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LAMONT GEOLOGICAL OBSERVATORY
PALISADES, NEW YORK

MIXING AND RESIDENCE TIME ON THE GREAT BAHAMA BANK

Report prepared by: J. Michael Costin

Technical Report No. CU-17-65 to the Atomic Energy Commission
Contract AT (30-1) 2663

January, 1965

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ABSTRACT

Many studies of regular and random motions have been made in the shallow waters of the Great Bahama Banks. Dye tracer experiments have been made near the edge of the shoal water-deep water boundary and at positions 20 and 50 miles in from the edge on the western banks in the lee of Andros Island. These studies are discussed with regard to circulatory patterns and flushing times.

Estimates are made of the magnitude of horizontal shearing stresses during the ebb and flood periods under conditions of wind and current moving in the same and opposing directions.

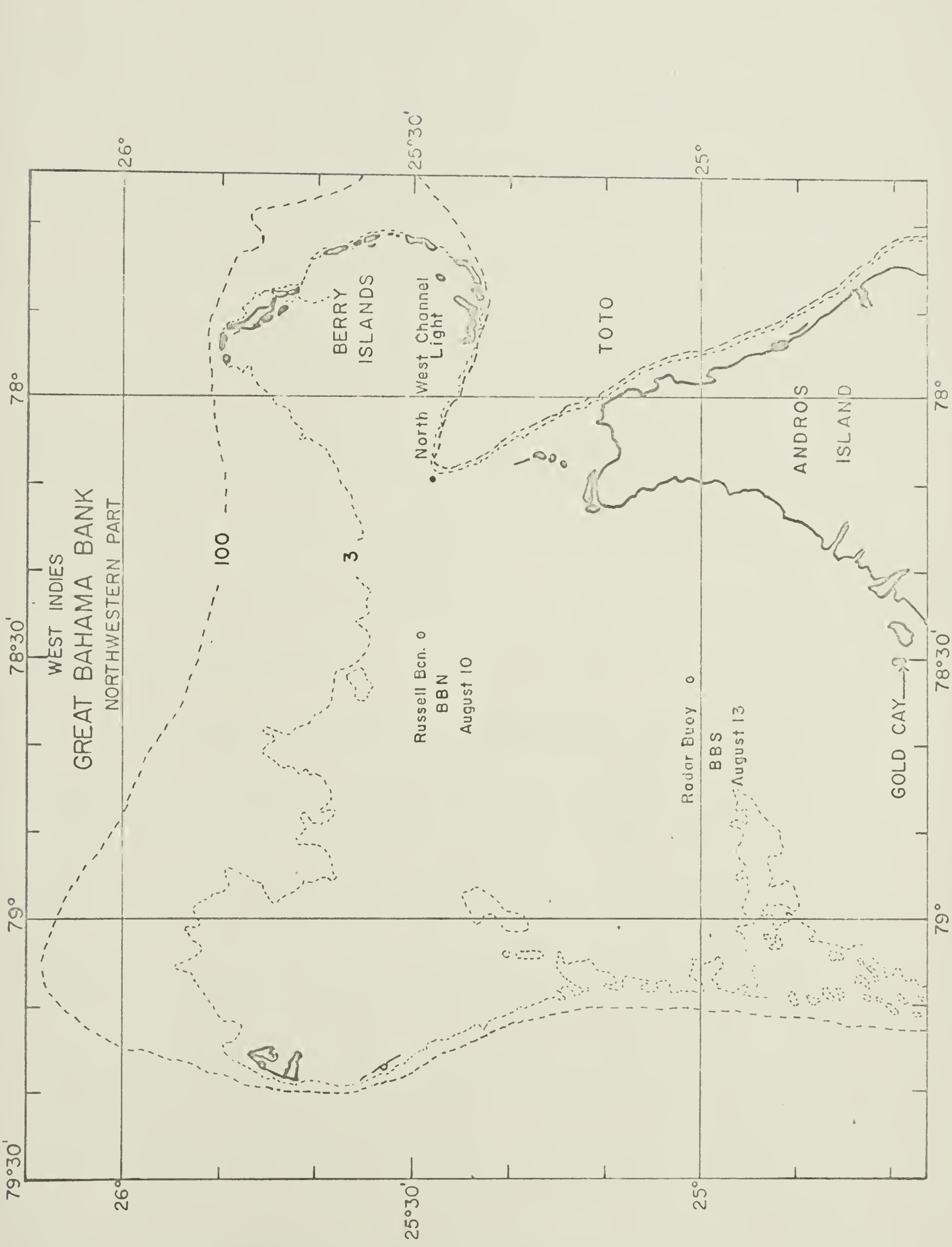
A two-dimensional model is proposed which gives the time in days for a particle to move from the experimental areas to the edge of the bank during the summer season. The computed values are in close agreement with the experimental results.

INTRODUCTION

Many studies of regular and random motions have been made in the shallow waters of the Great Bahama Banks. It is felt that, before any further work is initiated, a summary of the work to date would be useful.

Dye diffusion experiments (Costin, 1963) have shown that the level of turbulent energy on the western Great Bahama Banks is relatively low. Computed values of "the most probable diffusion velocity", after the method of Joseph and Sender, are on the order of 0.4 cm/sec in the area north of Russell Beacon (referred to as Bahama Banks North, BBN) and 0.2 cm/sec in an area some 50 km. to the south (BBS, see Fig. 1). Values of the flushing time, τ_N, were obtained by estimating the time in days it would take for a dyed water mass to leave the banks.

Measurements of the insitu C^{14}/C^{12} ratio (Broecker and Takahashi, January, 1965) have given estimates of a residence time, τ_C. The authors point out that since 1954 there has been an irregular but continuous increase in atmospheric C^{14} due to nuclear testing. The C^{14} falling on the open sea is mixed rapidly to an average depth of at least 100 meters, and therefore, the concentration of atmospheric carbon in the adjacent open ocean surface water is considerably depleted before this water moves onto the banks. The shallow water (2.5 fm) on the banks inhibits loss of C^{14} due to vertical mixing, and thus the corrected



