Essays on Digital Advertising

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Abstract

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Digital advertising has seen dramatic growth over the last decade. Total digital ad spending in the US has increased 6 times between 2010 and 2020, from $26 billion to $152 billion (eMarketer). This impressive development has in turn sparked a huge stream of literature studying all the different aspects of advertising in the digital media. My dissertation contributes to this literature via two essays. In the first essay, I consider a very important topic of ad blocking, that in the recent years has become a significant threat to advertising supported content. With a specific focus on consumer and total welfare, I show the detrimental role of the adblockers' current revenue model in decreasing content quality, consumer surplus and total welfare. In the second essay, I study demand learning in digital advertising markets, where firms learn over time how their advertising campaigns impact consumer demand by using their advertising campaign outcomes in earlier periods. By developing an analytic model, I demonstrate in several scenarios, such as monopoly and competition, that learning has an ambiguous effect on the key market parameters and, in particular, on the equilibrium advertising and quantities.

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Chapter 1: Introduction

This dissertation consists of 2 analytical papers each addressing a critical issue in the field of digital advertising. These 2 papers constitute Chapter 1 and Chapter 2 of the present document. In the first chapter, we talk about the phenomenon of ad blocking and how its emergence affects the market participants with a special emphasis on consumer welfare analysis. We build an analytic model where the ad blocker allows publishers to show only a limited number of ads in exchange for a payment ("whitelisting") and uses a threat of total ad block as a bargaining chip in the negotiations with the publishers. Since, as our model will show, ad blocking necessarily hurts publishers' incentives to invest in content quality this leads us to the following question: does (and if yes, then when) the direct positive effect of ad reduction on consumers outweigh the indirect negative effect of lower content quality and how the total welfare changes in the presence of ad blocking. Our findings show that, surprisingly, the negative effect of ad blocking prevails under a broad set of model parameters implying that consumer and total surplus usually decline when there is ad blocking. This means that the average consumer is generally hurt by ad blocking, even though there is always a segment of consumers who become better off. Ad blocking, thereby, is not an efficient market solution to regulate the amount of ads consumers are exposed to. Additionally, we show that increasing publisher's bargaining power still leads to a considerable decline in welfare even for very high values of the bargaining power. We also consider what happens when ad blocking is done by a platform (e.g. iOS, Google, Facebook) maximizing total surplus. Turns out the platform cannot improve upon the benchmark with no ad blocking unless it can commit to the maximum level of advertising before the publishers set quality, they will always under-invest under the threat of potential future ad blocking leading to a lower overall surplus. Finally, we consider a number of robustness checks.

In the second chapter, we analyze consumer demand learning in digital advertising markets. We study a scenario where firms use outcomes of their past advertising campaigns to better learn consumer demand in the future periods. We know that advertising is assumed to increase consumer demand but the scale of this increase is uncertain (ad effect measurement problem). But firms might decrease this uncertainty if they advertise more intensely. Hence, higher advertising early on leads to better demand estimates later on and then to better pricing decisions. We evaluate how learning affects the equilibrium outcomes and especially the total advertising shown to consumers. We build a two period model that incorporates this effect by allowing firms to use an informative signal about demand received in the first period to estimate their ad campaign efficiency in the second period. In our model we find that in most settings the effect of learning on advertising intensity is ambiguous and depends on market parameters. In particular, in the case of a single monopolist firm on the market, learning decreases total advertising when the advertising cost is small, because the firm's advertising strategy becomes more targeted instead of a blanket advertising strategy used if the firm cannot learn. But when the advertising cost is high, learning increases advertising intensity through increasing the return on additional advertising impression. We corroborate our findings studying a competitive model with two firms. The effect of learning on advertising intensity again turns out to be ambiguous but with an added layer of competitive effects making the dynamics more complicated. In the extensions we consider a competitive model where firms not only learn their own advertising effects on demand but also their competitor's signal. We find that, on the one hand, more information can decrease advertising because firms now will be able to avoid advertising simultaneously and decrease the competitive pressure. While, on the other hand, more information increases firms' returns to advertising making the advertising intensity higher. Finally, we look at how learning affects the equilibrium quantities produced by the firms. We discover that learning usually leads to higher quantities since it increases expected profits. For example, learning always leads a monopolist firm to choose a higher quantity. However, in the competitive scenario, learning might actually decrease equilibrium quantities for a certain range of values of the advertising cost because learning would increase competitive pressure.

Chapter 2: Ad Blocking

"Ad blocking" - the practice of installing a third-party software or app to block advertisements from loading on visited websites - has grown steadily since 2013 to reach hundreds of millions of Internet users worldwide. In the US alone, eMarketer estimates that in 2020, approximately 26.4% of U.S. Internet users installed ad blockers on their connected devices (Reyes, 2020). While this proportion is stabilizing, eMarketer notes that it still represents 76.1 million users and threatens about a quarter of publishers' $142 billion total U.S. online advertising revenues. The use of ad blockers is more popular in some other geographies (e.g. in the Asia-Pacific or Eastern Europe) but recent studies indicate that, in developed countries, usage rates are stabilizing. For example, in the UK, the Association for Online Publishing found that, by the end of 2019, ad blocking rates actually fell slightly compared to the previous year to 20.6% (Perrin, 2019). Importantly, in all countries, a significant number of consumers never use ad block software despite the fact that these applications are nominally free. Clearly, the use of ad blockers is not without cost to consumers.[1]

Ad blocking's benefits to consumers are obvious: ad blocking significantly reduces the intensity of advertising, which is a *direct* transfer of surplus to consumers. This direct positive effect is certainly one factor behind the public's generally favourable attitude towards adblockers and may also explain adblockers' favorable treatment by regulators. Indeed, in their PR, adblockers often cultivate a "Robin Hood" image, claiming to represent a consumer movement against intrusive online advertising.

In contrast, the practice is a major threat to content providers (publishers) whose revenue model is based on advertising. Publishers complain to be victims of "extortion" and liken ad blocking to

[1]Consumers using ad blockers face various nuisance costs (e.g. loss of content, frequent requests for upgrades), some consumers feel uncomfortable towards publishers when using ad blockers while others face difficulty of access (e.g. many corporate users cannot freely download browser extensions on their computers).

“highway robbery" (Katona and Sarvary, 2018). To respond to the threat to their revenues some publishers (e.g. Axel Springer, a large German publishing group, The Washington Post or the UK’s ITV and Channel 4) have denied content to their readers who blocked ads encouraging them to stop doing so or to purchase a subscription (Vasagar, 2015). These early strategies provided mixed results. In the case of Springer, rather than paying for a subscription, most users switched off their ad blocker but some of them simply left the site. It is also clear that such strategies are unlikely to work for lesser-known publishers who do not have enough resources to setup and manage a subscription system.

Publishers have also tried to bring adblockers to court. For example, Eyeo, the German owner of the world’s most popular adblocker, Adblock Plus faced (and won) half a dozen lawsuits in the past years. The question even reached the Supreme Court in Germany who ruled ad blocking to be legal.[2] It is clear that neither the courts nor regulators seem to take the side of publishers in their fight with adblockers.

There is no doubt that ad blocking hurts publisher profits and, since it reduces the returns to investing in quality content, *indirectly*, it results in lower content quality (Shiller, Waldfogel, and Ryan, 2018). Putting aside the externalities associated with a general decline in, say news quality, even a narrow focus on consumer welfare should ask: does the reduced intensity of advertising compensate consumers for the decline in content quality? Asked differently, is the current regulatory stance, which unambiguously supports ad blocking in its present form a wise approach to regulation?

The purpose of this paper is to shed light on these questions and examine the welfare implications of the current practice of ad blocking with a special focus on consumer welfare. Given regulators’ and courts’ unambiguously positive attitude towards adblockers, it is important to examine if indeed ad blocking increases or is, at least, neutral to welfare. Specifically, the paper builds an analytic model to explore if and when the *direct* positive effect of ad reduction on consumers outweighs the *indirect* negative effect of lower content quality and how total welfare changes in

[2] See, for example, https://www.reuters.com/article/us-germany-trial-adblocking/german-supreme-court-rules-ad-blockers-legal-in-defeat-for-springer-idUSKBN1HQ277.

the presence of ad blocking. Surprisingly, we find that, under a broad set of market conditions, total consumer surplus and *even total welfare* decline under ad blocking. While some consumers are always better off in the presence of ad blocking, for the *average* consumer, the negative impact of quality decline is larger than the positive effect of ad reduction.[3]

Importantly, beyond welfare implications our analyses also highlight the role of adblockers' unusual revenue model in sustaining the industry ecosystem. Indeed, while adblockers create value for consumers, they almost never charge consumers for their services. No doubt, a reason behind this approach is related to the relative technical difficulty of collecting small fees from a large and fragmented customer base, many of whom are used to and expect free digital products. Instead, adblockers negotiate with publishers for a fraction of their ad revenues in exchange for a limited amount of, presumably higher quality, ads to pass the blockade. This practice, which may affect up to 30% of the publishers' ad revenues is known as (partial) "whitelisting" in the trade (Maheshwari, 2016).[4]

In deciding the right amount of advertising volumes allowed, a key tradeoff for adblockers is how attractive they remain to consumers and how much surplus they can extract from publishers. The lower the ad volume, the more consumers are likely to download the adblocker app, which increases the adblocker's leverage over the publisher. However, a lower advertising volume also means that the potential surplus extracted from a publisher is also lower.[5] Making ad blocking software attractive to consumers is a key consideration. Recent trends of ad blocker usage indicate that the proportion of consumers using them (while different across countries) has stabilized. Importantly, the majority of consumers still do *not* use ad blockers.

Our model takes into account these complex strategic interactions in the ecosystem of publishers, adblockers and consumers. In particular, we pay special attention to consumer heterogeneity

[3]Our analysis is strictly based on economic surplus and does not take into account other negative externalities related to the reduction of content quality (e.g. its consequences for democracy).

[4]The term "whitelisting" is also used in other contexts (e.g. when consumers allow some publishers' advertising to pass the ad blocker). In this paper, we consistently refer to whitelisting as the practice by adblockers to allow a certain amount of qualified advertising from specific publishers to pass the blockade, often in return for a payment.

[5]Some adblockers may also allow consumers to block all ads although this option is discouraged and usually comes with additional costs to consumers. Nevertheless some consumers do opt for this option.

given the non-uniform behavior of consumers in using adblockers and the known differences across consumers in their sensitivity to advertising and content quality. Our analysis considers a single adblocker and one publisher that relies on an advertising revenue model. The publisher makes a costly investment in content quality. Consumers are assumed to be heterogeneous in three dimensions: (i) their sensitivity to quality, (ii) their disutility from advertising and, (iii) their cost of downloading the ad blocker app. We show that all three of these consumer characteristics play a critical role in understanding the complex dynamics of ad blocking. The adblocker sets an upper limit to the ad volume seen by consumers who downloaded its app. Furthermore, we assume that it can also reduce ad volume for these consumers to 0. This threat allows the adblocker to extract surplus from the publisher.

The solution of this simple model is consistent with the anecdotal evidence observed in the industry. In equilibrium, the adblocker does not block all ads and collects a fee from the publisher, which is reminiscent of the practice of 'whitelisting' in the industry. A fraction of consumers - those who are (i) sensitive to advertising, (ii) have high valuation for quality and (iii) have a relatively low cost of downloading - use the adblocker's app. We find that consumer heterogeneity is critical in explaining in which way consumer welfare changes in the presence of ad blocking. In equilibrium, adblocking always increases the surplus of *some* consumers, in fact, some consumers only consume content in the presence of an adblocker (i.e. the adblocker may expand the market). However, unless consumer heterogeneity is very limited *both* in advertising sensitivity and the valuation of quality, overall consumer welfare is lower with ad blocking. In the more realistic case of heterogeneous consumers, the publisher decreases its quality substantially enough to erase the benefit of lower ad exposure for most consumers. As a result, not only is the publisher worse off in the presence of ad blocking but total consumer surplus and overall welfare also decline. If consumers are homogeneous in both advertising disutility and the appreciation of quality, then overall consumer welfare can increase under ad blocking. In this case, without the adblocker, the publisher can extract almost all surplus from consumers. While ad blocking still reduces quality, the effect on consumers is small as they did not have much surplus to begin with. In contrast, advertising reduced by the adblocker provides a substantial benefit.

Overall, our first result suggests that the revenue model where adblockers charge publishers is more akin to the "highway robbery" narrative than the "Robin Hood" image that adblockers promote. It is true that adblockers transfer some surplus from publishers to consumers, they also collect substantial rents in the process. In an ecosystem with a standard publisher-consumer relationship where both sides walk away with healthy surpluses, the entry of an adblocker can be highly detrimental to publishers and, by extension, to consumers. The situation becomes even worse if adblockers provide a “full" ad blocking option to consumers. Hence, we conclude that, in contrast to prevailing regulatory practice, ad blocking is typically not an efficient market solution to regulate the amount of advertising shown on websites.

To further substantiate this conclusion we calculate welfare measures for a hypothetical model where adblockers directly charge consumers for ad blocking in a traditional, “fee for service” revenue model. We find that under realistic market conditions with heterogeneous consumers, this revenue model is better for publishers *as well as* consumers than the current practice of “publisher extorsion”. In the case of a heterogeneous consumer base, more surplus is left with the publisher, which limits the decline of content quality that, in turn also increases average consumer surplus. We conclude that restricting adblockers’ value extraction practices would be a relatively simple way to unambiguously increase publishers’ *and* consumers’ welfare.

In the second part of the paper, we analyze several variations in our setting to explore important institutional details. First, we explore the case when the publisher has more (and varying levels of) power in its negotiation with adblockers. This is a relevant practical consideration given the variety of investments publishers consider to protect themselves against ad blocking.[6] The results show that even at very high levels of publisher power ad blocking usually has a considerable negative effect on content quality and welfare. In fact, surprisingly, we find that if the adblocker could choose its negotiation power at no cost (and commit to it) the adblocker would exercise restraint by renouncing to extracting all potential surplus from the publisher. This suggests that

[6]We have already mentioned vigorous legal action but many publishers have also invested in, so-called “adblocker blockers” or introduced a subscription revenue model (a case we explicitly analyze).

an additional avenue for a more efficient regulation may be to help the coordination of incentives between publishers and adblockers for a better consumer (and general) welfare outcome.

Our final step is to discuss a variety of robustness checks that are based on additional analyses documented in the Web Appendix. Specifically, for the more common case of heterogeneous consumers, we extend the base model to consider multiple publishers with different levels of competition between them. Our main results concerning the practice of whitelisting, publishers' quality choice, consumer surplus and general welfare remain similar, although we also highlight a variety of mechanisms through which an adblocker common to multiple publishers may affect - typically increase - competition between them.

Finally, we look at the case of a transaction platform offering ad blocking on its own. This last setup is relevant for multiple reasons. First, more and more content consumption happens on platforms where consumers and publishers interact (e.g. iOS, Google or Facebook). Second, such platforms have tremendous power in "regulating" these interactions. We show that the platform tends to internalize competition between the adblocker and the publisher leading to better welfare outcomes. However, unless the platform can fully commit to the maximum level of advertising before the publisher sets quality, the platform cannot provide ad blocking services that would improve on our benchmark, that is, the case without any ad blocking.

The rest of the paper is organized as follows. In the next section, we survey the relevant literature. Section 2.2 describes and analyses the main model featuring a single publisher and an adblocker that may offer various levels of ad blocking to consumers. This section also compares adblockers current business practices to a more conventional revenue model. Section 2.3 explores the adblocker-publisher relationship and negotiations in more detail. Section 2.4 discusses extensions to gauge the model's robustness and discusses limitations. The paper ends with concluding remarks. To improve readability, all proofs are relegated to the Appendix and some extensions are detailed in a separate Web Appendix.

2.1 Relevant literature

A fundamental driving force behind ad blocking is that most consumers dislike ads. They try to mitigate the disutility caused by avoiding ads which raises a number of important questions. Anderson and Gans (2011) study the impact of ad avoidance technologies and their effects on content providers. They find that the reaction to ad avoidance leads to increased ad clutter and reduced content quality and decreases welfare. In another paper, Johnson (2013) considers how better targeted ads interact with ad avoidance, showing that consumers may underutilize ad avoidance tools. Eventually, consumers and content providers end up in a game where each player uses some methods to avoid or push through ads (Vratonjic et al., 2013).

On the empirical side, Goldstein et al. (2014) examine the costs of placing ads that are annoying. They find that such ads can cause users to abandon the website and negatively effect the process of consuming the site's content, potentially costing more than the revenue from the ads. Shiller, Waldfogel, and Ryan (2018) make the first attempt at thoroughly documenting the impact of ad blocking on online publishers. Using web traffic and ranking data for a broad set of sites, they show that, as larger proportion of consumers use ad blockers, publisher sites decline in their rankings (e.g. their rank becomes larger) and this effect is larger for worse-ranked sites. They conclude that "ad blocking poses a substantial threat to the ad-supported web". Our theoretical results are largely consistent with this general empirical finding.

More recent theoretical work in marketing challenges this general view. In particular, in a model with exogenous ad blocking Despotakis, Ravi, and Srinivasan (2020) show that under some conditions, ad blocking can benefit competing publishing platforms as they allow them to discriminate between consumers who have high and low ad sensitivity respectively.[7] They also find that under some conditions, content quality and overall welfare can also increase. Our model is fundamentally different from their setup in that it assumes the adblocker to be an active, profit maximizing agent. Our results attenuate some of the results by Despotakis, Ravi, and Srinivasan

[7] See also Chen and Liu (2021) on how ad blocking may help publishers in a context where advertising serves as a signaling device.

(2020) and reverse others. On the one hand, we do find that some consumers benefit from the presence of adblockers and, for fixed quality, the average consumer is better off. On the other hand, we show that quality is lower in the presence of an active adblocker and this leads to lower consumer, and overall welfare in equilibrium. We, therefore, argue that it is critical to model adblockers as active agents.

On the conceptual front, our paper relates to several streams of literature. The base model of publishers providing content to consumers in return for showing advertisements is reminiscent of typical two-sided markets (Rochet and Tirole, 2006), although for simplicity, we assume exogenous advertising prices. We need this simplification because with an adblocker, the model extends to a three-sided platform. On the consumer side, we assume a standard utility model with heterogeneity at multiple levels. In particular, in terms of consumers' valuation for content quality, we adopt the well-accepted approach of the vertical differentiation literature (see, Shaked and Sutton, 1982b and Moorthy, 1988). Furthermore, the ad blocking ecosystem exhibits characteristics of double marginalization well-documented in the literature (Spengler, 1950; Bresnahan and Reiss, 1985).

At the high level, ad blocking shows similarities with online music and software piracy as a third party (in our case the adblocker) allows users to consume content without proper compensation (advertising eyeballs in case of ad blocking). Piracy has been extensively studied in the marketing and economics literature. Prominent work includes Jain (2008) who investigates digital piracy from a competitive perspective, showing that weaker copyright protection may be beneficial to firms selling content, because it softens price competition by allowing price sensitive consumers to pirate the content. Danaher et al. (2010) show how reducing the availability of content on paid channels increases the demand for the pirated version of the same content, whereas Vernik, Purohit, and Desai (2011) look at Digital Rights Management, finding that piracy might decrease even if usage restrictions are removed from digital products.

A distinct, but fairly small body of literature shows interesting resemblances to our research questions. As adblockers essentially hold content publishers "hostage", there is a clear connection to the economic analysis of extortion. Several important details of ad blocking relate to the mechanics underlying extortion. For example, Konrad and Skaperdas (1997) examine the credibility of threats related to extortion. They find that an equilibrium with extortion only exists if there is a large enough number of victims. Choi and Thum (2004) investigate the dynamics of repeated extortion. In their setting, government officials demand bribes in a repeated fashion which shows similarities to ad blocking and they also find that a pure strategy equilibrium does not exist. In a fascinating setting, Olken and Barron (2009) study extortion in the field by analyzing the bribes demanded from truckers by corrupt officials. The findings demonstrate how multiple layers of extortion interact with higher amounts of bribes closer to the end of the journey. Overall, our contribution lies in uncovering the dynamics of the strategic interaction between adblockers, content publishers and consumers. Besides shedding light on some new industry practices, the paper examines the impact of ad blocking on content quality, consumer surplus and overall welfare.

2.2 Base model

Our base model has three types of active players: (i) a publisher, (ii) consumers and, (iii) an adblocker. The price of advertising (impression) is assumed to be exogenous and constant, denoted p_a.[8] The publisher chooses the quality level of its content q at cost $c(q) = cq^2$ and collects revenue through advertising by choosing the level of advertising volume, A.[9]

The adblocker offers a tool (e.g. an app or a browser plug-in) to consumers free of charge that limits the advertising on the publisher's site. We denote the amount of advertising the adblocker lets through by $\mathcal{V}$. We assume that consumers incur cost κ when downloading (using) the ad blocker app, which varies across consumers. The adblocker commits to a maximum advertising level V, such that $\mathcal{V} \leq V$. Setting a lower V results in more consumers downloading the app. For the consumers who download the app, the adblocker can also decide to block all ads by setting

[8]This is a reasonable assumption given the low proportion of people who use ad blockers and the vast inventory of online advertising available. Also, in the quasi-endogenous case (explored in the Web Appendix) when p_a is a function of the publisher's content quality, we find that our key findings remain similar.

[9]In the Web Appendix, we also analyze the case when the publisher directly charges consumers in a subscription revenue model.

$\mathcal{V} = 0$ if it wants to. The threat of committing to $\mathcal{V} = 0$ allows the adblocker to extract surplus from the publisher. The basic trade-off faced by the adblocker is to commit to a low enough V such that many consumers download the app, but not too low so that it can still extract sufficient surplus from the publisher. It is important to highlight that the setup described above is usually the default setting of large adblockers (e.g. the dominant adblocker, Adblock Plus). However, in practice, adblockers also provide consumers with the option to fully block all advertising at some additional (hassle) costs. In Section 2.2.3 further below, we analyze this practically relevant modification and show that, beyond reinforcing our main results it also provides key insights w.r.t. the dynamics of potential competition *between* adblockers.

A unit mass of consumers are distributed in a 3-dimensional cuboid with their total number normalized to 1.[10] Consumers get utility from visiting the publisher's site in the amount of

$$U = \max(\vartheta q - \gamma A, \vartheta q - \gamma \mathcal{V} - \kappa, 0),$$

where $\vartheta \sim U(t, 1)$ is consumers' sensitivity to quality, $\gamma \sim U(r, 1)$ measures their sensitivity to advertising and $\kappa \sim U(0, \overline{\kappa})$ is consumers' cost for downloading the ad blocker app. We assume that ϑ, γ and κ are independently distributed from each other. We also assume that $0 \leq t \leq 1$, $0 < r \leq 1$ and that $\overline{\kappa}$ is large enough.[11] The restriction on $\overline{\kappa}$ is similar to assuming that – in line with practice – there is a sufficiently high proportion of consumers who never download the ad blocker app.

The publisher's profit depends on whether consumers choose to visit its site and how much advertising they see. The publisher's profit is

$$D_A A p_a + D_{\mathcal{V}} \mathcal{V} p_a - cq^2 - T,$$

[10] We find that all three dimensions of consumer heterogeneity are necessary to explain the relevant dynamics of ad blocking. For additional clarity, the Web Appendix presents the analyses of cases when consumer heterogeneity is only along two dimensions.

[11] Setting r strictly above 0 is a purely technical assumption to avoid the case where advertising can be set to infinity for $r = 0$ consumers. r and t allow us to gauge the impact of the *level* of consumer heterogeneity on ad blocking.

where D_A is the amount of consumers who read without using the ad blocker and $D_{\mathcal{V}}$ is the amount that reads with the ad blocker installed. T denotes a payment the publisher makes to the adblocker. This payment represents the rent extracted by the adblocker from the publisher as the adblocker can threaten to lower the ads to zero for consumers who download the app. Since the payment may be decided through negotiations, the details of the negotiation process have a high impact on its outcome. In the basic model, we assume that the adblocker moves first and makes a take-it-or-leave-it offer for T combined with a promise of a low $\mathcal{V}$. We assume that the publisher always accepts when indifferent. We will see that in this setup, the adblocker extracts all advertising revenue generated from consumers who downloaded the app. In Section 2.3 below, we explore the case when the adblocker cannot extract all surplus from the publisher (for example, when the publisher has substantial negotiation power or when it can introduce a subscription model).

The timing of the game is the following: First, the publisher chooses the level of quality q. Then the adblocker sets V, the maximum advertising level when using the ad blocker. This timing reflects the fact that investing in quality (e.g. a brand or a large editorial staff) is a much larger commitment than setting the level of advertising, V. Given V, consumers decide whether to download the ad blocker app. Next, the adblocker makes a take-it-or-leave-it offer and the publisher accepts or rejects. Then the adblocker can set $\mathcal{V} \leq V$ and finally, the publisher decides on the volume of advertising, A. Given $\mathcal{V}$ and A, consumers decide on content consumption and firms' profits are realized.

2.2.1 Benchmark - No Adblocker

We first examine the case without the presence of an adblocker as a benchmark. For simplicity, we present a brief analysis for $t = 0$ and relegate $0 < t \leq 1$ to the Appendix. Consumers visit the publisher's website when $\vartheta q - \gamma A \geq 0$. Without the adblocker, κ is irrelevant, hence we can derive the consumer demand by considering only ϑ and γ. Figure 2.1 illustrates consumer choice by representing zero utility lines on the (γ, ϑ) graph. Consumer demand is the area above the line corresponding to a particular scenario, i.e. $A \leq q$ or $q \leq A$. There are three cases depending on the values of q and A.

Case 1: $A \leq q$. Demand equals the area of trapezoid (abdc) multuplied with density $\frac{1}{1-r}$:

$$D(q, A) = \frac{(1-r)\left(1 - \frac{A}{q} + 1 - \frac{A}{q} r\right)}{2(1-r)} = 1 - \frac{A}{2q}(1+r).$$

Case 2: $rA \leq q \leq A$. Demand equals the area of triangle (def) multuplied with density $\frac{1}{1-r}$:

$$D(q, A) = \frac{\left(1 - \frac{A}{q} r\right)\left(\frac{q}{A} - r\right)}{2(1-r)} = \frac{(q - Ar)^2}{2Aq(1-r)}.$$

Case 3: $q < rA$. In this case, there are no consumers visiting the publisher, hence $D(q, A) = 0$.

Summarizing the three cases, consumer demand faced by the publisher is:

$$D(q, A) = \begin{cases} 1 - \frac{A}{2q}(1+r) & \text{if } A \leq q, \\ \frac{(q-Ar)^2}{2Aq(1-r)} & \text{if } rA \leq q \leq A, \\ 0 & \text{if } q < rA. \end{cases} \tag{2.1}$$

Given this demand, for a given quality the publisher maximizes the following objective function

$$\max_A p_a A D(q, A) - cq^2,$$

where p_a is the price of an ad impression. We consider the two relevant cases with positive revenue:

Case 1: $A \leq q$. The publisher maximizes

$$p_a A \left(1 - \frac{A}{2q}(1+r)\right) - cq^2 \Rightarrow \max_A \quad s.t. \ A \leq q.$$

This is a concave second degree equation in A. The first order condition provides the following solution for A as $A^\star = \frac{q}{1+r}$ (the constraint is always satisfied).

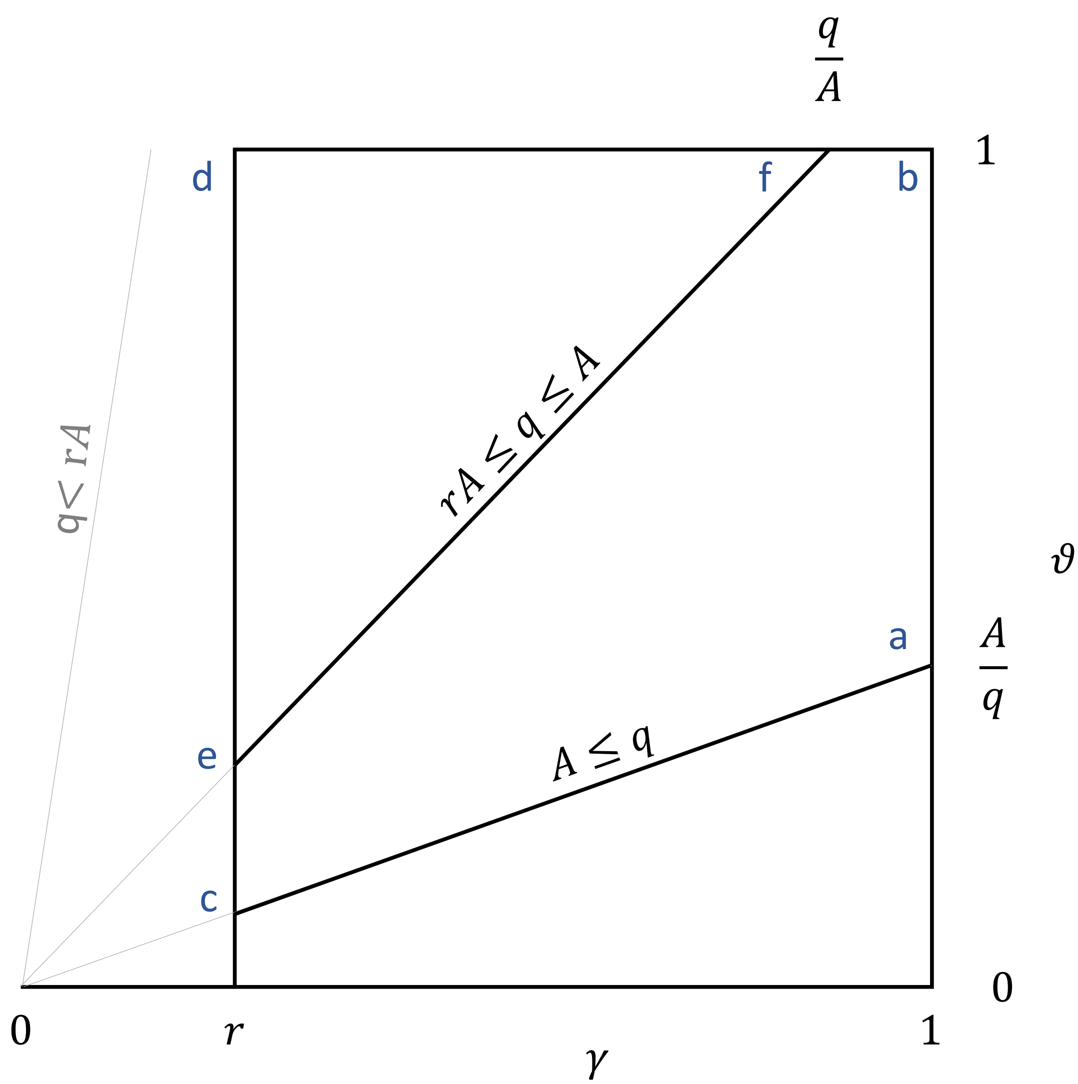

Figure 2.1: Consumer demand as a function of ϑ and γ for different values of A and q with $r > 0$ and $t = 0$. Demand is the area below the $\gamma = \frac{q}{A}\vartheta$ line.

Case 2: $rA \leq q < A$. Here, the publisher maximizes

$$p_a A \frac{(q - Ar)^2}{2Aq(1-r)} - cq^2 \Rightarrow \max_A \quad s.t.\ q < A \leq \frac{q}{r}.$$

The derivative of the objective function is always negative and implies that the optimal choice of advertising is $A_2^\star = q$ and, hence, Case 2 is never relevant. Conducting the same analysis for $0 < t \leq 1$ reveals the general solution for the optimal advertising level as:

$$A^\star = \begin{cases} \frac{q}{(1+r)} & \text{if } 0 \leq t \leq \frac{r}{1+r} \\ q(1 - r(1-t)) & \text{if } \frac{r}{1+r} \leq t \leq 1. \end{cases} \tag{2.2}$$

Solving for the optimal quality provides the equilibrium that we summarize below.

Lemma 1 *Without the presence of the adblocker, the publisher chooses the following quality and level of advertising:*

$$q^\star = q^\star_{NA} = \begin{cases} \dfrac{p_a}{4c(1+r)(1-t)} \\ \dfrac{p_a(1 - r(1-t) + t)}{4c} \end{cases} \quad \text{and} \quad A^\star = \begin{cases} \dfrac{q^\star}{(1+r)} & \textit{if } 0 \leq t \leq \frac{r}{1+r} \\ q^\star(1 - r(1-t)) & \textit{if } \frac{r}{1+r} \leq t \leq 1. \end{cases}$$

Both quality and advertising decrease in r and c and increase in t and p_a.

As expected, a higher cost reduces the quality investment, whereas a higher price at which ads can be sold increases it. Consumer preferences also have an impact. Equilibrium quality is higher when consumers value quality *more* or are *less* sensitive to advertising.

2.2.2 Analysis with Adblocker

In the presence of the adblocker there are three new stages added to the game. The first is when the adblocker sets the maximum advertising it lets through, V. The second is when consumers make a decision to download the ad blocker at cost κ and finally when the adblocker and the publisher negotiate.

Solving the game entails backwards induction. Details of the proofs are relegated to the Appendix; here we walk through the main steps and illustrate the intuition for $t = 0$. The last stage of setting the advertising is similar to the benchmark case and, given our assumption that $\overline{\kappa}$ is large enough we get that $A^* = \frac{q}{1+r}$ when t is small. The stage before that is the negotiation phase. The adblocker's main leverage here is the set of consumers who downloaded its app. The app limits advertising to a level of V for these consumers, but the adblocker can reduce that to 0. Note that a rejected offer by the publisher makes the adblocker indifferent in setting $\mathcal{V}$ anywhere within the $[0, V]$ interval. This serves as a threat and allows the adblocker to extract some of the publisher's revenue. Given the base model's assumption that the adblocker starts the negotiation with a take-it-or-leave-it offer and the publisher accepts if indifferent, the adblocker is able to extort a payment T equal to the entire advertising revenue the publisher generates from consumers who downloaded the ad blocker by threatening to block all ads. Hence the size of the segment who downloads the ad blocker becomes crucial and therein lies the key tradeoff: as the adblocker reduces V more consumers download the ad blocker, but at the same time its revenue, T decreases as the amount of ads V decreases.

It is important to realize that consumers are rational and they expect a deal between the adblocker and the publisher. Hence when they contemplate downloading the app, they know that it does not provide full ad blocking, rather it limits advertising to V.[12] Figure 2.2 illustrates the download decision for consumers with a given κ cost when $t = 0$. There are two constraints of interest. The vertical line $\gamma = \frac{\kappa(1+r)}{q-V(1+r)}$ represents the consumers who are indifferent between visiting the publisher's site while being exposed to advertising A^* or downloading the ad blocker, and being exposed to only V level of advertising. Hence consumers to the right of this line are potential downloaders, whereas consumers to the left never download. But we also have to make sure that consumers who download the ad blocker get positive utility (i.e. they want to visit the

[12]In its default setting, Adblock Plus, the largest adblocker, explicitly tells consumers that some, so-called "Acceptable Ads" will pass the blockade. With a few more clicks, consumers can also opt to block all ads, a case we will explicitly model further below.

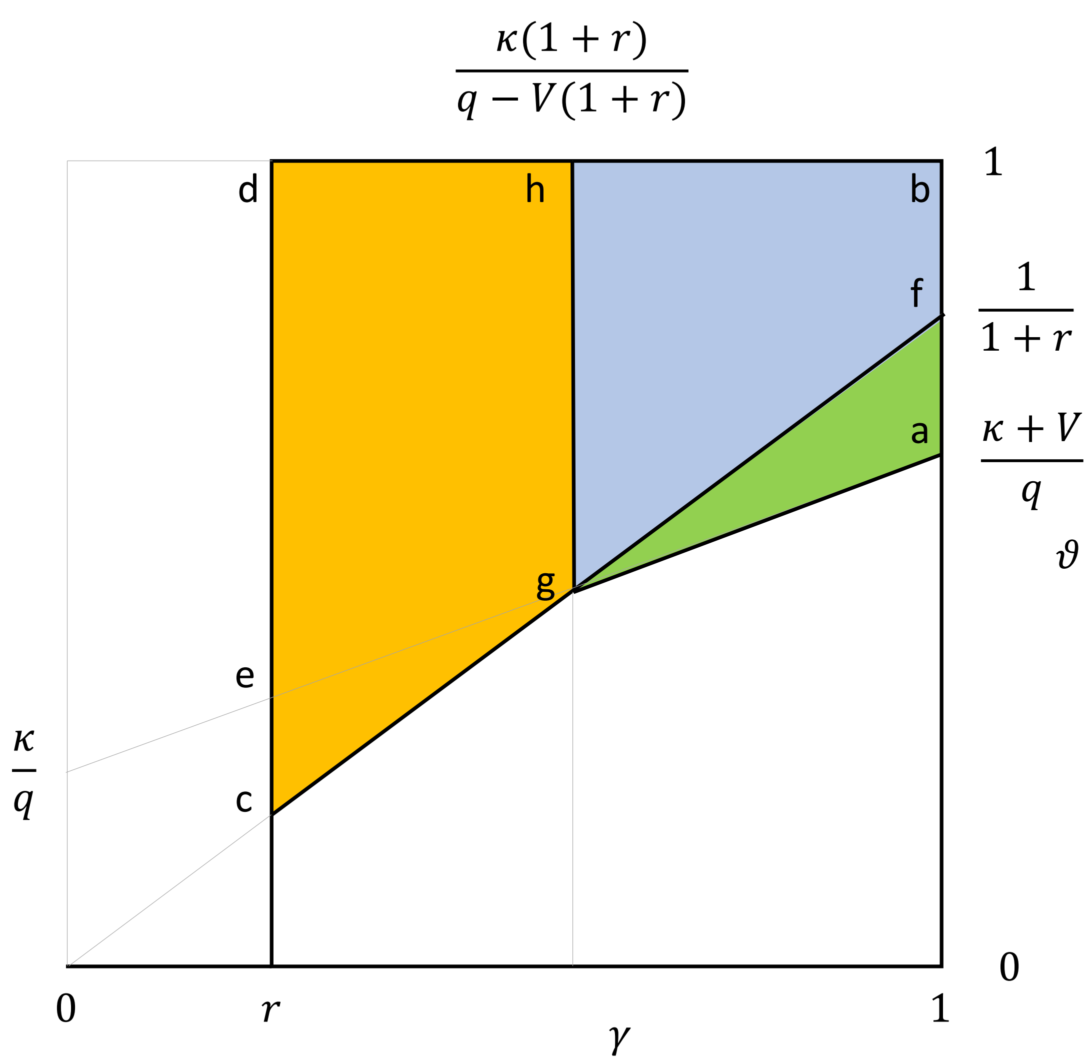

Figure 2.2: Download decision of consumers with cost κ in the (γ, ϑ) space when the adblocker sets maximum ad volume to V assuming $t = 0$.

site with V level of advertising). The indifference curve is given by the $\vartheta = \frac{\kappa+\gamma V}{q}$ line. Consumers below this line never download, whereas consumers above may. Hence exactly consumers in the (abhg) trapezoid download (and use) the ad blocker. As the (cg) line represent the threshold for consumers to visit the publisher's site in the absence of an adblocker, it is clear that consumers in the (cdhg) trapezoid continue to visit the site without the ad blocker. It is interesting to see that there is a segment of consumers (afg) who do not visit the site without the ad blocker but do visit with it.

Figure 2.2 illustrates the tradeoff clearly. As V decreases the (abhg) trapezoid grows, but the amount of rent to be extracted decreases, as it is exactly V for each consumer in this segment. The adblocker hence needs to find the optimal V that balances these two forces. The Figure shows our reasoning for a given κ, but to find the optimal V we need to integrate over all possible values of κ and solve

$$\max_V T(q,V) = \max_V V \cdot \int_0^{\overline{\kappa}} S_V(\kappa, q, V) d\kappa,$$

where $S_V(\kappa, q, V)$ is the area of the segment that downloads the ad blocker. Deriving the integral in the proof yields the following proposition.

Proposition 1 *For given quality, $V^\star = qF(r,t) \leq A^\star/2$, where $F()$ is defined in the Appendix. $F(r,t)$ is decreasing in r and increasing in t. $F(r,0) \xrightarrow{r \to 0} 1 - \frac{1}{\sqrt{3}}$ and $F(r,1) = \frac{1}{2}$ for any $r > 0$.*

This result captures the key idea behind how the ad blocking ecosystem works. The adblocker makes money by posing a threat to the publisher: 'pay us or we will completely block advertising'. Since adblockers typically do not charge consumers, this is their main source of revenue. In order for the threat to be real, the adblocker needs downloads. It entices consumers to download the app by filtering out at least some ads, that is limiting the total amount of disutility from advertising. The consumer segment that downloads consists of people who tend to be sensitive to advertising, appreciate quality, and have a low downloading cost.

It is important to point out that in equilibrium, the adblocker does not completely block ads. This is consistent with the practice of "whitelisting" that we observe in reality. Most adblockers let 'good' ads through and only block the intrusive ones, essentially limiting the amount of advertising. As our model demonstrates whitelisting is a central element of the ad blocking business model, in which the adblocker extracts rent from publishers.

Despite these whitelisting practices the adblocker still generates value for consumers. The maximum advertising is always less than half of what consumers without an adblocker encounter. The proposition also shows that consumer heterogeneity matters. The less heterogeneity there is on sensitivity to ads (increasing r), the lower $V^\star/A^\star$ will fall. On the contrary, if consumers are less heterogeneous with respect to their valuation of quality (increasing t), the more ads they will see. At the extreme, when all consumers have the same valuation for quality, exactly half of the ads are blocked.[13]

Overall, if publisher *quality is fixed*, the presence of the adblocker clearly benefits consumers.[14] Not only by making it less painful for existing visitors to consume the publisher's content, but by making it worthwhile for new visitors to come to the publisher's site. These latter are consumers who would not visit the site without the adblocker.

This brings us to the very first stage of the game. Up until now, we have treated the publisher's quality as given. However, many argue (Shiller, Waldfogel, and Ryan, 2018) that adblockers endanger the entire advertising supported publishing business model. If the adblocker can extort publishers and extract advertising revenues, the publisher will have no incentive to invest in content quality, which may ultimately hurt consumers. Our next result speaks to this issue.

Proposition 2 *There exists $G(r,t) > 0$, such that the optimal quality is*

$$q^\star = q^\star_{AB} = \begin{cases} \left(\dfrac{4c(1+r)(1-t)}{p_a} + \dfrac{G(r,t)}{\overline{\kappa}}\right)^{-1} & \text{if } 0 \le t \le \frac{r}{1+r} \\ \left(\dfrac{4c}{p_a(1-r(1-t)+t)} + \dfrac{G(r,t)}{\overline{\kappa}}\right)^{-1} & \text{if } \frac{r}{1+r} \le t \le 1. \end{cases}$$

We have $q^\star_{AB} < q^\star_{NA}$ and $q^\star_{AB}$ is increasing in $\overline{\kappa}$ and converges to $q^\star_{NA}$ as $\overline{\kappa} \to \infty$. The publisher's profit is also lower than in the benchmark case and increases with $\overline{\kappa}$.

The results confirm our intuition that quality decreases in the presence of the adblocker. This is driven by the reduction in advertising revenue the publisher can keep. At the extreme, if $\overline{\kappa}$ were 0, i.e. all consumers could download the ad blocker without any nuisance cost, the publisher would lose all incentive to invest in quality and the market would collapse. What protects the publisher's revenues is that downloading the ad blocker is costly. The more consumers with high download

[13] It is worth noting that if we change r and t, the mean sensitivity levels also change, but $V^\star/A^\star$ is invariant to the scaling of the means.

[14] Note that advertising levels do not change with the entry of the adblocker and this is driven by a large enough mass of consumers with high download costs. In the Web Appendix, we analyze a setting where all consumers have the same moderate level of download cost and we find that advertising levels may slightly increase with the entry of an adblocker.

costs, the less the publisher has to worry. To better illustrate how the download decision is made by different consumers, we next examine the size of the segment that downloads.

Corollary 1 *The size of the consumer segment who downloads the ad blocking software is increasing in the ad price, p_a and decreasing in the download cost $\overline{\kappa}$, converging to 0 as $\overline{\kappa} \to \infty$.*

The corollary is in line with some of our main intuition. A higher ad price leads to a higher quality level, which increases advertising levels. However, the gap between the advertising levels with and without the adblocker also increases. As a result, the benefit from downloading the ad blocker goes up with quality if the download costs are fixed. Conversely when download costs increase, fewer consumers download the ad blocker. This protects the publisher from the ad blocker's extortion threat yielding higher quality.

Clearly, the adblocker has an unambiguous negative effect on the publisher by extracting rent through the downloader segment. The effect on consumers is less clear. On one hand, content quality becomes lower in the presence of the adblocker, especially when the download costs are not very high. However, the consumers who downloaded the ad blocker see significantly less advertising. In fact, we can show that there are segments of consumers whose net benefit from the presence of the adblocker is positive.

Corollary 2 *Unless $r = t = 1$, there are consumers who are strictly worse off in the presence of an adblocker than in the benchmark case with no adblocker. There always are consumers who are better off. In particular, there is a segment of consumers who only visit the publisher's site and derive positive utility in the presence of the adblocker.*

This latter segment consists of consumers who are sensitive to advertising, but value quality moderately. They only visit the publisher's site if the adblocker is available. Importantly, this segment would *not* consume any content and therefore would obtain zero utility without the adblocker. In the presence of the adblocker they get positive utility even though quality decreases. Note that this particular segment is only part of the consumers who are better off with an adblocker. Consumers with very low download cost and high ad sensitivity are also better off with the adblocker, especially if most other consumers have high download costs and, thus, quality levels are not reduced.

Overall, adblocking has two opposing effects on consumers. The direct effect increases consumer utility by removing some ads and the associated disutility. The indirect, quality reduction effect reduces the utility consumers obtain from visiting the site which is proportional to the content quality. Depending on the heterogeneity in the consumer population either effect can dominate and total consumer surplus may increase or decrease. The following proposition compares total consumer surplus in the benchmark case with no adblocking (CS_{NA}) with that in the presence of an adblocker (CS_{AB}).

Proposition 3 *If either r or t is sufficiently low, then $CS_{AB} < CS_{NA}$. If both r and t are sufficiently high, then $CS_{AB} > CS_{NA}$ as long as κ is high enough.*

The main determinant of whether total consumer surplus declines or increases with the entry of the adblocker is the extent of consumer heterogeneity. The reason is that the impact of the quality reduction effect on consumers is determined by two factors: the extent of the quality reduction and the surplus consumers can extract relative to quality. The latter is strongly dependent on consumer heterogeneity. Consider the case when both r and t are close to 1. In this case the publisher is able to extract all surplus from consumers by setting a high advertising level. Hence in the case of no adblocking consumers walk away with almost no surplus. The entry of the adblocker changes the equation, because those with low download cost benefit and end up with a positive surplus. While the adblocker's entry does reduce quality levels, this does not hurt consumers as their surplus was close to zero to start with. Hence total consumer surplus increases.[15]

On the other hand, when consumer heterogeneity is sufficiently high (either r or t is low enough), consumers enjoy a good positive surplus in the absense of the adblocker. With its entry, consumers are hurt, because the extent to which the adblocker's rent seeking behavior reduces quality investment and content production counters its beneficial impact on advertising disutility.

[15] We would like to thank one of the AEs for drawing our attention to this case.

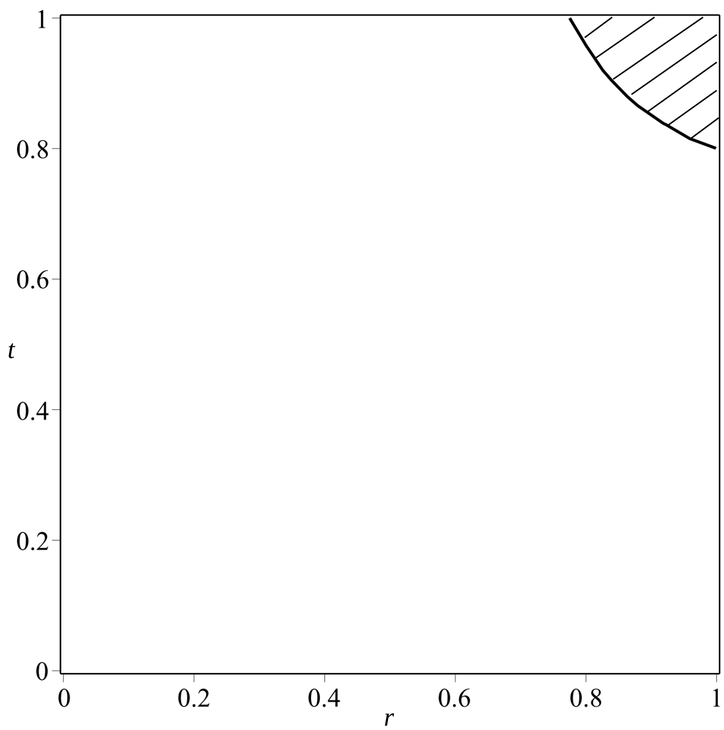

Figure 2.3: The shaded area represents the parameter values in the (r, t) space where total consumer surplus is higher with the adblocker than in the benchmark case as long as κ is high enough.

In our setting, consumer heterogeneity has to be fairly limited for the positive effects to dominate. Figure 2.3 depicts the region in the (r, t) parameter space where consumer surplus is higher than without an adblocker. The thresholds are close to 1. When $r = 1$, total consumer surplus increases if and only if $t > 4/5$ as long as κ is high enough. When $t = 1$, the threshold for r is $r > (\sqrt{141} - 1)/14$.

We next investigate how the total welfare of consumers, the publisher and the adblocker, (TW_{AB}) compares to the total welfare in the benchmark, where TW_{NA} is the sum of consumer and publisher surplus.

Corollary 3 *We have $TW_{AB} < TW_{NA}$, unless the conditions for $CS_{AB} > CS_{NA}$ hold and both p_a and c are sufficiently low.*

The results show that total welfare can only increase if consumer surplus increases. The reason is that the entry of the adblocker lowers the value that the publisher generates. Since $q^{AB} < q^{NA}$, the total surplus divided between the adblocker and the publisher is lower than that of the publisher in the benchmark case. Only consumer surplus can flip the inequality. When consumer surplus is higher with the adblocker than in the benchmark case, total welfare can also go up. But only if the increase in consumer surplus is high enough compared to the decrease due to lower quality. For the latter reduction to be low, both p_a and c have to be low. p_a has to be low so that the rate at which advertising space is converted into revenue is low relative to the consumer surplus. And c has to be low so that quality stays generally high enough for the consumer surplus increase to be substantial. Overall, we see that total welfare decreases with the entry of the adblocker unless a number of (quite demanding) conditions align.

2.2.3 Full Ad Blocking as an Option

In this section, we examine the case where, besides opting for partial ad blocking, consumers also have the option to use full ad blocking. This is a relevant case in practice, because in mainstream ad blocking software, while the default option is partial ad blocking, a full ad blocking option is also available if the consumer is willing to incur some extra cost.[16]

To study this scenario, we modify our basic model as follows. When consumers make the download decision, they now have three options: i) not downloading the ad blocker ii) downloading the ad blocker with some ads still displayed (V) at cost κ as before and iii), downloading the ad blocker and using it with full ad blocking at cost $\kappa(1 + \mu)$, where $\mu \geq 0$ is a fixed parameter. For simplicity, we also set $t = 0$. Note that this is a conservative assumption as total consumer surplus is lower with the adblocker for this parameter range. We investigate whether adding the full adblocking option changes that. All other aspects of the model are the same as in our basic model.

