Investigations of Nonlinear Optical Phenomenon and Dispersion in Integrated Photonic Devices

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ABSTRACT

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Integrated photonics is the field of shrinking and simplifying the fabrication of devices that guide and manipulate light. It not only offers to greatly lower the size and cost of systems used in optical communications it also offers a platform on which new physical phenomenon can be explored by being able to fabricate and manipulate structures on the scale of the wavelength of light.

One such platform in integrated photonics is that of two-dimensional slab photonic crystals. These structures exhibit a photonic band-gap, a band of optical frequencies that are prohibited from propagating within the medium, that can be used to guide and confine light. When used to create photonic crystal waveguides these waveguides exhibit unique dispersion properties that demonstrate very low optical group velocities, so called "slow-light".

This dissertation begins with the practical realization of design and fabrication of such waveguides using the silicon-on-insulator material system using conventional deep-UV photolithography fabrication techniques. It will detail and demonstrate the effect physical dimensions have on the optical transmission of these devices as well as their optical dispersion.

These photonic crystal waveguides will then be used to demonstrate the enhancement of nonlinear optical phenomenon due to the slow-light phenomenon they exhibit. First spontaneous Raman scattering will be theoretically demonstrated to be enhanced by slow-light and then experimentally shown to be enhanced in a practical realization. The process of four-wave mixing will be demonstrated to be enhanced in these devices and be shown to
be greatly affected by the unique optical dispersion within these structures.

Additionally, we will examine the dispersion that exists in silicon nitride microring resonators and the effect it has on the use of these devices to generate optical frequency combs. This is done by leveraging the dispersion measurement methods used to characterize photonic crystal waveguides.

We conclude this work by examining the avenues of future work that can be explored in the area of photonic crystal waveguides.
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Chapter 1

Introduction

1.1 Integrated Photonics

The modern technologies of today, such as radio and integrated circuits, very rarely have straightforward stories of invention. Their creation histories are ones of ideas contributed by multiple scientists and engineers spanning sometimes decades that make it difficult to assign definitive inventors. For example the invention of integrated circuit even though Jack Kilby was awarded the Nobel prize for his contributions to the invention there still remains a significant debate over how his contributions compare to the work of other early pioneers in the development such as Robert Noyce, Kurt Lehovec, and Jean Hoerni[1]. With this in mind we will bring to attention the important contributions in the relevant areas of research pertinent to this work and let it be known that these are meant to illuminate the development of the field and not to be exhaustive historical record.

Integrated photonics can be defined broadly as utilizing planar manufacturing techniques such as those used in integrated circuit fabrication to create devices that can guide and manipulate light. The idea of integrated photonics can be traced back to the seminal paper[2] by S.E. Miller from Bell Laboratories in 1969. Miller envisioned “laser circuitry” that could
be defined via photolithography and be used to create optical modulators and filters[3]. It is interesting to note that though this paper appears to be prescient from a modern vantage point, at the time Miller and Bell Labs were skeptical[4] of the possibility of creating low loss silica fibers such as the ones being investigated by Charles Kao[5] and others at the time. Of course, Charles Kao went on to be awarded the Nobel Prize in Physics for his work and low-loss silica optical fibers ushered in a new paradigm in telecommunications which invariably brought about the modern demand for integrated photonics.

When it comes to practical implementation a number of material systems have been successfully utilized in integrated optical platforms. The success of material as a platform for integrated photonics hinges on a number of factors and greatly depends on the application. However, in general three primary factors are essential: the ease with which it is to create passive waveguide structures using the material, the materials intrinsic electro-optical and nonlinear optical parameters in order to create active photonic devices such as lasers, photodetectors and modulators, and finally the materials economic viability in terms of scarcity and ease of fabrication. With these factors in mind here is a succinct summary of the most popular integrated photonics platforms.

Lithium Niobate (LiNbO$_3$) is one of the earliest successful integrated photonics materials owing to its high linear electro-optical effect coefficient[6], this has made it the material of choice for high speed optical modulators. As well, the ease with which diffused waveguides can be patterned in LiNbO$_3$ make it very suitable integrated photonic platform. Traditional fabrication techniques limited integration that could be achieved with LiNbO$_3$ due to high bending losses[7] however newer fabrication methods are able to achieve much more compact geometries[8][9].

Compound semiconductors, specifically the III-V group of semiconductors such as indium phosphide (InP), have been demonstrated as proven integrated photonic platforms[10]. The ability to epitaxially grow various semiconductor alloys on the same substrate in these material systems allows for monolithic integration of optical amplifiers and lasers alongside traditional passive photonic devices like waveguides, filters, and splitters. This capability
greatly increases the complexity and functionality of the integrated photonic devices capable with this material system as has been demonstrated commercially[11]. The cost of these materials when compared to less costly alternatives such as silicon is a major disadvantage. For example indium phosphide substrate costs can be over four times that of silicon[12].

Silicon nitride (Si$_3$N$_4$) has long been identified as an appropriate medium for integrated photonics[13]. Si$_3$N$_4$ films can be grown using chemical vapor deposition (CVD) techniques on silicon dioxide (SiO$_2$) films using silicon substrate wafers. This makes the Si$_3$N$_4$ system compatible with conventional silicon integrated circuit fabrication methods which is highly desirable for economy of scales and as well makes available the plethora of conventional silicon fabrication facilities. Si$_3$N$_4$ waveguides with SiO$_2$ cladding capable of very low propagation loss waveguides and the resulting resonators developed in this platform exhibit extremely large quality factors[14]. One key drawback of this material system is the low-index contrast between Si$_3$N$_4$ and SiO$_2$ limits the bending radius of waveguides and therefore the capability to create highly integrated photonic circuits is limited when compared to high-index contrast platforms like silicon-on-insulator.

The silicon-on-insulator (SOI) integrated photonics platform deserves special consideration not least of all because it is the platform used in this work but because it has a unique set of advantages and disadvantages.

### 1.2 Silicon-on-Insulator Integrated Photonics

The idea of using silicon as medium for integrated photonics goes back to the 1980s[15]. A key breakthrough was the adoption of silicon-on-insulator technologies by integrated circuit manufacturers. This enabled the development of new wafer fabrication techniques[16] that allow for thin (〜250 nm is a typical thickness) crystalline silicon layers to exist on thick (1-3 µm) SiO$_2$ layers. This arrangement allows for light to be confined within the higher refractive index silicon layer while being surrounded by a SiO$_2$ cladding. The silicon layer can then be patterned and etched with conventional integrated circuit fabrication methods to
create various photonic structures.

Silicon as a material has a number of advantages for its application in integrated photonics. It is optically transparent from 1.1 µm to about 6 µm. This makes it suitably compatible with the conventional wavelength ranges used in fiber optic communication systems (1.3 µm and 1.5 µm). Silicon has a large index of refraction ($n_{Si} \sim 3.48$ around 1.55 µm) which creates a large index of refraction contrast with its cladding material SiO$_2$ ($n_{SiO_2} \sim 1.46$). A large index of refraction contrast allows for tighter optical mode confinement in waveguiding structures which is important in number of aspects of integrated photonics not the least of which is it allows for tighter bend radii. Smaller bending radii allows for higher device integration and smaller overall device size.

However silicon has a number of well known disadvantages as an integrated photonics platform as well. Silicon is fundamentally an indirect band-gap material which means nonradiative recombination mechanisms dominate making silicon an unsuitable material for making optical amplifiers or lasers. Silicon is a centrosymmetric crystalline structure which means it exhibits no second-order nonlinearity (Pockels effect). This necessitates using other processes to produce electro-optical response for phase shifters and modulators in silicon photonic devices. Two such processes are the thermo-optic effect, used for slow tuning requirements, and free-carrier injection, capable of much higher speeds but still inefficient when compared to using the Pockels effect in materials like LiNbO$_3$.

A few of silicon’s advantages as an integrated photonics material can also be interpreted as disadvantages. The high-index of silicon while allowing for tighter integration makes the devices more susceptible to scattering losses due to imperfections in fabrication which inherently requires tighter tolerances in fabrication. The transparency of silicon can also be interpreted as a disadvantage when considering how to fabricate photodetectors which require some amount of absorption. However this has been circumvented with the development of germanium waveguide photodetectors that can be fabricated on silicon[17][18]. As mentioned above, silicon is transparent at the wavelengths that are of interest to traditional fiber optical data communication (1.3 µm and 1.5 µm) however it still experiences nonlin-
ear optical absorption due to two-photon absorption and free-carrier absorption at these wavelengths\cite{19} at high optical powers. These absorption mechanisms can have deleterious effects in certain applications such as silicon Raman lasers\cite{20}.

When extolling the advantages and disadvantages of silicon as an integrated optical medium one fact in particular stand above all others: the existing manufacturing infrastructure of the integrated circuit (IC) industry. The IC industry provides not only equipment and methods to fabricate the features required for integrated optical circuits it also provides a source of high quality and relatively cheap wafers. It is for this reason that so much effort has been put into working around the previously mentioned disadvantages of silicon as an integrated photonics platform. The result is a scaling in the complexity of SOI integrated photonic devices has been shown a similarity to that of the well known Moore’s Law scaling in integrated circuits\cite{21} that has driven modern technology for the last fifty years.

1.3 Photonic Crystals

In the investigation of two fundamental physical phenomena, the control of spontaneous emission\cite{22} and the observation of Anderson localization\cite{23}, two researchers independently put forth the idea of material that possessed an electromagnetic analog of the band-gap observed for electrons in atomic crystals. These so called photonic band-gap materials or photonic crystals are materials with periodic modulations of refractive index that create bands of optical frequencies that are inhibited from propagating in them. Although one-dimensional bands have been known since the late nineteenth century, the term photonic crystals refers specifically the materials with large index contrast that exhibit bands in either two or three dimensions\cite{24}.

A number of methods have been employed to create three-dimensional photonic crystals. Examples of the methods employed include conventional layer by layer fabrication\cite{25, 26}, holographic lithography\cite{27}, direct write two photo absorption lithography\cite{28}, and self-assembly via colloidal microspheres\cite{29–31}. Although these methods have successfully
been used to demonstrate photonic band-gaps these methods are cumbersome and lack the repeatability required to demonstrate practical applications of photonic crystals. As well, some of these methods lack the versatility required to create defect geometries required to create confined cavities and waveguide structures[32].

An alternative to the complexity of three-dimensional photonic crystals are two-dimensional photonic crystals that exhibit a photonic band-gap in two-dimensions while depending on conventional total internal confinement in the vertical dimension. So called two-dimensional slab photonic crystals [33, 34] were easily fabricated with the conventional lithography and etching processes used in integrated circuit manufacturing. This allows for these systems to more easily be used to demonstrate the unique aspects of photonic crystals including photonic band-gap defect lasers[35] and optical emission from photonic crystal defects[36].

The slab photonic crystal system allows for not just the creation of localized defect modes with exceptionally high quality factors[37] it also allows for waveguiding by linear defects that exhibit unique dispersion properties[38]. These so called photonic crystal waveguides exhibit very low optical group velocities due to their confinement by the bulk photonic crystal[39]. This low optical group velocity, often referred to as slow-light, results in enhanced field intensities within these waveguides when compared to conventional photonic waveguides which are confined via total internal reflection. These enhanced field intensities allow for an enhancement of the light-matter interaction within the waveguide and thus can be used to enhance nonlinear interactions[40]. Slow light in photonic crystal waveguides have a number of applications beyond nonlinear enhancement including all optical buffering and in areas of quantum information processing[41].

1.4 Scope of This Dissertation

The goal of this dissertation is to detail the design and practical realization of slow-light silicon photonic crystal waveguides and demonstrate their slow-light enhancement of spontaneous Raman scattering and four-wave mixing. This is accomplished by first introducing the
process of design, fabrication, and transmission characterization of silicon photonic crystal waveguides manufactured using deep-UV photolithography techniques in Chapter 2. Since the demonstration of slow-light in these waveguides is an integral part of this work, Chapter 3 is dedicated to the experimental methods used to characterize the optical dispersion of the fabricated structures and discuss their advantages and disadvantages.

With the photonic crystal waveguide devices introduced and characterized, we turn our attention to exploring the practical realization of enhancement of nonlinear phenomenon in these devices. We begin this by theoretically exploring the expected enhancement of spontaneous Raman scattering in these devices in Chapter 4. Encouraged by the numerical model showing an enhancement can be expected in these devices, Chapter 5 is the experimental realization of observing enhanced spontaneous Raman scattering in silicon photonic crystal waveguides. With the demonstration of enhanced Raman scattering we move on to experimentally demonstrate the enhancement of the four-wave mixing process in silicon photonic crystal waveguides and discuss how their unique dispersion affects this process in Chapter 6.

Additionally, this dissertation includes work on characterizing the dispersion of silicon nitride microring resonators used to generate optical frequency combs. The experimental methods used to characterize silicon photonic crystal waveguides is leveraged and used to analyze the dispersion of the resonant modes of these structures and analyze the effect of their interaction on the use of these devices in the generation of optical frequency combs in Chapter 7.

The dissertation then concludes with Chapter 8 which reviews the pertinent work demonstrated in the dissertation as well as offering avenues of future exploration in the topics discussed.
Chapter 2

Photonic Crystal Waveguides

2.1 Introduction

A photonic crystal is a material that exhibits a photonic bandgap, a range of frequencies of the electromagnetic spectrum for which propagation through the material is forbidden. The name photonic band gap and much of the nomenclature of photonic crystals borrows from and is analogous of the electron band gaps that occur naturally in atomic crystals. The existence of photonic crystals opened up a new paradigm in the study of light-matter interaction as well as a new method of optical confinement. Since photonic crystals limits the propagation of certain frequencies it can be used to great effect to spatially limit optical modes creating guided wave structures such as waveguides and localized modes such as cavities.

The goal of this chapter is to introduce the concept of the two-dimensional slab photonic crystal waveguide. These devices, owing to their photonic crystal confinement of light, exhibit unique propagation properties including tight optical confinement and very large optical dispersion. The design and fabrication by deep-UV photolithography will be described and their subsequent optical characterization. This chapter will serve as informative background for later in the thesis where the unique properties of the photonic crystal waveguides will be used to explore nonlinear optics.
2.2 Electromagnetic Theory

This thesis will make references to dispersion and optical bandstructures and so it is worth introducing how this information is obtained.

2.2.1 Maxwell’s Equations

Our goal is to understand how electromagnetic fields will distribute themselves in particular dielectric distributions as well to determine the dispersion relation that exists for waves in these distributions. Note that when we refer to dispersion we are referring to the relationship between wavevector and frequency as opposed to chromatic dispersion which is the derivatives of this relationship.

The electromagnetic fields in a dielectric medium are described by Maxwell’s equations:

\begin{align}
\nabla \cdot \mathbf{D} &= \rho \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 \\
\n\n\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}
\end{align}

Where \( \mathbf{E} \) is the electric field, \( \mathbf{H} \) is the magnetic field, \( \mathbf{D} \) is the electric displacement field, and \( \mathbf{B} \) is the magnetic displacement field. In our work we are considering sourceless mediums so the free charge and current densities are equal to zero (i.e. \( \rho = 0 \) and \( \mathbf{J} = 0 \)). The explicit spatial and temporal dependence of the fields is left out for clarity (e.g. \( \mathbf{E} \equiv \mathbf{E}(\mathbf{r}, t) \), where \( \mathbf{r} \) is a position vector in space).

If we restrict ourselves to isotropic and lossless materials then we can write the constitutive relations that relate \( \mathbf{D} \) to \( \mathbf{E} \) and \( \mathbf{B} \) to \( \mathbf{H} \) as
\[ D = \varepsilon_0 \varepsilon(r) E \]  
\[ B = \mu_0 \mu(r) H \]  

(2.2a)  
(2.2b)

where \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space respectively. The spatial dependence of the medium’s permittivity and permeability is explicitly stated in Eqn. 2.2 by \( \varepsilon(r) \) and \( \mu(r) \). The dielectric materials we will study are non-magnetic and so \( \mu(r) \approx 1 \). The relative permittivity is assumed to be independent of frequency \( (\varepsilon(r, \omega) = \varepsilon(r)) \) ignoring material dispersion. Any calculations will use the permittivity for the particular frequency range of interest.

2.2.2 Numerical Methods

2.2.2.1 Finite Difference

Given a particular permittivity distribution in space \( (\varepsilon(r)) \) our goal is to solve Eqn. 2.1 in order to discern the resulting mode field profiles as well as the dispersion relation for these modes. A common numerical technique to achieve this is the Finite-Difference Time-Domain method\[42\] which solves Eqn. 2.1 on a discrete grid space. A more detailed description of the FDTD is beyond the scope of this thesis and can be found elsewhere\[43\]. Though it is highly accurate and can be used to compute the dispersion of guided modes in photonic crystal structures it is typically more time efficient to utilize the plane-wave expansion method.

