Endogenous Firm Competition and the Cyclicality of Markups

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Abstract

The cyclicality of markups is crucial to understanding the propagation of shocks and the comovement of macroeconomic variables. I show that the degree of inertia in the response of output to shocks is a fundamental determining factor for the cyclicality of markups in a broad class of models. In particular, markups follow a forward looking law of motion in which they depend on firms' conditional expectations over the net present value of all future *changes* in output. I test this law of motion with data for firms' expectations and find that, across different types of microfounded models of cyclical markups, the behavior of firms is most consistent with implicit collusion models. Calibrating an implicit collusion model to the U.S. data, I find that markups are procyclical when the model matches the observed inertial response of output to shocks, as commonly found in the data.

Keywords: Cyclicality of markups, Implicit collusion, Customer-base frictions, Output inertia.

JEL Classification: E3

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1 Introduction

The cyclicality of markups have been emphasized as a key transmission mechanism in the business cycle literature. Countercyclical markups, for instance, are broadly viewed as a propagation mechanism within New Keynesian models[1], as well as a potential reason for positive comovement between hours and wages[2]. Empirical evidence on the cyclicality of markups, however, has been limited and open to discussion due to the difficulty in measuring marginal costs. Moreover, theoretical work has focused on microfounding models of markups, but the two most common frameworks – implicit collusion and customer base models – yield different cyclicalities: implicit collusion models are interpreted as implying countercyclical markups, while customer base models have been used to generate both procyclical and countercyclical markups.

This paper contributes to the literature in three dimensions. First, I show that the cyclicality of markups in both classes of models depends not on the response of output but rather on the response of output *growth* to shocks. In particular, by making more realistic assumptions on propagation of shocks into output, the same models can generate markups that move in the opposite direction of what previously perceived. For instance, within the implicit collusion model, oligopolistic firms always choose the highest markup from which no one has an incentive to deviate. Variable markups arise because these incentives are changing over time, in particular due to stochastic discount rates and expected changes in demand of firms. The reason for the former is that collusion is inherently a dynamic consideration, and how much players value future profits plays a key role in determining cheating incentives. The latter matters because firms care about the relative size of these profits over time. Industries that expect a relative increase in demand in the future are able to sustain a higher collusive markup now because none of the firms want to cheat and be punished later by their competitors in periods with higher demand. On the contrary, in industries where firms expect a relative decrease in demand, each firm wants to seize the day and cheat while demand is at its highest. Consequently, the oligopoly is forced to act more competitively and charge a lower markup than usual to eliminate cheating incentives. What determines the response of markups is the expected *changes* in relative demand of firms, which, in a general equilibrium setting, is equal to the output growth.

In fact, the reason that implicit collusion models are believed to imply countercyclical markups is

[1]See e.g. Christiano et al. (2011).
[2]See e.g. Rotemberg and Woodford (1992).

that in classic business cycle models output is always expected to decline towards its steady state level during a boom. The negative expected output growth increases firms' cheating incentives and forces the oligopoly to settle on lower than usual markups. However, if the model is rich enough to allow for a hump-shaped response of output, which is a well-documented empirical property for most identified shocks in the literature[3], then markups are procyclical during the periods that output is rising, even if it happens during a boom.

Furthermore, the inertial response of output not only changes the contemporaneous correlation of output and markups, but also is crucial in capturing the cross-correlation of the two as documented in the empirical literature. Specifically, Nekarda and Ramey (2013) show that lags of markups are procyclical while leads of them are countercyclical. Calibrating an implicit collusion model to the U.S. data, I show that while the traditional implicit collusion model fails to capture this feature of the data, the model almost completely captures it when we allow for inertial response of output to shocks.

Another class of models that microfound variable markups are customer base models. These models rely on the assumption that there is "stickiness" in demand of firms, which introduces dynamic considerations in their price setting decisions as they consider how their current prices will affect their future demand. While different versions of these models have been shown to produce both procyclical and countercyclical markups[4], using a simple reduced-form model of external habit formation on the side of the final good producer that incorporates both versions, I show that, similar to the implicit collusion models, output growth determines the direction of the cyclicality within each version. For instance, as long as future habits are positively sensitive to current relative price of firms, firms that expect a higher demand in future have an incentive to build a larger market base for those periods by immediately charging lower prices. In the setting of classic business cycle models, this would translate to procyclical markups as it is only in recessions that output is expected to grow over time. Nevertheless, a humped-shaped response for output will change the cyclicality of markups in this setting as well.

A second contribution of this paper is to formally show that, up to a first order approximation, both of these models yield the same reduced form expression for the dynamics of markups. Specifically, current markups depend on the net present value of all expected growth rates of output in the future, discounted by a stochastic discount factor. The two models differ, however, in terms of the sign restrictions that

[3]See e.g. Ramey (2011); Ramey and Shapiro (1998); Monacelli and Perotti (2008) for government spending shocks, Sims (2011); Smets and Wouters (2007) for productivity shocks, and Christiano et al. (2005) for monetary policy shocks.

[4]See, e.g., Phelps and Sidney (1970); Paciello et al. (2014); Ravn et al. (2006).

they imply for this reduced form representation of markup dynamics. Hence, one can differentiate between these two models by estimating the common expression and testing the restrictions implied by each model.

My third contribution is to implement this empirical test. Because current markups depend on firms' expectations of future economic conditions, measuring these expectations is crucial to the endeavor. To do so, I rely on a recent survey of firms' expectations from New Zealand, introduced by Coibion et al. (2015). Consistent with the insights from the calibrated model, the results favor the implicit collusion model, and therefore given the high levels of inertia observed in output, point toward acyclical or procyclical markups.

Both implicit collusion and customer-base models are used within macroeconomic and industrial organization settings to study the markup setting behavior of firms. In my analysis, I start by building the firm side of the implicit collusion model, and show that markups are determined by the joint distribution of expected growth of output and stochastic discount rates, so partial equilibrium settings, as are used in the industrial organization models, are not sufficiently equipped to give the right implications about the cyclicality of markups. The reason is that within such environments one needs to make certain assumptions about the stochastic processes of demand and discount rates, and these assumption are unlikely to reproduce their joint determination as equilibrium objects. Moreover, the IO models largely abstract from cyclical discount rates, and focus only on the implications of demand shocks. For instance, Rotemberg and Saloner (1986) assume that demand shocks are i.i.d., implicitly implying that the expected demand growth is countercyclical, and conclude that markups are countercyclical. On the other hand, Kandori (1991); Haltiwanger and Harrington Jr (1991); Bagwell and Staiger (1997), each by assuming alternative processes for demand shocks, find that these models can produce procyclical markups.

Rotemberg and Woodford (1991, 1992) are the first to study the implicit collusion model within a DSGE model. Contrary to the IO models, their general equilibrium setting endogenously pins down the joint distribution of output growth and stochastic discount rates; however, their result is not robust to the structure of the shocks. In fact, as I will show in later sections, the countercyclicality of markups in these papers is not a general result of the microfoundations, and is reversed by introducing a humped-shaped response for output.

Phelps and Sidney (1970) is the first paper that formalizes the idea for customer-base models, and show that firms will charge a lower markup than the static monopoly one. Various papers have used this idea to study the cyclical behavior of markups. Different versions of customer-base models have been shown to create either procyclical or countercyclical markups. For instance, among the most recent ones, by micro-founding the game between firms and customers, Paciello et al. (2014) find that markups are procyclical, but Ravn et al. (2006) argue that they are countercyclical.

In this paper, I do not take a stance on the micro-foundations of this friction. Instead, using a simple customer-base model with an exogenous habit formation process on the side of customers, that is rich enough to produce both procyclical and countercyclical markups, I show that there is a direct relationship between cyclicality of markups in these models and their implications for the relationship of average markups with the static monopoly one[5]. Then, I show that within each version inertia in response of output reverses the cyclicality of markups.

Due to the difficulty of measuring marginal costs, there is a wide range of results on the cyclicality of markups in the empirical literature. For instance, Bils et al. (2012) find markups to be countercyclical, while Nekarda and Ramey (2013) argue the opposite. The theoretical observations in this paper support findings of Nekarda and Ramey (2013). A calibrated version of the implicit collusion model closely matches the cross-correlation of markups and output that they document.

Section 2 provides a simple example of the implicit collusion model to illustrate the forces at work. Section 3 introduces the implicit collusion model in general equilibrium. Section 4 presents the results for the calibrated DSGE model and shows that markups are pro-cyclical when we account for the inertia of output response to shocks, and examines the robustness of the model to parameters that are difficult to calibrate. Section 5 derives the law of motion for the customer-base model. Section 6 presents the results from the New Zealand survey data, and section 7 concludes.

2 A Simple Example

I start with a very simple example of the implicit collusion model to illustrate the forces that affect markups. This provides intuition to interpret the results of the full model in later sections.

[5]For example, in the setting of Paciello et al. (2014) introducing customer-base frictions reduces the average markups in the steady state. Within a reduced-form model of customer-base frictions, I show that, in absence (presence) of a humped-shape response of output to shocks , this is a necessary and sufficient condition for getting procyclical (countercyclical) markups.

2.1 Environment

Consider an oligopoly with two identical firms producing two differentiated goods. In this example, assume that marginal cost is constant over time and normalized to one. Also, assume that firms discount future profits with a constant discount factor β. Demand of firm $i \in \{1, 2\}$ is given by $y_i = AD(\mu_i; \mu_{-i})$, where A is a constant level of demand, μ_i is the markup of the firm i, and μ_{-i} is the markup of its competitor. Moreover, assume that function $D(.;.)$ is homogeneous of degree $-\sigma$. Firms perfectly observe the sequence of previous markups of their competitor and are engaged in an infinitely repeated game. Following the literature, I focus on the following strategies: there are two sub-games; one for the collusion and the other for the punishment. The game starts in the collusion sub-game, where firms jointly choose a markup and each independently decide whether they want to commit to collusion or best respond to their competitor's markup. In the next period, if both firms committed to collusion last period, the game stays in the collusion sub-game and if not the game moves to the punishment sub-game where firms play a single period static competition[6] and then move back to the collusion sub-game the period after. An extensive representation of this game is shown in Figure 1.

In the class of these strategies, I will consider the one where at every period firms choose the largest markup from which none of them has an incentive to deviate, as this strategy is the one that yields the highest stream of profits to the firms, given the punishment strategy. It will be shown in next subsequent that such a strategy is a sub-game perfect Nash equilibrium. Accordingly, the problem of the oligopoly can be written as[7]

$$\max_{\{\mu_i\}_{i\in\{1,2\}}} (A_1 + \beta A_2) \sum_{i\in\{1,2\}} (\mu_i - 1) D(\mu_i; \mu_{-i})$$

$$s.t. \quad (A_1 + \beta A_2)(\mu_i - 1) D(\mu_i; \mu_{-i}) \geq$$

$$\max_{\tilde{\mu}} \{(\tilde{\mu} - 1) A_1 D(\tilde{\mu}; \mu_{-i})\} + \beta(\mu^{BR} - 1) A_2 D(\mu^{BR}; \mu^{BR}) \quad , \forall i$$

[6]meaning that they play the Nash equilibrium of the static game. Notice that this is not the worst possible punishment in general. The reason to use it is that we assume there is a regulation structure that prevents these firms from actively engaging in explicit collusion strategies, so absent an implicit collusion agreement they revert back to the static equilibrium.

[7]This formulation incorporates a slight abuse of notation as it assumes that firms are choosing a single markup, assuming that they will charge that markup in both the current and the future period. Nevertheless, since A_1 and A_2 are known, as long as $\frac{A_2}{A_1}$ is constant over time, the problem is well-defined. In other words, this formulation already takes into account the result that only the growth rate of demand matters for the equilibrium markup; hence, I could revert to using a single growth parameter defined as $\frac{A_2}{A_1}$. Nevertheless, since the goal of this example is to illustrate the effects of current and future demand shocks, I keep the levels A_1 and A_2.

where μ^{BR} is the best response of a firm when they are playing the one period static game. The constraint is nothing more than an incentive compatibility one, that requires each firm to weakly prefer collusion to cheating. Therefore, along the equilibrium path, the game never enters the punishment stage as by construction firms always choose incentive compatible markups. Also, firms only care about their current and one period ahead profits because the game after next period is independent of today's actions. From now on, I am going to focus on the symmetric equilibrium in which $\mu_1 = \mu_2 = \mu$.

