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Crustal Structure and Surface Wave Dispersion Part III
Theoretical Dispersion Curves for Sub-Oceanic Rayleigh Waves

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Crustal Structure and Surface Wave Dispersion

Part III. Theoretical Dispersion Curves for Sub-Oceanic Rayleigh Waves.

Technical Report # 18

by

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Frank Press Frank Press

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ABSTRACT

Theoretical Rayleigh wave dispersion curves for three different types of sub-oceanic basement layering are presented. Previous conclusions concerning the dispersion of Rayleigh waves across ocean basins are re-examined in the light of the new data.

I. Introduction

Recent investigations[1] have shown that the characteristics of propagation of Rayleigh waves across ocean basins can be explained by the effect of a layer of water overlying ultrabasic rock having an equivalent compressional velocity of slightly under 7.90 km/sec. In this earlier work the theoretical problem was reduced to its simplest form by treating the water and sediment as a single liquid layer and the basement rock as a single homogeneous layer of infinite thickness.

Since the theoretical dispersion curve from this idealized structure agreed remarkably well with the observed dispersion curve it was concluded that any layering present in the basement rocks involved only very moderate contrasts in elastic properties. It was pointed out that the small dispersion observed for Love waves along ocean paths demanded a superficial layer in the basement rocks, but that the elastic properties of the layer should be much closer to those of the substratum than is the case for the continents. Seismic refraction measurements [2,3,4] in the Atlantic and Pacific ocean basins have given the thickness and elastic constants of the basement layer in a number of places. The layer is roughly as thick as the ocean is deep and has a velocity for compressional waves varying between 6 1/4-7 km/sec, usually overlying more basic rock with a velocity range of about 7.9-8.2 km/sec.

In the present paper calculations of the effect of the basement layering on Rayleigh wave dispersion curves are presented for three different types of basement layering:

Case I $\alpha_0 = 1.52$ km/sec, $\alpha_1 = 5.5$ km/sec, $\alpha_2 = 8.1$ km/sec, $H_1 = H_0$, $H_2 = \infty$, $\rho_1 = 2.67\,\rho_0$, $\rho_2 = 3.0\,\rho_0$; Case II $\alpha_0 = 1.52$ km/sec. $\alpha_1 = 7.90$ km/sec, $H_1 = \infty$, $\rho_1 = 3.0\,\rho_0$; Case III $\alpha_0 = 1.52$ km/sec, $\alpha_1 = 6.9$ km/sec, $\alpha_2 = 8.1$ km/sec, $H_1 = H_0$, $H_2 = \infty$, $\rho_1 = 2.67\,\rho_0$, $\rho_2 = 3.0\,\rho_0$. The subscripts 0,1,2 refer respectively to the liquid layer, first and second basement layers, and Poisson's constant is taken as 1/4.

Comparison of the new theoretical curves with observed dispersion is made. Applications to microseism propagation will be discussed in a future paper.

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1. Maurice Ewing and Frank Press "Crustal Structure and Surface Wave Dispersion Part II, Solomon Islands Earthquake of 29 July 1950", in press Bull. Seism. Soc. Amer.

2. C. B. Officer, Jr., Maurice Ewing and Paul Wuenschel, "Seismic Refraction Measurements in the Atlantic Ocean Basin. Part IV, Bermuda, Bermuda Rise and Nares Deep, in press Bull. Geol. Soc. Amer.

3. Maurice Ewing, George Sutton, C.B. Officer, Jr., "Seismic Refraction Measurements in the Atlantic Ocean Basin, Part VI- Typical Deep Stations in the North Atlantic Basin, in preparation.

4. Russel W. Raitt, "Seismic Refraction Studies of the Pacific Ocean Basin", paper presented before annual meeting Seism. Soc. America, March 24, 1951.

II Derivation of Period Equation

In order to find the phase and group velocity in terms of the parameter kH (wave number x the depth of the liquid layer) for a system consisting of a liquid superposed on two solid layers, we consider the propagation of a train of plane waves of the Rayleigh type. We make use of notations represented in Figure 1, where ρ_i are the densities, λ_i and μ_i elastic constants, $\alpha_i = \sqrt{\frac{\lambda_i + 2\mu_i}{\rho_i}}$ and $\beta_i = \sqrt{\frac{\mu_i}{\rho_i}}$ the velocities of compressional and shear waves respectively.