2.2.2.2 Plane Wave Expansion

It is possible to express Eqn. 2.1 as a linear eigenproblem if we first isolate the \( H \) field:

\[ \nabla \times \frac{1}{\varepsilon(r)} \nabla \times H = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} H \]  

(2.3)
\[ \nabla \cdot \mathbf{H} = 0 \quad (2.4) \]

where \( c = 1/\sqrt{\varepsilon_0 \mu_0} \) is the speed of light in vacuum. If we limit ourselves to solving for periodic modes with real frequencies, \( H \) can be defined as

\[ H(r) = e^{i(k \cdot r - \omega t)} H_k(r) \quad (2.5) \]

where \( k \) is the Bloch wavevector of the periodic Bloch mode \( (H_k(r)) \). We can then write Eqn. 2.3 in the form of a linear eigenproblem:

\[ \hat{A}_k H_k = \left( \frac{\omega}{c} \right)^2 H_k \quad (2.6) \]

where \( \hat{A}_k \) is the Hermitian operator defined as

\[ \hat{A}_k = (\nabla + i k) \times \frac{1}{\varepsilon(r)} (\nabla + i k) \times \quad (2.7) \]

The result of Eqn. 2.7 is an eigenproblem that can be solved much more quickly than finite difference methods. The solution of Eqn. 2.7 is a discrete set of eigenfrequencies of the form \( \omega(k) \) which is the dispersion relation of the mode (sometimes referred to as the bandstructure).

The limitations of this method is that according to Eqn. 2.5 the modes are periodic and therefore the solution space defined by \( \varepsilon(r) \) must also be periodic in space. For bulk photonic crystals that exhibit translational symmetry this is to be expected but for structures that break the translational symmetry such as slab photonic crystals or photonic crystal waveguides a supercell must be used. The supercell must be chosen to be large enough that the calculated modes decay sufficiently so as not to interfere with each other. This typically is not an issue since the modes are highly confined such as within the slab in the case of slab photonic
crystal waveguides.

Throughout this thesis plane-wave expansion is used to calculate the bandstructure of photonic crystal waveguides and this is done using the MIT Photonic Bands[44] software package.

2.3 Two Dimensional Photonic Crystal Slabs

The practical realization of photonic bandgap materials is a daunting task. The creation of full three-dimensional photonic band gap requires the periodic modulation in refractive index in three dimensional space on the spatial resolution of wavelength. A number of unique fabrication methods have been used to create three-dimensional photonic band gap materials. One of the first demonstrations of a photonic band gap material at microwave frequencies was done by drilling into a dielectric medium[45]. The utilization of a drill is most definitely not practical for optical frequencies therefore alternative structures that are more easily fabricated are required.

A structure that is substantially simpler to fabricate and possesses a photonic band gap is the photonic crystal slab as shown in Fig. 2.1(a). This structure consists of a dielectric slab (refractive index \( n_{\text{slab}} \)) surrounded by a background material (refractive index \( n_{\text{air}} = 1 \)) that is patterned with a periodic hexagonal array of holes. The periodic lattice of holes is defined by two key physical properties: the lattice constant \( (a) \) and the radius of the holes \( (r) \). This structure supports guided modes which are confined to the slab via total internal refraction. The dispersion of these modes can be seen in the projected band structure shown in Fig. 2.1(b-c). The bandstructure is referred to as projected owing to the fact that the full wavevector, \( \mathbf{k} = (k_x, k_y, k_z) \), is projected into the plane of the photonic crystal therefore the wavevector of the band structure is the in-plane wavevector \( \mathbf{k}_\parallel = (k_x, k_y) \)[46, p. 136].

From the bandstructures (Fig. 2.1(b-c)) it can be seen that the photonic crystal slab possess a photonic band gap for the TE-like polarization while no such gap exists for the TM-like polarization. The bandstructures also illustrate the so called light-cone of
the slab by the shaded region surrounding the dispersion of the slabs guided modes. The light-cone represents the infinite continuum of radiation modes that exist in the background medium[47]. The light-cone is defined by the relation

\[ \omega = \frac{c}{n} k \]  

(2.8)

where \( n \) is the index of refraction of the background medium. Below the light cone, the modes of the slab cannot couple with the modes of the radiation continuum and inside the light cone the guided modes can couple to the radiation continuum and become highly leaky.

**Figure 2.1:** Two-dimensional photonic crystal slabs. (a) Hexagonal lattice two dimensional photonic crystal slab illustrating important physical dimensions (a : lattice constant, r : hole radius, h : slab thickness) (b) Brillouin zone of the hexagonal lattice and the irreducible Brillouin zone (defined by the symmetry points \( \Gamma \), M, and K) (c) Bandstructure for TE-like polarization (d) Bandstructure for TM-like polarization
2.4 Single Line Defects in Photonic Crystal Slabs

It is possible to create a guided mode defect in a photonic crystal slab by removing a single line of holes from the bulk photonic crystal lattice. We choose to examine a single line of holes removed in the $\Gamma - K$ direction and will do so for the totality of this work. This direction is chosen over the $\Gamma - M$ direction owing to the $\Gamma - K$ defect width more closely matches the width of conventional channel waveguides used for coupling into and out of fabricated structures.

The removal of a single line of holes in the $\Gamma - K$ from the bulk photonic crystal breaks the translational symmetry of the bulk crystal. To examine the modes of the resulting structure we can calculate the projected band-structure in which the calculated wavevector is now projected along the direction of the defect (the $\Gamma - K$ direction). The projected bandstructure of such a photonic crystal waveguide can be viewed in Fig. 2.2. It is common in literature for such a structure to be referred to as a "W1" waveguide indicating a single line defect as opposed to a "W3" waveguide which would be the removal of 3 rows of holes.

The projected bandstructure for the TE polarization of the W1 photonic crystal waveguide (See Fig. 2.2) exhibits a number of notable features. Within the photonic band gap of the bulk slab photonic crystal there exists two confined modes. One mode exhibits an even field parity with respect to the plane of symmetry while the other an odd parity which can be view from looking at the calculated mode profiles. Both these modes exhibit localized guided modes for a finite bandwidth under the light-line of the photonic crystal slab. As the modes approaches the edge of the Brillouin zone (at $k = 0.5$) the mode dispersion flattens. This region is typically referred to as the region of slow-light. A detailed plot of a waveguides TE even mode dispersion can be view in Fig. 2.3 which demonstrates how the group index increases asymptotically as the frequency approaches mode cutoff.

Even though the slab photonic crystal does not exhibit a photonic bandgap for the TM polarization, as can be seen in Fig. 2.2, the W1 waveguide still supports guided confined TM polarized modes which are strictly index guided. These modes exhibit a small stop band due to the folding of the fundamental mode at the Brillouin zone edge[48].
Figure 2.2: Projected Bandstructure of a W1 photonic crystal waveguide. The projected bandstructures of a W1 photonic crystal waveguide and the field distribution for bands of interest for (top) TE-like polarization and (bottom) TM-like polarization

2.5 Design of Photonic Crystal Waveguides

The design space of photonic crystal waveguides is dictated by a select set of physical dimensions, namely the photonic crystal lattice constant \(a\), the hole radius \(r\), and the slab height \(h\). These three design variables determine optical transmission properties of the resulting fabricated structures.

In order to accurately explore this design space we calculated the photonic bandstructures
of photonic crystal waveguides with dimensions that span the space. In other words we calculated a series of photonic bandstructures for various radii and various slab heights. We then plot the pertinent features with respect to the independent variable.

The result of these simulations for the case where the independent variable is the hole radius and slab height is kept constant can be view in Fig. 2.4. Key understandings of how spectral features shift with respect to radius can be understood from this plot. Particularly it can be seen that radius greater than 0.36a result in no transmission for the fundamental mode owing to the fact that it overlaps with the lower slab continuum band edge. It can also be seen that the TM spectral features have a much weaker dependence on on the hole radius than the TE features which is to be expected.

If the hole radius is kept fixed and the the slab height is varied the resulting shift in spectral features can be view in Fig. 2.5. In comparing Fig. 2.4 and Fig. 2.5 it can be seen that the TE features show much less dependence on the slab height than the TM spectral features.
Figure 2.4: Design space for silicon W1 photonic crystal waveguides (fixed slab thickness). Important spectral feature dependence on hole radius for a fixed slab thickness of 0.6a. The wavelength scale relates to an arbitrary lattice constant value of 420 nm.

With the design space characterized via numerical simulation it is now possible to design a photonic crystal waveguide which exhibits a particular spectral feature at a desired wavelength. This is important for experiments that require the slow-light region to exist within a specific bandwidth, such as that of a light source, for a particular application.

2.6 Layout and Fabrication

Once the design parameters for the required devices had been determined the device layout could then be created. The majority of the devices created for this thesis were laid out using Virtuoso from Cadence Design Systems. The Virtuoso platform is a full electronic design automation package meant for integrated circuit creation. We utilized only its layout functionality to create proper hierarchical designs that could be exported to the GDSII/OASIS format that our collaborators required.
Figure 2.5: Design space for silicon W1 photonic crystal waveguides (fixed hole radius). Important spectral feature dependence on slab thickness for a fixed hole radius of 0.3a. The wavelength scale relates to an arbitrary lattice constant value of 420 nm.

The layout process involved all the respective researchers in the Optical Nanostructures Laboratory dividing up the available fabrication area that our collaborators at IME had allocated to us. Once the area available for this work had been decided a spreadsheet was used to catalog all the devices that would be included in the layout, including any test structures. The devices were divided into groups with unique alphanumeric identifiers that were included in the subsequent layout that for easy device identification while under test.

A typical layout could consist of over 200 different photonic crystal waveguides designs which would then be composed of over 10 million individual holes. With that fact in mind the layout of devices was automated by utilizing a series of scripts, written in the Cadence SKILL scripting language, that would hierarchically create the desired layout from a device definition file. The definition file would be a list of the desired devices (PhCWG, channel waveguide, photonic crystal cavity designs, etc.) and the desired parameters (lattice constant, hole radius, length, etc.) This parametric method of device layout greatly reduced time
required to layout devices while being much less prone to error.

![Figure 2.6: Silicon-on-Insulator Integrated Photonic Chiplet Layout.](image)

An example of the full layout of the chiplet on which the photonic crystal waveguide devices are just one of many integrated photonic devices fabricated for investigation.

Once a designed layout had been created it was combined with other group members to create a total chiplet layout (See Fig. 2.6). This chiplet layout would then be shared with our collaborators at the Institute of Microelectronics in Singapore who would fabricate a photolithography mask based on our layout and go on to use deep-UV photolithography stepper to populate a 8-inch SOI wafer with our chiplet designs. For our fabricated devices a typical SOI wafer would have a 3 µm buried oxide layer with a 250 nm thick silicon device layer.

Upon delivery of a fabricated wafer from our collaborators it was inspected with an optical microscope (See Fig. 2.7) and scanning electron microscope to characterize the fabricated dimensions and quality. A chiplet was then selected (the wafer would arrive diced in a carrier from our collaborators) for further processing. First the selected chiplet was examined by optical microscope to ensure the devices of interest were not damaged. Once it was confirmed the devices of interest were intact the air-bridge fabrication process was then performed on the chip.

The goal of the air-bridge process was to remove the silicon oxide beneath select photonic
An optical microscope image of a fabricated chiplet containing photonic crystal waveguides. Photonic crystal waveguides of various lengths can be seen.

crystal structures on the chiplet. The first step of the air-bridge process was to thoroughly clean the chiplet using solvents (Acetone and Isopropyl alcohol) in an ultrasonic bath. The chip was then dehydrated on a hotplate set at 200°C for 10 minutes. Next the chiplet was first spin coated with hexamethyldisilazane (HMDS) and then coated with a photoresist (AZ4620). The HMDS acts as an adhesion promoter for the photoresist. An initial soft-bake was then performed on the hotplate to prepare the photoresist for exposure.

The photoresist covered chiplet was then exposed using a contact aligner photolithography system using a mask that had been designed to expose only the photonic crystal regions of the chiplet (See Fig. 2.8). After exposure the photoresist was developed using AZ400K developer and then hard baked on a hotplate. The chiplet was then etched using a hydrofluoric buffered oxide enchant (6:1 concentration). After etching the remaining photoresist was stripped using acetone and photoresist remover (AZ100). Care was taken post-etch to keep the chiplet submerged in liquid at all times due to the fragility of the air-bridged structures.

A final cleaning of the chiplet was then performed using a piranha clean procedure which consisted of 3:1 ratio of sulfuric acid to 30% hydrogen peroxide. After the final
cleaning the chiplet would be transferred to a beaker containing methanol and heated on a hotplate at 100°C until the methanol had evaporated. Methanol is specifically chosen due to its low surface tension and thus its evaporation would not put undue stress on the air-bridged photonic crystal regions. Early fabrication procedures utilized a supercritical drying procedure however this was found to be unnecessary for our devices and in fact the use of methanol resulted in cleaner devices owing to unknown contamination that could reside in the supercritical dyer. Scanning electron micrographs of our fabricated air-bridge photonic crystal waveguide can be seen in Fig. 2.9.

The final step in the fabrication process is the cleaving procedure. The chiplet would first be scribed either by hand using a diamond tipped scriber or by using a wafer scriber (Suss RA120M). Cleaving would then be done by hand by applying small amounts of force to the chiplet to cleave along the scribed line which resulted in high-quality optical facets.
Figure 2.9: Scanning Electron Microscope Images of Fabricated Photonic Crystal Waveguides. Scanning Electron Microscope images taken of a waveguide following the air bridge fabrication process. Note that this particular chiplet is from a process that included a cladding oxide as can be observed in the top most image.

2.7 Experimental Characterization

Once a chiplet had been fabricated, air-bridged, and cleaved it would then be mounted into an optical characterization setup. Mounting the chip onto an alignment stage (561D-YZ,
Newport Inc.) could be done a number of ways depending on the application. If quick mounting and dismounting was required double sided scotch tape could be used and if rigidity in mounting was required heat sensitive adhesives like Crystalbond could be used. Typically a mechanical method of mounting would be used (screw down clips) would be used because they were found to be athermal when compared to the previous two methods.

Once mounted light was coupled into the chip via aspheric lens (NA=0.61) mounted on precision stages (561D-XYZ, Newport Inc.). Light was collimated from fiber into the coupling aspherics and back out via fiber collimators. This free space method of coupling, in comparison to lensed fiber coupling, allowed for quick changes in incident polarization. It was also found to be a more robust method of coupling at high input powers which would typically cause lensed fibers to shift and therefore lose coupling.

### 2.8 Photonic Crystal Waveguide Transmission

The transmission of a series of fabricated photonic crystal waveguides can be seen in Fig. 2.10. These measurements were taken using a supercontinuum light-source which is a broadband lightsource generated by passing a Nd:YAG 1064 nm laser nanosecond pulse through a photonic crystal fiber. The transmission through the photonic crystal waveguide was then measured on an optical spectrum analyzer (Advantest Q8384).

The transmission measurements for the TE polarization in Fig. 2.10 show the distinctive mode onset edge at high wavelengths and waveguide transmission bandwidth limited by the intersection with the light line at lower wavelengths. With increasing hole radius it can be observed that the slow-light region of the mode onset shifts to lower wavelengths and that the bandwidth of the waveguide shrinks until finally for the $r = 0.38a$ case no TE transmission is present.

In the case of the TM polarization in Fig. 2.10 the identifiable stop gap of the index guided mode can be observed. The stop gap shifts to lower wavelengths with increasing hole radius and can be observed to shift with a different rate than that of the TE spectral
Figure 2.10: Measured transmission of W1 photonic crystal waveguides. The measured spectral transmission of for various hole sizes for both TE (left, blue) and TM (right, red) polarizations. Lattice constant was 440 nm and device length was 1.5 mm.

To explore fine grain control of the spectral location of the slow-light region of the photonic crystal waveguide we examined a the transmission of a series of waveguides that only differed in lattice constants. The TE transmission of this series of waveguides can be examined in Fig. 2.11.
Figure 2.11: Measured transmission cut-off wavelength of W1 photonic crystal waveguides. (left) The measured spectral transmission (offset for clarity) near the cut-off wavelength of the W1 photonic crystal waveguide for varying lattice constant (400 nm, 403 nm, 405 nm, 407 nm, 409 nm, 411 nm, 413 nm, 415 nm, 417 nm, 419 nm, 421 nm, 423 nm, 425 nm, 427 nm, 429 nm). (right) Measured cut-off wavelength versus designed lattice constant.

It must be noted that a shift in fabricated lattice constant is also equivalent to a shift in normalized slab thickness since the slab thickness remains constant. The measured transmissions shown in Fig. 2.11 show that the spectral features of the photonic crystal waveguides, specifically the region of slow-light, can be controlled on a fine grain level allowing for positioning of the slow-light region at specific wavelengths. The importance of this will become apparent in future chapters where experiments using erbium doped fiber amplifiers will be performed.

2.9 Conclusion

The goal of this chapter was to introduce photonic crystal waveguides and why one would be interested in studying such a structure. We outlined the design space consisting of lattice
constant, hole radius, and slab thickness which made it possible to fabricate waveguides that exhibit transmission in our bandwidth of interest. Our fabricated procedure was summarized along with a simple discussion and examination of the transmission of fabricated devices. Note that we have talked about the slow-light that these devices exhibit but have not discussed how we would go about experimentally measuring this property of the waveguide. That is the topic of the next chapter.
Chapter 3

Methods of Dispersion Measurement

3.1 Introduction

The measurement of optical dispersion is a broad field important in a number of applications. Chromatic dispersion plays an important factor in optical communication systems and so most work in this area relates to the measurement of optical fibers and most recently the characterization of dispersion in integrated optical devices.