Location of dye dumps on the Great Bahama Bank - August, 1962

depth in fathoms

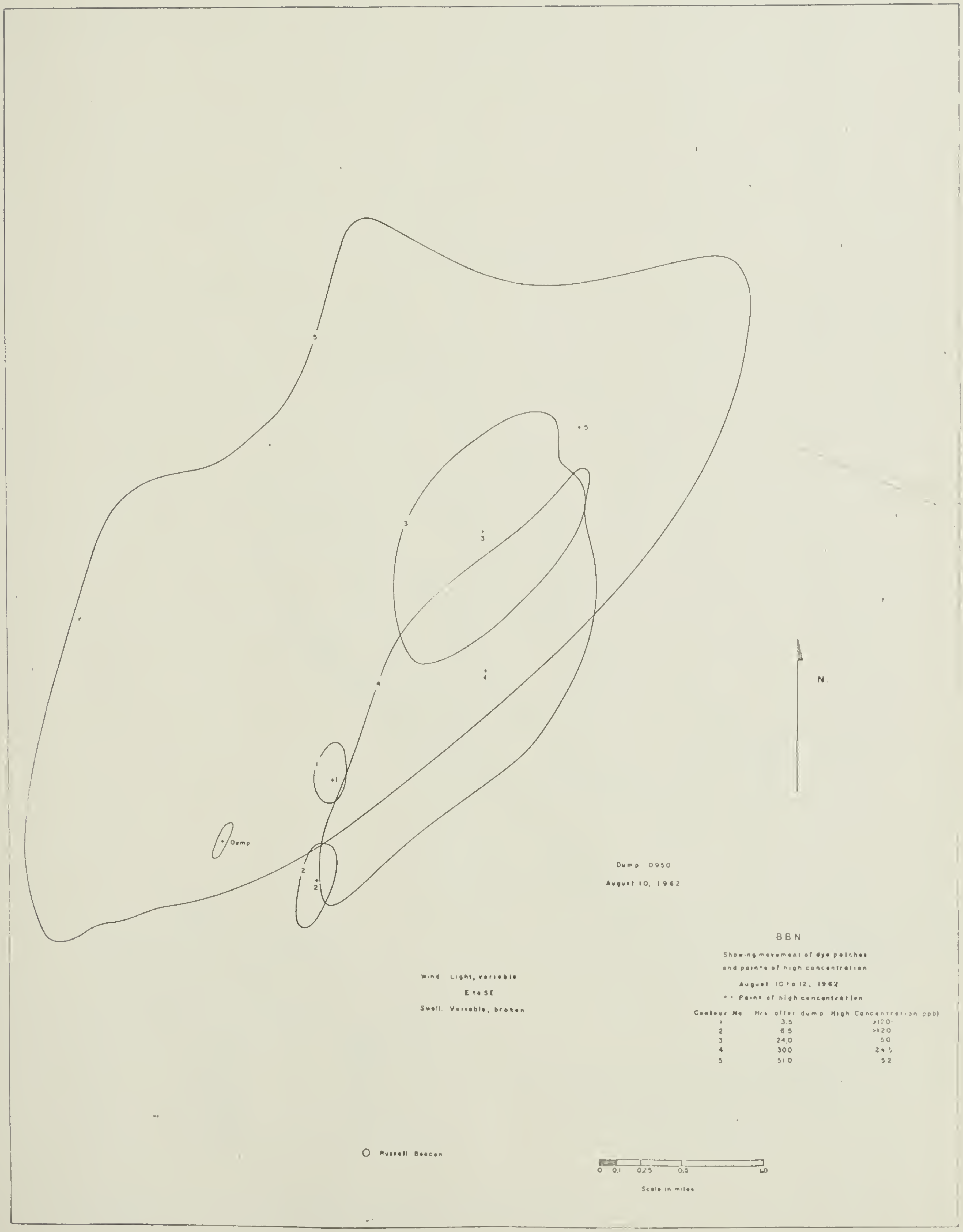
FIGURE 1

increase in C^{14} will be a measure of residence time, τ_C, defined as the mean time since the water parcel in a given sample entered the banks from the adjacent open ocean. The longer a sample remains on the banks, the higher the C^{14} concentration.

Although, by definition, there are differences in the physical meanings of τ_N and τ_C, it was observed that their respective values tend to increase by the same order of magnitude in the direction of decreasing diffusion velocities. This general trend has prompted a re-examination of these data in the light of more recent studies in an attempt to discover recurrent relationships of circulation and mixing on the banks.

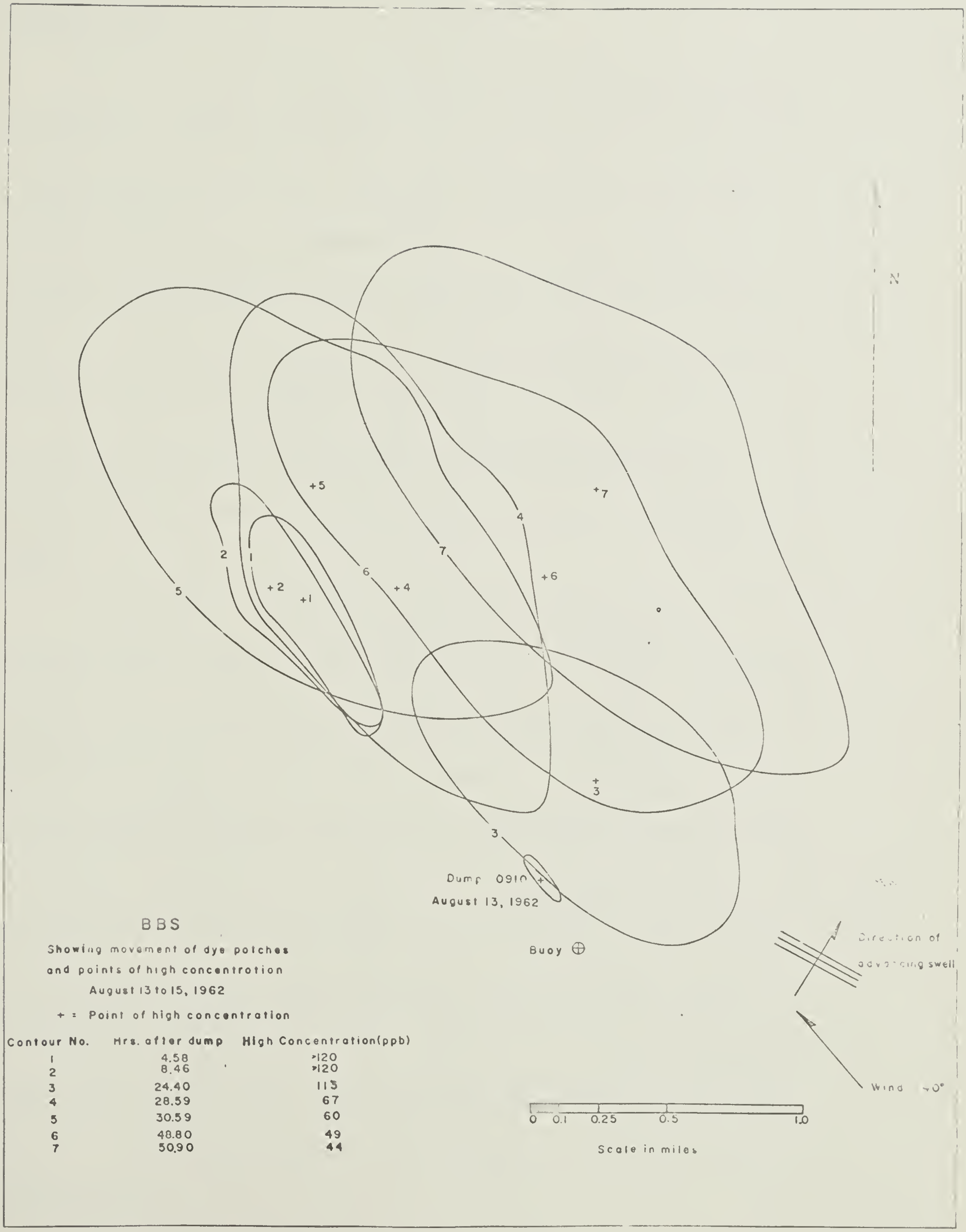
GENERAL CIRCULATION

The advective motions of the two dye patches, represented in outline form in Figures 2 and 3, are in close agreement with the general circulatory pattern discussed by Smith (1940). This pattern consists of tidal oscillations with major axes nearly perpendicular to the 100 fathom line near the edge of the bank and trending NW-SE on the central portion in the lee of Andros Island. These tidal oscillations are superimposed on a residual drift dependent on wind variation resulting in a northerly drift in summer, and a southerly drift in winter. Note: Dye patch outline #5 (Fig. 2, BBN) may be as much as 30% too large due to temporary difficulties in both the dye-sampling and navigational



Contour No	Hrs after dump	High Concentration ppb)
1	3.5	>120·
2	6 5	>120
3	24.0	50
4	300	24 5
5	510	52

FIGURE 2



BBS

Showing movement of dye patches
and points of high concentration
August 13 to 15, 1962