When deciding whether to commit to collusion or to cheat, a firm weighs the costs and benefits. The opportunity cost of collusion is the net profits that a firm loses by not best responding to other firm's action. In the setting of this example, given a collusion markup μ, this cost is represented by

$$C(\mu) \equiv A_1 \left[\max_{\tilde{\mu}} \{(\tilde{\mu} - 1)D(\tilde{\mu}; \mu)\} - (\mu - 1)D(\mu; \mu) \right]$$

which is a function of the chosen markup of the oligopoly. On the other hand the benefit from collusion is the present value of the net profits that the firm will gain by not being punished in future periods:

$$B(\mu) \equiv A_2 \beta[(\mu - 1)D(\mu; \mu) - (\mu^{BR} - 1)D(\mu^{BR}; \mu^{BR})]$$

As μ^{BR} is independent of firms' joint decisions in determining the markup, the problem of the oligopoly for the symmetric equilibrium can be rewritten as

$$\max_{\mu} B(\mu)$$
$$s.t. \quad B(\mu) \geq C(\mu)$$

where firms are maximizing their benefits from collusion subject to the constraint that those benefits should exceed the cost of collusion. If the constraint is not binding, firms are maximizing their joint benefit, acting as a monopoly. The homogeneity of the demand function then yields that if the constraint is not binding the optimal monopoly markup is just $\mu^{MON} = \frac{\sigma}{\sigma - 1}$. When the constraint is binding, where the collusion markup is given by $\mu = argmax\{x : B(x) \geq C(x)\}$. Therefore, in this case, the optimal markup is being implicitly defined by this incentive compatibility constraint. Figure 2a shows a depiction of the graphs of $B(\mu)$ and $C(\mu)$ for $\mu \in [\mu^{BR}, \mu^{MON}]$.

The concave curve depicts the gains of a single firm from committing to collusion as a function of the chosen markup of the oligopoly. This graph should be increasing at the best response markup point, because an oligopoly can earn more by increasing their markup at that point, but it should be flat at the monopoly markup because the best an oligopoly can do is to act like a monopoly. Hence, the gains from collusion should be concave in this region. On the other hand, the convex curve, depicts the gains of a single firm by best responding to the chosen markup of the oligopoly. This curve should be flat at the best response markup as the other firm is also best responding at that point; however, it should be increasing in monopoly markup because the higher the other firm's markup the higher the profit of a best responding competitor. Therefore, the cost of collusion should be convex in the chosen markup of the oligopoly.[8]

Another observation is that both curves should intersect at the best response markup, as both the cost and benefit of collusion are zero at that point. Hence, we have two curves, one convex and one concave, with at least one intersection at which they cannot be tangent. Therefore, there should be another intersection for these two curves. This second intersection should be at a higher markup than the best response one because of the shape of these curves, and if it falls below the monopoly markup, then it means that the incentive compatibility constraint is binding. In other words, either the monopoly markup is incentive compatible and firms are acting as a monopoly, or there exists a maximum markup within $[\mu^{BR}, \mu^{MON}]$ which is incentive compatible. Formally

$$\mu^* = \min\{\frac{\sigma}{\sigma-1}, \max\{x : B(x) \geq C(x)\}\}$$

From now on I will refer to this markup as the *collusion markup.*

2.2 Comparative Statics and Forces at Work

Rewriting the oligopoly's problem in terms of the benefit and the cost of collusion allows us to investigate the forces that affect firms' decision in an intuitive manner. The first and the most important observation is that while the cost of collusion is in terms of today's profits, the benefit of collusion is in terms of the present value of future profits. With this in mind, we can expect three forces to alter the collusion

[8]Formally, a sufficient condition for concavity of $B(\mu)$ is concavity of the aggregate profit with symmetric markup, $2(\mu-1)D(\mu;\mu)$, and a sufficient condition for convexity of $C(\mu)$ is $D_{22}(\tilde{\mu}(\mu);\mu) > 0$ where $\tilde{\mu}(\mu) = argmax_x\{(x-1)D(x;\mu)\}$.

markup by shifting one of these curves: an increase in current price of future profits (β), an increase in current demand, and an increase in future demand.

It is straight-forward to see that the current price of future profits only affects the benefit of collusion. These benefits are increasing in β as it determines how valuable future profits are for firms in the current period. Therefore, an increase in β will shift the benefit curve up, leaving the cost curve unchanged. Figure 2b depicts the effect of this increase on the collusion markup. The more patient firms are, the more they are able to sustain higher markups as incentive compatible agreements. In the full model, this force will affect markups through the stochastic discount factor of households who will own these firms.

Moreover, an increase in current demand, modeled as an increase in A_1, increases the cost of collusion and leaves the benefit of collusion unchanged. Accordingly each firm has a higher incentive to cheat as by committing to collusion they are now forgoing larger profits than before. Therefore, the oligopoly will have to settle on a lower markup to eliminate this higher cheating incentive. Figure 2c shows the effect of such an increase in the cost-benefit framework.

Finally, an increase in future demand, modeled as an increase in A_2, shifts the benefit curve up leaving the cost of collusion unchanged, as shown in Figure 2d. Knowing this, firms will now have lower incentives to cheat as they want to wait and collude in the future period when demand is going to high. This lower incentive to cheat, in turn, allows the oligopoly to sustain a higher collusion markup today.

Consequently, the relative increase in demand within periods, A_2/A_1, is crucial for determining the equilibrium markup. Even if current demand, A_1, goes up, which is the equivalent of a boom in this simple example, markups will also go up only if relative demand, A_2/A_1, goes up as well.

3 General Equilibrium

In this section, I embed the implicit collusion setup within a standard macro framework. While the general setup is similar to Rotemberg and Woodford (1991, 1992), I depart from their representation by deriving the law of motion for markups, and showing that markups depend on the expected *growth* of output. Then, I show that the cyclicality of markups is reversed once the model is calibrated to replicate the observed humped-shape response of output to shocks in the data, which I do by introducing investment adjustment costs

There is a final good of consumption in the economy which is produced using a large number of intermediate differentiated goods. There is a measure one of intermediate good sectors indexed by $i \in [0,1]$. In each sector, there are N identical firms producing differentiated goods, indexed by $ij, j \in \{1, \ldots, N\}$.

3.1 The Final Good Producer

The final good producer takes the price of consumption good, P_t, as given and produces with

$$Y_t = \left[\int_0^1 Y_{i,t}{}^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$$

where

$$Y_{i,t} = \Phi(Y_{i1}, \ldots, Y_{iN}) \equiv \left[N^{-\frac{1}{\eta}} \sum_{j=1}^{N} Y_{ij,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

Therefore, σ is the elasticity of substitution across sectors, and η is the elasticity of substitution within sectors. The profit maximization problem of this firm gives

$$\frac{P_{ij,t}}{P_t} = N^{-\frac{1}{\eta}} Y_{ij,t}^{-\frac{1}{\eta}} Y_{i,t}^{\frac{1}{\eta}-\frac{1}{\sigma}} Y_t^{\frac{1}{\sigma}}$$

which defines the inverse demand function of the firm ij. One can invert the set of these functions to get the demand function of each firm, given by

$$Y_{ij,t} = Y_t D(\frac{P_{ij,t}}{P_t}; \frac{P_{i-j,t}}{P_t}) \tag{1}$$

where

$$D(\frac{P_{ij,t}}{P_t}; \frac{P_{i-j,t}}{P_t}) \equiv \frac{1}{N} \left(\frac{P_{ij,t}}{P_t} \right)^{-\eta} \left[\frac{1}{N} \sum_{k=1}^{N} \left(\frac{P_{ik,t}}{P_t} \right)^{1-\eta} \right]^{\frac{\eta-\sigma}{1-\eta}}.$$

A general assumption in two layer CES models is that $\eta > \sigma > 1$. The economic intuition for this assumption is that goods within sectors are closer substitutes than goods across sectors, and it also implies that the demand of a single firm is increasing in the price of its competitors.

3.2 Intermediate Goods

Within sector i, there are N identical firms that use capital and labor to produce with a Cobb-Douglas production function, $Y_{ij,t} = Z_t^a K_{ij,t}^{\alpha} L_{ij,t}^{1-\alpha}$, where Z_t^a is an economy wide technology shock with an AR1 process:

$$\log(Z_t^a) = \rho_a \log(Z_{t-1}^a) + \sigma_a \varepsilon_{a,t}$$

$$\varepsilon_{a,t} \sim \mathcal{N}(0,1)$$

Also, intermediate firms know that their impact on total production is negligible and take Y_t and P_t as given. Moreover, I assume that there are competitive markets in place for renting labor and capital so that firms also take factor prices, W_t and R_t, as given.

3.2.1 The Repeated Game of Sector i

Let $Y_{i,t} \equiv \Phi(Y_{i1,t}, \ldots, Y_{iN,t})$ denote the output of sector i at time t. The N firms in sector i take the demand function of the final good producer, (1), as given, and play an infinitely repeated game. I assume that price (or alternatively, quantity) along with capital and labor demands are the only control variables of firms. The current price of future profits is a stochastic process, $\{Q_{0,t} : t \geq 0\}$, which firms take as given. In general equilibrium, these will be determined by the relative marginal utilities of households in different states.

As in every super-game, this repeated game has many potential equilibria. Although there is no rigorous way to rank the multiple equilibria of this game, the standard assumption in the literature is that given the structure of the game, firms choose the equilibrium that yields the highest possible profit stream. To this end, after fixing a punishment strategy for the firms, I will construct the best possible equilibrium in which firms always collude[9].

Following the literature on implicit collusion models, I assume that in every sector there exists a perfect monitoring system that detects any cheating with probability one. Therefore, the best cheating strategy for a firm would be to best respond to collusion outputs of their rivals, as they know that even

[9] Such an equilibrium is not necessarily the equilibrium with the highest net present value of profits; it may be the case that occasional deviations yield higher profits compared to staying in collusion forever. Therefore there might be an equilibrium with occasional collusion that dominates the best equilibrium in which firms always collude. I abstract from this case, following Rotemberg and Woodford (1991, 1992, 1999).

a slight deviation from collusion output will be noticed and the punishment strategy will be triggered[10].

3.2.2 Characterization of the Repeated Game Equilibrium

The equilibrium will be constructed as follows; before time 0 firms get together and layout a contingent plan for all possible states in the future. For every single state at every point of time, they assign a markup level for every firm such that it yields the highest profit for the sector and is incentive compatible with collusion relative to the following punishment strategy: in case a firm cheats from the agreement, the game will enter a punishment stage at which firms will charge the static best response markup forever after; however, at every period there is a possibility that the industry will renegotiate this with probability $1-\gamma$ and will move back to the collusion stage. This probability γ is in fact pinning down the expected punishment length such that after a firm cheats, the industry expects to remain in punishment stage for an average of $1/(1-\gamma)$ periods.

Therefore, firms within every sector maximize the discounted value of the industry's life time profits such that no firm in no state has an incentive to cheat. Note that incentive compatibility is the only restricting concern in this setting. Without it, firms would choose the monopoly markup for the industry at every state. However, a firm's incentive to cheat is at its highest level when the rest of the firms are committed to producing the monopoly output of the industry. This incentive declines as the markups of the other firms decrease towards the one in the static equilibrium. Also, notice that charging the best response markup at every state is trivially a feasible markup sequence for the industry in terms of incentive compatibility, and accordingly an equilibrium.

After choosing the markup sequence, firms then start the game at the collusion stage, denoted by C. Notice that at any time and any state firms would prefer to commit to collusion, so the game will stay in stage C forever. Furthermore, in the punishment stage, firms will play the static Nash equilibrium with probability γ at every period, and they will prefer to go back to collusion when industry renegotiates since collusion is at least as good as static best responding. Ergo, the proposed strategy is a sub-game perfect Nash equilibrium.

An alternative interpretation for this strategy, which justifies the name "implicit collusion", is that

[10] In the absence of such a system, however, static best responding may not be the best cheating strategy for a firm. If small deviations were unnoticeable with some probability, characterizing the best strategy is nontrivial. For instance, in an environment with imperfect monitoring, Green and Porter (1984) characterize equilibria in which firms switch to punishment when their price falls below a trigger price, even if it is caused by a negative demand shock rather than a cheating competitor.

there is an implicit agreement among firms according to which each firm chooses the highest markup from which no one in the industry has an incentive to deviate. However, if a firm observes that a competitor has deviated from this mutual understanding, the agreement breaks, and the oligopoly reverts back to static best responding.