The concept of slow-light was introduced in Chapter 2 in the context of photonic crystal waveguides and it is the goal of this chapter to introduce the methods used to experimentally measure this phenomenon in real devices. The methods demonstrated in this chapter are the ones used in this work and is not meant to be an exhaustive exploration of the field.

3.2 Optical Dispersion

The dispersion relation \( k(\omega) \) was introduced in Chapter 2 with reference to guided modes of an optical structure. Note here we use the nomenclature that is common with regard to photonic crystals where propagation constant and wavevector are defined as \( k \), this is in contrast to the nomenclature of optical fibers where this term, often referred to as the mode propagation constant, is represented by \( \beta(\omega) \). Here we will define various parameters
that can be extracted from the dispersion relation including group index and group velocity
dispersion (GVD). The dispersion relation can be expanded into a Taylor series expansion
to give:

\[ k(\omega) = k_0 + \frac{\partial k}{\partial \omega} (\omega - \omega_0) + \frac{1}{2} \frac{\partial^2 k}{\partial \omega^2} (\omega - \omega_0)^2 + \frac{1}{6} \frac{\partial^3 k}{\partial \omega^3} (\omega - \omega_0)^3 + \ldots \]  (3.1)

This expansion allows us to define the following terms

\[ k' = \frac{\partial k}{\partial \omega} = \frac{1}{v_g} \]  (3.2)

\[ k'' = \frac{\partial^2 k}{\partial \omega^2} = \frac{\partial}{\partial \omega} \left( \frac{1}{v_g} \right) \]  (3.3)

we can then define group index as

\[ n_g = \frac{c}{v_g} = n + \omega \frac{\partial n}{\partial \omega} \]  (3.4)

and group velocity dispersion (GVD) as

\[ GVD = \frac{1}{c} \left( 2 \frac{\partial n}{\partial \omega} + \omega \frac{\partial^2 n}{\partial \omega^2} \right). \]  (3.5)

Higher order dispersion parameters can be calculated from higher order terms of Eqn. 3.1
such as third order dispersion and fourth order dispersion\[49\].

A number of dispersion measurement methods actually measure the group delay \( (\tau_g) \)
through the medium being characterized. The group index can then be determined from the

group delay from the following equation:

\[ n_g = \frac{c \tau_g}{L} \]  (3.6)

where L is the length of the medium over which the delay was measured.
3.3 Fabry-Perot Method

The Fabry-Perot method of dispersion characterization makes use of the typically undesired modulation of the transmission of an optical system caused by optical interfaces. In the example of an integrated optical device such as a waveguide, the facets of the waveguide present a significant change in refractive index from which the reflectivity creates an undesired Fabry-Perot cavity.

The optical transmission of a Fabry-Perot cavity with respect to wavelength presents a periodic modulation that is dependent on the length of the cavity and the group index within the cavity. The periodic modulation is due to the resonant wavelengths of the cavity and the wavelength separation ($\Delta \lambda$) between adjacent resonances is called the free spectral range (FSR) and is approximately given by the equation:

$$\Delta \lambda = \frac{\lambda_0^2}{2n_g L}$$  \hspace{1cm} (3.7)

where $L$ is the length of the cavity. In the case of a waveguide, the length would be the distance between the input and output facet. With this equation it is possible to derive the group index of a device solely from the measured transmission.

An experimentally measured transmission of a photonic crystal waveguide can be seen in Fig. 3.1(a). This waveguide has a length of 100$a$ ($a = 440$ nm, $L = 44$ $\mu$m) and exhibits pronounced oscillation in its transmission from the photonic crystal:channel waveguide interface. The spectral features of the Fabry-Perot oscillations have been identified on the transmission in Fig. 3.1(a) and from these features the group index was calculated using Eqn. 3.7 and can be seen in Fig. 3.1(b).

The benefit of the Fabry-Perot method of group index measurement is that it can be done solely from the transmission measurement data of the device under test. This simplicity however comes at the cost that the resolution of the group index data is tied to the length of the device. It is typical in device design that efforts are made to reduce abrupt changes refractive index in order to avoid the creation of fringes in the transmission. These design
Figure 3.1: **Group index derived from Fabry-Perot oscillations.** (a) Measured transmission of a W1 photonic crystal waveguide with peak and troughs of Fabry-Perot oscillations marked. (b) Derived group index from Fabry-Perot oscillations.

efforts make the feature identification required by this method difficult.

An additional drawback of the Fabry-Perot method not illustrated in the example shown is the convolution of multiple Fabry-Perot responses that occurs in the presence of multiple interfaces. In the example demonstrated in Fig. 3.1 the photonic crystal device length was 44 µm which is much less than the total device length (∼3 mm). This large difference in lengths allows for easy identification of the Fabry-Perot response of the photonic crystal waveguide over those of those of the Fabry-Perot cavity created by the coupling channel waveguide facets. This limits the Fabry-Perot method to specific device geometries that lend themselves to easy feature identification in their transmission.

### 3.4 Phase Shift Method

The phase shift method (PSM) of dispersion measurement[50] is widely used in the characterization of optical fiber chromatic dispersion. A diagram of the experimental setup used in PSM is illustrated in Fig. 3.2. A tunable optical source, typically a tunable semiconductor laser, is modulated by an electro-optical modulator at a high frequency and then interrogates a device under test. The resulting phase shift between the transmitted optical signal detected by the photodetector and the modulating signal is can be measured using high-speed
sampling oscilloscope or vector network analyzer (VNA).

The temporal delay caused by the device under test is given by the equation:

\[
\tau = \frac{\Delta \phi}{360^\circ f_m}
\]  

(3.8)

where \(\Delta \phi\) is the measured phase difference and \(f_m\) is the frequency of modulation. Using the temporal delay from Eqn. 3.6 and the device length it is possible to calculate the group index.

**Figure 3.2: Experimental setup for phase delay measurement.** A tunable laser source is modulated at a frequency \(f_m\) using an electro-optical modulator (EOM) and transmits through the device under test (DUT). A high-speed photodetector is used to detect the transmitted signal and a high-speed sampling oscilloscope is used to measure the phase difference between the transmitted signal and modulating signal.

The group index of a photonic crystal waveguide measured using the PSM is shown in Fig. 3.3. In this experiment, a tunable laser (ANDO AQ4321A) was modulated by a lithium-niobate electro-optical modulator (JDSU-10Gb/s). The modulator was driven by an modulator driver amplifier (JDSU H301) at 5 GHz from a signal generator (Gigatronics 1026). The light was coupled into the photonic crystal waveguide and the transmitted light was collected back into fiber and detected using a high-speed photodetector (New Focus 1544-A). The modulating signal and photodetector signal were measured on a high speed sampling oscilloscope (Hewlett-Packard 11801C, SD-26 Sampling Head).

The phase shift method allows for direct measurement of the optical group delay of the
Figure 3.3: Example of group index measurement using the phase delay method. The group index (markers) of a photonic crystal waveguide measured using the phase-delay method and the device transmission (solid line).

device under test and therefore is a very versatile method. It is best suited for measuring the dispersion of optically broadband devices and physically long devices. The act of modulating the interrogating light creates modulation side-bands that create uncertainty in the actual wavelength at which the group delay was measured which can be detrimental in the characterization of narrow band optical features such as narrow resonance features or the mode onset of a photonic crystal waveguide.

Integrated optical devices also can exhibit rapid changes in delay and amplitude with respect to wavelength and since the phase-shift method attempts to recover the optical phase response by measuring the derivative of the phase response (group delay) the phase response is recovered by integration[51]. Another difficulty with the phase shift method is the limited dynamic range that can occur due to the need for high-speed photodetectors. This can make measurement of features such as the mode onset of a photonic crystal waveguide challenging.
and might require the use of optical amplifiers to overcome coupling losses.

### 3.5 Low Coherence Interferometry Method

Optical low coherence interferometry\[52\] is a method to deduce the optical impulse response of a system, both the amplitude and phase, and therefore can be used to measure the dispersion of an optical device. This is accomplished by using an interferometer, see Fig. 3.4, in which one arm contains the device under test and a second arm contains variable optical delay line. When this interferometer is illuminated using a low-coherence length sources interference within the interferometer will only occur on delays on the scale of this coherence length. As the delay is varied, the interferogram captured by the photodetector at the output of the interferometer contains information on the complex optical response of the device under test.

The signal detected by the photodetector \((U)\) with respect to the variable optical delay \((\tau)\) can be simplified to:

\[
U(\tau) \propto \text{Re} \{ \mathcal{F}^{-1} [S(\omega) \ast H(\omega)] \} = \text{Re} \{ s(\tau)h(\tau) \} \tag{3.9}
\]

where \(\mathcal{F} \{s(\tau)\} = S(\omega)\) is the power spectral density of the optical source and \(\mathcal{F} \{h(\tau)\} = H(\omega)\) is the field transfer function of the DUT (\(\mathcal{F}\) and \(\mathcal{F}^{-1}\) represent the Fourier and inverse Fourier transforms respectively). From the acquired interferogram signal, \(H(\tau)\), the transmission amplitude and phase can be derived:

\[
|H| = |\mathcal{F} \{U(\tau)\}| \tag{3.10a}
\]

\[
\phi = \text{Arg} (\mathcal{F} \{U(\tau)\}) \tag{3.10b}
\]

In order for the Fourier transforms to be performed in Eqn. 3.10 the acquired signal must be sampled uniformly with respect to the optical delay. In practice this is achieved by
using a second auxiliary Michelson interferometer to accurately track the optical delay path.

In a practical implementation of low coherence interferometry please refer to Fig. 3.4. The low-coherence source is an amplified spontaneous emission (ASE) source (Thorlabs ASE-FL7002) with emission from 1530-1610 nm. The source is split by a 50/50 fiber coupler into two arms of a Mach-Zehnder interferometer. The first arm, the measurement arm, contains the device under test and the corresponding coupling optics. The second arm, the delay arm, contains a free space optical delay line consisting of a retroreflector (HM-10-1, PLX Inc.) mounted on a precision translation stage (ALS130-150, Aerotech Inc.). The two arms are then combined by a second 50/50 fiber coupler and the resulting signal is detected by an InGaAs photodetector (PDA10CS, Thorlabs Inc.). The auxiliary interferometer consists of a plane mirror mounted on the optical delay stage and a polarization beam splitter, corner cube reflectors, and quarter wave plate assembly (PMA, Zygo Inc.). The use of the Zygo PMI allows for a two-pass Michelson interferometer which doubles the sampling resolution. The auxiliary interferometer uses a stabilized helium neon laser (5517B, Hewlett-Packard) and a silicon photoreceiver(10780A, Hewlett-Packard). Data acquisition is accomplished using a digital acquisition system (PCI-6132, National Instruments) triggered by the output of the auxiliary interferometer photodetector.

An example of the raw acquired interferogram from the low-coherence interferometer system can be seen in Fig. 3.5. The corresponding derived transmission amplitude and derived phase delay can be view in Fig. 3.6. The limitations of the OLCI method are coupled to the quality of the auxiliary interferometer[53], the resolution of which determines the resolution of the underlying derived data. As well the auxiliary interferometer can be significant source of noise from the mechanical translation of the delay line and ideally a configuration in which it traces the same optical path as the measurement interferometer should be used[54]. This is unfortunately not ideal for narrow bandwidth optical devices such as photonic crystal waveguides.
Figure 3.4: Experimental setup for low-coherence interferometry measurement. The device under test (DUT) is placed in one arm of a Mach-Zehnder interferometer. The other arm consists of a variable optical delay line which is interrogated by an auxiliary interferometer which utilizes a stabilized helium neon (HeNe) laser to generate a trigger for acquisition of the characterization interferogram. An amplified spontaneous emission (ASE) source is used as a low-coherence source in the characterization interferometer.

Figure 3.5: Example of a LCI acquired interferogram. An interferogram acquired from a photonic crystal waveguide using the apparatus in Fig. 3.4.
Figure 3.6: Optical transmission and delay of a photonic crystal waveguide measured using low coherence interferometry. (a) Transmission and (b) Optical Delay of a photonic crystal waveguide measured using the apparatus in Fig. 3.4.

3.6 Swept Wavelength Interferometry Method

The swept wavelength interferometric (SWI)[55][56] method of dispersion measurement is, as the name implies, another interferometer based measurement method similar to the optical low-coherence method. Like the OLCI method, the SWI method places the device to be measured within a Mach-Zehnder interferometer however in contrast to the OLCI the interferometer delay is constant. In order to characterize the interferometer a sweeping mode-hop free tunable laser source is utilized to interrogate the characterization interferometer with a continuous optical frequency sweep.

If we consider the tunable laser sweeping its optical frequency in time by a rate of $\gamma$ we can define the electric field of the laser as

$$E(t) = E_0 e^{2\pi j[\omega_0 + (\gamma/2)t]t} \hat{n}$$  \hspace{1cm} (3.11)
where $E_0$ is a constant amplitude and $\omega(t) = \omega_0 + \gamma t$ is the instantaneous optical frequency of the laser at time $t$, and $\hat{n}$ is a unit polarization vector. We can then consider the output of the Mach-Zehnder interferometer as

$$U(\omega) = U_0 \left[ 1 + |H(\omega)|^2 + 2 \Re \{ H(\omega) \} \cos(2\pi \omega \tau_0 + \psi) \right] \quad (3.12)$$

where $U_0$ is equal to $\rho E_0^2$ (the constant $\rho$ depends on photodetector sensitivity), $\tau_0$ is the group delay difference between the two interferometer paths, $\psi$ is a constant phase factor, and $H(\omega)$ is the scalar transfer function of the device under test in the measurement arm of the interferometer. Note that Eqn. 3.12 is defined not in time but in instantaneous optical frequency however it is an acquired signal in time. In order to accomplish this the photodetector signal is sampled by equal intervals of optical frequency.

It can be observed in Eqn. 3.12 that the desired device transfer function makes up an AC component of the acquired signal. In order to extract it from $U(\omega)$ the signal is transformed to the Fourier domain and a digital window filter function is applied to isolate only the single component of the cosine term. When the filtered data is transferred back via the inverse Fourier transform the result is a complex function of optical frequency that is equal to the transfer function with a constant amplitude and phase factor term. These constant terms can be removed by measuring the system transfer function, without the DUT present, and be calibrated out.

Our practical implementation of a swept-wavelength interferometry system can be seen in Fig. 3.7. The tunable laser source (AQ4321A, Ando Electric Co.) is split into three separate paths. The first path consists of the measurement interferometer which contains the DUT, the second path is the auxiliary interferometer which provides an optical frequency clock that triggers data acquisition, and lastly the third path contains a fiber coupled hydrogen cyanide gas cell (HCN-13, Wavelength References).

The measurement interferometer makes use of a custom built balanced photodetector (See Appendix B) which utilizes low-back reflection fiber coupled photodiodes. The system
Figure 3.7: Experimental setup for swept wavelength interferometry characterization. A tunable laser source (TLS) is split into three paths: the top path consists of the device under test (DUT) within a characterization interferometer, the middle path is an auxiliary interferometer which is used to produce a an acquisition clock, and a the bottom path interrogates a hydrogen cyanide (HCN gas cell for absolute wavelength calibration. A variable delay ($\tau$) is used to accurately align the acquisition clock to the transmission of the two other paths.

noise is increased by unwanted internal optical reflections within the system and so care was taken in construction to limit optical interfaces by splicing all possible fiber connections and using angled fiber connectors when splicing was not possible.

The transmission of the hydrogen cyanide gas cell can be seen in Fig. 3.8. The absorption lines provide 51 calibrated reference wavelength points (See Appendix A) that are measured with every sweep. During data processing each absorption feature is fit to achieve a subsample absolute wavelength reference for the interferogram data. For each measurement the gas cell transmission and interferogram data are collected simultaneously (See Fig. 3.9) sampled at equal intervals of optical frequency determined by the auxiliary interferometer.

The Fourier transform of the interferogram is shown in Fig. 3.10(a) as well as the filter window used to extract the complex transfer function. The computed insertion loss of the
Figure 3.8: Hydrogen cyanide gas cell transmission. The transmission of the hydrogen cyanide gas cell with the absorption features used for absolute wavelength calibration identified.

Figure 3.9: Raw data acquired from swept wavelength interferometry. (a) An example of the raw acquired signals from the digital acquisition system of the captured characterization interferogram and gas cell transmission. (b) Detail of acquired data showing the interferogram spacing.

3.7 Conclusion

In this chapter we have demonstrated four methods of measuring the slow-light properties of photonic crystal waveguides: the Fabry-Perot Method, the Phase-Shift Method, Optical
Figure 3.10: Photonic crystal waveguide transmission and group delay measured using swept wavelength interferometry. (a) The Fourier transform of the acquired interferogram with the complex optical response window highlighted. The corresponding (b) transmission and (c) group delay calculated from the complex optical response.

Low-Coherence Interferometry, and Swept Wavelength Interferometry.