As usual the horizontal and vertical components of the displacement, u and w, are represented by the formulae

$$u = \frac{\partial \varphi}{\partial x} + \frac{\partial^2 \psi}{\partial x \partial z} \quad , \quad w = \frac{\partial \varphi}{\partial z} + \frac{\partial^2 \psi}{\partial z^2} + \frac{\omega^2}{\beta^2}\psi \qquad (1)$$

and the functions φ and ψ should satisfy the wave equation, i.e.

$$\left(\nabla^2 + \frac{\omega^2}{\alpha^2}\right)\varphi = 0 \; , \quad \left(\nabla^2 + \frac{\omega^2}{\beta^2}\right)\psi = 0 \qquad (2)$$

The stresses are given by the equations

$$p_{zz} = \lambda \nabla^2 \varphi + 2\mu \frac{\partial w}{\partial z} \quad , \quad p_{zx} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \qquad (3)$$

Bearing in mind the assumed layering for cases I and III, we take $H_0 = H_1 = H$; the equation of the free surface is $z = -H$. It is obvious that the final results of this investigation may be easily changed in order to correspond to the more general case $H_0 \neq H_1$.

Now the boundary conditions of the problem may be written in the form

$$p = 0 \qquad at \quad z = -H \qquad (4)$$

$$\begin{matrix} (p_{zz})_1 = (p_{zz})_0 , \ (p_{zx})_1 = 0 \\ W_1 = W_0 \end{matrix} \qquad at \quad z = 0 \qquad (5)$$

$$\begin{matrix} (p_{zz})_2 = (p_{zz})_1 , \ (p_{zx})_2 = (p_{zx})_1 , \\ W_2 = W_1 , \quad u_2 = u_1 \end{matrix} \qquad at \quad z = H. \qquad (6)$$

We have 8 boundary conditions and the expressions for the functions φ_i and ψ_i will also contain 8 unknown coefficients. Omitting the time factor we write these functions in the form

$$\varphi_0 = \{B\, exp.(-\gamma_0 z) + C\, exp.(\gamma_0 z)\}\, exp(-i\kappa x) \qquad (7)$$

$$\varphi_1 = \{D\, exp(\gamma_1 z) + E\, exp(-\gamma_1 z)\}\, exp(-i\kappa x) \qquad (8)$$

$$\varphi_2 = F\, exp(-\gamma_2 z)\, exp(-i\kappa x) \qquad (9)$$

$$\psi_1 = \{M\, exp(\gamma_3 z) + N\, exp(-\gamma_3 z)\}\, exp(-i\kappa x) \qquad (10)$$

$$\psi_2 = P\, exp(-\gamma_4 z)\, exp(-i\kappa x) \qquad (11)$$

Upon eliminating the constants B,C,D,E,F,M,N,P between the eight boundary equations (4)-(6) we can obtain the period equation relating phase velocity to wave number. The method of elimination is similar to that described in a previous paper [5] in which the liquid layer was taken as infinitely thick.

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5 W. S. Jardetzky and Frank Press, "Rayleigh Wave Coupling to Atmospheric Compressional Waves", in press, Bull. Seism. Soc. Amer.

For briefness we make the following definitions

$$V_0 = \frac{c}{\alpha_0}\,, \qquad V = \frac{c}{\beta_1} \tag{12}$$

and

$$n_0 = \sqrt{1 - V_0^2}\,, \quad n_1 = \sqrt{1 - V_0^2\frac{\alpha_0^2}{\alpha_1^2}}\,, \quad n_2 = \sqrt{1 - V_0^2\frac{\alpha_0^2}{\alpha_2^2}}\,,$$
$$n_3 = \sqrt{1 - V_0^2\frac{\alpha_0^2}{\beta_1^2}}\,, \quad n_4 = \sqrt{1 - V_0^2\frac{\alpha_0^2}{\beta_2^2}} \tag{13}$$

