I refer to as signals' *commonalities*, represent the pairwise correlations between agents' signals.

Before describing the results, I note that this simple framework exhibits two distinct types of information frictions: (i) information frictions that arise due to incompleteness of information about the fundamental (as measured by signals' accuracies), and (ii) information frictions that arise due to heterogeneity of information and imperfect correlations between agents' signals (as measured by signals' commonalities). The former friction prevents agents from precisely estimating the value of the underlying fundamental, whereas the latter friction prevents them from arriving at a consensus estimate for it. Notably, both types of information frictions can result in welfare losses. My results focus on the interplay between the two welfare losses and study how changing the information structure in terms of the underlying accuracy and commonality affects the social welfare.

I start the analysis in Section 1.3 by characterizing the equilibrium and (constrained) efficient outcomes in terms of model primitives—namely, the economy's interaction network and the accuracy and commonality of agents' signals. As in Bergemann et al. [19], I find that equilibrium actions depend on the Hadamard product of the economy's interaction matrix and the matrix of pairwise signal commonalities. This characterization captures the fact that the extent to which a pair of agents try to coordinate with one another in equilibrium depends not only on the intensity of their interactions but also on the extent to which their signals are informative about each other's signals. I also obtain a similar characterization for the constrained-efficient outcome.

With these preliminary results in hand, I then characterize whether agents over- or under-react to their private signals in equilibrium (compared to the constrained efficient benchmark). Not surprisingly, I find that the extent of agents' over- and under-reaction to their private signals depends not only on the detailed nature of the economy's interaction network, but also on the underlying information structure. In particular, I find that it is possible for players with a fixed set of payoffs to over- and under-react in equilibrium (again, as benchmarked against the constrained-efficient solution) depending on the economy's information structure. Importantly, such a phenomenon is absent in economies with symmetric coordination motives, such as Angeletos and Pavan (2007) [17], who show that whether agents over- or under-react to their private signals only depends on the economy's payoff structure.

I then turn to the study of the social value of information as a function of the economy's information structure. Specifically, I provide a series of comparative static results for equilibrium welfare in terms of changes in signals' commonalities and accuracies. As my first main result, I find that if the economy is efficient under all information structures, social welfare is always increasing in signals' accuracy. This result follows from two observations. First, all else equal, more accurate signals enable agents to better match their actions with the underlying fundamental, thus reducing the friction due to imperfect signals. Second, when the economy is efficient under all information structures, the private value of information coincides with the social value of information, and hence more accuracy translates into higher equilibrium social welfare.

The comparative statics of welfare with respect to signal commonalities is less straightforward, even when the economy is efficient under all information structures.

I find that social welfare is increasing in signal commonalities if the "net degree of strategic complementarities", defined as the sum of all pairwise parameters of strategic complementarities, is positive. In other words, if pairwise actions over the network are on average more strategically complementary than substitutable, then more common signals translate into higher equilibrium welfare. Importantly what matters in this condition are only the pairwise direct interactions over the network: indirect interactions over the network do not appear in this condition. At the same time, I find that social welfare is decreasing in the commonalities if a weighted average of pairwise interactions terms is negative, where the weights are given by agents' centralities in the interaction network. Unlike the former condition, the dependence on the centralities in the latter condition means that whether welfare decreases in commonality of signals depends on both direct and indirect interactions over the network. The existence of a wedge between the two conditions also highlights that it may be possible for welfare to not have a monotonic behavior in signal commonalities. Such non-monotonic behavior, which is absent in the symmetric model of Angeletos and Pavan (2007) [17], is a consequence of the heterogeneity in interactions.

I then study the social value of information in economies that are inefficient under incomplete information. I find that the behavior of equilibrium social welfare as a function of the accuracy and commonality of signals depends on the extent and nature of the discrepancy between the equilibrium and constrained-efficient interaction matrices. I provide sufficient conditions under which the equilibrium social welfare is monotonic with respect to information commonality and accuracy. I also consider a special class of economies called *regular* economies in which I normalize the overall player interaction across all players. I provide results for regular economies that generalize the findings of Angeletos and Pavan (2007) [17].

Taken together, my results indicate that the social value of information depends on the intricate detail of how various agents interact with one another.

Related Literature: The work discussed in this chapter belongs to the literature that studies the social value of information in the presence of strategic interactions. One of the earliest works in this area is by Morris and Shin (2002) [46], who show that more precise public information may reduce social welfare when agents' actions are strategic complements. This result is sometimes interpreted as suggesting that central banks should exercise caution in how much information they reveal to the market Svensson [53]. On the other hand, Angeletos and Pavan (2004) [16] and Hellwig [34] find the seemingly contradictory result that more accurate public information is necessarily beneficial to social welfare. These various results are integrated in a unifying framework by Angeletos and Pavan (2007) [17], who provide a characterization of the social value of information in the class of quadratic-payoff games. They find that whether more precise public information increases or decreases social welfare is tightly linked to the wedge between the equilibrium and social values of coordination: if the social value of coordination exceeds its equilibrium value (and when actions are strategic complements), an increase in the quality of public information necessarily increases social welfare.

I contribute to this literature by extending the framework of Angeletos and Pavan (2007) [17] to economies with heterogenous strategic interactions. In the presence of such heterogeneities, the social and equilibrium values of coordination can no longer be represented by a pair of scalars. Rather, the detailed nature of pairwise interactions between agents would play a first-order role in determining the social value of information. More importantly, I also find that whether more commonality in agents' signals increases or decreases welfare depends not only on agents' payoffs (as is the case in the homogeneous-interaction economy of Angeletos and Pavan (2007) [17]), but also on the economy's information structure.

This work is also related to the broader literature on network games. Papers such as Ballester et al. [18], Bramouelle et al. [24], and Allouch [13] (among many others) study how the structure of the network of economic interactions shape economic outcomes.[2] With a few notable exceptions,[3] this literature mostly focuses on network games of complete information. In contrast, I am primarily interested in how the interaction network and the economy's information structure together determine equilibrium and efficient outcomes. Within this literature, my work is most closely related to Bergemann et al. [19], who, like me, study an incomplete-information network game with quadratic payoffs. I build on their work by providing comparative static results on how such network interactions determine the degree of equilibrium over- and under-reaction to private signals, as well as the social value of information.

While I consider a reduced-form framework, my model is general enough to nest a host of different micro-founded models in various contexts that study the value of information in the presence of strategic complementarities and substitutabilities. For example, the differentiated-product Cournot model of Myatt and Wallace (2015) [47] can be cast as a special case of my framework. Similarly, general equilibrium macro models such as Angeletos et al. [14], Angeletos and La'O [15], and La'O and Tahbaz-Salehi [45], which model nominal rigidities as an informational constraint on the firms' price-setting behavior, have reduced-form representations that coincide with that of my model. As a result, the insights developed in this chapter on the social value of information can be applied to these applications.

Outline of the Chapter: The rest of the chapter is organized as follows. In Section 1.2, I provide a description of the model. In Section 1.3, I characterize the equilibrium and constrained efficient solutions in terms of the model primitives and provide comparative static results with respect to the information structure. In Section 1.4, I study the social value of information. All proofs and technical details are presented as part of the appendix.

[2] See Jackson and Zenou [39] and Bramouelle and Kranton [23] for a survey of this literature.

[3] See Calvo-Armengol and Marti Beltran [25], Galeotti et al. [29], and Bergemann et al. [19].

1.2 Framework

1.2.1 Payoffs

Consider an economy consisting of n agents denoted by $N = \{1, 2, \ldots, n\}$. The payoff of agent i is given by a quadratic function that depends on the vector of actions $a = (a_1, \ldots, a_n) \in \mathbb{R}^n$ and a payoff state $\theta \in \mathbb{R}$, which I refer to as the economy's *fundamental*. More specifically, I assume that agent i's payoff is given by

$$u_i(a, \theta) = a_i\theta - \frac{1}{2}a_i^2 + a_i \sum_{j \neq i} q_{ij} a_j + \sum_{j,k \neq i} a_j a_k v_{jk}^{(i)} \tag{1.1}$$

where a_i denotes the action of agent i. Strategic interactions between agents $i \neq j$ are captured by parameter q_{ij}. More specifically, $q_{ij} > 0$ if i's and j's actions are strategic complements, $q_{ij} < 0$ if their actions are strategic substitutes, and $q_{ij} = 0$ if there are no direct strategic interactions between i and j. I summarize these coefficients by matrix $\mathbf{Q} = [q_{ij}]$, with the convention that $q_{ii} = 0$. I refer to $\mathbf{Q}$ as the economy's *interaction matrix*. As is standard in the literature, strategic interactions between various agents can alternatively be represented by a weighted and directed graph on n vertices. Each vertex in this graph—which I refer to as the economy's *interaction network*—corresponds to an agent and the directed edge from vertex j to vertex i has weight q_{ij}.

In addition to the strategic interactions captured by matrix $\mathbf{Q}$, agents exert direct payoff externalities on one another. The payoff externalities exerted on agent i are captured by the last term on the right-hand side of equation (1.1) and are parameterized by the $n \times n$ matrix $\mathbf{V}^{(i)} = [v_{jk}^{(i)}]$, with the convention that all elements on the i-th rows and columns of $\mathbf{V}^{(i)}$ are equal to zero (i.e., $v_{ij}^{(i)} = v_{ji}^{(i)} = 0$ for all j). I also assume that $v_{jj}^{(i)} = 0$ for all i. Note that, unlike the strategic interactions parameterized by $\mathbf{Q}$, these externalities do not affect i's marginal value of actions, and as a result do not impact equilibrium outcomes.

1.2.2 Information Structure

The fundamental θ is unknown to the agents and is drawn by nature according to a normal distribution with mean 0 and standard deviation σ. While agent i cannot observe the realization of the fundamental, she receives a private signal s_i that is potentially informative about θ. More specifically, I assume that the profile of signals $s = (s_1, \dots, s_n)$ and the fundamental are jointly normally distributed according to

$$\begin{bmatrix} \theta \\ \\ s \end{bmatrix} \sim \mathcal{N}\left(0, \sigma^2 \begin{bmatrix} 1 & \rho' \\ & \\ \rho & \mathbf{R} \end{bmatrix}\right), \tag{1.2}$$

where $\mathbf{R}$ denotes the correlation matrix between agents' private signals (with a typical element $r_{ij} = \text{corr}(s_i, s_j) \geq 0$) and ρ is the vector of pairwise correlations between the fundamental and private signals (with a typical element $\rho_i = \text{corr}(\theta, s_i) \geq 0$).

A few remarks are in order. First, note that I am assuming that all private signals and the fundamental have (unconditional) means that are equal to zero. This is a simple normalization with no bearing on my subsequent results.[4] Second, the formulation in (1.2) also assumes that all signals and the fundamental have identical variances. While somewhat restrictive, this assumption allows us to define the notions of accuracy and commonality of private signals in a transparent manner. More importantly, however, equation (1.2) implies that agent i's private signal is potentially informative about both the fundamental and other agents' private signals.

Indeed, it is easy to see that $\rho_i = \text{corr}(\theta, s_i)$ parameterizes the extent to which agent i's private signal is informative about the fundamental: given her private signal, i's uncertainty about the realization of the fundamental is given by $\text{var}_i(\theta) = (1 - \rho_i^2)\sigma^2$, where $\text{var}_i(\cdot) = \text{var}(\cdot|s_i)$. Thus, following the terminology of [17], I refer to ρ_i

[4] To be more specific, when the fundamental or the signals have non-zero means, the equilibrium and team efficient actions are shifted by constants that only depend on the interaction matrix $\mathbf{Q}$ but with no dependency on either $\mathbf{R}$ or ρ. Therefore, as long as I redefine the notion of equilibrium over- and under-reaction in terms of the weights placed on private signals, all my comparative static results with respect to $\mathbf{R}$ and ρ in Sections 1.3 and 1.4 remain unchanged.

as the *accuracy* of i's private signal. In the extreme case that $\rho_i = 0$, agent i's signal is completely uninformative about the underlying state, whereas when $\rho_i = 1$, agent i's signal fully reveals the fundamental (in fact, $s_i = \theta$). Similarly, since all private signals have the same standard deviation, r_{ij} parameterizes the extent to which agent i's private signal is informative about j's. More specifically, $\text{var}_i(s_j) = (1 - r_{ij}^2)\sigma^2$. Thus, in the remainder of the chapter, I refer to r_{ij} is the *commonality* of i and j's information sets.

Note that the above framework exhibits two types of information frictions. First, whenever agents have less than perfect private signals, they can only imperfectly match their actions to the realization of the fundamental θ. The vector of accuracies $\rho = (\rho_1, \ldots, \rho_n)$ thus parameterizes the extent of this information friction in the economy. Second, the fact that agents have asymmetric information means that they are also incapable of perfectly coordinating their actions with one another, despite the fact that they have an incentive to do so (whenever $\mathbf{Q} \neq 0$). This second type of information friction is parameterized by matrix of commonalities, $\mathbf{R}$. Note that the friction due to imperfect signals can be present even when there is no information asymmetry between the agents.

1.3 Equilibrium and Efficient Use of Information

With the framework in Section 1.2 in hand, in this section I study the equilibrium and efficient use of information. To this end, I first characterize the equilibrium and (constrained) efficient allocations in terms of model primitives. I then provide a series of comparative static results with respect to the commonality and accuracy parameters of the information structure, i.e., $\mathbf{R}$ and ρ, respectively.

1.3.1 Equilibrium Characterization

I start by defining the equilibrium. For the rest of the chapter, I use the notation $\mathbb{E}_i(\cdot) = \mathbb{E}(\cdot|s_i)$. Recall that each agent i has access to a potentially informative private signal s_i about the underlying state. Therefore, the Bayesian-Nash equilibrium of this game is defined in the usual manner:

Definition 1.1. *The equilibrium is a mapping $a : \mathbb{R}^n \to \mathbb{R}^n$ such that $a_i(s_i) \in \arg\max_{a'_i} \mathbb{E}_i[u_i\,(a'_i, a_{-i}, \theta)]$ for all i. Furthermore, $a : \mathbb{R}^n \to \mathbb{R}^n$ is a linear equilibrium if the equilibrium mapping $a_i(s_i)$ is an affine function of s_i for all i.*

To ensure the existence and uniqueness of a linear equilibrium, I impose the following standard assumption on the nature and extent of strategic interactions between agents.

Assumption 1.1. $\rho(|\mathbf{Q}|) < 1$*, where* ρ *denotes the spectral radius and* $|\mathbf{Q}|$ *denotes the element-wise absolute value of interaction matrix* $\mathbf{Q}$*.*

My first result characterizes the economy's unique equilibrium in terms of model primitives.

Proposition 1.1. *If Assumption 1.1 is satisfied, there exists a unique equilibrium in linear strategies of the form $a_i = \alpha_i s_i$ for all i, where*

$$\alpha = (\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1}\rho, \tag{1.3}$$

$\mathbf{Q}$ *is the economy's interaction matrix, and $\circ$ denotes the Hadamard (element-wise) product.*

The above proposition, which is in line with the results of [19], illustrates how the economy's interaction matrix and information structure shape equilibrium outcomes. In particular, it illustrates that the weight that each agent assigns to her private signal depends on the Hadamard product of the interaction matrix $\mathbf{Q}$ and the correlation matrix of agents' signals, $\mathbf{R}$.

To see the intuition underlying the above result, it is instructive to first consider the special case in which all agents have complete information about the fundamental. Recall that, under complete information, all agents have perfectly accurate signals about the fundamental and hence each others' signals, in particular, $s_i = \theta$ for all i. As a result, $\rho_i = r_{ij} = 1$. I therefore have the following corollary to Proposition 1.1:

Corollary 1.1. *Under complete information, the unique equilibrium is given by $a_i = \alpha_i \theta$ for all i, where*

$$\alpha = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1} \tag{1.4}$$

and $\mathbf{Q}$ is the matrix of relative interactions.

This corollary illustrates the well-known result that under complete information, equilibrium actions depend on the *Leontief inverse* matrix $(\mathbf{I} - \mathbf{Q})^{-1}$ corresponding to the economy's interaction matrix, $\mathbf{Q}$. This dependence captures the fact that, in equilibrium, agent i's action depends not only on the direct strategic interactions with her neighbors, but also on all indirect interactions over the network. To see this more explicitly, note that Assumption 1.1 guarantees that the Leontief inverse has an infinite geometric sum representation of the form $(\mathbf{I} - \mathbf{Q})^{-1} = \sum_{k=0}^{\infty} \mathbf{Q}^k$, in which $\mathbf{Q}^k$ captures indirect strategic interactions of order k over the network (see Stewart [52, p. 55]). Therefore, the equilibrium action of agent i is proportional to the sum of all direct and indirect strategic interactions over the network.

Another important observation is that equation (1.4) reduces to the well-known result of Ballester et al. [18], according to which the equilibrium action of agent i under complete information is proportional to her *Bonacich centrality* in the interaction network, defined as the column sums of the economy's Leontief inverse: $b = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1}$.

Contrasting equation (1.4) to equation (1.3) illustrates that once I introduce incomplete information, all direct and indirect strategic interactions are modulated by pairwise commonalities between agents' signals. In particular, in the special case that $\rho_i = \rho$ for all agents i, the vector of equilibrium weights α under incomplete information is proportional to the vector of equilibrium weights in an alternative complete-information economy, in which the strength of interaction between agents i and j is replaced by $q_{ij}r_{ij}$. These "modified interactions" reflect the fact that, under incomplete information, agent i's private signal not only informs her about the fundamental, but also about the private signals (and hence the actions) of other agents in the economy. Therefore, whether agent i assigns a higher weight on her private signal in response to stronger strategic compelementarties depends on the extent to which i can access whether j would raise her equilibrium action. In the extreme case that i's private signal is uninformative about other agents' private signals (i.e., when $r_{ij} = 0$ for all $j \neq i$), equation (1.3) implies that α_i is independent of the extent of strategic interactions in the economy. This is exactly because, irrespective of the value of q_{ij}, agent i's signal does not provide her with any information about other agents' equilibrium actions.

I clarify this mechanism using a simple example.

Example 1. Consider an economy consisting $n = 3$ agents and with interaction matrix

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 \\ q_{21} & 0 & 0 \\ 0 & q_{32} & 0 \end{bmatrix}, \tag{1.5}$$

where I assume that $q_{21}, q_{32} > 0$. Figure 1.1 depicts the corresponding interaction network, where an edge from vertex i to vertex j indicates that agent j views the action of agent i as strategic complement to her own. Also suppose $\mathbf{V}^{(i)} = 0$ for all agents $i \in \{1, 2, 3\}$, so that there are no pure externalities in the economy. A simple application of Proposition 1.1 implies that the weights agents assign to their private signals in equilibrium are given by

$$\alpha_1 = \rho_1$$
$$\alpha_2 = \rho_2 + (q_{21}r_{21})\rho_1$$
$$\alpha_3 = \rho_3 + (q_{32}r_{32})\rho_2 + (q_{32}r_{32})(q_{21}r_{21})\rho_1.$$

To interpret these equations, first consider agent 1. Since this agent does not face any strategic considerations, the weight she assigns to her private signal is simply equal to the accuracy of that signal (ρ_1): the more accurate her signal, the more responsive she is to that signal. An identical mechanism implies that similar terms also appear in the expressions for α_2 and α_3: a more accurate signal (i.e., a higher ρ_i) enables each agent i to better match her action to the underlying state.

The remainder of the terms in the expressions for α_2 and α_3 reflect the fact that agents 2 and 3 face strategic considerations. Take the expression for α_2. The fact that $q_{21} > 0$ means that agent 2 has an incentive to raise her action in response to an increase in 1's action. However, whether this strategic consideration translates into a higher weight on 2's private signal depends on (i) the extent to which 2's private signal is informative about 1's signal and (ii) the accuracy of 1's signal ρ_1, which determines 1's equilibrium action. This is why the dependence of α_2 on q_{21} is downweighted by the product $r_{21}\rho_1$.