The main difference in the analysis surfaces when we examine the decision of the adblocker to set V. As before, V has to be low enough to entice consumers to download the adblocker, but at the same time high enough to create advertising revenue for extortion potential. The novel force

[16]Beyond the somewhat trivial cost of a few additional clicks, many consumers experience a psychological cost for hurting publishers by blocking acceptable ads.

in our modified model is that there is a full ad blocking option which, even though at a higher cost, presents a potentially more attractive option for consumers. Moreover, those who choose the full ad blocking option will not contribute to the advertising revenue and thus will not increase the adblocker's potential revenue source. Our first result concerns the new equilibrium V:

Lemma 2 *For given quality q, we have $V^\star = qF_\mu(r)$, where $F_\mu(.)$ is defined in the Appendix. $F_\mu(r) < F(r,0)$ and $F_\mu(r)$ is decreasing in r and increasing in μ. Furthermore,*

$$F_0(r) = 0, \;\; F_\mu(r) \xrightarrow{\mu\to\infty} F(r,0) \text{ for } \forall r < 1; \;\; F_\mu(r) \xrightarrow{r\to 0} \frac{\mu}{1+\mu}\left(1-\sqrt{1/3}\right), \;\; \text{and} \;\; F_\mu(1) = 0.$$

The results are similar to those we obtain from our main model in Proposition 1, but there are a number of notable differences. V^* is still proportional to quality, q but the ratio V^*/q is smaller. Furthermore, the lower the extra cost of using the ad blocker to completely dispose of ads, μ the lower the multiplier: as it becomes easier for users to get rid of all ads, the more the adblocker is forced to let through only a small amount of ads. At the other extreme, when the extra cost of full ad blocking approaches infinity, we get a V that is identical to our basic results as long as $r < 1$. Figure 2.4 illustrates the consumer choice for a given κ. As before, consumers with low ad sensitivity (cdhg trapezoid) do not download the ad block app. The difference is that those who download it are split into two segments. Moderately ad sensitive consumers in the (ghik) trapezoid only incur the κ cost and download it for partial ad blocking. Highly ad sensitive consumers in the (kibj) rectangle incur the extra cost and use it for full ad blocking. This latter segment is the one that forces the adblocker to lower V further than before, especially if μ is small. Interestingly, when r approaches 1 the adblocker finds itself in a tough spot as it does not have enough heterogeneity in ad sensitivity to differentiate the partial ad blocking option from the full ad blocking option. Therefore, it is forced to set a low V reaching $V = 0$ as r approaches 1 (i.e. when there is no consumer heterogeneity in advertising disutility). In sum, the extortion-based business model is no longer feasible without heterogeneity in ad sensitivity and the adblocker faces similar problems with extracting revenue as in Section 2.2.4 even if μ is high. Overall, we see that the possibility of

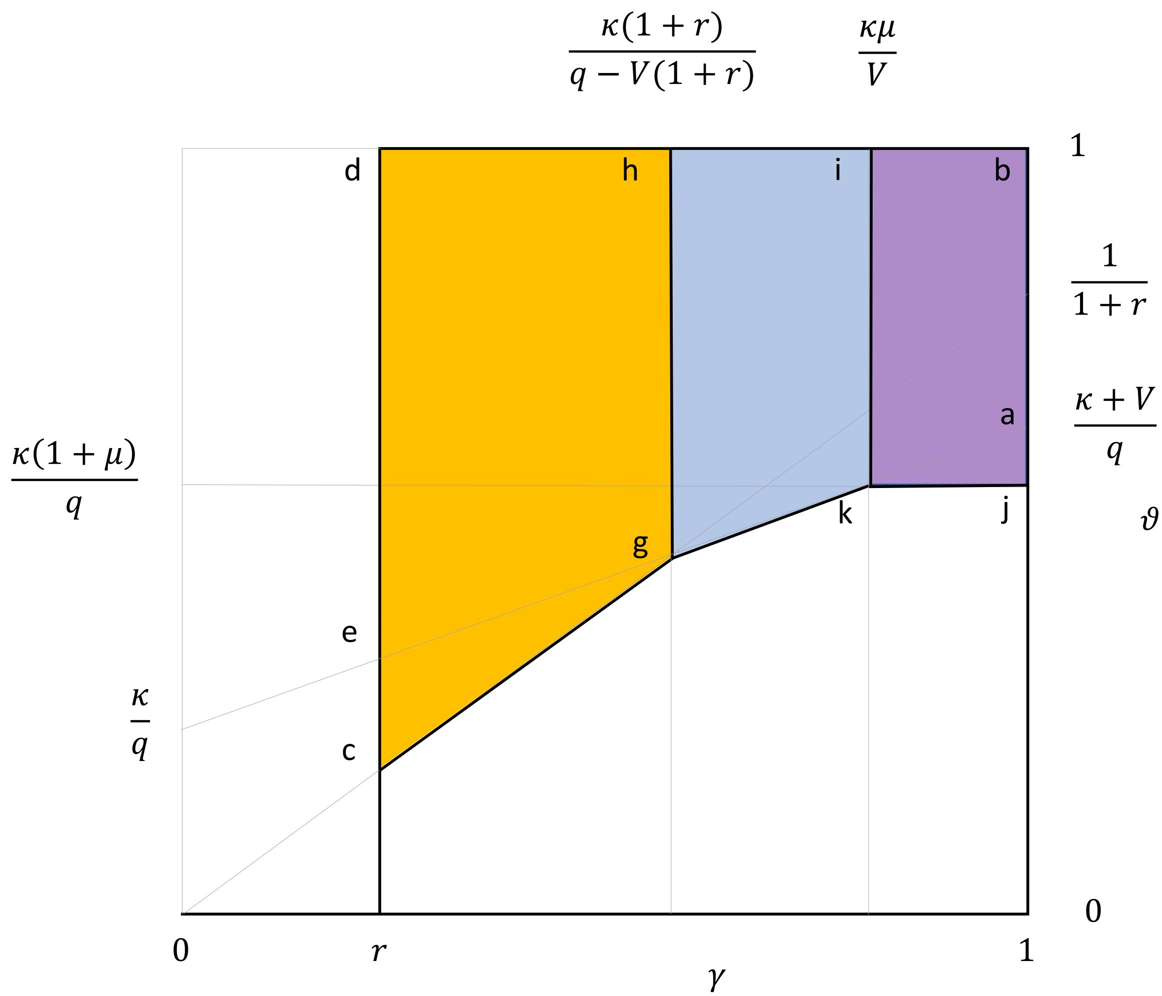

Figure 2.4: Download decision of consumers in the (γ, ϑ) space when the adblocker sets maximum ad volume to V. The cost of downloading and using the adblocker with a V level of advertising is κ, while using it for full blocking costs an additional $\mu\kappa$.

full ad blocking reduces the advertising level that the adblocker allows through. We next examine how the publisher's quality choice is affected by μ.

Proposition 4 *The optimal quality is* $q^\star_\mu = p_a\left(4c(1+r) + \frac{2p_a(r^2+4r+1)(1-(1+r)F_\mu(r))}{3\overline{\kappa}(1+r)^2}\right)^{-1}$. *Quality* $q^\star_\mu$ *is increasing in* μ *and* $q^\star_\mu \xrightarrow{\mu\to\infty} q^\star$. *As before,* $q^\star_\mu$ *is decreasing in* r *and increasing in* $\overline{\kappa}$ *and converges to the benchmark level as* $\overline{\kappa} \to \infty$.

Again, the results are similar to those in our main model, but the more stringent blocking of ads has a negative effect on the publisher. With the lower ad levels, the ad blocker is more attractive and more consumers download it, leaving the publisher with lower ad revenues for a given quality level. In turn, the incentives to invest in quality are further diminished resulting in a lower equilibrium quality. The lower the cost of full ad blocking the lower the quality.

Corollary 4 *For $t = 0$, total consumer surplus and total welfare is always lower with the full ad blocking option than in the benchmark with no adblocker for any μ value.*

Even though some of the very ad sensitive consumers benefit from full adblocking, overall consumer surplus is lower than without an adblocker for $t = 0$ just as in Proposition 3. No matter how easy it is to use the adblocker to fully block ads (even as μ becomes 0), consumers overall will always be worse off than without an adblocker. The reduction in quality is just too severe and the gains from blocked ads cannot counteract the disutility resulting from lower quality. Although we restricted this section to $t = 0$ for parsimony, it is instructive to check what happens when $t = 1$. Analyzing this case shows that a lower μ does, in fact, improve overall consumer surplus. There is a critical r above which consumer surplus is higher than in the benchmark case. This result parallels our previous finding that in the absence of consumer heterogeneity, ad blocking increases consumer surplus and overall welfare. In Proposition 3, $r > (\sqrt{141} - 1)/14 \approx 0.78$ ensured that consumers are better off. Here, as μ approaches 0, the threshold changes to $r > (\sqrt{33}-1)/8 \approx 0.59$. Overall, in this case, a lower μ does benefit consumers, but there is a wide range of r, t parameter values for which consumers are worse off in the presence of an adblocker even if $\mu = 0$.

2.2.4 Evaluating the Ad Blocking Business Model

The original benchmark we established was a world without ad blocking. We showed how the entry of an adblocker reduces quality and generally decreases consumer surplus and welfare in the practically relevant scenario when consumers exhibit some level of heterogeneity. The main reason is that the adblocker extorts "too much" surplus from the publisher, thereby strongly disincentivizing content quality investment. Consumers do not benefit much from the reduced advertising as the adblocker lets through a fair amount of ads to be able to put pressure on the publisher by wielding the threat of completely blocking ads. This equilibrium allows for the market to maintain a certain level of content and advertising, but reduces overall welfare and consumer surplus. The situation is even worse if adblockers also offer a full-ad blocking option as is often the case in practice.

There is an important regulatory debate whether this type of business model, often labeled "extortion" should be allowed or not. In some European countries these questions reached the supreme courts and in most cases the practice has been deemed legal.[17] Our model allows to speak to this issue, by comparing the practice to a world where ad blocking exists, but the adblocker is not allowed to extract a fee from the publisher who is hurt by ad blocking in the first place. The logical and more conventional alternative revenue model consists of the adblocker charging consumers, for whom ad blocking creates value. We have mentioned before that adblockers may shy away from charging consumers for practical reasons (i.e. the difficulty of collecting revenue from a large fragmented customer base with a generally negative attitude towards paying for digital products). Our analysis below shows that the reason behind adblockers' choice of an unconventional business model could be more fundamental, driven by the potential surplus they generate this way.

In implementing the adblockers' new revenue model we need to slightly alter the timing of the game. As before, the first stage is comprised of the publisher setting its quality q, followed by the adblocker setting V. At this point we add a step where the adblocker sets its price, p_b for consumers. Upon observing both V and p_b, consumers decide whether to download the ad blocker app. We remove the stage where the adblocker asks the publisher for a payment, and the last stage is where the publisher sets its advertising level, consumers make their choice to visit the site or not and profits are realized.

Below we compare this "charge consumers" (CC) case to our full model with the adblocker charging the publisher as is consistent with current practice (we continue to denote this case by AB) and our original benchmark of no-ad blocking (NA). For simplicity, we focus on two extreme cases that exhibit starkly different overall outcomes in the main model where the adblocker extorts the publisher. The first case represents the lower left corner of Figure 2.3, where both r and t

[17] https://www.reuters.com/article/us-germany-trial-adblocking/german-supreme-court-rules-ad-blockers-legal-in-defeat-for-springer-idUSKBN1HQ277.

are small and, therefore, consumer heterogeneity is maximal. This case is closest in relevance for practice. We also examine another case represented by the upper right corner of the Figure where both r and t are high and, thus heterogeneity is very low.

Proposition 5 *When r and t are sufficiently small, the equilibrium outcome in the case of charging consumers falls between the benchmark with no adblocker and the main model of charging publishers with respect to quality, publisher payoff, consumer surplus and total welfare:* $q^{\star}_{AB} < q^{\star}_{CC} < q^{\star}_{NA}$, $\Pi^{Pub\star}_{AB} < \Pi^{Pub\star}_{CC} < \Pi^{Pub\star}_{NA}$, $CS^{\star}_{AB} < CS^{\star}_{CC} < CS^{\star}_{NA}$, *and* $TW^{\star}_{AB} < TW^{\star}_{CC} < TW^{\star}_{NA}$. *Furthermore, the adblocker has lower profits when charging consumers:* $\Pi^{Adb\star}_{AB} > \Pi^{Adb\star}_{CC}$.

When $r = t = 1$, *we have* $q^{\star}_{CC} = q^{\star}_{AB} < q^{\star}_{NA}$, $\Pi^{Pub\star}_{CC} = \Pi^{Pub\star}_{AB} < \Pi^{Pub\star}_{NA}$, $TW^{\star}_{CC} = TW^{\star}_{AB} < TW^{\star}_{NA}$ *and* $CS^{\star}_{CC} = CS^{\star}_{AB} > CS^{\star}_{NA}$.

The results paint an interesting picture of how the business model affects profits and welfare. When consumer heterogeneity is high, surprisingly, both the publisher and consumers are better off when the adblocker charges the consumers compared to when it charges the publisher. Furthermore, total welfare is also higher in this case, but still lower than without the presence of the adblocker. Hence ad blocking is still harmful, but less so than in the case when the adblocker extorts the publisher. At the same time, the adblocker is worse off with the traditional business model of charging consumers. This analysis suggests that it is not a surprise that we do not see many adblockers employing such a business model in reality.

Before we describe the intuition behind the results, we note that although the decision to set $V > 0$ is still an option when charging consumers, the adblocker does not exercise this option for trivial reasons. The more ads the software blocks, the more valuable it is allowing the adblocker to collect more revenue. Hence in equilibrium V is always 0 when the adblocker charges consumers. Instead of adjusting V, the adblocker uses the price p_b to balance the revenue and demand for its software. Setting a higher p_b is similar to setting a higher V as both decrease the amount of consumers who download the ad blocker, but both increase the revenue that can be extracted from each consumer. Here, the revenue is extracted directly through a price, while in our main model the revenue for each downloader is extracted indirectly by collecting it from the publisher.

Charging a single price p_b allows for less discrimination than setting V as the latter affects consumers with different ad sensitivities to a different degree. As a consequence, charging consumers directly will lead to lower overall demand for the software and less revenue for the adblocker. On the flip side, fewer downloaders benefit the publisher as it can collect more ad revenue, which can then support a higher level of quality investment. Higher qualities lead to a higher total welfare.

Surprisingly, consumers are better off when they are charged directly. This happens for two reasons. First, a higher quality always benefits consumers. The other reason is that the adblocker is able to extract less surplus from consumers when charging them directly. Even though no surplus is extracted directly from consumers when the adblocker charges the publishers, a positive V causes consumers to lose utility by having to endure some advertising. When they are charged for the ad blocker directly, they do pay a price, but at least there is full ad blocking.

When consumer heterogeneity is very low, the dynamics are different. In the extreme case of $r = t = 1$, when consumers are only heterogeneous with respect to κ, charging the publisher and consumers results in very similar outcomes. The optimal p_b is $q/2$, which closely corresponds to a $V = q/2$ in our main model. Hence the direct price the consumers pay for purchasing the software is the same as the indirect price they pay in our main model, by enduring some advertising. As a result the segment of downloaders is exactly the same in both cases and hence the publisher's incentives are also the same in both cases, leading to the same quality, publisher profit, consumer welfare and total welfare.

Overall, we see that charging consumers makes a difference when consumer heterogeneity is significant. Such an ad blocking business model is less harmful to the ad-supported publishing industry and, importantly, it benefits consumers as well. However, adblockers typically do not want to use such a model as it hinders their ability to extract surplus and leads to lower revenues. While in practice there may be a variety of other factors that make it difficult to collect revenues from consumer, it is clear that adblockers have no incentive to do so even if these factors were mute.

2.3 Publisher-Adblocker Relations

So far, we have considered all negotiation power to be in the hands of the adblocker who presents a take-it-or-leave-it offer to the publisher. In practice, this assumption may apply to the case of small, powerless publishers. However, it is reasonable to assume that larger publishers have more negotiation power. The most obvious way a publisher can avoid adblockers is by erecting a paywall around its content - we formally explore this case in a Web Appendix to show that, indeed, if the publisher can switch to a subscription model, it is partially shielded from the adblocker's extorsion. But publishers may have many other (costly) means to avoid adblockers. For example, one option that often emerges in discussions about ad blocking is technology that is able to prevent ad blocking (so-called "ad blocker blockers").[18]

While adblockers' business model relies on extracting rent from publishers, there is no institutionalized system to determine and collect these payments. Hence, bilateral negotiations between the adblocker and the publishers and certain actions that each side can take play a crucial role. In this section, we explore this important issue.

Mathematically, we operationalize the negotiating power of a publisher as the share of the surplus from consumers who downloaded the ad blocking app that the publisher can keep. We define the publisher's share as $1 - \alpha$, with the adblocker's share being α. We first study the case of an exogenous α. For simplicity, we assume $t = 0$ throughout this section.

Several aspects of the analysis remain the same as in the base model. For a fixed V and q, the publisher chooses $A = \frac{q}{1+r}$ given our assumption that $\bar{\kappa}$ is high enough. The adblocker's problem of setting V also remains the same as its payoff is simply multiplied by α, but otherwise remains unchanged. Hence $V^{\star}(\alpha) = V^{\star} = qF(r, 0)$ remains the same. However, the publisher's quality choice problem is different since the payoff function now includes some portion of the ad revenue from the downloader segment as well. The analysis provides the following result.

[18]Publishers can also coordinate legal action against a powerful adblocker and have done so in European countries.

Proposition 6 *The optimal quality is*

$$q^*(\alpha) = p_a \left(4c(1+r) + \frac{2p_a[(r^2+4r+1)(1-(1+r)F(r)) - (1-\alpha)H(r)]}{3\overline{\kappa}(1+r)^2}\right)^{-1},$$

where $H(r) > 0$ only depends on r. Both $q^\star(\alpha)$ and the publisher's profit are decreasing in α, r and increasing in $\overline{\kappa}$.

Overall, from the publisher's perspective, having more negotiation power acts similarly to having a lower cost. The adblocker sets V at the same percentage of A regardless of α giving the same incentive to download the ad blocker, resulting in the exact same downloader population if quality were fixed. However, being able to collect and keep some portion of the ad revenue from the downloaders is a boost and results in a higher quality investment, which also benefits consumers.

Corollary 5 *Consumer surplus is decreasing in α. When $t = 0$ and $\overline{\kappa}$ is sufficiently high then $CS^\star(\alpha) > CS^\star_{NA}$ if and only if both r and α are sufficiently small.*

Since consumer behavior is only affected by α through the increased q, consumers are actually better off with a stronger publisher and weaker adblocker. So much so that when α becomes small enough then consumer surplus can be higher then in the benchmark with no adblocker. However, this can only happen when r is small enough. Numerically, the maximum thresholds for consumers to be better off are at about 0.12 for α and 0.009 for r. If either r or α are larger than these thresholds then consumers are worse off than without the adblocker.

Note that there is a slight difference in the outcome when $t = 1$. Consumers are still better off as α decreases, but the pattern reverses in r. Consumers are better off than in the benchmark case if and only if r is large. Recall that in Proposition 3, $r > (\sqrt{141} - 1)/14 \approx 0.78$ ensured that consumers are better off. Here, as α approaches 0, the threshold changes to $r > (\sqrt{33} - 1)/8 \approx 0.59$. Interestingly this is exactly the same threshold as what we find in Section 2.2.3 with the full adblocking option as μ approaches 0. Hence when $t = 1$, the outcome is equally good for consumers if they can easily use the adblocker for full adblocking or if there is limited adblocking, but the publisher keeps all the revenue from the adblocker. Note, however that the quality is higher in the latter case.

Overall, lowering α is beneficial to both the publisher and the consumers. However, for a wide range of parameters, consumers are still worse off than in the benchmark case with no adblocker. The reason is that the quality reduction effect dominates: even though the adblocker does not capture much surplus because of its low negotiation power, adblocking can simply act as a sub-optimal limit on ads (from both the publisher's and the consumers' perspectives), which reduces quality. This is similar to what happens when a platform limits advertising for publishers on its site, but there, the limit will be higher because the platform maximizes the total surplus in the entire ecosystem as opposed to only the adblocker's profit.[19]

2.3.1 Adblocker Restraint

Thus far, we have treated α as an exogenous variable. It is likely that both the publisher and the adblocker can take certain actions to influence α and prepare for the negotiation. It is plausible that both sides can invest in certain costly actions to tilt the scale, such as legal representation, better outside options and technology. As we have seen in Proposition 6, the publisher is certainly better off with a lower α and would potentially spend money to make it lower. Does that mean that the adblocker is always interested in the opposite, i.e. a higher α? Here, we examine this question by assuming that the adblocker can freely choose an α at the beginning of the game, but it has to (and can) commit to that α throughout the game.

Proposition 7 *The publisher chooses an optimal $\alpha^* > 0$ which is increasing in c, $\overline{\kappa}$ and decreasing in p_a. The optimal $\alpha^\star < 1$ if and only if $\overline{\kappa}c/p_a$ is sufficiently low.*

Thus, we get the interesting result that the adblocker wants to restrain itself from extracting all surplus from the publisher. The intuition is that even though a better negotiation power allows it to get a higher share of the pie, the pie itself decreases substantially due to the reduced incentive of the publisher to invest in quality. The adblocker's payoff turns out to be proportional to $q^2\alpha$.

[19] We analyze this specific case in a Web Appendix.

Clearly, setting a very low α cannot be optimal as the adblocker would get only a small share. But increasing α is only beneficial up to a point because quality decreases. Whether that point is more or less than $\alpha = 1$, depends on how sensitive the publisher is to α. The sensitivity depends on a combination of three variables: the download cost, the quality cost, and the ad price. An increase in the first two make the publisher more sensitive, whereas an increase in ad prices makes it less sensitive. Hence, in a situation where either costs are relatively low, or when ad prices are high, the optimal α is less than one and the adblocker wants to show restraint.

Whether the adblocker can commit to an upper limit on α depends on the circumstances. A plausible scenario is when both the adblocker and the publisher can invest in technology that ex post increases their relative negotiation power. In such a game (that is, unfortunately, intractable), it is easier for the adblocker to commit by not investing, for example. Similarly, the adblocker can voluntarily subject itself to a regulator, who can enforce ex post its commitment to an $\alpha < 1$.

2.4 Model extensions and robustness checks

In this section, we examine extensions to our base setup to speak to relevant market contexts and address a few limiting assumptions. First, we consider multiple publishers, which may compete to a varying degree. Next, we explore the increasingly common situation where publishers and consumers interact on a platform, which has broad powers to regulate (ban or promote, even provide) ad blocking. Finally, we discuss what happens if we relax various model assumptions. The detailed analyses and results of these model extensions are in the Web Appendix. Here, we only discuss the key findings.

2.4.1 Multiple publishers

We study two publishers that may have *different costs* of investing in quality. Importantly, we assume that the adblocker cannot discriminate between publishers and sets a common V for both, which is consistent with practice and is also needed if the adblocker wants consumers to develop rational expectations about the proportion of ads blocked when they download the ad blocker.[20] To vary the level of competition, we first consider two non-competing publishers with overlapping consumers, that is, consumers who visit either or both publishers. Then, we consider competition between two publishers where all consumers consider both publishers but visit at most one of them. For parsimony, we assume $t = 0$ throughout this section.

Non-competing publishers: Consumers visit a particular publisher if it provides positive utility. There is no publisher competition because consumers do not compare the utility they get from the two publishers. Publishers may have different costs of quality and, as a result will choose different quality levels. The rest of the model setup is identical to the basic model. All actions that publishers take are simultaneous with each other.

Even without competition there are two sources of strategic interaction between the publishers. First, the adblocker sets a common V, which is a function of both publisher qualities. Second, consumers in their overlapping consumer segment make the ad blocker download decision considering both publishers.

We find that the level of asymmetry between the players is important in explaining the strategic interaction between them and the qualities they set. Regardless of whether this asymmetry is high or low, the presence of another player has a negative effect on a publisher, but the mechanisms are slightly different. In the highly asymmetric case, although the adblocker focuses on the high-quality player, it also extracts rent from the low-quality publisher, reducing V compared to the basic model with just the high-quality player. This suppresses the high-quality as a lower V generally leads to more downloads of the ad blocker and generally hurts publishers. The low-quality player is also hurt. Even though V is higher than it would be with only the low-quality player in the market, V is low enough that it encourages to download the ad blocker with the purpose of using it for the high-quality site as well. As a result, a larger number of consumers download the ad blocker than if the low quality player were alone, which allows the adblocker to extract more surplus from even

[20] For completeness, in the Web Appendix, we also analyze the case when the adblocker can discriminate between publishers.

the low-quality player than if it were alone.

In the low asymmetry case, consumers use the ad blocker for both sites and consequently the adblocker uses roughly the average quality to set V. At the extreme, if the two qualities are equal, the adblocker will react similarly as if there were a single player. But an important difference from the basic model is that the consumer benefit from a single download doubles: for the same download cost κ, a consumer gets ad blocking on two sites. The larger downloading segment suppresses publisher profits and leads to lower quality investments.

Competing publishers: When publishers are substitutes, each consumer has capacity to visit at most one publisher's site, choosing the one that provides higher utility. To reach an analytical solution, we use the convention of the literature (Shaked and Sutton, 1982a) that quality investment is costless with a maximum quality of 1. Thus, here, there is no inherent heterogeneity in quality between publishers, although in equilibrium, quality choices differ. This feature allows us to isolate the strategic effect that the adblocker might have on competing publishers.[21] The rest of the model construction is identical to our previous setup.

In this case, a key insight is that the adblocker's V falls between the two firms' advertising levels. This is because the adblocker intends to undercut the ad level on the high-quality site, but not on the low-quality site. As before, this hurts the high-quality site, because advertising is reduced and the adblocker is able to extract more revenue. But interestingly, it also hurts the low-quality site because a segment of consumers switches to the high-quality site. These consumers previously could not 'afford' the high-quality site because of the higher advertising, but now with the ads limited by the adblocker they can enjoy the higher quality content. Furthermore, similarly to our basic model, there are some consumers who did not visit any site before, but now they do and they immediately jump to the high-quality content.

The second point brings forward an interesting strategic effect. Given our simplified cost structure, the higher quality publisher always chooses the maximum quality of 1 to differentiate. The

[21] If we use the same cost structure as in the basic model with different costs, we have to resort to numerical analysis to calculate the equilibrium qualities.

interesting question is how the lower quality changes with the adblocker. We find that it *increases*, therefore leading to *less differentiation*. The intuition relies on the notion that ad blocking makes consumers switch away from the low-quality site. The overall magnitude of switching and its impact on revenue depends on the quality differential. The biggest loss from switching happens at a quality level that is lower than the equilibrium quality without an ad blocker. The impact diminishes as the lower quality approaches the higher quality, because the overall pool of consumers that could switch decreases. Combining this effect with the basic forces that drive differentiation, we get that there is still differentiation, but less than without the adblocker.

In this case, the effect of the adblocker on consumers is mostly positive. One reason is that without quality costs, we do not observe a reduction in quality. In fact, with less differentiation the lower quality increases. Furthermore, ad blocking directly increases consumer utility and as we noted there is a segment of consumers who switch from the low-quality to the high-quality publisher and there is a segment that moves from no consumption to visiting a site. In addition to these segments, the reduced differentiation even benefits consumers who are not using the ad blocker as competition leads to lower ad levels. It is important to note that if we add back quality costs, ad blocking will have a primary effect of suppressing quality investment as in the base model. However, the key finding of this analysis is that competition between publishers may alleviate some of those negative factors.

2.4.2 Platform regulating ad blocking

In this section we will consider the case of a platform that does ad blocking itself instead of a third party adblocker. This setup is relevant for multiple reasons. First, more and more content consumption happens on platforms where consumers and publishers interact (e.g. iOS, Google or Facebook). For example, Google's Chrome browser offers to block certain ads. Second, multi-sided platforms have tremendous power in "regulating" the interactions between the different sides and can potentially internalize competition between the adblocker and the publisher leading to better welfare outcomes. Indeed, the platform is interested in maximizing the total surplus generated by the transactions while a third-party adblocker is only interested in maximizing its own profit. In this sense, the incentives of the platform and the adblocker are not necessarily aligned, leading to different outcomes. This tension shows in practice, where adblockers are criticizing Google, for example, for not blocking enough ads. At the same time, Google is attempting to prevent ad blocking software from operating properly in its system. As we see a trend of greater platform control, especially in the mobile content ecosystem, it is important to understand how different entities can and should block or limit ads.

To speak to this issue, we analyze a model in the Web Appendix, in which the platform itself does the ad blocking with the objective to maximize total welfare. Interestingly, we find that in such a setup, total surplus is always lower compared to the no-adblocker (benchmark) case. Even though for a fixed value of q the platform indeed sets V such that the total surplus is higher than in the benchmark, the platform's choice of V lowers the publisher's profit for a given q. This implies that the equilibrium q in the benchmark model is higher than in the model where the platform is the adblocker. Moreover, the total surplus in the latter case can be sometimes even lower than with a third party adblocker. Depending on the value of advertising an adblocker may be less harmful to quality levels than the platform itself. Despite the drop in quality, consumer surplus is mostly positively affected. While the combination of reduced advertising and reduced quality hurts total surplus, the former factor often dominates when considering consumers' payoffs.

2.4.3 Additional robustness checks

We conduct various other robustness checks in the Web Appendix, to address some limitations of our model. We start by alleviating the assumption that the price of advertising is exogenous. Solving the model with a fully endogenous p_a is intractable but we can analyze a quasi-endogenous case, where advertising price is an increasing (concave) function of quality. We find that the structure of the analysis and the results are similar to the main model's.

We also explore the case when the publisher can switch to a subscription revenue model, which essentially puts a lower bound on the publisher's profit. In turn, this provides more negotiation power to the publisher. While this limits the decline in the quality level, our main findings concerning consumer surplus and welfare remain valid.

Throughout the analysis we consider a monopolist adblocker. This assumption is driven by the fact that, while one can observe a large number of adblockers, the market is highly concentrated.[22] Indeed, while the cost of entry is relatively low, reliably operating ad blocking requires scale. As such, new entrants typically shut down because they cannot generate revenues. Nevertheless, it is relevant to ask how the presence of competing entrants, often offering full ad blocking, affects our results. Fortunately, the version of our model in which the adblocker also provides a 'full ad blocking' option (see Section 2.2.3) is essentially the same as this setup as long as full-adblockers represent higher downloading costs to consumers, which is safe to assume. Furthermore, in the Web Appendix, we also explore the possibility that two symmetric adblockers compete. If such competition only concerns the limit of advertising, V then the dynamics are similar to Bertrand competition. However, if adblockers compete in V as well as in consumers' downloading cost, a differentiated outcome with positive profits is possible, with one adblocker offering lower V at a higher downloading cost, while the other doing the opposite.

2.5 Concluding remarks

We have developed a model to evaluate the impact of ad blocking on publishers' content quality and, in turn, how the decline in quality affects consumers compared to the beneficial effect of reduced ad exposure. Consistent with practice, we assumed that the adblocker uses the unconventional revenue model, in which it charges the publisher in exchange for allowing a certain amount of ads through its blockade, a practice often qualified as "extorsion" by publishers. Our analysis reveals that this practice, while beneficial to adblockers, is typically an inefficient way to limit consumers' advertising exposure that ends-up leading to lower overall consumer surplus. By directly charging publishers - whose business is damaged by ad blocking in the first place - the adblocker forces publishers to reduce quality investment to such a degree that the resulting

[22] Adblock Plus, the most popular adblocker is estimated to represent about 60% of the desktop browser market, for example, with no comparable second player (Maheshwari, 2016).

lower content quality has more negative effect on consumers than the positive impact of lower advertising exposure due to ad blocking. We show that the conventional revenue model of directly charging consumers would alleviate this problem, although it is not in the interest of adblockers. This insight may provide some guidance to policy makers who, at present seem to largely support adblockers' current, so-called "whitelisting" business practice.

We have explored a number of model extensions to address important institutional details and describe additional business practices. These include the possibility for adblockers to offer a full-adblocking option, publishers capability to introduce a subscription revenue model and the possibility of negotiation between the adblocker and the publisher. We have also found that the model is robust to various structural factors such as, for example, the existence of multiple publishers, quasi-endogenous advertising prices, adblocker competition and the case where a transaction platform provides the ad blocking service.

The surge of ad blocking in 2016 created a small 'crisis' among publishers predicting the end of advertising supported content creation. Today, although ad blocking has stabilized, it still represents a loss of almost 25% of total digital advertising revenues for publishers. Understanding its drivers and its impact on welfare is crucial for the media industry and policy makers. The present paper intends to be a first step in this direction.

Chapter 3: Demand Learning in Digital Advertising Markets

With the dramatic increase of consumers' time spent online digital advertising has experienced rapid growth in the last decades. Indeed, eMarketer forecasts global digital ad spend to reach $455 billion in 2021, surpassing TV advertising to reach roughly 61% of total global ad spend.[1]

The digital advertising environment has a few characteristics. First, the response and conversion rates are often low which frustrates advertisers due to low return on their advertising spend (Dalessandro et al., 2015). This happens partially because firms often have little understanding of how their advertising campaigns affect consumer demand. Second, advertising is relatively cheap due to the large inventory available on the Internet. This potentially allows firms to use this extensive online inventory as a testing ground to gauge the impact their advertising has on consumer demand.

The goal of this paper is to shed some light on how firms' ability to learn the impact their advertising campaigns have on consumer demand affects these firms' equilibrium behavior. As such, this paper studies firms' advertising and quantity decisions when higher advertising intensity allows firms to estimate the effectiveness of advertising more accurately, thereby allowing them to make better future advertising decisions.

Advertising is costly and serves two distinct roles. First, it increases awareness about the product. Second, it can increase consumer demand. This latter effect is uncertain but firms can decrease this uncertainty if they advertise more intensely. Specifically, we assume that each firm gets a private signal about the unknown demand parameter, the accuracy of which is increasing in that firm's advertising intensity (in the model we will assume for simplicity that advertising intensity is either 0 or 1, that is the firm either advertises or not). We consider a model where each firm conducts two successive advertising campaigns such that more active advertising in period 1

[1] https://forecasts-na1.emarketer.com/5a4d341cd8690c01349716dc/5a4d1b48d8690c01349716ad

might help the firm advertise more efficiently in period 2 by refining the estimate of the unknown parameter.

Overall, our results show that, indeed, demand learning plays an important role in how firms choose their advertising and quantity. Moreover, the learning effects turn out to be ambiguous: sometimes they lead to higher advertising and quantities, sometimes they lead to lower advertising and quantities. Starting with a simple model with only one firm (a monopolist), we compare the market outcomes, and especially the advertising choices, depending on whether this monopolist firm can or cannot learn through advertising. We find that the effect of learning in our model depends on the interplay between the cost of advertising (the cost to conduct an advertising campaign) and the efficiency of learning (how much information learning provides about the demand parameter). When the cost is small, learning can only lead to a decrease in advertising intensity. Indeed, if the firm cannot learn, the best strategy it can employ is to advertise indiscriminately in all of the periods (the low costs allows this strategy to be profitable), while if the firm can learn it will employ a more granular advertising strategy declining to advertise if the signal received is low. When the cost is high, on the other hand, learning can only increase advertising intensity. If the firm cannot learn, it will advertise very little (in our model it will not advertise at all) because indiscriminate advertising is no longer profitable. If the firm can learn however, learning still keeps advertising profitable due to the fact that it increases expected second period profits, even though advertising in the first period brings pure losses. Hence, the effect of learning depends on the actual market characteristics. The equilibrium quantity, though, is always higher when the monopolist firm can learn due to the fact that learning always increases expected profits and, thereby, the incentives to invest in quantity.

We corroborate the findings above by further studying the model with two competitive firms. The influence of learning on advertising intensities here is also ambiguous, even though the dynamics are more complicated. In addition to the effects described above, namely allowing the firms to go from indiscriminate advertising to more granular advertising, learning here also has an effect on the number of firms actually being on the market. When the firms cannot learn, as the cost increases one of the firms may drop out of competition since the market cannot accommodate two firms anymore. While if the firms can learn, the higher second period expected profits might make both firms stay and compete leading, thereby, to increased advertising intensity. Moreover, as opposed to the monopoly case, the effect of learning on the equilibrium quantities is also ambiguous. Even though, the quantities generally increase due to higher expected profits, for some values of the advertising cost they might actually fall. This, again, has to do with the fact that learning sometimes leads to higher competitive pressure because it allows firms to stay in the market even when advertising costs are relatively high.

Finally, in an extension, we study a special competitive case where the firms can also learn each other's demand in addition to their own demand. Additional learning, once again, has an ambivalent effect on advertising intensity compared to the baseline competition model. On the one hand, more information can decrease advertising because firms now will be able to avoid advertising simultaneously and decrease the competitive pressure. On the other hand, more information increases the firms' returns to advertising making the advertising intensity higher. The effect of the additional information on the equilibrium quantities is always one-way though. Reduction in competitive pressure always leads firms to choose higher equilibrium quantities.

The paper is organized as follows. Section 1 describes the model structure. Section 2 provides the analysis of the model in a special case of a single monopolist firm. We first solve the model when the firm can learn, then the model where the firm cannot learn and, then, compare the results of the two. In section 3, we move to the model of Cournot competition between two firms. We again solve the model where the firms can learn and the model where the firms cannot learn and, then, compare the results. Finally, in Section 4, we consider an extension to the competitive model of Section 3, where the competitive firms are able to also learn their rival's demand parameters and compare the results to the baseline competition model. We end the paper by a general discussion and concluding remarks. For ease of readability proofs are relegated to the Appendix.

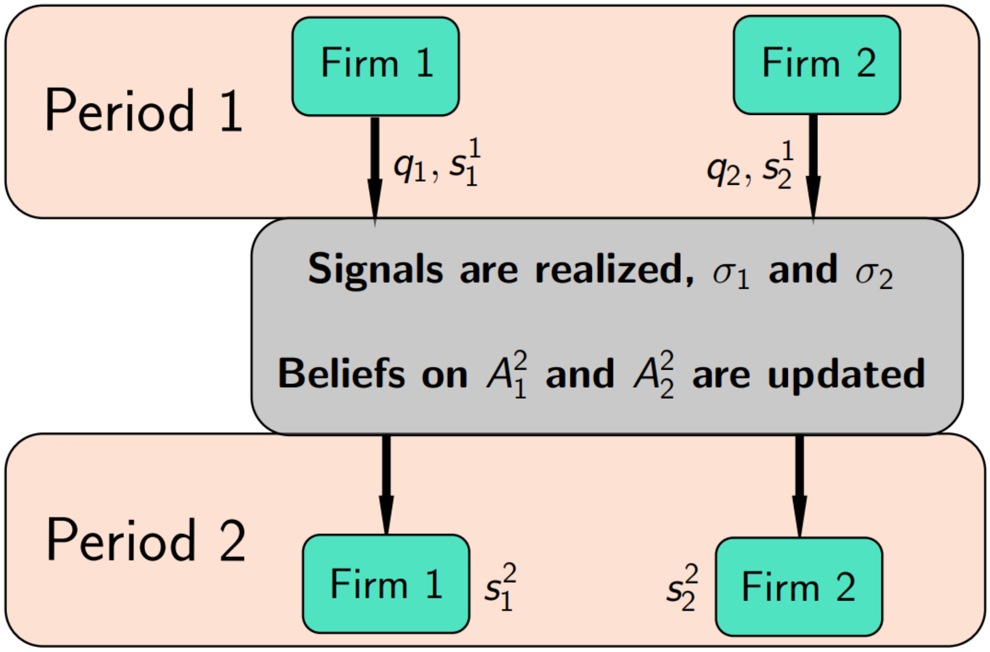

Figure 3.1: Game Structure

3.1 Model

We consider a competitive model with two completely symmetric firms, 1 and 2. The structure of the game is schematically shown on Figure 3.1. The game proceeds in two periods. In period 1, firms simultaneously decide on quantities, q_1 and q_2 (which will be fixed until the end of the game), and on whether they do an advertising campaign (whether $s_i^1 = 0$ or $s_i^1 = 1$, where $i \in \{1; 2\}$ is a firm). If a firm conducts an advertising campaign, it gains access to a segment of consumers. These consumers have an inherent random parameter $A_i^j \in \{0; 1\}$ that describes whether this segment of consumers is responsive to the firm i advertising at time period j, that is, if $A_i^j = 0$ then consumers never consider a product of firm i in period j even if firm i advertised, while if $A_i = 1$, then consumers do consider buying from firm i. Parameters A_i^1 and A_i^2 can be correlated. For simplicity, we only consider two cases: perfectly correlated A_i^1 and A_i^2 (baseline competition model) and uncorrelated A_i^1 and A_i^2 (a competition model considered in the extension). Parameters A_i^j are independent in between the firms. Parameters A_i^j are a priori equal to 1 with probability $\frac{1}{2}$ and 0 with probability $\frac{1}{2}$. In order to advertise a firm has to pay a cost of k. Additionally, advertising to this segment allows the firm to better estimate the distribution of A_i^2. Having conducted the advertising campaign, each firm receives a private signal such that:

$$Pr(\sigma_i = 1|A_i^2 = 1) = Pr(\sigma_i = 0|A_i^2 = 0) = \frac{1}{2} + \gamma,$$

$$Pr(\sigma_i = 1|A_i^2 = 0) = Pr(\sigma_i = 0|A_i^2 = 1) = \frac{1}{2} - \gamma.$$

where $0 \leq \gamma \leq \frac{1}{2}$.

For tractability concerns, we keep γ only in the benchmark case with a monopolist firm. With two competing firms we set $\gamma = \frac{1}{2}$, so if a firm advertises it knows its real A_i for sure. If the firm does not do the advertising campaign no informative signal is received. In stage 2, the firms again decide on conducting their advertising campaigns as in stage 1. This time, though, they can use the signals they received in period 1 (if they had advertised in period 1) to refine their estimates of A_i to better understand whether the advertising campaign is worthwhile. Additionally, in period 2 both firms know all the quantities set in period 1 and whether their competitor advertised. Also, in some cases firms can indirectly observe each other's A_i^j's. For instance, if both firms conducted advertising campaigns in period j then their own sales give them information on whether their competitor's advertising worked or not (that is, if they observe a high demand for their own product then they can deduce that their competitor's advertising did not work and vice versa). This fact is especially important in case of perfectly correlated A_i^1 and A_i^2 since if firm i can deduce the value of A_{-i}^1, then it also knows the value of A_{-i}^2 and can use this information to better set its s_i^2.

In both of these periods, firms compete for consumers via quantities in a Cournot fashion. This means that if $A_1 = A_2 = 1$, the clearing price is $p = 1 - q_1 - q_2$, where q_i is the quantity set by firm i; if $A_1 = A_2 = 0$, consumers do not buy any of the products; while if $A_i = 1; A_{-i} = 0$ consumers only consider product i and, hence, the clearing price is $p = 1 - q_i$. The profit of firm i given a specific realization of quantities and advertising choices is :

$$\pi_i = q_i s_i A_i \left(s_{-i}\left[(1 - A_{-i})(1 - q_i) + A_{-i}(1 - q_i - q_{-i})\right] + (1 - s_{-i})(1 - q_i)\right). \tag{3.1}$$

where q_i and q_{-i} are the quantities set in period 1 and fixed for the rest of the game.

We will solve the game using backward induction starting from period 2.

3.2 Monopoly

In this section, we will consider a benchmark case of a monopolist firm. The following proposition summarizes the monopolist's choices of both periods' advertising campaigns, the optimal quantity and the price.

Proposition 8 *Define $s_2(\sigma = \bar{\sigma})$ as the optimal choice of advertising in period 2 given that the realization of the signal, σ, happened to be $\bar{\sigma}$. Then, the monopolist's optimal quantity, the market price and the two periods' optimal decisions on advertising are the following:*

1. *When $k < \frac{1}{8} - \frac{\gamma}{4}$:*

$$q^{\star} = \frac{1}{2},$$

$$p^{\star} = \frac{1}{2},$$

$$s_1 = s_2(\sigma = 1) = s_2(\sigma = 0) = 1.$$

2. *When $\frac{1}{8} - \frac{\gamma}{4} \leq k \leq \frac{1}{8} + \frac{\gamma}{12}$:*

$$q^{\star} = \frac{1}{2},$$

$$p^{\star} = \frac{1}{2},$$

$$s_1 = s_2(\sigma = 1) = 1,$$

$$s_2(\sigma = 0) = 0.$$

3. *When $k > \frac{1}{8} + \frac{\gamma}{12}$:*

$$q^{\star} = \textit{arbitrary},$$

$$p^{\star} = \textit{arbitrary},$$

$$s_1 = s_2(\sigma = 1) = s_2(\sigma = 0) = 0.$$

What we see is that when the cost of advertising is very small, the monopolist firm advertises in period 1 and in period 2 regardless of the signal realization. Even if the firm receives a negative signal (unless the signal is perfect, *i.e.* $\gamma = \frac{1}{2}$), it would still advertise because the potential payoff of advertising is large which offsets a low probability of success. When the cost is very high the firm would never advertise since the potential profit of learning is not large enough to compensate the cost of launching an advertising campaign. When the cost is neither very high nor very low, this is where learning plays the biggest role. Here it is always profitable for the firm to advertise in period 1 because the signal it receives actually determines whether the second period advertising campaign is worthwhile or not. If it receives a positive signal it will advertise in period 2 knowing that the probability of success is high and if it receives a low signal it will not advertise in period 2 knowing that the probability of success is low. The quantity (and price) is $\frac{1}{2}$ for low-to-middle advertising costs and is not defined for the high costs (since for high costs the firm never advertises and its quantity is irrelevant).

In the next two lemmas we will describe the two extreme learning cases, no learning (that is, $\gamma = 0$) and perfect learning (that is, $\gamma = \frac{1}{2}$). We are going to use these lemmas to quantify the impact learning exerts upon the expected advertising by the monopolist firm. The lemmas are special cases of Proposition 8 for $\gamma = 0$ and $\gamma = \frac{1}{2}$.

Lemma 3 *Assume $\gamma = 0$, that is, any signal is uninformative. Then, the monopolist's optimal quantity, the market price and the two periods' optimal decisions on advertising are the following:*

1. *When $k < \frac{1}{8}$:*

$$q^\star = \frac{1}{2},$$

$$p^\star = \frac{1}{2},$$

$$s_1 = s_2 = 1.$$

2. *When $k > \frac{1}{8}$:*

$$q^\star = \textit{arbitrary},$$

$$p^\star = \textit{arbitrary},$$

$$s_1 = s_2 = 0.$$

When learning is uninformative, the two periods are identical and independent from the point of view of the monopolist firm. Hence, either the firm advertises in both periods when the cost is small (hence, the expected return is positive) or it never advertises when the cost is high (hence, the expected return is negative). The optimal quality choice is similar to the general model with $\gamma \in \left[0; \frac{1}{2}\right]$. The quality is either $q^\star = \frac{1}{2}$ when the firm advertises or is not defined when the firm never advertises.

Lemma 4 *Assume $\gamma = \frac{1}{2}$, that is, any signal is perfectly informative. Then, the monopolist's optimal quantity, the market price and the two periods' optimal decisions on advertising are the following:*

1. *When $k \leq \frac{1}{6}$:*

$$q^\star = \frac{1}{2},$$

$$p^\star = \frac{1}{2},$$

$$s_1 = s_2(\sigma = 1) = 1,$$

$$s_2(\sigma = 0) = 0.$$

2. *When $k > \frac{1}{6}$:*

$$q^\star = \textit{arbitrary},$$

$$p^\star = \textit{arbitrary},$$

$$s_1 = s_2(\sigma = 1) = s_2(\sigma = 0) = 0.$$

When learning is perfectly informative, the monopolist would never advertise in the second period if it receives a negative signal because now he knows for sure that his next campaign is going to be unsuccessful. For high values of the advertising cost it is also not going to ever advertise in period 1 because even a very strong learning signal is not enough to cover the cost of the advertising campaign. For small values of the cost the firm will always advertise in period 1 and then advertise in period 2 if it received a strong positive signal. The optimal quality is again either $q^{\star} = \frac{1}{2}$ when the firm advertises and not defined when the firm never advertises as in the general model.

Now, knowing equilibrium advertising as a function of learning, we can compare the results to see how learning impacts advertising decisions by the monopolist and what it does to equilibrium advertising. In what follows we will be comparing the general learning case for $\gamma \neq 0$ with the uninformative learning case, $\gamma = 0$. We will first compare the advertising choices period by period (*e.g.* period 1 with learning vs. period 1 with no learning) and then compare the expected aggregate advertising over period 1 and period 2. Next lemma shows the results for the period by period comparison.

Lemma 5 *Define s_j^L and s_j^{NL} as the advertising campaign choices in period j in the "learning" ("L", $\gamma \neq 0$) and "no-learning" ("NL", $\gamma = 0$) cases respectively. Then:*

$$s_1^L \geq s_1^{NL},$$

$$s_2^L(\sigma = 1) \geq s_2^{NL},$$

$$s_2^L(\sigma = 0) \leq s_2^{NL}.$$

As expected, the first period advertising is never lower when learning is informative, that is, learning always increases the firm's incentives to advertises when it conducts its first advertising campaign. Also, predictably, if the firm receives a positive informative signal in period 2 it advertises more than if no informative signal is received since the probability of a successful advertising campaign is higher. And the opposite is true when the informative signal received is negative. Comparing quantities (and prices) is tough since for small values of the advertising costs the quantities are not well defined.

Let us now see how the expected aggregate advertising by the firm over periods 1 and 2 depends on learning. We define the total expected amount of advertising seen by the consumers as (we call it advertising intensity or AI):

$$AI = \mathbb{E}[(s_1)^\star + (s_2)^\star] = 1 * Pr[(s_1)^\star = 1] + 1 * Pr[(s_2)^\star = 1]. \tag{3.2}$$

where $(s_j)^\star$ is equilibrium advertising by the firm in period j (which might depend on signal in the general learning case).

We calculate AI^L (the advertising intensity with learning) and AI^{NL} (the advertising intensity with no learning) and compare them. If there is no learning (see Lemma (3)), the two periods are independent and in each period the monopolist firm advertises only when $k \leq \frac{1}{8}$, otherwise the monopolist does not advertise. Hence, when $k \leq \frac{1}{8}$ the total consumed advertising over the two periods is 2 and when $k > \frac{1}{8}$ the total consumed advertising over the two periods is 0. The advertising intensity of the monopolist with no learning is, therefore:

$$AI^{NL} = \begin{cases} 2, & \text{if } k \leq \frac{1}{8} \\ 0, & \text{if } k > \frac{1}{8}. \end{cases} \tag{3.3}$$

When the monopolist firm is able to learn from advertising, the equilibrium outcomes are given in Proposition 8. When the cost is very small (that is, $k \leq \frac{1}{8} - \frac{\gamma}{4}$), the monopolist always advertises in both periods, hence the total two period advertising shown is 2. When the cost is in the intermediate range the monopolist always advertises in the first period and, then advertises in period 2 only if the signal is positive (which happens with probability $\frac{1}{2}$). Hence, if $\frac{1}{8} - \frac{\gamma}{4} \leq k \leq \frac{1}{8} + \frac{\gamma}{12}$, the total two period advertising is $1 + \frac{1}{2} * 1 + \frac{1}{2} * 0 = \frac{3}{2}$. Finally, if the cost is high (that is, $k \geq \frac{1}{8} + \frac{\gamma}{12}$), the monopolist never advertises, hence the total two period advertising is 0. To summarize, the advertising intensity in the case of a monopolist with learning is:

$$AI^L = \begin{cases} 2, & \text{if } k \leq \frac{1}{8} - \frac{\gamma}{4}, \\ \frac{3}{2}, & \text{if } \frac{1}{8} - \frac{\gamma}{4} < k \leq \frac{1}{8} + \frac{\gamma}{12}, \\ 0, & \text{if } k > \frac{1}{8} + \frac{\gamma}{12}. \end{cases} \tag{3.4}$$

Comparing (3.3) and (3.4) we can observe 4 distinct regions, see Figure (3.2). Consider first the lower half of the figure (the red and green regions). In this half, the cost of advertising k is low enough such that the expected one period profit for the firm when learning is not possible is positive, which implies that the firm would always advertise in both periods if it cannot learn. If the firm can learn, on the other hand, the situation is more complicated. When the learning parameter γ is low (that is, we are in the red region), the value of learning is low so that any signal the firm receives does not warrant the change in behavior. But when γ is high (the green region), the negative signal the firm can receive provides strong enough information such that the firm would lower its advertising in period 2 and, hence, its total two period advertising. This means that when the cost of advertising is low, additional strong information can only dampen advertising intensity. Consider, now, the top half of the picture (the blue and the orange regions). In this half, the cost of advertising k is on the opposite high enough such that the one period expected profit for the firm without learning is negative meaning that the firm would never advertise if it cannot learn. If the firm can learn, it will also never advertise if the value of learning is low (low γ), but if the value of learning is high, it can actually increase its advertising intensity compared to no learning. When γ is high (the orange region), the firm will advertise in both periods even though its first period payoff is negative because learning provides strong enough information that the firm can compensate for its lost profits in period 2. That is, when the cost of advertising is high, additional information can only increase advertising intensity.