+ = Point of high concentration

Contour No.	Hrs. after dump	High Concentration(ppb)
1	4.58	>120
2	8.46	>120
3	24.40	115
4	28.59	67
5	30.59	60
6	48.80	49
7	50.90	44

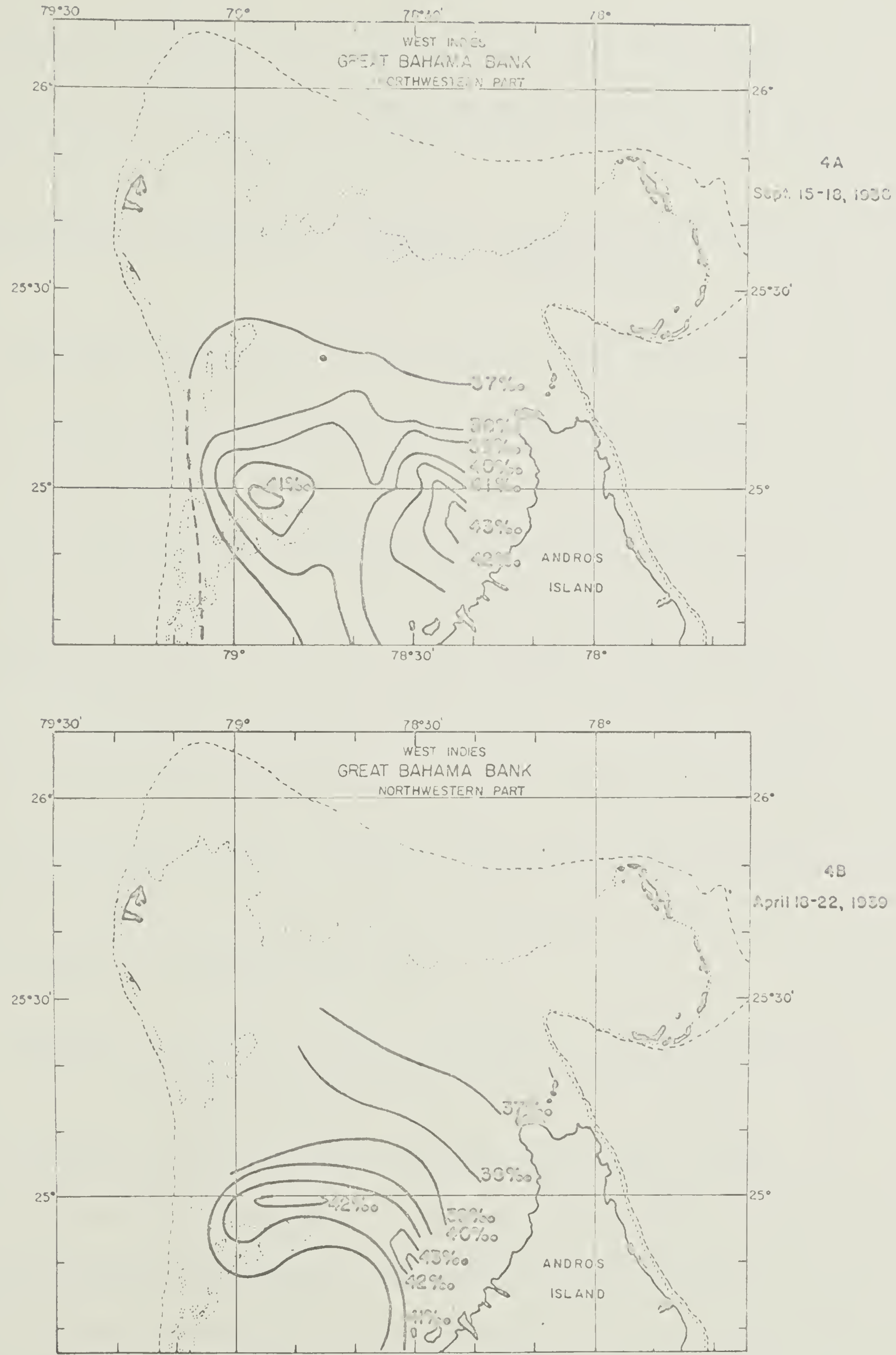
FIGURE 3

systems, but has been included to show the northernmost position of the dye patch.

In such shallow water as encountered on the Bahama Banks, the direction of tidal current flow may be described as being nearly normal to the cotidal lines. Due to the rotation of the earth, the water particles move in elliptic orbits, but because of frictional stresses over the shallow bottom, the ratio of the major to minor axes of the tidal ellipse is greater than in the case of an unlimited ocean. The narrow elliptical trajectories of the dye patches are due both to the earth's deflecting force and the interference of tidal streams crossing the banks from north to south and west to east.

Smith also indicates that during most of the summer season, there is a high salinity wedge with an axis lying along 25°N which is being pinched off by an invasion of less saline water on both sides (Fig. 4). The salinity gradient is consistently sharper on the northern side of the high salinity wedge which indicates that there may be a net flow in that direction. Figure 4A was constructed from measurements made during September 15th to 18th, 1938 when the average wind was about 4 knots from ENE. The isohalines are displaced somewhat to the south of those in Fig. 4B April 18th to 22nd, 1939, when the average wind was from the east at over 7 knots, illustrating the local dependence of drift on the wind direction.

The dye survey was conducted during a period of generally



4A
Sept. 15-18, 1938

4B
April 18-22, 1939

Distribution of Isohalines after Smith

(1940)

