Essays on Market Microstructure

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Submitted in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy
under the Executive Committee
of the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2021

ABSTRACT

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In this doctoral dissertation, I study markets in which the private information held by various agents may be reflected in prices, and as a result may be leaked to other market participants. Specifically, I study how the market microstructure interacts with the price discovery process, the market efficiency, agents' market power, and social welfare. This dissertation consists of two chapters.

The first chapter studies the implications of leakage of information through prices for the efficient operation of markets with heterogeneous agents. Focusing on uniform-price double auctions, I first characterize how the presence of heterogeneity (e.g., in terms of agents' trading costs, information precision, or risk attitudes) can shape the information content of prices and hence the market's informational efficiency. I find that price informativeness decreases with the extent of heterogeneity in the market. I then establish that such reductions in price informativeness can in turn manifest themselves as an informational externality: in the presence of heterogeneity, agents do not internalize the impact of their trading decisions on the information revealed to others via prices. This chapter also shows that the welfare implications of this heterogeneity-induced informational externality depends on the intricate details of the market. The results thus indicate that accounting for the possibility of information leakage should be an important consideration in designing markets with asymmetric information. I conclude by exploring the welfare implications of market segmentation in the presence of heterogeneous agents and information leakage.

The second chapter studies how information asymmetry shapes price impact in the presence of strategic interactions, i.e., agents' actions being strategic substitutes or strategic complements. Focusing on demand-function competition with strategic interactions, I first establish the existence and characterize the equilibrium. The char-

acterization indicates that strategic interactions have a direct impact on the weights agents put on their private information: as strategic interaction increases, agents put less weight on their private information. I also characterize the relation between price impact, strategic interaction, and information asymmetry. While price impact decreases as the level of information asymmetry decreases, the relation between price impact and strategic interaction is more subtle, and it depends on whether agents submit upward- or downward-sloping demand schedules. When agents submit downward-sloping demand curves, price impact decreases as the extent of strategic substitutability increases, and increases as the extent of strategic complementarity increases. Furthermore, strong interaction may mitigate or exacerbate the effect of information asymmetry on agents' price impact, depending on the slope of the inverse supply curve. The results in this chapter thus emphasize the importance of accounting for strategic interactions between market participants, when assessing their price impact in markets with asymmetric information.

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Acknowledgements

My years as a graduate student at Columbia Business School have been one of the most influential experiences of my life so far. I was given the opportunity to expand my knowledge and skills, and gain new perspectives on life. I have grown as a researcher and as a person. Many people have helped in shaping these years of my life, to whom I am grateful for.

I would like to express my deepest gratitude to my advisor, Alireza Tahbaz-Salehi, for his dedicated support and guidance. Alireza taught me how to think through research problems, dig deeper, and be precise in my writing and presentations. He is a person I always enjoy talking to on research and life in general. I will always be grateful to him, for guiding me throughout this journey, from start to end.

I would also like to thank Xavier Vives, who I had the honor to work with as a co-author. Xavier's immense amount of knowledge and understanding has contributed a lot to my research.

I would like to thank Agostino Capponi for giving me the opportunity to work with him on one of his projects. It was a very interesting experience and I enjoyed working with him. Agostino has always been kind and supportive, and willing to help.

I am very thankful to the staff and faculty of the DRO division. I feel incredibly lucky to have been surrounded by such smart and caring people for the past several years. Special thanks to Assaf Zeevi and Gur Huberman, for their help in making it possible to join the DRO department; to Gabriel Weintraub, for all his advice and support during my first year; to Carri Chan for her tremendous support and caring, it meant a lot to me; to Omar Besbes for all his help and support throughout the years; to Paul Glasserman, who I enjoyed TA'ing for, and who was always willing to help; to Costis Maglaras, Awi Federgruen, Yash Kanoria, Ciamac Moallemi, Jacob Leshno, Fanyin Zheng, and Jing Dong for their helpful conversations and advice. Many thanks to the department staff, Clara Magram, Cristina Melo-Moya, Winnie Leung and Maria Micheles, for all their help and fun conversations. Also, a big thank you to the business school PhD office staff, Elizabeth Elam and Dan Spacher, for their tremendous help along the years.

I am extremely thankful for the great cohort of students I've had a chance to interact with. In particular, Yoni, who helped me survive my first year. Daniela, Daniel, Joan, Davide and Cinar for their help and support during my first years. My two great friends, Vashist and Gowtham, for all the stimulating conversations, laughs and studies. Also Francisco, Zhe and Fei whose interactions in the office have made my grad school life fun.

I have no words to express how thankful I am to my friends and family in Israel. They have been my pillar of strength. I am especially thankful to my parents, who have been nothing but supportive and loving through every step of my way, I could have never done this without them. Also, my two best friends, Ilanit and Oneg, who were always there to encourage me when times were rough, and be happy for me when times were good, even from 9135 km away and 7 hours ahead.

Last but not least, I am most thankful to my life partner Amnon, who encouraged me to go on this adventure and came along with me to an unknown future. He has been by my side for the past fifteen years, supporting me patiently in everything I try and do. To our daughters, Lilach and Netta, who made this impossible and possible at the same time. Thank you.

To my parents, Ruhama and Efraim,

my husband Amnon,

and our daughters, Lilach and Netta

with gratitude and love

Introduction

> *"Fundamentally, in a system in which the knowledge of the relevant facts is dispersed among many people, prices can act to coördinate the separate actions of different people."*
>
> — Friedrich A. Hayek, *The Use of Knowledge in Society* (1945)

The availability and distribution of information in a market has an important role in shaping the market's operations and welfare. As a result, a growing literature has focused on the role information plays in various financial and economic settings. It is often the case that information is dispersed throughout the market. That is, many market participants hold different pieces of private information regarding the goods that are being traded. In financial markets, for example, traders have private information regarding the value of various financial assets. An important question thus emerges on whether and how markets aggregate information in a world where information is dispersed.

It is by now conventional wisdom that prices convey important information to market participants. The seminal idea on the role of prices in aggregating and efficiently transmitting information is due to Hayek [32]. The Vice Chair for Supervision of the Board of Governors of the Federal Reserve System observes in his speech at [18], that "Hayek emphasized that the signals transmitted by the various individual prices in the economy could, together, serve as a useful means of guiding overall resource allocation. The reason is that prices convey messages to consumers and producers even when the information that drives prices is not aggregated or directly observed." According to this perspective, prices allow market participants to take advantage of the information that would not have otherwise been available to them. Prices, therefore, serve a dual role as an index of scarcity and conveyor of information, or in other words, they clear the market and aggregate information.

If one is to take the Hayekian perspective seriously, then the information aggregation process should be an important parameter for design and regulation of markets. Indeed, in its policy report on the emergence of "dark pools" for equity trading, the International Organization of Securities Commissions [36] argued that "[...] where regulators consider permitting different market structures [...] they should consider the impact of doing so on price discovery [...]." Another prominent example is the class of emissions permits markets in the various "cap-and-trade" systems implemented around the globe. In brief, a cap is set on total amount of permitted pollution, while emission allowances within the cap are distributed to (potential) emitters. The allowances can then be exchanged on a secondary market. One of the key design objectives of such a scheme is for the price in the secondary market to reflect the social costs of emissions, thus inducing firms to internalize the impact of their production decisions. An inefficient price discovery process can lead to suboptimal budgeting decisions for firms and inefficient reduction of emissions (e.g., see [21]).

Designing markets that take into account the value of information and its interaction with other factors in the market requires the analysis and the understanding of the market operations and the underlying forces that impact the market outcomes. The aims of this dissertation is to take a step towards understanding the impact of the market microstructure on the information aggregation process by prices and on the overall performance of the market.

Chapter 1 of this dissertation studies how the extent to which the price reveals information can shape the informational and allocative efficiency of the market. Furthermore, it explores how the way and extent the information is revealed depends on the characteristics of market participants and on the market structure. By informational efficiency, we mean that prices are able to fully aggregate all dispersed information. In other words, a trader observing the price and her private information is able to trade, as if she knew all the available information in the market. Allocative efficiency, on the other hand, refers to an efficient allocation of resources. In a financial market setting, for example, this means that the quantity of the asset traded by each trader coincides with the socially optimal value.

Specifically, we study how heterogeneity in the market (e.g., in terms of agents' trading costs, information precision, valuations, and risk attitudes) may interact with the market price's role as an endogenous source of information. We consider a uniform-price double auction consisting of an arbitrary number of agents trading a single good with interdependent valuations. Importantly, each agent has access to some private information about the good's underlying value. For example, in the context of the example mentioned above, these agents may correspond to various carbon-producing firms trading emissions permits, who may be privately informed about their abatement costs.

We start by investigating the market's informational efficiency, defined as the extent to which prices are able to aggregate the information dispersed throughout the market. In this sense, a market is informationally efficient if the price serves as a sufficient statistic for all the information dispersed throughout the market.

Our first set of results establish that the market's informational efficiency is highly sensitive to the characteristics of market participants. In particular, we find that the market is less informationally efficient the more heterogeneous the agents are. This is a consequence of the fact that agents' characteristics determine the intensity of their market activity and hence the extent to which their private information will be reflected in the price. This result thus indicates that the extent of information leakage can vary with the intricate details of the market structure.

As our second set of results, we find that this informational inefficiency also translates into an informational externality: in a heterogeneous market, agents do not internalize how their actions shape the information content of prices and hence the information available to other agents. This finding indicates that the characteristics of market participants, and as a result, the extent to which the price reveals information, may severely undermine the workings of the market. Interestingly, we find that the direction of this informational externality depends on whether the informational or the allocative role of prices dominate. When the informational role dominates — i.e., when the main role of the price is as an endogenous public signal in the market — agents with relatively high trading costs under-react to their private information, whereas agents with relatively low trading costs over-react, resulting in an equilibrium price that is not informative enough. In contrast, when the main role of the price is as an index of scarcity — i.e., to match supply and demand — agents with high trading costs over-react to their private information, and agents with low trading costs under-react, resulting in an equilibrium price that is too informative. In particular, whether an agent is over-reacting or under-reacting depends on (i) how her private signal covaries with the asset's payoff estimation error of other traders; and (ii) the slope of agents' demand curves. The covariance in (i) captures whether other agents are over-estimating or under-estimating the true underlying value of the good. Then, the slope of their demand curves determine how they react to a change in the price resulting from the agent's action.

Finally, to assess the policy implications of the mechanisms identified in this work, we investigate how changes in the market architecture can impact the market's informational and allocative efficiencies. Importantly, we find that policies that shape the distribution of agents that participate in the market can have a first-order effect on the efficient operations of the market.

Chapter 2 of this dissertation studies how information asymmetry shapes price impact in the presence of strategic interactions. Strategic interactions may arise due to a verity of reasons, such as competition in other markets, and result in agents' actions being strategic substitutes or strategic complements. While prior work has explored the interaction between information asymmetry and price impact, the interaction between information asymmetry, price impact and strategic interactions has been largely unexplored.

Specifically, we investigate how the type and extent of strategic interactions — whether it is strategic complementarity or strategic substitutability — shape the equilibrium actions of market participants and as a result, their price impact, in the presence of information asymmetry. We consider a model similar to the one in Chapter 1, but with an additional parameter representing the nature of strategic interactions between agents. We use a reduced-form approach that allows us to represent both strategic substitutability and strategic complementarity, that may result from agents' activities outside of the market (e.g., spill-overs and externalities).

Our first set of results establishes the existence of a Bayes-Nash equilibrium, and we characterize the equilibrium in terms of model primitives. We then turn to a comparative statics analysis. In order to obtain a tractable representation, we look at the large market limit as the number of agents in the market grows. We find that the existence of strategic interactions between agents has a direct impact on the weight agents put on their private information. Specifically, agents put less weight on their private information as the extent of strategic interaction between them increases. From the view point of agent i, when the actions of all other agents $j \neq i$ have a dominant effect on her profit, she is less incentivized to pay attention to her private information. Put differently, there is a coordination motive (in the case where agents' actions are strategic complements), or a miss-coordination motive (in the case where agents' actions are strategic substitutes) that offsets the desire to match the fundamentals.

Our second set of results establishes the relationship between price impact, information asymmetry and strategic interactions. We find that there is a monotonic relation between price impact and information asymmetry: as the extent of information asymmetry decreases, price impact decreases. Specifically, as agents' private information becomes biased towards the average valuation of the good, the level of information asymmetry in the market decreases, which results in a decrease in agents' price impact. We also characterize the relation between price impact and the extent and nature of strategic interactions. Importantly, this relation depends on the the slope of agents' demand curves. When agents submit downward-sloping demand curves, price impact decreases with the extent of strategic susbstitutability and increases with the extent of strategic complementarity. However, when agents submit upwords-sloping demand curves, price impact decreases with the extent of strategic complementarity. This is a consequence of the fact that strategic complements offset the "price effect", which induces agents to decrease their demand as price increases. However, when agents' actions are strategic substitutes, the strategic effect reinforces the "price effect".

Finally, we study the trade-offs of the effects of strategic interaction and information asymmetry on agents' price impact. We find that strategic interaction may mitigate or exacerbate the effect of information asymmetry on price impact, and it depends on the elasticity of the supply of the asset.

Chapter 1

Heterogeneous Markets

1.1 Motivation and Overview of Results

There is, by now, unanimity amongst researchers that the design of markets has an important role in improving market performance. As a result, a growing literature in the economics and operations management has focused on optimal design of market in various contexts, such as auctions, two-sided markets, and on-line marketplaces. However, one aspect that has been over-looked is the possibility of information leakage, its interaction with the market design, and the impact it may have on the market outcomes.

With the advance in technology, information leakage has become ever more relevant. The digitalization of information has made the process of observing and processing agents' actions — by the market maker or the market participants — fairly easy and fast, resulting in a high possibility of their private information — on which they rely on when taking their actions — being leaked. This possibility for information leakage, in turn, may affect the actions of market participants – whether they are the agents observing the leaked information or if it is their information that is being leaked to others. Understanding such effects, their interaction with the market structure, and their impact on the outcomes is crucial for policy makers and market designers, who want to guarantee a smooth and efficient operation of the market.

The possibility of information leakage is one of the central features of financial markets. It is by now conventional wisdom that the private information held by various market participants (even anonymous ones for that matter) can be reflected in a security's price. Such a possibility has made the role of information in the "price discovery" process as one of the main concerns of policymakers in designing market regulations. For instance, in its policy report on the emergence of "dark pools" for equity trading, the International Organization of Securities Commissions [36] argued that "[...] where regulators consider permitting different market structures [...] they should consider the impact of doing so on price discovery [...]." Similarly, Commissioner Troy A. Paredes from the U.S. Securities and Exchange Commission [64] observed that "price discovery matters because investors would be less willing to invest if the contrarian views of short sellers were not fully incorporated into securities prices" and that "when price discovery is compromised, we run the risk that our securities markets allocate capital inefficiently."

Another prominent example is the class of emissions permits markets in the various "cap-and-trade" systems implemented around the globe. In brief, a cap is set on total amount of permitted pollution, while emission allowances within the cap are distributed to (potential) emitters. The allowances can then be exchanged on a secondary market. One of the key design objectives of such a scheme is for the price in the secondary market to reflect the social costs of emissions, thus inducing firms to internalize the impact of their production decisions. But this means an inefficient price discovery process can lead to suboptimal budgeting decisions for firms and inefficient reduction of emissions. Thus, not surprisingly, policymakers consider an accurate price discovery to be an important concern when designing and implementing these systems. For instance, according to the [26], "[...]maintaining the functioning and integrity of the secondary markets as lead venues for price discovery and efficient allocation should continue to enjoy highest priority when designing a comprehensive auctioning scheme for the trading period 2013–2020 (Phase III)." Despite this emphasis, studies conducted on data from the European Union Emissions Trading

System (the world's first and so far largest cap-and-trade system) indicate that in the first phase of the implementation in 2005–2007, prices failed to aggregate information effectively ([24], [61]), resulting in suboptimal market operations.

In this chapter, we take a step towards understanding how the quality of price discovery in the presence of information leakage can shape the informational and allocative efficiency of the market. We further try to understand how the extent of information leakage, and in turn, the quality of price discovery depends on the market architecture. While building on a large prior literature, we explore a novel aspect: how heterogeneity in the agent-level (e.g., in terms of agents' trading costs, information precision, valuations, and risk attitudes) may interact with the market price's role as an endogenous source of public information. We then explore the implications of price informativeness on allocative efficiency by comparing welfare across various market architectures.

We base our analysis on a standard model of a uniform price double auction. More specifically, following [66], we focus on a competitive market consisting of finitely many agents who trade a single asset. Agents have interdependent valuations for the asset, but are uncertain about the underlying state that determines the asset's payoff. Instead, each agent observes a potentially informative private signal about the asset fundamental. As in the rational expectations tradition (e.g., [29] and [25]), the price serves as an endogenous public signal with the ability to (fully or partially) convey each agent's private information to other market participants. As our key assumption, we allow for ex ante heterogeneity in the precision of private signals and the agents' preferences, by assuming that agents face potentially heterogeneous trading costs.

Our first set of results, which serves as the basis for the rest of our analysis, establishes that the distribution of trading costs has a direct impact on the informativeness of the price. In particular, we show that, in markets with more than two agents, the market is informationally efficient if and only if all trading costs coincide. This result is a consequence of the fact that introducing a second dimension of heterogeneity (i.e., heterogeneity in trading costs in addition to heterogeneity in information sets) leads to a secondary motive for trade that may be orthogonal to agents' private information: all else equal, agents with higher trading costs trade less intensely on the same information than those with lower trading costs. This preference-induced heterogeneity, in turn, biases the price towards the private signals of agents with lower trading costs compared to the benchmark with identical traders. In fact, we show that price informativeness decreases in the (weighted) variance of agents' trading costs, with the weights given by the precision of each agents' private signal. In summary, higher preference heterogeneity leads to less informative prices.

Given the above observation, our second set of results then establishes that the reduction in price informativeness also manifests itself as an informational externality. More specifically, we show that agents do not internalize the impact of their trading decisions on price informativeness for other traders. Crucially, this informational externality only exists when agents are heterogeneous: we show the equilibrium is constrained efficient when all agents have identical trading costs. In contrast, when agents have heterogeneous trading costs, a subset of agents over-react to their private signals (compared to the constrained efficient benchmark), whereas the remainder of the agents under-react, where the sets of over-reacting and under-reacting agents depend on the underlying model parameters. In particular, whether an agent is over-reacting or under-reacting depends on (i) how her private signal covaries with the asset's payoff estimation error of other traders; and (ii) the slope of agents' demand curves. The covariance in (i) captures whether other agents are over-estimating or under-estimating the true underlying value of the good. Then, the slope of their demand curves determine how they react to an increase or decrease of the price resulting from the agent's action. These two factors, in turn, depend on agents' trading costs and on the dominant role played by the price, respectively. When the informational role of price dominates — i.e., when the main role of the price is as an endogenous public signal in the market — agents with relatively high trading costs under-react to their private information, whereas agents with relatively low trading costs over-react. In contrast, when the main role of the price is as an index of scarcity — i.e., to match supply and demand — agents with high trading costs over-react to their private information, and agents with low trading costs under-react.

With the above results in hand, we then leverage the heterogeneity-induced informational externality identified above to study how the interaction of private information and market architecture determines social welfare. More specifically, we use our framework to compare a centralized market architecture to a segmented market in which agents can only trade with a subset of other individuals. As our main result, we show that, depending on the distribution of trading costs, a segmented market architecture can achieve a higher welfare compared to the centralized market. This is despite the fact that a centralized market provides more trading opportunities and — at least in principle — should lead to higher levels of price informativeness as the price can aggregate the private information of a larger set of individuals. Nonetheless, our results establish that if market centralization leads to sufficiently high levels of heterogeneity, not only price informativeness may decline, but also this decline in the quality of information aggregation and the corresponding informational externality may reduce the welfare in the centralized market below that in the segmented market architecture. Thus, policies that shape the distribution of agents that participate in the market can have a first-order effect on the efficient operations of the market.

Overall, our theoretical findings provide insight on the role of information leakage through prices in shaping market outcomes, and how it may depend on the intricate details of the market. They also suggest that information leakage may have first-order effect on welfare, and as a result should be an important policy concern when designing markets.

1.2 Related Literature

Our theoretical framework is related to the literature on rational expectations equilibrium with a Gaussian information structure, such as [33], [29], [3], [25] and [46], among many others[1]. Within this literature, our work is most closely related to [57], who study a market consisting of traders with identical trading costs but heterogeneous pairwise correlations in valuations. In line with our findings for a model with heterogeneous trading costs, they establish that heterogeneity in pairwise correlations can break informational efficiency. However, unlike our framework, the failure of information aggregation in their model does not translate into an informational externality: even though the market cannot fully reveal the information to all traders, neither can a social planner who has to respect the decentralized information structure of the economy.[2] The disparity between the results of [57] and our findings is driven by the distinct origins of informational inefficiencies in the two models. More specifically, in the presence of heterogeneous pairwise correlations, the price cannot fully aggregate information because there is no single one-dimensional statistic that can serve as a sufficient statistic for all market participants simultaneously. In contrast, our model admits such a common sufficient statistic. Yet, heterogeneity in trading costs leads to an equilibrium price that does not coincide with this statistic.

Information leakage effects have also been studied by the supply chain management literature. [47] studies the possibility that confidentially-shared information between a retailer and a manufacturer may be leaked to other retailers as they observe the manufacturer's actions. The leaked information, in turn, may affect the

[1] For an extensive review of the early models see [65], chapters 3-4, and for a more recent literature review see [59].

[2] We formally establish this claim in Section 1.7.

strategies of the other retailers, even though they were not part of the information sharing agreement. Relatedly, [8] emphasize the importance of "strategic information management," according to which firms take the possibility of information leakage to competitors into account, while [44] study revenue-sharing contracts that can mitigate the negative effect of information leakage in the supply chain.

Our work is also related to the literature on optimal information revelation in disclosure policies in the context of platforms and queues. [17] focus on the optimal information disclosure policy of a contest designer regarding the competitors' progress. Relatedly, [53] study the problem of optimal information provision of online platforms that collect and disseminate consumers' experiences, while [20] study the problem of optimal information revelation in a setting of a social networking platform facing the trade-off between engagement and misinformation. In the context of queues, [30], [40], and [5], among others study the effect of different information revelation methods on customers and on the overall performance of the system. In contrast to this literature, where there exists a platform or a service system who controls the nature of the information provision, in our setting, there is no entity who controls for the amount of information revealed, but rather it is determined by the way that prices incorporate and convey information, from and to market participants.

A growing literature has been studying another channel for endogenous public information — ratings and reviews — through which customers can learn about the value of different products and services. For example, [62], [16], [1], and [35] investigate how successful is the learning process in terms of learning the true value of the products. While related, our work departures from this literature as we allow for learning from prices.

Our work is also related to the literature on informational efficiency and allocative efficiency of markets. For example, [55] studies the general tension between the two notions in a setting of auctions. In a multi-unit auction model with a finite number of bidders, the more sensitive bids are to private information, the more information is aggregated in the price but also the greater is the allocative inefficiency. However, in the limit (of the number of items and bidders) [55] shows that both are attained – full information aggregation and allocative efficiency. [37] look at predictions markets in a dynamic setting. They show that when all traders are risk-averse, although prices reflect risk-adjusted probabilities, under some smoothness condition, the allocation is ex-post Pareto efficient. In addition, they show that information is aggregated in the sense that an uninformed observer of the market, sharing only the common knowledge of market participants can infer the true probabilities.

Also related is the literature on efficient use of public versus private information. [52] show that in a game with strategic complementarities, agents might over-react to public information, and so releasing more public information can reduce social welfare. [9] generalize the model in [52] and study for different economies the efficient use of public information. As opposed to our model, they consider exogenous public information, and so the weight agents put on their private information does not affect the content of the public signal.

Our work is also related to the literature that studies the implications of learning from endogenous signals and the resulting informational externalities. For instance, [6] illustrates how in the presence of strategic complementaries, providing agents with more precise signals can reduce the informational efficiency of the price system. Relatedly, [11] show that over-the-counter trade can lead to an informational externality, as traders do not internalize how their trading decisions impact the information set of other market participants.[3]

Finally, our results on the welfare implications of various market architectures are related to the work of [49], who argue that when agents can exert market power, frag-

[3]Other examples include [7] and [67], among others.

mentation of centralized markets may increase aggregate welfare.[4] In contrast to this paper, we assume that all traders are competitive, but instead allow them to learn from endogenous public signals, i.e., prices. This channel creates an informational externality, whose magnitude is closely tied to the market architecture. As such, a transition from centralized to segmented markets impacts equilibrium price informativeness, thus leading potentially higher aggregate welfare. Also related is the recent work of [38] who look at the impact of introducing dark pools to financial markets on welfare. In other words, what are the welfare implications of having these "closed" markets in addition to the open market (i.e., an exchange). They show the answer is ambiguous and depends on the intrinsic value of traders and the mass of speculators. Thus, similar to our work, they show that a centralized open market may be inferior to a more decentralized market with respect to welfare.