Summarizing the above, we can conclude that for the monopolist firm the effect of learning is ambiguous on its advertising intensity. For small advertising costs, learning can only lead the firm

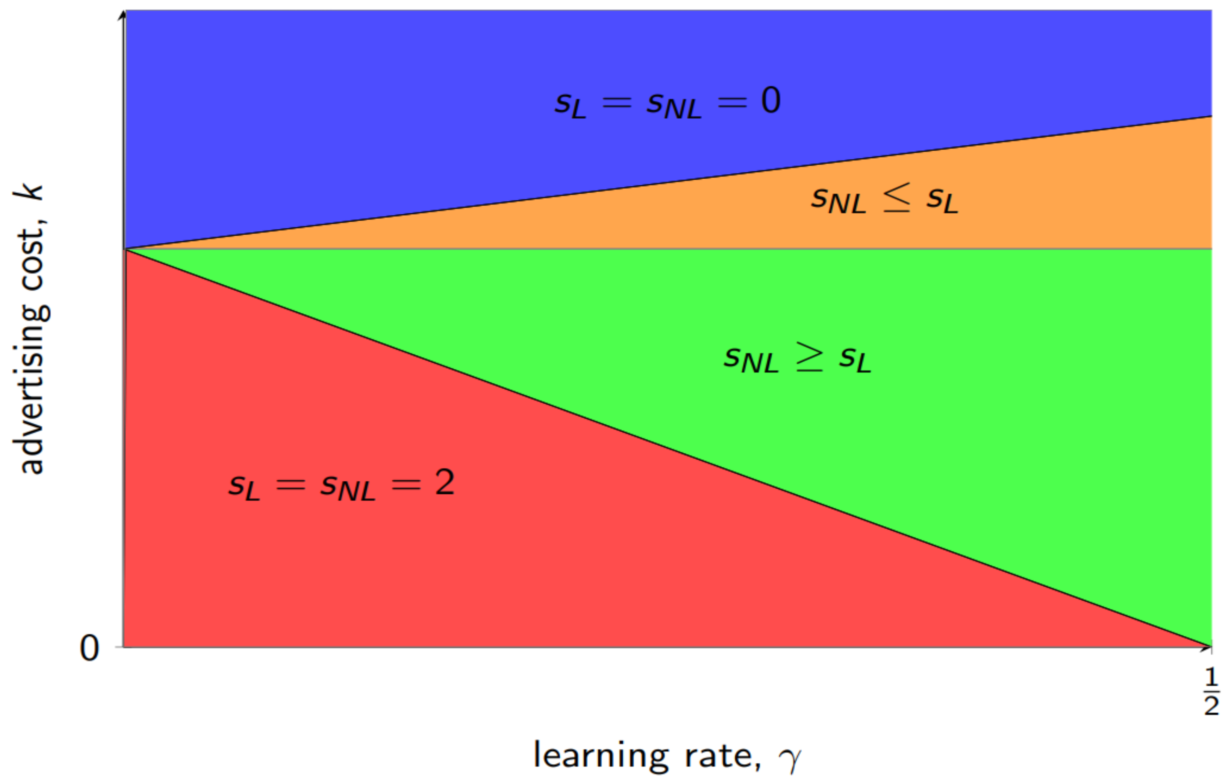

Figure 3.2: Monopolist: Learning vs. No Learning

to advertise less (when $\frac{1}{8} - \frac{\gamma}{4} \leq k \leq \frac{1}{8}$), while the for high advertising costs, learning can only lead the firm to advertise more (when $\frac{1}{8} \leq k \leq \frac{1}{8} + \frac{\gamma}{12}$). Hence, whether learning incentivizes or dampens the advertising intensity depends on the market conditions.

Let us finally look at how the equilibrium quantity set by the monopolist depends on learning. The equilibrium quantities in the learning scenario are given in Proposition 8 and the equilibrium quantities in the no-learning scenario are given in Lemma 3. When the equilibrium quantity is arbitrary we will assume that the firm sets $q^{\star} = 0$, since the monopolist is indifferent between any level of q, and, therefore, there is no reason for him to set a positive q. The following proposition summarizes the effect of learning on quantity.

Proposition 9 *Define q^L and q^{NL} as the equilibrium quantity choices of the monopolist firm in the "learning" ("L", $\gamma \neq 0$) and "no-learning" ("NL", $\gamma = 0$) cases respectively. Then, it is always true that $q^L \geq q^{NL}$.*

We see that in the monopoly case, learning always has a non-decreasing effect on the equilibrium quantity. This happens because learning always increases the first period advertising as shown in Lemma 5. Indeed, the equilibrium quantity changes exactly in line with the equilibrium first period advertising strategy (*i.e.* if $s_1 = 0 \Rightarrow q_1 = 0$ and if $s_1 = 1 \Rightarrow q_1 = \frac{1}{2}$). Since the expected first period profit always weakly increases in learning and the quantity and first period advertising strategy choices are made simultaneously, these two variables act the same way. Interestingly, it means that learning has an effect on quantity only through its effect on the advertising campaign choice, that is, there is no direct effect of learning on the quantity.

3.3 Competition

3.3.1 Baseline Model

In this section we will consider the competition between two firms. The firms compete in a Cournot fashion, that is, they set their q_i's in the beginning of Period 1 until the end of the game with the equilibrium price being set as $p_1 = p_2 = P(q_1 + q_2) = 1 - q_1 - q_2$ (that is, the demand is linear). We assume for tractability that the learning parameter is $\gamma = \frac{1}{2}$, meaning that by advertising firm i perfectly learns its second period unknown demand parameter A_i^2. We will assume also that the random demand parameters A_i^t are uncorrelated over time periods, that is over t. This means that, for instance, even if firm 1 can perfectly learn its own second period A_1^2, it cannot directly deduce its competitor's value of A_2^2 (because A_2^2 might be different from A_2^1). In the extensions, we will consider a case of perfectly time correlated A_i^t's where firm i can deduce its competitor's A_{-i}^2 just by observing the first period realized demand.

The following Proposition summarizes the equilibrium quantities, profits and advertising campaign choices.

Proposition 10 *Assume that $\gamma = \frac{1}{2}$. Then, in any pure strategy Nash equilibria, $s_i^2(\sigma_i = 1) = 1$ and $s_i^2(\sigma_i = 0) = 0$. Furthermore:*

1. *When $k \leq \frac{8}{75}$, there is a unique symmetric equilibrium such that:*

$$s_1^1 = s_2^1 = 1,$$

$$q_1 = q_2 = \frac{2}{5},$$

$$\mathbb{E}\pi_1 = \mathbb{E}\pi_2 = \frac{4}{25} - \frac{3k}{2}.$$

2. *When $\frac{3}{32} \leq k \leq \frac{1}{6}$, there are two asymmetric equilibria (for $i = 1$ and for $i = 2$) such that:*

$$s_i^1 = 1 \text{ and } s_{-i}^1 = 0,$$

$$q_i = \frac{1}{2} \text{ and } q_{-i} = \text{arbitrary},$$

$$\mathbb{E}\pi_i = \frac{1}{4} - \frac{3k}{2} \text{ and } \pi_{-i} = 0.$$

3. *When $k > \frac{1}{6}$, there is a unique symmetric equilibrium such that:*

$$s_1^1 = s_2^1 = 0,$$

$$q_1 \text{ and } q_2 \text{ are arbitrary},$$

$$\pi_1 = \pi_2 = 0.$$

For small values of the advertising cost (that is, $k \leq \frac{8}{75}$), both firms advertise in equilibrium in period 1 because the immediate first period payoff is large. In the second period though, it might happen that none of the firms, only one firm or both firms advertise depending on the signal realizations. In short, if firm i receives a negative signal it never advertises in period 2 (remember the signals are perfect), while if it receives a positive signal, it always advertises. The equilibrium quantities of the firms are symmetric and equal to $q_1^\star = q_2^\star = \frac{2}{5}$, which is smaller than the optimal quantity set by the monopolist. This is not surprising since having a competitor that also supplies some quantity of the good to the market lowers the equilibrium price. Therefore, a price a firm would get for a unit of its own good is smaller than under monopoly thereby reducing incentives to produce additional units. When the advertising cost is too large (that is, $k > \frac{1}{6}$), none of the firms ever advertise (neither in period 1 nor in period 2) due to the prohibitively large cost. Finally, when the advertising cost is in the intermediate range (that is, $\frac{3}{32} \leq k \leq \frac{1}{6}$), only one firm will ever advertise with the other firm never advertising. The cost in this case is too high to accommodate both firms and leave them non-negative profits. The one firm that advertises is effectively a monopolist and, therefore, the monopoly results carry over. This means that this firm will always advertise in period 1 and will advertise in period 2 if and only if it receives a positive signal. The quantity this firm sets is also the same as in the monopoly case and equal to $\frac{1}{2}$. We should also note that there is a small region in the advertising cost ($\frac{8}{75} \leq k \leq \frac{3}{32}$) where multiple equilibria exists: one where both firms advertise and one where only one firm advertises.

In the next lemma we will describe a competitive case with no learning in order to quantify the impact of learning on the advertising decision choices by the competitive firms on the market. The proof of the lemma is relegated to Appendix.

Lemma 6 *Assume $\gamma = 0$. Then this competitive game with no learning has the following pure strategy Nash equilibria:*

1. *When $k \leq \frac{3}{50}$, there is a unique symmetric equilibrium such that:*

$$s_1^1 = s_2^1 = 1,$$

$$q_1 = q_2 = \frac{2}{5},$$

$$\mathbb{E}\pi_1 = \pi_2 = \frac{4}{25} - 2k.$$

2. *When $\frac{1}{16} \leq k \leq \frac{1}{8}$, there are two asymmetric equilibria (one for $i = 1$ and for $i = 2$) such that:*

$$s_i^1 = 1 \text{ and } s_{-i}^1 = 0,$$

$$q_1 = q_2 = \frac{1}{2},$$

$$\mathbb{E}\pi_i = \frac{3}{16} - \frac{3k}{2} \text{ and } \mathbb{E}\pi_{-i} = \frac{1}{16} - \frac{k}{2}.$$

3. When $k \geq \frac{1}{8}$ there is a unique symmetric equilibrium such that:

$$s_1^1 = s_2^1 = 0,$$

$$q_1 \text{ and } q_2 \text{ are arbitrary},$$

$$\pi_1 = \pi_2 = 0.$$

When the learning is uninformative the two periods are independent of each other from the firms' point of view. Therefore, when the advertising costs are very small (that is, $k \leq \frac{3}{50}$), both firms advertise both in period 1 and in period 2 since the expected one period profits are positive for both firms. The equilibrium quantities in this case are the same as in the perfect learning competitive case and equal to $q_1^\star = q_2^\star = \frac{2}{5}$ and are less than the monopoly quantity. When the advertising costs are very high (that is, $k \geq \frac{1}{8}$), neither of the firms advertises in any of the periods since the expected one period profits are negative. Or, if the advertising costs are neither very high nor very low (that is, $\frac{1}{16} \leq k \leq \frac{1}{8}$), only one firm advertises in each period (might be different firms in each period). The quantities set by the firms equal to $\frac{1}{2}$ since the firm that advertises is essentially a monopolist in its period. There is also a small region in k (that is, $\frac{3}{50} \leq k \leq \frac{1}{16}$) where no pure strategy Nash equilibrium exists.

Let us now see how the total aggregate advertising shown to consumers by both firms depends on learning just as we did in the section on monopoly. To this goal, we modify the advertising intensity formula (3.2) we have already used to accommodate it for two firms:

$$AI = \mathbb{E}\left[(s_1^1)^\star + (s_2^1)^\star + (s_1^2)^\star + (s_2^2)^\star\right] =$$

$$= 1 * Pr[(s_1^1)^\star = 1] + 1 * Pr[(s_2^1)^\star = 1] + 1 * Pr[(s_1^2)^\star = 1] + 1 * Pr[(s_2^2)^\star = 1]. \tag{3.5}$$

where s_i^t is the advertising choice of firm i in period j.

We calculate AI^{PL} (the advertising intensity with perfect learning)) and AI^{NL} (the advertising intensity with no learning) and compare them. If there is no learning, the two periods are independent and the equilibrium outcomes are given in Lemma 6. When, k is very small (that is, $k \leq \frac{3}{50}$), both firms always advertise in both periods. Therefore, the total two period advertising is 4. When k is in the intermediate range (that is, $\frac{1}{16} \leq k \leq \frac{1}{8}$), one and only one firm advertises in each period (though a different one might advertise in period 2 compared to period 1), so the total two period advertising is 2. Finally, when k is high (that is, $k \geq \frac{1}{8}$), none of the firms advertise, so the total two period advertising is 0. Also, there is a small region where no pure strategy equilibria exist (that is, $\frac{3}{50} \leq k \leq \frac{1}{16}$). The advertising intensity in the case of competition with no learning is, therefore:

$$AI^{NL} = \begin{cases} 4, & \text{if } k \leq \frac{3}{50}, \\ 2, & \text{if } \frac{1}{16} < k \leq \frac{1}{8}, \\ 0, & \text{if } k > \frac{1}{8}. \end{cases} \tag{3.6}$$

When the firms are able to perfectly learn from advertising, the equilibrium outcomes are given in Proposition 10. When, k is very small (that is, $k \leq \frac{8}{75}$), both firms always advertise in period 1 and either none of the firms advertise in period 2 (in case they both received negative signals, which happens with probability $\frac{1}{4}$), only one of them advertises (in case exactly one firm received a positive signal, which happens with probability $\frac{1}{2}$) or both firms advertise (in case both firms received a positive signal, which happens with probability $\frac{1}{4}$). Therefore, the total two period advertising is $2 + 2 * \frac{1}{4} + 1 * \frac{1}{2} = 3$. When k is in the intermediate range (that is, $\frac{3}{32} \leq k \leq \frac{1}{6}$), one firm never advertises, while the other one always advertises in period 1 and advertises in period 2 only if it received a positive signal, that is, the total two period advertising is $1 + 1 * \frac{1}{2} = \frac{3}{2}$. Since there exist multiple equilibria in the region of $\frac{3}{32} \leq k \leq \frac{8}{75}$ (one where both firms advertise in period 1 with $AI = 3$ and the one where only one firm advertises in period 1 with $AI = \frac{3}{2}$), we have to choose an equilibrium. In what follows, we assume that in this region the advertising intensity is $AI = 3$. Finally, when k is high (that is, $k \geq \frac{1}{6}$), none of the firms advertise, so the total two period advertising is 0. Therefore, the advertising intensity in the case of competition with perfect learning is:

$$AI^{PL} = \begin{cases} 3, & \text{if } k \leq \frac{8}{75}, \\ \frac{3}{2}, & \text{if } \frac{8}{75} < k \leq \frac{1}{6}, \\ 0, & \text{if } k > \frac{1}{6}. \end{cases} \tag{3.7}$$

Comparing (3.6) and (3.7) we can observe 5 distinct regions, see Figure (3.3). For the very high cost of advertising (the red region, $k \geq \frac{1}{6}$) the expected profits are way too low that none of the firms would ever advertise regardless of whether they can learn or not and the total two period advertising shown is equal to 0 in both cases. When the advertising cost is not too large, the effect of learning on advertising intensity is ambiguous and can either increase or decrease the amount of ads shown compared to the no learning benchmark. For the very small values of the cost (the blue region, $k \leq \frac{3}{50}$), the expected one period payoff for each firm is large which means that they both will be advertising in both periods if they cannot learn. When they can learn though, learning can only exert downward pressure on advertising intensity because firms would never advertise if they discovered that their second period advertising campaign is going to be unsuccessful (due to negative signal received). For a slightly higher value of the cost (the orange region, $\frac{1}{16} \leq k \leq \frac{8}{75}$), the expected one period payoff in the no learning case decreases enough such that the market can no longer support both firms simultaneously advertising. At the same time, when firms can learn, the payoff does not fall as sharply because learning improves efficiency in the second period and so the firms are still able to simultaneously coexist. So it turns out that in the orange region, the advertising intensity with learning is actually higher due to the fact that more firms advertise in equilibrium. When the cost is a bit higher still (the purple region, $\frac{8}{75} \leq k \leq \frac{1}{8}$), the one period payoff from advertising becomes too low even in the learning case that two firms can no longer profitably coexist on the market and one of them drops out. It means that in this region, both in the learning and no learning cases only one firm advertises in each period (as in the monopoly

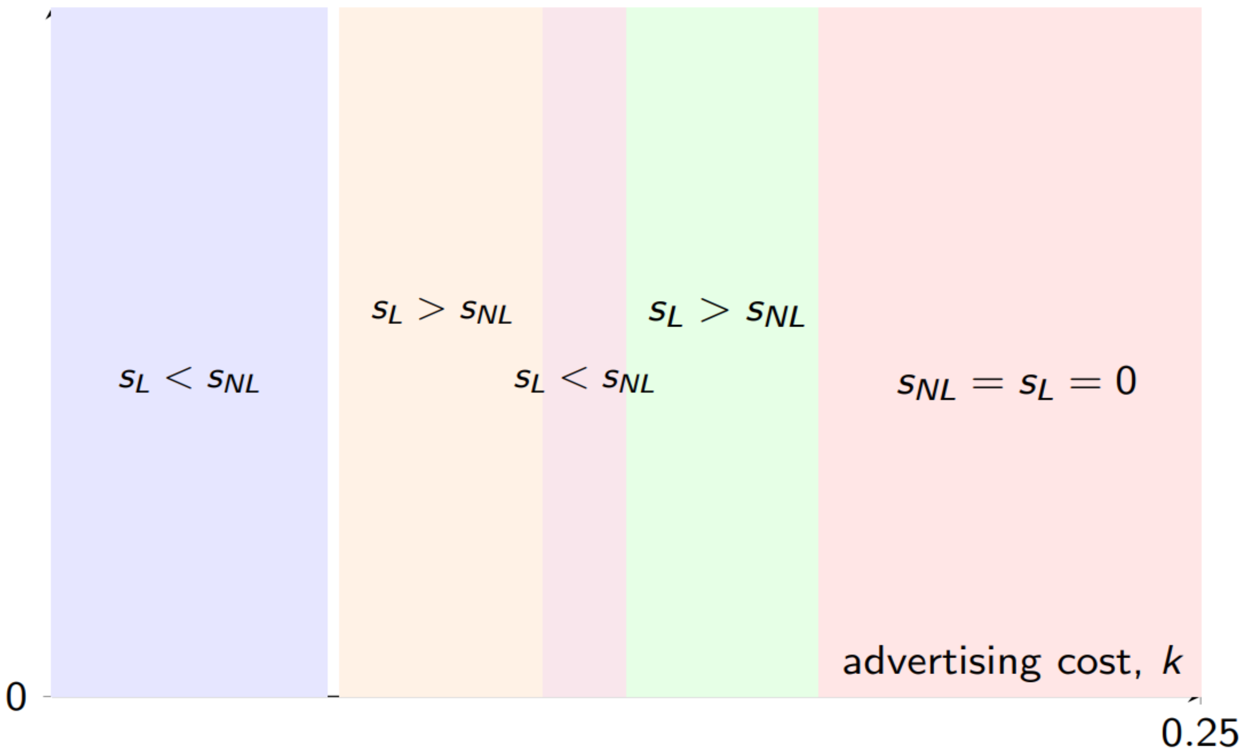

Figure 3.3: Competition: Perfect Learning vs. No Learning

case). Therefore, learning can only dampen advertising intensity in this region, because a negative signal would cause a firm to discontinue its second period advertising campaign if it can learn, while if the firm cannot learn it will keep advertising in both periods. Finally, in the green region ($\frac{1}{8} \leq k \leq \frac{1}{6}$), the advertising cost is way too high in the no learning case that all of the firms drop out of the market and none of them advertise, while learning still allows one firm to stay afloat due to better information in the second period even though its first period profit becomes negative. Also, it should be mentioned, that there is a small region where no pure strategy equilibrium exists in the no learning case, $\frac{3}{50} \leq k \leq \frac{1}{16}$, shown as a thin white bar on the figure. No analysis is possible when this happens, so we cannot compare the advertising intensities in this region.

Overall, we can say that the impact of learning on advertising intensity is ambiguous and depends on the advertising cost in a complicated way. On the one hand, it can decrease advertising because firms' second period advertising strategies do not have to involve blanket advertising like they did in the no learning scenario, that is, the firms now can choose not to advertise in period

2 if they received bad information. On the other hand, learning increases expected second period profits because firms know they will not lose money in the second period if the actual A_i^t realization turns out to be 0. This means that it makes them more eager to advertise in the first period which increases their incentives to advertise and, hence, the advertising intensity. Therefore, the actual impact learning exerts depends on the market conditions.

As a side note, now that we have the results for both the monopoly and competition cases, we can compare the advertising intensities in these two models (even though the monopoly model involves only one firm while the competitive model involves two) to see how competition impacts total advertising. We will compare the following pairs of situations: monopoly with learning (see (3.4) for $\gamma = \frac{1}{2}$) vs. competition with learning (see (3.7)); monopoly with no learning (see (3.3)) vs. competition with no learning (see (3.6)). Looking at the expressions we can quickly see that the total two period advertising in the competitive case is never lower than in the monopoly case (and usually higher). Competition provides strictly more advertising when $k \leq \frac{8}{75}$ if we compare the perfect learning models and if we compare the no learning models, competition provides strictly more advertising when $k \leq \frac{3}{50}$. This happens because for smaller k's both firms advertise in the competition case meaning the advertising intensity will be higher, while for high k's only one firm advertises in the competition case leading to the same intensity. If, on the other hand, we use a per firm advertising intensity in the competition case (*i.e.* if both firms advertise in the competition case than we divide the advertising intensity by 2), we can actually see that the advertising intensities are identical between these model comparisons so, interestingly, competition does not bring down the amount of advertising shown on average by a single firm.

Let us now see the effect of learning on the equilibrium quantity choice of firm i as we did in the monopoly section. The equilibrium quantities in the learning scenario are given in Proposition 10 and the equilibrium quantities in the no learning scenario are given in Lemma 6. As before, we assume that if the equilibrium quantity stated in the Proposition (or Lemma) is arbitrary, then $q_i = 0$. Moreover, to make the results comparable (since for the middle range of k, there are always two symmetric equilibria where one firm advertises and the other does not), we choose the same symmetric equilibrium in both the learning and no learning cases, that is, $s_1^1 = 1$ and $s_2^1 = 0$. The following Proposition summarizes the results of the equilibrium quantity comparison.

Proposition 11 *Define q_i^L and q_i^{NL} as the equilibrium quantity choices of firm i in the "learning" ("L", $\gamma \neq 0$) and "no-learning" ("NL", $\gamma = 0$) cases respectively. Then:*

- *If $\frac{1}{16} < k < \frac{8}{75}$, then $q_1^L < q_1^{NL}$ and $q_2^L < q_2^{NL}$,*
- *Otherwise, $q_1^L \geq q_1^{NL}$ and $q_2^L \geq q_2^{NL}$.*

We see that, interestingly, as opposed to the monopoly case, the impact of learning on quantity choice is ambiguous. For the high values of the advertising cost ($k \geq \frac{8}{75}$), there is only one firm ever advertising, and so the monopoly results on the impact of learning on quantity carry over (that is, learning never decreases the equilibrium quantity). For the very low values of the cost ($k \leq \frac{1}{16}$), the learning and no learning cases lead to identical equilibrium quantity choices, because firms always advertise in period 1 regardless of learning. For the intermediate advertising cost ($\frac{1}{16} < k < \frac{8}{75}$) though, learning actually leads to lower quantity. When the cost is in the middle range, when learning is possible both firms advertise simultaneously in the first period while when learning is impossible only firm 1 advertises in period 1 (and in period 2, firms play a mixed advertising strategy where one random firm advertises and the other one does not). This means that the competitive pressure on demand is higher in the learning case leading the firms to reduce the quantities they offer. That is, the lack of learning mitigates competitive forces.

3.3.2 Competition with $A_i's$ Perfectly Correlated over Time

In the next section, we will consider an extension where the parameters describing the consumers' willingness to include products in their consideration set, A_1 and A_2, are perfectly correlated between the time periods. That is, the parameters A_i^t, where $i \in \{0; 1\}$ is the the firm and $t \in \{0; 1\}$ is the time period, are such that A_i^1 and A_i^2 are perfectly correlated. In this case, just as in the main competitive model described earlier, if firm i advertises in period 1, it is going to perfectly know the value of A_i^2 in period 2. The difference now is that firm i can potentially observe its competitor's second period parameter A^2_{-i}. Indeed, since $A^2_{-i} = A^1_{-i}$, the true value of A^2_{-i} can be deduced from observing the first period demand. For example, if firm 1 advertises and observes that its demand is $p_1 = 1 - q_1 - q_2$ (as opposed to $p_1 = 1 - q_1$), it knows that $A^1_2 = 1$ and, therefore, $A^2_2 = 1$.

Mathematically, the above peculiarity will change the solution of the period 2 subgame when the firms advertised in period 1. Turns out that the solutions to the other period 2 subgames (the ones where neither of the firms had advertised in period 1 or only one of the firms had advertised in period 1) do not change. Indeed, from the point of view of period 2, the only information that matters to firm i in the aforementioned subgames is its own signal about A^2_i. Firm i would never know the realization of A^1_{-i} regardless of whether A^t_i are perfectly correlated over time or are uncorrelated.

The following Proposition summarizes the equilibrium quantities, profits and advertising campaign choices.

Proposition 12 *Assume $\gamma = \frac{1}{2}$. Then, in any pure strategy Nash equilibria, equilibrium $s^2_i(\sigma_i = 0) = 0$. Furthermore:*

1. *When $k \leq \frac{5}{44}$ there is a unique symmetric equilibrium such that:*

$$s^1_1 = s^1_2 = 1,$$

$$s^2_1(\sigma_1 = 1) = 1 - s^2_1(\sigma_2 = 1),$$

$$q_1 = q_2 = \frac{1}{2},$$

$$\mathbb{E}\pi_1 = \mathbb{E}\pi_2 = \frac{5}{32} - \frac{11k}{8}.$$

2. *When $\frac{5}{44} < k \leq \frac{1}{6}$ there are two asymmetric equilibria (for $i = 1$ and for $i = 2$) such that:*

$$s^1_i = 1 \textit{ and } s^1_{-i} = 0,$$

$$s_i^1(\sigma_i = 1) = 1,$$

$$q_i = \frac{1}{2} \textit{ and } q_{-i} = \textit{arbitrary},$$

$$\mathbb{E}\pi_i = \frac{1}{4} - \frac{3k}{2} \textit{ and } \pi_{-i} = 0.$$

3. When $k > \frac{1}{6}$, there is a unique symmetric equilibrium such that:

$$s_1^1 = s_2^1 = 0,$$

$$q_1 \textit{ and } q_2 \textit{ are arbitrary},$$

$$\pi_1 = \pi_2 = 0.$$

The results are mostly similar to the baseline competition model described in Proposition 10. When the advertising cost is very small (that is, $k \leq \frac{5}{44}$), both firms advertise in period 1 because the payoff from advertising is large. In period 2 though, as opposed to the baseline competition model, at most one firm advertises. Indeed, if firm i receives a negative signal, it will never advertise in period 2 (remember that the signal is perfect). Also, if firm i knows that its competitor's $A_{-i}^2 = 1$ (because it could observe its competitor's $A_{-i}^2 = 1$ in period 1), it will not advertise either because it knows that its second period demand will be $q_1(1 - q_1 - q_2) = \frac{1}{2}\left(1 - \frac{1}{2} - \frac{1}{2}\right) = 0$ in case if it advertises. Interestingly, as well, is that the equilibrium quantities in this model are actually larger ($q_1^\star = q_2^\star = \frac{1}{2}$) than in the baseline model ($q_1^\star = q_2^\star = \frac{2}{5}$) and are equal to the monopoly quantities, that is, better information (about the competitor's A_{-i}^2) allows firms to avoid advertising simultaneously, and, therefore, to maintain the monopoly level of the quantities. When the advertising cost is very large (that is $k \geq \frac{1}{6}$), none of the firms ever advertise in any period due to the prohibitively large cost. Finally, when the advertising cost is in the intermediate range (that is, $\frac{5}{44} \leq k \leq \frac{1}{6}$), only one firm will ever advertise with the other firm never advertising. The cost in this case is too high to accommodate both firms and leave them non-negative profits. The one firm that advertises is effectively a monopolist and, therefore, the monopoly results carry over. This means that this firm will always advertise in period 1 and will advertise in period 2 if and only if it receives a positive signal. The quantity this firm sets is also the same as in the monopoly case and equal $\frac{1}{2}$.

Compared to the baseline competition model, in this extension there is no special sub-case with no learning. Indeed, just by advertising the firm infers its first period A_i^1 (and, hence, second period A_i^2) by observing the realized demand, so there cannot be a situation where advertising does not provide the firm with information as to the second period realization of A_i^2. Therefore, we cannot do the same exercise as we did in the section on the baseline monopoly case and compare the learning case to the no learning case. What we can do though, is compare this competition model to the baseline competition model with learning and see whether there are any differences.
To this goal, we compute the advertising intensity AI^{PC} (that is, advertising intensity in the perfectly correlated competition model) and compare it to the advertising intensity (3.7) from the baseline model. To compute AI^{PC} we use an already familiar formula (3.5):

$$AI = \mathbb{E}\left[(s_1^1)^\star + (s_2^1)^\star + (s_1^2)^\star + (s_2^2)^\star\right] =$$

$$= 1 * Pr[(s_1^1)^\star = 1] + 1 * Pr[(s_2^1)^\star = 1] + 1 * Pr[(s_1^2)^\star = 1] + 1 * Pr[(s_2^2)^\star = 1].$$

The outcomes are given in Proposition 12. When, k is very small (that is, $k \leq \frac{5}{44}$), both firms always advertise in period 1 and either none of the firms advertise in period 2 (in case they both received negative signals, which happens with probability $\frac{1}{4}$) or only one of them advertises (in all other cases, that is, with probability $\frac{3}{4}$). Therefore, the total two period advertising is $2+1*\frac{3}{4} = \frac{11}{4}$. When k is in the intermediate range (that is, $\frac{5}{44} \leq k \leq \frac{1}{6}$), one firm never advertises, while the other one always advertises in period 1 and advertises in period 2 only if it received a positive signal, that is, the total two period advertising is $1 + 1 * \frac{1}{2} = \frac{3}{2}$. Finally, when k is high (that is, $k \geq \frac{1}{6}$), none of the firms advertise, so the total two period advertising is 0. To summarize, the advertising intensity in the case of competition with perfectly correlated A_i^t's is:

$$AI^{PC} = \begin{cases} \frac{11}{4}, & \text{if } k \leq \frac{5}{44}, \\ \frac{3}{2}, & \text{if } \frac{5}{44} < k \leq \frac{1}{6}, \\ 0, & \text{if } k > \frac{1}{6}. \end{cases} \tag{3.8}$$

Comparing (3.7) and (3.8) we can observe 4 distinct regions, see Figure (3.4). When the advertising cost is very large (the red region, $k \geq \frac{1}{6}$), none of the firms ever advertise in any of the periods due to the large costs of doing so. Therefore, the advertising intensities are identical and equal to 0 in both cases. The models also behave identically for a slightly smaller cost of advertising (the green region, $\frac{5}{44} \leq k \leq \frac{1}{6}$) because in this region only one firm is advertising in both periods. Therefore, since the second firm does not participate, the correlation structure of A_i^t's does not matter. When the cost is even smaller the correlation structure starts to matter though. For the very small values of the cost (the blue region, $k \leq \frac{8}{75}$), the firms advertise more when they cannot observe their rival's A_i^2. Indeed, when the cost is very small, the firms would always advertise in the second period when they cannot observe the other firm's A_i^2. If they can observe A_i^2 though, it can only dampen the incentives to advertise because if the firm knows that its rival's $A_i^2 = 1$, then it might not want to advertise knowing that the rival is strong. When the cost is slightly higher (the orange region, $\frac{8}{75} \leq k \leq \frac{5}{44}$) though, the opposite is true. Even though, the information on the other firm's A_i^2 exerts downward pressure on the advertising intensity, this information also allows the firm to better advertise in the second period raising its expected period 2 profits. These higher second period profits also raise incentives to advertise in period 1 and in the orange region this effect wins.

Overall, having more information on the rival's outcomes has an ambivalent effect on total aggregate advertising by the competing firms. On the one hand, having more information can decrease advertising intensity due to the fact that the firms now will be able to avoid advertising simultaneously and, hence, avoid advertising competition, which means that the firms would ad-

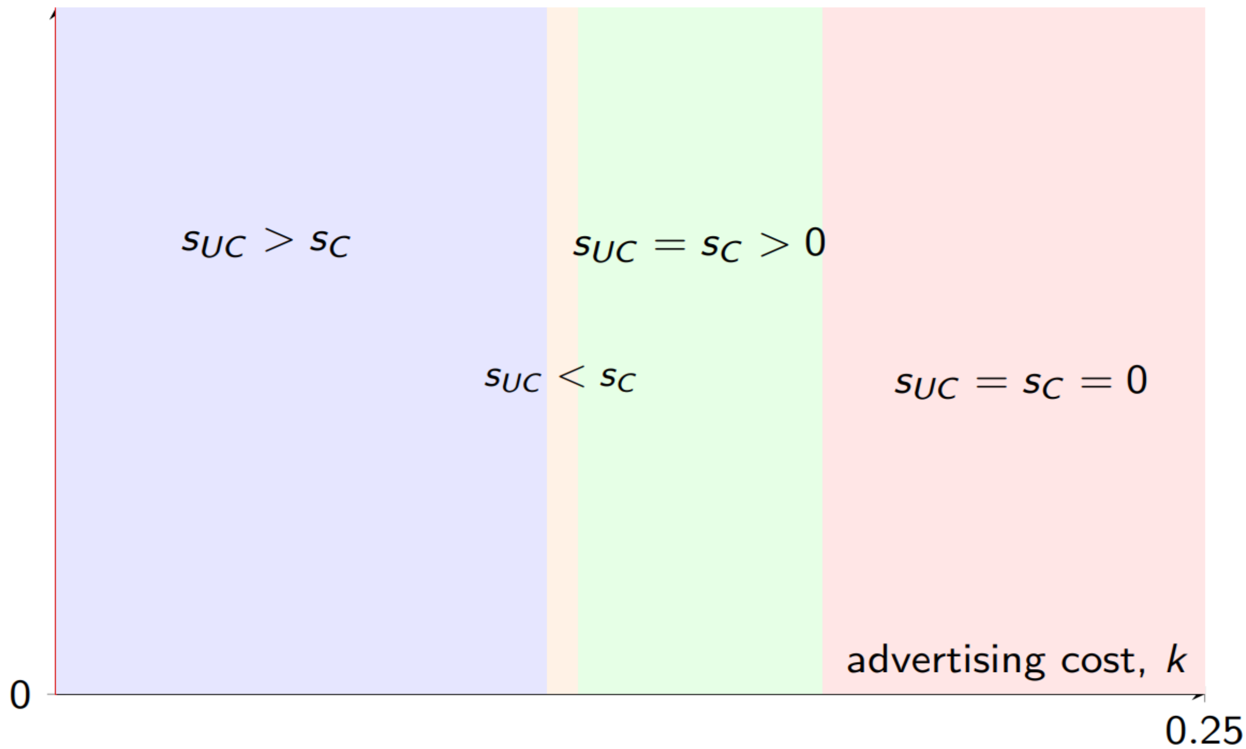

Figure 3.4: Competition with Perfect Learning: A_i^t perfectly correlated over time vs. A_i^t uncorrelated over time

vertise less often. But, on the other hand, having more information increases the expected second period profits, which also increases incentives to advertise in period 1. Whichever of these effects prevails depends on the actual market situation.

Finally, let us compare the equilibrium quantities between the two competitive models to see how the additional information on the opponent's A_i^2 impacts the equilibrium quantity choices of the competitors. Just as in the previous section on the baseline competitive model, we assume that if the equilibrium quantity is arbitrary, than the firms choose $q_i = 0$, and the same symmetric equilibrium is played in both competitive cases for the middle range of k, that is, $s_1^1 = 1$ and $s_2^1 = 0$. The following Proposition summarizes the results of the equilibrium quantity comparison.

Proposition 13 *Define q_i^C and q_i^{UC} as the equilibrium quantity choices of the firms in the "perfect correlation" ("C", A_i's are perfectly correlated over time) and "no correlation" ("UC", A_i's are uncorrelated over time) cases respectively. Then, it is always true that $q_i^C \geq q_i^{UC}$.*

The additional information on the competitor's parameter A_i^2 never leads to a lower equilibrium quantity. This result is due to the fact that in the perfect correlation case firms use their additional knowledge to reduce the intensity of competition as we mentioned earlier (via avoiding to advertise simultaneously when possible). This reduction in competitive pressure leads firms to choose a higher equilibrium quantity.

3.4 Concluding Remarks

In this paper we have developed a model to analyze how firms might learn the impact of their advertising on consumer demand by using the results of their previous advertising campaigns and what it means for the equilibrium outcomes, especially advertising. We assumed that firms are able to use their first period advertising campaign results as a signal of the potential success of the second period campaign, that is, use advertising as a learning tool for future periods. We have built a model that incorporates this effect by allowing the firms to receive an informative signal about their future advertising campaign effectiveness by providing more advertising in the current period. Using the model we study how this additional information affects the firms' equilibrium advertising choices and the overall aggregate level of advertising shown in different market situations.

We show that the effect of learning on advertising intensities is ambiguous in all the studied contexts. When there is only one monopolist firm on the market, the model's learning effects depend on the interplay between the advertising cost parameter and the learning efficiency parameter. On the one hand, learning can decrease advertising intensity because the firms' ability to learn discourages them from using indiscriminate advertising strategies in favor of more granular strategies. On the other hand, learning increases expected profits, and, hence, the returns to advertising, making the firms more eager to increase advertising intensity. When there is competition between firms, the learning effects stay ambiguous as well. On top of the learning effects shown in the monopolistic setting, there are additional effects pertaining to the fact that learning influences the number of firms advertising in an equilibrium. As the advertising cost increases, some firms may stop advertising because the market can no longer accommodate two firms simultaneously. Learning though, may lead to the firms still staying because it increases returns to advertising, thereby, increasing the advertising intensity. In the extension, we consider a situation where firms receive a signal not only about their own advertising campaign efficiency but also about their competitor's advertising campaign efficiency. In this model, the additional information can also either increase or decrease the advertising intensity compared to the main competition model. More information, on the one hand, might lead to lower advertising intensity due to the fact that firms knowing each other's signals can use it to avoid direct advertising competition and, on the other, more information leads to higher returns on advertising making the advertising intensity stronger. Finally, we show that the equilibrium quantities chosen by the firms usually increase when the firms can learn (*e.g.* the monopolist always sets a higher quantity when it can learn). However, there is a range of values of the advertising cost such that the quantities fall in the competitive case due to learning increasing competitive pressure.

In future research, there are several directions along which the paper can be improved. First, our competitive results might be extended to different types of competition. Especially interesting would be a pricing competition between firms instead of a quantity competition between firms. Second, a smoother version of the model with continuous advertising might provide additional insights as to the real world behavior of the firms. Third, we can relax the assumption that the firms only conduct two advertising campaigns since in reality the advertising dynamics are a lot more long term, so a model with an infinite horizon might be more realistic. Finally, it is interesting to see how the number of firms on the market affects advertising intensity.

Conclusion

In this dissertation proposal I have presented two chapters covering a broad topic of digital advertising. Chapter 1 ("Ad Blocking" with Miklos Sarvary and Zsolt Katona) is a study on the effects of ad blocking upon the digital advertising market participants, notably, on consumers. In this paper, we have analyzed how the current model of "whitelisting" employed by most ad blocking apps affects the key market outcomes such as content quality, consumer welfare and total welfare. We show that, the content quality predictably falls due to extortionary behavior by the ad blocker. However, surprisingly, consumer and total welfare also generally decline despite the fact that consumers see fewer ads than before. Turns out that the positive effect of ad reduction on consumer welfare is usually more than offset by the negative effect of declining quality. Nevertheless, there is always a segment of consumers who are better of under ad blocking, in particular, those who never consumed the publisher's content before they could block ads. The above effects are not sufficiently mitigated even if the publisher possess high bargaining powers. This implies that the current arrangement where regulatory institutions unambiguously support ad blocking should perhaps be reconsidered. Additionally, we consider a case of a total surplus maximizing platform (e.g. iOS, Google, Facebook) providing ad blocking services instead of a third party ad blocker. We show that unless this platform can fully commit to the maximum level of advertising beforehand, the platform cannot perform better in terms of consumer and total welfare than the benchmark with no ad blocking, because the publishers have to invest in quality upfront and given that the platform will now have incentives to over-restrict advertising since the quality cannot be changed anymore.

In Chapter 2 ("Demand Learning in Digital Advertising Markets" with Kinshuk Jerath and Miklos Sarvary) we study demand learning in digital advertising markets. We analyze how firms learn over time how their consumers respond to their advertising by using the outcomes of their previous advertising campaigns. We build a model that incorporates this learning effect by allowing the firms to receive a signal from their previous advertising campaigns in order to better tailor their advertising strategies in the next periods. We find that in all of our studied settings, learning has an ambiguous effect on advertising intensity. In a monopoly model we show that the monopolist will advertise more when he can learn but only if the cost of advertising is relatively large due to a larger return on advertising. On the other hand, though, when the cost is small the advertising intensity might fall because the monopolist firm will use a more complicated advertising strategy instead of blanket advertising. We strengthen our results by considering a model with two competing firms, where the effects of learning again turn out to be ambiguous. This is partially true due to the same dynamics that make the learning effects ambiguous in the monopolistic model and partially due to the new layer of competitive effects, in particular, due to the fact that the advertising cost now also determines the number of firms that advertise in any given period (*e.g.* as the cost increases one of the firms might drop out of the market). Additionally, we consider a different competitive model in an extension, where the firms do not only receive a signal as to their own advertising campaign efficiency but also about their competitors' advertising campaign efficiency. We show that, on the one hand, more information can decrease advertising because firms now will be able to avoid advertising simultaneously and decrease the competitive pressure. While, on the other hand, more information increases firms' returns to advertising making the advertising intensity higher. Finally, we analyze how learning affects the equilibrium quantity choices by the firms. We show that usually learning leads to higher equilibrium quantities since it increases expected profits. Nevertheless, learning might actually lead to lower quantity choices in the competitive case for a certain range of the advertising cost parameter. This happens because learning could result in an increase in competitive pressure.

In terms of future research, there are several directions along which the paper could be improved. First, our competitive results might be extended to different types of competition. Especially interesting would be a pricing competition between firms instead of a quantity competition between firms. Second, a smoother version of the model with continuous advertising might provide additional insights as to the real world behavior of the firms. Third, we can relax the assumption that the firms only conduct two advertising campaigns since in reality the advertising dynamics are a lot more long term, so a model with an infinite horizon might be more realistic. Finally, it is interesting to see how the number of firms on the market affects advertising intensity.

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Appendix A: Essay 1

A.1 Proof of Lemma 1

The paper already covers the case of $t = 0$. The exact same analysis applies to $0 < t < \frac{r}{1+r}$. When $\frac{r}{1+r} \le t \le 1$, the steps are similar.

Summarizing the three cases, consumer demand faced by the publisher is:

$$D(q, A) = \begin{cases} 1 & \text{if } 0 \le A \le qt \\ 1 - \frac{(A-tq)^2}{2Aq(1-r)(1-t)} & \text{if } qt \le A \le q \\ \frac{q(1+t)-2Ar}{2A(1-r)} & \text{if } q \le A \le t\frac{q}{r} \\ \frac{(q-Ar)^2}{2Aq(1-r)(1-t)} & \text{if } tq/r \le A \le q/r \\ 0 & \text{if } q/r \le A. \end{cases} \tag{A.1}$$

The objective function $p_a AD(q, A)$ is increasing in A in the first case and decreasing in A in the last three. In the second case, we maximize

$$\max_A \left(p_a A - \frac{p_a (A - tq)^2}{2q(1-r)(1-t)} - cq^2 \right),$$

which yields $A^\star = (1 - r(1 - t))q$. Substituting into the profit function results in a quadratic function of q:

$$\max_q \left(\frac{1 - r(1-t) + t}{2} \cdot p_a q - cq^2 \right),$$

Maximizing gives the stated formula.

A.2 Proof of Proposition 1

First, we determine the optimal A in the last stage of the game, given V and the set of consumers who download the ad blocker app. If the publisher optimizes only for consumers with a high enough κ who never download the app, the optimal A is the same as in the benchmark case, that is, $A^* = \max\left(\frac{q}{1+r}, q(1-r(1-t))\right)$. Setting a different level of advertising would result in a discrete downward jump in revenue for the segment that never downloads. Therefore, our assumption of a high enough $\overline{\kappa}$ ensures that the mass of consumers with high enough κ to never download the app is high enough for the overall optimum A to remain the same.[1]

In the negotiation stage, the adblocker first makes a take-it-or-leave it offer. As long as this offer results in a non-negative payoff for the publisher, the publisher will accept. Therefore, the adblocker always has an incentive to increase the demanded payment until it extracts all possible surplus. When this payment reaches the maximum the publisher becomes indifferent and our tie-braking assumption dictates that the publisher accepts. We discuss relaxing the tie-braking rule and the assumption around the power balance of negotiations in Section 2.3. Since the publisher will always accept the adblocker's offer, consumers will expect an advertising level of V when making the decision to download. Consumers who have a cost κ will download, if and only if, their $\gamma > \frac{(1+r)\kappa}{q-V(1+r)}$ and $\vartheta > \frac{\kappa}{q} + \gamma\frac{V}{q}$. The first constraint ensures that a consumer is better off with an ad level of V instead of $A = \frac{q}{1+r}$ and the second constraint guarantees a positive utility. When $\kappa > \max\left(\frac{q}{1+r}, q(1-r(1-t))\right) - V$, no consumer downloads the adblocker. When $\kappa < \frac{qr}{1+r} - rV$, all consumers who visit the publisher's site download the adblocker. When κ is between the two, we have a binding $\hat{\gamma}$ cutoff with $r < \hat{\gamma} = \frac{(1+r)\kappa}{q-V(1+r)} < 1$.