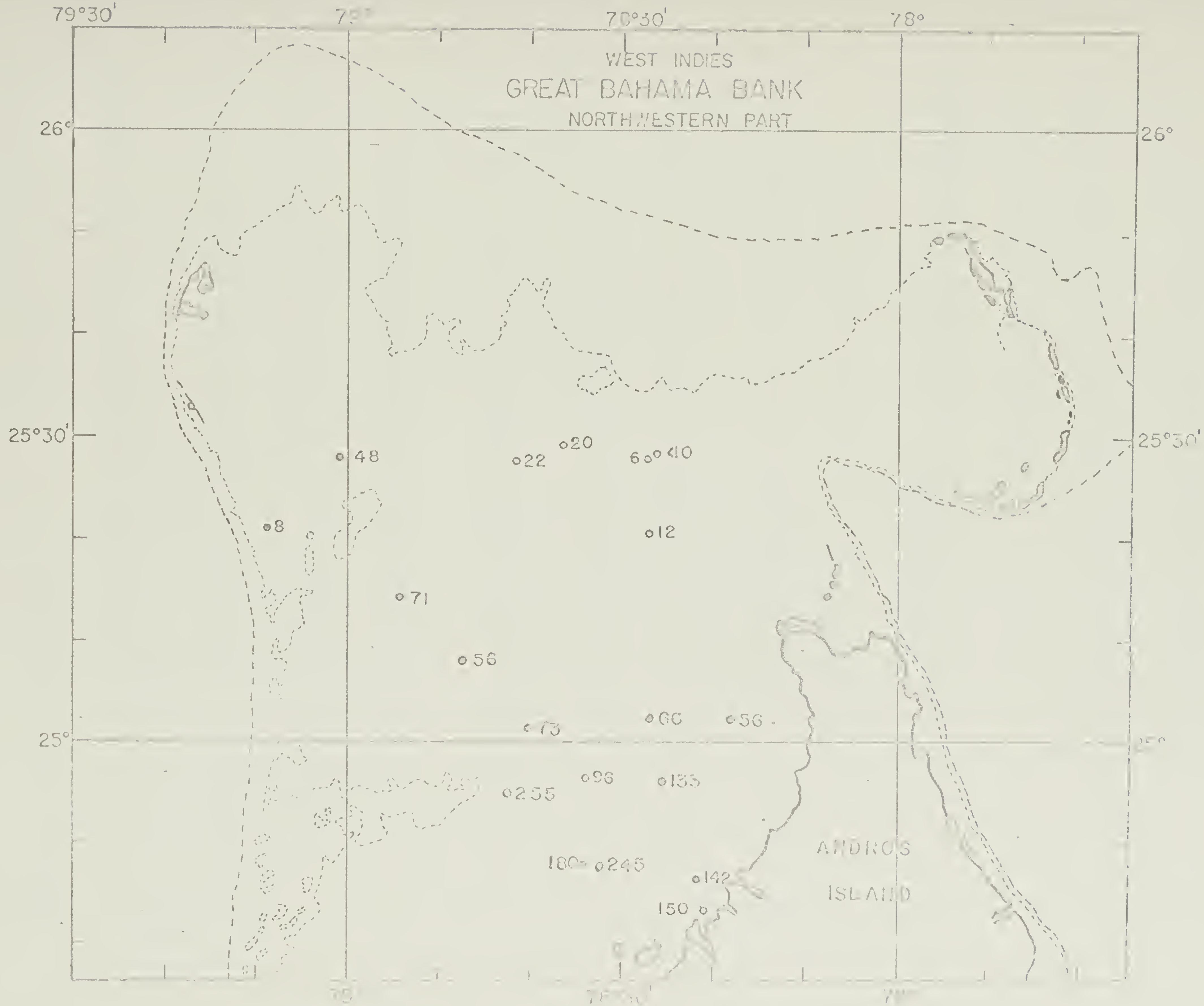
FIGURE 4

light southeasterly winds, and, as expected, the dye patches showed a net drift toward the north with average residual velocities of 0.07 knots for BBN, and 0.03 knots for BBS.

EXPERIMENTAL RESULTS (1): Residence times from dye studies and C^{14} measurements

Estimates of the time it would take for the dye patches to leave the northern edge of the bank were made by projecting the respective motions of the leading edges of the dye patches through time. It is seen that if the southeasterly winds were to prevail for a given length of time, the dye patch at BBN would have reached the 100 fathom line in 6 to 8 days, and that of BBS in 40 to 50 days. However, the winds are constantly shifting and in the southern area where the tidal currents are quite weak, the above figures could be in error by as much as 20 or 25 days. If the experiments had been made under conditions of steady northerly winds, the above values of τ_N would be considerably different.

Figure 5 (after Broecker and Takahashi, 1964) illustrates values of τ_C made from measurements of insitu C^{14} during June, 1962 (open circles) and June, 1963 (closed circles). These charted values indicate that water north of Andros (about 25° 10' N) had been in residence for a period of less than 20 days and that of the BBS area between 50 and 100 days. The reference states that the absolute residence times could be in error by as much as a factor of two, and that the ratios of residence times for various



Values of Residence Times τ_c after Broecker & Takahashi

(1964)

FIGURE 5

samples should be good to ± 20%.

The results of the C^{14} study also indicate that increased values of τ_C will generally be found in areas of higher salinity, although salinities of June, 1962 were lower than average because of heavy rains during that period.

EXPERIMENT RESULTS (2): Circulation near the shoal water-deep water boundary

It has long been felt that, due to density considerations in the area, water leaving the banks on the ebb current generally sinks as it moves out into deeper water. In the Tongue of the Ocean basin (TOTO) to the east of Andros, water having the same density as the shoal water is found to depths of about 200 meters (Busby and Dick, 1964). In April, 1964, a small-scale experiment was conducted in TOTO in an attempt to observe circulatory patterns near the boundary of the shoal water.

Figure 6 shows the result of that study. The measurements were made along the line E-W (Fig. 6A) which is nearly parallel to the direction of the ebb current. Figures 6B and 6C are temperature and salinity profiles along E-W during the ebb. The measurements were made with a small portable insitu salinometer, and, although a high order of precision may be lacking, the results are valid in defining a circulatory trend. A small patch of yellow dye was introduced at point W just after high slack water and a patch of red dye at point E about 20 minutes later. The wind was moderate from

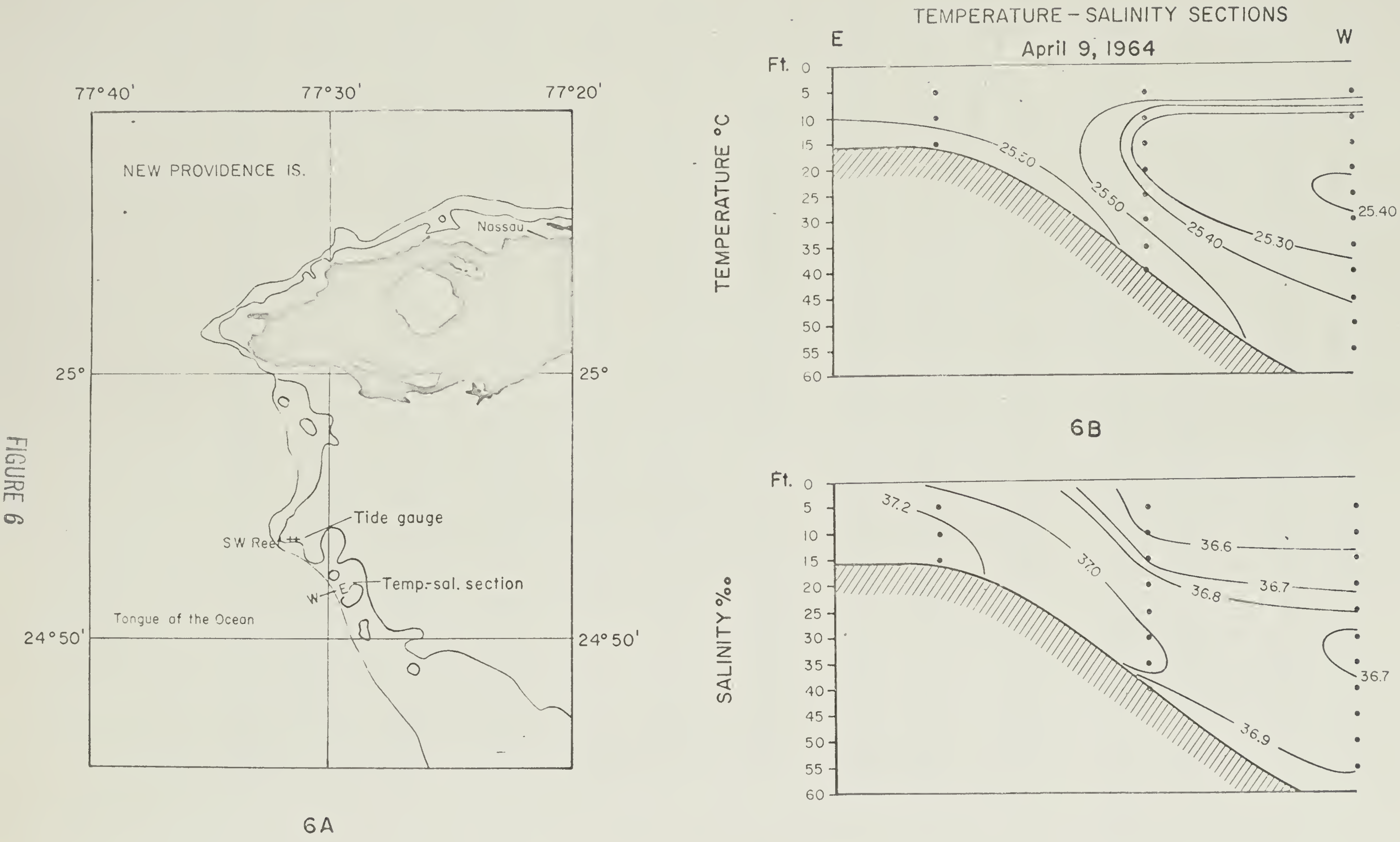

77°40'
77°30'
77°20'
NEW PROVIDENCE IS.
Nassau
25°
25°
Tide gauge
SW Reef
Temp.-sal. section
W
E
Tongue of the Ocean
24°50'
24°50'
6A
TEMPERATURE – SALINITY SECTIONS
April 9, 1964
E
W
Ft.
0
5
10
15
20
25
30
35
40
45
50
55
60
TEMPERATURE °C
25.30
25.50
25.40
25.30
25.40
6B
SALINITY ‰
37.2
37.0
36.6
36.7
36.8
36.9
36.7
6C

FIGURE 6

the southeast and both dye patches showed a slight northerly drift.

