# Chemistry Teachers’ Pedagogical Content Knowledge and Belief on Integrating Proportional Reasoning in Teaching Stoichiometry

Min Jung Lee

Submitted in partial fulfillment of the

requirements for the degree of

Doctor of Philosophy

under the Executive Committee

of the Graduate School of Arts and Sciences

COLUMBIA UNIVERSITY

2020

**Abstract**

Chemistry Teachers' Pedagogical Content Knowledge and Belief on Integrating Proportional Reasoning in Teaching Stoichiometry

Min Jung Lee

Proportional reasoning refers not only to the ability to manipulate the proportions, but also to detect, express, analyze, explain, and provide evidence in support of assertions about proportional relationships. Students' understanding of the proportional relationships that they encounter in science can be improved through well-designed instruction. In other words, teacher practice is key to the successful learning of both proportional reasoning and science. Stoichiometry is a basic topic in chemistry that focuses on the proportional relationship between the amount of reactants and/or that of products in a chemical reaction. This study explored 10 chemistry teachers' knowledge and beliefs on integrating proportional reasoning in teaching stoichiometry mainly through interview, survey, and lesson materials. The framework of pedagogical content knowledge was used to examine key dimensions of teacher knowledge that were triggered as they teach stoichiometry. Moreover, teachers' problem-solving strategies were sorted by using the proportional reasoning strategies framework. Three representative case studies allowed a deep analysis and the relation of each component of pedagogical content knowledge with implications for teacher education and professional development design.

Table of Contents

List of Tables

List of Figures

## Chapter I: Introduction

After Galileo Galilei successfully demonstrated the fall of bodies to the earth in mathematical terms, mathematics started to play a prominent role in science (de Berg, 1992; National Research Council [NRC], 2012). According to Norman Robert Campbell, mathematics is used in three different ways in science: "(a) establish the systems of derived measurement, such as density, (b) calculate in the form of combining numerical relationships to produce new numerical relations, such as Ideal Gas Law, which is a combination of other gas laws, and (c) formulation of theories, such as the development of kinetic-molecular theory of gases" (as cited in de Berg, 1992, p. 80). Also, mathematics is a powerful model for predicting natural phenomena, such as atomic structure and climate change. Likewise, mathematics and science have a long history together and, now, it is hard to think of these two as separate disciplines.

Science is not just a body of knowledge about the natural world. It is a set of practices used to establish and refine that knowledge (NRC, 2012). Thus, engaging in those practices is important in order to understand how scientists work and develop scientific knowledge authentically. As mathematics play a significant role in science, mathematical thinking is a daily practice that scientists engage while establishing and refining knowledge. For example, Einstein conducted a mathematical analysis that revealed the concept of the equivalence of mass and energy. Therefore, it is important for K-12 students to engage in mathematical practices associated with science in order to have a deeper understanding of quantitative-based science concepts and how scientists work.

The developers of the most recent science standards asserted the critical role of mathematics in learning science. As a result, they constantly emphasized and implemented mathematical knowledge and practice in a K-12 science curriculum. More recently, *A*

*Framework for K-12 Science Education* (NRC, 2012; referred to henceforth as the *Framework*) and Next Generation Science Standards, or simply NGSS (NGSS Lead States, 2013), suggested integrating 'Using Mathematics and Computational Thinking' Science and Engineering Practices (SEP) and 'Scale, Proportion, and Quantity' Crosscutting Concepts (CCCs) with a corresponding Core Ideas in the major Disciplines of natural science (DCIs) with an expectation to familiarize students with the role of mathematics in science. Mathematically thinking about the concepts of ratio and proportion refers to proportional reasoning (Lamon, 2007; Taylor & Jones, 2009; Tourniaire & Pulos, 1985). In the CCSSM (Common Core State Standards Initiative, 2012), 'Ratios and Proportional Relationship' is listed as one of the eleven content domains known to be a critical concept for understanding later concepts in mathematics, such as functions, graphing, algebraic equations, and measurement (Karplus, Pulos, & Stage, 1983; Lobato & Ellis, 2010). As a result, the *Framework* (NRC, 2012) committee explicitly suggested aligning science standards with Common Core State Standards in Mathematics (Common Core State Standards Initiative, 2012; referred to henceforth as the CCSSM) because it will bring effective learning for both subject areas. Practically, the NGSS (NGSS Lead States, 2013) development team worked with CCSSM to ensure science learning was coordinated with learning of mathematics.

Proportional reasoning is an important cognitive milestone (Lamon, 1993) as it prepares students to be science and mathematics literate citizens. For example, proportional reasoning is required for calculating tax and bank interest, understanding and reading a map, adjusting recipes, or creating various concentrations of mixtures and solutions. Thus, proportional reasoning is an essential ability for K-12 students not only for success in their K-12 science and mathematics learning (Wagner, 2001), but also for preparing them to be citizens for the 21st century.