1.3 Model

Consider a market consisting of n agents, denoted by $\{1, 2, \ldots, n\}$, who trade a divisible good. These agents may correspond to firms trading emissions permits in a secondary market in a cap-and-trade scheme or traders buying and selling assets in financial markets. The realized payoff of agent i who obtains x_i units of the good is given by

$$\pi_i(x_i) = \theta_i x_i - \frac{1}{2}\lambda_i x_i^2 - p x_i, \tag{1.1}$$

where p denotes the price of the good and θ_i, which we refer to as i's valuation, is a random variable that is drawn from the standard normal distribution. We allow for interdependence in traders' valuations by assuming that $\text{corr}(\theta_i, \theta_j) = \rho$ for all pairs of agents $i \neq j$, where $\rho \in [0, 1)$. This formulation thus nests the cases with

[4] Other recent contributions on the welfare consequences of market architecture include [27], who argue that the introduction of trade frictions can increase predictability in trading encounters.

independent ($\rho = 0$) and common ($\rho \to 1$) valuations as special cases. We refer to parameter λ_i in (1.1) as agent i's trading cost and treat the collection of parameters $(\lambda_1, \ldots, \lambda_n)$ as a primitive of the model, which we assume to be commonly known to all agents.

In the context of financial markets, θ_i represents the dividend of the traded asset, x_i is the quantity of the asset purchased by trader i, and the trading cost λ_i can arise due to transaction taxes, inventory costs for holding the asset, or other costs incurred as a consequence of trade. Alternatively, to interpret equation (1.1) in the context of the emissions permits market, suppose each polluter i can produce one unit of output per one unit of pollution permit. Thus, to produce x_i units of output, which results in a revenue of $\theta_i x_i$, polluter i incurs a cost px_i to obtain the required permits as well as a quadratic production cost $\lambda_i x_i^2/2$. Regardless of the interpretation, equation (1.1) represents a market consisting of agents with interdependent valuations and potentially heterogeneous costs.

Prior to trading, each agent i observes a noisy private signal $s_i = \theta_i + \epsilon_i$ about her valuation, where $\epsilon_i \sim N(0, \sigma_i^2)$ are mutually independent and σ_i parametrizes i's uncertainty about θ_i. Under this specification, all signals $(s_1, \ldots, s_n)$ are informative about agent i's valuation as long as $\rho \neq 0$ and $\sigma_i > 0$.

The good/asset is supplied by a competitive market of outside agents, represented by the inverse aggregate supply function $p = \alpha + \beta \sum_{i=1}^{n} x_i$, where α and β are non-negative constants and $\sum_{i=1}^{n} x_i$ is the (inside) agents' aggregate demand for the good. Such an inverse supply function can arise by assuming that, in addition to the n traders discussed above, the market contains a representative outside agent, indexed 0, with payoff

$$\pi_0(y) = \alpha y - \beta y^2/2 - py, \tag{1.2}$$

where y is the total units of the good purchased by the outside agent.[5] In the context

[5]In parallel with traders indexed 1 through n, one can interpret α and β as the outside agent's

of the emissions permits market, the outside agent can be though of as the government or regulator supplying the asset, with $\alpha y - \beta y^2/2$ capturing the social cost of y units of emissions. Market clearing requires that the traders' aggregate demand and the demand of the outside agent satisfy $y + \sum_{i=1}^{n} x_i = 0$.

Trade occurs via a one-shot, uniform-price double auction mechanism, according to which all agents simultaneously submit demand schedules that specify their demand for the asset as a function of the price p. Under such a trading mechanism, the strategy of trader $i \in \{1, \ldots, n\}$ is a mapping $x_i(s_i, p)$ from her private information and the price to a quantity, whereas the strategy of the representative outside agent is a function $y(p)$ that specifies his demand at any given price p. The price is then determined by the submitted demand functions and the market-clearing condition.

The competitive equilibrium of this market is defined in the usual way: it consists of a collection of demand schedules $x_i(s_i, p)$ and $y(p)$ such that (i) each trader $i \in \{1, \ldots, n\}$ maximizes her expected payoff conditional on her information set $\{s_i, p\}$ while taking the price as given, (ii) the representative outside agent maximizes his payoff given the price, and (iii) the market clears. Throughout, we restrict our attention to equilibria in linear strategies, according to which each agent i's demand schedule is an affine function of her private signal s_i and the market price p.

Before presenting our results, a few remarks are in order. First, note that the assumption that agents submit price-contingent demand schedules enables them to take the information content of the price into account, thus paving the way for the possibility of information leakage in the market: the price can serve as an endogenous public signal with the ability to (fully or partially) convey agents' private information to one another. Second, the absence of noise traders in our framework enables us to perform a well-defined welfare analysis. Such an analysis will be instrumental in

valuation and trading cost, respectively.

disentangling the market's informational inefficiency from its allocative inefficiency. Finally, our assumption that agents are price takers ensures that the inefficiencies identified by the welfare analysis are not driven by market power or other departures from the competitive benchmark. Our main results on information leakage and the market's informational inefficiency extend to settings where agents exert market power.

We have the following result:

Proposition 1.1. *There exists an equilibrium in linear strategies $x_i = a_i s_i + b_i - c_i p$, where the coefficients corresponding to trader i's strategy depend on the price via*

$$\lambda_i a_i = \frac{\mathrm{var}(p) - \mathbb{E}[ps_i]\mathbb{E}[p\theta_i]}{\mathbb{E}[s_i^2]\,\mathrm{var}(p) - \mathbb{E}^2[ps_i]} \tag{1.3}$$

$$\lambda_i b_i = \frac{\mathbb{E}[ps_i] - \mathbb{E}[s_i^2]\mathbb{E}[p\theta_i]}{\mathbb{E}[s_i^2]\,\mathrm{var}(p) - \mathbb{E}^2[ps_i]}\mathbb{E}[p] \tag{1.4}$$

$$\lambda_i c_i = 1 + \frac{\mathbb{E}[ps_i] - \mathbb{E}[s_i^2]\mathbb{E}[p\theta_i]}{\mathbb{E}[s_i^2]\,\mathrm{var}(p) - \mathbb{E}^2[ps_i]} \tag{1.5}$$

and the price depends on the equilibrium strategies via

$$p = \frac{\alpha + \beta \sum_{k=1}^{n}(a_k s_k + b_k)}{1 + \beta \sum_{k=1}^{n} c_k}. \tag{1.6}$$

Furthermore, coefficients $(a_1, \dots, a_n)$ are independent of parameters α and β.

The above result, which will serve as the basis for the rest of our analysis, provides an implicit characterization of agents' equilibrium strategies and market-clearing price as a function of trading costs and signal precisions.[6] Despite the implicit nature of Proposition 1.1, a few observations are immediate. First, equation (1.6) establishes that the price is an affine function of all traders' private signals, thus formalizing the idea that the equilibrium price is an endogenous public signal, with the weighted average $\sum_{k=1}^{n} a_k s_k$ serving as a sufficient statistic for the information content of the

[6]Even though not explicit, the traders' signal precisions are reflected in the various variance and covariance terms between θ_i, s_i, and p. We explore these relationships in detail in subsequent sections.

price. Second, the fact that coefficients $(a_1, \dots, a_n)$ are independent of α and β implies that even though the price level depends on the characteristics of the outside agent, its information content does not. Third and most importantly, the expression in (1.6) illustrates that various agents' private signals do not impact the information content of the price symmetrically. Rather, the price is biased towards the private signals of agents who assign larger weights on their signals in equilibrium. In view of equations (1.3)–(1.5), this observation implies that, in general, equilibrium price informativeness may depend on the entire profile of trading costs $(\lambda_1, \dots, \lambda_n)$ and signal precisions $(\sigma_1^{-1}, \dots, \sigma_n^{-1})$, an issue which will be the main focus of Section 1.4.

As a final remark, we note that the expression in (1.5) underscores the trade-off between the two roles played by the price: (i) as a measure of the opportunity cost of obtaining an extra unit of the asset and (ii) as a potentially informative endogenous public signal about the asset's underlying payoff. In particular, when the price is uninformative about the underlying state (i.e., when $\sigma_i = 0$ or $\rho = 0$), equation (1.5) implies that $c_i = 1/\lambda_i$ (the second term on the right-hand side of (1.5) equals zero), reflecting the opportunity cost for agent i. In this case, agents' demand slopes depend only on their trading costs, and so the demand of agents with lower trading costs is more elastic than the demand of agents with higher trading costs (the latter trade more conservatively). On the other hand, the informational role of the price is captured by the second term on the right-hand side of (1.5): if the price contains some information about θ_i above and beyond agent i's private information, she infers that a higher p reflects a higher payoff, thus reducing her opportunity cost of obtaining the asset. This reduction in opportunity cost is reflected as a smaller coefficient c_i in equilibrium. Put differently, the slope of the demand curve submitted by agent i not only reflects i's opportunity cost of trade, but also her desire to utilize the information contained in the price in her demand. Importantly, the relative importance of the two roles played by the price depends on the slope of the inverse aggregate supply function β. For small values of β, the price level is insensitive to the aggregate demand $\sum_{k=1}^{n} x_k$. Thus, while a small increase in the price does not change the marginal cost of acquiring the asset, such an increase is interpreted by the market as a strong positive signal about the asset's underlying value. As a result, the informational role of the price dominates, inducing the agents to submit upward-sloping demand curves ($c_i < 0$). In contrast, when β is large, the price is very sensitive to the aggregate demand $\sum_{k=1}^{n} x_k$. As a result, an increase in demand by an agent in the market — say, due to a positive signal —results in a sharp increase in the price, which induces other agents to purchase less of the asset. In other words, the role of the price as a measure of opportunity cost of the asset dominates its informational role, inducing downward-sloping demand curves ($c_i > 0$).

1.4 Information Leakage and Informational Efficiency

With the equilibrium characterization in Proposition 1.1 in hand, we now turn to studying how model primitives, and in particular, the profile of trading costs and signal precisions, shape the informational content of the price and hence the market's informational efficiency. Throughout, we rely on the following notion of informational efficiency:

Definition 1.1. *The equilibrium is fully privately revealing to trader i if* $\mathbb{E}[\theta_i|s_i, p] = \mathbb{E}[\theta_i|s_1, ..., s_n]$.

In other words, under full private revelation, the price coupled with agent i's private signal serve as a sufficient statistic for all the information dispersed throughout the market. We say the market is *informationally efficient* if the equilibrium is fully privately revealing to all agents simultaneously. Thus, in an informationally efficient market, the leakage of information via the price is complete. We have the following result:

Proposition 1.2. *Suppose $\rho \neq 0$ and $\sigma_i > 0$ for all traders i. The market is informationally efficient if and only if either*

(i) there are only two agents in the market (i.e., $n = 2$); or

(ii) all trading costs coincide (i.e., $\lambda_1 = \cdots = \lambda_n$).

The above result thus establishes that when all agents have identical trading costs, the leakage of information is complete, in the sense that all agents behave as if they had access to all the private information held by other agents in the market. This is the case regardless of the profile of signal precisions $(\sigma_1, \ldots, \sigma_n)$ and hence how private information is initially distributed among the agents. Additionally, Proposition 1.2 also shows that in a market consisting of $n \geq 3$ agent, any heterogeneity in trading costs would make the equilibrium price to be less than fully revealing to at least one market participant.

To see the intuition underlying this result, consider the special case in which all signals are of equal precision, that is, $\sigma_i = \sigma$ for all i. In such an environment, it is immediate that full private revelation requires the equilibrium price to be a sufficient statistic for the unweighted average of all traders' private signals, i.e., $p = d_0 + d_1 \sum_{k=1}^{n} s_k$ for some constants d_0 and d_1, where $d_1 \neq 0$. Yet, as we established in Proposition 1.1, the equilibrium price reflects $\sum_{k=1}^{n} a_k s_k$, where the coefficient a_k depends on the entire profile of trading costs $(\lambda_1, \ldots, \lambda_n)$. Thus, as long as there are a pair of traders i and j with non-identical trading costs, the equilibrium price would reflect a weighted average of private signals, making the extraction of the unweighted average of the signals and hence full revelation impossible. Note, however, that this argument is no longer valid if $n = 2$. In that case, each trader can back out the private signal of the other trader from the price (which reveals $a_1s_1 + a_2s_2$) and her own signal, irrespective of the coefficients a_1 and a_2.

We remark that the failure of information aggregation established in Proposition 1.2 is distinct from the reasons behind partial revelation in [39] and [57]. [39] illustrates that equilibrium is generically inconsistent with the efficient market hypothesis when "the dimension of the signal space is larger than the number of assets," or in other words, when the dimension of payoff-relevant variables exceeds the number of prices. On the other hand, [57] construct a class of models under which there is no single statistic that can simultaneously serve as a sufficient statistic for all agents in the market. In contrast to these papers, in our environment, the (single-dimensional) linear combination $\bar{S} = \sum_{k=1}^{n} s_k/(1-\rho+\sigma_k^2)$ is a sufficient statistic for all the information in the market for all traders simultaneously. In particular, it is easy to verify that $\mathbb{E}[\theta_i|s_i,\bar{S}] = \mathbb{E}[\theta_i|s_1,...,s_n]$ for all i. Yet the equilibrium price does not coincide with this statistic. This failure of information aggregation is a consequence of agents' equilibrium actions: the heterogeneity in trading costs induces a dispersion in agents' trading intensity that is orthogonal to their private signals, thus biasing the information content of the price towards the private information of agents with lower trading costs.

Proposition 1.2 thus illustrates that the nature and extent of information leakage in the market is highly sensitive to the distribution of agents' trading costs. Our next result provides a refinement of this observation by relating the extent of informational inefficiency in the market to the distribution of agents' trading costs. For each trader i, define *the information revelation gap* as

$$\phi_i = \frac{\mathrm{var}(\theta_i|s_i,p) - \mathrm{var}(\theta_i|s_1,\ldots,s_n)}{\mathrm{var}(\theta_i|s_i) - \mathrm{var}(\theta_i|s_1,\ldots,s_n)}. \tag{1.7}$$

This index, which is always a number between 0 and 1 measures the extent to which the price reduces agent i's uncertainty relative to a benchmark with no informational asymmetry. More specifically, $\phi_i = 0$ whenever the equilibrium is fully privately revealing to agent i, whereas $\phi_i = 1$ if the price does not provide agent i with any new information above and beyond her private signal s_i.[7]

Our next result relates each agent's information revelation gap to the distribution of trading costs in the market.

Proposition 1.3. *The information revelation gap of trader i satisfies*

$$\phi_i = \frac{\Sigma_i^2}{\Sigma_i^2 + (1/\Lambda_i)^2} + o(\rho), \tag{1.8}$$

where

$$\Lambda_i = \left(\frac{\sum_{k\neq i} w_k \lambda_k^{-1}}{\sum_{k\neq i} w_k}\right)^{-1} \quad , \qquad \Sigma_i^2 = \frac{\sum_{k\neq i} w_k \left(1/\lambda_k - 1/\Lambda_i\right)^2}{\sum_{k\neq i} w_k}$$

are, respectively, the weighted harmonic mean and weighted variance of the reciprocal of trading costs of agents $k \neq i$ with weights $w_k = 1/\mathrm{var}(s_k)$.

The above result thus provides a refinement of Proposition 1.2 (for small values of ρ) by linking the information content of the price to the distribution of trading costs in the market. More specifically, it illustrates that, keeping the harmonic mean of trading costs Λ_i constant, an increase in the heterogeneity Σ_i in the trading costs of agents $k \neq i$ widens i's information revelation gap. In contrast, when agents $k \neq i$ have identical trading costs, equation (1.8) implies that the equilibrium is fully privately revealing to agent i ($\phi_i = 0$), thus recovering Proposition 1.2(ii) as a special case. Also note that, in line with condition (i) of Proposition 1.2, the above result implies that $\phi_i = 0$ when there are only two agents in the market, irrespective of their trading costs.

The characterization in Proposition 1.3 also underscores that the extent of information leakage depends on the joint distribution of agents' trading costs and signal

[7] Our notion of information revelation gap as a measure of price informativeness is distinct, but closely related to what [57] refer to as the index of price informativeness. More specifically, their index, ψ_i, measures the contribution of the price signal to the reduction of i's uncertainty relative to the *complete information* benchmark with no uncertainty. In contrast, ϕ_i in equation (1.7) measures i's residual uncertainty relative to the benchmark of *full revelation*. Formally, the two indices are related to one another via $\psi_i = (1-\phi_i)(1-\mathrm{var}(\theta_i|s_1,\ldots,s_n)/\mathrm{var}(\theta_i|s_i))$.

precisions. In particular, equation (1.8) establishes that the information revelation gap ϕ_i depends not on the dispersion of trading costs, but rather on a *weighted* variance of the reciprocal of trading costs of agents $k \neq i$ with weights $w_k = 1/\text{var}(s_k)$. This expression captures the idea that agent k's trading cost matters for revelation only to the extent that she posses informative signals, with the trading cost of agents with uninformative signals assigned a weight $w_k = 0$.

1.4.1 Large Markets

Our results thus far focused on a market consisting of finitely many agents. We conclude this section by studying the model's behavior at its continuum limit and illustrating that, as long as all agents are informationally small, the market is informationally inefficient unless the distribution of trading costs is degenerate.

Formally, we consider a sequence of markets indexed by the number of agents n and focus on the limit as $n \to \infty$. Let λ_{in} and σ_{in} respectively denote the trading cost and the standard deviation of noise in agent i's signal in the market consisting of n agents, with their joint empirical distribution denoted by $\mathbf{F}_n(\lambda, \sigma)$. We use $\mathbf{F}_n(\lambda)$ and $\mathbf{F}_n(\sigma)$ to denote the corresponding marginals. Furthermore, we assume that

$$\lim_{n \to \infty} \mathbf{F}_n(\lambda, \sigma/\sqrt{n}) = \mathbf{F}(\lambda, \sigma) \tag{1.9}$$

for all λ and σ, with $\mathbf{F}(0, \sigma) = 0$ for all σ. This assumption serves a dual purpose. First, it ensures that the limiting market is well-defined, with almost all agents exhibiting a non-zero trading cost. Second and more importantly, as in [15], the normalization constant $1/\sqrt{n}$ on the left-hand side of (1.9) guarantees that each agent is informationally small as $n \to \infty$: the variance of noise σ_{in}^2 in each agent i's signal grows linearly in n. Intuitively, this normalization implies that the aggregate amount of information dispersed among all agents remains bounded even as $n \to \infty$. More specifically, it guarantees that $\liminf_{n \to \infty} \text{var}(\bar{\theta}_n | s_{1n}, \ldots, s_{nn}) > 0$, where $\bar{\theta}_n = \frac{1}{n}\sum_{i=1}^{n}\theta_{in}$ is the average of agents' valuations and s_{kn} denotes the private signal of agent k in the market with n agents. We have the following result:

Proposition 1.4. *Let ϕ_{in} denote the information revelation gap of agent i in the market consisting of n traders. Then, $\phi^* = \lim_{n\to\infty}\phi_{in} = 0$ if and only if the marginal distribution $\mathbf{F}(\lambda)$ is degenerate.*

The above result thus implies that even when the number of agents participating in the market is very large, as long as the aggregate amount of information dispersed throughout the market is bounded, the equilibrium is fully privately revealing if and only if all agents have identical trading costs.

1.5 Informational Externality

Propositions 1.2–1.4 in the previous section illustrate that, as long as $n \geq 3$, the equilibrium is not fully privately revealing to all market participants simultaneously unless all agents have identical trading costs. These results, however, are silent about the (in)efficiency of the equilibrium *allocation*. In this section, we study the welfare implications of the market's failure to aggregate information and show that heterogeneity in the market can lead to the emergence of an informational externality, whereby traders do not internalize how their actions shape the information content of the price. This analysis will serve as the basis of our results in Section 1.6 on the welfare implications of various market architectures with endogenous public signals.

We consider the constrained efficiency benchmark of [9, 10], according to which the social planner maximizes total expected surplus in the market

$$\mathbb{E}[W] = \mathbb{E}[\pi_0] + \sum_{i=1}^{n}\mathbb{E}[\pi_i]$$

subject to the same informational constraints faced by the agents in equilibrium, where recall that π_i denotes trader i's payoff and π_0 is the payoff of the outside agent. Under this specification, the action prescribed to each agent cannot depend on the private information of other agents. Thus, this formulation ensures that the planner internalizes any potential externality that agents may impose on one another while respecting the decentralized information structure of the market.[8] As in the equilibrium, we restrict the planner to affine strategies in the form of $x_i = a_i s_i + b_i - c_i p$, while imposing the market-clearing condition $y + \sum_{i=1}^{n} x_i = 0$.

We have the following result:

Proposition 1.5. *Suppose $\sigma_i^2 > 0$ for all i. The equilibrium is constrained efficient if either*

(i) there are only two agents in the market (i.e., $n = 2$);

(ii) agents have private valuations (i.e., $\rho = 0$); or

(iii) all traders have identical trading costs (i.e., $\lambda_1 = \cdots = \lambda_n$).

If the above conditions are violated, then the equilibrium is constrained inefficient for almost all β.

The above result thus provides a necessary and sufficient condition for constrained efficiency of the equilibrium allocation. More specifically, Proposition 1.5 establishes that if either condition (i)–(iii) is satisfied, then the social planner cannot improve on the equilibrium allocation without violating the decentralized information structure of the market. Contrasting this observation with Proposition 1.2 illustrates that these conditions are identical to the conditions that guarantee the market's informational efficiency: in a market with $n \geq 3$ agents, the equilibrium attains both informational and allocative efficiency when all agents have identical trading costs.[9]

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[8] This concept bypasses the details of specific policy instruments and instead directly identifies the strategy that maximizes welfare under the restriction that information cannot be centralized.

[9] The equivalence between informational and allocative efficiency does not hold in general. See Section 1.7 for a slight variation of the model along the lines of [57], in which the equilibrium is

More importantly for our purposes, however, the juxtaposition of Propositions 1.2 and 1.5 also establishes a converse implication: within our environment, any heterogeneity in trading costs not only leads to an informational inefficiency, but also a constrained inefficient allocation. In other words, as long as there is a pair of agents i and j with $\lambda_i \neq \lambda_j$, the market exhibits an externality that is not fully internalized by the market participants in equilibrium. Crucially, this externality is absent if agents have either perfect information ($\sigma_i = 0$) or private valuations ($\rho = 0$). Under either scenario, the price cannot provide the agents with any useful information. This simple observation thus implies that the heterogeneity-induced externality identified in Proposition 1.5 is an *informational externality*: agents do not internalize how their actions shape the information content of the endogenous public signal.

To see the intuition for the relationship between heterogeneity and the emergence of the informational externality, first consider the case in which all agents have identical trading costs. Since such a market is informationally efficient, any deviation from the equilibrium strategies can only reduce price informativeness, thus implying that the social planner cannot improve on the equilibrium allocation. In contrast, when the distribution of trading costs is non-degenerate, a marginal deviation by agent i away from her equilibrium strategy results in a second-order loss in i's payoff, but potentially a first-order gain in price informativeness for other market participants and hence a first-order increase in aggregate welfare. Our next result captures how this informational externality manifests itself:

Proposition 1.6. *A marginal deviation by agent i away from the equilibrium weight she assigns on her private signal leads to a first-order change in aggregate welfare*

constrained efficient even though the price is not fully privately revealing to any of the market participants, i.e., $\phi_i > 0$ for all i. In other words, even though informational efficiency in a competitive market implies allocative efficiency (as argued by [28]), the converse is not generally true. This means that taking informational efficiency as a proxy for allocative efficiency — without performing a proper welfare analysis — may result in misleading conclusions.

given by

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \gamma \sum_{k=1}^{n} \frac{\partial x_k}{\partial p} \operatorname{cov}\big(s_i, \underbrace{\theta_k - \mathbb{E}[\theta_k|s_k, p]}_{e_k}\big) \tag{1.10}$$

where $\gamma > 0$ *is some positive constant.*

The above result provides a characterization for how changes in the equilibrium strategy of agent i shapes the total surplus in the market as a function of slope of demand curves submitted by any given agent k ($\partial x_k/\partial p$) and the covariance between agent i's private signal and k's estimation error $e_k = \theta_k - \mathbb{E}[\theta_k|s_k, p]$. Before exploring the intuition underlying equation (1.10), we first note that the right-hand side of this equation is equal to zero whenever the market is informationally efficient. This is a consequence of the fact that when the price is fully privately revealing to agent k, then there cannot be a systematic relationship between k's estimation error and i's private signal (as otherwise the equilibrium could not have been privately revealing to k). To see this formally, note that, under full private revelation to agent k, the covariance between i's private signal and k's estimation error is given by

$$\operatorname{cov}(s_i, e_k) = \mathbb{E}[s_i\theta_k] - \mathbb{E}[s_i\mathbb{E}[\theta_k|s_k, p]] = \mathbb{E}[s_i\theta_k] - \mathbb{E}[\mathbb{E}[s_i\theta_k|s_k, p]] = 0,$$

where the second equality is a consequence of the fact that the price is fully revealing to agent k (and hence already reflects agent i's private signal), whereas the last equality is a consequence of the law of iterated expectations. Thus, Proposition 1.6 substantiates the relationship between Propositions 1.2 and 1.5 discussed earlier in this section: the same conditions that guarantee informational efficiency also guarantee an efficient allocation in the market.

More importantly, the characterization in Proposition 1.6 also illustrates that when the equilibrium is not fully privately revealing, the nature of the informational externality depends on the *interaction* between the above covariance and the slopes of the equilibrium demand curves. To see this in the most transparent manner, consider a scenario with incomplete information leakage and suppose that $\text{cov}(s_i, e_k) > 0$ between a pair of agents $i \neq k$. This means that whenever agent i has higher signals, agent k tends to underestimate the true underlying value of the asset. If, in addition, agent k submits upward-sloping demand curves (so that $\partial x_k/\partial p > 0$), then a marginal increase in agent i's trading intensity would raise the price, thus inducing agent k to acquire more of the asset exactly in the states of the world in which k was underestimating its value. This increases k's utility. In contrast, if agent k submits downward-sloping demand curves (i.e., $\partial x_k/\partial p < 0$), then by putting a marginally higher weight on her private signal, agent i increases the price and induces agent k to acquire less of the good exactly when the latter was underestimating the asset's value. This reduces k's utility. An analogous argument shows that when $\text{cov}(s_i, e_k) < 0$, a marginal increase in a_i decreases k's utility when k's demand curve is upward sloping, whereas it increases k's utility when k's demand is downward sloping. Finally, note that the equilibrium is constrained efficient as long as traders' strategies are not indexed to the price (i.e., $\partial x_k/\partial p = 0$), in which case the model reduces to a competition in quantities as opposed to schedules[10].