The yellow patch developed a rather sharp face along its eastern boundary (nearest the shallow water), and moved very slowly to the west and north. The eastern boundary of the patch was nearly coincident with a long line of seaweed which indicated the presence of the offshore convergence.

The shallow-water patch at point E moved very rapidly to the west and sank beneath the deep-water patch. At the end of the ebb current, the dye had become too diffuse for visual tracking, but the line of seaweed which had marked the convergence began to drift on to the banks.

Due to instrumental limitations, we were not able to sample dye concentrations, but the dye patch motions point to the existence of a thin wedge of warmer and more saline bank water descending over the edge of the bank and into deep water during the ebb current.

If this situation can be considered typical of the entire bank area, we see that during each tidal cycle, water on the banks sinks over the edge during the ebb current and is partially or wholly replaced by lower salinity water (and thus water of lower C^{14} content) during the flood. Through turbulent mixing processes, some lower salinity ocean water is thus able to work its way south onto the western banks in the summer, although the net flow is to the north during that season.

EXPERIMENTAL RESULTS (3): Local turbulent intensities

Since the estimates of local turbulent intensities have been made from averaged results of a small number of dye experiments, we are able to make only general comments concerning the character of short-term diffusion at this time.

The rate at which a contaminant field is diffused by fluid turbulence should be proportional to the shearing stresses over the bottom, and the surface wind stresses. In very shallow water, the flow is greatly retarded in a thin frictional layer above the bottom. Various authors have suggested that the thickness of this layer will be approximately 1/10 of the total depth, which corresponds to the bottom 1/2 meter on the Bahama Banks. Above this lower boundary layer, the velocity is commonly described as uniform, or at least as a linear function of depth above the bottom.

In a shallow area where stratification is locally non-existent, the shearing stresses generated by flow over the bottom are transmitted throughout the water column. Disregarding short-term wind stresses, the intensity of local turbulent energy should be proportional to the current velocity, and thus, in a tidal area, should change with the changing phases of the tidal motion.

The data from the dye dumps of August, 1962 is inconclusive regarding this point, as dye sampling runs were not made according to a schedule which would permit determination of differential diffusion intensities during ebb or flood periods.

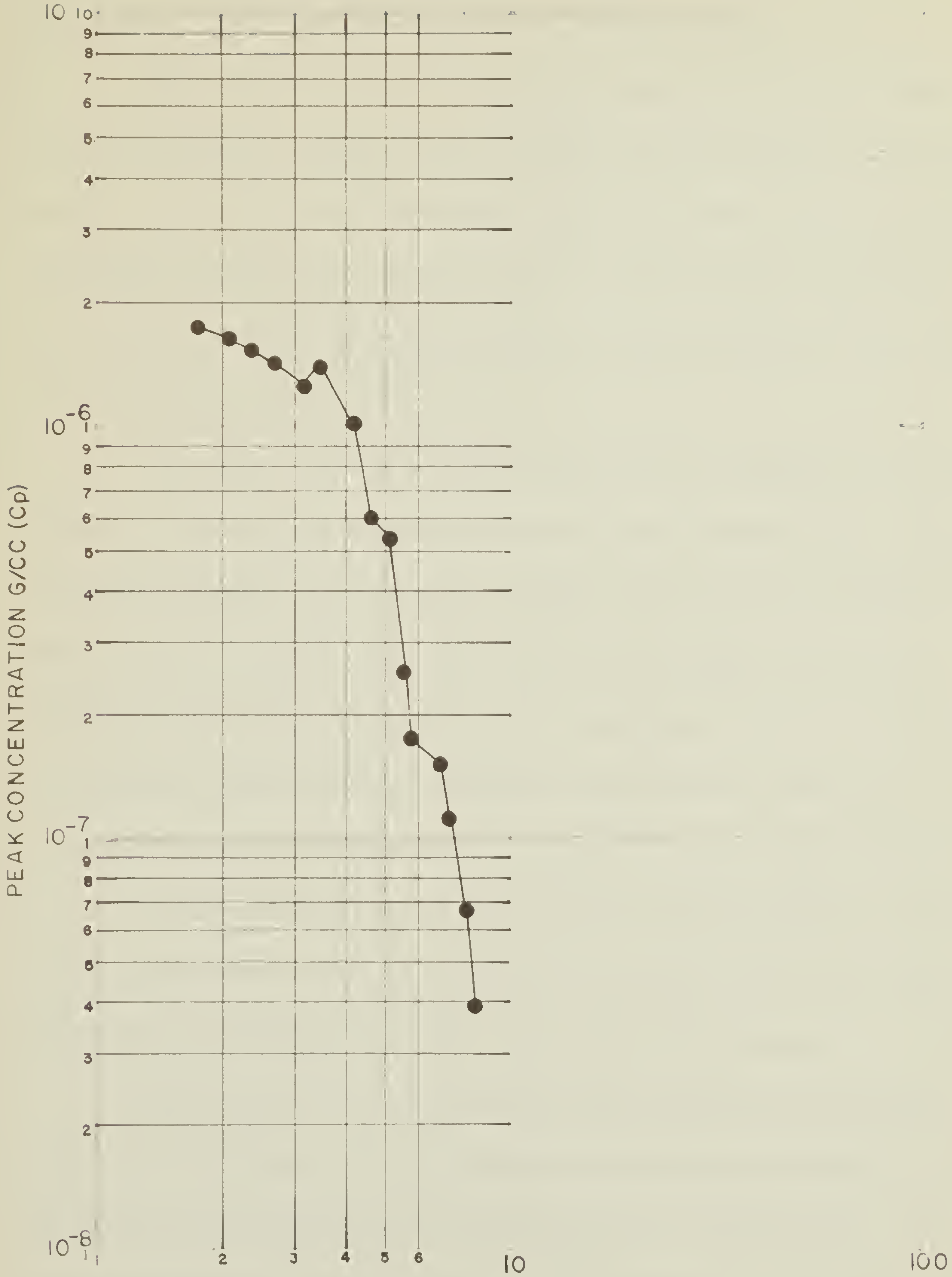
Figure 3 (BBS) indicates that the greatest observed rate of

decrease of maximum dye concentration was during the ebb period occurring from about 24.5 to 31 hours after time of introduction. The wind direction was parallel to the major axis of the tidal ellipse and positive (at 2 to 3 knots) in the direction of the ebb.

Figure 7 reproduced from an unpublished field report (1963) is a plot of C_{max} versus time for a dye dump made on the eastern bank a few miles to the southeast of line E-W (Fig. 6A) in November, 1963. The dye was dumped during the ebb current and sampled continuously for the next nine hours. During the first four hours, the dye moved in a westerly direction toward the edge of the bank, and then during the next five hours moved off to the east with the flood current. The wind was from the east (also positive in the direction of the ebb) at 8 knots. The sharp acceleration of the rate of decrease of peak dye concentration (at 3.5 hrs. after dump, see Figure 7) is coincident with the time of current reversal and probably due in part to increased mixing after the reversal of the current.