The logic behind the expression for α_3 is similar, with indirect strategic interactions between agents 1 and 3 resulting in a third term. Importantly, note that the weight agent 3 assigns to her private signal does not depend on the commonality between the information sets of agents 1 and 3: α_3 is independent of r_{13}. This is because a change in agent 1's action impacts 3's marginal benefit of raising her action only to the extent that it results in a change in 2's action. Therefore, what matters is the informational commonality between agents 1 and 2 (which determines 2's equilibrium

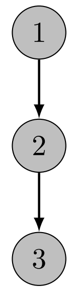


Figure 1.1: A simple economy consisting of 3 agents

action) and between agents 2 and 3 (which determines 3's best response to 2's action).

The next result, which is an immediate corollary to Proposition 1.1, formalizes the idea that commonality between agents matters only to the extent that they face direct strategic interactions over the network.

Corollary 1.2. *Suppose there are no direct strategic interactions between agents i and j, i.e., $q_{ij} = q_{ji} = 0$. Then, the equilibrium is independent of the commonality between i and j's signals, r_{ij}.*

I conclude with two simple comparative static results on how equilibrium actions respond to the information structure.

Corollary 1.3. *Suppose all pairwise actions are strategic complements, i.e., $q_{ij} \geq 0$ for all $i \neq j$. Then,*

(a) the weights agents assign to their private signals are increasing in all signal accuracies;

(b) the weights agents assign to their private signals are increasing in the extent of strategic complementarities;

(c) the weights agents assign to their private signals are increasing in all pairwise commonalities.

Statement (a) is a fairly straightforward result: all else equal, a more accurate signal about the fundamental translates into higher equilibrium weights on that signal. Statement (b) mirrors standard results for the benchmark of network games of complete information: an increase in the extent of strategic complementarities increases the coordination motive between agents and their neighbors, and hence, raises their incentives to respond more aggressively to their private signal. These strategic effects then spill over the network due to indirect interactions between agents. Finally, statement (c) of Corollary 1.3 illustrates the role of agents' private signals in the above-mentioned mechanism: a higher commonality between s_i and s_j means that agent i has a more precise estimate for j's private signal and hence her equilibrium action. Therefore, an increase in commonality, coupled with the underlying strategic interactions, translates into a higher weight on i's private signal.

Corollary 1.4. *There exists $\hat{q} < 0$ such that if $q_{ij} \in (\hat{q}, 0)$ for all $i \neq j$ then*

(a) the weight agent i assigns to her signal is increasing in ρ_i and decreasing in ρ_j for all $j \neq i$.

(b) the weight agent i assigns to her signal is decreasing in r_{ik} and is increasing in r_{jk} when $i \notin \{j, k\}$;

This result is the counterpart to Corollary 1.3 when all actions are strategic substitutes, though under the stronger assumption that there exists an upper bound $|\hat{q}|$ on the intensity of strategic substitutabilities between actions. When the substitutions in the economy are small, the first order impact of changing the accuracy of player's signal on themselves is due to the convexity of their own payoff. Hence their equilibrium weight is necessarily increasing in their own accuracy. However, the first order impact of changing a player's accuracy on a neighboring player must be due to the first order interaction between them, which is substitutive. Hence the second player's equilibrium weight is decreasing in the first player's accuracy. In a similar fashion, changing a player's commonality with their neighbor necessarily impacts the neighbor through their first order substitutive interaction. Hence the neighbor's equilbrium weight is decreasing in the player's commonality. However, changing the commonality of two players' necessarily impacts a third player through the second order interaction which is complementary. Therefore, the third player's equilibrium weight is increasing in the commonality of the first two players.

1.3.2 Constrained-Efficient Solution

I next turn to defining and characterizing the (constrained) efficient outcome in my economy. Such a result provides me with a benchmark to define the extent to which agents' equilibrium actions over- or under-react to their private information. I start with the following standard definition:

Definition 1.2. *A strategy profile $a^* : \mathbb{R}^n \to \mathbb{R}^n$ is constrained-efficient if*

$$a_i^*(s_i) \in \arg\max_{a_i'} \sum_{j=1}^{n} \mathbb{E}_i[u_j(a, \theta)]$$

for all i.

The above definition implies that in the constrained-efficient solution, the social planner assigns actions to individual agents in order to maximize the social welfare, without transferring information between them. Put differently, the planner maximizes the social welfare while being subject to the same informational constraints as the agents. As a result, under such an efficiency benchmark, the planner internalizes any (payoff or informational) externalities between the agents.

In order to characterize the constrained-efficient solution, first note that social welfare in this economy is given by $W(a, \theta) = \sum_{i=1}^{n} u_i(a, \theta)$, which, in view of equation (1.1), can be expressed as

$$W(a, \theta) = \theta \sum_{i=1}^{n} a_i - \frac{1}{2} \sum_{i=1}^{n} a_i^2 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j \neq i} q_{ij}^* a_i a_j, \tag{1.6}$$

where q^*_{ij} is the (i,j) element of matrix

$$\mathbf{Q}^* = \mathbf{Q} + \mathbf{Q}' + \mathbf{V} + \mathbf{V}' \tag{1.7}$$

and $\mathbf{V}$ is an $n \times n$ matrix given by $\mathbf{V} = \sum_{i=1}^{n} \mathbf{V}^{(i)}$. I impose the following assumption on $\mathbf{Q}^*$ for the rest of the chapter:

Assumption 1.2. $\mathbf{Q}^* \prec \mathbf{I}$. *Furthermore,* $\rho(|\mathbf{Q}^*|) < 1$.

The first part of the assumption guarantees that the social welfare $W(a, \theta)$ in equation (1.6) is concave in the vector of actions a. The second part of the assumption is the counterpart to Assumption 1.1 for the planner's problem: it guarantees the existence of a unique constrained-efficient solution for all information structures. My next result formalizes this claim:

Proposition 1.2. *There exists a unique constrained-efficient strategy profile in linear strategies of the form* $a^*_i = \alpha^*_i s_i$ *for all* i*, where*

$$\alpha^* = (\mathbf{I} - \mathbf{Q}^* \circ \mathbf{R})^{-1} \rho, \tag{1.8}$$

$\mathbf{Q}^*$ *is given by* (1.7)*, and* $\circ$ *denotes the Hadamard (element-wise) product.*

Comparing Proposition 1.2 to Proposition 1.1 illustrates that the constrained-efficient strategy profile has the same exact functional form as the equilibrium profile in equation (1.3), except that the interaction matrix $\mathbf{Q}$ is replaced by $\mathbf{Q}^*$. This disparity is due to the fact that the social planner internalizes the impact of agents' actions on one another.

Once again, it is instructive to first consider the benchmark with complete information. From equation (1.8), it is immediate that, under complete information, the weight that the planner assigns to agent i's private signal is given by $\alpha^*_i = b^*_i$, where $b^* = (\mathbf{I} - \mathbf{Q}^*)^{-1}\mathbf{1}$ is the vector of Bonacich centralities of agents in a network defined by $\mathbf{Q}^*$. In contrast, recall from Corollary 1.1 that the weight that agent i assigns to her private signal in equilibrium is equal to her Bonacich centrality b_i constructed from matrix $\mathbf{Q}$. Therefore, the wedge between centrality vectors b and b^* captures the extent to which payoff externalities (including any strategic interactions) result in an inefficient equilibrium under complete information.

Importantly, equations (1.3) and (1.8) also illustrate that whether an equilibrium is efficient depends not only on matrices $\mathbf{Q}$ and $\mathbf{Q}^*$, but also on the economy's information structure, as summarized by the commonality and accuracy parameters $\mathbf{R}$ and ρ. In particular, they imply that an equilibrium is efficient if and only if $(\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1}\rho = (\mathbf{I} - \mathbf{Q}^* \circ \mathbf{R})^{-1}\rho$. Thus, holding agents' payoffs constant, the economy may be efficient under one particular information structure, even though it is inefficient under others. This is a consequence of the mechanism discussed in Subsection 1.3.1 and Example 1: how strategic interactions manifest themselves depends on what agents can infer about others' actions from their private signals. Therefore, starting from an efficient economy, a change in the information structure can result in an inefficient economy.

1.3.3 Equilibrium Over- and Under-reaction

Having characterized the equilibrium and the constrained-efficient outcomes, I now study how model primitives shape the wedge between two. I am particularly interested in the extent to which agents' equilibrium actions over- or under-react to their private signals—as compared to the efficiency benchmark—under different information structures.

I start with the following result, which characterizes the conditions under which equilibrium and efficient actions coincide irrespective of the economy's information structure.

Proposition 1.3. *The equilibrium is constrained efficient under all information structures if and only if* $\mathbf{Q} = \mathbf{Q}^*$.

Therefore, in view of equation (1.7), the equilibrium is constrained efficient for all information structures if $\mathbf{V} + \mathbf{V}' = -\mathbf{Q}$. If this joint restriction on agents' payoffs is violated, then there is at least one information structure under which agents do not fully internalize the impact of their actions on other agents, and hence, over- or under-react to their private signals. My next result provides sufficient conditions on agents' payoffs and information structure for equilibrium over- and under-reaction. As in the previous section, I use $b = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1}$ and $b^* = (\mathbf{I} - \mathbf{Q}^*)^{-1}\mathbf{1}$ to denote the vectors of agents' Bonacich centralities in networks corresponding to $\mathbf{Q}$ and $\mathbf{Q}^*$, respectively.

Proposition 1.4. *Suppose agents' private signals are informative about the fundamental, i.e., $\rho_i \neq 0$ for all i. There exist constants $\underline{r}, \overline{r} \in (0,1)$ such that*

(a) if $r_{ij} \in (0, \underline{r})$ for all $i \neq j$, then all agents overreact to their signals if $q_{ij} > q^*_{ij}$ for all $i \neq j$ and underreact to their signals $q_{ij} < q^*_{ij}$ for all $i \neq j$.

(b) if $r_{ij} \in (\overline{r}, 1)$ for all $i \neq j$, then all agents overreact to their signals if $b_i > b^*_i$ for all i and underreact to their signals if $b_i < b^*_i$ for all i.

As anticipated, the above result highlights the fact that, in general, whether agents over- or under-react in equilibrium depends not only on their payoffs (as summarized by matrices $\mathbf{Q}$ and $\mathbf{Q}^*$), but also on the economy's information structure. This can be seen more explicitly by contrasting statements (a) and (b): the restrictions on $\mathbf{Q}$ and $\mathbf{Q}^*$ to induce equilibrium over- and under-reaction change depending on the pairwise commonalities r_{ij} in agents' signals.

To see the intuition underlying Proposition 1.4, first consider statement (a), which establishes that when all pairwise commonalities are below some threshold $\underline{r}$, a sufficient condition for whether agents over- or under-react to their private signals is in terms of the element-wise wedge between $\mathbf{Q}$ and $\mathbf{Q}^*$. This is because when commonalities are small, agents' private signals are not particularly useful coordination devices beyond agents' direct neighbors. As a result, in both equilibrium and planner's solution, the marginal cost of increasing the weight on the signal is dominated by the direct interactions, while the indirect interaction terms become irrelevant. Importantly, this observation also implies that the details of the interaction network—above and beyond first-order interactions—become irrelevant.

Statement (b) of the proposition considers the polar opposite case, in which large commonalities in signals imply that each can forecast the realizations of all other agents' signals with a high precision. As a result, agents' signals inform them not only of the realization of the fundamental but also of other agents' actions. Consequently, agents' actions—whether in equilibrium or the planner's solution—would also depend on all direct and indirect interactions over the network. This is indeed reflected by the condition in statement (b), which establishes that when pairwise commonalities exceed some threshold $\bar{r}$, whether agents over- or under-react in equilibrium depends on the wedge between agents' centralities b and b^*.

I conclude this section by a brief remark on how my results thus far compare to an economy in which (i) pairwise strategic interactions are homogeneous and (ii) the information structure is symmetric across all agents. In my framework, these symmetry assumptions can be stated as $q_{ij} = q$, $v_{ij}^{(k)} = v$, and $r_{ij} = \hat{r}$ for all pairs of agents $i \neq j$ and $\rho_i = \hat{\rho}$ for all i. In such economies, whether agents' overreact or under-react to their private signals only depends on the economy's payoff structure. Indeed, one can readily see that by imposing the homogeneity and symmetry assumptions on the expressions in (1.3) and (1.8): when strategic interactions are homogeneous and the information structure is symmetric, equations (1.3) and (1.8) imply that all agents overreact to their private signals if and only if $q > q^*$, where q and q^* only depend on the agents' payoff. In contrast, the two parts of Proposition 1.4 illustrate that, in the presence of heterogenous interactions, whether an agent i over- or under-reacts to her private signal also depends on the details of the information structure.

1.4 Social Value of Information

My results in the previous section focused on how model primitives shape the wedge between equilibrium and constrained-efficient outcomes. In this section, I provide comparative static results on how changes in the economy's information structure impact equilibrium social welfare. As in Angeletos and Pavan (2007) [17], I show that whether changes in accuracy and commonality increase welfare depends on the degree of equilibrium over- and under-reaction (i.e., the relationship between α and α^*). However, my results also indicate that the heterogeneity in interactions (as captured by matrix $\mathbf{Q}$) plays a non-trivial role in determining the social value of information.

I start by focusing on economies that are efficient under all information structures. I then extend the analysis to economies that may be inefficient for certain information structures.

1.4.1 Efficient Economies

Recall from Proposition 1.3 that the equilibrium and constrained-efficient outcomes coincide with one another irrespective of the information structure if and only if the corresponding interaction matrices are equal, i.e., $\mathbf{Q} = \mathbf{Q}^*$. This condition guarantees that agents' equilibrium coordination motives align with those of the social planner. I have the following result:

Proposition 1.5. *Suppose the economy is efficient under all information structures.*

(a) If $q_{ij} \geq 0$ for all $i \neq j$, then social welfare is increasing in agents' signal accuracy and commonality.

(b) There exists $\hat{q} < 0$ such that if $q_{ij} \in (\hat{q}, 0)$ for all $i \neq j$, then social welfare is decreasing in agents' signal commonalities and increasing in their accuracy.

The above result establishes that when the economy is efficient and all actions are strategic complements, an increase in either the accuracy or the commonality of signals increases expected social welfare. To see the intuition underlying this result, recall from the discussion in Subsection 1.2.2 that my framework exhibits two types of information frictions: (i) a friction that arises due to the fact that agents have imperfect signals about the realization of the fundamental; and (ii) a friction that arises because agents cannot perfectly coordinate their actions with one another. An increases in either information accuracy or commonality reduces these frictions. As a result, in the presence of strategic complementarities, the social value of information—which, by assumption, coincides with the private value of information—is always positive. In other words, welfare necessarily increases in both ρ_i and r_{ij}. This result thus generalizes the results of Angeletos and Pavan (2007) [17] for the social value of information in the presence of strategic complementarities to economies with heterogeneous interactions.

The social value of information has a less straightforward characterization when actions are strategic substitutes, as the detailed nature of the economy's interaction network also becomes relevant. To capture how network interactions shape the social value of information in the presence of strategic substitutabilities in a transparent manner, my next result imposes a symmetry assumption on the economy's information structure.

Proposition 1.6. *Suppose the economy is efficient under all information structures. Also, suppose the information structure is symmetric, with $r_{ij} = \hat{r}$ for all $i \neq j$ and $\rho_i = \hat{\rho}$ for all i.*

(a) Social welfare is always increasing in $\hat{\rho}$.

(b) If $\sum_{i,j} q_{ij} > 0$, then social welfare is increasing in $\hat{r}$.

(c) If $\sum_{i,j} b_i b_j q_{ij} < 0$, then social welfare is decreasing in $\hat{r}$, where b_i is the Bonacich centrality of i.

(d) If $\sum_{i,j} b_i b_j q_{ij} > 0 > \sum_{i,j} q_{ij}$, then social welfare first decreases and then increases in $\hat{r}$.

Statement (a) of the above result establishes that social welfare is necessarily increasing in the common accuracy parameter $\hat{\rho}$. This is to be expected: all else equal, more accurate signals enable agents to better match their actions with the underlying fundamental, thus reducing the friction due to imperfect signals. When coupled with the assumption that the private value of information coincides with the social value of information ($\mathbf{Q} = \mathbf{Q}^*$), more accuracy translates into higher equilibrium social welfare.

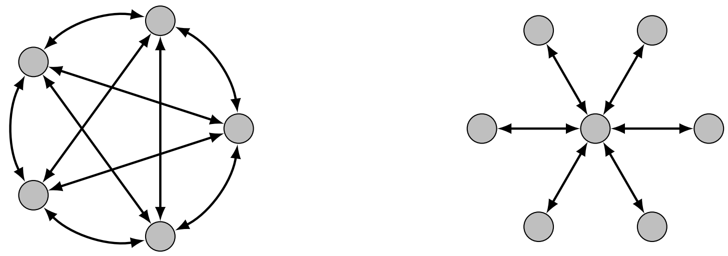
While an increase in accuracy unambiguously increases social welfare, the welfare impact of an increase in commonalities depends on the nature and intensity of strategic interactions. Statements (b)–(d) of Proposition 1.6 provide a full taxonomy of conditions for the range of possible outcomes as a function of the underlying interaction network.

Statement (b) of the proposition illustrates that social welfare is increasing in the commonality parameter $\hat{r}$ if the *net degree of strategic complementarities*, given by $\sum_{i,j} q_{ij}$ is strictly positive. In other words, if pairwise actions over the network are on average more strategically complementary than substitutable, then more common signals translate into higher equilibrium welfare. Importantly what matters in this condition are only the pairwise direct interactions over the network: indirect interactions over the network do not appear in this condition. Also, note that when all actions are strategic complements (i.e., $q_{ij} \geq 0$ for all i, j), the condition in statement (b) is automatically satisfied, guaranteeing that welfare is increasing in commonality of signals (consistent with Proposition 1.5).

Statement (c) of Proposition 1.6 provides a condition under which social welfare is decreasing in the commonality parameter $\hat{r}$. Note that, in contrast to the previous statement, this condition is not in terms of the net degree of strategic complementarities. Rather, the degree of strategic complementarity between agents i and j is weighted by their respective Bonacich centralities: welfare decreases in $\hat{r}$ if $\sum_{i,j} b_i b_j q_{ij} < 0$. This has two important consequences. First, unlike the condition in statement (b), the dependence on the centralities means that the condition in statement (c) depends on both direct and indirect interactions over the network. Second, Proposition 1.6(c) establishes that welfare decreases with informational commonality if actions of more central agents are strategic substitutes with those of other central agents.

The wedge between the conditions in statements (b) and (c) of Proposition 1.6 highlights that it may be possible for welfare to not have a monotonic behavior in $\hat{r}$. Indeed, statement (d) of the proposition establishes that when both conditions are violated (so that $\sum_{i,j} b_i b_j q_{ij} > 0 > \sum_{i,j} q_{ij}$), then social welfare is first decreasing and then increasing in $\hat{r}$. This non-monotonic behavior, which is absent in the symmetric model of Angeletos and Pavan (2007) [17], is entirely driven by the heterogeneity in interactions. To see the intuition underlying this result, note that when $\hat{r}$ is small—so agents have fairly imprecise information about others' signals—they base their actions solely on direct interactions over the network. Therefore, the relevant condition is the condition on first-order interactions. However, as $\hat{r}$ increases and gets closer to 1, higher-order interactions become important as well and therefore the condition in terms of centralities appears.

Example 2. Consider the fully symmetric economy depicted in Figure 1.2(a), in which all pairwise interactions are identical, i.e., $q_{ij} = q$ for all $i \neq j$. This means that the interaction matrix can be written as $\mathbf{Q} = q(\mathbf{1}\mathbf{1}' - \mathbf{I})$. This economy is therefore the discrete counterpart of the symmetric-interaction economy of Angeletos



(a) (b)

Figure 1.2: Interaction networks corresponding to two economies. Each vertex corresponds to an agent, with a directed edge present from vertex j to vertex i if $q_{ij} \neq 0$.

and Pavan (2007) [17].