Our next result explores the implications of Proposition 1.6 by relating the market's allocative efficiency to the distribution of agents' trading costs in the market.

Proposition 1.7. *Let a_i^{eq} and a_i^{eff} denote the weights that i assigns to her private signal in equilibrium and constrained efficient allocations, respectively. There exist $\bar{\rho} > 0$ and functions $\underline{\beta}(\rho) < \overline{\beta}(\rho)$ such that*

(a) if $\rho < \bar{\rho}$ and $\beta < \underline{\beta}$, then $a_i^{\text{eq}} < a_i^{\text{eff}}$ if and only if

$$\frac{1}{\lambda_i} < \frac{\sum_{k \neq i} \frac{1 - w_k}{\lambda_k} \left(\frac{\sum_{j \neq k} w_j/\lambda_j}{\sum_{j \neq k} w_j/\lambda_j^2} \right)}{\sum_{k \neq i} \frac{1 - w_k}{\lambda_k} \left(\frac{\sum_{j \neq k} w_j/\lambda_j}{\sum_{j \neq k} w_j/\lambda_j^2} \right)^2};$$

[10] We formally establish this in Section 1.8.

(b) if $\rho < \bar{\rho}$ and $\beta > \overline{\beta}$, then $a_i^{\text{eq}} < a_i^{\text{eff}}$ if and only if

$$\frac{1}{\lambda_i} > \frac{\displaystyle\sum_{k \neq i} \frac{1 - w_k}{\lambda_k}}{\displaystyle\sum_{k \neq i} \frac{1 - w_k}{\lambda_k} \left(\frac{\sum_{j \neq k} w_j / \lambda_j}{\sum_{j \neq k} w_j / \lambda_j^2} \right)},$$

where $w_k = 1/(1 + \sigma_k^2)$.

The above result therefore characterizes the set of traders that over- and under-react to their private signals. It shows that not all departures from the efficient strategy profile are in the same direction: while some traders over-react to their private signals in equilibrium, others under-react relative to the constrained efficient benchmark. Importantly, proposition 1.7 also illustrates that whether any given agent i over- or under-reacts to her private signal also depends on the value of β. For example, agents with large trading costs under-react to their private signals when β is small, the same agents over-react to their signals when β is large.

It is instructive to interpret Proposition 1.7 through the prism of Proposition 1.6. To this end, suppose β is small and consider an agent i with the largest trading cost. Our discussion in Section 1.4 indicates that the private signal of such an agent is reflected in the price with a small weight a_i relative to the sufficient statistic that would have resulted in full private revelation to all agents. This under-reflection means that when agent i has a strong positive signal, other agents tend to underestimate the value of the asset, thus implying that $\text{cov}(s_i, e_k) > 0$. On the other hand, recall from the discussion following Proposition 1.1 that a small β means that the informational role of the price dominates its role as an index of scarcity and as a result leads to upward-sloping demand curves, as agents interpret higher prices as strong signals in favor of the asset's underlying value. Thus, by Proposition 1.6, an increase in a_i induces all other agents to acquire more of the asset when they underestimate its value, thus increasing the overall welfare in the market. This is indeed the statement of Proposition 1.7(a). In contrast, for large values of β, the informational role of the price is weakened, resulting in downward-sloping demand curves. Hence, equation (1.10) indicates that a marginal increase in a_i would reduce the welfare of all other agents, consistent with Proposition 1.7(b).

Taken together, Propositions 1.6 and 1.7 illustrate that, in the presence of information leakage, the efficient operation of the market is highly sensitive to (i) the extent of market's informational efficiency and (ii) the relative importance of the price's informational and allocative roles, parameterized by parameter β in our setting.

1.6 Information Leakage and Market Architecture

Our results in Section 1.5 illustrate that the price's role as an endogenous public signal leads to the emergence of an informational externality whenever the distribution of trading costs is non-degenerate. In this section, we study how this externality can lead to non-trivial implications by comparing welfare across various market architectures. More specifically, we consider two market architectures, one centralized and one segmented, and show that market centralization may reduce price informativeness by strengthening the informational externality, thus resulting in a reduction of aggregate welfare compared to a segmented market architecture.

To this end, fix the set of traders $\{1,\ldots,n\}$ with profile of trading costs $(\lambda_1,\ldots,\lambda_n)$ and signal precisions $(\sigma_1^{-1},\ldots,\sigma_n^{-1})$ and consider two different market architectures: a centralized architecture in which all trade occurs on the same exchange with a single market-clearing price — as in the model studied thus far — and a segmented market architecture in which each trader can only trade in one of the multiple exchanges with a specific subset of other market participants. Formally, the segmented market architecture is defined as a partition $\mathcal{S}=\{S_1,\ldots,S_m\}$ of the set of traders $\{1,\ldots,n\}$

for some $m \geq 2$, with trader $i \in S_k$ only capable of trading with other traders $j \in S_k$. Thus, as in [49], each segment S_k in the segmented market architecture has a separate market-clearing price. To ensure consistency between the centralized and decentralized architectures, we also assume that a fraction $\zeta_k \in [0,1]$ of outside traders are also active in segment S_k, with $\sum_{k=1}^{m} \zeta_k = 1$.

We start with the following benchmark result:

Proposition 1.8. *Expected welfare in the centralized market architecture is higher than the segmented architecture, if either of the following conditions are satisfied:*

(i) all traders have complete information about their valuations ($\sigma_i = 0$);

(ii) traders' valuations are independent ($\rho = 0$);

(iii) all trading costs are identical.

Proposition 1.8 provides a benchmark for the comparison between various market architectures. In particular, it establishes that as long as the market is informationally efficient — which occurs either because market participants have no use for the private information of other traders or when they all have identical trading costs — then market centralization leads to a higher aggregate welfare. This increase in welfare operates via two distinct channels. First, market centralization enables each agent to trade in a market consisting of a larger number of participants, thus leading to further realization of gains from trade. This is the channel that underlies the gains from centralization in cases (i) and (ii) above. Second, in the case that traders can benefit from other market participants' private information — as in case (iii) above — market centralization means that the price aggregates the private signals of a larger number of traders, thus increasing price informativeness for all agents and hence welfare.

With Proposition 1.8 as the benchmark, our next result provides a comparison between the two market architectures in the presence of trader heterogeneity. For any trader i in the centralized architecture, define $m_i^{\text{cen}} = \sum_{j\neq i} 1/\text{var}(s_j)$. Similarly for the segmented architecture, let $m_i^{\text{seg}} = \sum_{j\in S(i)\setminus\{i\}} 1/\text{var}(s_j)$, where $S(i)$ denotes the segment that agent i can trade in. We have the following result:

Proposition 1.9. *There exist constants $\underline{\rho} > 0$ and $\underline{\beta} > 0$ such that if $\rho < \underline{\rho}$ and $\beta < \underline{\beta}$, then expected welfare in the centralized market structure is higher than the segmented market structure only if*

$$\sum_{i=1}^{n} \frac{1}{\lambda_i} \left(\frac{\sigma_i^2}{1+\sigma_i^2}\right)^2 \left((m_i^{\text{cen}} - m_i^{\text{seg}}) - (\phi_i^{\text{cen}} m_i^{\text{cen}} - \phi_i^{\text{seg}} m_i^{\text{seg}}) \right) \geq 0, \tag{1.11}$$

where ϕ_i^{cen} and ϕ_i^{seg} are trader i's information revelation gaps in the centralized and segmented markets, respectively.

To see the intuition of the above result, consider agent i. The term $m_i^{\text{cen}} - m_i^{\text{seg}}$, which is always positive, measures how much more information is available to agent i post centralization if the information is fully aggregated. But the extent of information aggregation depends on the market structure. The second term on the left-hand side of (1.11) creates a countervailing force that may reduce and even reverse the welfare gains from centralization. The juxtaposition of the above result with Proposition 1.3 relates welfare gains in centralizing the markets to the heterogeneity in each segment and the market in general. Furthermore, the above result reduces to part (iii) of Proposition 1.8 when all trading costs are identical, in which case $\phi_i^{\text{cen}} = \phi_i^{\text{seg}} = 0$ for all i, implying that $\mathbb{E}[W^{\text{cen}}] \geq \mathbb{E}[W^{\text{seg}}]$. On the other hand, when (1.11) is violated, we get the opposite. Finally, each term is also weighed by trader i's trading cost: clearly traders with high trading costs will trade less and hence matter less for aggregate welfare.

The result above thus implies that policies that shape the distribution of agents that participate in the market, which in turn, shapes price informativeness can have a first-order effect on the efficient operations of the market.

1.7 Informationally-Inefficient Efficient Markets

Our results in this chapter establish that when trading costs are heterogeneous, (i) the price signal does not fully aggregate the information in the market and (ii) the equilibrium is constrained inefficient, as traders do not internalize the impact of their trading decisions on the information content of the price. In other words, the equilibrium is both *informational* and *allocative* inefficient. In this section, we show that, in general, incomplete aggregation of information does not necessarily imply allocative inefficiency. We illustrate this by contrasting our results to a variation of the model of [57], where traders have homogeneous trading costs but are asymmetric in the correlation between their private valuations. More specifically, we show that even though such heterogeneity leads to an incomplete aggregation of information, the equilibrium is constrained efficient, in the sense that the social planner cannot improve on the allocation.

As in the baseline model in Section 1.3, consider a market consisting of n price-taking traders with payoffs given by (1.1) and private signals $s_i = \theta_i + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \sigma_i^2)$. As in our baseline model, the assumption that traders take the price as given guarantees that any potential inefficiency is not driven by traders' market power. In a departure from the baseline model, however, suppose that the interdependencies in private valuations can be heterogeneous among different pairs of traders. More specifically, suppose $\text{corr}(\theta_i, \theta_j) = \rho_{ij}$, with the assumption that

$$\frac{1}{n-1}\sum_{j \neq i} \rho_{ij} = \bar{\rho} \tag{1.12}$$

for some $\bar{\rho} \in (0,1)$ and all traders i. This assumption ensures that all traders face the same average interdependencies in the market. We have the following result:

Proposition 1.10. *Suppose pairwise correlations satisfy* (1.12)*. Also suppose all trading costs and signal precisions coincide. Then,* *(i) The equilibrium is fully privately revealing to all traders if and only if $\rho_{ij} = \bar{\rho}$ for all $i \neq j$.*

(ii) The equilibrium is constrained efficient, regardless of the pairwise correlations.

The first statement of the above proposition, which is in line with Proposition 3 of [57], illustrates that heterogeneity in pairwise correlations prevents full private revelation in the sense of Definition 1.1. This is a consequence of the fact that full private revelation for trader i requires the price to be equal to a specific weighted average of private signals. But the presence of heterogeneous correlations means that this weight average may be different for different traders, implying that at least one trader cannot fully extract the sufficient statistic of other traders' private signals by observing the price.

More importantly for our purposes however, part (ii) of Proposition 1.10 illustrates that the failure of informational efficiency highlighted in part (i) may not translate into allocative inefficiency: no matter what the pairwise correlations are, all traders internalize the impact of their actions on others and no policy can improve upon the equilibrium allocation. This result thus underscores that equating informational efficiency with allocative efficiency — without performing a proper welfare analysis — can lead to misleading conclusions.

1.8 Comparison with Cournot Competition

We conclude this chapter with a comparison to a market in which agents engage in Cournot competition. More specifically, we consider an economy, in which agents compete by choosing quantities instead of submitting demand schedules, as in Section 1.3. We will show that while the later market structure results in an inefficient market, the former results in an efficient one. This emphasizes that the source for inefficiency in our model is an informational externality, resulting from the fact that agents use the price as an endogenous public signal.

To that end, consider an economy consisting of n traders competing in quantities contingent only on their private signals. Thus, x_i is of the form $x_i = a_i s_i + b_i$ for $i \in \{1, ..., n\}$. In equilibrium, traders maximize their expected profit, given by

$$\mathbb{E}[\pi_i(x_i)|s_i] = \mathbb{E}[\theta_i x_i - \frac{1}{2}\lambda_i x_i^2 - p x_i | s_i] \tag{1.13}$$

and the price is such that the market clears, i.e., $p = \alpha + \beta \sum_{i=1}^{n} x_i$. Similar to the analysis of the demand function competition, we find the equilibrium action of agent i by solving the first-order condition, resulting in:

$$x_i = \frac{1}{\lambda_i}(\mathbb{E}[\theta_i - p|s_i]), \tag{1.14}$$

Together with market clearing, the above equation leads to the following characterization of equilibrium:

Proposition 1.11. *Under a Cournot competition, there exist an equilibrium* $x_i = a_i s_i - c_i p$, $i \in \{1, ..., n\}$, *where constants* $(a_1, ..., a_n)$ *and* $(b_1, ..., b_n)$ *satisfy*

$$a_i = \frac{1 - \beta\rho \sum_{j \neq i} a_j}{(\lambda_i + \beta)(1 + \sigma_i^2)} \tag{1.15}$$

$$b_i = \frac{-\alpha - \beta \sum_{j \neq i} b_j}{\lambda_i + \beta} \tag{1.16}$$

Next we characterize the constrained-efficient benchmark, where a social planner maximizes total surplus:

$$\mathbb{E}[TS] = \sum_{i=1}^{n} \mathbb{E}\left[\theta_i x_i - \frac{1}{2}\lambda_i x_i^2 - p x_i\right] + \mathbb{E}[u(y) - py]$$

by choosing quantities $(x_1, ..., x_n)$, while subject to the same informational constraints as the agents. By taking the first-order condition of maximizing the expected total surplus, we have the following proposition:

Proposition 1.12. *Under a Cournot competition, the team-efficient actions are given by*

$$a_i = \frac{1 - \beta\rho \sum_{j \neq i} a_j}{(\lambda_i + \beta)(1 + \sigma_i^2)} \tag{1.17}$$

$$b_i = \frac{-\alpha - \beta \sum_{j \neq i} b_j}{\lambda_i + \beta} \tag{1.18}$$

From Propositions 1.11 and 1.12 it is immediate that the economy is efficient when traders compete á la Cournot. This is in contrast to our results in Section 1.5, where we showed that an economy in which traders compete in schedules is efficient if and only if traders have identical trading costs (which results in a fully revealing price), or when there is no incentive for the traders to learn from the price, i.e., their private information is complete, or their valuations are independent. Thus, it is implied that the inefficiency we identified in Section 1.5 is a consequence of an inefficient price discovery process, resulting from the market characteristics.

1.9 Conclusions

This chapter investigates how heterogeneity of market participants can shape the information content of the price, in the presence of information leakage. We find that price informativeness is highly sensitive to the characteristics of market participants. In particular, we find that the price is less informative the more heterogeneous the agents are. This is a consequence of the fact that agents' characteristics determine the intensity of their market activity and hence the extent to which their private information will be leaked via prices.

Moreover, we find that the market's *informational* inefficiency translates into an informational externality, resulting in an *allocative* inefficient market, as agents do not internalize how their actions shape the information that is leaked via prices and hence the information available to other agents. Two factors shape the extent of this externality: (i) how private signals covary with others' estimation error of the asset's payoff; and (ii) the slope of agents' demand curves. These two factors, in turn, depend on agents' trading costs and on the dominant role played by the price, respectively. The sign of the covariance between agent i's private signal and the estimation error of other agents determines whether other agents are overestimating or underestimating the true underlying value of the good. Then, the slope of their demand curves determine how they react to a change in the price resulting from agent i's action. Taken together, we find that when the informational role of price dominates, agents submit upward-sloping demand curves, in which case, agents with high trading costs (compared to other agents with similar private information precision) help others by placing a higher weight on their private signal. Thus, implying, that in equilibrium they are under-reacting to their private information. Agents with low trading costs, on the other hand (again, compared to other agents with similar private information precision), help others by placing less weight on their private signal. Thus, implying that in equilibrium they are over-reacting to their private information. However, when the allocative role of price dominates, agents submit downward-sloping demand curves, and the opposite is true – agents with high trading costs are over-reacting to their private information, whereas agents with low trading costs are under-reacting.

We further conclude that the extent of information leakage and its effect on market performance is tightly related to the market architecture. As opposed to conventional belief, we find that welfare in a centralized market — where, potentially, there are more gains from trade and more information to be aggregated — may be low compared to a segmented market (i.e., agents are allowed to trade only within one segment of the market). This result emphasizes the potential impact that the heterogeneity-induced informational externality may have on market welfare.

Our findings suggest that the extent of information leakage via prices may vary with the intricate details of the market structure, and the extent of information leakage can have a first-order effect on the efficient operations of the market. Thus, accounting for the possibility and extent of information leakage should be a central pillar of optimal market design, specially in environments with highly dispersed information.

Chapter 2

Market Power and Strategic Interaction

2.1 Motivation and Overview of Results

In a perfectly competitive market, prices equal the social marginal cost and they are a signal for efficient allocation of society's resources. But perfectly competitive, and thus efficient, markets rarely exist. Most markets are marred by "market failures". When there are market failures, prices no longer reflect the social marginal cost and the resulting allocation of resources in the economy may no longer be efficient. In such circumstances, regulatory intervention may be needed.

Market power and asymmetric information are two main sources of market failures that may distort the efficiency of prices. Market power arises when sellers (or buyers) have power to increase the market price above (or below) competitive levels. Information asymmetry exists when buyers or sellers in a market have an informational advantage that they can exploit to their benefit. Importantly, these two frictions may also interact with one another, as informational asymmetries exacerbate market power issues, even in large markets (e.g., see [14]). These sources of friction have received significant attention in the literature, as well as by governments and regulators[1].

In addition to the previous two frictions, there may also be externalities and strategic spillovers that are not mediated through markets. Firms often compete in more than one market, e.g., financial firms investing in a diversified portfolio, or

[1] The financial crisis in 2008 has created new demand for understanding the effects of imperfect competition in financial markets.

electric companies competing in different geographical regions. Then, a change in firm i's action in one market may affect the actions taken by it's competitors in other markets they compete in, which in turn affects firm i's profits in the first market. The change in profit is the result of the strategic interaction between the firms, and it depends on whether competition is with strategic complements or with strategic substitutes[2]. However, theoretical work in this area lacks the analysis of markets that take into account the presence of a "strategic effect", while incorporating price impact and information asymmetry.

The focus of this work is precisely to understand the interplay between the payoff environment and the information structure in the determination of the equilibrium outcome. Specifically, we investigate how the presence and nature of strategic interaction — whether it is strategic complementarity or strategic substitutability — affects the equilibrium actions of market participants and as a result, their price impact, in the presence of information asymmetry. Our aims are to characterize the equilibrium and to perform comparative statics.

To that end, we analyze a demand function competition model where agents have incomplete information about the value of the good that is being traded. More specifically, following [14], we focus on a non-competitive market consisting of finitely many agents who trade a single divisible asset. Agents have interdependent valuations for the asset, but are uncertain about the underlying state that determines the asset's payoff. Each agent observes a private signal about the asset's fundamental value. In addition, the price serves as an endogenous public signal with the ability to (fully or partially) convey each agent's private information to other market participants. We use a reduced-form approach that allows us to represent both strategic substitutability and strategic complementarity, in order to account for the possible strategic effect, resulting from agents activities outside of the market (e.g., spill-overs

[2]This is referred to as a "strategic effect" in [19].

and externalities). When there is no strategic effect, our model reduces to the one in [14].

We now provide an overview of our main results. Our first set of results establishes the existence of an equilibrium, and a closed-form characterization of the market Bayes-Nash equilibrium. For interpretability reasons, we look at the large market equilibrium characterization, where the number of market participants goes to infinity. Although price impact monotonically decreases to zero as the size of the market increases, this limit allows us to analyze the market and perform comparative statics analyses. We find that the existence of strategic interactions between agents has a direct impact on the weight agents put on their private information. Specifically, agents place less weight on their private information as the intensity of strategic interactions between them becomes stronger. From the view point of agent i, when the actions of all other agents $j \neq i$ have a dominant impact on her profit, she is less incentivized to pay attention to her private information. Thus, there is a coordination (miss-coordination) motive that offsets the desire to match the fundamentals.

Our second set of results establishes the relationship between price impact, information asymmetry and strategic interactions. We find that there is a monotonic relation between price impact and information asymmetry: as the extent of information asymmetry decreases, price impact decreases. Specifically, as agents' private information becomes biased towards the average valuation of the good, the level of information asymmetry in the market decreases, which results in a decrease in agents' price impact. We also characterize the relation between price impact and strategic interaction. Importantly, this relation depends on the the slope of agents' demand curves. When agents submit downward-sloping demand curves, price impact decreases with the extent of strategic susbstitutability and increases with the extent of strategic complementarity. However, when agents submit upwords-sloping demand curves, price impact decreases with the extent of strategic complementarity. This is a consequence of the fact that the strategic effect, when agents' actions are strategic complements, offsets the "price effect", which induces agents to decrease their demand as price increases. However, when agents' actions are strategic substitutes, the strategic effect reinforces the "price effect".

With the above results in hand, we then leverage the relations identified above to study the trade-offs of the effects of strategic interaction and information asymmetry on agents' price impact. We find that when strategic interaction is strong enough, it is able to mitigate the effect of information asymmetry on price impact, and price impact approaches zero. However, in other cases, strategic interaction may exacerbate the effect of information asymmetry on price impact. Importantly, this depends on the elasticity of the supply of the good.

Overall, our theoretical findings provide insight on the role of strategic interaction in shaping market outcomes, and the impact it has on agents' price impact with different levels of uncertainty. They also suggest that it is important for policy makers to take strategic interaction into account when discussing possible price impact of market participants.

2.2 Literature Review

Our theoretical framework is related to the literature on rational expectations equilibrium (REE) with a Gaussian information structure, such as [33], [29], [3], [25] and [46], among many others[3]. Within this literature, our work is most closely related to [14] who study market power and price volatility in demand function competitions with a general incomplete information structure. They show that any degree of market power can arise in the unique equilibrium under an information structure that is arbitrarily close to complete information. In particular, market power may be arbi-

[3]For an extensive review of the early models see [65], chapters 3-4, and for a more recent literature review see [59].

trarily close to zero or arbitrarily large. The payoff function in our model generalizes the one used in [14], by accounting for the possibility of additional strategic interaction between agents, originating from outside the market. In contrast to their results, our findings show that when there is a high enough interaction between agents, price impact approaches zero for any level of information asymmetry.

The informational role of prices has received significant attention in the REE literature with asymmetric information (see, among others, [29], and [28]). REE have been implemented with auctions ([50], [56]) or with competition in schedules ([46], and [66]). [57] extend the model in [66] to asymmetrically correlated valuations and show that the equilibrium may fail to be privately revealing. In contrast to these papers, the price in our framework is always fully privately revealing. Nonetheless, the incompleteness of information has non-trivial implications for market power.

Competition in supply or demand schedules has a long tradition in the literature. It was studied in the absence of uncertainty by [28] and [31], who showed a great multiplicity of equilibria. Similar results in a complete information setting were obtained by [68] in a share auction model. [43] showed how adding uncertainty in the supply function model can reduce the range of equilibria and even pin down a unique equilibrium provided the uncertainty has unbounded support. [46] introduced private information into a double auction for a risky asset of unknown liquidation value and derived a unique symmetric linear Bayesian equilibrium in demand schedules. [66] studied a model of strategic competition in schedules with an information structure that encompasses private and common values, avoiding the need for introducing noise traders or noisy supply. While we define a similar model to [66], non of these paper consider the strategic effect between agents and it's impact on the equilibrium actions as well as on the price impact.

Also related is the literature on the use of information in quadratic-payoff games, e.g., [52], [9], [63]. This literature studies the efficient use of public versus private information. [52] show that in a game with strategic complementarities, agents might over-react to public information, and so releasing more public information can reduce social welfare. [9] generalize the model in [52] and study the efficient use of public information for different economies. As opposed to our model, they consider exogenous public information, and so the weight agents put on their private information does not affect the content of the public signal. [63] provide a necessary and sufficient condition for welfare to increase with public or private information for given precision of information. While these papers consider an exogenous public signal, our work considers an endogenous public signal (the price). Also, we are interested in comparative statistics of the equilibrium outcome, as opposed to a welfare analysis that is done in these papers.

Information frictions have been incorporated in market competition models starting from the pioneering work of [48] in macroeconomics, [29] and [45] in finance, [68] and [50] in auctions, [66] in models of competition in schedules, and many others[4]. In a lab study, [12] examine both information frictions and market power: the authors study whether cost interdependence leads to greater market power in relation to when costs are uncorrelated. In contrast to these papers we look at the interaction between information asymmetry and price impact, in the presence of strategic effects.

Price impact has been studied in different economic settings. In financial markets, [34], [22], [23], [41] and [42] show the significance of institutional investors' price impact. [60] initiated the estimation of the slope of aggregate market demand, showing that the demand curve for US equities is downward sloping and thus there is a price impact when large orders are executed. In [66], price impact (as measured by the slope of a trader's residual supply, otherwise known as "Kyle's lambda" in market microstructure models) is monotone in the number of bidders, and as the number of bidders grows large, the allocation converges to a Walrasian equilibrium

[4] For an extensive literature review see [54].

with price-taking behavior. [58] show that with heterogeneous correlation in valuations, the result in [66] need not hold. [14] show that when considering a general information structure, price impact may take any value from zero to infinity (as well as negative values). In contrast, our work offers a theoretical analysis which takes into account agents' price impact, but also considers strategic interactions between agents, and we are interested in characterizing the relation between price impact, strategic interaction and information asymmetry.