The Fabry-Perot Method is the simplest method demonstrated since it requires no other data except the optical transmission data. This simplicity however limits this method only to devices of certain dimensions and that exhibit strong undesirable Fabry-Perot oscillations.

The Phase Shift Method is often used for dispersion measurement of optical fibers and is a well established method. The apparatus required to perform this measurement method is relatively simple. However, this method is not ideally suited for the measurement of integrated devices which typically have optical delays that changes quickly with respect to wavelength and fairly short in length and so exhibit small delays.
The Optical Low-Coherence Interferometry method is the first of two interferometric methods demonstrated. Interferometric methods allow for direct measurement of the optical transfer function from which the group delay can be calculated from. OLCI only requires low cost broadband source making it suitable for situations where no suitable for wavelength regions where tunable laser is available. The simple source requirements however are offset by the requirement of a high-precision optical delay line. The wavelength resolution and accuracy of this method is fundamentally tied to the precision of the auxiliary interferometer which can be difficult to improve upon.

The final method discussed was Swept Wavelength Interferometry which like OLCI is an interferometric method that directly measures the optical transfer function of the device under test. SWI trades the precision delay line requirement of OLCI for a requirement of a mode hop free swept tunable laser source. This limits this methods to those wavelengths for which such lasers can be procured however it is found that this method provides high resolution in both wavelength and group delay. With the addition of concurrent acquisition of the transmission of a calibrated gascell the SWI method can also achieve very good absolute wavelength accuracy. The absolute wavelength accuracy also allows for accurate alignment of successive measurements which makes it possible to further improve the signal-to-noise ratio of the method through averaging.

Each of the methods described in this chapter offer their own benefits and challenges. Typically the method used is determined by what equipment is available, most significantly the availability of applicable optical sources, and the trade off between resolution and experimental complexity.
Chapter 4

Raman Scattering in Silicon Photonic Crystal Waveguides: Theory

4.1 Introduction

Subwavelength silicon nanostructures such as photonic crystals and high-index-contrast photonic integrated circuits offer the opportunity to manipulate the propagation of light at subwavelength scales. Moreover, the inherent ease of integrating the silicon photonics platform with complementary metal-oxide semiconductor foundry integrated circuits offers unprecedented bandwidth per unit cost and distance in optical data communications.

Silicon, however, is at an intrinsic disadvantage for optical amplification and lasing due to its indirect bandgap and nonexistent second-order nonlinear response. Recent work has demonstrated that stimulated Raman scattering (SRS) in single-crystal silicon channel waveguides is a feasible means to achieve amplification and lasing via optical pumping[20, 57–60]. This is due to the intrinsically large Raman gain coefficient in silicon (being $10^3 - 10^4$ times greater than for silica), and silicon nanostructures offering the benefit of high optical confinement due to the high-index contrast of silicon with air or silicon oxide. While still requiring an optical pump and possessing limited gain bandwidth, SRS can serve as a compact on-chip gain medium at desired telecommunications frequencies. In order to
enhance the intrinsic Raman gain of silicon, SRS in optical nanostructures exhibiting slow group velocities is currently being explored. Enhanced Raman scattering has been observed in bulk hollow-core slow-light guided-wave structures[61], and has also recently been suggested for photonic crystal (PhC) defect nanocavities[62]. In addition, a semiclassical model of Raman scattering in bulk photonic crystals has been introduced[63].

4.1.1 Raman Scattering

Raman scattering is an optical scattering process by which the optical wave interacts with the vibrational modes of the medium and therefore altering the frequency of the scattered light. This process is named for Chandrashekhara Venkata Raman who was the first observer of this phenomenon [64] and was subsequently awarded the Nobel prize for physics in 1930 for his work.

Raman scattering is an inelastic process, as opposed to elastic light scattering such as Rayleigh scattering (See Fig. 4.1(a)), meaning the scattering process alters the optical frequency upon scattering. In Raman scattering the nature of this change in frequency is due to the interaction with vibrational modes of the medium via phonon emission or absorption. In Stokes emission Raman scattering the incident light with energy $\hbar \omega_0$ is scattered into the Stokes emission ($\hbar \omega_s$) by the emission of the phonon with energy $\hbar \omega_p$ (See Fig. 4.1(b)). Due to the conservation of energy the relationship between $\omega_0$, $\omega_s$, and $\omega_p$ for Stokes emission is given by the relation:

$$\hbar \omega_0 = \hbar \omega_s + \hbar \omega_p$$

(4.1)

Raman scattering can also involve the absorption of phonon energy by the scattered light, in which case the scattered emission is called anti-Stokes ($\hbar \omega_{as}$) (See Fig. 4.1(c)). The anti-Stokes emission energy relates to the incident energy and phonon energy by the equation:
To understand Raman scattering in silicon we start with the incident light electric field, $E(k_i, \omega_i)$, which induces a polarization $P(k_i, \omega_i)$ in the atoms the silicon proportional to an electric susceptibility $\chi(k_i, \omega_i)$:

$$P(k_i, \omega_i) = \chi(k_i, \omega_i)E(k_i, \omega_i)$$  \hfill (4.3)

We can model the fluctuating susceptibility due to atomic thermal fluctuations caused by quantized plane-wave vibrations (phonons):

$$Q(r, t) = Q(q, t)\cos(q \cdot r - \omega_0 t)$$  \hfill (4.4)

These displacements are small compared to the crystal lattice parameter and therefore $\chi$ can be expanded into a Taylor series in terms of $Q(r, t)$:
\[
\chi (k_i, \omega_i, Q) = \chi_0 (k_i, \omega_i) + \sum_m \left( \frac{\partial \chi}{\partial Q_m} \right) Q_m (r, t) + \frac{1}{2} \sum_{m,n} \left( \frac{\partial^2 \chi}{\partial Q_m \cdot \partial Q_n} \right) (Q_m \cdot Q_n) + \ldots
\] (4.5)

where \( \chi_0 (k_i, \omega_i) \) is the susceptibility with no thermal displacements. We can substitute Eqn. 4.5 into Eqn. 4.3 to get

\[
P (r, t, Q) = P_0 + P_1 + P_2 + \ldots
\] (4.6)

where \( P_0 \) is Rayleigh scattering component of the polarizability defined as :

\[
P_0 (r, t) = \chi_0 (k_i, \omega_i) E_i (k_i, \omega_i) \cos (k_i \cdot r - \omega_i t)
\] (4.7)

\( P_1 \) represents the first-order Raman process :

\[
P_1 (r, t, Q) = \frac{1}{2} \left( \frac{\partial \chi}{\partial Q} \right)_0 Q (q, \omega_0) E_i (k_i, \omega_i) \times
\]

\[
(\cos ((k_i + q) \cdot r - (\omega_i + \omega_0) t) + \cos ((k_i - q) \cdot r - (\omega_i - \omega_0) t))
\] (4.8)

The terms corresponding to Anti-Stokes \( (\omega_i + \omega_0) \) and the Stokes \( (\omega_i - \omega_0) \) scattering can be easily identified. Since momentum must be conserved in this process the wavevector cannot exceed \( 2k_i \) which dictates that only phonons at the center of crystalline Brillouin zone participate in first-order Raman scattering. The center of Brillouin zone for silicon is the \( \Gamma \) point (See Fig. 4.2(a)) and from the experimentally measured phonon dispersion relation in Fig. 4.2(b) it can be seen that at the \( \Gamma \) point the acoustic phonon branch have energies close to zero and the optical phonon branch exhibit energies of 15.6 THz.

In Eqn. 4.6 \( P_2 \) represents the second-order Raman scattering process (and so on to higher orders) and involves the scattering of two phonons. This process can involve phonons
from the entire crystalline Brillouin zone but is significantly weaker than the first-order process. This process is not studied in this work.

![Brillouin zone and phonon dispersion of silicon](image)

**Figure 4.2: Brillouin zone and phonon dispersion of silicon.** (a) The first Brillouin zone of crystalline silicon showing the high symmetry points (b) The phonon dispersion relation of crystalline silicon[65].

### 4.2 Photonic Crystal Waveguides for Raman Scattering

The silicon PhCWG studied here, made by removing a single row in a hexagonal lattice of holes, denoted as “W1 PhCWG,” and its projected band structure can be seen in Fig. 1. This structure supports two tightly confined modes with small group velocities, as illustrated by the two bands within the bandgap, with frequencies below the light line. The field distribution of these two modes, as computed through the plane wave expansion method[44], is illustrated in Fig. 2. The strong subwavelength modal confinement of the high-index contrast PhCWG leads to increased field intensities in the silicon gain media, permitting increased nonlinear interactions. In addition to increased field intensities from high-index confinement, there is additional SRS enhancement from the small group velocities of the PhCWG propagating modes. Physically this enhancement originates from the effective long light–matter interaction times at small group velocities. Photon localization is observed at the band edge; the photon experiences multiple scattering processes and moves very slowly through the material structure. The guided bands of a 2D PhCWG can be designed to be as flat as desired ($v_g \equiv d\omega/dk$) for slowlight behavior, and group velocities as low as $10^{-2}c - 10^{-3}c$ have been demonstrated[39, 66].
Figure 4.3: Proposed Raman pumping scenarios for a W1 photonic crystal waveguide.  
Three proposed pumping scenarios for achieving group index enhanced Raman Stokes emission in photonic crystal waveguides. (Open circles : pump frequency. Closed circles : generated Stokes emission frequency) (a) Pump : TE\textsubscript{odd}, Stokes : TE\textsubscript{even}, (b) Pump : TE\textsubscript{even}, Stokes : TE\textsubscript{even}, (c) Pump : TM, Stokes : TE\textsubscript{even}.

4.3 Spontaneous Raman Scattering Theory for Silicon

In SRS for silicon, an incident photon interacts with the LO and TO phonons. The strongest Stokes peak arises from the single first-order Raman phonon at the center of the Brillouin zone. The generation of the Stokes photons can be understood classically as a third-order nonlinear effect; this formalism has been used to model SRS in silicon-on-insulator (SOI) waveguides, both in continuous-wave\cite{67} and pulsed\cite{68} operations. It can be modeled in bulk materials as a degenerate four-wave-mixing problem involving the pump and Stokes beams. The important material parameter is the third-order nonlinear Raman susceptibility, $\chi^R$. For silicon, at resonance, $\chi^R$ is defined by the components $\chi^R_{ijij} = -i\chi^R = -i11.2 \times 10^{-18} \text{ m}^2/\text{V}^2 (i, j = 1, 2, 3)$\cite{67}. An additional symmetry, imposed by the crystal point group (m3m for Si), is $\chi^R_{ijij} = 0.5\chi^R_{ijij}$ \cite{69}. These components and their
permutations as defined by the crystal point group define the SRS in a silicon crystal. For our purpose we shall consider scattering in silicon along the [1\bar{1}0] direction since practical devices are fabricated along this direction due to the favorable cleaving of silicon along this direction.

For bulk silicon, the evolution of the Stokes beam is defined by the following equation:

\[
\frac{dI_s}{dz} = -\frac{3\omega_s Im(\chi^R_{eff})}{\epsilon_0 c^2 n_p n_s} I_p I_s
\]

(4.9)

where the effective third order nonlinear susceptibility is defined as:

\[
\chi^R_{eff} = \sum_{ijkl} \chi^R_{ijkl} \hat{\alpha}_i \hat{\beta}_j \hat{\beta}_k \hat{\alpha}_l
\]

(4.10)

Here \(\hat{\alpha}\) and \(\hat{\beta}\) are unit vectors along the polarization directions of the pump and Stokes beams, respectively. Eqn.4.9 describes the gain of the Stokes intensity, \(I_s\). It shows an intrinsic dependence on the polarization and the phonon selection rules through \(\chi^R\), and the intensity of the pump beam by \(I_p\). The bulk solution also describes SRS in dielectric waveguides, where \(\chi^R_{eff}\) is averaged over the waveguide mode field distribution.

Figure 4.4: Computed mode profiles of PhCWG for proposed Raman pumping scenarios. Calculated bound states of a hexagonal lattice W1 PhCWG with defect modes separated by the LO–TO optical phonon (TE_odd-TE_even). (a) Stokes. (b) Pump
4.4 Raman Scattering Theory for Photonic Crystal Waveguides

A PhCWG presents a very different field distribution from the bulk or dielectric waveguide case. As shown in the computed modal profiles of Fig. 4.4, the mode differs from that of a conventional channel waveguide in that it exhibits a periodic variation in the direction of propagation. We introduce the modal distribution of the pump and Stokes modes in a Bloch–Floquet formalism,

\[ E_{n,k_n}(r,\omega_n) = E_{n,k_n} e^{ik(\omega_n)\cdot r} \]  

(4.11)

where \( n \) is a mode index \((n = p, s)\), \( k_n = k(\omega_n) \) is the mode wave vector, \( E_{n,k_n}(r,\omega_n) \) is the modal distribution within a unit cell of the PhC, defined in Fig. 4.4, and obeys the Bloch boundary condition \( E_{n,k_n}(r + \Delta,\omega_n) = E_{n,k_n}(r,\omega_n) \). \( \Delta \) defines the length of the unit cell in the direction of propagation; for a W1 waveguide this equals the PhC lattice constant \( a \). To develop the evolution, we employ the Lorentz reciprocity theorem\[68, 70],

\[ \frac{\partial}{\partial z} \int_A \left[ E_{n,k_n}^* \times \mathbf{H} + \tilde{E} \times H_{n,k_n}^* \right] \cdot \hat{e}_z dA = i\omega \int_A P^R \cdot E_{n,k_n} dA \]  

(4.12)

This relates the unperturbed PhCWG modes of the pump or Stokes wavelengths, \( \{E_{n,k_n}, H_{n,k_n}\} \), to those of the nonlinearly induced fields. The envelopes of the fields are defined as

\[ \tilde{E}(r) = u_s(z)E_{s,k_s}(r,\omega_s) + u_p(z)E_{p,k_p}(r,\omega_p), \]  

(4.13a)

\[ \tilde{H}(r) = u_s(z)H_{s,k_s}(r,\omega_s) + u_p(z)H_{p,k_p}(r,\omega_p), \]  

(4.13b)

with the assumption that the change in the pump and Stokes field amplitudes, \( u_p(z) \) and \( u_s(z) \), respectively, over the length of the unit cell of the waveguide is very small.
Taking the fields as defined in Eq. 4.13 and substituting into Eq. 4.12, we derive the dependence of the Stokes amplitude on the longitudinal distance, \( z \),

\[
\frac{du_s(z)}{dz} = \frac{i\omega_s}{4P_s\Delta} \int_{V_0} P^R(r, \omega_s) \cdot E_{s,k_s}(r, \omega_s) dV
\]  

(4.14)

where \( P_s \) is the mode power and \( P^R(r, \omega_s) = 6\epsilon_0 R_s^c E_{p,k_p}(r) E_{s,k_s}(r) |u_p|^2 u_s \).

The integral in Eq. (4.14) is taken over the volume \( (V_0) \) of the unit cell of the PhCWG mode. Furthermore, the group velocity of the modes can expressed by the following equation [70]

\[
v_{g}^{p,s} = \frac{P_{p,s}}{2 \epsilon_0 \int_{V_0} \epsilon(r)|E_{p,s}(r, \omega_{p,s})|^2 dV}
\]  

(4.15)

With Eqs. (4.13) and (4.15), and by rewriting Eq. (4.14) in terms of the modes intensity, an equation for the intensity of the Stokes mode inside the PhCWG is obtained,

\[
\frac{dI_s}{dz} = -\frac{3\omega_s}{\epsilon_0 v_g^{p,s} v_g^{s}} \kappa I_p I_s
\]  

(4.16)

where

\[
\kappa = \frac{\Delta A_{eff} \text{Im} \left( \int_{V_0} E^*(\omega_s) \cdot \chi^R: E^*(\omega_p) E(\omega_p) E(\omega_s) dV \right)}{\left( \frac{1}{2} \int_{V_0} \epsilon(r)|E(\omega_p)|^2 dV \right) \left( \frac{1}{2} \int_{V_0} \epsilon(r)|E(\omega_s)|^2 dV \right)}
\]  

(4.17)

is the effective susceptibility. Here, the effective area \( A_{eff} \) is defined as the average modal area across the volume \( V_0 \),

\[
A^2_{eff} = \frac{\left( \int_{V_0} x^2 |E(\omega_s)|^2 dV \right) \left( \int_{V_0} y^2 |E(\omega_s)|^2 dV \right)}{\left( \int_{V_0} |E(\omega_s)|^2 dV \right)^2}
\]  

(4.18)
The final equation, Eq. (4.16), shows the explicit inverse dependence the Stokes mode amplification has on the group velocities of the pump and Stokes modes. When compared to Eq. (4.9), which shows an inverse dependence on $c^2$, it can be seen that equivalent Raman gains at lower pump powers ($I_p$) can be achieved in a PhCWG at frequencies with low group velocities.