However, since the highest rate of decrease of dye concentration in the BBS experiment was observed when the wind and tidal current were moving in the same direction, while the highest rate of concentration decrease in the TOTO work was under conditions of winds moving in opposition to the current, something may be said concerning the relative importance of surface wind stress.

Bowden et al (1959) have defined the horizontal shear stress, F_{zx}, at some depth z, by combining the surface and bottom stresses, F_s and F_b, respectively, in the following expression:



CONCENTRATION (Cp) VERSUS TIME

FIGURE 7

$$F_{zx} = F_{zx}(1 + z/h) - F_{bx}\, z/h + \rho \int_{0}^{z} A_{x}\, dz \qquad (1)$$

where $z = 0$ at the surface and $z = -h$ at the bottom.

Since we will deal with average current and wind velocities over one-half of a tidal period, we shall assume that the acceleration term, A_x, will be zero and thus, F_{zx} is a linear function of depth z. The depth in both experimental areas was approximately 5 meters, and the sampling depth 2 meters, so that the shearing stress, F_{zx}, will be computed at $-z/h = 2/5$.

The bottom stress was computed from the equation, $F_b = k\rho u^2$, where u is the mean current above the lower boundary layer. Since we shall compute F_{bx} from the average velocity over one-half of a tidal cycle, we shall assume the velocity to be uniform from the surface to the bottom and will use the mean value $k = 3.5 \times 10^{-3}$ as reported by the authors. Similarly, the surface stress F_s is computed from the equation $F_s = k_a \rho a W^2$ where $k_a = 2.5 \times 10^{-3}$.

For the sake of convenience, we will examine only the ideal case, approached in both experimental situations, where the wind direction is parallel to the major axis of the tidal ellipse.

The average current during the BBS experiment was estimated at 0.25 knots. The greatest possible difference in the absolute value of the ebb and flood velocities is equal to the residual velocity, 0.03 kt., which, since positive to the north, will be added to the ebb velocity. The winds varied from calm, during the night and morning hours, to 3 to 5 knots during parts of

the day. The average wind over a tidal cycle would be from 2 to 3 knots. Computations of F_{zx} for BBS using the above values of wind and current are shown in Table 1.

TABLE 1

Values of F_{zx} (dynes/cm^2)

Wind Vel positive in dir of ebb	Ebb Vel u = 0.28 k	Flood Vel u = 0.25 k
2.0 k	0.280	0.257
2.5 k	0.270	0.267
3.0 k	0.256	0.281

The table indicates that at wind velocities equal to or less than 2 kts. the current velocity would be the dominant factor in generating shearing stresses. At velocities higher than 3 knots, the wind becomes dominant (assuming the current velocity does not change) and greatest shearing stresses would be expected when the wind and current are in opposition.

In the work done near the edge of TOTO, the currents were estimated at 0.5 knot during both the ebb and flood periods. Computations of F_{zx} with a 0.5 knot current, and an 8 knot wind positive in the direction of ebb, give $F_{zx} = 0.818$ dynes/cm^2 for the ebb and $F_{zx} = 1.428$ dynes/cm^2 for the flood, indicating that shearing stress during the time of maximum rate of decrease of dye concentration was roughly twice that of the alternate period.

If the residual drift of 0.07 kt. observed in the BBN experiment had been used in the computation, adding 0.035 kt. to the

ebb and subtracting 0.035 kt. from the flood, the resultant values of F_{zx} would be 0.792 dynes/cm^2 and 1.127 dynes/cm^2, respectively, also indicating greater shear when the currents and wind are opposed. In the latter case, the wind stress would become the dominant factor in producing directional shear stress at a wind velocity of about 6 knots.

SUMMER RESIDENCE TIMES:

Previous studies of exchange rates in shallow tidal areas have made extensive use of the tidal prism method, whether in original or modified form. The applicability of a tidal prism model to the Bahama Banks is limited because of the great areal extent of the banks and the natural deviation from estuarine geometry.

Modifications of the tidal prism method in estuarine areas commonly use the rate of supply of fresh water as a conservative property on which to base diffusion rates. During the summer months, the net flow in the area under consideration is in the direction of decreasing salinity. However, salinity decrease with distance or time is a poor conservative property for use in the above method because evaporation rates or rainfall dilution rates are not clearly defined for the area.

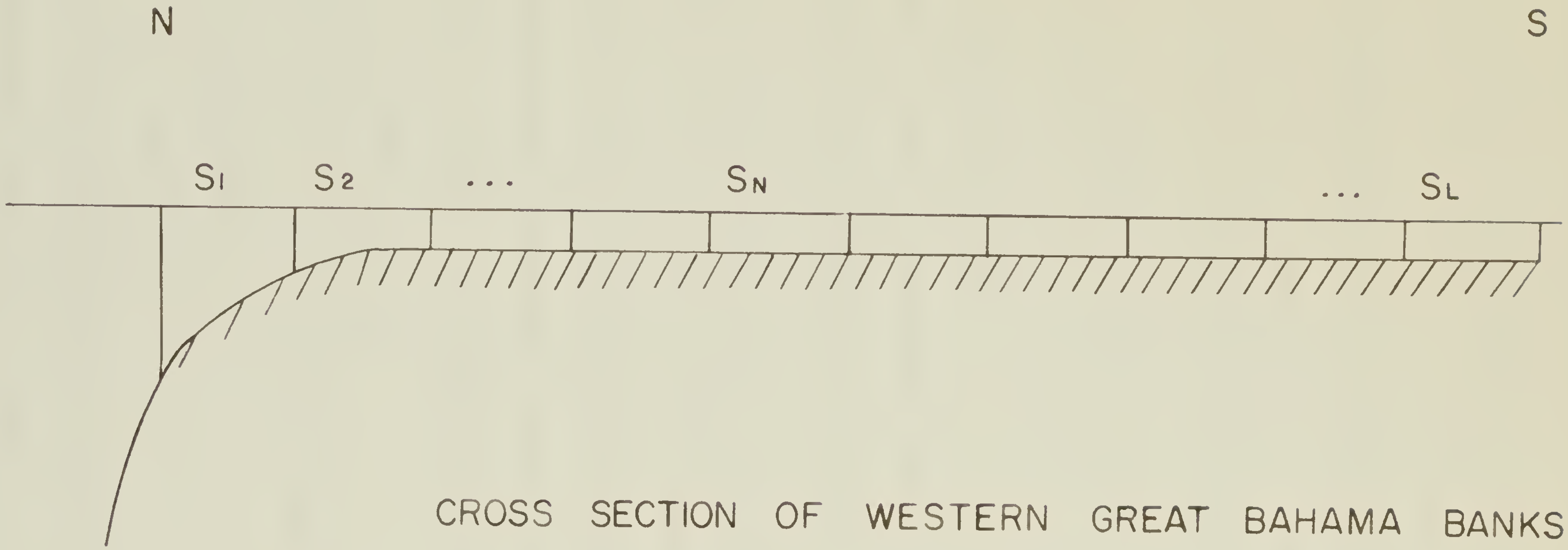
The tidal prism model also assumes complete mixing in segments which are defined by the average length of the excursion of a particle on the rising tide. Mean and high water volumes are used to calculate mixing coefficients. It is evident from the dye

studies that there is a decrease of tidal velocities with increasing distance from the northern edge of the banks. If the wave period remains the same and the current velocity decreases, it follows that the local tidal excursion also decreases.