Though proportional reasoning has been emphasized in K-12 curriculum, especially in middle school years, Lamon (2012) found that over 90% of high school students have limited understanding of it, which holds them back from learning high school mathematics (e.g., calculus or statistics) and science (e.g., biology or physics) and applying it in daily situations (e.g., bank interest). Moreover, adults were found to have a limited understanding of this concept (Lamon, 2012; Tourniaire & Pulos, 1985). This aligns with the literature that reports teachers' insufficient knowledge of proportional reasoning (Brown et al., 2016; Lobato, Orrill, Druken, & Jacobson, 2011). Furthermore, mathematics teachers have expressed their challenges to create instructional interventions and provide productive questions to support students' proportional reasoning (Watson, Callingham, & Donne, 2008). However, compared to studies that explored students' understanding of proportional reasoning, there are few studies about how teachers understand and teach proportional reasoning (Lamon, 2007). Within those few studies, teachers' understanding of proportional reasoning is often bootstrapped with that of students and reports that the teachers have similar conceptual difficulties as the students do (Lobato et al., 2011). In addition, it was found that teachers' algorithmic approach often does not resemble the way in which the students approach it (Kastberg, D'Ambrosio, & Lynch-Davis, 2012; Lobato & Ellis, 2010; Lobato et al., 2011), largely because the algorithmic approach is shaped by an adult understanding of the problem. Therefore, teachers' understanding should be captured separately from that of the students.

In the *Framework* (NRC, 2012), several DCIs are suggested to integrate 'Using Mathematics and Computational Thinking' SEP and 'Scale, Proportion, and Quantity' CCC. This study will specifically pay attention to DCI of chemical reactions (Physical Science 1), or often known as stoichiometry, not only because it requires those SEPs and CCCs, but also because it is a fundamental topic in the practice of chemistry. Chemistry is a study that explains the quantitative and qualitative aspects of substances. Stoichiometry especially focuses on the quantitative relationship among reactants and products in a chemical change. More specifically, it explains the relationships between the amounts of reactant and product based on a chemical reaction. Scientists, even engineers, use this concept in their everyday work. For example, chemists predict the amount of product that will be produced with a given amount of reactant. Therefore, in order to understand the quantitative aspect of substances and the practice of chemistry, stoichiometry is a core concept that must be understood in learning chemistry.

Stoichiometry is an important topic for high school students for practical reasons. It is listed in most U.S. high school chemistry curricula; as a result, it is always included at the end of year state examinations. Moreover, students who are planning to major in chemistry or other related science disciplines are required to take the SAT chemistry subject. The guideline of this test specifically lists stoichiometry as one of the eight topics, which covers 14% of the test. The concept of stoichiometry is also required in the topic of the State of Matter, which takes up 13% of the test. Despite the importance of stoichiometry, most secondary students perceive it as a difficult topic because it requires many scientific concepts, as well as mathematical skills (Schmidt, 1990). The meaning and understanding of stoichiometry are shielded by the mathematical practice involved in solving the related problems. For example, past studies have found that students depend on the algorithm to solve stoichiometry problems without conceptually understanding it (Gabel, Sherwood, & Enochs, 1984; Nurrenbern & Pickering, 1987). This results from how textbooks represent and how teachers teach stoichiometry problems. DeMeo (2008) found that most chemistry textbooks and science teachers greatly depend on the algorithm when teaching stoichiometry (DeMeo, 2008). As a result, students also heavily rely on the algorithm and, moreover, they consider stoichiometry problems one kind of mathematics problem. Their only concern is setting up the right equation for the right answer without visiting the underlying scientific concept in the problem.

As mentioned earlier, stoichiometry is an area that studies the relationships between the amounts of reactant and product based on a chemical reaction, which is fundamentally a statement of proportionality between the reactants and the products and between the elements. This is where the gist of proportionality lies in stoichiometry (Ramful & Narod, 2014). Therefore, proportional reasoning becomes an essential mathematical practice for conceptually understanding of stoichiometry. However, it is found that adolescent performance on proportional reasoning depends largely on the familiarity of the problem (Akatugba & Wallace, 2009; Saunders & Jesunathadas, 1988). Students scored significantly higher when the proportion problems were directly related to their everyday, real-life experiences compared to solving unfamiliar problems from science textbooks. This implies that even the students with good proportional reasoning ability need support from the science teachers in order to transfer their proportional reasoning ability that they have acquired in mathematics context to science context. Therefore, science educators should put effort into improving students' proportional reasoning ability in the context of science to help them not only to solve stoichiometry problems but also to transfer the mathematical practices required for solving similar science problems that involve a proportional relationship.

According to NGSS (NGSS Lead States, 2013), "Mathematics is a tool that is key to understanding science. As such, classroom instruction must include critical skills of mathematics" (Appendix F, p.58). As classroom instruction heavily depends on the teachers' professional competency, how they implement these skills in the instruction is critical in students understanding the role of mathematics in science. However, proportional reasoning is a complex mathematical skill to teach. Therefore, the teacher's role is crucial in developing a student's proportional reasoning ability. They must have a deep understanding of proportional reasoning and as well know how to present it in an accessible way for students while pushing them to develop more sophisticated forms of reasoning. Though it is not very clear how teacher's teaching is related to students' learning of proportional reasoning, it is generally known that two factors attribute to student learning: teacher's knowledge and beliefs (Lobato & Lester Jr, 2010).

As Shulman (1986) suggested, there is a specific professionally grounded knowledge that is important for teachers, which is known as pedagogical content knowledge (PCK). It refers to a unique knowledge that only teachers possess for teaching specific content. This category of knowledge was spotlighted by science educators. Though there are only a few studies that examined teacher's PCK in teaching stoichiometry and, even less, with a specific focus on proportional reasoning, those have found that science teachers lack sufficient PCK to teach it (Kastberg et al., 2012; Lobato et al., 2011). This is problematic because it is impossible to employ explicit teaching strategies without their understanding of the various aspects of teaching proportional reasoning (Hilton, Hilton, Dole, & Goos, 2016).