Table 4.1 shows the results of Eq. (4.16) as being applied to two different PhCWG scenarios for SRS. The group velocities are calculated from the slope of the projected band structure. The first (TE_odd-TE_even) involves utilizing both the guided modes of the W1 waveguide; odd-parity is the pump mode and even-parity is the Stokes mode. The wavelength separation of the modes at the edge of the Brillouin zone is matched to the LO/TO frequency separation of the pump and Stokes beams (15.6 THz in Si [71]). The second (TE_even-TE_even) utilizes a wide bandwidth PhCWG[48], in order for the Stokes and pump modes to exist both in the fundamental mode and below the light line. The third scenario (TM-TE_even) makes use of the TM mode of PhCWG for the pump field and stokes emission is then generated in the even TE mode. As we will see this method is most conducive to experimental measurement since it does not require the excitation of the odd mode required by the TE_odd-TE_even scenario or the fabrication of a waveguide with small hole radius as required by the TE_even-TE_even scenario. The circles in Fig. 4.3 indicate the pump and Stokes frequency locations for all scenarios.

From the results of Table 4.1, the Raman gain, which is proportional to $\kappa / v_g^p v_g^s$, is enhanced by up to approximately $10^4$ (TE_odd-TE_even: 66000; TE_even-TE_even: 86, TM-TE_even: 20) times compared to bulk silicon based on a comparison of the respective group velocities. The results in Table 4.1 also show a $\kappa$ value of the same order with a conventional SOI.

**Table 4.1: Group Velocity and Effective Susceptibility in Photonic Crystal Waveguide Raman Pumping Scenarios.**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$v_g^s$</th>
<th>$v_g^p$</th>
<th>$\kappa \times 10^{-19}$ $[m^2 \cdot V^{-2}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE_odd-TE_even</td>
<td>0.00017c</td>
<td>0.0077c</td>
<td>0.55</td>
</tr>
<tr>
<td>TE_even-TE_even</td>
<td>0.0041c</td>
<td>0.24c</td>
<td>2.02</td>
</tr>
<tr>
<td>TM-TE_even</td>
<td>0.017</td>
<td>0.0067c</td>
<td>0.63</td>
</tr>
</tbody>
</table>
waveguide[68]. In addition, we note a reduction in $\kappa$ in $\text{TE}_{\text{odd}}$-$\text{TE}_{\text{even}}$ as compared to $\text{TE}_{\text{even}}$-$\text{TE}_{\text{even}}$, due to the lower modal overlap. However, the single mode ($\text{TE}_{\text{even}}$-$\text{TE}_{\text{even}}$) operation has the disadvantage that only the Stokes mode is at low group velocities for enhanced SRS.

The above results highlight the benefits of SRS enhancement through slow-light interactions in compact PhCWG scenarios. This approach can be readily extended to include two photon and bulk free carrier absorption effects[68] by the addition of loss terms to Eq. (4.16), which may limit the effective Raman gain in PhCWGs. These effects, in the experimental realization of silicon SRS amplification and lasing in slow-light PhCWGs, can be surmounted with pulsed-laser operation[60] or p-i-n diodes[20] to sweep out the free carriers.

### 4.5 Conclusion

In addition, we note recent theoretical[72] and experimental[73] studies of PhCWGs, which show that slow group velocity modes exhibit increased scattering losses. These losses are from coupling and intrinsic (backscatter) reflection. Coupling into slow-light modes is currently the dominant loss experimentally, although this can in principle be reduced through careful adiabatic coupling between the PhCWGs and input–output channel bus waveguides. Moreover, with thorough attention to fabrication disorder, reflection losses in PhCWG are suggested to be comparable with index-guided waveguides[74]. These scattering losses can thus potentially be smaller than the enhanced SRS gain discussed, permitting the possibility for compact silicon Raman amplifiers and lasers. We also note that, for the same desired Raman gain, the device length is reduced by $(c/v_g)^2$, allowing compact integration for high-density photonic circuits.
Chapter 5

Raman Scattering in Silicon Photonic Crystal Waveguides : Experiment

5.1 Introduction

The prospect of silicon acting as an active optical material with the possibility of amplification and lasing has been the driving force behind the research of Raman scattering in silicon-on-insulator waveguides. This evolved from the observations of spontaneous Raman scattering in silicon rib waveguides[75] through the observations of amplification[76] and finally to lasing[20, 60]. In addition to these achievements in relatively large mode area rib waveguides, the observation of spontaneous scattering[77] and amplification[57] has been made in submicron channel waveguides. In order to reduce the threshold power of Raman lasers, racetrack cavity[78] lasers have been studied experimentally. It has been shown theoretically that the high confinement and unique dispersion properties of photonic crystal cavities[62, 79] and waveguides[80] can be utilized to further reduce threshold values and enhance Stokes emission. Spontaneous Raman scattering has been observed in GaAs photonic crystal slab waveguides[81]; however no wavelength dependence of the Stokes emission was reported.

The enhancement of Raman scattering in photonic crystal waveguides when compared
to the aforementioned rib waveguide structures is due to two mechanisms: higher modal confinement and larger group indices. The waveguides studied here have a modal area of 0.13 $\mu m^2$ (averaged over one unit cell in the direction of propagation), which put them close in value to the nanowire waveguide devices studied previously[57, 77]. Such small modal areas lead to large optical intensities and increase the probability of Raman scattering. The periodic lattice of the photonic crystal membrane leads to a Bragg-reflection-like lateral confinement (total internal confinement in the vertical direction) for transverse electric (TE) optical mode. This Bragg confinement leads to a flat dispersion curve at the fundamental mode onset, as seen in Fig. 5.1a. These slow-light frequencies, which have been observed experimentally[39], are frequencies where the optical mode experiences large amounts of Bragg reflections from the bulk photonic crystal lattice on the either side of the waveguide. For transverse magnetic (TM) polarized light, no photonic band gap exists[47]; however there is still a periodic modulation of the effective index in the direction of propagation due to the lattice[48]. This modulation creates a one-dimensional photonic crystal, creating a distinctive Bragg stop gap at the Brillouin zone edge. Of course, the corresponding edges of the stop gap exhibit flat dispersion curves, indicative of slow light. For both of these areas of the dispersion curve, the TE mode onset and the TM stop gap edges, slow light offers the possibility of increased light matter interaction, effectively increasing the probability for Raman scattering to take place. The higher power density within the waveguide due to the increased multiple Bragg reflection at the slow-light region is the phenomenon we are exploiting here. It has been shown theoretically that the scattered Stokes intensity in silicon PhCWGs is inversely proportional to the product of the pump and Stokes group velocities[80].

5.2 Waveguide Design for Raman Scattering

In our experiments the pump laser is TM polarized and tuned in the vicinity of the upper edge of the Bragg stop gap. By careful design of the PhCWG parameters (lattice constant, hole
radius, and slab thickness), the resulting Stokes emission is scattered into the fundamental TE mode onset. This results in both the pump and Stokes wavelength operating at wavelengths of high group index.

The waveguides used in our experiment are fabricated utilizing deep-UV photolithography on silicon-on-insulator wafers with a 250 nm thick layer of silicon and a 1 \( \mu m \) layer of SiO\(_2\) on silicon substrate. The holes are etched using a plasma dry-etch process. After the dry etch, a wet etch of hydrofluoric acid is used to remove the underlying SiO\(_2\), creating a suspended silicon membrane. The devices are then cleaved manually. The fabricated photonic crystal has a lattice constant of 480 nm and holes with a radius of 0.34\( a \) (163 nm). From cutback measurements the minimum waveguide loss for the TM-like mode was found to be (1.1±0.2) dB/mm and (2.9±1.0) dB/mm for the TE-like mode. In agreement with previous studies\cite{82, 83} of PhCWG loss, the loss in our waveguides was seen to increase
dramatically when approaching the TE mode onset. The transmission properties of the waveguide were experimentally verified using a supercontinuum source and can be seen in Fig. 5.1b.

![Figure 5.2: Transmission and Group Index of the TE and TM modes of the photonic crystal waveguide. Transmission measurements (solid line) and derived group index (open circles). (Top) TM polarization. (Bottom) TE polarization.](image)

### 5.3 Waveguide Characterization

If any claim of Stokes emission enhancement due to slow group velocity is to be made, the group index of both pump and Stokes modes must be measured. In order to do this, high spectral resolution transmission measurements (wavelength step of 1 pm) were taken of the PhCWG (see Fig. 5.2). Utilizing the method outlined in Ref. [39], the transmission data of the waveguide were analyzed and the Fabry–Pérot oscillation spacing was used to determine the group index. This method allows us to measure the group index of both Stokes and pump modes in situ. The maximum group indices of TM-like and the TE-like modes of the waveguide were measured to be 57 and 149, respectively, similar orders of
magnitude to those observed in Refs. [39], [82], and [83]. The high resolution transmission data show that there is a 2.2 THz (2.9 nm) difference between the optical phonon frequency spacing (15.6 THz or 135 nm at the pump wavelength of maximum TM group index) and the frequency spacing between the wavelength of maximum TM and TE group indices (17.8 THz or 137.9 nm). This difference reduces the expected maximum Raman enhancement, which is proportional to the product of the Stokes index \( n_s \) and pump index \( n_p \) [80] since the peak \( n_s \) does not occur at the Stokes wavelength that would be generated by the peak \( n_p \) wavelength.

### 5.4 Raman Scattering Measurement

![Experimental Setup Used to Characterize SRS in Photonic Crystal Waveguides](image)

**Figure 5.3: Experimental Setup Used to Characterize SRS in Photonic Crystal Waveguides.** PC : Polarization Controller, EDFA : Erbium Doped Fiber Amplifier, BPF : Bandpass Filter, PBS : Polarization Beam Splitter, P : Polarizer, WDM : Wavelength Division Multiplexer, OSA : Optical Spectrum Analyzer

To characterize the spontaneous Raman scattering properties of these waveguides the experimental setup shown in Fig. 5.3 is used. The pump source was a tunable laser source (Ando AQ4321D) that was amplified using an erbium doped fiber amplifier (IPG EAD-3K-C, IPG Photonics). A fiber bandpass filter (BPF-1530/1560, OEQuest) was then used to reduce the amplified spontaneous emission noise of the pump. Fiber based wavelength division
multiplexers (MWDMG16150, Oplink Communications LLC) were used to separate pump and Raman wavelengths for detection. A polarization beam splitter is used to ensure that the pump is TM polarized and to allow collection of the backscattered TE Stokes emission. The Stokes output is then measured using an optical spectrum analyzer (Advantest Q8384, resolution at 0.5 nm) or optical power meter (Newport 2832-C with 818-F-IR module).

Figure 5.4: Pump dependence of Spontaneous Raman Scattering in a photonic crystal waveguide. Measured spontaneous emission versus pump wavelength. (Red line) backscattered Stokes emission. (Black line) forward scattered Stokes emission. (Inset) Measured forward scatter Stokes spectrum for \( \lambda_{\text{pump}} = 1535 \, \text{nm} \) and \( \lambda_{\text{pump}} = 1544.24 \, \text{nm} \). Coupled pump power \( \sim 15 \, \text{mW} \).

Fig. 5.4 shows the total Stokes power for both forward scattered and backscattered emissions with respect to the pump wavelength. Since the pump wavelength is far from any electronic resonances, there should only be a \( (1/\lambda_s) \) (Ref. [20]) variation in the Stokes output with respect to the pump wavelength. However, as can be seen in Fig. 5.4, the Raman emission of the waveguide has a strong pump wavelength dependence. As the pump wavelength approaches the TM stop gap edge, the Raman emission increases. If the Raman emission variation with pump wavelength was solely dependent on loss, one would expect it to drop as it approaches the TM stop gap, as loss in both the TE and TM modes increases as
they approach their regions of large group index. The fact that the Raman emission does not decrease as the pump approaches the stop gap edge, but in fact increases, is a sign of group index dependence. In order to characterize the Stokes emission enhancement, we introduce a simple model

\[
\frac{dP_s^\pm}{dz} = \pm \alpha_s P_s^\pm \pm \kappa P_p^+ \quad (5.1)
\]

where \( P_s^+ \) and \( P_s^- \) are the forward scattered and backscattered Stokes powers, respectively. \( \kappa \) is the spontaneous Raman scattering coefficient, which has been shown\[80\] to be inversely dependent on the Stokes and pump group velocities. \( P_s^+ \) is the pump power and is defined as

\[
P_s^+ = P_0 e^{-\alpha_p z} \quad (5.2)
\]

where \( \alpha_s \) and \( \alpha_p \) are the wavelength dependent losses in the Stokes and pump modes, respectively. They are derived by scaling the measured transmission to the minimum measured losses which were determined by cutback measurements on a similar photonic crystal waveguide. We use this model to fit the measured Stokes power in order to derive the pump wavelength dependent value of \( \kappa \) (see Fig. 5.5). Knowing that \( \kappa \) is directly proportional to the product of the Stokes and pump group indices, we also plot in Fig. 5.5 the product of the experimentally derived TE and TM mode group indices (utilizing a constant fitting parameter). It can be seen that the fitted \( \kappa \) values follow the experimentally measured product of the Stokes and pump group indices closely and are shown to be enhanced by six times (within the error bars) in the region of high group index in comparison with the region of normal group index away from the TM stop band edge.
Figure 5.5: Spontaneous Raman scattering coefficient with respect to pump wavelength. (Black squares) derived Raman scattering coefficient ($\kappa$) from model. (Red circles and line) Product of experimentally determined Stokes and pump group velocities (see Fig. 5.2) multiplied by a fitting constant.

5.5 Conclusion

This simplified model does not take into account any nonlinear absorption such as two-photon absorption (TPA) or free-carrier absorption (FCA). For our experiment this is a valid assumption since the pump power utilized $P_0=15$ mW is very low compared to the expected power (30 W[77]) where TPA would become non-negligible for a device with the length and modal area equal to ours. No deviation from a linear relationship between pump power in and pump power out was observed in our measurements. It has been experimentally verified that TPA is group index dependent[84]; however no observations of such an increase in absorption were observed in our measurements due to the low pump powers used in this experiment.
Chapter 6

Four-Wave Mixing in Silicon Photonic Crystal Waveguides

6.1 Introduction

The investigation of nonlinear optical processes in silicon-on-insulator (SOI) waveguides has reached a level of maturity that has allowed researchers to move beyond the proof-of-concept phase and into the practical application realm. In particular, the third-order nonlinear process of four-wave mixing (FWM) in silicon integrated waveguides has recently received particularly intense investigation due to the large third-order susceptibility ($\chi^{(3)}$) of silicon and a wavelength-tunable method of signal amplification, conversion, and regeneration is an extremely attractive capability in the silicon-on-insulator platform for integrated photonic circuits.

Initial experimental investigations into FWM in silicon waveguides [85, 86] were successful due to the large pump powers and small modal areas of the waveguides used. The tight optical confinement of these waveguides ensured large optical intensities resulting in a measurable nonlinear response despite the lack of dispersion optimization. Subsequent investigations [87, 88] demonstrated the ability to optimize not only the conversion efficiency but also the available bandwidth. This was accomplished by dispersion engineering the
waveguide: tuning the waveguide modal dispersion to optimize phase matching at the wavelengths of interest. With the modal dispersion optimized, maximum conversion efficiencies of up to -9.6 dB and usable bandwidths greater than 150 nm have been demonstrated [87]. In addition to these optimizations, further enhancement of the FWM conversion efficiency has been demonstrated utilizing ring resonators [85, 89, 90], hybrid silicon-organic slot waveguide structures [91], and chalcogenide-based planar waveguides [92]. These promising proof-of-concept experiments have paved the way for the demonstration of practical applications of FWM in silicon. These applications include: wavelength conversion at 40 [93] and 160 GB/s [94], all optical signal regeneration [95], spectral phase conjugation [96], and waveform compression [97]. The utility of four-wave mixing in silicon waveguides has branched out beyond applications grounded in optical communications system and into a number of other areas. The realization that FWM could be utilized as an on-chip optical time lens [98] has led to a number of interesting new applications including temporal [99] and spectral magnification [100] and the demonstration of an ultrafast optical oscilloscope [101].

For the majority of applications of FWM in silicon waveguides, the length of the waveguide needed to achieve large conversion efficiency is typically on the order of several centimeters, occupying a large footprint on the photonic integrated circuit. Photonic crystal waveguides with slow-light modes, however, offer an approach to significantly increase the light-matter interaction with significantly (100x or more) smaller footprints. The modes of a photonic crystal waveguide exhibit unique dispersion properties that have been demonstrated to exhibit large group indices [39, 102–104]. Slow-light in two-dimensional, hexagonal photonic crystal single line-defect waveguides has been demonstrated to enhance a number of nonlinear phenomenon including Raman scattering [105, 106] self-phase modulation [84, 107–110], third harmonic generation [111], in the presence of nonlinear absorption such as two-photon (TPA), three-photon, and free carrier absorption (FCA) [112].

In the specific case of slow-light enhancement of FWM in photonic crystal waveguides, the conversion efficiency has been theoretically suggested to be enhanced due to the increase
in the effective length of the waveguide \[113, 114\]. However, due to the large group velocity dispersion (GVD) of these waveguides in the slow-light regime, the conversion efficiency bandwidth is typically reduced. Four-wave mixing has been observed in silicon \[115\] and GaInP \[116\] photonic crystal waveguides previously. In addition, chalcogenide-based photonic crystal waveguides have observed conversion efficiencies 13x higher than in silicon wire waveguides \[117\]. In this chapter we experimentally demonstrate the slow-light enhancement of the FWM conversion efficiency and corresponding shrinking of the conversion bandwidth due to the increase in GVD at large group index in W1 silicon photonic crystal waveguides.