It is easy to verify that, in this economy, all agents have identical Bonacich centralities given by $b_i = \frac{1}{(n-1)q-1}$. Consequently, the net and centrality-weighted degrees of strategic complementarities in the economy are given by

$$\sum_{i,j} q_{ij} = n(n-1)q$$
$$\sum_{i,j} b_i b_j q_{ij} = \frac{n(n-1)q}{((n-1)q-1)^2},$$

respectively. This, coupled with Proposition 1.6, shows that when there is no heterogeneity in the intensity of interactions, the sign of q determines whether welfare increases or decreases with informational commonality between various agents: when $q > 0$ (so that actions are strategic complements) ex ante welfare is increasing in commonality, whereas when $q < 0$ (so that actions are strategic substitutes) welfare is decreasing in commonality. These observations are, of course, in line with the results of Angeletos and Pavan (2007) [17] for homogeneous-interaction economies.

Example 3. Next, consider the economy with the star interaction network depicted in Figure 1.2(b), where $q_{1i} = q_{i1} = q$ for all $i \neq 1$ and $q_{ij} = 0$ otherwise. As in the previous example, scalar $q \in (-\frac{1}{\sqrt{n-1}}, \frac{1}{\sqrt{n-1}})$ parameterizes the intensity of strategic interactions, where the restrictions on q are imposed to ensure that Assumptions 1.1 and 1.2 are satisfied. However, in contrast to Example 2, strategic interactions in this economy are asymmetric. Once again, I assume that $\mathbf{V} + \mathbf{V}' = -\mathbf{Q}$ to ensure that the economy is efficient under all information structures.

It is easy to verify that the net and centrality-weighted degrees of strategic complementarities in this economy are given by

$$\sum_{i,j} q_{ij} = 2(n-1)q$$

$$\sum_{i,j} b_i b_j q_{ij} = \frac{2(n-1)q(1+q)(1+(n-1)q)}{(1-(n-1)q^2)^2},$$

respectively. Therefore, by statement (b) of Proposition 1.6, welfare is increasing in $\hat{r}$ if $q > 0$, which is the case when all actions are strategic complements. Of course, this is also consistent with the prediction of Proposition 1.5. On the other hand, by Proposition 1.6(c), welfare in this economy is decreasing in $\hat{r}$ if $-\frac{1}{n-1} < q < 0$. Finally, statement (d) of the proposition implies that if the degree of strategic substitutabilities becomes stronger (and in particular, when $q < \frac{1}{n-1}$), then welfare first decreases and then increases in $\hat{r}$. In other words, the comparative static results ith respect to $\hat{r}$ change as the degree of strategic substitutabilities.

To see why, note that when q is negative, all pairwise actions between neighboring agents are strategic substitutes. But this means that second-order indirect interactions exhibit strategic complementarities.[5] When q is negative but small in magnitude, the direct interactions always dominate the effect of indirect, higher-order interactions. As a result, an increase in informational commonality unambiguously reduces welfare. In contrast, once the degree of strategic substitutabilities becomes stronger (in the sense that q becomes more negative), then the indirect (and complementarity) interactions also become important. Whether these higher-order interactions dominate the direct interactions then depends on the commonality parameter $\hat{r}$. For smaller values of $\hat{r}$, the direct first-order strategic substitutabilities dominate

[5] More generally, all odd- and even-ordered interactions exhibit strategic substitutabilities and complementarities, respectively.

and so the ex ante social welfare is decreasing. As $\hat{r}$ increases, the higher-order complementary interactions become more prominent and the social welfare eventually increases in $\hat{r}$.

1.4.2 Inefficient Economies

Propositions 1.5 and 1.6 in the previous subsection provide comparative static results with respect to the information structure in efficient economies. These results, however, rely on the efficiency assumption, which guarantees that the social and private values of information coincide with one another irrespective of the economy's information structure. In the remainder of this section, I extend my earlier results to the class of economies that may be inefficient under some or all information structures.

I start with the generalization of Proposition 1.5.

Proposition 1.7. *If $q^*_{ij} \geq q_{ij} \geq 0$ for all pairs of agents $i \neq j$, then social welfare is increasing in agents' signal accuracy and commonality.*

In other words, as long as pairwise actions are strategic complements and the social value of coordination between any pair of agents is at least as high as the private value of coordination, an increase in either the accuracy or the commonality of signals increases expected social welfare. The intuition behind this result mirrors that of Proposition 1.5. Recall from the discussion in Subsection 1.2.2 that my framework exhibits two types of information frictions: (i) a friction that arises due to the fact that agents have imperfect signals about the realization of the fundamental; and (ii) a friction that arises because agents cannot perfectly coordinate their actions with one another. Therefore, an increases in either information accuracy or commonality reduces these frictions and facilitates coordination between the agents. Finally, the assumption that the planner values coordination at least as much as the agents guarantees that such an increase also translates into higher social welfare. This result thus generalizes the results of Angeletos and Pavan (2007) [17] for the social value of information in the presence of strategic complementarities to economies with heterogeneous interactions.

My next result provides a series of sufficient conditions under which social welfare is decreasing in the commonality parameter.

Proposition 1.8. *Consider an economy with a symmetric information structure, in the sense that* $r_{ij} = \hat{r}$ *for all* $i \neq j$ *and* $\rho_i = \hat{\rho}$ *for all* i.

(a) Suppose $q_{ij} \geq 0 \geq q^*_{ij}$ *for all pairs of agents* $i \neq j$*. Then, there exists* $\bar{r} < 1$ *such that social welfare is decreasing in* $\hat{r}$ *for all* $\hat{r} \in (\bar{r}, 1)$.

(b) There exist $\underline{q} < 0$ *and* $\bar{r} < 1$ *such that if* $\underline{q} \leq q_{ij} \leq q^*_{ij} \leq 0$ *for all* $i \neq j$*, then social welfare is decreasing in* $\hat{r}$ *for all* $\hat{r} \in (\bar{r}, 1)$.

To see the intuition underlying the above result, note that when the commonality parameter $\bar{r}$ is sufficiently close to 1, agents have fairly precise information about each others' signals (and hence, actions). Therefore, when all pairwise actions are strategic complements (i.e., $q_{ij} \geq 0$ for all $i \neq j$ as in statement (a)), in response to an increase in the commonality of signals, $\hat{r}$, agents assign higher weights on their private signals (as established in Corollary 1.3(c)), which in turn implies that the equilibrium actions of any pair of agents $i \neq j$ become more coordinated. But the assumption that $q^*_{ij} \leq 0$ means that such an increase in coordinations is socially inefficient from the planner's perspective. Therefore, an increase in $\hat{r}$ reduces social welfare.

Next consider the condition in part (b). In this case, the planner values coordination at least as much as agents. However, the fact that all actions are strategic substitutes from both the agents' and planner's perspectives guarantees that an increase in commonality translates into lower social welfare when these substitutions are sufficiently small. Consequently, the ex ante equilibrium social welfare is decreasing in $\hat{r}$.

My next result provides a counterpart to Proposition 1.6 for the class of inefficient economies. As before, let $b = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1}$ denote the vector of Bonacich centralities corresponding to the interaction matrix $\mathbf{Q}$.

Proposition 1.9. *Consider an economy with a symmetric information structure, in the sense that $r_{ij} = \hat{r}$ for all $i \neq j$ and $\rho_i = \hat{\rho}$ for all i.*

(a) There exists $\underline{r}$ such that social welfare is increasing in $\hat{r}$ in $(0, \underline{r})$ if and only if $\sum_{i,j} q^*_{ij} > 0$.

(b) Suppose the economy is efficient under complete information. Then there exists $\bar{r}$ such that social welfare is increasing in $\hat{r}$ in $(\bar{r}, 1)$ if and only if $\sum_{i,j} q^*_{ij} b_i b_j > 0$.

This result thus (partially) generalizes Proposition 1.6 to the class of inefficient economies. In particular, statement (a) states that the behavior of the social welfare in the small-commonality regime depends only on the net degree of complementarity between agents—much like in statement (b) of Proposition 1.6—but from the planner's perspective. Similarly, statement (b) of Proposition 1.9 establishes that when the economy is efficient under complete information, the behavior of the social welfare depends on the weighted sum $\sum_{i,j} b_i b_j q^*_{ij}$ of the planner's complementarities, where the weights are given by the agents' Bonacich centralities. A point of note is that since $\mathbf{Q} \neq \mathbf{Q}^*$ in inefficient economies, I do not necessarily have that $\sum_{i,j} q^*_{ij} b_i b_j > \sum_{i,j} q^*_{ij}$. Therefore, unlike in completely efficient economies, it is possible for social welfare to be increasing with respect to the commonality in the small-commonality regime but decreasing in the large-commonality regime.

1.4.3 Regular Economies

While intuitive, Propositions 1.7 and 1.8 impose fairly stringent assumptions on *all* direct pairwise interactions simultaneously. For example, Proposition 1.7 requires that (i) all pairwise interactions are strategically complementarity over the network and (ii) the social value of coordination, q^*_{ij}, is at least as high as the private value of coordination, q_{ij}, for all pairs of agents $i \neq j$. I conclude this section by relaxing these assumptions and illustrating that the same forces that underpin the results in Propositions 1.7 and 1.8 lead to similar results under more general conditions.

In order to obtain such a generalization, I focus on the class of regular economies, defined as follows:

Definition 1.3. *An economy is regular if* $\sum_{j \neq i} q_{ij} = \sum_{j \neq k} q_{kj}$ *for all pairs of agents* i *and* k.

In a regular economy, the aggregate degree of strategic interactions of any given agent with the rest of the agents in the economy is constant across all agents i. The symmetric-interaction economy of Angeletos and Pavan (2007) [17]—studied in Example 2 and depicted in Figure 1.2(a)—clearly belongs to the class of regular economies. However, this class also contains more general economies with heterogenous pairwise interactions across agents. Also note that the regularity condition does not impose any sign restrictions on pairwise interactions q_{ij}. For example, an economy may be regular even if $q_{ij} > 0 > q_{kl}$ for pairs of agent (i, j) and (k, l). I have the following result:

Proposition 1.10. *Consider a regular economy with a symmetric information structure in the sense that* $r_{ij} = \hat{r}$ *for all* $i \neq j$ *and* $\rho_i = \hat{\rho}$ *for all* i.

(a) If $\sum_{i,j} q^*_{ij} > \sum_{i,j} q_{ij} > 0$*, then social welfare is increasing in* $\hat{r}$.

(b) If $\sum_{i,j} q^*_{ij} < \sum_{i,j} q_{ij} < 0$*, then there exists* $\underline{q} < 0$ *such that if* $\underline{q} < \sum_{i,j} q_{ij} < 0$*, social welfare is decreasing in* $\hat{r}$.

Proposition 1.10 is a generalization of the analogous result in Angeletos and Pavan (2007) [17] for inefficient economies. In part (a), as previously seen in Proposition

1.7, the planner values coordination at least as much as the agents. As a result, when there is a net degree of strategic complementarity in the economy, an increase in the commonality causes an increase in the social welfare. Part (b) of the above result then establishes the flip side of the condition in part (a): agents coordinate to a greater than optimal degree. As a result, when strategic substitutabilities are not too large, any increase in the commonality necessarily results in a decrease in the social welfare. Compared to Proposition 1.7, the regularity of the interaction network in Proposition 1.10 allows me to impose the sufficient conditions on the aggregate degree of complementarity instead of imposing the same on every pairwise degree of coordination.

1.5 Conclusion and Future Work

I find that heterogeneity of strategic interactions in an economy affects both the private and social value of information. Specifically, in equilibrium, I find that a player within an economy with a fixed set of payoffs can potentially both overreact and underreact with respect to the constrained-efficient action as one changes the underlying information structure. This behavior primarily results due to the heterogeneous nature of interactions and is absent when the interactions are homogeneous. I find that in the presence of strategic complementarities, a player's signal becomes more prominent to their respective equilibrium action as the information commonality and accuracy increase. However, the opposite is not necessarily true when the economy exhibits heterogeneous substitutions – this is because the even-numbered higher order effects due to first-order substitutions are always complementary and when interaction heterogeneities are introduced, one can potentially encounter a situation in which these even-numbered complementary interactions dominate over the odd-numbered substitutive interactions. This can give rise to novel social-welfare behavior that is unseen in homogeneous economies. Specifically, I find that social welfare in efficient economies can be non-monotonic with respect to the information commonality. I also provide sufficient conditions under which social welfare is monotonic in inefficient economies and the give the intuition behind these results.

The insights that I develop in this work can be useful when studying the social welfare considerations in problems such as the peer-effect model in Ballester et al. [18], the differentiated-product Cournot model of Myatt and Wallace (2015) [47], the generalized beauty-contest payoff economy in Myatt and Wallace (2019) [48] and price-setting behavior in production networks seen in La'O and Tahbaz-Salehi [45], Angeletos et al. [14] and Angeletos and La'O [15]. Going forward, a natural extension would be to incorporate my framework in the pricing of information products discussed in Bimpikis et al. (2019) [21]. Specifically, one could study how heterogeneous interactions among buyers affect the offer and pricing decisions faced by the monopolistic seller. Another extension would be to apply my framework to Candogan, Bimpikis and Ozdaglar ([20] which considers the problem of optimally pricing products with network externalities where interaction heterogeneities can arise endogenously due to the optimal price discrimination employed by the seller.

Chapter 2

Are the Federal Reserve's Stress Test Results Predictable?

2.1 Introduction

Regulatory stress tests have become a central tool for enhancing the resilience of the banking system. The current era of stress testing began with the 2009 Supervisory Capital Assessment Program (SCAP), which played an important role in turning around the financial crisis in the United States. The SCAP tested the ability of the largest U.S. bank holding companies to withstand a further worsening of economic conditions, and it combined this test with a government backstop for banks needing additional capital. In a major departure from customary supervisory practice, results of the SCAP were made public. The release of this information is credited with helping to restore market confidence by reducing uncertainty about the state of the financial system and by making the government's response transparent. See Hirtle, Schuermann, and Stiroh [37] for further discussion of the SCAP's information content and its importance to the program.

The success of stress testing in the SCAP led to the Federal Reserve's Comprehensive Capital Analysis and Review (CCAR) and the Dodd-Frank Act Stress Testing (DFAST) program. I will provide background on these programs in the next section, but their supervisory stress testing components are broadly similar to the SCAP's. The Federal Reserve defines a small number of scenarios through economic and financial variables, and the banks and their supervisors evaluate bank losses resulting from these scenarios. In several but not all cases, summary information on stress losses by bank and asset category have then been made public.

The results of these stress tests are pivotal in setting bank capital levels and allowed distributions. Stress testing has overshadowed the use of internal bank models to calculate risk-weighted assets, which drove capital requirements for the largest banks prior to the financial crisis. The annual execution of the CCAR/DFAST process has become an enormous undertaking for the banks covered by these programs and their supervisors.

Despite the complexity of this process, using results made public thus far across various stress tests I find that projected losses by bank and loan category are fairly predictable and are becoming increasingly so. In particular, losses for DFAST 2019 and 2020 are nearly perfectly correlated for banks that participated both years. Most of this article documents these findings.

That stress losses would become predictable from one year to the next should not be surprising. If a bank's portfolio and the Fed's scenarios remain reasonably consistent over time, so should the bank's stress test results. In its first year of participation in the stress tests, a bank needs to make major investments in staff and information technology; over time, the process matures and becomes more routine. Indeed, consulting firms and software vendors have made a business of trying to simplify and standardize the stress testing process for banks to make it more routine. The models used by the Fed to define scenarios and project losses have also been refined and should change less over time. Banks have incentives to avoid investments that will attract high capital requirements through the stress tests. As discussed in Schuermann [51], they also face incentives to align their internal risk assessments with the Federal Reserve's. All of these factors contribute to making outcomes more predictable over time.

But whereas the results of stress tests may be predictable, the results of actual shocks to the financial system are not, and herein lies the concern. The process of maturation that makes stress test results more predictable may also make the stress tests less effective. One should be careful in extrapolating from the early success of the SCAP and its immediate successors to assume that the same process will continue to be effective in the future. The SCAP worked, in part, by providing new information. To the extent that stress test results become more predictable, they become less informative.

Several authors including Acharya, Engle, and Pierret [12], Covas, Lump, and Zakrajsek [26], Guerrieri and Welch [33], Hirtle et al. [36], and Kapinos and Mitnik [41], have developed models for bank stress testing using public data. These models seek to forecast bank vulnerability as an alternative or complement to supervisory stress tests. My focus is different: I am interested in the predictability of the outcomes of the supervisory stress tests themselves, rather than in more accurate forecasting of bank vulnerability. Hirtle et al. [36] find significant correlation between their forecasts and DFAST outcomes, consistent with the predictability of the outcomes.

One way to reduce predictability is to increase the number and diversity of scenarios evaluated in a stress test. I compare results for the two stress scenarios used in DFAST 2014 – 2019 and find an oddly high degree of correlation across scenarios by bank and by loan category. This pattern is particularly surprising given the large number of variables used to define the scenarios. The pattern suggests a missed opportunity to diversify the types of stresses tested.

The next section provides additional background on supervisory stress tests. I then compare the results for the two stress scenarios in DFAST 2014 to DFAST 2019. The next two sections compare predictability across time, first for loss levels and then for loss rates. I then examine the stock market reaction to announcements of stress test results; consistent with the predictability of the results, in most years, I find no significant correlation between the severity of a bank's reported stress losses and the change in its stock price relative to the market. I conclude the chapter with comments on how the limitations of predictable stress tests might be countered.

2.2 Background on Supervisory Bank Stress Tests

My analysis draws on results from the ten rounds of stress tests conducted so far: SCAP 2009, CCAR 2012, DFAST 2013, DFAST 2014, DFAST 2015, DFAST 2016, DFAST 2017, DFAST 2018, DFAST 2019 and DFAST 2020. For background, I review the essential features of these programs.

2.2.1 SCAP

The SCAP was launched in February 2009 and its results announced that May. It had the following features:

- *Scope:* The program applied to the 19 largest U.S. bank holding companies (BHCs): American Express, Bank of America, Bank of New York/Mellon, BB&T, Capital One Financial, Citigroup, Fifth Third, Goldman Sachs, GMAC, JPMorgan Chase, KeyCorp, MetLife, Morgan Stanley, PNC Financial, Regions Financial, State Street, SunTrust, US Bancorp, and Wells Fargo. MetLife exited banking in 2013 and has not been covered by the bank stress tests since then. The remaining group of 18 are the common participants across all rounds of stress tests. GMAC changed its name to Ally Financial after the SCAP.

- *Scenarios:* The SCAP used a baseline scenario and a more adverse scenario defined through a two-year decline in GDP and house prices and an increase in unemployment. A separate market shock similar to the second half of 2008 was applied to trading portfolios.

- *Asset categories:* Projected losses were calculated for five loan categories: first-lien mortgages, junior-lien mortgages, commercial and industrial (C&I) loans, commercial real estate (CRE) loans, and credit cards. Projected losses were calculated for banks' securities portfolios and for the trading portfolios of the five banks with large trading positions.

- *Disclosure:* Projected loss amounts and loss rates under the more adverse scenario were disclosed by asset category for each bank in Board of Governors [1].

The SCAP report also disclosed projected capital levels by bank, but I focus exclusively on projected losses.

2.2.2 CCAR

There was no supervisory stress test in 2010. CCAR, launched in 2011, differed from the SCAP in putting greater emphasis on the capital planning process and on the robustness of the processes employed by the participating BHCs in their internal risk management; see Board of Governors [3]. I focus exclusively on the stress testing component of the review.

CCAR 2011 applied to the same bank holding companies as the SCAP. The adverse scenario was enriched to specify a path of the economy over nine quarters for nine economic and financial variables. The scenario was made public, but no bank-specific results were disclosed. CCAR 2011 is therefore not part of my analysis.