To determine the optimal V, we need to calculate how much surplus the adblocker can extract during the negotiations. This surplus is equal to the difference between the revenue of the publisher under 'light' adblocking (V) and full adblocking. This difference is simply the product of Vp_a and the amount of consumers who visit the site and download the adblocker. We begin with the case of $t \leq \frac{r}{1+r}$ when $A^\star = \frac{q}{1+r}$ and we also assume $V \geq \frac{qt}{r}$. For $\kappa < \frac{qr}{1+r} - rV$, the size of the segment

[1] We discuss the case of a fixed, potentially lower κ in the Web Appendix

is all consumers with $\vartheta > \frac{\kappa}{q} + \gamma\frac{V}{q}$, i.e.

$$S_{V1}(\kappa, q, V, r) = \frac{1}{1-t}\left(1 - \frac{\kappa}{q} - \frac{(1+r)V}{2q}\right). \tag{A.2}$$

For $\frac{qr}{1+r} - rV < \kappa < \frac{q}{1+r} - V$, the $\gamma > \frac{(1+r)\kappa}{q-V(1+r)}$ constraint is also binding and the size of the segment becomes

$$S_{V2}(\kappa, q, V, r) = \frac{1}{(1-r)(1-t)}\left(1 - \frac{(1+r)\kappa}{q-V(1+r)}\right)\left(1 - (1/2)\frac{\kappa}{q-V(1+r)} - (1/2)\frac{\kappa+V}{q}\right). \tag{A.3}$$

To determine the objective function of the adblocker, that is, the total surplus it can extract, we have to integrate over κ for each segment.

$$\begin{aligned} T(q,V) = T_1(q,V) &= p_a V \int_0^{\frac{qr}{1+r}-Vr} \frac{1}{\overline{\kappa}} S_{V1}(\kappa, q, V, 0)d\kappa + V p_a \int_{\frac{qr}{1+r}-Vr}^{\frac{q}{1+r}-V} \frac{1}{\overline{\kappa}} S_{V2}(\kappa, q, V, 0)d\kappa = \\ &= p_a V \frac{(q - V(1+r))(q(2+5r+2r^2) - V(1+r)(1+r+r^2))}{6q(1+r)^2(1-t)\overline{\kappa}}. \end{aligned} \tag{A.4}$$

There are two more cases we need to examine if $t \leq \frac{r}{1+r}$. When $\frac{qt}{r} \geq V \geq qt$, we get

$$T_2(q,V) = p_a \frac{V^2(1+r)^2(V-3q(1+r^2(1-t))) + Vq^2(2+3r-3r^2+3rt^2-2r^3+6r^2t^2+3r^3t^2) - q^3t^3(1+r)^2}{6q(1+r)^2(1-r)(1-t)\overline{\kappa}}$$

and when $qt \geq V \geq 0$, we have

$$T_3(q,V) = p_a V \frac{q(r+(2-3t^2)(1+r)^2) - 3V(1+r)^3(1-t)}{6(1+r)^2(1-t)\overline{\kappa}}.$$

All three of these functions are either cubic or quadratic in V, hence we can maximize them in V. Depending on the value of t, the optimal V can fall in either of the three segments. When $t \geq \frac{r}{1+r}$, we have $A^\star = q(1 - r(1-t))$ and we can similarly integrate to get the adblocker's payoff functions as

$$T_4(q,V) = p_a \frac{(q(1-r(1-t))-V)(V(1-r(1-t))(q(2-rt-3r^2(1-t))-V) - q^2t^3)}{6q(1-r)(1-t)(1-r(1-t))\overline{\kappa}}$$

when $v \geq tq$. If $tq \geq V \geq 0$ then

$$T_5(q,V) = p_a V \frac{q(3r^3t^2-6r^3t+2r^2t^2+3r^3+2r^2t+2rt^2-4r^2+2rt-t^2-r+2t+2)-3V(1+r)(1-r(1-t))}{6(1-r(1-t))\overline{\kappa}}$$

Again, these are cubic and quadratic in V. Overall maximization yields that $V^* = qF(r,t)$, where

$$F(r,t) = \begin{cases} \text{i)} & \frac{1+r}{1+r+r^2} - \frac{\sqrt{1+5r+9r^2+5r^3+r^4}}{\sqrt{3}(1+r)(1+r+r^2)} \\ \text{ii)} & \frac{3(1+r)(1-r^2(1-t)-\sqrt{3(1-r)(1-3r^5t^2+6r^5t-9r^4t^2-3r^5+18r^4t-12r^3t^2-9r^4+18r^3t-9r^2t^2-6r^3+6r^2t-3rt^2+4r^2+4r)}}{3(1+r)} \\ \text{iii)} & \frac{2+5r-3t^2+2r^2-6rt^3-3r^2t^2}{6(1+r)^3(1-t)} \\ \text{iv)} & \frac{3(1-r(1-t))(1-r^2(1-t))-\sqrt{3(1-r(1-t)(1-r)(1-t)(3r^4t^2-6r^4t+3r^4+3r^3t+r^2t^2-3r^3+r^2t+rt^2-2r^2+rt+t^2+r+t+1)}}{3(1-r(1-t))} \\ \text{v)} & \frac{3r^3t^2-6r^3t+2r^2t^2+3r^3+2r^2t+2rt^2-4r^2+2rt-t^2-r+2t+2}{6(1+r)(1-r(1-t))}. \end{cases} \tag{A.5}$$

$$\text{if} \begin{cases} \text{i)} & 0 \leq t \leq r\frac{3(1+r)^2-\sqrt{3r^4+15r^3+27r^2+15r+3}}{3(1+r)(1+r+r^2)} \\ \text{ii)} & r\frac{3(1+r)^2-\sqrt{3r^4+15r^3+27r^2+15r+3}}{3(1+r)(1+r+r^2)} \leq t \leq \min\left(\frac{r}{1+r}, \frac{3(1+r)^2-\sqrt{9r^4+24r^3+18r^2+9r+3}}{3(1+r)(1+2r)}\right) \\ \text{iii)} & \frac{3(1+r)^2-\sqrt{9r^4+24r^3+18r^2+9r+3}}{3(1+r)(1+2r)} \leq t \leq \frac{r}{1+r} \\ \text{iv)} & \frac{r}{1+r} \leq t \leq \frac{(3r^2-r-2+\sqrt{3(1+r)(3r+2)})(1-r)}{1+4r+4r^2-3r^3}, \\ \text{v)} & \max\left(\frac{r}{1+r}, \frac{(3r^2-r-2+\sqrt{3(1+r)(3r+2)})(1-r)}{1+4r+4r^2-3r^3}\right) \leq t \leq 1. \end{cases}$$

A.3 Proof of Proposition 2

In the first stage, the publisher maximizes its profit given V. The publisher's revenue is exclusively from consumers who did not download the adblocker as all the surplus generated by the downloaders is extracted by the adblocker. We begin with the case of $t \leq \frac{r}{1+r}$. We calculate the lost revenue compared to the benchmark case of no adblocker. The lost revenue for consumers in $\frac{qr}{1+r} - Vr < \kappa < \frac{q}{1+r} - V$ is:

$$\frac{qp_a}{(1+r)(1-r)(1-t)}\left(1 - \frac{\kappa}{2(q-V(1+r))} - \frac{1}{2(1+r))}\right)\left(1 - \frac{\kappa(1+r)}{(q-V(1+r))}\right). \tag{A.6}$$

For $\kappa > \frac{q}{1+r} - V$, the lost revenue is 0, whereas for $\kappa < \frac{qr}{1+r} - Vr$ it is $\frac{qp_a}{2(1+r)(1-t)}$. Integrating over κ yields

$$\frac{qp_a}{(1+r)(1-t)\overline{\kappa}}\left(\int_{\frac{qr}{1+r}-Vr}^{\frac{q}{1+r}-V}\frac{1}{1-r}\left(1-\frac{\kappa}{2(q-V(1+r))}-\frac{1}{2(1+r))}\right)\left(1-\frac{\kappa(1+r)}{(q-V(1+r))}\right)d\kappa+\right.$$
$$\left.+\int_0^{\frac{qr}{1+r}-Vr}1d\kappa\right)=\frac{(r^2+4r+1)(q-(1+r)V)qp_a}{6\overline{\kappa}(1+r)^3(1-t)}. \quad \text{(A.7)}$$

Substituting the equilibrium V and subtracting from the benchmark, we get the objective function of

$$\frac{qp_a}{2(1+r)(1-t)}-\frac{(r^2+4r+1)(1-(1+r)F(r))q^2p_a}{6\overline{\kappa}(1+r)^3(1-t)}-cq^2$$

Since this is a simple quadratic function in q, the optimal quality is

$$q^*=\left(\frac{4c(1+r)(1-t)}{p_a}+\frac{2(r^2+4r+1)(1-(1+r)F(r))}{3\overline{\kappa}(1+r)^2}\right)^{-1},$$

which is clearly an increasing function of $\overline{\kappa}$ and converges to $\frac{p_a}{4c(1+r)(1-t)}$ as $\overline{\kappa}\to\infty$. For $t \geq \frac{r}{1+r}$, similar calculations yield the objective function as

$$\frac{qp_a(1-r(1-t)+t)}{2}-\frac{((3r+1)(1-r)^2+(1+r+t+rt+4r^2+r^2t-6r^3+3r^3t)t)(1-r(1-t)-F(r))q^2p_a}{6\overline{\kappa}(1-r(1-t))}-cq^2$$

Maximizing in q yileds the formula stated in the Proposition where

$$G(r,t)=\begin{cases}\frac{2(r^2+4r+1)(1-(1+r)F(r,t))}{3(1+r)^2} & \text{if} \quad t\leq\frac{r}{1+r}\\ \frac{2((3r+1)(1-r)^2+(1+r+t+rt+4r^2+r^2t-6r^3+3r^3t)t)(1-r(1-t)-F(r,t))}{3(1-r(1-t))(1-r(1-t)+t)} & \text{if} \quad \frac{r}{1+r}\leq t\end{cases} \quad \text{(A.8)}$$

Given the quadratic profit function, it is easy to calculate the equilibrium profit, yielding the comparative statics results:

$$\Pi^{\star}_{PUB} = \begin{cases} \frac{q^{\star} p_a}{2(1+r)(1-t)} & \text{if} \quad t \leq \frac{r}{1+r} \\ \\ \frac{q^{\star} p_a (1-r(1-t)+t)}{4} & \text{if} \quad \frac{r}{1+r} \leq t \end{cases}$$

A.4 Proof of Corollary 1

In (A.4), and the following equations, we have essentially calculated the size of the downloader segment which is $T(q, V)/(V p_a)$. Plugging in $V^{\star} = q^{*} F(r)$ yields that in equilibrium the size of this segment is a which is a linear function of $\frac{q^{*}}{\overline{\kappa}}$ and its multiplier only depends on r and t. Since q^{*} is an increasing function of p_a so is the segment size. Furthermore, $\frac{q^{*}}{\overline{\kappa}}$ is a decreasing function of $\overline{\kappa}$.

A.5 Proof of Corollary 2

Consumers who do not download the adblocker are worse off, because the adblocker causes the publisher quality to decline, reducing their surplus. In particular, a consumer with $\gamma = r$ and $\vartheta = 1$ has a surplus of $q - rA = q(\min\left(\frac{1}{1+r}, 1 - r + r^2(1-t)\right)) > 0$ if $r < 1$. This is surplus clearly goes down as the adblocker enters and quality decreases. On the other hand, there are consumers, who only visit the publisher's site if there is an adblocker. Consumers who are below the $\vartheta = \gamma \max\left(\frac{1}{1+r}, 1 - r(1-t)\right)$ line on the (ϑ, γ) space do not visit the publisher's site if there is no adblocking and hence get zero utility in the absence of the adblocker. However, in the presence of an adblocker, consumers with ϑ in the range $\gamma \max\left(\frac{1}{1+r}, 1 - r(1-t)\right) > \vartheta > \frac{\kappa + \gamma V}{q}$ visit the site get positive utility their κ is low enough.

A.6 Proof of Proposition 3

Total consumer utility can be calculated as

$$CS_{AB} = q^\star C_0(r,t) + (q^\star)^2 \frac{C_\Delta(r,t)}{\overline{\kappa}}. \tag{A.9}$$

The first component is the benchmark consumer surplus. The second component is positive and represents the additional surplus consumers enjoy from adblocking when quality is fixed. The benchmark consumer surplus multiplier is

$$C_0(r,t) = \begin{cases} \frac{1+r+r^2}{6(1+r)^2} & \text{if } \; t \leq \frac{r}{1+r} \\ \frac{(1-t)^2(1+r+r^2)+3((t-r(1-t))(1-r+r^2(1-t))}{6(1-r(1-t))} & \text{if } \; \frac{r}{1+r} \leq t \end{cases}$$

The second component is a longer formula, but we can calculate it for certain parameter regions:

$$C_\Delta(r,0) = \frac{(2(r+1) - F(r,t)(1+r^2))(1 - F(r,t)(1+r))^2}{24(1+r)}.$$

$$C_\Delta(r,1) = \frac{(1+r+r^2)(1-F(r,t))^2}{6}.$$

$$C_\Delta(0,t) = \begin{cases} \frac{(F(r,t)(2-F(r,t))-t^4)(1-F(r,t))^2}{24F(r,t)(1-t)} & \text{if } \; t \leq \sqrt{6}-2 \\ \frac{2(1+t+t^2-t^3)+F(r,t)(4(F(r,t)-5-5t+t^2+t^3)}{24} & \text{if } \; \sqrt{6}-2 \leq t \end{cases}$$

$$C_\Delta(1,t) = \begin{cases} \frac{(2-F(r,t))(1-2F(r,t))^2}{24} & \text{if } \; t \leq \frac{4-\sqrt{7}}{6} \\ \frac{4(1-t)(3F(r,t)(F(r,t)-1-t)+t^2)-3F(r,t)-2+4t^2}{24(1-t)} & \text{if } \; \frac{4-\sqrt{7}}{6} \leq t \leq \frac{r}{1+r} \\ \frac{(t-F(r,t))^2}{2} & \text{if } \; \frac{r}{1+r} \leq t \end{cases}$$

To compare to CS_{NA}, we examine CS_{AB} as a function of $\overline{\kappa}$. As $q^\star$ converges to $q^\star_{NA}$ when $\overline{\kappa} \to \infty$, we get the same for CS_{AB}. What we need to determine is whether it converges increasingly or decreasingly. Plugging in the formula for $q^\star$ from Proposition 2 and differentiating with respect to $\overline{\kappa}$, we find that there are two distinct cases. The derivative is always positive for small values of $\overline{\kappa}$. However, depending on whether $C_\Delta(r,t) > C_0(r,t)G(r,t)$ or not, the derivative either crosses into negative at some threshold of $\overline{\kappa}$ or stays positive for all $\overline{\kappa}$. Hence consumer surplus is increasing for all $\overline{\kappa}$ values if $C_\Delta(r,t) \leq C_0(r,t)G(r,t)$ or it is first increasing and then decreasing after reaching a maximum if $C_\Delta(r,t) > C_0(r,t)G(r,t)$. The function $G(r,t)$ is given in (A.8) and, as we can see, the condition only depends on r, t. Figure 2.3, presents the contour line for $C_\Delta(r,t) = C_0(r,t)G(r,t)$ which is the solution of a sixth degree polynomial, hence it has no closed form solution. For the purposes of the corollary, all we need to show is that $C_\Delta(0,t) \leq C_0(0,t)G(0,t)$ for any t, that $C_\Delta(r,0) \leq C_0(r,0)G(r,0)$, for any r and that $C_\Delta(1,1) \leq C_0(1,1)G(1,1)$. With the formulas given above, this is straightforward to verify. Since both sides of the inequality change continuously, we get that the left side is smaller when either r or t is low and that the left side is larger when both r and t are high, yielding the stated result.

A.7 Proof of Corollary 3

To see that total welfare is lower (even if we include the adblocker's surplus), we need to establish that the sum of the publisher surplus and the adblocker surplus is lower in the presence of the adblocker. There are two effects. First, the adblocker extracts some of the surplus generated the by the publisher. Keeping the quality fixed, this is a zero-sum game. The second effect of the adblocker is that quality decreases. But with a lower quality, the publisher surplus decreases. The combination of the two effects reduces the sum of the publisher and adblocker surpluses. Clearly if consumer surplus is also reduced by the adblocker's entry the total welfare decreases as well.

When $CS_{AB} > CS_{NA}$, we need to check whether the increase can counteract the decrease in the sum of the publisher and adblocker's surplus. p_a plays a crucial role here as both the publisher's and the adblocker's surplus are scaled by it. $q^\star$ is also dependent on p_a, but only as a function of p_a/c. Hence if p_a is reduced while keeping p_a/c constant, the sum of the losses of the publisher and the adblocker can be reduced compared to the gains in consumer surplus. On the contraery, if either p_a or c is too high, total welfare is reduced.

A.8 Proof of Lemma 2

We determine the optimal V just as in the proof of Proposition 1 by first calculating the amount of people who download the adblocker and use it for partial adblocking. In the typical case depicted in Figure 2.4 this segment is in the (ghik) trapezoid with a size of

$$S_{V2}(\kappa,\mu,q,V,r)=\frac{1}{1-r}\left(\frac{\mu\kappa}{V}-\frac{(1+r)\kappa}{q-V(1+r)}\right)\left(1-\frac{\kappa}{2(q-V(1+r))}-\frac{(1+\mu)\kappa}{2q}\right).$$

This formula applies for $\frac{qr}{1+r}-rV<\kappa<\frac{V}{\mu}$. For smaller $\kappa<\frac{qr}{1+r}-rV$, we have

$$S_{V1}(\kappa,\mu,q,V,r)=\frac{1}{1-r}\left(\frac{\mu\kappa}{V}-r\right)\left(1-\frac{(1+\mu)\kappa}{2q}-\frac{\kappa+rV}{2q}\right).$$

representing the (deki) trapezoid. And for larger $\frac{V}{\mu}<\kappa<\frac{q}{1+r}-V$, we obtain

$$S_{V3}(\kappa,\mu,q,V,r)=\frac{1}{1-r}\left(1-\frac{(1+r)\kappa}{q-V(1+r)}\right)\left(1-\frac{\kappa}{2(q-V(1+r))}-\frac{\kappa+V}{2q}\right).$$

for the (abik) trapezoid. Assuming $r<1$ and integrating over κ yields the publisher's payoff function:

$$\begin{aligned}T(q,V)=Vp_a&\int_0^{\frac{qr}{1+r}-Vr}\frac{1}{\overline{\kappa}}S_{V1}(\kappa,q,V,0)d\kappa+\\&+Vp_a\int_{\frac{qr}{1+r}-Vr}^{\frac{V}{\mu}}\frac{1}{\overline{\kappa}}S_{V2}(\kappa,q,V,0)d\kappa+Vp_a\int_{\frac{V}{\mu}}^{\frac{q}{1+r}-V}\frac{1}{\overline{\kappa}}S_{V3}(\kappa,q,V,0)d\kappa=\\&=Vp_a\left(\frac{(q-V(1+r))(q(2+5r+2r^2)-V(1+r)(1+r+r^2))}{6q(1+r)^2\overline{\kappa}}-\frac{3\mu qV-(1+2\mu)V^2}{6q\mu^2(1-r)\overline{\kappa}}\right)\end{aligned}$$

if $V < \frac{\mu q}{(1+\mu)(1+r)}$ and 0 otherwise. Maximizing $T(q, V)$ yields

$$V^* = q\mu\left(\frac{1-\mu-\mu r^2}{1-2\mu-\mu^2-\mu^2 r^3} - \frac{\sqrt{\mu^2(1-r)^2(1+5r+9r^2+5r^3+r^4)+2\mu(1-r)(3r^3+7r^2+4r+1)+2r^3+6r^2+3r+1}}{\sqrt{3}(1+r)(1-2\mu-\mu^2-\mu^2 r^3)}\right) = qF_\mu(r).$$

Analyzing $F_\mu(r)$ at different values yields the stated results.

A.9 Proof of Proposition 4

The analysis is identical to that in the proof of Proposition 2, but with $F_\mu(r)$ instead of $F(r)$.

A.10 Proof of Corollary 4

Total consumer utility can be calculated as $CS_\mu = q^\star C_{\mu 0}(r, 0) + (q^\star)^2 \frac{C_{\mu\Delta}(r,0)}{\overline{\kappa}}$ where

$$C_0(r, 0) = \frac{1 + r + r^2}{6(1 + r)^2} \quad \text{and} \quad C_{\mu\Delta}(r, 0) = \frac{(2(r + 1) - F_\mu(r)(1 + r^2))(1 - F_\mu(r)(1 + r))^2}{24(1 + r)}$$

Using $G_\mu(r, 0) = \frac{2(r^2+4r+1)(1-(1+r)F_\mu(r))}{3(1+r)^2}$, we can see that $C_{\mu\Delta}(r, 0) < C_0(r, 0)G_\mu(r, 0)$, yielding that consumer surplus increases in $\overline{\kappa}$ as it approaches the benchmark. We have shown in the proof of Corollary 3 that for $t = 0$ the sum of the publisher's and the adblocker's surplus is lower in the full model than in the benchmark case. Both the publisher's and the adblocker's surplus are lower in this version of the model with the full adblocking option as the quality is lower and the adblocker collects a lower revenue per consumer from a smaller segment.

When $t = 1$, we have

$$F^\star_\mu(r, 1) = \frac{\mu}{2(1 + \mu)}, \; G_\mu(r, 1) = \frac{1 + r}{2} \cdot \frac{2 + \mu}{1 + \mu}, \; C_{\mu 0}(r, 1) = \frac{1 - r}{2}, \; C_{\mu\Delta}(r, 1) = \frac{(4 + \mu)(1 + r + r^2)}{24(1 + \mu)}$$

Comparing $C_{\mu\Delta}(r, 1)$ with $C_0(r, 1)G_\mu(r, 1)$ reveals that the left hand side is larger if and only if $r > \hat{r}(\mu) = \frac{\sqrt{141\mu^2+552\mu+528}-4-\mu}{14\mu+32}$, where $\hat{r}(\mu)$ is increasing in μ.

A.11 Proof of Proposition 5

As in the main model, the advertising levels are not affected by the adblocker because there are enough consumers with high download cost who do not use the adblocker and thus ad levels are the same as given in Lemma 1 as a funciton of q. Specifically, for $0 \le t \le \frac{r}{r+1}$, we have $A = \frac{q}{1+r}$. Given the ad levels and p_b, it is straightforward to calculate the number of downloaders, which yields the optimal price. Assuming $0 \le t \le \frac{r}{r+1}$, and $r \le \frac{\sqrt{57}-7}{2}$ we get

$$p_b^* = \frac{9(1+r)+1-\sqrt{81r^2+84r+36}}{16(1+r)}q$$

Given this price, consumers with $\gamma > \frac{\kappa+p_b^*}{q}(1+r)$ and $\vartheta > \frac{\kappa+p_b^*}{q}$ will download and use the adblocker. Hence, the publisher will only be able to generate revenue on the remaining consumers. The publisher's payoff can be calculated as

$$\frac{qp_a}{2(1+r)(1-t)} - \frac{(3-r-6r^2)(3-r)^2q^2p_a}{384\overline{\kappa}(1+r)^2(1-r)(1-t)} - cq^2$$

as long as $r < \frac{\sqrt{57}-7}{\sqrt{57}-5}$. Since this is a simple quadratic function in q, the optimal quality is

$$q^* = \left(\frac{4c(1+r)(1-t)}{p_a} + \frac{(3-r-6r^2)(3-r)^2}{96\overline{\kappa}(1+r)^2(1-r)(1-t)}\right)^{-1}$$

When $t=0$, this yields $q^*_{CC} \xrightarrow{r\to 0} \left(\frac{4c}{p_a} + \frac{9}{32\overline{\kappa}}\right)^{-1} < q^*_{NA} = \frac{p_a}{4c}$. Calculating $q^*_{AB} \xrightarrow{r\to 0} \left(\frac{4c}{p_a} + \frac{2\sqrt{3}}{9\overline{\kappa}}\right)^{-1}$ yields $q^*_{AB} < q^*_{CC}$ for small values of r, specifically for $r < \frac{\sqrt{57}-7}{\sqrt{57}-5}$. The publisher's profit is then simply $\Pi^{pub\star}_{CC} \xrightarrow{r\to 0} p_a^2\left(16c + \frac{9p_a}{8\overline{\kappa}}\right)^{-1} < \Pi^{pub\star}_{NA} = \frac{p_a^2}{16c}$. and $\Pi^{pub\star}_{AB} \xrightarrow{r\to 0} p_a^2\left(16c + \frac{8\sqrt{3}}{9\overline{\kappa}}\right)^{-1}$ in the two cases, yielding $\Pi^{pub\star}_{AB} < \Pi^{pub\star}_{CC} < \Pi^{pub\star}_{NA}$ for small r values. Next, we calculate the adblocker's profit as a function of q for the same range of small r values:

$$\Pi^{Adb}_{CC} = \frac{\left(9(1+r)+1-\sqrt{81r^2+84r+36}\right)\left(67r^2+124r-20+(21r+18)\sqrt{81r^2+84r+36}\right)}{12288\overline{\kappa}(1+r)^3(1-t)}q^2$$

We then have $\Pi^{Adb}_{CC} \xrightarrow{r\to 0} \frac{11}{384\overline{\kappa}(1-t)}q^2$ and from our previous analysis, we have $\Pi^{Adb}_{AB} \xrightarrow{r\to 0} \frac{\sqrt{3}}{18\overline{\kappa}(1-t)}q^2$. Thus, for the same quality level charging consumers leads to more than three times lower profit. However, the quality is slightly higher when charging consumers, but $\frac{q^*_{CC}}{q^*_{AB}} < \frac{64\sqrt{3}}{81} < 1.369$ for small r, hence $\Pi^{Adb\star}_{CC} < \Pi^{Adb\star}_{AB}$. Moving on to the consumer surplus, we start from (A.9) to establish that

$$CS_{AB} \xrightarrow{r\to 0} q^\star_{AB}C_0(0,t) + (q^\star_{AB})^2\frac{3+\sqrt{3}}{216\overline{\kappa}},$$

where $CS_{NA} = q^\star_{NA}C_0(r,t)$. We can similarly calculate

$$CS_{CC} \xrightarrow{r\to 0} q^\star_{CC}C_0(0,t) + (q^\star_{CC})^2\frac{27}{1024\overline{\kappa}}$$

and a simple comparison yields $CS^\star_{AB} < CS^\star_{CC} < CS^\star_{NA}$ for small r values. Adding the resulting consumer suprlus to publisher and adblocker profits further yields $TW^\star_{AB} < TW^\star_{CC} < TW^\star_{NA}$.

When $r = t = 1$, the analysis becomes simpler as consumers are only heterogenous with respect to the download costs. As before, the publisher sets the advertising to q, regardless of who downloads the adblocker. Then any consumer for whom $\kappa + p_b < q$ will download and use the adblocker. This yields a profit function of $p_b\frac{q-p_b}{\overline{\kappa}}$ for the adblocker with an optimal price of $p^\star_b = q/2$. Recall that this is the same value as $V^\star = q/2$ in the main model. As a result in equilibrium, the exact same consumers will download the adblocker regardless of whether the publisher or consumers are charged. The publisher only makes money on consumers who do not download the adblocker, hence publisher profits will be the same as well. Given the equal cost to the downloading consumers – whether it is through a direct price or non-zero advertising, – consumer surplus will be the same, as well as adblocker profits and also total welfare.

A.12 Proof of Proposition 6

We have calculated the size of the segment who does not download the adblocker in (A.7) as

$$SNDL(r) = \frac{1}{2} - \frac{(r^2+4r+1)(1-(1+r)F(r))q}{6\overline{\kappa}(1+r)^2}$$

The publisher gets $\frac{qp_a}{1+r}$ for each consumer in this segment. The size of the segment who downloads was calculated in (A.4). After substituting $V = qF(r)$, we get

$$SDL(r) = \frac{q(1-(1+r)F(r))(2+5r+2r^2-(1+r)(1+r+r^2)F(r))}{6(1+r)^2\overline{\kappa}}$$

The publisher gets $(1-\alpha)Vp_a = (1-\alpha)F(r)qp_a$ for each consumer in this segment. Hence the publisher maximizes

$$\frac{qp_a}{1+r}SNDL(r) + (1-\alpha)F(r)qp_aSDL(r) - cq^2$$

in q, yielding

$$q^\star(\alpha) = \left(\frac{4c(1+r)}{p_a} + \frac{2(1-(1+r)F(r))}{3\overline{\kappa}(1+r)} \cdot \frac{r^2+4r+1-(1-\alpha)(1+r)F(r)(2+5r+2r^2-(1+r)(1+r+r^2)F(r))}{1+r}\right)^{-1}$$

which is the same as stated in the proposition with $H(r) = (1+r)F(r)(1-(1+r)F(r))(2+5r+2r^2-(1+r)(1+r+r^2)F(r))$. Since $H(r) > 0$ is increasing, we get the comparative statics results.

A.13 Proof of Corollary 5

We use the same steps as in the proof of Proposition 3 and check whether $C_\Delta() > C_0()G()$. The particular formulas for $t = 0$ are

$$C_0(r,0) = \frac{1+r+r^2}{6(1+r)^2},$$

$$C_\Delta(r,0) = \frac{(2(r+1)-F(r,0)(1+r^2))(1-F(r,0)(1+r))^2}{24(1+r)},$$

$$G_\alpha(r,0) = \frac{2(1-(1+r)F(r,0))\left(r^2+4r+1-(1-\alpha)(1+r)F(r,0)(2+5r+2r^2-(1+r)(1+r+r^2)F(r,0))\right)}{3(1+r)^2},$$

We find that when $r = \alpha = 0$, we get $C_\Delta(r, 0) > C_0(r, 0)G_0()$. For positive α and r values, the inequality quickly reverses. When $t = 1$, we have

$$C_0(r, 1) = \frac{1 - r}{2}, \quad C_\Delta(r, 1) = \frac{1 + r + r^2}{24}, \quad G_\alpha(r, 1) = \frac{(1 + r)(1 + \alpha)}{4},$$

yielding that consumer surplus can go up if and only if $1 + r + r^2 > 3(1 - r)(1 + r)(1 + \alpha)$.

A.14 Proof of Proposition 7

The adblocker maximizes $(1-\alpha)F(r)qp_aSDL(r)$ in α. This function takes the form of $const \cdot q^\star(\alpha)^2\alpha$, where $q^\star(\alpha) = const' \cdot (D + \alpha)^{-1}$ with

$$D = \frac{(c\overline{\kappa}/p_a) \cdot 2(1 + r)^3 + (r^2 + 4r + 1)(1 - (1 + r)F(r)) - H(r)}{H(r)},$$

and the constanst not depending on α. Maximizing $(D + \alpha)^{-2}\alpha$ yields a single maximum at $\alpha = \min(D, 1)$. Given the expression for D, we see that it is increasing in $(c\overline{\kappa}/p_a)$. Furthermore, setting $c\overline{\kappa}/p_a = 0$ gives $0 < D < 1$, and since D is linear in $(c\overline{\kappa}/p_a)$, we get that there is a single threshold where $D = 1$.

A.15 Web Appendix

In this web appendix we conduct a number of robustness checks and additional analysis to support the findings of our paper. First, in Section A.15.1, we explore what happens when the publisher has the option to switch to a subscription model. In Section A.15.2, we study a content platform that conducts adblocking itself instead of allowing third party adblockers. In Section A.15.3, we explore the possibility of multiple publishers that are either competing with each other directly or just interact indirectly, whereas in Section A.15.4, we discuss the nature of competition between different adblockers. Next, in Section A.15.5, we examine the nature of consumer heterogeneity looking at different models with heterogeneity along only two dimensions and in Section

A.15.6, we examine a setting in which some consumers like ads. Finally, Section A.15.7 analyzes a variation of the model where the ad price increases with quality.

A.15.1 Subscription model

Assume that the publisher has the option of using a subscription model instead of relying on ads for revenues.[2] Under the subscription model, consumers need to pay a fee to the publisher for accessing its site. We assume that in the last stage of the game, the publisher can switch to a subscription model if advertising is not appealing due to the adblocker's rent seeking. This can be used as a threat in the negotiation by the publisher. For the sake of parsimony, we set $t = 0$ throughout this section.

First, we analyze the benchmark case without ad blocking. If the publisher chooses the subscription model then consumers buy a subscription when $\vartheta q - s \geq 0$, i.e. when $\vartheta \geq s/q$, where s is the subscription fee. This implies that the publisher's demand is:

$$D_S = \begin{cases} 1 - \frac{s}{q}, & \text{if } s/q \leq 1 \\ 0, & \text{if } s/q > 1. \end{cases}$$

Hence, the publisher solves:

$$s\left(1 - \frac{s}{q}\right) \Rightarrow \max_{s} \quad s.t. \quad \frac{s}{q} \leq 1,$$

which yields the following optimal s and corresponding profits (for a given q):

$$s^\star = \frac{q}{2}, \quad \pi_{sub}^{pub} = \frac{q}{4} - cq^2.$$

Comparing this profit to the profit resulting from advertising (2.2), it is evident that even without the adblocker the publisher never chooses the advertising revenue model if $p_a < \frac{1+r}{2}$, i.e. when

[2] For simplicity, we assume that the publisher cannot discriminate between consumers and offer different revenue models to consumers who did or did not download the adblocker's app.

the price of an ad impression is too low.[3] Therefore, to make the case of subscription relevant, we assume that $p_a > \frac{1+r}{2}$. For ease of exposition, we fix $r = 1/2$, which translates this condition to: $p_a > 3/4$ (the analysis considers a general r).

To analyze the case with the adblocker, we follow similar steps as in the main model (for details see the proof of Proposition 14). We can write the total profit generated by the publisher as:

$$\pi^{pub}_{adv} = \frac{p_a q(54\bar{\kappa} - 26q + 39V)}{162\bar{\kappa}} + \frac{(1-\alpha)p_a V(40q - 21V)(2q - 3V)}{216\bar{\kappa}q} - cq^2. \tag{A.10}$$

The first term denotes the profit generated from the non-downloaders (and, therefore, cannot be expropriated by the adblocker) whereas the last term is the surplus generated by the downloaders, therefore, can be - at least partially - expropriated by the adblocker. Denote by α the share of the last term that is actually captured by the adblocker. The adblocker's problem, hence, becomes to maximize its share of the surplus while keeping the publisher away from subscription. More precisely:

$$\max_V \frac{p_a V(40q - 21V)(2q - 3V)}{216\bar{\kappa}q}\alpha$$

$$s.t.\ \frac{p_a q(54\bar{\kappa} - 26q + 39V)}{162\bar{\kappa}} + \frac{(1-\alpha)p_a V(40q - 21V)(2q - 3V)}{216\bar{\kappa}q} \geq \frac{q}{4}.$$

Since we have assumed that $p_a \geq \frac{3}{4}$, this problem always has a non-trivial solution, that is, there is a combination of α and V such that the publisher chooses to advertise. The following proposition summarizes the equilibrium quality and optimal V.

Proposition 14 *Assume that $p_a \geq \frac{3}{4}$ and the publisher can offer a subscription to consumers. Then, in the presence of the adblocker, the publisher chooses the advertising revenue model and sets quality to:*

$$q^\star = \begin{cases} \frac{1}{8c}, & \text{if } c < \frac{13(\sqrt{309}-6)p_a}{189\bar{\kappa}(16p_a^2-9)}, \\ \frac{567\bar{\kappa}p_a}{3402c\bar{\kappa}+26(\sqrt{309}-6)p_a}, & \text{if } c \geq \frac{13(\sqrt{309}-6)p_a}{189\bar{\kappa}(16p_a^2-9)}. \end{cases}$$

[3] It is straightforward to show that a mixed revenue model is never optimal.

The maximum level of advertising that the adblocker lets through is:

$$V^{\star} = \begin{cases} \frac{1}{12c} - \frac{9\bar{\kappa}(4p_a - 3)}{26p_a}, & \textit{if } c < \frac{13(\sqrt{309}-6)p_a}{189\bar{\kappa}(16p_a^2-9)}, \\ \frac{9(27-\sqrt{309})\bar{\kappa}p_a}{1701c\bar{\kappa}+13(\sqrt{309}-6)p_a}, & \textit{if } c \geq \frac{13(\sqrt{309}-6)p_a}{189\bar{\kappa}(16p_a^2-9)}. \end{cases}$$

and some consumers download the ad blocker app.

PROOF:

We have seen that when the publisher chooses the subscription revenue model then the optimal s and the corresponding profits are:

$$s^{\star} = \frac{q}{2}, \quad \pi_{sub}^{pub} = \frac{q}{4} - cq^2.$$

If the publisher chooses the advertising revenue model, then given that $\bar{\kappa}$ is high enough, its choice of the advertising volume is $A^{\star} = \frac{q}{1+r}$ as in the analysis of the base model. In the presence of the adblocker, the regions to explore are the same as in the main model.

Region 1: $\kappa < \frac{qr}{1+r} - rV$.

All of the consumers download the adblocker. The publisher's profit in this region is $p_a V$ times the size of the consumer segment $S_{V1}(\kappa, q, V, r)$ (see (A.2)):

$$\pi_1^{pub} = p_a V S_{V1}(\kappa, q, V, r) = p_a V \left(1 - \frac{\kappa}{q} - \frac{1+r}{2}\frac{V}{q}\right). \tag{A.11}$$

Region 2: $\frac{qr}{1+r} - rV \leq \kappa \leq \frac{q}{1+r} - V$.

In this case some consumers download the adblocker. The publishers's profit from people who did *not* download the app is given by (A.6), which is the following:

$$\pi_{noadb}^{pub} = \frac{qp_a}{(1-r)(1+r)}\left(1 - \frac{\kappa}{2(q - V(1+r))} - \frac{r}{2(1+r)}\right)\left(\frac{\kappa(1+r)}{q - V(1+r)} - r\right). \tag{A.12}$$

Again, the adblocker cannot extract this profit from the publisher. The publisher's profit that comes from people who downloaded the app is:

$$\pi_{adb}^{pub} = p_a V S_{V2}(\kappa, q, V, r) = \frac{p_a V}{1-r}\left(1 - \frac{\kappa}{q - V(1+r)}\right)\left(1 - \frac{\kappa(1+r)}{2(q - V(1+r))} - \frac{\kappa + V}{2q}\right), \quad \text{(A.13)}$$

where $S_{V2}(\kappa, q, V, r)$ is the size of the segment withing Region 2, that downloads the app (see (A.3) above). Since this profit comes from people who download the app, the adblocker can extract some part of it.

Region 3: $\kappa > \frac{q}{1+r} - V$.

None of the consumers download the adblocker. The publisher's profit is as in the benchmark case (see (2.2); note $-cq^2$ is a sunk cost so we omit it until the quality setting decision):

$$\pi_1^{pub} = \frac{q p_a}{2(1+r)}, \quad \text{(A.14)}$$

and since this profit entirely comes from people who did not download the app, the adblocker cannot extract any of it. The total profit of the publisher is the integral over κ of the profits in Regions 1, 2 and 3:

$$\pi^{pub} = \int_0^{\frac{qr}{1+r} - rV}\left[p_a V\left(1 - \frac{\kappa}{q} - \frac{1+r}{2}\frac{V}{q}\right)\right]\frac{1}{\bar{\kappa}}d\kappa + \int_{\frac{q}{1+r} - V}^{\bar{\kappa}} \frac{q p_a}{2(1+r)}\frac{1}{\bar{\kappa}}d\kappa +$$

$$+ \int_{\frac{qr}{1+r} - rV}^{\frac{q}{1+r} - V}\left[\frac{q p_a}{(1-r)(1+r)}\left(1 - \frac{\kappa}{2(q - V(1+r))} - \frac{r}{2(1+r)}\right)\left(\frac{\kappa(1+r)}{q - V(1+r)} - r\right)\right]\frac{1}{\bar{\kappa}}d\kappa +$$

$$+ \int_{\frac{qr}{1+r} - rV}^{\frac{q}{1+r} - V}\left[\frac{p_a V}{1-r}\left(1 - \frac{\kappa}{q - V(1+r)}\right)\left(1 - \frac{\kappa(1+r)}{2(q - V(1+r))} - \frac{\kappa + V}{2q}\right)\right]\frac{1}{\bar{\kappa}}d\kappa =$$

$$= \frac{p_a V r(q - V(1+r))((2+r)q - V(1+r))}{2\bar{\kappa} q(1+r)^2} + \frac{p_a q((1+r)\bar{\kappa} - q + V(1+r))}{2\bar{\kappa}(1+r)^2} +$$

$$+ \frac{p_a q(2 - r - r^2)(q - V(1+r))}{6\bar{\kappa}(1+r)^3} + \frac{p_a V(q - V(1+r))(q(4 + 3r - r^3) - (1 - r^2)(2+r)V)}{12\bar{\kappa} q(1+r)^3} =$$

$$= \frac{p_a q(3\bar{\kappa}(1+r)^2 - (1 + r(4+r))(q - V(1+r)))}{6\bar{\kappa}(1+r)^3} +$$

$$+\frac{p_a V(q-V(1+r))(q(2+5r+2r^2)-(1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^2}, \tag{A.15}$$

where the second term in (A.15) comes from the people who use the app. Hence, the adblocker can extract only this part of the profit. Denote by α the part of this term that the adblocker actually extracts (it might not be possible to extract all of it because the publisher would switch to subscription). Then in order for the publisher not to switch to subscription, the profit from advertising should be higher than the profit from subscription. Hence, the adblocker maximizes his share of profits subject to the publisher not switching to subscription:

$$\frac{p_a V(q-V(1+r))(q(2+5r+2r^2)-(1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^2}\alpha \Rightarrow \max_V \tag{A.16}$$

$$s.t.\ \frac{p_a q(3\bar{\kappa}(1+r)^2-(1+r(4+r))(q-V(1+r)))}{6\bar{\kappa}(1+r)^3}+$$

$$+\frac{p_a V(q-V(1+r))(q(2+5r+2r^2)-(1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^2}(1-\alpha) \geq \frac{q}{4}.$$

We can divide this problem into 3 sub-problems: (i) for V's high enough, the publisher will not switch to advertising even for $\alpha = 1$; (ii) for V's small enough, the publisher will switch to advertising even for $\alpha = 0$; (iii) for middle values of V, the adblocker will have to set $\alpha \in (0,1)$ so that the publisher's profit form advertising exactly equals the publisher's profit form subscription. We consider these cases separately.

Case 1: $V \geq \frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))}$.

The publisher does not switch to subscription even for $\alpha = 1$, because:

$$\frac{p_a q(3\bar{\kappa}(1+r)^2-(1+r(4+r))(q-V(1+r)))}{6\bar{\kappa}(1+r)^3} \geq \frac{q}{4} \Rightarrow V \geq \frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))}.$$

Note, that if $\frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))} < 0$, then even for $V = 0$, the publisher will not switch. Thus, the adblocker maximizes:

$$\pi_1^{adb} = \frac{p_a V(q-V(1+r))(q(2+5r+2r^2)-(1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^2} \Rightarrow \max_V$$

$$s.t.\ V \geq \frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a - 1 - r)(1+r)}{2p_a(1+r(4+r))}.$$

The optimal solution to this is:

$$V^\star = \begin{cases} qF(r) & \text{if } q \leq \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))}, \\ \frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))} & \text{if } q > \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))}. \end{cases} \tag{A.17}$$

Case 2: $\frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))} > 0$ and $V < \hat{V}$,

where $\hat{V}$ is the unique solution to the following equation (provided that $q > \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1+r(4+r))}$):

$$2p_a(1+r)^3(1+r+r^2)x^3 - 6p_a q(1+r)^4 x^2 + 6p_a q^2(1+r)(1+r(3+r))x + q^2(3\bar{\kappa}(2p_a-1-r)(1+r)^2 -$$

$$-2p_a q(1+r(4+r))) = 0.$$

The publisher switches to subscription even if $\alpha = 0$ because:

$$\frac{p_a q(3\bar{\kappa}(1+r)^2 - (1+r(4+r))(q - V(1+r)))}{6\bar{\kappa}(1+r)^3} +$$

$$+\frac{p_a V(q - V(1+r))(q(2+5r+2r^2) - (1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^2} < \frac{q}{4} \Rightarrow$$

$$\Rightarrow \frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a - 1 - r)(1+r)}{2p_a(1+r(4+r))} > 0 \text{ and } V < \hat{V}.$$

The adblocker's profit in this case is 0, so the adblocker never sets V in this region.

Case 3: $\frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))} > 0$ and $\hat{V} \leq V < \frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))}$.

In this case, there exists an optimal $\alpha \in (0,1)$ such that the publisher's profit form advertising exactly equals the publisher's profit from subscription. This α is found from:

$$\frac{p_a q(3\bar{\kappa}(1+r)^2 - (1+r(4+r))(q - V(1+r)))}{6\bar{\kappa}(1+r)^3} +$$

$$+\frac{p_a V(q-V(1+r))(q(2+5r+2r^2)-(1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^2}(1-\alpha)=\frac{q}{4}\Rightarrow$$

$$\alpha=\frac{3\bar{\kappa}q^2(2p_a-1-r)(1+r)^2-2p_a(q-(1+r)V)^2(q(1+r(4+r))-(1+r)(1+r+r^2)V)}{2p_a(1+r)V(q^2(2+r)(1+2r)-3q(1+r)^3V+(1+r)^2(1+r+r^2)V^2)}.$$

Substituting this α back into the adblocker's profit (A.16), we obtain the following maximization problem:

$$\frac{q(2p_a-1-r)}{4(1+r)}-\frac{p_a(q-(1+r)V)^2(q(1+r(4+r))-(1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^3}\Rightarrow\max_V$$

$$s.t.\ \hat{V}\le V<\frac{q}{1+r}-\frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))}.$$

This objective function is strictly increasing in V, therefore, the optimal choice of V is $V^\star=\frac{q}{1+r}-\frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))}$. It implies that Case 1 always dominates Case 3 since the adblocker can always replicate $V=\frac{q}{1+r}-\frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))}$ in Case 1.

The optimal $V^\star$ is therefore like in (A.17):

$$V^\star=\begin{cases}qF(r) & \text{if } q\le\frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))},\\ \frac{q}{1+r}-\frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))} & \text{if } q>\frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))}.\end{cases}$$

To get the corresponding publisher's profit we need to substitute $V=V^\star$ and $\alpha=1$ into (A.15):

$$\pi_{pub}=\begin{cases}\frac{qp_a}{2(1+r)}-\frac{(r^2+4r+1)(1-(1+r)F(r))q^2p_a}{6\bar{\kappa}(1+r)^3}-cq^2 & \text{if } q\le\frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))},\\ \frac{q}{4}-cq^2 & \text{if } q>\frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))}.\end{cases}$$

To solve for the optimal quality, since $V^\star$ is a piecewise function in q, the publisher has to solve two sub-problems: (i) maximize q subject to $q\le\frac{3\sqrt{3}\bar{\kappa}(2p_a-1)}{2p_a}$ and (ii) maximize q subject to $q\ge\frac{3\sqrt{3}\bar{\kappa}(2p_a-1)}{2p_a}$. Then we can compare the profits obtained in these two sub-problems to find the final solution for q.

Case 1: $q\le\frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))}$.

The adblocker chooses $V^\star = qF(r)$ and, therefore, the publisher solves:

$$\frac{qp_a}{2(1+r)} - \frac{(r^2+4r+1)(1-(1+r)F(r))q^2 p_a}{6\bar{\kappa}(1+r)^3} - cq^2 \Rightarrow \max_q$$

$$s.t.\ q \leq \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))}.$$

This leads to the optimal choice of quality:

$$q_1^\star = \begin{cases} \frac{3\bar{\kappa}p_a(1+r)^2}{12c\bar{\kappa}(1+r)^3+2p_a(1-(1+r)F(r))(1+r(4+r))} & \text{if } c \geq \frac{p_a(1+r-p_a)(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(2p_a-1-r)(1+r)^3} \\ \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))} & \text{if } c < \frac{p_a(1+r-p_a)(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(2p_a-1-r)(1+r)^3}. \end{cases}$$

and the corresponding profits:

$$\pi_1^\star = \begin{cases} \frac{3\bar{\kappa}p_a^2(1+r)}{48c\bar{\kappa}(1+r)^3+8p_a(1-(1+r)F(r))(1+r(4+r))} & \text{if } c \geq \frac{p_a(1+r-p_a)(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(2p_a-1-r)(1+r)^3} \\ \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2(p_a(1-(1+r)F(r))(1+r(4+r))-6c\bar{\kappa}(2p_a-1-r)(1+r)^2)}{8p_a^2(1-(1+r)F(r))^2(1+r(4+r))^2} & \text{if } c < \frac{p_a(1+r-p_a)(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(2p_a-1-r)(1+r)^3}. \end{cases} \tag{A.18}$$

Case 2: $q \geq \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))}$.

The adblocker chooses $V^\star = \frac{q}{1+r} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))}$ and, therefore, the publisher solves:

$$\frac{q}{4} - cq^2 \Rightarrow \max_q$$

$$s.t.\ q \geq \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))}.$$

This leads to the optimal choice of quality:

$$q_2^\star = \begin{cases} \frac{1}{8c} & \text{if } c \leq \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{12\bar{\kappa}(2p_a-1-r)(1+r)^2} \\ \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2}{2p_a(1-(1+r)F(r))(1+r(4+r))} & \text{if } c > \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{12\bar{\kappa}(2p_a-1-r)(1+r)^2}. \end{cases}$$

and the corresponding profits:

$$\pi_2^\star = \begin{cases} \frac{1}{64c} & \text{if } c \leq \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{12\bar{\kappa}(2p_a-1-r)(1+r)^2} \\ \frac{3\bar{\kappa}(2p_a-1-r)(1+r)^2(p_a(1-(1+r)F(r))(1+r(4+r))-6c\bar{\kappa}(2p_a-1-r)(1+r)^2)}{8p_a^2(1-(1+r)F(r))^2(1+r(4+r))^2} & \text{if } c > \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{12\bar{\kappa}(2p_a-1-r)(1+r)^2}. \end{cases} \tag{A.19}$$

Comparing (A.18) and (A.19), the optimal quality in this model turns out to be:

$$q^\star = \begin{cases} \frac{1}{8c} & \text{if } c < \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)} \\ \frac{3\bar{\kappa}p_a(1+r)^2}{12c\bar{\kappa}(1+r)^3+2p_a(1-(1+r)F(r))(1+r(4+r))} & \text{if } c \geq \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)}. \end{cases} \tag{A.20}$$

The corresponding publisher's profits are:

$$\pi_{pub}^\star = \begin{cases} \frac{1}{64c} & \text{if } c < \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)} \\ \frac{3\bar{\kappa}p_a^2(1+r)}{48c\bar{\kappa}(1+r)^3+8p_a(1-(1+r)F(r))(1+r(4+r))} & \text{if } c \geq \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)}. \end{cases} \tag{A.21}$$

The optimal advertising restriction, $V^\star$, is computed by substituting (A.20) into (A.17):

$$V^\star = \begin{cases} \frac{1}{8c(1+r)} - \frac{3\bar{\kappa}(2p_a-1-r)(1+r)}{2p_a(1+r(4+r))}, & \text{if } c < \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)} \\ \frac{3\bar{\kappa}p_a(1+r)^2F(r)}{12c\bar{\kappa}(1+r)^3+2p_a(1-(1+r)F(r))(1+r(4+r))}, & \text{if } c \geq \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)}. \end{cases}$$

and the corresponding adblocker profits are computeted by substituting the optimal $V^\star$ and $q^\star$ into (A.16)). Substituting $r = 1/2$) completes the proof:

$$\pi_{adb}^\star = \begin{cases} \bar{\pi}_{adb} & \text{if } c < \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)} \\ \frac{3F(r)\bar{\kappa}p_a^3(1+r)^2(1-(1+r)F(r))\left((2+r)(1+2r)-F(r)(1+r)\left(1+r+r^2\right)\right)}{8\left(p_a(1+r(r+4))(1-(1+r)F(r))+6c\bar{\kappa}(1+r)^3\right)^2} & \text{if } c \geq \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)}, \end{cases} \tag{A.22}$$

where $\bar{\pi}_{adb} = \frac{(2p_a-1-r)\left(p_a(1+r(r+4))-12c\bar{\kappa}(1+r)^2(2p_a-1-r)\right)\left(p_a(1+r(r+4))^2+12c\bar{\kappa}\left(1+r+r^2\right)(1+r)^2(2p_a-r-1)\right)}{32cp_a^2(1+r)(1+r(r+4))^3}$.

□

Thus, in equilibrium, when c is high, the solution is the same as in the model with no subscription option. The reason is that when costs are high, quality levels and consequently advertising levels are generally lower. This limits the adblocker's ability to undercut the advertising and eat into the publisher's profit, thus the gain from switching to a subscription model diminishes. Hence there is no need for the adblocker to restrict its choice of V to avoid pushing the publisher into subscription. On the other hand, when c is small, the adblocker cannot set its V in an unrestricted manner and V is calibrated such that the publisher becomes indifferent between advertising and subscription. Therefore, the choice of quality is the same as in the case where subscription is chosen, because the option to switch to a subscription model protects the publisher. In fact, in this case, publishers are indifferent and may switch to the subscription model if the negotiation breaks down. This is consistent with reality where we see some of the higher quality sites increasingly shifting to charging their users.

Comparing the consumer and total surplus under the possibility of subscription and without subscription we obtain the following.

Corollary 6 *Assume that $p_a \geq \frac{3}{4}$. If*

$$c \leq \frac{13(\sqrt{309}-6)p_a}{189\bar{\kappa}(16p_a^2-9)},$$

then both the consumer and total surplus is higher with the option of a subscription model than without it. If

$$c > \frac{13(\sqrt{309}-6)p_a}{189\bar{\kappa}(16p_a^2-9)},$$

both the consumer and total surplus generated are identical.

PROOF: We calculate the consumer and total surpluses for the subscription model for both possible cases outlined in Proposition 14: $c < \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)}$ and $c \geq \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)}$.

Case 1: $c < \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)}$.

The optimal quality is, $q^\star = \frac{1}{8c}$ and the optimal advertising restriction is $V^\star = \frac{1}{8c} - 3\bar{\kappa} + \frac{3\bar{\kappa}}{2p_a}$.

The expression for the consumer surplus is the same as derived in the proof of Proposition 15 (see (A.25)) with the optimal $q^\star$ and $V^\star$ substituted into it, that is:

$$CS_1 = \frac{9c\bar{\kappa}(1-r)(1+r)^4(2p_a-1-r)^2\left(12c\bar{\kappa}(1+r^2)(1+r)^2(2p_a-1-r)+p_a(1+r(r+4))^2\right)}{96cp_a^3(1+r)^2(1+r(r+4))^3}+$$

$$+\frac{2(1-r^3)p_a^3(1+r(r+4))^3}{96cp_a^3(1+r)^2(1+r(r+4))^3}.$$

The optimal publisher's profit is given in (A.21):

$$\pi_1^{pub} = \frac{1}{64c}.$$

The optimal adblocker's profit is given in (A.22):

$$\pi_1^{adb} = \bar{\pi}_{adb}.$$

The total surplus is calculated as the sum of the consumer surplus, publisher's profits and adblocker's profits:

$$TS_1 = 1-r(6+r(4r+3)) - \frac{216c^2\bar{\kappa}^2(1+r)^5(2p_a-1-r)^3\left(4p_a\left(1+r+r^2\right)+r^4-1\right)}{192cp_a^3(1+r)^2(1+r(r+4))^3}+$$

$$+\frac{6(1+r)\left(2p_a^3(1+r(r+4))^2-3c\bar{\kappa}(1+r)^2(2p_a-1-r)^2\left(r\left(12p_a+(r+4)r^2-4\right)-1\right)\right)}{192cp_a^2(1+r)^2(1+r(r+4))^2}.$$

Case 2: $c \geq \frac{p_a(1-(1+r)F(r))(1+r(4+r))}{6\bar{\kappa}(1+r)(4p_a^2-(1+r)^2)}$.