According to Jones and Carter (2006) and Keys and Bryan (2001), teachers' attitudes and beliefs, including knowledge acquisition and interpretation and choices of assessment, are strongly linked with teaching practices. Related to proportional reasoning, chemistry teachers, in general, believe that students should have acquired this skillset from previous mathematics and science classes (Deters, 2003; Hilton & Hilton, 2016). This expectation might have been drawn from the fact that most high schools require students to take at least Algebra 1 or Geometry before taking a chemistry course. However, such requirements can limit the student's opportunity to learn chemistry, which interrupts one of the big themes in science education, i.e., "science for all."

While reviewing the literature related to teaching stoichiometry and proportional reasoning, several gaps were noticed. First, there are very few studies that explored teachers' understanding and the teaching of stoichiometry. Most of the studies on stoichiometry focused on students' learning, such as their conceptual understanding and problem-solving strategies. Second, despite the importance of proportional reasoning in science, most studies investigating proportional reasoning have been carried out in the field of mathematics education. However, as the *Framework* (NRC, 2012) pointed out, the concept of ratio used in science, such as density (ratio of mass to volume), is quite different from a ratio of numbers describing fractions of a pie. Thus, students' mathematical understanding of proportion can be extended by the way that ratio and proportionality are used in science. Thus, how proportional reasoning is taught in science class through the related scientific language is studied differently from mathematics education. In this way, additional misunderstanding of mathematics and science use in understanding proportional reasoning may offer additional challenges. In addition, most studies related to proportional reasoning in science education are from the 1970s and 1980s. As we are living in a time where there is more exposure to data and numbers compared to the time period of the 1970s-1980s, we need an updated understanding of how proportional reasoning is conveyed in the science classroom. Finally, compared to other SEPs and CCCs, there are very few studies that investigated 'Using mathematical and computational thinking' and 'Scale, proportion, and quantity' after the *Framework* (NRC, 2012) and NGSS (NGSS Lead States, 2013) was introduced. Based on these gaps, this study explores the chemistry teacher's understanding and teaching of proportional reasoning of stoichiometry in the chemistry context.

## Purpose and Research Questions

Though proportional reasoning and stoichiometry are such important topics in mathematics and science education, an exploration of the literature on these topics revealed limited studies on exploring teachers' teaching of proportional reasoning, especially, in science education. In other words, there appears to be a lack of certainty with respect to how the concepts of proportional reasoning are being taught in chemistry classrooms, especially for the topic of stoichiometry. This lack of certainty further suggests exploring chemistry teachers' competency in teaching proportional reasoning, specifically related to their knowledge and beliefs. Therefore, the purpose of this study is to explore a group of chemistry teacher's knowledge of and beliefs about integrating proportional reasoning in their teaching of stoichiometry, which is appropriate to help their students understand and apply these concepts. In relation to this purpose, the following research questions are addressed:

1. What beliefs does a group of chemistry teachers hold about teaching proportional reasoning and stoichiometry?
2. What problem-solving strategies were used by a group of chemistry teachers when solving proportional reasoning used in stoichiometry problems?
3. What pedagogical content knowledge (PCK) does a group of chemistry teachers bring when asked about teaching proportional reasoning in stoichiometry?
4. What characteristics do three of the chemistry teachers show based on the evidence of Likert scale survey, proportion problems, and interview?

## Chapter II: Literature Review

The purpose of this study was to explore a group of chemistry teachers' knowledge and beliefs on integrating proportional reasoning in teaching stoichiometry, particularly as it helps students understand and apply both concepts in their daily lives. In this chapter, the literature covers three major areas: (1) proportional reasoning, (2) teaching and learning stoichiometry, and (3) teachers' knowledge and beliefs on teaching practice. These three areas that are most relevant to this study were examined.

The first topic, proportional reasoning, is an important concept in the secondary school mathematics curriculum. It is also a common scientific practice that students should engage in to understand the work of scientists. However, there are fewer studies that revealed how it is taught in science class compared to that in mathematics class. Therefore, the studies regarding proportional reasoning in mathematics education will be first introduced to understand why it is important, how it is developed among learners, and how it is related to the K-12 curriculum. Followed by proportional reasoning, literature that explores teaching and learning of stoichiometry will be reviewed. This topic is relevant because the study explored proportional reasoning as a practice while teaching stoichiometry. Finally, the literature on teachers' knowledge and beliefs on teaching proportional reasoning was investigated because those two factors are generally known to contribute to students' learning. These three topics are expected to provide baseline information of this study, namely the rationale and significance of this study and a context for the reader to understand.

### Proportional Reasoning

According to Lamon (2012), "proportional reasoning refers to detecting, expressing, analyzing, explaining, and providing evidence in support of assertions about proportional relationships (p.4).” A proportion is a relationship of equality between two ratios, a/b = c/d (Lobato & Ellis, 2010). One fraction is a scale up or down version of another ratio (Lamon, 2012). Proportional reasoning involves understanding the multiplicative relationship between rational quantities (a and b or a and c in a/b = c/d). More broadly, proportional reasoning is the ability to manipulate proportions, or the two equivalent fractions (Lamon, 2007; Taylor & Jones, 2009; Tourniaire & Pulos, 1985).