Strategic interactions have been studied in different settings. In a complete information network setting in [2], where firms interact through input-output linkages under complete information, and it is the heterogeneous interactions between firms that cause aggregate volatility to not vanish. In particular, the shocks of some firms have disproportionate high impact on aggregate volatility due to the interaction matrix. The equilibrium behavior in arbitrary networks and in arbitrary information structures are tightly linked, as argued by [51]. Most related to our work is [13], who study the equilibrium outcome when agents have asymmetric information and there is heterogeneous interaction between agents. In contrast, we consider homogeneous interaction between agents, and we are interested in the impact it has on agents' price impact in a setting with asymmetric information.

2.3 Model

Consider an economy consisting of n agents, denoted by $\{1, 2, \ldots, n\}$, who trade a divisible asset. These agents may correspond to investors in a financial market. The supply of the good is given by an exogenous supply function $S(p)$, represented by a linear inverse supply function $p = \alpha + \beta \sum_i x_i$, where α and β are non-negative constants and $\sum_i x_i$ is the total demand. Agents choose the quantity the quantity of the good x_i to acquire.. The realized payoff of agent i, who obtains x_i units of the asset, is given by

$$\pi_i(x_i) = \theta_i x_i - p x_i - \frac{1}{2}x_i^2 + \gamma \sum_{j \neq i} x_i x_j \tag{2.1}$$

where θ_i is i's valuation of the good, p denotes the price of the good, and $\gamma \in \mathbb{R}$ is a parameter that captures strategic interactions between agents. When $\gamma > 0$, agents' actions are strategic complements, and when $\gamma < 0$, it implies that agents' actions are strategic substitutes. Such strategic interaction can arise due to a verity of reasons, such as competition in other markets or spillover effects. A negative γ may also be a result of a congestion cost. While central to the analysis, we find it more convenient to model these interactions in a reduced-form matter. Notice that in the special case that $\gamma = 0$, our framework reduces to that of [14]. The payoff shocks θ_i are symmetrically and normally distributed across the agents, with a correlation coefficient ρ. That is, $\theta_i \sim N(\mu, \sigma^2)$, and $corr(\theta_i, \theta_j) = \rho$ for all pairs of agents $i \neq j$, where $\rho \in [0, 1)$. Thus, our model includes the special cases of common values ($\rho \to 1$) and independent values ($\rho = 0$).

In the context of financial markets, θ_i represents the dividend of the traded asset, x_i is the quantity of the asset purchased by trader i, and the extra cost (benefit) γ can arise due to competition in other markets, with firms' actions being strategic substitutes (complements). In the context of electricity generators, θ_i represents the uncertain cost of generating electricity, x_i is the quantity generated by firm i, and the extra cost (benefit) γ can arise due to competition in other regional markets, with firms' actions being strategic substitutes (complements).

2.3.1 Information Structure

Prior to trading, each agent i observes a one-dimensional signal that is a deterministic function of both her idiosyncratic shock and the aggregate shock. More specifically, following [14], we assume that: $s_i = \Delta\theta_i + \delta \cdot \bar{\theta}$, $\delta \in \mathbb{R}$, where $\Delta\theta_i := \theta_i - \bar{\theta}$, and

$\bar{\theta} = \frac{1}{n}\sum_{i=1}^{n}\theta_i$. That is, private signals are a compound of an agent's own valuation and other agents' valuations. While there is no noise in agents' signals, they are nonetheless imperfect because they leave the agent uncertain about the size of the idiosyncratic and aggregate shocks. The shocks are confounded in the agent's signal. Under our normality assumption on agents' private valuations, we have that s_i is also normally distributed with mean $\delta\mu$ and variance $\frac{\sigma^2}{n}\left(n + (\delta^2 - 1)(1 + (n-1)\rho)\right)$. Notice that under this information structure, all signals $(s_1, ..., s_n)$ are informative about agent i's valuation θ_i.

2.3.2 Demand Function Competition

Agents compete in demand functions, according to which all agents simultaneously submit demand schedules that specify their demand for the asset as a function of the price p. Each agent i submits a demand function $x_i(s_i, p)$ specifying the demanded quantity as a function of her private signal s_i and the market price p. The price is then determined by the submitted demand schedules for given realizations of signals $s_i, i = 1, ..., n$, and the market-clearing condition:

$$p = \alpha + \beta \sum_{i=1}^{n} x_i(s_i, p) \tag{2.2}$$

The assumption that agents submit price-contingent demand schedules enables them to take the information content of the price into account. In other words, the price can serve as an endogenous public signal with the ability to (fully or partially) convey agents' private information to one another.

2.4 Equilibrium

We study the Bayes-Nash equilibrium of the demand function competition. The equilibrium is defined as a collection of demand schedules $x_i(s_i, p)$, where (i) each agent $i \in 1, ..., n$ maximizes her expected payoff conditional on her information set $\{s_i, p\}$, and (ii) the market clears.

Formally, every agent i chooses $x_i(s_i, p)$ to maximize her expected profit, conditional on her information set $\{s_i, p\}$, given by:

$$\mathbb{E}[\pi_i | s_i, p] = \mathbb{E}[\theta_i x_i - p x_i - \frac{1}{2} x_i^2 + \gamma \sum_{j \neq i} x_i x_j | s_i, p] \tag{2.3}$$

such that the market clears, or in other words, p satisfies equation (2.2).

As is standard in the literature, we restrict our analysis to equilibria in linear strategies of the form $x_i = a_i s_i + b_i - c_i p$ for constants (a_i, b_i, c_i). That is, agent i's demand schedule x_i is an affine function of her private signal s_i and the market price p. Since agents are symmetric, we are interested in symmetric linear equilibria of the form $x_i = a s_i + b - c p$. Then, the market clearing-condition implies

$$p = \frac{\alpha + \beta \left(nb + a \sum_i s_i \right)}{1 + \beta n c} \tag{2.4}$$

Given linear strategies of all other agents $x_j(s_j, p) = a s_j + b - c p$, $j \neq i$, agent i faces a residual inverse supply: $S(p) - \sum_{j \neq i} x_j(s_j, p)$. This implies:

$$\begin{aligned} p &= \alpha + \beta x_i + \beta \sum_{j \neq i} x_j \\ &= \alpha + \beta x_i + \beta a \sum_{j \neq i} s_j + \beta b(n-1) - \beta c(n-1) p \end{aligned}$$

Rearranging the above expression, we get that the residual inverse supply is a linear combination of $\sum_{j \neq i} s_j$ and x_i.

We define the *price impact* of agent i by

$$\lambda_i = \frac{\partial p}{\partial x_i}$$

The price impact determines the rate at which the price changes as the quantity bought by agent i changes[5]. Thus, it determines how much demand agent i withholds

[5] This is known as Kyle's Lambda.

to decrease/increase the equilibrium price of the good. Since agents are symmetric, we have $\lambda_i = \lambda \; \forall i$, in equilibrium. Notice that the slope of the residual inverse supply for an agent i is exactly i's price impact. Thus, we can rewrite the equilibrium price impact as follows:

$$\lambda = \frac{\partial p}{\partial x_i} = \frac{\beta}{1 + \beta c(n-1)} \tag{2.5}$$

Another consequence of equation (2.4) is that the information provided by the price to agent i about the signals of others is subsumed in the intercept of i's residual inverse supply. Thus, the information available to agent i, $\{s_i, p\}$, is equivalent to $\{s_i, \sum_{j \neq i} s_j\}$.

Throughout our analysis, we assume that $\alpha = 0$, for simplicity.

We have the following result:

Proposition 2.1. *There exists a symmetric equilibrium in linear strategies of the form $x_i = as_i - cp$ with*

$$a = \frac{1}{1 + \lambda + \gamma(2 - \lambda/\beta)} \tag{2.6}$$

$$c = \frac{1}{n-1}\left(\frac{1}{\lambda} - \frac{1}{\beta}\right) \tag{2.7}$$

where λ is given by

$$\begin{aligned} \lambda = {} & \frac{\beta}{2(\beta - \gamma)}\left(-\frac{\beta n\left(\delta(n-1) - 1\right)}{\delta(n-1)+1} - 1 - \gamma\left(1 + \frac{2n}{\delta(n-1)+1}\right)\right) \\ & + \frac{\beta}{2(\beta-\gamma)}\left(\sqrt{\left(\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right)^2 + 2\beta n + 1 + g(\gamma)}\right) \end{aligned} \tag{2.8}$$

and $g(\gamma) = \gamma^2\left[1 + \frac{4n(1-\delta)(n-1)}{(\delta(n-1)+1)^2}\right] + 2\gamma\left[1 + \frac{\beta n(\delta(n-1)-1)}{\delta(n-1)+1}\left(1 + \frac{2n}{\delta(n-1)+1}\right)\right]$.

Furthermore, the equilibrium price is given by

$$p = \frac{\beta n \bar{\theta}\left(1 + \gamma \cdot \frac{\delta(n-1)}{1+\lambda+\gamma(2-\lambda/\beta)}\right)}{1 + \lambda + \beta n + \gamma \cdot \frac{(\beta-\lambda)(n\beta+\lambda)}{\lambda\beta}} \tag{2.9}$$

The above result, which will serve as the basis for the rest of our analysis, provides an explicit characterization of agents' equilibrium strategies and market-clearing price as a function of the model primitives, β, δ and γ, where recall that δ parameterizes the informativeness of each agent's signal about her valuation, whereas γ measures the extent and nature of strategic interaction between agents.

Equations (2.6) and (2.7), which characterize the weight agents put on their private information and on the price, respectively, depend on the endogenous parameter λ, which is characterized in equation (2.8). Despite the complex dependence of λ on model primitives, there are still a few observations one can make from Proposition 2.1. First, from Equation (2.6), we see that when agents are price takers, i.e., $\lambda = 0$, then γ is the only model primitive that affects the weight agents put on their private information. Specifically, as strategic interaction between agents increases, agents become less sensitive to their own private signal. This is true for both types of strategic interaction (i.e., strategic substitutability and strategic complementarity). As the interaction between agents increases, it has a higher impact on agent i's payoff, and agent i puts less weight on her private information. This is somewhat similar to agents behavior in Keynes's beauty contest example. In our model, a large population of agents have access to public and private information on the underlying fundamentals, and aim to take actions appropriate to the underlying state. But they also engage in a competition where agent i's payoff depends on the actions of the other agents. Depending on whether agents' actions are strategic complements or strategic substitutes, their payoff may increase or decrease, respectively, as their actions are more aligned with the other agents. In other words, there is a coordination (or miscoordination) motive to the agents as well as the desire to match the fundamentals. As $|\gamma|$ increases, the coordination (or miscoordination) motive becomes more dominant than the motive to align the true state fundamentals [6].

[6] This result is different than in [52], who conclude that when actions are strategic substitutes,

Second, from Equation (2.7), we find that as price impact increases (while keeping β constant), the weight agents put on the price decreases. This may imply that agents prefer to decrease the level of dependence their actions' have on the price, when the market experiences high levels of price impact. This holds for both cases, when price impact is positive or negative. We further discuss this in Section 2.5, where we analyzes a large market approximation model.

2.4.1 Information Revelation

An important concept in the REE literature is information revelation by prices. Information related to the valuation of a good that is being traded is often dispersed throughout the market. The market price aggregates the information held by different market participants and may transmit it back, fully or partially, to the agents participating in the market. Following [4], we have the following definition:

Definition 2.1. *The equilibrium is fully privately revealing to agent i if* $\mathbb{E}[\theta_i|s_i, p] = \mathbb{E}[\theta_i|s_1, ..., s_n]$.

In other words, under full private revelation, the price coupled with agent i's private signal serve as a sufficient statistic for all the information dispersed throughout the market. We say the market is *informationally efficient* if the equilibrium is fully privately revealing to all agents simultaneously.

From Proposition 2.1, we observe that in our model the equilibrium is fully privately revealing to all agents $i \in \{1, ..., n\}$. Formally,

Lemma 2.1. *In equilibrium, the market is informationally efficient.*

This means that in equilibrium, every agent $i \in \{1, ..., n\}$ is able to learn from the price p, together with her own private information s_i, all the information held

agents put more weight on their private information than on the public. But, they refer to exogenous public information, whereas we have endogenous public information (the price of the good).

by the other agents $j \neq i$, that is relevant to her private valuation θ_i. This is the case regardless of the value of δ and hence how agents' private signals weigh the idiosyncratic and the common components.

To see why this is true, notice from equation (2.9), that the equilibrium price p is an affine function of the average valuation of all agents, $\bar{\theta}$. As a result, by observing the price p, agent i can infer the value of $\bar{\theta}$. Obviously, this information is also available to agent i when she knows all other agents private information. Given our noise-free information structure, combining the knowledge of $\bar{\theta}$ with her private signal s_i, agent i can infer her exact valuation of the good, θ_i.

Notice, that under the noise-free information structure, private revelation to agent i is equivalent to full information of agent i's private valuation. This means that in equilibrium, all agents have full information of their own private valuations. However, agent i does not know the value of the other agents' private valuations, as opposed to a complete information economy.

2.5 Large Markets

The equilibrium characterization in Proposition 2.1 is difficult to interpret, and thus we turn to studying the equilibrium in its continuum limit. Although price impact monotonically decreases as the number of agents participating in the market increases (in fact, the price impact basically disappears as n goes to infinity), taking the limit of $n \to \infty$ will allow us to simplify the equilibrium characterization. As a result, we will be able to better understand and analyze this market, which is the goal of this work.

In this section we therefore continue our analysis for large markets, where $n \to \infty$[7].

[7] One needs to be careful in defining the information agents hold in a large market limit, when there is a possibility to learn from prices other agents' information. However, in our setting, agents already have full information of their private valuations in equilibrium in the finite market setting, and thus there is no change in the information agents have when looking at the limit with infinitely

An immediate result from Proposition 2.1 is an equilibrium characterization for the large market model:

Corollary 2.1. *In the limit case where $n \to \infty$ we have*

$$\lim_{n\to\infty} n\lambda = \frac{\beta}{\delta(\beta-\gamma)} \tag{2.10}$$

$$\lim_{n\to\infty} a = \frac{1}{1+2\gamma} \tag{2.11}$$

$$\lim_{n\to\infty} c = \frac{\delta(\beta-\gamma)}{\beta} \tag{2.12}$$

There are five immediate observations from Corollary 2.1. First, from equation (2.11), we see that the weight agents put on their private signal does not depend on δ. However, from equation (2.12), the weight agents put on the price p does depend on δ. By changing the private signal's weight on the agent's own payoff shock θ_i relative to the other agents' payoff shocks, we change the weight an agent puts on the price in equilibrium. This is made clear from looking at the equation for agent i's expected valuation of the good conditional on her private signal and on the price:

$$\mathbb{E}[\theta_i|s_i, p] = s_i + \frac{1-\delta}{\delta n} \cdot \sum_j s_j \tag{2.13}$$

The parameter δ determines the weight an agent places on all agents' signals together, which, in turn, is a sufficient statistic for the price. As a result, δ affects the perceived (agents' conditional expectation) degree of the interdependence between agents' valuations of the good.

Second, from equation (2.11), we see that as strategic interaction between agents increases, agents become less sensitive to their private signal. This holds for both types of strategic interaction, i.e., strategic substitutability and strategic complementarity. Thus, even in the large market, there exist these two forces that work in opposite direction in shaping the weight agents' place on their private signal: a coordination (or miscoordination) motive and a desire to match the fundamentals. As

many agents.

$|\gamma|$ increases, the coordination (or miscoordination) motive becomes more dominant than the motive to align the true state fundamentals.

Third, the weight agents put on the price, which is the slope of the demand schedule, has the same sign as the price impact. That is, when the slope of agent i's demand schedule is positive, then her price impact is positive, and when the slope of her demand schedule is negative, then her price impact is negative. To see the intuition underlying this result, consider a case where agent i has a downward-sloping demand curve and a negative price impact. In this case, the more agent i buys, the more the price decreases, and then the more she continues to buy. That means, agent i can potentially buy infinitely many units in a price close to zero. This, of course, is not realistic. Similarly, if agent i has an upward-sloping demand curve and a positive price impact, then she would potentially buy more and more, increasing the price to infinity. But, this scenario is also not realistic.

Fourth, equation (2.10) implies that the effect of strategic interaction on price impact is ambiguous. It is not enough to know the type of interaction between agents, i.e., strategic complements or strategic substitutes, in order to determine the sign of agents' price impact and the way it changes with the intensity of interaction. However, in Propositions 2.2 and 2.3 we show that by fixing some of the model parameters, we can determine a relation between γ and price impact.

Finally, we can characterize agents' demand behavior in terms of the model parameters. In other words, we can determine when is agent i's demand schedule downward-sloping and when is it upward-sloping, as a function of the model primitives. We formulate this observation in the following corollary:

Corollary 2.2. *When $\delta > 0$*[8] *we have:*

[8] This corollary can be characterized for $\delta < 0$ as well (in fact, the proof includes this characterization). The difference in these two cases is in the way agents interpret a change in the price. This is a consequence of the fact that when $\delta > 0$, good news for agent i means bad news for agent j, whereas when $\delta < 0$, good news for agent i means bad news for agent j. This results in a opposite characterization of agents' demand schedules.

(i) if $\gamma < \beta$, the equilibrium demand schedule is downward-sloping.

(ii) if $\gamma > \beta$, the equilibrium demand schedule is upward-sloping.

The corollary above tells us under which conditions on the model parameters: δ, γ and β, agents' demand schedules are upward-sloping in equilibrium, and under which conditions they are downward-sloping in equilibrium. In particular, the parameter γ can be divided into two regions, determined by β, where in one region agents' demand schedules are upward-sloping, and in the other, agents' demand schedules are downward-sloping.

To understand why agents increase or decrease their demand as a result from a change in the price, it is helpful to examine agent i's payoff function. Recall that agent i's payoff function, given by equation (2.3), has four components: (1) the valuation of the good; (2) the price; (3) a fixed cost; and (4) a strategic interaction term. Agent i computes the conditional expectation of her valuation, given her private information and the information inferred from the price. In Section 2.4.1 we proved that in our model, agents are able to compute their true value. Importantly, the true valuation of the good is not affected by the price or by other agents' actions. The second component, however, is effected by the price explicitly, but also by other agents' actions, implicitly, as by definition $p = \alpha + \beta \sum_i x_i$. In general, as the price increases, agent i's payoff decreases. Thus, keeping everything else equal, an increase in the price decreases agent i's demand. The third component is fixed and thus is not affected by any other factors. Finally, the forth component is determined by the value of γ and by the actions of the other agents. We conclude that the two terms which are relevant in determining agent i's reaction to a change in the price, are the second and forth components, which we will refer to as the price effect and the strategic effect, respectively.

When agents' actions are strategic substitutes (i.e., $\gamma < 0 < \beta$), their actions offset one another: agent i decreases her demand as other agents increase theirs. This results in downward-sloping demand curves: when other agents increase their demand, they induce a price increase. The strategic effect (agents' actions being strategic substitutes) from other agents' increased demand results in a decrease in agent i's demand, and it is in fact reinforced be the price effect, which also induces agent i to decrease her demand in response to the increase in price.

When agents' actions are strategic complements (i.e., $\gamma > 0$), their actions reinforce one another: agent i increases her demand as other agents increase their demand. In this case we have two forces working in opposite direction: as before, when other agents increase their demand, they cause an increase in the price, which, in turn reduces agent i's demand (this is the price effect). But, in the case with strategic complements, the strategic effect induces an increase in agent i's demand. Thus, the direction of agents' demand curves depends on which force dominates: if the strategic effect is stronger, then agents will submit upward-sloping demand curves. However, if the price effect is stronger, then agents will submit downward-sloping demand curves. Which force is stronger then depends on the parameter β. Indeed, when β is relatively small, an increase (decrease) in agents' demand results in only a small increase (decrease) in the price. This implies that the price effect is small, whereas the strategic effect remains the same (it is independent of β). Therefore, when β is small relative to γ, the strategic effect dominates the price effect, which in turn results in agents having upward-sloping demand curves. On the other hand, when β is relatively large with respect to γ, then the price effect dominates the strategic effect, and agents have downward-sloping demand curves[9].

Determining the direction of agents' demand schedules is crucial, as we will see in the next following sections. Now, with the equilibrium characterization for the

[9]This is somewhat similar to our results in Chapter 1, regarding the relation between β and the slope of the demand curve.

large market in hand, we are ready to analyze agents' price impact. We will show that the way agent i's price impact interacts with the level of information asymmetry between agents, and with the intensity of agents' strategic interaction, highly depends on whether agents submit upward- or downward-sloping demand schedules.

2.5.1 Price Impact and Information Asymmetry

In this section we explore the interaction between price impact and information asymmetry. In their paper, [14], the authors study what are possible range of market power in an environment with asymmetric information between agents. Their main result is that, in general, there is no robust prediction of the expected market power for all possible information structures. In our model, we allow for a strategic effect between agents, and we are interested to see how this affects the possible interaction between price impact and information asymmetry. The following proposition characterizes the relationship between price impact and the level of information asymmetry, while allowing for strategic complementarity or strategic substitutability. We first state the following Lemma, which will be helpful in understanding the proposition following it.

Lemma 2.2. *For fixed values of β and γ,*

(i) if $\gamma < \beta$, then λ decreases with δ;

(ii) if $\gamma > \beta$, then λ increases with δ.

The Lemma above implies that for any pair of fixed values of β and γ, the parameter λ changes monotonically with δ. Whether it monotonically increases or monotonically decreases depends on the relation between β and γ.

The characterization in Lemma 2.2 is the consequence of Corollaries 2.1 and 2.2. Corollary 2.1 characterizes λ as a function of the model primitives: β, γ and δ. Also, it implies that the price impact is positive whenever agents have downward-sloping demand curves, and it is negative whenever agents have upward-sloping demand curves. Corollary 2.2 determines if agents have upward- or downward-sloping demand curves, as a function of the model primitives: β, γ and δ. Taken together, one can derive the relation in Lemma 2.2 (the proof requires showing the the parameter λ is uniformly continuous as a function of the parameter δ).

Notice, that when price impact is negative, an increase in the value of λ is in fact a decrease in the value of $|\lambda|$. Thus, an immediate result from Lemma 2.2 is the following proposition characterizing the relation between price impact and information asymmetry.

Proposition 2.2. *For fixed values of β and γ, price impact decreases with $|\delta|$.*

Proposition 2.2 implies that agent i's price impact decreases as the private signals place a higher weight on the average valuation. To understand the underlying intuition of Proposition 2.2, notice that when $|\delta|$ is large, even though agents are incompletely informed, there is only a small level of information asymmetry. That is, all agents know the common component, but they do not know their private valuation. This means that no agent has an informational advantage on any other agent, implying also that agent i's private information is not very valuable to the other agents. On the other hand, when $|\delta|$ is small, agents' private signal depends more heavily on the idiosyncratic component, implying a high level of information asymmetry between agents. In general, as the level of asymmetry between agents decreases we expect the price impact to decrease. Indeed, that is the underlying implication of Proposition 2.2.

This relation is illustrated in Figures 2.1 and 2.2. In Figure 2.1, we can see how agent i's price impact changes with δ, for fixed values of β and γ, where $\gamma < \beta$ (particularly, $\gamma = 0$, that is, there is no additional strategic interaction between agents). We see that the price impact decrease with δ in this case. Furthermore, we notice that when $\delta < 0$, then the price impact is negative, and when $\delta > 0$, then the

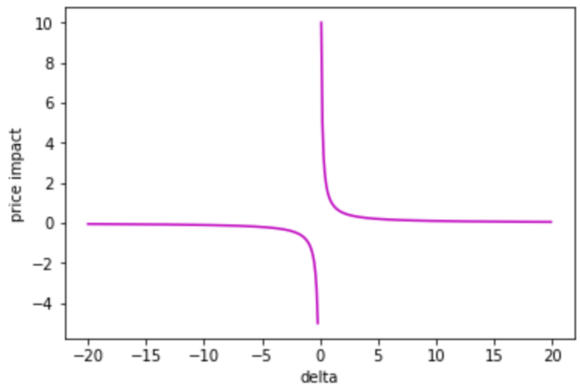

Figure 2.1: Equilibrium price impact in a market with no strategic interaction ($\gamma = 0 < \beta = 10$).

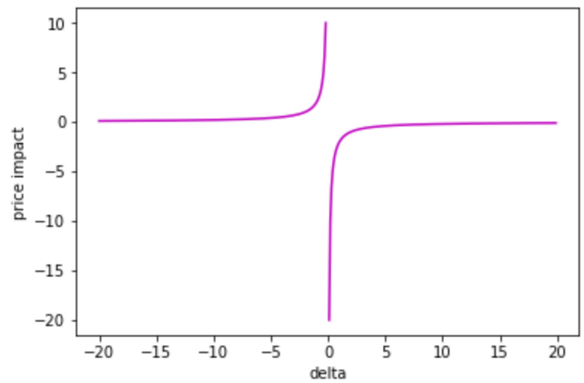

Figure 2.2: Equilibrium price impact in a market when $\gamma > \beta$ ($\gamma = 15$, $\beta = 10$).

price impact is positive. Thus, we can conclude that when agent i's price impact is negative, it approaches zero as δ decreases to $-\infty$, and when agent i's price impact is positive, it approaches zero as δ increases to ∞. Similarly, in Figure 2.2, we can see how price impact changes with δ in the case when $\gamma > \beta$ (particularly,$\gamma = 15$ and $\beta = 10$). In this case, the shape of agent i's price impact flips, and it increases with δ. Now, price impact is positive when $\delta < 0$ and negative when $\delta > 0$. Thus resulting in a increase as δ increases to ∞, and a decreases as δ decreases to $-\infty$. Put together, we see that the magnitude of agent i's price impact decreases with $|\delta|$, as expected.

