Long-term versus Short-term Contracting in Salesforce Compensation

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ABSTRACT

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This dissertation investigates multi-period salesforce incentive contracting. The first chapter is an overview of the problems as well as the main findings. The second chapter continues with a review of the related literatures. The third and fourth chapters address a central question in salesforce contracting: how frequently should a firm compensate its sales agents over a long-term horizon? Agents can game the long-term contract by varying their effort levels dynamically over time, as discussed in Chapter 3, or by altering between a “bold” action and a “safe” action dynamically over time, as discussed in Chapter 4.

Chapter 3 studies multi-period salesforce incentive provisions when agents are able to vary their demand-enhancing effort levels dynamically. I establish a stylized agency-theory model to analyze this central question. I consider salespeople’s dynamic responses in exerting effort (often known as “gaming”). I find that long time horizon contracts weakly dominate short time horizon contracts, even though they enable gaming by the agent, because they allow compensation to be contingent on more extreme outcomes — this not only motivates the salesperson more, but also leads to lower expected payment to the salesperson. A counterintuitive observation that my analysis provides is that under the optimal long time horizon contract, the firm may find it optimal to induce the agent to not exert high effort in every period. This provides a rationale for effort exertion patterns that are often interpreted as suboptimal for the firm (e.g., exerting effort only in early periods, often called “giving up”; exerting effort only in later periods, often called “postponing effort”). I also discuss the implication of sales pull-in and push-out, and dependence of periods (through limited inventory) upon the structure of the optimal contracting.

Chapter 4 examines multi-period salesforce incentive contracting, where sales agents can dynamically choose between a bold action with higher sales potential but also higher variance,
and a safe action with limited sales potential but lower variance. I find that the contract format is determined by how much the firm wants later actions to depend on earlier outcomes. Making later actions independent of earlier demand outcomes reduces agents’ gaming, but it also reduces an agent’s incentive to take bold actions. When the two periods are independent, an extreme two-period contract with a hard-to-achieve quota, or a polarized two-period contract allowing agents to make up sales, can strictly dominate a period-by-period contract, because they induce more bold actions in earlier periods by making later actions dependent on earlier outcomes. However, when the two periods are dependent through a limited inventory to be sold across two periods, the period-by-period contract can strictly dominate the two-period contract, by allowing the principal more flexibility in adjusting the contract.
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Chapter 1

Introduction

1.1 Introduction

Salesforce expenditures account for 10%-40% of the revenues of US firms (Albers and Mantrala 2008); this is of the order of hundreds of billions of dollars annually and is about three times the amount spent on advertising (Zoltners et al. 2008). Consequently, how to best motivate salespeople is of prime importance to most firms, and salesforce compensation design problems have drawn significant attention from economists and marketing researchers, prominent early papers in each area being Hölmstrom (1979) and Basu et al. (1985), respectively. In most cases, demand outcomes are uncertain and sales activities (e.g., how much effort the agent exerts, what actions the agent takes) are not fully observable by the firm; this makes the determination of compensation, which is to reimburse for sales agents’ costly actions, a difficult problem.

Typical compensation contracts used by firms are comprised of a fixed part (e.g., base salary) and a variable part (e.g., commissions on sales or discrete bonuses awarded for achieving a quota of sales during a specified time period). Based on a survey of Fortune 500 firms conducted by Joseph and Kalwani (1998), 58% of firms use commissions and over 90% use
quota-based contracts in their compensation plans. Quota-based plans provide stronger incentives for salespeople to reach higher levels of sales, as variable compensation is awarded only when certain sales goals are met. However, if the quota is either too difficult or too easy to achieve, the actions taken by the salesperson will be suboptimal. This issue does not exist with a commission-based contract, since every additional sales unit brings in additional reward. Determining whether a firm should utilize quota-based bonuses or commissions to incentivize their salesforce is an important problem.

Furthermore, firms employ salespeople for extended periods of time and they have to determine how frequently salespeople are evaluated and rewarded, e.g., monthly, quarterly, semi-annually, annually, a combination of these, etc. Coughlan and Joseph (2012) refer to this as the time horizon over which rewards are offered to the salesperson, and state that essentially all firms face this problem. When quota-based incentives are used in such a multi-period setting, the issue of dynamic gaming arises. This is because, in a multi-period scenario, agents may strategically adjust their actions over time, based on the way in which uncertain outcomes are realized and the way the contract determines their reward during current and future periods.

Indeed, dynamics gaming and the associated time horizon problem can extend beyond the salesforce compensation context, and have a broader implication for many aspects of firms’ decision-making. In financial markets, venture capitalists need to decide how often they should finance entrepreneurs in order to maximize financial return (Diamond 1993). Individual option investors make decisions about when to exercise their options, by trading off between realizing current gains and forgoing future profits (Merton et al. 1973). In supply chain management, understanding how often firms should replenish their inventory when faced with uncertain demand over a long-term horizon, is a fundamental topic of study (Scarf 1960). However, despite the importance of salesforce compensation, analytical research on optimal horizons of salesforce compensation remains scarce.
In this thesis, I attempt to narrow this gap by comparing long-term and short-term contracting in Salesforce compensation. I consider agents’ dynamic gaming within two different contexts, in Chapters 3 and 4. In Chapter 3, I discuss how agents can game the long-term contract by varying their effort levels dynamically over time. As a canonical example, consider a scenario in which the salesperson is paid a bonus if a particular sales quota is met in six months. To meet their six-month quota with minimum effort, the agent may strategically shirk work in the first quarter hoping for a high demand outcome without much effort, and exert greater effort only in the second quarter in case of low demand realization in the first quarter. For similar strategic reasons, sales agents who have already achieved the quota in early sales cycles may not have the incentive to put in extra effort later. This phenomenon of agents dynamically adjusting their sales effort — both postponement of effort exertion and shirking after achieving the quota — has been widely documented (e.g., Oyer 1998, Chen 2000, Steenburgh 2008, Misra and Nair 2011, Jain 2012, Kishore et al. 2013). Specifically, postponement of effort exertion is considered an especially bad outcome from the firm’s point of view because it involves not exerting effort before the sales quota is reached; it is sometimes known as the “hockey stick” pattern (because effort exertion is flat in early periods and increases sharply in later periods, thus taking the shape of a hockey stick; Chen 2000). Such perverse gaming incentives are, however, not present in commission-based contracts where every additional unit of demand brings in additional compensation.

In Chapter 4, I discuss how agents can game the long-term contract by altering between taking a “bold” action and a “safe” action dynamically over time. A bold action has both a larger upside potential, and a larger downside risk in generating sales, compared to a safe action. Unlike Chapter 3 in which agents’ effort exertion only increases the upside potential, in Chapter 4, effort exertion increases both the upside potential and the downside risk, with the former being greater than the latter. For example, when faced with a sales target, the salesperson can spend time reaching out to new potential customers, which is considered
bold since the reach might be unsuccessful; however if it is successful, it can bring higher profits to the firm. Or, a salesperson can follow up with existing customers (Godes 2004, Rubel and Prasad 2015), which has limited sales potential but is considered safer, since the sales relationship has already been established. Such agents’ dynamic gaming behaviors induced by a long-term incentive plan have been empirically documented by Chevalier and Ellison (1997a). They find that, when faced with a yearly performance review, mutual fund managers have an incentive later in the year to invest in riskier assets if they are close to a performance target; they may also have an incentive to invest in safer assets and act more passively if they have already met the performance target. The incentives to alter the riskiness of investment described above are derived from management fees as a nonlinear function of the calendar-year return.

In response to agents’ dynamic gaming, a firm has multiple options for structuring the time horizon for a contract. A firm could offer a short time horizon contract that evaluates and rewards an agent over a short-term horizon (e.g., every quarter). Alternatively, the firm could offer a longer time horizon contract that evaluates and rewards the agent over a long-term horizon (e.g., six months); in this case, the firm would also have to determine the reward for a larger set of possible realizations. In each of these cases, the firm would have to determine the structure of the contract, for instance, whether it should include only commissions or whether it should include rewards for reaching quotas. If the firm chooses to reward over a longer time horizon, with a quota-based contract, it would be more exposed to gaming.

In Chapter 3 and Chapter 4, I ask the central question that firms face when employing salespeople for an extended period of time: how often should sales agents be awarded? Should the compensation be granted over a long-term period or be based on agents’ short-term performance? Agents can game the long-term contract by varying their actions over time; Chapter 3 focuses on agents’ effort exertion, and Chapter 4 focuses on agents’ dynamic
engagement of bold and safe actions.

In Chapter 3, I examine multi-period salesforce incentive contracting in a two-period scenario, involving a risk neutral agent with limited liability. In this setting, sales agents can choose to vary their levels of effort dynamically. I show that two often ignored factors—the agent’s outside option and level of limited liability—are important determinants of the optimal time horizon and contract form, and of the agent’s effort-making. While it is elementary that a fully flexible two-period contract will weakly dominate a period-by-period contract, I find that a two-period contract can strictly dominate a period-by-period contract, because it allows the reward to be contingent upon a more extreme sales outcome, even while it allows the agent to game effort exertion.

I show that this insight continues to hold when the agent can borrow or postpone sales between periods. However, if the time periods are dependent (e.g., a fixed amount of inventory has to be sold across the two time periods), then a period-by-period contract can strictly dominate given certain conditions. This is under the assumption that under the period-by-period contract, the agent chooses his action only based on the current period’s contract. However, if the agent is fully forward looking under the period-by-period contract, and can predict how the second period’s contract may change based on the outcome of the first period when there is a limited inventory, then a long-term contract still weakly dominates a short-term contract. I also derive implications for the agent’s multi-period effort profile and show that various profiles, including effort postponement, are induced under the firm’s optimal contract.

Chapter 4 looks at multi-period salesforce incentive contracting when sales agents can shift between a “safe” action and a “bold” action over two periods. I assume that the bold action has a greater probability of generating extreme sales (both high and low) relative to the safe action. Furthermore, the bold action’s upside potential is greater than its downside risk, and thus, is preferred by the principal to the safe action ceteris paribus. I focus on
the case where taking the bold action is more costly to the agent than the safe action, since otherwise, there would be no conflict of interest between the principal and the agent.

I find that the optimal contract for inducing a bold action in Chapter 4, can be structured differently from the optimal contract for inducing effort exertion in Chapter 3. The contract format is determined by how much the firm wants later actions to depend on earlier outcomes. The optimal two-period contract for the principal is an “account-balance contract”, an “extreme contract”, or a “polarized contract”, depending on the parameter space. The “account-balance” contract compensates the agent based on how many times the agent obtains high demand realization, and induces later actions that are independent of earlier demand outcomes. The “extreme” contract incentivizes bold actions via a hard-to-achieve quota, and induces later actions that are heavily dependent on earlier demand outcomes. The “polarized” contract allows agents to act bold to make up sales if demand in the first period is low, and induces later actions that are moderately dependent on earlier demand outcomes.

In choosing an optimal contract, the principal faces the tradeoff between making later actions dependent on earlier outcomes to incentivize more bold actions earlier on, and reducing losses from an agent’s gaming. Making later actions independent of earlier demand outcomes reduces agents’ gaming, but it also reduces incentives to take bold actions. Therefore, if providing incentives is of a higher order, then either the extreme contract or the polarized contract that induces later actions to be heavily or moderately dependent on earlier outcomes respectively, is optimal for the firm. If reducing gaming losses is of a high order, then the account-balance contract is optimal for the firm. Furthermore, it is weakly less costly to induce bold actions in earlier periods than later periods. As a result, the principal prefers to encourage bold actions from the agent in the early period (given agents’ limited liability). However, the optimal action precipitated during the second period is conditional on the first period’s demand outcome.
With independent periods, the long time horizon contracting weakly dominates short time horizon contracting, since the “account-balance” long term horizon contract is a replication of the short term contract. With dependent periods (i.e., a fixed amount of inventory has to be sold across two time periods), a period-by-period contract can strictly dominate under certain conditions, as when a bold action is induced in the early period but is not induced in the later period upon a successful first period. This result holds under the assumption that an agent chooses his action under the period-by-period contract only based on the current period’s contract.

Across the two chapters I find general insights regarding the optimal action profile to be encouraged by the principal. I find that given agents’ limited liability, in the presence of agents’ dynamic gaming, the principal prefers to elicit the desirable action (i.e., the action that leads to greater expected demand) from the agent in the early period. In the later period, however, the principal may prefer to implement the desirable action only upon demand realization that is more likely to occur in association with the desirable action in the earlier period. Specifically, in Chapter 3, effort exertion is induced in the later period only when the first period has a high demand realization. In Chapter 4, a bold action is induced in the later period when the first period has a high or low demand realization (rather than medium).

However, given different gaming behaviors in the agent (namely, effort variation in Chapter 3, and dynamic shifting between bold and safe actions in Chapter 4), the optimal contract structure may change. First, in Chapter 4, a polarized contract, rather than an extreme contract, may be the most effective at inducing the agent to take a bold action. This is because taking a bold action increases the probability of obtaining both high and low demand realizations. Then, compensating the agent at the end of the second period if the earlier demand realizes at extreme levels can motivate the agent to choose a bold action in the first period. Second, given agents’ limited liability, the optimally chosen two-period contract is
path-independent in Chapter 3, but may be path-dependent in Chapter 4, if the optimally chosen contract is the polarized two-period contract.

To address whether a firm should reward their sales agents over a long-term horizon or a short-term horizon, the major insights from the two chapters are that agents’ dynamic gaming, in terms of effort exertion or dynamic shifting between bold and safe actions, can benefit the principal if optimally induced. Overall, the two-period contract outperforms the period-by-period contract with independent periods. However, the period-by-period contract which gives the principal more flexibility in adjusting the contract may outperform the two-period contract, when the two periods are not independent, under the assumption that an agent chooses his action under the period-by-period contract only based on the current period’s contract.

Finally, I want to highlight the constraints that I impose on the contract structure and their impacts. First, I do not allow renegotiation under long-term contracting. This is critical to my results, since commitments to the two-period contract at the beginning of the first period are necessary for the two-period contract to outperform the period-by-period contract for the principal. As Fudenberg et al. (1990) state, long-term contracting is valuable only if optimal contracting requires commitment to a plan today that would not otherwise be adopted tomorrow. This is because rational agents who anticipate renegotiation at the time of exerting effort would have no incentive to exert more effort earlier on under long-term contracting relative to short-term contracting (conditional on the same bonus payment). Second, I assume that under short-term contracting, an agent chooses his action only based on the current period’s contract. This would not change my results with independent periods. However, with inter-dependent two periods, if the agent acts fully strategically under the period-by-period contract, and chooses the first period’s action by anticipating how the second period’s contract may change based on the outcome of the first period, then a long-term contract still weakly dominates a short-term contract.
Chapter 2

Literature Review

2.1 Literature Review

My research adds to the body of work on dynamic incentives with repeated moral hazard. One line of academic inquiry assumes the firm to be risk neutral but the agent to be risk averse, which leads to contracting frictions. Within this paradigm, Hölmstrom and Milgrom (1987) show that a linear contract is optimal for the principal when a number of other assumptions hold. I note that the gradual two-period contract in Chapter 3, and the account-balance contract in Chapter 4, that I derive as optimal for the firm under certain conditions, can be interpreted as a linear contract as well. However, these are only optimal for intermediate values of effort effectiveness (recall that I assume the agent to be risk neutral). A number of papers in the risk aversion paradigm revisit the assumptions of Hölmstrom and Milgrom (1987), and demonstrate the optimality of non-linear contracts (Rogerson 1985, Spear and Srivastava 1987, Schättler and Sung 1993, Sung 1995, Hellwig and Schmidt 2002, Sannikov 2008, Rubel and Prasad 2015). Fudenberg et al. (1990) analyze a variant of my question — when might a sequence of short-term contracts, similar to linear contracts, replicate a long-term contract? My work differs in two perspectives. First, the
“decreasing utility frontier” assumption in Fudenberg et al. (1990) is violated in my model due to agents’ limited liability. Second, I counter-intuitively show when a long-term contract cannot perfectly replicate, and may even perform worse, than a short-term contract.

A second line of inquiry on dynamic incentives assumes agents to be risk neutral and have limited liability, which leads to a different kind of contracting friction (my dissertation falls under this paradigm). Bierbaum (2002) studies how to elicit great effort from the agent in each of the two periods (which may not be profit-maximizing for the principal), while I allow different effort profiles to be elicited by optimal contracts, under varying conditions. Schmitz (2005) studies the question of whether to employ the same or different agents in two periods, given interdependence between the outcomes of the periods. Kräkel and Schöttner (2016) study the firm’s choice between offering commissions and offering bonuses, and determines conditions under which one or the other (or a combination) is optimal, when the reward must be paid at the end of multiple periods. Schöttner (2016) studies optimal contracting when the agent’s effort costs change over time. None of these papers, however, considers whether long time horizon or short time horizon contracts are optimal. Relatedly, they do not permit the agent to strategically borrow or postpone sales between periods, neither do they consider the case of limited inventory to be sold across two periods, which creates a particular form of interdependence between periods. In addition, these papers normalize the values of the outside option and the limited liability, and are unable to study comparative statics with respect to these quantities in association with the optimal contract and the induced effort profile.

My work is also related to both the quantitative and behavioral study of agents’ gaming by varying their actions over time under a long time horizon contract. Empirical literature has consistently documented that salespeople tend to shirk in the early periods and increase effort as they reach quota. Oyer (1998) and Steenburgh (2008) both document that firms sales increase at the end of fiscal year, suggesting that sales people postpone effort exertion until
the end of a compensation window to meet quota and get bonus. Misra and Nair (2011) finds evidence for shirking by agents in the early part of the compensation cycle. Furthermore, a significant increase in sales at the end of quarters suggest that agents tend to increase effort as they reach closer to the end of a compensation window. Existing explanations on the “hockey-stick” phenomenon where sales agents postpone effort exertion focus on suboptimal behaviors from either the principal or the agent. Chen (2000) shows that if quotas are not in line with an agent’s productivity, the salesperson may find it optimal to wait to resolve uncertainty over the realization of early demand shocks. On the other hand, Chung et al. (2014) focus on suboptimal gaming behaviors committed by the agent. They discover from a counterfactual analysis that effort concentration in later periods can arise from agents’ myopic behaviors. A forward-looking agent would smooth out efforts over time to take into account the uncertainty in future demand shocks. Goal literature, adopts a behavioral perspective to explain agents’ procrastination. Kivetz et al. (2006) propose the goal gradient hypothesis, and Heath et al. (1999) use goal-serving as a reference point to explain effort postponement in agents. The idea is that goals have diminishing returns, and thus combining multiple short term goals into a long term goal will result in less effort exertion earlier on. Also writing from the behavior standpoint, Jain (2012) studies the scenario where agents lack in self-control. Then, the principal can take advantage of agents’ lack of self-control to maximize her profits by paying a single bonus at the end, which essentially encourages effort postponement.

A second type of agents’ dynamic gaming predicted by my model — the “give-up” pattern where agents reduce effort after realizing that they have no chance to reach at a bonus also have been documented empirically. For instance, Steenburgh (2008) shows that for agents who are unlikely to make quota, they give up if they feel that they cannot make the quota. Jain (2012) argues that long time horizon quotas can sometimes lead to the salesperson decreasing effort in later periods, if he believes that he cannot make the quota. Chung and
Narayandas (2017) find empirical evidences that under a monthly quota plan, salespeople who had a series of bad luck early in the month may decide to give up late in the month because there is no chance that they can meet or exceed the quota set by the firm. An earlier paper from Chung et al. (2014) shows similar results that weak performers may give up if they realize that sales quotas under the long time horizon contract become unachievable. Also in this paper, Chung et al. (2014) present that the best performers will reduce productivity after attaining quotas. On a similar note, Misra and Nair (2011) show from their structural analyses that agents may shirk after they already bring in enough sales to meet a quota. These evidences correspond to a third type of dynamic gaming predicted by my model—the “relaxing” pattern where salespeople relax after they meet their quota. Separate from the above explanations for agents’ effort gaming patterns, my work in Chapter 3 provides the novel insight that agents’ gaming behaviors can indeed be optimally induced by the firm to improve the firm’s profits.

Most of the literature mentioned focuses on the demand-enhancing impact of effort, where effort does not directly affect variance in sales outcomes. In Chapter 4, I study a different problem where agents’ actions affect both the mean and the variance of demand outcomes. With risk averse agents, this can lead to contracting friction due to the associated risk-return tradeoff. Godes (2004) reveals that when risk can be endogenously determined by the agent, compared with exogenous risk, fixed salaries fail to provide sufficient incentives. As a result, the principal needs to increase the commission to incentivize effort. Another related paper comes through Rubel and Prasad (2015), who consider agents’ risk-shifting in a dynamic setting from the perspective of a carryover effect between two periods. Distinct from this stream of literature, the friction in my work stems from agents’ dynamic gaming rather than risk aversion.

Chapter 4 is also related to the literature on innovation in economics (Holmstrom 1989, Aghion and Tirole 1994, Manso 2011). Manso (2011) studies a juxtaposition of the moral-
hazard problem with the exploration-exploitation problem, where the “exploitation” action leads to a high failure rate in early periods, but can reveal information about potentially superior actions. The optimal contract in Manso (2011) has the common feature of early failure being tolerated, and long-term contracting being preferred over short-term contracting. However, in Manso (2011), the results are driven by interdependence between the two periods: the principal is more willing to reward short-term failure in order to inform the later period’s successful rate. In my work, there is no learning and the two periods are independent; the principal still benefits from aggregating contracts to provide the most effective incentives for agents with limited liability.

Many papers document agents’ dynamic gaming in altering between acting bold or playing safe under a long time horizon contract. Brown et al. (1996), and Chevalier and Ellison (1997b) find that fund managers appear to act bold late in the year by investing on portfolios with greater volatility and try to catch the market, if they are a few points behind. They also found that fund managers may also have an incentive to play it safe and act more like an index fund if they are ahead of the market. In Ederer and Manso (2012)’s experiment, subjects choose between following tips from the previous manager, or explore different locations to discover a more profitable strategy, which is considered bold since the new strategy may or may not be as profitable as the previous manager’s strategy. The variability of action choices significantly declines over the course of their experiment, suggesting that subjects tend to act bold in early periods and play safe in later periods.

My results that setting a hard-to-achieve quota by combining compensation periods, or providing makeup opportunities, can be more effective in inducing bold actions are also supported empirically. Chung and Narayandas (2017) collaborate with a retail chain at Sweden that sells electronic goods. They find that less frequent quotas encourage salespeople to sell more high-margin products and pursue fewer incremental sales. Lerner and Wulf (2007) suggest that the shift from compensating corporate R&D heads using short term
contracting to long term contracting is associated with more heavily cited patents. Both papers suggest that delayed rewards incentivize more bold actions. Furthermore, Azoulay et al. (2011), and Ederer and Manso (2013) show that protecting agents from earlier downside risk encourages bold actions at earlier periods. An article from Wall Street Journal \(^1\) reported that a growing number of startup companies are explicitly rewarding employees who acted bold but failed by giving cash prizes or trophies in order to encourage creativity. This evidence aligns with my result that offering agents makeup opportunities in later periods motivates agents to act bold in earlier periods.

My research also adds to the extensive literature on salesforce incentives in marketing which, in addition to the papers already cited, includes Raju and Srinivasan (1996), Simester and Zhang (2010) and Zhang (2016), among many others. Finally, insights from my dissertation are related to the contract design literature in operations research and finance. My extension into limited inventory is related to the work on salesforce compensation when operational considerations are important (Chen 2000, Plambeck and Zenios 2003, Dai and Jerath 2013, Saghafian and Chao 2014, Dai and Jerath 2016, Dai and Jerath 2018a, Dai and Jerath 2018b). The optimality of the quota-bonus contract in motivating sales agents’ effort is related to the optimality of the target rebate contract in incentivizing retailers’ ordering in supply chain management (Taylor 2002), in which the rebate is paid for each unit sold beyond a specified target. The implication of the quota-bonus contract in inducing effort exertion and risk seeking from the sales agent also has its counterpart in finance—debt structure induces effort and risk seeking behaviors among entrepreneurs by maximizing the agent’s residual value upon a positive outcome.

\(^1\)https://www.wsj.com/articles/SB10001424052970204010604576594671572584158
Chapter 3

Multi-Period Incentives and Effort Dynamics

3.1 Introduction

As I have introduced in Chapter 1, the problems of determining the time horizon of compensation and determining the optimal compensation structure are inter-related. This is an issue that essentially every company that uses a salesforce must resolve—non-linear quota-based incentive contracts lead to stronger incentives but invite gaming, while linear commission-based incentive contracts reduce gaming but also weaken incentives.

Previous empirical research has studied this tradeoff and has not reached a clear answer regarding which factor—the incentive effect or the gaming effect—dominates in a multi-period dynamic incentives scenario under which conditions. Oyer (1998) analyzes aggregate data from the Survey of Income and Program Participation (SIPP) for the years 1984–1988 spanning scores of industries in which quota-based plans are used and detects dynamic gaming effects, and suggests that this gaming hurts more than the incentive effect helps. Steenburgh (2008) analyzes individual salesperson-level data from a Fortune 500 company
that sells durable office products and uses quota-based plans, and determines that stronger incentives dominate the downside from gaming (and also states that analyzing these data in aggregate would produce results similar to those reported in Oyer (1998)). Misra and Nair (2011) uses a dynamic structural model to analyze data from a Fortune 500 contact lens manufacturer and shows that a plan that uses only commissions performed better than a quota-based plan (that was originally in use at the company); using only commissions makes the time horizon decision irrelevant. Kishore et al. (2013) studies this question using data from a large pharmaceutical firm in an emerging market and finds that commissions do better than quotas by preventing gaming, but this comes at the cost of neglecting non-incentivized tasks. Chung et al. (2014) uses a dynamic structural model to analyze data from a Fortune 500 office durable goods manufacturer and determines that quotas, through higher effort motivation, perform better than plans without quotas in spite of gaming effects being present; it also finds that both short-term and long-term quotas have roles to play. Across these studies, choosing a better (even if not “optimal”) compensation plan can lead to very significant increases in revenues and profits, of the order of 5% to 20%. These papers also carefully document the effort exertion profiles of agents induced by different types of contracts in a multi-period scenario. They consistently report effort postponement as an issue of concern in long time horizon contracts. Overall, existing empirical research has found the problem of determining the optimal time horizon (and contract form) to be highly relevant across a wide variety of scenarios but has reached mixed conclusions regarding this.

In this chapter, I conduct a theoretical investigation to shed light on this fundamental question that, arguably, any firm in any industry that employs a salesforce faces (and, in a recent review article, Coughlan and Joseph (2012) list as a very important yet under-researched issue in salesforce management): What time horizon should the firm use to evaluate and compensate the salesperson, and what should be the associated contract? Should the firm offer multiple sequential short time horizon contracts (which enables the firm to
have more control over the effort exertion of the salesperson in every period) or should it offer a long time horizon contract (which allows the salesperson more freedom to adjust his effort profile to “game” the system but also allows the firm to make variable compensation contingent on an outcome that is more difficult to achieve)? What are some key factors that influence this decision? Furthermore, what effort profile(s) will be induced by the optimal incentive contract, and does effort postponement by the agent always hurt the principal?

To answer these questions, I build a stylized principal-agent model in which a firm interacts with a salesperson for two time periods. In this context, using short time horizon evaluation implies offering two period-by-period contracts where each contract is determined at the start of a period and pays at the end of the period based on the outcome of the period. On the other hand, using long time horizon evaluation implies offering a two-period contract that is determined at the start of the first period and pays once at the end of the second period based on the outcomes of the two periods. I do not allow renegotiation under long-term contracting, and I will show that the ability to commit to a long-term contract makes the optimally chosen long-term contract outperform the optimally chosen short-term contract for the principal. (In the rest of the dissertation, I will use “long time horizon contracting” interchangeably with “two-period contract,” and “short time horizon contracting” interchangeably with “period-by-period contracts.”)

I assume the demand outcome in each period to be stochastically dependent on the effort exerted in that period, and assume the demand outcomes in the two periods to be independent of each other. In the two-period contract, the agent can dynamically adjust his effort level in the later period based on the early period’s demand outcome which also influences his first-period effort exertion decision. I assume that the firm and the salesperson are risk neutral, and that the agent has limited liability. Limited liability can be thought of as protection from downside risk for the salesperson, i.e., he will be guaranteed a minimum payment even in the case of an unfavorable market outcome (which is a robust feature of
real-world compensation plans). I assume that the agent’s limited liability can be lower or higher than his outside option; the latter can happen, for instance, when the salespeople’s skills are most valuable in a sales context and they cannot expect comparable compensation in other professions (Kim 1997, Oyer 2000).

My analysis shows that, for the firm, a fully flexible two-period contract weakly dominates a period-by-period contract, as expected. Interestingly, however, I find that the two-period contract, even though it allows gaming of effort by the agent, strongly dominates the period-by-period contract under certain conditions. In the optimal two-period contract it is sufficient to determine compensation based on the cumulative sales for the two periods and, under different conditions (discussed shortly), the optimal two-period contract is either an “extreme” contract that concentrates the reward only at the highest cumulative output level, or a “gradual” contract with rewards at all cumulative output levels. In fact, similar to Hölmstrom and Milgrom (1987), the optimal gradual two-period contract can be interpreted as identical to a commission contract. Furthermore, I obtain an interesting equivalence result that states that the optimal two-period gradual (commission) contract is identical in all ways (i.e., in terms of expected effort exertion, sales outcomes and total compensation) to the optimal period-by-period contract which is quota based; in other words, a long time horizon contract with commissions achieves the same outcomes as a short time horizon contract with quotas.