CCAR 2012 again applied to the same nineteen bank holding companies as the SCAP. The Fed's adverse scenario was further expanded to define paths for 25 variables, including more international factors. The loan categories were expanded to include consumer loans and a category called Other Loans. Projected stress losses were disclosed by bank and category under the Fed's adverse scenario. Banks were also required to define their own scenarios and estimate stress losses in these scenarios, but those results were not made public.

2.2.3 DFAST

The Dodd-Frank Act, passed by Congress in July 2010, includes requirements for annual regulatory stress tests, commonly referred to as DFAST. The act requires at least three scenarios — a baseline, an adverse scenario, and a severely adverse scenario.

DFAST 2013 applied to the same bank holding companies as the SCAP, except for MetLife. DFAST 2014 covered all bank holding companies with over $50 billion in consolidated assets, bringing the number of reporting BHCs to 30. The group will continue to grow because the Dodd-Frank Act's stress testing provisions apply to all banks (and certain other financial companies) with over $10 billion in consolidated assets. The number of participating banks has mostly been stable from 2014 onwards, with a few additions in between. For example, DB USA Corporation and MUFG Americas Holdings Corporation were added to the list in 2015, TD Group US Holdings LLC in 2016, RBC US Group Holdings LLC in 2018 and Truist Financial Corporation in 2020. The most significant change to bank participation criteria occured in 2019 when the Federal Reserve provided relief to less-complex firms with total consolidated assets between $100 billion and $250 billion by moving them to an two-year stress test cycle. Consequently, there were only 18 banks participating in DFAST 2019. This change in bank participating criteria reflects the smaller risk that these firms present. See Federal Reserve's reports Board of Governors [10**?**] for a more detailed discussion.

In DFAST 2013 and DFAST 2020, results were disclosed for the severely adverse scenario only. In DFAST 2014 to DFAST 2019, results were disclosed for both the adverse and severely adverse scenarios. I will compare results for the two scenarios in the next section.

Projected losses under DFAST are used as inputs to the CCAR capital planning process, so the two programs operate in parallel. DFAST provides public information on the capital strength of large banks, but CCAR is much more comprehensive and determines a large bank's ability to pay dividends or repurchase shares. I focus on the results of the stress tests.

DFAST requires banks to run and disclose two types of stress tests, in addition to the results calculated and reported by supervisors. Banks must disclose their own loss projections under the Federal Reserve's scenarios, and they are also required to run mid-cycle stress tests using their own scenarios and loss projections. See Hirtle, Kovner, and McKay [35] for a comparison of the Fed's and bank's results for the Fed's scenarios. I will use only the annual supervisory results in my analysis; I expect that the mid-cycle disclosures by individual companies would only enhance the predictability of the supervisory tests.

Data

I compiled data on projected loss amounts and loss rates by bank and by loan category from the Federal Reserve's reports Board of Governors [1, 2, 4, 5, 6, 7, 8, 9, 10, 11]. Tables 2.1 and 2.2 report summary statistics on projected losses (in billions of dollars) and Tables 2.3 and 2.4 report summary statistics on loss rates (in percent).

I focus primarily on projected losses on loans. Trading and counterparty shocks apply only to a subset of the participating bank holding companies, and this part of the stress test operates differently from the rest of the program. The details of the market shock were not made public prior to 2013.

Figure 2.1 compares the loss rate distributions across the CCAR 2012 and DFAST 2013 – 2020. For each histogram, I pool loss rates for all loan categories and all banks. The distributions for all the rounds of stress tests are similar to each other, consistent

Year	Category	Mean	Median	Std Dev	Min	Max
	First Liens	7.3	1.9	10.4	0.1	32.4
	Junior Liens	6.4	1.7	7.8	0.6	21.4
2009	C&I	3.5	1.5	4.6	0.0	15.7
	CRE	3.3	2.9	2.7	0.2	9.4
	Credit Cards	6.9	3.2	8.4	0.0	21.2
	All Loans	32.8	12.6	41.6	5.4	136.6
	First Liens	3.8	1.5	5.6	0.0	17.7
	Junior Liens	3.5	1.2	5.1	0.0	16.0
	C&I	3.7	1.9	4.5	0.0	12.3
2012	CRE	1.4	0.8	1.7	0.0	6.7
	Credit Cards	7.1	3.2	9.1	0.1	27.0
	Other Consumer	1.4	0.6	2.1	0.0	8.1
	Other Loans	0.9	0.3	1.3	0.0	4.8
	All Loans	18.9	7.2	24.8	0.3	70.1
	First Liens	3.8	1.2	5.5	0.0	15.3
	Junior Liens	2.3	1.0	3.1	0.0	9.4
	C&I	3.4	1.7	3.5	0.0	11.1
2013	CRE	1.9	0.9	2.5	0.0	9.6
	Credit Cards	6.7	3.2	8.0	0.1	23.3
	Other Consumer	1.5	0.6	1.9	0.0	6.5
	Other Loans	0.7	0.3	0.8	0.0	2.9
	All Loans	17.6	6.7	21.3	0.3	57.5
	First Liens	3.2	0.9	5.0	0.0	15.7
	Junior Liens	2.4	0.8	3.5	0.0	9.9
	C&I	3.1	1.5	3.2	0.0	9.4
2014	CRE	2.2	1.1	2.6	0.0	9.4
	Credit Cards	6.5	2.8	8.0	0.1	24.8
	Other Consumer	1.5	0.7	1.8	0.0	6.1
	Other Loans	1.1	0.4	1.7	0.0	5.8
	All Loans	17.4	5.5	21.4	0.5	55.5
	First Liens	1.3	0.4	2.1	0.0	7.3
	Junior Liens	1.1	0.3	2.1	0.0	8.2
	C & I	2.2	0.9	2.9	0.0	10.9
2015	CRE	1.7	1.2	2.1	0.0	10.3
	Credit Cards	2.7	0.1	5.3	0.0	20.9
	Other Consumer	1.1	0.3	1.7	0.0	5.9
	Other Loans	0.9	0.3	1.7	0.0	7.1
	All Loans	11.0	5.1	15.2	0.6	49.7

Table 2.1: Summary statistics for loss levels (in billions of dollars) across loan categories for SCAP 2009, CCAR 2012, DFAST 2013, DFAST 2014 and DFAST 2015.

with the view that the overall process has stabilized over time. For illustration, I have superimposed on the histograms a probability density estimated from the 2012–2020 data.[1] The consistency in the distributions is surprising given the increasing complexity of the underlying stress scenarios and the expansion in the number of participating banks starting in 2014.

[1] I fit a gamma distribution with shape parameter 2.3 and scale parameter 3.1 to the 2012–2020 data. The density is multiplied by three in the charts for ease of comparison.

Year	Category	Mean	Median	Std Dev	Min	Max
2016	First Liens	1.2	0.4	1.9	0.0	7.5
	Junior Liens	1.0	0.3	2.0	0.0	10.1
	C & I	2.8	1.4	3.6	0.0	13.7
	CRE	1.6	1.0	2.0	0.0	10.0
	Credit Cards	2.8	0.1	5.6	0.0	21.1
	Other Consumer	1.1	0.5	1.6	0.0	6.5
	Other Loans	1.2	0.4	2.0	0.0	8.0
	All Loans	11.7	5.2	15.8	0.4	53.7
2017	First Liens	0.8	0.3	1.2	0.0	4.9
	Junior Liens	0.5	0.2	0.8	0.0	3.5
	C & I	2.9	1.7	3.7	0.0	15.9
	CRE	1.6	1.2	2.1	0.0	11.4
	Credit Cards	2.9	0.1	6.0	0.0	21.7
	Other Consumer	1.2	0.5	1.7	0.0	6.8
	Other Loans	1.3	0.4	2.2	0.0	8.6
	All Loans	11.3	5.4	15.0	0.5	54.0
2018	First Liens	1.0	0.4	1.6	0.0	6.3
	Junior Liens	0.4	0.2	0.7	0.0	2.8
	C & I	3.2	1.8	4.3	0.0	18.1
	CRE	1.8	1.4	2.4	0.0	12.9
	Credit Cards	3.2	0.1	6.4	0.0	23.1
	Other Consumer	1.1	0.4	1.5	0.0	5.1
	Other Loans	1.5	0.5	2.6	0.0	10.5
	All Loans	12.3	5.7	16.3	0.1	61.8
2019	First Liens	0.8	0.3	1.1	0.0	3.2
	Junior Liens	0.3	0.0	0.4	0.0	1.3
	C & I	4.0	1.8	5.2	0.0	17.3
	CRE	1.8	1.0	2.5	0.0	10.0
	Credit Cards	6.0	0.6	9.3	0.0	26.5
	Other Consumer	1.2	0.5	1.6	0.0	5.2
	Other Loans	2.4	0.9	3.1	0.1	10.8
	All Loans	16.4	6.5	20.1	0.1	60.2
2020	First Liens	0.6	0.3	0.9	0.0	3.4
	Junior Liens	0.2	0.1	0.3	0.0	1.0
	C & I	3.5	1.9	4.5	0.0	19.0
	CRE	1.4	1.0	1.9	0.0	10.0
	Credit Cards	4.4	0.3	8.2	0.0	27.5
	Other Consumer	1.5	0.8	1.9	0.0	6.9
	Other Loans	1.6	0.6	2.6	0.0	11.1
	All Loans	13.1	5.7	16.6	0.1	64.4

Table 2.2: Summary statistics for loss levels (in billions of dollars) across loan categories for DFAST 2016, DFAST 2017, DFAST 2018, DFAST 2019 and DFAST 2020.

2.3 Comparison Across Scenarios

As I noted earlier, DFAST 2014 was the first stress test to disclose loss projections for both an adverse and severely adverse scenario. Before investigating predictability across time, I compare results from the two scenarios.

Figuret 2.2 shows results for the 30 BHCs that participated in DFAST 2014. For each BHC, I plot the severely adverse loss rate on the vertical scale and the adverse loss rate on the horizontal scale. Loss rates are measured in percent. In

Year	Category	Mean	Median	Std Dev	Min	Max
	First Liens	7.7	8.1	2.8	3.4	11.9
	Junior Liens	13.2	13.2	4.6	6.3	21.2
2009	C&I	6.8	5.8	4.8	1.2	22.8
	CRE	15.2	10.9	11.9	5.5	45.2
	Credit Cards	22.6	22.3	5.4	17.4	37.9
	All Loans	7.9	8.7	3.7	0.4	14.3
	First Liens	6.3	7.1	2.8	0.0	9.5
	Junior Liens	12.5	12.1	3.3	7.8	21.1
	C&I	6.5	7.4	3.2	0.0	10.9
2012	CRE	6.5	5.6	4.3	2.1	20.1
	Credit Cards	17.7	18.5	3.4	10.0	22.4
	Other Consumer	5.3	3.7	5.3	0.0	23.4
	Other Loans	2.4	2.5	1.3	0.0	4.7
	All Loans	6.5	7.3	3.2	0.9	11.4
	First Liens	6.1	6.3	2.6	0.6	10.3
	Junior Liens	10.3	9.7	3.6	6.1	21.1
	C&I	8.8	6.5	10.5	0.0	49.8
2013	CRE	8.7	8.0	2.9	4.8	18.3
	Credit Cards	17.2	17.3	2.8	12.0	22.2
	Other Consumer	5.0	4.1	4.2	0.0	16.5
	Other Loans	2.1	1.8	0.9	0.8	4.5
	All Loans	6.6	6.7	2.8	2.0	13.2
	First Liens	4.6	4.9	2.0	1.0	7.5
	Junior Liens	9.0	9.9	2.7	4.8	13.5
	C&I	6.2	5.4	2.2	3.8	11.4
2014	CRE	9.5	8.9	4.7	4.8	26.2
	Credit Cards	15.6	16.2	2.7	10.6	20.5
	Other Consumer	4.5	3.9	3.6	0.0	14.0
	Other Loans	2.6	2.6	1.0	1.0	4.5
	All Loans	5.9	5.4	2.6	1.6	11.8
	First Liens	3.5	3.1	2.2	0.0	12.5
	Junior Liens	7.3	6.8	4.3	0.0	22.3
	C & I	5.7	4.8	2.5	3.0	14.0
2015	CRE	10.2	8.3	6.3	0.0	31.6
	Credit Cards	9.6	12.7	6.5	0.0	18.5
	Other Consumer	6.6	5.8	4.4	0.6	17.2
	Other Loans	3.2	2.7	2.2	0.0	12.7
	All Loans	5.8	5.0	2.2	2.3	12.2

Table 2.3: Summary statistics for loss rates (in percent) across loan categories for SCAP 2009, CCAR 2012, DFAST 2013, DFAST 2014 and DFAST 2015.

most cases, this gives us seven points for each bank, corresponding to the seven loan categories used to report loss projections. Some banks have little or no lending in some categories, resulting in fewer than seven points.

The results are striking. Across all 30 banks, I see a nearly perfect linear relationship between the losses in the two scenarios. This visual impression is quantified in Table 2.5, which shows the results of bank-specific regressions of the form

$$\mathit{SevereLossRate}_{b,c} = \text{Intercept}_b + \text{Slope}_b \times \mathit{AdverseLossRate}_{b,c}, \tag{2.1}$$

where the intercept and slope depend on the bank b but not on the loan category c.

Year	Category	Mean	Median	Std Dev	Min	Max
	First Liens	4.7	3.4	8.3	0.0	50.1
	Junior Liens	6.7	6.3	3.5	0.0	16.3
	C & I	6.5	5.5	2.7	2.6	15.5
2016	CRE	7.4	6.8	3.6	0.0	22.9
	Credit Cards	10.4	12.8	6.8	0.0	19.3
	Other Consumer	6.8	6.1	4.1	0.6	16.5
	Other Loans	3.9	3.7	1.8	1.2	9.4
	All Loans	5.9	5.6	2.1	1.9	12.4
	First Liens	3.6	2.2	8.7	0.0	52.3
	Junior Liens	4.5	4.5	2.1	0.0	10.4
	C & I	6.8	6.0	2.5	3.4	13.8
2017	CRE	7.3	7.4	2.7	0.0	15.5
	Credit Cards	10.2	13.1	6.6	0.0	20.3
	Other Consumer	6.5	6.2	4.1	0.0	17.5
	Other Loans	4.1	3.5	2.3	0.0	11.1
	All Loans 5.9	5.5	2.3	2.5	13.0	
	First Liens	3.8	2.6	7.6	0.0	46.9
	Junior Liens	5.0	5.1	3.0	0.0	14.8
	C & I	8.2	7.2	4.4	0.0	24.4
2018	CRE	8.3	8.2	3.3	0.0	18.8
	Credit Cards	11.3	13.9	6.6	0.0	21.2
	Other Consumer	7.0	5.9	4.4	0.6	18.0
	Other Loans	4.1	4.3	1.7	0.6	7.8
	All Loans	6.4	6.1	2.7	0.6	14.2
	First Liens	2.5	1.5	5.1	0.0	22.9
	Junior Liens	3.1	3.3	2.4	0.0	8.7
	C & I	7.2	6.0	5.1	0.0	22.6
2019	CRE	6.8	6.9	2.8	0.0	14.2
	Credit Cards	10.6	15.1	8.4	0.0	23.0
	Other Consumer	6.9	6.5	4.9	0.6	14.0
	Other Loans	3.4	3.1	1.7	0.6	6.3
	All Loans	5.4	4.7	3.3	0.6	15.1
	First Liens	2.3	1.7	4.3	0.0	25.9
	Junior Liens	3.7	3.8	2.4	0.0	10.0
	C & I	8.0	6.7	4.3	0.0	20.9
2020	CRE	6.8	6.3	3.7	0.0	20.9
	Credit Cards	14.6	18.1	8.1	0.0	26.4
	Other Consumer	8.2	7.1	5.1	0.6	17.3
	Other Loans	4.2	4.0	2.0	0.6	11.1
	All Loans	6.4	5.7	3.3	0.9	17.0

Table 2.4: Summary statistics for loss rates (in percent) across loan categories for DFAST 2016, DFAST 2017, DFAST 2018, DFAST 2019 and DFAST 2020.

The average R^2 across the 30 BHCs is 0.96. The slopes vary by bank but are mostly between 1.1 and 1.3. Few of the intercepts are significantly different from zero.

To put these patterns in perspective, consider that each scenario in DFAST 2014 is defined by the paths over nine quarters of 28 economic variables, so each scenario is a $9 \times 28 = 252$ dimensional object. This leaves a lot of room for differences across scenarios. I might expect different loan categories to respond differently to two such scenarios, given the complexity of the model. Yet the results say otherwise. Consider Bank of America, for example. The results say that its projected losses across all

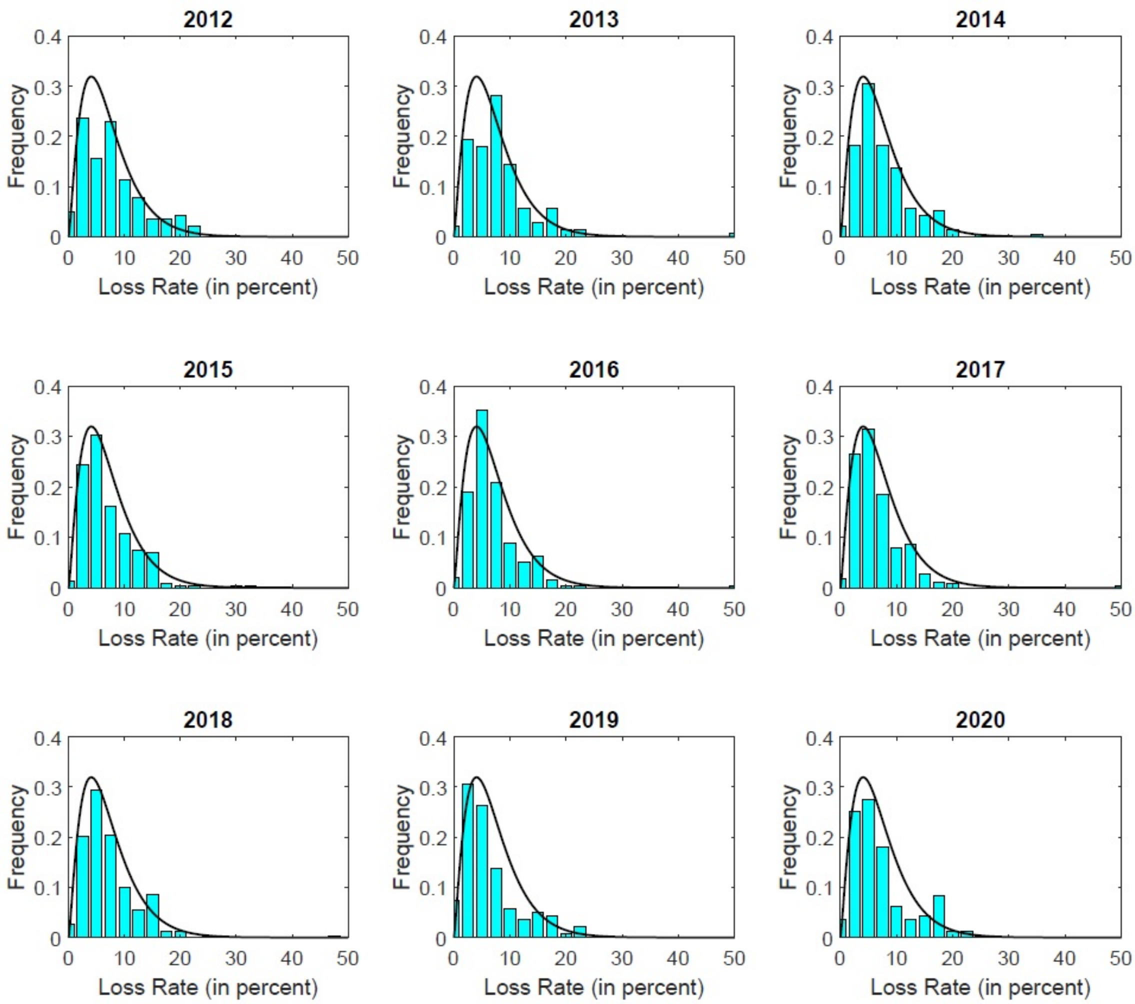


Figure 2.1: Distribution of loss rates across all loan categories and all banks in DFAST 2012–2020. The continuous curve in the figures is a gamma density estimated from the 2012–2020 data.

seven loan categories are a little more than 25 percent worse in one scenario than the other, effectively reducing stress severity to a single dimension.