With a CRS production function, firms' capital to labor ratios are independent of their output level. This can be interpreted as firms having a constant marginal cost of production in a given period that is pinned down by factor prices:

$$MC_t = \frac{1}{Z_t^a}\left(\frac{R_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$$

Given firms' price taking behavior for factor prices, we can treat them as only choosing their markup for every possible state, with input demands being determined automatically given the ratio induced by prices.

Since I only focus on symmetric equilibria, in characterizing the strategy of a firm, I assume that others are charging the same markup which is going to be the collusion markup in the equilibrium. Therefore, ij's profit from the action profile $\mu_i^t \equiv (\mu_{ij,t}; \mu_{it})$, where μ_{it} is the collusion markup chosen by the industry and μ_t is the average markup in the economy, is given by

$$\Pi_{ij,t}(\mu_{ij,t}; \mu_{i,t}) = P_t Y_t \left(\frac{\mu_{ij,t}}{\mu_{it}} - \frac{1}{\mu_{it}}\right)\mu_{i,t}^{1-\sigma}\mu_t^{\sigma-1} D\left(\frac{\mu_{ij,t}}{\mu_{i,t}}; 1\right)$$

The following Proposition formalizes the equilibrium.

Proposition 1. *Each firm in sector i, maximizes its net present value of future profits subject to no other firm having an incentive to undercut them:*

$$\max_{\{\mu_{i,t}\}_{t=0}^{\infty}} \frac{1}{N}\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta\gamma)^t Q_{0,t} Y_t \left(1 - \frac{1}{\mu_{it}}\right)\mu_{i,t}^{1-\sigma}\mu_t^{\sigma-1}$$

$$s.t. \quad \max_{\rho_{i,t}} \left\{\left(\rho_{i,t} - \frac{1}{\mu_{i,t}}\right) D(\rho_{i,t}; 1)\right\} - \frac{1}{N}\left(1 - \frac{1}{\mu_{it}}\right) \leq \beta\gamma \mathbb{E}_t Q_{t,t+1} \frac{Y_{t+1}}{Y_t}\left(\frac{\mu_{t+1}/\mu_{i,t+1}}{\mu_t/\mu_{i,t}}\right)^{\sigma-1}\Gamma_{it+1} \quad \forall t \qquad (2)$$

$$\Gamma_{it} \equiv \frac{1}{N}\left[\left(1 - \frac{1}{\mu_{it}}\right) - \mu_{COU}^{-\sigma}(\mu_{COU} - 1)\mu_t^{\sigma-1}\right] + \beta\gamma \mathbb{E}_t Q_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\frac{\mu_{t+1}/\mu_{i,t+1}}{\mu_t/\mu_{i,t}}\right)^{\sigma-1}\Gamma_{it+1}$$

where $\beta^{\tau} Q_{t,t+\tau}$ is the time t price of a claim that pays a unit of consumption at $t+\tau$, and $\mu_{COU} \equiv \frac{(N-1)\eta+\sigma}{(N-1)\eta+\sigma-N}$ is the equilibrium markup of static best responding for firms at any state. $\eta > \sigma$ guarantees that $\mu_{COU} < \mu_{MON} \equiv \frac{\sigma}{\sigma-1}$. The solution to this problem $\{\mu_{i,t}\}_{t=0}^{\infty}$ exists, and it is a Sub-game Perfect

Nash Equilibrium for the repeated game in sector i, in which firms always collude.

Note that (2) is the incentive compatibility constraint which requires that all firms in a sector prefer collusion to cheating in every possible state. Accordingly, such a sequence of assigned collusion markups are incentive compatible by construction and therefore form an equilibrium.

Now, suppose that the model is calibrated such that the constraint binds in the steady state, then for small perturbations around that steady state, a first order approximation yields

$$\hat{\mu}_t = \psi_1 \mathbb{E}_t \left[\Delta \hat{y}_{t+1} + \hat{q}_{t,t+1}\right] + \psi_2 \mathbb{E}_t \left[\hat{\mu}_{t+1}\right] \tag{3}$$

where hats denote percentage deviations from the steady state level, and $\Delta \hat{y}_{t+1} \equiv \hat{y}_{t+1} - \hat{y}_t$.

$$\psi_1 \equiv \gamma\beta \frac{\bar{\mu}\bar{\Gamma}}{D(\bar{\rho};1) - 1/N} \geq 0$$
$$\psi_2 \equiv \gamma\beta \frac{D(\bar{\rho};1) - (\sigma-1)(\mu_C - 1)(\frac{\bar{\mu}}{\mu_C})^{\sigma}/N}{D(\bar{\rho};1) - 1/N} \lesseqgtr 0$$

Equation 3 gives the law of motion for average markups in the partial equilibrium of the firm side. This is the key equation in this paper that will underlie all the results in later sections. Therefore, the following subsection is devoted to discussing this result.

3.2.3 Interpretation

Implicit collusion implies that markups are forward looking variables that depend on the expected change in demand in the next period, the changes in the price of future profits, and the expected change of markup in the next period. ψ_1, which is the coefficient on the first two, is a positive number that is increasing in steady state gains from collusion ($\gamma\beta\bar{\mu}\bar{\Gamma}$) and decreasing in the marginal revenue that a firm makes by cheating in the steady state ($D(\bar{\rho};1) - 1/N$). The intuition behind this equation is the key to understanding the main results of this paper. Two things between current period and the period ahead affect the current period's markup: first, the current price of next period's profit, which is the discount factor of the firms. The more patient the firms are in an industry, the higher their collusion markup will be today as they value future profits more. Second, the expected growth in demand from current period to next period. If firms expect that demand tomorrow will be higher than today, then they do not want to lose the chance of cheating tomorrow by cheating today. Basically, firms want to wait until demand is at its highest to take advantage of cheating, as in that case they will collect the highest cheating gains. This incentive to wait diminishes firms' cheating incentives in the current period, allowing the industry to sustain a higher collusion markup. Therefore, when firms expect output to grow, they will charge markups that are closer to the monopoly one.

ψ_2, however, can theoretically be positive or negative based on the calibration of the model. The reason is that there are two opposite forces that affect the firms' cheating incentives based on their expectation of the future markup. Before explaining these two forces, it is useful to recall that what ultimately determines the sign and the magnitude of the change in the markup is how hard it is to sustain the collusion markup, or in other words, how motivated firms are to cheat given a level of markup for the industry.

Suppose that firms expect that the markup next period will be higher than its steady state level. On one hand, they know that their industry is going to collude on a higher markup tomorrow, so they do not want to miss that chance by cheating today and pushing the industry to the punishment stage. On the other hand, since firms know that all other industries will also charge high markups, they expect to have a very high demand shift towards their industry from the final good producer, if industry as a whole charge the static best response markup. This second force gives an incentive to every single firm to push the industry to punishment stage by cheating today. Obviously the magnitude of this effect depends on how elastic the final good producer's demand for the industry is; as seen in the expression of ψ_2, when σ is close to 1, this force is negligible because the firms do not expect to get a large demand shift if the industry moves to the static Nash equilibrium.

The previous results in the IO literature can be seen as special cases of equation 3. For example, Rotemberg and Saloner (1986) follows a case where $\hat{q}_{t,t+1} = 0$ due to a constant discount rate, and $\mathbb{E}_t \left[\Delta \hat{y}_{t+1}\right] = -\hat{y}_t$ as shocks are assumed to be i.i.d. over time. Therefore, in their model

$$\hat{\mu}_t = -\psi_1 \hat{y}_t$$

which is a demonstration of their result that markups should be counter-cyclical. But as (3) implies, assuming other processes for these variables can give rise to different results. With two different random processes, $\Delta \hat{y}_{t+1}$ and $\hat{q}_{t,t+1}$, that are potentially correlated, the spectrum of possibilities for their underlying distribution is large enough to allow for *any* type of result in terms of the cyclicality of markups.

Therefore, we need to pin down this joint distribution, which in the case of this paper will be done by introducing a household side for the model.

Finally, ψ_1 and ψ_2 are completely pinned down by the firm side parameters σ, η, N, γ plus β which is going to be the subjective discount factor of the households in the general equilibrium.

3.3 Households, the Government and Market Clearing

There is a representative household that solves the following standard problem with investment adjustment costs.

$$\max_{\{(C_t, N_t, I_t, K_{t+1})\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\theta}}{1-\theta} - \phi \frac{L_t^{1+1/\epsilon}}{1+1/\epsilon} \right]$$

$$s.t. \quad P_t C_t + P_t I_t \leq W_t L_t + R_t K_t + \int_0^1 \sum_{j=1}^{N} \Pi_{ij,t} di - T_t$$

$$K_{t+1} = (1-\delta) K_t + (1 - S(\tfrac{I_t}{I_{t-1}})) I_t$$

$$S(\tfrac{I_t}{I_{t-1}}) \equiv \tfrac{a}{2}(1 - \tfrac{I_t}{I_{t-1}})^2$$

The investment adjustment cost is included to allow for inertia in the response of output to shocks. As I will discuss later the extent of this inertia will be crucial in determining the cyclicality of markups.

There is also a government that uses lump-sum taxes from households to conduct fiscal policy, G_t. I assume that G_t follows an AR(2) stochastic process

$$G_t = \bar{G} Z_t^g$$

$$\log(Z_t^g) = \rho_1^g \log(Z_{t-1}^g) + \rho_2^g \log(Z_{t-2}^g) + \sigma_g \varepsilon_{g,t}$$

$$\varepsilon_{g,t} \sim \mathcal{N}(0,1)$$

Again, the AR(2) assumption on government spending process is to allow for a humped-shape response of output to a fiscal policy shock.

Finally, the market clearing conditions are

$$C_t + I_t + G_t = Y_t$$

$$K_t = \int_0^1 \sum_{j=1}^N K_{ij,t} di$$

$$L_t = \int_0^1 \sum_{j=1}^N L_{ij,t} di$$

4 Calibration and Simulation

In this section, by simulating a log-linearized version of the model around a steady state in which the incentive compatibility constraint binds, I will show (1) why these models are typically interpreted as implying counter-cyclical markups and (2) that markups are actually pro-cyclical once the model is calibrated to generate realistic amounts of inertia in economic activity.

4.1 Parameters

I have set $\beta = 0.993$ to match the a steady state annual real interest rate of 3 percent, $\alpha = 0.35$ to match a steady state share of capital income of 35 percent, $\delta = 0.025$ to match a 10 percent annual rate of depreciation on capital, $\phi = 8$ to match a steady state labor supply of 0.3, $\bar{G} = 0.2$ to match a steady state G/Y of 20 percent, and $a = 2.48$ following Christiano et al. (2005). I also set the Frisch labor supply elasticity, ϵ, to 2.5.

Moreover, I have set the elasticity of substitution across sectoral goods, σ, equal to 4, and the elasticity of substitution within sectoral goods,η, equal to 20. I set $\gamma = 0.8$[11] and $N = 15$ to match a steady state markup level of 20 percent. Although these are calibrated in an arbitrary fashion, as I will show later in a series of robustness checks, for the given levels of σ and η the model is not very sensitive to these parameters, and reasonable variations in them do not affect the main results of my analysis. The qualitative results in terms of direction of cyclicality of markups are robust to any calibration as long as $\eta > \sigma$.

Finally, I have set the persistence of the technology shock to 0.95. For the persistence parameters of the government spending shock, I run the following regression on the quarterly data for real government

[11] Given σ and η, this is the highest level for γ for which the incentive compatibility constraint binds, and the Blanchard Kahn condition for the law of motion for markups holds.

consumption expenditures and gross investment from 1947Q1 to 2014Q1:

$$\log(G_t) = Constant + \rho_1^g \log(G_{t-1}) + \rho_2^g \log(G_{t-2}) + \varepsilon_t$$

which gives the estimates $\rho_1^g = 1.51$ and $\rho_2^g = -0.52$. I will also consider alternative persistence parameters for robustness checks in Section 4.4.

4.2 Impulse Response Functions

First, consider the case of no investment adjustment cost ($a = 0$). The dashed curves in Figure 3a show the impulse responses of this model to a 1 percent technology shock. The key observation is that in this setting, output jumps up on impact and converges back to zero as the effect of the transitory shock fades away. Moreover, the response of stochastic discount rate, which is given by $Q_{t,t+1} = \beta \frac{u'(C_{t+1})}{u'(C_t)}$, is countercyclical given that households are able to smooth their consumption without being restricted by costly investment. The fact that consumption has an inertial response to the technology shock is a crucial element to the countercyclicality of stochastic discount rates. On impact, households expect that their consumption will peak later in the expansion; therefore, they are not really concerned about future states as they know they will have a higher consumption.