Whether the extreme long time horizon contract or the gradual long time horizon contract (equivalently, a sequence of short time horizon extreme contracts) is optimal can be explained by understanding the two familiar countervailing effects at play. The first is the beneficial “incentive effect,” which is that, given the agent’s limited liability, an extreme plan provides a larger incentive to work compared to a gradual plan because any output lower than the highest possible does not provide any additional reward. However, the extreme plan also leads to a negative “gaming effect,” that is, dynamic gaming of effort in the second period.
based on the outcome of the first period hurts the principal. The extreme contract is optimal when the incentive effect is stronger than the gaming effect, and this is the case when the effectiveness of the agent’s effort is either low or high. This is because in the extreme contract the loss in demand due to the gaming effect is larger for higher effort effectiveness, but in the optimally designed contract the probability that this loss will happen is lower for higher effort effectiveness. Therefore, the expected demand loss due to the gaming effect in the extreme contract is highest for intermediate effectiveness levels, and this loss is large enough to offset the incentive effect, so that in this region the extreme contract is not optimal. As limited liability decreases (fixing the agent’s outside option) the friction from moral hazard becomes smaller and the incentive effect becomes less important, so that the gradual contract becomes optimal in a larger parameter space.

In terms of the agent’s effort exertion, we find that multiple effort exertion profiles are possible under different conditions under the optimal contract—effort exertion in both periods; effort exertion in the first period and conditional effort exertion in the second period; and no effort exertion in the first period and conditional effort exertion in the second period. The last pattern is especially interesting as it implies that in the optimal contract the firm induces effort postponement (or “hockey stick” effort profile). This effort postponement is typically interpreted negatively (Chen 2000), and as something to avoid; our analysis shows that it indeed can be generated under an optimal contract even with independent periods, and this happens when limited liability is intermediate. This implies that one has to carefully understand and consider the setting and environmental factors when making inferences about contract efficiency from dynamic effort profiles of agents.

Next, I extend my basic model such that the two time periods are not completely independent. Specifically, I introduce the idea of an exogenous and limited amount of product inventory that has to be sold in the two periods, such that the contract design decisions for the principal in the two time periods become dependent. (Note that demand outcomes in
the two periods are still independent.) I assume that under the period-by-period contract, an agent chooses his action in a period only based on the current period’s contract. Under this assumption, in this scenario the principal may find it optimal to use a period-by-period contract in which the second-period contract is decided based on the outcome of the first period. Such a period-by-period contract can strongly dominate the two-period contract because it gives the principal more flexibility in adjusting the contract. This cannot be reproduced by a two-period contract under the assumption that contract terms do not depend on inventory levels. Furthermore, with limited inventory, the principal’s incentive to induce effort in the first period is lesser, i.e., the principal may optimally desire effort postponement by the agent in a larger parameter space.

A number of papers, including Oyer (1998), Steenburgh (2008), Misra and Nair (2011), Jain (2012), Chung et al. (2014) document another kind of gaming (in addition to effort gaming) in a dynamic incentives setting—they show that in a multi-period setting with non-linear contracts, sales agents pull in orders from future periods if they would otherwise fall short of a sales quota in one cycle, whereas they push out orders to the future if quotas are either unattainable or have already been achieved. I extend my basic model to study such strategic sales pull in and push out behavior, which also introduces dependence between the periods. Allowing this affects period-by-period contracts because it gives the agent more freedom to game the system. In accordance with this insight, I find that if sales pull in and push out is possible then a long time horizon contract becomes more attractive to the principal, because it evaluates the agent only for the output at the end of the two periods.

The rest of the chapter is organized as follows. In Section 3.2, I present the basic model with independent time periods. In Section 3.3, I analyze this model and obtain my key insights regarding the different forces at play, and the comparison between period-by-period and two-period contracts. In Section 3.4, I allow for periods to be dependent by assuming that the principal has limited inventory to be sold in the two periods. In Section 4.6, I
conclude with a discussion. The proofs for the results in Section 3.3 are provided in the Appendix, and those for the results in Section 3.4 are provided in the Online Appendix.

### 3.2 Model

I develop a simple agency theoretic model in which a firm (the principal) hires a salesperson (the agent) to exert demand-enhancing effort. There are two time periods denoted by \( t \in \{1, 2\} \). Demand in both periods is uncertain and independent. Let \( D_t \) be the demand realization in period \( t \), which can be either \( H \) or \( L \) with \( H > L > 0 \). The agent’s effort increases the probability of realizing high demand levels. The effort level in period \( t \), denoted by \( e_t \), can be either 1 or 0, i.e., the agent either “works” or “shirks” in each period; however, the principal does not observe the effort level. We can think of effort level 0 as a salesperson making a client visit (which is observable and verifiable) and effort level 1 as the salesperson’s additional effort spent in talking to and convincing the client to make the purchase (which the firm cannot observe or verify). Without effort exertion \((e_t = 0)\) demand is realized as \( H \) with a probability of \( q \), and with effort exertion \((e_t = 1)\) this probability increases to \( p \) \((0 < q < p < 1)\). A larger \( p \) implies greater effectiveness of the salesperson’s effort, while \( q \) can be interpreted as the natural market outcome. I assume that all the demand created can be met and each unit sold gives a revenue of 1 and has a marginal cost of zero. The cost of effort is given by \( \phi > 0 \) for \( e_t = 1 \) and is normalized to zero for \( e_t = 0 \).

I assume that both the firm and the salesperson are risk neutral. Unlike the firm, however, the salesperson has limited liability, implying that he must be protected from downside risk. Specifically, I assume that the salesperson has a limited liability of \( K \) in each period, i.e., to employ the agent for one period, the principal must guarantee a compensation of at least \( K \) under any demand outcome. Limited liability is a widely observed feature of salesforce contracts in the industry, and this assumption is a standard one in the literature
(cf. Laffont and Martimort 2009; examples in the salesforce literature include Sappington 1983, Park 1995, Kim 1997, Oyer 2000, Simester and Zhang 2010, Dai and Jerath 2013). The limited liability assumption also implies the existence of a wage floor to the salesperson, which is aligned with industry practice. I assume that the salesperson’s reservation utility is \( U \) for each period, and that the limited liability can be either lower or higher relative to the agent’s reservation utility. For instance, if the salesperson’s alternative employment opportunities are attractive, then limited liability can be relatively low compared with reservation utility, but if salespeople’s skills are most valuable in a sales context and they cannot expect comparable compensation in other professions, then limited liability can be relatively high compared with reservation utility (as also discussed in Kim 1997, Oyer 2000).

The agent is reimbursed for effort using an incentive contract. Effort is unobservable to the firm and demand is random but can be influenced by effort, so the firm and the agent sign an outcome-based contract. The firm can propose a disaggregate contract, i.e., two period-by-period contracts, where each contract is determined at the start of each period and pays at the end of the period based on the outcome of the period. Alternatively, the firm can propose a single aggregate two-period contract that is determined at the beginning of the first period and pays once at the end of the second period based on the outcomes of the two periods.\(^1\) I assume that under a period-by-period contract, an agent chooses his effort level during a period only based on the current period’s contract, and I do not allow renegotiation under the two-period contract.

\(^1\)The discrete demand distribution that I have assumed ensures that effort will not change the support of the demand distribution; otherwise, the principal may be able to infer the agent’s effort from the demand outcome and would induce the agent to work by imposing a large penalty for demand outcomes that cannot be obtained under equilibrium effort but can be obtained under off-equilibrium efforts, as argued in Mirrlees (1976).
3.3 Analysis

3.3.1 First-Best Scenario

I start by presenting the first-best solution (for instance, if the agent’s effort is observable). In this case, the two periods are independent and equivalent and it is sufficient to study just one period. The firm can implement any effort level $e_t$ in either period, by reimbursing the agent a fixed salary $s_t$ which must be at least $K$ while ensuring the agent’s participation. The principal’s problem in each period is the following.

$$\max_{s_t} \ E[D_t|e_t] - E[s_t|e_t]$$

s.t. 

$U_A(e_t) \geq U$ \hspace{1cm} (PC_t)

$s_t \geq K$ \hspace{1cm} (LL_t)

Here, $(PC_t)$ is the agent’s participation constraint, where $U_A(e_t)$ stands for the salesperson’s expected net utility on exerting effort $e_t$, which is equal to $s_t - \phi$ if the agent exerts effort and is equal to $s_t$ if the agent does not exert effort. It states that to employ the sale agent, the principal needs to provide a fixed salary that makes the agent’s expected net utility from exerting effort $e_t$ no less than his outside option, which simplifies as $s_t \geq U + \phi$ if effort is exerted, and as $s_t \geq U$ if effort is not exerted. $(LL_t)$ stands for the agent’s limited liability constraint, which ensures that the agent receives a fixed salary $s_t$ no less than his limited liability $K$.

If the contract specifies effort exertion in period $t \in \{1, 2\}$, i.e., $e_t = 1$, the principal’s expected profit is equal to the expected market demand subject to the agent’s effort exertion, $pH + (1 - p)L$, minus the minimal salary to ensure effort exertion, $\max\{U + \phi, K\}$, i.e., $pH + (1 - p)L - \max\{U + \phi, K\}$. If $e_t = 0$, the principal gets the natural market outcome and pays the minimal salary to employ the salesperson, i.e. $qH + (1 - q)L - \max\{U, K\}$. 

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This leads to the following first-best solution (the proof is in Section A1.1 in the Appendix).

**Proposition 3.1 (Optimal First-Best Solution)** The first-best contract and outcomes are as per the following table.

<table>
<thead>
<tr>
<th>$U - K$</th>
<th>$H - L$</th>
<th>$e_{FB}^*$</th>
<th>$s_{FB}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U - K \geq 0$</td>
<td>$H - L \geq \frac{\phi}{p-q}$</td>
<td>$U + \phi$</td>
<td>$U + \phi$</td>
</tr>
<tr>
<td>$-\phi \leq U - K &lt; 0$</td>
<td>$H - L \geq \frac{\phi}{p-q} + \frac{U-K}{p-q}$</td>
<td>1</td>
<td>$U + \phi$</td>
</tr>
<tr>
<td>$U - K &lt; -\phi$</td>
<td>$H - L \geq 0$</td>
<td>$K$</td>
<td>$K$</td>
</tr>
<tr>
<td>$U - K \geq 0$</td>
<td>$H - L &lt; \frac{\phi}{p-q}$</td>
<td>0</td>
<td>$U$</td>
</tr>
<tr>
<td>$-\phi \leq U - K &lt; 0$</td>
<td>$H - L &lt; \frac{\phi}{p-q} + \frac{U-K}{p-q}$</td>
<td>$K$</td>
<td>$K$</td>
</tr>
</tbody>
</table>

In the table in Proposition 3.1, the first column gives the condition on $U - K$, the second column gives the condition on $H - L$, the third column gives the effort exertion under the optimal salary, and the fourth column gives the optimal salary. Figure 3.1 depicts the first-best solution with respect to the range of the demand distribution ($H - L$), the agent’s effectiveness parameter ($p - q$), and the agent’s outside option relative to his limited liability ($U - K$). From Figure 3.1, we can infer that the principal would like the agent to exert effort
when the upside market potential is large, or when the effectiveness of the agent’s effort is high, or when the agent’s limited liability is large relative to his outside option.

Intuitively, the firm would like to direct the salesperson to work hard if and only if the increase in the expected demand subject to the agent’s effort exertion (given by \((p-q)(H-L)\)) outweighs the marginal cost for soliciting effort (given by \(\max\{U + \phi, K\} - \max\{U, K\}\)). When limited liability is low relative to the agent’s outside option (given by \(K \leq U\)), the principal only needs to compensate the agent for his outside option plus cost of effort. Therefore the additional cost for soliciting effort is \(\phi\), and the principal solicits effort exertion if and only if \(H - L \geq \frac{\phi}{p-q}\). When limited liability is intermediate (i.e., \(U < K \leq U + \phi\)), even if the principal does not solicit effort, she still has to pay the agent his limited liability, so the additional cost for soliciting effort becomes \(\phi + U - K\). In this case as \(K\) increases, the additional cost for soliciting effort decreases, thus the principal solicits effort in a larger parameter space. When limited liability increases beyond \(U + \phi\), the principal pays the agent his limited liability regardless of effort levels and there is no additional cost for soliciting effort, therefore, the principal instructs the agent to exert effort given any \(H \geq L\). The above arguments give the following counterintuitive result.

**Corollary 3.1** In the first-best scenario, as limited liability increases the principal solicits effort in a weakly larger parameter space.

### 3.3.2 Period-by-Period Contract

In this scenario, the principal specifies a one-period contract at the beginning of the first period, and then specifies another one-period contract at the beginning of the second period. The effort for each period is rewarded separately, and therefore I call this a disaggregate contract. As the two periods are identical and independent, it is sufficient to study just one period.
Consider the problem for period $t \in \{1, 2\}$. Since demand follows a binomial distribution, the principal offers quota-bonus contracts with quota levels $\chi_t \in \{H, L\}$ and bonuses $b_{\chi_t,t} \geq 0$, where the bonus $b_{\chi_t,t}$ is paid to the salesperson if and only if the sales reach the quota $\chi_t$, together with a fixed salary of $s_t$. Indeed, it suffices for the principal to consider only two of the decision variables. Without loss of generality, I normalize $b_{L,t}$ to 0 and simplify the notation of $b_{H,t}$ as $b_t$, i.e., the principal does not issue bonus when the demand outcome is $L$ and issues bonus $b_t$ when the demand outcome is $H$. The principal’s problem in each period is the following.

$$\max_{s_t, b_t} E[D_t|e_t] - E[s_t + b_t|e_t]$$  

subject to:

$$U_A(e_t) > U_A(\tilde{e}_t) \quad (IC_t)$$

$$U_A(e_t) \geq U \quad (PC_t)$$

$$s_t, s_t + b_t \geq K \quad (LL_t)$$

The participation constraint $(PC_t)$ and the limited liability constraint $(LL_t)$ can be interpreted in a similar way as in the first-best scenario. In addition, the contract needs to satisfy an incentive compatibility constraint $(IC_t)$, which states that to induce effort $e_t$, the principal needs to ensure that the agent gains a higher net utility by exerting effort $e_t$ compared with a different effort level $\tilde{e}_t$.

Before solving the optimal contract for the principal, I first derive the best contract for the principal to induce any given effort level. To implement $e_t = 1$, from the incentive compatibility constraint $(IC_t)$, the principal needs to set $b_{H,t}$ satisfying $s_t + pb_t - \phi \geq s_t + qb_t$, which simplifies into $b_t \geq \phi \frac{p - \phi}{p - q}$. The participation constraint $(PC_t)$ requires that the agent’s expected utility from exerting effort no lower than his reservation utility, that is, $s_t + p\frac{\phi}{p - q} - \phi \geq U$. To meet the limited liability constraint $(LL_t)$ we need the guaranteed salary no less than the agent’s limited liability, i.e., $s_t \geq K$. The solution is that to implement
Figure 3.2: Optimal Period-by-period Contract

$e_t = 1$, the principal offers a fixed salary $s_t = \max\{K, U - q\frac{\phi}{p-q}\}$, and a bonus $b_t = \frac{\phi}{p-q}$ if the demand outcome is high. To implement $e_t = 0$, it is enough for the principal to only offer the agent a fixed salary $s_t = \max\{K, U\}$. The overall solution to the optimal period-by-period contract is specified in the following proposition (the proof is in Section A1.2 in the Appendix).

**Proposition 3.2 (Optimal Period-by-Period Contract)** The optimal period-by-period contract and outcomes are as per the following table.

<table>
<thead>
<tr>
<th>$U - K$</th>
<th>$H - L$</th>
<th>$e_t^*$</th>
<th>$s_t^*$</th>
<th>$b_t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U - K \geq \frac{q}{p-q} \phi$</td>
<td>$H - L \geq \frac{\phi}{p-q}$</td>
<td>$U - \frac{q}{p-q} \phi$</td>
<td>$\frac{\phi}{p-q}$</td>
<td></td>
</tr>
<tr>
<td>$0 \leq U - K &lt; \frac{q}{p-q} \phi$</td>
<td>$H - L \geq \frac{p}{(p-q)^2} \phi - \frac{U - K}{p-q}$</td>
<td>1</td>
<td>$K$</td>
<td>$\frac{\phi}{p-q}$</td>
</tr>
<tr>
<td>$U - K &lt; 0$</td>
<td>$H - L \geq \frac{p}{(p-q)^2} \phi$</td>
<td>$K$</td>
<td>$\frac{\phi}{p-q}$</td>
<td></td>
</tr>
<tr>
<td>$U - K \geq \frac{q}{p-q} \phi$</td>
<td>$H - L &lt; \frac{\phi}{p-q}$</td>
<td>$U$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$0 \leq U - K &lt; \frac{q}{p-q} \phi$</td>
<td>$H - L &lt; \frac{p}{(p-q)^2} \phi - \frac{U - K}{p-q}$</td>
<td>0</td>
<td>$U$</td>
<td>0</td>
</tr>
<tr>
<td>$U - K &lt; 0$</td>
<td>$H - L &lt; \frac{p}{(p-q)^2} \phi$</td>
<td>$K$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3.2 depicts the optimal period-by-period contract with respect to the range of the demand distribution ($H - L$), the agent’s effectiveness parameter ($p - q$), and the agent’s
reservation utility relative to his limited liability \((U - K)\). In Region I, the principal does not want to induce effort even in the first-best scenario. In Region II, the principal wants to induce effort in the first-best scenario but not in the period-by-period contracting scenario. In Region III, the principal wants to induce effort in the period-by-period scenario. Note that when limited liability is relatively small \((K \geq U - \frac{q}{p-q} \phi)\), even if effort is unobservable, the principal can still achieve the first-best solution by penalizing the agent for low demand realization and rewarding the agent for high demand realization. As limited liability increases beyond \(U - \frac{q}{p-q} \phi\), the principal cannot pay the agent less than his limited liability when demand realization is \(L\), therefore the first-best solution is no longer achievable. This leads the principal to induce effort in a smaller parameter space as limited liability increases. When limited liability exceeds \(U\), the agent needs to be guaranteed his limited liability, with or without a bonus to induce effort. Therefore, the principal induces effort if and only if the extra cost for inducing effort \(\frac{p}{(p-q)^2} \phi\) is offset by the increase in expected demand from exerting effort \((H - L)(p - q)\). From Figure 3.2, we can see that as limited liability increases, while in the first-best scenario the principal solicits effort in a weakly larger parameter space (as per Corollary 3.1; represented by the dashed line), with unobservable effort she will induce effort in a weakly smaller parameter space (represented by the solid line).

### 3.3.3 Two-Period Contract

In this scenario, the firm proposes a two-period contract at the beginning of the first period and pays once at the end of the second period based on the outcomes of the two periods. The timeline of the game is as follows. At the beginning of period 1, i.e., \(T = 1\), the principal proposes the contract and the agent decides whether or not to accept the offer. If accepted, the agent then decides on his effort in the first period, \(e_1\). At the end of \(T = 1\), the agent and the principal observe the demand outcome for the first period, \(D_1\). The agent then chooses his second period effort \(e_2\). At the end of \(T = 2\), the agent and the principal observe the
second period demand outcome $D_2$. The agent then gets paid according to the contract.

A key feature of this scenario introduced due to unobservability of effort and the contract paying at the end of two periods is that the agent can “game” the system — the agent can choose effort in period 2 based on the outcome of period 1 (and, realizing this, can also choose the effort in period 1 strategically). I denote the two-period effort profile by $(e_1, e_1^H, e_1^L, e_2^H, e_2^L)$, where the second period’s effort $e_2^{D_1}$ is contingent on the first period’s demand realization, $D_1$.

In full generality, this contract involves a guaranteed salary for employing the agent for two periods, plus a bonus issued at the end of the two periods that is contingent on the whole history of outputs. I denote the fixed salary as $S$, and denote the bonus paid at the end of $T = 2$ by $b_2(D_1, D_2)$. Such a contract thus stipulates four possible bonuses, $b_2(L, L), b_2(L, H), b_2(H, L)$ and $b_2(H, H)$. To prevent the agent from restricting sales to $L$ when demand is $H$, I impose a constraint on the two-period contract given by $b(H, H) \geq \max\{b(H, L), b(L, H)\}$, i.e., the bonus paid when demand in both periods is realized as $H$ should be no lower than that paid when demand in only one of the periods is realized as $H$.

Under this constraint, I obtain the following lemma (the detailed proof is in Section A1.3.1 in the Appendix).

**Lemma 3.1** When the two periods are independent of each other, in the weakly dominant two-period contract, $b_2(H, L) = b_2(L, H)$.

Lemma 3.1 implies that it is sufficient for the principal to pay the agent at the end of two periods a bonus according to cumulative sales (which can be $2L, H + L$ or $2H$) and independent of the sales history.$^{2,3}$ I denote the fixed salary by $S$, normalize the bonus

---

$^2$Only the contract to induce $(0, 1 - q)$ is history-dependent, but I find it suboptimal for the principal when the two periods are independent. However, in Section 3.4, I will show that such a history-dependent contract can be optimal when the two periods become dependent.

$^3$The lemma holds without discounting and with risk neutral agents. As shown by Spear and Srivastava
payment when the total sales are $2L$ as 0, denote the bonus payment when the total sales across two periods are $H + L$ by $B_1$, and denote the bonus payment when the total sales are $2H$ by $B_2$. I formulate the principal’s problem as follows.

$$\max_{S,B_1,B_2} \ E[D|e_1, e_2^H, e_2^L] - E[S + B_1 + B_2|e_1, e_2^H, e_2^L]$$

s.t.  \[ U_A(e_2^H) > U_A(\tilde{e}_2^H) \quad (IC_2^H) \]

\[ U_A(e_2^L) > U_A(\tilde{e}_2^L) \quad (IC_2^L) \]

\[ U_A(e_1|e_2^H, e_2^L) > U_A(\tilde{e}_1|e_2^H, e_2^L) \quad (IC_1) \]

\[ U_A(e_1, e_2^H, e_2^L) \geq 2U \quad (PC) \]

\[ S, S + B_1, S + B_2 \geq 2K \quad (LL) \]

$(IC_2^H)$ stands for the agent’s incentive compatible constraint in the second period following $D_1 = H$, where $U_A(e_2^H)$ represents the agent’s net payoff in Period 2 upon exerting effort $e_2^H$. If the agent exerts effort, he will get $S + B_2 - \phi$ with probability $p$ and $S + B_1 - \phi$ otherwise; without exerting effort, he will get $S + B_2$ with probability $q$ and $S + B_1$ otherwise. To induce $e_2^H$, the principal needs to ensure that the agent gets a higher payoff upon exerting effort $e_2^H$, compared with a different effort level $\tilde{e}_2^H$. Similarly, $(IC_2^L)$ stands for the incentive compatible constraint for inducing effort level $e_2^L$ in the second period following $D_1 = L$.

Then, $(IC_1)$ represents the incentive compatible constraint in the first period. $U_A(e_1|e_2^H, e_2^L)$ denotes the agent’s net payoff across two periods upon exerting $e_1$ in the first period, given that the agent is induced to exert effort $(e_2^H, e_2^L)$ in the second period. If $e_1 = 1$, his total net payoff will be $U_A(e_2^H) - \phi$ with probability $p$ and $U_A(e_2^L) - \phi$ otherwise; if $e_1 = 0$, his total net payoff will be $U_A(e_2^H)$ with probability $q$ and $U_A(e_2^L)$ otherwise. To induce $e_1$, the principal needs to ensure that the agent gets a higher total net payoff on exerting $e_1$, (1987) and Sannikov (2008), if agents discount their future utility, or if the agent is risk averse, a path-dependent contract can be optimal.
compared with a different effort level $\tilde{e}_1$. The participation constraint ($PC$) and the limited liability constraint ($LL$) are similar to that in the period-by-period case, except for that I multiply the right-hand sides by two under a two-period contracting.

To arrive at an optimal contract for the principal, it is crucial to understand how the agent’s effort profile in the two periods changes with the bonuses $B_1$ (provided for $H + L$) and $B_2$ (provided for $2H$) in the two-period contract. The following lemma describes this effort profile (the proof is immediate from the proof of Lemma 3.1).\(^4\) Note that since $e_2$ depends on $D_1$, which is random, I write $e_2$ in terms of its expectation value. For instance, if the agent exerts effort in period 1 and will exert effort in period 2 only if the outcome in period 1 is $H$, then $e_2 = 1$ with probability $p$, so I write this effort profile as $(1, p)$.

**Lemma 3.2 (Agent’s Response to Two-period Contract)** Given $B_1$ and $B_2$, the agent’s expected effort profile $(e_1, E[e_2])$ is as per the following table.

<table>
<thead>
<tr>
<th>$(B_1, B_2)$</th>
<th>$(e_1, E[e_2])$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1 \geq \frac{\phi}{p-q},$ $B_2 - B_1 \geq \frac{\phi}{p-q}$</td>
<td>$(1, 1)$</td>
</tr>
<tr>
<td>$0 \leq B_1 &lt; \frac{\phi}{p-q},$ $pB_2 + (1 - p - q)B_1 \geq \frac{\phi}{p-q} + \phi$</td>
<td>$(1, p)$</td>
</tr>
<tr>
<td>$B_2 - B_1 &lt; \frac{\phi}{p-q},$ $qB_2 + (1 - p - q)B_1 \geq \frac{\phi}{p-q} - \phi$</td>
<td>$(1, 1 - p)$</td>
</tr>
<tr>
<td>$B_2 - B_1 \geq \frac{\phi}{p-q},$ $pB_2 + (1 - p - q)B_1 &lt; \frac{\phi}{p-q} + \phi$</td>
<td>$(0, q)$</td>
</tr>
<tr>
<td>$B_1 \geq \frac{\phi}{p-q},$ $qB_2 + (1 - p - q)B_1 &lt; \frac{\phi}{p-q} - \phi$</td>
<td>$(0, 1 - q)$</td>
</tr>
<tr>
<td>$0 \leq B_1 &lt; \frac{\phi}{p-q},$ $0 \leq B_2 - B_1 &lt; \frac{\phi}{p-q}$</td>
<td>$(0, 0)$</td>
</tr>
</tbody>
</table>

Figure 3.3 illustrates Lemma 3.2 graphically. The $x$-axis, $B_1$, is the incremental reward when total sales increase from $2L$ to $H + L$; the $y$-axis, $B_2 - B_1$, is the incremental reward when total sales increase from $H + L$ to $2H$. If both rewards are small, there is no effort exertion in either period, denoted by $e = (0, 0)$, which is Region I. If both rewards are large,

\(^4\)I make the assumption that when the agent is indifferent between exerting effort or not, he will choose to exert effort.
the agent will put in effort in both periods, i.e., \( e = (1, 1) \), which is Region IV. For other regions, the effort exertion decisions are more involved. If the agent does not secure the bonus \( B_1 \) after period 1 with the demand outcome \( L \), he will not expend additional effort if \( B_1 \leq \frac{\phi}{p-q} \). If the agent secures the bonus \( B_1 \) after period 1 with the demand outcome \( H \), he will not expend additional effort if \( B_2 - B_1 \leq \frac{\phi}{p-q} \). In other words, \( B_1 \) and \( B_2 - B_1 \) motivate the agent to exert effort in the second period if demand in the first period turns out to be \( L \) and \( H \), respectively. Furthermore, the agent’s effort exertion at \( T = 1 \) depends on the values of both \( B_1 \) and \( B_2 - B_1 \). In Regions II and VI, the agent does not work in period 1 and chooses to “ride his luck” in period 1. However, in Region II, he works in period 2 if the demand outcome is unfavorable, i.e., \( L \), in period 1, and in Region VI, he works in period 2 if the demand outcome is favorable, i.e., \( H \), in period 1. In Regions III and V, the agent works in period 1. However, in Region III, he works in period 2 if the demand outcome is unfavorable, i.e., \( L \), in period 1, and in Region V, he works in period 2 if the demand outcome is favorable, i.e., \( H \), in period 1.

I now determine the optimal compensation plan for the firm. I find the optimal contract by balancing the expected revenue \( E[D] \) less the expected compensation cost \( E[S + B_1 + B_2] \). Proposition 3.3 characterizes the optimal two-period contract for the principal (the detailed

Figure 3.3: Agent’s Response to Two-period Contract
Proposition 3.3 (Optimal Two-period Contract) The optimal two-period contract and outcomes are as per the following table.