6.2 Theory

In this experiment we examine the nonlinear optical process of partially degenerate four-wave mixing (FWM). In this process a pump wave (\(\omega_{\text{pump}}\)) and a signal wave (\(\omega_{\text{signal}}\)) couple power into an idler wave with an optical frequency determined by 
\[
\omega_{\text{idler}} = 2\omega_{\text{pump}} - \omega_{\text{signal}}
\]
via the third-order nonlinear susceptibility (\(\chi^{(3)}\)) of silicon (See Fig. 6.1). The efficiency of this exchange of energy is dependent on the level of coherence between all the frequencies involved (\(\omega_{\text{pump}}, \omega_{\text{signal}}, \omega_{\text{idler}}\)) in order to maintain conservation of momentum. For optical waves confined to a guided mode structure, such as in a optical waveguide, the coherence is determined by the nonlinear and linear phase mismatch of the propagating modes. The lower the phase mismatch, the more efficiently the power transfers to \(\omega_{\text{idler}}\).

The goal of this work is to characterize the wavelength dependence of this conversion efficiency within a silicon W1 photonic crystal waveguide (PhCWG). The W1 waveguide is defined as a single line defect in a 2D photonic crystal created by a hexagonal array of holes in a silicon membrane. These waveguides exhibit very large anomalous dispersion owing to the optical mode being confined laterally by the photonic bandgap. Consequently, the group index of the waveguide increases monotonically with wavelength as it approaches the waveguide cut-off wavelength \(\lambda_c\), at which point the Bragg condition is satisfied allowing
Figure 6.1: Spectral diagram of partially degenerate four-wave mixing. Illustration of the generation of cascaded idler wavelengths from consecutive partially degenerate four-wave mixing processes. In this process the input to the medium is the \( \lambda_{\text{Pump}} \) and \( \lambda_{\text{Signal}} \) fields.

the forward and backward modes to couple and form a standing wave. This “slow-light” behavior allows for the enhancement of nonlinear optical phenomenon [40] due to the increased optical path length the larger group index introduces or more specifically: the lightmatter interaction length is increased by the coherent Bragg reflections the optical mode experiences the closer the wavelength is to \( \lambda_c \). The group index enhancement does not only limit itself to desired nonlinear effects therefore nonlinear absorption and scattering loses will be enhanced as well. However, it can be imagined that if the processes responsible for a desired nonlinear process and the unwanted process scale unequally, then the group index enhancement allows for achieving a desired nonlinear performance within a shorter
length device. In addition to the enhancement of nonlinear absorption, the photonic crystal waveguides being investigated in this work have the additional unwanted characteristic, in terms of FWM, of having large group velocity dispersion (GVD). The GVD of the waveguide is the primary source of phase mismatch within the FWM process and its effect will be directly observed in this experiment. It is expected that GVD will be the limiting factor on the applicability of any observed enhancement as it will limit the maximum signal and pump wavelength separation that FWM can efficiently occur for.

In order to gauge the veracity of any observed FWM enhancement a simple numerical model will be employed. By extending previous work done in fibers [118], it has been shown [113] that for a photonic crystal waveguide of length L, the conversion efficiency is given by:

\[
G_{\text{idler}} = \frac{P_{\text{out}}^{\text{idler}}}{P_{\text{in}}^{\text{signal}}} = \frac{\gamma^* P_p e^{\alpha^*_\text{idler} L}}{g}
\] (6.1)

The efficiency is governed by an effective coupled pump power:

\[
P_p = P(0) \frac{(1 - e^{-\alpha^*_\text{idler} L})}{\alpha^*_\text{idler} L}
\] (6.2)

which accounts for the losses experienced by the pump wave during propagation through the waveguide. In order to account for the slow-light effect of the PhCWG, the nonlinear parameter (\(\gamma\)) and the waveguide linear propagation loss (\(\alpha\)) have been substituted with group index dependent versions of themselves (\(\gamma^*\), \(\alpha^*\)). For the third-order nonlinear parameter the group index scaling is defined as [107] \(\gamma^* = \gamma_0 (n_g/n_0)^2\), where \(\gamma_0 = \omega_{\text{pump}} n_2 / c A_{\text{eff}}\). In the simulations presented within this thesis the value of \(n_2\) used was \(4.5 \times 10^{-16} \text{ m}^2/\text{W}\) [19] and the effective mode area (\(A_{\text{eff}}\)) of the PhCWG mode, (averaged over a single unit volume of the waveguide [80]) was calculated to range from 0.24 to 0.4 \(\mu\text{m}^2\). The group index scaling of the propagation loss is defined as \(\alpha^* = \alpha (n_g/n_0)\), where the propagation loss scales linearly with the group index. The justification for this choice of
scaling will be addressed later in the thesis.

In Eqn. 6.1 the parametric gain, $g$, contains the phase dependent variables and is defined as:

$$g = \sqrt{\left(\gamma^* \bar{P}_p\right)^2 - \left(\frac{\Delta k_L + \Delta k_{NL}}{2}\right)^2}$$  \hspace{1cm} (6.3)

The linear phase mismatch between the interacting optical frequencies is $\Delta k_L = 2k_{pump} - k_{signal} - k_{idler}$. In order to examine the wavelength dependence of $\Delta k_L$ it can be approximated using a Taylor expansion about the pump frequency given by:

$$\Delta k_L \approx (\Delta \omega)^2 \beta_2(\lambda_{pump}) + \frac{1}{12}(\Delta \omega)^4 \beta_4(\lambda_{pump})$$  \hspace{1cm} (6.4)

where $\Delta \omega = |\omega_{pump} - \omega_{signal}|$. The dispersion dependent parameters, $\beta_2(\lambda)$ and $\beta_4(\lambda)$, are derived [49] from the numerically computed [44] dispersion relation of the guided mode of the PhCWG. In this work all wavelengths propagate in the fundamental quasi-TE guided mode of the PhCWG. The other phase term in Eqn.6.3, $\Delta k_{NL}$, is the nonlinear phase mismatch and is defined as:

$$\Delta k_{NL} = 2\gamma^* \bar{P}_p$$  \hspace{1cm} (6.5)

This simple model neglects nonlinear absorption effects such as two-photon and free-carrier absorption. However due to the relatively short length of our waveguide and the low coupled powers used it will be shown that these effects are minimal in these particular measurements and the model provides a confident representation of the basic trends of the underlying FWM.
6.3 Experiment

The W1 waveguide utilized in this experiment was fabricated using deep-UV photolithography on a silicon-on-insulator wafer with a silicon thickness of 250 nm and a buried oxide thickness of 1 µm. The lattice constant (a) for the hexagonal photonic crystal membrane was 440 nm and the radius of the fabricated holes was $0.36a \sim 158$ nm. The total length of the PhCWG was $1000a$ (0.44 mm). The photonic crystal region was subjected to a hydrofluoric acid etch in order to remove the oxide beneath it, creating a suspended silicon membrane with air cladding above and below. The input and output of the PhCWG are butt coupled to channel waveguides. The input (output) channel waveguide tapers linearly from an initial width of $3a \sim 0.76$ µm to a width of $4a \sim 0.8$ µm, over a length of 1.3 mm (0.8 mm). The transmission of TE polarized light through the device can be seen in Fig. 1(a). The significant drop in transmission, which corresponds to the fundamental quasi-TE mode cut off wavelength ($\lambda_c$), can be seen to be near 1545.5 nm.

In order to determine the group index of the waveguide with respect to wavelength we employed the phase-delay method [119] in which the light from a tunable laser was modulated at a frequency of 5 GHz using a lithium niobate modulator and microwave synthesizer. The light was then transmitted through the waveguide, collected, and fed into a high-speed photoreceiver (New Focus 1544-A). The wavelength dependent time delay ($\tau(\lambda)$) between the transmitted waveform and the modulator driving signal was measured using a high speed sampling oscilloscope. The measured delay was used to compute the group index by from the relation $n_g(\lambda) = n_{wg} + \Delta n(\lambda) = n_{wg} + c\Delta \tau(\lambda)/L_{PhCWG}$, where $\Delta \tau(\lambda)$ is the measured time delay difference between the PhCWG and a channel waveguide of the same length with a known group index ($n_{wg}$), derived from numerical simulation. The group index is observed to increase rapidly as the wavelength approaches $\lambda_c$, confirmation that this is indeed the cutoff of the quasi-TE mode.

Degenerate four-wave mixing in the PhCWG was observed using the experimental setup in Figure 6.3. Two tunable continuous-wave lasers were employed, one acting as a pump and the other as a signal probe, are both amplified using separate erbium doped fiber amplifiers
Figure 6.2: Experimental setup used to measure four-wave mixing in photonic crystal waveguides. Two separate tunable lasers (TL) pass through polarization controllers (PC) and are amplified by an EDFA then filtered using a band pass filter (BPF) and combined and coupled into the photonic crystal waveguide. The output light of the waveguide is coupled back into fiber and measured using an optical spectrum analyzer (OSA).

and then subsequently optically filtered (3-dB bandwidth ~0.2 nm) of amplified spontaneous emission noise. The pump and signal were combined using a 50:50 fiber coupler and the combined wavelengths were coupled into the waveguide using an aspheric lens (NA = 0.6). The light exiting the waveguide was collimated back into fiber and analyzed using an optical spectrum analyzer (OSA, Advantest Q8384) with 10 pm resolution. Taking into account filter loss, collimator loss, and an 8.5 dB per facet coupling loss, the estimated coupled pump power and signal power was +14 dBm and +11 dBm respectively. All values reported for measured idler power are the powers measured at the OSA and therefore include the output coupling loss, as will be expanded upon when conversion efficiency is discussed.

Experimentally observed four-wave mixing in a silicon photonic crystal waveguide for two different wavelengths ($\lambda_{pump} = 1535.00$ nm and $\lambda_{pump} = 1545.75$ nm) can be seen in
Figure 6.3: Transmission and group index of waveguide used in four-wave mixing experiment. Optical transmission (blue, solid line), measured group index (red, open circles), and numerically calculated group index (black, solid line) for the photonic crystal waveguide used in the four-wave mixing experiment.

In both measured spectra the difference between the pump and signal is $\sim 0.1$ nm, the pump being defined as the high wavelength laser (the peak at $+0.05$ nm). As can be seen from this data, the generated idler power for the case of the high group index (1545.75 nm, $n_g \approx 80$) is greater than that of lower group index (1535.00 nm, $n_g \approx 10$). As further evidence of increased FWM conversion efficiency at the higher group index wavelength is that the higher order idler (small peaks at $\pm 0.24$ nm), which are generated from cascaded parametric mixing of the signal and pump with their fundamental idler. Also of note in Figure 6.4(a) is the attenuation of the transmitted pump wavelength at slow-light which is due in part to it experiencing a higher group index enhanced propagation loss and also due to the strong Fabry-Perot oscillations at these wavelengths. In order to gauge whether any nonlinear absorption could be observed, the pump power dependence of the idler for $\lambda_{pump} = 1535.00$ nm was measured for varying pump input powers and plotted in Figure.
Figure 6.4: Observation of four-wave mixing in a photonic crystal waveguides and pump-idler power dependence. (a) Optical spectrum of degenerate four-wave mixing in a W1 silicon photonic crystal waveguide. $\lambda_{\text{pump}} = 1535.00 \text{ nm}$, $n_g \approx 10$ (black), $\lambda_{\text{pump}} = 1545.75 \text{ nm}$, $n_g \approx 80$ (red) (b) Measured idler output for varying pump power ($\lambda_{\text{pump}} = 1535.00 \text{ nm}$); measurement (points) linear fit, $m = 1.92$ (solid line).

The resulting linear fit gives a slope of 1.92, close to the expected 2 the quadratic dependence would display on such a log-log plot.

In order to investigate the wavelength dependence of the four wave mixing process within the waveguide, the generated idler power at $2\omega_{\text{pump}} - \omega_{\text{signal}}$ was measured for signal wavelengths -5 nm to +5 nm detuned from the pump wavelength in 0.1 nm steps. This was done for pump wavelengths from 1535.0 to 1545.5 nm in 0.5 nm steps. The resulting two-dimensional map of measured idler powers can be seen in Fig. 6.5a. To corroborate the measured results the model outlined was used with the numerically computed dispersion; the result of which can be seen in Fig. 6.5b. Since the value being plotted here is the power measured at the OSA, the value actually measured is $\eta P_{\text{idler}}^{\text{out}}$, where $\eta$ is the output coupling loss. In the model, in order to compare with measurements, we numerically computed $\eta P_{\text{idler}}^{\text{out}} G_{\text{idler}}$ and in order to be consistent, the model is computed with the same wavelength steps as the experimental data ($\Delta \lambda_{\text{signal}} = 0.1 \text{ nm}$, $\Delta \lambda_{\text{pump}} = 0.5 \text{ nm}$). Slices from the 2D figures that directly compare the measured results and the numerical model for $\lambda_{\text{pump}} = 1535.0 \text{ nm}$ and $\lambda_{\text{pump}} = 1545.5 \text{ nm}$ are plotted in Fig. 6.5c. The maximum measured idler power for each pump wavelength is plotted in Fig. 6.5d.

The free parameters used to fit the numerical model with the experimental data were
Figure 6.5: Mapping generated idler power in a photonic crystal waveguide. (a) Measured idler power wavelength dependence ($\Delta_{\text{pump}} = 0.5 \text{ nm}$, $\Delta_{\text{signal}} = 0.1 \text{ nm}$) (b) Simulated idler power wavelength dependence (c) Idler power dependence (measured : circles, simulated : solid line) for $\Delta_{\text{pump}} = 1535.0 \text{ nm}$ (black) and $\Delta_{\text{pump}} = 1545.5 \text{ nm}$ (red) (d) Maximum measured idler power versus pump wavelength; measurement (circles), calculation (solid line)
an absolute wavelength shift of the numerically simulated group index data from which \( \beta_2(\lambda) \) and \( \beta_4(\lambda) \) are derived from \( n_g^{\text{model}} = n_g^{\text{sim}}(\lambda + \lambda_0 \Delta) \), where \( \lambda_0 = 1535 \) nm. The group index shift was determined by finding the value of \( \Delta \) that produced the best match to experimental results for the \( \lambda_{\text{pump}} = 1535 \) nm data plotted in Fig. 6.5c. The best fit was achieved for \( \Delta = -2.6\% \), equivalent to a shift of \( \sim 40 \) nm in the absolute wavelength of the numerically simulated dispersion. This blue shift of the numerically calculated dispersion, when compared to experimental results, can be attributed to the thinning of the silicon slab thickness during the wet oxide etch[48]. A 2.6% blue-shift in wavelength corresponds to a reduction in the silicon slab thickness by approximately 24 nm (9.6% of the slab thickness).

It is worth noting that this method of utilizing the idler power dependence on signal detuning at a wavelength far from the \( \lambda_c \) has advantages over conventional dispersion fitting methods. The method allows for direct examination of dispersion difference of the PhCWG and the simulated dispersion and is not dependant on exactly where \( \lambda_c \) occurs [120], which can be impossible for long waveguides where \( \lambda_c \) and the minimum transmission might not coincide or for waveguides with large disorder which can cause a broadening of \( \lambda_c \). In addition, the method appears immune to the broadening of the phase-delay response (See Fig. 6.3) of the waveguide which is due to the etalon effects of the waveguide facets. The minimum propagation loss which gave the best reproduction of the experiment was 10 dB/cm. This resulting value of \( \alpha_0 \) is lower than what we have measured using the cut-back method for similarly fabricated waveguides (~20 dB/cm), but this value of \( \alpha_0 \) is not out of the realm of possibility, devices fabricated in a similar manner (deep-UV photolithography) have shown losses as low 14 dB/cm [121].

The calculated wavelength dependence of the generated idler power matches closely with that of the measured results and the idler power is observed to increase by over 12 dB when the pump wavelength is translated to a wavelength in the vicinity of \( \lambda_c \), while at the same time the bandwidth over which a measurable idler signal is observed decreases appreciably. This is mostly in good agreement with previously reported theoretical analysis [113], except for a noticeable asymmetry of the generated idler power about the pump.
wavelength in the experimental results. This asymmetry is attributed to the fact that for signal wavelengths blue shifted from the pump wavelength (signal detuning < 0 nm) generate idlers at wavelengths closer to $\lambda_c$ or even beyond it, leading to larger propagation losses for these wavelengths. This becomes more apparent as the pump wavelength approaches $\lambda_c$, as can be seen in the plot for $\lambda_{pump} = 1545.5$ nm in Fig. 6.5c where the measured idler power is seen to drop by 14 dBm from -0.1 nm to -0.2 nm. This asymmetry is captured in the model by ensuring the proper group index is used to scale the loss at the wavelength that it is being applied.