**Importance of Proportional Reasoning**

Proportional reasoning is an important ability in one’s cognitive development (Lamon, 1993) because it helps them to become a science and mathematics literate citizen. For example, proportional reasoning is required when calculating taxes and interest, understanding and drawing maps, adjusting recipes, or creating various concentrations of mixtures. However, because proportional reasoning is prevalent in our daily lives, it seems as if it could be mastered without a formal education. Some argued that proportional reasoning is unnecessary to be taught in schools; however, Schliemann and Carraher (1992) addressed this issue and concluded that schooling could deepen and advance one’s proportional reasoning ability by linking seemingly unrelated situations into a mathematical formula.

Since proportional reasoning is an important cognitive development skill, it became a critical and prevalent topic and practice in mathematics education, as well as in science education. As a result, CCSSM (Common Core State Standards Initiative, 2012) has listed “ratios and proportional reasoning” as its own content domain for grades 6 and 7. The science education community not only noticed the importance of proportional reasoning but also realized how it is important in learning science. Thus, “Using Mathematics and Computational Thinking” and “Scale, Proportion, and Quantity” are listed as one of the SEPs and CCCs, respectively, in the *Framework* (NRC, 2012) and the NGSS. Moreover, the *Framework* (NRC, 2012) committee suggested working with the CCSSM team to align NGSS with their standards for coherence in K-12 education. Therefore, students' proportional reasoning must be developed with an effort of both mathematics and science subject areas in order to enhance the understanding of both areas that associate proportion and ratio, such as linear equation, density, speed, and gas laws used in the sciences (Karplus et al., 1983; Lobato & Ellis, 2010).

**Types of Ratio in Proportional Reasoning**

According to Freudenthal (1978), there are two types of multiplicative relationships based on the two entities that are proportionally related to each other: within-ratio and between-ratio. While within-ratio operates with the same entities, between-ratio deals with different entities. For example, for solving questions such as: "Four cars can hold 12 people. How many people will 40 cars hold?" These two situations hold the proportional relationship constant, and the two entities that are involved are cars and people. A within-ratio will compare the same entity and set an equation, such as 4 people : 20 people = 1 car : 5 cars, whereas a between-ratio sets a relationship, such as 4 people : 1 car = 20 people : 5 cars, by comparing the two different entities. However, the author found that there was no preference for these two types of ratios among students.

**Types of Proportion Problems**

Solving a variety of problems that is an ability that proportional reasoners should possess (Lamon, 2012). Thus, previous studies often determined proportional reasoning ability by solving certain types of problems (Tourniaire & Pulos, 1985). However, methodologies and tasks that have been employed were different. Traditionally, two types of problems were used by educational researches: comparison problems and missing-value problems. A comparison problem presents two complete ratios, a/b and c/d, with four specific numbers (a, b, c, and d). Then, the two ratios are compared to determine whether those are equal to each other, or whether they form a proportion (a/b = c/d). However, the missing value problem provides only three of the four numbers (a, b, and c) in the proportion (a/b = c/d) and asks to find the missing value (d). In the science context, missing value problems are more common than comparison problems. For example, when solving the gas law problems, the pressure of the gas is predicted based on the other given variables: temperature, volume, and number of gas particles. Though comparing two ratios is important in a mathematical perspective, finding a missing value is more common in science.

Historically, four contexts of proportional reasoning tasks were commonly used in educational studies: (a) physical tasks, (b) rate problems, (c) mixture problems, and (d) probability task (Tourniaire & Pulos, 1985). However, physical tasks have a limitation on measuring one's proportional reasoning ability. For example, the most known physical task is a Balance Beam task. It asks to predict the position of a fulcrum to balance the beam. In order to find this point, one has to know the relation between the weight of the pendulum and the distance from the fulcrum. Thus, because physical tasks require physical knowledge in addition to the concept of proportion, it is hard to understand one's proportional reasoning ability based on whether it was solved accurately. The most frequently used tasks in proportional reasoning studies are rate and mixture problems. Rate problems compare the ratio that has different objects, such as assigning patients to doctors. The most widely known mixture problem is Noelting's Orange Juice Puzzle (1980), in which the concentrations of two mixtures of orange juice and water have to be compared. Though rate problems and mixture problems look similar, they differ mostly in terms of the unit; the quantities used in mixture problems are expressed in the same unit of measurement (e.g., ounces over ounces) while rate problems use different units of measurement (e.g., ounces per dollars).

These different contexts of proportion problems are important because it is known that adolescent performance of proportional reasoning depends largely on the familiar to the context of the problem (Akatugba & Wallace, 2009; Saunders & Jesunathadas, 1988). Saunders and Jesunathadas (1988) conducted a study with 72 freshman college students and found that they scored significantly higher when the proportional reasoning tasks were directly related to everyday, real-life experiences (i.e., when familiar and concrete examples were used). However, when students were required to solve unfamiliar problems (i.e., science), their performance decreased significantly. Based on this result, they concluded that the context familiarity becomes an important performance variable in proportional reasoning. This study reflects that the science teachers' role is not limited to supporting students with poor proportional reasoning ability, but expands to helping those who are good at it so that they can smoothly transfer it to the science context. Such transferring is important because proportional reasoners should not be affected by the context in which the problem is posed. Also, it will add coherence in K-12 education as the *Framework* (NRC, 2012) committee expected from working with the CCSSM team.