By equation 3, the combination of counter-cyclical output growth and discount rates gives rise to counter-cyclical markups. The interpretation from the firm side is that on impact, firms know that demand is at its highest. This expectation along with the low price of future profits increases their cheating incentives to its highest, as they know that now is the best time to cheat and steal the market share of their rivals. The oligopoly, knowing this, is forced to settle on a low markup to eliminate this high cheating incentive.

A similar exercise can be done with the government spending shock. Suppose that Z_t^g is an AR(1) process with persistence 0.95. The impulse response functions of the model to such a shock is illustrated by the dashed curves in Figure 3b. On impact, government spending is at its highest, which means that private consumption is at its lowest. First, since private consumption will increase to its steady state level, such a shock would give rise to counter-cyclical stochastic discount rates. Moreover, the income effect of G is at its highest on impact, so that Y will peak immediately due to a jump in labor supply and converge back to its steady state as the shock fades away. Again, the combination of counter-cyclical discount rates and output growth will translate into counter-cyclical markups.

However, empirical evidence on TFP shocks and government spending shocks suggests that the response of output to these shocks is inertial such that the peak effect happens not on impact but in later periods[12]. To allow for such inertia, I introduce investment adjustment costs and an AR(2) process for government spending. Solid curves in Figure 3a show the IRFs of the model to a 1% technology shock when $a = 2.48$. With positive adjustment costs, two things happen: first, investment does not jump on impact and has an inertial response, which translates to an inertial response in output, and second, households now face a stronger trade-off in smoothing their consumption because they face costly investment, which gives rise to pro-cyclical stochastic discount rates. Therefore, on impact firms expect their demand to increase in future periods, which gives them the incentive to avoid cheating until demand peaks as they do not want to force the oligopoly to the punishment stage of the game when demand is at its highest. This lower incentive to cheat allows the oligopoly to settle on higher incentive compatible markups. Hence, on impact one would expect a higher markup than the one in the steady state, making markups pro-cyclical.

A similar exercise can be done with the government spending shock by assuming an AR(2) process for Z_t^g. Figure 3b depicts the IRFs of the model to such a shock. The inertial implementation of the fiscal policy translates to an inertial output and consumption responses, as shown by solid curves in Figure 3b, which in turn produce procyclical markups for similar reasons to the case of the technology shock with $a > 0$.

4.3 Cross Correlation of Markups and Output

Another set of results that are emphasized in the literature is the cross correlation of markups and output over time. For instance, Nekarda and Ramey (2013) empirically document that in addition to the positive contemporaneous correlation of the two, lags of markups are pro-cyclical with output while the leads of it are counter-cyclical, as illustrated by the dashed curve with no markers in Figure 4. The goal of this section is to show that the model without inertia completely fails to match this evidence; however, the model with inertia is highly consistent with it.

[12] For empirical evidence on humped-shape response of output to productivity and government spending shocks, see, for example, Sims (2011); Smets and Wouters (2007); Ramey (2011); Ramey and Shapiro (1998); Christiano et al. (2005); Monacelli and Perotti (2008).

The dashed curve with circle markers in Figure 4 shows the simulated correlation of lags and leads of markups with output conditional on a TFP shock when there is no inertia in the response of output; and the solid curve in the same figure depicts the same graph for the model with inertia. While the model without inertia misses the direction of cyclicality for the most part, the model with inertia in the output response matches the empirical evidence closely.[13]

4.4 Robustness

In this section, I check the robustness of the predictions of the model with respect to different variables.

Figures 5a and 5c show the simulated correlations of leads and lags of markups with GDP conditional on a technology shock and a government spending shock respectively, for values of γ between 0.4 and 0.8, such that darker curves correspond to higher levels of γ. Aside from the fact that lower γ's create lower steady state markups because of the higher impatience of firms, they also produce lower correlations between GDP and markups. The reason for the latter is that variations in current markup are a weighted sum of all expected output changes and stochastic discount rate changes in the future, and as γ gets smaller, they put lower weights on future values. Nevertheless, all values of γ yield the same structure of correlations of lags and leads of the markup with the output.

Moreover, Figures 5b and 5d, respectively, show the correlation of leads and lags of markups with GDP conditional on a technology shock and government spending shock for values of N between 5 and 25. Again darker curves represent higher values of N. Variation in number of competitors does not change the structure of correlations and has very small level effects. The reason is that what ultimately determines the cheating incentives of firms, and hence markups, is the elasticity of demand for a single firm which is equal to $\eta - \frac{\eta-\sigma}{(N-1)\rho^{\eta-1}+1} \in [\sigma, \eta]$. Note that for small amounts of η ($\eta \leq 20$), which corresponds to a relatively high differentiation among within industry goods, the effect of N on the structure and level of correlations is negligible.

In the model, investment adjustment costs are the mechanism generating the humped-shape response of output to technology shocks. While I have calibrated this parameter to the estimated value of Christiano et al. (2005), this section examines the question of how large this parameter needs to be for

[13]The same exercise can be done with government spending shocks, and while the results qualitatively remain the same – meaning that correlations in the inertial model are larger than in the model with no inertia in the output response – the inertia created by the AR(2) process is not enough to make the conditional correlation positive.

markups to be procyclical. Figure 6a depicts the number of periods that markups are procyclical after a 1% technology shock given different values of $a \in [0, 5]$. As soon as a is larger than 0, markups are procyclical on impact. Also, the duration of procyclicality increases as a gets larger. For my calibration of this parameter, markups are pro-cyclical for 5 quarters after the shock hits the economy.

Moreover, Figure 6b shows the contemporaneous correlation of the markup with GDP conditional on a 1% technology shock, which shows that the conditional correlation is increasing in a, and for $a > 1.2$, it is positive. Hence, any empirically reasonable value of investment adjustment costs will generate procyclical markup in this model.

In the baseline calibration with inertia, I use estimated parameters for the AR(2) process of government spending to create the humped-shape response of the output to a G shock. Now, I consider a wider range of persistence parameters to check for robustness of results in previous section. Consider the set $\{(\rho_1^g, \rho_2^g) | \rho_2^g \in [-0.7, 0], \rho_1^g + \rho_2^g = \rho_G\}$, where ρ_G is the persistence of government spending shocks, fixed to an estimated value of 0.98. Therefore, this set defines a locus for persistence parameters of G such that when $\rho_2^g = 0$ the process is AR(1) and when $\rho_2^g < 0$ the process is AR(2) with highest inertia achieved when $\rho_2^g = -0.7$. In fact, the magnitude of this parameter, $|\rho_2^g|$, determines the degree of inertia in the response of output. Figure 6c shows the number of periods that markups are procyclical after a 1% government spending shock given different values of $|\rho_2^g|$. Again, for the most part ($|\rho_2^g| > 0.1$), the inertia causes the markups to be procyclical on impact. For my estimate of persistence parameters, markups are procyclical for 2 periods after the impact. However, as Figure 6d shows, the inertia is not enough to make the conditional correlation of markup and GDP positive. Nevertheless, the correlation is still increasing with inertia.

5 Customer Base Models

Another class of models that micro-found variable markups is based on the notion that it is costly for customers to switch among firms. Accordingly, firms' pricing decisions in the current period affect their market share in future periods. In contrast to implicit collusion models, customer base models have been shown to imply both procyclical and countercyclical markups.

The method through which customer-base frictions are modeled in the literature is by a habit formation process on the side of customers. This habit component makes it costly for customers to switch to other products, and hence creates dynamic considerations for firms as their future market share changes with their current decisions. The dynamics of markups, however, depend on how customers are reacting to pricing of the firms over time. Hence, the habit formation process is crucial for their dynamics.

In this section I build a simple reduced form customer-base model, with a rich enough external habit-formation process that could support both procyclical and countercyclical markups, and then show that introducing a humped-shape response for output changes the cyclicality of markups within each version. The reason is identical to the implicit collusion model; regardless of how the habit formation process works, the dynamic considerations that it creates for the firms are not in terms of the level of demand but its relative changes over time. Hence, similar to the implicit collusion model, since a humped-shape response of output changes the expectations of firms about changes in future demand over the business cycle, it also changes the cyclicality of markups.

5.1 Model Specification

Consider the final good producer of Section 3. To incorporate the customer base model, I assume that this final good producer has an external habit formation over the goods within industries, meaning that

$$\begin{aligned} Y_t &= \left[\int_0^1 Y_{it}^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} \qquad (4) \\ Y_{it} &\equiv \Phi\left(\frac{Y_{i1,t}}{S_{i1,t}}, \ldots, \frac{Y_{iN,t}}{S_{iN,t}}\right) \\ \Phi(x_1, \ldots, x_N) &= \left[N^{-\frac{1}{\eta}} \sum_{j=1}^N x_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

Where $S_{ij,t}$ is the external habit of producer in using the input $Y_{ij,t}$, which is taken as given by the final good producer at time t. I assume that $S_{ij,t}$ has the following general law of evolution

$$S_{ij,t} = h\left(\frac{\mu_{ij,t}}{\mu_{it}}\right)(\gamma S_{ij,t-1} + 1 - \gamma)$$

where $h(.)$ is differentiable, $h(1) = 1$, and $\gamma \in [0, 1)$. Therefore, the problem of the final good producer is

$$\max_{Y_{ij,t}} P_t Y_t - \int_0^1 \sum_{j=1}^N P_{ij,t} Y_{ij,t} di$$

which implies that the demand for $Y_{ij,t}$ is then given by

$$Y_{ij,t} = Y_t D(P_{ij,t} S_{ij,t}; P_{i-jt} S_{i-j,t})$$

where $D(.;.)$ is defined exactly as in section 3. Firm ij takes demand as given and maximizes the net present value of all its future profits by choosing a relative markup $\frac{\mu_{ij,t}}{\mu_{it}}$, and $S_{ij,t}$, where S_{it} is the the final good producer's habit for the others in the sector, in the symmetric equilibrium. Therefore, firm ij's dynamic problem is

$$\begin{aligned} & \max_{\{\mu_{ij,t}, S_{ij,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} Q_{0,t} Y_t P_t^{-\sigma} \left(\frac{\mu_{it}}{\mu_t}\right)^{1-\sigma} \left(\frac{\mu_{ij,t}-1}{\mu_{it}}\right) D\left(\frac{\mu_{ij,t}}{\mu_{it}} \frac{S_{ij,t}}{S_{it}}; 1\right) \\ s.t. \quad & S_{ij,t} = h\left(\frac{\mu_{ij,t}}{\mu_{it}}\right)(\gamma S_{ij,t-1} + 1 - \gamma) \end{aligned}$$

Proposition 2. *In a symmetric equilibrium where all firms identically solve the problem above,*

1. *The law of motion for markups, up to a first order approximation, takes the same form as the implicit collusion model, i.e.*

$$\hat{\mu}_t = \psi_1 E_t\{\hat{q}_{t,t+1} + \Delta \hat{y}_{t+1}\} + \psi_2 E_t\{\hat{\mu}_{t+1}\}$$

where $\psi_1 \equiv \beta\gamma \frac{\mu\mu_C^{-1} - 1}{1 + h'(1)} \lesseqgtr 0$, $\psi_2 \equiv \frac{\beta\gamma}{1+h'(1)} > 0$, $\mu_C \equiv \frac{(N-1)\eta + \sigma}{(N-1)\eta + \sigma - N} > 1$ *is the static best response markup in absence of the customer base friction, and* μ *is the steady state markup in presence of the friction.*

2. *Markups move in the opposite direction of firms' expectations of output growth, meaning that* $\psi_1 < 0$*, if and only if the existence of customer base frictions reduce the average markup, i.e.* $\mu < \mu_C$.

3. *Existence of customer base frictions reduce the average markup if and only if* $h'(1) > 0$.