Figure 2.3 shows corresponding results grouped by asset category and pooled across banks for DFAST 2014. For the seven loan categories, I show loss rates rather than dollar losses to put BHCs of different sizes on a consistent scale. In the lower right panel of the chart, I have included trading and counterparty losses. These are reported in billions of dollars because the Federal Reserve does not report rates for this category. Only eight of the 18 BHCs participate in this part of the stress test,

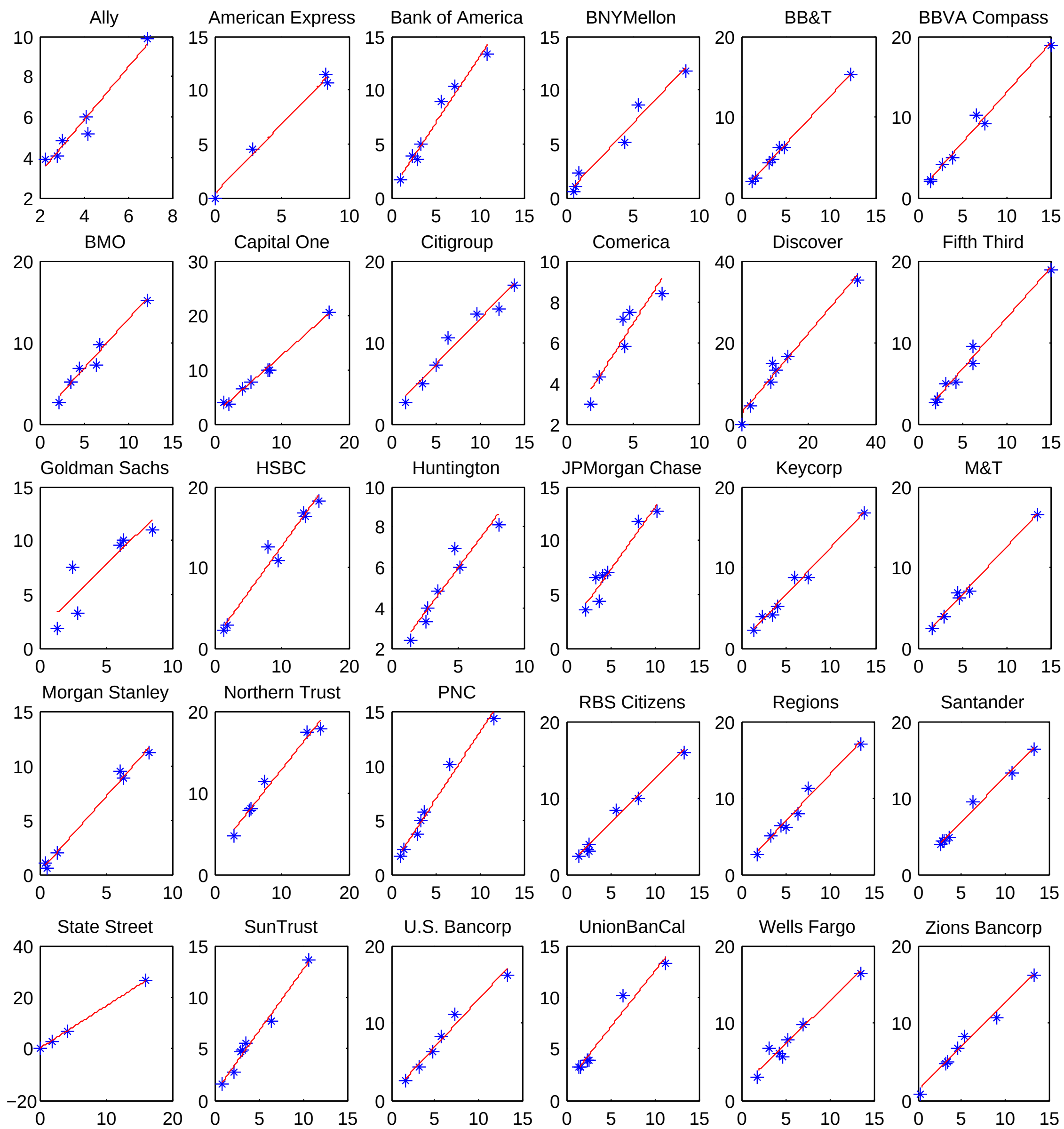


Figure 2.2: The plot for each BHC shows the severely adverse loss rate on the vertical scale and the adverse loss rate on the horizontal scale. Loss rates are in percent. Values shown are for DFAST 2014.

and these are all among the largest BHCs, so size discrepancy is less of a concern in this category.

Again, I see a striking linear relationship between the two scenarios across all categories. The corresponding regression is now

$$SevereLossRate_{b,c} = \text{Intercept}_c + \text{Slope}_c \times AdverseLossRate_{b,c}, \qquad (2.2)$$

with coefficients that depend on the loan category c but not the bank b. Table 2.6 quantifies the pattern in the scatter plots. The results are surprising, even given the results of Figure 2.2: because the banks have different slopes and intercepts, there is no reason to expect that pooling the bank-specific linear relationships would produce category-specific linear relationships in loss rates. I repeat this exercise for DFAST 2015–2019 and present the results of the regression in Tables 2.5 – 2.16. Recall that loss rates for the adverse scenario in DFAST 2020 was not made public. The strong linear relationships continue to hold with a few exceptions, notably Discover Financial Services in DFAST 2018 where the outlier in Commercial Real Estate loans drives the value of R^2 down to 0.53. Once the outlier is removed, the R^2 increases to 0.83.

These patterns are puzzling. I would expect to see a more complex relationship between adverse and severely adverse outcomes, reflecting a nonlinear response of bank portfolios to economic shocks. The patterns appear to be an artifact of the stress testing process rather than an accurate reflection of potential bank losses. They suggest an opportunity to get more information out of the stress tests through greater diversity in the scenarios used.

In what follows, I confine myself to the predictability of stress losses in the severely adverse scenario.

Bank Holding Company	Slope_b	Intercept_b	R^2
Ally Financial Inc.	1.31^{***}	0.56	0.96
American Express Company	1.25^{**}	0.33	0.96
Bank of America Corporation	1.25^{***}	0.65	0.97
The Bank of New York Mellon Corporation	1.31^{***}	0.25	0.97
BB&T Corporation	1.19^{***}	0.62^{**}	1.00
BBVA Compass Bancshares, Inc.	1.24^{***}	0.50	0.99
BMO Financial Corp.	1.20^{***}	0.66	0.97
Capital One Financial Corporation	1.08^{***}	1.63^{***}	0.99
Citigroup Inc.	1.12^{***}	1.48	0.96
Comerica Incorporated	1.02^{***}	1.74	0.87
Discover Financial Services	0.98^{***}	2.35^{*}	0.97
Fifth Third Bancorp	1.25^{***}	0.34	0.99
The Goldman Sachs Group, Inc.	1.21^{**}	1.62	0.79
HSBC North America Holdings Inc.	1.15^{***}	1.05	0.97
Huntington Banchsares Incorporated	0.89^{***}	1.48^{**}	0.91
JPMorgan Chase & Co.	1.15^{***}	1.49	0.93
KeyCorp	1.18^{***}	0.60	0.98
M&T Bank Corporation	1.18^{***}	0.62^{*}	0.99
Morgan Stanley	1.39^{***}	0.12	0.99
Northern Trust Corporation	1.04^{***}	2.37^{**}	0.98
The PNC Financial Services Group, Inc.	1.24^{***}	0.62	0.98
RBS Citizens Financial Group, Inc.	1.16^{***}	0.73	0.99
Regions Financial Corporation	1.23^{***}	0.61	0.98
Santander Holdings USA, Inc.	1.16^{***}	0.94^{*}	0.99
State Street Corporation	1.64^{***}	-0.13	1.00
SunTrust Banks, Inc.	1.20^{***}	0.66	0.98
U.S. Bancorp	1.22^{***}	0.69	0.97
UnionBanCal Corporation	1.09^{***}	1.57^{**}	0.97
Wells Fargo & Company	1.11^{***}	1.62^{*}	0.96
Zions Bancorporation	1.14^{***}	1.12^{*}	0.98

Table 2.5: Results of regression (2.1) by bank holding company of DFAST 2014 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Category	Slope_c	Intercept_c	R^2
First Liens	1.22^{***}	0.98^{***}	0.90
Junior Liens	1.23^{***}	0.51	0.95
C &I	1.30^{***}	0.58^{***}	0.98
CRE	1.02^{***}	3.07^{***}	0.94
Credit Cards	1.28^{***}	-0.71	0.97
Other Consumer	1.14^{***}	0.21^{**}	1.00
Other Loans	1.51^{***}	0.02	0.97
All Loans	1.15^{***}	1.08^{***}	0.96

Table 2.6: Results of regression (2.2) by loan category of DFAST 2014 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Bank Holding Company	Slope_b	Intercept_b	R^2
Ally Financial Inc.	1.68^{***}	-0.39	0.95
American Express Company	1.22^{***}	0.15	0.99
Bank of America Corporation	1.29^{***}	0.83	0.95
The Bank of New York Mellon Corporation	1.38^{***}	0.38	0.95
BB&T Corporation	1.21^{***}	0.88^{**}	0.98
BBVA Compass Bancshares, Inc.	1.25^{***}	0.76	0.93
BMO Financial Corp.	1.20^{***}	0.99	0.94
Capital One Financial Corporation	1.14^{***}	1.43^{***}	0.99
Citigroup Inc.	1.12^{***}	1.70	0.93
Citizens Financial Group, Inc.	1.25^{***}	0.75	0.93
Comerica Incorporated	1.34^{***}	0.70	0.88
Deutsche Bank Trust Corporation	1.73^{***}	-0.38	0.98
Discover Financial Services	1.61^{***}	-2.09	0.91
Fifth Third Bancorp	1.40^{***}	0.18	0.95
The Goldman Sachs Group, Inc.	1.74^{***}	-0.47	0.97
HSBC North America Holdings Inc.	1.08^{***}	1.69^{*}	0.98
Huntington Bancshares Incorporated	1.20^{***}	0.70	0.98
JPMorgan Chase & Co.	1.22^{***}	1.30	0.90
KeyCorp	1.17^{***}	1.00	0.96
M&T Bank Corporation	1.17^{***}	0.97^{*}	0.98
Morgan Stanley	1.83^{***}	-0.32	1.00
MUFG Americas Holdings Corporation	1.19^{***}	1.39^{*}	0.95
Northern Trust Corporation	1.28^{***}	0.81	0.97
The PNC Financial Services Group, Inc.	1.30^{***}	0.64	0.96
Regions Financial Corporation	1.36^{***}	0.29	0.93
Santander Holdings USA, Inc.	1.20^{***}	1.03	0.98
State Street Corporation	1.74^{***}	-0.07	1.00
SunTrust Banks, Inc.	1.24^{***}	0.66	0.97
U.S. Bancorp	1.26^{***}	0.95	0.95
Wells Fargo & Company	1.14^{***}	1.57^{**}	0.96
Zions Bancorporation	1.14^{***}	1.46	0.97

Table 2.7: Results of regression (2.1) by bank holding company of DFAST 2015 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Category	Slope_c	Intercept_c	R^2
First Liens	1.20^{***}	0.38^{***}	0.99
Junior Liens	1.17^{***}	1.32^{***}	0.95
C & I	1.34^{***}	1.01^{***}	0.95
CRE	1.73^{***}	-0.05	1.00
Credit Cards	1.24^{***}	0.05	1.00
Other Consumer	1.21^{***}	0.22	0.99
Other Loans	1.82^{***}	-0.12^{*}	0.99
All Loans	1.16^{***}	1.26^{***}	0.98

Table 2.8: Results of regression (2.2) by loan category of DFAST 2015 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Bank Holding Company	Slope_b	Intercept_b	R^2
Ally Financial Inc.	1.55^{***}	0.16	0.97
American Express Company	1.21^{***}	0.47	0.97
BancWest Corporation	1.11^{***}	2.04^{**}	0.97
Bank of America Corporation	1.22^{***}	1.20	0.94
The Bank of New York Mellon Corporation	1.28^{***}	0.88	0.90
BB&T Corporation	1.20^{***}	1.33	0.94
BBVA Compass Bancshares, Inc.	1.16^{***}	1.40	0.93
BMO Financial Corp.	1.14^{***}	1.49^{*}	0.94
Capital One Financial Corporation	1.18^{***}	1.14^{*}	0.99
Citigroup Inc.	1.05^{***}	2.41^{*}	0.87
Citizens Financial Group, Inc.	1.14^{***}	1.32	0.93
Comerica Incorporated	1.22^{***}	1.14	0.88
Deutsche Bank Trust Corporation	1.49^{***}	0.43	0.79
Discover Financial Services	1.49^{**}	0.20	0.66
Fifth Third Bancorp	1.34^{***}	0.92	0.88
The Goldman Sachs Group, Inc.	1.04^{***}	2.07^{*}	0.99
HSBC North America Holdings Inc.	1.07^{***}	1.93^{**}	0.97
Huntington Bancshares Incorporated	1.18^{***}	1.16^{*}	0.96
JPMorgan Chase & Co.	1.25^{***}	1.32^{*}	0.94
KeyCorp	1.14^{***}	1.41^{*}	0.94
M&T Bank Corporation	1.10^{***}	1.68^{**}	0.96
Morgan Stanley	1.48^{***}	0.40	0.92
MUFG Americas Holdings Corporation	1.18^{***}	1.53^{**}	0.96
Northern Trust Corporation	1.29^{***}	1.15	0.96
The PNC Financial Services Group, Inc.	1.23^{***}	1.03	0.91
Regions Financial Corporation	1.28^{***}	1.05	0.91
Santander Holdings USA, Inc.	1.25^{***}	1.14^{**}	0.99
State Street Corporation	2.06^{***}	-0.13	0.95
SunTrust Banks, Inc.	1.24^{***}	0.97^{*}	0.97
TD Group US Holdings LLC	1.15^{***}	1.68	0.93
U.S. Bancorp	1.23^{***}	1.49	0.89
Wells Fargo & Company	1.12^{***}	1.87^{**}	0.95
Zions Bancorporation	1.15^{***}	1.63^{**}	0.97

Table 2.9: Results of regression (2.1) by bank holding company of DFAST 2016 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Category	Slope_c	Intercept_c	R^2
First Liens	1.06^{***}	1.01^{***}	1.00
Junior Liens	1.28^{***}	0.73^{***}	0.96
C & I	1.32^{***}	1.12^{***}	0.96
CRE	2.16^{***}	0.16	0.98
Credit Cards	1.27^{***}	0.10	1.00
Other Consumer	1.22^{***}	0.42^{**}	0.98
Other Loans	1.72^{***}	0.04	0.99
All Loans	1.17^{***}	1.49^{***}	0.97

Table 2.10: Results of regression (2.2) by loan category of DFAST 2016 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Bank Holding Company	Slope_b	Intercept_b	R^2
Ally Financial Inc.	1.50^{***}	0.18	0.99
American Express Company	1.35^{***}	−0.01	1.00
BancWest Corporation	1.15^{***}	1.69^{**}	0.95
Bank of America Corporation	1.25^{***}	1.08	0.92
The Bank of New York Mellon Corporation	1.34^{***}	0.52	0.89
BB&T Corporation	1.23^{***}	0.98^{*}	0.97
BBVA Compass Bancshares, Inc.	1.18^{***}	1.27	0.96
BMO Financial Corp.	1.14^{***}	1.39	0.92
Capital One Financial Corporation	1.19^{***}	1.20^{*}	0.98
CIT Group Inc.	1.35^{***}	0.83	0.91
Citigroup Inc.	1.08^{***}	1.96^{*}	0.93
Citizens Financial Group, Inc.	1.22^{***}	1.02	0.92
Comerica Incorporated	1.14^{***}	1.06	0.89
Deutsche Bank Trust Corporation	1.62^{***}	−0.13	0.82
Discover Financial Services	1.17^{**}	1.85	0.73
Fifth Third Bancorp	1.39^{***}	0.65	0.91
The Goldman Sachs Group, Inc.	1.02^{***}	2.06^{*}	0.99
HSBC North America Holdings Inc.	1.14^{***}	1.39^{*}	0.95
Huntington Bancshares Incorporated	1.21^{***}	1.07^{*}	0.97
JPMorgan Chase & Co.	1.24^{***}	0.96^{**}	0.98
KeyCorp	1.24^{***}	1.03	0.87
M&T Bank Corporation	1.19^{***}	1.11	0.94
Morgan Stanley	1.49^{***}	0.26	0.96
MUFG Americas Holdings Corporation	1.13^{***}	1.80^{**}	0.97
Northern Trust Corporation	1.21^{***}	1.12	0.95
The PNC Financial Services Group, Inc.	1.27^{***}	0.80	0.96
Regions Financial Corporation	1.29^{***}	0.86	0.93
Santander Holdings USA, Inc.	1.27^{***}	0.92	0.98
State Street Corporation	1.88^{***}	−0.14	0.95
SunTrust Banks, Inc.	1.27^{***}	0.91	0.96
TD Group US Holdings LLC	1.18^{***}	1.08^{*}	0.98
U.S. Bancorp	1.24^{***}	1.22	0.93
Wells Fargo & Company	1.16^{***}	1.56^{*}	0.95
Zions Bancorporation	1.21^{***}	1.31	0.95

Table 2.11: Results of regression (2.1) by bank holding company of DFAST 2017 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Category	Slope_c	Intercept_c	R^2
First Liens	1.04^{***}	0.83^{***}	1.00
Junior Liens	1.18^{***}	0.86^{***}	0.94
C & I	1.37^{***}	0.58^{***}	0.98
CRE	2.08^{***}	0.07	0.98
Credit Cards	1.28^{***}	0.07	1.00
Other Consumer	1.26^{***}	0.23	0.99
Other Loans	1.52^{***}	0.16^{**}	0.99
All Loans	1.21^{***}	1.23^{***}	0.97

Table 2.12: Results of regression (2.2) by loan category of DFAST 2017 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Bank Holding Company	Slope_b	Intercept_b	R^2
Ally Financial Inc.	1.52^{***}	0.51	0.94
American Express Company	1.30^{***}	0.33	0.98
Bank of America Corporation	1.21^{***}	1.77	0.90
The Bank of New York Mellon Corporation	1.37^{**}	1.15^{*}	0.73
Barclays US LLC	1.18^{***}	1.06	0.97
BB&T Corporation	1.21^{***}	1.74^{**}	0.94
BBVA Compass Bancshares, Inc.	1.11^{***}	2.41	0.86
BMO Financial Corp.	1.19^{***}	1.80	0.87
BNP Paribas USA, Inc.	1.23^{***}	2.05^{**}	0.94
Capital One Financial Corporation	1.20^{***}	1.75^{*}	0.96
Citigroup Inc.	0.91^{**}	3.50^{*}	0.71
Citizens Financial Group, Inc.	1.16^{***}	1.94	0.80
Credit Suisse Holdings (USA), Inc.	1.27^{***}	-0.01	1.00
DB USA Corporation	1.49^{**}	0.96	0.65
Discover Financial Services	1.07^{*}	4.33	0.53
Fifth Third Bancorp	1.43^{***}	1.13	0.87
The Goldman Sachs Group, Inc.	1.11^{***}	2.91^{*}	0.97
HSBC North America Holdings Inc.	1.08^{***}	2.59^{*}	0.83
Huntington Bancshares Incorporated	1.29^{***}	1.38	0.94
JPMorgan Chase & Co.	1.23^{***}	1.63^{**}	0.96
KeyCorp	1.23^{***}	1.58	0.85
M&T Bank Corporation	1.22^{***}	1.90	0.87
Morgan Stanley	1.66^{***}	0.66	0.89
MUFG Americas Holdings Corporation	1.00^{***}	3.37^{***}	0.95
Northern Trust Corporation	1.20^{***}	2.03	0.85
The PNC Financial Services Group, Inc.	1.30^{***}	1.20	0.92
RBC USA Holdco Corporation	1.14^{***}	2.79^{**}	0.91
Regions Financial Corporation	1.23^{***}	1.77	0.87
Santander Holdings USA, Inc.	1.11^{***}	2.34^{**}	0.97
State Street Corporation	1.89^{***}	0.06	0.89
SunTrust Banks, Inc.	1.24^{***}	1.69	0.90
TD Group US Holdings LLC	1.20^{***}	1.59	0.96
UBS Americas Holding LLC	1.37^{***}	0.99	0.95
U.S. Bancorp	1.24^{***}	1.89	0.87
Wells Fargo & Company	1.11^{***}	2.44^{*}	0.87