**Development of Proportional Reasoning**

"The development of proportional reasoning is an area of inquiry that focuses on how it evolves, develops, and unfolds over time" (Al-wattban, 2001, p. 30). Many scholars agree that proportional reasoning requires a long-term developmental process, as the understanding at one level forms a foundation of higher levels of understanding. However, "there is disagreement regarding its developmental time course" among educators (Boyer, Levine, & Huttenlocher, 2008, p. 2). Some studies indicate that proportional reasoning is a late achievement, while others report that children as young as five-years-old can also successfully reason proportionally (Boyer et al., 2008). The issue of age in the development of proportional reasoning arose from Piaget's Cognitive Developmental Theory (Piaget & Inhelder, 1958). Piaget suggested that proportional reasoning is only possible after the child enters the formal operational stage, which is a final stage of cognitive development known to be entered by the age of 12-years and finish by the age of 15. However, some researchers argue that proportional reasoning is evident in early childhood, far before the formal operations stage than Piaget proposed. For example, Singer-Freeman and Goswami (2001) found that even the students in the age of years 3 to 4 can understand proportional reasoning when incorporating an analogy. However, Schliemann and Carraher (1992) addressed this issue by suggesting that the presence of proportional reasoning in early childhood can be discovered only when researchers use simple test problems. In other words, a young child's performance on proportional reasoning can only be found when the problems are simple enough for them to solve, which typically requires a low level of proportional reasoning ability.

**Learning Proportional Reasoning**

In general, proportional reasoning begins to develop around the late elementary school years of children older than ten years. Nevertheless, studies have found that over 90% of students who enter high school cannot reason proportionally well enough to learn mathematics and science and are unprepared for real applications in statistics, biology, geography, or physics (Lamon, 2007). According to Piaget's theory, proportional reasoning, an indicator of the formal operational stage, should have been acquired by late adolescent years in a child's development. However, it was found that college-bound adolescents had difficulty incorporating proportional reasoning for complex tasks (Farrell & Farmer, 1985; Lamon, 2007; Tourniaire & Pulos, 1985).

Moreover, it has been revealed that even adults have a limited understanding of proportional reasoning (Lamon, 2012; Tourniaire & Pulos, 1985).

Studies have identified reasons for having difficulties in understanding proportional reasoning. According to Al-wattban (2001) and Hart (1988), proportional reasoning is a complex concept because it requires: (a) understanding the concept of ratio, (b) understanding the concept of two or more ratios being equal, (c) being able to extract relevant information from a problem while at the same time ignoring irrelevant information; and (d) handling the number format, especially non-integer, in the problem. In addition, high school students' difficulties with learning proportional reasoning are not only linked with these complex concepts, but also with the challenges they face when transferring mathematical reasoning strategies to a new science context (Akatugba & Wallace, 2009). And, a student's performance can be affected by the types of questions used on the test (Saunders & Jesunathadas, 1988).

**Strategies for Solving Proportion Problems**

Research studies that examined students' performance on solving proportional reasoning revealed common strategies that were either successful or those that were erroneous (Cramer & Post, 1993; Tourniaire &Pulos, 1985). These strategies are important in terms of theoretical and practical implications. Theoretically, the strategies can help to uncover the mental processes behind solving problems. Practically, based on adolescents' successful or unsuccessful strategies, more effective curricula and educational teaching practices can be designed to increase students' proportional reasoning abilities (Al-wattban, 2001). Many successful strategies that students used were identified in the literature. For example, Cramer and Post (1993) identified four strategies for successfully solving proportion problems: unit-rate, factors-of-change, fractions, and cross products algorithms. Lamon (2007) focused on unitizing and norming strategies in her studies about ratio and proportion. She defined a unitizing strategy as the construction of a reference unit or unit whole, which is similar to unit-rate from Cramer and Post (1993).

Tourniaire and Pulos (1985) introduced two types of strategies that were most popularly used for solving proportion problems: (a) building-up strategies, and (b) multiplicative strategies. The buildings-up strategies are very basic and commonly used by both elementary students and adolescents. This strategy incorporates "establishing a relationship within a ratio and extending it to the second larger ratio by repetitively addition" (Tourniaire & Pulos, 1985, p. 184). For example, when solving the following problem: "the candy store sells 2 pieces of candy for 8 cents. How much do 6 pieces of candy cost?" (Tourniaire & Pulos, 1985, p. 184). A student can solve it as "8 cents for 2, 8 more is 16 cents for 4 pieces, and 8 more is 24 cents for 6 pieces" (Tourniaire & Pulos, 1985, p. 185). This strategy is considered as a building-up strategy because a ratio is built up by adding a constant number. Studies have identified this strategy being commonly used by young adolescents and by older adolescents as a backup strategy for solving complex proportion problems (Hart, 1984; Steinthorsdottir, 2003; Tourniaire, 1984; Tourniaire & Pulos, 1985). Moreover, Guckin and Morrison (1991) found that a considerable percentage of older adolescents retained this iterative additive approach instead of using multiplicative approaches.

Multiplicative strategies use multiplicative relationships between two ratios. This relationship can be identified in three ways based on the types of ratios involved in proportional reasoning: *within, between*, and *other*. *Within-ratio* type uses a relationship within *one ratio* of the two ratios in a proportion (relationship between a and b in $a/b=c/d$). For example, when solving proportion $6/2 = x/3$, one can use the relationship between 6 and 2 to find x. In other words, as 6 is a multiplication of 2 to 3, students can find x by multiplying 3 to 3. Secondly, *between-ratio* type establishes the relationship between the *two ratios* in a proportion (relationship between a and b in a/b=c/d). For example, for solving proportion 4/2 = 8/x, a student can use the relationship between 4 and 8 to solve for x. Finally, the relationship between the numerators and denominators across the two ratios can be established by reducing them into lower terms (2 for the example problem), which is called unitizing or unit-rate. Given the problem that asks to compare the two ratios, 75/100 and 6/8, a student can reduce the two ratios into the simplest form by dividing the first ratio's numerator and denominator by 25 and that of the second ratio by 2 (Al-wattban, 2001).