We continue by studying the effect of having strategic interaction between agents.

2.5.2 Price Impact and Strategic Interaction

In the previous section we proved that as the extent of information asymmetry decreases, price impact decreases. In this section we further explore the interaction between price impact and strategic interaction. First, we study the effect of strategic interaction between agents on agent i's price impact, regardless of the existence or level of information asymmetry. We have the following proposition which characterizes the relationship between agent i's price impact and the parameter γ.

Proposition 2.3. *For any fixed value of $\delta > 0$,*

(i) when demand schedules are downward-sloping, price impact increases with γ.

(ii) when demand schedules are upward-sloping, price impact decreases with γ.

The proposition above implies that when demand is downward-sloping, strategic complementarity increases agents' price impact, whereas strategic substitutability decreases it. When agents' demand schedules are upward-sloping, strategic complementarity decreases agents' price impact.

This brings us back to the importance of Corollary 2.2, which identifies the conditions on the model primitives under which agents have downwards or upwards-sloping demand curves. Corollary 2.1 implies that the price impact is positive whenever agents have downward-sloping demand curves, and it is negative whenever agents have upward-sloping demand curves. Taken together, we find that when agents have positive price impact, having strategic complementarity increases agents price impact. However, this is only true while demand curves are downward-sloping. From Proposition 2.2, we know that when strategic complemetarity is strong (with respect to β), then agents submit upward-sloping demand curves, and then we find that price impact decreases with the extent of the interaction.

Figure 2.3 explains the underlying intuition of Proposition 2.3. Consider the case when agent i has positive price impact. That implies that by buying more units of the good, agent i is able to increase the price of the good. That, in turn, means there is a shift to the right in the demand curve (illustrated by a shift from point (q_{eq}, p_{eq}) to point (q'_{eq}, p'_{eq}) in Figure 2.3). From Corollary 2.1, we know that price impact is positive whenever agents' demand is downward-sloping. So, on the one hand, we have downward-sloping demand, which implies that a price increase reduces demand, and in turn reduces the price. But, on the other hand, if agents are strategic complements, then by buying more units of the good, agent i induces

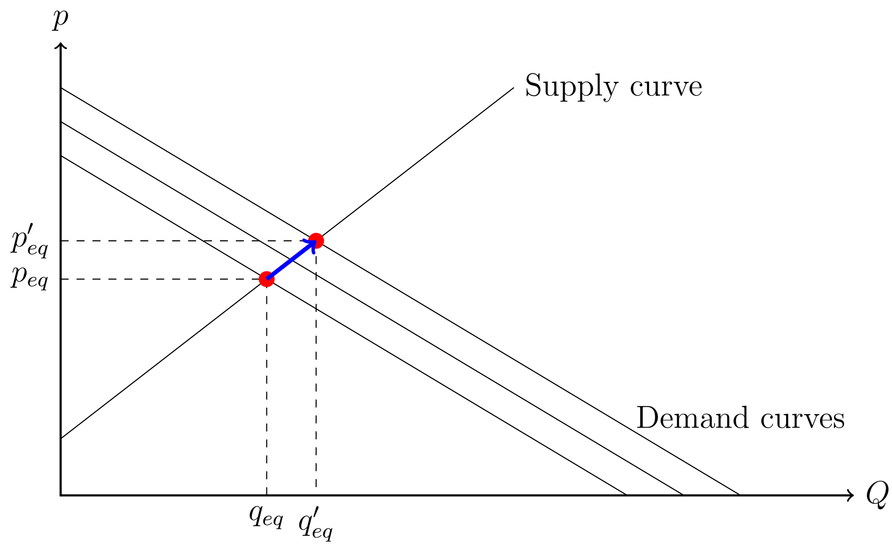

Figure 2.3: Positive price impact

the other agents to increase their demand. Thus, downward-sloping demand and strategic complementarity have an opposite effect on the total demand curve, and as a result on the price. The existence of strategic complementarity decreases the effect of the downward-sloping demand, and so, as the interaction increases, the price impact increases. A similar explanation holds when agent i has a negative price impact, and agents submit upwards-sloping demand schedules.

Our results in Sections 2.5.1 and 2.5.2 illustrate that both information and interaction have a first-order effect on agents' price impact. We saw that by keeping one of these variables fixed, the price impact changes monotonically in the other variable, with the direction of change depending on the sign of the slope of the demand curve.

2.5.3 Price Impact, Information and Interaction

In this section, we study the effect of strategic interaction on price impact, while changing the level of information asymmetry. In other words, we compare an economic environment with high levels of information asymmetry with one that has a low level of information asymmetry, by changing the weight agents' private signals place on the common component relative to the idiosyncratic component of the good's valuation. In particular, high values of $|\delta|$ imply a higher weight on the common component, which in turn, implies a move towards information symmetry, whereas small values of $|\delta|$ imply a higher weight on the idiosyncratic component, which in turn, implies a move towards a high level of information asymmetry.

From our characterization of the price impact λ in equation 2.10 in Corollary 2.1, we can see the monotonic relation between the price impact and the parameter δ. Most importantly, we find that as $|\delta|$ increases, and agents' information asymmetry decreases, price impact decreases. We also find that there is a monotonic relation between price impact and the parameter γ, which depends on the slope of the demand curve. Putting together our results in Section 2.5, we can analyze the trade-off between the effects of information asymmetry and strategic interaction on price impact. We do so by comparing the change in price impact as a function of γ for different values of $delta$. We also incorporate the case of agents having full information of their private valuation by setting $\delta = 1$ (implying that each agent i receives a private signal $s_i = \theta_i$).

In Figures 2.4 and 2.5 we can see how price impact changes with γ in the case where agents have full information of their private valuations (blue curve) and in the case where agents' private information leans towards the common component of the good's valuation, i.e., high value of $|\delta|$ (red curve). As agents' private signals become more symmetric, their price impact decreases. However, we can see that as the strategic interaction between agents increases (either it's strategic complementary or strategic substitutability), it diminishes the effect of asymmetric information on price impact, and the price impact goes to zero, even when the level of information asymmetry is high.

In Figures 2.6 and 2.7 we can see how price impact changes with γ in the case when agents have full information of their private valuations (blue curve) and in the

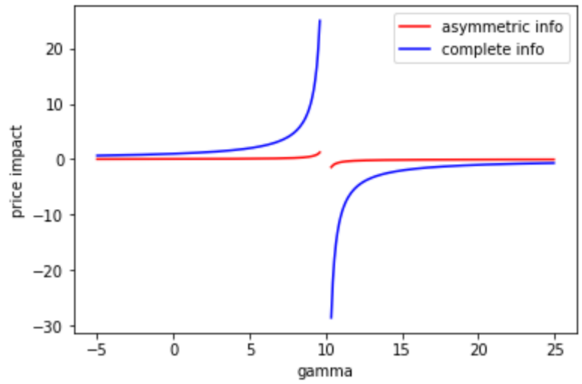

Figure 2.4: Equilibrium price impact with strategic interaction γ for large values of δ. Specifically, $\beta = 10$ and $\delta = 20$.

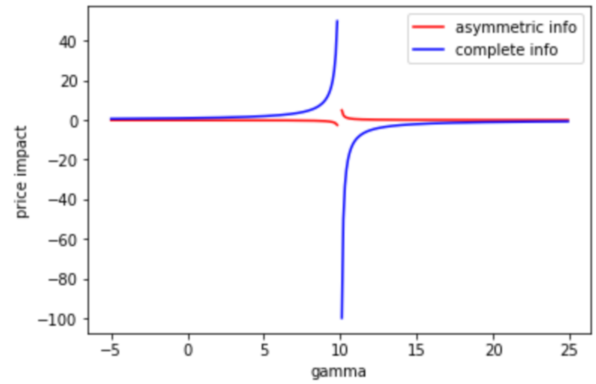

Figure 2.5: Equilibrium price impact with strategic interaction γ for large values of δ. Specifically, $\beta = 10$ and $\delta = -20$

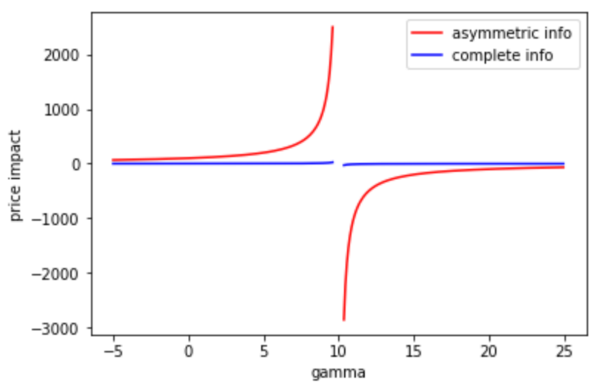

Figure 2.6: Equilibrium price impact with strategic interaction γ for small values of δ. Specifically, $\beta = 10$ and $\delta = 0.1$

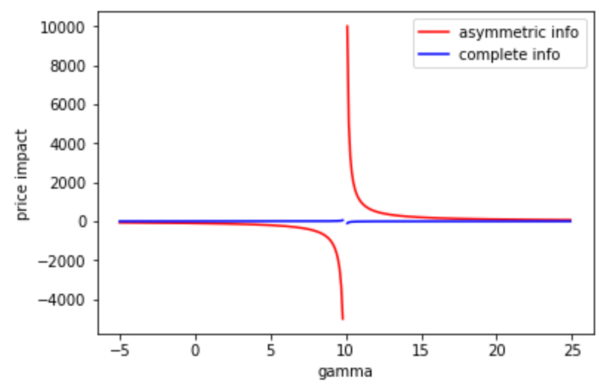

Figure 2.7: Equilibrium price impact with strategic interaction γ for small values of δ. Specifically, $\beta = 10$ and $\delta = -0.1$

case when agents' private information leans towards the idiosyncratic component of the value of the good, i.e., small value of $|\delta|$ (red curve). When there is uncertainty about the private valuation of the good and a high level of asymmetry, then price impact may be very high, especially with respect to the case where agents have full information of their private valuation. However, we can see that as strategic interaction between agents increases (either it's strategic complementary or strategic substitutability), it diminishes the effect of asymmetric information on price impact, and the price impact goes to zero.

2.6 Conclusions

In this work, we investigate the implications of incomplete information and learning from prices on agents' price impact in the presence of strategic interaction (strategic substitutability or strategic complementarity). We find that strategic interaction has a direct impact on agents' price impact. In particular, we find that when demand schedules are downward-sloping, the price impact increases as the extent of strategic complementarity increases, but when demand schedules are upward-sloping, the price impact decreases as strategic complementarity gets stronger. This is a consequence of the fact that when demand schedules are downward-sloping, agents have positive price impact, which is enforced by strategic complementarity. Strategic complementarity induces agents to buy more even if there is an increase in the price, which offsets the reduction in demand induced by the price effect. When demand schedules are upward-sloping, agents have negative price impact, which is weakened by strategic complementarity.

Moreover, we find that when private information is a compound of an agent's own valuation and other agents valuations, the relative weight between these two components has a monotonic effect on price impact. We find that as agents' private information puts more weight on the average valuation, agents' price impact decreases. This is a consequence of the fact that as the private information puts lower weight on the idiosyncratic component, and higher weight on the common component, agents' private signals become more symmetric. A decrease in the level of information asymmetry between agents, decreases their price impact.

Finally, we studied the trade-offs of the effects of strategic interaction and information asymmetry on agents' price impact. We find that strategic interaction may mitigate or exacerbate the effect of information asymmetry on price impact, and it depends on the slope of the inverse supply function. But, when strategic interaction is strong enough, it is able to mitigate the effect of information asymmetry on price impact, and price impact approaches zero.

Our findings suggest that accounting for agents strategic interaction, whether it is strategic complements or strategic substitutes, is crucial when analyzing agents' price impact in markets with asymmetric information. Policies that shape the interaction of agents that participate in the market may have direct effect on price impact, and as such, on the efficient operations of the market. Thus, accounting for the possibility of strategic substitutability and/or strategic complementarity should be a central pillar of optimal market design, specially in environments with highly dispersed information.

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Appendix A

Heterogeneous Markets

A.1 CARA Utility Function

A common utility function in the finance literature is the constant absolute risk aversion (CARA) utility. However, the CARA utility function is not as tractable as the quadratic - Gaussian framework when solving for the team-efficient benchmark, which is necessary for doing a welfare analysis. While we use a quadratic utility function in our analysis (which is also common in the literature), we show that our main characterization of the market being informationally efficient if and only if all trading costs λ_i are identical, still holds when using a CARA utility function. This implies that the insights from Chapter 1 are robust and are not specific to the quadratic utility framework.

Consider an economy consisting of n competitive traders, denoted by $\{1, 2, \ldots, n\}$, who trade a divisible asset. Suppose instead of a quadratic utility, agents now have a CARA utility. The realized payoff of trader i who obtains x_i units of the asset is thus given by

$$\pi_i(x_i) = -\exp\{-\lambda_i(x_i(\theta_i - p))\} \tag{A.1}$$

In equilibrium, agent i maximizes her profit, such that the market clears, which results in:

$$x_i = \frac{\mathbb{E}[\theta_i|s_i, p] - p}{\lambda_i \operatorname{var}[\theta_i|s_i, p]} \tag{A.2}$$

Considering linear actions of the form $x_i = a_i s_i - c_i p$, imply:

$$a_i = \frac{\sum_{j\neq i}(1-\rho+\sigma_j^2)a_j^2 + \rho(1-\rho)(\sum_{j\neq i} a_j)^2 - \rho\sigma_i^2 a_i \sum_{j\neq i} a_j}{\lambda_i\left(\sigma_i^2 \sum_{j\neq i}(1-\rho+\sigma_j^2)a_j^2 + \sigma_i^2\rho(1-\rho)(\sum_{j\neq i} a_j)^2\right)} \tag{A.3}$$

Recall that by using a quadratic utility function, we had:

$$a_i = \frac{\sum_{j\neq i}(1-\rho+\sigma_j^2)a_j^2 + \rho(1-\rho)(\sum_{j\neq i} a_j)^2 - \rho\sigma_i^2 a_i \sum_{j\neq i} a_j}{\lambda_i\left((1+\sigma_i^2) \sum_{j\neq i}(1-\rho+\sigma_j^2)a_j^2 + \rho(1-\rho+\sigma_i^2)(\sum_{j\neq i} a_j)^2\right)}$$

From equation (A.3), the weight a_i agents put on their private information is given by the solution to the following fixed point equation:

$$a_i = \frac{\sum_{j\neq i}(1-\rho+\sigma_j^2)a_j^2 + \rho(1-\rho)(\sum_{j\neq i} a_j)^2}{\lambda_i\left(\sigma_i^2 \sum_{j\neq i}(1-\rho+\sigma_j^2)a_j^2 + \sigma_i^2\rho(1-\rho)(\sum_{j\neq i} a_j)^2\right) + \rho\sigma_i^2 \sum_{j\neq i} a_j} \tag{A.4}$$

We will show that if the equilibrium is fully privately revealing, it must be the case that all trading costs are identical (the only if part is immediate). Suppose all signals are ex-ante symmetric, i.e. $\sigma_i = \sigma$ for all i. In this case, if the equilibrium is fully revealing it must be the case that $a_i = Q$, where Q is some constant. Plugging that back in equation (A.4) gives,

$$Q = \frac{(1-\rho+\sigma^2)(n-1)Q^2 + \rho(1-\rho)(n-1)^2Q^2}{\lambda_i\sigma^2\left((1-\rho+\sigma^2)(n-1)Q^2 + \rho(1-\rho)(n-1)^2Q^2\right) + \rho\sigma^2(n-1)Q}$$

Simplifying the above implies that the above equation holds only if

$$\lambda_i Q = \frac{1}{\sigma^2} - \frac{\rho}{1-\rho+\sigma^2+(n-1)\rho(1-\rho)}$$

Thus, λ_i must be constant for all i.

A.2 Proofs

Proof of Proposition 1.1

Recall that the (ex ante) expected profit of trader i is given by $\mathbb{E}[\pi_i] = \mathbb{E}[\theta_i x_i] - \frac{1}{2}\lambda_i\mathbb{E}[x_i^2] - \mathbb{E}[px_i]$ and suppose trader i follows a linear strategy given by $x_i = a_i s_i +$

$b_i - c_i p$, where a_i, b_i, and c_i are coefficients that only depend on model parameters. Plugging this expression into i's expected profit implies that

$$\mathbb{E}[\pi_i] = a_i - c_i \mathbb{E}[\theta_i p] - \frac{1}{2}\lambda_i(1+\sigma_i^2)a_i^2 - \frac{1}{2}(\lambda_i c_i^2 - 2c_i)\mathbb{E}[p^2] + a_i(\lambda_i c_i - 1)\mathbb{E}[ps_i] \\ - \frac{1}{2}\lambda_i b_i^2 + b_i(\lambda_i c_i - 1)\mathbb{E}[p].$$

Trader i's objective is to maximize her expected profit while taking the price as given. As a first observation, note that i's objective function is jointly concave in (a_i, b_i, c_i). Therefore, the first-order conditions with respect to these parameters are both necessary and sufficient for optimality. Hence, the best-response strategy of trader i satisfies the following relationships:

$$1 - \lambda_i(1+\sigma_i^2)a_i + (\lambda_i c_i - 1)\mathbb{E}[ps_i] = 0 \tag{A.5}$$

$$-\lambda_i b_i + (\lambda_i c_i - 1)\mathbb{E}[p] = 0 \tag{A.6}$$

$$-\mathbb{E}[\theta_i p] - (\lambda_i c_i - 1)\mathbb{E}[p^2] + \lambda_i a_i \mathbb{E}[ps_i] + \lambda_i b_i \mathbb{E}[p] = 0. \tag{A.7}$$

On the other hand, market clearing requires that $y + \sum_{i=1}^n x_i = 0$, where y is the quantity demanded by the outside trader. Hence,

$$\alpha - p + \beta \sum_{i=1}^{n}(a_i s_i + b_i - c_i p) = 0,$$

where we are using the fact that the first-order condition of the outside trader is given by $\alpha - p + \beta y = 0$. Rearranging the above terms therefore implies that the equilibrium price is given by (1.6).

Equations (A.5)–(1.6) provide a system of equations that relate traders' equilibrium strategies to the model fundamentals. Plugging in the expression for the price (1.6) in equations (A.5)–(A.7) (followed by some tedious calculations) then implies that

$$\lambda_i a_i = \frac{\sum_{k\neq i}(1-\rho+\sigma_k^2)a_k^2 + \rho(1-\rho)(\sum_{k\neq i} a_k)^2 - \rho\sigma_i^2 a_i \sum_{k\neq i} a_k}{(1+\sigma_i^2)\sum_{k\neq i} a_k^2(1-\rho+\sigma_k^2) + \rho(1-\rho+\sigma_i^2)(\sum_{k\neq i} a_k)^2} \tag{A.8}$$

$$\beta\lambda_i b_i = -\frac{\rho\sigma_i^2(\sum_{k\neq i} a_k)(\alpha+\beta\sum_{k=1}^n b_k)}{(1+\sigma_i^2)\sum_{k\neq i} a_k^2(1-\rho+\sigma_k^2) + \rho(1-\rho+\sigma_i^2)(\sum_{k\neq i} a_k)^2} \tag{A.9}$$

$$\beta(1-\lambda_i c_i) = \frac{\rho\sigma_i^2(\sum_{k\neq i} a_k)(1+\beta\sum_{k=1}^n c_k)}{(1+\sigma_i^2)\sum_{k\neq i} a_k^2(1-\rho+\sigma_k^2) + \rho(1-\rho+\sigma_i^2)(\sum_{k\neq i} a_k)^2}. \tag{A.10}$$

The proof is complete once we show that the system of equations (A.8)–(A.10) has a solution $(a_i, b_i, c_i)_{i=1}^n$. We first establish that there exists a vector $a = (a_1, \ldots, a_n)$ that satisfies (A.8) for all i. To this end, define the mapping $\Phi : \mathbb{R}_{++}^n \to \mathbb{R}_{++}^n$ as

$$\Phi_i(a) = \frac{\sum_{k\neq i}(1-\rho+\sigma_k^2)a_k^2 + \rho(1-\rho)\left(\sum_{k\neq i} a_k\right)^2}{\lambda_i\left((1+\sigma_i^2)\sum_{k\neq i} a_k^2(1-\rho+\sigma_k^2) + \rho(1-\rho+\sigma_i^2)(\sum_{k\neq i} a_k)^2\right) + \rho\sigma_i^2\sum_{k\neq i} a_k}.$$

Note that a satisfies equilibrium condition (A.8) if and only $\Phi(a) = a$. Define the set $A = \prod_{i=1}^n[\underline{a}_i, \overline{a}_i]$, where $\underline{a}_i = \lambda_{\max}^{-1}(1-\rho)/(1-\rho+\sigma_i^2)$ and $\overline{a}_i = \lambda_{\min}^{-1}(1-\rho)/(1-\rho+\sigma_i^2)$, with $\lambda_{\max}$ and $\lambda_{\min}$ denoting the largest and smallest trading costs, respectively. It is easy to verify that $\Phi_i(a) \geq \underline{a}_i$ whenever $\rho\sigma_i^2\sum_{k\neq i} a_k(\lambda_{\max}(1-\rho+\sigma_k^2)a_k - (1-\rho)) \geq 0$, which holds trivially as long as $a_k \geq \underline{a}_k$ for all $k \neq i$. Similarly, $\Phi_i(a) \leq \overline{a}_i$ as long as $\rho\sigma_i^2\sum_{k\neq i} a_k(\lambda_{\min}(1-\rho+\sigma_k^2)a_k - (1-\rho)) \leq 0$, an inequality that is satisfied when $a_k \leq \overline{a}_k$ for all $k \neq i$. These observations therefore imply that Φ maps the compact and convex set A to itself. Thus, by Brouwer's fixed point theorem, there exists $a \in A$ such that $\Phi(a) = a$, hence guaranteeing that there exist coefficients $a_1, \ldots, a_n$ that satisfy equation (A.8) for all i simultaneously.

Next, consider (A.9). This system of equations has a trivial solution of $b_i = 0$ for all i when $\alpha = 0$. We therefore consider the case that $\alpha \neq 0$. Dividing both sides of the equation by λ_i and summing over all i leads to

$$\beta\sum_{i=1}^n b_i = -\left(\alpha + \beta\sum_{i=1}^n b_i\right)\sum_{i=1}^n \frac{\rho\sigma_i^2}{\lambda_i\delta_i}\sum_{k\neq i} a_k, \tag{A.11}$$

where

$$\delta_k = (1+\sigma_k^2)\sum_{j\neq k} a_j^2(1-\rho+\sigma_j^2) + \rho(1-\rho+\sigma_k^2)(\sum_{j\neq k} a_j)^2. \tag{A.12}$$

Since $a_i > 0$ for all i, it must be the case that $\sum_{i=1}^n \frac{\rho\sigma_i^2}{\lambda_i\delta_i}\sum_{k\neq i} a_k \neq -1$. Therefore, given coefficients $a_1, \ldots, a_n$, there exists a unique $\sum_{i=1}^n b_i$ that satisfies (A.11). Plugging back this solution into (A.9) then implies that there exists a collection of constants $(b_1, \ldots, b_n)$ that satisfy the equilibrium condition.

Finally, consider (A.10). This equation implies that

$$\beta\sum_{i=1}^n c_i = \beta\sum_{i=1}^n \frac{1}{\lambda_i} - \left(1+\beta\sum_{i=1}^n c_i\right)\sum_{i=1}^n \frac{\rho\sigma_i^2}{\lambda_i\delta_i}\sum_{k\neq i} a_k.$$

Once again, the fact that $\sum_{i=1}^n \frac{\rho\sigma_i^2}{\lambda_i\delta_i}\sum_{k\neq i} a_k \neq -1$ guarantees that the exists a unique $\sum_{i=1}^n c_i$ that satisfies the above equation. Plugging back this solution into (A.10) then implies that there exists a collection $(c_1, \ldots, c_n)$ that satisfies the equilibrium conditions. □

Proof of Proposition 1.2

Lemma A.2.1. $a_i(1-\rho+\sigma_i^2) = a_k(1-\rho+\sigma_k^2)$ *for all pairs* i *and* k *if and only if all trading costs coincide.*

Proof. First suppose all trading costs are identical, i.e., $\lambda_i = \lambda$ for all i. Under such an assumption, it is immediate to verify that $\lambda a_i = (1-\rho)/(1-\rho+\sigma_i^2)$, thus implying that $a_i(1-\rho+\sigma_i^2) = a_k(1-\rho+\sigma_k^2)$ for all pairs of traders i and k.