The optimal quality is, $q^\star = \frac{3\bar{\kappa}p_a(1+r)^2}{12c\bar{\kappa}(1+r)^3+2p_a(1-(1+r)F(r))(1+r(4+r))}$ and the optimal advertising restriction is, $V^\star = \frac{3\bar{\kappa}p_a(1+r)^2F(r)}{12c\bar{\kappa}(1+r)^3+2p_a(1-(1+r)F(r))(1+r(4+r))}$. Consumer surplus, publisher's profits, adblocker profits and total surplus are calculated or referenced the same way as in Case 1. After simplification we get:

$$CS_2 = -\frac{\bar{\kappa}p_a(1-r)H(r,p_a,\bar{\kappa},c)}{32\left(p_a(1+r(r+4))(1-(1+r)F(r))+6c\bar{\kappa}(1+r)^3\right)^2},$$

where:

$$H(r, p_a, \bar{\kappa}, c) = 48c\bar{\kappa}(1+r)^3\left(1+r+r^2\right) + p_a(1-(1+r)F(r))\times$$

$$\times\left(3F(r)^2\left(1+r^2\right)(1+r)^4 - 3F(r)(r(3r+4)+3)(1+r)^3 + 2(r(r(r(7r+32)+42)+32)+7)\right).$$

$$\pi_2^{pub} = \frac{3\bar{\kappa}p_a^2(1+r)}{48c\bar{\kappa}(1+r)^3 + 8p_a(1-(1+r)F(r))(1+r(4+r))},$$

$$\pi_2^{adb} = \frac{3F(r)\bar{\kappa}p_a^3(1+r)^2(1-(1+r)F(r))\left((2+r)(1+2r) - F(r)(1+r)\left(1+r+r^2\right)\right)}{8\left(p_a(1+r(r+4))(1-(1+r)F(r)) + 6c\bar{\kappa}(1+r)^3\right)^2},$$

$$TS_2 = \frac{\bar{\kappa}p_a L(r, p_a, \bar{\kappa}, c)}{32\left(p_a(1+r(r+4))((1+r)F(r)-1) - 6c\bar{\kappa}(1+r)^3\right)^2},$$

where:

$$L(r, p_a, \bar{\kappa}, c) =$$

$$24c\bar{\kappa}(1+r)^3\left(3p_a(1+r) - 2r^3 + 2\right) - p_a(1-(1+r)F(r))(3F(r)^2(1+r)^3\left(4p_a\left(1+r+r^2\right) + r^4 - 1\right)) -$$

$$p_a(1-(1+r)F(r))(-3F(r)(1+r)^2\left(4p_a(2+r)(1+2r) + (4+3r)r^3 - 4r - 3\right))$$

$$-p_a(1-(1+r)F(r))\,(2((1-r)(r(r(r(7r+32)+42)+32)+7) - 6p_a(1+r)(1+r(r+4)))).$$

The consumer and total surpluses of the model without subscription are exactly the same as Case 2 in the model with subscription since in Case 2 subscription turns out not to be binding.
The rest of the proof is just a direct comparison between the consumer and total surpluses. □

When there is a possibility to introduce subscription consumers are never worse off. This is because the possibility of a subscription does not allow the adblocker to behave too aggressively and expropriate too much profit from the publisher. This, in turn, leads to higher equilibrium quality. Total surplus also turns out to increase due to both consumers and the publisher being better off.[4]

[4] Importantly, these results are further strengthened if the publisher can discriminate between consumers who downloaded the ad blocker app or not. In that case, the publisher can successfully threaten the adblocker without the cost of sacrificing the advertising revenue model for the non-downloading consumers.

A.15.2 Platform Operating the Adblocker

In this section we will consider the case of the platform that does ad blocking itself instead of a third party adblocker. This setup is something that Google offers in the Chrome browser by blocking certain ads. The difference is that the platform is interested in maximizing the total surplus generated while a third party adblocker is only interested in maximizing its own profits. As we show below the incentives of the platform and the adblocker are not necessarily aligned, leading to different outcomes. This tension shows in practice, where adblockers are criticizing Google, for example, for not blocking enough ads. At the same time, Google is attempting to prevent ad blocking software from operating properly in its system. As we see a trend of greater platform control, especially in the mobile content ecosystem, it is important to understand how different entities can and should block or limit ads. For simplicity, we assume $t = 0$ in this section.

Assuming, as before, that $\bar{\kappa}$ is high enough, the publisher chooses the optimal advertising level of $A^\star = \frac{q}{1+r} = \frac{2q}{3}$. Given this we can calculate the consumer and total surpluses generated in the model as a function of V and then consider the choice of V by the platform to maximize total surplus. The total surplus, i.e. the sum of consumer surplus and publisher profits is:

$$TS(V) = CS + \pi^{pub} = \frac{q(7 + 36p_a)}{108} - \frac{(2q - 3V)^2(4(104p_a - 27)q + (45 - 336p_a)V)}{10368\bar{\kappa}q} - cq^2.$$

The platform problem is to maximize the total surplus, $TS(V)$ above with respect to V. Note that there is no reason to set $V > \frac{2q}{3}$ because setting $V = \frac{2q}{3}$ achieves exactly the same result. Hence, the problem becomes:

$$\max_V \; TS(V) \quad s.t.\; V \leq \frac{2q}{3}.$$

The objective function is a second degree polynomial in V, therefore, it is easy to show that the optimal choice of V is:

$$V^\star = \begin{cases} 0, & \text{if } p_a < \frac{41}{176}, \\ \frac{2(176p_a-41)q}{336p_a-45}, & \text{if } \frac{41}{176} \le p_a \le \frac{13}{32}, \\ \frac{2q}{3}, & \text{if } p_a > \frac{13}{32}. \end{cases} \tag{A.23}$$

Given the optimal $V^\star$, we can calculate the optimal quality yielding

$$q^\star = \begin{cases} \frac{27\bar{\kappa}p_a}{162c\bar{\kappa}+26p_a}, & \text{if } p_a < \frac{41}{176}, \\ \frac{27\bar{\kappa}p_a(112p_a-15)^3}{162c\bar{\kappa}(112p_a-15)^3+32p_a(32p_a-13)^2(23+56p_a)}, & \text{if } \frac{41}{176} \le p_a \le \frac{13}{32}, \\ \frac{p_a}{6c}, & \text{if } p_a > \frac{13}{32}. \end{cases} \tag{A.24}$$

By substituting $q^\star$ and $V^\star$ into the expressions for the consumer and total surplus, we can find their equilibrium values and compare them to corresponding consumer and total surplus in the model with no adblocker and third-party adblocker.

Proposition 15 *Assume that $\bar{\kappa}$ is high enough. Then there is a $\underline{p_a}$ such that we have*

1. $CS_{platfrom} \ge CS_{benchmark}$ *iff* $p_a \ge \frac{6907}{18128} \approx 0.381$*,*
2. $TS_{platform} \le TS_{benchmark}$ *always,*
3. $CS_{platform} \ge CS_{adb}$ *always,*
4. $TS_{platform} \ge TS_{adb}$ *iff* $p_a \le \frac{59589\sqrt{309}-976292}{3411408} \approx 0.021$ *or* $p_a \ge \underline{p_a} \approx 0.248$.

PROOF: As stated in the main text, we compute the consumer surplus (CS) and profits in 3 possible regions: (i) $\kappa \le \frac{qr}{1+r} - rV$ - everyone downloads the adblocker; (ii) $\frac{qr}{1+r} - rV < \kappa < \frac{q}{1+r} - V$ - some people download the adblocker while some do not; (iii) $\kappa \ge \frac{q}{1+r} - V$ - nobody downloads the adblocker. We analyze these regions using Figure 2 in the main paper.

In **region 1** ($\kappa < \frac{qr}{1+r} - rV$), all the people who end up visiting the publisher's website download the adblocker. These people are located in trapezoid (abhg) and their utility is $U(q, V) = \vartheta q -$

$\gamma V-\kappa$. The trapezoid above can be represented as a union of a triangular region and a rectangular region. Hence, we will divide the consumer surplus into the sum of the consumer surplus over the triangular region plus the consumer surplus over the rectangular region:

$$CS_1=\iint_{(\vartheta,\gamma)\in(abhg)}(\vartheta q-\gamma V-\kappa)d\vartheta d\gamma=\int_{\frac{\kappa+rV}{q}}^{\frac{\kappa+V}{q}}d\vartheta\int_{r}^{\frac{\vartheta q-\kappa}{V}}(\vartheta q-\gamma V-\kappa)d\gamma+\int_{\frac{\kappa+V}{q}}^{1}d\vartheta\int_{r}^{1}(\vartheta q-\gamma V-\kappa)d\gamma=$$

$$\int_{\frac{\kappa+rV}{q}}^{\frac{\kappa+V}{q}}\frac{(\vartheta q-rV-\kappa)^2}{2V}d\vartheta+\int_{\frac{\kappa+V}{q}}^{1}\left(\frac{1-r}{2}(2\vartheta q-(1+r)V-2\kappa)\right)d\vartheta=\frac{(1-r)^3V^2}{6q}+$$

$$+\frac{(1-r)(q-V-\kappa)(q-rV-\kappa)}{2q}.$$

In **region 2** ($\frac{qr}{1+r}-rV\leq\kappa<\frac{q}{1+r}-V$), the consumer surplus equals the sum of the consumer surplus obtained by the downloaders (located in trapezoid (abgh)) and non-downloaders (located in trapezoid (cdhg)). Each of these trapezoids can be represented as a union of a triangular region and a rectangular region. Hence, we will break each integral into a sum of the integral over a triangular region plus the integral over a rectangular region:

$$CS_2=\iint_{(\vartheta,\gamma)\in(abgh)}(\vartheta q-\gamma V-\kappa)d\vartheta d\gamma+\iint_{(\vartheta,\gamma)\in(cdhg)}(\vartheta q-\gamma A)d\vartheta d\gamma=$$

$$=\int_{\frac{\kappa}{q-(1+r)V}}^{\frac{\kappa+V}{q}}d\vartheta\int_{\frac{\kappa(1+r)}{q-(1+r)V}}^{\frac{\vartheta q-\kappa}{V}}(\vartheta q-\gamma V-\kappa)d\gamma+\int_{\frac{\kappa+V}{q}}^{1}d\vartheta\int_{\frac{\kappa(1+r)}{q-(1+r)V}}^{1}(\vartheta q-\gamma V-\kappa)d\gamma+$$

$$+\int_{\frac{r}{1+r}}^{\frac{\kappa}{q-(1+r)V}}d\vartheta\int_{r}^{(1+r)\vartheta}\frac{q}{1+r}((1+r)\vartheta-\gamma)d\gamma+\int_{\frac{\kappa}{q-(1+r)V}}^{1}d\vartheta\int_{r}^{\frac{\kappa(1+r)}{q-(1+r)V}}\frac{q}{1+r}((1+r)\vartheta-\gamma)d\gamma=$$

$$=\int_{\frac{\kappa}{q-(1+r)V}}^{\frac{\kappa+V}{q}}\frac{q^2(\kappa-\vartheta(q-(1+r)V))^2}{2V(q-(1+r)V)^2}d\vartheta+$$

$$+\int_{\frac{\kappa+V}{q}}^{1}\left(\vartheta q-\frac{V}{2}-\kappa-\frac{\kappa(1+r)(2q\vartheta(q-(1+r)V)-\kappa(2q-(1+r)V))}{2(q-(1+r)V)^2}\right)d\vartheta+$$

$$+q\int_{\frac{r}{1+r}}^{\frac{\kappa}{q-(1+r)V}}\frac{(\vartheta-r(1-\vartheta))^2}{2(1+r)}d\vartheta+$$

$$+q\int_{\frac{\kappa}{q-(1+r)V}}^{1}\left(\frac{((1+r)\kappa-r(q-(1+r)V))((2(1+r)\vartheta-r)(q-(1+r)V)-(1+r)\kappa)}{2(1+r)(q-(1+r)V)^2}\right)d\vartheta=$$

$$=\frac{V^2(q-(1+r)V-(1+r)\kappa)^3}{6q(q-(1+r)V)^3}+\frac{(q-V-\kappa)(q-(1+r)V-\kappa)(q-(1+r)V-(1+r)\kappa)}{2(q-(1+r)V)^2}+$$

$$+\frac{q((1+r)\kappa-r(q-(1+r)V))^3}{6(1+r)^3(q-(1+r)V)^3}+\frac{q(q-(1+r)V-\kappa)((1+r)\kappa-r(q-(1+r))V)}{2(1+r)(q-(1+r)V)^2}.$$

In **region 3** ($\kappa \geq \frac{q}{1+r}-V$), nobody downloads the app. Hence, the consumer surplus equals the integration of $U(q,A)=\vartheta q-\gamma A$ over ϑ and γ such that the constraint on non-negativity of utility holds, that is, $\vartheta q-\gamma A\geq 0 \Rightarrow q(\vartheta-\frac{\gamma}{1+r})\geq 0 \Rightarrow (1+r)\vartheta\geq\gamma$. Therefore, consumer surplus is:

$$CS_3=\iint\limits_{(1+r)\vartheta\geq\gamma}(\vartheta q-\gamma A)d\vartheta d\gamma=\iint\limits_{(1+r)\vartheta\geq\gamma}\frac{q}{1+r}((1+r)\vartheta-\gamma)d\vartheta d\gamma=$$

$$=\frac{q}{1+r}\left[\int_{\frac{r}{1+r}}^{\frac{1}{1+r}}d\vartheta\int_{r}^{(1+r)\vartheta}((1+r)\vartheta-\gamma)d\gamma+\int_{\frac{1}{1+r}}^{1}d\vartheta\int_{r}^{1}((1+r)\vartheta-\gamma)d\gamma\right]=$$

$$=\frac{q}{1+r}\left[\int_{\frac{r}{1+r}}^{\frac{1}{1+r}}\left(\frac{1}{2}(\vartheta-r(1-\vartheta))^2\right)d\vartheta+\int_{\frac{1}{1+r}}^{1}\left(\frac{1}{2}(1-r^2)(2\vartheta-1)\right)d\vartheta\right]=$$

$$=\frac{q}{1+r}\left[\frac{(1-r)^3}{6(1+r)}+\frac{r(1-r)}{2(1+r)}\right]=\frac{q(1-r^3)}{6(1+r)^2}.$$

The total consumer surplus is calculated as the integral over κ of the consumer surplus in regions 1, 2 and 3:

$$CS=\int_{0}^{\frac{qr}{1+r}-rV}\left(\frac{(1-r)^3V^2}{6q}+\frac{(1-r)(q-V-\kappa)(q-rV-\kappa)}{2q}\right)\frac{1}{\bar{\kappa}}d\kappa+\int_{\frac{q}{1+r}-V}^{\bar{\kappa}}\frac{q(1-r^3)}{6(1+r)^2}\frac{1}{\bar{\kappa}}d\kappa+$$

$$+\int_{\frac{qr}{1+r}-rV}^{\frac{q}{1+r}-V}\left(\frac{V^2(q-(1+r)V-(1+r)\kappa)^3}{6q(q-(1+r)V)^3}+\frac{(q-V-\kappa)(q-(1+r)V-\kappa)(q-(1+r)V-(1+r)\kappa)}{2(q-(1+r)V)^2}\right)\frac{1}{\bar{\kappa}}+$$

$$+\int_{\frac{qr}{1+r}-rV}^{\frac{q}{1+r}-V}\left(\frac{q((1+r)\kappa-r(q-(1+r)V))^3}{6(1+r)^3(q-(1+r)V)^3}+\frac{q(q-(1+r)V-\kappa)((1+r)\kappa-r(q-(1+r))V)}{2(1+r)(q-(1+r)V)^2}\right)\frac{1}{\bar{\kappa}}d\kappa=$$

$$=\frac{r(1-r)(q-(1+r)V)(2q^2(3+r(3+r))-q(1+r)(6+r(3+r))V+(2+r(3+r+r^2+r^3))V^2)}{12\bar{\kappa}q(1+r)^3}+$$

$$+\frac{q(1-r^3)((1+r)\bar{\kappa}-q+(1+r)V)}{6\bar{\kappa}(1+r)^3}+$$

$$+\frac{(1-r)^2(2q^3(3+r(2+r))-q^2(1+r)(9+r(2+r))V+4q(1-r)(1+r)^2V^2-(1-r)^2(1+r)^3V^3)}{24\bar{\kappa}q(1+r)^3}=$$

$$=\frac{4\bar{\kappa}q^2(1-r^3)+(1-r^2)(q-(1+r)V)^2(2q(1+r)-(1+r^2)V)}{24\bar{\kappa}q(1+r)^2}. \tag{A.25}$$

The total publisher's profit is the sum of (A.11), (A.12), (A.13) and (A.14) minus cq^2 integrated over κ:

$$\pi^{pub}=\int_0^{\frac{qr}{1+r}-rV}\pi_1^{pub}(\kappa)\frac{1}{\bar{\kappa}}d\kappa+\int_{\frac{qr}{1+r}-rV}^{\frac{q}{1+r}-V}\pi_2^{pub}(\kappa)\frac{1}{\bar{\kappa}}d\kappa+\int_{\frac{q}{1+r}-V}^{\bar{\kappa}}\pi_3^{pub}(\kappa)\frac{1}{\bar{\kappa}}d\kappa-cq^2=$$

$$=\int_0^{\frac{qr}{1+r}-rV}\left[p_aV\left(1-\frac{\kappa}{q}-\frac{(1+r)V}{2q}\right)\right]\frac{1}{\bar{\kappa}}d\kappa+\int_{\frac{q}{1+r}-V}^{\bar{\kappa}}\frac{qp_a}{2(1+r)}\frac{1}{\bar{\kappa}}d\kappa-cq^2+$$

$$+\int_{\frac{qr}{1+r}-rV}^{\frac{q}{1+r}-V}\frac{qp_a}{(1-r)(1+r)}\left(1-\frac{\kappa}{2(q-(1+r)V)}-\frac{r}{2(1+r)}\right)\left(\frac{\kappa(1+r)}{q-(1+r)V}-r\right)\frac{1}{\bar{\kappa}}d\kappa+$$

$$+\int_{\frac{qr}{1+r}-rV}^{\frac{q}{1+r}-V}\frac{p_aV}{1-r}\left(1-\frac{\kappa(1+r)}{q-(1+r)V}\right)\left(1-\frac{\kappa}{2(q-(1+r)V)}-\frac{\kappa+V}{2q}\right)\frac{1}{\bar{\kappa}}d\kappa=$$

$$=\frac{qp_a}{2(1+r)}-\frac{p_a(q-(1+r)V)^2(q(1+r(4+r))-(1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^3}-cq^2. \tag{A.26}$$

The total surplus is calculated as the sum of CS from (A.25) and the publisher's profit from (A.26):

$$TS=\frac{4\bar{\kappa}q^2(1-r^3)+(1-r^2)(q-(1+r)V)^2(2q(1+r)-(1+r^2)V)}{24\bar{\kappa}q(1+r)^2}+$$

$$+\frac{qp_a}{2(1+r)}-\frac{p_a(q-(1+r)V)^2(q(1+r(4+r))-(1+r)(1+r+r^2)V)}{6\bar{\kappa}q(1+r)^3}-cq^2=$$

$$=\frac{q(1-r^3+3p_a(1+r))}{6(1+r)^2}-cq^2+$$

$$+\frac{(q-(1+r)V)^2((1+r)(4p_a(1+r+r^2)+r^4-1)V-2q(2p_a(1+r(4+r))-(1-r)(1+r)^3))}{24\bar{\kappa}q(1+r)^3}. \tag{A.27}$$

The platform sets the optimal V by maximizing $TS(V)$ from (A.27). Note that there are no incentives to set $V > \frac{q}{1+r}$ because setting $V = \frac{q}{1+r}$ achieves exactly the same result. Hence, the problem is:

$$TS(V) \Rightarrow \max_{V},$$

$$s.t.\ V \leq \frac{q}{1+r}.$$

The objective function is a second degree polynomial in V, therefore, it is easy to show that the optimal choice of V is:

$$V^{\star} = \begin{cases} 0, & \text{if } p_a < \frac{5+8r-8r^3-5r^4}{12(1+3r+r^2)}, \\ q * G(r, p_a), & \text{if } \frac{5+8r-8r^3-5r^4}{12(1+3r+r^2)} \leq p_a \leq \frac{1+4r-4r^3-r^4}{12r}, \\ \frac{q}{1+r}, & \text{if } p_a > \frac{1+4r-4r^3-r^4}{12r}. \end{cases} \tag{A.28}$$

where $G(r, p_a) = \frac{(12p_a(1+3r+r^2)+r^3(8+5r)-8r-5)}{3(1+r)(4p_a(1+r+r^2)+r^4-1)}$.

Given the optimal $V^{\star}$, we can calculate the optimal quality by substituting $V^{\star}$ into (A.26) and maximizing over q. The objective function of the publisher is a simple second order polynomial, so the equilibrium quality is:

$$q^{\star} = \begin{cases} \frac{3\bar{\kappa}p_a(1+r)^2}{2p_a(1+4r+r^2)+12c\bar{\kappa}(1+r)^3}, & \text{if } p_a < \frac{5+8r-8r^3-5r^4}{12(1+3r+r^2)}, \\ \frac{3\bar{\kappa}p_a(1+r)^2}{12c\bar{\kappa}(1+r)^3+2p_a(1-(1+r)G(r,p_a))^2(1+4r+r^2-(1+r)(1+r+r^2)G(r,p_a))}, & \text{if } \frac{5+8r-8r^3-5r^4}{12(1+3r+r^2)} \leq p_a \leq \frac{1+4r-4r^3-r^4}{12r}, \\ \frac{p_a}{4c(1+r)}, & \text{if } p_a > \frac{1+4r-4r^3-r^4}{12r}. \end{cases} \tag{A.29}$$

Once we have computed the equilibrium values of $V^{\star}$ and $q^{\star}$ as shown in the main text, then the equilibrium values of the consumer and total surpluses are found by substituting the optimal $V^{\star}$ from (A.23) and optimal $q^{\star}$ from (A.24) into $(A.25)$ and (A.27), respectively. Since depending on p_a and r, the optimal quality and upper bound on advertising are piecewise functions, we need to compute the equilibrium CS and TS for each region separately and compare to the benchmark model and the model with third-party adblocker. Since this is just a straightforward comparison, we can state the results right away.

Assume that $\bar{\kappa}$ is very high. Define p_a^1 as:

$$p_a^1 = \frac{5r^2}{26} - \frac{1-r}{4(1+r+r^2)} + \frac{587r}{676} - \frac{1}{1+r} + \frac{2+7r}{1+r(r+4)} + \frac{6r(r(22387r-2152)+24025)+61464}{2197(r(r(r(13r+44)+30)+44)+13)} - \frac{27193}{8788} +$$

$$+\frac{3\sqrt{3}(1-r)(1+r)(r(r(r(r(r(3r+20)+41)+64)+41)+20)+3)\sqrt{(r(1+r)(r(r+4)+5)+1)}}{4(r^2+r+1)(r(r+4)+1)(r(r(r(13r+44)+30)+44)+13)}.$$

Define p_a^2 as:

$$p_a^2 = \frac{-19r^8 - 42r^7 - 134r^6 - 106r^5 + 106r^3 + 134r^2 + 42r + 19}{36r^6 + 84r^5 + 276r^4 + 360r^3 + 276r^2 + 84r + 36}.$$

Define p_a^+ and p_a^- as (depending on the sign in front of the second term):

$$p_a^{\pm} = \frac{-r^7 - r^6 - 5r^5 + 3r^4 - 3r^3 + 5r^2 + r + 1}{24r\left(r^3 + 2r^2 + 2r + 1\right)} \pm$$

$$\pm \frac{\sqrt{C}}{24r(r^3 + 2r^2 + 2r + 1)}.$$

where:

$$C = r^{14} - 74r^{13} - 233r^{12} - 700r^{11} - 859r^{10} - 214r^9 + 1155r^8 + 1848r^7 +$$

$$+1155r^6 - 214r^5 - 859r^4 - -700r^3 - 233r^2 - 74r + 1.$$

Define p_a^5 as the unique solution to the following equation in the region $\left(\frac{5+8r-8r^3-5r^4}{12(1+3r+r^2)}; \frac{1+4r-4r^3-r^4}{12r}\right)$:

$$TS_{platform}(p_a^5) = TS_{adb}(p_a^5).$$

Then:

1. $CS_{platfrom} \geq CS_{benchmark}$ when $p_a \geq p_a^2$,

2. $TS_{platform} \geq TS_{benchmark}$ when $p_a^- \leq p_a \leq p_a^+$ or $p_a \geq \frac{1+4r-4r^3-r^4}{12r}$,

3. $CS_{platform} \geq CS_{adb}$ always,

4. $TS_{platform} \geq TS_{adb}$ when $p_a \leq p_a^1$ or $p_a \geq p_a^5$.

□

We see that the total surplus is always lower in this model compared to the no adblocker benchmark model. Even though for a fixed value of q the platform indeed sets V such that the total surplus is higher than in the benchmark, the platform's choice of V lowers the publisher's profit for a given q. This implies that the equilibrium q in the benchmark model is higher than in this platform model. Interestingly, the total surplus under this model is sometimes even lower than with a third party adblocker. Depending on the value of advertising an adblocker may be less harmful to quality levels than the platform itself. Despite the drop in quality, consumer surplus is mostly positively affected. While the combination of reduced advertising and reduced quality hurts total surplus, the former factor often dominates when considering consumers' payoffs.

Zero Cost of Downloading the Ad Blocker

In this subsection, we explore the case where consumers do not have to incur a cost to download the ad blocking app. This means that every consumer downloads or uses the ad blocker. A typical example is when ad blocking is built into a browser and is turned on by default. Clearly, in this case, the user does not have to take any costly action to use the ad blocking feature.

Since everyone uses the app, there is no incentive for the publisher to set its A higher than V. Given that $A \leq V$, consumers visit the publisher's website when $\vartheta q - \gamma A \geq 0 \Rightarrow \gamma \leq \vartheta \frac{q}{A}$. This constraint is exactly the same as in the benchmark model with no adblocker, the only difference between these two problems being the upper bound on A, with the publisher never setting $A > V$. The optimal solution for $A^\star$ (in case of $r = 1/2$) is:

$$A^\star = \min\left(\frac{2q}{3}; V\right). \tag{A.30}$$

Consider the optimal choice of the platform maximizing the total surplus. Clearly, there is no reason to set $V > \frac{2q}{3}$ since any such V achieves the same total surplus as $V = \frac{2q}{3}$, therefore, the optimal V always satisfies: $V \leq \frac{2q}{3}$ and as a result, the publisher sets the highest possible advertising, $A^\star = V$. The total surplus and the platform's maximization problem are:

$$TS = \frac{1}{48}\left(12q(1-4cq) + \frac{(7-36p_a)V^2}{q} + 6(8p_a-3)V\right),$$

$$\max_V \; TS(V) \quad s.t.\ V \leq \frac{2q}{3}.$$

When $p_a < \frac{3}{8}$, the objective function is strictly decreasing in V and, therefore, $V^\star = 0$. Otherwise, the optimal V is defined by the solution of the first order conditions yielding

$$V^\star = \begin{cases} 0, & \text{if } p_a < \frac{3}{8}, \\ \frac{3(8p_a-3)q}{36p_a-7}, & \text{if } p_a \geq \frac{3}{8}. \end{cases} \tag{A.31}$$

This allows us to calculate optimal quality set by the publisher by substituting (A.31) into the publisher's profits and maximizing over q:

$$q^\star = \begin{cases} 0, & \text{if } p_a < \frac{3}{8}, \\ \frac{3p_a(8p_a-3)(72p_a-1)}{8c(36p_a-7)^2}, & \text{if } p_a \geq \frac{3}{8}. \end{cases}$$

Our main focus in this section is to derive the welfare levels and compare them to the previous cases. Assuming that $\bar{\kappa}$ is large enough, we can formulate the following proposition.

Proposition 16 *There exist $\bar{p}_a \approx 0.412$ and $\hat{p}_a \approx 0.551$ such that if $\bar{\kappa}$ is high enough.*

1. *The consumer surplus in the model with the platform providing ad blocking at zero cost of downloading is* higher *than the consumer surplus generated in the benchmark model and in the model with a third-party adblocker iff $p_a \geq \bar{p}_a$.*

2. *The total surplus in the model with the platform providing ad blocking at zero cost of down-*

loading is higher *than the total surplus generated in the benchmark model and in the model with a third-party adblocker iff* $p_a \geq \hat{p}_a$.

PROOF: Let us consider first the publisher's advertising level setting problem. As we said in the main text, this problem looks exactly like in the benchmark model, the only difference being that there is additional constraint $A \leq V$ above which the publisher has no reason to go. Therefore, the publisher solves:

$$p_a A \left(1 - \frac{A}{2q}(1+r)\right) - cq^2 \Rightarrow \max_A \quad s.t.\ A \leq \min(q, V). \tag{A.32}$$

which implies the optimal $A^\star = \frac{q}{1+r}$.

Consider the optimal choice of the platform maximizing the total surplus. Clearly, there is no need to set $V > \frac{q}{1+r}$ since any such V achieves the same total surplus as $V = \frac{q}{1+r}$ (that is, it does not restrict the publisher in his optimal choice). Therefore, the optimal V always satisfies: $V \leq \frac{q}{1+r}$. The publisher's profit in this case is (substitute $A^\star = V$ into (A.32)):

$$\pi = p_a V \left(1 - \frac{V}{2q}(1+r)\right) - cq^2 \tag{A.33}$$

And, the consumer surplus is:

$$CS = \iint_{\vartheta q - \gamma V \geq 0} (\vartheta q - \gamma V) d\vartheta d\gamma = \int_{\frac{rV}{q}}^{\frac{V}{q}} d\vartheta \int_r^{\frac{\vartheta q}{V}} (\vartheta q - \gamma V) d\gamma + \int_{\frac{V}{q}}^{1} d\vartheta \int_r^1 (\vartheta q - \gamma V) d\gamma =$$

$$= \frac{1}{6}\left(\frac{(1-r^3)V^2}{q} - 3(1-r^2)V + 3(1-r)q\right) \tag{A.34}$$

The total surplus is therefore the sum of (A.33) and (A.34):

$$TS = \frac{1}{6}\left(\frac{(1-r^3-3p_a(1+r))V^2}{q} - 3(1-r^2-2p_a)V + 3(1-r-2cq)q\right) \tag{A.35}$$

The platform, hence, maximizes this total surplus subject to $V \leq \frac{q}{1+r}$:

$$\frac{1}{6}\left(\frac{(1-r^3-3p_a(1+r))V^2}{q}-3(1-r^2-2p_a)V+3(1-r-2cq)q\right) \Rightarrow \max_V,$$

$$s.t.\ V \leq \frac{q}{1+r}.$$

When $p_a < \frac{1-r^2}{2}$, the objective function is strictly decreasing in V and, therefore, $V^\star = 0$. Otherwise, the optimal V is defined by the solution of the first order conditions (easy to check that the constraint is not binding). Hence:

$$V^\star = \begin{cases} 0, & \text{if } p_a < \frac{1-r^2}{2}, \\ \frac{3q(2p_a+r^2-1)}{2(3p_a(1+r)+r^3-1)}, & \text{if } p_a \geq \frac{1-r^2}{2}. \end{cases} \tag{A.36}$$

We can finally calculate the optimal quality set by the publisher by substituting (A.36) into (A.33) and maximizing over q:

$$q^\star = \begin{cases} 0, & \text{if } p_a < \frac{1-r^2}{2}, \\ \frac{3p_a(2p_a+r^2-1)(6p_a(1+r)-(1-r)^3)}{16c(3p_a(1+r)+r^3-1)^2}, & \text{if } p_a \geq \frac{1-r^2}{2}. \end{cases}$$

Having computed the equilibrium values of the variables, we can calculate the consumer and total surpluses of the model with zero cost of downloading the adblock and compare them to the corresponding consumer and total surpluses of the benchmark model, the model with a third party adblocker and the platform model with non-zero cost of downloading the adblock. Assuming that $\bar{\kappa} \longrightarrow +\infty$, we obtain the following to complete the proof by comparing the surpluses. Define $\bar{p}_a$ as the unique solution to the following equation:

$$216(1+r)^3x^3 - 216(1+r)^2(1-r^3)x^2 + 6(1-r)^2(1+r)(11+r(16+r(18+r(16+11r))))x-$$

$$-(1-r)^3(1+r+r^2)(7+r(8+r(6+r(8+7r)))) = 0.$$

Then, the consumer surplus generated in the model with the platform and zero cost of downloading the adblocker is higher than the consumer surpluses generated in the benchmark model, in the model with a third-party adblocker and in the model with the platform and non-zero cost of downloading the adblocker if and only if

$$p_a \geq \bar{p}_a.$$

Define $\hat{p}_a$ as the unique solution to the following equation:

$$216(1+r)^3x^3 - 288(1+r)^2(1-r^3)x^2 + 3(1-r)^2(1+r)(37+r(56+r(66+r(56+37r))))x-$$

$$-2(1-r)^3(1+r+r^2)(7+r(8+r(6+r(8+7r)))) = 0.$$

The total surplus generated in the model with the platform and zero cost of downloading the adblocker is higher than the total surpluses generated in the benchmark model, in the model with a third-party adblocker and in the model with the platform and non-zero cost of downloading the adblocker if and only if

$$p_a \geq \hat{p}_a.$$

□

We again see that welfare can be negatively affected by the platform limiting advertising, even if consumers do not have to incur any costs. The reasons are similar to those in the previous case: the platform limits advertising after qualities are set. If the platform were able to commit an ad limit in the beginning, before quality levels are set, it could naturally achieve the highest possible total surplus. But in this case the players are hurt through the reduced incentives to invest in quality. The quality decision is especially sensitive to limiting advertising when selling ads is *not* very lucrative, i.e., when p_a is relatively low. We should note that the thresholds are actually lower than the threshold for switching to a subscription model. That is, when the advertising model is lucrative enough to be chosen by a publisher, ad limiting by the platform does improve both the total and consumer surplus.

A.15.3 Multiple Publishers

In this section, we extend our basic model from a single publisher to explore the case with multiple publishers. We consider two publishers which may have *different costs* of investing in quality. A key factor is the nature of the competition between the publishers. We will explore two separate settings. In one, we consider two non-competing publishers with overlapping consumers, that is, consumers who visit either or both publishers, but there is no competition for consumers. In the other, we consider competition between two publishers where all consumers consider both publishers but visit at most one of them. For parsimony, we assume $t = 0$ throughout this section. In the first subsection we assume that the adblocker cannot discriminate between two non-competing publishers and sets a common V for both, which is needed if consumers develop rational expectations about the proportion of ads blocked when they download the ad blocker, irrespective of publishers. Next, we relax the common V assumption and allow the adblocker to set separate V levels for the two non-competing publishers to demonstrate that our results are robust. Finally, we examine the case of competing publishers, but for parsimony we only look at the case of a common V level for both competing publishers.

Non-Competing Publishers

The main difference between our basic model and this generalization lies in the structure of consumer demand. As in the basic model, we assume that there is a unit mass of consumers with heterogeneity in ϑ, γ, and κ. All of these consumers have the option of visiting either one or both publishers' sites. They will visit a particular publisher if it provides positive utility. Importantly, there is no competition here, consumers do not compare the utility they get from the two publishers. Publishers may have different costs of quality. The rest of the model setup is identical to the basic model. All actions that publishers take are simultaneous with each other.

While there is no competition between the publishers, there are two sources of strategic interaction between them. First, the adblocker sets a common V, which is a function of both qualities (which may be different as publishers have different costs). Second, consumers in the overlapping segment make the ad blocker download decision considering both publishers.

We begin the analysis with the last stage. We use $q_H \geq q_L$ to denote the quality levels set by the two publishers in the first stage. Since there is no competitive interaction, a benchmark case without an adblocker is identical to our basic model. The publishers set $A_L = \frac{q_L}{1+r}$ and $A_H = \frac{q_H}{1+r}$ for the advertising levels. Using the same logic as in the basic model, we show that even in the presence of the adblocker the advertising levels are the same.

The decision by the adblocker to set the optimal V and the resulting decision by consumers to download the ad blocker becomes more complex. Given the ad levels, consumers have to make a decision to download the ad blocker considering both publishers. In the simplest case when qualities are equal or very similar, consumers will essentially consider the sum of qualities as they visit either, both or no site(s). However, when qualities are markedly different each consumer has to examine whether the ad blocker will be useful for either, both or neither site(s). Consider the case of a relatively low V as shown on Figure A.1. Consumers in the upper right corner, in the (abhgi) polygon will download the ad blocker for usage on both sites. However, consumers who value quality moderately, but are sensitive to the ads, in the (afi) triangle will download it only for the higher quality site. Consequently the boundaries for the latter (e.g the (fi) line) will only include q_H and not q_L.

When the adblocker sets V it also has to consider what consumers do, because it affects the negotiation power with the publishers. The adblocker can extract all the surplus V from *both* publishers for consumers in (abhgi), but it can only extract the surplus from the *high quality* publisher for consumers in (afi). The Figure only depicts one case, but it well illustrates the forces that the adblocker has to consider when setting V. It also explains why we get two qualitatively different outcomes depending on the difference between the quality levels. The following proposition summarizes the results.

Proposition 17 *There exists a $\tilde{q}$ threshold such that*

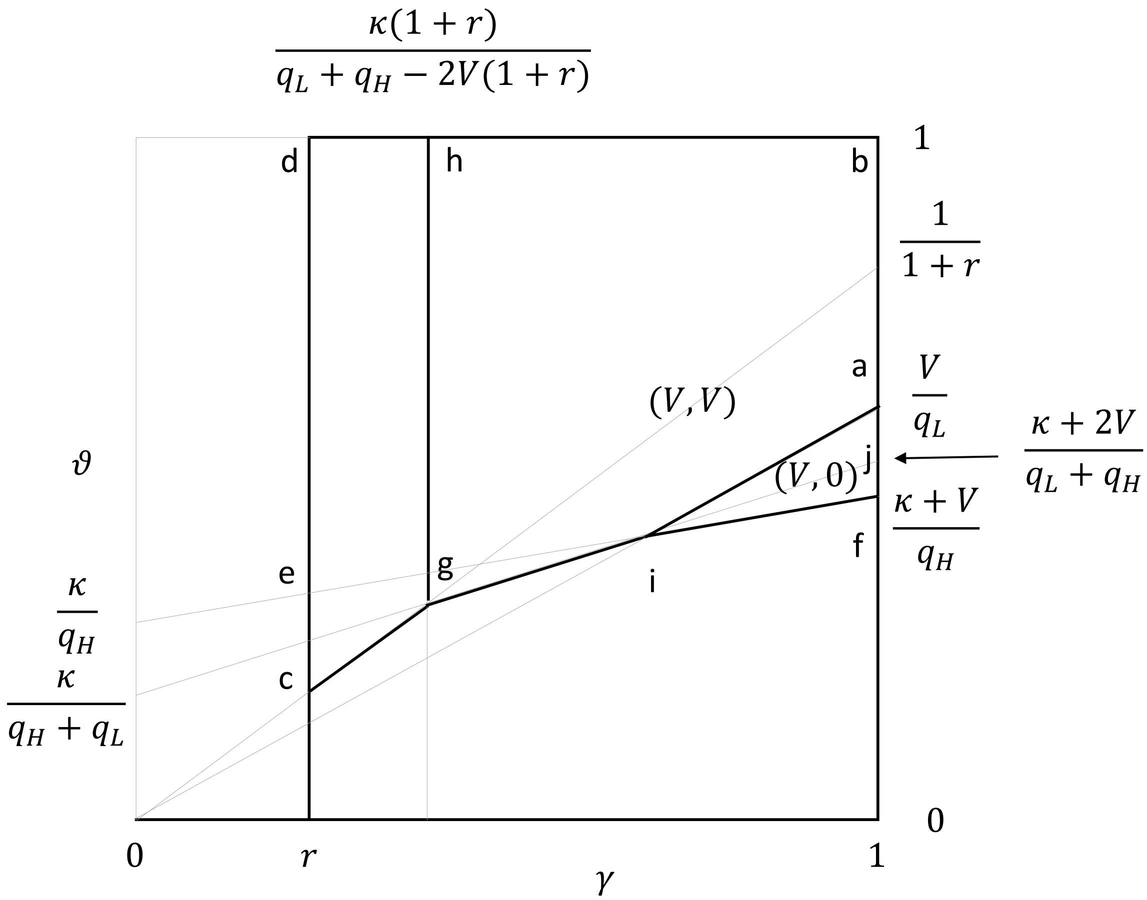

Figure A.1: Download decision of consumers with cost κ in the (γ, ϑ) space when the adblocker sets maximum ad volume to V. Publishers' qualities are q_H and q_L.

1. *If $q_H/q_L < \tilde{q}$ then $V^* < A_L$ and the adblocker blocks ads for both publishers*

2. *If $q_H/q_L > \tilde{q}$ then $A_L < V^* < A_H$ and the adblocker only blocks ads for one publisher.*

In both cases, the adblocker is able to extract a payment from both publishers.

PROOF: When determining the size of the segment that downloads the adblocker and the payment the adblocker can extract, we have to distinguish between two cases.

Case 1: $V \leq A_L = \frac{q_L}{1+r} \leq A_H = \frac{q_H}{1+r}$. Using $T(q_L, q_H, V)$ to denote the total payment from the two publishers, we get that

$$T(q_L, q_H, V) = T(q_L + q_H, 2V) + T_\Delta(q_L, q_H, V),$$

where $T(q, V)$ is the payment from the basic model and $T_\Delta(q_L, q_H, V)$ is a differential term. As illustrated on Figure A.1, the segment that downloads is similar to the segment that would download in the basic model if the quality were $q_L + q_H$ and the adblocker set the maximum advertising to $2V$. On the figure this is represented by the trapezoid (bjgh). The difference here, is that the adblocker only collects V from one firm in the triangle (aif). Hence, we need to account for the difference, this is the area of (jif) minus the area of (aij) multiplied by Vp_a. Integrating over κ yields

$$T_\Delta(q_L, q_H, V) = \frac{Vp_a}{\overline{\kappa}} \int_0^{rV(q_H/q_L-1)} \left(\frac{2\kappa + 2(1+r)V}{q_L + q_H} - \frac{(1+r)V}{2q_L} - \frac{2\kappa + (1+r)V}{2q_H} \right) d\kappa +$$
$$+ \frac{Vp_a}{\overline{\kappa}} \int_{rV(q_H/q_L-1)}^{V(q_H/q_L-1)} \frac{1 - \frac{\kappa}{V(q_H/q_L)-1}}{2(1-r)} \left(\frac{2\kappa + 4V}{q_L + q_H} - \frac{V}{q_L} - \frac{\kappa + V}{q_H} \right) d\kappa = -V^3 p_a \frac{(q_H - q_L)^3 (r^2 + r + 1)}{6\overline{\kappa} q_L^2 q_H (q_L + q_H)}. \tag{A.37}$$

If we optimize $T(q_L, q_H, V)$ in this region only, we get

$$V_1^* = \frac{12q_L^2 q_H(1+r)}{3(4q_H q_L - q_H^2 + q_L^2)(1+r+r^2)} - \frac{q_L\sqrt{2q_H((2+7r+9r^2+7r^3+2r^4)(q_H^3 - 3q_H^2 q_L + 7q_H q_L^2 - q_L^3) + 12rq_H q_L^2(1+3r+r^2))}}{\sqrt{3}(4q_H q_L - q_H^2 + q_L^2)(1+r)(1+r+r^2)}. \tag{A.38}$$

Case 2: $A_L = \frac{q_L}{1+r} \leq V \leq A_H = \frac{q_H}{1+r}$. In this case, the adblocker only extracts V from the high quality publisher and the amount is the same as in the basic case since the same people download. However, the adblocker extracts $A_L = \frac{q_L}{1+r}$ from the low quality publisher for consumers who download the adblocker and visit the low quality publisher. This yields

$$T(q_L, q_H, V) = T(q_H, V) + T'_\Delta(q_L, q_H, V),$$

where

$$\begin{aligned} T_\Delta(q_L, q_H, V)' &= \frac{q_L p_a}{1+r}\int_0^{r(\frac{q_H}{1+r}-V)}\left(1 - \frac{r}{2(1+r)} - \frac{1}{2(1+r)}\right)d\kappa + \\ &+ \frac{q_L p_a}{1+r}\int_{r(\frac{q_H}{1+r}-V)}^{(\frac{q_H}{1+r}-V)} \frac{1 - \frac{k}{V(q_H/(1+r))-V}}{1-r}\left(1 - \frac{\kappa}{2(q_H - V(1+r))} - \frac{1}{2(1+r)}\right)d\kappa = \\ &= \frac{q_L p_a}{1+r}\frac{(q_H-(1+r)V)(r^2+4r+1)}{6(1+r)^2}. \end{aligned} \tag{A.39}$$

If we optimize $T(q_L, q_H, V)$ in this region only, we get

$$V_2^* = q_H\left(\frac{1+r}{1+r+r^2} - \frac{\sqrt{(q_L/q_H+1)(1+5r+6r^2+5r^3+r^4)+3r^2}}{\sqrt{3}(1+r)(1+r+r^2)}\right).$$

Having determined the maximum in each region, we can determine the combined maximum by comparing $T(q_L, q_H, V)$ in V_1^* and V_2^*. The difference of the optima is monotone in q_H/q_L, hence $\tilde{q}$ will denote the value where the two are equal. Consequently, the global optimum V^* is V_1^*

if $q_H/q_L < \tilde{q}$ and V_2^* otherwise. □

The first point of the Proposition covers the case when the quality levels are not very different, i.e. when there is not too much asymmetry between the firms in terms of costs. The threshold for the ratio of qualities is between 3 and 4 depending on r. That is, when the higher quality is less than three times the lower quality we get the first case. Here, the adblocker focuses on both publishers. It sets V by considering roughly the average quality and V will be lower than both advertising levels. Hence most consumers will also consider both sites when making the download decision as the adblocker reduces advertising on both sites.

The second point describes the case when qualities are very asymmetric. In this case, the adblocker will set V *between* the two advertising levels, because setting it too low (below the low quality site's ad level) would hinder it's ability to extract surplus from the high quality site. Hence, in this case the adblocker will focus on the high quality site and mostly leave the low quality site alone. However, not completely. Interestingly, even though there is no ad blocking in equilibrium on the low quality site (as V is higher than the ad level), the adblocker can still extract surplus from the low quality site. The reason is that some consumers who visit both sites will download the ad blocker, hence the adblocker has power to negotiate a transfer even from the low quality site by threatening to block all ads.

While each of the two cases resembles what happens in the basic model with either replacing quality with the average (first case) or the higher quality (second case), there are some important interactions between the quality levels even when the adblocker only blocks one site. The following proposition summarizes the results on the optimal qualities.

Proposition 18 *In equilibrium, both publishers will set a lower quality level than without the presence of another publisher.*

1. *If the difference in costs is sufficiently high then the qualities will be different enough such that the adblocker only blocks ads for one publisher.*

2. *If the difference in costs is sufficiently low then the adblocker will block ads for both pub-*

lishers.

PROOF:

When the costs are highly asymmetric, i.e. c_H is sufficiently low and c_L is sufficiently high, the equilibrium qualities must be highly asymmetric as well. To prove this, one can set c_L high enough that publisher profits become negative for any small positive q_L. Given the high asymmetry in the qualities, the asymmetric case is triggerred in the previous proof. Publisher $i's$ revenue then becomes

$$q_i p_a \left(\frac{1}{2(1+r)} - \frac{(r^2+4r+1)(q_H - (1+r)V)}{6\overline{\kappa}(1+r)^3} \right).$$

The first order condition for the high quality (low cost) publisher's maximization problem is

$$\frac{1}{2(1+r)} + V\frac{r^2+4r+1}{6\overline{\kappa}(1+r)^2} = q_H \left(2c_H + \frac{r^2+4r+1}{3\overline{\kappa}(1+r)^3} - \frac{\partial V}{\partial q_H}\frac{r^2+4r+1}{6\overline{\kappa}(1+r)^2} \right),$$

where we use $V = V_2^*$ from the previous proof. When $q_L = 0$, V_2^* equals to that in the basic model and V_2^* is decreasing in q_L, while $\frac{\partial V}{\partial q_H}$ is 0 for $q_L = 0$, hence the optimal q_H has to be lower than in the basic model. The first order condition for the low quality (high cost) publisher's maximization problem is

$$\frac{1}{2(1+r)} + ((1+r)V - q_H)\frac{r^2+4r+1}{6\overline{\kappa}(1+r)^3} = q_L \left(2c_L - \frac{\partial V}{\partial q_L}\frac{r^2+4r+1}{6\overline{\kappa}(1+r)^2} \right).$$

Compared to the basic model, the $((1+r)V - q_H)$ is higher, resulting in a lower q_L.

To analyze the case when costs are similar we first look at the case when the two costs are identical. In this case, publisher $i's$ revenue is

$$q_i p_a \left(\frac{1}{2(1+r)} - \frac{(r^2+4r+1)(\overline{q} - (1+r)V)}{3\overline{\kappa}(1+r)^3} \right),$$

where $\overline{q}$ is the average quality and V is V_1^* as the two qualities are equal. The first order condition for publisher i's maximization problem is

$$\frac{1}{2(1+r)} + V\frac{r^2+4r+1}{3\overline{\kappa}(1+r)^2} = q_i \left(2c + \frac{r^2+4r+1}{2\overline{\kappa}(1+r)^3} - \frac{\partial V}{\partial q_i}\frac{r^2+4r+1}{3\overline{\kappa}(1+r)^2}\right).$$

The difference from the first order condition in the basic case is

$$\frac{r^2+4r+1}{6\overline{\kappa}(1+r)^3}((1+r)V - q_i\left(1-(1+r)\frac{\partial V}{\partial q_i}\right).$$

Since $1-(1+r)\frac{\partial V}{\partial q_i} > (1+r)V$ when the two qualities are equal, the optimal quality is lower than in the benchmark case. The same result holds when the two costs are sufficiently close as that leads to quality levels that are sufficiently close as well, allowing us to conduct the same analysis in the limit. □

The main results of this Proposition resemble our previous findings. Quality decreases with cost and asymmetry between the two players leads to the familiar pattern: when the two publishers are different enough the adblocker will only focus on one of them. Clearly, there is strategic interaction between the two players and the qualities they set. In both cases the presence of another player has a negative effect on the a publisher, but the mechanisms are slightly different. In the asymmetric case, although the adblocker focuses on the high quality player it also extracts rent from the low quality publisher, reducing V compared to the basic model with just the high quality player. This suppresses the high quality as a lower V generally leads to more downloads of the ad blocker and generally hurts publishers. The low quality player is also hurt. Even though V is higher than it would be with only the low quality player in the market, V is low enough that it encourages to download the ad blocker with the purpose of using it for the high quality site. However, a larger number of consumers download the ad blocker than if the low quality player were alone, which allows the adblocker to extract more surplus from even the low quality player than if it were alone.

In the symmetric case, consumers use the ad blocker for both sites and consequently the adblocker uses roughly the average quality to set V. At the extreme, if the two qualities are equal the adblocker will react similarly as if there were a single player. But an important difference from the basic model is that the consumer benefit from a single download doubles: for the same download cost κ, a consumer gets ad blocking on two sites. The larger downloading segment suppresses publisher profits and leads to lower quality investments.