Making use of Lee's [6] notation we obtain

$$\begin{aligned}
X &= \frac{\rho_2}{\rho_1} V_0^2 \frac{\alpha_0^2}{\beta_1^2} - 2\left(\frac{\mu_2}{\mu_1} - 1\right) \\
Y &= V_0^2 \frac{\alpha_0^2}{\beta_1^2} + 2\left(\frac{\mu_2}{\mu_1} - 1\right) = V_0^2 \frac{\alpha_0^2}{\beta_1^2} + W \\
Z &= \frac{\rho_2}{\rho_1} V_0^2 \frac{\alpha_0^2}{\beta_1^2} - V_0^2 \frac{\alpha_0^2}{\beta_1^2} - 2\left(\frac{\mu_2}{\mu_1} - 1\right) = X - V_0^2 \frac{\alpha_0^2}{\beta_1^2} \\
W &= 2\left(\frac{\mu_2}{\mu_1} - 1\right)
\end{aligned} \tag{14}$$

Putting (see equations 34-36 of the previous paper[5])

$$G_1 = XZ - n_2 n_4 WY, \quad G_2 = Z^2 - n_2 n_4 Y^2, \quad G_3 = n_2 n_4 W^2 - X^2 \tag{15}$$

and

$$\begin{aligned}
l_0 &= 4\left(2 - V_0^2 \frac{\alpha_0^2}{\beta_1^2}\right) G_1 \\
l_1' &= \left(2 - V_0^2 \frac{\alpha_0^2}{\beta_1^2}\right)^2 \frac{1}{n_1 n_3} G_2 - 4 n_1 n_3 G_3 \\
l_2' &= -\left(2 - V_0^2 \frac{\alpha_0^2}{\beta_1^2}\right)^2 \frac{\rho_2}{\rho_1} \frac{n_2}{n_1} V_0^4 \frac{\alpha_0^4}{\beta_1^4} + 4 \frac{\rho_2}{\rho_1} n_1 n_4 V_0^4 \frac{\alpha_0^2}{\beta_1^2} \\
l_3' &= -\left(2 - V_0^2 \frac{\alpha_0^2}{\beta_1^2}\right)^2 \frac{\rho_2}{\rho_1} \frac{n_4}{n_3} V_0^4 \frac{\alpha_0^4}{\beta_1^4} + 4 \frac{\rho_2}{\rho_1} n_2 n_3 V_0^4 \frac{\alpha_0^2}{\beta_1^2} \\
l_4' &= \left(2 - V_0^2 \frac{\alpha_0^2}{\beta_1^2}\right)^2 G_3 - 4 G_2
\end{aligned} \tag{16}$$

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6 A.W. Lee, "The effect of Geologic Structure upon Microseismic Disturbance", M.N.R.A.S. (Geophys. Suppl.) Vol. 3, pp. 83-105, 1932

$$
\begin{aligned}
l_1'' &= -\frac{\rho_0\rho_2}{\rho_1^2}\,\frac{n_1 n_4}{n_0 n_3}\,V_0^8\,\frac{\alpha_0^8}{\beta_1^8}\tanh(n_0\kappa H)\\
l_2'' &= \frac{\rho_0}{\rho_1}\,\frac{n_1}{n_0}\,V_0^4\,\frac{\alpha_0^4}{\beta_1^4}\,G_3\tanh(n_0\kappa H)\\
l_3'' &= \frac{\rho_0}{\rho_1}\,\frac{1}{n_0 n_3}\,V_0^4\,\frac{\alpha_0^4}{\beta_1^4}\,G_2\tanh(n_0\kappa H)\\
l_4'' &= -\frac{\rho_0\rho_2}{\rho_1^2}\,\frac{n_2}{n_0}\,V_0^8\,\frac{\alpha_0^8}{\beta_1^8}\tanh(n_0\kappa H)
\end{aligned}
\tag{17}
$$

the period equation takes the form

$$
\begin{aligned}
&l_0 + l_1'\sinh(n_1\kappa H)\sinh(n_3\kappa H) + l_2'\sinh(n_1\kappa H)\cosh(n_3\kappa H)\\
&+ l_3'\cosh(n_1\kappa H)\sinh(n_3\kappa H) + l_4'\cosh(n_1\kappa H)\cosh(n_3\kappa H)\\
&+\Big[l_1''\sinh(n_1\kappa H)\sinh(n_3\kappa H) + l_2''\sinh(n_1\kappa H)\cosh(n_3\kappa H)\\
&+ l_3''\cosh(n_1\kappa H)\sinh(n_3\kappa H) + l_4''\cosh(n_1\kappa H)\cosh(n_3\kappa H)\Big]\tanh(n_0\kappa H) = 0.
\end{aligned}
\tag{18}
$$

The period equation for the more general case, where the depth of the liquid layer differs from that of the upper solid layer, will have the same form as equation (18), the factor H in tanh $(n_0 kH)$ being replaced by a new value, say H_0.