Table 2.13: Results of regression (2.1) by bank holding company of DFAST 2018 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Category	Slope_c	Intercept_c	R^2
First Liens	1.13^{***}	1.43^{***}	0.99
Junior Liens	1.70^{***}	0.73^{**}	0.87
C & I	1.25^{***}	1.60^{***}	0.96
CRE	2.50^{***}	0.35	0.93
Credit Cards	1.34^{***}	0.11	1.00
Other Consumer	1.23^{***}	0.22	0.99
Other Loans	1.77^{***}	-0.22^{***}	0.99
All Loans	1.21^{***}	1.66^{***}	0.95

Table 2.14: Results of regression (2.2) by loan category of DFAST 2018 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

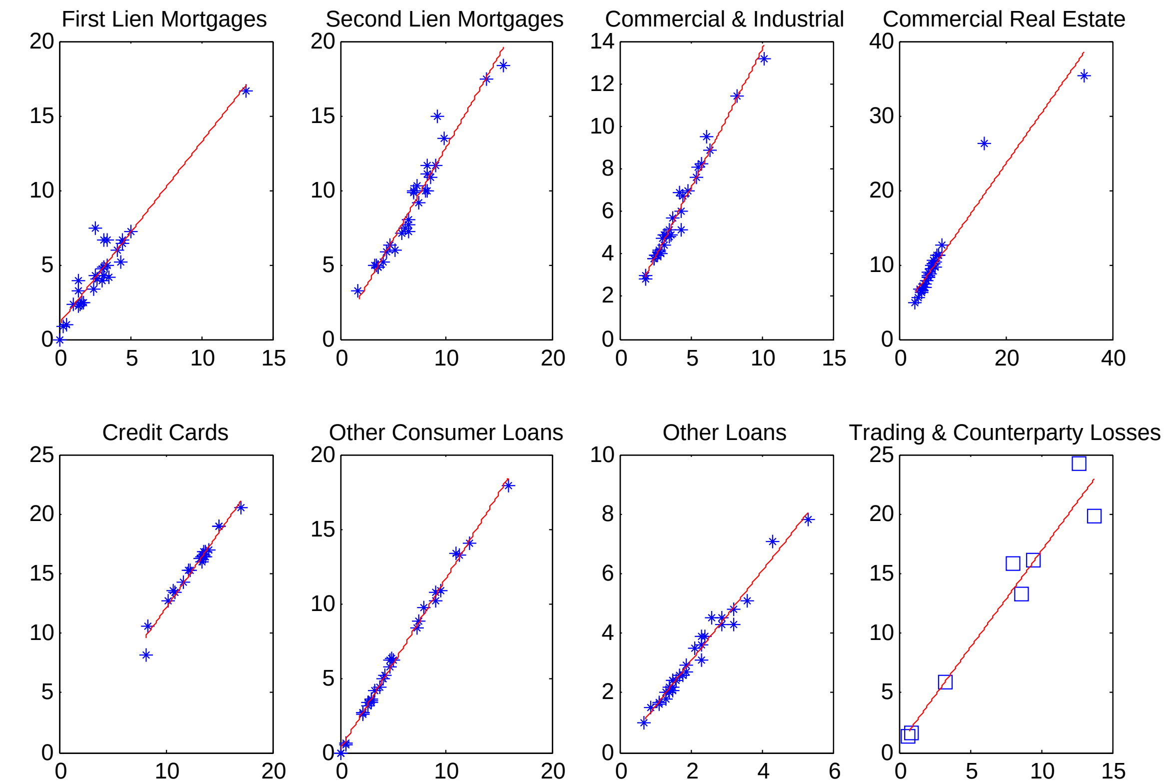


Figure 2.3: The plot for each loan category shows the severely adverse loss rate on the vertical scale and the adverse loss rate on the horizontal scale. Loss rates are in percent. Trading and counterparty losses are in billions of dollars. Values shown are for DFAST 2014.

Bank Holding Company	Slope$_b$	Intercept$_b$	R^2
Bank of America Corporation	1.33^{***}	0.90	0.97
The Bank of New York Mellon Corporation	1.28^{***}	0.97	0.87
Barclays US LLC	1.21^{***}	0.85	0.98
Capital One Financial Corporation	1.25^{***}	1.38^{**}	0.99
Citigroup Inc.	1.16^{***}	1.54	0.91
Credit Suisse Holdings (USA), Inc.	1.25^{***}	0.00	1.00
DB USA Corporation	1.40^{**}	0.59	0.71
The Goldman Sachs Group, Inc.	1.07^{***}	2.86	0.85
HSBC North America Holdings Inc.	1.26^{***}	1.46	0.90
JPMorgan Chase & Co.	1.36^{***}	0.64^{*}	0.99
Morgan Stanley	1.73^{***}	0.10	0.86
Northern Trust Corporation	1.18^{***}	1.27^{*}	0.95
The PNC Financial Services Group, Inc.	1.40^{***}	0.70	0.96
State Street Corporation	1.83^{***}	0.01	0.90
TD Group US Holdings LLC	1.28^{***}	0.84	0.99
UBS Americas Holding LLC	1.44^{***}	0.59	0.98
U.S. Bancorp	1.31^{***}	0.96	0.95
Wells Fargo & Company	1.27^{***}	1.15	0.95

Table 2.15: Results of regression (2.1) by bank holding company of DFAST 2019 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

Category	Slope_c	Intercept_c	R^2
First Liens	1.18^{***}	0.56^{***}	1.00
Junior Liens	1.46^{***}	0.19	0.96
C & I	1.24^{***}	1.24^{***}	0.98
CRE	2.50^{***}	-0.02	0.97
Credit Cards	1.37^{***}	0.11	1.00
Other Consumer	1.21^{***}	0.12	1.00
Other Loans	1.70^{***}	-0.17	0.99
All Loans	1.32^{***}	0.76^{***}	0.98

Table 2.16: Results of regression (2.2) by loan category of DFAST 2019 severely adverse loss rates versus adverse loss rates. Asterisks indicate statistical significance at the 10% level (*), 5% level (**), and 1% level (***).

T	Slope_c	Intercept_c	R^2
2013	0.93^{***}	0.02	0.93
2014	0.97^{***}	0.00	0.94
2015	0.95^{***}	0.01	0.96
2016	0.97^{***}	0.05^{***}	0.95
2017	0.99^{***}	-0.02^{*}	0.95
2018	0.99^{***}	0.06^{***}	0.98
2019	0.99^{***}	-0.06^{***}	0.96
2020	1.00^{***}	0.03^{***}	0.98

Table 2.17: Results of regression (2.3) for all DFAST tests from $T = 2013$ to $T = 2020$. Asterisks indicate statistical significance at 10% level (*), 5% level (**), and 1% level (***).

2.4 Predictability in Loss Levels

In this section, I examine the relationship between the projected losses S_T and S_{T-1} from stress tests run in years T and $T-1$, respectively. To do so, I need to limit myself to those BHCs that participated in both the years.

I start by pooling losses across all banks (that are common to years T and $T-1$) and all loan categories. Because the BHCs vary widely in size, I take logarithms of the losses to put them on a more consistent scale. I run the following regression:

$$\log_{10}(S_T^{b,c}) = \text{Intercept}_T + \text{Slope}_T \times \log_{10}(S_{T-1}^{b,c}). \tag{2.3}$$

I run the regressions from $T = 2013$ to $T = 2020$. I summarize the results in Table 2.17.

Table 2.17 clearly shows that losses by bank and loan category are highly persistent from one year to the next. In every case, the R^2 is greater than 0.90 and is generally increasing with time, suggesting a trend towards increasing predictability. The slopes are all highly significant and close to, but less than one, suggesting that losses are decreasing with time and the intercepts are all close to zero. Table 2.17 pools losses across all banks and categories. To examine individual loan types, I run the following regression for each category c using losses for each bank b:

$$\log_{10}(S_T^{b,c}) = \text{Intercept}_c + \text{Slope}_c \times \log_{10}(S_{T-1}^{b,c}). \tag{2.4}$$

I run the regressions from $T = 2013$ and $T = 2020$. Tables 2.18–2.20 summarize the results. The R^2 is very high in almost all cases. The intercepts are close to zero, and the slopes are close to one. The results indicate a high degree of predictability in a bank's projected losses in an individual loan category from one year to the next.

Category	$T = 2013$			$T = 2014$			T = 2015		
	Slope	Intercept	R^2	Slope	Intercept	R^2	Slope	Intercept	R^2
First Liens	0.98^{***}	-0.02	0.96	1.03^{***}	-0.12^{**}	0.93	0.82^{***}	-0.08^{***}	0.95
Junior Liens	0.84^{***}	-0.08	0.85	1.13^{***}	-0.11^{**}	0.94	0.93^{***}	-0.08^{***}	0.99
C & I	0.83^{***}	0.08	0.84	1.04^{***}	-0.09^{**}	0.95	1.01^{***}	0.03	0.95
CRE	1.05^{***}	0.12^{**}	0.93	0.93^{***}	0.08^{**}	0.95	0.85^{***}	0.07^{**}	0.89
Credit Cards	0.96^{***}	0.03	0.99	0.99^{***}	-0.02^{*}	1.00	1.01^{***}	-0.06^{**}	0.99
Other Consumer	0.89^{***}	0.09^{**}	0.94	1.04^{***}	-0.03	0.97	0.91^{***}	0.06^{***}	0.98
Other Loans	0.89^{***}	-0.11^{**}	0.89	0.92^{***}	0.16^{*}	0.75	1.03^{***}	0.06^{**}	0.96
All Loans	0.82^{***}	0.21^{***}	0.93	0.99^{***}	-0.01	0.98	0.91^{***}	0.08^{***}	0.98

Table 2.18: Regression estimates for log losses on lagged log losses, by loan category, as in (2.4) for $T = 2013, 2014$ and 2015. Asterisks indicate results statistically significantly different from zero at the 10% level (*), 5% level (**) and 1% level (***).

Category	$T = 2016$			$T = 2017$			T = 2018		
	Slope	Intercept	R^2	Slope	Intercept	R^2	Slope	Intercept	R^2
First Liens	0.88^{***}	-0.02	0.89	0.88^{***}	-0.16^{***}	0.80	0.97^{***}	0.10^{***}	0.92
Junior Liens	0.97^{***}	-0.05	0.94	0.82^{***}	-0.23^{***}	0.87	0.92^{***}	-0.03	0.95
C & I	0.98^{***}	0.14^{***}	0.96	1.03^{***}	0.02	0.98	1.00^{***}	0.06^{***}	0.98
CRE	0.96^{***}	-0.05^{*}	0.93	0.97^{***}	0.03^{*}	0.96	0.97^{***}	0.09^{***}	0.96
Credit Cards	0.96^{***}	0.07^{***}	0.99	1.02^{***}	0.02^{***}	1.00	1.02^{***}	0.02^{***}	1.00
Other Consumer	0.94^{***}	0.04	0.94	1.04^{***}	0.02	0.98	0.86^{***}	0.02	0.94
Other Loans	0.97^{***}	0.13^{***}	0.91	0.90^{***}	0.04	0.91	1.02^{***}	0.08^{***}	0.97
All Loans	0.98^{***}	0.06^{*}	0.96	0.99^{***}	0.02	0.98	1.00^{***}	0.06^{***}	0.99

Table 2.19: Regression estimates for log losses on lagged log losses, by loan category, as in (2.4) for $T = 2016, 2017$ and 2018. Asterisks indicate results statistically significantly different from zero at the 10% level (*), 5% level (**) and 1% level (***).

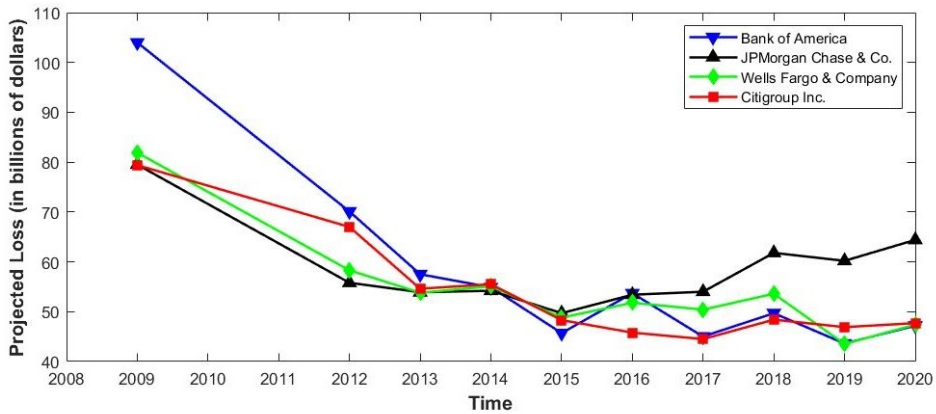


Figure 2.4: Projected stress loan losses across the SCAP 2009, the CCAR 2012, and DFAST 2013 – 2020 for Bank of America, Citigroup Inc., JPMorgan Chase & Co and Wells Fargo & Company. The chart shows loan losses only and does not include trading and counterparty losses.

Category	$T = 2019$			$T = 2020$		
	Slope	Intercept	R^2	Slope	Intercept	R^2
First Liens	0.91^{***}	-0.22^{***}	0.98	1.00^{***}	0.01	0.93
Junior Liens	0.72^{***}	-0.26^{***}	0.97	0.92^{***}	-0.05	0.95
C & I	1.01^{***}	-0.06^{***}	0.99	1.03^{***}	0.01	0.97
CRE	0.96^{***}	-0.08^{*}	0.95	0.94^{***}	-0.01	0.98
Credit Cards	0.92^{***}	0.13^{***}	0.99	0.89^{***}	0.16^{***}	1.00
Other Consumer	0.88^{***}	0.06	0.95	0.96^{***}	0.05^{*}	0.98
Other Loans	0.99^{***}	-0.04	0.97	0.99^{***}	0.04^{**}	0.99
All Loans	0.99^{***}	-0.03	0.99	1.00^{***}	0.03^{**}	1.00

Table 2.20: Regression estimates for log losses on lagged log losses, by loan category, as in (2.4) for $T = 2019$ and 2020. Asterisks indicate results statistically significantly different from zero at the 10% level (*), 5% level (**) and 1% level (***).

Figure 2.4 summarizes projected stress loan losses from 2009 SCAP to DFAST 20 for the four largest banks – Bank of America Corporation, Citigroup Inc., JPMorgan Chase & Co. and Wells Fargo & Company. The chart shows a striking convergence for three of the four banks in the projected loss levels. The loss levels for JPMorgan Chase & Co. start diverging around 2017 in an increasing fashion. This is consistent with news reported that JPMorgan Chase & Co. had to resubmit their capital plans in 2019 after failing the first time (see Lang [44]). The chart shows loan losses only and does not include trading and counterparty losses.

2.5 Predictability in Loss Rates

The projected loss rate for a given bank in a given category is the corresponding projected loss level divided by the pre-stress value of the bank's assets in that category. Loss rates are more sensitive to small changes than are loss levels, particularly when the denominator is small. For example, Goldman Sachs's C&I loss rate in DFAST 2013 is huge at 49.8%. But its projected loss in that category is only $1.4 billion, below the median projected C&I loss that year and much smaller than Goldman Sachs's projected trading and counterparty loss of $24.9 billion.

Tables 2.21 and 2.22 show the correlation in loss rates across consecutive years broken down by loan category across all BHCs that participated in both years. All Loans corresponds to the overall loss rate which is the size-weighted average of a BHC's loss rates across the seven loan categories. These overall rates are included in the Federal Reserve's CCAR and DFAST reports. I find a general trend towards increased predictability across the years. One notable exception is the Commercial Real Estate loan 2019–2020 correlation of -0.15, which is primarily driven by the Credit Suisse Holdings (USA), Inc. without which the correlation increases to 0.78.

The correlations vary widely across categories, which makes the predictability of the overall rates in Table 2.21 even more surprising. The pattern suggests that banks' overall loss rates are much more stable than their loss rates in individual categories; higher loss rate projections for a bank in one category tend to be offset by lower projections in another category.

Category	2012–2013	2013–2014	2014–2015	2015–2016
First Liens	0.53	0.70	0.84	0.25
Junior Liens	0.10	0.41	0.94	0.85
C & I	0.57	0.46	0.91	0.91
CRE	0.80	0.88	0.90	0.60
Credit Cards	0.66	0.98	0.57	0.97
Other Consumer	0.78	0.98	0.66	0.98
Other	0.17	0.55	0.42	0.64
All Loans	0.86	0.96	0.95	0.89

Table 2.21: Correlations in loss rates for consecutive years by loan category from $T = 2012$ to $T = 2016$.

Category	2016–2017	2017–2018	2018–2019	2019–2020
First Liens	0.99	1.00	1.00	1.00
Junior Liens	0.55	0.86	0.58	0.77
C & I	0.93	0.92	0.99	0.95
CRE	0.85	0.93	0.74	−0.15
Credit Cards	0.56	0.96	0.98	0.41
Other Consumer	0.80	0.82	0.97	0.99
Other	0.49	0.65	0.93	0.91
All Loans	0.96	0.99	0.98	0.99

Table 2.22: Correlations in loss rates for consecutive years by loan category from $T = 2016$ to $T = 2020$.

I have experimented with using other variables to forecast loss rates, including actual charge-offs reported by the BHCs, stock returns and stock return volatility for individual BHCs. In some cases, actual charge-offs appear to have some forecasting power: BHCs reporting higher loan losses in the prior year often experience higher loss rate projections in the subsequent stress test. However, none of the variables I tested adds much in forecasting stress loss rates compared with using a bank's prior year's stress loss rate.

2.6 Stock Market Reaction to Stress Test Results

In this section, I examine the stock market's response to the Federal Reserve's announcement of the DFAST results. The stock market's response is a measure of the informativeness of the results. I carry out this analysis for the banks that participated in consecutive rounds of stress tests and are publically traded.

Peristiani, Morgan, and Savino [49] analyze the market's response to the SCAP, the first of the Fed's stress tests. They find that the results of the SCAP were highly informative for the banks that were found to require additional capital but not for the banks that "passed" the stress test. Glasserman and Wang [31] find a significant correlation between the value of the SCAP's government backstop and the market's response to the announcement of the terms of the program.

The DFAST results were always announced at the end of the respective business day. I evaluate the stock market's response by calculating the return for each bank from its closing price on the date of announcement to its closing price on the next business day. To remove the overall effect of the market on that day, for each bank I run a regression

$$BankReturn_{b,t} = \alpha_b + \beta_b MarketReturn_t \tag{2.5}$$

using daily returns for one year prior to the date of the announcement and using the CRSP value-weighted index for the market return. The unexpected component of the stock market response for each bank is the difference

$$\eta_{b,T} = BankReturn_{b,T} - (\alpha_b + \beta_b MarketReturn_T) \tag{2.6}$$

evaluated on the next business day after the date of the announcement. For example, DFAST 2014 was announced on March 20 and so the impact of the stock market is evaluated on March 21, 2014.