To prove the converse implication, suppose $a_i(1-\rho+\sigma_i^2) = a_k(1-\rho+\sigma_k^2)$ for all pairs $i \neq k$. This means that there exists a constant $S > 0$ such that $(1-\rho+\sigma_k^2)a_k = S$

for all k. Plugging this expression into equilibrium condition (A.8) leads to

$$S\lambda_i\left((1+\sigma_i^2)+\rho(1-\rho+\sigma_i^2)\sum_{k\neq i}(1-\rho+\sigma_k^2)^{-1}\right)+\rho\sigma_i^2 =$$
$$(1-\rho+\sigma_i^2)\left(1+\rho(1-\rho)\sum_{k\neq i}(1-\rho+\sigma_k^2)^{-1}\right).$$

Solving for the constant S from the above expression implies that $S=(1-\rho)/\lambda_i$ for all i, which can hold only if $\lambda_i=\lambda_k$ for all i and k. □

We now turn to the proof of Proposition 1.2. As a first observation, note that when $n=2$, it is immediate that the equilibrium is fully privately revealing to both traders. Hence, in the rest of the proof we assume that there are at least three traders in the market. Suppose that the equilibrium is fully privately revealing to all traders, where recall from Definition 1.1 that this is equivalent to assuming that $\mathbb{E}[\theta_i|s_i,p]=\mathbb{E}[\theta_i|s_1,...,s_n]$ for all i, where

$$\mathbb{E}[\theta_i|s_1,\ldots,s_n]=\left(\frac{1-\rho}{1-\rho+\sigma_i^2}\right)s_i$$
$$+\frac{\rho\sigma_i^2}{(1-\rho+\sigma_i^2)\left(1+\rho\sum_{j=1}^n(1-\rho+\sigma_j^2)^{-1}\right)}\sum_{k=1}^n\left(\frac{1}{1-\rho+\sigma_k^2}\right)s_k.$$

On the other hand, the fact that market-clearing price satisfies (1.6) means that

$$\mathbb{E}[\theta_i|s_i,p]=\frac{1}{\delta_i}\left(\sum_{k\neq i}(1-\rho+\sigma_k^2)a_k^2+\rho(1-\rho)\Big(\sum_{k\neq i}a_k\Big)^2-\rho\sigma_i^2a_i\sum_{k\neq i}a_k\right)s_i$$
$$+\frac{1}{\delta_i}\left(\rho\sigma_i^2\sum_{j\neq i}a_j\right)\sum_{k=1}^n a_ks_k,$$

where δ_i is given by (A.12). Hence, full private revelation requires that the coefficient on signal s_k in the above two expressions coincide for all k. Hence, as long as there are at least three traders in the market, full private revelation to all traders i implies that $a_j/a_k=(1-\rho+\sigma_k^2)/(1-\rho+\sigma_j^2)$ for all $j,k\neq i$. Consequently, by Lemma A.2.1, all trading costs have to coincide. □

Proof of Proposition 1.3

As a first observation, note that

$$\operatorname{var}(\theta_i|s_1,\dots,s_n) = \frac{\sigma_i^2}{1-\rho+\sigma_i^2}\left(1-\frac{\rho(1-\rho+\sigma_i^2)^{-1}+\rho^2\sum_{k\neq i}(1-\rho+\sigma_k^2)^{-1}}{1+\rho\sum_{k=1}^{n}(1-\rho+\sigma_k^2)^{-1}}\right). \tag{A.13}$$

Furthermore, recall from (1.6) that $\operatorname{var}(\theta_i|s_i,p) = \operatorname{var}(\theta_i|s_i,\sum_{k\neq i}a_k s_k)$. Therefore,

$$\operatorname{var}(\theta_i|s_i,p) = \frac{\sigma_i^2}{1-\rho+\sigma_i^2}\left(1-\frac{\rho(1-\rho+\sigma_i^2)^{-1}\sum_{k\neq i}a_k^2(1-\rho+\sigma_k^2)+\rho^2(\sum_{k\neq i}a_k)^2}{(1+\rho(1-\rho+\sigma_i^2)^{-1})\sum_{k\neq i}a_k^2(1-\rho+\sigma_k^2)+\rho(\sum_{k\neq i}a_k)^2}\right).$$

Finally, note that $\operatorname{var}(\theta_i|s_i) = \sigma_i^2/(1+\sigma_i^2)$. Combining the above expressions implies that trader i's information revelation gap, defined in (1.7), is given by

$$\phi_i = \left(\frac{1+\sigma_i^2}{1-\rho+\sigma_i^2}\right)\frac{\sum_{k\neq i}a_k^2(1-\rho+\sigma_k^2)-(\sum_{k\neq i}a_k)^2(\sum_{k\neq i}(1-\rho+\sigma_k^2)^{-1})^{-1}}{(1+\rho(1-\rho+\sigma_i^2)^{-1})\sum_{k\neq i}a_k^2(1-\rho+\sigma_k^2)+\rho(\sum_{k\neq i}a_k)^2}. \tag{A.14}$$

On the other hand, equation (A.8) implies that $\lim_{\rho\to 0}a_i = w_i/\lambda_i$ for all traders i, where $w_i = 1/(1+\sigma_i^2)$. Taking the limit as $\rho\to 0$ from both sides of the above equation implies that

$$\lim_{\rho\to 0}\phi_i = \frac{\left(\sum_{k\neq i}w_k/\lambda_k^2\right)-\left(\sum_{k\neq i}w_k/\lambda_k\right)^2\Big/\left(\sum_{k\neq i}w_k\right)}{\left(\sum_{k\neq i}w_k/\lambda_k^2\right)}.$$

Dividing both the numerator and the denominator by $\sum_{k\neq i}w_k$ then complete the proof. $\square$

Proof of Proposition 1.4

The implication that $\phi_i^* = \lim_{n\to\infty}\phi_{in} = 0$ whenever $\mathbf{F}(\lambda)$ is degenerate is trivial. We therefore only provide the proof of the converse implication. In particular, suppose that $\phi_i^* = 0$. Recall from the proof of Proposition 1.3 that trader i's information revelation gap satisfies equation (A.14). Therefore,

$$\phi_i^* = \frac{\displaystyle\int a^2\sigma^2 d\mathbf{G} - \left(\int a d\mathbf{G}\right)^2 \Big/ \int \sigma^{-2} d\mathbf{G}}{\displaystyle\int a^2\sigma^2 d\mathbf{G} + \rho\left(\int a d\mathbf{G}\right)^2}, \tag{A.15}$$

where $\mathbf{G}(a,\lambda,\sigma) = \lim_{n\to\infty} \mathbf{G}_n(na,\lambda,\sigma/\sqrt{n})$ and $\mathbf{G}_n(a,\lambda,\sigma)$ denotes the joint empirical distribution of the weight that traders assign to their private signals, their trading costs, and the signal precisions. Note that whereas the joint distribution of λ_i and σ_i, denoted by $\mathbf{F}_n(\lambda,\sigma)$ is exogenous, the weights that traders assign to their private signals are equilibrium objects that are determined endogenously. Nonetheless, $\mathbf{G}_n$ can always be expressed in terms of the model primitive $\mathbf{F}_n$ using equation (A.8).[1]

Since $\phi_i^* = 0$, (A.15) implies that

$$\int a^2\sigma^2 d\mathbf{G} \int \sigma^{-2} d\mathbf{G} = \left(\int a d\mathbf{G}\right)^2.$$

But by he Cauchy-Schwarz inequality, the above equality can hold only if $a_i\sigma_i^2 = a_j\sigma_j^2$ for almost all pairs i and j. Hence, by equation (A.8), it must be the case that $\lambda_i = \lambda_j$, which means that $\mathbf{F}(\lambda)$ is degenerate. □

Proof of Proposition 1.5

Lemma A.2.2. *Let $x_i = a_i s_i + b_i - c_i p$ denote traders' equilibrium strategies. Then,*

$$\frac{\beta c_k}{1+\beta\sum_{j=1}^n c_j} = \frac{\beta Q_k - M_k}{\lambda_k(1+\beta\sum_{j=1}^n 1/\lambda_j)}, \tag{A.16}$$

where Q_k and M_k are independent of the value of β.

Proof. Recall that equilibrium coefficients (a_i, b_i, c_i) satisfy equations (A.8)–(A.10). Summing both sides of (A.10) over all traders i and solving for $1+\beta\sum_{i=1}^n c_i$ implies

[1]The normalization constant n in the definition $\mathbf{G}$ is a consequence of the assumption that all traders are informationally small as $n\to\infty$. More specifically, the fact that σ_{in} grows at rate $\sqrt{n}$ implies that the weight a_{in} has to decay to zero at rate n.

that

$$1+\beta\sum_{j=1}^{n}c_j=\frac{1+\beta\sum_{j=1}^{n}1/\lambda_j}{1+\rho\sum_{j=1}^{n}\sum_{r\neq j}a_r\sigma_j^2/(\lambda_j\delta_j)}, \tag{A.17}$$

where δ_k is given by (A.12). Plugging the above back into the expression for c_k in (A.10) then establishes (A.16), where Q_k and M_k are given by
$Q_k = 1+\rho\sum_{j\neq k}\left(\sum_{r\neq j}\frac{\sigma_j^2 a_r}{\lambda_j\delta_j}-\sum_{r\neq k}\frac{\sigma_k^2 a_r}{\delta_k\lambda_j}\right)$ and $M_k=\frac{\rho\sigma_k^2}{\delta_k}\sum_{j\neq k}a_j$. Finally, to establish that Q_k and M_k are independent of β, recall that the coefficients $(a_1,\dots,a_n)$ are solutions to the system of equations given by (A.8), which does not depend on β. Hence, Q_k and M_k are independent of β. □

With the above lemma in hand, we now return to the proof of Proposition 1.5. We prove this result by determining the conditions under which the equilibrium strategies identified in Proposition 1.1 satisfy the optimality conditions of the planner's problem.

Recall that the total ex ante surplus in the market is given by $\mathbb{E}[W]=\mathbb{E}[\pi_0]+\sum_{i=1}^{n}\mathbb{E}[\pi_i]$, where π_0 is the surplus of the outside trader and π_i is the profit of trader i. Therefore, the market-clearing condition $y+\sum_{i=1}^{n}x_i=0$ implies that

$$\mathbb{E}[W]=\sum_{i=1}^{n}\mathbb{E}[\theta_i x_i]-\frac{1}{2}\sum_{i=1}^{n}\lambda_i\mathbb{E}[x_i^2]+\alpha\mathbb{E}[y]-\frac{\beta}{2}\mathbb{E}[y^2]. \tag{A.18}$$

When agents follow linear strategies in the form of $x_i=a_is_i+b_i-c_ip$, the expected total surplus is given by

$$\mathbb{E}[W]=\sum_{i=1}^{n}\mathbb{E}[(\theta_i-\alpha)(a_is_i+b_i-c_ip)]-\frac{1}{2}\sum_{i=1}^{n}\lambda_i\mathbb{E}[(a_is_i+b_i-c_ip)^2]$$
$$-\frac{\beta}{2}\mathbb{E}\left[\sum_{i=1}^{n}(a_is_i+b_i-c_ip)\right]^2, \tag{A.19}$$

where once again we are using the market-clearing condition. Thus the social planner chooses the constants a_i, b_i, and c_i to maximize the total expected surplus in (A.19). We now determine the conditions under which the equilibrium strategies identified in Proposition 1.1 satisfy the first-order conditions corresponding to the planner's problem.

First, consider the planner's first-order condition with respect to coefficients $(b_1, \dots, b_n)$. Differentiating (A.19) with respect to b_i and using the fact that the market-clearing price satisfies (1.6) implies that

$$\frac{d\mathbb{E}[W]}{db_i} = -\lambda_i b_i + \lambda_i c_i \mathbb{E}[p] - \frac{\alpha + \beta \sum_{k=1}^n b_k}{1 + \beta \sum_{k=1}^n c_k} + \frac{\beta}{1 + \beta \sum_{k=1}^n c_k} \sum_{k=1}^n c_k \left(\lambda_k b_k + (1 - \lambda_k c_k)\mathbb{E}[p]\right). \quad \text{(A.20)}$$

On the other hand, recall from equation (A.6) that equilibrium coefficients satisfy $(\lambda_i c_i - 1)\mathbb{E}[p] = \lambda_i b_i$. Consequently, the first-order condition of the planner's problem with respect to b_i evaluated at the equilibrium strategies is given by

$$\left.\frac{d\mathbb{E}[W]}{db_i}\right|_{\text{eq}} = \mathbb{E}[p] - \frac{\alpha + \beta \sum_{k=1}^n b_k}{1 + \beta \sum_{k=1}^n c_k}.$$

But note that (1.6) implies that the right-hand side of the above expression is equal to zero, thus implying that equilibrium strategies always satisfy the planner's first-order conditions with respect to b_i for all parameter values.

Next, consider the social planner's first-order condition with respect to coefficients $(c_1, \dots, c_n)$. Differentiating (A.19) with respect to c_i leads to

$$\begin{aligned}\frac{d\mathbb{E}[W]}{dc_i} &= \lambda_i a_i \mathbb{E}[ps_i] + \lambda_i b_i \mathbb{E}[p] - \lambda_i c_i \mathbb{E}[p^2] - \mathbb{E}[\theta_i p] \\ &\quad + \frac{\alpha + \beta \sum_{k=1}^n b_k}{1 + \beta \sum_{k=1}^n c_k}\mathbb{E}[p] + \frac{\beta \sum_{k=1}^n a_k \mathbb{E}[s_k p]}{1 + \beta \sum_{k=1}^n c_k} \\ &\quad + \frac{\beta}{1 + \beta \sum_{k=1}^n c_k} \sum_{k=1}^n c_k \left(\mathbb{E}[\theta_k p] + (\lambda_k c_k - 1)\mathbb{E}[p^2] - \lambda_k a_k \mathbb{E}[ps_k] - \lambda_k b_k \mathbb{E}[p]\right),\end{aligned} \quad \text{(A.21)}$$

where once again we are using the fact that the market-clearing price satisfies (1.6). On the other hand, recall that equilibrium coefficients satisfy equation (A.7). Therefore, the first-order condition of the planner's problem with respect to c_i evaluated at equilibrium strategies is equal to

$$\left.\frac{d\mathbb{E}[W]}{dc_i}\right|_{\text{eq}} = -\mathbb{E}[p^2] + \frac{\alpha + \beta \sum_{k=1}^n b_k}{1 + \beta \sum_{k=1}^n c_k}\mathbb{E}[p] + \frac{\beta \sum_{k=1}^n a_k \mathbb{E}[s_k p]}{1 + \beta \sum_{k=1}^n c_k}.$$

Equation (1.6) then implies that right-hand side of the above equation is equal to zero. In other words, no matter the parameter values, the equilibrium strategies always satisfy the planner's first-order conditions with respect to $(c_1, \ldots, c_n)$.

Finally, we consider the planner's first-order condition with respect to $(a_1, \ldots, a_n)$. Differentiating (A.19) with respect to a_i and using the fact that the market-clearing price satisfies (1.6) implies that

$$\frac{d\mathbb{E}[W]}{da_i} = 1 - \lambda_i(1+\sigma_i^2)a_i + (\lambda_i c_i - 1)\mathbb{E}[ps_i] + \frac{\beta}{1+\beta\sum_{j=1}^n c_j}\sum_{k=1}^n c_k\left(\lambda_k a_k \mathbb{E}[s_i s_k] - \mathbb{E}[\theta_k s_i] + (1-\lambda_k c_k)\mathbb{E}[s_i p]\right). \quad \text{(A.22)}$$

Recall that we have already established that $d\mathbb{E}[W]/db_i = d\mathbb{E}[W]/dc_i = 0$ at the equilibrium strategies. Therefore, the equilibrium is constrained efficient if only if the above expression is equal to zero when evaluated at the equilibrium strategies. Furthermore, recall that equilibrium strategies satisfy equations (A.5)–(A.7). Hence, by (A.5), it is immediate that

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \frac{\beta}{1+\beta\sum_{j=1}^n c_j}\sum_{k=1}^n c_k\left(\lambda_k a_k \mathbb{E}[s_i s_k] - \mathbb{E}[\theta_k s_i] + (1-\lambda_k c_k)\mathbb{E}[s_i p]\right),$$

which by using (A.5) one more time further simplifies to

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \frac{\beta}{1+\beta\sum_{j=1}^n c_j}\sum_{k\neq i} c_k\left(\rho(\lambda_k a_k - 1) + (1-\lambda_k c_k)\mathbb{E}[s_i p]\right).$$

Replacing for coefficients a_i and c_i from equations (A.8) and (A.10) and using the fact that the market-clearing price satisfies (1.6) implies that

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \frac{\beta\rho}{1+\beta\sum_{j=1}^n c_j}\sum_{k\neq i}\frac{c_k\sigma_k^2}{\delta_k}\sum_{j\neq k} a_j\Big(a_i(1-\rho+\sigma_i^2) - a_j(1-\rho+\sigma_j^2)\Big),$$

where δ_k is defined in (A.12). Thus, by Lemma A.2.2,

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \frac{\rho}{1+\beta\sum_{r=1}^n 1/\lambda_r}\sum_{k\neq i}\frac{\sigma_k^2}{\delta_k\lambda_k}(\beta Q_k - M_k)\sum_{j\neq k} a_j\Big(a_i(1-\rho+\sigma_i^2) - a_j(1-\rho+\sigma_j^2)\Big). \quad \text{(A.23)}$$

We now use (A.23) to prove Proposition 1.5. As a first observation, note that when $\rho = 0$, the right-hand side of the above equation is equal to zero, thus implying that the equilibrium is constrained efficient for all profiles of trading costs. Next, consider the case that $n = 2$. With only two traders, it is immediate that the right-hand side of (A.23) is also equal to zero for all parameter values, thus once again implying constrained efficiency. To establish that the equilibrium is constrained efficient when all trading costs coincide, recall from Lemma A.2.1 that $\lambda_i = \lambda$ guarantees that $a_i(1-\rho+\sigma_i^2) = a_j(1-\rho+\sigma_j^2)$ for all i and j. Therefore, when all trading costs are identical, the right-hand side of (A.23) is equal to zero, thus guaranteeing constrained efficiency.

Finally, we show that as long as $n \geq 3$, trading costs are heterogeneous, and $\rho > 0$, the equilibrium is constrained inefficient for almost all values of β. We establish this by contradiction. Suppose there exist $\beta \neq \tilde{\beta}$ for which the equilibrium is constrained efficient. Hence, the right-hand side of (A.23) is equal to zero for both β and $\tilde{\beta}$ and all traders i. Since $\rho \neq 0$, this implies that

$$\sum_{k \neq i} \frac{\sigma_k^2}{\delta_k \lambda_k}(\beta Q_k - M_k) \sum_{j \neq k} a_j \Big(a_j(1-\rho+\sigma_j^2) - a_i(1-\rho+\sigma_i^2) \Big) = 0$$

$$\sum_{k \neq i} \frac{\sigma_k^2}{\delta_k \lambda_k}(\tilde{\beta} Q_k - M_k) \sum_{j \neq k} a_j \Big(a_j(1-\rho+\sigma_j^2) - a_i(1-\rho+\sigma_i^2) \Big) = 0,$$

where recall that the coefficients $(a_1, \ldots, a_n)$ are the solution to the fixed point equation (A.8) and hence are independent of the value of β. Subtracting the above two equations from one another and using the fact that $\beta \neq \tilde{\beta}$ leads to

$$\sum_{k \neq i} \frac{\sigma_k^2 M_k}{\delta_k \lambda_k} \sum_{j \neq k} a_j \Big(a_j(1-\rho+\sigma_j^2) - a_i(1-\rho+\sigma_i^2) \Big) = 0 \tag{A.24}$$

for all traders i. Since not all trading costs are identical, Lemma A.2.1 in the proof of Proposition 1.2 guarantees that there exists a i such that $a_i(1-\rho+\sigma_i^2) \leq a_j(1-\rho+\sigma_j^2)$ for all j, with at least one inequality holding strictly. But since $M_k > 0$, this means that the left-hand side of (A.24) has to be strictly negative, leading to a contradiction. $\square$

Proof of Proposition 1.6

Recall that the total ex ante surplus in the market is given by $\mathbb{E}[W] = \mathbb{E}[\pi_0] + \sum_{i=1}^{n} \mathbb{E}[\pi_i]$, where π_0 is the surplus of the outside trader and π_i is the profit of trader i. For ease of notation, denote $\pi_i(x_i) = u_i(x_i) - px_i$ and $\pi_0 = u_0(y) - py$. Differentiating the total surplus with respect to a_i implies

$$\frac{d}{da_i}\mathbb{E}[W] = \sum_{k=1}^{n} \mathbb{E}\left[\left(\frac{\partial u_k}{\partial x_k} - p\right)\left(\frac{dx_k}{da_i} + \frac{\partial x_k}{\partial p}\frac{dp}{da_i}\right) - x_k\frac{dp}{da_i}\right] + \mathbb{E}\left[\left(\frac{\partial u_0}{\partial y} - p\right)\frac{dy}{da_i}\right] - \mathbb{E}\left[y\frac{dp}{da_i}\right]$$

Therefore, the market-clearing condition $y + \sum_{i=1}^{n} x_i = 0$ implies that

$$\frac{d}{da_i}\mathbb{E}[W] = \mathbb{E}\left[\left(\frac{\partial u_i}{\partial x_i} - p\right)\left(\frac{dx_i}{da_i}\right)\right] + \sum_{k=1}^{n}\mathbb{E}\left[\left(\frac{\partial u_k}{\partial x_k} - p\right)\left(\frac{\partial x_k}{\partial p}\frac{dp}{da_i}\right)\right] \tag{A.25}$$

Recall that in equilibrium the first-order condition is given by

$$\mathbb{E}\left[\left(\frac{du_i}{dx_i} - p\right)\frac{dx_i}{da_i}\right] = 0$$

Writing the same expression in the ex post form, results in

$$\mathbb{E}\left[\frac{\partial u_k}{\partial x_k}|s_k, p\right] - p = 0.$$

Therefor, plugging the equilibrium action in equation (A.25) results in

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \sum_{k=1}^{n}\mathbb{E}\left[\left(\frac{\partial u_k}{\partial x_k} - \mathbb{E}\left[\left.\frac{\partial u_k}{\partial x_k}\right| s_k, p\right]\right)\left(\frac{\partial x_k}{\partial p}\frac{dp}{da_i}\right)\right]$$

By the law of iterated expectations,

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \sum_{k=1}^{n}\mathbb{E}\left[\mathbb{E}\left[\left.\left(\frac{\partial u_k}{\partial x_k} - \mathbb{E}\left[\left.\frac{\partial u_k}{\partial x_k}\right| s_k, p\right]\right)\left(\frac{\partial x_k}{\partial p}\frac{dp}{da_i}\right)\right]\right| s_1, \ldots, s_n\right]$$

Note that all equilibrium variables have to be measurable with respect to the collection of all the signals in the market. Consequently, we have,

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \sum_{k=1}^{n} \mathbb{E}\left[\frac{\partial x_k}{\partial p}\frac{dp}{da_i}\left(\mathbb{E}\left[\left.\frac{\partial u_k}{\partial x_k}\right| s_1,\ldots,s_n\right] - \mathbb{E}\left[\left.\frac{\partial u_k}{\partial x_k}\right| s_k, p\right]\right)\right]$$

Now, plugging back the marginal utility functions $u_k' = \theta_k - \lambda_k x_k$ and the linear strategies $x_i = a_i s_i - c_i p$, implies that

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \sum_{k=1}^{n} \frac{\partial x_k}{\partial p}\mathbb{E}\left[\frac{dp}{da_i}\Big(\mathbb{E}\left[\theta_k \mid s_1,\ldots,s_n\right] - \mathbb{E}\left[\theta_k \mid s_k, p\right]\Big)\right]$$

From equation (1.6) we have $\partial p/\partial a_i = \gamma s_i$, where $\gamma = \beta/(1+\beta\sum_{k=1}^{n} c_k)$. On the other hand, recall from equation (A.17) that $\gamma > 0$. Thus, by the law of iterated expectations,

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \gamma\sum_{k=1}^{n} \frac{\partial x_k}{\partial p}\operatorname{cov}\big(s_i, \mathbb{E}\left[\theta_k \mid s_1,\ldots,s_n\right] - \mathbb{E}\left[\theta_k \mid s_k, p\right]\big).$$

Finally, using the law of iterated expectations one more time to establish that $\operatorname{cov}\left(s_i, \mathbb{E}\left[\theta_k \mid s_1,\ldots,s_n\right] - \mathbb{E}\left[\theta_k \mid s_k, p\right]\right) = \operatorname{cov}(s_i, \theta_k - \mathbb{E}\left[\theta_k \mid s_k, p\right])$ then completes the proof. $\square$

Proof of Proposition 1.7

Proof of part (a) Recall from equation (A.23) that

$$\lim_{\beta\to 0}\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \rho^2\sum_{k\neq i}\frac{\sigma_k^4}{\delta_k^2\lambda_k}\left(\sum_{j\neq k} a_j\right)\left(\sum_{j\neq k} a_j\Big(a_j(1-\rho+\sigma_j^2) - a_i(1-\rho+\sigma_i^2)\Big)\right),$$

where δ_k is given by (A.12). The above expression therefore implies that

$$\lim_{\rho\to 0}\lim_{\beta\to 0}\frac{1}{\rho^2}\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} > 0 \tag{A.26}$$

if and only if

$$\lim_{\rho\to 0} a_i(1-\rho+\sigma_i^2) < \lim_{\rho\to 0}\frac{\sum_{k\neq i}\frac{\sigma_k^4}{\delta_k^2\lambda_k}(\sum_{j\neq k} a_j)(\sum_{j\neq k} a_j^2(1-\rho+\sigma_j^2))}{\sum_{k\neq i}\frac{\sigma_k^4}{\delta_k^2\lambda_k}(\sum_{j\neq k} a_j)^2}.$$

On the other hand, equation (A.5) implies that $\lim_{\rho\to 0} \lambda_i a_i = 1/(1+\sigma_i^2)$. Consequently, replacing for a_i in the above equation implies that inequality (A.26) holds if and only if

$$\frac{1}{\lambda_i} < \frac{\sum_{k\neq i} \frac{\sigma_k^4}{\lambda_k(1+\sigma_k^2)^2} \frac{\sum_{j\neq k} \frac{1}{\lambda_j(1+\sigma_j^2)}}{\sum_{j\neq k} \frac{1}{\lambda_j^2(1+\sigma_j^2)}}}{\sum_{k\neq i} \frac{\sigma_k^4}{\lambda_k(1+\sigma_k^2)^2} \left(\frac{\sum_{j\neq k} \frac{1}{\lambda_j(1+\sigma_j^2)}}{\sum_{j\neq k} \frac{1}{\lambda_j^2(1+\sigma_j^2)}} \right)^2}.$$

$\square$

Proof of part (b) Next, consider the case that $\beta \to \infty$. In this case, we have

$$\lim_{\rho\to 0} \lim_{\beta\to\infty} \frac{1}{\rho} \left. \frac{d\mathbb{E}[W]}{da_i} \right|_{\text{eq}} = \frac{1}{\sum_{r=1}^n 1/\lambda_r} \sum_{k\neq i} \frac{\sigma_k^2}{\lambda_k(1+\sigma_k^2)} \left(\frac{1}{\lambda_i} \frac{\sum_{j\neq k} \frac{1}{\lambda_j(1+\sigma_j^2)}}{\sum_{j\neq k} \frac{1}{\lambda_j^2(1+\sigma_j^2)}} - 1 \right).$$

Therefore,

$$\lim_{\rho\to 0} \lim_{\beta\to\infty} \frac{1}{\rho} \left. \frac{d\mathbb{E}[W]}{da_i} \right|_{\text{eq}} > 0$$

if and only if

$$\frac{1}{\lambda_i} > \frac{\sum_{k\neq i} \frac{\sigma_k^2}{\lambda_k(1+\sigma_k^2)}}{\sum_{k\neq i} \frac{\sigma_k^2}{\lambda_k(1+\sigma_k^2)} \frac{\sum_{j\neq k} \frac{1}{\lambda_j(1+\sigma_j^2)}}{\sum_{j\neq k} \frac{1}{\lambda_j^2(1+\sigma_j^2)}}}$$

$\square$

Proof of Proposition 1.8

Before presenting the proof, we state and prove two simple lemmas.