Proof. See Appendix. □

The first part of the proposition formalizes the idea that the law of motion implied by this model takes the same form as that of the implicit collusion model; however, the implications of the model on signs of ψ's are different. While ψ_2 is unambiguously positive, the second part of the proposition relates the sign of ψ_1 to the nature of the customer base friction. If the customer base friction is such that it reduces the average markups compared to the frictionless economy, then markups should move in the opposite direction of expected output growth, and vice versa. In the context of models without inertia, the former would translate into procyclical markups. The latter, which predicts that markups should move in the same direction as the expected output growth if the average markup is higher than the frictionless one, is also consistent with the other side of the literature which predicts that markups are countercyclical. The third part gives a necessary and sufficient condition between the shape of the function $h(.)$ and the relationship between the average and frictionless markups.

Although they have opposing predictions, both versions of the model follow the same intuition. The reason that in this model markups are variable over time is that by altering their relative markup, and through the law of motion for habit, firms can change their future market shares. The fact that the law of motion depends on the net present value of all output growths in the future, implies that firms want to build high market share for periods in which they expect the output to be *higher*. However, how this incentive affects the markups, which is determined by the sign of ψ_1, depends on how potentially opposing forces aggregate together: there are two forces through which the relative markup affects firms' market shares. The first one is the direct relationship between price and demand; that a higher relative markup reduces current demand for firms. The second one affects market share through law of the motion for the habit: higher relative markup can either increase or decrease the habit of the final good producer for a particular firm[14]. While the first one always goes in one direction, the second force can work in either direction, causing a different law of motion for markups. For instance, if decreasing the relative markup increases the habit of the final good producer[15], then markups should move in the opposite direction of the firms' expectations on output growth.

More importantly, even if we fix the sign of ψ_1, similar to the implicit collusion model, a humped-shaped response for the output would change the cyclicality of the markups. Figures 7a and 7b, shows

[14]By assuming a reduced form law of motion for final producer's habit, I abstract away from microfounding the slope of the $h(.)$ function which determines the sign of ψ_1. Nevertheless, the equivalence between slope of $h(.)$ and the relationship between average and frictionless markup, derived in Proposition 2, offers some intuition. The nature of the friction in terms of whether it decreases or increases the average markups is sufficient for identifying the sign of ψ_1. For instance, Paciello et al. (2014) mention that in their version of the customer base model, the friction reduces the average markups and they find that markups are procyclical, which implies $\psi_1 < 0$.

[15]It is important to distinguish between how relative markup affects the habit of the final good producer, and how it affects the market share. The market share is determined by the two forces combined, and the assumption that a higher markup increases the habit, does not imply that the market share is also increasing in relative markup.

the impulse responses of a version of the model with $\psi_1 < 0$[16], to a 1% technology shock and government spending shock with and without inertial response for the output. Humped-shaped response of output changes the cyclicality of markups, even with a fixed ψ_1.

6 Testing the Law of Motion for Markups

The two models considered in the previous sections both imply a law of motion for markups of the form

$$\hat{\mu}_t = \psi_1 \mathbb{E}_t\{\Delta \hat{y}_{t+1} + \hat{q}_{t,t+1}\} + \psi_2 \mathbb{E}_t\{\hat{\mu}_{t+1}\}$$

where the implication of each model for the signs of the coefficients are different. The following table summarizes these implications:

	Implicit Collusion	Customer Base
ψ_1	> 0	$\lesseqgtr 0$
ψ_2	$\lesseqgtr 0$	> 0

Table 1: Sign of Coefficients in the Law of Motion for Markups in Different Models

The goal of this section is to, first, empirically test this law of motion, and second, determine which model is more suited to explain the data based on the sign implications above. Because the question inherently relates to firms' expectations of future outcomes of the economy, I use a quantitative survey of firms' expectations from New Zealand introduced by Coibion et al. (2015). Hence, if one could estimate the above parameters, rejecting that $\psi_1 > 0$ would reject the implicit collusion model; while rejecting the hypothesis that $\psi_2 > 0$ would imply the rejection of the customer base model.

6.1 Identification

The survey includes data that allows me to directly test the law of motion for markups. Firms were asked to provide information about their number of competitors, average markup, current markup, their expected growth in sales, and their next expected price change. The following Proposition states that this cross-sectional data are enough to partially identify the coefficients in the law of motion up to the elasticity of substitution across industry goods.

[16]The calibration is such that the frictionless markup, μ_C, is 11.5%, and the average markup, μ, is 10%. In terms of parameters, the only change compared to the previous section is $\eta = 10$.

Proposition 3. *Let industries be indexed by i and firms within them be indexed by j. Consider the following regression*

$$\begin{aligned}\hat{\mu}_{ij} - \sum_i \sum_j \hat{\mu}_{ij} &= Industry\_FE_i + \beta_1 \{Ex\Delta Sales_{ij} - \sum_i \sum_j Ex\Delta Sales_{ij}\} \\ &+ \beta_2 \{Ex\Delta Price_{ij} - \sum_i \sum_j Ex\Delta Price_{ij}\} + \varepsilon_{ij}\end{aligned}$$

where $\hat{\mu}_{ij}$ is the deviation of current markup of firm ij from its average level, $Ex\Delta Sales_{ij}$ is the expected growth in sales for firm ij, and $Ex\Delta Price_{ij}$ is its next expected price change. Now consider the following decomposition of firms' errors in expecting stochastic discount rates and changes in marginal costs:

$$\mathbb{E}_t^{ij}\{\hat{q}_{t,t+1}\} - \sum_i \sum_j \mathbb{E}_t^{ij}\{\hat{q}_{t,t+1}\} = u_{1,t}^i + u_{2,t}^{ij}$$
$$\mathbb{E}_t^{ij}\{\Delta \hat{mc}_{t+1}\} - \sum_i \sum_j \mathbb{E}_t^{ij}\{\Delta \hat{mc}_{t+1}\} = v_{1,t}^i + v_{2,t}^{ij}$$

where $u_{1,t}^i$ and $v_{1,t}^i$ are industry specific errors that are orthogonal to the firm level errors $v_{2,t}^{ij}$ and $u_{2,t}^{ij}$. Assuming that $v_{2,t}^{ij}$ and $u_{2,t}^{ij}$ are independent across firms and are orthogonal to the other terms in the above regression, ψ_1 and ψ_2 can be identified from β_1 and β_2 up to the elasticity of substitution across sectors, σ.

Proof. See Appendix. □

6.2 Results

A distinct characteristic of implicit collusion and customer base models is that they are derived for oligopolistic firms. Hence, one would expect that in the data, while oligopolistic firms should conform to the above law of motion, competitive ones should not. With that in mind, to test the validity of the the law of motion, I divide the sample into two sub-samples, firms with more than 20 competitors, i.e. competitive firms, and firms with fewer than 20 competitors[17]. The prediction is that coefficients of the law of motion, identified through the regression in Proposition 3, should be significant for the second group, and not significant for the first. Table 2 shows the results of these two regressions, and this hypothesis cannot be rejected at a 95% confidence level.

[17] I also drop firms with less than 2 competitors considering the possibility of a non-binding incentive compatibility constraint for these firms.

Moreover, as the coefficients are significant for the oligopolistic firms, we can use the results of Proposition 3 to infer the implied coefficients on the law of motion for markups, and hence identify the model that is more consistent with the data. Specifically, rejecting the null hypothesis that $\psi_2 > 0$ would lead to rejection of the customer base model, and similarly rejecting the null that $\psi_1 > 0$ would imply the rejection of the implicit collusion model. The following Proposition shows that while the first hypothesis is rejected by the data, the latter is not.

Proposition 4. *Given the results in Table 2, and imposing the theoretical bound $|\psi_2| < 1$[18], the 0.95% confidence intervals for ψ_1 and ψ_2 are*

$$\psi_2 \in (-1, -0.06]$$

$$\psi_1 \in [0.003, 0.64]$$

Therefore, we can reject the null that $\psi_2 > 0$, which implies the rejection of the customer base models. On the other hand, the null hypothesis that $\psi_1 > 0$ (or $\psi_1 = 0$) cannot be rejected.

Proof. See Appendix. □

Therefore, these results imply a rejection of the customer base models but are consistent with the implicit collusion models. Given that this class of models points towards procyclical markups for any empirically realistic level of inertia in output, these results suggest that countercyclical markups are unlikely to be a key propagation mechanism of business cycles.

	$2 < N \leq 20$	$N > 20$
Expected growth in sales (demeaned)	**0.16**** (0.08)	-0.03 (0.11)
Expected size of next price change (demeaned)	**-0.18**** (0.06)	0.02 (0.09)
Observations	495	200

Table 2: The table reports the coefficients for the regression specified in Proposition 3, allowing for industry fixed effects. The first column reports the coefficients for firms that report less than 20 but more than 2 competitors. Second column reports the coefficients for firms that report more than 20 competitors.

[18]So that the law of motion is not divergent.

7 Conclusion

In this paper, I revisit the implicit collusion and customer base models and show they both imply a forward looking law of motion for markups in which they depend on the firms' expectations of output growth and stochastic discount rates. Because markups are related to the expected output growth, and not to its level, the conditional expectations of firm for the dynamics of output are the key component of the cyclicality of markups. In particular, if firms expect a humped-shape response for output during the business cycle, the predictions of these models are reversed.

Previous work using these models has not allowed for sufficiently rich dynamics in output which has lead to the conclusion that implicit collusion models lead to counter-cyclical markups. I show that this prediction is overturned once empirically realistic dynamics of output are incorporated into the model. Doing so, also helps the implicit collusion model to match the empirical evidence on the dynamic cross correlation of output and markups documented in Nekarda and Ramey (2013).

Furthermore, while different versions of customer base models have been used in the literature to model both procyclical and countercyclical markups, I show that the same result extends to each version of these models; meaning that, given a customer base model, introducing inertia in response of the output will reverse the cyclicality of markups.

Therefore, matching the humped-shape response of output to shocks is crucial for choosing a proper model for understanding the cyclicality of markups. The common law of motion that I derive in this paper boils down this problem to estimating the signs of its two reduced-form parameters. Using survey data on firms' expectations from New Zealand, I find that firms increase their markup when they expect that their demand will be higher in the future. Combined with the evidence that output responds to shocks with inertia, as firms expect output to keep growing on the impact of an expansionary shock, this then implies that markups are procyclical.

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A Figures

Figure 1: Extensive Representation for the Repeated Game of Section 2

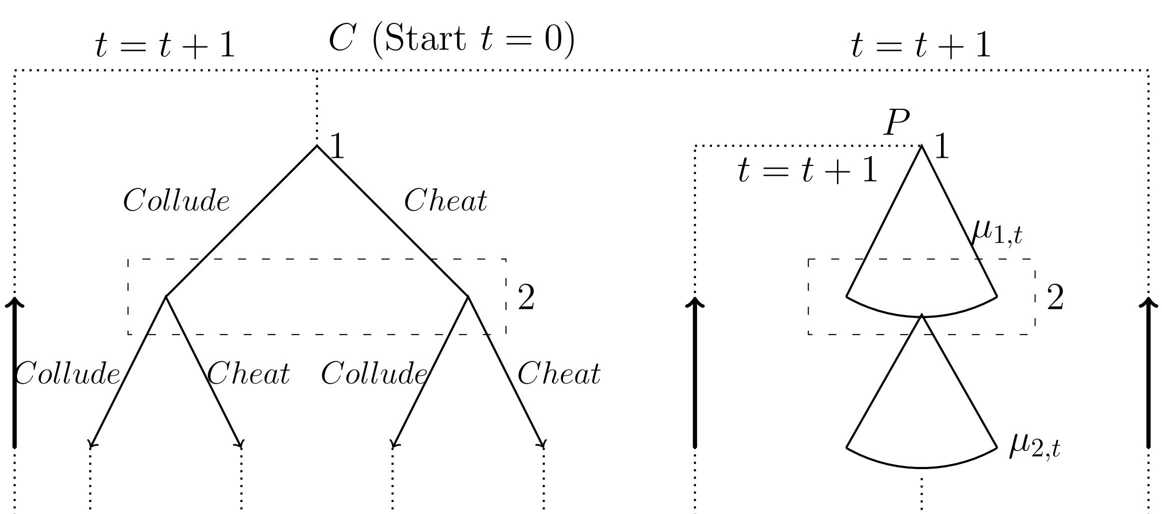
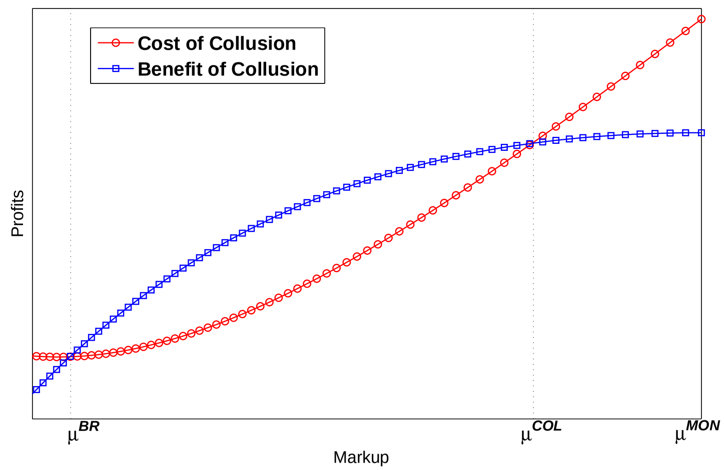
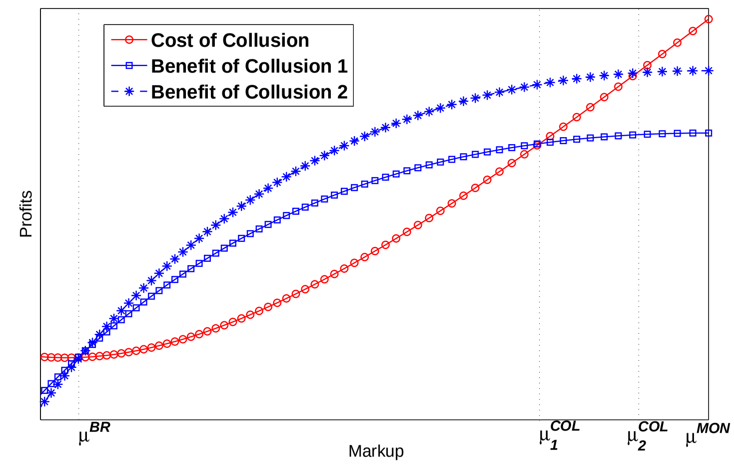