It may be noted that the period equation for the present problem can be obtained from that for an infinite liquid layer[5] by replacing ρ_0 with $\rho_0 \tanh(n_0 kH)$.

The numerical procedure for computing kH as a function of $V_0 = c/\alpha_0$ consists of assuming values of V_0 and obtaining kH from (18) by successive approximations. Group velocity is obtained by a graphical differentiation of the phase velocity curves. Periods are related to kH through the equation $T = 2\pi H/\alpha_0 kH\, c/\alpha_0$.

The derivation of the period equation and a description of methods of computation for case II were given in previous papers.[7]

Haskell[8] recently derived in matrix form appropriate equations for computing Rayleigh dispersion in three layers.

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7. Frank Press, Maurice Ewing, and Ivan Tolstoy, "The Airy Phase of Shallow Focus Submarine Earthquakes", Bull. Seism. Soc. Vol. 40 pp 111-148, 1950.

8. Norman A. Haskell, "The Dispersion of Surface Waves on Multi-Layered Media", Geophysical Research Paper No. 9, Air Force Cambridge Research Center, Cambridge, Mass., 1951.

III Discussion

The numerical results for phase velocity are presented for the lowest modes of cases I, II and III in Tables 1 and 2. Group velocity curves for these three cases are presented in dimensionless form in Figure 2. Group velocity curves for periods between 15 and 40 sec are presented for cases I and III in Figure 3 for an ocean depth H = 5.57 km. As in previous papers we have taken this value of H to include the thickness of unconsolidated deep sea sediments.

From examination of these curves the following conclusions can be reached.

1- Cases II and III are experimentally indistinguishable from each other in the range 15-40 sec. For $T > 40$ sec the two curves will diverge. Previous results(1) based on Rayleigh wave dispersion indicated that the Pacific Ocean basin is underlain by rock having an equivalent velocity of slightly under 7.90 km/sec. It is now seen that a layered basement approximating the properties assumed for case III with H = 5.57 km explains the observed dispersion equally well. This is consistent with refraction measurements in the Atlantic and Pacific Oceans.

2- The sensitivity of the technique of comparing observed Rayleigh wave dispersion curves with theoretical curves to ascertain suboceanic basement structure can be estimated from Figure 3. Since changes in H simply shift the group velocity curve to the right or left, it can be seen that these curves are more sensitive to changes in velocity of the first basement layer in the range $23 < T < 40$ sec. For a well located earthquake having a predominatly oceanic path[2], the scatter of observed Rayleigh wave velocities in this period range is less than 0.1 km/sec. One can conclude that differences in structure of the first basement layer from the conditions assumed in case III with H = 5.57 km, can be detected by analysis of Rayleigh wave dispersion providing the layer thickness is at least as great as the ocean is deep when the velocity contrast is as great as that between cases I and III.

It is to be hoped that further investigations will reveal techniques for correcting for variations in water depth along paths, refraction at the continental border, so that determination of crustal structure can be made over most oceanic areas from the records of existing seismological stations.

We are grateful to Professor Maurice Ewing for many helpful discussions on this problem.

Table I - Phase Velocity Values

Case I

c/α_0	kH
1.00	10.667
1.05	4.204
1.075	3.483
1.10	2.990
1.20	2.130
1.30	1.726
1.40	1.471
1.50	1.289
1.60	1.156
1.65	1.099
1.70	1.049
1.75	1.003
1.80	.961
1.85	.921
1.90	.883
1.95	.850
2.00	.818
.205	.789
.210	.759
.220	.704
.230	.656
.240	.602
.250	.537
.260	.442
.270	.283
.273	.222
.275	.177
.277	.123
.280	.061

Case III

c/α_0	kH
1.00	47.206
1.05	4.431
1.075	3.646
1.10	3.168
1.20	2.220
1.30	1.782
1.40	1.513
1.50	1.326
1.60	1.185
1.65	1.126
1.70	1.073
1.75	1.026
1.80	.982
1.85	.942
1.90	.906
1.95	.871
2.00	.839
2.05	.808
2.10	.781
2.20	.728
2.30	.678
2.40	.629
2.50	.579
2.60	.510
2.70	.380
2.73	.323
2.75	.268
2.77	.204
2.80	.098