Non-competing publishers with different adblocking levels

We now relax the assumption that the adblocker sets a common V for both publishers and allow it to set different levels of V. We start the analysis in the stage where the adblocker sets V given quality levels of $q_H \geq q_L$. Let V_H and V_L denote the respective ad limits for the two publishers. We first examine the case when $V_L/q_L \geq V_H/q_H$. Since the adblocker has control over two separate V values, there is no reason to set any of them above the respective advertising levels, hence $V_L \leq \frac{q_L}{1+r}$ and $V_H \leq \frac{q_H}{1+r}$. We can thus proceed similarly as in case 1 of the proof of Proposition 17. Using $T(q_L, q_H, V_L, V_H)$ to denote the total payment from the two publishers, we get that

$$T(q_L, q_H, V_L, V_H) = T(q_L + q_H, V_L + V_H) + T_\Delta(q_L, q_H, V_L, V_H),$$

where $T(q, V)$ is the payment from the basic model and $T_\Delta(q_L, q_H, V)$ is a differential term. The difference can be calculated by accounting for the segment that downloads the ad blocker only to consume the higher quality site's content:

$$\begin{aligned}
T_\Delta(q_L, q_H, V_L, V_H) = \frac{V_H p_a}{\overline{\kappa}} & \int_0^{r(V_L \frac{q_H}{q_L} - V_H)} \left(\frac{2\kappa + (1+r)(V_L + V_H)}{2(q_L + q_H)} - \frac{2\kappa + (1+r)V_H}{2q_H} \right) d\kappa \\
& + \frac{V_H p_a}{\overline{\kappa}} \int_{r(V_L \frac{q_H}{q_L} - V_H)}^{V_L \frac{q_H}{q_L} - V_H} \frac{1 - \frac{\kappa}{V_L(q_H/q_L) - V_H}}{2(1-r)} \left(\frac{\kappa + V_L + V_H}{q_L + q_H} - \frac{\kappa + V_H}{q_H} \right) d\kappa \\
& - \frac{V_L p_a}{\overline{\kappa}} \int_0^{r(V_L \frac{q_H}{q_L} - V_H)} \left(\frac{(1+r)V_L}{2q_L} - \frac{2\kappa + (1+r)(V_L + V_H)}{2(q_L + q_H)} \right) d\kappa \\
& - \frac{V_L p_a}{\overline{\kappa}} \int_{r(V_L \frac{q_H}{q_L} - V_H)}^{V_L \frac{q_H}{q_L} - V_H} \frac{1 - \frac{\kappa}{V_L(q_H/q_L) - V_H}}{2(1-r)} \left(\frac{V_L}{q_L} - \frac{\kappa + V_L + V_H}{q_L + q_H} \right) d\kappa \\
& = -p_a \frac{(V_L q_H - V_H q_L)^3 (r^2 + r + 1)}{6 q_L^2 q_H (q_L + q_H)}. \qquad \text{(A.40)}
\end{aligned}$$

If we optimize $T(q_L, q_H, V_L, V_H)$ in this region only, we get

$$V_L^\star = q_L F(r) \text{ and } V_H^\star = q_H F(r).$$

Similarly, if we assume $V_L/q_L \leq V_H/q_H$, we get the same optimum. That is the adblocker sets the same ad limit levels as if it was facing two different publishers without an overlapping consumer-base. It is worth noting that $V_L^\star/q_L = V_H^\star/q_H$, the proportion of ads blocked is the same on both sites. Using i and j to denote the two sites, we can write the profit function for site i as

$$\frac{q_i p_a}{2(1+r)} - \frac{(r^2+4r+1)(1-(1+r)F(r))q_i(q_i+q_j)p_a}{6\overline{\kappa}(1+r)^3} - cq_i^2.$$

Maximizing yields

$$q_i^\star = \frac{1 - \frac{2(r^2+4r+1)(1-(1+r)F(r))}{3\overline{\kappa}(1+r)^2} q_j^\star}{\frac{4c_i(1+r)}{p_a} + \frac{2(r^2+4r+1)(1-(1+r)F(r))}{3\overline{\kappa}(1+r)^2}}.$$

Quality decisions are strategic substitutes. A higher quality set by the other publisher leads to a lower quality choice.

$$q_i^\star = \left(\frac{4c_i(1+r)}{p_a} + \frac{2(r^2+4r+1)(1-(1+r)F(r))}{3\overline{\kappa}(1+r)^2} \cdot \frac{c_i + c_j}{c_j}\right)^{-1}.$$

The equilibrium quality is thus lower than without the presence of another site and it increases in the other site's cost. Overall, this outcome exhibits patterns similar to what we find in the second point of Proposition 18 where costs are similar. The difference here is that when the adblocker can set different levels of V for the publishers, we do not replicate what we found in the first point of Proposition 18 where costs were sufficiently different and the adblocker only blocked ads on one site.

Competing Publishers

In this section, we analyze the case of competing publishers. Publishers are substitutes, so each consumer has capacity to visit at most one publisher's site, choosing the one that provides higher utility. In order to provide an analytical solution, we use the convention of the literature (Shaked and Sutton, 1982) that quality investment is costless with a maximum quality of 1. Thus, here, there is no inherent heterogeneity in quality between publishers. This allows us to isolate the strategic effect that the adblocker might have on competing publishers.[5] The rest of the model construction is identical to our previous setup.

We first analyze a benchmark without the adblocker. Publishers can adjust both quality and advertising levels to win over customers. We begin by analyzing the last stage, in which they set the advertising levels given the qualities (denoted by $q_L \leq q_H$). Denoting the two advertising levels by A_L, A_H and determining the boundaries reveals that consumers with $\vartheta \geq \gamma \frac{A_H - A_L}{q_H - q_L}$ choose the high quality site, consumers with $\gamma \frac{A_H - A_L}{q_H - q_L} \geq \vartheta \geq \gamma \frac{A_L}{q_L}$ choose the low quality site and the rest will not visit any site. Calculating the size of the demand for each site and differentiating yields

$$A_H^* = \frac{4q_H(q_H - q_L)}{(4q_H - q_L)(1+r)}, \quad A_L^* = \frac{2q_L(q_H - q_L)}{(4q_H - q_L)(1+r)}.$$

Calculating the revenues and differentiating yields that one publisher will select the maximum quality of $q_H = 1$ and its competitor will set $q_L = 4/7$ to differentiate.

We are now ready to analyze how the adblocker changes the nature of competition. The analysis flows similarly to that in the previous versions of the model. One important difference emerges when we look at consumers' decision to download the ad blocker. Unlike in the non-competing case, here, consumers pick a single site to visit. Consequently, they will only use the ad blocker for a single site. With the ad blocker this site will necessarily be the higher quality site because the only cost to the consumer was the higher ad level which is irrelevant with the ad blocker. That is, consumers who use the ad blocker will only visit the high quality site and they will only take

[5] If we use the same cost structure as in the basic model with different costs, we have to resort to numerical analysis to calculate the equilibrium qualities.

into account the higher quality level when making the download decision. This in turn leads the adblocker to also focus on the high quality publisher when setting V and the optimal V will be the same as if the high quality site were alone (but setting a competitive advertising level). The following proposition summarizes the results together with the effects on quality. We assume $r \to 0$ in the quality analysis to be able to determine the equilibrium.

Proposition 19

1. *In equilibrium $A_L < V^* < A_H$ and only ads on the high quality site are blocked. As a result, a segment of consumers switches from the low quality publisher to the high quality publisher.*
2. *If $r \to 0$, the equilibrium q_L^* is always higher in the presence of the adblocker. Furthermore q_L^* decreases with $\overline{\kappa}$ and approaches $4/7$ as $\overline{\kappa} \to \infty$.*

PROOF:

We calculate the segment that downloads the adblocker along similar lines as before. Here, all consumers consider only the high quality site when making the download as they will all visit that given the same level of (V) advertising. The transfer that the adblocker maximizes can be calculated as

$$T(q_L, q_H, V, r) = V \int_0^{r(A_H-V)} \left(1 - \frac{\kappa}{q_H} - \frac{(1+r)V}{2q_H}\right) d\kappa +$$
$$+V \int_{r(A_H-V)}^{A_H-V} \frac{1 - \frac{k}{A_H-V}}{1-r} \left(1 - \frac{\kappa A_H}{2(A_H-V)q_H} - \frac{k+V}{2q_H}\right) d\kappa = V((V(1+r) - q_H)(4q_H - ql) + 3q_H q_L)$$
$$\cdot \frac{((V(1+2r+2r^2+r^3) - q_H(2+5r+2r^2))(4q_H - ql) - 3(1+r+r^2)q_H q_L)}{6q_H(4q_H - qL)^2(1+r)^2}. \quad \text{(A.41)}$$

Maximizing this in V yields

$$V^* = q_H \frac{1+r}{1+r+r^2}$$
$$- q_H \frac{\sqrt{3(16q_H^2 + 4q_H q_L + 7q_L^2)(1+5r+9r^2+5r^3+r^4) - 15q_L^2 r(7+9r+7r^2) - 108 q_L q_H r^2}}{3(4q_H - qL)(1+r)(1+r+r^2)}, \tag{A.42}$$

which falls between the two advertising levels. Specifically, for $r = 0$, we get

$$V^* = q_H \left(1 - \frac{\sqrt{16q_H^2 + 4q_H q_L + 7q_L^2}}{\sqrt{3}(4q_H - qL)} \right).$$

Since the high quality players payoff is increasing in its quality, we can set $q_H = 1$ and calculating the low quality players payoff yields

$$\frac{2(1-q_L)q_L}{(4-q_L)^2} - \frac{4(q_L)(1-q_L)\left(16 + 7q_L + 10q_L^2 + 6q_L^3 - 3\sqrt{3(16+4q_L+7q_L^2)}\right)}{3(4-q_L)^3\left(\sqrt{3(16+4q_L+7q_L^2)} - 3q_L\right)\overline{\kappa}}.$$

The first term is equal to the revenue without the adblocker, hence it is maximal for $q_L = 4/7$. The second term captures the loss from the adblocker and it is inverse U-shaped. Since its derivative is negative in $q_L = 4/7$, the overall derivative is positive there. As a consequence the overall optimum must be $q_L^* > 4/7$. Finally, it is clear that the second term converges to 0 as $\overline{\kappa} \to \infty$, hence the optimization problem converges to the no adblocker case if the download cost goes to infinity. □

The first point of this Proposition illustrates our main result that V falls between the two advertising levels. This is because the adblocker intends to undercut the ad level on the high quality site, but not on the low quality site. As before, this hurts the high quality site, because advertising is reduced and the adblocker is able to extract revenue. However, it also hurts the low quality site, because a segment of consumers switches to the high quality site. These consumers previously could not 'afford' the high quality site because of the higher advertising, but now with the ads limited by the adblocker they can enjoy the higher quality content. Furthermore, similarly to our basic model, there are some consumers who did not visit any site before, but now they do and they immediately jump to the high quality content.

The second point highlights a strategic effect. Given our simplified cost structure, the higher quality publisher always chooses the maximum quality of 1 to differentiate. The interesting question is how the lower quality changes with the adblocker. We find that it *increases*, therefore leading to *less differentiation*. The intuition relies on the notion that the ad blocker makes consumers switch away from the low quality site. The overall magnitude of switching and its impact on revenue depends on the quality differential. The biggest loss from switching happens at a quality level that is lower than the equilibrium quality without an ad blocker. The impact diminishes as the lower quality approaches the higher quality, because the overall pool of consumers that could switch decreases. Combining this effect with the basic forces that drive differentiation, we get that there is still differentiation, but less than without the adblocker. Naturally, as $\overline{\kappa}$ increases, the main differentiating forces take over and the differentiation approaches the level we have without the adblocker.

The effect of the adblocker on consumers is mostly positive. One reason is that without quality costs, we do not observe a reduction in quality. In fact, with less differentiation the lower quality increases. Furthermore, ad blocking directly increases consumer utility and as we noted there is a segment of consumers who switch from the low quality to the higher quality publisher and there is a segment that moves from no consumption to visiting a site. In addition to these segments, the reduced differentiation even benefits consumers who are not using the ad blocker as competition leads to lower ad levels. It is important to note that if we add back quality costs, ad blocking will have a primary effect of suppressing quality investment as in the base model. However, the competitive forces we identify above may alleviate some of those negative factors.

A.15.4 Competing Adblockers

The paper assumes a monopolist adblocker thorughout. Here, we explore modeling competition between adblockers.

Perfect Competition

In a simple extension of the model, we assume that there are two competing adblockers that are identical. The decision variable in which they can compete is V. Consumer preferences are the same and consumers simply choose to download at most one of the adblocking software. Since the adblockers only differ in V, consumers will simply download the adblocker with the lower V. This, however leads to a Bertrand-like competition with an only equilibrium of $V_i = V_j = 0$, i.e. full adblocking. Both adblockers make zero payoffs in this equilibrium. From the publisher and consumer perspective, we compare this case of “perfect competition" (PC) to our main model with a single adblocker that charges the publisher (AB). As we show below, consumers are not necessarily better off, because even though adblocker competition has a direct utility increasing effect, it also hurts publisher, reducing quality and consequently diminishing consumer surplus.

Proposition 20 *Quality and publisher profits are always lower in the “perfect competition" case than in the main model:* $q^\star_{PC} < q^\star_{AB}$, $\Pi^{Pub\star}_{PC} < \Pi^{Pub\star}_{AB}$. *Total consumer surplus can be either higher or lower. We get* $CS^\star_{PC} < CS^\star_{AB}$ *when* $\frac{p_a}{c}$ *is relatively high and* $CS^\star_{AB} < CS^\star_{PC}$ *when* $\frac{p_a}{c}$ *is low. For total welfare we get* $TW^\star_{PC} < TW^\star_{AB}$ *unless* p_a *is sufficiently small and* $\frac{c}{p_a}$ *is also small.*

PROOF: From the publisher and consumer perspective it does not matter how many adblockers there are if they are all equivalent and all set $V = 0$. Consumers who download an adblocker choose randomly. Hence, for the purposes of the proof, we assume that there is a single adblocker with $V = 0$. In this case, the publisher's payoff from users who did not download the adblocker can be calculated as in (A.7), by simply substituting $V = 0$. As a result, we get

$$q^\star = q^\star_{PC} = \begin{cases} \left(\dfrac{4c(1+r)(1-t)}{p_a} + \dfrac{G_{\{V=0\}}(r,t)}{\overline{\kappa}}\right)^{-1} & \text{if } 0 \le t \le \frac{r}{1+r} \\ \left(\dfrac{4c(1+r)}{p_a(1-r(1-t)+t)} + \dfrac{G_{\{V=0\}}(r,t)}{\overline{\kappa}}\right)^{-1} & \text{if } \frac{r}{1+r} \le t \le 1. \end{cases}, \quad \text{where}$$

$$G_{\{V=0\}}(r,t) = \begin{cases} \frac{2(r^2+4r+1)}{3(1+r)^2} & \text{if } \ t \le \frac{r}{1+r} \\ \frac{2((3r+1)(1-r)^2+(1+r+t+rt+4r^2+r^2t-6r^3+3r^3t)t)}{3(1-r(1-t))(1-r(1-t)+t)} & \text{if } \ \frac{r}{1+r} \le t \end{cases}$$

Since $G_{\{V=0\}}(r,t) > G(r,t)$, we obtain that $q^\star_{PC} < q^\star_{AB} < q^\star_{NA}$. At a lower quality, publisher profits are also lower. Note that the decrease in quality is relatively low if the first term is large, that is when p_a/c is small. Total consumer utility can be calculated as

$$CS^\star_{PC} = q^\star_{PC} C_0(r,t) + (q^\star_{PC})^2 \frac{C_{\Delta\{V=0\}}(r,t)}{\overline{\kappa}}. \tag{A.43}$$

Comparing $CS^\star_{PC}$ in (A.43) with $CS^\star_{AB}$ in (A.9) shows that $CS^\star_{PC} - CS^\star_{AB}$ is a linear function of $\frac{q^2}{\overline{\kappa}}$ for a fixed q and the multiplier is positive and only depends on r and t, not on p_a or c. If the reduction from $q^\star_{AB}$ to $q^\star_{PC}$ is relatively small because $\frac{p_a}{c}$ is high only then the positive multiplier results in an overall increase and $CS^\star_{PC} > CS^\star_{AB}$. Since both the adblocker's and the publisher's profit is lower in the PC case than in the AB case, the total welfare is also lower unless the increase in consumer surplus is sufficiently higher. This can only happen when p_a is sufficiently small in addition to $\frac{p_a}{c}$ being large.

□

Adblocker Differentiation

It is possible for adblockers to differentiate. Theoretically, it would be possible for adblockers to horizontally differentiate either by blocking different types of ads or working better on certain sites, realistically the potential to differentiate seems more vertical. We change our model by allowing the competing adblockers not only to choose V, but also to be able to change the download costs. As before, the baseline is that download costs are distributed uniformly between 0 and $\overline{\kappa}$. However, each adblocker is able to lower the download cost, by setting a $\lambda_i \leq 1$ multiplier. Thus, the download costs will be uniformly distributed between 0 and $\lambda_i \overline{\kappa}$ for each adblocker. Note that these two distributions are not independent, the ratio of download costs of the adblocker is the same for each individual adblocker. Naturally, adblockers generally want lower download costs, but this presumably takes effort. In a simple setup we assume that the adblocker is free to set any $\lambda_i \geq \underline{\lambda}$. The timing of the game is changed by inserting a stage where each adblocker selects λ_i *between* the stage when the adblocker selects quality and the stage when the adblockers select V_i. Hence the adblockers first simultaenously set λ_i and then simultaneously select V_i.

The subgame of choosing V_i is very similar to the analysis we have conducted in Section 2.2.3 of the paper. There, we had two different download costs and $V_j = 0$. Extending the analysis of Lemma 2 reveals the response functions to each other's V_i. The only equilibrium where both adblockers make positive profits is one where $V_i > V_j$ if $\lambda_i < \lambda_j$. That is, the adblocker with the lower download cost sets a higher ad pass-through level. Unfortunately, the equilibrium V levels are the roots of a forth-degree polynomial, making a close-form solution intractable. However, it is clear that when $\lambda_i = \lambda_j$, a Bertrand-like competition ensues and both publishers walk away with zero payoffs. Therefore, it is optimal for one publisher to differentiate by setting a download cost level which is more than the minimum. That is, in equilibrium $\lambda_i = \underline{\lambda}$ and $\lambda_j > \underline{\lambda}$, resulting in vertical differentiation. This outcome is consistent with what we observe in reality: the most visible adblockers that make a conscious effort to reduce consumer frictions and facilitite downloads do set a higher ad pass-through rate than less media-savvy adblockers who often offer closer to full adblocking.

A.15.5 Reducing Dimensions of Heterogeneity

In this section, we reduce the dimensionality of our main model presented in Section 2.2. The base model has heterogeneity across three parameters. Here, we fix each parameter one by one while keeping heterogeneity in the other dimensions and examine how the results change.

Fixed Ad Sensitivity

We first examine a model with a fixed γ instead of γ being distributed in the $[r,1]$ interval. Note that our main model already includes such a setting as setting $r=1$ is equivalent to assuming $\gamma=1$. Below, we examine the case of $\gamma<1$ which is very similar.

In the benchmark case without an adblocker, the publisher maximizes $A\frac{1}{1-t}\left(1-\max\left(t,\frac{A\gamma}{q}\right)\right)$ in A, yielding $A^\star=\max\left(\frac{q}{2\gamma},\frac{qt}{\gamma}\right)$. The resulting optimal quality is then $q^*=\max\left(\frac{p_a}{8c(1-t)\gamma},\frac{p_a t}{2c\gamma}\right)$.

When an ad blocker enters the market the optimal advertising does not change, just as in the main model. A consumer downloads the ad blocking software if and only if $\gamma\geq\frac{\kappa}{A-V}=\frac{\kappa\gamma}{q/2-\gamma V}$ *and* $\vartheta\geq\frac{\kappa+\gamma V}{q}$. Consider $t=0$. For a given κ, the size of the consumer segment who downloads is $S_V(\kappa,q,V)=1-\frac{\kappa+\gamma V}{q}$ if $\kappa<\gamma(A-V)=q/2-\gamma V$ and 0 otherwise. By integrating over κ we get the overall size of the downloaders and consequently the transfer the adblocker can extract from the publisher:

$$T(q,V)=\frac{Vp_a}{\overline{\kappa}}\int_0^{q/2-\gamma V}\left(1-\frac{\kappa+\gamma V}{q}\right)d\kappa=\frac{Vp_a}{\overline{\kappa}}\left(\frac{q}{2}-\gamma V\right)\left(\frac{3}{4}-\frac{\gamma V}{2q}\right)$$

Maximizing in V yields $V^*=\frac{q}{\gamma}\left(\frac{2}{3}-\frac{\sqrt{7}}{6}\right)$ for $t=0$. The same exercise for $t>0$ yields

$$V^*=\begin{cases}\frac{q}{\gamma}\left(\frac{2}{3}-\frac{\sqrt{7}}{6}\right) & \text{if } \; t\leq\frac{2}{3}-\frac{\sqrt{7}}{6}\\ \frac{q}{\gamma}\left(\frac{3-2t^2-2t^3}{16(1-t)}\right) & \text{if } \; \frac{2}{3}-\frac{\sqrt{7}}{6}\leq t\leq\frac{1}{2}\\ \frac{qt}{2\gamma} & \text{if } \; \frac{1}{2}\leq t\end{cases}$$

Comparing this quantity to Proposition 1, shows that we simply reproduce those results (assuming $r=1$) with a $1/\gamma$ multiplier.

Given the transfer to the adblocker, the publisher can only keep revenue from the consumers who do not download the ad blocker, but visit the publisher's site. Integrating over κ gives the publisher's revenue and accounting for the quality investment costs and maximizing in q yields

$$q^\star = \begin{cases} \left(\frac{8\gamma c(1-t)}{p_a} + \frac{\sqrt{7}-1)}{3\overline{\kappa}}\right)^{-1} & \text{if} \quad t \le \frac{2}{3} - \frac{\sqrt{7}}{6} \\ \left(\frac{8\gamma c(1-t)}{p_a} + \frac{4(1-t)^2+1}{8(1-t)\overline{\kappa}}\right)^{-1} & \text{if} \quad \frac{2}{3} - \frac{\sqrt{7}}{6} \le t \le \frac{1}{2} \\ \left(\frac{4\gamma c}{p_a t} + \frac{t}{\overline{\kappa}}\right)^{-1} & \text{if} \quad \frac{1}{2} \le t \end{cases}$$

This result again closely matches those in Proposition 2, yielding essentially the same pattern as in our main model.

Despite the similarities with the main model we lose some features compared to the main model with three dimensions of heterogeneity. In particular, this model is less realistic because consumers with a given download cost are always either all better off using a site with the ad blocker or without. That is, the adblocker does not create two segments based on ad sensitivity. Intuitively, ad sensitivity is a key reason for consumers to use ad blockers and it is important to understand the implications of heterogeneity along this dimension.

Fixed Quality Sensitivity

We now study the model where $0 < \vartheta \le 1$ is fixed instead of being uniformly distributed in the $[0, 1]$ interval. Note that our main model already includes such a setting as setting $t = 1$ is equivalent to assuming $\vartheta = 1$. Below, we examine the case of $\vartheta < 1$ which is very similar. In the absence of an ad blocker, the publisher maximizes $A(\frac{\vartheta q}{A} - r)$ in A as long as $A \ge \vartheta q$, yielding $A^\star = \vartheta q$ as the publisher chooses the lowest ad level that results in full consumption. The resulting optimal quality is then $q^* = \frac{\vartheta p_a}{2c}$.

In the presence of an adblocker, the optimal advertising does not change just as before. A consumer downloads the ad blocking software if and only if $\gamma \ge \frac{\kappa}{A-V} = \frac{\kappa}{\vartheta q - V}$ *and* $\gamma \le \frac{\vartheta q - \kappa}{V}$. However, the lower and upper bound of downloaders cannot both be less than 1. The lower bound is less than one if $\kappa < \vartheta q - V$ and it is r whenever $\kappa < r(\vartheta q - V)$. Therefore, the total amount of downloaders is

$$\int_0^{r(\vartheta q-V)} \frac{1}{\overline{\kappa}} d\kappa + \int_{r(\vartheta q-V)}^{\vartheta q-V} \frac{1}{(1-r)\overline{\kappa}} \left(1 - \frac{\kappa}{\vartheta q - V}\right) d\kappa = \frac{1+r}{2\overline{\kappa}}(\vartheta q - V),$$

yielding $T(q,V) = \frac{(1+r)Vp_a}{2}\overline{\kappa}(\vartheta q - V)$ and thus $V^\star = \frac{\vartheta q}{2}$.

The segment who does not download the ad blocker and visits the publisher's site is between r and $\min\left(\frac{\kappa}{\vartheta q - V}, 1\right)$. The overall size is

$$\int_{r(\vartheta q-V)}^{\vartheta q-V} \frac{1}{(1-r)\overline{\kappa}} \left(\frac{\kappa}{\vartheta q - V} - r\right) d\kappa + \int_{\vartheta q-V}^{\overline{\kappa}} \frac{1}{\overline{\kappa}} d\kappa = \frac{2\overline{\kappa} - (1+r)(\vartheta q - V)}{2\overline{\kappa}} = \frac{4\overline{\kappa} - (1+r)\vartheta q}{4\overline{\kappa}}$$

Thus, the publisher maximizes $\vartheta q p_a \frac{4\overline{\kappa} - (1+r)\vartheta q}{4\overline{\kappa}} - cq^2$, yielding

$$q^* = \frac{\vartheta}{\frac{2c}{p_a} + \frac{(1+r)\vartheta^2}{2\overline{\kappa}}}.$$

Although the overall results of this simplified model are similar to those of the main model, we lose an important segment. Those consumers who are better off with the presence of the adblocker. The reason is that we do not have ad sensitive consumers who are moderately quality sensitive. In our main model, these are the consumers who visit the publisher only with an adblocker. In this two dimensional model the publisher sets the advertising at a level such that the most ad sensitive consumers always visit the publisher even if there is no adblocker present.

Fixed Download Cost

In this last subsection we fix $\kappa > 0$ instead of having it distributed in the $[0, \overline{\kappa}]$ interval. In the absence of the adblocker we obtain the exact same results as in our main model. However, with the adblocker the analysis becomes substantially more complicated. When κ is sufficiently high, nobody downloads the software and we reproduce the benchmark results. But when κ is relatively low, the game becomes more complex with an additional moving part. Unlike in the main model, where the mass of high download cost consumers ensures that the publisher does not change its advertising levels, here, there is an incentive to react. Intuitively, as more and more consumers who are ad sensitive download the ad blocker, the publisher can focus more on the less ad sensitive consumers and raise advertising levels. This in turn allows the adblocker to set a higher V to increase profits, but then at some point the publisher has an incentive to drastically lower advertising to undercut the ad blocker. The result is the lack of a pure-strategy equilibrium. Next, we determine the conditions under which a pure strategy equilibrium exists and then we prove the non-existence of an equilibrium when those conditions are not satisfied.

An equilibrium where no consumer downloads the adblocker exists when all consumers are better off visiting the site without adblocker then with. This holds when even the most ad sensitive consumer (with $\gamma = 1$) prefers $A^\star = \frac{q}{1+r}$ advertising as opposed to V with a download cost of κ. The necessary and sufficient condition is $V + \kappa \geq \frac{q}{1+r}$. That is, if the combination of download costs and ads let through by the ad blocker are sufficiently high.

Let us now assume that $V + \kappa < \frac{q}{1+r}$. In this case the most ad sensitive consumers will always want to download the ad blocker if they expect the benchmark level of advertising, hence there is no zero-download equilibrium. Recall that the timing of our game places the decision of advertising levels at the last stage, after V has been set and the download decisions have been made by consumers. However, consumers have rational expectations about the ad levels when they decide to download or not. Mathematically, this is equivalent to the download and advertising decision being made simultaneously, but after V has been set. Let us now fix V and suppose there is an equilibrium with advertising at $\hat{A}$. As before, there are two constraints for a consumer to download the ad blocker: one ensuring that visiting with the site using the ad blocker provides non-negative utility and one ensuring that the utility is not less with an ad blocker. These correspond to lines ga and gh in Figure 2.2. Let $\hat{\gamma}$ denote the critical ad sensitivity level corresponding to the second constraint. Given V and $\hat{A}$, we have $\hat{\gamma} = \frac{\kappa}{\hat{A}-V}$. That is, a consumers with $\gamma > \hat{\gamma}$ who also have $\vartheta > \frac{\kappa+\gamma V}{q}$ all download the ad blocker and nobody else does. If this were an equilibrium $A = \hat{A} = V + \kappa/\hat{\gamma}$ would have to be the optimal advertising level for the publisher to set. However, there is always a better option for the publisher. If $V \leq \frac{\hat{\gamma}}{\hat{\gamma}+r} - \hat{\gamma}\kappa$ then $A = \frac{q}{(1+r)\hat{\gamma}}$ is a better response, otherwise

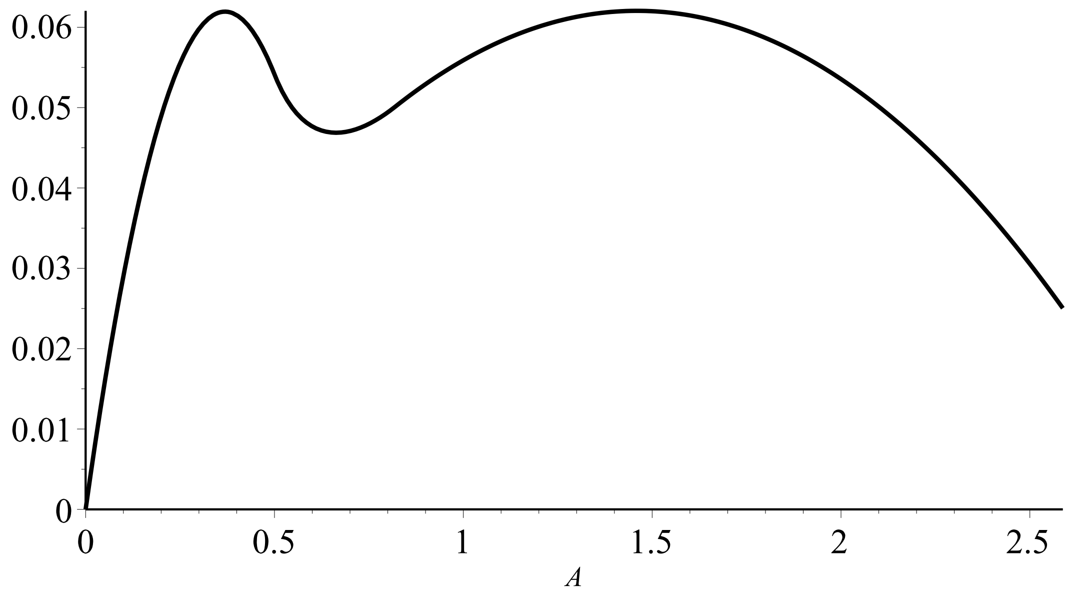

Figure A.2: Advertiser payoff as a function of A when $q = 1, \kappa = 0.2, V = 0.3, r = 0.3, \hat{\gamma} = 0.385$

$A = \frac{2\hat{\gamma}(q-\kappa-r)+2\kappa+V(1-\hat{\gamma}^2)}{2(1-r^2)} < \hat{A}$ is a better response. Hence, there is no equilibrium in pure strategies.

The above proof suggests an intuition for a mixed strategy equilibrium that may exist. For a given level of expected advertising level and corresponding download boundaries, the publisher has dueling incentives. On one hand, it wants to increase advertising to focus only on consumers with low ad sensitivity. On the other hand it may want to undercut the adblocker's V advertising level and also satisfy the very ad sensitive population who did not download the ad blocker because even with $V < \hat{A}$ they do not get positive utility from visiting the site. When both of these local optima are equally attractive, the publisher could mix between these two ad levels with probabilities that put the expected ad level at $\hat{A}$.

Unfortunately, the analysis become intractable for us to solve it. Figure A.2 shows a numerical example with $q = 1, \kappa = 0.2, V = 0.3, r = 0.3$. We find that under these parameters, $\hat{\gamma} \approx 0.385$ yields a publisher payoff function that has two equal local maxima. One is at $\underline{A} \approx 0.37$ and the other is at $\overline{A} \approx 1.46$. The expected advertising level that yields the above $\hat{\gamma}$ threshold is $\hat{A} = V + \kappa/\hat{\gamma} \approx 0.82$, thus in equilibrium the publishers mixes between $\underline{A} \approx 0.37$ and $\overline{A} \approx 1.46$ by choosing the former with probability ≈ 0.59 and choosing the latter with probability ≈ 0.41.

Note that the expected advertising level of ≈ 0.82 is higher than the benchmark of $\frac{1}{1+r} \approx 0.77$. Thus, we observe a slight overall increase in advertising as a response to the adblocker, because the publisher tends to shift focus on the less ad sensitive consumers. The increase does not always happen though as the publisher can sometimes lower advertising to capture ad sensitive consumers who are dissatisfied with the adblocker due to it not blocking all ads.

A.15.6 Consumers who like ads

There may be consumer who do not derive a disutility from ads. On the contrary, they may be matched with an advertiser that gives them positive utility eventually. While the likelihood of a positive match is possibly positive for all consumers, there is heterogeneity in the overall dislike of ads. In our basic model γ measures this and we assume that it is always positive since it is distributed uniformly between $r > 0$ and 1. However, for consumers who actually like ads in expectation, γ maybe negative. A straightforward way to incorporate this into our model is to allow for a negative r. The reason we assume that r is positive in our main model is that we placed no limit on A, the amount of advertising. If there are consumers who like ads, the publisher has an incentive to infinitely increase ads in our model. In reality this does not happen because there is a limit to how much advertising can be placed on a certain site. Thus, to allow for a negative r, we assume that A has an upper limit. For simplicity, we assume here that $A \leq q$ and $t = 0$, but other upper bounds and positive t values could yield similar results. Otherwise our model is identical to the base model presented in the paper.

The analysis is also similar to what we do in the proofs of Propositions 1 and 2. Given the negative r, the publisher will set the highest possible advertising level, $A = q$. With that, the size of the segment that downloads the adblocker and the adblocker's objective function can be calculated as

$$T(q,V) = p_a V \int_0^{q-V} \frac{1}{\overline{\kappa}} \cdot \frac{1}{1-r}\left(1 - \frac{\kappa}{q-V}\right)\left(1 - \frac{\kappa}{2(q-V)} - \frac{\kappa + V}{2q}\right) = p_a V \frac{(q-V)(2q-V)}{6q(1-r)\overline{\kappa}},$$

similar to (A.4), resulting in $V^\star = (1 - 1/\sqrt{3})q$. Given that, the publisher's objective function becomes

$$\max_q \left(p_a q \frac{1-2r}{2(1-r)} - \frac{q^2 p_a}{6\sqrt{3}\overline{\kappa}(1-r)} \right).$$

Consequently, the equilibrium quality becomes

$$q^* = \left(\frac{4c(1-r)}{p_a(1-2r)} + \frac{2}{3\sqrt{3}\overline{\kappa}(1-r)} \right)^{-1}.$$

This is similar to what we obtain in Proposition 2. This $q^\star$ is increasing in $\overline{\kappa}$ and is converging to the benchmark $q^\star_{NA}$ as $\overline{\kappa} \to \infty$. The same holds true for the total consumer surplus. Our main interest here is to see how the results change as r becomes smaller (more negative). Since even the baseline $q^\star_{NA}$ is changing (increasing) in r, we calculate the proportion of the two qualities with and without adblocker:

$$\frac{q^\star}{q^\star_{NA}} = \frac{\frac{4c(1-r)}{p_a(1-2r)}}{\frac{4c(1-r)}{p_a(1-2r)} + \frac{2}{3\sqrt{3}\overline{\kappa}(1-r)}} = \left(1 + \frac{p_a(1-2r)}{6\sqrt{3}\overline{\kappa}(1-r)^2}\right)^{-1}.$$

This proportion is *decreasing* in r, that is it *increases* as r becomes more negative and eventually converges to 1 as $r \to -\infty$. In other words, the impact of the adblocker diminishes as more consumers like ads.

In summary, our results are similar to those of our main model with the adblocker having a negative effect on the publishers and consumers through reduced quality. However, the damage caused by the adblocker is smaller if more consumers like ads.

A.15.7 Ad price increasing in quality

Throughout the paper we assume that the price of an ad impression, p_a is exogenously given and is fixed for all publishers. In reality there is a variation which is mostly driven by the type of consumers who visit the site. With the emergence of programmatic ad transactions and real-time bidding, the price can be fully personalized and tailored to the individual visiting the site. Whereas modeling these targeting mechanisms is beyond the scope of our paper, we examine a scenario which does not involve personalized ad pricing, but involves a plausible scenario. Publishers with higher quality or quantity of content are overall likely able to attract an audience that is more valuable to advertisers. Thus, it's plausible to assume that p_a is an increasing, concave function of q. In particular, we assume a specific family of functions, where each ad impression can be sold for $p_a q^\beta$. Clearly, $\beta = 0$ corresponds to our basic model. Below, we examine our main model for $0 \leq \beta < 1$ with all other parts of the model unchanged.

When there is no adblocker present, the ad level remains $\frac{q}{1+r}$ and the ad price only affects the publisher's decision making at the quality choice stage, where it maximizes

$$\begin{cases} \frac{q^{1+\beta} p_a}{2(1+r)(1-t)} - cq^2 \\ \frac{q^{1+\beta} p_a (1-r(1-t)+t)}{2} - cq^2 \end{cases} \quad \text{yielding} \quad q^\star_{NA} = \begin{cases} \left(\frac{(1+\beta)p_a}{4c(1+r)(1-t)}\right)^{\frac{1}{1-\beta}} & \text{if } 0 \leq t \leq \frac{r}{1+r} \\ \left(\frac{(1+\beta)p_a(1-r(1-t)+t)}{4c}\right)^{\frac{1}{1-\beta}} & \text{if } \frac{r}{1+r} \leq t \leq 1. \end{cases}$$

As expected $q^\star_{NA}$ is increasing in β, because as higher quality leads to a higher price, the more incentive there is to invest.

In the presence of an adblocker, we can also see that β only has an effect at the quality choice stage. Even though the adblocker's payoff also depends on the ad price, it is only a mere multiplier and does not effect the optimal choice of V. Given that $V^\star$ remains the same as in the basic model, we can calculate the objective function of the publisher. For example, when $t = 0$, we have

$$\frac{q^{1+\beta} p_a}{2(1+r)} - \frac{(r^2+4r+1)(1-(1+r)F(r))q^{2+\beta} p_a}{6\overline{\kappa}(1+r)^3} - cq^2.$$

There is typically no closed-form solution for the optimum as the derivative includes terms with

q^{β}, q and $q^{1+\beta}$. Only when $\beta = 1/2$ do we obtain a closed-form solution of

$$q^{\star} = \begin{cases} \left(\dfrac{\frac{4\sqrt{2}(1+r)(1-t)c}{p_a} - \sqrt{\frac{32(1+r)^2(1-t)^2c^2}{p_a^2} + 15G(r,t)}}{5G(r,t)} \right)^2 & \text{if } 0 \leq t \leq \frac{r}{1+r} \\ \left(\dfrac{\frac{4\sqrt{2}c}{(1-r(1-t)+t)p_a} - \sqrt{\frac{32c^2}{(1-r(1-t)+t)^2 p_a^2} + 15G(r,t)}}{5G(r,t)} \right)^2 & \text{if } \frac{r}{1+r} \leq t \leq 1. \end{cases}$$

However, we can show for any $0 < \beta < 1$ that there is a unique optimal $q^{\star}$ which is decreasing in c, and increasing in both $\overline{\kappa}$ and p_a. Hence the structure of the results, we obtained in the main model does not change.

Extending our analysis carries through for most of the paper. One notable example is the case of competing publishers in Section A.15.3. The main effect here is reduced differentiation as β increases. In the benchmark case without the adblocker, the publishers set qualities of $4/7$ and 1 when $\beta = 0$. We can calculate the equilibrium qualities when $\beta > 0$ and we still get that one publisher sets a high quality of q. The other one sets

$$q_L^{\star} = \frac{5\beta + 7 - \sqrt{9\beta^2 + 54\beta + 49}}{2\beta} > 4/7,$$

which is increasing in β. When the adblocker is present, β only enters the publishers' decision making at the quality choice stage and the forces are the same as in Section A.15.3. Overall, publishers still differentiate but less so if β is higher. The strategic effects introduced by the adblocker are similar to what we have derived before.

Appendix B: Essay 2

B.1 Proof of Proposition 8

We first start with finding out the optimal period 2 advertising strategies given the fixed choice of period 1 advertising strategy and the quantity.

Period 2

First, let us assume that the firm advertised in period 1. Then, it receives a signal about the actual value of A. Using the Bayes' formula, we can revise the a posteriori probabilities of A as:

$$Pr(A = \hat{A}|\sigma = \hat{\sigma}) = \frac{Pr(\sigma = \hat{\sigma}|A = \hat{A})Pr(A = \hat{A})}{Pr(\sigma = \hat{\sigma}|A = 1)Pr(A = 1) + Pr(\sigma = \hat{\sigma}|A = 0)Pr(A = 0)}.$$

where $\hat{A} \in \{0; 1\}$ and $\hat{\sigma} \in \{0; 1\}$.

If $\hat{\sigma} = 1$, then the distribution of A becomes:

$$A = \begin{cases} 1, \textit{w/p: } \frac{1}{2} + \gamma, \\ 0, \textit{w/p: } \frac{1}{2} - \gamma. \end{cases}$$

The firm's profit therefore becomes the following (we are taking the expectation of (3.1)). If it advertises:

$$\pi_2^H(s = 1) = q(1 - q)\left(\frac{1}{2} + \gamma\right) + 0 * \left(\frac{1}{2} - \gamma\right) - k = q(1 - q)\left(\frac{1}{2} + \gamma\right) - k.$$

If it does not:

$$\pi^H(s = 0) = 0.$$

The firm chooses to advertise when:

$$q(1-q)\left(\frac{1}{2}+\gamma\right)-k \geq 0 \Rightarrow q(1-q)\left(\frac{1}{2}+\gamma\right) \geq k.$$

If $\hat{\sigma}=0$, then the distribution of A becomes:

$$A=\begin{cases} 1, \textit{w/p: } \frac{1}{2}-\gamma, \\ 0, \textit{w/p: } \frac{1}{2}+\gamma. \end{cases}$$

with the corresponding profits:

$$\pi_2^L(s=1)=q(1-q)\left(\frac{1}{2}-\gamma\right)+0*\left(\frac{1}{2}+\gamma\right)-k=q(1-q)\left(\frac{1}{2}-\gamma\right)-k.$$

$$\pi^L(s=0)=0.$$

The firm chooses to advertise when:

$$q(1-q)\left(\frac{1}{2}-\gamma\right) \geq k.$$

Since in period 1 the firm, when it decides on the advertising strategy in period 1, does not yet know what the signal realization is going to be, it has to base its decision on the expected period 2 profit, that is:

$$\mathbb{E}\pi_2=\pi_2^H(s_2=\hat{s_2}|\hat{\sigma}=1)Pr(\hat{\sigma}=1)+\pi_2^L(s_2=\hat{s_2}|\hat{\sigma}=0)Pr(\hat{\sigma}=0).$$

Now, since the period 2 decisions on the choice of s_2 as a function of the signal realizations depend on the parameters of the model, there are several parameter regions we need to consider (note that a priori $Pr(\hat{\sigma}=1)=Pr(\hat{\sigma}=0)=\frac{1}{2}$).

Region 1: $k \leq q(1-q)\left(\frac{1}{2}-\gamma\right)$.

The firm advertises regardless of the signal realization. Hence, $\mathbb{E}\pi_2$ becomes:

$$\mathbb{E}\pi_2 = \frac{1}{2}\left[q(1-q)\left(\frac{1}{2}+\gamma\right)-k\right]+\frac{1}{2}\left[q(1-q)\left(\frac{1}{2}-\gamma\right)-k\right] =$$

$$= \frac{1}{2}q(1-q)-k.$$

Region 2: $k \geq q(1-q)\left(\frac{1}{2}+\gamma\right)$.

The firm never advertises regardless of the signal realization. Hence, $\mathbb{E}\pi_2$ becomes:

$$\mathbb{E}\pi_2 = 0.$$

Region 3: $q(1-q)\left(\frac{1}{2}-\gamma\right) < k < q(1-q)\left(\frac{1}{2}+\gamma\right)$.

The firm advertises only if it received a positive signal realization. Hence, $\mathbb{E}\pi_2$ becomes:

$$\mathbb{E}\pi_2 = \frac{1}{2}\left[q(1-q)\left(\frac{1}{2}+\gamma\right)-k\right].$$

Finally, let us assume that the firm did not advertise in period 1. Then, the distribution of A is the same as the a priori distribution, that is, $Pr(A=1) = Pr(A=0) = \frac{1}{2}$. If the firm advertises in period 2, then:

$$\mathbb{E}\pi_2(s=1) = q(1-q)*\frac{1}{2}+0*\frac{1}{2}-k = \frac{1}{2}q(1-q)-k.$$

If it does not advertise, then:

$$\mathbb{E}\pi_2(s=0) = 0.$$

The firm chooses to advertise when:

$$\frac{1}{2}q(1-q)-k \geq 0 \Rightarrow \frac{1}{2}q(1-q) \geq k. \tag{B.1}$$

Period 1

The firm anticipates the outcomes of period 2 and makes the advertising decision and the quantity decision that takes future profits into account. Since in the case of a monopolist firm, it does not matter whether s_1 and q are set simultaneously or sequentially (there is no new information gained), we will solve the model as if the firm first set the quantity, q, and then the advertising decision, s_1, for simplicity.

Choice of s_1

When setting s_1, the firm knows its optimal choices of s_2 as a function of the signal realization in period 2. The expected profits in period 2 as a function of the period 1 choice of s_1 are, therefore, as given in the previous period 2 section. These period 2 anticipated profits depend on the parameter regions, so we will again consider the choice of s_1 separately for the 3 regions.

Region 1: $k \leq q(1-q)\left(\frac{1}{2}-\gamma\right)$.

Assuming that in period 1 $s_1 = 1$ has been chosen, then in period 2 the firm advertises regardless of the signal realization. If, on the other hand, we assume that the firm chose $s_1 = 0$ in period 1, than the firm still chooses to advertise in period 2 since, according to (B.1), $k \leq q(1-q)\left(\frac{1}{2}-\gamma\right) \leq \frac{1}{2}q(1-q)$.

If the firm chooses $s_1 = 1$, then its profits are:

$$\mathbb{E}\pi = \mathbb{E}\pi_1 + \mathbb{E}\pi_2 = \frac{1}{2}q(1-q) - k + \frac{1}{2}q(1-q) - k = q(1-q) - 2k.$$

If the firm chooses $s_1 = 0$, then its profits are:

$$\mathbb{E}\pi = \mathbb{E}\pi_1 + \mathbb{E}\pi_2 = 0 + \frac{1}{2}q(1-q) - k = \frac{1}{2}q(1-q) - k.$$

Hence, it is optimal to choose $s_1 = 1$, when:

$$q(1-q) - 2k \geq \frac{1}{2}q(1-q) - k \Rightarrow \frac{1}{2}q(1-q) \geq k$$

It can be shown that in Region 1 this inequality is always satisfied. Therefore, in Region 1, it is always true that $s_1 = 1$.

Region 2: $k \geq q(1-q)\left(\frac{1}{2}+\gamma\right)$.

In period 2, the firm never advertises regardless of whether the firm has chosen $s_1 = 1$ or $s_1 = 0$ in period 1.

If the firm chooses $s_1 = 1$, then its profits are:

$$\mathbb{E}\pi = \frac{1}{2}q(1-q) - k + 0 = \frac{1}{2}q(1-q) - k.$$

If the firm chooses $s_1 = 0$, then its profits are:

$$\mathbb{E}\pi = 0 + 0 = 0.$$

Hence, it is optimal to choose $s_1 = 1$, when:

$$\frac{1}{2}q(1-q) - k \geq 0 \Rightarrow \frac{1}{2}q(1-q) \geq k.$$

It can be shown that in Region 2 this inequality is never satisfied. THerefore, in Region 2, it is always true that $s_1 = 0$.

Region 3: $q(1-q)\left(\frac{1}{2}-\gamma\right) < k < q(1-q)\left(\frac{1}{2}+\gamma\right)$.

Assuming that the firm chooses $s_1 = 1$ in period 1, the firm advertises in period 2 only if it receives a positive signal. If the firm chooses $s_1 = 0$ in period 1, the firm advertises in period 2 only when $k \leq \frac{1}{2}q(1-q)$ (as in (B.1)).

If the firm chooses $s_1 = 1$, then its profits are:

$$\mathbb{E}\pi = \frac{1}{2}q(1-q) - k + \frac{1}{2}q(1-q)\left(\frac{1}{2}+\gamma\right) - \frac{k}{2} = \frac{1}{2}q(1-q)\left(\frac{3}{2}+\gamma\right) - \frac{3k}{2}.$$

If the firm chooses $s_1 = 0$, then it profits are the following. If $q(1-q)\left(\frac{1}{2}-\gamma\right) < k \leq \frac{1}{2}q(1-q)$, then:

$$\mathbb{E}\pi = 0 + \frac{1}{2}q(1-q) - k = \frac{1}{2}q(1-q) - k.$$

If $\frac{1}{2}q(1-q) < k < q(1-q)\left(\frac{1}{2}+\gamma\right)$, then:

$$\mathbb{E}\pi = 0 + 0 = 0.$$

First, the firm would choose $s_1 = 1$ in the sub-region of $q(1-q)\left(\frac{1}{2}-\gamma\right) < k \leq \frac{1}{2}q(1-q)$, when:

$$\frac{1}{2}q(1-q)\left(\frac{3}{2}+\gamma\right) - \frac{3k}{2} \geq \frac{1}{2}q(1-q) - k \Rightarrow q(1-q)\left(\frac{1}{2}+\gamma\right) \geq k.$$

It can be shown that in this sub-region the inequality always holds. Therefore, in this sub-region, it is always true that $s_1 = 1$.

Second, the firm would choose $s_1 = 1$ in the sub-region of $\frac{1}{2}q(1-q) < k < q(1-q)\left(\frac{1}{2}+\gamma\right)$, when:

$$\frac{1}{2}q(1-q)\left(\frac{3}{2}+\gamma\right) - \frac{3k}{2} \geq 0 \Rightarrow \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right) \geq k.$$

Hence, combining the results from these 2 sub-regions we can show that it is optimal to choose $s_1 = 1$ when:

$$\frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right) \geq k.$$

Interestingly, if there was no concern for learning for the next period, the firm would only advertise in period 1 when $k \leq \frac{1}{2}q(1-q)$. It means that when:

$$\frac{1}{2}q(1-q) < k \leq \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right)$$

the firm advertises in period 1 even though the immediate period 1 payoff from doing so is negative.

Finally, if we summarize all the 3 regions, we can see the following. When $k \leq q(1-q)\left(\frac{1}{2}-\gamma\right)$ the firm advertises in both periods regardless of signals. When $k \geq \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right)$ the firm never advertises in either period. When $q(1-q)\left(\frac{1}{2}-\gamma\right) < k < \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right)$ the firm always advertises in period 1 and advertises in period 2 only when it receives a high signal.

Given the results above, the firm's expected profit over the two periods as a function of q is:

$$\mathbb{E}\pi = \begin{cases} 0, & \text{if } k \geq \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right) \\ q(1-q) - 2k, & \text{if } k \leq q(1-q)\left(\frac{1}{2}-\gamma\right) \\ \frac{1}{2}q(1-q)\left(\frac{3}{2}+\gamma\right) - \frac{3k}{2}, & \text{if } q(1-q)\left(\frac{1}{2}-\gamma\right) < k < \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right). \end{cases} \tag{B.2}$$

Choice of q

Now let us move on to the quantity choice. The firm maximizes profits (B.2) over q. Since profit is piece-wise in q, we will maximize the profits separately for all the 3 pieces and then compare them to find the global solution.

Note first, that if $k > \frac{1}{8} + \frac{\gamma}{12}$, then $k > \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right)$ for any q. This means that for any choice of q, the profit $\pi^\star = 0$. So the optimal quantity is arbitrary and the profits are zero. Clearly, when $k \leq \frac{1}{8}+\frac{\gamma}{12}$, the firm would never choose q in such a way as to have $k > \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right)$ since by setting a high enough q it can get a strictly positive profit of at least $\mathbb{E}\pi = \frac{1}{2}q(1-q)\left(\frac{3}{2}+\gamma\right) - \frac{3k}{2}$. Now let us assume that $k \leq \frac{1}{8} + \frac{\gamma}{12}$. In this case, the optimal q can only lie either in $0 \leq k \leq q(1-q)\left(\frac{1}{2}-\gamma\right)$ (with the corresponding profits of $\mathbb{E}\pi = q(1-q) - 2k$) or in $q(1-q)\left(\frac{1}{2}-\gamma\right) < k < \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right)$ (with the corresponding profits of $\mathbb{E}\pi = \frac{1}{2}q(1-q)\left(\frac{3}{2}+\gamma\right) - \frac{3k}{2}$). Therefore, we will consider these two cases one by one.

Case 1: $k < q(1-q)\left(\frac{1}{2}-\gamma\right)$ (or, if rearranged, $q(1-q) > \frac{2k}{1-2\gamma}$).

The firm solves the following problem:

$$\mathbb{E}\pi = q(1-q) - 2k \Rightarrow \max_q,$$

$$s.t.\ q(1-q) \geq \frac{2k}{1-2\gamma}.$$

which implies the choice of q:

$$q_1^\star = \begin{cases} \frac{1}{2}, & \text{if } k \leq \frac{1}{8} - \frac{\gamma}{4}, \\ \text{undefined}, & \text{if } k > \frac{1}{8} - \frac{\gamma}{4}. \end{cases}$$

and the profits:

$$\pi_1^\star = \begin{cases} \frac{1}{4} - 2k, & \text{if } k \leq \frac{1}{8} - \frac{\gamma}{4}, \\ \text{undefined}, & \text{if } k > \frac{1}{8} - \frac{\gamma}{4}. \end{cases} \tag{B.3}$$

Note that for high values of k the constraint can never be satisfied and, therefore, the optimal q and the profits are not defined.