Erroneous strategies were acknowledged in the problems that are more appropriate for using the multiplicative concept (Tourniaire & Pulos, 1985). One common error is ignoring the given data in the problem. Such partial use of the data can be divided into two types of error: (a) quantitative and (b) qualitative (Al-wattban, 2001; Langrall & Swafford, 2000). The quantitative error involves the use of mathematics, but the numbers, operations, or strategies are randomly used rather than logically. The qualitative error does not involve the use of mathematics; rather, it involves the use of visual clues, pictures, models, or manipulatives when making sense of situations in the problem. Another common error is the use of a constant difference for the wrong entities, which is known as additive reasoning (Tourniaire & Pulos, 1985). For example, for solving 6/4 = 8/x, a student might say that the difference between 6 and 4 is 2 by subtracting the two so that x will be 6 by subtracting 2 from 8. Luke (1991) tested 266 ninth-grade students and found that one of the most common mistakes was that they mistakenly applied constant difference for solving ratio problems and geometric problems that require multiplicative thinking.

**Proportional Reasoning in Science Education**

Proportional reasoning is an important mathematical tool for students' success in learning and understanding many science concepts, such as density, concentration, speed, power, and pressure (Akatugba & Wallace, 1999). When Piaget's theory was introduced, it was a popular topic in the 1970s and 1980s in science education (DeBoer, 1991). However, it was dealt as one subdomain of formal reasoning ability required for learning science rather than as one topic in science education. For example, traditional assessments for measuring scientific reasoning ability, such as Classroom Test of Scientific Reasoning (CTSR), Test of Logical Thinking (TOLT), and Group Assessment of Logical Thinking (GALT), included proportional reasoning as one type of formal reasoning.

Though proportional reasoning plays such a significant role in different disciplines of science, there are only a few proportional reasoning studies in science education. For example, McBride and Chiappetta (1978) investigated 136 ninth-grade physical science students understanding of simple machines, the structure of matter, and equivalent fractions with its relation to proportional thinking. They discovered that proportional reasoning ability was associated with the achievement of simple machines, the structure of matter, and equivalent fractions concepts. Mitchell and Lawson (1988) also concluded that a lack of proportional reasoning skills limited students' ability to comprehend genetics and apply Mendelian principles in the solution of the problems. Hwang and Liu (1994) revealed that student conceptions of concentration were related to proportional reasoning. They concluded that the development of proportional reasoning ability could be an indicator of students' success in learning chemistry. This finding aligns with Johnson and Lawson's (1998) study, which argued that proportional reasoning ability is a possible predictor of biology achievement. Moreover, they found that proportional reasoning ability is a greater factor than the prior knowledge on college students' success in biology, no matter what type of biology instruction they received. Finally, Taylor and Jones' study (2009) examined whether proportional reasoning ability is correlated to a student's ability to understand surface area to volume relationships. Their findings revealed a correlation between proportional reasoning scores and the test scores on that topic. Overall, these few studies on proportional reasoning show a significant correlation between performance on proportional reasoning tasks and student's achievement in science.

## Stoichiometry

Chemistry is a discipline that studies the composition, structure, properties, and change of matter. It focuses on both the quantitative and qualitative properties of substances. Stoichiometry is a basic chemistry topic that focuses on the quantitative relationship between the amount of reactants used or products formed by a chemical reaction. It usually requires scientific skills and knowledge of several subtopics, such as writing chemical equations, balancing chemical equations, mass percent, empirical formulae, molecular formulae, percent yield, limiting reagent, mole concept, conservation of mass, and stoichiometric ratio (Gulacar, Overton, Bowman, & Fynewever, 2013). Also, it requires mathematical skills of manipulating two quantities (DeMeo, 2008; Gulacar, Overton, & Bowman, 2013). Therefore, stoichiometry requires not only a conceptual understanding of the subtopics but also a mathematical skill to understand the quantitative aspect of a chemical reaction.

### Importance of Stoichiometry

Stoichiometry is important not only because it is a basic concept in chemistry but also because it is a common science practice that scientists engage while working in laboratories. For example, one of the basic tasks for a scientist is to predict how much product will be produced with a given amount of reactants and to analyze how efficient the reaction is. When predicting and analyzing the amount of product, stoichiometry is used. Therefore, secondary students' proper understanding of stoichiometry at the end of K-12 education is necessary for their understanding of the work of scientists. It is also important for understanding and succeeding later in chemistry class. For example, Deters (2003) surveyed postsecondary chemistry professors to list the topics that should be mastered in high school in order to be successful in college chemistry. Seven significant topics and skills were found to be needed for college chemistry: basic skills (i.e., units, significant figures, graphing, etc.), moles, dimensional analysis, stoichiometry, naming and writing formulas, atomic structure, and balancing equations. This result implies that stoichiometry itself and subtopics are important in students' success in college chemistry. Even before considering success in college chemistry, stoichiometry plays a critical role in high school chemistry. In state exams at the end of chemistry class and college entry tests, such as SAT chemistry subject test, stoichiometry is included as one of the major topics covered in the assessment. Thus, a sufficient understanding of stoichiometry is required for student success in both high school and college chemistry.