<table>
<thead>
<tr>
<th>Region</th>
<th>$U - K$</th>
<th>$H - L$</th>
<th>$(e_1, E[e_2])$</th>
<th>$S^*$</th>
<th>$B_1^*$</th>
<th>$B_2^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$U - K \geq \frac{\phi}{p-q} \phi$</td>
<td>$H - L &lt; \frac{\phi}{p-q}$</td>
<td>$(0,0)$</td>
<td>$2U$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\frac{p^2}{1+p+q(p-q)} \phi \leq U - K &lt; \frac{p^2}{2p-q} \phi$</td>
<td>$H - L &lt; \frac{p}{1+p+q(p-q)} \phi - \frac{U - K}{2q-p}$</td>
<td>2U</td>
<td>$0 \quad 0$</td>
<td>2K</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0 \leq U - K &lt; \frac{p^2}{21p+q(p-q)} \phi$</td>
<td>$H - L &lt; \frac{\phi}{21p+q(p-q)} \phi$</td>
<td>2K</td>
<td>$0 \quad 0$</td>
<td>2K</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>$\frac{p^2}{2q-p} \phi \leq U - K &lt; \frac{p}{2q-p} \phi$</td>
<td>$\frac{\phi}{p-q} \leq H - L &lt; \frac{p(1+p-q)}{1+p+q(p-q)} \phi - \frac{2U-K}{1+p+q(p-q)} \phi$</td>
<td>$(0, q)$</td>
<td>$2U - \frac{p^2}{p-q} \phi$</td>
<td>0</td>
<td>$\frac{1}{p-q} \phi$</td>
</tr>
<tr>
<td></td>
<td>$\frac{p^2}{2q-p} \phi \leq U - K &lt; \frac{p}{2q-p} \phi$</td>
<td>$\frac{\phi}{p-q} \leq H - L &lt; \frac{p(1+p-q)}{1+p+q(p-q)} \phi - \frac{2U-K}{1+p+q(p-q)} \phi$</td>
<td>$(0, q)$</td>
<td>$2U - \frac{p^2}{p-q} \phi$</td>
<td>0</td>
<td>$\frac{1}{p-q} \phi$</td>
</tr>
<tr>
<td></td>
<td>$0 \leq U - K &lt; \frac{p^2}{21p+q(p-q)} \phi$</td>
<td>$H - L &lt; \frac{\phi}{21p+q(p-q)} \phi$</td>
<td>2K</td>
<td>$0 \quad 0$</td>
<td>2K</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>$U - K \geq \frac{\phi}{p-q} \phi$</td>
<td>$H - L \geq \frac{p(1+p-q)}{1+p+q(p-q)} \phi - \frac{2U-K}{1+p+q(p-q)} \phi$</td>
<td>$(1, p)$</td>
<td>$2U - \frac{1}{p-q} \phi$</td>
<td>2K</td>
<td>$0 \quad 0$</td>
</tr>
<tr>
<td></td>
<td>$\frac{p^2}{2q-p} \phi \leq U - K &lt; \frac{p}{2q-p} \phi$</td>
<td>$\frac{\phi}{p-q} \leq H - L &lt; \frac{p(1+p-q)}{1+p+q(p-q)} \phi - \frac{2U-K}{1+p+q(p-q)} \phi$</td>
<td>$(1, p)$</td>
<td>$2U - \frac{1}{p-q} \phi$</td>
<td>2K</td>
<td>$0 \quad 0$</td>
</tr>
<tr>
<td></td>
<td>$0 \leq U - K &lt; \frac{p^2}{21p+q(p-q)} \phi$</td>
<td>$H - L &lt; \frac{\phi}{21p+q(p-q)} \phi$</td>
<td>2K</td>
<td>$0 \quad 0$</td>
<td>2K</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>$U - K \geq \frac{\phi}{p-q} \phi$</td>
<td>$H - L \geq \frac{p(1+p-q)}{1+p+q(p-q)} \phi - \frac{2U-K}{1+p+q(p-q)} \phi$</td>
<td>$(1, 1)$</td>
<td>$2U - \frac{2}{p-q} \phi$</td>
<td>2K</td>
<td>$\frac{1}{p-q} \phi$</td>
</tr>
<tr>
<td></td>
<td>$\frac{p^2}{2q-p} \phi \leq U - K &lt; \frac{p}{2q-p} \phi$</td>
<td>$\frac{\phi}{p-q} \leq H - L &lt; \frac{p(1+p-q)}{1+p+q(p-q)} \phi - \frac{2U-K}{1+p+q(p-q)} \phi$</td>
<td>$(1, 1)$</td>
<td>$2U - \frac{2}{p-q} \phi$</td>
<td>2K</td>
<td>$\frac{1}{p-q} \phi$</td>
</tr>
</tbody>
</table>

I illustrate the result with the aid of Figure 3.4. The optimal contract is either a “gradual contract” (in which $B_1 > 0$, i.e., it rewards bonuses at both $H + L$ and $2H$) or an “extreme contract” (in which $B_1 = 0$, i.e., it rewards bonuses only at $2H$). In Region I, the principal does not want to motivate effort. In Region II, the principal finds it optimal to use the extreme contract to motivate the effort profile $(0, q)$ by giving a bonus $B_2 = \frac{\phi}{p-q}$. In Region III, the principal finds it optimal to use the extreme contract to motivate the effort profile $(1, p)$ by giving a bonus $B_2 = \frac{1+p-q}{p(p-q)} \phi$ (which is larger than $\frac{\phi}{p-q}$). In Region IV, the principal finds it optimal to use the optimal gradual two-period contract to motivate the effort profile $(1, 1)$.

To develop the intuition behind these results, I first focus on the case when limited liability is sufficiently high. Specifically, I assume $K = U$, in which case the principal pays a fixed salary of $S = 2K$ for inducing any effort profile. From Figure 3.4, I can see that in this case the optimal contract is either the extreme two-period contract with $B_2 = \frac{1+p-q}{p(p-q)} \phi$
to implement \( e = (1, p) \), or the gradual two-period contract with bonus \( B_1 = \frac{\phi}{p-q}, B_2 = 2 \frac{\phi}{p-q} \) to implement \( e = (1, 1) \). To understand why, I discuss two effects that are operative, namely the “incentive effect” and the “gaming effect.”

First, I discuss the incentive effect. In Figure 3.5, I vary \( p - q \), the effectiveness of the agent’s effort, keeping \( H - L \) fixed. Generally speaking, more effective agents require lower incentives to work because the outcome is a better signal of effort exerted. In line with this, the expected bonus payments under the extreme contract, \((p^2 + p - pq)\frac{\phi}{p-q}\), and under the gradual contract, \(2p\frac{\phi}{p-q}\), both decrease with \( p \). However, the difference between them, \( E[B]_{\text{gradual}} - E[B]_{\text{extreme}} = p\left(\frac{1}{p-q} - 1\right)\phi \), is always positive, as shown by the solid line in Figure 3.5. This means that the principal always pays a smaller expected bonus under the extreme contract than under the gradual contract. Therefore, on the positive side, the extreme contract benefits from the incentive effect: it provides more effective incentives for an agent with limited liability, thus saving on the bonus payment for the principal. The reason behind this is that under limited liability, the principal concentrates compensation at a high level of sales. In a period-by-period contract the highest level of sales at which reward can be given is \( H \) while in a two-period contract this level is \( 2H \); this can lead to higher
incentive provision in a two-period contract (even though the reward is given only once). Another interesting observation from Figure 3.5 is that the incentive effect, as measured by the solid line, shrinks as $p$ increases. This is because as moral hazard frictions decrease with more effective agents, so will the comparative advantage of the extreme contract on saving incentive costs.

However, in a dynamic setting, such a non-linear reward structure will suffer from the agent’s gaming. As a consequence, on the negative side, the principal obtains less demand under the extreme contract, as the dashed line in Figure 3.5 illustrates. Mathematically, $E[D]_{\text{gradual}} - E[D]_{\text{extreme}} = (1 - p)(p - q)(H - L)$ is always positive. As I have mentioned, due to the non-linear structure of the extreme contract, an agent will game the system by varying his effort in a dynamic setting. Specifically, the agent exerts effort in the first period, but if the first period outcome turns out to be $L$, the agent will give up on effort exertion in the second period, leading to a demand loss for the principal. Interestingly, as $p$ gets larger, agents under both contracts generate higher sales, but the difference between the sales they generate, caused by the gaming effect, takes an inverse-U shape. This is because when $p$ increases, the demand loss, if it happens, $(p - q)(H - L)$, gets larger, but the probability of its happening, $(1 - p)$, decreases.

Combining the incentive effect and the gaming effect, we can see from Figure 3.5 that if $p - q$ is small, the incentive effect dominates and the extreme plan outperforms the gradual
plan—the gaming loss under the extreme contract is relatively small compared with its advantage in providing incentives. Above a threshold of $p - q$, the gaming loss becomes dominant and the gradual contract is preferred by the principal. However, as $p - q$ continues to increase, the gaming loss begins to decline, rendering the extreme contract better again. Overall, when $p - q$ is either very small or very large, the incentive effect will be more significant than the gaming effect and the extreme plan outperforms the gradual plan.

The above analysis is based on the premise that limited liability is sufficiently high. Now I discuss the optimal contract as limited liability decreases. I fix $H - L$ and $p - q$ at a low level so that when limited liability is sufficiently high the principal does not want to induce effort. As limited liability decreases, the friction due to moral hazard becomes smaller, and the principal starts to motive effort using the extreme two-period contract, which provides more effective incentives than the period-by-period contract. Since limited liability is still relatively high in this scenario, the principal only induces $e_2^H = 1$ through a low ultimate bonus and the full effort profile is $e = (0, q)$—that is, there is no early effort exertion in the first period, and there is effort exertion in the second period if the early period realizes as high. As limited liability continues to decreases further, the principal implements $e_1 = 1$ through a high ultimate bonus and the full effort profile is $e = (1, p)$—that is, the agent exerts effort in the first period, and he will continue exerting effort in the second period if the early period realizes as high. When limited liability becomes small enough, the principal will implement effort $e = (1, 1)$ using the gradual two-period contract.

Put together, the preceding discussion explains the patterns in Figure 3.4. When limited liability is not too small (relative to the agent’s outside option), in a market with small upside demand potential, and with either very inefficient or very efficient salespeople, the extreme contract performs best for the principal. In other circumstances, it is profitable for firms to propose a gradual contract to motivate hard work in both periods. Next, I state an interesting corollary.
**Corollary 3.2** Under a two-period contract with independent sales periods, when the upside demand potential, the agent’s effort effectiveness and limited liability are all intermediate, the principal does not induce early effort, and will induce late effort only when the first period’s demand outcome is high. This leads to a “hockey stick” effort profile from the agent.

The corollary states that, in terms of the agent’s effort profile, we may observe a “hockey stick” pattern $e = (0, q)$ in the agent’s effort profile in equilibrium, that is, the agent exerts higher effort in the second period compared with the first period in expectation. This happens when the limited liability, the demand upside potential, and agent’s effort effectiveness are all at intermediate levels. In this case, the principal would like to induce effort using an extreme two-period contract with a low ultimate bonus, which provides the most effective incentives. In other parameter spaces, early effort exertion is preferred by the principal under a two-period contract, i.e., $e^*_1 \geq E[e^*_2]$, as happens when the optimal two-period contract induces either $e = (1, 1)$ or induces $e = (1, p)$. In Section 3.4, I introduce dependence between the two periods and show that the “hockey stick” effort profile can be preferred even in a larger parameter space by the principal in that case.

### 3.3.4 Comparison between Two-Period and Period-by-Period Contracts

I now compare the outcomes in the period-by-period contract scenario and the two-period contract scenario from the point of view of the principal. I find, not surprisingly, that the principal weakly prefers the two-period contract to the period-by-period contract. However, more interestingly, my analysis shows that, under certain conditions, the principal strongly prefers a two-period contract over a period-by-period contract (even though the latter gives the principal more control over the agent’s action while the former allows the agent the freedom to exert effort to game the contract). Furthermore, with independent periods, a
two-period contract that rewards bonuses on the basis of the total sales in the two periods suffices, i.e., achieves the same outcome as a contract that rewards for the full sequence of outcomes. I obtain the following proposition.

**Proposition 3.4** In Regions II and III as defined in Proposition 3.3, the principal strongly prefers a two-period contract over a period-by-period contract.

The reason is that the gradual contract with \( B_1 = \frac{\phi}{p-q}, B_2 = 2 \frac{\phi}{p-q} \) is essentially a replicate of the period-by-period contract. Therefore, whenever the principal prefers the extreme contract over the gradual contract in the two-period contract, she strongly prefers the two-period contract over the period-by-period contract. This happens when the effectiveness of effort of the salesperson is either very high or very low (but high enough that it is worthwhile to have effort exertion). Also, as the limited liability decreases, the strong preference for the two-period contract reduces. I also note that the preferred extreme two-period contract may be the one that pays a small bonus for high sales in both periods, which induces effort only in the second period if the outcome in the first period (without effort exertion) is high, or it may be the one that pays a large bonus for high sales in both periods, which induces effort in the first period and in the second period only if the outcome in the first period is high.

It is noteworthy that I do not allow renegotiation under long-term contracting. If renegotiation is allowed, rational agents will anticipate that when the first period demand outcome is low, the principal will renegotiate the contract at the beginning of the second period to avoid agents giving up in the second period. This will eliminate the value of long-term contracting in inducing more effort exertion in the first period (conditional on the same amount of bonus payment) relative to short-term contracting. This aligns with Fudenberg et al. (1990)’s result that long-term contracting outperforms short-term contracting for the principal only if optimal contracting requires commitment to a plan today that would not otherwise be adopted tomorrow.
3.3.5 Extension: Sales Push-out and Pull-in between Periods

Salespeople working under quota-based plans may resort to modifying demand in particular periods to meet quotas in those periods. Oyer (1998) empirically demonstrates the existence of demand pull-in and push-out between fiscal cycles when salespeople face non-linear contracts. In particular, Oyer (1998) reveals that sales agents will pull in orders from future periods if they would otherwise fall short of a sales quota in one cycle, whereas they push out orders to the future if quotas are either unattainable or have already been achieved. I ignore such sales push-out and pull-in phenomena in previous sections, by assuming that agents cannot shift sales between two periods. In this section, I relax this assumption and allow the agents to push extra sales to (or borrow sales from) the later period. While the two-period optimal contract, which pays at the end, is not affected, the period-by-period contract, which pays in the interim, is subject to sales push-out and pull-in effects, and thus has to be reanalyzed. I provide a sketch of the analysis below, with details provided in Section A1.4 in the Appendix.

In the period-by-period contract, at the end of the first period the agent observes the actual sales $D_1$ ahead of the principal. He can then strategically push out sales to (or pull in sales from) the second period, if necessary. The principal only observes the sales level after the agent’s manipulation, which I denote by $D'_1$, and pays the agent according to $D'_1$. For instance, the principal will observe $D'_1 = H$ if $D_1 = H$ or if $D_1 = L$, but the agent pulls in $H - L$ from the second period. Likewise, observing $D'_1 = L$ may imply that $D_1 = L$ or that $D_1 = H$ and the agent pushed out the demand $H - L$ to the second period.

To derive the optimal contract, consider first the principal’s problem at $T = 2$. Distinct from Section 3.3.2, the problems of the two periods are dependent. At the beginning of the

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5I assume that the agent can pull in at most $L$ from the second period to the first, and I focus on the case when $H < 2L$. This ensures that even if $D_1 = L$, the agent can manage to report $D'_1 = H$ by pulling in $H - L < L$ from $T = 2$. 

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second period, a new contract is initiated. To induce a specific effort level in the second period, the principal will set the second period’s quota level and bonus value based on the observed earlier outcome $D_1'$, which I denote by $\chi_2(D_1')$ and $b_2(D_1')$, respectively. While I relegate the details to Appendix A1.4, the result is that to induce $(e^H_2, e^L_2) = (1, 0)$ the principal sets $\chi_2(D_1') = 2H - D_1'$, and to induce $(e^H_2, e^L_2) = (0, 1)$, $\chi_2(D_1')$ is set to be $H + L - D_1'$. In both cases, $b_2(D_1') = \frac{\phi}{p - q}$. This implies that the principal readjusts the quota level but not the bonus amount to achieve a desired effort profile in the second period. Anticipating this, if the first period’s quota level is not high enough, agents prefer to pull in sales to achieve the first-period quota and obtain the first period bonus rather than exerting effort. To induce $e_1 = 1$, in turn, the principal then sets $\chi_1$ high enough (for instance $H + L$), such that it becomes impossible for the agent to simply secure early bonuses by pulling in sales if $D_1 = L$, but it is still achievable in case $D_1 = H$. The fixed salary at each period is chosen such that each of them is no lower than the agent’s limited liability, and the two combined together ensures the agent’s participation. In Table 3.1, I summarize the optimal contract to induce different effort profiles.

Indeed, comparing with Section 3.3.3, the period-by-period contract, in the presence of sales push-out and pull-in, still performs weakly worse for the principal than the two-period contract. Namely, it performs the same as the two-period contract for inducing $e = (1, p)$, $e = (1, 1 - p)$, $e = (0, q)$ and $e = (0, 0)$, but it performs worse then the two-period contract for inducing $e = (0, 1 - q)$ and it fails to induce $e = (1, 1)$. In other words, when the principal anticipates that agents may push-out and pull-in sales, and agents also realize

<table>
<thead>
<tr>
<th>$(e_1, E[e_2])$</th>
<th>$s_1 + s_2$</th>
<th>$\chi_1$</th>
<th>$b_1$</th>
<th>$\chi_2(D_1')$</th>
<th>$b_2(D_1')$</th>
<th>$E[b]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(0, 0)$</td>
<td>$\max{2K, 2U}$</td>
<td>0</td>
<td>0</td>
<td>$2L$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(0, 1 - q)$</td>
<td>$\max{2K + \frac{\ell}{p - q}, 2U + (1 - q)\phi}$</td>
<td>0</td>
<td>0</td>
<td>$H + L - D_1'$</td>
<td>$\frac{\phi}{p - q}$</td>
<td>$(p + q - pq)\frac{\phi}{p - q}$</td>
</tr>
<tr>
<td>$(0, q)$</td>
<td>$\max{2K + \frac{\ell}{p - q}, 2U + q\phi}$</td>
<td>0</td>
<td>0</td>
<td>$2H - D_1'$</td>
<td>$\frac{\phi}{p - q}$</td>
<td>$pq\frac{\phi}{p - q}$</td>
</tr>
<tr>
<td>$(1, 1 - p)$</td>
<td>$\max{2K + \frac{2p - \phi}{p - q}, 2U + (2 - p)\phi}$</td>
<td>$H + L$</td>
<td>$\frac{\phi}{p - q}$</td>
<td>$H + L - D_1'$</td>
<td>$(2p - p^2 + pq)\frac{\phi}{p - q}$</td>
<td></td>
</tr>
<tr>
<td>$(1, p)$</td>
<td>$\max{2K + \frac{p - \phi}{p - q}, 2U + (1 + p)\phi}$</td>
<td>$H + L$</td>
<td>$(1 - q)\frac{\phi}{p - q}$</td>
<td>$2H - D_1'$</td>
<td>$(p^2 + p - pq)\frac{\phi}{p - q}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Optimal Period-by-period Contract with Sales Push-out and Pull-in
that the principal will respond optimally to it, the principal is unable to induce consistently high efforts. As a result, the period-by-period contract is still dominated by the two-period contract (which is unaffected by sales push-out and pull-in).

3.4 Interdependent Periods: Limited Inventory

Until now, I have assumed that the two time periods are independent of each other (except for the extension in Section 3.3.5 in which the salesperson’s actions can influence the period-by-period contracting decision). In this section, I allow the two periods to be dependent on each other. There may be many ways due to which the periods can be interdependent. I consider one such way, in which I assume that the principal has a limited amount of product to sell, such that the demand outcome in the first period can change incentive provision for inducing demand in the second period. In the model until now, the agent had the incentive to dynamically adjust his effort; in the model with limited inventory, the principal also has the incentive to dynamically adjust the contract in the two time periods.

I extend the model by assuming that the principal has limited inventory, denoted by $\Omega$, to be sold across two periods. The inventory cannot be replenished before period 2 starts and any demand more than $\Omega$ is lost, i.e., actual sales $\bar{D} = \min\{D_1 + D_2, \Omega\}$. Therefore, the two periods become dependent through $\Omega$. I assume zero inventory costs for simplicity. I keep everything else in the model the same as before. To focus on the interesting cases, I only consider the case when $0 \leq U - K \leq \frac{q}{p-q} \phi$ so that the optimal contract varies with the agent’s limited liability, and $H - L \geq \frac{p}{(p-q)^2} \phi$ so that the market upside potential is large enough to justify effort induction in both periods given unlimited inventory.

I assume that under the period-by-period contract, the agent chooses his action only based on the current period’s contract. As I will show later, under this assumption, short term horizon contracting can strictly dominate long term horizon contracting in certain
parameter spaces. However, if the agent is fully forward looking under the period-by-period contract, and can predict how the second period’s contract may change based on the outcome of the first period, then a long-term contract still weakly dominates a short-term contract.

3.4.1 Period-by-Period Contract with Limited Inventory

Recall that in Section 3.3.2, independence across the two periods implies that the optimal contract stays the same for \( t = 1 \) and \( t = 2 \). However, in the presence of limited inventory, the principal’s decision at \( T = 2 \), after observing \( D_1 \), is affected by the remaining inventory level \( \Omega - D_1 \). In other words, with limited inventory, the principal’s decision variables become \((s_1, s_2^D_1, b_1^D_1, b_2^D_1)\), where she will dynamically adjust contract terms at period 2 depending on the realization of \( D_1 \) as \( H \) or \( L \), and the effort levels induced correspondingly are \((e_1, e_2^D_1)\). I obtain the following proposition (the proof is in Section A2.2.1 in the Online Appendix).

**Proposition 3.5 (Optimal Period-by-period Contract with Limited Inventory)**

With limited inventory, the optimal period-by-period contract and the outcomes are as per the following table.\(^6\)

<table>
<thead>
<tr>
<th>Region</th>
<th>( \Omega )</th>
<th>((e_1, E[e_2]))</th>
<th>((s_1, s_2^H, s_2^L))</th>
<th>((b_1, b_2^H, b_2^L))</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>([2L, \omega_3 - \frac{U-K}{p\phi}))</td>
<td>((0,0))</td>
<td>((K,K,K))</td>
<td>((0,0,0))</td>
</tr>
<tr>
<td>II</td>
<td>([\omega_3 - \frac{U-K}{p\phi}, \text{max}{\omega_2 - \frac{1+q-p}{q(p-q)}(U-K), \omega_2' - \frac{1+q-p}{1-p(p-q)}(U-K)}])</td>
<td>((0,1-q))</td>
<td>((U,U,K))</td>
<td>((0,0,\frac{\phi}{p\phi}))</td>
</tr>
<tr>
<td>III</td>
<td>([\text{max}{\omega_2 - \frac{1+q-p}{q(p-q)}(U-K), \omega_2' - \frac{1+q-p}{1-p(p-q)}(U-K)}, \omega_1 - \frac{U-K}{p\phi}}])</td>
<td>((1,1-p))</td>
<td>((K,U,K))</td>
<td>((\frac{\phi}{p\phi}, 0, \frac{\phi}{p\phi}))</td>
</tr>
<tr>
<td>IV</td>
<td>([\omega_1 - \frac{U-K}{p\phi}, 2H])</td>
<td>((1,1))</td>
<td>((K,K,K))</td>
<td>((\phi, 0, \frac{\phi}{p\phi}))</td>
</tr>
</tbody>
</table>

When the inventory level is high enough (Region IV) and does not lead to a bottleneck, the principal induces \( e = (1,1) \), consistent with the case without inventory concerns. For a

\[\begin{align*}
\omega_1 &= H + L + \frac{p}{(p-q)^2} \phi, \\
\omega_2 &= H + L - \frac{1-p}{q}(H-L) + \frac{p-p^2+pq}{q(p-q)^2} \phi, \\
\omega_2' &= 2L + \frac{p-p^2+pq}{(1-p)(p-q)^2} \phi, \\
\omega_3 &= 2L + \frac{p}{(p-q)^2} \phi, \\
\omega_4 &= \frac{1+q-p}{(1-p)(p-q)} \phi
\end{align*}\]
smaller $\Omega$ (Region III), although $e_1 = 1$ remains, $e_2$ becomes contingent on $D_1$; a successful first period will cause an inventory shortage later and no extra effort is needed. The expected effort in the second period thus is the probability of realizing $D_1$ as $L$, which is $1 - p$, and the resulting effort profile is $e = (1, 1 - p)$. If $\Omega$ is further below a threshold (Region II), the principal abandons early effort induction. This leads to an equilibrium effort profile $e = (0, 1 - q)$. For a yet smaller $\Omega$ (Region I), inventory levels are too low to justify any effort induction, i.e. $e = (0, 0)$. It is noteworthy that the set of optimal effort profiles excludes $e = (0, 1)$ and $e = (1, 0)$. I state the following corollary.

**Corollary 3.3** Under a period-by-period contract with limited inventory, when $\Omega$ is intermediate, the principal does not induce early effort, and will induce late effort only when the first period’s demand outcome is low. This leads to a “hockey stick” effort profile from the agent.

A key insight is that when the working environment is easy enough for the agent, i.e., the total amount of product to be sold, $\Omega$, is small, or $H$ or $p$ is large, the principal has incentive to induce agents to work only in the late period, i.e., postpone effort.

### 3.4.2 Two-Period Contract with Limited Inventory

In this case, I solve the firm’s contracting problem by replacing the total demand $D$ by its truncated value $\bar{D} = \min\{D_1 + D_2, \Omega\}$. Note that I assume that compensation cannot be decreasing in total sales. I obtain the following proposition (the proof is provided in the Online Appendix).

**Proposition 3.6** (Optimal Two-Period Contract with Limited Inventory) With limited inventory, the optimal two-period contract and the outcomes are as per the following
Table 7

<table>
<thead>
<tr>
<th>Region</th>
<th>( \Omega )</th>
<th>( U - K )</th>
<th>( (c_1,E[c_2]) )</th>
<th>Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>([p_2, \frac{1}{p-q}])</td>
<td>([2L, \omega_2 - \frac{2(U-K)}{q(A-y)}])</td>
<td>(0,0)</td>
<td>( S = 2U, B_1 = 0, B_2 = 0 )</td>
</tr>
<tr>
<td></td>
<td>([0, p_2])</td>
<td>([2L, \omega_2 - \frac{2(U-K)}{q(A-y)}])</td>
<td></td>
<td>( S = 2U, B_1 = 0, B_2 = 0 )</td>
</tr>
<tr>
<td>III</td>
<td>([\frac{1}{p-q}, p_3])</td>
<td>([H + L + \frac{1}{A-y}, 2H])</td>
<td>(1,p)</td>
<td>( S = 2U - \frac{1}{p-q}\phi, B_1 = 0, B_2 = \frac{1}{p-q}\phi )</td>
</tr>
<tr>
<td></td>
<td>([p_2, \frac{1}{p-q}])</td>
<td>([H + L + \frac{1}{A-y}, 2H])</td>
<td></td>
<td>( S = 2K, B_1 = 0, B_2 = \frac{1}{p-q}\phi )</td>
</tr>
<tr>
<td>IV</td>
<td>([p_3, \frac{1}{p-q}])</td>
<td>([H + L + \frac{1}{A-y}, 2H])</td>
<td>(1,1)</td>
<td>( S = 2K, B_1 = \frac{a}{p-q}, B_2 = 2\frac{a}{p-q} )</td>
</tr>
<tr>
<td>V</td>
<td>([\frac{1}{p-q}, p_4])</td>
<td>([\max {\omega_2 - \frac{2(U-K)}{q(A-y)}, \omega'_2 - \frac{2(U-K)}{q(A-y)}}, H + L + \frac{1}{A-y}\phi])</td>
<td>(1,1)</td>
<td>( S = 2K, B_1 = \frac{a}{p-q}, B_2 = 2\frac{a}{p-q} )</td>
</tr>
<tr>
<td>VI</td>
<td>([p_3, \frac{1}{p-q}, p_5])</td>
<td>([2L + \frac{1}{A-y}\phi, \max {\omega_2 - \frac{2(U-K)}{q(A-y)}, \omega'_2 - \frac{2(U-K)}{q(A-y)}}, H + L + \frac{1}{A-y}\phi])</td>
<td>(0,1-q)</td>
<td>( S = 2U - \frac{1}{p-q}\phi, b(L,H) = \frac{a}{p-q}, b(L,H) = 0, b(H,H) = \frac{a}{p-q} )</td>
</tr>
<tr>
<td></td>
<td>([\frac{1}{p-q}, \phi, p_1])</td>
<td>([\omega_2 - \frac{2(U-K)}{q(A-y)}, H + L + \frac{1}{A-y}\phi])</td>
<td></td>
<td>( S = 2U - \frac{1}{p-q}\phi, b(L,H) = \frac{a}{p-q}, b(L,H) = 0, b(H,H) = \frac{a}{p-q} )</td>
</tr>
<tr>
<td></td>
<td>([p_2, \frac{1}{p-q}, \phi])</td>
<td>([\omega_2 - \frac{2(U-K)}{q(A-y)}, H + L + \frac{1}{A-y}\phi])</td>
<td></td>
<td>( S = 2U - \frac{1}{p-q}\phi, b(L,H) = \frac{a}{p-q}, b(L,H) = 0, b(H,H) = \frac{a}{p-q} )</td>
</tr>
</tbody>
</table>

Figures 3.6 illustrates the parametric regions with the different effort profiles under the optimal contract. In Section 3.3.3, I showed that without limited inventory, the optimal contract is either a gradual contract inducing \( e = (1,1) \) or an extreme contract inducing \( e = (1,p) \) or \( e = (0,q) \). In this scenario, if \( \Omega \) is relatively high, we are in Region IV in which a gradual contract induces \( e = (1,1) \) or in Region III in which an extreme contract induces \( e = (1,p) \). For a small \( \Omega \), we are in Region V in which a gradual contract induces effort \( e = (1,1-p) \). In this case, the agent still exerts early effort, but will exert effort in the second period only when the first period’s outcome is \( L \). For a yet smaller \( \Omega \), we are in Region VI in which a history-dependent contract inducing effort \( e = (0,1-q) \) is optimal for the principal. Under this contract, the principal offers \( b_2(L,H) = 0 \) and \( b_2(L,H) = b_2(H,H) = \frac{\phi}{p-q} \). In this case, the bonus payment is not affected by the first period’s demand outcome, and will be issued if the second period realizes as \( H \).