I compare the unexpected returns η_b with the DFAST stress test results in two ways. First, I measure the correlation ρ_{overall} between the unexpected returns and the overall loss rates reported. Next, I form a simple forecast of the stress test results. Suppose I intend to estimate the stock market reaction for year T. Then, I regress overall loss rates announced in year $T-1$ on loss rates announced in year $T-2$ to estimate a_0 and a_1 in the equation

$$LossRate_{b,T-1} = a_0 + a_1 \times LossRate_{b,T-2} \tag{2.7}$$

where b indexes the BHCs that have participated in all three rounds of stress tests. I use this equation to forecast the loss rates in year T as

$$\widehat{LossRate}_{b,T} = a_0 + a_1 \times LossRate_{b,T-1} \tag{2.8}$$

I take the differences between the actual and predicted loss rates,

$$\epsilon_{b,T} = LossRate_{b,T} - \widehat{LossRate}_{b,T} \tag{2.9}$$

as the unexpected component in the stress test results and then measure the correlation $\rho_{\text{unexpected}}$ with the abnormal stock returns calculated in equation (2.6). I measure the statistical significance of the correlation in terms of the $p-$value of the $F-$test for the respective regression. I report the observed correlations in Table 2.23.

Year	ρ_{overall}	$\rho_{\text{unexpected}}$
2014	-0.08	0.29
2015	0.00	0.18
2016	0.34^{*}	0.32^{*}
2017	0.15	-0.16
2018	0.09	-0.02
2019	-0.39	-0.43^{*}
2020	-0.09	0.70^{***}

Table 2.23: The correlations of the unexpected stock returns on the day after announcement and the announced overall and unexpected loss rates, ρ_{overall} and $\rho_{\text{unexpected}}$. Asterisks indicate results statistically significantly different from zero at the 10% level (*), 5% level (**) and 1% level (***).

The correlations observed in years 2014 and 2015 are very small and statistically not significant. This is consistent with the view that the loss rates reported did not inform the market. In both these years, I find that $\rho_{\text{unexpected}} > 0$ which is surprising: if the unexpected losses were informative, I would expect them to be negatively correlated with the excess returns. But the market likely forms a better forecast of the overall results using additional current information not captured in my simple forecast.

I observe a significant stock market impact in 2016 for both the overall and unexpected loss rate regressions. However, this may not necessarily imply that the effect is solely due to the Federal Reserve's stress test announcements because the results of the Brexit Referendum were announced on the exact same day as the DFAST 2016 results, on June 23. One cannot ignore the effect of this major political event on the observed correlations.

I see no statistically significant stock market reaction in the years 2017 and 2018. However, for both these years I have $\rho_{\text{unexpected}}$ smaller than ρ_{overall} and negative, which is consistent with the intuition outlined above.

For both 2019 and 2020, I observe a statistically significant $\rho_{\text{unexpected}}$ but a statistically insignificant ρ_{overall}. However, this may not necessarily mean that the stock market had a significant reaction to the stress test announcements. Recall that starting in 2019, the Federal Reserve decided to move less-complex banks to a two-year stress testing cycle. This has resulted in some data availability issues when it comes to performing the regression (2.7). Consequently, I end up considering a much smaller number of BHCs when evaluating the stock market impact, which could potentially throw the significance of the observed correlation into question. This is somewhat supported by the fact that ρ_{overall} is not significant for both these years.

The Federal Reserve's DFAST and CCAR announcements include much more than the stress test results. For example, in 2014, the CCAR results were announced a week after DFAST and the biggest surprise at the announcement was that Citigroup had "failed" for shortcomings in its internal processes that were not directly related to its projected stress losses. These other, simultaneous announcements make it difficult to isolate the effect of the stress test results, but there is no indication of a significant market reaction to these results.

2.7 Stock Market Reaction to Stress Test Announcement

In this section, I try to ascertain the fixed effect of the Federal Reserve's stress test announcements on the stock and option market. This is in contrast to the previous section where I examined the relationship between the stock market reaction and the loss rates reported in the DFAST announcement. For this section, I follow the analysis presented in Flannery, Hirtle and Kovner [28] and estimate the market reaction in terms of abnormal return (AR), abnormal volume (AV) and change in the option implied volatility (Δ VOL). However, in order to be consistent with Section 2.6 and compare results, I employ methodologies different from those in Flannery, Hirtle and Kovner [28] to estimate these quantities. I extend the analysis in Flannery, Hirtle and Kovner [28] to the years 2016 – 2020. In addition to the measures above, I follow the methodology outlined in Kelly, Pastor and Veronesi [42] and calculate and report the implied volatility slope difference (ΔVOLSLOPE). I elaborate on the specifics of the methodologies employed below.

2.7.1 Abnormal Return (AR)

I calculate the abnormal return η_b as before using regression (2.6) in Section 2.6 for every participating bank b. AR is calculated as the average of η_b for all banks. I then perform a $t-$test on this collection of abnormal returns to see if there is a statistically significant effect of the announcement on the stock market returns.

2.7.2 Abnormal Volume (AV)

Let τ denote the date of the announcement. To calculate the abnormal volume (AV), I first perform the regression

$$Vol_{b,t} = \mu_b + \nu_b MarketVol_t \tag{2.10}$$

for each bank b over the time window of one year prior to τ where Vol_b and Vol_{Market} are calculated as follows

$$\begin{aligned} Vol_{b,t} &= \frac{\text{No. of shares of } b \text{ traded on } t}{\text{Total Outstanding Shares}} \\ MarketVol_t &= \frac{\text{Sum of all shares traded of CRSP constituents on } t}{\text{Sum of total outstanding shares traded of CRSP constituents}} \end{aligned}$$

The abnormal volume for bank b is then calculated as

$$\epsilon_b = Vol_{b,\tau+1} - (\mu_b + \nu_b MarketVol_{\tau+1}) \tag{2.11}$$

AV is calculated as the average of ϵ_b for all banks. I then perform a $t-$test on this collection of abnormal volumes to see if there is a statistically significant effect of the announcement on the stock volume traded.

2.7.3 ΔVOL

I calculate the change of the implied volatility between the dates τ and $\tau+1$ for each bank b. ΔVOL is the average of the changes in implied volatility. I perform a $t-$test and report the values.

2.7.4 ΔVOLSLOPE

I follow the methodology outlined in Kelly, Pastor and Veronesi [42] to calculate ΔVOLSLOPE. I collect put options with option delta Δ satisfying $-0.5 < \Delta < -0.1$ for each bank b on dates τ and $\tau + 1$. I then perform the regressions

$$\sigma_{b,t} = \kappa_{b,t} + \lambda_{b,t}\Delta_{b,t}$$

on the dates $t = \tau$ and $t = \tau + 1$, where $\sigma_{b,t}$ and $\Delta_{b,t}$ are respectively the put option implied volatility and the corresponding Δ for bank b on date t. I then calculate the difference $\lambda_{b,\tau+1} - \lambda_{b,\tau}$ for each bank b. ΔVOLSLOPE is the average of these slope differences. I perform a $t-$test on slope differences and report the values. Since the Δ for put options is negative, a positive value of the slope difference indicates that deep out-of-the-money put options were particularly expensive before the stress test announcements indicating that investors were willing to pay a premium to hedge against potential market downfall.

2.7.5 Results and Discussion

I calculate AR, AV, ΔVOL and ΔVOLSLOPE for all the stress test announcements in the period 2009–2020. The results are summarized in Table 2.24.

Test Name	AR (%)	AV	VOL (%)	VOLSLOPE ($\times 10^{-2}$)
SCAP 2009	2.34	44.45	-4.05^{**}	-4.82
CCAR 2011	-1.68^{***}	8.02	-3.79^{***}	-1.31
CCAR 2012	1.46^{**}	19.50^{***}	-2.45	18.00
CCAR 2013	-0.13	-0.20	0.01	-0.07
DFAST 2013	0.59	3.69^{*}	-0.89^{***}	-2.52
CCAR 2014	-0.70^{**}	2.22	-0.64^{***}	-2.00
DFAST 2014	0.09	7.67^{**}	0.01	0.80
CCAR 2015	0.86^{***}	2.25	-3.45	9.87
DFAST 2015	1.98^{***}	1.02	0.47	-2.00
CCAR 2016	0.06	8.81^{***}	2.00	3.07
DFAST 2016	-0.76^{*}	-1.46	10.51^{***}	1.19
CCAR 2017	1.54^{***}	1.86	-0.87^{***}	-7.58
DFAST 2017	0.26	-8.79^{***}	-1.56	11.00
CCAR 2018	-0.11	-0.28	1.51	-0.10
DFAST 2018	-0.55^{**}	-13.01^{**}	4.49	-39.00
CCAR 2019	0.52	-12.47^{*}	-1.66^{***}	1.34
DFAST 2019	-0.17	-9.30	0.80	-1.72
DFAST2020	-0.41	-18.22^{**}	2.29	7.31
Pooled	0.09	0.26	0.21	-0.36

Table 2.24: The values of AR, AV, ΔVOL and ΔVOLSLOPE across all stress test announcements in the period 2009–2020. Asterisks indicate results statistically significantly different from zero at the 10% level (*), 5% level (**) and 1% level (***).

Overall, I find instances of stress test announcements having a signficant impact on the stock market, but no general pattern seems to emerge for AR and AV. For AR, the earlier stress test announcements in 2011 and 2012 seem to have a statistically significant stock market impact while the more recent announcements of 2019 and 2020 have no significant impact. On the other hand, for recent years 2018 and 2019, AV is statistially significant with several years in between where it isn't. I see similar behavior in terms of ΔVOL as well. The presence of statistically significant instances is surprising. The announcement date for the stress tests is known well in advance. It would seem that investors do not react to the new information provided by stress announcements (as seen in the Section 2.6) but do react to the event itself. This would suggest that most activity on those dates is driven by speculation.

However, the effect in terms of ΔVOLSLOPE is consistent over time. The reported value of ΔVOLSLOPE is statistically insignificant for every single stress test announcement so far. That means that investors do not anticipate the stress test announcement to result in a significant market downfall. This is consistent with my finding in Section 2.6 that the stress test announcements do not seem to add any new information to the market. Also, note that when I pool the observations across all the stress test announcement dates, I find no overall significant impact on the market for all the four measures. This too, is consistent with my finding in Section Section 2.6.

2.8 Discussion and Conclusion

The results of the Federal Reserve's bank stress tests suggest a trend toward greater predictability. In this final section, I discuss implications and possible responses to this trend. I see two primary options.

One option is to accept greater predictability as a consequence of the maturing of the stress testing process. If bank portfolios change slowly, then their capital levels should arguably change slowly as well. And a predictable process still has value: the stress tests require banks to invest in resources for thorough risk assessment with overall benefits for financial stability. The CCAR process includes much more than stress testing, and the other dimensions of the CCAR review may take on greater relative importance than the stress test over time.

The main concern with a routinized stress test is the danger that it will lead banks to optimize their choices for a particular supervisory hurdle and implicitly create new, harder to detect risks in doing so. This concern applies to any fixed supervisory scheme, including one based on risk-weighted assets. (To counter this effect, Glasserman and Kang [30] propose risk weights that adapt to changes in bank portfolios.) One should not expect stress testing to be immune to this concern once the element of surprise is lost. A further concern is that predictability in stress testing may lead to pressures to weaken the process, given the costs involved in its implementation.

A second option is to resist the trend toward predictability. There are at least three ways this might be done, in increasing order of difficulty. First, the adverse and severely adverse scenarios required by DFAST could be differentiated qualitatively to bring greater diversity to the stress testing process, even without increasing the cost of the process. Second, the overall number of scenarios could be significantly expanded to help plug holes inevitably left by just two or three scenarios. Third and most ambitious, the stress testing process could be expanded, as discussed in Bookstaber et al. [22], to include knock-on and feedback effects between institutions, and interactions between solvency and liquidity, leading to a richer set of outcomes than can be achieved through a fixed set of stress scenarios applied separately to each bank. Such a process, though difficult to implement, would respond to changes in the financial and economic environment and would be less likely to get stuck in a predictable outcome.

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Appendix A

Network Interactions and the Social Value of Information

A.1 Proofs

Proof of Proposition 1.1

The first-order condition of agent i's problem implies that

$$\mathbb{E}_i[\theta] - a_i + \sum_{j \neq i} q_{ij} \mathbb{E}_i[a_j] = 0. \tag{A.1}$$

By definition, in any linear equilibrium, equilibrium actions are given by $a_i = \alpha_i s_i + \beta_i$ for all i, where α_i and β_i are constants that do not depend on the realization of signals. Plugging this expression into the above equation implies that

$$\left(\rho_i - \alpha_i + \sum_{j \neq i} q_{ij} r_{ij} \alpha_j \right) s_i + \left(-\beta_i + \sum_{j \neq i} q_{ij} \beta_j \right) = 0$$

with the convention that $r_{ii} = 1$. In deriving the above, I am using the fact that $\mathbb{E}_i[\theta] = \rho_i s_i$ and $\mathbb{E}_i[a_j] = \alpha_j r_{ij} s_i + \beta_j$. Since the above equality has to hold for all agents i and all realizations of s_i, it is therefore immediate that $(\mathbf{I} - \mathbf{Q})\beta = 0$ and that $(\mathbf{I} - \mathbf{Q} \circ \mathbf{R})\alpha = \rho$.

Note that Assumption 1.1 then guarantees that $\mathbf{I} - \mathbf{Q}$ is invertible. Therefore, $(\mathbf{I} - \mathbf{Q})\beta = 0$ implies that $\beta = 0$.

The proof is therefore complete once I show that $\mathbf{I} - \mathbf{Q} \circ \mathbf{R}$ is also invertible, as this would imply that $\alpha = (\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1}\rho$. To this end, note that,

$$-|\mathbf{Q}|^k \leq (\mathbf{Q} \circ \mathbf{R})^k \leq |\mathbf{Q}|^k$$

element-wise for all positive integers k, where I am using the fact that the diagonal elements of $\mathbf{R}$ are all equal to 1. Taking the limit $k \to \infty$ of all sides of the above inequality and using the fact that $\rho(|\mathbf{Q}|) < 1$ (as guaranteed by Assumption 1.1) then implies that $\lim_{k\to\infty}(\mathbf{Q} \circ \mathbf{R})^k = 0$. Thus, by Theorem 4.20 of Stewart [52], matrix $\mathbf{I} - \mathbf{Q} \circ \mathbf{R}$ is invertible. □

Proof of Corollary 1.3

Recall from Proposition 1.1 that the vector of equilibrium weights that agents assign to their private signals is given by $\alpha = (\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1}\rho$, where $\mathbf{Q}$ is the economy's interaction matrix. In the proof of Proposition 1.1, I already established that $\lim_{k\to\infty}(\mathbf{Q} \circ \mathbf{R})^k = 0$. Thus, by Theorem 4.20 of Stewart [52],

$$\alpha = \sum_{k=0}^{\infty}(\mathbf{Q} \circ \mathbf{R})^k \rho. \tag{A.2}$$

Since $\mathbf{Q}$, $\mathbf{R}$, and ρ are element-wise non-negative, the above equation implies that α is element-wise increasing in all off-diagonal elements of $\mathbf{Q}$. This establishes part (b) of the result.

Similarly, since matrices $\mathbf{Q}$ and $\mathbf{R}$ are element-wise positive, it follows that α is element-wise increasing in ρ_i for all i. Thus equilibrium weights are increasing in signal accuracies, establishing statement (a). Finally, since in addition ρ is element-wise positive by assumption, the same equation also implies that α is element-wise increasing in all elements of $\mathbf{R}$, that is, the weight each agent assigns to her private signal is increasing in signal commonalities. This establishes part (c) of the result. □

Proof of Corollary 1.4

Define

$$\mathbf{L} = [l_{ij}] = (\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-\mathbf{1}} = \mathbf{I} + \sum_{k=1}^{\infty}(\mathbf{Q} \circ \mathbf{R})^k \tag{A.3}$$

From Proposition 1.1, one can readily see the following:

$$\frac{d\alpha_i}{d\rho_j} = l_{ij} \tag{A.4}$$

$$\frac{d\alpha_i}{dr_{jk}} = q_{jk}\alpha_k l_{ij} + q_{kj}\alpha_j l_{ik} \tag{A.5}$$

for all players k. Suppose $q_{jk} \in (\hat{q}, 0)$ for all $j \neq k$ for some $\hat{q} < 0$. It is easy to see from Equation (A.3) that $l_{ii} = 1 + O(\hat{q})$ and $l_{ij} = q_{ij}r_{ij} + O(\hat{q}^2)$ for all $j \neq i$. Thus for a value of $\hat{q}$ sufficiently close to 0, from Equation (A.4), it is immediate that $d\alpha_i/d\rho_i > 0$ and $d\alpha_i/d\rho_j < 0$ for all $j \neq i$. In other words, α_i is increasing in ρ_i and decreasing in ρ_j for all $j \neq i$. This establishes part (a) of the result.

To establish part (b), once again suppose $q_{jk} \in (\hat{q}, 0)$ for all $j \neq k$ for some $\hat{q} < 0$. Note that $\alpha_i = \rho_i + O(\hat{q})$ for all agents i. Therefore, from Equation (A.5), I have $d\alpha_i/dr_{ik} = q_{ik}\alpha_k l_{ii} + q_{ki}\alpha_i l_{ik} = q_{ik}\rho_k + O(\hat{q}^2)$ for all $k \neq i$ and $d\alpha_i/dr_{jk} = q_{ij}q_{jk}r_{ij}\rho_k + q_{ik}q_{kj}r_{ik}\rho_j + O(\hat{q}^3)$ for all $j \neq k$ such that $i \notin \{j, k\}$. Thus for a value of $\hat{q}$ sufficiently close to 0, it is immediate that $d\alpha_i/dr_{ik} < 0$ and $d\alpha_i/dr_{jk} > 0$. In other words, α_i is decreasing in r_{ik} for all $k \neq i$ and increasing in r_{jk} for all $j \neq k$ such that $i \notin \{j, k\}$. □

Proof of Proposition 1.2

Recall that the social welfare function in this economy is given by equation (1.6). Therefore, the first-order condition with respect to agent i's action is given by

$$\mathbb{E}_i[\theta] - a_i^* + \sum_{j \neq i} q_{ij}^* \mathbb{E}_i[a_j^*] = 0. \tag{A.6}$$

The first order conditions for the planner given by Equation (A.6) are identical in structure to the equilibrium first order conditions given by Equation (A.1). Thus, following the outline of the proof for Proposition 1.1, the planner admits a solution of the form $a_i^* = \alpha_i^* s_i$ where $\alpha^* = \mathbf{I} - \mathbf{Q}^* \circ \mathbf{R}$. Since the problem of constrained efficiency posed in Definition 1.2 is concave, this is the unique constrained efficient solution across all possible strategies. □

Proof of Proposition 1.3

First suppose $\mathbf{Q} = \mathbf{Q}^*$. Propositions 1.1 and 1.2 then imply that $\alpha = \alpha^*$ irrespective of the value of $\mathbf{R}$ and ρ. Thus, the equilibrium is constrained efficient for all information structures.