Table II. Phase Velocity Values Case II

c/α_0	kH	c/α_0	kH
1.00	32.863	1.92	.89932
1.04	5.0945	1.96	.87201
1.08	3.6337	2.00	.84611
1.12	2.96086	2.04	.82144
1.16	2.54963	2.08	.79786
1.20	2.26415	2.12	.77521
1.24	2.05061	2.16	.75337
1.28	1.88272	2.20	.73218
1.32	1.74606	2.24	.71149
1.36	1.63187	2.28	.69114
1.40	1.53451	2.32	.67094
1.44	1.45016	2.36	.65065
1.48	1.37609	2.40	.62998
1.52	1.31033	2.44	.60850
1.56	1.25141	2.48	.58559
1.60	1.19847	2.52	.56029
1.64	1.14975	2.56	.53097
1.68	1.10542	2.60	.49463
1.72	1.06461	2.64	.44532
1.76	1.02685	2.68	.36854
1.80	.99179	2.72	.23092
1.84	.95896	2.74	.12196
1.88	.92825		

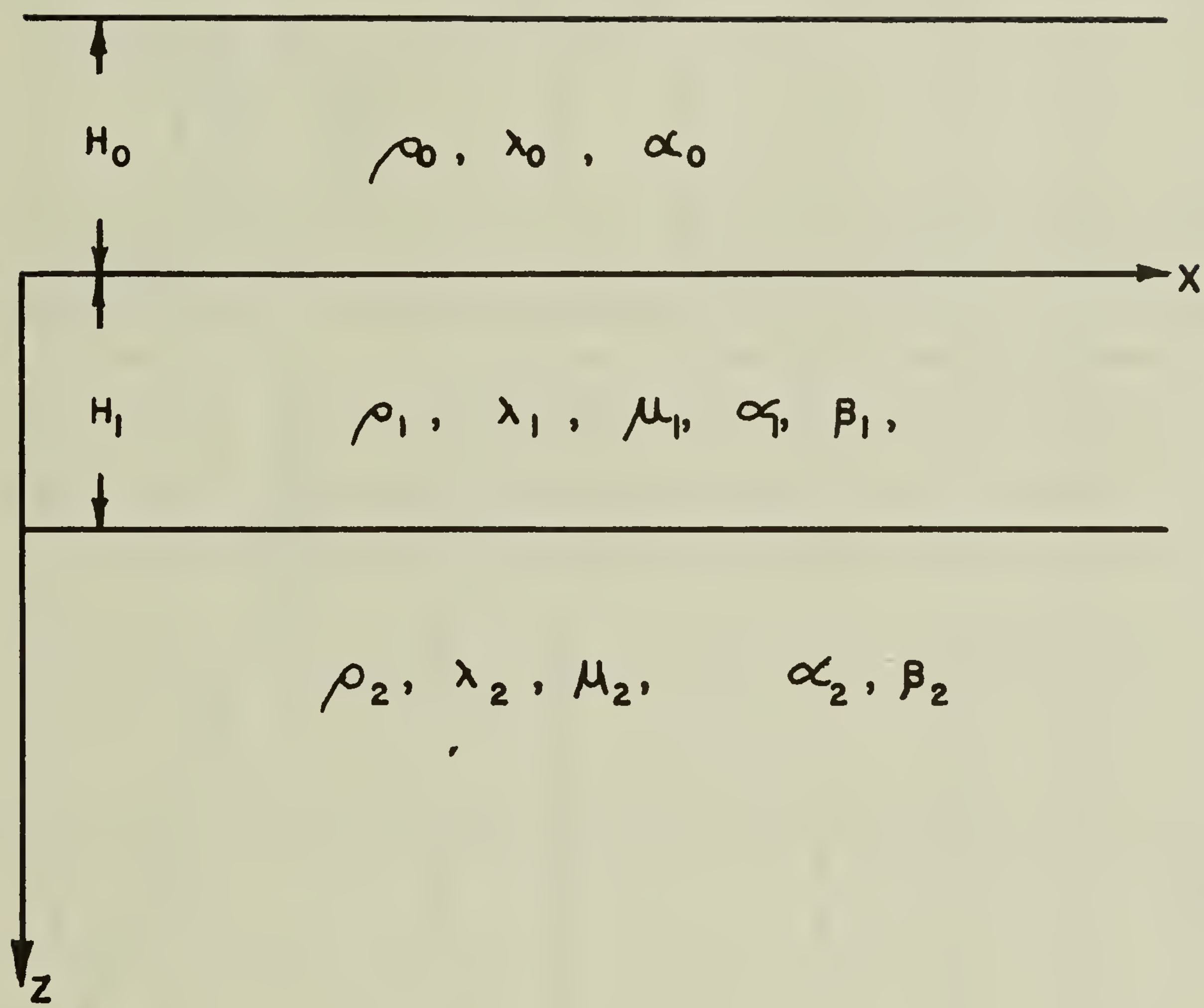

Figure 1. Nomenclature and coordinate system.

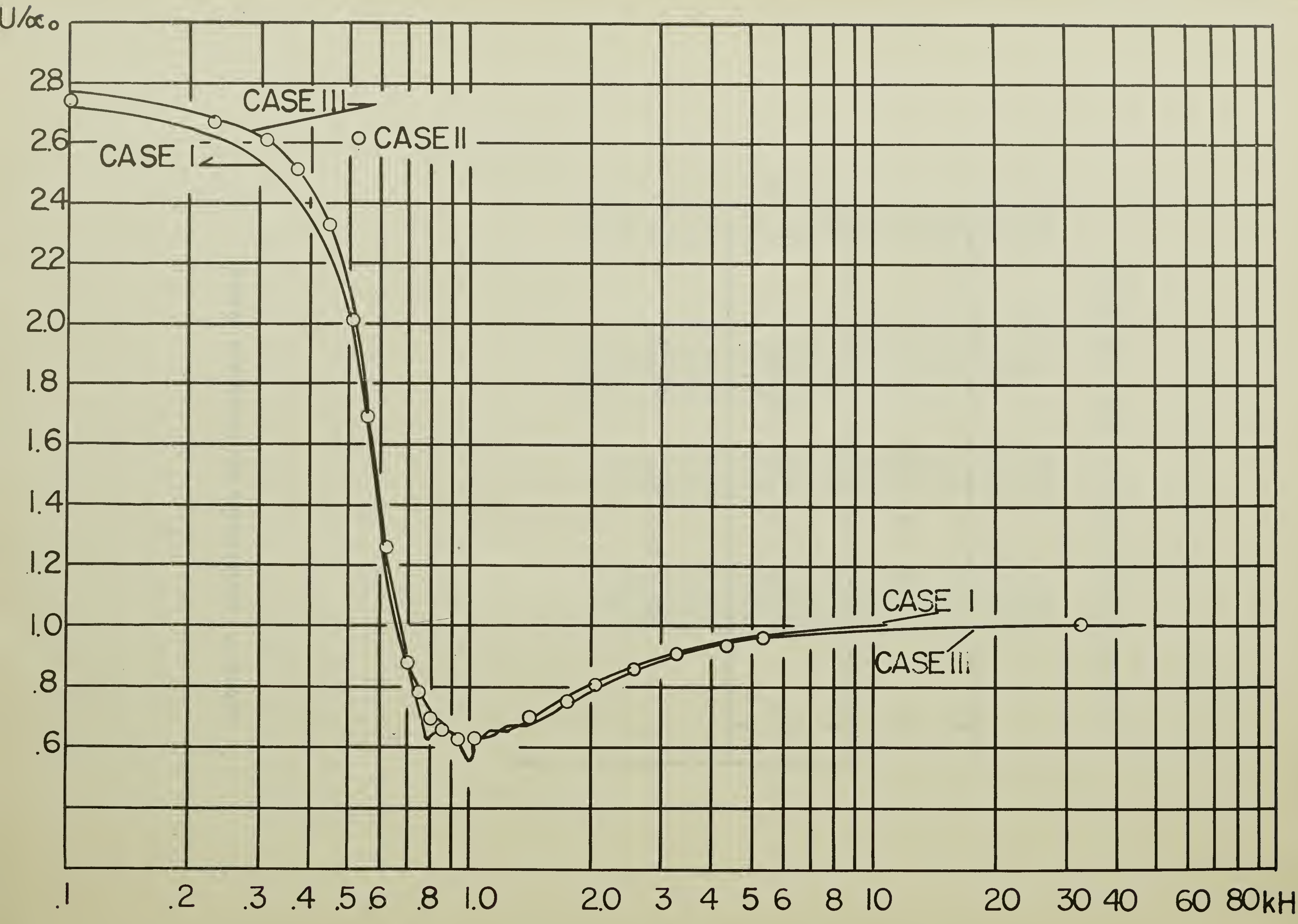

Figure 2. Dimensionless group velocity curve for cases I, II and III.