The principal offers \( b_2(L,H) = 0 \) and \( b_2(L,H) = b_2(H,H) = \frac{\phi}{p-q} \). In this case, the bonus payment is not affected by the first period’s demand outcome, and will be issued if the second period realizes as \( H \).

\[
\omega_6 = H + L - \frac{1-p}{p+q}(H - L) + \frac{p^2 + p -pq}{(p-q)(p-q^2)} \phi, \omega_7 = 2L + \frac{p^2 + p - pq}{(1-p)(p-q)} \phi, \mu_1 = \frac{1}{2} \left[ \frac{p+q-p^2-q^2}{p-q} \phi - (1-p)(p-q)(H - L) \right], \mu_2 = \frac{1}{2} \left[ \frac{p+q-p^2-q^2}{p-q} \phi - (1-p)(p-q)(H - L) \right].
\]

Note that this is the only case where the non-decreasing constraint (that compensation should not be decreasing in sales) binds in the optimal contract. In particular, to induce \( e_2^2 = 1 \), we need \( b_2(L,H) \) is at least \( \phi \cdot \frac{1}{p-q} \). Given \( b_2(L,H) = \frac{\phi}{p-q} \), \( b_2(H,H) \) cannot be less than \( \phi \cdot \frac{1}{p-q} \) due to the non-decreasing constraint \( b_2(H,H) \geq b_2(L,H) \).
effort in the first period, and will exert effort in the second period only if the outcome of the first period is $L$. This is again the interesting case of the “hockey stick” effort profile with effort postponement.\footnote{Note that with limited inventory the effort profile $(0, q)$ is not induced under the optimal contract, while without limited the effort profile $(0, 1 - q)$ is not induced under the optimal contract.}

### 3.4.3 Comparison between Period-by-Period and Two-Period Contracts with Limited Inventory

As a result of limited inventory, the conclusion from the basic model that firms weakly prefer two-period contracting over period-by-period contracting does not always hold true. Overall, the period-by-period contract outperforms the two-period contract when limited liability is very small or very large, since it gives the principal more flexibility in adjusting the contracts (note that I maintain the assumption that compensation is non-decreasing in sales). I state the following proposition (the proof is in the Online Appendix).

**Proposition 3.7** *In the presence of limited inventory, the period-by-period contract and the*
two-period contract compare as per the following table; specifically, the principal prefers the period-by-period contract to the two-period contract in Regions III and IV. \(^{10}\)

<table>
<thead>
<tr>
<th>Region</th>
<th>(U - K)</th>
<th>(\Omega)</th>
<th>(e = (e_1, e_2))</th>
<th>Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(2L \leq \Omega \leq \omega_1 - \frac{1}{2(1 + q)(p-q)})</td>
<td>(0 \leq U - K &lt; \frac{\phi}{\mu} &lt; \frac{1}{\phi})</td>
<td>(1, p)</td>
<td>Period-by-period / Two-period</td>
</tr>
<tr>
<td>II</td>
<td>(2L \leq \Omega \leq \omega_1 - \frac{1}{2(1 + q)(p-q)})</td>
<td>(\omega = \frac{1}{2(1 + q)(p-q)} U - K) (&lt; 2H)</td>
<td>(\pi_3 \leq U - K &lt; \frac{\phi}{\mu})</td>
<td>(1, 1)</td>
</tr>
<tr>
<td>III</td>
<td>(\omega_1 - \frac{1}{2(1 + q)(p-q)} (U - K) &lt; \Omega \leq 2H)</td>
<td>(\pi_3 \leq U - K &lt; \frac{\phi}{\mu})</td>
<td>(0.1 - q)</td>
<td>Period-by-period</td>
</tr>
<tr>
<td>IV</td>
<td>(\max(\omega_0 - \frac{2(1 + q)(p-q)}{\phi} (U - K), \omega_1 - \frac{1}{2(1 + q)(p-q)} (U - K)) &lt; \Omega \leq \min(\omega_0 + \frac{2(1 + q)(p-q)}{\phi} (U - K), \omega_1 - \frac{1}{2(1 + q)(p-q)} (U - K)))</td>
<td>(\min(\omega_0, \omega_1) \leq U - K &lt; \frac{\phi}{\mu})</td>
<td>(1, 1 - q)</td>
<td>Period-by-period / Two-period</td>
</tr>
<tr>
<td>V</td>
<td>(\omega_0 - \frac{1}{2(1 + q)(p-q)} (U - K) &lt; \Omega \leq 2H)</td>
<td>(\omega_1 - \frac{1}{2(1 + q)(p-q)} (U - K) &lt; \Omega \leq 2H)</td>
<td>(0.1 - q)</td>
<td>Two-period</td>
</tr>
<tr>
<td>VI</td>
<td>(\omega_1 - \frac{1}{2(1 + q)(p-q)} (U - K) &lt; \Omega \leq 2H)</td>
<td>(\omega_1 - \frac{1}{2(1 + q)(p-q)} (U - K) &lt; \Omega \leq 2H)</td>
<td>(0.1)</td>
<td>Two-period</td>
</tr>
</tbody>
</table>

Figure 3.7 illustrates the results of the proposition. I now discuss the above results in greater detail with respect to the agent’s limited liability, keeping \(\Omega\) fixed at \(\Omega = H - L\). In Region III, where the agent’s limited liability is large, the period-by-period contract implements \(e = (0, 1 - q)\) and performs the best for the principal. This is because, under

\[
\omega_1 \equiv H + L + \frac{p^2}{(p + q)(p - q)} \phi, \omega_2 \equiv 2L + \frac{p+q+pq-p^2-q^2}{(1-p)(p-q)} \omega_1, \omega_3 \equiv H + L - \frac{1-p}{q} (H - L) + \frac{p+q+pq-p^2-q^2}{q(p-q)} \omega_2, \omega_4 = H + L + \frac{1-p}{p} (H - L) - \frac{p+q+pq-2p^2}{p(p-q)^2} \phi, \mu_4 = \frac{(p-q)(1-p)}{2-p}(H - L).
\]

---

\(^{10}\)
the constraint that compensation cannot be decreasing in sales (to prevent the agent from restricting sales) the two-period contract rewards the agent more than the period-by-period contract when demand outcomes at both periods are $H$. In Region IV, where the agent’s limited liability is small, the period-by-period contract implements $e = (1, 1-p)$ and performs the best for the principal. The two-period contract cannot replicate the period-by-period contract for inducing $e = (1, 1-p)$, because it suffers from the agent’s dynamic gaming. To ensure early effort exertion, the principal pays higher bonuses under the two-period contract when demand outcomes at both periods are $H$, compared with under the period-by-period contract. In those scenarios, a period-by-period contract that gives the principal flexibility to adjust quota levels performs better than a two-period contract. In Region V where the limited liability is intermediate, the two-period contract induces $e = (0, 1-q)$ performs better than the period-by-period contract, since it can provide more effective incentives by paying the agent for the aggregate at the end of the second period. I also state the following corollary.

**Corollary 3.4** With limited inventory, if the agent’s limited liability is large enough compared to the outside option, the principal induces the agent to delay effort exertion under the optimal contract, i.e., the “hockey stick” effort profile is optimally induced by the principal.

### 3.5 Conclusions

Firms employ and reward salespeople over multiple time periods. I address a fundamental question that arises in this context: Should salespeople be rewarded using period-by-period contracts that reward for the outcome of each period, or should they be rewarded using a multi-period contract that rewards for the outcomes over multiple periods? I employ a two-period repeated moral hazard framework with stochastic demand and unobservable effort, and assume the agent to be risk neutral with limited liability. I allow the agent’s limited
liability to be greater than or smaller than his outside option.

I find that a multi-period contract weakly dominates a period-by-period contract but, interestingly, I find that the former can do strictly better than the latter. The reason is that in a multi-period contract the firm can reward the salesperson only for more extreme outcomes as compared to a period-by-period contract, which allows it to incentivize the salesperson more strongly (the “incentive effect”). Even though the salesperson has the ability to game the contract by adjusting effort levels strategically across periods (the “gaming effect”), the incentive effect dominates under certain conditions. I find that the above result holds when the effectiveness of the salesperson’s effort in terms of inducing demand is either low or high, but not when it is intermediate. When the effort effectiveness is low, the principal strongly prefers a multi-period contract that, for a low level of limited liability awards a small bonus to induce no effort in the early period and conditional effort only in the later period, while for a higher level of limited liability awards a large bonus to induce effort exertion in the early period and conditional effort exertion in the later period.

I extend my analysis to a case in which a fixed amount of inventory has to be sold across multiple periods — this introduces dependence between periods as the principal’s preferred effort exertion in the later period depends on the outcome of the early period. In this case, I find that, under the assumption that compensation cannot be decreasing in sales, a period-by-period contract can strongly dominate a multi-period contract.

I also study the effort exertion profile of the agent. I show that, under different conditions, the principal may optimally induce different effort profiles which may or may not include effort exertion in early period. In other words, effort postponement, which is often called a “hockey stick” effort profile and is typically interpreted as shirking by the agent and suboptimal for the principal, may actually be optimal for the principal. With independent periods, this happens when limited liability is small compared to the outside option, which is a reasonable condition to hold in reality. With dependent periods, effort postponement
happens even for a larger parameter space (including cases in which limited liability is larger than the outside option) because, given that only a limited amount of inventory can be sold, the principal wants to wait for the first period outcome to benefit from the eventuality that it is high without paying for effort exertion. The optimal effort exertion in the second period is typically conditional on the outcome of the first period, and may be conditional on demand being high or low in the first period, depending on the parameters. In summary, I show that a number of different effort profiles are possible under the optimal contract, and high effort exertion in every period is actually not always desired by the principal (even without inventory constraints). Therefore, one has to be careful in making inferences about contract efficiency from effort profiles of agents (which firms sometimes monitor, or can back out from data as in Misra and Nair (2011)).

I obtain an interesting and useful interpretation of my model if I impose the restriction that in a multi-period contract only the total sales at the end of the multiple periods can be measured. Similar to Kräkel and Schöttner (2016), this models a situation in which a salesperson, within a time period, has the opportunity to sell to multiple potential consumers one at a time; however, the outcome of each interaction is not observable and only the total sales at the end are observable. I show that a gradual contract (that rewards for intermediate levels of sales) or an extreme contract (that rewards only for reaching a large enough level of sales) may be optimal for the principal in this setting, and that effectiveness of the salesperson’s effort, degree of limited liability, inventory constraints, etc., will all influence this. Furthermore, multiple effort profiles may be observed here too in the optimal contract, including that in which effort is exerted only in the later part of this time period (Chen 2000, Misra and Nair 2011).
Chapter 4

Dynamically Motivating a Bold Action

4.1 Introduction

When faced with a selling task, salespeople are often able to balance pursuing a “bold” action with higher sales potential but also higher variance, and maintaining a “safe” action with limited sales potential but lower variance. For example, a salesperson may make an effort to reach out to new customers as well as follow up with existing customers (Godes 2004, Rubel and Prasad 2015). The former is considered bold for the salesperson, but if successful, can bring in higher profits for the firm. The latter is considered the safer route, since an established relationship with customers already exists. This agents’ dynamic shifting between bold and safe actions can be easily extended to other contexts as well. Fund managers who need to reach a certain threshold of rate of return at the end of the year may re-balance their portfolios — between more-risky assets with higher return, and less-risky assets with lower return — several times throughout the year. Brown et al. (1996) and Chevalier and Ellison (1997b) find that fund managers appear to act bold late in the year by investing on
portfolios with greater volatility and try to catch the market, if they are a few points behind. They also found that fund managers may also have an incentive to play it safe and act more like an index fund if they are ahead of the market. Researchers may assess whether to keep pursuing an innovative project, or switch to a mundane project, in order to meet promotion requirements. In Ederer and Manso (2012)’s experiment, subjects choose between following tips from the previous manager (i.e., taking a safe action), or explore different locations to discover a more profitable strategy (i.e., taking a bold action). The variability of action choices significantly declines over the course of their experiment, suggesting that subjects act more bold in earlier periods than in later periods.

In spite of many papers that document agents’ dynamic gaming in altering between acting bold or playing safe under a long time horizon contract, previous research has not reached a consensus about what format the optimal contract takes to dynamically induce bold actions. A few papers show evidence that long time horizon contracting with delayed rewards is more effective in inducing bold actions than short time horizon contracting. Chung and Narayandas (2017) collaborate with a retail chain at Sweden that sells electronic goods. They find that less frequent quotas encourage salespeople to act bold and sell more high-margin products, and pursue fewer incremental sales that are considered safe. Lerner and Wulf (2007) suggest that the shift from compensating corporate R&D heads using short term contracting to long term contracting is associated with more heavily cited patents. In other words, long term incentives encourage researchers to take radical approaches and explore untested technologies, rather than playing safe and applying existing techniques. Protecting agents from earlier downside risks under a long time horizon contract can further facilitate incentivizing bold actions based on both industry practices and empirical evidence. An article from Wall Street Journal \(^1\) reported that a growing number of companies are explicitly

\(^1\)https://www.wsj.com/articles/SB10001424052970204010604576594671572584158
rewarding failure — giving cash prizes or trophies to people who foul up, in order to encourage creativity. Azoulay et al. (2011) show that under a research grant that tolerates earlier failures, researchers take more radical inquiries and produce higher-impact work measured by the number of citations, than a research grant that does not tolerate earlier failures.

Firms usually employ agents for an extended period of time, and a recognized issue is that if the long-term incentive plan is inherently nonlinear (for instance, the widely popular quota-bonus plan), agents can dynamically engage in different sales activities based on the salesperson’s past performance. In other words, with a long-term compensation plan, according to the salesperson’s current sales status, a salesperson can decide whether to pursue the bold transaction that could mean higher sales, or the safe transaction with limited sales expectation. In this chapter, I ask: How frequently should a firm compensate its sales agents over the long-term, when the agent can shift between bold and safe actions dynamically over time? What is the structure of the optimal contract, and what action profiles are induced by the optimal incentive contract? Finally, does an agent’s dynamic shifting between bold and safe actions always hurt the principal?

I build a two-period model under the principal-agent framework to approach these questions. Same as in Chapter 3, a risk neutral firm (principal) hires a risk neutral salesperson (agent) for two periods. In this context, using a short-term horizon evaluation implies offering two period-by-period contracts, where each contract is determined at the start of a period and pays at the end of the period based on the outcome of the period. On the other hand, using a long-term horizon evaluation implies offering one two-period contract that is determined at the start of the first period and pays once at the end of the second period based on the outcomes of the two periods. I further assume that the agent has limited liability, an assumption that has been widely made in previous salesforce literature.

However, unlike Chapter 3, demand in each period is uncertain and can exist at any of three levels (high, medium, and low). At the beginning of each period, the agent can
choose to take either the bold action or the safe action. Compared with the safe action, the bold action has an increased probability of achieving both high and low demand realizations. Furthermore, the upside potential of taking the bold action is more greater than its downside risk (relative to the safe action). I focus on the parameter space where the bold action is more costly for the agent than the safe action, so that the agent’s and the principal’s preferences over the the bold action (relative to the safe action) are misaligned *ceteris paribus*. The agent’s action is unobservable to the principal, and the principal can only observe the sales outcome in each period.

In general, the principal has three possible ways of inducing the agent to perform the bold action — rewarding the agent for high demand realization, penalizing the agent for medium demand realization, or protecting the agent from low demand realization. I find that under the optimal period-by-period contract, the principal induces the bold action by providing only an upside reward (i.e., the principal issues a bonus upon high demand realization).

I also find that there are three possible optimal two-period contracts, given different conditions (discussed shortly). The contract format is determined by how much the firm wants later actions to depend on earlier outcomes. The “account-balance” contract compensates the agent based on how many times the agent obtains high demand realization, and induces later actions that are independent of earlier demand outcomes. The “extreme” contract incentivizes bold actions via a hard-to-achieve quota, and induces later actions that are heavily dependent on earlier demand outcomes. The “polarized” contract allows agents to “act bold” and make up sales if demand in the first period is low, and induces later actions that are moderately dependent on earlier demand outcomes.

My analysis shows that, for the firm, a two-period contract weakly dominates a period-by-period contract, as expected. Interestingly, however, I find that the two-period contract, even though it allows for dynamic gaming by the agent, strongly dominates the period-by-period contract under certain conditions. This can be explained by understanding the two
countervailing effects at play—the expected bold actions induced, and the expected bonus payment to induce each bold action. First, making later actions heavily dependent (under an extreme two-period contract) or moderate dependent (under a polarized two-period contract) on earlier demand outcomes, pays less bonus to induce a bold action on average, than making later actions independent of earlier outcomes (under an account-balance two-period contract, or a period-by-period contract). This is because given the same expected bonus payment, making later actions heavily and moderately dependent on earlier outcomes incentivizes more bold actions earlier on. However, making later actions independent of earlier demand outcomes reduces gaming losses and induces more bold actions.

Therefore, when providing incentives is of a higher order than reducing gaming losses for the principal, an extreme two-period contract or a polarized two-period contract that pays less for inducing each bold action leads to higher profits for the principal, compared with a period-by-period contract (or an account-balance two-period contract). In terms of agents’ action profiles, the firm structures the contract to induce the bold action in the first period, since it is weakly less costly to induce bold actions in earlier periods than in later periods.

However, if the two periods become independent, for example through a limited level of inventory to be sold across these two periods, then the period-by-period contract can strictly outperform the two-period contract, under the assumption that an agent chooses his action under the period-by-period contract based on the current period’s contract. This is because, with limited inventory, the principal may not want to induce a bold action in the latter period, if the first period has a high demand realization. However, taking a bold action increases the probability of achieving a high demand realization. As a result, the principal has to compensate the agent more, compared with the period-by-period contract, if she wants to induce a bold action in the earlier period. This suggests the fully-flexible two-period contract which compensates the agent based on any possible sales histories cannot perfectly replicate the period-by-period contract in certain scenarios, if agents are not completely
forward looking under period-by-period contracting.

This chapter is organized as following. In Section 4.2, I present the basic model together with key assumptions. In Section 4.3, I first establish the first-best benchmark case, assuming that the firm can observe the agent’s actions. I then derive the optimal period-by-period contract and the optimal two-period contract for the principal, respectively. In Section 4.4, I compare the optimal period-by-period contract and the optimal two-period contract, with both independent and dependent periods. Section 4.4 demonstrates a scenario in which the principal cannot perfectly observe the sales outcomes. In Section 4.6, I summarize.
4.2 Model

In my model, a firm (the principal, referred as “she”) hires a salesperson (the agent, referred as “he”) for two time periods denoted by $t \in \{1, 2\}$. Demand in both periods is uncertain and independent, and can exist at any of three levels (high, medium, and low). For simplicity, I normalize the medium level of the demand outcome to 0, and keep the high and low levels of demand outcomes symmetric around the middle level, as $d$ and $-d$ respectively. Let $D_t$ be the demand realization in period $t$, then $D_t$ can be $d$, 0, or $-d$, corresponding to the high, medium, and low levels of the demand outcome, respectively.

The agent’s action in period $t$, denoted by $e_t$, can be either 1 or 0, i.e. the agent either takes the bold action or the safe action in each period. However, the principal does not observe the agent’s action. We can think of taking action $e = 1$ as a salesperson reaching out to new customers, and talking to and convincing the client to make the purchase, and taking action $e = 0$ as a salesperson following up with existing customers.

If the agent takes the safe action ($e_t = 0$), demand is realized as $d$ or $-d$, each with a probability of $p$ ($0 \leq p \leq \frac{1}{2}$), and is realized as 0 with a probability of $1 - 2p$. If the agent takes the bold action ($e_t = 1$), compared with taking the safe action, $D_t$ is more likely to realize as $d$ or $-d$, and less likely to realize as 0. Specifically, the probability that demand realizes as $d$ increases by $h$ to $p + h$, the probability that demand realizes as $-d$ increases by $l$ to $p + l$, with $0 < l < h < \frac{1}{2} - p$, and the probability that demand realizes as 0 decreases by $h + l$ to $1 - 2p - h - l$. I summarize demand outcomes under the agent’s two possible
actions as below,

\[ D_t(e = 0) = \begin{cases} 
  d & \text{w.p. } p \\
  0 & \text{w.p. } 1 - 2p \\
  -d & \text{w.p. } p 
\end{cases}, \quad D_t(e = 1) = \begin{cases} 
  d & \text{w.p. } p + h \\
  0 & \text{w.p. } 1 - 2p - h - l \\
  -d & \text{w.p. } p + l 
\end{cases}. \]

Here, \( h \) and \( l \) can be interpreted as the upside potential and the downside risk, respectively, of taking the bold action in generating sales (compared with the safe action). Taking action \( e = 1 \) is considered more bold than taking action \( e = 0 \) for generating demand, since it has a larger upside potential and downside risk. A larger \( h \) relative to \( l \) also implies that taking the bold action entails larger upside potential than downside risk, and leads to a higher expected sales outcome than taking the safe action.

I assume that both the principal and the agent are risk neutral. Unlike the firm, however, the salesperson has limited liability, implying that he must be protected from downside risk. Limited liability is a widely observed feature of salesforce contracts in the industry, and has been widely assumed in salesforce literature. The limited liability assumption also implies the existence of a wage floor for the salesperson, which is aligned with industry practice. I normalize the agent’s limited liability to 0 in each period, i.e., to employ the agent for one period, the principal must guarantee a compensation of at least 0 for any demand outcome. For simplicity, I also normalize the agent’s outside option to 0. To employ the sales agent, the agent’s expected net utility from engaging in sales activities with the firm should be no less than his outside option \( u_0 \).

Finally, I focus on the interesting case that the agent needs to exert more effort in taking the bold action than the safe action. This is also a natural assumption since the bold action generates high demand in expectation compared with the safe action. If taking the bold action is less costly for the agent, then there is no conflict of interest between the principal
and the agent. The principal will pay the agent a fixed salary equal to his limited liability, 0, to induce the safe action in equilibrium given any parameter space. For this purpose, the cost of taking the bold action is given by $\phi > 0$ for $e_t = 1$ and the cost of taking the safe action is normalized to zero for $e_t = 0$. Furthermore, I assume that all of the demand can be met, and each unit sold provides a revenue of 1 and has a marginal cost of zero.

The agent is reimbursed for his action using an incentive contract. The agent’s action is unobservable to the firm, and demand is random but can be influenced by the agent’s action, so the firm and the agent sign an outcome-based contract. The firm can propose a disaggregate contract, i.e., two period-by-period contracts, where each contract is determined at the start of each period and pays at the end of the period based on the outcome of the period. Alternatively, the firm can propose a single aggregate two-period contract that is determined at the beginning of the first period and pays once at the end of the second period based on the outcomes of the two periods. Similar to Chapter 3, I assume that under a period-by-period contract, an agent chooses his effort level during a period only based on the current period’s contract, and I do not allow renegotiation under the two-period contract.

4.3 Analysis

4.3.1 First-Best Scenario

I first establish the first-best solution, assuming that the agent’s action is observable. As the two periods are independent and equivalent, it suffices to study just one period. Because

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2 The discrete demand distribution that I have assumed ensures that action will not change the support of the demand distribution; otherwise, the principal may be able to infer the agent’s action from the demand outcome and would induce the agent to work by imposing a large penalty for demand outcomes that cannot be obtained under equilibrium action but can be obtained under off-equilibrium actions, as argued in Mirrlees (1976).
moral hazard is absent in the first-best scenario, the firm can implement any action by the agent \( e_t \) in either period, by reimbursing the agent a fixed salary \( s_t \) which must be at least 0, while ensuring the agent’s participation. The principal’s problem in each period is the following.

\[
\max_{s_t} \ E[D_t|e_t] - E[s_t|e_t]
\]

s.t. \( U_A(e_t) \geq 0 \) \( (PC_t) \)
\( s_t \geq 0 \) \( (LL_t) \)

\( (PC_t) \) stands for the agent’s participation constraint, where \( U_A(e_t) \) denotes the salesperson’s expected net utility on taking action \( e_t \), which is equal to \( s_t - \phi \) if the agent takes the bold action, and is equal to \( s_t \) if the agent takes the safe action. To employ the sales agent, the principal needs to provide a fixed salary that causes the agent’s expected net utility from taking action \( e_t \) to be no less than his outside option. This simplifies as \( s_t \geq \phi \) if \( e_t = 1 \), and as \( s_t \geq 0 \) if \( e_t = 0 \). \( (LL_t) \) is the agent’s limited liability constraint. It states that the fixed salary \( s_t \) that the agent receives is no less than his limited liability.

If the contract specifies that the agent takes the bold action in period \( t \in \{1, 2\} \), i.e., \( e_t = 1 \), the principal’s expected profit is equal to the expected market demand associated with the bold action, \((h - l)d\), minus the minimal salary to ensure the bold action, \( \phi \), i.e., \((h - l)d - \phi\). If the contract specifies that the agent takes the safe action, the principal garners the market outcome associated with the safe action, 0, and also pays the minimal salary, 0, to employ the salesperson. This leads to the following first-best solution.

**Result 4.1** The first-best solution, attainable if the agent’s choice of action is costless observable, would entail instructing the agent to choose the bold action, and paying a fixed salary equal to the agent’s cost \( \phi \), if and only if \( h - l \geq \frac{\phi}{d} \). Otherwise, the principal directs the agent to choose the safe action, and pays 0.

Based on Result 4.1, the firm will direct the salesperson to choose a bold action if and only
if the increase in the expected demand subject to taking the bold action (given by \((h - l)d\)) outweighs the marginal cost of soliciting it (given by \(\phi\)). Intuitively, the principal will want the agent to take a bold action when its upside potential, \(h\) is large enough compared with its downside risk, \(l\). Result 4.1 suggests that if \(h = l\), the principal will not have incentives to induce the risky action in any parameter space. To rule out the trivial case in which the firm is not interested in motivating the bold action in the first-best scenario, I only consider when \(h - l \geq \frac{\phi}{d}\) for the remainder of this chapter.

### 4.3.2 Period-by-Period Contract

In this scenario, the principal initiates a one-period contract at the beginning of each period. The agent’s action in each period is rewarded separately — I call this a disaggregate contract. Again, it suffices to study just one period, when the two periods are identical and independent.

Before specifying the principal’s problem, I derive a general condition for inducing the bold action in a certain period. I denote the agent’s continuation payoff at a certain time as \(v(d)\), \(v(0)\), and \(v(-d)\), corresponding to the agent’s net utility as derived from backward induction if demand in the current period realizes as \(d\), \(0\), and \(-d\), respectively. To induce the agent to take the bold action, the principal needs to propose a contract that satisfies

\[
(p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq pv(d) + (1 - 2p)v(0) + pv(-d),
\]

i.e. the agent would obtain a higher expected net payoff from taking the bold action compared with taking the safe action. Lemma 4.1 summarizes this result.

**Lemma 4.1** The agent will take the bold action if the continuation payoffs he is faced with satisfy \(v(d) - v(0) \geq \frac{1}{h}(v(0) - v(-d)) + \frac{\phi}{h}\).

The constraint in Lemma 4.1 can be satisfied either by increasing \(v(d)\), reducing \(v(0)\), or increasing \(v(-d)\). In other words, in order to induce the bold action, the principal can
choose to: reward the agent upon high demand realization, penalize the agent upon medium demand realization, or protect the agent against low demand realization.

Next, I develop the optimal strategy for the principal to incentivize the bold action under the period-by-period contract. Since demand follows a discrete distribution, the principal offers quota-bonus contracts with a quota level $\chi_t \in \{d, 0, -d\}$ and bonus $b_t(\chi_t) \geq 0$, where bonus $b_t(\chi_t)$ is paid to the salesperson if and only if the sales reach the quota $\chi_t$, together with a fixed salary of $s_t$. The principal’s problem in each period is the following.

$$\max_{s_t, b_t(d), b_t(0), b_t(-d)} E[D_t|e_t] - E[s_t + b_t(D_t)|e_t]$$

s.t.

$$U_A(e_t) > U_A(\tilde{e}_t) \quad (IC_t)$$

$$U_A(e_t) \geq 0 \quad (PC_t)$$

$$s_t, s_t + b_t(d), s_t + b_t(0), s_t + b_t(-d) \geq 0 \quad (LL_t)$$

In addition to the participation constraint $(PC_t)$ and the limited liability constraint $(LL_t)$ which can be interpreted in a similar way to the first-best scenario, the contract needs to satisfy an incentive compatibility constraint $(IC_t)$. It states that in order to induce action $e_t$, the principal needs to ensure that the agent gains a higher net utility in taking action $e_t$ compared with a different action $\tilde{e}_t$.

To implement $e_t = 1$, from the incentive compatibility constraint $(IC_t)$, the principal needs to set $b_t(d) - b_t(0) \geq \frac{1}{h}(b_t(0) - b_t(-d)) + \frac{\phi}{h}$ (based on Lemma 4.1). The participation constraint $(PC_t)$ requires that the agent’s expected utility from choosing the safe action is no lower than his reservation utility, that is, $(p + h)b_t(d) + (1 - 2p - h - l)b_t(0) + (p + l)b_t(-d) - \phi \geq 0$. To meet the limited liability constraint $(LL_t)$ we need the guaranteed salary to be no less than the agent’s limited liability. The solution is that, to implement the bold action $e_t = 1$, the principal must offer a fixed salary $s_t = 0$, and a bonus $b_t(d) = \frac{\phi}{h}$ if and only if the demand outcome is $d$. 

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This suggests that, under the period-by-period contract, in order to induce the bold action, the principal is better off only rewarding the agent for high demand realization, rather than penalizing the agent for medium demand realization, or protecting the agent against low demand realization. In other words, rewarding the agent upon a high demand outcome is the most efficient in motivating a bold action, compared with the other two alternatives. This is because a relaxed MLRP constraint such that \( \frac{p + h}{p} > \max \left\{ \frac{p + l}{p}, \frac{1 - 2p - h - l}{1 - 2p} \right\} \) is satisfied\(^{3}\). The relaxed MLRP property essentially states that the highest demand outcome is the most reliable indicator of the salesperson taking a bold action. This is sufficient to ensure that the principal will concentrate all bonus payment on the high demand outcome in order to induce a bold action in a single period. This important insight is consistent with recent literature, e.g., Dai and Jerath (2018b).

In order to implement the safe action \( e_t = 0 \), it is enough for the principal to offer the agent a fixed salary \( s_t = 0 \). Weighing the expected demand versus the compensation cost for the principal to induce the bold and safe actions, I present the optimal period-by-period contract in the following proposition.

**Proposition 4.1 (Optimal Period-by-Period Contract)** Under the optimal period-by-period contract, when \( h - l \geq \left( 1 + \frac{p}{h} \right) \frac{\phi}{d} \), the firm induces the bold action by paying a fixed salary \( s_t = 0 \) and rewarding the agent \( b_t(d) = \frac{\phi}{h} \) upon high demand realization. Otherwise, the firm induces the safe action by the salesperson and simply offers him a fixed salary of \( s_t = 0 \).