Our analysis of the experimental data and modeling ignores the contribution to the measured idler that the input and output tapered waveguides contribute, making the assumption that the measured idler power is primarily generated in the PhCWG. This may seem like a strange assumption considering the PhCWG makes up only 20% of the total length of the device however the measured data shows that this assumption is valid by displaying two distinct features as the pump wavelength approaches $\lambda_c$: a decreasing bandwidth over which an idler signal is measured and an increase in idler power while incident pump and signal powers are constant. The first observation, the decrease in bandwidth, is a direct result of the fact that the GVD of the PhCWG is at least two orders of magnitude larger than what would be found in waveguides with widths experienced in the tapered waveguides. The $\beta_2(\lambda)$ and $\beta_4(\lambda)$ for the PhCWG used in this experiment is plotted in Fig. 6.6(a), along with the $\beta_2(\lambda)$ of conventional waveguides, calculated using the plane wave expansion method [44] incorporating the material dispersion of silicon, with widths within the range of the input/output tapered waveguides for comparison. Consequently, if the idler was being primarily generated inside the input/output tapered waveguides it would show little variation in idler output power for $\pm 5$ nm signal detunings. A 3.5 mm long silicon waveguide has been shown [85] to have a flat response for over $\pm 20$ nm pump-signal detuning. As our total device length is only 2.5 mm, it would be expected to have an even better detuning performance, yet it only has a maximum flat response over $\pm 2.5$ nm, significantly smaller and an indication that the idler is being generated in the PhCWG, where there is a much
higher GVD. In addition to this bandwidth reduction, the decaying oscillatory peaks of Eq. 6.1 are resolved in the experimental data of Fig. 6.5a and 6.5c. If there was a strong detuning independent (over the $\pm 5$ nm detuning examined here) idler being generated in the channel waveguides it would be expected these oscillations would be washed out and unable to be resolved.

Figure 6.6: $\beta_2$, $\beta_4$, $\gamma L$, and $A_{eff}$ of the photonic crystal waveguide. (a) $\beta_2$ (solid line) and $\beta_4$ (dotted line) for the photonic crystal waveguide used in this experiment (inset) $\beta_2$ for conventional silicon waveguides for various widths (0.76 $\mu$m, 1.0 $\mu$m, 1.5 $\mu$m, 2.0 $\mu$m, 3.0 $\mu$m, 4.0 $\mu$m) (b) Calculated $\gamma L$ value (solid line) and the $A_{eff}$ (dotted line) versus waveguide width. Arrow indicates the expected minimum ($\lambda_{pump} = 1535$ nm) value of $\gamma^* L$ for a 0.44 mm PhCWG (channel waveguide length = 2.1 mm, height = 0.25 $\mu$m)

In addition to the observed bandwidth contraction, the measured idler power is observed to increase as the pump wavelength approaches $\lambda_c$ for fixed pump and signal powers, a telling sign of group index enhancement of the nonlinear parameter. Especially when one considers that the transmission of the waveguide experiences a 5 dB drop in transmission between these two wavelengths. In order to justify that the 0.44 mm long PhCWG by itself is responsible we compare it to a theoretical 2.1 mm (the total length of the device minus the length of the PhCWG) long channel waveguide by plotting the $\gamma L = \omega_{pump} n_2 / c A_{eff} f$ parameter value for a for widths within the range of the taper used in our device, as well as the minimum $\gamma^* L$ value expected from the 0.44 mm PhCWG. The 0.44 mm long PhCWG shows a larger nonlinear response than the majority of the 2.1 mm long waveguides with widths experienced in the tapered. It is only overtaken by the conventional waveguides when
their effective area becomes small enough to compensate for the group index enhancement in the PhCWG. However, the effective area of the conventional waveguide remains relatively constant over the wavelengths investigated here while the PhCWG sees an eight fold increase in group index. Therefore the pump wavelength dependence of the channel waveguides nonlinear parameter can be expected to be flat over the wavelengths examined, while the PhCWGs nonlinear parameter is expected to scale with the square of the group index. Additionally, the actual input/output channel waveguides used in the experiment have a linear taper in width going from the minimum of 0.76 \( \mu \)m to the maximum of 4 \( \mu \)m, therefore a more realistic value we should compare to would be the average \( \gamma L \) value of the widths, which is \( \sim 14 \) and is half the minimum value expected from the PhCWG. Of course this simple comparison ignores the losses incurred from the PhCWG that occurs between the two channel waveguides in the actual experimental device, which if considered would further show that the idler power measured in the experiment is generated primarily within the PhCWG.

### 6.4 Results

In order to quantify the FWM process in the PhCWG, the measured idler powers should be normalized with respect to the coupled signal power in order to gauge the conversion efficiency. The conversion efficiency is derived experimentally by taking the ratio of the measured output idler power and measured output signal power (\( \eta P_{\text{idler}}^{\text{out}} / \eta P_{\text{signal}}^{\text{out}} \)) in order to cancel any uncertainty in the coupling efficiency. It has recently been demonstrated [122] that using this definition can inadvertently overestimate conversion efficiency in silicon channel waveguides due to loss experienced by the signal within the waveguide, either through conventional linear propagation loss or by nonlinear loss due to two-photon and free-carrier absorption. In order to avoid this, the conversion efficiency can be defined as \( P_{\text{idler}}^{\text{out}} / P_{\text{signal}}^{\text{out}} \). Using this definition the maximum conversion efficiency of our waveguide for this pump power is derived from the maximum measured idler power max \( \eta P_{\text{idler}}^{\text{out}} = \).
-35.6 dBm, (see Fig. 6.5d) and knowing that $P_{\text{signal}}^{\text{in}}$ is +11.5 dBm and that the coupling loss is estimated to be 8.5 dB a maximum measured conversion efficiency of -38.8 dB is found. We can conclude that this value is the lower bound estimate of the maximum conversion efficiency of our waveguide for this coupled pump power. The assumption made with this definition is that the coupling to the waveguide of both the pump and signal is wavelength independent, which is valid for channel waveguides for the bandwidths considered here but not for a photonic crystal waveguide which exhibits increased coupling losses as the group index of the PhCWG mode increases.

With the lower bound conversion efficiency of the experiment stated, we will now present the results for the conventional definition ($\eta P_{\text{idler}}^{\text{out}}/\eta P_{\text{signal}}^{\text{out}}$) (where $\eta P_{\text{signal}}^{\text{out}}$ is measured with the pump off), in order to make an attempt at correcting for this group index dependent coupling loss. In order for this definition to be valid the conversion efficiencies will be derived from the maximum idler power generated by a signal detuned $\pm 0.1$ nm from the pump wavelength in order to minimize any difference in the group index dependent coupling loss experienced by the idler and signal. The pump wavelength dependence of this conversion efficiency is plotted in Fig. 6.7(a). The maximum measured conversion efficiency, using the $(\eta P_{\text{idler}}^{\text{out}}/\eta P_{\text{signal}}^{\text{out}})$ definition, is -35.7 dB; a difference of $\sim 3$ dB compared with the lower bound. This difference stems from the fact that the assumption that the idler and signal are experiencing equivalent coupling and propagation losses begins to break down at large group indices. For these pump wavelengths the maximum generated idler occurs when the signal is red shifted from the pump and consequently experiencing a higher group index and higher losses. This artificial increase in conversion efficiency could be mitigated by reducing the wavelength separation between the pump and signal however we are limited by our filter bandwidths and OSA resolution. In order to explicitly display that the increase in the group index is responsible for the enhancement of the conversion efficiency it is plotted versus group index in Fig. 6.7(b). In plotting the experimentally measured conversion efficiency, the measured group index is used (red circles in Fig. 6.3) and the wavelength shifted numerically computed group index is used to plot the model conversion efficiency.
(a) Four-wave mixing conversion efficiency dependence on pump wavelength ($|\lambda_{signal} - \lambda_{pump}|=0.1\text{nm}$). (b) Conversion Efficiency dependence on $n_g$ at $\lambda_{pump}$; measurement (points), calculation (solid line).

The numerical model appears to underestimate the maximum conversion efficiency by 5 dB, which would appear counterintuitive since the simulation does not take into account the decrease in the amount of pump power coupling into the waveguide as the group index increases. It would be expected that because of this fact the numerical model should show higher maximum conversion efficiencies than the experiment. This discrepancy can be attributed to the fact that this simple numerical model does not account for the effect the finite size of the PhCWG has on its transmission, which is responsible for the increasing strength and frequency of the Fabry-Perot oscillations as the cutoff wavelength is approached [39]. A decrease in the pump and signal wavelength step sizes ($\Delta \lambda_{signal}, \Delta \lambda_{pump}$) would be expected to reveal the measured data points oscillating about a trend more in line with what is expected. The strength of the Fabry-Perot oscillations can be reduced through proper impedance matching of the channel waveguide and the PhCWG [123]. The high resolution data would also make it possible to accurately probe the dispersion and observe the wavelength/group index at which point disorder can no longer be considered perturbative [124].

As was discussed previously, the large dispersion of the PhCWG is responsible for the reduction in the bandwidth over which an appreciable idler power is measured as the pump
wavelength gets closer to $\lambda_c$. The increasing values of $\beta_2(\lambda)$ and $\beta_4(\lambda)$ (See Fig. 6.6(a)) cause a larger phase mismatch between the interacting wavelengths within the propagation length of the PhCWG, resulting in the efficiency of conversion falling off more quickly with respect to pump and signal detuning. This phenomenon can be directly observed in the experimental data in Fig. 6.5a by observing the shrinking signal detuning bandwidth over which a measurable idler power is observed as the pump wavelength moves towards $\lambda_c$, giving the plot a distinctive triangular appearance. This limitation of the W1 waveguide has been reported previously [113] and can be mitigated by the introduction of regions of low-GVD into the waveguide dispersion by structural optimization [84, 125].

To characterize the decrease in the FWM bandwidth as the pump is moved closer to $\lambda_c$ the bandwidth of the measured conversion efficiency is plotted in Fig. 6.8(a). Due to the strong Fabry-Perot oscillations and limited resolution of the experimental data a conventional -3dB bandwidth could not be accurately gauged so we define the bandwidth as the total detuning range over which the conversion efficiency is within -10 dB of the maximum value for each pump wavelength. The same definition is applied to the simulated data to derive the bandwidth from the model. The measured results agree well with the model except for the last two measured pump wavelengths. This can be attributed to the imperfect match between the measured group index and the shifted numerically derived group index used for the model (illustrated in Fig. 6.3) at these wavelengths. However, when the measured bandwidth is plotted with respect to group index, as it is in Fig. 6.8(b), this discrepancy is removed. The remaining deviations between experiment and model here are attributable to the finite signal wavelength step (0.1 nm) and the Fabry-Perot induced uncertainty in both the group index when using the phase shift method [82] and transmission when determining the -10 dB wavelengths.

It was stated previously that the model employed in this work uses a linear group index ($n_g$) scaling of the linear propagation loss: $\alpha^* = \alpha_0(n_g/n_0)$, where $\alpha_0$, the minimum propagation loss, was 10 dB/cm. A linear scaling of loss with respect to group index was chosen because it was found to give the best fit to the experimental data. This fact is
Figure 6.8: Four-Wave Mixing bandwidth of the photonic crystal waveguide. (a) The -10-dB bandwidth versus pump wavelength and (b) the -10-dB bandwidth versus $n_g$. Measurement (circles) and calculation (solid line).

illustrated in Fig. 6.8 where the data presented in Fig. 6.8(b) is shown with the conversion efficiency of the model for different propagation loss $n_g$ scaling factors with all other variables being equal. A number of recent experiments and theoretical investigations on the exact scaling of loss, both scattering loss due to fabrication disorder and loss caused by coupling to the backward propagating mode, with respect to group index in photonic crystal waveguides has provided a number of differing opinions on the subject [72, 83, 124, 126, 127], with several reports concluding that the fabrication disorder-induced loss in photonic crystal waveguides, like that of regular channel waveguides[128], scaling linearly with group index and that the loss from scattering of the forward propagating mode into the backward propagating mode scaling quadratically with group index. However, fabrication disorder can cause coherent scattering within and between unit cells of the waveguide along the direction of propagation[129]. In addition, there is a modification of the propagating Bloch mode shape with respect to frequency which has the effect of increasing the disorder sampling. These two factors have been used to suggest that the Beer-Lambert law and the simplistic view of relating group index scaling of loss fail for large group indices[130]. In regards to this experiment, since the majority of the data collected was in the $n_g$ less than 40 regime, the dominant scaling of propagation loss observed in this experiment was linear.
Figure 6.9: Four-Wave Mixing conversion efficiency dependance on group index. Conversion Efficiency dependence on $n_g$ at $\lambda_{pump}$ for different propagation loss $n_g$ scaling factors: $\alpha^* = \alpha_0(n_g/n_0)^\delta$, $\delta = \sqrt{2}/2$ (dash), $\delta = 1$ (solid), $\delta = \sqrt{2}$ (dot), $\delta = 2$ (dash-dot), measurement (circles).

6.5 Conclusion

We have experimentally investigated the group index enhancement of four-wave mixing in a W1 silicon photonic crystal waveguides. A 0.44 mm long waveguide exhibited a maximum conversion efficiency of -36 dB using a coupled pump power of 14 dBm. Over the wavelengths examined a group index enhancement of the third-order nonlinearity resulted in an increase of the conversion efficiency over 12 dB from a $\Delta n_g \approx 55$. A corresponding decrease in the conversion efficiency bandwidth, from 5 nm to 0.5 nm, was also observed, which would severely limit the utility of the device in regard to most known applications of FWM in conventional silicon waveguide. Both of these experimental observations match well with a simple numerical model of four-wave mixing in photonic crystal waveguides which accounts for group index scaling of the propagation loss and nonlinear. The results presented here reinforce the slow-light nonlinear enhancement possible within silicon photonic crystal waveguides.
Chapter 7

Dispersion in Frequency Comb

Generating Silicon Nitride Microrings

7.1 Introduction

Silicon nitride ($\text{Si}_3\text{N}_4$) has long been identified as a viable platform for integrated photonic devices[13, 131]. It is a material system that is compatible with modern CMOS manufacturing much like the SOI photonics platform but brings an interesting alternative set of advantages/disadvantages when compared to the SOI platform. Notably $\text{Si}_3\text{N}_4$ has lower refractive index than silicon which allows for the creation of waveguides with lower loss[132]. However this comes with the trade-off in terms of integration since the lower index requires larger waveguide dimensions and large bending radii. Another advantage of $\text{Si}_3\text{N}_4$ is it large bandgap (5 eV) which means it is free of two-photon and free carrier nonlinear absorption at the common telecom wavelength region which is in contrast to silicon which suffers from both these phenomenon at those wavelengths.

One of the specific areas where $\text{Si}_3\text{N}_4$ integrated photonics has shown very promising application is the use of $\text{Si}_3\text{N}_4$ microrings to generate parametric frequency combs[133, 134]. The low propagation loss of waveguides and large quality factor resonators that can be fabricated using the $\text{Si}_3\text{N}_4$ platform have provided a unique integrated platform for the
Figure 7.1: Four-wave mixing comb generation in a microring resonator. Degenerate FWM from the pump resonance produces a cascade of processes that generates products occupying the other resonances of the microring.

generation of frequency combs.

Parametric frequency combs can be generated in microresonators due to a series of cascaded four-wave mixing processes aligned to the resonances of the resonator[135] as shown in Fig. 7.1. The pump laser tuned to a resonance generates sidebands due to degenerate FWM and these sidebands can themselves undergo degenerate FWM to occupy successive resonance modes. Non-degenerate FWM can occur between resonances to fill in subsequent resonant modes to create a dense spectrum of optical frequencies. A major limiting factor to the capability to continue this cascade of generated products is the deviation of the cavity resonances from the ideal free spectral range of the resonator[136] due to dispersion within the resonator.

7.2 Silicon Nitride Microrings

The fabrication of our Si$_3$N$_4$ microring resonators was, like our SOI integrated photonic devices, performed at the Institute of Microelectronics in Singapore. The Si$_3$N$_4$ fabrication process involves the growth of a 3µm thick SiO$_2$ layer using plasma-enhanced chemical vapor deposition (PECVD) on p-type 8 inch silicon carrier wafers. On this oxide layer a 725 nm thick Si$_3$N$_4$ layer is deposited using low-pressure chemical vapor deposition
(LPCVD). The Si$_3$N$_4$ was then patterned using 248 nm deep-ultraviolet photolithography and reactive ion dry etching. After etching the wafers were then covered with a 3 µm SiO$_2$ cladding using LPCVD for the first 500 nm and then PECVD for the remaining 2500 nm.

Figure 7.2: Photo and dimensions of Si$_3$N$_4$ microring. (a) Optical microscope image of the ring resonator and bus waveguide. (b) Diagram of important ring and waveguide dimensions.

The microring examined in this work has radius of 200 µm. The waveguide width is 2 µm and height is 0.725 µm. The waveguide and ring gap is designed to be 0.5 µm. The microring couples to a waveguide of equal cross-section that has spot-size mode converters[137] to reduce insertion loss that have dimensions optimized for the Si$_3$N$_4$ platform. The spot-size converter consists of an inverse taper from 0.2 µm to 2.0 µm over a length of 600 µm.