**Proportional Reasoning in Stoichiometry**

Stoichiometry bases its concept on a chemical reaction, which is a fundamental statement of proportionality between the reactants and products or the elements of a reaction, shown as an equation. This is where the gist of proportionality lies in stoichiometry (Ramful & Narod, 2014). The importance of proportional reasoning for understanding and solving stoichiometric problems has been observed in several studies (Akatugba & Wallace, 1999; Bird, 2010; Chandran, Treagust, & Tobin, 1987; Ozcan Gulacar et al., 2013). For example, Seminara's (1996) study, which listed 21 of chemistry content knowledge outcomes required for successfully solving stoichiometry problems, found two that is related to proportional reasoning: “A balanced equation provides proportional information about all reactants and products in the equation” (p. 186) and “Use proportionality in some form to solve for an unknown variable” (p. 187). In addition, Glazar and Devetak (2002) listed nine types of efforts that teachers should make in order to improve students' understanding of stoichiometry, and developing proportional reasoning was included. This literature reflects that students should not only understand the proportional relation between the reactants and products but also be able to manipulate such relation to solve stoichiometry problems.

**Problem Solving in Stoichiometry**

A key feature of understanding stoichiometry is the ability to solve numerical problems (Dahsah & Coll, 2007). Surprisingly, before 1867 none of the textbooks contained numerical problems (Jensen, 2003). It was only after when Josiah Parsons Cooke, an American chemist, published the booklet to the public in 1868 that included numerical stoichiometry problems. He believed that the textbooks used by that time were insufficient for teaching college chemistry.

Before going deeper into the literature about problem-solving in chemistry, the word “problem” should be defined clearly because its definition is different between students and chemists. According to Hayes (1980), the problem is defined as “whenever there is a gap between where you are now and where you want to be, and you don’t know how to find a way to cross that gap, you have a problem” (p. 236, as cited in Bodner & Herron, 2002). However, as if there is a difference in the way students and chemists solve numerical problems, the gap each encounter is different. The gap that chemists encounter is the one that they do not know how to cross. However, the gap that K-12 students encounter in class is manageable and solvable, which is already a routine exercise for chemists. Therefore, the problems that I refer to in this study are the ones that chemists consider as a routine exercise, yet are what students are doing as exercises in learning stoichiometry within the chemistry classroom.

**Types of Stoichiometry Problems**

Three types of stoichiometry problems are commonly introduced in chemistry textbooks: stoichiometric mole-to-mole, stoichiometric mole-to-quantity, and vice versa stoichiometric quantity-to-quantity. Chemistry teachers often introduce these three types in this order because the difficulty increases with more steps to solve problems. For example, *stoichiometric mole-to-mole* requires one step: (a) converting the known number of moles to an unknown number of moles by using the coefficients in a chemical reaction. *Stoichiometric mole-to-quantity* problems require two steps: (a) adding on to the mole-to-mole type of problems, which is (b) converting an unknown number of moles to quantity, such as grams, by using the molar mass of the unknown substance. Finally, *stoichiometric quantity-to-quantity* problems include one more step at the end of stoichiometric mole-to-quantity problems: (c) converting the known mass to the number of moles.

There are studies that have examined stoichiometry problems from a diverse point of view. For example, Tang, Kirk, and Pienta (2014) analyzed the complexity factors of stoichiometry problems by its number format, equation, and unit. Ramful and Narod (2014) categorized stoichiometry problems in five levels based on their proportional structure.

**Conceptual and Procedural Knowledge in Problem-Solving**

Procedural knowledge is about knowing how to apply concepts to solve problems (Heyworth, 1999; Wolfer, 2000). It is often referred to as knowing the algorithm, which is a "carefully developed procedure for getting the right answers to exercises and routine tasks within problems" (Herron, 1996, p. 64). Problem-solving is "a contextual dependent process that is guided by relevant conceptual knowledge" (DeMeo, 2008, p. 73). To solve problems, both conceptual knowledge and procedural knowledge are required.

According to the literature, many chemistry educators equated successful problem solving with good command of chemistry content knowledge (Nakhleh & Mitchell, 1993; Nurrenbern & Pickering, 1987; Sawrey, 1990). Seminara (1996) proposed a list of content knowledge required for solving stoichiometry problems. Two of them were related to an understanding of proportional relationship: "the coefficients [in the chemical equation] indicate the molar amounts in proportional ratios of reactants to produce and representing the proportionality of the equation must be used in some form to solve for an unknown variable" (pp. 186-187).

It is generally known that students struggle with solving stoichiometry problems. Several researchers have found that such difficulties in problem-solving are linked to lack of conceptual knowledge (Bodner & Herron, 2002; Cracolice, Deming, & Ehlert, 2008; Gabel et al., 1984; Nakhleh & Mitchell, 1993; Nurrenbern & Pickering, 1987; Sanger, Campbell, Felker, & Spencer, 2007; Sawrey, 1990). For example, BouJaoude and Barakat (2003) found a correlation between a conceptual understanding of stoichiometry and the success of problem-solving among 11$^{th}$ grade students. They concluded that conceptual understanding became the most important factor in succeeding in problem-solving. As a result, students' conceptual understanding became an important theme in solving stoichiometry problems. However, conceptual understanding does not guarantee the correct solutions. It has been also observed that a few students have difficulty formulating algorithms even though they have conceptual understanding of the subject (Nakhleh & Mitchell, 1993). Moreover, it has been found that students over-relying on an algorithm for solving stoichiometry problems without a conceptual understanding of the concept. For instance, Gabel, Sherwood, and Enochs (1984) and Nurrenbern and Pickering (1987) found that many of the students were able to successfully solve the problems by application of algorithms without understanding the chemical concepts. There were also students who were not able to solve the stoichiometry problems even with appropriate content knowledge and mathematical abilities because they had difficulty linking the knowledge and skills in different topics (Gulacar et al., 2013).