---

\(^{3}\)The regular MLRP is not satisfied in my setting since \( \frac{p + h}{p} > \frac{p + l}{p} > \frac{1 - 2p - h - l}{1 - 2p} \).
Figure 4.1: Optimal Period-by-period Contract

Note: $\phi = 0.01$, $d = 1$, $p = 0.3$ in the left figure. $\phi = 0.2$, $d = 1$, $l = 0.05$ in the right figure.

Figure 4.1 illustrates the optimal period-by-period contract with respect to the upside potential $h$, the downside risk $l$ and the baseline outcome $p$. In Region I, the principal wants to induce the safe action in the first-best scenario but not in the period-by-period contracting scenario. In Region II, the principal wants to induce the bold action in the period-by-period scenario. From Figure 4.1, we can see that the firm implements the bold action in a smaller parameter space than in the first-best scenario, conforming to the standard result when moral hazard exists.

4.3.3 Two-Period Contract

I derive the optimal two-period contract for the firm in this section. A two-period contract is proposed at the beginning of the first period that pays at the end of the second period according to the sales realizations across the two periods. The timeline of the two-period contract is the same as that in Chapter 3. At $T = 1$, the principal specifies the contract and then the agent decides whether to accept the offer. If the contract is accepted, the agent
decides on his action in the first period, \(e_1\), and then determines his second period action \(e_2^{D_1}\) based on the commonly observed demand realization in the first period \(D_1\). At the end of \(T = 2\), the demand outcome \(D_2\) is observed by the agent and the principal, and the agent gets paid based on the contract.

In the dynamic setting featuring unobservable agents’ actions, the contract paying at the end of two periods implies that the agent can “game” the system by choosing his action in the later period according to the outcome of the earlier period, and, expecting this, strategically choose his action in the earlier period. I denote the two-period action profile by \((e_1, \{e_2^d, e_2^0, e_2^{-d}\})\), where the second period’s action \(e_2^{D_1}\) is contingent on the first period’s demand realization, \(D_1\).

In full generality, this contract involves a guaranteed salary to employ the agent for two periods, plus a bonus issued at the end of the two periods that is contingent on the entire history of output. I denote the fixed salary as \(S\), and the bonus paid at the end of \(T = 2\) with \(B(D_1, D_2)\). Such a contract thus stipulates nine possible bonuses, with \(D_1, D_2 \in \{d, 0, -d\}\).

The principal’s problem is similar to that in the period-by-period case, but the principal has ten decision variables in this scenario. I formulate the principal’s problem in the following,

\[
\begin{align*}
\max_{s, \ B(D_1, D_2)} & \quad E[D_1 + D_2|(e_1, \{e_2^d, e_2^0, e_2^{-d}\})] - E[S + B|(e_1, \{e_2^d, e_2^0, e_2^{-d}\})] \\
\text{s.t.} & \quad U_A(e_2^{D_1}) > U_A(\tilde{e}_2^{D_1}), \ \forall \ D_1 \in \{d, 0, -d\} \quad (IC^{D_1}_2) \\
& \quad U_A(e_1|\{e_2^d, e_2^0, e_2^{-d}\}) > U_A(\tilde{e}_1|\{e_2^d, e_2^0, e_2^{-d}\}) \quad (IC_1) \\
& \quad U_A((e_1, \{e_2^d, e_2^0, e_2^{-d}\})) \geq 0 \quad (PC) \\
& \quad S, S + B(D_1, D_2) \geq 0, \ \forall \ D_1, D_2 \in \{d, 0, -d\} \quad (LL)
\end{align*}
\]

\((IC^{D_1}_2)\) stands for the agent’s incentive-compatible constraint in the second period following the realization of the first period outcome \(D_1\), where \(U_A(e_2^{D_1})\) represents the agent’s net payoff in period 2 upon exerting action \(e_2^{D_1}\). To induce \(e_2^{D_1}\), the principal needs to ensure
that the agent gets a higher payoff upon exerting action $e_2^{D_1}$, compared with a different action $\tilde{e}_2^{D_1}$. $U_A(e_1|e_2^d, e_2^0, e_2^{-d})$ denotes the agent’s net payoff across two periods upon exerting $e_1$ in the first period, given that the agent is induced to exert action $\{e_2^d, e_2^0, e_2^{-d}\}$ in the second period. To induce $e_1$, the principal needs to ensure that the agent obtains a higher total net payoff upon exerting $e_1$, compared with a different action $\tilde{e}_1$. The participation constraint (PC) and the limited liability constraint (LL) are similar to that in the period-by-period case.

I now determine the optimal two-period compensation plan for the firm. First, for each action profile $(e_1, \{e_2^d, e_2^0, e_2^{-d}\})$ that the firm wants to motivate, I find an optimal scheme for the firm by minimizing the expected payment. Since the agent needs to decide for four actions, each of which can take a value of either 0 or 1, there are 16 combinations in total. I then find the optimal contract by balancing the expected revenue $E[D_1 + D_2]$ less the expected compensation cost $E[S + B]$. Proposition 4.2 characterizes the optimal two-period contract for the principal (the detailed proof is in Section A2.1 in the Appendix).

**Proposition 4.2 (Optimal Two-period Contract)**

The optimal two-period contract for the principal is an “account-balance” contract, an “extreme” contract, or a “polarized” contract.
Based on Proposition 4.2, the optimal two-period contract can take three formats depending on parameter spaces (which I will specify later) — an “account-balance contract”, an “extreme contract”, or a “polarized contract”. I illustrate the structure of each contract and the action profile induced with the aid of Figure 4.2.

Under the account-balance contract, the principal finds it optimal to motivate a bold action in both periods, and does so by rewarding the agent based on the number of times that $D_t$ realizes as $d$. Figure 4.2(a) illustrates the account-balance contract. If demand in only one of the two periods realizes as high, the firm issues bonus $B(-d,d) = B(0,d) = B(d,-d) = B(d,0) = \frac{\phi}{h}$. If demand in both periods realizes as high, the firm issues bonus $B(d,d) = 2\frac{\phi}{h}$, which is twice bonus acquired when demand in only one of the two periods realizes as high. As a result, the firm manages to induce the agent to take a bold action in the second period, independent of the first period’s demand outcome.

Under the extreme contract, the principal sets a hard-to-achieve quota $2d$ and concentrates the bonus at $B(d,d) = \frac{\phi}{h}$. That is, the agent is awarded if and only if demand in both periods realizes as $d$, as Figure 4.2(b) illustrates. Facing a hard-to-achieve quota, the
agent is incentivized to act bold earlier on, since if the first period does not have a high demand realization, the bonus at the end of the second period will become unattainable. Nevertheless, if demand in the first period realizes as medium or low and the bonus becomes unattainable, the agent will take a safe action in the later period. As a result, under the extreme contract, the action induced in the second period is heavily dependent on the first period’s outcome.

Finally, the polarized contract gives the agent an opportunity to make up sales if the first period’s demand realizes as $-d$. Figure 4.2(c) presents that bonus $B(-d, d) = \frac{\phi}{h}$ is issued if demand in the earlier period realizes as $-d$ and demand in the second period realizes as $d$. Meanwhile, she issues a higher bonus $B(d, d) = \frac{1 + h - p h}{p + h} \frac{\phi}{h}$ if demand in both periods realizes as $d$. This is a polarized contract since the agent obtains a positive surplus in the second period if demand in the first period realizes as $d$ or $-d$, but he obtains zero surplus if demand in the first period realizes as 0. Under this contract, the agent is incentivized to act bold in the second period if and only if demand in the first period is not 0. In other words, the polarized contract induces an action in the second period moderately dependent on the first period’s outcome.

Before discussing how the optimal contract structure is decided, it is also interesting to discuss the action induced by the principal under the optimally chosen contract.

**Corollary 4.1** Under the optimal two-period contract, the bold action is always induced by the principal during the first period, i.e., $e_1 = 1$, with independent sales periods.
The above corollary states that, while under a two-period contract, the principal always prefers the agent to take a bold action in the earlier period. However, unlike the period-by-period contract, the optimal two-period contract may not always induce the bold action in the second period. The optimal contract structure is determined by how much the action induced in the second period depends on the first period’s sales outcomes. If the action induced in the second period is independent of the first period’s sales outcomes, then an account-balance contract is optimal. If the action induced in the second period is heavily dependent on the first period’s sales outcomes, then an extreme contract is optimal. If the action induced in the second period is moderately dependent on the first period’s sales outcomes, then a polarized contract is optimal.

In order to explore the intuition behind these results, I discuss the tradeoffs for the principal in choosing the long time horizon contract. We can write the principal’s profits as $E[D] \left(1 - \frac{E[B]}{E[D]} \right)$, where $E[D]$ is the expected demand associated with a contract, and $\frac{E[B]}{E[D]}$ stands for the bonus payment that is expected to induce each unit of demand. Since the expected demand is in proportion to the expected number of bold actions induced, a contract that induces more bold actions and pays less bonus for inducing each bold action, renders greater profits for the principal.
First, I find that either the extreme contract, or the polarized contract, pays the smallest possible bonus payment for inducing each bold action among the three contracts, and the account-balance contract pays the most among the three contracts. To further understand this, given a contract, I decompose the final bonus paid into the bonus paid to induce each bold action within the two periods. The results are given in Figure 4.4. Under the account-balance contract, the second period’s action is independent of the first period’s outcome. Therefore, to induce a bold action in each period, the contract pays the same expected bonus as a period-by-period contract does (given by $E[b]$). However, under the extreme contract, the principal pays less bonus to induce the bold action in the first period (given by $(1 - p)E[b]$). This is because the first period’s demand outcome has a heavy influence on the second period’s action. With a hard-to-achieve quota, the agent is more incentivized to act bold in the first period. This makes it less costly for the principal to induce an early bold action. Under the polarized contract, the principal pays even less bonus to induce a bold action in the first period than an extreme contract does (given by $(1 - p - p_h)E[b]$). The reason is that as the agent is offered a makeup opportunity if the first period demand is $-d$, he has more incentives to take a bold action earlier on.

Figure 4.4 highlights that it is weakly less costly to induce bold actions in earlier periods than in the later periods, since agents’ dynamic gaming can in turn be used by the principal to save on incentive cost. This is the reason behind why the principal induces the bold action in the first period under any optimal contract, as Corollary 4.1 states. Furthermore, although the polarized contract pays the least bonus to induce a bold action in the first period, it induces more bold actions in the second period (which is more costly than inducing bold actions in the first period) than the extreme contract. On average, either the extreme contract or the polarized contract pays the least bonus to induce a bold action. When $h$ is not significantly large relative to $l$ (given by $h/l < (1 + p)/p$), the polarized contract, relative to the extreme contract, save more on bonus payment for inducing bold actions in the first
period (given by $p \frac{1}{h}$ based on Figure 4.4(a)). When $p$ is small, the polarized contract induces fewer bold actions in expectation in the second period. In both case, the polarized contract induces the most bold actions given a fixed level of expected bonus payments. When $h$ is far greater than $l$, or when $p$ is large (given by $\frac{h}{l} > \frac{1+p}{p}$), the extreme contract induces the most bold actions given a fixed level of expected bonus payments.

Next, I compare the three contracts in terms of the number of bold actions induced. It is straightforward that the account-balance contract induces more bold actions than the polarized contract does, which induces more bold actions than the extreme contract does. Because the bold action increases variance in sales outcomes, making later actions independent on earlier outcomes suffers the least from agents’ gaming. To summarize, the extreme contract or the polarized contract pays the least bonus in inducing a bold action on average, by making the second period’s action dependent on the first period’s outcome. The account-balance contract, on the other hand, induces the most bold actions by making the second period’s action independent of the first period’s outcome, thus reducing agents’ dynamic gaming.

![Figure 4.4: Expected Bonus Payment to Induce Each Bold Action](image)

I am now ready to discuss under what conditions each contract appears as optimal for the principal. I first present the result with the aid of Corollary 4.2 and Figure 4.5. In Region I of Figure 4.5, the principal does not want to motivate a bold action. In Region II, the
principal finds it optimal to adopt an extreme contract than makes the agent’s action in the second period heavily dependent on the sales outcome in the first period. In Region III, the principal finds it beneficial to adopt a polarized contract that makes the agent’s action in the second period moderately dependent on the sales outcome in the first period. In Region IV, the principal finds it optimal to adopt an account-balance contract that makes the agent’s action in the second period independent of the sales outcome in the first period.

**Corollary 4.2 (Optimal Two-period Contract)**

The optimal two-period contract and outcomes are as per the following table.

<table>
<thead>
<tr>
<th>Region</th>
<th>Condition</th>
<th>$\alpha$</th>
<th>Contract</th>
<th>Principal’s profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$h &lt; \phi$ &amp; $h - l &lt; \frac{1}{3} \leq K_1$, $h &lt; \frac{1}{2}$ &amp; $h - l &lt; K_1$</td>
<td>0</td>
<td>$0(h, p, \phi)$</td>
<td>$S = 0$</td>
</tr>
<tr>
<td>II</td>
<td>$h &gt; \frac{1}{2}$ &amp; $h - l &lt; \frac{1}{3} \leq K_1$, $h &lt; \frac{1}{2}$ &amp; $h - l &lt; K_1$</td>
<td>1</td>
<td>$1(h, p, \phi)$</td>
<td>$(1 + p + h)(b - (1 - (1 + h)p + h))$</td>
</tr>
<tr>
<td>III</td>
<td>$h &gt; \frac{1}{2}$ &amp; $h - l &lt; \frac{1}{3} \leq K_1$, $h &lt; \frac{1}{2}$ &amp; $h - l &lt; K_1$</td>
<td>2</td>
<td>$2(h, p, \phi)$</td>
<td>$(1 + p + h)(b - (1 - (1 + h)p + h))$</td>
</tr>
<tr>
<td>IV</td>
<td>$h &lt; \phi$</td>
<td>3</td>
<td>$3(h, p, \phi)$</td>
<td>$3(h, p, \phi)$</td>
</tr>
</tbody>
</table>

**Table 4.1: Optimal Two-period Contract**

Note: $K_1 \equiv \frac{p^2(h-l)+p(2h^2-h)+h^2(h+l-1)}{(2p+h+l-1)(h-l)h^2}$, $K_2 \equiv \frac{p^2(h-l)+ph^2+[h^2]}{(h-l)(p+h)h^2}$, $K_3 \equiv \frac{p^2(h-l)+p(2h^2+h)+h^2(h+l+1)}{(2p+h+l+1)(h-l)h^2}$, $K_4 \equiv \frac{(1+h)(p+h)}{(h-l)h(1+p+h)}$.

**Figure 4.5: Optimal Two-period Contract**

Note: The regions are defined by the solid lines. $\phi = 0.01$, $d = 1$, $p = 0.3$ in the left figure. $\phi = 0.2$, $d = 1$, $l = 0.05$ in the right figure.
To understand the comparative statics, let’s first focus on the scenario where $h$ is small relative to $l$, or $p$ is small (i.e., the areas below the dashed lines in Figure 4.5). In this scenario, the polarized contract pays the least bonus for inducing each bold action, whereas the account-balance contract induces the most bold actions. The former contract is optimal when $h$ is extreme, whereas the latter contract is optimal when $h$ is intermediate, since the gaming loss from the polarized contract is the largest when $h$ is intermediate. Note that the extreme contract is suboptimal for the principal in this scenario because it does not pay the least bonus for inducing each bold action, and it suffers the most from agents’ gaming.

When $h$ is large relative to $l$, or when $p$ is large (i.e., the areas above the dashed lines in Figure 4.5), the extreme contract pays the least bonus for inducing each bold action. It becomes the optimal contract for the principal, when the value of $h$ is small or $p$ is large so that the gaming loss is minimal. Altogether, we can see from Figure 4.5 that the polarized contract is optimal for the principal when (1) the difference between $h$ and $l$ is small or $p$ is small, and $h$ is extreme, or (2) when the difference between $h$ and $l$ is large or $p$ is large, and $h$ is relatively small or large. The extreme contract is optimal for the principal when the difference between $h$ and $l$ is large or $p$ is large, and $h$ is extremely small. Finally, the account-balance contract is optimal when $l$ or $p$ is small, and $h$ is intermediate.

The optimal contract I derived above aligns with the contract structure observed in practice to dynamically induce bold actions. The extreme contract corresponds to a long time horizon contract with delayed rewards and hard-to-achieve quotas, which is documented as a higher-powered incentive plan for inducing bold actions (Chung and Narayandas 2017). The optimality of a polarized contract gives theoretical explanations for why startups and research entities benefit from protecting agents from earlier low outcomes in order to encourage bold actions (Tian and Wang 2011, Azoulay et al. 2011).

The implication of agents’ dynamic gaming between bold and safe actions in this chapter differs from that of effort dynamics in Chapter 3, in two ways. First, since a bold action
also entails a larger downside risk, a polarized two-period contract that offers makeup opportunities can be the most effective in motivating the agent to take the bold action in a certain parameter space. Second, unlike in Chapter 3, the optimal contract in this chapter can be history-dependent. For example, the polarized contract does not issue a bonus when demand in the first period realizes as \( d \) and demand in the second period realizes as \(-d\), but it issues a bonus when demand in the first period realizes as \(-d\) and demand in the second period realizes as \( d \).

In the next section, I compare the outcomes in the period-by-period contract scenario and the outcomes in the two-period contract scenario, from the point of view of the principal.

### 4.4 Comparison between Two-Period and Period-by-Period Contracts

Having derived both the optimal period-by-period and the optimal two-period contract for the principal, I now discuss whether firms benefit from the disaggregate contract or the aggregate contract, when the salesperson can game the aggregate contract by varying the action he takes over time.

#### 4.4.1 Independent Periods

I now compare the principal’s profits under the optimal period-by-period contract and the optimal two-period contract. I find, not surprisingly, that the principal weakly prefers the two-period contract to the period-by-period contract. The reason is that account-balance contract is essentially a replicate of the period-by-period contract. However, further analysis shows that, under special conditions, the principal strongly prefers a two-period contract over a period-by-period contract (even though the latter gives the principal more control over the
agent’s actions while the former allows the agent the freedom to game the contract). I obtain the following proposition.

**Proposition 4.3** In Regions II and III as defined in Proposition 4.2, the principal strongly prefers a two-period contract over a period-by-period contract.

In other words, when the action the principal prefers to induce in the second period is contingent on the first period outcome in association with the two-period contract, the principal strongly prefers the two-period contract over the period-by-period contract. This way, the principal can obtain, higher profits than with the period-by-period contract under certain parametric conditions by setting a hard-to-achieve quota, or offering makeup opportunities. This happens when the upside potential of the bold action, $h$ is extreme, when the baseline outcome, $p$ is low, or when the downside risk of the bold action, $l$ is low.

This result is qualitatively the same as the result in Chapter 3, namely, the two-period contract strictly dominates the period-by-period contract when the effectiveness of an agent’s effort is extreme. The difference is that in most of the parameter space, the optimally chosen two-period contract will be a polarized contract that offers makeup opportunities, rather than an extreme contract with a hard-to-achieve quota (as the extreme contract in Chapter 3 does). I do not explicitly model the sales push-out and pull-in phenomena, as their impacts will be similar to that in Chapter 3. In particular, if the agents are allowed to push extra sales to (or borrow sales from) the later period, the two-period optimal contract, which pays at the end, is not affected. However, the period-by-period contract, which pays in the interim, is subject to sales push-out and pull-in effects, and will perform even worse relative to the two-period contract for the principal.
4.4.2 Interdependent Periods

In the above subsection, I arrived at the conclusion that the principal weakly prefers an aggregate contract with independent periods. However, this seems contrary to the widely popular disaggregate contract we observe in practice. In this section, I allow the two periods to be dependent on each other by introducing the condition that the principal has limited inventory to sell during the two periods. In the presence of limited inventory, the principal has an incentive to dynamically adjust the contract within the two time periods. This interacts with the agent’s incentive to dynamically adjust his actions within the two time periods. I show that, because of this interaction, there are scenarios where a period-by-period contract cannot be replicated by the two-period contract, and will be strictly preferred by the principal.

My analysis in this section is based on the assumption that under the short time horizon contract, an agent chooses his action in a period only based on the current period’s contract. However, if the agent acts fully strategically under the period-by-period contract, and chooses the first period’s action by anticipating how the second period’s contract may change based on the outcome of the first period, then a long-term contract still weakly dominates a short-term contract.

Period-by-period Contract with Limited Inventory

I extend my model by assuming that the principal has limited inventory, denoted by $\Omega$, to sell over the two periods. The inventory cannot be replenished before the end of the second period and any demand beyond $\Omega$ is lost, i.e., actual sales $\bar{D} = \min\{D_1 + D_2, \Omega\}$. As a result, the two periods become dependent through $\Omega$. For simplicity, I assume zero inventory cost. In order to focus on the interesting cases, I only consider cases when $h - l > (1 + \frac{p}{\phi}) \frac{\phi}{d}$, so that a bold action generates enough upside potential to be induced in both periods given
unlimited inventory.

Unlike in Section 4.3.2, where independence across the two periods leads to the same optimal contract for the two periods, in the presence of limited inventory, the principal’s decision at the beginning of the second period, after observing \( D_1 \), will depend on the remaining inventory level \( \Omega - D_1 \). Mathematically speaking, given limited inventory, the principal’s decision variables become \((b_1(d), b_2^{D_1}(d))\). In the above notation, the bonus offered in the second period will by dynamically adjusted depending on the realization of \( D_1 \) as \( d, 0 \), or \(-d\), and the corresponding action induced is \((e_1, e_2^{D_1})\). The following proposition presents the optimal period-by-period contract for the principal (see Section A2.2.1 for the proof).

**Proposition 4.4 (Optimal Period-by-period Contract with Limited Inventory)**

With limited inventory, the optimal period-by-period contract and outcomes emerge as per the following table.

<table>
<thead>
<tr>
<th>Condition</th>
<th>((e_1, {e_2^H, e_2^M, e_2^L}))</th>
<th>Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega \geq \Omega_1 )</td>
<td>((1, {1^H, 1^M, 1^L}))</td>
<td>(s = 0, b_1(d) = b_2^H(d) = b_2^M(d) = b_2^L(d) = \frac{\phi}{h} )</td>
</tr>
<tr>
<td>( \max{d, \Omega_4} \leq \Omega &lt; \Omega_1 ) or ( \max{\Omega_5, \Omega_2} \leq \Omega &lt; d )</td>
<td>((1, {0^H, 1^M, 1^L}))</td>
<td>(s = 0, b_1(d) = b_2^0(d) = b_2^M(d) = \frac{\phi}{h} )</td>
</tr>
<tr>
<td>( \max{0, \Omega_6} \leq \Omega &lt; \Omega_2 ) or ( \Omega &lt; 0 )</td>
<td>((1, {0^H, 0^M, 1^L}))</td>
<td>(s = 0, b_1(d) = b_2^0(d) = \frac{\phi}{h} )</td>
</tr>
<tr>
<td>( d \leq \Omega &lt; \Omega_4 ) or ( \Omega_2 \leq \Omega &lt; \min{d, \Omega_5} )</td>
<td>((0, {0^H, 1^M, 1^L}))</td>
<td>(s = 0, b_2^0(d) = b_2^L(d) = \frac{\phi}{h} )</td>
</tr>
<tr>
<td>( 0 \leq \Omega &lt; \min{\Omega_6, \Omega_2} ) or ( \Omega_3 \leq \Omega &lt; \min{0, \Omega_7} )</td>
<td>((0, {0^H, 0^M, 1^L}))</td>
<td>(s = 0, b_2^0(d) = \frac{\phi}{h} )</td>
</tr>
<tr>
<td>( \Omega &lt; \Omega_3 )</td>
<td>((0, {0^H, 0^M, 0^L}))</td>
<td>(s = 0 )</td>
</tr>
</tbody>
</table>

Table 4.2: Optimal Period-by-period Contract with Limited Inventory

Note: \( \Omega_1 \equiv (1 + \frac{l}{h})d + (1 + \frac{p}{h^2})\frac{\phi}{h}, \Omega_2 \equiv \frac{l}{h}d + (1 + \frac{p}{h})\frac{\phi}{h}, \Omega_3 \equiv (\frac{l}{h} - 1)d + (1 + \frac{p}{h})\frac{\phi}{h}, \Omega_4 \equiv \frac{dh((1 + \frac{l}{h})1 + p(h + l)) + (1 + \phi)\phi}{h^2 + phl + h(-1 + 2p + l)}, \Omega_5 \equiv \frac{dh((-1 + h - l) l + p(h + l)) - (p + h) (1 + l) \phi}{h((-1 + 2p)h + pl)}, \Omega_6 \equiv \frac{-dh((-1 + h - l) l + p(h + l)) + (p + h) (1 + l) \phi}{h((-1 + h)l + p(h + l))}, \Omega_7 \equiv \frac{-dh((-1 + h - l) l + p(h + l)) + (p + h) (1 + l) \phi}{h((-1 + h)l + p(h + l))} \)
Figure 4.6: Optimal Period-by-period Contract with Limited Inventory

When the inventory level is high enough (Region VI) and does not lead to a bottleneck, the principal induces $e = (1, 1)$, consistent with cases without inventory concerns. For a smaller $\Omega$, when $h$ is relatively large (Region V in Figure 4.6), although a bold action is still induced in the earlier period, $e_2$ becomes contingent on $D_1$; high demand in the first period will cause an inventory shortage later and no bold action is needed. When $h$ is relatively small (Region III), the principal further abandons inducing a bold action in the early period. As $\Omega$ reduces further, when $h$ is relatively large (Region IV), a bold action is still induced in the earlier period, but is only induced in the later period if the first period has low demand realization. When $h$ is relatively small (Region II), the principal abandons inducing a bold action even in the early period. For a yet smaller $\Omega$ (Region I), inventory levels are too low to justify any bold action. It is noteworthy that the set of optimal action profiles includes that bold action not be taken in the early period, but may be taken in the second period. I state the following corollary.

**Corollary 4.3** Given a period-by-period contract with limited inventory, when $\Omega$ is intermediate and $h$ is small, the principal does not induce a bold action in the first period, and may
induce a bold action in the second period, if the first period’s demand outcome is medium or
low (Region II and III in Figure 4.6).

Two-Period Contract with Limited Inventory

In a similar vein, the firm’s contracting problem can be solved by substituting the total
demand \( D \) with its truncated value \( \hat{D} = \min\{D_1 + D_2, \Omega\} \). I focus on the parameter space
with \( h > \frac{p}{1+p} l \) and \( h - l > (1 + \frac{p}{h})\hat{\phi} \), so that \( e = (1, \{1^d, 0^d, 0^d\}) \) is not within the optimal
action profiles with unlimited inventory. I obtain the following proposition (the proof is in
Appendix A2.2.2).

**Proposition 4.5** (Optimal Two-Period Contract with Limited Inventory) With
limited inventory, the optimal two-period contract and outcomes emerge as per the following
table.

<table>
<thead>
<tr>
<th>Condition</th>
<th>((c_1, (c_1', c_1'')))</th>
<th>Contract</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_1 &lt; \frac{h}{2} ) and ( \Omega &gt; \omega_1 )</td>
<td>((1, {1^d, 1^d}))</td>
<td>( S = 0, B(-d,d) - B(0,d) = B(d,-d) - B(d,0) = \hat{\phi}, B(d,d) = 2\hat{\phi} )</td>
</tr>
<tr>
<td>( K_1 &gt; \frac{h}{2} ) and ( \Omega &gt; \omega_1 )</td>
<td>((1, {1^d, 1^d}))</td>
<td>( S = 0, B(-d,d) - B(0,d) = \hat{\phi}, B(d,-d) - B(d,0) = (1+p)\hat{\phi} )</td>
</tr>
<tr>
<td>( K_1 &gt; \frac{h}{2} ) and ( \max{d, \omega_1} \leq \Omega &lt; \omega_1 )</td>
<td>((1, {0^d, \vartheta^d}))</td>
<td>( S = 0, B(-d,d) - B(0,d) = \hat{\phi} )</td>
</tr>
<tr>
<td>( K_1 &gt; \frac{h}{2} ) and ( \max{d, \omega_1} \leq \Omega &lt; \omega_1 )</td>
<td>((0, {0^d, 1^d}))</td>
<td>( S = 0, B(-d,d) - B(0,d) = \hat{\phi} )</td>
</tr>
<tr>
<td>( d \leq \Omega &lt; \omega_1 ) or ( \min{\omega_1, \omega_2} \leq \Omega &lt; d )</td>
<td>((0, {0^d, 1^d}))</td>
<td>( S = 0, B(-d,d) - B(0,d) = \hat{\phi} )</td>
</tr>
<tr>
<td>( 0 \leq \Omega &lt; \min{\omega_1, \omega_2} ) or ( \Omega &gt; \Omega_3 )</td>
<td>((0, {0^d, 0^d}))</td>
<td>( S = 0 )</td>
</tr>
</tbody>
</table>

Table 4.3: Optimal Two-period Contract with Limited Inventory

Note: \( \omega_1 = \frac{d(h+l)+\phi}{h} \), \( \omega_2 = \frac{d^2l(2p-h+I)+(p+h)(2p-h+l)\phi}{ph^2l} \), \( \omega_3 = -\frac{d^2l(-h+l)(p+h)h}{h^2(-h+l)(l+p(h+2l))} \), \( \omega_4 = \frac{d^2l(2p-h+I)(-h+l)(p-h)\phi}{ph^2l} \), \( \omega_5 = \frac{d^2l(2p-h+I)(-h+l)(p-h)\phi}{ph^2l} \), \( \omega_6 = \frac{d^2l(2p-h+I)(-h+l)(p-h)\phi}{ph^2l} \), \( \omega_7 = \frac{d^2l(2p-h+I)(-h+l)(p-h)\phi}{ph^2l} \), \( \omega_8 = \frac{d^2l(-h+l)(p+h)h}{h^2(-h+l)(l+p(h+2l))} \).
The optimal two-period contracts for inducing different action profiles in the presence of limited inventory are illustrated by Figure 4.7. In this scenario, if $\Omega$ is relatively high, we are in Region VI or VII, where $e = (1, \{1d, 10, 1-d\})$ or $e = (1, \{1d, 00, 1-d\})$ is induced. For an intermediate $\Omega$, we are in Region II, III, IV, V, where no bold action is induced upon a realization of high demand in the first period. Furthermore, when $h$ is relatively large (as in Region IV or Region V), a bold action is induced in the first period. When $h$ is relatively small (as in Region II or Region III), a bold action is not induced in the first period.