Case 2: $q(1-q)\left(\frac{1}{2}-\gamma\right) \leq k \leq \frac{1}{3}q(1-q)\left(\frac{3}{2}+\gamma\right)$ (or, if rearranged, $\frac{6k}{3+2\gamma} \leq q(1-q) \leq \frac{2k}{1-2\gamma}$).

The firm solves the following problem:

$$\mathbb{E}\pi = \frac{1}{2}q(1-q)\left(\frac{3}{2}+\gamma\right) - \frac{3k}{2} \Rightarrow \max_q,$$

$$s.t.\ \frac{6k}{3+2\gamma} \leq q(1-q) \leq \frac{2k}{1-2\gamma}.$$

which implies the choice of q:

$$q_2^\star = \begin{cases} \frac{1}{2}, & \text{if } k \geq \frac{1}{8} - \frac{\gamma}{4}, \\ \frac{2k}{1-2\gamma}, & \text{if } k < \frac{1}{8} - \frac{\gamma}{4}. \end{cases}$$

and the profits:

$$\pi_2^\star = \begin{cases} \frac{1}{8}\left(\frac{3}{2}+\gamma\right) - \frac{3k}{2}, & \text{if } k \geq \frac{1}{8} - \frac{\gamma}{4}, \\ \frac{4\gamma k}{1-2\gamma}, & \text{if } k < \frac{1}{8} - \frac{\gamma}{4}. \end{cases} \tag{B.4}$$

By directly comparing the profits (B.3) and (B.4) (and recalling that when $k > \frac{1}{8} + \frac{\gamma}{12}$ the optimal q is undefined and the profits are 0), we can find the global optimum of q and the global optimal profits:

$$q^{\star} = \begin{cases} \text{arbitrary}, & \text{if } k > \frac{1}{8} + \frac{\gamma}{12}, \\ \frac{1}{2}, & \text{if } k \leq \frac{1}{8} + \frac{\gamma}{12}. \end{cases}$$

$$\pi^{\star} = \begin{cases} 0, & \text{if } k > \frac{1}{8} + \frac{\gamma}{12}, \\ \frac{1}{8}\left(\frac{3}{2} + \gamma\right) - \frac{3k}{2}, & \text{if } \frac{1}{8} - \frac{\gamma}{4} \leq k \leq \frac{1}{8} + \frac{\gamma}{12}, \\ \frac{1}{4} - 2k, & \text{if } k < \frac{1}{8} - \frac{\gamma}{4}. \end{cases}$$

To verbally describe the solution, the firm either chooses arbitrary q when k is very high and never advertises afterwards (neither in period 1 nor in period 2), or chooses $q^{\star} = \frac{1}{2}$ when k is small and advertises in both periods (in period 2 regardless of the signal received), or chooses $q^{\star} = \frac{1}{2}$ when k is in the intermediate range (*i.e.* $\frac{1}{8} - \frac{\gamma}{4} \leq k \leq \frac{1}{8} + \frac{\gamma}{12}$) and always advertises in period 1 but only advertises in period 2 the signal realization is high. Exactly in this last parameter region it is possible that the firm would advertise in period 1 even though the immediate period 1 payoff might be negative. This happens when $\frac{1}{8} \leq k \leq \frac{1}{8} + \frac{\gamma}{12}$.

In the learning is perfect, that is, $\gamma = \frac{1}{2}$, the equilibrium quantity and profits simplify to:

$$q^{\star} = \begin{cases} \text{arbitrary}, & \text{if } k > \frac{1}{6}, \\ \frac{1}{2}, & \text{if } k \leq \frac{1}{6}. \end{cases}$$

$$\pi^{\star} = \begin{cases} 0, & \text{if } k > \frac{1}{6}, \\ \frac{1}{4} - \frac{3k}{2}, & \text{if } k \leq \frac{1}{6}. \end{cases}$$

B.2 Proof of Lemma 5

Here, since there is no spillover from period 1 to period 2, the firm advertises in these periods if and only if its one period immediate profit is positive, that is, $\mathbb{E}\pi = \frac{1}{2}q(1-q) - k \geq 0 \Rightarrow k \leq$

$\frac{1}{2}q(1-q)$. The aggregate profit over periods 1 and 2 is therefore:

$$\mathbb{E}\pi = \begin{cases} q(1-q) - 2k, & \text{if } k \le \frac{1}{2}q(1-q), \\ 0, & \text{if } k > \frac{1}{2}q(1-q). \end{cases}$$

If $k > \frac{1}{8}$, then it is true that $k > \frac{1}{2}q(1-q)$ for any value of q. Hence, q is arbitrary and the profits are 0. Clearly, when $k \le \frac{1}{8}$, the firm would never set q in such a way that $k > \frac{1}{2}q(1-q)$ since by setting a high enough q it can get a strictly positive profit of $\mathbb{E}\pi = q(1-q) - 2k$.

If $k \le \frac{1}{8}$, the optimal q is found, therefore, as a solution to the following maximization problem:

$$\mathbb{E}\pi = q(1-q) - 2k \Rightarrow \max_q$$

$$s.t.\ k \le \frac{1}{2}q(1-q).$$

Which gives the following optimal q and the optimal profits:

$$q^\star = \frac{1}{2},$$

$$\pi^\star = \frac{1}{4} - 2k.$$

Combining the $k > \frac{1}{8}$ case and the $k \le \frac{1}{8}$ case, we get:

$$q^\star = \begin{cases} \text{arbitrary}, & \text{if } k > \frac{1}{8}, \\ \frac{1}{2}, & \text{if } k \le \frac{1}{8}. \end{cases}$$

$$\pi^\star = \begin{cases} 0, & \text{if } k > \frac{1}{8}, \\ \frac{1}{4} - 2k, & \text{if } k \le \frac{1}{8}. \end{cases}$$

Comparing the benchmark quantities to the quantities in the learning model is a bit tough since for certain regions the quantities are not well defined. Nevertheless, we can say that when there is learning, the region where the quantity is well defined (and $q = \frac{1}{2}$) is wider than with no learning. Profits, though, are always higher with learning than without.

B.3 Proof of Proposition 10

We solve the model using backwards induction starting from period 2 going back to period 1. In period 2 both firms know each others quantities and first period advertising strategy choices.

Period 2

Without loss of generality assume that $q_1(1 - q_1) \leq q_2(1 - q_2)$. In what follows, we will assume some choice of the period 1 advertising strategies (there are 4 possible combinations) and solve for period 2 equilibria. Then, we will find conditions under which the firms actually choose these period 1 strategies. We consider all 4 of the first period combinations one by one. Denote by s^i_j the advertising campaign choice of firm i in period j.

Case 1: $s^1_1 = s^1_2 = 1$.

Due to perfect learning, both firms know their actual personal A^2_i's. At the same time, firm i never knows the value of A^2_{-i} and uses a prior distribution to estimate it. Therefore, from the point of view of firm i, there can be two situations depending on the signal received: either firm i observes $A^2_i = 0$ or it observes $A^2_i = 1$.

If firm i observes $A^2_i = 0$, then it always true that:

$$s^2_i = 0.$$

If firm i observes $A^2_i = 1$, then it chooses between $s^2_i = 1$ and $s^2_i = 0$ based on the expected profits from advertising (with expectation being taken over its competitor's A^2_{-i} not known to firm

i). Firm i's profits in this case are (as a function of s_i^2 and s_{-i}^2):

$$\mathbb{E}\pi_i^2(s_i^2 = 1; s_{-i}^2 = 1) = \frac{1}{2}q_i(1-q_i) + \frac{1}{2}q_i(1-q_i-q_{-i}) - k = q_i(1-q_i) - \frac{1}{2}q_iq_{-i} - k,$$

$$\mathbb{E}\pi_2^1(s_i^2 = 1; s_{-i}^2 = 0) = q_i(1-q_i) - k,$$

$$\mathbb{E}\pi_i^2(s_i^2 = 0; s_{-i}^2 = 1) = \pi_2^1(s_i^2 = 0; s_{-i}^2 = 0) = 0.$$

Firm $-i$'s profits are symmetric. Denote, by s_i^{2H} the advertising choice of firm i given that it observed $A_i^2 = 1$ and by s_i^{2L} the advertising choice of firm i given that it observed $A_i^2 = 0$. This subgame, therefore, has the following pure strategy Nash equilibria:

$$(s_1^{2H}; s_1^{2L}; s_2^{2H}; s_2^{2L}) = \begin{cases} (1;0;1;0), & \text{if } k \leq q_1(1-q_1) - \frac{1}{2}q_1q_2, \\ (1;0;0;0), & \text{if } q_2(1-q_2) - \frac{1}{2}q_1q_2 \leq k \leq q_1(1-q_1), \\ (0;0;1;0), & \text{if } q_1(1-q_1) - \frac{1}{2}q_1q_2 \leq k \leq q_2(1-q_2), \\ (0;0;0;0), & \text{if } k \geq q_2(1-q_2). \end{cases}$$

As we can see, $(1;0;0;0)$ and $(0;0;1;0)$ equilibria overlap (the region where the $(1;0;0;0)$ equilibrium exists is fully located inside the region where the $(0;0;1;0)$ equilibrium exists). We will assume that the equilibrium that actually realizes is the one that is Pareto dominant, that is, in our case the one for which $q_2(1-q_2) \geq q_1(1-q_1)$ or $(0;0;1;0)$ (if $q_1(1-q_1) = q_2(1-q_2)$, than one of these two equilibria realizes randomly with probability $\frac{1}{2}$). Hence, with this refinement, the equilibria of this sub-game are:

$$(s_1^{2H}; s_1^{2L}; s_2^{2H}; s_2^{2L}) = \begin{cases} (1;0;1;0), & \text{if } k \leq q_1(1-q_1) - \frac{1}{2}q_1q_2, \\ (0;0;1;0), & \text{if } q_1(1-q_1) - \frac{1}{2}q_1q_2 \leq k \leq q_2(1-q_2), \\ (0;0;0;0), & \text{if } k \geq q_2(1-q_2). \end{cases}$$

Before period 2 begins, firms do not yet know their signal realizations. So, to make a decision on first period strategies, they need to use an expectation (over possible signal realizations) as to their second period profits. From the point of view of period 1 both signal realizations ($A_i^2 = 1$ or $A_i^2 = 0$) are equally likely, so the expected second period profit is $\frac{1}{2}$ times the profit obtained if the realization is $A_i^2 = 1$ plus $\frac{1}{2}$ times the profit if the realization turned out to be $A_i^2 = 0$ (which turns out to be 0). These profits turn out to be the following:

$$\mathbb{E}\pi_2^1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{k}{2}, & \text{if } k \leq q_1(1-q_1) - \frac{1}{2}q_1q_2, \\ 0, & \text{if } k > q_1(1-q_1) - \frac{1}{2}q_1q_2. \end{cases} \tag{B.5}$$

$$\mathbb{E}\pi_2^2 = \begin{cases} \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 - \frac{k}{2}, & \text{if } k \leq q_1(1-q_1) - \frac{1}{2}q_1q_2, \\ \frac{1}{2}q_2(1-q_2) - \frac{k}{2}, & \text{if } q_1(1-q_1) - \frac{1}{2}q_1q_2 < k < q_2(1-q_2), \\ 0, & \text{if } k \geq q_2(1-q_2). \end{cases} \tag{B.6}$$

Case 2: $s_1^1 = 1$ and $s_2^1 = 0$.

In this case, firm 1 perfectly knows its A_1^2 (and does not know A_2^2) while firm 2 knows neither A_1^2 nor A_2^2. Consider the profits of firm 1. If firm 1 observes $A_1^2 = 0$, then it always chooses $s_1^2 = 0$.

If firm 1 observes $A_1^2 = 1$, then its profits are (here we take an expectation over the unknown A_2^2):

$$\mathbb{E}\pi_1^2(s_1^2 = 1; s_2^2 = 1) = q_1(1-q_1) - \frac{1}{2}q_1q_2 - k,$$

$$\mathbb{E}\pi_1^2(s_1^2 = 1; s_2^2 = 0) = q_1(1 - q_1) - k,$$

$$\mathbb{E}\pi_1^2(s_1^2 = 0; s_2^2 = 1) = \mathbb{E}\pi_1^2(s_1^2 = 0; s_2^2 = 0) = 0.$$

The profits of firm 2 are (here we take an expectation over both A_1^2 and A_2^2 which are unknown):

$$\mathbb{E}\pi_2^2(s_1^2 = 1; s_2^2 = 1) = \frac{1}{2}q_2(1 - q_2) - \frac{1}{4}q_1q_2 - k,$$

$$\mathbb{E}\pi_2^2(s_1^2 = 0; s_2^2 = 1) = \frac{1}{2}q_2(1 - q_2) - k,$$

$$\mathbb{E}\pi_2^2(s_1^2 = 1; s_2^2 = 0) = \mathbb{E}\pi_2^2(s_1^2 = 0; s_2^2 = 0) = 0.$$

Denote by s_1^{2H} the firm 1 choice of advertising when it observes $A_1^2 = 1$ and by s_1^{2L} the firm 1 choice of advertising when it observes $A_1^2 = 0$. We already said that it is always true that $s_1^{2L} = 0$. Nevertheless, we add s_1^{2L} into the description of the equilibria for completeness. The equilibria of Case 2 are therefore described as:

$$(s_1^{2H}; s_1^{2L}; s_2^2) = \begin{cases} (1;0;1), & \text{if } k \leq \min\left[q_1(1-q_1) - \frac{1}{2}q_1q_2; \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2\right], \\ (1;0;0), & \text{if } \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 \leq k \leq q_1(1-q_1), \\ (0;0;1), & \text{if } q_1(1-q_1) - \frac{1}{2}q_1q_2 \leq k \leq \frac{1}{2}q_2(1-q_2), \\ (0;0;0), & \text{if } k \geq \max\left[q_1(1-q_1); \frac{1}{2}q_2(1-q_2)\right]. \end{cases}$$

The equilibrium profits of the firms corresponding to the strategies above are:

$$\mathbb{E}\pi_2^1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{k}{2}, & \text{if } k \le \min\left[q_1(1-q_1) - \frac{1}{2}q_1q_2; \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2\right], \\ \frac{1}{2}q_1(1-q_1) - \frac{k}{2}, & \text{if } \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 \le k \le q_1(1-q_1), \\ 0, & \text{otherwise.} \end{cases} \tag{B.7}$$

$$\mathbb{E}\pi_2^2 = \begin{cases} \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 - k, & \text{if } k \le \min\left[q_1(1-q_1) - \frac{1}{2}q_1q_2; \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2\right], \\ \frac{1}{2}q_2(1-q_2) - k, & \text{if } q_1(1-q_1) - \frac{1}{2}q_1q_2 \le k \le \frac{1}{2}q_2(1-q_2), \\ 0, & \text{otherwise.} \end{cases} \tag{B.8}$$

Case 3: $s_1^2 = 0$ and $s_2^2 = 1$.

In this case, firm 2 perfectly knows its A_2^2 (and does not know A_1^2) while firm 1 knows neither A_1^2 nor A_2^2. Consider the profits of firm 2. If firm 2 observes $A_2^2 = 0$, then it always chooses $s_2^2 = 0$. If firm 2 observes $A_2^2 = 1$, then its profits are (here we take an expectation over the unknown A_1^2):

$$\mathbb{E}\pi_2^2(s_1^2 = 1; s_2^2 = 1) = q_2(1-q_2) - \frac{1}{2}q_1q_2 - k,$$

$$\mathbb{E}\pi_2^2(s_1^2 = 0; s_2^2 = 1) = q_2(1-q_2) - k,$$

$$\mathbb{E}\pi_2^2(s_1^2 = 1; s_2^2 = 0) = \mathbb{E}\pi_1^2(s_1^2 = 0; s_2^2 = 0) = 0.$$

The profits of firm 1 are (here we take an expectation over both A_1^2 and A_2^2 which are unknown):

$$\mathbb{E}\pi_1^2(s_1^2 = 1; s_2^2 = 1) = \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k,$$

$$\mathbb{E}\pi_1^2(s_1^2 = 1; s_2^2 = 0) = \frac{1}{2}q_1(1 - q_1) - k,$$

$$\mathbb{E}\pi_1^2(s_1^2 = 0; s_2^2 = 1) = \mathbb{E}\pi_2^2(s_1^2 = 0; s_2^2 = 0) = 0.$$

The equilibria of Case 3 are therefore described as:

$$(s_1^2; s_2^{2H}, s_2^{2L}) = \begin{cases} (1; 1; 0), & \text{if } k \leq \frac{1}{2}q_1(1 - q_1) - \frac{1}{4}q_1 q_2, \\ (0; 1; 0), & \text{if } \frac{1}{2}q_1(1 - q_1) - \frac{1}{4}q_1 q_2 < k < q_2(1 - q_2), \\ (0; 0; 0), & \text{if } k \geq q_2(1 - q_2). \end{cases}$$

The equilibrium profits from the point of view of period 1 are:

$$\mathbb{E}\pi_1^2 = \begin{cases} \frac{1}{2}q_1(1 - q_1) - \frac{1}{4}q_1 q_2 - k, & \text{if } k \leq \frac{1}{2}q_1(1 - q_1) - \frac{1}{4}q_1 q_2, \\ 0, & \text{if } k > \frac{1}{2}q_1(1 - q_1) - \frac{1}{4}q_1 q_2. \end{cases} \tag{B.9}$$

$$\mathbb{E}\pi_2^2 = \begin{cases} \frac{1}{2}q_2(1 - q_2) - \frac{1}{4}q_1 q_2 - \frac{k}{2}, & \text{if } k \leq \frac{1}{2}q_1(1 - q_1) - \frac{1}{4}q_1 q_2, \\ \frac{1}{2}q_2(1 - q_2) - \frac{k}{2}, & \text{if } \frac{1}{2}q_1(1 - q_1) - \frac{1}{4}q_1 q_2 < k \leq q_2(1 - q_2), \\ 0, & \text{if } k > q_2(1 - q_2). \end{cases} \tag{B.10}$$

Case 4: $s_1^2 = s_2^2 = 0$.

In this case, none of the firms has any information about A_1^2 or A_2^2. Consider the profits of firm i:

$$\mathbb{E}\pi_i^2(s_i^2 = 1; s_{-i}^2 = 1) = \frac{1}{2}q_i(1 - q_i) - \frac{1}{4}q_i q_{-i} - k,$$

$$\mathbb{E}\pi_i^2(s_i^2 = 1; s_{-i}^2 = 0) = \frac{1}{2}q_i(1 - q_i) - k,$$

$$\mathbb{E}\pi_i^2(s_i^2 = 0; s_{-i}^2 = 1) = \mathbb{E}\pi_i^2(s_i^2 = 0; s_{-i}^2 = 0) = 0.$$

The above leads us to the following equilibria:

$$(s_1^2; s_2^2) = \begin{cases} (1;1), & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ (1;0), & \text{if } \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 \leq k \leq \frac{1}{2}q_1(1-q_1), \\ (0;1), & \text{if } \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 \leq k \leq \frac{1}{2}q_2(1-q_2), \\ (0;0), & \text{if } k \geq \frac{1}{2}q_2(1-q_2). \end{cases}$$

As we can see, $(1;0)$ and $(0;1)$ equilibria overlap (the region where the $(1;0)$ equilibrium exists is fully located inside the region where the $(0;1)$ equilibrium exists). We will assume that the equilibrium that actually realizes is the one that is Pareto dominant, that is, in our case the one for which $q_2(1-q_2) \geq q_1(1-q_1)$ or $(0;1)$ (if $q_1(1-q_1) = q_2(1-q_2)$, then one of these two equilibria realizes randomly with probability $\frac{1}{2}$). Hence, with this refinement, the equilibria of this sub-game are:

$$(s_1^2; s_2^2) = \begin{cases} (1;1), & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ (0;1), & \text{if } \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 < k < \frac{1}{2}q_2(1-q_2), \\ (0;0), & \text{if } k \geq \frac{1}{2}q_2(1-q_2). \end{cases}$$

The expected profits corresponding to these equilibria are:

$$\mathbb{E}\pi_1^2 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2. \end{cases} \tag{B.11}$$

$$\mathbb{E}\pi_2^2 = \begin{cases} \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 - k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ \frac{1}{2}q_2(1-q_2) - k, & \text{if } \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 < k \leq \frac{1}{2}q_2(1-q_2), \\ 0, & \text{if } k > \frac{1}{2}q_2(1-q_2). \end{cases} \tag{B.12}$$

Now that we have solved for the period 2 strategies, we can move on with backwards induction towards the first period.

Period 1

In period 1, firms choose both q_i and s_i^1 simultaneously at once. We will consider all possible pure strategy equilibria. To that end, since there is a limited number of possible pure strategy choices for s_1^1 and s_2^1 (namely, as in period 2, there are 4 possible combinations of them), we will proceed in the following way. We fix a certain advertising strategy combination, $(\hat{s_1^1}; \hat{s_2^1})$, (*e.g.* $s_1^1 = 1$ and $s_2^1 = 0$) and look at whether there is a combination of q_1 and q_2 such that $\{(\hat{s_1^1}; \hat{s_2^1}); (\hat{q_1}; \hat{q_2})\}$ is an equilibrium, and if it is, find the conditions when it holds.

To simplify further analysis we will make several observations:

1. The biggest one period profit a firm can obtain is $q_i(1-q_i)-k$ (when $A_i^2 = 1$ and $s_{-i}A_{-i}^2 = 0$). This means that if $k > \frac{1}{4} = \max_{q_i} q_i(1-q_i)$, then no firm ever advertises, and therefore, $s_1^1 = s_2^1 = s_1^2 = s_2^2 = 0$ with q_1, q_2 being undefined.

2. As a consequence of (1), if in any equilibrium $s_i^1 = 1$, then it must be true that $k \leq q_i(1-q_i)$.

3. If firm i chooses $s_i^1 = 0$, then for any equilibrium choice of q_1 and q_2 it must be true that $k \geq \frac{1}{2}q_i(1-q_i) - \frac{1}{4}q_iq_{-i}$. Indeed, if this was not true then, $\mathbb{E}\pi_i^1(s_i^1 = 0) = 0$ while $\mathbb{E}\pi_i^1(s_i^1 = 1) \geq \frac{1}{2}q_i(1-q_i) - \frac{1}{4}q_iq_{-i} - k \geq 0$. This would mean that the choice of $s_i^1 = 1$ dominates $s_i^1 = 0$.

Having reiterated the above, let us consider all the possible pure strategy Nash equilibria of the game one by one.

1: $s_1^1 = 1$ and $s_2^1 = 1$.

Consider the problem of firm 1 (the problem of firm 2 is symmetric). It takes $s_2^1 = 1$ and q_2 as given. Assuming that $s_1^1 = 1$, we will look at possible combinations of q_1 and q_2 that could support an equilibrium.

If $q_1(1-q_1) < q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.5) with the one-shot first period expected profits of $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k$):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{2}q_1q_2 - \frac{3k}{2}, & \text{if } k \leq q_1(1-q_1) - \frac{1}{2}q_1q_2, \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k > q_1(1-q_1) - \frac{1}{2}q_1q_2. \end{cases} \tag{B.13}$$

If $q_1(1-q_1) < q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.6) with the one-shot first period expected profits of $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k$):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{2}q_1q_2 - \frac{3k}{2}, & \text{if } k \leq q_2(1-q_2) - \frac{1}{2}q_1q_2, \\ q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{3k}{2}, & \text{if } q_2(1-q_2) - \frac{1}{2}q_1q_2 < k < q_1(1-q_1), \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \geq q_1(1-q_1). \end{cases} \tag{B.14}$$

Solving the problem explicitly for the best response is complicated, so instead we will show that some values of q_1 and q_2 can never sustain an equilibrium and can be ruled out.

First, as we mentioned in the begging of the proof, $k > q_1(1-q_1)$ can never happen in an equilibrium where $s_1^1 = 1$. Otherwise, firm 1 would have incentives to deviate to $s_1^1 = 0$.

Second, it cannot happen in an equilibrium that $q_1(1-q_1) > q_2(1-q_2)$ and $k > q_2(1-q_2) - \frac{1}{2}q_1q_2$ (or, symmetrically, $q_2(1-q_2) > q_1(1-q_1)$ and $k > q_1(1-q_1) - \frac{1}{2}q_1q_2$) because the corresponding profit of firm 2 would be $\mathbb{E}\pi_2 = \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k < 0$ and firm 2 would have incentives to deviate to $s_2^1 = 0$.

Third, it cannot happen in an equilibrium that $q_1(1-q_1) = q_2(1-q_2) \neq \frac{1}{4}$ and $q_1(1-q_1) -$

$\frac{1}{2}q_1q_2 = q_2(1-q_2) - \frac{1}{2}q_1q_2 < k < q_1(1-q_1) = q_2(1-q_2)$. Indeed, if this was true than firm 1 (and firm 2) would have incentives to set q_1 in such a way that $q_1(1-q_1) = q_2(1-q_2)+\epsilon$ and get a discontinuous increase in profits.

Fourth, it cannot happen in an equilibrium that $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{4}$ (or, $q_1 = q_2 = \frac{1}{2}$). To show this, consider the choice of firm 1. Firm 1 takes as given that firm 2 plays an equilibrium strategy of $q_2 = \frac{1}{2}$. The profit of firm 1, if it plays $q_1(1-q_1) < q_2(1-q_2) = \frac{1}{4}$ is (substitute $q_2 = \frac{1}{2}$ into (B.13)):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{4}q_1 - \frac{3k}{2}, & \text{if } k \le q_1(1-q_1) - \frac{1}{4}q_1, \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1 - k, & \text{if } k > q_1(1-q_1) - \frac{1}{4}q_1. \end{cases} \tag{B.15}$$

Maximizing the above piece-wise with respect to q_1 (we omit calculations) leads to the following optimal profits:

$$\mathbb{E}\pi_1 = \begin{cases} \frac{9}{64} - \frac{3k}{2}, & \text{if } k \le \frac{9}{64}, \\ \frac{9}{128} - k, & \text{if } k > \frac{9}{64}. \end{cases} \tag{B.16}$$

The profit of firm 1 if it plays $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{4}$ is (substitute $q_2 = \frac{1}{2}$ in (B.13) and (B.14) and take the average of these profits):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{8} - \frac{3k}{2}, & \text{if } k \le \frac{1}{8}, \\ \frac{1}{8} - \frac{5k}{4}, & \text{if } \frac{1}{8} < k < \frac{1}{4}, \\ \frac{1}{16} - k, & \text{if } k \ge \frac{1}{4}. \end{cases} \tag{B.17}$$

Comparing the profits in (B.16) and (B.17) with each other, we can conclude that setting $q_1 = \frac{1}{2}$ is optimal when $\frac{1}{8} \le k \le \frac{7}{32}$ with the corresponding expected profit being $\mathbb{E}\pi_1 = \frac{1}{8} - \frac{5k}{4}$. But if $\frac{1}{8} \le k \le \frac{7}{32}$, then $\mathbb{E}\pi_1 = \frac{1}{8} - \frac{5k}{4} < 0$. That means that firm 1 will never play $s_1^1 = 1$ and $q_1 = \frac{1}{2}$ if firm 2 chooses $s_2^1 = 1$ and $q_2 = \frac{1}{2}$. Hence, $q_1 = q_2 = \frac{1}{2}$ cannot be an equilibrium.

The above points lead us to conclude that the only possible equilibrium combination of q_1 and q_2 must satisfy $q_1(1-q_1) \le q_2(1-q_2)$ and $k \le q_1(1-q_1) - \frac{1}{2}q_1q_2$ or, symmetrically,

$q_2(1-q_2) \leq q_1(1-q_1)$ and $k \leq q_2(1-q_2) - \frac{1}{2}q_1q_2$. Notice now that the following constraint, $k \leq q_1(1-q_1) - \frac{1}{2}q_1q_2$ (or its symmetric counterpart $k \leq q_2(1-q_2) - \frac{1}{2}q_1q_2$), cannot hold with equality in equilibrium. Indeed, if $k = q_1(1-q_1) - \frac{1}{2}q_1q_2$ ($k = q_2(1-q_2) - \frac{1}{2}q_1q_2$), then $\mathbb{E}\pi_1 = q_1(1-q_1) - \frac{1}{2}q_1q_2 - \frac{3k}{2} = k - \frac{3k}{2} = -\frac{k}{2} < 0$ ($\mathbb{E}\pi_2 = q_2(1-q_2) - \frac{1}{2}q_1q_2 - \frac{3k}{2} = k - \frac{3k}{2} = -\frac{k}{2} < 0$) and firm 1 (firm 2) profits are negative. Therefore, in equilibrium it must be true that $k < q_1(1-q_1) - \frac{1}{2}q_1q_2$ ($k < q_2(1-q_2) - \frac{1}{2}q_1q_2$). Hence, the only possible equilibrium combination must be an internal solution for q_1 (and q_2, symmetrically) to the following maximization problem:

$$q_1(1-q_1) - \frac{1}{2}q_1q_2 - \frac{3k}{2} \Rightarrow \max_{q_1}.$$

Solving the first order conditions we get that $q_1 = \frac{1}{2} - \frac{q_2}{4}$ and $q_2 = \frac{1}{2} - \frac{q_1}{4}$. This system of equations implies the only possible equilibrium combination of $(q_1; q_2) = \left(\frac{2}{5}; \frac{2}{5}\right)$ with the corresponding profits being $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = \left(\frac{4}{25} - \frac{3k}{2}; \frac{4}{25} - \frac{3k}{2}\right)$. We also should not forget that $k < q_1(1-q_1) - \frac{1}{2}q_1q_2 = q_2(1-q_2) - \frac{1}{2}q_1q_2 = \frac{4}{25}$ must be satisfied in equilibrium.

Now that we have found out the only possible equilibrium strategy in q_1 and q_2, let us check under which conditions neither firm has profitable deviations to either $q_i \neq \frac{2}{5}$ or $s_i^1 \neq 1$. Let us begin with exploring possible profitable deviations (we concentrate on firm 1 since the firms are symmetric) to $s_1^1 = 1$ and $q_1 \neq \frac{2}{5}$. Taking the strategies of firm 2, $s_2^1 = 1$ and $q_2 = \frac{2}{5}$, as given, the profits of firm 1 when $q_1(1-q_1) \leq q_2(1-q_2) = \frac{6}{25}$ are (substitute $q_2 = \frac{2}{5}$ into (B.13)):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{5}q_1 - \frac{3k}{2}, & \text{if } k \leq q_1(1-q_1) - \frac{1}{5}q_1, \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1 - k, & \text{if } k > q_1(1-q_1) - \frac{1}{5}q_1. \end{cases}$$

Maximizing the above with respect to q_1 (we omit calculations), we can show that when $k \leq \frac{4}{25}$ there is no profitable deviation for firm 1.

The profits of firm 1 when $q_1(1-q_1) \geq q_2(1-q_2)$ are (substitute $q_2 = \frac{2}{5}$ into (B.14)):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{5}q_1 - \frac{3k}{2}, & \text{if } k \leq \frac{6}{25} - \frac{1}{5}q_1, \\ q_1(1-q_1) - \frac{1}{10}q_1 - \frac{3k}{2}, & \text{if } \frac{6}{25} - \frac{1}{5}q_1 < k < q_1(1-q_1), \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1 - k, & \text{if } k \geq q_1(1-q_1). \end{cases}$$

Maximizing the above with respect to q_1 (we omit calculations again), the optimized profits of firm 1 are (assuming that $k \leq \frac{4}{25}$):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{81}{400} - \frac{3k}{2}, & \text{if } k \geq \frac{3}{20}, \\ -\frac{1}{25}(3-25k)^2, & \text{if } \frac{3}{25} < k < \frac{3}{20}, \\ \frac{4}{25} - \frac{3k}{2}, & \text{if } k \leq \frac{3}{25}. \end{cases} \tag{B.18}$$

Looking at (B.18), we see that firm 1 chooses $q_1 = \frac{2}{5}$ (with the corresponding profit of $\mathbb{E}\pi_1 = \frac{4}{25} - \frac{3k}{2}$) when $k \leq \frac{3}{25}$. Hence, in order for firm 1 not to have incentives to deviate away from $q_1 = \frac{2}{5}$, it has to be true that $k \leq \frac{3}{25}$.

Finally, let us consider possible profitable deviations to $s_1^1 = 0$ and any q_1 taking as given that $s_2^1 = 1$ and $q_2 = \frac{2}{5}$. If firm 1 deviates to $s_1^1 = 0$, then its profits are the following. If $q_1(1-q_1) < q_2(1-q_2) = \frac{6}{25}$, then (substitute $q_2 = \frac{2}{5}$ into (B.9)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1 - k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1, \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1. \end{cases} \tag{B.19}$$

If $q_1(1-q_1) > q_2(1-q_2) = \frac{6}{25}$, then the profits of firm 1 are (substitute $q_2 = \frac{2}{5}$ into (B.8) and also notice that a second period equilibrium with firm 1 advertising is Pareto dominated by an equilibrium with firm 2 advertising):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1 - k, & \text{if } k \le \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1, \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1. \end{cases} \tag{B.20}$$

We can see that (B.19) and (B.20) are identical. Maximizing them with respect to q_1, we obtain the optimal profits firm 1 can get from deviating to $s_1^1 = 0$:

$$\mathbb{E}\pi_1 = \max\left[0; \frac{2}{25} - k\right].$$

In equilibrium it must be true that:

$$\frac{4}{25} - \frac{3k}{2} \ge \max\left[0; \frac{2}{25} - k\right].$$

Simplifying the above inequality, we get that the equilibrium holds whenever:

$$k \le \frac{8}{75} < \frac{3}{25}.$$

Therefore, the only pure strategy Nash equilibrium is the following one: $(s_1^1; s_2^1) = (1; 1)$, $(q_1; q_2) = \left(\frac{2}{5}; \frac{2}{5}\right)$, $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = \left(\frac{4}{25} - \frac{3k}{2}; \frac{4}{25} - \frac{3k}{2}\right)$ and $k \le \frac{8}{75}$.

2: $s_1^1 = 0$ and $s_2^1 = 1$ (or, symmetrically, $s_1^1 = 1$ and $s_2^1 = 0$).

Let us begin by looking at the maximization problem of firm 1. If $q_1(1-q_1) \le q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.9) with the one-shot first period expected profits of 0):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \le \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ 0, & \text{if } k \ge \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2. \end{cases} \tag{B.21}$$

If $q_1(1-q_1) \ge q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.8) with the one-shot first period expected profits of 0):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k < \min\left[q_2(1-q_2) - \frac{1}{2}q_1q_2; \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2\right], \\ \frac{1}{2}q_1(1-q_1) - k, & \text{if } q_2(1-q_2) - \frac{1}{2}q_1q_2 \leq k \leq \frac{1}{2}q_1(1-q_1), \\ 0, & \text{otherwise.} \end{cases}$$

Looking at the above we can notice the following as described in the observations stated earlier in the proof (preface to Period 1). First, from observation (3), we know that in any equilibrium where firm 1 chooses $s_1^1 = 0$, it must be true that equilibrium values of q_1 and q_2 satisfy $k \geq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$. This means that in the above-stated profit functions, the equilibrium profits of firm 1 cannot lie in the following regions: $k < \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$ or $k < \min\left[q_2(1-q_2) - \frac{1}{2}q_1q_2; \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2\right]$. Second, the equilibrium profits of firm 1 cannot lie in the region of $q_2(1-q_2) - \frac{1}{2}q_1q_2 \leq k \leq \frac{1}{2}q_1(1-q_1)$. This is true because it can be shown that either this region does not exist at all (i.e. its right boundary is smaller than its left boundary), is Pareto dominated by a second period equilibrium where firm 2 advertises instead (*i.e.* when $q_2(1-q_2) \geq \frac{1}{2}q_1(1-q_1)$) or implies that the corresponding equilibrium profit of firm 2 (*i.e.* $\mathbb{E}\pi_2 = \frac{1}{2}q_2(1-q_2) - k$) is negative. Hence, in any equilibrium where $s_1^1 = 0$ and $s_2^1 = 1$ it must be true that the equilibrium profits of firm 1 are exactly 0 and, therefore, the corresponding equilibrium profits of firm 2 are $\mathbb{E}\pi_2 = q_2(1-q_2) - \frac{3k}{2}$. Third, if firm 1 instead sets $s_1^1 = 1$, the worst expected profits it can get (assuming that firm 2 plays $s_2^1 = s_2^2 = 1$ and $q_2 = 1$) are $\max_{q_1}\left[\frac{1}{2}q_1(1-q_1) - \frac{3k}{2}\right] = \frac{1}{8} - \frac{3k}{2}$. These profits must be smaller than 0 to possibly sustain an equilibrium with $s_1^1 = 0$, or, $\frac{1}{8} - \frac{3k}{2} \leq 0 \Rightarrow k \geq \frac{1}{12}$. Fourth, it must be true that when firm 2 sets $q_2 = \frac{1}{2}$, the following constraint is always satisfied: $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 \leq k \leq q_2(1-q_2)$. Indeed, $k \leq q_2(1-q_2) = \frac{1}{4}$ by assumption made in the preface to the analysis of period 1. Also, $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 = \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1 \leq \frac{9}{128} < \frac{1}{12} \leq k$.

Knowing the above, consider the combined two-period profits of firm 2 if $q_2(1-q_2) \geq q_1(1-$

q_1) (these are obtained by summing the second period expected profits in (B.10) with one-shot first period profits of $\frac{1}{2}q_2(1-q_2)-k$):

$$\mathbb{E}\pi_2 = \begin{cases} q_2(1-q_2) - \frac{1}{4}q_1q_2 - \frac{3k}{2}, & \text{if } k < \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ q_2(1-q_2) - \frac{3k}{2}, & \text{if } \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 \leq k \leq q_2(1-q_2), \\ \frac{1}{2}q_2(1-q_2) - k, & \text{if } k > q_2(1-q_2). \end{cases}$$

We can show that under the assumption that $q_2(1-q_2) \geq q_1(1-q_1)$, $q_2 = \frac{1}{2}$ is a dominant strategy of firm 2. When $q_2 = \frac{1}{2}$ it is clearly true that, indeed, $\frac{1}{2} * \frac{1}{2} = q_2(1-q_2) \geq q_1(1-q_1)$ for any possible q_1. Also, in any equilibrium the following constraint is always satisfied (remember that in any equilibrium it must be true that $k \geq \frac{1}{12}$) when $q_2 = \frac{1}{2}$: $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 \leq k \leq q_2(1-q_2)$. This comes from observation 4 made several paragraphs above. This means that the strategy $q_2 = \frac{1}{2}$ is always available to firm 2 and, moreover, the profits from playing this strategy have to be equal to $\mathbb{E}\pi_2\left(q_2 = \frac{1}{2}\right) = q_2(1-q_2) - \frac{3k}{2} = \frac{1}{4} - \frac{3k}{2}$. Finally, it is easy to see that this profit is always bigger than any other possible profit obtainable given that $q_2(1-q_2) \geq q_1(1-q_1)$. Indeed, $\frac{1}{4} - \frac{3k}{2}$ is bigger than any unrestricted maximum of the other possible profit function forms, let alone, their restricted maxima.

We can also show that the strategy $q_2 = \frac{1}{2}$ with the corresponding profits of $\mathbb{E}\pi_2 = \frac{1}{4} - \frac{3k}{2}$ always dominates any strategy of firm 2 where $q_2(1-q_2) \leq q_1(1-q_1)$. The combined two-period profits of firm 2 if $q_2(1-q_2) \leq q_1(1-q_1)$ are (these are obtained by summing the second period expected profits in (B.7) with one-shot first period profits of $\frac{1}{2}q_2(1-q_2)-k$):

$$\mathbb{E}\pi_2 = \begin{cases} q_2(1-q_2) - \frac{1}{4}q_1q_2 - \frac{3k}{2}, & \text{if } k \leq \min\left[q_2(1-q_2) - \frac{1}{2}q_1q_2; \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2\right], \\ q_2(1-q_2) - \frac{3k}{2}, & \text{if } \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 \leq k \leq q_2(1-q_2), \\ \frac{1}{2}q_2(1-q_2) - k, & \text{otherwise.} \end{cases}$$

Again, $\mathbb{E}\pi_2 = \frac{1}{4} - \frac{3k}{2}$ is bigger than any possible unrestricted maxima of the profits above, let alone their restricted counterparts. Therefore, in equilibrium firm 2 always must play $q_2 = \frac{1}{2}$.

Now, let us come back to the maximization problem of firm 1. Firm 1 takes for granted that firm 2 chooses $q_2 = \frac{1}{2}$. Knowing this, firm 1's profits are the following (these are profits shown in (B.21) with q_2 replaced by $\frac{1}{2}$):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1 - k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1, \\ 0, & \text{if } k \geq \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1. \end{cases}$$

We have already shown that in any equilibrium it must be true that $k \geq \frac{1}{12}$. Under this assumption, $k \geq \frac{1}{12} \geq \max_{q_1}\left[\frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1\right] = \frac{9}{128}$. Therefore, the profits of firm 1 are always equal to 0 regardless of the choice of q_1.

This means that the only equilibrium candidate in this case is $(q_1; q_2) = \left(\text{arbitrary}; \frac{1}{2}\right)$ with $(\pi_1; \pi_2) = \left(0; \frac{1}{4} - \frac{3k}{2}\right)$. To make sure this is an equilibrium we need to consider firm 1 deviating to $s_1^1 = 1$ and firm 2 deviating to $s_2^1 = 0$.

Let us first consider a possible deviation of firm 1 to $s_1^1 = 1$ and some arbitrary q_1. Firm 1 takes the strategy of firm 2 as granted, that is, $s_2^1 = 1$ and $q_2 = \frac{1}{2}$. We have already calculated the profit functions of firm 1 under these assumptions while we were solving the Case 1.

The combined two-period profits of firm 1 when $q_1(1-q_1) < q_2(1-q_2) = \frac{1}{4}$ are (see (B.15)):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{4}q_1 - \frac{3k}{2}, & \text{if } k \leq q_1(1-q_1) - \frac{1}{4}q_1, \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1 - k, & \text{if } k > q_1(1-q_1) - \frac{1}{4}q_1. \end{cases}$$

Maximizing the above piece-wise with respect to q_1 leads to the following optimal profits (see (B.16)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{9}{64} - \frac{3k}{2}, & \text{if } k \leq \frac{9}{64}, \\ \frac{9}{128} - k, & \text{if } k > \frac{9}{64}. \end{cases} \tag{B.22}$$

The combined two period profits of firm 1 when $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{4}$ are (see (B.17)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{8} - \frac{3k}{2}, & \text{if } k \leq \frac{1}{8}, \\ \frac{1}{8} - \frac{5k}{4}, & \text{if } \frac{1}{8} < k < \frac{1}{4}, \\ \frac{1}{16} - k, & \text{if } k \geq \frac{1}{4}. \end{cases} \tag{B.23}$$

Combining (B.22) and (B.23), we get the following best response profit of firm 1 to $s_2^1 = 1$ and $q_2 = \frac{1}{2}$:

$$\mathbb{E}\pi_1 = \begin{cases} \frac{9}{64} - \frac{3k}{2}, & \text{if } k \leq \frac{1}{8}, \\ \frac{1}{8} - \frac{5k}{4}, & \text{if } \frac{1}{8} < k < \frac{7}{32}, \\ \frac{9}{128} - k, & \text{if } k \geq \frac{7}{32}. \end{cases}$$

Firm 1 prefers not to deviate when the profit above is smaller than 0. This happens when $k \geq \frac{3}{32}$. Therefore, in equilibrium it must be true that $k \geq \frac{3}{32}$.

Finally, consider firm 2 deviating to $s_2^1 = 0$ taking as given the strategy of firm 1. As will be shown in the next case, the best firm 2 can do is to get $\mathbb{E}\pi_2 = 0$ if $k > \frac{1}{8}$ or $\mathbb{E}\pi_2 = \frac{1}{16} - \frac{k}{2}$ if $\left\{k \leq \frac{1}{8} \text{ and } q_2 = \frac{1}{2}\right\}$ or $\mathbb{E}\pi_2 = \frac{1}{8} - k$ if $\left\{k \leq \frac{1}{8} \text{ and } q_2 \neq \frac{1}{2}\right\}$. We can see that $\mathbb{E}\pi_2 = \frac{1}{4} - \frac{3k}{2}$ is bigger than any of these deviation options when $k \leq \frac{1}{6}$. Therefore, firm 2 would never want to deviate to $s_2^1 = 0$ when $k \leq \frac{1}{6}$.

To combine all the results above, the only equilibrium here is the one where the following hold (or, the symmetric version of it): $(s_1^1; s_2^1) = (0; 1)$, $(q_1; q_2) = \left(\text{arbitrary}; \frac{1}{2}\right)$, $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = \left(0; \frac{1}{4} - \frac{3k}{2}\right)$ and $\frac{3}{32} \leq k \leq \frac{1}{6}$.

3: $s_1^1 = 0$ and $s_2^1 = 0$.

Let us begin by looking at the profit maximization problem of firm 1. If $q_1(1-q_1) < q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.11) with one-shot first period profits of 0):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ 0, & k > \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2. \end{cases}$$

If $q_1(1-q_1) > q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.12) with one-shot first period profits of 0):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \leq \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2, \\ \frac{1}{2}q_1(1-q_1) - k, & \text{if } \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k \leq \frac{1}{2}q_1(1-q_1), \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1). \end{cases}$$

Seeing the above, we can make the following observations. First, as in case 2, in any equilibrium it cannot be that $k < \frac{1}{2}q_i(1-q_i) - \frac{1}{4}q_iq_{-i}$ for $i \in \{1; 2\}$. Otherwise, firm i's first period profits from playing $s_i^1 = 1$ would be strictly positive implying that there is a profitable deviation away from $s_i^1 = 0$. Second, from the first point it follows that then either the equilibrium profits of both firms are equal to 0 or the profits one of the firms are equal to 0 (while the profits of the other firm are equal to $\frac{1}{2}q_i(1-q_i) - k$). This means that again as in case 2, in any equilibrium it must be true that $k \geq \frac{1}{12}$. Otherwise, even it the worst case scenario, firm i would be able to obtain a strictly positive profit, and hence, there would be a deviation away from $s_i^1 = 0$. Third, if $k > \frac{1}{8} = \max_{q_i}\left[\frac{1}{2}q_i(1-q_i)\right] \geq \frac{1}{2}q_i(1-q_i)$, then the best any firm can do given that $s_1^1 = s_2^1 = 0$ is a profit of 0 for any arbitrary q_i. Therefore, when $k > \frac{1}{8}$, the only possible equilibrium in case 3 is $(s_1^1; s_2^1) = (0; 0)$, $(q_1; q_2) = (\text{arbitrary}; \text{arbitrary})$ and $(\pi_1; \pi_2) = (0; 0)$.

Now let us focus on what happens when $\frac{1}{12} \leq k \leq \frac{1}{8}$. In this case, we can show that firm 1 has a dominant strategy of playing $q_1 = \frac{1}{2}$. When $q_1 = \frac{1}{2}$, it is clear that $\frac{1}{2} * \frac{1}{2} = q_1(1-q_1) \geq q_2(1-q_2)$. Also, the following constraint is always satisfied: $\frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 \leq k \leq \frac{1}{2}q_1(1-q_1)$. Indeed, $k \leq \frac{1}{2}q_1(1-q_1) = \frac{1}{8}$ by assumption that $k \leq \frac{1}{8}$ and $k \geq \frac{1}{12} \geq \max_{q_2}\left[\frac{1}{2}q_2(1-q_2) - \frac{1}{8}q_2\right] = \frac{9}{128}$ by assumption that $k \geq \frac{1}{12}$. Hence, $\mathbb{E}\pi_1 = \frac{1}{8} - k$ when $q_2 \neq \frac{1}{2}$ and $\mathbb{E}\pi_1 = \frac{1}{16} - \frac{k}{2}$ when $q_2 = \frac{1}{2}$

(when firms' q_i's are equal one of the will randomly play $s_i^2 = 1$ and so the profit is cut in half). In both cases it can be shown, as in case 2, that playing $q_i = \frac{1}{2}$ is strictly better than playing any other unrestricted, let alone restricted, possible piece-wise maxima. Therefore, the only possible equilibria in case 3 when $\frac{1}{12} \le k \le \frac{1}{8}$ is: $(s_1^1; s_2^1) = (0; 0)$, $(q_1; q_2) = \left(\frac{1}{2}; \frac{1}{2}\right)$ and $(\pi_1; \pi_2) = \left(\frac{1}{16} - \frac{k}{2}; \frac{1}{16} - \frac{k}{2}\right)$.

To see whether any of these equilibrium candidates are indeed equilibria we need to check for profitable deviations towards $s_i^1 = 1$.

Consider a possible equilibrium where $\frac{1}{12} \le k \le \frac{1}{8}$, $(s_1^1; s_2^1) = (0; 0)$, $(q_1; q_2) = \left(\frac{1}{2}; \frac{1}{2}\right)$ and $(\pi_1; \pi_2) = \left(\frac{1}{16} - \frac{k}{2}; \frac{1}{16} - \frac{k}{2}\right)$. Firm 1 takes the strategy of firm 2 as given. In this case, if firm 1 deviates to $s_1^1 = 1$ it has a dominant strategy of playing $q_1 = \frac{1}{2}$ with the corresponding profit of $\frac{1}{4} - \frac{3k}{2}$ (see case 2). Firm 1 would not want to deviate when $\frac{1}{4} - \frac{3k}{2} \le \frac{1}{16} - \frac{k}{2} \Rightarrow \frac{3}{16} \le k$. Since by assumption we know that $k \le \frac{1}{8}$, then the constraint $k \ge \frac{3}{16}$ is impossible to satisfy. Therefore, firm 1 would always want to deviate to $s_1^1 = 1$ and, hence, this equilibrium candidate is not actually an equilibrium.

Consider now a possible equilibrium where $k > \frac{1}{8}$, $(s_1^1; s_2^1) = (0; 0)$, $(q_1; q_2) = (\text{arbitrary}; \text{arbitrary})$ and $(\pi_1; \pi_2) = (0; 0)$. Firm 1 takes the strategy of firm 2 as given. In this case if firm 1 deviates to $s_1^1 = 1$ it again has a dominant strategy of playing $q_1 = \frac{1}{2}$ with the corresponding profits of $\frac{1}{4} - \frac{3k}{2}$ (see case 2). Firm 1 would not want to deviate when $\frac{1}{4} - \frac{3k}{2} \le 0 \Rightarrow k \ge \frac{1}{6}$. This means that $(s_1^1; s_2^1) = (0; 0)$, $(q_1; q_2) = (\text{arbitrary}; \text{arbitrary})$ and $(\pi_1; \pi_2) = (0; 0)$ is indeed an equilibrium when $k \ge \frac{1}{6}$. To sum up, the only equilibrium in this case is $k \ge \frac{1}{6}$, $(s_1^1; s_2^1) = (0; 0)$, $(q_1; q_2) = (\text{arbitrary}; \text{arbitrary})$ and $(\pi_1; \pi_2) = (0; 0)$.

B.4 Proof of Lemma 6

Period 2

If firms cannot learn any information about their A_i^t's (or their competitors' A_i^t's) from period 1 advertising decisions, then in period 2 both firms will still have to use prior probabilities of A_1^t and A_2^t (i.e. $Pr(A_i = 1) = \frac{1}{2}$ and $Pr(A_i = 0) = \frac{1}{2}$) to decide on second period strategies.

The second period equilibrium profits of the firms look exactly the same as the second period equilibrium profits of the competitive game with learning, when in the first period the firms chose $(s_1^1; s_2^1) = (0; 0)$. Indeed, when $(s_1^1; s_2^1) = (0; 0)$, then in period 2 both firms receive no informative signals about any A_i's and have to use the prior probabilities. Therefore, we will reference these profits from the previous proofs below.