The decrease in these tidal components is associated with the dissipation of wave energy by bottom friction. This phenomenon has been discussed by Sverdrup et al (1942) and other authors, and in general, it is assumed that the amount of energy dissipated is proportional to the kinetic energy of the wave. If this is so, the friction leads to a logarithmic decrease of the wave amplitude where the depth is constant. The current should be proportional to the amplitude and should also decrease logarithmically with increasing distance.

In the extreme case in which the incoming wave is completely destroyed by friction, the tide has the character of a progressive wave that is subject to damping. This latter case is similar to the situation on the banks, as Smith has observed that with increasing distance from the northern edge of the banks, the influence of N-S tidal components becomes less and less, and that at about 24°20' N the tide floods east by south.

In order to obtain a first estimate of the order of magnitude of summer residence times, we will assume that the banks area under consideration is represented in cross-sectional form by the illustration in Figure 8. We shall disregard lateral mixing and



CROSS SECTION OF WESTERN GREAT BAHAMA BANKS

FROM NORTH TO SOUTH

VERTICAL EXAGGERATION: 1:50000

FIGURE 8

will, therefore, concern ourselves with the motion of a particle in some segment, S_N, in a plane parallel to the line N-S. A coordinate system is superimposed, such that $x = 0$ at the deep water boundary of S_1 and x is positive to the right (to the south).

We shall assume that the entering tidal wave decreases as a logarithmic function of increasing x; from 1 meter (an average tidal fluctuation at the edge of the banks) at $x = 0$, to 0 at $x = L$. The point $x = 0$ coincides with the 100 fm. line (see figure 1), and $x = L$ at 24°20' N, or the point at which, according to Smith, the N-S tidal components vanish.

The smaller S_N becomes, the more precisely are we able to describe the motion within the segment. Since we are now interested only in describing the relative importance of the various components of the motion as well as establishing the order of magnitude of local residence time, we shall divide the distance from $x = 0$ to $x = L$ (approximately 90 miles) into ten equal-length segments such that any $S_N = 9$ miles. The BBN dye experiment was conducted within S_3 and BBS within S_6.

The number of tidal cycles (or days) it will take a particle to traverse S_N will equal

$$\tau_S = S_N / U_N \qquad (2)$$

where U_N is the average distance a particle travels during a tidal cycle. The time it takes for a particle to go from S_N to $x = 0$ will then be

$$\tau_N = \Sigma_N^0 \; \tau_S \qquad (3)$$

The velocity, U_N, depends on: (1) the average length of the local tidal excursion, E_N; (2) the proportionate distance of E_N a particle travels in a tidal cycle, or $E_N r_N$ where r_N is the exchange ratio; and (3) the distance the entire water mass moves as a result of residual wind drift, R_N.

The average length of E_N will be a logarithmic function of increasing x, where $E_1 = 6.2$ miles at $x = 0$, and approaches 0 at $x = L$. The lengths used in the computation are given below:

Average Length	S_1	2	3	4	5	6	7	8	9	10
of E_N in Miles	6.0	5.1	3.84	3.22	2.46	2.0	1.53	1.18	1.0	0.76

The maximum distance a particle can travel during a tidal cycle is E_N and the minimum is 0. As stated above, the average distance a particle travels in S_N due to tidal exchange will become a fraction, r_N of E_N. We shall use the relationship for r_N as previously described by Ketchum (1951) where

$$r_N = \frac{P_N}{P_N + V_N} \tag{4}$$

where V_N is the mean low water depth (5 meters) and P_N is the water available for mixing during a tidal cycle or the inter-tidal depth. P_N will, therefore, be the amplitude of the incoming tide wave, and will decrease as a log function of the distance x .

This relationship (4) is not strictly valid in S_1 and S_2 because of the sinking of bank water over the edge during the ebb current (see Experimental Results (2) and Figure 6). If we assume that the

bank water flowing over the edge is wholly replaced during a tidal cycle, the exchange ratio becomes simply P_N / V_N, where P_N is the amount of water replaced and V_N is the total amount of water within S_N. P_1 will be the area under the length of the first excursion in S_1, and V_1 the total area under S_1. The exchange ratio for S_1 then has the value $r_1 = 0.85$.

The same reasoning applied to S_2, also in contact with the open ocean over the lip of the bank, gives the value $r_2 = 0.45$. The remaining values of r_N computed from (4) are given below:

	S_1	2	3	4	5	6	7	8	9	10
r_N	0.85	0.45	.117	.098	.076	.062	.048	.040	.032	.025

We shall further assume steady wind conditions, and will use the value $R_N = 0.03k$ (positive in the direction of decreasing x) as observed in the BBS experiment. In a tabulation of daily wind directions at Nassau (Smith), 2/3 of the summer wind directions are southerly in nature, with the remaining 1/3 being north or northwesterly. For purpose of comparison then, we will also compute τ_N where $R_N = 2/3\ (0.03k) = 0.02k$.

The distance a particle travels in S_N during a tidal cycle will, therefore, equal

$$U_N = E_N r_N + R_N \tag{5}$$

where, as stated above, $E_N r_N$ is the motion due to tidal exchange, and R_N is the residual motion of the entire water mass due to wind drift.

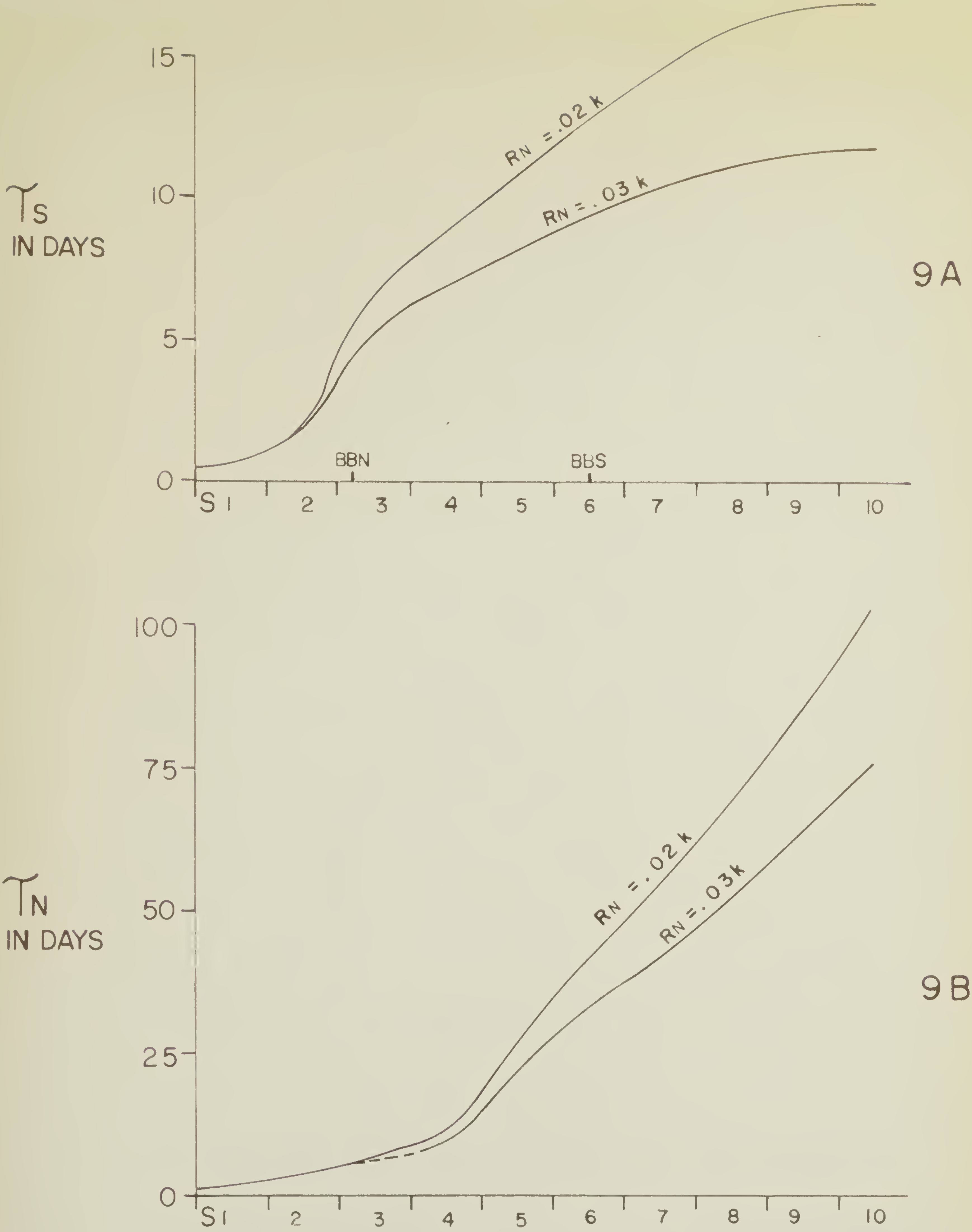
The computed values of τ_S and τ_N are illustrated in Figures 9A and B, respectively. Although the absolute value of any point on the curves is subject to error because of the greatly simplified model, we feel that the shape of the curves is significant.