Comparison between Two-period and Period-by-period Contracts with Limited Inventory

I first summarize a comparison between the two-period contract and the period-by-period contract given limited inventory through Proposition 4.6.

**Proposition 4.6** The period-by-period contract may outperform the two-period contract in the presence of limited inventory, when the principal has an intermediate need and the upside potential of the bold action is intermediate.
To understand the results, we must consider all possible action profiles. Compared with the period-by-period contract, the two-period contract pays a greater bonus when inducing $e = (1, \{1^d, 1^0, 0^{-d}\})$, $e = (1, \{0^d, 1^0, 1^{-d}\})$, and $e = (1, \{0^d, 1^0, 0^{-d}\})$. To summarize, the two-period contract pays no less of a bonus relative to the period-by-period contract when both $e_1 = 1$ and $e_0^2 = 1$ are induced. This is because in order to induce a bold action in the second period when demand in the first period realizes as medium (i.e., $e_0^2 = 1$), the two-period contract needs to reward the agent (in expectation) upon $D_1 = M$. However, taking a bold action in the first period reduces the probability of obtaining medium demand realization in the first period. As a result, when the principal aggregates the bonus payment for the two-period contract, she needs to offer more of a bonus to incentivize taking the bold action in the first period.

In contrast, the two-period contract pays less of a bonus when inducing $e = (1, \{1^d, 0^0, 1^{-d}\})$, $e = (1, \{1^d, 0^0, 0^{-d}\})$, and $e = (1, \{0^d, 0^0, 1^{-d}\})$ compared with the period-by-period contract. In other words, the two-period contract pays no more bonus relative to the period-by-period contract, when both $e_1 = 1$ and $e_0^2 = 0$ are induced. This is because in order to induce the
safe action when demand in the first period realizes as medium (i.e., \( e_2^0 = 0 \)), the two-period contract does not reward the agent when \( D_1 = 0 \). As taking a bold action in the first period reduces the probability of obtaining medium demand realization in the first period, this provides greater incentive for the agent to take a bold action in the first period.

Since inducing \( e = (1, \{0^d, 1^0, 1^{-d}\}) \) can be optimal for the principal when the inventory level is intermediate (based on Proposition 4.4 and Proposition 4.5), the period-by-period contract can strictly outperform the two-period contract for the principal when \( e = (1, \{0^d, 1^0, 1^{-d}\}) \) is induced (Region I in Figure 4.8). In particular, under the optimal period-by-period contract to induce \( e = (1, \{0^d, 1^0, 1^{-d}\}) \), \( b_1(d) = \frac{\phi}{h} \), suggesting that when demand in the first period realizes as \( d \), the agent gets rewarded \( \frac{\phi}{h} \). Under the optimal two-period contract to induce \( e = (1, \{0^d, 1^0, 1^{-d}\}) \), \( B(d, d) = B(d, 0) = B(d, d) = (1 + p)\frac{\phi}{h} \), suggesting that when demand in the first period realizes as \( H \), the agent gets rewarded \( (1 + p)\frac{\phi}{h} \), which is higher than his reward under the period-by-period contract, given by \( \frac{\phi}{h} \).

Similarly, the two-period contract can strictly outperform the period-by-period contract for the principal when \( e = (1, \{0^d, 0^0, 1^{-d}\}) \) (which can be optimal for the principal to induce based on Proposition 4.4 and Proposition 4.5) is induced, as Region III in Figure 4.8 presents. In particular, to induce \( e = (1, \{0^d, 0^0, 1^{-d}\}) \), under the optimally chosen period-by-period contract, we have \( b_1(d) = \frac{\phi}{h} \), implying that upon a high demand realization in the first period, the agent gets rewarded \( \frac{\phi}{h} \). The optimal two-period contract, on the other hand, has \( B(d, -d) = B(d, 0) = B(d, d) = (1 - p\frac{1}{h})\frac{\phi}{h} \), implying that upon a high demand realization in the first period, the agent gets rewarded \( (1 - p\frac{1}{h})\frac{\phi}{h} \). This is lower than the agent’s reward under the period-by-period contract, given by \( \frac{\phi}{h} \). Finally, in Region II where the inventory level is relatively high, again, the two-period contract may outperform the period-by-period contract for inducing \( e = (1, \{1^d, 0^0, 1^{-d}\}) \), similar to cases with independent periods.

To summarize, due to dynamic gaming on the part of the agent, the two-period contract cannot replicate the period-by-period contract when a bold action is induced in the first
period, and is induced again in the second period upon a medium level of demand realization in the first period. This makes the period-by-period contract perform strictly better than the two-period contract in the presence of limited inventory. To emphasize, the above analysis is based on the premise that under the short time horizon contract, an agent chooses his action in a period only based on the current period’s contract. If the agent acts fully strategically under the period-by-period contract, and chooses the first period’s action by anticipating how the second period’s contract may change based on the outcome of the first period, then a long-term contract still weakly dominates a short-term contract.

The above analyses identify the tradeoffs for a firm to consider in choosing between a long time horizon contract and a short time horizon contract. A long time horizon contract provides higher-powered incentives for an agent to take bold actions by inducing later actions dependent on earlier demand outcomes. Therefore, the firm is better off under a long time horizon contract if providing incentives is of a higher order. On the other hand, a short time horizon contract reduces agents’ gaming and provides the principal more flexibility in adjusting the contracts at interim. As a result, if reducing gaming losses is of a higher order or there are external factors making the two periods dependent, then a short time horizon contract may improve the firm’s profits.

Compared with the discussion in Chapter 3, Chapter 4 enables us to gain a full picture of when the two-period contract can outperform the period-by-period contract and when it cannot. The assumption that the bold action not only increases the upside potential but also increases the downside risk is critical in generating this full picture.

### 4.5 Consideration of Non-Decreasing Constraint

Up to this point, I assume that the principal can perfectly observe sales outcomes. In this section, I discuss a scenario where the agent can destroy sales, and as a result, the salesperson
would under-report sales if there is a possibility that this will lead them larger bonuses.

Mathematically speaking, I need to impose a non-decreasing constraint that the agent’s continuation payoffs at the end of the first period, i.e., after the sales outcome has been realized but before the agent reports sales to the principal, should satisfy \( v(d) \geq v(0) \geq v(-d) \). The non-decreasing constraint ensures that the agent does not have an incentive to under-report the sales outcome in the first period. \(^4\) While the optimal period-by-period contract is not affected, the two-period contract is subject to the non-decreasing constraint, and thus has to be reanalyzed. I provide a sketch of the analysis below, with details provided in Section A2.3.1 in the Appendix.

The principal only observes the sales level after the agent’s manipulation, which I denote by \( D'_1 \), and pays the agent according to \( D'_1 \). To derive the optimal contract, consider first the optimal contracts I developed in Section 4.3.3. I find that the polarized contract derived in Section 4.3.3 to induce \( e = (1, \{1^d, 0^0, 1^{-d}\}) \) does not meet the non-decreasing constraint. When a makeup opportunity \( B(-d,d) = \frac{\phi}{h} \) is offered, the principal needs to increase the bonus payment corresponding to \( D_1 = 0 \) (compared with that in absence of the non-decreasing constraint), to restrict the agent from reporting sales \( D'_1 = -d \) when the demand realization is actually \( D_1 = 0 \). Due to the increase in the expected bonus payment when \( D_1 = 0 \), in order to induce the bold action in the first period, the principal needs to further increase the upper side reward \( B(d,d) \), from \( \frac{1+p+h}{p+h} \frac{\phi}{h} \) (as in Section 4.3.3) to \( \frac{1+p+h}{p+h} \frac{\phi}{h} \). It therefore becomes more costly for the principal to induce \( e = (1, \{1^d, 0^0, 1^{-d}\}) \) in the presence of the non-decreasing constraint.

Indeed, due to the non-decreasing constraint, it becomes suboptimal for the principal to induce \( e = (1, \{1^d, 0^0, 1^{-d}\}) \). In equilibrium, the principal either induces \( e = (1, \{1^d, 0^0, 1^{-d}\}) \) or \( e = (1, \{1^d, 0^0, 0^{-d}\}) \). The optimal two-period contract and outcomes are defined as per

\(^4\)The non-decreasing constraint at the end of the second period will automatically be satisfied under the optimal contract.
the following table.

<table>
<thead>
<tr>
<th>Region</th>
<th>Condition</th>
<th>$c_l$</th>
<th>$(c_1^e, c_0^e, c_2^e)$</th>
<th>Contract</th>
<th>Principal’s profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$h - l &lt; \frac{d}{2} &lt; K_1$</td>
<td>0</td>
<td>$(0^e, 0^e, 0^e)$</td>
<td>$S = 0$</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>$K_1 &lt; \frac{d}{2} &lt; K_1'$</td>
<td>1</td>
<td>$(0^e, 0^e, 0^e)$</td>
<td>$S = 0$, $B(d, d) = \frac{1 + b + h}{1 + p + h}$</td>
<td>$(1 + p + h)(h - l)d - (1 + h)(p + h)\frac{d}{2}$</td>
</tr>
<tr>
<td>IV</td>
<td>$K_1' &lt; \frac{d}{2}$</td>
<td>1</td>
<td>$(1^e, 1^e, 1^e)$</td>
<td>$S = 0$, $B(-d, d) = B(0, d) = B(d, -d) = B(d, 0) = \frac{d}{h}$, $B(d, d) = 2\frac{d}{h}$</td>
<td>$2(h - l)d - 2(p + h)\frac{d}{2}$</td>
</tr>
</tbody>
</table>

Table 4.4: Optimal Two-period Contract with Non-decreasing Constraint

Note: $K_1' \equiv \frac{(1-h)(p+h)}{(h-l)h(1-p-h)}$.

Figure 4.9: Optimal Two-period Contract with Non-decreasing Constraint

Note: $\phi = 0.01$, $d = 1$, $p = 0.3$ in the left figure. $\phi = 0.2$, $d = 1$, $l = 0.02$ in the right figure.

It is also worth mentioning that, with the non-decreasing constraint, the optimal contract would be a history-independent contract that rewards the agent based on the number of high demand realizations. The extreme contract II rewards the agent if and only if both periods have high demand realization, and the gradual contract IV rewards the agent the same amount for each incremental realization of high demand.
To summarize, in the presence of limited inventory, I find that when the non-decreasing constraint is imposed, the period-by-period contract outperforms the two-period contract when the inventory level becomes a bottleneck. The two-period contract outperforms the period-by-period contract only when the inventory level is high and the action profile $e = (1, \{1^H, 0^M, 0^L\})$ is induced. When the inventory level becomes a bottleneck, the principal strictly prefers the period-by-period contract over the two-period contract. Figure 4.10 presents this result. Briefly speaking, with the non-decreasing constraint, the two-period contract is preferred by the principal in a smaller parameter space.

### 4.6 Conclusions

It is common for firms to employ salespeople for an extended period of time and compensate salespeople based on their performance during a window extending across multiple periods. Many sales agents facing a selling task choose between bold actions and safe actions to generate sales. A bold action has greater potential to lead to extreme sales compared with
a safe action. An inherent problem with paying salespeople over a long-time horizon is that, when the agent gets to observe his performance before proceeding to the next period, he can vary his action in response to past outcomes. Hölmstrom (1979) prescribes an account-balance plan rewarding agents based on how many times each possible demand is realized over a period of time, as most efficient for providing incentives. Interestingly, my analysis suggests a history-dependent plan that rewards the agent for high-demand realization and also protects the agent against low-demand realization in an earlier period, can be optimal under certain conditions.

I develop a principal-agent framework in which a risk neutral firm employs a risk neutral salesperson with limited liability. I focus my analysis on two periods. Both outcome levels and salespeople’s actions are discrete. The firm can either propose a one-period contract followed by another one-period contract, or propose a two-period contract that pays only at the end. Under the period-by-period contract, the two periods are independent, and following a standard moral hazard problem, the firm would induce a bold action if and only if the upside potential of the bold action offsets its downside risk.

In the two-period contract, the firm designs an optimal compensation plan taking into account the salesperson’s gaming behaviors. I find that the principal may find it optimal to adopt an “extreme” contract concentrating the reward at a hard-to-achieve quota level, a “polarized” contract that provides makeup opportunities, or an “account-balance” plan that compensates the agent based on how many times he obtains a high demand outcome. The extreme plan has inherent pros and cons. On the positive side, it provides larger incentives to motivate agents with limited liability to take bold actions. The flip side is that the agent may game his action choice during the later period. I find that the optimal two-period contracts vary with the upside potential and downside risk brought on by the bold action.

I then proceed to compare the optimal period-by-period contract with the optimal two-period contract. I find that while it is intuitive that the two-period contract is weakly
preferred, the firm may strictly prefer the two-period plan, when the upside potential of the bold action is very small or very large. Under such a scenario, allowing an agent to adjust his actions dynamically over time may serve the principal, by balancing providing incentive with generating demand. This result, however, may not hold true when the two periods become dependent through a limited inventory level across the two periods. Under the assumption that agents choose their actions under a period-by-period contract based on the current period’s contract, the principal may benefit from motivating a bold action in the later period only if demand in the first period is not high. The period-by-period contract — which suffers less from agents’ dynamic gaming — performs well in inducing such action profiles, compared with the two-period contract.
Chapter 5

Conclusions and Discussions

5.1 Conclusions and Discussions

This dissertation studies multi-period salesforce incentive provisions. I address a fundamental question that arises in this context: Should salespeople be rewarded using period-by-period contracts that reward for the outcome of each period, or should they be rewarded using a multi-period contract that rewards for the outcomes over multiple periods? I consider agents’ dynamic gaming within two different contexts. In Chapter 3, agents are able to vary their demand-enhancing effort levels dynamically. In Chapter 4, sales agents can dynamically choose between a bold action with higher sales potential but also higher variance, and a safe action with limited sales potential but lower variance.

I employ a two-period repeated moral hazard framework with stochastic demand and unobservable actions, and assume the agent to be risk neutral with limited liability. I find that with independent two periods, a multi-period contract can perform strictly better than a repetition of single-period contracts. The intuition is that a multi-period contract rewards the salesperson only for more extreme outcomes as compared to a period-by-period contract, which allows it to incentivize the salesperson more strongly to take the desired action (i.e.,
the action which leads to high expected demand) — I call this the “incentive effect”. The downside is that the salesperson has the ability to game the contract by adjusting effort levels strategically across periods — I call this ”the “gaming effect”. The former incentive effect dominates the latter gaming effect when the effectiveness of the desired action is extreme. It is noteworthy that the two-period contract that strictly outperforms the period-by-period contract is not renegotiation-proof. In other words, commitments to a two-period contract are necessary for it to outperform the period-by-period contract for the principal.

However, with interdependent two periods, the two-period contract may not perfectly replicate the period-by-period contract, and can even preform worse than the period-by-period contract, under the assumption that agents choose their actions in response to their current contracts. I extend to a scenario in which there is a fixed amount of inventory to be sold across multiple periods — this introduces dependence between periods as the principal’s preferred action in the later period depends on the outcome of the early period. If the desired action is induced in the first period, and is induced again in the second period, upon demand realization in the first period that is less likely to happen under the desired action, the period-by-period contract performs strictly better than the two-period contract in the presence of limited inventory.

Another scenario where a two-period contract can be less preferred by the principal is when firms cannot perfectly observe sales outcomes and agents can choose to destroy sales. Under the additional constraint that compensation cannot be decreasing in sales, with independent periods, a two-period contract performs strictly better than a period-by-period contract in a smaller parameter space. When the two periods become interdependent through a limited inventory, a two-period contract performs strictly worse then a period-by-period contract in a larger parameter space.

The insights from Chapter 4 can be built on top of the insights from the effort-exertion dynamic in Chapter 3. The similarity is that in both cases, the desired action is induced by
the principal in the first period. This is because under the multi-period contract, motivating
the desired action in earlier periods is weakly less costly than in later periods. Also, an
extreme contract that pays bonuses on the highest possible sales outcomes in all periods can
be most effective in providing incentives in certain parameter space. This is driven by the
agent’s limited liability. If the agent cannot be penalized significantly, aggregating bonus
payment at extreme levels results in less bonus payment for inducing each unit of demand.

However, depending on the contexts of agents’ dynamic gaming, the structure of the
optimal contract can be different. In Chapter 3, an extreme contract that concentrates
rewards at the highest outcome level is the most effective in providing incentive. In Chapter
4, a polarized contract that offers a makeup opportunity can be most effective in providing
incentive in some parameter spaces. This is because taking a bold action increases the
probability of obtaining both high and low demand realizations. Then, compensating the
agent at the end of the second period if the earlier demand realizes at extreme levels can
motivate the agent to choose a bold action in the first period. Another difference of the
two essays are that given agents’ limited liability, with independent periods, the optimal
contract is history-independent in Chapter 3, but it can be history-dependent in Chapter
4. Finally, Chapter 4 enables us to gain a full picture of when the two-period contract can
outperform the period-by-period contract and when it cannot. The assumption that the
bold action not only increases the upside potential but also increases the downside risk is
critical in generating this full picture.

I conclude with a brief discussion of some of my assumptions and limitations. I have
restricted my analysis to the case in which the principal chooses between a two-period con-
tract and a period-by-period contract, but not allowed a mixed contract. I note that the
principal cannot do better by mixing these two types of contracts, as using the better of the
period-by-period and the two-period contracts is a weakly dominant strategy for the princi-
This is because the tradeoff facing the principal remains the same—providing larger incentives or avoiding gaming losses. The mixed contract cannot provide higher incentives compared with the two-period extreme contract, and it cannot further avoid gaming losses compared with the gradual two-period contract (or the period-by-period contract). I have assumed inventory to be exogenous and have assumed away inventory costs for simplicity. However, it is straightforward to make the inventory decision endogenous by incorporating inventory costs such as marginal cost of goods, inventory holding cost for holding leftover inventory across periods, salvage costs for leftover inventory, etc. These costs will determine the choice of Ω, and given Ω the results and insights that I have derived will hold.

I have assumed demand outcome and agents’ actions take discrete levels, and in Chapter 4, the outcome distribution is symmetric against the middle level. However, since my main results are driven by the tradeoff in offering an extreme contract between providing incentive and inducing gaming losses, I expect them to hold in a general setting as well. By the same token, if I allow periods to be dependent in other ways (e.g., a high demand outcome in the early period makes a high demand outcome in the second period more or less likely), my key insights will hold. Finally I only detail an analysis within two periods, and leave the optimal scheme with multi-periods for future investigation. I hope my work will cast light on analytical study in the optimal frequency of dynamic salesforce compensation problems and encourage further studies.

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1A formal proof is available on request.


Scarf, Herbert. 1960. The optimality of (s, s) policies in the dynamic inventory problem. kj arrow, s. karlin, p. suppes, eds. mathematical methods in the social sciences.


A1 Appendix for Chapter 3

A1.1 First-Best Solution

We first list the expected payment to the agent in different regions of the parameter space.

- When \( U - K \geq 0 \), if the principal instructs the agent to exert effort, she pays the agent a fixed salary of \( U + \phi \), and if there is no effort exertion, the principal pays the agent a fixed salary of \( U \).

- When \( -\phi \leq U - K < 0 \), if the principal instructs the agent to exert effort, she pays the agent a fixed salary of \( U + \phi \), and if there is no effort exertion, the principal pays the agent a fixed salary of \( K \).

- When \( U - K < -\phi \), regardless of whether the principal instructs the agent to exert effort or not, she pays the agent a fixed salary of \( K \).

Now we compare the principal’s profits when instructing the agent to exert effort or not. The incremental payment to the agent when instructing the agent to exert effort, compared with not exerting effort, is given by

\[
E[s_t | e_t = 1] - E[s_t | e_t = 0] = \begin{cases} 
\phi, & \text{if } U - K \geq 0, \\
\phi + (U - K), & \text{if } -\phi \leq U - K < 0, \\
0, & \text{if } U - K < -\phi. 
\end{cases}
\]

Compared with not instructing the agent to exert effort, instructing him to exert effort can increase the expected demand by

\[
E[D_t | e_t = 1] - E[D_t | e_t = 0] = (p - q)(H - L).
\]
The principal would like to instruct the agent to exert effort if the increase in expected demand offsets the increase in payment to the agent, i.e., when \( \mathbb{E}[D_t|e_t = 1] - \mathbb{E}[D_t|e_t = 0] \geq \mathbb{E}[s_t|e_t = 1] - \mathbb{E}[s_t|e_t = 0] \). This is equivalent to the following condition:

\[
H - L \begin{cases} \frac{\phi}{p-q}, & \text{if } U - K \leq 0, \\ \frac{\phi}{p-q} + \frac{U-K}{p-q}, & \text{if } -\phi \leq U - K < 0, \\ 0, & \text{if } U - K < -\phi. \end{cases}
\]

A1.2 Period-by-Period Contract

To induce effort \( e = 1 \), the expected payment to the agent is \( \max\{K, U - \frac{q}{p-q}\phi\} + \frac{p}{p-q}\phi = \max\{\frac{p}{p-q}\phi + K, \phi + U\} \). To induce effort \( e = 0 \), the payment to the agent is \( \max\{K, U\} \).

Comparing the two cases, the increase in expected payment to the agent when inducing effort (compared with not inducing effort) can be simplified as

\[
\mathbb{E}[s_t + b_t|e_t = 1] - \mathbb{E}[s_t + b_t|e_t = 0] = \begin{cases} \phi, & \text{if } U - K > \frac{q}{p-q}\phi, \\ \frac{p}{p-q}\phi - (U - K), & \text{if } 0 \leq U - K < \frac{q}{p-q}\phi, \\ \frac{p}{p-q}\phi, & \text{if } U - K \leq 0. \end{cases}
\]

Furthermore, inducing effort from the agent, compared with not inducing effort, increases the expected demand by

\[
\mathbb{E}[D_t|e_t = 1] - \mathbb{E}[D_t|e_t = 0] = (p-q)(H - L).
\]

The principal would like to induce effort exertion from the agent if the increase in expected demand offsets the increase in expected payment, i.e., \( \mathbb{E}[D_t|e_t = 1] - \mathbb{E}[D_t|e_t = 0] \geq \mathbb{E}[D_t|e_t = 1] - \mathbb{E}[D_t|e_t = 0] \).
\[ E[s_t + b_t|e_t = 1] - E[s_t + b_t|e_t = 0]. \] This is equivalent to the following condition:

\[
H - L \geq \begin{cases} 
\frac{1}{p-q} \phi, & \text{if } U - K \geq \frac{q}{p-q} \phi, \\
\frac{p}{(p-q)^2} \phi - \frac{U-K}{p-q}, & \text{if } 0 \leq U - K < \frac{q}{p-q} \phi, \\
\frac{p}{(p-q)^2} \phi, & \text{if } U - K < 0.
\end{cases}
\]

A1.3 Two-Period Contract

A1.3.1 General Two-Period Contract

By enumerating the optimal contract to incentivize any possible effort profile, we show that under the weakly-dominant long-term contract, \( b_2(L, H) = b_2(H, L) \). Therefore, it is sufficient for the principal to focus on the long-term contract that pays at the end according to cumulative sales.

In the following, we use the labels \((IC_H^H - ge)\), \((IC_H^H - l)\), \((IC_L^H - ge)\), \((IC_L^H - l)\), \((IC_1 - ge)\), \((IC_1 - l)\) and \((LL)\) to denote the following constraints:

\begin{align*}
(IC_H^H - ge) & \text{ denotes } b_2(H, H) - b_2(H, L) \geq \frac{\phi}{p-q}; \\
(IC_H^H - l) & \text{ denotes } b_2(H, H) - b_2(H, L) < \frac{\phi}{p-q}; \\
(IC_L^H - ge) & \text{ denotes } b_2(L, H) \geq \frac{\phi}{p-q}; \\
(IC_L^H - l) & \text{ denotes } b_2(L, H) < \frac{\phi}{p-q}; \\
(IC_1 - ge) & \text{ denotes } U_H - U_L \geq \frac{\phi}{p-q}; \\
(IC_1 - l) & \text{ denotes } U_H - U_L < \frac{\phi}{p-q}; \\
(LL) & \text{ denotes } S + b_2(H, H), S + b_2(H, L), S + b_2(L, H) \geq 2K.
\end{align*}
• To induce $e = (1, 1)$, the principal’s problem is:

$$\min_{S, b_2(H, H), b_1(H, L), b_2(L, H)} S + p^2b_2(H, H) + q(1 - p)(b_2(H, L) + b_2(L, H))$$

s.t. $$S + p^2b_2(H, H) + q(1 - p)(b_2(H, L) + b_2(L, H)) - 2\phi \geq 2U \quad (PC)$$

and

$$(IC_2^H{-}ge), (IC_2^L{-}ge), (IC_1{-}ge), (LL)$$

In $(IC_1{-}ge)$, $U_H = S + pb_2(H, H) + (1 - p)b_2(H, L) - \phi$ is the agent’s expected utility in the second period given $D_1 = H$, and $U_L = S + pb_2(L, H) - \phi$ is the agent’s expected utility in the second period given $D_1 = L$.

The optimal contract to induce this effort profile is given by: $S = 2\max\{K, U - \frac{\phi}{p-q}\}$, $b_2(L, H) = \frac{\phi}{p-q}$, $b_2(H, L) \leq \frac{\phi}{p-q}$, $pb_2(H, H) + (1 - p)b_2(H, L) = (1 + p)\frac{\phi}{p-q}$.

The following history-independent contract lies within the optimal contract set: $S = 2\max\{K, U - \frac{\phi}{p-q}\}$, $b_2(L, H) = \frac{\phi}{p-q}$, $b_2(H, L) = \frac{\phi}{p-q}$, $b_2(H, H) = 2\frac{\phi}{p-q}$. The expected payment to the agent is $\max\{2\frac{\phi}{p-q} + 2K, 2\phi + 2U\}$.

• To induce $e = (1, p)$, the principal’s problem is:

$$\min_{S, b_2(H, H), b_1(H, L), b_2(L, H)} S + p^2b_2(H, H) + p(1 - p)b_2(H, L) + p(1 - q)b_2(L, H)$$

s.t. $$S + p^2b_2(H, H) + p(1 - p)b_2(H, L) + p(1 - q)b_2(L, H) - (1 + p)\phi \geq 2U \quad (PC)$$

and

$$(IC_2^H{-}ge), (IC_2^L{-}ge), (IC_1{-}ge), (LL)$$

In $(IC_1{-}ge)$, $U_H = pb_2(H, H) + (1 - p)b_2(H, L) - \phi$, $U_L = qb_2(L, H)$. The optimal contract to induce this effort profile is given by: $S = 2\max\{K, U - \frac{1}{2}\frac{\phi}{p-q}\}$, $b_2(L, H) = 0$, $0 \leq b_2(H, L) \leq \frac{1+\frac{\phi}{p-q}}{1-p}pb_2(H, H) + (1 - p)b_2(H, L) = (1 + p - q)\frac{\phi}{p-q}$. The following history-independent contract lies within the optimal contract set: $S = 2\max\{K, U - \frac{1}{2}\frac{\phi}{p-q}\}$, $b_2(L, H) = 0$, $b_2(H, L) = 0$, $b_2(H, H) = (1 + \frac{\phi}{p})\frac{\phi}{p-q}$.
• To induce \( e = (1, 1 - p) \), the principal’s problem is:

\[
\min_{S,b_2(H,H),b_2(H,L),b_2(L,L)} \quad S + pqb_2(H,H) + p(1 - q)b_2(H,L) + (1 - p)pb_2(L,H) \\
\text{s.t.} \quad S + pqb_2(H,H) + p(1 - q)b_2(H,L) + (1 - p)pb_2(L,H) - (2 - p)\phi \geq 2U \quad \text{(PC)} \\
\text{and} \quad (IC_2^H-\text{ge}), (IC_2^L-\text{ge}), (IC_1-\text{ge}), (LL)
\]

In \((IC_1-\text{ge})\), \( U_H = qb_2(H,H) + (1 - q)b_2(H,L), U_L = pb_2(L,H) - \phi \).

The optimal contract to induce this effort profile is given by: \( S = 2 \max\{K, U - \frac{q}{p-q} \phi\} \), \( b_2(L,H) = \frac{\phi}{p-q}, 0 \leq b_2(H,L) \leq \frac{1+q}{1-q} \frac{\phi}{p-q}, qb_2(H,H) + (1 - q)b_2(H,L) = (1 + q) \frac{\phi}{p-q} \). The following history-independent contract lies within the optimal contract set:

\( S = 2 \max\{K, U - \frac{q}{p-q} \phi\} \), \( b_2(L,H) = \frac{\phi}{p-q}, b_2(H,L) = \frac{\phi}{p-q}, b_2(H,H) = 2 \frac{\phi}{p-q} \).