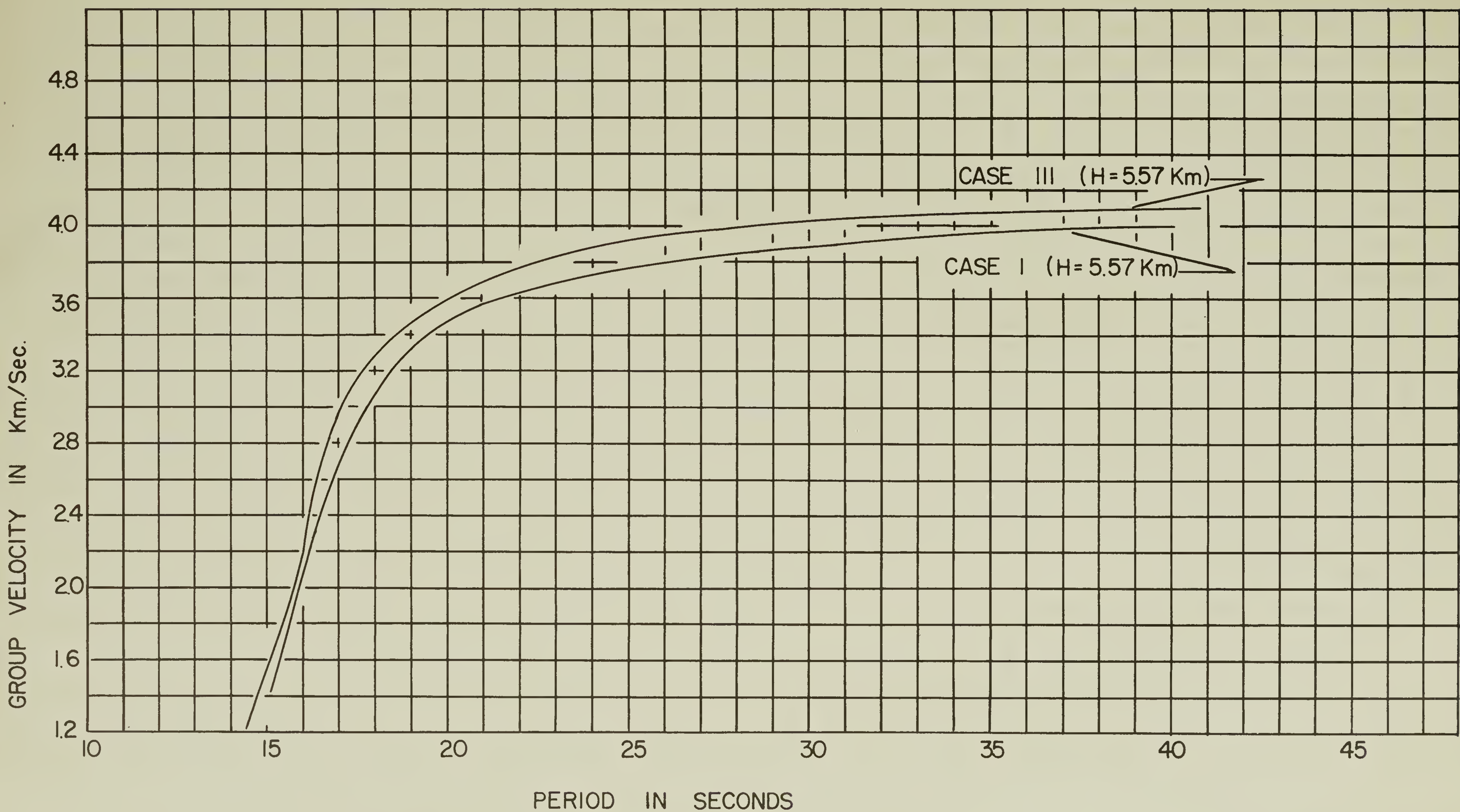

Figure 3. Group velocity curves for cases I and III with H = 5.57 km.