If $q_1(1-q_1) < q_2(1-q_2)$, then the second period expected profits of firm 1 are (see (B.11)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \le \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2. \end{cases} \tag{B.24}$$

If $q_1(1-q_1) > q_2(1-q_2)$, then the second period expected profits of firm 1 are (see (B.12)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \le \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2, \\ \frac{1}{2}q_1(1-q_1) - k, & \text{if } \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k \le \frac{1}{2}q_1(1-q_1), \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1). \end{cases} \tag{B.25}$$

Period 1

In period 1, firms choose both q_i and s_i^1 simultaneously at once. We consider all pure strategy Nash equilibria of the game. As earlier, the solution strategy is the following. Since there is a limited number of possible pure strategy choices for s_1^1 and s_2^1 (namely, as in period 2, there are 4 possible combinations of them), we will proceed in the following way. We fix a certain advertising strategy combination, $(\hat{s_1^1}; \hat{s_2^1})$, (*e.g.* $s_1^1 = 1$ and $s_2^1 = 0$) and look at whether there is a combination of q_1 and q_2 such that $\{(\hat{s_1^1}; \hat{s_2^1}); (\hat{q}_1; \hat{q}_2)\}$ is an equilibrium, and if it is, find the conditions when it holds.

To simplify further analysis we will make several observations similarly as to how we did it in earlier proofs:

1. The biggest one period expected profit a firm can obtain is $\frac{1}{2}q_i(1-q_i) - k$ (when $s_{-i} = 0$). This means that if $k > \frac{1}{8} = \max_{q_i}\left[\frac{1}{2}q_i(1-q_i)\right]$, then no firm ever advertises, and, therefore,

$s_1^1 = s_2^1 = s_1^2 = s_2^2 = 0$ with q_1, q_2 being undefined.

2. As a consequence of (1), if in any equilibrium $s_1^1 = 1$, then it must be true that $k \leq \frac{1}{2}q_i(1-q_i)$.

3. If firm i chooses $s_i^1 = 0$, then for any equilibrium choice of q_1 and q_2 it must be true that $k \geq \frac{1}{2}q_i(1-q_i) - \frac{1}{4}q_i q_{-i}$. Indeed, if this was not true, then $\mathbb{E}\pi_i^1(s_i^1 = 1) = 0$ while $\mathbb{E}\pi_i^1 \geq \frac{1}{2}q_i(1-q_i) - \frac{1}{4}q_i q_{-i} - k > 0$. This would mean that the choice of $s_i^1 = 1$ dominates $s_i^1 = 0$.

Having stated all of the above, let us consider all the possible pure strategy Nash equilibria of the game one by one.

1: $s_1^1 = 1$ and $s_2^1 = 1$.

Consider the problem of firm 1 (the problem of firm 2 is symmetric). It takes $s_2^1 = 1$ and q_2 as given. Assuming that $s_1^1 = 1$, we will look at possible combinations of q_1 and q_2 that could support an equilibrium.

If $q_1(1-q_1) < q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.24) with the one-shot first period expected profits of $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{2}q_1q_2 - 2k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2. \end{cases} \tag{B.26}$$

If $q_1(1-q_1) > q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.25) with the one-shot first period expected profits of $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{2}q_1q_2 - 2k, & \text{if } k \leq \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2, \\ q_1(1-q_1) - \frac{1}{4}q_1q_2 - 2k, & \text{if } \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k \leq \frac{1}{2}q_1(1-q_1), \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k > \frac{1}{2}q_1(1-q_1). \end{cases} \tag{B.27}$$

Solving the problem explicitly for the best response is complicated, so instead we will show that some values of q_1 and q_2 can never sustain an equilibrium and can be ruled out.

First, as we mentioned in the beginning of the proof, $k > \frac{1}{2}q_1(1-q_1)$ can never happen in an equilibrium where $s_1^1 = 1$. Otherwise, firm 1 will deviate to $s_1^1 = 0$.

Second, it cannot happen in an equilibrium that $q_1(1-q_1) > q_2(1-q_2)$ and $k > \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2$ (or, symmetrically, $q_2(1-q_2) > q_1(1-q_1)$ and $k > \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$) because the corresponding profit of firm 2 would be $\mathbb{E}\pi_2 = \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < 0$ and firm 2 would have incentives to deviate to $s_2^1 = 0$.

Third, it cannot happen in equilibrium that $q_1(1-q_1) = q_2(1-q_2) \neq \frac{1}{4}$ and $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 = \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k < \frac{1}{2}q_1(1-q_1) = \frac{1}{2}q_2(1-q_2)$. Indeed, if this was true than firm 1 (and firm 2) would have incentives to set q_1 in such a way that $q_1(1-q_1) = q_2(1-q_2) + \epsilon$ and get a discontinuous increase in profits.

Fourth, it cannot happen in an equilibrium that $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{4}$ (or, $q_1 = q_2 = \frac{1}{2}$). To show this, consider the choice of firm 1. Firm 1 takes as given that firm 2 plays an equilibrium strategy of $q_2 = \frac{1}{2}$. The profit of firm 1 if it plays $q_1(1-q_1) < q_2(1-q_2) = \frac{1}{4}$ is (substitute $q_2 = \frac{1}{2}$ into (B.26)):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{4}q_1 - 2k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1, \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1 - k, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1. \end{cases}$$

Maximizing the above piece-wise with respect to q_1 (we omit calculations) leads to the following optimal profits:

$$\mathbb{E}\pi_1 = \begin{cases} \frac{9}{64} - 2k, & \text{if } k \leq \frac{9}{128}, \\ \frac{9}{128} - k, & \text{if } k > \frac{9}{128}. \end{cases} \tag{B.28}$$

The profits of firm 1 if it plays $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{4}$ are (substitute $q_1 = q_2 = \frac{1}{2}$ into

(B.26) and (B.27) and take the average of these profits):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{8} - 2k, & \text{if } k \leq \frac{1}{16}, \\ \frac{1}{8} - \frac{3k}{2}, & \text{if } \frac{1}{16} < k \leq \frac{1}{8}, \\ \frac{1}{16} - k, & \text{if } k > \frac{1}{8}. \end{cases} \tag{B.29}$$

Comparing the profits in (B.28) and (B.29) with each other, we can conclude that setting $q_1 = \frac{1}{2}$ is optimal when $\frac{1}{16} \leq k \leq \frac{7}{64}$ with the corresponding expected profits of being $\mathbb{E}\pi_1 = \frac{1}{8} - \frac{3k}{2}$. If firm 1 deviates to $s_1^1 = 0$ and $q_1 = \frac{1}{2}$, then the best profit it can get is $\mathbb{E}\pi_1(s_1^1 = 0) = \frac{1}{2}\left(\frac{1}{2}q_1(1-q_1) - k\right) + \frac{1}{2} * 0 = \frac{1}{16} - \frac{k}{2}$ (in period 2, if $\frac{1}{16} < k < \frac{1}{8}$, then in any equilibrium of period 2 one and only one firm advertises; and since both firms have identical q_i's each firm advertises with probability $\frac{1}{2}$). Comparing $\mathbb{E}\pi_1(s_1^1 = 1) = \frac{1}{8} - \frac{3k}{2}$ and $\mathbb{E}\pi_1(s_1^1 = 0) = \frac{1}{16} - \frac{k}{2}$, we can see that $\mathbb{E}\pi_1(s_1^1 = 0) = \frac{1}{16} - \frac{k}{2}$ is always bigger. Therefore, $q_1 = q_2 = \frac{1}{2}$ and $s_1^1 = s_2^1 = 1$ can never be an equilibrium.

The above considerations lead us to conclude that the only possible equilibrium combination of q_1 and q_2 must satisfy $q_1(1-q_1) \leq q_2(1-q_2)$ and $k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$, or, symmetrically, $q_2(1-q_2) \leq q_1(1-q_1)$ and $k \leq \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2$. Notice that the following constraint, $k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$, cannot hold with equality in equilibrium. Otherwise, firm 1 would have incentives to set $q_1(1-q_1) = q_2(1-q_2) + \epsilon$ and get a discontinuous increase in profits.

Therefore, if an equilibrium exists, it must be an internal solution to the following problem:

$$\mathbb{E}\pi_1 = q_1(1-q_1) - \frac{1}{2}q_1q_2 - 2k \Rightarrow \max_{q_1}$$

First order conditions imply the following optimal $q_1 = \frac{1}{2} - \frac{q_2}{4}$ and, symmetrically, $q_2 = \frac{1}{2} - \frac{q_1}{4}$. This system of equations implies the only possible equilibrium combination of $(q_1; q_2) = \left(\frac{2}{5}; \frac{2}{5}\right)$ with the corresponding equilibrium profits of $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = \left(\frac{4}{25} - 2k; \frac{4}{25} - 2k\right)$. We should also not forget that $k < \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 = \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 = \frac{2}{25}$.

Now that we have found out the only possible equilibrium strategy in q_1 and q_2, let us check under which condition neither firm has profitable deviations to either $q_i \neq \frac{2}{5}$ or $s_i^1 \neq 1$. Let us begin with exploring possible profitable deviations to (we concentrate on firm 1 since the firms are symmetric) $s_1^1 = 1$ and $q_1 \neq \frac{2}{5}$. Taking the strategies of firm 2, $s_2^1 = 1$ and $q_2 = \frac{2}{5}$, as given, the profits of firm 1 when $q_1(1-q_1) \leq q_2(1-q_2) = \frac{6}{25}$ are (substitute $q_2 = \frac{2}{5}$ into (B.26)):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{5}q_1 - 2k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1, \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1 - k, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1. \end{cases}$$

Maximizing the above with respect to q_1 (we omit calculations), we can show that when $k \leq \frac{2}{25}$ there is no profitable deviation for firm 1.

The profits of firm 1 when $q_1(1-q_1) \geq q_2(1-q_2) = \frac{6}{25}$ are (substitute $q_2 = \frac{2}{5}$ into (B.27)):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{5}q_1 - 2k, & \text{if } k \leq \frac{6}{50} - \frac{1}{10}q_1, \\ q_1(1-q_1) - \frac{1}{10}q_1 - 2k, & \text{if } \frac{6}{50} - \frac{1}{10}q_1 < k \leq \frac{1}{2}q_1(1-q_1), \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k > \frac{1}{2}q_1(1-q_1). \end{cases}$$

Maximizing the above with respect to q_1 (we omit calculations again), the optimized profits of firm 1 are (assuming that $k \leq \frac{2}{25}$):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{81}{400} - 2k, & \text{if } k \geq \frac{3}{40}, \\ k(13 - 100k) - \frac{9}{25}, & \text{if } \frac{3}{50} < k < \frac{3}{40}, \\ \frac{4}{25} - 2k, & \text{if } k \leq \frac{3}{50}. \end{cases} \tag{B.30}$$

Looking at (B.30), we see that firm 1 chooses $q_1 = \frac{2}{5}$ (with the corresponding profit of $\mathbb{E}\pi_1 = \frac{4}{25} - 2k$) when $k \leq \frac{3}{50}$. Hence, for firm 1 in order not to have incentives to deviate away from $q_1 = \frac{2}{5}$, it has to be true that $k \leq \frac{3}{50}$.

Finally, let us consider a possible profitable deviation to $s_1^1 = 0$ and any q_1 taking as given

$s_2^1 = 1$ and $q_2 = \frac{2}{5}$. If firm 1 deviates to $s_1^1 = 0$, the its profits are the following. If $q_1(1-q_1) < q_2(1-q_2) = \frac{6}{25}$, then (substitute $q_2 = \frac{2}{5}$ into (B.24)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1 - k, & \text{if } k \le \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1, \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1. \end{cases} \tag{B.31}$$

Maximizing the above with respect to q_1 we can show that there is no profitable deviation for firm 1.

If $q_1(1-q_1) > q_2(1-q_2) = \frac{6}{25}$, then the profits of firm 1 are (substitute $q_2 = \frac{2}{5}$ into (B.25)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{10}q_1 - k, & \text{if } k \le \frac{6}{50} - \frac{1}{10}q_1, \\ \frac{1}{2}q_1(1-q_1) - k, & \text{if } \frac{6}{50} - \frac{1}{10}q_1 < k \le \frac{1}{2}q_1(1-q_1), \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1). \end{cases}$$

Maximizing the above with respect to q_1, the optimized profits of firm 1 are:

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{8} - k, & \text{if } k > \frac{7}{100}, \\ 6k - 50k^2 - \frac{3}{25}, & \text{if } \frac{3}{50} < k \le \frac{7}{100}, \\ \frac{2}{25} - k, & \text{if } k \le \frac{3}{50}. \end{cases} \tag{B.32}$$

Comparing the equilibrium profit of $\mathbb{E}\pi_1 = \frac{4}{25} - 2k$ to (B.32) we can see that there is no profitable deviation when $k \le \frac{3}{50}$.

To sum up, the only equilibrium in this case is $k \le \frac{3}{50}$, $(s_1^1; s_2^1) = (1; 1)$, $(q_1; q_2) = \left(\frac{2}{5}; \frac{2}{5}\right)$ and $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = \left(\frac{4}{25} - 2k; \frac{4}{25} - 2k\right)$.

2: $s_1^1 = 0$ and $s_2^1 = 1$ (or, symmetrically, $s_1^1 = 1$ and $s_2^1 = 0$).

Let us begin by looking at the maximization problem of firm 1. If $q_1(1-q_1) \le q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.24) with the one-shot first period profits of 0):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2, \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2. \end{cases} \tag{B.33}$$

If $q_1(1-q_1) > q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.25) with the one-shot first period profits of 0):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \leq \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2, \\ \frac{1}{2}q_1(1-q_1) - k, & \text{if } \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k \leq \frac{1}{2}q_1(1-q_1), \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1). \end{cases} \tag{B.34}$$

Looking at the above we can notice the following as described in the observations stated earlier in the proof (Preface to Period 1). First, from observation (3), we know that in any equilibrium where firm 1 chooses $s_1^1 = 0$, it must be true that equilibrium values of q_1 and q_2 satisfy $k \geq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$. That means that in the above stated profit functions, the equilibrium profits of firm 1 cannot lie in the following regions: $k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$ and $k \leq \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2$. Second, the equilibrium profits of firm 1 cannot lie in the region of $\frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k \leq \frac{1}{2}q_1(1-q_1)$ when $q_1(1-q_1) > q_2(1-q_2) \neq \frac{1}{4}$ (or, $q_2 \neq \frac{1}{2}$). Indeed, when $\frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k \leq \frac{1}{2}q_1(1-q_1)$, firm 2's combined two period profits are $\mathbb{E}\pi_2 = \frac{1}{2}q_2(1-q_2) - k$ and firm 2 has a profitable deviation to $q_2 = \frac{1}{2}$ (since $q_2 = \frac{1}{2}$ maximizes this profit). Third, it cannot be true in an equilibrium that $q_1(1-q_1) = q_2(1-q_2) \neq \frac{1}{2}$ and $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 = \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k < \frac{1}{2}q_1(1-q_1) = \frac{1}{2}q_2(1-q_2)$. Indeed, for firm 1 (and, symmetrically for firm 2) there is a profitable deviation to $q_1(1-q_1) = q_2(1-q_2) + \epsilon$ that brings a discontinuous increase in profits. Fourth, it cannot be true that $q_1(1-q_1) < q_2(1-q_2) \neq \frac{1}{4}$ and $k \geq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$. Indeed, if this was true than the equilibrium profits of firm 1 would be $\mathbb{E}\pi_1 = 0$. But consider firm 1 deviating to

$s_1^1 = 0$ and $q_1(1 - q_1) = \frac{1}{4}$, then its profits would be (substitute $q_1 = \frac{1}{2}$ into (B.25)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{8} - \frac{1}{8}q_2 - k, & \text{if } k \leq \frac{1}{2}q_2(1-q_2) - \frac{1}{8}q_2, \\ \frac{1}{8} - k, & \text{if } \frac{1}{2}q_2(1-q_2) - \frac{1}{8}q_2 < k \leq \frac{1}{8} \\ 0, & \text{if } k > \frac{1}{8}. \end{cases}$$

In the above we can see that the profits of firm 1 cannot be equal to 0 because we have assumed that $k \leq \frac{1}{8}$. So, either $\mathbb{E}\pi_1 = \frac{1}{8} - k > 0$ (because $k \leq \frac{1}{8}$) or $\mathbb{E}\pi_1 = \frac{1}{8} - \frac{1}{8}q_2 - k > 0$ (because $k \leq \frac{1}{2}q_2(1-q_2) - \frac{1}{8}q_2$). Therefore, any profit firm 1 could get from a deviation to $q_1 = \frac{1}{2}$ is bigger than 0. Hence, there are always incentives for this deviation. Fifth, finally, in any equilibrium where $q_1(1-q_1) < q_2(1-q_2)$ and $k \geq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$ it must be true that $k \geq \frac{1}{16}$. Indeed, when $q_1(1-q_1) < q_2(1-q_2)$ and $k \geq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$, the profits of firm 1 are $\mathbb{E}\pi_1 = 0$. But the lowest possible profit firm 1 can obtain from playing $s_1^1 = 1$ is (when $s_2^1 = 1$ and $q_2 = 1$) $\mathbb{E}\pi_1 = \max_{q_1}\left[\frac{1}{2}q_1(1-q_1) - 2k\right] = \frac{1}{8} - 2k$. In equilibrium the latter profit must be smaller than the former, that is, $\frac{1}{8} - 2k \leq 0 \Rightarrow k \geq \frac{1}{16}$.

Therefore, from the points above, in any equilibrium one of the following must hold: either $q_1(1-q_1) < q_2(1-q_2) = \frac{1}{4}$ (that is, $q_1 = q_2 = \frac{1}{2}$) and $k \geq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$; or, $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{4}$ and $\frac{1}{16} = \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 = \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 < k \leq \frac{1}{2}q_1(1-q_1) = \frac{1}{2}q_2(1-q_2) = \frac{1}{8}$. Notice from the above, that in any equilibria it must hold that $q_2(1-q_2) = \frac{1}{4}$ (or, $q_2 = \frac{1}{2}$).

Knowing that it is always true that an equilibrium $q_2 = \frac{1}{2}$, we can easily calculate the best response of firm 1 to $q_2 = \frac{1}{2}$. When $q_1(1-q_1) < q_2(1-q_2) = \frac{1}{4}$, the combined two period profits of firm 1 are (substitute $q_2 = \frac{1}{2}$ into (B.33)):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1 - k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1, \\ 0, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1. \end{cases}$$

Maximizing the above with respect to q_1, we get the following optimal profits:

$$\mathbb{E}\pi_1 = \begin{cases} \frac{9}{128} - k, & \text{if } k \leq \frac{9}{128}, \\ 0, & \text{if } k > \frac{9}{128}. \end{cases} \tag{B.35}$$

If $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{4}$, then the combined two period profits of firm 1 are (substitute $q_2 = \frac{1}{2}$ into (B.33) and (B.34) and take the average of them):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{16} - k, & \text{if } k \leq \frac{1}{16}, \\ \frac{1}{16} - \frac{k}{2}, & \text{if } \frac{1}{16} < k \leq \frac{1}{8}, \\ 0, & \text{if } k > \frac{1}{8}. \end{cases} \tag{B.36}$$

Combining (B.35) and (B.36), we get the following best response profit of firm 1 to $s_2^1 = 1$ and $q_2 = \frac{1}{2}$:

$$\mathbb{E}\pi_1 = \begin{cases} \frac{9}{128} - k, & \text{if } k < \frac{1}{16}, \\ \frac{1}{16} - \frac{k}{2}, & \text{if } \frac{1}{16} \leq k \leq \frac{1}{8}, \\ 0, & \text{if } k > \frac{1}{8}. \end{cases} \tag{B.37}$$

Taking into account that in any equilibrium it must be true that $\frac{1}{16} \leq k \leq \frac{1}{8}$, firm 1 always has a dominant strategy of playing $q_1 = \frac{1}{2}$. This means that in equilibrium it can never be true that $q_1(1-q_1) < q_2(1-q_2) = \frac{1}{4}$ and $k \geq \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2$. Hence, in any equilibrium it has to be true that $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{2}$ and $\frac{1}{16} \leq k \leq \frac{1}{8}$. We have already shown that for firm 1 $q_1 = \frac{1}{2}$ is indeed the best response to $q_2 = \frac{1}{2}$. Now let us see when $q_2 = \frac{1}{2}$ is the best response of firm 2 to firm 1 playing $q_1 = \frac{1}{2}$.

If $q_2(1-q_2) < q_1(1-q_1) = \frac{1}{4}$, then the combined two period profits of firm 2 are (these are obtained by summing the second period expected profits in (B.24) with the one shot first period expected profits of $\frac{1}{2}q_2(1-q_2)-k$ and substituting $q_1=\frac{1}{2}$):

$$\mathbb{E}\pi_2=\begin{cases} q_2(1-q_2)-\frac{1}{8}q_2-2k, & \text{if } k\leq \frac{1}{2}q_2(1-q_2)-\frac{1}{8}q_2,\\ \frac{1}{2}q_2(1-q_2)-k, & \text{if } k>\frac{1}{2}q_2(1-q_2)-\frac{1}{8}q_2. \end{cases} \tag{B.38}$$

Maximizing the above with respect to q_2, we get the following optimal profits:

$$\mathbb{E}\pi_2=\begin{cases} \frac{49}{256}-2k, & \text{if } k\leq \frac{17}{256},\\ \frac{1}{8}-k, & \text{if } k>\frac{17}{256}. \end{cases} \tag{B.39}$$

If $q_2(1-q_2)=q_1(1-q_1)=\frac{1}{4}$, then the combined two period profits of firm 2 are (these are obtained by summing the average of the second period expected profits between (B.24) and (B.25) with the one shot first period expected profits of $\frac{1}{2}q_2(1-q_2)-k$ and substituting $q_1=\frac{1}{2}$):

$$\mathbb{E}\pi_1=\begin{cases} \frac{3}{16}-2k, & \text{if } k\leq \frac{1}{16},\\ \frac{3}{16}-\frac{3k}{2}, & \text{if } \frac{1}{16}<k\leq \frac{1}{8},\\ \frac{1}{8}-k, & \text{if } k>\frac{1}{8}. \end{cases} \tag{B.40}$$

Comparing (B.39) and (B.40) we can see that firm always chooses to play $q_2=\frac{1}{2}$ with the corresponding profits of $\mathbb{E}\pi_2=\frac{3}{16}-\frac{3k}{2}$ when $\frac{1}{16}\leq k\leq\frac{1}{8}$. Now, the only thing that is left to be checked is whether firm 1 has a profitable deviation to $s_1^1=1$ and whether firm 2 has a profitable deviation to $s_2^1=0$.

Consider firm 2 deviating to $s_2^1=0$ taking as given that firm 1 plays $s_1^1=0$ and $q_1=\frac{1}{2}$. As will be shown in the next later on in the next case, the best firm 2 can do is to get $\mathbb{E}\pi_2=0$ if $k>\frac{1}{8}$ and $\mathbb{E}\pi_2=\frac{1}{8}-k$ if $k\leq\frac{1}{8}$. We can clearly see that $\mathbb{E}\pi_2=\frac{3}{16}-\frac{3k}{2}$ is bigger than any of these deviation options when $k\leq\frac{1}{8}$. Therefore, firm 2 would never want to deviate to $s_2^1=0$ when $k\leq\frac{1}{8}$.

Now, consider firm 1 deviating to $s_1^1=1$ taking as given the strategy of firm 2: $s_2^1=1$ and $q_2=\frac{1}{2}$. If $q_1(1-q_1)<q_2(1-q_2)=\frac{1}{4}$, then the combined two period profits of firm 1 are (these

are obtained by summing the expected second period profits in (B.24) with the expected one-shot first period profit of $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k$ and setting $q_2 = \frac{1}{2}$):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{4}q_1 - 2k, & \text{if } k \leq \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1, \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1 - k, & \text{if } k > \frac{1}{2}q_1(1-q_1) - \frac{1}{8}q_1. \end{cases}$$

Maximizing the above with respect to q_1, we get the optimal profits of firm 1:

$$\mathbb{E}\pi_1 = \begin{cases} \frac{9}{64} - 2k, & \text{if } k \leq \frac{9}{128}, \\ \frac{9}{128} - k, & \text{if } k > \frac{9}{128}. \end{cases} \tag{B.41}$$

If $q_1(1-q_1) = q_2(1-q_2) = \frac{1}{4}$, then the combined two period profits of firm 1 are (these are obtained by summing the average of the second period expected profits between (B.24) and (B.25) with the one shot first period expected profits of $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k$ and substituting $q_2 = \frac{1}{2}$):

$$\mathbb{E}\pi_1 = \begin{cases} \frac{1}{8} - 2k, & \text{if } k \leq \frac{1}{16}, \\ \frac{1}{8} - \frac{3k}{2}, & \text{if } \frac{1}{16} < k < \frac{1}{8}, \\ \frac{1}{16} - k, & \text{if } k \geq \frac{1}{8}. \end{cases} \tag{B.42}$$

Combining (B.41) and (B.42), we get the following best response profit of firm 1 to $s_2^1 = 1$ and $q_2 = \frac{1}{2}$:

$$\mathbb{E}\pi_1 = \begin{cases} \frac{9}{64} - 2k, & \text{if } k \leq \frac{1}{16}, \\ \frac{1}{8} - \frac{3k}{2}, & \text{if } \frac{1}{16} < k < \frac{7}{64}, \\ \frac{9}{128} - k, & \text{if } k \geq \frac{7}{64}. \end{cases}$$

Firm 1 would not deviate when the profit above is smaller than $\mathbb{E}\pi_1 = \frac{1}{16} - \frac{k}{2}$. This happens when $\frac{1}{16} \leq k \leq \frac{1}{8}$.

To combine all the results above, the only equilibrium here is the one where the following hold (or, the symmetric version of it): $(s_1^1; s_2^1) = (0;1)$, $(s_1^2; s_2^2) = (1;1)$, $(q_1; q_2) = \left(\frac{1}{2}; \frac{1}{2}\right)$, $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = \left(\frac{1}{16} - \frac{k}{2}; \frac{3}{16} - \frac{3k}{2}\right)$ and $\frac{1}{16} \leq k \leq \frac{1}{8}$.

3: $s_1^1 = 0$ and $s_2^1 = 0$.

In this case, if firm i not to advertise in period 1, it must be true that the one-shot first period profit is smaller than 0 (where 0 is the profit firm i gets if it does not advertise) for any q_i, that is:

$$\max_{q_i} \mathbb{E}\pi_i^1(s_i^1 = 1) = \max_{q_i} \left[\frac{1}{2}q_i(1-q_i) - k\right] = \frac{1}{8} - k \leq 0 \Rightarrow k \geq \frac{1}{8}.$$

The above condition is a necessary and sufficient one for $s_i^1 = 0$: when $k \geq \frac{1}{8}$ no deviations are ever profitable and when $k < \frac{1}{8}$ there is always a profitable deviation to $s_i^1 = 1$ and $q_i = \frac{1}{2}$. Therefore, the only equilibrium here is the one where the following hold: $(s_1^1; s_2^1) = (0;0)$; $(q_1; q_2) = (\text{arbitrary}; \text{arbitrary})$. $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = (0;0)$ and $k \geq \frac{1}{8}$.

B.5 Proof of Proposition 12

As in earlier proofs, we will use backwards induction to solve the model starting from period 2 and then going back to period 1. In period 2 both firms know each others' quantities and first period advertising strategy choices.

Period 2

Without loss of generality assume that $q_1(1-q_1) \leq q_2(1-q_2)$. As earlier, we will first assume some first period choice of advertising strategies (there are 4 possibilities) and solve for period 2 equilibria. Then, we will find conditions under which these first period strategies are actually chosen by the firms. Interestingly, only one (out of 4) second period sub-games (namely, when $s_1^1 = s_2^1 = 1$) differs from the second period sub-games in the model where A_i^t's are uncorrelated over time. Indeed, regardless of whether A_i^t are correlated over time or not, both firms will know their respective second period A_i^2 under perfect learning given that they advertised in period 1; and they will not know their second period A_i^2 if they did not advertise in period 1. The only difference between the two models arises from the fact that when A_i^t are perfectly correlated over time, firm

i can infer firm $-i$ A^t_{-i} from observing the firms' first period interaction. But this interaction only happens when both firms advertised, that is $s_1^1 = s_2^1 = 1$. Therefore, we will only explicitly consider the case $s_1^1 = s_2^1 = 1$ in what follows (it is called case 1). The results of all other cases will be referenced from earlier proofs.

Case 1: $s_1^1 = s_2^1 = 1$.

In this case both firms know perfectly their real A_i^2. Moreover, if $A_i^2 = 1$ firm i also knows the other firm's A^2_{-i}, since if $A^2_{-i} = A^1_{-i} = 1$, then firm i should have observed that in period 1 the price was the following: $p = 1 - q_1 - q_2$, while if $A^2_{-i} = A^1_{-i} = 0$, firm i would have observed the price: $p = 1 - q_i$. If $A_i^2 = 0$, firm i cannot distinguish between $A^2_{-i} = 1$ and $A^2_{-i} = 0$ but it does not care since $A_i^2 = 0$ implies it would never advertise regardless of firm $-i$.

Consider firm i's choice of s_i^2. If it happened to be that $A_i^2 = A^2_{-i} = 1$:

$$\mathbb{E}\pi_i^2(s_i^2 = 1; s^2_{-i} = 1) = q_i(1 - q_i - q_{-i}) - k,$$

$$\mathbb{E}\pi_2^1(s_i^2 = 1; s^2_{-i} = 0) = q_i(1 - q_i) - k,$$

$$\mathbb{E}\pi_i^2(s_i^2 = 0; s^2_{-i} = 1) = \mathbb{E}\pi_2^1(s_i^2 = 0; s^2_{-i} = 0) = 0.$$

We get the following equilibria in this sub-case:

$$(s_1^2; s_2^2) = \begin{cases} (1;1), & \text{if } k \leq q_1(1 - q_1 - q_2), \\ (1;0), & \text{if } q_2(1 - q_1 - q_2) \leq k \leq q_1(1 - q_1), \\ (0;1), & \text{if } q_1(1 - q_1 - q_2) \leq k \leq q_2(1 - q_2), \\ (0;0), & \text{if } k \geq q_2(1 - q_2). \end{cases}$$

As we can see, $(1;0)$ and $(0;1)$ equilibria overlap (the region where the $(1;0)$ equilibrium exists is fully located inside the region where the $(0;1)$ equilibrium exists). We will assume that the equilibrium that actually realizes is the one that is Pareto dominant, that is, in our case the one for which $q_2(1-q_2) \geq q_1(1-q_1)$ or $(0;1)$ (if $q_1(1-q_1) = q_2(1-q_2)$, then one of these two equilibria realizes randomly with probability $\frac{1}{2}$). Hence, with this refinement, the equilibria of this sub-case are:

$$(s_1^2; s_2^2) = \begin{cases} (1;1), & \text{if } k \leq q_1(1-q_1-q_2), \\ (0;1), & \text{if } q_1(1-q_1-q_2) \leq k \leq q_2(1-q_2), \\ (0;0), & \text{if } k \geq q_2(1-q_2). \end{cases}$$

If it happened to be that $A_i^2 = 1$ and $A_{-i}^2 = 0$ (also applies to a symmetric case of $A_i^2 = 0$ and $A_{-i}^2 = 1$ by replacing i with $-i$), then the firms' profits are:

$$\mathbb{E}\pi_i^2(s_i^2 = 1; s_{-i}^2 = 1) = \mathbb{E}\pi_i^2(s_i^2 = 1; s_{-i}^2 = 0) = q_i(1-q_i) - k,$$

$$\mathbb{E}\pi_i^2(s_i^2 = 0; s_{-i}^2 = 1) = \mathbb{E}\pi_i^2(s_i^2 = 0; s_{-i}^2 = 0) = 0,$$

$$\mathbb{E}\pi_{-i}^2(s_i^2 = 1; s_{-i}^2 = 1) = \mathbb{E}\pi_{-i}^2(s_i^2 = 0; s_{-i}^2 = 1) = -k,$$

$$\mathbb{E}\pi_{-i}^2(s_i^2 = 1; s_{-i}^2 = 0) = \mathbb{E}\pi_{-i}^2(s_i^2 = 0; s_{-i}^2 = 0) = 0,$$

We get the following equilibria in this sub-case:

$$(s_i^2; s_{-i}^2) = \begin{cases} (1;0), & \text{if } k \leq q_i(1-q_i), \\ (0;0), & \text{if } k \geq q_i(1-q_i). \end{cases}$$

If it happened to be that $A_1^2 = A_2^2 = 0$, then the only equilibrium is:

$$s_1^2 = s_2^2 = 0.$$

Before period 2 begins, firms do not yet know their signal realizations. So, to make a decision on first period strategies, they need to use an expectation (over possible signal realizations) as to their second period profits. From the point of view of period 1, all four signal realizations ($A_1^2 = A_2 = 1$; $A_1^2 = A_2^2 = 0$; $A_1^2 = 1$, $A_2^2 = 0$; $A_1^2 = 0$, $A_2^2 = 1$) are equally likely, so the expected second period profit is the average profit obtained over these 4 combinations. The functional form of this second period expectation depends on parameter values and period one choices of q_i. Overall, there are 4 regions in k and q_1, q_2 that we discuss below.

Region 1: $k \leq q_1(1 - q_1 - q_2)$.

In this region, the equilibrium second period sub-case strategies are the following: if $A_1^2 = A_2^2 = 1$, then $(s_1^2; s_2^2) = (1; 1)$; if $A_1^2 = 1$ and $A_2^2 = 0$, then $(s_1^2; s_2^2) = (1; 0)$; if $A_1^2 = 0$ and $A_2^2 = 1$, then $(s_1^2; s_2^2) = (0; 1)$; if $A_1^2 = A_2^2 = 0$, then $(s_1^2; s_2^2) = (0; 0)$. Taking the average of the profits each firm obtains in these 4 scenarios, we get the following Region 1 expected equilibrium profits:

$$\mathbb{E}\pi_1^2 = \frac{1}{2}q_1(1 - q_1) - \frac{1}{4}q_1 q_2 - \frac{k}{2},$$

$$\mathbb{E}\pi_2^2 = \frac{1}{2}q_2(1 - q_2) - \frac{1}{4}q_1 q_2 - \frac{k}{2}.$$

Region 2: $q_1(1 - q_1 - q_2) < k < q_1(1 - q_1)$.

In this region, the equilibrium second period sub-case strategies are the following: if $A_1^2 = A_2^2 = 1$, then $(s_1^2; s_2^2) = (0; 1)$; if $A_1^2 = 1$ and $A_2^2 = 0$, then $(s_1^2; s_2^2) = (1; 0)$; if $A_1^2 = 0$ and $A_2^2 = 1$, then $(s_1^2; s_2^2) = (0; 1)$; if $A_1^2 = A_2^2 = 0$, then $(s_1^2; s_2^2) = (0; 0)$. Taking the average of the profits each firm obtains in these 4 scenarios, we get the following Region 2 expected equilibrium profits:

$$\mathbb{E}\pi_1^2 = \frac{1}{4}q_1(1 - q_1) - \frac{k}{4},$$

$$\mathbb{E}\pi_2^2 = \frac{1}{2}q_2(1 - q_2) - \frac{k}{2}.$$

Region 3: $q_1(1-q_1) \le k < q_2(1-q_2)$.

In this region, the equilibrium second period sub-case strategies are the following: if $A_1^2 = A_2^2 = 1$, then $(s_1^2; s_2^2) = (0;1)$; if $A_1^2 = 1$ and $A_2^2 = 0$, then $(s_1^2; s_2^2) = (0;0)$; if $A_1^2 = 0$ and $A_2^2 = 1$, then $(s_1^2; s_2^2) = (0;1)$; if $A_1^2 = A_2^2 = 0$, then $(s_1^2; s_2^2) = (0;0)$. Taking the average of the profits each firm obtains in these 4 scenarios, we get the following Region 3 expected equilibrium profits:

$$\mathbb{E}\pi_1^2 = 0.$$

$$\mathbb{E}\pi_2^2 = \frac{1}{2}q_2(1-q_2) - \frac{k}{2}.$$

Region 4: $k \ge q_2(1-q_2)$.

In this region, the equilibrium second period sub-case strategies are the following: if $A_1^2 = A_2^2 = 1$, then $(s_1^2; s_2^2) = (0;0)$; if $A_1^2 = 1$ and $A_2^2 = 0$, then $(s_1^2; s_2^2) = (0;0)$; if $A_1^2 = 0$ and $A_2^2 = 1$, then $(s_1^2; s_2^2) = (0;0)$; if $A_1^2 = A_2^2 = 0$, then $(s_1^2; s_2^2) = (0;0)$. Taking the average of the profits each firm obtains in these 4 scenarios, we get the following Region 4 expected equilibrium profits:

$$\mathbb{E}\pi_1^2 = 0.$$

$$\mathbb{E}\pi_2^2 = 0.$$

We can combine the above regions to express them more succinctly in one expression:

$$\mathbb{E}\pi_2^1 = \begin{cases} \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{k}{2}, & \text{if } k \le q_1(1-q_1-q_2), \\ \frac{1}{4}q_1(1-q_1) - \frac{k}{4}, & \text{if } q_1(1-q_1-q_2) < k < q_1(1-q_1), \\ 0, & \text{if } k \ge q_1(1-q_1). \end{cases} \tag{B.43}$$

$$\mathbb{E}\pi_2^2 = \begin{cases} \frac{1}{2}q_2(1-q_2) - \frac{1}{4}q_1q_2 - \frac{k}{2}, & \text{if } k \le q_1(1-q_1-q_2), \\ \frac{1}{2}q_2(1-q_2) - \frac{k}{2}, & \text{if } q_1(1-q_1-q_2) < k < q_2(1-q_2), \\ 0, & \text{if } k \ge q_2(1-q_2). \end{cases} \tag{B.44}$$

Period 1

In period 1, firms choose both q_i and s_i^1 simultaneously at once. We will consider all possible pure strategy equilibria. As earlier, the solution strategy is the following. Since there is a limited number of possible pure strategy choices for s_1^1 and s_2^1 (namely, as in period 2, there are 4 possible combinations of them), we will proceed in the following way. We fix a certain advertising strategy combination, $(\hat{s}_1^1; \hat{s}_2^1)$, (*e.g.* $s_1^1 = 1$ and $s_2^1 = 0$) and look at whether there is a combination of q_1 and q_2 such that $\{(\hat{s}_1^1; \hat{s}_2^1); (\hat{q}_1; \hat{q}_2)\}$ is an equilibrium, and if it is, find the conditions when it holds.

To simplify further analysis we will make several observations.

1. The biggest one period profit a firm can obtain is $q_i(1-q_i) - k$ (when $A_i = 1$ and $s_{-i}A_{-i} = 0$). This means that if $k > \frac{1}{4} = \max_q q_i(1-q_i)$, then no firm ever advertises, and therefore, $s_1^1 = s_1^2 = s_2^1 = s_2^2 = 0$ with q_1, q_2 being undefined.

2. As a consequence of (1), if in any equilibrium $s_1^1 = 1$, then it must be true that $k \le q_i(1-q_i)$.

3. If firm i chooses $s_i^1 = 0$ then for any equilibrium choice of q_1 and q_2 it must be true that $k \ge \frac{1}{2}q_i(1-q_i) - \frac{1}{4}q_iq_{-i}$. Indeed, if it was not true then, $\mathbb{E}\pi_i^1(s_i^1 = 0) = 0$ while $\mathbb{E}\pi_i^1(s_i^1 = 1) \ge \frac{1}{2}q_i - \frac{1}{4}q_iq_{-i} - k \ge 0$. This would mean that the choice of $s_i^1 = 1$ dominates $s_i^1 = 0$.

Having said the above, let us consider all the possible pure strategy Nash equilibria of the game one by one.

1: $s_1^1 = 1$ and $s_2^1 = 1$.

Consider the problem of firm 1 (the problem of firm 2 is symmetric). It takes $s_2^1 = 1$ and q_2 as given. Assuming that firm 1 sticks with $s_1^1 = 1$, we will find the best possible choice of q_1 under the assumption that $s_1^1 = 1$.

If $q_1(1-q_1) < q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.43) with the one-shot first period expected profits of $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k$):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{2}q_1q_2 - \frac{3k}{2}, & \text{if } k \leq q_1(1-q_1-q_2), \\ \frac{3}{4}q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{5k}{4}, & \text{if } q_1(1-q_1-q_2) < k < q_1(1-q_1), \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \geq q_1(1-q_1). \end{cases}$$

If $q_1(1-q_1) > q_2(1-q_2)$, then firm 1's combined two period profits are (these are obtained by summing the second period expected profits in (B.44) with the one-shot first period expected profits of $\frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k$):

$$\mathbb{E}\pi_1 = \begin{cases} q_1(1-q_1) - \frac{1}{2}q_1q_2 - \frac{3k}{2}, & \text{if } k \leq q_2(1-q_1-q_2), \\ q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{3k}{2}, & \text{if } q_2(1-q_1-q_2) < k < q_1(1-q_1), \\ \frac{1}{2}q_1(1-q_1) - \frac{1}{4}q_1q_2 - k, & \text{if } k \geq q_1(1-q_1). \end{cases} \tag{B.45}$$

Solving the above problem explicitly for the best response is somewhat complicated. Instead, we will show that some values of q_1 can never sustain an equilibrium and so can be ruled out. First, as we already mentioned before, $k > q_1(1-q_1)$ can never happen in equilibrium because firm 1 will want to deviate to $s_1^1 = 0$ to avoid getting negative profits.

Second, we are going to show that in equilibrium it is impossible that an equilibrium q_1 satisfies $q_1(1-q_1) < q_2(1-q_2) < \frac{1}{4}$ and $q_1(1-q_1-q_2) < k < q_1(1-q_1)$. Notice, that when $q_1(1-q_1) < q_2(1-q_2)$ there is no reason for firm 2 to choose $q_2 > \frac{1}{2}$. Indeed, an unconstrained optimum of any form of the profit function $\mathbb{E}\pi_2$ is strictly smaller than $q_2 = \frac{1}{2}$ and the only thing increasing q_2 achieves is lowering the profit of firm 2. Hence, $q_2 < \frac{1}{2}$ (we will consider a case $q_2 = \frac{1}{2}$ separately). Now, consider firm 1 deviating to $q_1 = q_2 + \epsilon$ for an infinitely small ϵ . If

$q_2(1-2q_2) = q_1(1-q_1-q_2) < k < q_2(1-q_2)$ still holds then the profit of firm 1 switches from $\mathbb{E}\pi_1 = \frac{3}{4}q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{5k}{4}$ to $\mathbb{E}\pi_1 = q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{3k}{2} = q_2(1-q_2) - \frac{1}{4}q_2^2 - \frac{3k}{2}$ (because now $q_1(1-q_1) > q_2(1-q_2)$). Comparing the two, we can show that either the latter profit is bigger and, hence, firm 1 has a profitable deviation to $q_1 = q_2 + \epsilon$ or the former profit is negative in which case firm 1 has a profitable deviation to $s_1^1 = 0$. That means the only remaining possibility is $k > q_2(1-2q_2)$ but in this case it can be shown that $\mathbb{E}\pi_1 = \frac{3}{4}q_1(1-q_1) - \frac{1}{4}q_1q_2 - \frac{5k}{4}$ is negative. So, to sum up, the above means that there is no q_1 and q_2 such that in equilibrium $q_1(1-q_1) < q_2(1-q_2) < \frac{1}{4}$ and $q_1(1-q_1-q_2) < k < q_1(1-q_1)$.

Therefore, equilibrium q_1 and q_2 must satisfy $q_1(1-q_2) \leq q_2(1-q_2) < \frac{1}{4}$ and $k \leq q_1(1-q_1-q_2)$ (and, the other symmetric case, $q_2(1-q_2) \leq q_1(1-q_1) < \frac{1}{4}$ and $k \leq q_2(1-q_1-q_2)$) or $q_1 = q_2 = \frac{1}{2}$ (the case we have temporarily excluded previously from consideration). Start with the former. Consider the profit maximization problem of firm 1 if $q_1(1-q_2) \leq q_2(1-q_2)$ and $k \leq q_1(1-q_1-q_2)$:

$$q_1(1-q_1) - \frac{1}{2}q_1q_2 - \frac{3k}{2} \Rightarrow \max_{q_1},$$

$$s.t.\ k \leq q_1(1-q_1-q_2),$$

$$q_1(1-q_1) \leq q_2(1-q_2).$$

Note that in equilibrium it can never be true that $k = q_1(1-q_1-q_2)$ holds with equality. Otherwise, firm 2 would have incentives to increase q_2 by ϵ which would discontinuously increase $\mathbb{E}\pi_2$ from $q_2(1-q_2) - \frac{1}{2}q_1q_2 - \frac{3k}{2}$ to $q_2(1-q_2) - \frac{1}{4}q_1q_2 - \frac{3k}{2}$. This means that an equilibrium q_1 (and symmetrically q_2) has to satisfy the first order conditions to $q_1(1-q_1) - \frac{1}{2}q_1q_2 - \frac{3k}{2} \Rightarrow \max_{q_1}$. Hence, $q_1 = \frac{1}{2} - \frac{q_2}{4}$ and $q_2 = \frac{1}{2} - \frac{q_1}{4}$. Solving these two equations simultaneously we obtain that in equilibrium it must be that $(q_1; q_2)$ equals $(\frac{2}{5}; \frac{2}{5})$ with the corresponding profits being $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = \left(\frac{4}{25} - \frac{3k}{2}; \frac{4}{25} - \frac{3k}{2}\right)$.

However, the above strategy $(q_1; q_2) = \left(\frac{2}{5}; \frac{2}{5}\right)$ cannot be an equilibrium because firm 1 has a profitable deviation to $q_1 = \frac{3}{5}$. Given that firm 2 plays $s_2^1 = 1$ and $q_2 = \frac{2}{5}$, firm 1's profits are the following (substitute $q_1 = \frac{3}{5}$ and $q_2 = \frac{2}{5}$ to (B.45) and note that $0 = \frac{2}{5}\left(1 - \frac{3}{5} - \frac{2}{5}\right) = q_2^\star(1 - q_1 - q_2^\star) < k < q_1(1 - q_1) = \frac{3}{5} * \frac{2}{5} = \frac{6}{25}$ always holds): $\mathbb{E}\pi_1\left(s_1^1 = 1; q_1 = \frac{3}{5}\right) = \frac{9}{50} - \frac{3k}{2} > \frac{4}{25} - \frac{3k}{2} = \mathbb{E}\pi_1\left(s_1^1; q_1 = \frac{2}{5}\right)$. Hence, $(q_1; q_2) = \left(\frac{2}{5}; \frac{2}{5}\right)$ cannot be an equilibrium.

Therefore, the only possible equilibrium candidate left is $(q_1; q_2) = \left(\frac{1}{2}; \frac{1}{2}\right)$. In this case, by directly comparing the strategy of $q_1 = \frac{1}{2}$ case to $q_1 \neq \frac{1}{2}$, we can show that if $\frac{5}{32} - \frac{11k}{8} \geq 0 \Rightarrow k \leq \frac{5}{44}$, then choosing $q_1 = \frac{1}{2}$ is a strictly dominant strategy for firm 1. If $k > \frac{5}{44}$, then firm 1 would have a profitable deviation to $s_1^1 = 0$ and $q_1 = 0$ with the corresponding profit of $\mathbb{E}\pi_1 = 0$. This implies that the only equilibrium candidate in this case is $(q_1; q_2) = \left(\frac{1}{2}; \frac{1}{2}\right)$, $(\pi_1; \pi_2) = \left(\frac{5}{32} - \frac{11k}{8}; \frac{5}{32} - \frac{11k}{8}\right)$ and $k \leq \frac{5}{44}$.

The last thing we need to check is whether firm 1 has a profitable deviation to $s_1^1 = 0$ and some $q_1 = q_1^{'}$. As will be shown in the next case, given that firm 2 plays $s_2^1 = 1$ and $q_2 = \frac{1}{2}$, the best firm 1 can do given that $s_1^1 = 0$ is to set $q_1 = \frac{1}{2}$ and achieve a profit of 0. Firm 1 would not want to deviate when $\frac{5}{32} - \frac{11k}{8} \geq 0 \Rightarrow k \leq \frac{5}{44}$ implying that the only equilibrium in this case is $k \leq \frac{5}{44}$, $(s_1^1; s_2^1) = (0; 0)$, $(q_1; q_2) = \left(\frac{1}{2}; \frac{1}{2}\right)$ and $(\pi_1; \pi_2) = \left(\frac{5}{32} - \frac{11k}{8}; \frac{5}{32} - \frac{11k}{8}\right)$.

2: $s_1^1 = 0$ and $s_2^1 = 1$ (or, symmetrically, $s_1^1 = 1$ and $s_2^1 = 0$).

As we have already said in the beginning of this proof, the only period 2 sub-game that changes compared to the competitive model we solved before is the one where $s_1^1 = s_2^1 = 1$. This means that the sub-game we are currently considering does not change compared to the previous model and, hence, the results carry over. Therefore, the only possible pure strategy Nash equilibrium in this case is $(s_1^1; s_2^1) = (0; 1)$ and $(q_1; q_2) = \left(\text{arbitrary}; \frac{1}{2}\right)$ with the corresponding profits of $(\mathbb{E}\pi_1; \mathbb{E}\pi_2) = \left(0; \frac{1}{4} - \frac{3k}{2}\right)$. The only thing that changes in the extension and is important to our proof is that possible deviations change. To make sure this is an equilibrium we need to consider firm 1 deviating to $s_1^1 = 1$ and firm 2 deviating to $s_2^1 = 0$.

Consider a possible deviation of firm 2 to $s_2^1 = 0$ and some arbitrary q_2. Since when firm 2 deviates to $s_2^1 = 0$ we now end up in the sub-game where $(s_1^1; s_2^1) = (0; 0)$, it means that the constraint on non-deviation does not change compared to the competitive model we considered earlier (remember that neither the $(s_1^1; s_2^1) = (0; 1)$ nor the $(s_1^1; s_2^1) = (0; 0)$ sub-games change compared to the earlier competitive model). To recap, firm 2 does not deviate from playing $s_2^1 = 1$ to playing $s_2^1 = 0$ when $k \leq \frac{1}{6}$.

Now, consider firm 1 deviating to $s_1^1 = 1$ taking as given the strategy of firm 2: $s_2^1 = 1$ and $q_2 = \frac{1}{2}$. We have already shown in the previous case that the best firm 1 can do under this scenario is $s_1^1 = 1$ and $q_1 = \frac{1}{2}$ with the profits being $\frac{5}{32} - \frac{11k}{8}$. Comparing $\mathbb{E}\pi_1(s_1^1 = 0; q_1 = \text{arbitrary}) = 0$ and $\mathbb{E}\pi_1\left(s_1^1 = 1; q_1 = \frac{1}{2}\right) = \frac{5}{32} - \frac{11k}{8}$, we see that there is no deviation when $k \geq \frac{5}{44}$.

Therefore, combining all of the above, we can state that the only equilibrium in this case is a set of strategies satisfying the following conditions (or their symmetric counterparts): $(s_1^1; s_2^1) = (0; 1)$, $\frac{5}{44} \leq k \leq \frac{1}{6}$, $(q_1; q_2) = \left(\text{arbitrary}; \frac{1}{2}\right)$ and $(\pi_1; \pi_2) = \left(0; \frac{1}{4} - \frac{3k}{2}\right)$.

3: $s_1^1 = s_2^1 = 0$.

In this case, neither this current sub-game (*i.e.* $(s_1^1; s_2^1) = (0; 0)$) nor any sub-game that could arise from firms' deviating (i.e. $(s_1^1; s_2^1) = (1; 0)$ or $(s_1^1; s_2^1) = (0; 1)$) differ from the competitive model we considered earlier in the paper. This means the the earlier results carry over. That is, the only equilibrium is characterised by the following: $(s_1^1; s_2^1) = (0; 0)$, $(q_1; q_2) = (\text{arbitrary}; \text{arbitrary})$, $(\pi_1; \pi_2) = (0; 0)$ and $k \geq \frac{1}{6}$.