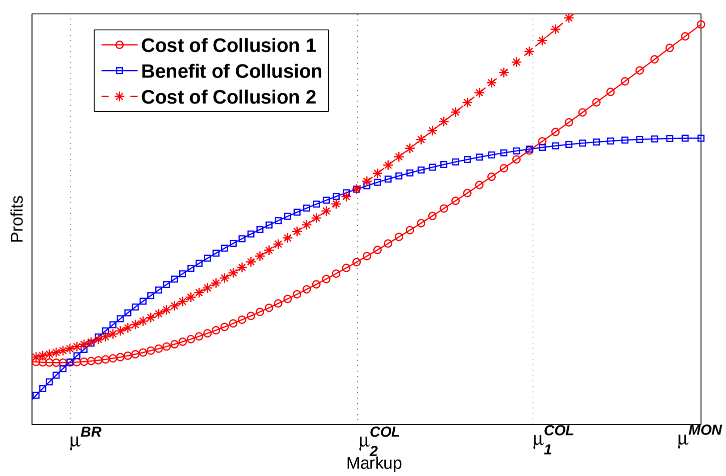
Figure 2: Benefit and Cost of Collusion



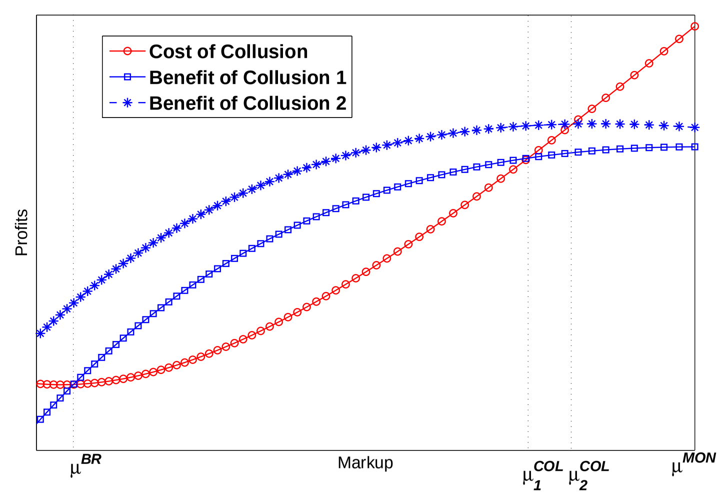
(a) This figure plots benefit and cost of collusion for a single firm, as a function of the oligopoly markup. See Section 2 for details.



(b) Comparative statics with respect to an increase in the discount rate β. More patient firms have a higher valuation of the benefit of collusion and are able to sustain higher markups in the equilibrium.

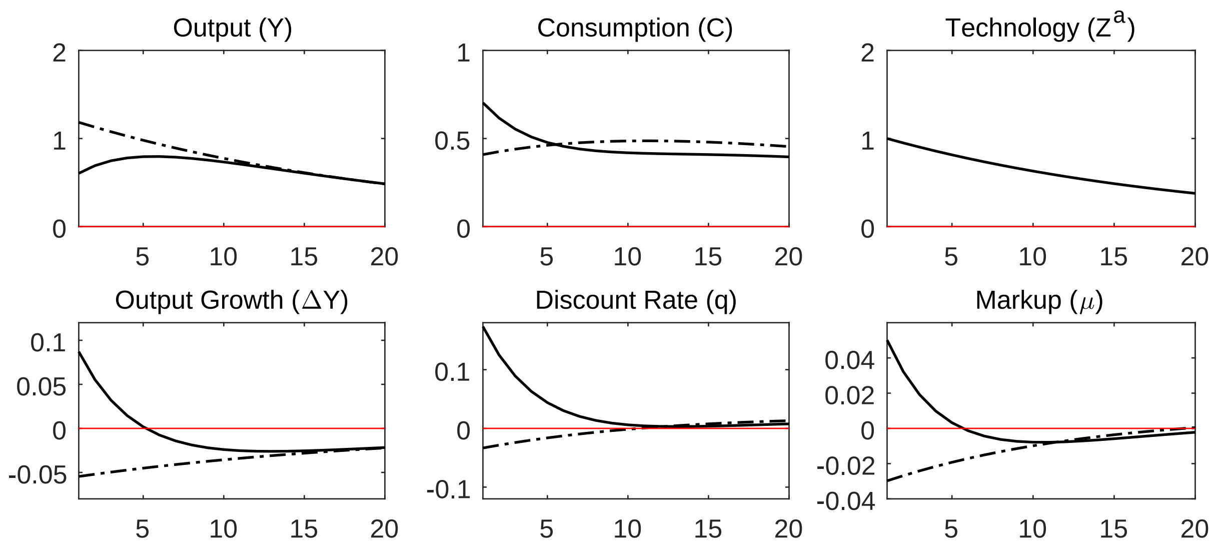


(c) Comparative statics with respect to an increase in current demand. Higher current demand increases the [opportunity] cost of collusion, and shrink the set of incentive compatible markups. Firms are forced to collude on lower markups.

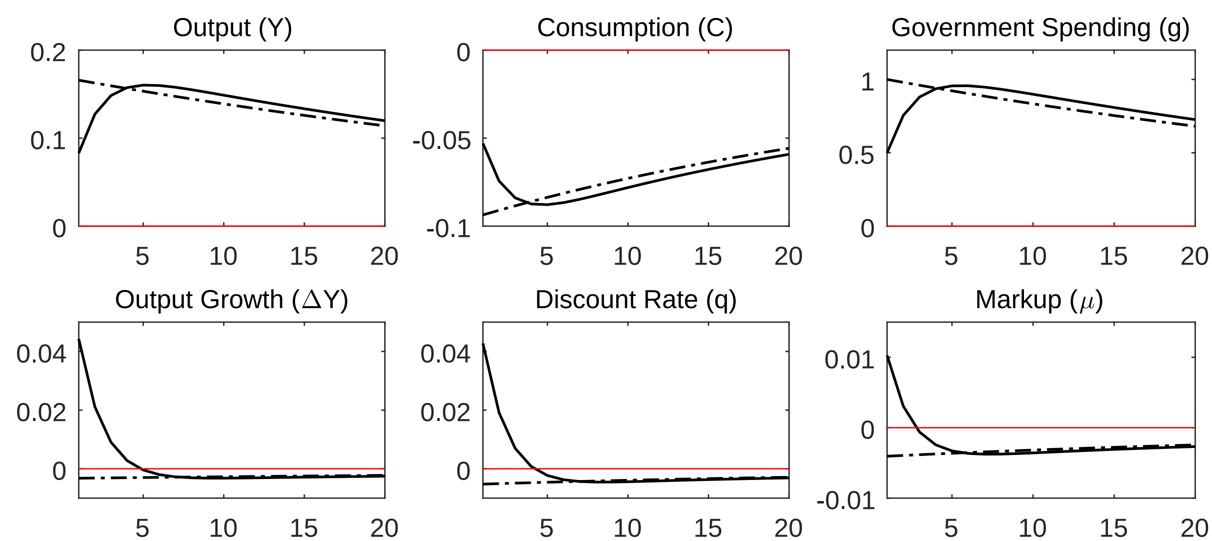


(d) Comparative statics with respect to an increase in future demand. Higher demand in future increases the benefit of collusion, and expands the set of incentive compatible markups. Firms are able to collude on higher markups as cheating incentives diminish.

Figure 3: Impulse Response Functions: Implicit Collusion Model



(a) The dashed curves plot the impulse response functions of the implicit collusion model to a 1% technology shock with <u>no</u> adjustment cost in which markups are counter-cyclical as output growth and stochastic discount rates are counter-cyclical. Solid curves illustrate the impulse response functions of the same model to a 1% technology shock <u>with</u> investment adjustment cost. Markups are pro-cyclical as long as firms expect output to grow. See Section 4.2 for details.



(b) The dashed curves plot the impulse response functions of the implicit collusion model to a 1% government spending shock <u>without</u> inertia in which markups are counter-cyclical as output growth is negative during the expansion. Solid curves illustrate the impulse response functions of the same model to an inertial government spending shock that peaks at 1%. Markups are pro-cyclical on impact as output growth and stochastic discount rates are pro-cyclical. See Section 4.2 for details.