Lemma A.2.3. *Suppose $\zeta_1, \ldots, \zeta_m \geq 0$ and $\sum_{k=1}^m \zeta_k = 1$. Then,*

$$\sum_{k=1}^m \frac{y_k}{\zeta_k + z_k} \geq \frac{(\sum_{k=1}^m \sqrt{y_k})^2}{1 + \sum_{k=1}^m z_k} \tag{A.27}$$

for any collection of non-negative numbers $y_1, \ldots, y_m$ and $z_1, \ldots, z_m$.

Proof. We establish the lemma by showing that $\min_\zeta f(\zeta)$ subject to the constraint that $\sum_{k=1}^n \zeta_k = 1$ is equal to the right-hand side of (A.27), where $f(\zeta) = \sum_{k=1}^m y_k/(\zeta_k + z_k)$. First, note that $f(\zeta)$ is convex in ζ, thus implying that the first-order condition is a sufficient for optimality. This implies that $\eta = y_k/(\zeta_k + z_k)^2$, where η is the Lagrange multiplier corresponding to the constraint. Plugging this expression into the constraint implies that the optimal value of ζ_k is given by

$$\zeta_k = \left(\frac{1 + \sum_{j=1}^m z_j}{\sum_{j=1}^m \sqrt{y_j}} \right) \sqrt{y_k} - z_k.$$

Evaluating $f(\zeta)$ at the above values leads to the right-hand side of (A.27), thus completing the proof. $\square$

Lemma A.2.4. *Suppose $\alpha = 0$. The equilibrium welfare in a market consisting of n traders is*

$$\mathbb{E}[W] = \sum_{i=1}^n \frac{1}{2\lambda_i}(1 - \mathrm{var}(\theta_i|s_i, p)) - \left(\frac{1}{2\beta} + \sum_{i=1}^n \frac{1}{2\lambda_i} \right) \mathbb{E}[p^2].$$

Proof. Recall from the proof of Proposition 1.5 that the expected welfare in the market is given by (A.18). Furthermore, note that the first-order condition of trader i is given by $x_i = (\mathbb{E}[\theta_i|s_i, p] - p)/\lambda_i$, whereas that of the outside trader is given by $y = (\alpha - p)/\beta$. Plugging these expressions into (A.18) therefore implies that

$$\begin{aligned} \mathbb{E}[W] = \sum_{i=1}^n \frac{1}{\lambda_i} \mathbb{E}[\mathbb{E}^2[\theta_i|s_i, p]] - \sum_{i=1}^n \frac{1}{\lambda_i} \mathbb{E}[\theta_i p] - \sum_{i=1}^n \frac{1}{2\lambda_i} \mathbb{E}[(\mathbb{E}[\theta_i|s_i, p] - p)^2] \\ + \frac{\alpha}{\beta} \mathbb{E}[\alpha - p] - \frac{1}{2\beta} \mathbb{E}[(\alpha - p)^2]. \end{aligned}$$

Consequently,

$$\begin{aligned} \mathbb{E}[W] &= \sum_{i=1}^n \frac{1}{2\lambda_i} \mathbb{E}[\mathbb{E}^2[\theta_i|s_i, p]] - \left(\frac{1}{2\beta} + \sum_{i=1}^n \frac{1}{2\lambda_i} \right) \mathbb{E}[p^2] + \frac{\alpha^2}{2\beta} \\ &= \sum_{i=1}^n \frac{1}{2\lambda_i} \left(\mathrm{var}(\theta_i) - \mathbb{E}[\mathrm{var}(\theta_i|s_i, p)] \right) - \left(\frac{1}{2\beta} + \sum_{i=1}^n \frac{1}{2\lambda_i} \right) \mathbb{E}[p^2] + \frac{\alpha^2}{2\beta}, \end{aligned}$$

where the second equality is a consequence of the fact that $\mathbb{E}[\mathbb{E}[\theta_i|s_i, p]] = \mathbb{E}[\theta_i] = 0$ and the law of total variance. Noting that $\mathrm{var}(\theta_i) = 1$ and $\mathbb{E}[\mathrm{var}(\theta_i|s_i, p)] =$

$\mathrm{var}(\theta_i|s_i,p)$, which is a consequence of normality, and setting $\alpha = 0$ completes the proof. $\square$

With the above lemmas in hand, we now proceed to proving Proposition 1.8.

Proof of part (a). Suppose traders face no uncertainties about their private valuations, i.e., $\sigma_i = 0$ for all i. This means that $\mathrm{var}(\theta_i|s_i,p) = 0$ for all traders regardless of the market structure. Thus, by Lemma A.2.4, expected welfare in the centralized market is given by

$$\mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \left(\frac{1}{2\beta} + \sum_{i=1}^{n} \frac{1}{2\lambda_i}\right) \mathbb{E}[p^2].$$

On the other hand, equations (A.8)–(A.10) imply that when $\sigma_i = 0$, equilibrium strategies satisfy $b_i = 0$ and $a_i = c_i = \lambda_i^{-1}$. Thus, by equation (1.6), the market clearing price in the centralized market is equal to $p = \beta \sum_{i=1}^{n} s_i \lambda_i^{-1} / (1 + \beta \sum_{i=1}^{n} \lambda_i^{-1})$. Therefore,

$$\mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \frac{\beta}{2} \left(\frac{(1-\rho)\sum_{i=1}^{n} 1/\lambda_i^2 + \rho(\sum_{i=1}^{n} 1/\lambda_i)^2}{1 + \beta \sum_{i=1}^{n} 1/\lambda_i}\right). \tag{A.28}$$

Following similar steps implies that expected welfare in the segmented architecture is given by

$$\mathbb{E}[W^{\text{seg}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \frac{\beta}{2} \sum_{S_k \in \mathcal{S}} \frac{(1-\rho)\sum_{i\in S_k} 1/\lambda_i^2 + \rho(\sum_{i\in S_k} 1/\lambda_i)^2}{\zeta_k + \beta \sum_{i\in S_k} 1/\lambda_i},$$

where S_k denotes the set of traders in the k-th segment and ζ_k is the fraction of outside traders that are active in that segment. Applying Lemma A.2.3 to the second term on the right-hand side above and noting that $\sum_{S_k\in\mathcal{S}} \zeta_k = 1$ leads to

$$\mathbb{E}[W^{\text{seg}}] \le \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \frac{\beta/2}{1 + \beta\sum_{i=1}^{n} 1/\lambda_i} \left(\sum_{S_k\in\mathcal{S}} \sqrt{(1-\rho)\sum_{i\in S_k} 1/\lambda_i^2 + \rho\Big(\sum_{i\in S_k} 1/\lambda_i\Big)^2}\right)^2,$$

which in turn implies that

$$\mathbb{E}[W^{\text{seg}}] \leq$$

$$\sum_{i=1}^{n} \frac{1}{2\lambda_i} - \frac{\beta/2}{1+\beta\sum_{i=1}^{n} 1/\lambda_i} \left((1-\rho)\sum_{i=1}^{n} \frac{1}{\lambda_i^2} + \rho \sum_{S_k \in \mathcal{S}} \left(\sum_{i \in S_k} \frac{1}{\lambda_i} \right)^2 + \rho \sum_{S_k \neq S_j} \left(\sum_{i \in S_k} \frac{1}{\lambda_i} \right) \left(\sum_{i \in S_j} \frac{1}{\lambda_i} \right) \right).$$

Equation (A.28) implies that the right-hand side of the above inequality coincides with $\mathbb{E}[W^{\text{cen}}]$, thus establishing that expected welfare is weakly higher in the centralized market structure. □

Proof of part (b). Suppose $\rho = 0$. This means that $\text{var}(\theta_i|s_i, p) = \text{var}(\theta_i|s_i) = \sigma_i^2/(1+\sigma_i^2)$ regardless of the market structure. Consequently, Lemma A.2.4 implies that expected welfare in the centralized architecture is equal to

$$\mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i(1+\sigma_i^2)} - \left(\frac{1}{2\beta} + \sum_{i=1}^{n} \frac{1}{2\lambda_i} \right) \mathbb{E}[p^2].$$

Equations (A.8)–(A.10) imply that when $\rho = 0$, the coefficients corresponding to equilibrium strategies satisfy $a_i = \lambda_i^{-1}/(1+\sigma_i^2)$, $b_i = 0$, and $c_i = \lambda_i^{-1}$. Replacing for p from (1.6) leads to

$$\mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i(1+\sigma_i^2)} - \frac{\beta \sum_{i=1}^{n} \frac{1}{\lambda_i^2(1+\sigma_i^2)}}{2(1+\beta\sum_{i=1}^{n} \lambda_i^{-1})}. \tag{A.29}$$

Following similar steps for the segmented market structure implies that

$$\mathbb{E}[W^{\text{seg}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i(1+\sigma_i^2)} - \sum_{S_k \in \mathcal{S}} \frac{\beta \sum_{i \in S_k} \frac{1}{\lambda_i^2(1+\sigma_i^2)}}{2(\zeta_k + \beta\sum_{i \in S_k} \lambda_i^{-1})},$$

and as a result,

$$\mathbb{E}[W^{\text{seg}}] \leq \sum_{i=1}^{n} \frac{1}{2\lambda_i(1+\sigma_i^2)} - \sum_{S_k \in \mathcal{S}} \frac{\beta \sum_{i \in S_k} \frac{1}{\lambda_i^2(1+\sigma_i^2)}}{2(1 + \beta\sum_{i=1}^{n} \lambda_i^{-1})}.$$

Note that, by (A.29), the right-hand side of the above inequality is equal to $\mathbb{E}[W^{\text{cen}}]$, thus implying that expected welfare in the centralized architecture is higher than the segmented architecture. □

Proof of part (c). Suppose all traders have identical trading costs, i.e., $\lambda_i = \lambda$ for all i. Since all trading costs are identical, Proposition 1.2 implies that the equilibrium in the centralized market structure is fully privately revealing to all traders, i.e., $\text{var}(\theta_i|s_i, p) = \text{var}(\theta_i|s_1, \ldots, s_n)$ for all i. A similar argument also guarantees that the equilibrium of the segmented market structure is also fully privately revealing to all traders within that segment. Therefore, Lemma A.2.4 implies that the expected welfare gain from market centralization is given by

$$\begin{aligned}\mathbb{E}[W^{\text{cen}}] - \mathbb{E}[W^{\text{seg}}] &= \frac{1}{2\lambda}\sum_{i=1}^{n}\big(\text{var}(\theta_i|s_k : k \in S_{(i)}) - \text{var}(\theta_i|s_1, \ldots, s_n)\big) \\ &\quad + \frac{1}{2}\sum_{i=1}^{n}\left(\frac{\zeta_{(i)}}{\beta|S_{(i)}|} + \frac{1}{\lambda}\right)(\mathbb{E}[p_{(i)}^2] - \mathbb{E}[p^2]),\end{aligned}$$

where $S_{(i)}$ denotes the set of traders that belong to the same segment as trader i and $p_{(i)}$ is the market-clearing price in that segment. Since $S_{(i)} \subseteq \{1, \ldots, n\}$, it is immediate that the first term on the right-hand side above is always non-negative. It is therefore sufficient to establish that the second term is also non-negative. To this end, first consider the centralized market structure. Recall that when all trading costs are identical (A.8) implies that

$$a_i = \frac{1-\rho}{\lambda(1-\rho+\sigma_i^2)},$$

whereas (A.17) leads to

$$1 + \beta\sum_{j=1}^{n} c_j = \frac{(1-\rho)(1+n\beta/\lambda)(1+\rho\sum_{k=1}^{n}(1-\rho+\sigma_k^2)^{-1})}{1+(n-1)\rho}.$$

Plugging the above two expressions into the expression for the market-clearing price in (1.6), we obtain

$$\left(\frac{1}{2\beta} + \sum_{i=1}^{n}\frac{1}{2\lambda}\right)\mathbb{E}[p^2] = \frac{\beta(1+(n-1)\rho)^2\sum_{k=1}^{n}(1-\rho+\sigma_k^2)^{-1}}{2\lambda(\lambda+n\beta)(1+\rho\sum_{k=1}^{n}(1-\rho+\sigma_k^2)^{-1})}.$$

Following similar steps for the segmented market structure implies that

$$\frac{1}{2}\sum_{i\in S_k}\left(\frac{\zeta_{(i)}}{\beta|S_{(i)}|} + \frac{1}{\lambda}\right)\mathbb{E}[p_{(i)}^2] = \frac{\beta(1+(|S_{(i)}|-1)\rho)^2\sum_{k=1}^{n}(1-\rho+\sigma_k^2)^{-1}}{2\lambda(\lambda+|S_{(i)}|\beta)(1+\rho\sum_{k=1}^{n}(1-\rho+\sigma_k^2)^{-1})}.$$

Finally, defining the expression above as a function of n: $f(n) = \frac{\beta(1+(n-1)\rho)^2 \sum_{k=1}^{n}(1-\rho+\sigma_k^2)^{-1}}{2\lambda(\lambda+n\beta)(1+\rho\sum_{k=1}^{n}(1-\rho+\sigma_k^2)^{-1})}$ and applying subadditivity completes the proof.

Proof of Proposition 1.9

First consider the centralized market architecture. By Lemma A.2.4,

$$\lim_{\beta\to 0} \mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i}(1 - \text{var}(\theta_i|s_i, p)),$$

where we are using the fact that, by equation (1.6), $\lim_{\beta\to 0} p^2/\beta = 0$. Replacing for $\text{var}(\theta_i|s_i, p)$ in terms of the information revelation gap defined in (1.7) leads to

$$\lim_{\beta\to 0} \mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i}\left(1 - \phi_i^{\text{cen}}\,\text{var}(\theta_i|s_i) - (1-\phi_i^{\text{cen}})\,\text{var}(\theta_i|s_1,\ldots,s_n)\right).$$

On the other hand, recall that $\text{var}(\theta_i|s_i) = \sigma_i^2/(1+\sigma_i^2)$, whereas (A.13) implies that $\text{var}(\theta_i|s_1,\ldots,s_n) = \frac{\sigma_i^2}{1+\sigma_i^2} - \frac{\rho^2\sigma_i^4}{(1+\sigma_i^2)^2}\sum_{j\neq i}(1+\sigma_j^2)^{-1} + o(\rho^2)$. Consequently,

$$\lim_{\beta\to 0} \mathbb{E}[W^{\text{cen}}] =$$
$$\sum_{i=1}^{n} \frac{1}{2\lambda_i}\left(1 - \phi_i^{\text{cen}}\frac{\sigma_i^2}{1+\sigma_i^2} - (1-\phi_i^{\text{cen}})\left(\frac{\sigma_i^2}{1+\sigma_i^2} - \frac{\rho^2\sigma_i^4}{(1+\sigma_i^2)^2}\sum_{j\neq i}(1+\sigma_j^2)^{-1}\right)\right) + o(\rho^2).$$

Following similar steps for the segmented market structure implies that

$$\lim_{\beta\to 0} \mathbb{E}[W^{\text{seg}}] =$$
$$\sum_{S_k\in\mathcal{S}}\sum_{i\in S_k} \frac{1}{2\lambda_i}\left(1 - \phi_i^{\text{seg}}\frac{\sigma_i^2}{1+\sigma_i^2} - (1-\phi_i^{\text{seg}})\left(\frac{\sigma_i^2}{1+\sigma_i^2} - \frac{\rho^2\sigma_i^4}{(1+\sigma_i^2)^2}\sum_{\substack{j\in S_k\\ j\neq i}}(1+\sigma_j^2)^{-1}\right)\right) + o(\rho^2),$$

where ϕ_i^{seg} is trader i's information revelation gap in the segmented market structure. Subtracting the above two equations from one another implies that

$$\lim_{\beta\to 0} \left(\mathbb{E}[W^{\text{cen}}] - \mathbb{E}[W^{\text{seg}}]\right) =$$
$$\rho^2 \sum_{S_k\in\mathcal{S}}\sum_{i\in S_k} \frac{\sigma_i^4}{2\lambda_i(1+\sigma_i^2)^2}\left((1-\phi_i^{\text{cen}})\sum_{j\neq i}\frac{1}{1+\sigma_j^2} - (1-\phi_i^{\text{seg}})\sum_{\substack{j\in S_k\\ j\neq i}}\frac{1}{1+\sigma_j^2}\right) + o(\rho^2),$$

thus completing the proof. $\square$

Proof of Proposition 1.10

Proof of part (i) The proof of part (i) is similar to that of Proposition 3 of [57]. First suppose that $\rho_{ij} = \rho$ for all $i \neq j$. Since all trading costs coincide, then Proposition 1.2 guarantees that the equilibrium is fully privately revealing to all traders simultaneously.

To prove the converse implication, suppose the price is fully privately revealing to all traders. That is, $\mathbb{E}[\theta_i|s_i, p] = \mathbb{E}[\theta_i|s_1, ..., s_n]$ for all i. In addition, recall that when traders follow linear strategies in the form of $x_i = a_i s_i + b_i - c_i p$, the corresponding coefficients satisfy (A.5)–(A.7). Consequently,

$$\lambda a_i = \frac{\operatorname{var}(p) - \mathbb{E}[ps_i]\mathbb{E}[p\theta_i]}{(1+\sigma^2)\operatorname{var}(p) - \mathbb{E}^2[ps_i]},$$

where we are using the fact that all traders have identical trading costs and signal precisions. Also recall that the market-clearing price satisfies (1.6). Replacing for the price in the above expression therefore implies that coefficients $(a_1, \ldots, a_n)$ are the solution to the following system of equations:

$$\lambda a_i = \frac{\sum_{k\neq i} a_k^2(1+\sigma^2) + \sum_{j,k\neq i}\rho_{kj}a_k a_j - (\sum_{k\neq i}\rho_{ik}a_k)^2 - a_i\sigma^2\sum_{k\neq i}\rho_{ik}a_k}{(1+\sigma^2)\sum_{k\neq i} a_k^2(1+\sigma^2) + (1+\sigma^2)\sum_{j,k\neq i}\rho_{kj}a_k a_j - (\sum_{k\neq i}\rho_{ik}a_k)^2}. \quad \text{(A.30)}$$

It is easy to verify that the solution to the above system of equations is given by

$$a_i = \frac{1-\bar{\rho}}{\lambda(1-\bar{\rho}+\sigma^2)}, \quad \text{(A.31)}$$

where $\bar{\rho}$ is defined (1.12). Since $a_i = a_j$ for all pairs of traders i and j, equation (1.6) implies that the price is a sufficient statistic for the unweighted average of traders' signals, namely, $(1/n)\sum_{k=1}^n s_k$. Therefore,

$$\begin{aligned}\mathbb{E}[\theta_i|s_1, \ldots, s_n] &= \mathbb{E}[\theta_i|s_i, p] \\ &= \left(\frac{1-\bar{\rho}}{1-\bar{\rho}+\sigma^2}\right)s_i + \frac{\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+\sigma^2+\bar{\rho}(n-1))}\sum_{k=1}^n s_k,\end{aligned}$$

where we are using the fact that the equilibrium is fully privately revealing to trader i. Consequently,

$$\mathbb{E}[\theta_i s_j] = \left(\frac{1-\bar{\rho}}{1-\bar{\rho}+\sigma^2}\right)\rho_{ij} + \frac{\bar{\rho}\sigma^2}{(1-\bar{\rho}+\sigma^2)(1+\sigma^2+\bar{\rho}(n-1))}\left(1+\sigma^2+\sum_{k\neq j}\rho_{jk}\right)$$

for any $j \neq i$. Replacing the left-hand side of the above equation with ρ_{ij} and noting that $\sum_{k\neq j}\rho_{jk} = (n-1)\bar{\rho}$ implies that the above equality is satisfied for all $i \neq j$ only if $\rho_{ij} = \bar{\rho}$ for all pairs of traders $i \neq j$. $\square$

Proof of part (ii) Recall from the proof of Proposition 1.1 that equilibrium strategies satisfy equations (A.5)–(A.7). Furthermore, recall from the proof of Proposition 1.5 that the first-order conditions of the planner's problem with respect to coefficients a_i, b_i, and c_i are given by (A.22), (A.20), and (A.21), respectively. As in the proof of Proposition 1.5, it is immediate to verify that, as long as (A.6) is satisfied, the right-hand side of (A.20) is equal to zero, thus implying that equilibrium strategies satisfy the planner's first-order condition with respect to b_i. Similarly, using (A.7) to simplify (A.21) implies that the right-hand side of the latter equation is also equal to zero for all parameter values, which establishes that equilibrium strategies satisfy the planner's first-order condition with respect to c_i.