• To induce \( e = (0, q) \), the principal’s problem is:

\[
\min_{S,b_2(H,H),b_2(H,L),b_2(L,L)} \quad S + qpb_2(H,H) + q(1 - p)b_2(H,L) + (1 - q)qb_2(L,H) \\
\text{s.t.} \quad S + qpb_2(H,H) + q(1 - p)b_2(H,L) + (1 - q)qb_2(L,H) - q\phi \geq 2U \quad \text{(PC)} \\
\text{and} \quad (IC_2^H-\text{ge}), (IC_2^L-\text{ge}), (IC_1-\text{ge}), (LL)
\]

In \((IC_1-\text{ge})\), \( U_H = pb_2(H,H) + (1 - p)b_2(H,L) - \phi, U_L = qb_2(L,H) \).

The unique optimal contract to induce this effort profile is given by: \( S = 2 \max\{K, U - \frac{1}{2} \frac{q^2}{p-q} \phi\} \), \( b_2(L,H) = 0, b_2(H,L) = 0, b_2(H,H) = \frac{\phi}{p-q} \). The above contract is clearly history-independent.

• To induce \( e = (0, 1 - q) \), the principal’s problem is:

\[
\min_{S,b_2(H,H),b_2(H,L),b_2(L,L)} \quad S + qb_2(H,H) + q(1 - q)b_2(H,L) + (1 - q)qb_2(L,H) \\
\text{s.t.} \quad S + qb_2(H,H) + q(1 - q)b_2(H,L) + (1 - q)qb_2(L,H) - (1 - q)\phi \geq 2U \quad \text{(PC)} \\
\text{and} \quad (IC_2^H-\text{ge}), (IC_2^L-\text{ge}), (IC_1-\text{ge}), (LL)
\]

In \((IC_1-\text{ge})\), \( U_H = qb_2(H,H) + (1 - q)b_2(H,L), U_L = pb_2(L,H) - \phi \).
The unique optimal contract to induce this effort profile is given by: 

\[ S = 2 \max \{ K, U - \frac{1}{2} \frac{q}{p-q} \phi \}, \quad b_2(L, H) = \frac{\phi}{p-q}, \quad b_2(H, L) = 0, \quad b_2(H, H) = \frac{\phi}{p-q}. \]

Here, the principal offers 

\[ b_2(H, H) = \frac{\phi}{p-q} \]

due to the non-decreasing constraint. \( b_2(L, H) \) needs to be at least \( \frac{\phi}{p-q} \) to induce \( e_L^2 = 1 \). Because \( b_2(H, H) \) cannot be lower than \( b_2(L, H) \) according to the non-decreasing constraint, we will also have \( b_2(H, H) = \frac{\phi}{p-q} \). Indeed, the non-decreasing constraint matters only when inducing \( e = (0, 1-q) \). We will reach at the same contract for inducing any other profile regardless of imposing the non-decreasing constraint or not.

Note that the optimal contract for inducing \( e = (0, 1-q) \) turns out to be history-dependent. However, we can prove that inducing \( e = (0, 1-q) \) is suboptimal for the principal when the two periods are independent (see Appendix A1.3.2 for details), thus is out of consideration.

- The cases when the principal would like to induce effort \( e = (1, 0) \) or \( e = (1, 0) \) are trivially dominated by the case when she would like to induce \( e = (1, 1) \) so there is no need for consideration.

We observe that, although the contract to induce \( e = (0, 1-q) \) is history dependent, it is suboptimal for the principal; in all other cases, the optimal contract is characterized by \( b_2(L, H) = b_2(H, L) \). Therefore, when the two periods are independent, it suffices for the principal to pay the agent the same at the end of the second period based on the cumulative sales across two periods.

We summarize in Table A1 the expected sales and payments to the agent for different effort profiles that the principal induces.
\begin{center}
\begin{tabular}{|c|c|c|}
\hline
$(e_1, E[e_2])$ & $E[D]$ & $S + E[B]$ \\
\hline
$(0, 0)$ & $2qH + (2 - 2q)L$ & $\max\{2K, 2U\}$ \\
\hline
$(0, 1 - q)$ & $p + q^2 + q - pq)H + (1 - q)(2 - p + q)L$ & $\max\{2K + \frac{p+q^2-pq}{p-q} \phi, 2U + (1 - q) \phi\}$ \\
\hline
$(0, q)$ & $(pq + 2q - q^2)H + (2pq - 2q + q^2)L$ & $\max\{2K + \frac{pq}{p-q} \phi, 2U + q \phi\}$ \\
\hline
$(1, 1 - p)$ & $(2p - p^2 + pq)H + (2 - 2p + p^2 - pq)L$ & $\max\{2K + \frac{2p-p^2+pq}{p-q} \phi, 2U + (2 - p) \phi\}$ \\
\hline
$(1, p)$ & $(1 + p)H + (2 - p^2 - q + pq)L$ & $\max\{2K + \frac{p^2+p-pq}{p-q} \phi, 2U + (1 + p) \phi\}$ \\
\hline
$(1, 1)$ & $2pH + (2 - 2p)L$ & $\max\{2K + \frac{2p}{p-q} \phi, 2U + 2 \phi\}$ \\
\hline
\end{tabular}
\end{center}

Table A1: Two-period Contract

A1.3.2 Optimal Two-Period Contract

We first rule out the optimality of inducing $e = (0, 1 - q)$ and $e = (1, 1 - p)$ for the principal. Inducing $e = (0, 1 - q)$ is suboptimal for the principal due to the following reasons. The saving in expected payment in inducing $e = (0, 1 - q)$ compared with inducing $e = (1, p)$ is given by $(p + q) \phi$, regardless of the value of $U - K$. (This is because when $K < -\frac{q}{2(p-q)} \phi$, the expected payment to the agent for inducing $e = (0, 1 - q)$ is $(1 - q) \phi + 2U$, and the expected payment to the agent for inducing $e = (1, p)$ is $(1 + p) \phi + 2U$. When $K > -\frac{q}{2(p-q)} \phi$, the expected payment to the agent for inducing $e = (0, 1 - q)$ is $\frac{p+q^2-pq}{p-q} \phi + 2K$, and the expected payment to the agent for inducing $e = (1, p)$ is $\frac{p^2+p-pq}{p-q} \phi + 2K$.) In addition, the loss in expected demand in inducing $e = (0, 1 - q)$ compared with inducing $e = (1, p)$ is given by $(p^2 - q^2)(H - L)$. Therefore, inducing $e = (0, 1 - q)$ is dominated by inducing $e = (1, p)$ if $H - L \geq \frac{\phi}{p-q}$. This is the parameter space we consider when $U - K \geq 0$. For $U - K < 0$, inducing $e = (0, 1 - q)$ generates a lower profit for the principal compared with inducing $e = (0, 0)$ when $H - L < \frac{p+q^2-pq}{(1-q)(p-q)^2} \phi$. Since $\frac{1}{p-q} \phi < \frac{p+q^2-pq}{(1-q)(p-q)^2} \phi$, inducing $e = (0, 1 - q)$ will be either dominated by inducing $e = (1, p)$ or inducing $e = (0, 0)$ under the parameter space we are considering.

We can rule out the optimality of inducing $e = (1, 1 - p)$ using a similar rationale. In
particular, when $U - K \geq 0$, inducing $e = (1, 1 - p)$ is dominated by inducing $e = (1, 1)$ given $H - L \geq \frac{\phi}{p-q}$, the parameter space we focus on. When $U - K < 0$, inducing $e = (1, 1 - p)$ is dominated by inducing either $e = (1, 1)$ or $e = (0, 0)$ for the principal and is also suboptimal. In particular, when $H - L < \frac{2p - p^2 + pq}{(2-p)(p-q)} \frac{\phi}{p-q}$, we can prove that the incremental expected demand when inducing $e = (1, 1 - p)$ compared with inducing $e = (0, 0)$, given by $(2 - p)(p - q)(H - L)$, is no more than the incremental expected payment, given by $\frac{2p - p^2 + pq}{p-q} \phi$. Therefore inducing $e = (0, 0)$ dominates inducing $e = (1, 1 - p)$ for the principal when $H - L < \frac{2p - p^2 + pq}{(2-p)(p-q)} \frac{\phi}{p-q}$. Since $\frac{2p - p^2 + pq}{(2-p)(p-q)} \frac{\phi}{p-q} > \frac{\phi}{p-q}$, inducing $e = (1, 1 - p)$ will be dominated either by inducing $e = (1, 1)$ or inducing $e = (0, 0)$ for the principal and thus is suboptimal.

Next, we compare the principal’s profits for inducing the remaining effort profiles, i.e., $e = (0, 0), e = (0, q), e = (1, p)$, and $e = (1, 1)$. We now solve for the optimal two-period contract.

- **Case 1:** $U - K \leq 0$.

In this case, under the first-best scenario it is in the principal’s interest to induce effort if and only if $H - L > \frac{\phi + (U - K)}{p-q}$.

$$E[S + B] = \begin{cases} 2K, & \text{if } e = (0, 0) \\ 2K + \frac{pq}{p-q} \phi, & \text{if } e = (0, q) \\ 2K + \frac{p^2 + pq}{p-q} \phi, & \text{if } e = (1, p) \\ 2K + \frac{2p}{p-q} \phi, & \text{if } e = (1, 1) \end{cases}$$

Under this scenario, we find that inducing $e = (0, q)$ is suboptimal, since the principal gets a lower profit by inducing $e = (0, q)$ compared with inducing $e = (1, p)$ when $H - L \geq \frac{p(1+p-2q)}{(1+p-q)(p-q)^2} \phi$, and the principal gets a lower profit inducing $e = (0, q)$ compared with inducing $e = (0, 0)$ when $H - L < \frac{p}{(p-q)^2} \phi$. Since $\frac{p(1+p-2q)}{(1+p-q)(p-q)^2} \phi < \frac{p}{(p-q)^2} \phi < \frac{2p}{p-q} \phi$, the principal prefers inducing $e = (0, 0)$. Therefore, we can conclude that when $U - K \leq 0$, the principal prefers inducing $e = (0, 0)$.
\[
\frac{p}{(p-q)\phi}, \text{ it is suboptimal for the principal to induce } e = (0, q) \text{ under this scenario. Comparing the expected demand and payments for inducing } e = (0, 0), e = (1, p) \text{ and } e = (1, 1), \text{ we get the optimal contract for the principal as follows:}
\]

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{p(p+q)}{(1-p)(p-q)^2} \phi, \\
(1, p), & \text{if } \frac{p(p+q)}{(1-p)(p-q)^2} \phi \leq H - L < \frac{p(p+q)}{(1-p)(p-q)^2} \phi, \\
(0, 0), & \text{o.w.}
\end{cases}
\]

- **Case 2:** \(0 < U - K \leq \frac{q^2}{2(p-q)} \phi\).

Case 2 is the same with Case 1, except that the expected payment to the agent for inducing \(e = (0, 0)\) becomes \(2U\), rather than \(2K\). The other payments are the same as in Case 1. Comparing the principal’s profit for inducing the other effort profiles, we get the solution under this case as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{p(p+q)}{(1-p)(p-q)^2} \phi, \\
(1, p), & \text{if } 0 \leq U - K < \frac{p^2}{(1+p-q)(p-q)} \phi, H - L > \frac{p^2 + p(q - \frac{U-K}{p-q}) \phi}{(1+p-q)(p-q)}, \text{ or,} \\
(0, q), & \text{if } \frac{p^2}{(1+p-q)(p-q)} \phi \leq U - K < \frac{q^2}{2(p-q)} \phi, H - L \leq \frac{p^2 + p(q - \frac{U-K}{p-q}) \phi}{(1+p-q)(p-q)}, \text{ or,} \\
(0, 0), & \text{if } 0 \leq U - K < \frac{p^2}{(1+p-q)(p-q)} \phi, H - L \leq \frac{p^2 + p(q - \frac{U-K}{p-q}) \phi}{(1+p-q)(p-q)}, \text{ or,}
\end{cases}
\]

- **Case 3:** \(\frac{q^2}{2(p-q)} \phi < U - K \leq \frac{q}{2(p-q)} \phi\).

The expected payment for inducing \(e = (0, q)\) becomes \(q \phi + 2U\) in this case. The other payments are the same as in Case 2. The solution for this case is as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{p(p+q)}{(1-p)(p-q)^2} \phi, \\
(1, p), & \text{if } \frac{p(p+q)}{(1+p-q)(p-q)} \phi - \frac{2(U-K)}{(1+p-q)(p-q)} \leq H - L < \frac{p(p+q)}{(1-p)(p-q)^2} \phi, \\
(0, q), & \text{if } \frac{\phi}{p-q} \leq H - L < \frac{p(p+q)}{(1+p-q)(p-q)^2} \phi - \frac{2(U-K)}{(1+p-q)(p-q)} \\
(0, 0), & H - L < \frac{\phi}{p-q}.
\end{cases}
\]
• Case 4: \( \frac{q}{2(p-q)} \phi < U - K \leq \frac{q}{p-q} \phi \).

The expected payment for inducing \( e = (1, p) \) becomes \((1 + p) \phi + 2U\) in this case. The other payments are the same as in Case 3. The solution for this case is as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)}, \\
(1, p), & \text{if } \frac{\phi}{p-q} \leq H - L < \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)}, \\
(0, 0), & \text{if } H - L \leq \frac{\phi}{p-q}.
\end{cases}
\]

• Case 5: \( U - K > \frac{q}{p-q} \phi \).

The expected payment for inducing \( e = (1, 1) \) becomes \( 2\phi + 2U \) in this case. The other payments are the same as in Case 4. The solution for this case is as follows:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } H - L \geq \frac{1}{p-q}, \\
(0, 0), & \text{if } H - L < \frac{1}{p-q}.
\end{cases}
\]

**Combined together**, the optimal two-period contract, represented by their effort profiles being induced, is given by:

\[
(e_1, E[e_2])^* = \begin{cases} 
(1, 1), & \text{if } U - K \geq \frac{q}{p-q} \phi, H - L \geq \frac{\phi}{p-q} \phi, \\
 & \text{if } \frac{\phi}{p-q} \phi \leq U - K < \frac{q}{p-q} \phi, H - L \geq \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
 & \text{if } U - K < \frac{q}{p-q} \phi, H - L \geq \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
(1, p), & \text{if } \frac{\phi}{p-q} \phi \leq U - K < \frac{q}{p-q} \phi, \frac{\phi}{p-q} \phi \leq H - L < \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
 & \text{if } \frac{\phi}{p-q} \phi \leq U - K < \frac{q}{p-q} \phi, \frac{\phi}{p-q} \phi \leq H - L < \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
 & \text{if } 0 \leq U - K < \frac{q}{p-q} \phi, \frac{\phi}{p-q} \phi \leq H - L < \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
 & \text{if } 0 \leq U - K < \frac{q}{p-q} \phi, \frac{\phi}{p-q} \phi \leq H - L < \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
 & \text{if } 0 \leq U - K < \frac{q}{p-q} \phi, \frac{\phi}{p-q} \phi \leq H - L < \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
(0, 0), & \text{if } U - K \geq \frac{q}{p-q} \phi, H - L \leq \frac{\phi}{p-q} \phi, \\
 & \text{if } U - K \geq \frac{q}{p-q} \phi, H - L \leq \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
 & \text{if } U - K \geq \frac{q}{p-q} \phi, H - L \leq \frac{p(1-p+q)}{(1-p)(p-q)} \phi - \frac{2(U-K)}{(1-p)(p-q)} \phi, \\
 & \text{if } U - K < 0, H - L < \frac{p(1-p+q)}{(1-p)(p-q)} \phi.
\end{cases}
\]
A1.4 Period-by-Period Contract with Sales Push-out and Pull-in

We derive the optimal period-by-period contract using backward induction. We start from the principal's problem of inducing a specified effort profile in the second period.

- **Case 1:** Consider the case when the principal induces $e^H_2 = 1$, i.e., effort is exerted by the agent when the first period’s sales realization is $D_1 = H$.

  Given the reported sales level $D'_1$, even though the principal cannot observe the real realization of $D_2$ but can only observe the reported sales level $D'_2$, she can still infer $D_2$ from $D'_2$ by readjusting the quota level at the second period and setting bonus value high enough to ensure that if $D_2 = H$ the agent will not restrict sales. In particular, given $D_1 = H$, no matter what sales level the agent reports for the early period $D'_1$, if $D_2$ also realizes as $H$, the principal expects to observe the second period’s sales level as $D'_2 = 2H - D'_1$ (conditional on the bonus being high enough).

  Consequently, to induce $e^H_2 = 1$, it suffices to set the quota level $\chi^H_2$ equal to $2H - D'_1$, and to provide bonus $b_2$ equal to $\frac{\phi}{p-q}$. Additionally, since $\chi_2 = 2H - D'_1 \geq H + L - D'_1$, in case $D_1 = L$, no matter how much the agent reports, the later quota level will never be met. This implies that inducing $e^H_2 = 1$ will lead to $e^L_2 = 0$.

  Combined together, by setting $\chi_2 = 2H - D'_1 \geq H$ and $b_2 = \frac{\phi}{p-q}$, the principal induces $(e^H_2, e^L_2) = (1, 0)$. We will discuss the level of the fixed salary later, as the agent decides whether to accept the principal’s contract at the beginning by weighing his utilities across two periods, anticipating that the principal may readjust quota levels later.

- **Case 2:** Inducing $e^L_2 = 1$, i.e., motivating effort exertion when the first period’s sales realization is $D_1 = L$.

  In a similar way as the case above, we have that the principal needs to set the quota level at $\chi^L_2 = H + L - D'_1$ and the bonus level at $b_2 = \frac{\phi}{p-q}$ to induce $e^L_2 = 1$. Also,
since $\chi_2 = H + L - D_1'$, the quota level in the second period will always be met in case $D_1 = H$ so that there will be no effort exerted. This implies $e_2^L = 1$ will lead to $e_2^H = 0$. Combined together, by setting $\chi_2 = H + L - D_1' \geq H$ and $b_2 = \frac{\phi}{p-q}$, the principal induces $(e_2^H, e_2^L) = (0, 1)$.

To summarize, to induce a specific effort level, the principal can adjust the quota level in the second period $\chi_2$ based on reported sales $D_1'$ in the first period. To induce $(e_2^H, e_2^L) = (1, 0)$, the principal sets $\chi_2(D_1') = 2H - D_1'$. To induce $(e_2^H, e_2^L) = (0, 1)$, the principal sets $\chi_2(D_1') = H + L - D_1'$. In both cases, the principal offers a bonus of $b_2 = \frac{\phi}{p-q}$ once the sales meet the quota level. Furthermore, $(e_2^H, e_2^L) = (1, 1)$ is not incentive compatible when a salesperson can push out or pull in sales since the range of quota levels required to motivate $e_2^H = 1$ has no overlap with that to induce $e_2^L = 1$.

Now we move to the principal’s problem in the first period. To induce $e_1 = 1$, the principal needs to set the corresponding quota level $\chi_1$ at such a level that, after accounting for sales push out and pull in, when $D_1 = L$ the agent cannot meet the quota, and when $D_1 = H$ he can meet the quota; this is derived as $2L < \chi_1 \leq H + L$. To see this, given $\chi_1 > 2L$, when $D_1 = L$, the agents cannot make $\chi_1$ even by pulling in all available sales $L$ from the second period. Given $\chi_1 \leq H + L$, when $D_1 = H$, the agent can meet the quota by pulling in $\chi - H < L$. Without loss of generality, it is enough to consider $\chi_1 = H + L$. The early bonus level $b_1$ as well as the two fixed wage levels $s_1$ and $s_2$ are chosen by accounting for the second period’s effort profile. Now we discuss all the possible scenarios.

(1) To induce $e = (1, p)$, the principal needs to provide sufficient $b_1$ for agents to exert effort, which is given by $(1 - q)\frac{\phi}{p-q}$. This is because, if $D_1 = H$, the agent earns $s_1 + b_1 + s_2 + p\frac{\phi}{p-q} - \phi$; if $D_1 = L$, the agent earns $s_1 + s_2$. To induce $e_1 = 1$, the principal needs to make sure $b_1 + p\frac{\phi}{p-q} - \phi \geq \frac{\phi}{p-q}$, which simplifies into $b_1 \geq (1 - q)\frac{\phi}{p-q}$. Following this, the principal pays the agent $s_1 + s_2 + p(b_1 + p\frac{\phi}{p-q}) = s_1 + s_2 + p(1 + p - q)\frac{\phi}{p-q}$ in expectation. Fixed wages are chosen such that the fixed wage in each period is no lower than the limited
liability, and the two fixed wages combined can ensure the agent’s participation, namely, 
\[ s_1 \geq K, s_2 \geq K, s_1 + s_2 \geq 2U + p(1 + p - q)\frac{\phi}{p-q} - (1 - p)\phi = 2U + \frac{q}{p-q}\phi. \]

To summarize, to induce \( e = (1, p) \), at \( T = 1 \), the principal sets \( \chi_1 = H + L \) and \( b_1 = (1 - q)\frac{\phi}{p-q} \). Under this contract, the agent exerts effort at \( T = 1 \). If \( D_1 = H \), the agent pulls in \( L \) from \( T = 2 \) to meet the early quota. Then at \( T = 2 \), the principal readjusts the quota level to \( \chi_2 = H - L \) and sets \( b_2 = \frac{\phi}{p-q} \) to encourage effort exertion. If \( D_1 = L \), the agent reports \( D_1 = L \) as it is. The principal then readjusts the quota level at the second period to \( \chi_2 = 2H - L \) which is higher than \( H \) so the quota level is not achievable and the agent gives up.

(2) Similarly, to induce \( e = (1, 1 - p) \), the principal sets \( \chi_1 = H + L \) and offers \( b_1 = q\frac{\phi}{p-q} \). To see this, if \( D_1 = H \), the agent gets paid \( s_1 + b_1 + s_2 + \frac{\phi}{p-q} \); otherwise, the agent gets paid \( s_1 + s_2 + p\frac{\phi}{p-q} - \phi \). It requires \( b_1 + \frac{\phi}{p-q} - (p\frac{\phi}{p-q} - \phi) \geq \frac{\phi}{p-q} \) to motivate \( e_1 = 1 \), which is equivalent to \( b_1 \geq q\frac{\phi}{p-q} \). The principal pays the agent \( s_1 + s_2 + (2p - p^2 + pq)\frac{\phi}{p-q} \) on expectation. Finally, the fixed wages are set such that each period’s wage is no lower than the limited liability and the two wages in combination will ensure the agent’s participation, i.e., \( s_1 \geq K, s_2 \geq K, s_1 + s_2 \geq 2U + \frac{2q}{p-q}\phi \).

In this scenario, if \( D_1 = H \), agents again pull in \( L \) from the second period to earn early bonuses; the principal later sets \( \chi_2 = 0 \) inducing no effort. If \( D_1 = L \) instead, agents cannot make early bonuses and simply report \( D_1 = L \); the principal will set \( \chi_2 = H \) to induce effort.

(3) In other circumstances when the principal does not want to induce early effort, the contract is straightforward to derive and is shown in Table 3.1.
A2 Appendix for Chapter 4

A2.1 Two-Period Contract

A2.1.1 General Two-Period Contract

In the following, I use the labels \((IC_2^d - ge)\), \((IC_2^d - l)\), \((IC_0^o - ge)\), \((IC_0^o - l)\), \((IC_2^- - ge)\), \((IC_2^- - l)\), \((IC_1 - ge)\), \((IC_1 - l)\) and \((LL)\) to denote the following constraints:

\[(IC_2^d - ge)\] denotes \(B(d, d) - B(d, 0) \geq \frac{1}{h}((B(d, 0) - B(d, -d)) + \frac{\phi}{h});\]
\[(IC_2^d - l)\] denotes \(B(d, d) - B(d, 0) < \frac{1}{h}((B(d, 0) - B(d, -d)) + \frac{\phi}{h});\]
\[(IC_0^o - ge)\] denotes \(B(0, d) - B(0, 0) \geq \frac{1}{h}((B(0, 0) - B(0, -d)) + \frac{\phi}{h});\]
\[(IC_0^o - l)\] denotes \(B(0, d) - B(0, 0) < \frac{1}{h}((B(0, 0) - B(0, -d)) + \frac{\phi}{h});\]
\[(IC_2^- - ge)\] denotes \(B(-d, d) - B(-d, 0) \geq \frac{1}{h}((B(-d, 0) - B(-d, -d)) + \frac{\phi}{h});\]
\[(IC_2^- - l)\] denotes \(B(-d, d) - B(-d, 0) < \frac{1}{h}((B(-d, 0) - B(-d, -d)) + \frac{\phi}{h});\]
\[(IC_1 - ge)\] denotes \(v(d) - v(0) \geq \frac{1}{h}(v(0) - v(-d)) + \frac{\phi}{h};\]
\[(IC_1 - l)\] denotes \(v(d) - v(0) < \frac{1}{h}(v(0) - v(-d)) + \frac{\phi}{h};\]
\[(LL)\] denotes \(B(d, d), B(d, 0), B(d, -d), B(0, d), B(0, 0), B(0, -d), B(-d, d), B(-d, 0) \geq 0.\)

- To induce \(e = (1, 1, 1, 1)\), the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} & \quad (p + h)(v(d) + \phi) + (1 - 2p - h - l)(v(0) + \phi) + (p + l)(v(-d) + \phi) \\
\text{s.t.} & \quad (IC_2^d - ge), (IC_0^o - ge), (IC_2^- - ge), (IC_1 - ge), (LL) \\
\text{and} & \quad (p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq 0 \quad \text{(PC)}
\end{align*}
\]

Under this scenario, \(v(d) = (p + h)B(d, d) + (1 - 2p - h - l)B(d, 0) + (p + l)B(d, -d) - \phi\) is
the agent’s expected utility in the second period given \( D_1 = H \), \( v(0) = (p + h)B(0, d) + (1 - 2p - h - l)B(0, 0) + (p + l)B(0, -d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = M \), and \( v(-d) = (p + h)B(-d, d) + (1 - 2p - h - l)B(-d, 0) + (p + l)B(-d, -d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = L \).

The optimal contract to induce this effort profile is given by: 

\[
B(-d, d) = \frac{\phi}{h}, \quad B(0, d) = \frac{\phi}{h}, \quad (p + h)B(d, d) + (1 - 2p - h - l)B(d, 0) + (p + l)B(d, -d) = (1 + p + h)\frac{\phi}{h}, \quad B(d, d) - B(d, 0) \geq \frac{l}{h} (B(d, 0) - B(d, -d)) + \frac{\phi}{h}.
\]

The following history-independent contract lies within the optimal contract set: 

\[
B(-d, d) = B(0, d) = B(d, -d) = B(d, 0) = \frac{\phi}{h}, \quad B(d, d) = 2\frac{\phi}{h}.
\]

Such a contract is history-independent since the bonus payment only depends on how many times the demand in each period realizes as \( H \). If both periods’ demand realizes as \( H \), the agent gets a bonus of \( 2\frac{\phi}{h} \); if the demand in only one of the two periods realizes as \( H \), the agent gets a bonus of \( \frac{\phi}{h} \); if the demand in neither period realizes as \( L \), the agent gets zero bonus. The expected payment to the agent is \( 2(p + h)\frac{\phi}{h} \).

- To induce \( e = (0, 1, 1, 1) \), the principal’s problem is:

\[
\min_{B(D_1, D_2), \; D_1 \text{ and } D_2 \in \{d, 0, -d\}} p(v(d) + \phi) + (1 - 2p)(v(0) + \phi) + p(v(-d) + \phi)
\]

s.t. 

\[
(\text{IC}_{2}, \text{ge}), (\text{IC}_{0}, \text{ge}), (\text{IC}_{-d}, \text{ge}), (\text{IC}_{1}, \text{l}), (\text{LL})
\]

and

\[
pv(d) + (1 - 2p)v(0) + pv(-d) \geq 0 \quad (PC)
\]

Under this scenario, \( v(d) \), \( v(0) \), and \( v(-d) \) is the same as in the above case with inducing \( e = (1, 1, 1, 1) \).

The optimal contract to induce this effort profile is given by: 

\[
B(-d, d) = B(0, d) = B(d, d) = \frac{\phi}{h}, \quad \text{The expected payment to the agent is } (p + h)\frac{\phi}{h}.
\]
• To induce \( e = (1, 1, 1, 0) \), the principal’s problem is:

\[
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} \quad (p + h)(v(d) + \phi) + (1 - 2p - h - l)(v(0) + \phi) + (p + l)v(-d)
\]

\[
\text{s.t.} \quad (lC_2^d\text{-ge}), (IC_2^0\text{-ge}), (HC_2^-d\text{-l}), (IC_1\text{-l}), (LL)
\]

and

\[
(p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq 0 \quad (PC)
\]

Under this scenario, \( v(d) = (p + h)B(d, d) + (1 - 2p - h - l)B(d, 0) + (p + l)B(d, -d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = H \), \( v(0) = (p + h)B(0, d) + (1 - 2p - h - l)B(0, 0) + (p + l)B(0, -d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = M \), and \( v(-d) = pB(-d, d) + (1 - 2p)B(-d, 0) + pB(-d, -d) \) is the agent’s expected utility in the second period given \( D_1 = L \).

The optimal contract to induce this effort profile is given by: \( B(0, d) = \frac{\phi}{h}, (p + h)B(d, d) + (1 - 2p - h - l)B(d, 0) + (p + l)B(d, -d) = (1 + p + h + \frac{l}{h})\phi, B(d, d) - B(d, 0) \geq \frac{l}{h}(B(d, 0) - B(d, -d)) + \frac{\phi}{h}. \) The following history-independent contract lies within the optimal contract set: \( B(0, d) = B(0, 0) = \frac{\phi}{h}, \)

\[
B(d, d) = \frac{p(3h + h) + 2h + l}{p + h} \frac{\phi}{h}. \]

The expected payment to the agent is \( (2 - \Delta)h^2 + p(2h - l)(1 - l) + (2 - l)(h - l)^2 - p^2(h - l)\phi \).