Figure 10 illustrates the percentage of total motion due either to tidal exchange or the residual wind drift. At $x = L$, wind drift dominates the motion. As a particle moves toward the edge of the bank, wind drift becomes less and less important, until in S_6, tidal motion becomes dominant. Still nearer the edge in S_2, the increased exchange rate results in the sharp change in the slope of the curves.

The horizontal displacement of the point where the magnitude of the two components of motion (tidal exchange and wind drift) is equal is dependent on the wind force: Increasing winds will displace the point more and more to the left (as well as reducing the area under the curves).

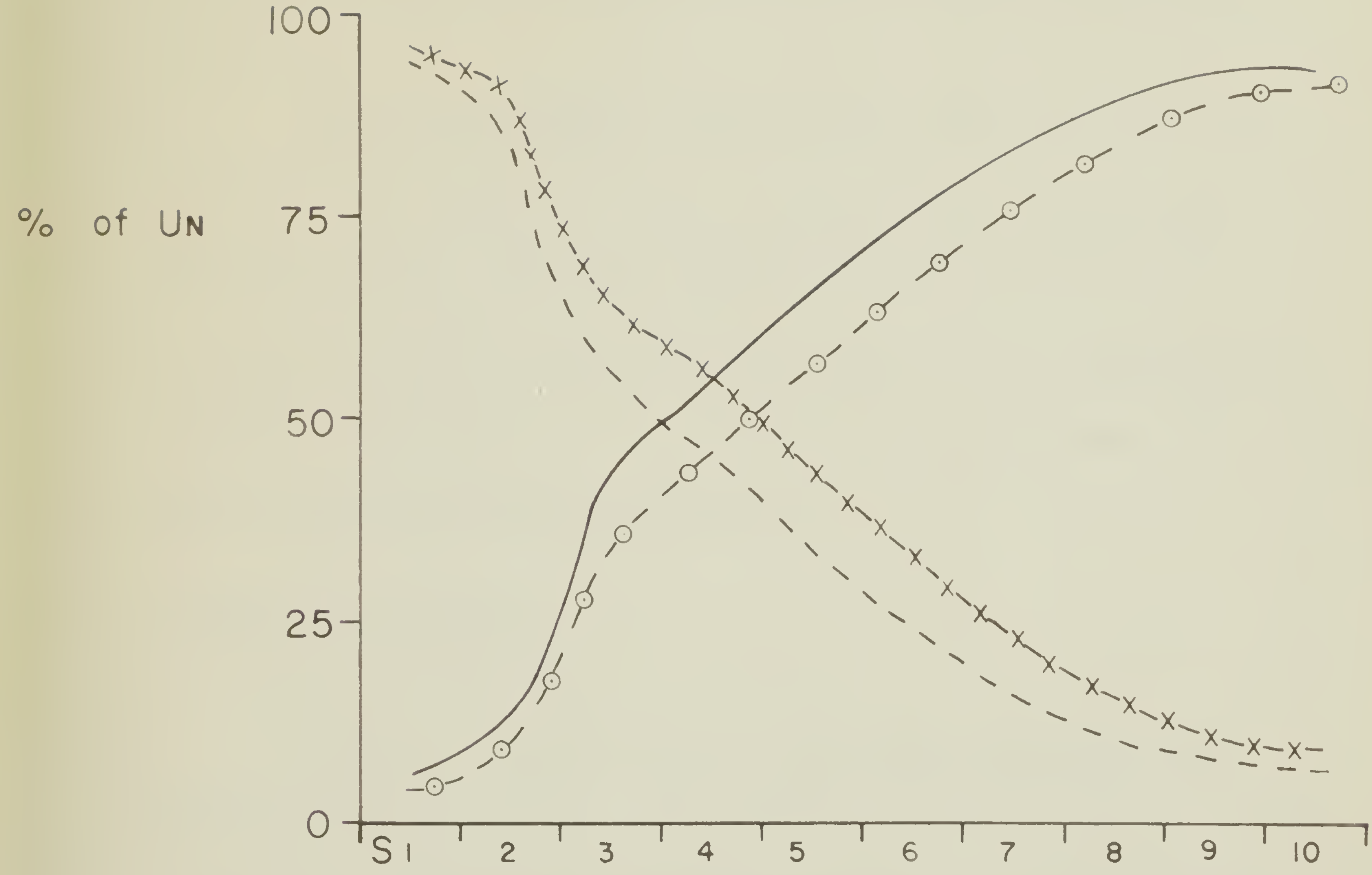
From the motions of the dye patches, we estimated that the patch at BBN would leave the banks in 6 to 8 days, and that of BBS in 40 to 50 days ± 20 to 25 days. The values of τ_N computed in the above model are in close agreement with values of 6 days for BBN and 32 to 40 days for BBS.

Since the estimated values of τ_N were made under assumed steady wind conditions in both cases, the error will increase with increasing distance from the edge of the bank in proportion to the



τ_S IN DAYS
15
10
5
0
$R_N = .02\,k$
$R_N = .03\,k$
BBN
BBS
S 1
2
3
4
5
6
7
8
9
10
9A
τ_N IN DAYS
100
75
50
25
0
$R_N = .02\,k$
$R_N = .03\,k$
S 1
2
3
4
5
6
7
8
9
10
9B

FIGURE 9



PER CENT OF U_N DUE TO

RESIDUAL VELOCITY, Rn, OR TIDAL EXCHANGE, T_E

AT $R_N = 0.03k$: AT $R_N = 0.02k$:

R_N/U_N = ——— R_N/U_N = —⊙—⊙—

T_E/U_N = - - - T_E/U_N = —x—x—

FIGURE 10

variability of the wind.

As more is learned about the actual effect of bottom drag on shallow water tide waves, this type of model will become more realistic. In those areas where residual wind drift dominates the motion, it is evident that a water mass may be shifted about for long periods of time without making much headway in any one direction.

An extended series of relationships which would take into account lateral mixing, as well as residual wind drift over the period of a year, would give approximations of absolute local residence times, the values of which would have the same physical meanings as the values of τ_C obtained as a result of the C^{14} study (see Introduction and Figure 5).

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