To prove the converse implication, suppose that the equilibrium is constrained efficient for all information structures. Consequently, Propositions 1.1 and 1.2 imply that

$$(\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1}\rho = (\mathbf{I} - \mathbf{Q}^* \circ \mathbf{R})^{-1}\rho \tag{A.7}$$

for all $\mathbf{R}$ and all ρ. Fix an arbitrary pair of agents i and j and consider the information structure $(\mathbf{R}, \rho)$ in which (i) $r_{kl} = 0$ for all agent pairs $\{k, l\} \neq \{i, j\}$ and (ii) $\rho_k = 0$ for all $k \neq i$. Under such an information structure, agent i is the only agent with an informative signal about the fundamental ($\rho_i \neq 0$) and agents i and j are the only pairs of agents with non-zero informational commonality ($r_{ij} = r_{ji} \neq 0$). Note that for this to be a valid information structure, the resulting covariance matrix in (1.2) has to be positive semidefinite. It is easy to verify that the covariance matrix

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho' \\ & \\ \rho & \mathbf{R} \end{bmatrix}$$

has $n-2$ zero eigenvalues, one eigenvalue equal to 1, and a pair of eigenvalues given by $1 \pm \sqrt{\rho_i^2 + r_{ij}^2}$. Therefore, as long as $\rho_i^2 + r_{ij}^2 < 1$, the resulting information structure is a valid information structure. For this choice of information structure, equation (A.7) implies that

$$\frac{\rho_i}{1 - q_{ij}q_{ji}r_{ij}^2} = \frac{\rho_i}{1 - q_{ij}^*q_{ji}^*r_{ij}^2}$$
$$\frac{q_{ji}r_{ij}\rho_i}{1 - q_{ij}q_{ji}r_{ij}^2} = \frac{q_{ji}^*r_{ij}\rho_i}{1 - q_{ij}^*q_{ji}^*r_{ij}^2}$$

Since $\rho_i \neq 0$ and $r_{ij} \neq 0$ by assumption, the only way for the above two equations to be satisfied simultaneously is that $q_{ij} = q^*_{ij}$. Since the pair (i, j) was chosen arbitrarily, this implies that $q_{ij} = q^*_{ij}$ for all pairs of agents i and j, thus guaranteeing that $\mathbf{Q} = \mathbf{Q}^*$. □

Proof of Proposition 1.4

Proof of part (a) Recall from Propositions 1.1 and 1.2 that the equilibrium and efficient weights on agents signals are given by $\alpha = (\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1}\rho$ and $\alpha^* = (\mathbf{I} - \mathbf{Q}^* \circ \mathbf{R})^{-1}\rho$, respectively. Therefore,

$$\alpha - \alpha^* = \left[\sum_{k=1}^{\infty} (\mathbf{Q} \circ \mathbf{R})^k - \sum_{k=1}^{\infty} (\mathbf{Q}^* \circ \mathbf{R})^k \right] \rho. \tag{A.8}$$

Consider the limit as all pairwise commonalities converge to 0, i.e., $r_{ij} \downarrow 0$ for all $i \neq j$. In matrix notation, I have $\mathbf{R} \downarrow \mathbf{I}$. Consequently,

$$\lim_{\mathbf{R} \downarrow \mathbf{I}} (\alpha - \alpha^*) - ((\mathbf{Q} - \mathbf{Q}^*) \circ \mathbf{R})\, \rho = 0.$$

Therefore, in the limit, if $\mathbf{Q} > \mathbf{Q}^*$ element-wise, then agent i overreacts to her private signal (i.e., $\alpha_i > \alpha^*_i$), whereas she underreacts if the inequality is reversed. Since these relationships hold strictly in the limit, there exists a $\underline{r} > 0$ small enough such that they are also satisfied if $r_{ij} < \underline{r}$ for all $i \neq j$. □

Proof of part (b) Note that when $\mathbf{R} = \mathbf{1}\mathbf{1}'$, I must necessarily have that $\rho_i = \overline{\rho}$ for some $\overline{\rho} \in (0, 1)$. Suppose $r_{ij} \in (\overline{r}, 1)$ for all $i \neq j$. A first order Taylor expansion implies that $\alpha - \alpha^* = (\alpha - \alpha^*)|_{\mathbf{R}=\mathbf{1}\mathbf{1}'} + O(1 - \overline{r}) = \overline{\rho}(b - b^*) + O(1 - \overline{r})$. Thus, for a value of $\overline{r}$ sufficiently close to 1, if $b_i > b^*_i$ then agent i overreacts to their signal and if $b_i < b^*_i$ then agent i underreacts to their signal. This concludes the proof. □

Proof of Proposition 1.5

Recall that when agents' action profile is given by $(a_1, \ldots, a_n)$, the social welfare in the economy is given by equation (1.6). On the other hand, Proposition 1.1 implies that agents' equilibrium actions are given by $a_i = \alpha_i s_i$, where $\alpha = (\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1}\rho$. As a result, ex ante equilibrium social welfare is given by

$$\mathbb{E}[W] = \sigma^2 \sum_{i=1}^{n} \alpha_i \rho_i - \frac{1}{2}\sigma^2 \sum_{i=1}^{n} \alpha_i^2 + \frac{1}{2}\sigma^2 \sum_{i,j=1}^{n} q_{ij}^* \alpha_i \alpha_j r_{ij} = \sigma^2 \alpha' \rho - \frac{1}{2}\sigma^2 \alpha' (\mathbf{I} - \mathbf{Q}^* \circ \mathbf{R})\alpha. \tag{A.9}$$

Furthermore, since by assumption the economy is efficient under all information structures, Proposition 1.3 implies that $\mathbf{Q}^* = \mathbf{Q}$. Consequently, the ex ante social welfare is given by

$$\mathbb{E}[W] = \frac{1}{2}\sigma^2 \rho' (\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1} \rho, \tag{A.10}$$

In the proof of Proposition 1.1, I established that $\lim_{k \to \infty} (\mathbf{Q} \circ \mathbf{R})^k = 0$. Therefore, by Theorem 4.20 of Stewart [52],

$$\mathbb{E}[W] = \frac{1}{2}\sigma^2 \sum_{k=0}^{\infty} \rho' (\mathbf{Q} \circ \mathbf{R})^k \rho. \tag{A.11}$$

Since matrix $\mathbf{Q}$ is element-wise nonnegative, all matrices and vectors on the right-hand side of the above equation are non-negative. Therefore, increasing ρ and $\mathbf{R}$ element-wise can only increase the expected social welfare. □

Proof of part(b) Suppose $q_{ij} \in (\hat{q}, 0)$ for some $\hat{q} < 0$. Differentiating both sides of equation (A.11) with respect to r_{ij}, I have

$$\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}r_{ij}} = \frac{1}{2}\sigma^2 (q_{ij} + q_{ji}) \rho_i \rho_j + O(\hat{q}^2)$$

Thus, for a sufficiently small value of $\hat{q}$, the derivative of $\mathbb{E}[W]$ with respect to $\hat{r}$ is decreasing and hence the result follows for agents' signal commonalities. For the comparative statics of social welfare with respect to accuracy, I differentiate both sides of equation (A.11) with respect to ρ_i. I have

$$\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\rho_i} = \sigma^2 \rho_i + O(\hat{q})$$

I see that for a sufficiently small value of $\hat{q}$, the derivative of $\mathbb{E}[W]$ with respect to ρ_i is increasing and hence the result follows. □

Proof of Proposition 1.6

Proof of part (a) Recall from the proof of Proposition 1.5 that when the economy is efficient under all information structures, expected social welfare in equilibrium is given by equation (A.10). Therefore, in the special case that the information structure is symmetric (i.e, when $\rho_i = \hat{\rho}$ for all i and $r_{ij} = \hat{r}$ for all distinct pairs of agents i and j),

$$\mathbb{E}[W] = \frac{1}{2}\sigma^2 \hat{\rho}^2 \mathbf{1}'(\mathbf{I} - \hat{r}\mathbf{Q})^{-1}\mathbf{1}. \tag{A.12}$$

By Assumption 1.2, $\mathbf{I} - \mathbf{Q} \succ \mathbf{0}$. Therefore, $\mathbf{I} - \hat{r}\mathbf{Q} = (1-\hat{r})\mathbf{I} + \hat{r}(\mathbf{I} - \mathbf{Q}) \succ \mathbf{0}$. It is then immediate that the right-hand side of (A.12) is increasing in the accuracy parameter $\hat{\rho}$. □

Proof of part (b) Recall that when the information structure is symmetric, equilibrium social welfare is given by equation (A.12). Differentiating this expression with respect to $\hat{r}$ implies that

$$\frac{1}{\sigma^2\hat{\rho}^2}\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}} = \frac{1}{2}\mathbf{1}'(\mathbf{I} - \hat{r}\mathbf{Q})^{-1}\mathbf{Q}(\mathbf{I} - \hat{r}\mathbf{Q})^{-1}\mathbf{1}. \tag{A.13}$$

Taking a second derivate, I obtain

$$\frac{1}{\sigma^2\hat{\rho}^2}\frac{\mathrm{d}^2\mathbb{E}[W]}{\mathrm{d}\hat{r}^2} = z'(\mathbf{I} - \hat{r}\mathbf{Q})^{-1}z, \tag{A.14}$$

where, utilizing the fact that $\mathbf{Q} = \mathbf{Q}^*$ is symmetric, the vector z is given by $z = \mathbf{Q}(\mathbf{I} - \hat{r}\mathbf{Q})^{-1}\mathbf{1}$. Since $\mathbf{I} - \hat{r}\mathbf{Q}$ is positive definite, it then follows that the right-hand side of (A.14) is always positive. This means that the expression on the right-hand side of (A.13) is increasing in $\hat{r}$. Setting $\hat{r} = 0$, I have,

$$\frac{1}{\sigma^2\hat{\rho}^2}\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}} \geq \frac{1}{2}\mathbf{1}'\mathbf{Q}\mathbf{1}.$$

Thus, if the right-hand side of the above inequality is positive, expected social welfare is increasing in the commonality parameter $\hat{r}$. □

Proof of part (c) In the previous part, I established that the right-hand side of (A.13) is increasing in $\hat{r}$. Setting $\hat{r} = 1$, I have,

$$\frac{1}{\sigma^2\hat{\rho}^2}\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}} \leq \frac{1}{2}\mathbf{1}'(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{Q}(\mathbf{I} - \mathbf{Q})^{-1}\mathbf{1} = \frac{1}{2}b'\mathbf{Q}b.$$

Therefore, if the right-hand side of the above inequality is negative, then expected social welfare is decreasing in $\hat{r}$. □

Proof of part (d) Finally, suppose

$$b'\mathbf{Q}b > 0 > \mathbf{1}'\mathbf{Q}\mathbf{1}.$$

Coupled with equation (A.13) this implies that

$$\left.\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}}\right|_{\hat{r}=1} > 0 > \left.\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}}\right|_{\hat{r}=0}.$$

This immediately guarantees that the social welfare is non-monotone in $\hat{r}$. In particular, it is first decreasing and then increasing in $\hat{r}$. □

Proof of Proposition 1.7

Recall that the ex ante equilbrium social welfare is given by equation (A.9), which can be rewritten as

$$\mathbb{E}[W] = \frac{1}{2}\sigma^2\rho'(\mathbf{I} - \mathbf{Q} \circ \mathbf{R})^{-1}\rho + \frac{1}{2}\sigma^2\alpha'((\mathbf{Q}^* - \mathbf{Q}) \circ \mathbf{R})\alpha. \qquad \text{(A.15)}$$

The first term above is mathematically identical to the right hand side of equation (A.10). Thus, from the argument presented in the proof of Proposition 1.5, it is easy to see that the first term is increasing with respect to r_{ij} for all $i \neq j$ and ρ_i for all i.

Consider the second term. Recall that Corollary 1.3 implies that when $\mathbf{Q} \geq \mathbf{0}$ element-wise, α is increasing in r_{ij} for all $i \neq j$ and ρ_i for all i. Thus, when $\mathbf{Q}^* \geq \mathbf{Q} \geq \mathbf{0}$ element-wise, the second term is a matrix product of positive increasing functions and hence is always increasing in r_{ij} for all $i \neq j$ and ρ_i for all i. Consequently, the ex ante equilibrium welfare is increasing in r_{ij} for all $i \neq j$ and ρ_i for all i. This concludes the proof. □

Proof of Proposition 1.8

Proof of part (a) Recall that the ex ante equilbrium social welfare $\mathbb{E}[W]$ is given by equation (A.9). A first order Taylor series expansion of $\mathbb{E}[W]$ around $\hat{r} = 1$ is given by

$$\mathbb{E}[W] = \mathbb{E}[W]|_{\hat{r}=1} + \left.\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}}\right|_{\hat{r}=1} (\hat{r}-1) + O\left((\hat{r}-1)^2\right)$$

Thus, the behavior of the ex ante equilibrium social welfare with respect to $\hat{r}$ in a neighborhood of 1 is dictated by the derivative $\mathrm{d}\mathbb{E}[W]/\mathrm{d}\hat{r}|_{\hat{r}=1}$ which is given by

$$\begin{aligned} \left.\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}}\right|_{\hat{r}=1} &= \sigma^2 \left(\left.\frac{\mathrm{d}\alpha}{\mathrm{d}\hat{r}}\right|_{\hat{r}=1}\right)' \rho - \sigma^2 \left(\left.\frac{\mathrm{d}\alpha}{\mathrm{d}\hat{r}}\right|_{\hat{r}=1}\right)' (\mathbf{I}-\mathbf{Q}^*)\, \alpha|_{\hat{r}=1} + \frac{1}{2}\sigma^2\, \alpha'\mathbf{Q}^*\alpha|_{\hat{r}=1} \\ &= \sigma^2 \left(\left.\frac{\mathrm{d}\alpha}{\mathrm{d}\hat{r}}\right|_{\hat{r}=1}\right)' (\mathbf{Q}^*-\mathbf{Q})\, \alpha|_{\hat{r}=1} + \frac{1}{2}\sigma^2\, \alpha'\mathbf{Q}^*\alpha|_{\hat{r}=1} \end{aligned} \tag{A.16}$$

where I use the fact that $\rho = (\mathbf{I}-\mathbf{Q})\, \alpha|_{\hat{r}=1}$ from Proposition 1.1. Note that when $\mathbf{Q} \geq \mathbf{0}$ element-wise, $\alpha|_{\hat{r}=1} = \hat{\rho} b \geq 0$ and $\mathrm{d}\alpha/\mathrm{d}\hat{r}|_{\hat{r}=1} = (\mathbf{I}-\mathbf{Q})^{-1}\mathbf{Q}b \geq 0$ element-wise. If $\mathbf{Q}^* \leq 0$ element-wise, then clearly the right hand side of equation (A.16) is non-positive. This implies that in neighborhood around $\hat{r} = 1$, the equilibrium ex ante social welfare is decreasing in $\hat{r}$. □

Proof of part (b) Suppose $\underline{q} < q_{ij} \leq 0$ for all $i \neq j$. From equation (A.2), one can readily see that $\alpha = \rho + O(\underline{q})$. Thus, for a value of $\underline{q}$ sufficiently close to 0, $\alpha_i \geq 0$ for all i. Further, suppose $q_{ij} < q^*_{ij} < 0$ for all $i \neq j$. Under these values of $\mathbf{Q}$ and $\mathbf{Q}^*$, it is easy to see $\alpha|_{\hat{r}=1} = \hat{\rho} b \geq 0$, $\mathrm{d}\alpha/\mathrm{d}\hat{r}|_{\hat{r}=1} = (\mathbf{I} - \mathbf{Q})^{-1}\mathbf{Q} b \leq 0$ and $\mathbf{Q}^* - \mathbf{Q} \geq \mathbf{0}$ element-wise. Consequently, the right hand side of equation (A.16) is non-positive. Thus, the result follows. □

Proof of Proposition 1.9

Proof of part (a) Recall that the ex ante equilbrium social welfare $\mathbb{E}[W]$ is given by equation (A.9). A first order Taylor series expansion of $\mathbb{E}[W]$ around $\hat{r} = 0$ is given by

$$\mathbb{E}[W] = \mathbb{E}[W]|_{\hat{r}=0} + \left.\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}}\right|_{\hat{r}=0} \hat{r} + O\left(\hat{r}^2\right)$$

Thus, the behavior of the ex ante equilibrium social welfare with respect to $\hat{r}$ in a neighborhood of 1 is dictated by the derivative $\mathrm{d}\mathbb{E}[W]/\mathrm{d}\hat{r}|_{\hat{r}=0}$ which is given by

$$\begin{aligned} \left.\frac{\mathrm{d}\mathbb{E}[W]}{\mathrm{d}\hat{r}}\right|_{\hat{r}=0} &= \sigma^2 \left(\left.\frac{\mathrm{d}\alpha}{\mathrm{d}\hat{r}}\right|_{\hat{r}=0} \right)' \rho - \sigma^2 \left(\left.\frac{\mathrm{d}\alpha}{\mathrm{d}\hat{r}}\right|_{\hat{r}=0} \right)' \alpha|_{\hat{r}=0} + \frac{1}{2}\sigma^2\, \alpha'\mathbf{Q}^*\alpha|_{\hat{r}=0} \\ &= \frac{1}{2}\sigma^2\hat{\rho}^2\mathbf{1}'\mathbf{Q}^*\mathbf{1} \end{aligned} \tag{A.17}$$

where I use the fact that $\alpha|_{\hat{r}=0} = \hat{\rho}\mathbf{1}$ from Proposition 1.1. Thus, the result in part (a) follows. □

Proof of part (b) The proof of part (b) follows directly from equation (A.16). When the economy is efficient under complete information, I must have $\alpha|_{\hat{r}=1} = \alpha^*|_{\hat{r}=1} = b$. Therefore, I have $(\mathbf{Q}^* - \mathbf{Q})\, \alpha|_{\hat{r}=1} = (\mathbf{I} - \mathbf{Q})\, \alpha|_{\hat{r}=1} - (\mathbf{I} - \mathbf{Q}^*)\, \alpha^*|_{\hat{r}=1} = 0$, where the final equality follows from Propositions 1.1 and 1.2. Thus, when the economy is efficient under complete information, $\mathrm{d}\mathbb{E}[W]/\mathrm{d}\hat{r}|_{\hat{r}=1} = \frac{1}{2}\sigma^2\hat{\rho}^2 \sum_{i,j} q^*_{ij} b_i b_j$. This completes the proof of part (b). □

Proof of Proposition 1.10

Proof of part (a) Recall that the ex ante equilibrium social welfare is given by equation (A.15). For convenience, suppose $\sum_{j\neq i} q_{ij} = \hat{q}$ for all i. Then, from Proposition 1.1, it is straightforward that $\alpha = \frac{\hat{\rho}}{1-\hat{q}\hat{r}}\mathbf{1}$. Using that the fact that the economy is regular and the information structure is symmetric, equation (A.15) can be simplified to

$$\mathbb{E}[W] = \frac{n}{2}\sigma^2\hat{\rho}^2\left(\frac{1}{1-\hat{r}\hat{q}} + \frac{\left(\sum_{i,j} q^*_{ij} - \sum_{i,j} q_{ij}\right)\hat{r}}{(1-\hat{r}\hat{q})^2}\right) \tag{A.18}$$

When $\hat{q} > 0$ (or equivalently $\sum_{i,j} q_{ij} = n\hat{q} > 0$), the first term in equation (A.18) is increasing in $\hat{r}$. Further, when $\sum_{i,j} q^*_{ij} > \sum_{i,j} q_{ij}$, the second term is also increasing in $\hat{r}$. Thus, when $\sum_{i,j} q^*_{ij} > \sum_{i,j} q_{ij} > 0$, both terms are increasing in $\hat{r}$ and thus, the social welfare is increasing in $\hat{r}$. This concludes the proof of part (a). □

Proof of part (b) Note that the ex ante equilibrium social welfare in a regular economy is given by equation (A.18). Suppose, $\hat{q} < 0$ (or equivalently $\sum_{i,j} q_{ij} = n\hat{q} < 0$). Then the first term in equation (A.18) is decreasing in $\hat{r}$. Consider the function $h(\hat{r}) = \frac{\hat{r}}{(1-\hat{r}\hat{q})^2}$ which is proportional to the second term in equation (A.18). I have $\mathrm{d}h(\hat{r})/\mathrm{d}\hat{r} = \frac{1+\hat{q}\hat{r}}{(1-\hat{r}\hat{q})^2}$. Therefore, $\mathrm{d}h(\hat{r})/\mathrm{d}\hat{r} > 0$ for all $\hat{r}$ if and only if $\hat{q} > -1$ (where I utilize the fact that $\hat{q} < 0$). If I suppose that $\sum_{i,j} q^*_{ij} < \sum_{i,j} q_{ij}$ and $\sum_{i,j} q_{ij} = n\hat{q} > -n$, then both terms in equation (A.18) are decreasing in $\hat{r}$ and thus, the social welfare is decreasing in $\hat{r}$. This concludes the proof of part (b). □