• To induce \( e = (0, 1, 1, 0) \), the principal’s problem is:

\[
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} \quad p(v(d) + \phi) + (1 - 2p)(v(0) + \phi) + p(v(-d) + \phi)
\]

\[
\text{s.t.} \quad (lC_2^d\text{-ge}), (IC_2^0\text{-ge}), (HC_2^-d\text{-l}), (IC_1\text{-l}), (LL)
\]

and

\[
pv(d) + (1 - 2p)v(0) + pv(-d) \geq 0 \quad (PC)
\]

Under this scenario, \( v(d), v(0), \) and \( v(-d) \) is the same as in the above case with inducing \( e = (1, 1, 1, 0) \).

The optimal contract to induce this effort profile is given by: \( B(d, d) = B(0, d) = \frac{\phi}{h} \).
The optimal contract to induce this effort profile is given by:

\[
(1 - p)(p + h)^{\frac{\phi}{h}}.
\]

- To induce \( e = (1, 1, 0, 1) \), the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} & \quad (p + h)(v(d) + \phi) + (1 - 2p - h - l)v(0) + (p + l)(v(-d) + \phi) \\
\text{s.t.} & \quad (IC_2^{d-ge}), (IC_2^0-1), (IC_2^{-d-ge}), (IC_1-ge), (LL) \\
\text{and} & \quad (p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq 0 \quad (PC)
\end{align*}
\]

Under this scenario, \( v(d) = (p + h)B(d, d) + (1 - 2p - h - l)B(d, 0) + (p + l)B(d, -d) - \phi \)

is the agent’s expected utility in the second period given \( D_1 = H \), \( v(0) = pB(0, d) + (1 - 2p)B(0, 0) + pB(0, -d) \) is the agent’s expected utility in the second period given \( D_1 = M \), and \( v(-d) = (p + h)B(-d, d) + (1 - 2p - h - l)B(-d, 0) + (p + l)B(-d, -d) - \phi \)

is the agent’s expected utility in the second period given \( D_1 = L \).

The optimal contract to induce this effort profile is given by:

\[
B(-d, d) = \frac{\phi}{h}, \quad (p + h)B(d, d) + (1 - 2p - h - l)B(d, 0) + (p + l)B(d, -d) = (1 + h - pL)^{\frac{\phi}{h}}, \quad B(d, d) - B(d, 0) \geq \frac{1}{h}(B(d, 0) - B(d, -d)) + \frac{\phi}{h}. \]

The following contract lies within the optimal contract set:

\[
B(-d, d) = \frac{\phi}{h}, \quad B(d, d) = \frac{1 + h - pL}{p + h} \frac{\phi}{h}. \quad \text{The expected payment to the agent is,}
\]

\[
(p + h)\left(1 + h + l + p\frac{b - l}{p + h}\right)\frac{\phi}{h}.
\]

- To induce \( e = (0, 1, 0, 1) \), the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} & \quad p(v(d) + \phi) + (1 - 2p)v(0) + p(v(-d) + \phi) \\
\text{s.t.} & \quad (IC_2^{d-ge}), (IC_2^0-1), (IC_2^{-d-ge}), (IC_1-1), (LL) \\
\text{and} & \quad pv(d) + (1 - 2p)v(0) + pv(-d) \geq 0 \quad (PC)
\end{align*}
\]

Under this scenario, \( v(d), v(0), \) and \( v(-d) \) is the same as in the above case with inducing \( e = (1, 1, 0, 1) \).

The optimal contract to induce this effort profile is given by:

\[
B(-d, d) = B(d, d) = \frac{\phi}{h}.
\]
The expected payment to the agent is $2p(p + h)\frac{\phi}{h}$.

- To induce $e = (1, 1, 0, 0)$, the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} & \quad (p + h)(v(d) + \phi) + (1 - 2p - h - l)v(0) + (p + l)v(-d) \\
\text{s.t.} & \quad (IC^d_{-2} - ge), (IC^0_{-2} - l), (IC^{-d}_{-1}), (LL) \\
\text{and} & \quad (p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq 0 \quad (PC)
\end{align*}
\]

Under this scenario, $v(d) = (p + h)B(d, d) + (1 - 2p - h - l)B(d, 0) + (p + l)B(d, -d) - \phi$ is the agent’s expected utility in the second period given $D_1 = H$, $v(0) = pB(0, d) + (1 - 2p)B(0, 0) + pB(0, -d)$ is the agent’s expected utility in the second period given $D_1 = M$, and $v(-d) = pB(-d, d) + (1 - 2p)B(-d, 0) + pB(-d, -d)$ is the agent’s expected utility in the second period given $D_1 = L$.

The optimal contract to induce this effort profile is given by: $(p + h)B(d, d) + (1 - 2p - h - l)B(d, 0) + (p + l)B(d, -d) - \phi = (1 + h)\frac{\phi}{h}$, $B(d, d) - B(d, 0) \geq \frac{l}{h} (B(d, 0) - B(d, -d)) + \frac{\phi}{h}$. The following history-independent contract lies within the optimal contract set: $B(d, d) = \frac{1 + h \phi}{p + h}$. The expected payment to the agent is $(1 + h)(p + h)\frac{\phi}{h}$.

- To induce $e = (0, 1, 0, 0)$, the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} & \quad p(v(d) + \phi) + (1 - 2p)v(0) + pv(-d) \\
\text{s.t.} & \quad (IC^d_{2} - ge), (IC^0_{2} - l), (IC^{-d}_{-1}), (IC_{1} - l), (LL) \\
\text{and} & \quad pv(d) + (1 - 2p)v(0) + pv(-d) \geq 0 \quad (PC)
\end{align*}
\]

Under this scenario, $v(d)$, $v(0)$, and $v(-d)$ is the same as in the above case with inducing $e = (1, 1, 0, 0)$.

The optimal contract to induce this effort profile is given by: $B(d, d) = \frac{\phi}{h}$. The expected payment to the agent is $p(p + h)\frac{\phi}{h}$.
• To induce \( e = (1, 0, 1, 1) \), the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1,D_2), \ D_1 \text{ and } D_2 \in \{d,0,-d\}} & \quad p(v(d) + (1 - 2p - h - l)v(0) + p + l)v(-d + \phi) \\
\text{s.t.} & \quad (IC_2^d \text{-} l), (IC_2^d \text{-} ge), (IC_2^{-d} \text{-} ge), (IC_1 \text{-} l), (LL) \\
\text{and} & \quad (p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq 0 \quad (PC)
\end{align*}
\]

Under this scenario, \( v(d) = pB(d,d) + (1 - 2p)B(d,0) + pB(d,-d) \) is the agent’s expected utility in the second period given \( D_1 = H \), \( v(0) = (p + h)B(0,d) + (1 - 2p - h - l)B(0,0) + (p + l)B(0,-d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = M \), and \( v(-d) = (p + h)B(-d,d) + (1 - 2p - h - l)B(-d,0) + (p + l)B(-d,-d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = L \).

The optimal contract to induce this effort profile is given by: \( B(-d,d) = \frac{\phi}{h}, \ B(0,d) = \frac{\phi}{h}, (p + h)B(d,d) + (1 - 2p - h - l)B(d,0) + (p + l)B(d,-d) = (1 + p)\frac{\phi}{h}, B(d,d) - B(d,0) < \frac{1}{h}(B(d,0) - B(d,-d)) + \frac{\phi}{h} \). The following contract lies within the optimal contract set: \( B(-d,d) = B(0,d) = \frac{\phi}{h}, B(d,d) = B(d,0) = B(d,d) = (1 + p)\frac{\phi}{h} \). The expected payment to the agent is \( (2 - h)(p + h)\frac{\phi}{h} \).

• To induce \( e = (0, 0, 1, 1) \), the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1,D_2), \ D_1 \text{ and } D_2 \in \{d,0,-d\}} & \quad p(v(d) + \phi) + (1 - 2p)v(0) + p(v(-d) + \phi) \\
\text{s.t.} & \quad (IC_2^d \text{-} l), (IC_2^0 \text{-} ge), (IC_2^{-d} \text{-} ge), (IC_1 \text{-} l), (LL) \\
\text{and} & \quad pv(d) + (1 - 2p)v(0) + pv(-d) \geq 0 \quad (PC)
\end{align*}
\]

Under this scenario, \( v(d), v(0), \) and \( v(-d) \) is the same as in the above case with inducing \( e = (1, 0, 1, 1) \).

The optimal contract to induce this effort profile is given by: \( B(-d,d) = B(0,d) = \frac{\phi}{h} \). The expected payment to the agent is \( (1 - p)(p + h)\frac{\phi}{h} \).
• To induce \( e = (1, 0, 1, 0) \), the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} & \quad (p + h)v(d) + (1 - 2p - h - l)(v(0) + \phi) + (p + l)v(-d) \\
\text{s.t.} & \quad (IC_2^d), (IC_2^d \text{-ge}), (IC_2^{-d}) , (IC_1 \text{-ge}), (LL) \\
\text{and} & \quad (p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq 0 \quad (PC)
\end{align*}
\]

Under this scenario, \( v(d) = pB(d, d) + (1 - 2p)B(d, 0) + pB(d, -d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = H \), \( v(0) = (p + h)B(0, d) + (1 - 2p - h - l)B(0, 0) + (p + l)B(0, -d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = M \), and \( v(-d) = pB(-d, d) + (1 - 2p)B(-d, 0) + pB(-d, -d) - \phi \) is the agent’s expected utility in the second period given \( D_1 = L \).

The optimal contract to induce this effort profile is given by: \( B(0, d) = \frac{\phi}{h} \), \( pB(d, d) + (1 - 2p)B(d, 0) + pB(d, -d) = (1 + p + p\frac{l}{h})\frac{\phi}{h} \), \( B(d, d) - B(d, 0) < \frac{l}{h} (B(d, 0) - B(d, -d)) + \frac{\phi}{h} \). The following contract lies within the optimal contract set: \( B(0, d) = \frac{\phi}{h} \), \( B(d, -d) = B(d, 0) = B(d, d) = (1 + p + p\frac{l}{h})\frac{\phi}{h} \). The expected payment to the agent is \( \left(2 - 2l - \frac{(h-l)(p+h)}{h}\right) (p + h)\frac{\phi}{h} \).

• To induce \( e = (0, 0, 1, 0) \), the principal’s problem is:

\[
\begin{align*}
\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} & \quad pv(d) + (1 - 2p)(v(0) + \phi) + pv(-d) \\
\text{s.t.} & \quad (IC_2^d), (IC_2^0 \text{-ge}), (IC_2^{-d}) , (IC_1 \text{-l}), (LL) \\
\text{and} & \quad pv(d) + (1 - 2p)v(0) + pv(-d) \geq 0 \quad (PC)
\end{align*}
\]

Under this scenario, \( v(d) \), \( v(0) \), and \( v(-d) \) is the same as in the above case with inducing \( e = (1, 0, 1, 0) \).

The optimal contract to induce this effort profile is given by: \( B(0, d) = \frac{\phi}{h} \). The expected payment to the agent is \( (1 - 2p)(p + h)\frac{\phi}{h} \).
To induce $e = (1, 0, 0, 1)$, the principal’s problem is:

$$\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} (p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)(v(-d) + \phi)$$

s.t. 

$$(IC_2^d - l), (IC_0^d - l), (IC_{-d}^d - ge), (IC_1^d - l), (LL)$$

and 

$$(p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq 0 \quad (PC)$$

Under this scenario, $v(d) = pB(d, d) + (1 - 2p)B(d, 0) + pB(d, -d)$ is the agent’s expected utility in the second period given $D_1 = H$, $v(0) = pB(0, d) + (1 - 2p)B(0, 0) + pB(0, -d)$ is the agent’s expected utility in the second period given $D_1 = M$, and $v(-d) = (p + h)B(-d, d) + (1 - 2p - h - l)B(-d, 0) + (p + l)B(-d, -d) - \phi$ is the agent’s expected utility in the second period given $D_1 = L$.

The optimal contract to induce this effort profile is given by: $B(-d, d) = \frac{\phi}{h}$, $pB(d, d) + (1 - 2p)B(d, 0) + pB(d, -d) = (1 - p\frac{l}{h})\frac{\phi}{h}$, $B(d, d) - B(d, 0) < \frac{1}{h}(B(d, 0) - B(d, -d)) + \frac{\phi}{h}$.

The following contract lies within the optimal contract set: $B(-d, d) = \frac{\phi}{h}$, $B(d, -d) = B(d, 0) = B(d, d) = (1 - p\frac{l}{h})\frac{\phi}{h}$. The expected payment to the agent is $\frac{1}{h}(1 + l + (h - l)(1 + l + p))(p + h)\frac{\phi}{h}$.

To induce $e = (0, 0, 1, 1)$, the principal’s problem is:

$$\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} pv(d) + (1 - 2p)v(0) + p(v(-d) + \phi)$$

s.t. 

$$(IC_2^d - l), (IC_0^d - l), (IC_{-d}^d - ge), (IC_1^d - l), (LL)$$

and 

$$pv(d) + (1 - 2p)v(0) + pv(-d) \geq 0 \quad (PC)$$

Under this scenario, $v(d), v(0)$, and $v(-d)$ is the same as in the above case with inducing $e = (1, 0, 0, 1)$.

The optimal contract to induce this effort profile is given by: $B(-d, d) = \frac{\phi}{h}$. The expected payment to the agent is $p(p + h)\frac{\phi}{h}$. 

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• To induce $e = (1, 0, 0, 0)$, the principal’s problem is:

$$\min_{B(D_1, D_2), D_1 \text{ and } D_2 \in \{d, 0, -d\}} \quad (p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d)$$

s.t. $$(IC_d^2 - 1), (IC_0^2 - 1), (IC_{-d}^2 - 1), (IC_1 - ge), (LL)$$

and $$(p + h)v(d) + (1 - 2p - h - l)v(0) + (p + l)v(-d) - \phi \geq 0 \quad (PC)$$

Under this scenario, $v(d) = pB(d, d) + (1 - 2p)B(d, 0) + pB(d, -d)$ is the agent’s expected utility in the second period given $D_1 = H$, $v(0) = pB(0, d) + (1 - 2p)B(0, 0) + pB(0, -d)$ is the agent’s expected utility in the second period given $D_1 = M$, and $v(-d) = pB(-d, d) + (1 - 2p)B(-d, 0) + pB(-d, -d)$ is the agent’s expected utility in the second period given $D_1 = L$.

The optimal contract to induce this effort profile is given by: $pB(d, d) + (1 - 2p)B(d, 0) + pB(d, -d) = \frac{\phi}{h}$, $B(d, d) - B(d, 0) < \frac{l}{h}(B(d, 0) - B(d, -d)) + \frac{\phi}{h}$. The following contract lies within the optimal contract set: $B(d, -d) = B(d, 0) = B(d, d) = \frac{\phi}{h}$. The expected payment to the agent is $(p + h)\frac{\phi}{h}$.

I summarize in Table A1 the expected sales and payments to the agent for different effort profiles that the principal induces.
\[
\begin{array}{|c|c|c|c|}
\hline
(c_1, e_{\theta}) & (c_1, E[\theta]) & \text{Contract} & E[B] \\
\hline
(1, 1, 1, 1) & (1, 1) & B(-d, d) - B(0, d) - B(d, 0) - B(d, d) = \frac{p}{h}, \ B(d, d) = \frac{2p}{h} & 0 \\
(1, 1, 1, 0) & (1, 1 - p - h) & B(0, d) - B(d, 0) = \frac{p}{h}, \ B(d, d) = \frac{p + h}{h} & (2 - p - h)(h - l)d \\
(1, 1, 0, 1) & (1, 2p + h + l) & B(-d, d) = \frac{p}{h}, \ B(d, d) = \frac{1 + p + h}{h} & (1 + 2p + h + l)(h - l)d \\
(1, 0, 0, 0) & (1, p) & B(d, d) = \frac{p}{h} & (1 + p)(h - l)d \\
(0, 1, 1, 1) & (0, 1) & B(-d, d) - B(0, d) - B(d, 0) - B(d, d) = \frac{p}{h}, \ B(d, d) = \frac{p + h}{h} & (1 - p)(h - l)d \\
(0, 1, 0, 1) & (0, 2p) & B(-d, d) - B(0, d) - B(d, 0) - B(d, d) = \frac{p}{h}, \ B(d, d) = \frac{p + h}{h} & 2p(h - l)d \\
(0, 0, 0, 0) & (0, 0) & B(d, d) = \frac{p}{h} & p(h - l)d \\
\hline
\end{array}
\]

Table A2: Two-period Contract

A2.1.2 Optimal Two-Period Contract

I first rule out the optimality of inducing \((e^1, e^2, e^M, e^L) = (1, 1, 1, 0)\) for the principal. I can show that the principal gets a lower profit by inducing \((1, 1, 1, 0)\) compared with inducing \((1, 1, 1, 1)\) when \(d \geq \frac{p^2(h-l)+ph^2+lh^2}{(h-l)(p+l)h^2} \phi\). Moreover, the principal gets a lower profit inducing \((1, 1, 1, 0)\) compared with inducing \(e = (0, 0, 0, 0)\) when

\[
d < \frac{(p^2(h-l)+(-2+l)h^2 + p(h-l)^2 + 2(h-l)(-1+l) + (-2+l) l)}{(h-l)(-2+p+l)h^2} \phi.
\]

Since

\[
\frac{p^2(h-l)+ph^2+lh^2}{(h-l)(p+l)h^2} < \frac{(p^2(h-l)+(-2+l)h^2 + p(h-l)^2 + 2(h-l)(-1+l) + (-2+l) l)}{(h-l)(-2+p+l)h^2} \phi,
\]

inducing \((1, 1, 1, 0)\) will be dominated either by inducing \((1, 1, 1, 1)\) or inducing \((0, 0, 0, 0)\) for the principal and thus is suboptimal. Following a similar rationale, I can prove the suboptimality of inducing the following effort profiles for the principal:

- Inducing \((1, 0, 1, 1)\) is dominated by inducing \((1, 1, 1, 1)\) or inducing \((0, 0, 0, 0)\) for the principal and thus is suboptimal.
- Inducing \((1, 0, 1, 0)\) is dominated by inducing \((1, 1, 1, 1)\) or inducing \((0, 0, 0, 0)\) for the principal and thus is suboptimal.
principal and thus is suboptimal.

- Inducing \((1, 0, 0, 1)\) is dominated by inducing \((1, 1, 0, 1)\) or inducing \((0, 0, 0, 0)\) for the principal and thus is suboptimal.

- Inducing \((1, 0, 0, 0)\) is dominated by inducing \((1, 1, 1, 1)\) or inducing \((0, 0, 0, 0)\) for the principal and thus is suboptimal.

- Inducing \((0, e_H^2, e_L^2)\), for any \(e_H^2, e_L^2 \in \{0, 1\}\) is dominated by inducing \((1, 1, 0, 0)\) or inducing \((0, 0, 0, 0)\) for the principal and thus is suboptimal.

Combined together, the possible effort profiles to be induced by the principal at equilibrium are \((1, 1, 1, 1)\), \((1, 1, 0, 1)\), \((1, 1, 0, 0)\), \((0, 0, 0, 0)\). Define

\[
K_1 = \frac{p^2(h-l)+p(2h^2-h)+h^2(h+l-1)}{(2p+h+l-1)(h-l)h^2}, \quad K_2 = \frac{p^2(h-l)+ph^2+lh^2}{(h-l)(p+l)h^2},
\]

\[
K_3 = \frac{p^2(h-l)+p(2h^2+h)+h^2(h+l+1)}{(2p+h+l+1)(h-l)h^2}, \quad K_4 = \frac{(1+h)(p+h)}{(h-l)h(1+p+h)}.
\]

Comparing the principal’s net profit under these effort profiles, I get the optimal contract for the principal as below,

- Inducing \((1, 1, 1, 1)\) is optimal when \(K_1 < \frac{d}{\phi}\),

- Inducing \((1, 1, 0, 1)\) is optimal when \(h > \frac{1+p}{p} l\) and \(K_2 < \frac{d}{\phi} < K_1\), or when \(h < \frac{1+p}{p} l\) and \(K_3 < \frac{d}{\phi} < K_1\)

- Inducing \((1, 1, 0, 0)\) is optimal when \(h > \frac{1+p}{p} l\) and \(K_4 < \frac{d}{\phi} < K_2\);

- Inducing \((0, 0, 0, 0)\) is optimal when \(h > \frac{1+p}{p} l\) and \(h - l < \frac{d}{\phi} < K_4\), or when \(h < \frac{1+p}{p} l\) and \(h - l < \frac{d}{\phi} < K_3\).

### A2.2 Incentive Contract with Limited Inventory

#### A2.2.1 Optimal Period-by-Period Contract with Limited Inventory

I first solve the subgame at \(T=2\) when \(Y_1 = H\). In this case, the remaining inventory is \(\Omega - H\).
• If $\Omega - d > d$, inventory does not serve as a bottleneck and following the case without inventory concerns, the principal prefers $e_2^d = 1$ under the condition $(h - l)d \geq (1 + \frac{p}{h})\phi$.

• If $0 < \Omega - d \leq d$, even if $D_1 = d$, the sales cannot exceed $\Omega - d$, thus the principal will prefer effort $\Omega \geq \Omega_1 \equiv (1 + \frac{l}{h})d + (1 + \frac{p}{h})\phi$.

• Following a similar argument, when $D_1 = 0$, the remaining inventory is $\Omega$ and if and only if $\Omega \geq \Omega_2 \equiv l\phi + \frac{(1 + \frac{p}{h})\phi}{h}$, will the principal prefer to motivate effort, i.e. $e_0^d = 1$.

• When $D_1 = -d$, the remaining inventory is $\Omega + d$. If and only if $\Omega \geq \Omega_3 \equiv (1 - 1)\phi + \frac{(1 + \frac{p}{h})\phi}{h}$, the principal prefers to motivate effort, i.e. $e_{-d}^d = 1$.

Next consider the game at $T=1$.

• When $\Omega_1 < \Omega \leq 2d$, $e_{d}^d = e_0^0 = e_{-d}^d = 1$. To have $e_1^* = 1$, we need

$$\Omega \geq \frac{dh((-1+2p)h+2h^2+l)+(p+h)\phi}{h^2(p+h)}.$$ Since the above cutoff value on $\Omega$ is smaller than $\Omega_1$, we arrive that when $\Omega_1 < \Omega \leq 2d$, the principal always prefer to induce effort at the first period.

• When $\Omega_2 < \Omega \leq \Omega_1$, $e_{-d}^d = e_0^d = 1$ and $e_2^d = 0$. Under this condition,

- If $\Omega \geq d$, the constraints to make the principal motivate $e_1 = 1$ is $\Omega \geq \max\{d, \Omega_4 \equiv \frac{dh((-1+2p)h+2h^2+l)+(p+h)\phi}{h^2(p+h)}\}$.

- If $\Omega < d$, the constraints to make the principal motivate $e_1 = 1$ is $\Omega \geq \max\{\Omega_2, \Omega_5 \equiv \frac{dh((-1+2h)h+l)+(p+h)\phi}{h^2(p+h)+l} \}$.

• When $\Omega_3 \equiv (1 - 1)d + (1 + \frac{p}{h})\phi < \Omega < \Omega_2$, $e_{-d}^d = 1$ and $e_0^0 = e_2^d = 0$. Under this condition,

- If $\Omega \geq 0$, the constraints to make the principal motivate $e_1 = 1$ is $\Omega \geq \max\{0, \Omega_6 \equiv \frac{dh((-1+h-l)l+p(h+l)+(p+h)(1+l)\phi}{h((-1+2p)h+l)}\}$.
If $\Omega < 0$, the constraints to make the principal motivate $e_1 = 1$ is $\Omega \geq \max\{\Omega_3, \Omega_7 \equiv -dh((-1+l-h) t+p(h+l))+(p+h) (1+l)\phi \}.$

- When $-d < \Omega \leq \Omega_3$, $e_{2-d} = e_2^0 = e_2^d = 0$. Conditional on the second period’s outcomes and to ensure the principal would like to motivate effort at the first period, $\Omega$ needs to satisfy $\Omega \geq -dh((-1+l-h) t+p(h+l))+(p+h) \phi \frac{h}{(h-1+l)(l+p(h+2l))}$. This, however, contradicts with the assumption that $-d < \Omega \leq \Omega_3$. As a result, if the principal has no incentive to induce effort at period 2, she will neither incentivize early effort. This is intuitive since the possible impact of limited inventory is to push effort towards the later period.

### A2.2.2 Optimal Two-Period Contract with Limited Inventory

Compared with the period-by-period contract, the two-period contract pays more bonus when inducing $e = (1, \{e_2^d, 1^0, e_2^{-d}\})$ compared with the period-by-period contract. This includes $e = (1, \{1^d, 1^0, 0^{-d}\})$, $e = (1, \{0^d, 1^0, 1^{-d}\})$, and $e = (1, \{0^d, 1^0, 0^{-d}\})$. This is because to induce $e_2^0 = 1$, under the period-by-period contract, the agent expects a positive bonus payment in the second period. However, exerting $e_1 = 1$ reduces the probability of getting $D_1 = 0$. Thus when the principal aggregates bonus payment by the two-period contract, she needs to offer more bonus to incentivize early effort. In contrast, the two-period contract pays less bonus when inducing $e = (1, \{e_2^d, 0^0, e_2^{-d}\})$ compared with the period-by-period contract, due to the opposite reason. This includes $e = (1, \{1^d, 0^0, 1^{-d}\})$, $e = (1, \{1^d, 0^0, 0^{-d}\})$, and $e = (1, \{0^d, 0^0, 1^{-d}\})$.

Therefore, in terms of the optimal effort profiles, besides these induced under the period-by-period contract in Table *, the principal may also induce $e = (1, \{1^d, 0^0, 1^{-d}\})$ and $e = (1, \{1^d, 0^0, 0^{-d}\})$. Furthermore, we need to update the boundary conditions for inducing $e = (1, \{0^d, 1^0, 1^{-d}\})$ and $e = (1, \{0^d, 0^0, 1^{-d}\})$. The optimal contract is listed in Table *. 

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A2.3 Two-Period Contract with Non-Decreasing Constraint

A2.3.1 General Two-Period Contract with Non-Decreasing Constraint

For the contracts listed in Table A2, the non-decreasing constraint is not met when inducing \( e = (1, 1, 0, 1), e = (0, 1, 0, 1), e = (0, 0, 1, 1), e = (0, 1, 0, 1), e = (0, 0, 1, 1) \). As a result, under the non-decreasing constraint, we need to recalculate the optimal two-period contract for inducing the above effort profiles, while the optimal two-period contract for inducing other profiles remain the same. Without specifying the details, I re-summarize the optimal two-period contract for inducing any given effort profile that meets the non-decreasing constraint in Table A3.

<table>
<thead>
<tr>
<th>( r_0, r_1, r_2 )</th>
<th>Contract</th>
<th>( E[l] )</th>
<th>( E[h] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.1, 1, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>0</td>
<td>2(( p + b ))</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((2 - p)(1 - b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = 1, B(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + 2b)(1 - b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + b)(p + 2b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((2 - p)(1 - b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + b)(p + 2b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + b)(p + 2b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + b)(p + 2b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + b)(p + 2b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + b)(p + 2b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + b)(p + 2b))</td>
<td>0</td>
</tr>
<tr>
<td>(1.1, 0, 1)</td>
<td>( B(-d, d) = h(0, 0) - B(0, 0) = 2B(0, 0) - B(0, 0) = 2 )</td>
<td>((1 + b)(p + 2b))</td>
<td>0</td>
</tr>
</tbody>
</table>

Table A3: Two-period Contract with Non-decreasing Constraint

A2.3.2 Optimal Two-Period Contract with Non-Decreasing Constraint

Since the optimal contracts for inducing \((1, 1, 1, 1), (1, 1, 0, 1)\) and \((0, 0, 0, 0)\) remain the same as in Section A2.3.1, we only need to prove the suboptimality of \((1, 1, 0, 1)\) and \((1, 0, 1, 1)\). This is because only these two effort profiles are not dominated by \((1, 1, 1, 1), (1, 1, 0, 1)\) or \((0, 0, 0, 0)\) in Section A2.3.1. Indeed, I can show that under the non-decreasing constraint, inducing \((1, 1, 0, 1)\) is always dominated by inducing \((1, 1, 1, 1)\) for the principal when \( h - l >
\( \frac{\phi}{d} \), and is thus suboptimal. Similarly, inducing \((1, 0, 0, 1)\) is always dominated by inducing \((1, 1, 0, 0)\) for the principal when \(h - l > \frac{\phi}{d}\), and is thus suboptimal.

To summarize, it can be optimal for the principal to induce \((1, 1, 1, 1)\), \((1, 1, 0, 0)\), \((0, 0, 0, 0)\), depending on the parameter space. Define \(K_1' \equiv \frac{(1-h)(p+h)}{(h-l)h(1-p-h)}\). Comparing the principal’s net profit under these effort profiles, I get the optimal contract for the principal as below,

- Inducing \((1, 1, 1, 1)\) is optimal when \(K_1' < \frac{d}{\phi}\);
- Inducing \((1, 1, 0, 0)\) is optimal when \(K_4 < \frac{d}{\phi} < K_1'\);
- Inducing \((0, 0, 0, 0)\) is optimal when \(\frac{d}{\phi} < K_4\).

A2.3.3 Optimal Non-Decreasing Two-Period Contract with Limited Inventory

\[
\begin{align*}
e^* &= (1, \{0, 0, 1-d, 1\}) \\
e^* &= (1, \{1, 1, 0, 0\}) \\
e^* &= (1, \{0, 0, 1-d, 1\}) \\
e^* &= (1, \{1, 0, 0, 0\}) \\
e^* &= (0, \{0, 0, 1-d, 1\}) \\
e^* &= (0, \{0, 0, 0, 0\})
\end{align*}
\]

Figure A1: Optimal Two-period contract with Non-decreasing Constraint

Note: \(\phi = 1, \ d = 50, \ l = 0.1, \ p = 0.26.\)