Having established $d\mathbb{E}[W]/db_i = d\mathbb{E}[W]/dc_i = 0$ for all i, it is therefore sufficient to verify that the right-hand side of (A.22), when evaluated at equilibrium strategies, is equal to zero. The fact that equilibrium strategies satisfy (A.5) implies that

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \frac{\beta}{1+\beta\sum_{j=1}^n c_j}\sum_{k\neq i} c_k\left(\rho_{ik}(\lambda a_k - 1) + (1-\lambda(1+\sigma^2)a_k)\frac{\mathbb{E}[s_i p]}{\mathbb{E}[s_k p]}\right),$$

where we are using the fact that all traders have identical trading costs and signal precisions. Plugging for equilibrium actions from (A.31) and noting that equilibrium strategies are symmetric lead to

$$\left.\frac{d\mathbb{E}[W]}{da_i}\right|_{\text{eq}} = \frac{\beta}{1+n\beta c}\frac{c\sigma^2}{1-\bar{\rho}+\sigma^2}\sum_{k\neq i}\left(\bar{\rho}\left(\frac{1+\sigma^2+\sum_{j\neq i}\rho_{ij}}{1+\sigma^2+\sum_{j\neq i}\rho_{jk}}\right)-\rho_{ik}\right)$$
$$= \frac{\beta}{1+n\beta c}\frac{c\sigma^2}{1-\bar{\rho}+\sigma^2}\sum_{k\neq i}\left(\bar{\rho}-\rho_{ik}\right).$$

The definition of $\bar{\rho}$ in (1.12) now guarantees that the right-hand side of the above equality is equal to zero, thus completing the proof. □

Proof of Proposition 1.11

In equilibrium, traders maximize their expected profit, given by

$$\mathbb{E}[\pi_i(x_i)|s_i] = \mathbb{E}[\theta_i x_i - \frac{1}{2}\lambda_i x_i^2 - p x_i|s_i] \tag{A.32}$$

such that the market clears, i.e., $p = \alpha + \beta\sum_{i=1}^n x_i$. By taking the FOC we get:

$$x_i = \frac{1}{\lambda_i}(\mathbb{E}[\theta_i - p|s_i]), \tag{A.33}$$

From the projection theorem for Gaussian random variables, this simplifies to:

$$x_i = \frac{1}{\lambda_i(1+\sigma_i^2)}s_i - \frac{1}{\lambda_i}\mathbb{E}[p|s_i] \tag{A.34}$$

Market clearing implies,

$$x_i = \frac{1}{\lambda_i(1+\sigma_i^2)}s_i - \frac{1}{\lambda_i}\mathbb{E}[\alpha+\beta\sum_{i=1}^n x_i|s_i]$$
$$= \left(\frac{1}{\lambda_i(1+\sigma_i^2)} - \frac{\beta a_i}{\lambda_i} - \frac{\beta\rho\sum_{j\neq i}a_j}{\lambda_i(1+\sigma_i^2)}\right)s_i - \frac{1}{\lambda_i}(\alpha+\beta\sum_{j=1}^n b_i)$$

This, in turn, implies

$$a_i = \frac{1}{\lambda_i(1+\sigma_i^2)} - \frac{\beta a_i}{\lambda_i} - \frac{\beta\rho\sum_{j\neq i}a_j}{\lambda_i(1+\sigma_i^2)}$$
$$b_i = -\frac{1}{\lambda_i}(\alpha+\beta\sum_{j=1}^n b_j)$$

Or equivalently,

$$a_i = \frac{1 - \beta\rho \sum_{j \neq i} a_j}{(\lambda_i + \beta)(1 + \sigma_i^2)} \tag{A.35}$$

$$b_i = \frac{-\alpha - \beta \sum_{j \neq i} b_j}{\lambda_i + \beta} \tag{A.36}$$

In order to find a closed-form solution for b_i we can sum over i for the equation for b_i resulting in

$$\sum_{i=1}^{n} b_i (1 - \beta \sum_{i=1}^{n} \frac{1}{\lambda_i}) = - \alpha \sum_{i=1}^{n} \frac{1}{\lambda_i}$$

implying

$$\begin{aligned} b_i &= - \frac{\alpha}{\lambda_i} \left(1 + \frac{\beta \sum_{i=1}^{n} \frac{1}{\lambda_i}}{1 - \beta \sum_{i=1}^{n} \frac{1}{\lambda_i}} \right) \\ &= - \frac{\alpha}{\lambda_i (1 - \beta \sum_{i=1}^{n} \frac{1}{\lambda_i})} \end{aligned}$$

□

Proof of Proposition 1.12

In the constrained-efficient maximization problem, we maximize the total surplus given by

$$\mathbb{E}[TS] = \sum_{i=1}^{n} \mathbb{E} \left[\theta_i x_i - \frac{1}{2} \lambda_i x_i^2 - p x_i \right] + \mathbb{E}[u(y) - py]$$

By plugging in x_i, y and p in terms of a_i, b_i, α and β, we get:

$$\sum_{i=1}^{n} a_i \mathbb{E}[\theta_i s_i] + \sum_{i=1}^{n} b_i \mathbb{E}[\theta_i] - \frac{1}{2} \sum_{i=1}^{n} \lambda_i \mathbb{E}[a_i^2 s_i^2 + 2 a_i b_i s_i + b_i^2] + \mathbb{E} \left[-\alpha \sum_{i=1}^{n} x_i - \beta \frac{(\sum_{i=1}^{n} x_i)^2}{2} \right]$$

which further simplifies to:

$$\begin{aligned} &\sum_{i=1}^{n} a_i - \frac{1}{2} \sum_{i=1}^{n} \lambda_i a_i^2 (1 + \sigma_i^2) - \frac{1}{2} \sum_{i=1}^{n} \lambda_i b_i^2 + -\alpha \sum_{i=1}^{n} b_i \\ &\qquad - \frac{\beta}{2} \sum_{i=1}^{n} (1 - \rho + \sigma_i^2) a_i^2 - \frac{\beta}{2} \rho \left(\sum_{i=1}^{n} a_i \right)^2 - \frac{\beta}{2} \left(\sum_{i=1}^{n} b_i \right)^2 \end{aligned}$$

Differentiating with respect to a_i and b_i results in

$$\begin{aligned}
\frac{\partial \mathbb{E}[TS]}{\partial a_i} =& 1 - \lambda_i(1+\sigma_i^2)a_i - \beta(1-\rho+\sigma_i^2)a_i - \beta\rho\sum_{j=1}^{n} a_j \\
\frac{\partial \mathbb{E}[TS]}{\partial b_i} =& -\lambda_i b_i - \alpha - \beta\sum_{j=1}^{n} b_j
\end{aligned}$$

Equating the derivatives to zero results in

$$a_i = \frac{1-\beta\rho\sum_{j\neq i} a_j}{(\lambda_i+\beta)(1+\sigma_i^2)} \tag{A.37}$$

$$b_i = \frac{-\alpha-\beta\sum_{j\neq i} b_j}{\lambda_i+\beta} \tag{A.38}$$

□

Appendix B

Market Power and Strategic Interaction

B.1 Proofs

Proof of Proposition 2.1

Recall that the expected profit of trader i is given by,

$$\mathbb{E}[\pi_i|s_i, p] = \mathbb{E}[\theta_i x_i - p x_i - \frac{1}{2}x_i^2 + \gamma \sum_{j \neq i} x_i x_j | s_i, p] \tag{B.1}$$

and suppose trader i follows a linear strategy given by $x_i = a_i s_i + b_i - c_i p$, where a_i, b_i, and c_i are coefficients that only depend on model parameters. Since we assume agents are symmetric, we have $x_i = a s_i + b - cp$ for every agent i. Trader i's objective is to maximize her expected profit. The first order condition with respect to x_i implies,

$$\mathbb{E}[\theta_i|s_i, p] - p - \lambda x_i - x_i + \gamma \sum_{j \neq i} \mathbb{E}[x_j|s_i, p] + \gamma x_i \sum_{j \neq i} \frac{\partial \mathbb{E}[x_j|s_i, p]}{\partial x_i} = 0, \tag{B.2}$$

and hence, the best-response strategy of trader i satisfies the following relationships:

$$x_i = \frac{\mathbb{E}[\theta_i|s_i, p] - p + \gamma(n-1)(b - cp) + \gamma a \sum_{j \neq i} \mathbb{E}[s_j|s_i, p]}{1 + \lambda + \gamma(n-1)c\lambda} \tag{B.3}$$

Since $\frac{\partial p}{\partial x_i} = \lambda$, the second order condition with respect to x_i implies that a maximum exists if and only if:

$$-1 - 2\lambda - 2\gamma(n-1)c\lambda \leq 0 \tag{B.4}$$

We will find the expression of c that satisfies equation B.3 and plug in the second-order condition above.

Given our normal information structure, we have the following Variance-Covariance matrix:

$$\operatorname{cov}\begin{bmatrix}\theta_i\\ s_i\\ \sum_j s_j\end{bmatrix} =$$
$$\begin{bmatrix}\sigma^2 & \frac{\sigma^2}{n}\left(n+(\delta-1)(1+(n-1)\rho)\right) & \sigma^2\delta\left(1+(n-1)\rho\right)\\ \frac{\sigma^2}{n}\left(n+(\delta-1)(1+(n-1)\rho)\right) & \frac{\sigma^2}{n}\left(n+(\delta^2-1)(1+(n-1)\rho)\right) & \delta^2\sigma^2(1+(n-1)\rho)\\ \sigma^2\delta\left(1+(n-1)\rho\right) & \delta^2\sigma^2(1+(n-1)\rho) & n\delta^2\sigma^2(1+(n-1)\rho)\end{bmatrix}$$

$$\operatorname{cov}\begin{bmatrix}s_j\\ s_i\\ \sum_j s_j\end{bmatrix} =$$
$$\begin{bmatrix}\frac{\sigma^2}{n}\left(n+(\delta^2-1)(1+(n-1)\rho)\right) & \frac{\sigma^2}{n}\left(n\rho+(\delta^2-1)(1+(n-1)\rho)\right) & \sigma^2\delta\left(1+(n-1)\rho\right)\\ \frac{\sigma^2}{n}\left(n\rho+(\delta^2-1)(1+(n-1)\rho)\right) & \frac{\sigma^2}{n}\left(n+(\delta^2-1)(1+(n-1)\rho)\right) & \delta^2\sigma^2(1+(n-1)\rho)\\ \sigma^2\delta\left(1+(n-1)\rho\right) & \delta^2\sigma^2(1+(n-1)\rho) & n\delta^2\sigma^2(1+(n-1)\rho)\end{bmatrix}$$

which in turn results in the following conditional expectations:

$$\mathbb{E}[\theta_i|s_i,p] = s_i + \frac{1-\delta}{\delta n}\cdot\sum_j s_j \tag{B.5}$$

and

$$\mathbb{E}[s_j|s_i,p] = -\frac{1}{n-1}\cdot s_i + \frac{1}{n-1}\cdot\sum_j s_j (= \frac{1}{n-1}\sum_{j\neq i} s_j). \tag{B.6}$$

Notice that in this noise-free information structure $\mathbb{E}[\theta_i|s_i,p] = \theta_i$. Thus, in equilibrium, every agent i is perfectly informed about her true value θ_i[1]. Then, by comparing the coefficients of $x_i = as_i + b - cp$ and equation (B.3), we get:

$$a = \frac{1}{1+\lambda+\gamma[1+(n-1)\lambda c]} \tag{B.7}$$

$$b = 0 \tag{B.8}$$

[1]When $\delta = -\frac{1}{n-1}$, $\mathbb{E}[\theta_i|s_i,p]$ has zero weight on agent i's private signal s_i.

$$\frac{c^2\lambda\gamma(n-1)}{\delta} + \frac{c}{\delta}\left[1+\lambda+\gamma\left(1+\frac{\lambda(n-1)(1-\delta)}{\beta n}\right)\right] = 1 - \frac{(1+\lambda)(1-\delta)}{\delta\beta n} - \gamma\frac{1+(n-1)\delta}{\delta\beta n} \quad \text{(B.9)}$$

Rewriting equation (2.5) in terms of c results in

$$c = \frac{1}{n-1}\left(\frac{1}{\lambda} - \frac{1}{\beta}\right) \quad \text{(B.10)}$$

Thus, we have two equations (B.10) and (B.9) with two variables c and λ. Plugging equation (B.10) in equation (B.9) results in

$$\frac{(\beta-\lambda)^2\gamma}{\delta\beta^2\lambda(n-1)} + \frac{\beta-\lambda}{\delta\beta\lambda(n-1)}\left[1+\lambda+\gamma\left(1+\frac{\lambda(n-1)(1-\delta)}{\beta n}\right)\right] = 1 - \frac{(1+\lambda)(1-\delta)}{\delta\beta n} - \gamma\frac{1+(n-1)\delta}{\delta\beta n}$$

Rearranging the equation above and multiplying by $\lambda\delta\beta(n-1)$ results in

$$\lambda^2\left[\frac{(\gamma-\beta)(1+\delta(n-1))}{\beta n}\right] + \lambda\left[\beta\left(1-\delta(n-1)\right) - (1+\gamma)\frac{1+\delta(n-1)}{n} - 2\gamma\right] = -\beta - 2\gamma\beta$$

Multiplying again by $\frac{n}{\delta(n-1)+1}$ simplifies to

$$\lambda^2\left(\frac{\gamma-\beta}{\beta}\right) + \lambda\left[\frac{\beta n\left(1-\delta(n-1)\right)}{1+\delta(n-1)} - 1 - \gamma - \frac{2\gamma n}{1+\delta(n-1)}\right] + \frac{\beta n + 2\gamma\beta n}{1+\delta(n-1)} = 0 \quad \text{(B.11)}$$

Solving this quadratic equation then completes the proof for λ.

Now, we can also substitute c in the second order condition, given by equation (B.4), with the expression given by equation (B.10). Thus the second order conditions are satisfied if and only if:

(i) if $\beta > \gamma$, then $\lambda \geq -\frac{\beta(1+2\gamma)}{2(\beta-\gamma)}$; and

(ii) if $\beta < \gamma$, then $\lambda \leq -\frac{\beta(1+2\gamma)}{2(\beta-\gamma)}$.

where λ is the solution to equation B.11.

To find the equilibrium price we use equations (B.3), (B.5), and (B.6) to conclude that

$$x_i(s_i, p) = \frac{\theta_i - p + \gamma(n-1)(b - cp) + \gamma a(\sum_{j \neq i} \theta_j + (n-1)(\delta - 1)\bar{\theta}}{1 + \lambda + \gamma(n-1)c\lambda}$$

Substituting a and c with the expressions in equations (B.7) and (B.10) and summing over i results in

$$\sum_i x_i(s_i, p) = \frac{-np\left(1 + \gamma(\frac{1}{\lambda} - \frac{1}{\beta})\right) + n\bar{\theta}\left(1 + \frac{\gamma\delta(n-1)}{1+\lambda+\gamma(2-\lambda/\beta)}\right)}{1 + \lambda + \gamma(1 - \lambda/\beta)} \tag{B.12}$$

Then, market clearing, $p = \beta \sum_i x_i(s_i, p)$, implies

$$p = \frac{\beta n\bar{\theta}\left(1 + \gamma \cdot \frac{\delta(n-1)}{1+\lambda+\gamma(2-\lambda/\beta)}\right)}{1 + \lambda + \beta n + \gamma \cdot \frac{(\beta-\lambda)(n\beta+\lambda)}{\lambda\beta}} \tag{B.13}$$

□

Proof of Lemma 2.1

To prove that the market is informationally efficient in equilibrium, we will show that $\mathbb{E}[\theta_i | s_i, p] = \mathbb{E}[\theta_i | s_1, ..., s_n]$ for all i, simultaneously.

From equation (B.5), we get:

$$\mathbb{E}[\theta_i | s_i, p] = s_i + \frac{1-\delta}{\delta n} \cdot \sum_j s_j = \theta_i + (\delta - 1)\bar{\theta} + \frac{1-\delta}{\delta n}(n\bar{\theta} + (\delta - 1)n\bar{\theta}) = \theta_i$$

Obviously, the information set $\{s_i, p\}$ is contained in the information set $\{s_1, ..., s_n\}$. Combining that with the equation above results in:

$$\mathbb{E}[\theta_i | s_1, ..., s_n] = \theta_i$$

□

Proof of Corollary 2.1

In order to characterize the equilibrium in the limit, we look at the limit $\lim_{n\to\infty} \lambda n$, where λ is given by equation (2.8):

$$\lim_{n\to\infty} \lambda \cdot n = \lim_{n\to\infty} \frac{\beta n}{2(\beta-\gamma)} \left[-\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1} - 1 - \gamma\left(1+\frac{2n}{\delta(n-1)+1}\right)\right]$$
$$\pm \frac{\beta n}{2(\beta-\gamma)} \left[\sqrt{n^2\left(\frac{\beta\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right)^2 + 2\beta n + 1 + 2\gamma + \gamma^2 + f(n)}\right]$$

where $f(n) = \gamma^2\left[\frac{4n^2(1-\delta)}{(\delta(n-1)+1)^2} - \frac{4n(1-\delta)}{(\delta(n-1)+1)^2}\right] + 2\gamma\left[\frac{\beta n(\delta(n-1)-1)}{\delta(n-1)+1} + \frac{2\beta n^2(\delta(n-1)-1)}{(\delta(n-1)+1)^2}\right]$.

In order to solve the equation, we rewrite the expression in the square root:

$$\lim_{n\to\infty}\Bigg(n^2\left(\frac{\beta\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right)^2 + 2\beta n\left(1+\frac{\gamma\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right)$$
$$+4\gamma\beta n\frac{n\left(\delta(n-1)-1\right)}{(\delta(n-1)+1)^2} + v(n) + C\Bigg)^{1/2} \tag{B.14}$$

where $v(n) = \gamma^2\left[\frac{4n^2(1-\delta)}{(\delta(n-1)+1)^2} - \frac{4n(1-\delta)}{(\delta(n-1)+1)^2}\right]$ and $C = 1 + 2\gamma + \gamma^2$.

Equation (B.14) can then be rewritten as:

$$\lim_{n\to\infty} n\cdot K_0\sqrt{1+\frac{2\beta}{nK_0^2}\left(1+\frac{\gamma\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right) + \frac{4\gamma\beta}{K_0^2}\frac{\left(\delta(n-1)-1\right)}{(\delta(n-1)+1)^2} + \frac{v(n)}{K_0^2n^2} + \frac{C}{K_0^2n^2}} \tag{B.15}$$

where $K_0 = \frac{\beta(\delta(n-1)-1)}{\delta(n-1)+1}$ and $\frac{v(n)}{n^2} = \gamma^2\left[\frac{4(1-\delta)}{(\delta(n-1)+1)^2} - \frac{4(1-\delta)}{n(\delta(n-1)+1)^2}\right]$.

Recall that the first order approximation of $f(x) = \sqrt{1+ax+bx^2+cx^3}$ when $x\to 0$ is given by $1+\frac{a}{2}x$. Thus, a first order approximation of equation (B.15) is given by:

$$\lim_{n\to\infty} n\cdot K_0\left(1+\frac{\beta}{nK_0^2}\left(1+\frac{\gamma\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right) + \frac{2\gamma\beta}{K_0^2}\frac{\left(\delta(n-1)-1\right)}{(\delta(n-1)+1)^2}\right)$$
$$= \lim_{n\to\infty}\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1} + \frac{\delta(n-1)+1}{\delta(n-1)-1} + \gamma + \frac{2\gamma n}{\delta(n-1)+1}$$

Plugging this back into the equation of $\lim_{n\to\infty}\lambda n$ results in

$$\lim_{n\to\infty}\lambda\cdot n = \lim_{n\to\infty}\frac{\beta n}{(\beta-\gamma)\delta(n-1)} = \frac{\beta}{\delta(\beta-\gamma)} \tag{B.16}$$

Taking the negative root of the quadratic equation for λ would result in:

$$\lim_{n\to\infty} \lambda \cdot n =$$
$$\lim_{n\to\infty} \frac{\beta n}{(\beta-\gamma)} \left[-\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1} - \frac{\delta(n-1)}{\delta(n-1)-1} - \gamma \left(1 + \frac{2n}{\delta(n-1)+1}\right)\right] \to \pm\infty \tag{B.17}$$

Since we know that the limit is finite for $\gamma = 0$, we conclude that only the positive root is a valid solution.

Now we can plug equation (B.16) in equations (2.6) and (2.7) to get the expressions for a and c, respectively, in the limit. □

Proof of Corollary 2.2

In order to determine whether agents submit downward-sloping demand curves or upward-sloping demand curves, we need to determine the sign of the parameter c, which is the slope of agents demand curve.

From Corollary 2.1, we have equation (2.12): $\lim_{n\to\infty} c = \frac{\delta(\beta-\gamma)}{\beta}$ characterizing the slope of the demand schedule, by a closed form expression in terms of the model primitives: δ, γ, and β. Since we assume $\beta > 0$, the parameter c is positive if and only if $\delta(\beta-\gamma)$ is positive, implying the conditions for having a downward-sloping demand curve (remember that agent i's schedule is given by $x_i = as_i - cp$). Similarly, the the parameter c is negative if and only if $\delta(\beta-\gamma)$ is negative, implying the conditions for having an upward-sloping demand curve.

□

Proof of Lemma 2.2

We want to find how the price impact λ changes with the parameter δ. From Corollary 2.1, we have the expression for λ in the large market equilibrium, given by equation

(2.10):

$$\lim_{n\to\infty} \lambda \cdot n = \frac{\beta}{\delta(\beta-\gamma)}$$

Since the equation above is for the limit as $n \to \infty$, we cannot simply take the derivative with respect to δ, in order to find the relation we are interested in. If we can show that the equilibrium price impact in the finite market, given by equation (2.8) is uniformly continuous in δ, only then we can use the derivative of the price impact in the large market given by equation (2.10). Indeed, suppose $\lambda(\delta)$ is uniformly continuous. Then, from equation (2.10), we know that the derivative of $\lim_{n\to\infty} \lambda \cdot n$ with respect to δ is given by,

$$\frac{\partial \lim_{n\to\infty} \lambda \cdot n}{\partial \delta} = -\frac{\beta}{\delta^2(\beta-\gamma)}$$

implying that when $\gamma < \beta$, then price impact is decreasing as a function of δ (the derivative is negative), and when $\gamma < \beta$, then price impact is increasing as a function of δ (the derivative is positive).

Now, it is left to prove that $\lambda(\delta)$ is uniformly continuous, where $\lambda(\delta)$ is given by,

$$\begin{aligned}\lambda(\delta) = &\frac{\beta}{2(\beta-\gamma)}\left(-\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1} - 1 - \gamma\left(1+\frac{2n}{\delta(n-1)+1}\right)\right)\\ &+\frac{\beta}{2(\beta-\gamma)}\left(\sqrt{\left(\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right)^2 + 2\beta n + 1 + g(\gamma)}\right)\end{aligned}$$

and $g(\gamma) = \gamma^2\left[1+\frac{4n(1-\delta)(n-1)}{(\delta(n-1)+1)^2}\right] + 2\gamma\left[1+\frac{\beta n(\delta(n-1)-1)}{\delta(n-1)+1}\left(1+\frac{2n}{\delta(n-1)+1}\right)\right]$.

We will show that the derivative of $\lambda(\delta)$ with respect to δ is bounded, implying that $\lambda(\delta)$ is a Lipschitzian function, and thus uniformly continuous. The derivative is given by,

$$\begin{aligned}\frac{\partial\lambda(\delta)}{\partial\delta} = &\frac{\beta}{\beta-\gamma}\left(\frac{(\gamma-\beta)n(n-1)}{(\delta(n-1)+1)^2}\right)\\ &+\frac{\beta}{4(\beta-\gamma)}\left(\left(\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right)^2 + 2\beta n + 1 + g(\gamma)\right)^{-1/2}\\ &\cdot\left(\frac{4\beta^2 n^2(n-1)(\delta(n-1)-1)}{(\delta(n-1)+1)^3}\right)\cdot\frac{\partial g(\gamma)}{\partial\delta}\end{aligned}$$

After some tedious calculations, We conclude that

$$\lim_{\delta \to 0} \frac{\partial \lambda(\delta)}{\partial \delta} < K$$

where K is a finite constant, and

$$\lim_{\delta \to \pm\infty} \frac{\partial \lambda(\delta)}{\partial \delta} = 0$$

which completes the proof. □

Proof of Proposition 2.2

Proposition 2.2 is an immediate consequence of Lemma 2.2 together with Corollaries 2.1 and 2.2. Indeed, from Corollary 2.1 we know that the price impact is negative when $\gamma > \beta$ and $\delta > 0$ or when $\gamma < \beta$ and $\delta < 0$. From Lemma 2.2 we know that when $\gamma > \beta$, then λ increases with δ. Thus, an increase in $\delta > 0$ will imply a decrease in the price impact. Similarly, when $\gamma < \beta$, then λ decreases with $\delta < 0$. Thus, an increase in $\delta < 0$, which is equivalent to a decrease in $|\delta|$, will imply an increase in the price impact. A similar explanation follows when price impact is positive.

□

Proof of Proposition 2.3

Now, we want to find the relation between price impact and interaction. The proof of this proposition takes the same approach as the proofs for Lemma 2.2 and Proposition 2.2. In Corollary 2.1 we characterize the price impact λ in terms of the model parameters: γ, β and δ, as given by the equation: $\lim_{n\to\infty} n\lambda = \frac{\beta}{\delta(\beta-\gamma)}$. Thus, we can compute the derivative with respect to γ:

$$\frac{\partial \lim_{n\to\infty} \lambda \cdot n}{\partial \gamma} = \frac{\beta}{\delta(\beta-\gamma)^2}$$

In Corollary 2.2, we identified when demand schedules are upward-sloping and when they are downward-sloping, in terms of the model parameters: γ, β and δ. We know that demand schedules are upward-sloping whenever both $\delta > 0$ and $\beta < \gamma$, or both $\delta < 0$ and $\beta > \gamma$. Similar to the proof of Proposition 2.2, we then need to look at the different cases when the price impact is negative and when it is positive, and remember that an increase in λ when $\lambda < 0$ is in fact a decrease in the price impact.

Now, it is left to prove that the derivative of $\lambda(\gamma)$ with respect to γ is bounded, implying that $\lambda(\gamma)$ is a Lipschitzian function, and thus uniformly continuous. The derivative is given by,

$$\begin{aligned}\frac{\partial \lambda}{\partial \gamma} = & \frac{\beta}{2(\beta-\gamma)^2}\left(-\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1} - 1 - \gamma\left(1+\frac{2n}{\delta(n-1)+1}\right)\right. \\ & \left. \pm\sqrt{\left(\frac{\beta n\left(\delta(n-1)-1\right)}{\delta(n-1)+1}\right)^2 + 2\beta n + 1 + g(\gamma)}\right) \\ & + \frac{\beta}{2(\beta-\gamma)}\left(-1-\frac{2n}{\delta(n-1)+1}\right. \\ & \left. \pm\frac{\gamma\left[1+\frac{4n(1-\delta)(n-1)}{(\delta(n-1)+1)^2}\right] + \left[1+\frac{\beta n(\delta(n-1)-1)}{\delta(n-1)+1}\left(1+\frac{2n}{\delta(n-1)+1}\right)\right]}{\sqrt{\left(\frac{\beta n(\delta(n-1)-1)}{\delta(n-1)+1}\right)^2 + 2\beta n + 1 + g(\gamma)}}\right)\end{aligned}$$

where $g(\gamma) = \gamma^2\left[1+\frac{4n(1-\delta)(n-1)}{(\delta(n-1)+1)^2}\right] + 2\gamma\left[1+\frac{\beta n(\delta(n-1)-1)}{(\delta(n-1)+1)}\left(1+\frac{2n}{\delta(n-1)+1}\right)\right]$.
After some tedious calculations, We conclude that

$$\lim_{\gamma\to 0}\frac{\partial\lambda(\gamma)}{\partial\gamma} < K$$

where K is a finite constant, and

$$\lim_{\gamma\to\pm\infty}\frac{\partial\lambda(\gamma)}{\partial\gamma} = 0$$

which completes the proof.

$\square$