### 7.3 Characterization of Silicon Nitride Microrings

The characterization of the SiN microring resonators required accurate and repeatable transmission measurements. The goal is to accurately locate the ring resonances with respect to wavelength so that the underlying waveguide dispersion can be discerned. In order to achieve this in situ wavelength calibration during each measurement is done by measuring the transmission of a gas cell exhibiting calibrated absorption features simultaneously with the transmission of the microring. The acquisition of both the microring transmission and
gascell transmission is clock using an auxiliary Mach-Zehnder interferometer as an optical frequency acquisition clock for the data acquisition system. This experimental setup and procedure shares much in common with the swept wavelength dispersion measurement system discussed in Chapter 3.

**Figure 7.3: Experimental setup for Si$_3$N$_4$ microring characterization.** A tunable laser is split into three separate paths. The first path is used to interrogate the microring. A second path passes through a Hydrogen Cyanide (HCN) gas cell to provide an absolute wavelength reference. A third path is passed through a fiber Mach-Zehnder interferometer to provide a wavelength reference clock that is used to trigger the acquisition system to acquire the photodetector signals of the other two paths.

The experimental apparatus for measuring the microring transmission is shown in Fig. 7.3. The microresonator transmission, from which quality factor and FSR values are determined, was measured using a tunable laser (Ando AQ4321A, Ando AQ4321D) swept through its full wavelength tuning range (AQ4321A: 1480-1580 nm, AQ4321D: 1520-1620 nm) at a tuning rate of 40 nm/s. For absolute wavelength calibration, the microresonator and gas cell transmission were recorded during the laser sweep by a data acquisition system (PCI-6132, National Instruments) whose sample clock was derived from a photodetector monitoring the laser transmission through an unbalanced fiber Mach-Zehnder Interferometer (MZI). The MZI has a path length difference of approximately 40 m, making the measurement optical frequency sampling resolution 5 MHz (~0.14 pm).
The absolute wavelength of each sweep was determined by fitting 51 absorption features present in the gas cell transmission to determine their subsample position, assigning them known traceable wavelengths[138] and calculating a linear fit in order to determine the full sweep wavelength information. Each resonance was fit with a Lorentzian lineshape unless a cluster of resonances were deemed too close to achieve a conclusive fit with a single Lorentzian. In the case resonances were deemed too close together a N-Lorentzian fit was utilized where N is the number of resonances being fit.

**Figure 7.4: Transmission of a Si$_3$N$_4$ microring resonator.** a) Broadband transmission of microring resonator with identified TE resonances marked (blue: TE$_{11}$, yellow: TE$_{21}$, green: TE$_{31}$) b) Resonance families detail, also visible is the TM$_{11}$ mode due to some amount of polarization conversion within the microring. c) Single TE$_{11}$ resonance showing Lorentzian fit and calculated quality factor.

A full transmission measurement of a Si$_3$N$_4$ microring resonator can be seen in Fig. 7.4(a). The detected mode families have been highlighted by markers in the figure with a detail of closely spaced resonances show in Fig. 7.4(b). It can be observed that even though this was a TE polarization measurement, determined by the input polarization, a TM
resonance was measured due to some amount of polarization rotation that can occur in ring resonator structures[139, 140] when the resonances overlap spectrally. An example fit of an isolated resonance from the $TE_{11}$ family is shown in 7.4(c) showing a measured quality factor of $1.2 \times 10^6$.

![Figure 7.5: Quality factor of $TE_{11}$ mode resonances.](image)

**Figure 7.5: Quality factor of $TE_{11}$ mode resonances.** Quality factor for TE resonances of Si$_3$N$_4$ microring. The large drop in quality factor at 1520 nm can be attributed to the N-H bond absorption.

Focusing on the fundamental TE mode family ($TE_{11}$) we examine how its measured quality factor varies with respect to wavelength in Fig. 7.5. A local minimum in measured quality factor is observed at 1520 nm with quality factors seeming to increase with wavelength. This observed reduction in Q-value is attributed to residual hydrogen in the Si$_3$N$_4$ film, the resulting N-H bonds are well known to sources of absorption in Si$_3$N$_4$ films[13, 132, 141]. In order to drive out the hydrogen and achieve a more stoichiometric film, high temperature annealing has been shown to be effective[133].
7.4 Frequency Comb Generation from Silicon Nitride Microrings

In order to generate a Kerr frequency comb using the described a tunable laser (Ando AQ4321D, Santec TSL-510C) was amplified using an erbium doped fiber amplifier (EDFA) (IPG EAD-3K-C, Manlight HWT-EDFA-B-SC-L30-FC/APC). The input power was 25-30 dBm and the fiber to fiber coupling loss was measured to be 6 dB. The amplified laser light was coupled into the microring resonator and the wavelength would be tuned into a resonance while monitoring the output spectrum on an optical spectrum analyzer (Advantest Q8384). It was observed that for some resonances only limited cascaded four-wave mixing was observed and no parametric Kerr comb was generated.

![Frequency Comb](image)

**Figure 7.6:** Si$_3$N$_4$ microring generated frequency comb. Frequency comb generated in the Si$_3$N$_4$ microring by pumping a resonance near 1566 nm.

A small subset of resonances did manage to produce viable frequency combs (See Fig. 7.6 and Fig. 7.7). It was found that if a resonance did manage to produce a comb it was
not guaranteed the surrounding resonances would also produce a frequency comb. The uniformity of the generated comb was also found to depend on which resonance was used for pumping. The comb generated by pumping a resonance near 1566 nm (See Fig. 7.7) was found to be much more uniform than the comb generated by pumping a resonance near 1566 nm (See Fig. 7.6). It was initially thought the capability of a resonance to produce a Kerr comb was to be dominated by the quality factor but since resonances with similar quality factors, and in some cases larger quality factors, were failing to produce combs a more thorough analysis of the resonance dispersion of our microrings was undertaken to understand its contribution to comb generation.

### 7.5 Dispersion Analysis of Ring Resonator

In order to retrieve a though analysis of the resonance distribution in our Si$_3$N$_4$ microrings, high resolution transmission measurements were performed for both TE and TM transmission in order to identify all resonances within the wavelength region of interest. In analyzing the transmission five mode families of resonances were identified, three for the...
TE polarization (TE\textsubscript{11}, TE\textsubscript{21}, TE\textsubscript{31}) and two for the TM polarization (TM\textsubscript{11}, TM\textsubscript{21}). We are interested in examining how the free spectral range (FSR) of these resonance families change with respect to wavelength. We define FSR as

$$FSR(\lambda_n) = v_n - v_{n-1}$$

(7.1)

where $v_n$ is the optical frequency of the measured resonance. We plot the measured FSR for the various mode families in Fig. 7.8

![Free spectral range of identified resonances](image)

**Figure 7.8: Free spectral range of identified resonances.** Measured free spectral range of identified resonances from both polarizations.

We are most interested in the effect the other mode families have on disrupting the resonance distribution of the TE\textsubscript{11} family since it these resonances that are responsible for comb generation. In Fig. 7.9 we have plot the frequency difference between each mode families resonances and the TE\textsubscript{11} resonances as well as the TE\textsubscript{11} FSR. It can be seen that there are significant deviations, up to 50 MHz, to the TE\textsubscript{11} FSR caused by interaction with other mode families. Vertical lines have been added to Fig. 7.9 to show where the frequency difference between the TE\textsubscript{11} resonance and the resonance of another modal family the TE\textsubscript{11} FSR deviates. Significant deviations can be seen around 1555 nm due to overlap with the TM\textsubscript{21} resonance and near 1590 nm due to overlap with the TE\textsubscript{21} and TE\textsubscript{31} resonances.
Figure 7.9: Free spectral range of TE$_{11}$ mode compared to the resonances of the other mode families. (Top) Frequency difference between other mode families and the fundamental TE mode (TE$_{11}$) (bottom) FSR of the TE$_{11}$ mode, vertical bars highlight large disruptions in the FSR of the TE$_{11}$ family are due to overlap with the resonances of the other families (close to zero frequency difference on top plot).

7.6 Conclusion

This chapter details the effort to understand the unique mode structure of Si$_3$N$_4$ microring resonators by leveraging some of the methods that were developed to characterize silicon photonic crystal waveguides. The Si$_3$N$_4$ microring resonators have the capability of generating wideband Kerr parametric frequency combs and therefore are of great interest. By using high resolution swept wavelength transmission measurements that make use of absolute wavelength calibration via a simultaneous gas cell transmission measurement it was possible to measure to a high precision the various families resonances that exist in these microring resonators.

Analysis of the distribution of these resonance families it was possible to identify not only the underlying waveguide/ring dispersion of the devices but also the deviations caused by resonance interaction. Understanding these resonance interactions is fundamental in
optimizing these structures for improving frequency comb generation.
Chapter 8

Summary and Future Outlook

8.1 Summary

The goal of this thesis was to detail the design, fabrication, and characterization of photonic crystal waveguides as well as explore and demonstrate the enhancement of nonlinear optical effects such as Raman scattering and Four-Wave Mixing that can occur in these devices due to their unique dispersion. Additionally, we leveraged some of the methods used to characterize photonic crystal waveguides to examine the dispersion of Si$_3$N$_4$ microring resonators and explore the effect it has on their ability to generate parametric frequency combs.

In Chapter 2 we introduced the design, fabrication, and characterization of silicon photonic crystal waveguides. The design parameter space was detailed by examining how physical dimensions such as lattice constant, hole size, and slab thickness effect the optical transmission properties. We then detailed our layout and fabrication process including the procedure for creating air-bridged photonic devices. Finally, we demonstrated and examined the transmission of our fabricated photonic crystal waveguides with the goal of understanding the limits of our fabrication accuracy. These examinations allows us to accurately fabricate waveguides with specific transmission properties that were required for our nonlinear optical studies.
The optical transmission measurement of Chapter 2 were sufficient to identify the spectral features of interest, such as the waveguide cutoff region, however in order to thoroughly explore the slow-light dispersion of these devices further characterization was required. Chapter 3 details our experimental characterization methods of measuring the dispersion of slow-light photonic crystal waveguides. We discuss the four methods of measurement that were examined in our studies: the Fabry-Perot Method, the Phase-Shift method, Optical Low Coherence Interferometry, and Swept-Wavelength Interferometry.

The Fabry-Perot method was demonstrated and praised for its simplicity since the group index is derived solely from the transmission spectrum. However this simplicity comes at the cost of its limitation to the non-ideal device exhibiting pronounced Fabry-Perot transmission oscillations as well as the resolution of the measurement being tied to the device geometry.

The Phase-Shift method was introduced and demonstrated as a well known method of dispersion measurement commonly used in the characterization of fiber optics. The relative benefits of the Phase-Shift method were discussed and demonstrated by measuring the phase delay through a photonic crystal waveguide. This method has an increased wavelength resolution over the Fabry-Perot method however difficulties with measurement dynamic range as well as the required resolution for small device geometries limits it to longer structures.

Optical Low-Coherence Interferometry was introduced as a method that could directly measure the complex optical transfer function of a photonic crystal waveguide. Experimental realization of this technique was demonstrated and the difficulties in improving on these initial results was discussed. Lastly, Swept-Wavelength Interferometry was introduced as a method that can have high wavelength and group delay resolution while also exhibiting a large measurement dynamic range. In the demonstrated implementation it also exhibits built in high absolute wavelength repeatability through the use of a gas-cell absorption feature calibration. As well, the experimental apparatus required for the measurement was simplified when compared to Optical Low-Coherence Interferometry. The obvious difficulty
with this method is the requirement of a high quality mode hop free tunable laser source for the wavelength of interest.

With the design, fabrication and characterization of photonic crystal waveguides demonstrated, we then turned our attention to examining the effect of the slow-light behavior of these devices has on nonlinear optical interactions. This examination begins in Chapter 4 with an examination of the theoretical underpinning of Spontaneous Raman Scattering in silicon photonic crystal waveguides. Since these waveguides have unique wavelength dependent modal distributions that deviate largely from conventional waveguide structures especially in the region of slow-light near the mode cutoff we examined numerically the effective Raman susceptibility. We also preemptively examined the viability of various pumping schemes for experimentally examining these effects. The conclusion of this analysis was that for the proposed silicon photonic crystal waveguides it is viable to observe an enhancement of the Spontaneous Raman Scattering in the region of slow-light.

With a viable theoretical underpinning realized we then explored an experimental realization of observing enhanced spontaneous Raman scattering in silicon photonic crystal waveguides in Chapter 5. This realization began with the design and fabrication of photonic crystal waveguide that exhibited the specific transmission properties required by our experimental setup. The resulting measurement showed an enhancement of the spontaneous Raman scattering to increase by as much as six times in the wavelength region of slow-light.

After successfully demonstrating an enhancement of spontaneous Raman scattering due to the slow-light effect in silicon photonic crystal waveguides we next examined the enhancement of Four-Wave Mixing in these waveguides in Chapter 6. Although the spectral requirements for the waveguides to demonstrate enhanced Four-Wave Mixing were less exacting than in the Raman case since there was no need to ensure transmission at both the pump and Stokes wavelengths the experimental procedure for the measurement of Four-Wave mixing was notably more complex. We successfully demonstrated a 12-dB enhancement of the Four-Wave Mixing conversion efficiency in the wavelength region of increased group index.
Finally, we leveraged some of the methods utilized in the dispersion analysis of photonic crystal waveguides to examine the unique dispersion of silicon nitride microrings. These microrings had exhibited the capability to generate broadband parametric frequency combs the capability of which depends greatly on the dispersion properties of the underlying waveguide and materials. Utilizing our optically clocked, gas-cell calibrated characterization setup we were able to take high resolution transmission measurements of the microrings. Analysis of these transmission measurements allowed for the identification of the supported mode families for these resonators and the effect the interaction of these mode families have on the underlying dispersion of the device. Understanding the interaction of these mode families is critical in developing resonators that can be used to generate ultra-broadband frequency combs.

8.2 Future Outlook

The study of photonic crystal waveguides and their capability to enhance nonlinear optical phenomenon is still a very active area of research. A number of alternative device structures have been put forth\[84, 142\] in the hope to enlarge the bandwidth of the slow-light region making the devices more suitable for practical applications.

The practical applications of photonic crystal waveguides predominantly reside in two major areas: optical telecommunications and optical sensing. We have explored some initial investigations in the application of photonic crystal waveguides to the transmission of 10 Gb/s signals[143] and others have examined much higher bit rates[144]. The underlying limitation for photonic crystal waveguides in this particular application is the bandwidth of the slow-light region. In this application area the demonstration of compact all-optical logic gates has been demonstrated[145] in dispersion engineered photonic crystal waveguides. This a direct application of slow-light enhanced Four-Wave mixing that was demonstrated in this work.

The majority of studies on slab photonic crystal waveguides has been in the silicon-on-
insulator material system owing to reasons touched upon in the introduction of this thesis. However a number of recent investigations have explored photonic crystal waveguides in more exotic material systems such as chalcogenides[146]. Some chalcogenide glasses can exhibit nonlinear optical parameters 20 times greater than silicon[147]. Another material system being explored for photonic crystal waveguides is silicon rich silicon nitride[148]. The benefits of the silicon nitride photonics platform were discussed in Chapter 7 however the goal of silicon rich silicon nitride is to take advantage of the benefits of silicon nitride while increasing the nonlinear optical performance of the material[149]. Another interesting avenue of research is the enhancement of the conventional silicon-on-insulator platform by the addition of graphene. It has been demonstrated that graphene can enhance the Four-Wave Mixing conversion efficiency of photonic crystal waveguides[150].
References


Appendix A

Hydrogen Cyanide (H^{13}C^{14}N) Gas Cell

**Figure A.1: Hydrogen cyanide gas cell transmission.** The measured transmission of the Wavelength References Inc. HCN-13-100 gas cell. Numbers indicate the calibrated wavelength reference lines listed in Table A.1
Table A.1: Calibrated absorption wavelengths for the hydrogen cyanide gas cell. Calibration wavelengths of the Wavelength References Inc. HCN-13-100 Gas Cell. Wavelength uncertainty is ±0.0015 nm for all lines except for ones marked with * which have ±0.0006 nm uncertainty.

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Appendix B

Balanced Photodetector

Figure B.1: Fiber coupled balanced photodetector circuit. This is the photodetector used for the swept-wavelength interferometry. PD1 and PD2 are JDSU EPM 606LL InGaAs photodiodes.

The swept-wavelength interferometry apparatus required a balanced photodetector circuit (See Fig. B.1) that utilized fiber coupled, low back reflection photodiodes (JDSU EPM 606LL). The first operational amplifier (A1) is used in the traditional transimpedance amplifier (TIA) configuration. The feedback capacitance was accomplished using a "gimmick capacitor"; a short twisted pair of enamel wire cut to length in order to achieve a flat gain.
response from the TIA. The variable resistor was included to offset any differences in dark currents that can occur in unmatched photodiodes however it was found unnecessary during construction if the photodiodes were matched by selection. The second operational amplifier (A2) acts as a buffer and cable driver to a BNC connector. Not shown in Fig. B.1 is the supply decoupling capacitors and linear voltage regulation for the photodiode bias voltage supplies and operational amplifier supplies.
Appendix C

Published Work

C.1 Publications


**C.2 Conference Proceedings**


C.3 Posters