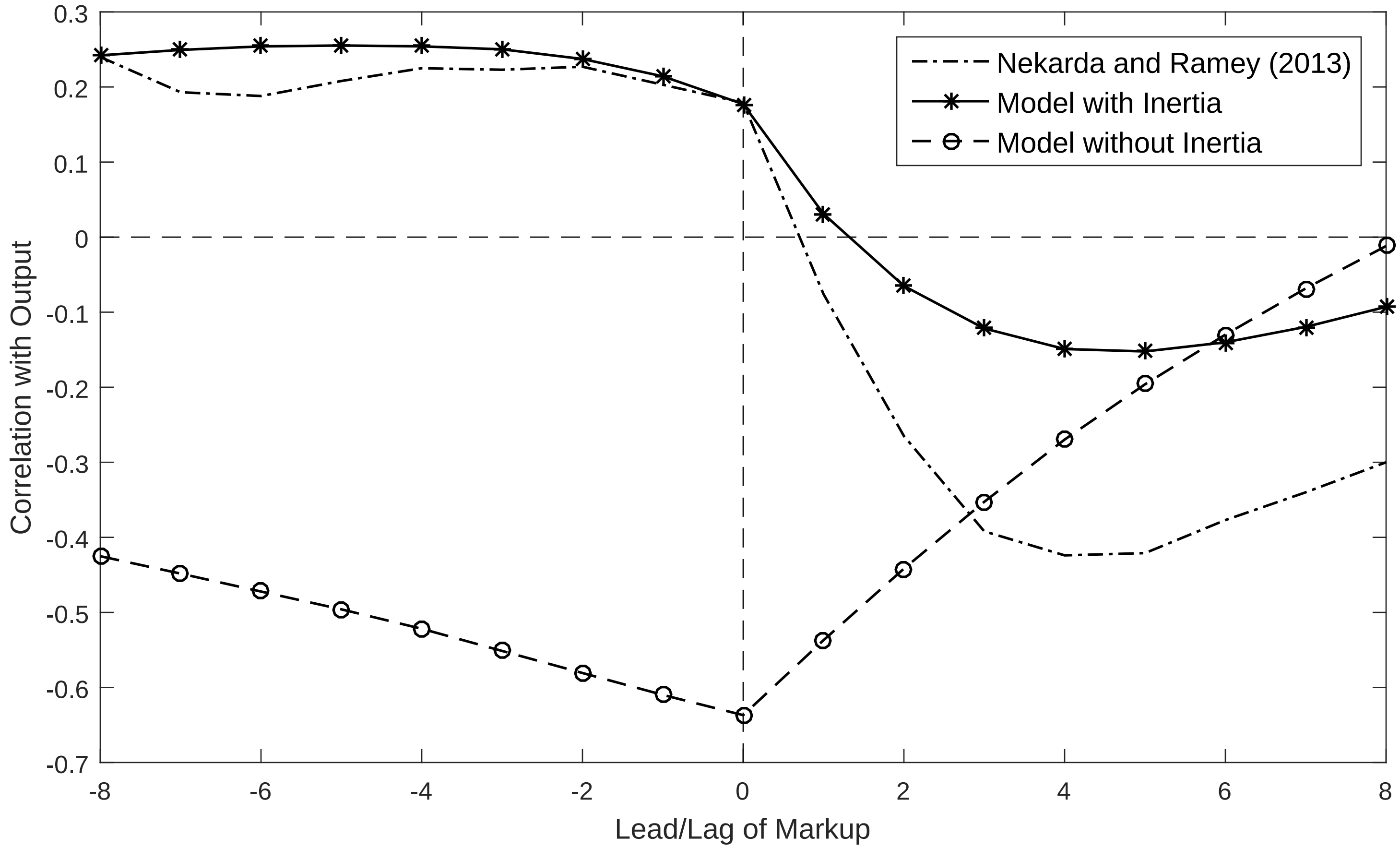
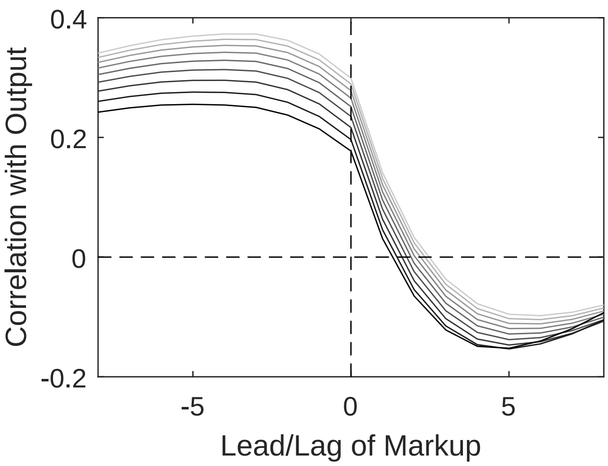
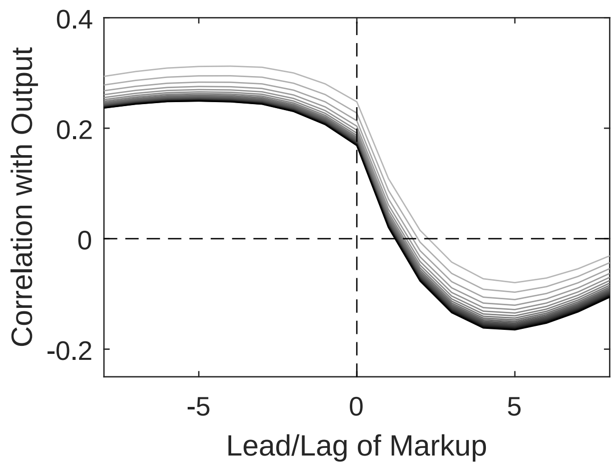


Figure 4: The black curve with square markers depicts correlation of μ_{t+j} with Y_t from the simulated implicit collusion model without inertial response of output conditional on a TFP shock. The dotted curve shows the unconditional cross-correlation of the cyclical components of markups with real GDP from Nekarda and Ramey (2013). The black curve with circle markers illustrate this cross correlation from the simulated implicit collusion model with inertial response of output conditional on a TFP shock. Inertia is crucial in matching the data. See Section 4.3 for details.

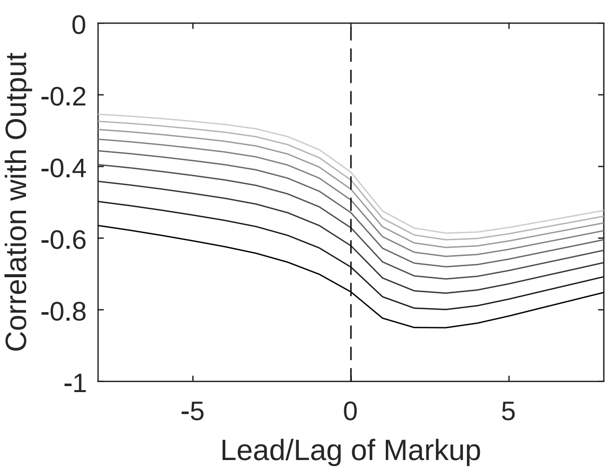
Figure 5: Robustness to number of firms in each sector N, and the renegotiation probability γ



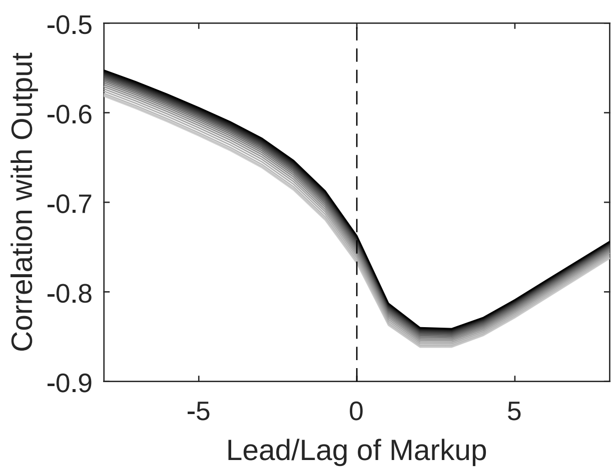
(a) Simulated correlation of μ_{t+j} with Y_t conditional on a TFP shock for $\gamma \in [0.4, 0.8]$. See section 4.4 for details.



(b) Simulated correlation of μ_{t+j} with Y_t conditional on a TFP shock for $N \in \{5, \ldots, 25\}$. See section 4.4 for details.

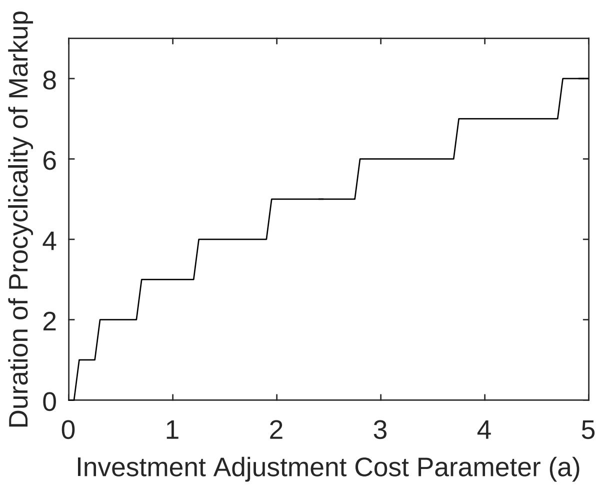


(c) Simulated correlation of μ_{t+j} with Y_t conditional on a government spending shock for $\gamma \in [0.4, 0.8]$. See section 4.4 for details.

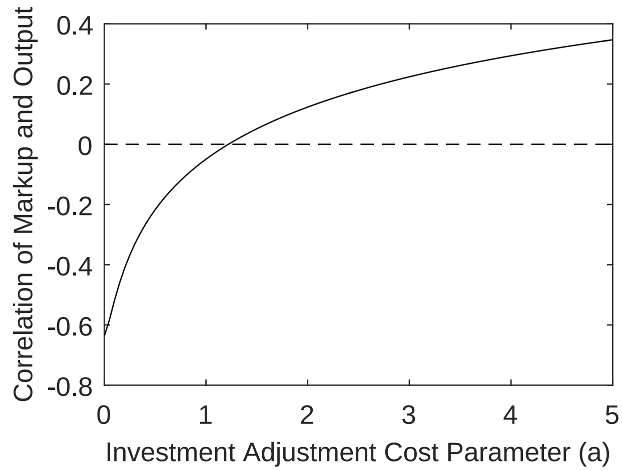


(d) Simulated correlation of μ_{t+j} with Y_t conditional on a government spending shock for $N \in \{5, \ldots, 25\}$. See section 4.4 for details.

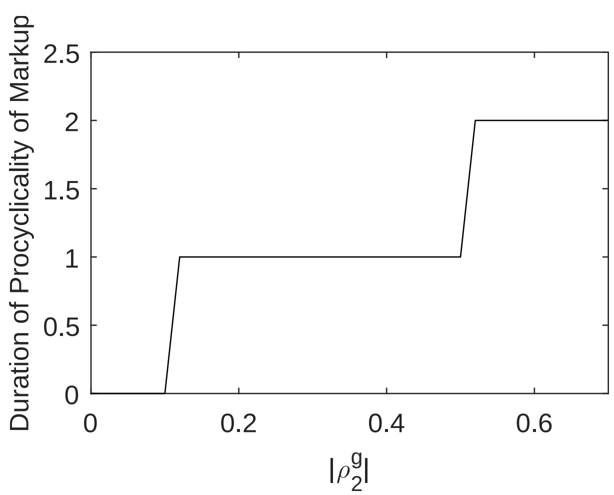
Figure 6: Robustness to inertia



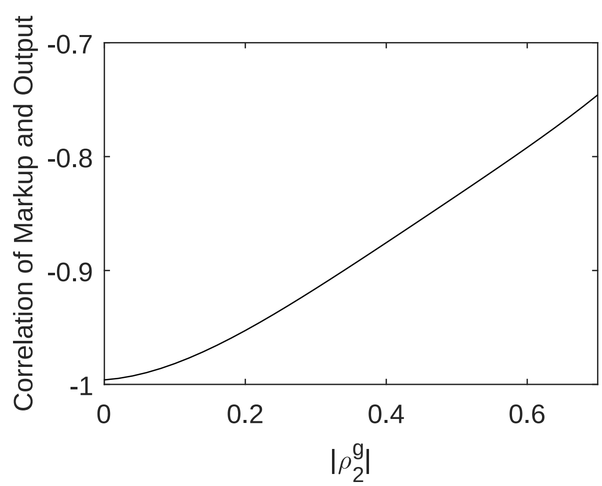
(a) Duration of procyclicality of markup after a 1% TFP shock for different value of investment adjustment cost parameter, $a \in [0, 5]$. See Section 4.4 for details.



(b) Simulated correlation of μ_t with Y_t conditional on a TFP shock for different value of investment adjustment cost parameter, $a \in [0, 5]$. See Section 4.4 for details.

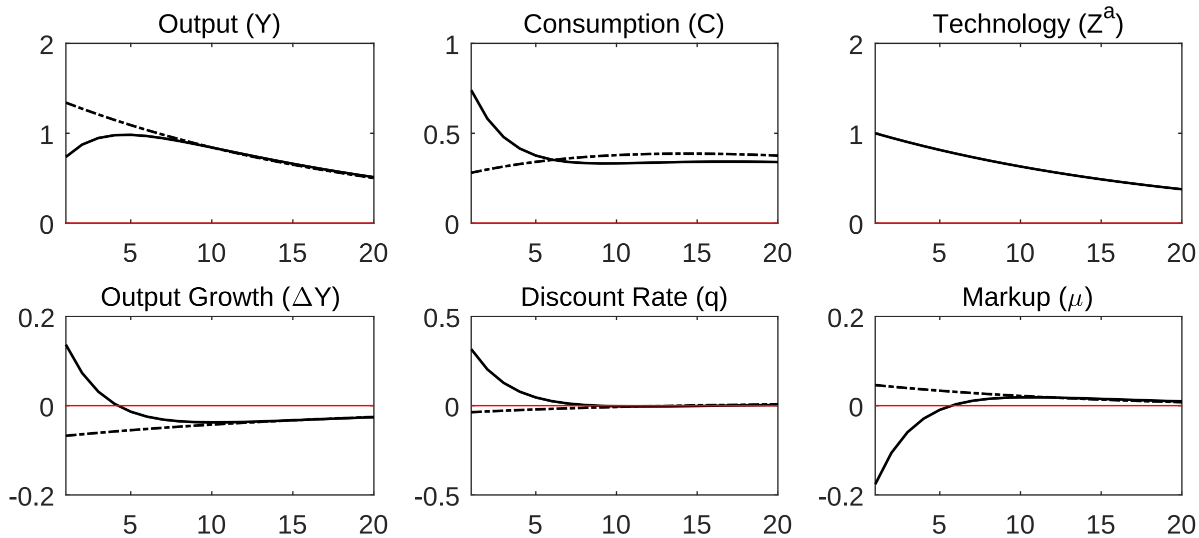


(c) Duration of procyclicality of markup after a 1% government spending shock for different values of the inertia parameter in the AR(2) process, $|\rho_2^g| \in [0, 0.7]$. See Section 4.4 for details.

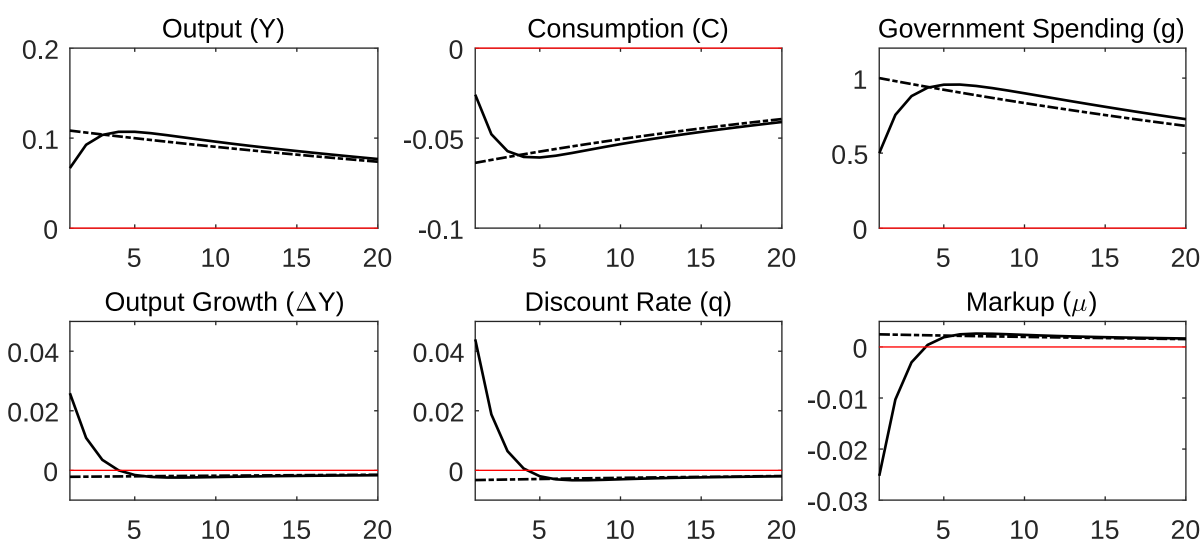


(d) Simulated correlation of μ_t with Y_t conditional on a government spending shock for different values of the inertia parameter in the AR(2) process, $|\rho_2^g| \in [0, 0.7]$. See Section 4.4 for details.

Figure 7: Impulse Response Functions: Customer-Base Model



(a) The dashed curves plot the impulse response functions of the customer base model to a 1% technology shock with no adjustment cost in which markups are pro-cyclical as output growth and stochastic discount rates are counter-cyclical. Solid curves illustrate the impulse response functions of the same model to a 1% technology shock with investment adjustment cost. Markups are counter-cyclical as long as firms expect output to grow. See Section 5.1 for details.



(b) The dashed curves plot the impulse response functions of the customer base model to a 1% government spending shock without inertia in which markups are pro-cyclical as output growth is negative during the expansion. Solid curves illustrate the impulse response functions of the same model to an inertial government spending shock that peaks at 1%. Markups are counter-cyclical on impact as output growth and stochastic discount rates are pro-cyclical. See Section 5.1 for details.

B Proofs.

Proof of Proposition 1.

First, observe that the set of solutions is not empty as $\mu_{it} = \mu_{COU}, \forall t$ satisfies the constraint for all periods. Moreover, if the constraint is not binding, the firms will simply act like a monopoly and choose $\mu_{it} = \mu_{MON}$, as it maximizes their joint profits. Hence, the choice set of firms can be compactified so that $\mu_{it} \in [\mu_{COU}, \mu_{MON}]$, and as the usual assumption of continuity holds, the problem has a solution. Finally, for the solution to be a sub-game Nash equilibrium, two conditions have to hold: first, that firms do not have an incentive to deviate from the chosen markups in the equilibrium path, which is true by construction, and second, that if ever the game were to go to punishment stage, firms would have an incentive to revert back to this strategy, which is also true as collusion is always at least as good as best responding.

Proof of Proposition 2.

The first order conditions, after imposing the symmetric equilibrium conditions $\frac{\mu_{ijt}}{\mu_{it}} = 1, S_{ij,t} = 1$ and normalizing $P_t = 1$ (as all firms solve the same problem), are

$$(1 + h'(1))(\mu_t^{-1} - \mu_C^{-1}) - h'(1)(1 - \mu_C^{-1}) = \gamma E_t\{Q_{t,t+1}\frac{Y_{t+1}}{Y_t}(\mu_{t+1}^{-1} - \mu_C^{-1})\}$$

Hence, in the steady state

$$\mu = \frac{1 - \beta\gamma + h'(1)}{1 - \beta\gamma + \mu_C h'(1)}\mu_C$$

Notice that $\mu \geq 1$ if and only if

$$1 - \beta\gamma + \mu_C h'(1) \geq 0 \Leftrightarrow h'(1) \geq -\frac{1 - \beta\gamma}{\mu_C} \tag{5}$$

also observe that

$$\mu - \mu_C = -h'(1)\frac{(\mu_C - 1)\mu_C}{1 - \beta\gamma + \mu_C h'(1)} < 0 \Leftrightarrow h'(1) > 0 \tag{6}$$

meaning that customer base frictions reduce markups on average if and only if $h'(1)$ is positive.

Finally, a first order approximation to the above equation around the steady state yields:

$$\hat{\mu}_t = \psi_1 \mathbb{E}_t\{\hat{q}_{t,t+1} + \Delta\hat{y}_{t+1}\} + \psi_2 \mathbb{E}_t\{\hat{\mu}_{t+1}\}$$

such that $\psi_1 \equiv \beta\gamma\frac{\mu\mu_C^{-1}-1}{1+h'(1)}$ and $\psi_2 \equiv \frac{\beta\gamma}{1+h'(1)}$. This verifies that the customer base model, up to a first order approximation, implies the same law of the motion for markups as the implicit collusion model. To infer the signs of ψ_1 and ψ_2, observe that 5 implies that $h'(1)+1$ as $-\frac{1-\beta\gamma}{\mu_C} > -1$. Thus, $\psi_2 > 1$ for any parametrization. Moreover, $\psi_1 > 0$ if and only if $\mu_C^{-1}\mu - 1 > 0$ which based on 6 happens if and only if $h'(1) < 0$, and vice versa.

Proof of Proposition 3.

Notice that

$$\begin{aligned} Ex\Delta Sales_{ij,t} &= E_t^{ij}\frac{P_{ij,t+1}Y_{ij,t+1} - P_{ij,t}Y_{ij,t}}{P_{ij,t}Y_{ij,t}} \\ &\approx E_t^{ij}\left[(1-\sigma)\Delta\hat{p}_{ij,t+1} + \Delta\hat{y}_{t+1}\right] \\ &= (1-\sigma)Ex\Delta Price_{ij,t} + E_t^{ij}\left[\Delta\hat{y}_{t+1}\right] \end{aligned}$$

where the second line is derived using the demand structure $Y_{ij,t} = Y_{i,t} = Y_t D(P_{it}; P_{it})$. Now, rewriting the law of motion

$$\begin{aligned} \hat{\mu}_{ij,t} &= \frac{\psi_1}{1-\psi_2}\mathbb{E}_t^{ij}\left\{\Delta\hat{y}_{t+1} + \hat{q}_{t,t+1}\right\} + \frac{\psi_2}{1-\psi_2}\mathbb{E}_t^{ij}\{\Delta\hat{\mu}_{it+1}\} \\ &= \frac{\psi_1}{1-\psi_2}\mathbb{E}_t^{ij}\left\{Ex\Delta Sales_{ij,t} + (\sigma-1)\Delta\hat{p}_{it+1} + \hat{q}_{t,t+1}\right\} + \frac{\psi_2}{1-\psi_2}\mathbb{E}_t^{ij}\{\Delta\hat{p}_{ij,t+1} - \Delta\hat{mc}_{t+1}\} \\ &= \frac{\psi_1}{1-\psi_2}\mathbb{E}_t^{ij}\left\{\hat{q}_{t,t+1}\right\} + \frac{\psi_1}{1-\psi_2}Ex\Delta Sales_{ij,t} + \frac{(\sigma-1)\psi_1 + \psi_2}{1-\psi_2}Ex\Delta Price_{ij,t} - \frac{\psi_2}{1-\psi_2}\mathbb{E}_t^{ij}\{\Delta\hat{mc}_{t+1}\} \end{aligned}$$

Now sum over i and j and subtract the two to get

$$\begin{aligned}
\hat{\mu}_{ij} - \sum_i \sum_j \hat{\mu}_{ij} &= \frac{\psi_1}{1-\psi_2}\{Ex\Delta Sales_{ij} - \sum_i \sum_j Ex\Delta Sales_{ij}\} \\
&+ \frac{(\sigma-1)\psi_1+\psi_2}{1-\psi_2}\{Ex\Delta Price_{ij} - \sum_i \sum_j Ex\Delta Price_{ij}\} \\
&+ \frac{\psi_1}{1-\psi_2}(\mathbb{E}_t^{ij}\{\hat{q}_{t,t+1}\} - \sum_i \sum_j \mathbb{E}_t^{ij}\{\hat{q}_{t,t+1}\}) - \frac{\psi_2}{1-\psi_2}(\mathbb{E}_t^{ij}\{\Delta \hat{mc}_{t+1}\} - \sum_i \sum_j \mathbb{E}_t^{ij}\{\Delta \hat{mc}_{t+1}\}) \\
&= \frac{\psi_1}{1-\psi_2}\{Ex\Delta Sales_{ij} - \sum_i \sum_j Ex\Delta Sales_{ij}\} \\
&+ \frac{(\sigma-1)\psi_1+\psi_2}{1-\psi_2}\{Ex\Delta Price_{ij} - \sum_i \sum_j Ex\Delta Price_{ij}\} + Industry_FE_i + \varepsilon_{ij,t}
\end{aligned}$$

where

$$Industry\_FE_i \equiv \tfrac{\psi_1}{1-\psi_2}u_{1,t}^i + \tfrac{\psi_2}{1-\psi_2}v_{1,t}^i$$
$$, \qquad \varepsilon_{ij,t} \equiv \tfrac{\psi_1}{1-\psi_2}u_{2,t}^{ij} + \tfrac{\psi_2}{1-\psi_2}v_{2,t}^{ij}$$

Since $u_{2,t}^{ij}$ and $v_{2,t}^{ij}$ are independent of $Indsutry\_FE_i$ by construction and the other two terms by assumption, we have,

$$\psi_1 = \frac{\hat{\beta}_1}{1+\hat{\beta}_2-(\sigma-1)\hat{\beta}_1}, \ \psi_2 = 1 - \frac{1}{1+\hat{\beta}_2-(\sigma-1)\hat{\beta}_1}$$

Proof of Proposition 4.

From the proof of last proposition, observe that

$$\begin{aligned}
&\tfrac{1}{1-\psi_2} = 0.82 - 0.16(\sigma-1) \pm 1.96 \times (0.06 + 0.08(\sigma-1)) \\
\Rightarrow \quad &\tfrac{1}{1-\psi_2} \in [0.7 - 0.32(\sigma-1), 0.94]
\end{aligned}$$

Also, since $|\psi_2| < 1$, we have

$$\frac{1}{1-\psi_2} \in (\frac{1}{2}, \infty)$$

combining the two we get

$$\frac{1}{1-\psi_2} \in (0.5, 0.94], \forall \sigma > 1$$

which implies that

$$\psi_2 \in (-1, -0.06], \forall \sigma > 1$$

Also

$$\frac{\psi_1}{1 - \psi_2} = \hat{\beta}_1 = 0.16 \pm 1.96 \times 0.08$$

Hence,

$$\begin{aligned} \psi_1 &\in (1 - \psi_2) \times [0.003, 0.32] \\ &\subset [0.003, 0.64], \forall \sigma > 1. \end{aligned}$$