### Dimensional Analysis and Proportional Reasoning Method

Students' over-reliance on algorithm results from two important findings from Piaget's developmental theory: (a) many high school students were not at the formal operational level, and (b) most science concepts require a formal operational level of reasoning (DeMeo, 2008). According to these findings, it seemed impossible for high school students to learn science. Thus, educators started to find solutions to the problem. One recommendation was to remove content that required formal reasoning. As a result, Herron (1975) suggested dimensional analysis (DA) as a method for solving numerical problems because it does not require formal reasoning ability. DA is also known as unit analysis or unit-factor analysis because the 'unit' guides setting up an equation when solving a problem; "determine the unit of the quantity required. Then string together the available quantities so that all units cancel, except the required one" (Canagaratna, 1993, p. 40). If the units do not match on both sides of the equation, then one can easily notice that there was an error in setting up the equation. Thus, this method is most legitimate in unit conversions (Canagaratna, 1993).

DA became the most favorable and most used problem-solving strategy in chemistry education. DeLorenzo (1994) advocated for the use of DA for solving stoichiometry problems. Though he recognized the general concern over DA, overlooking the underlying concept, he believed that students would eventually understand the underlying meaning as if multiplication tables are first memorized, and then eventually understood. Chemistry college professors even listed DA as one of the topics that students must acquire in their high school years to be successful in college chemistry (Deters, 2003).

Educators advocate the DA method for multiple reasons (DeLorenzo, 1994; Herron, 1975). First, because the DA method is coherently utilized in K-12 education, students are already familiar with this method. Thus, teachers can easily implement it in their lessons. Second, it is beneficial for students who are weak in mathematics. Because the unit of measurement (referred to henceforth as *unit* for unit of measurement) is the only component that should be considered in the DA method to get the right answer, other underlying concepts do not need to be visited while solving problems. Thus, students who are weak in mathematics can also get the correct answer quickly and accurately. Finally, it is fast and simple because the equation is set up in one single line. No multiple steps or equations are required. One line of the equation leads to the correct answer.

However, there are some limitations to the DA method. The overall impression of the DA method is that it is a foolproof procedure of solving problems (Herron, 1975).The biggest challenge of the DA method is that it does not require one to consider the underlying concepts in the problems. The correct answer can be derived by canceling units until the unit for the answer remains. It does not involve the thinking process of why and how the units are being canceled (Canagaratna, 1993; Cook & Cook, 2005; DeMeo, 2008).Thus, Cook and Cook (2005) concede that though DA is a "powerful and highly efficient method" (p.1187), it is unsuitable for introductory students because of its absence of the conceptual knowledge. Canagaratna (1993) also pointed out the limitation of DA for its ambiguities. First, when the equation uses similar units, students can easily be confused with operating the equation. For example, when calculating a volume of a cylinder, students guess which component to square, either the radius or height of a cylinder, because both components uses the same unit, centimeter. Unless they see the that the volume is a sum of planes, they cannot make a right decision to solve the problem. Second, when the different physical quantities have the same units, the students get confused deciding which formula to use and, even worse, the concept of the two different physical entities might be fuzzy. For example, Joule could relate to a variety of quantities such as heat, work, internal energy, enthalpy, and Gibbs free energy, which all are different concepts that use different equations. Finally, though DA seems like a safe pass for solving numerical problems, it should be recognized that it only works with a direct proportionality relationship.

Dimensional analysis (DA) and proportional reasoning (PR) are the two most popular strategies for solving proportion problems across science disciplines (DeMeo, 2008). Before the DA method was introduced, the PR was more prevalent for solving problems because it delivers the proportional relationship conceptually. However, there was a sudden shift from the PR to the DA method around 1960s (DeMeo, 2008). Three possible reasons for such a change was suggested by DeMeo (2008). First, it might be caused by a sudden change in textbooks, study aids, and video for use on the secondary level by CHEM study in the 1960s. Though he did not specify explicit reasons for the change, he assumed that sudden change in the textbook might have turned the whole trend of using PR to DA. Second, standardization of the metric system, the international system of measurement (SI), suggested in 1960 at the international meeting might have caused this change. This new standardization of units for the physical quantities aligns with DA method in that it emphasizes unit. Thus, DA might have been highlighted as a need to this change. The final reason for the change comes from the difficulty level of the problems. As students have trouble solving problems with the PR method, teachers look for other instructional strategies and new ways to explain the content. This might have caused the teachers to use DA rather than the PR method for solving a problem.

Both DA and PR are the most popularly used methods in solving science problems because both are similar in that it is "reliable, easy to use, understandable, and applicable to all four science disciplines that compose a secondary and introductory college science curriculum" (Demeo, 2008, p.12). The difference between the DA and PR method can be noticed from Table 1. First, the PR method usually requires more steps than the DA method. Second, when you see the first step of DA and the second step of PR, the equation eventually becomes the same. Thus, it might seem like there is hardly any difference other than the mathematical skills that are required. However, the information that the two methods convey is very different. While PR explicitly recognizes the quantities and the proportional relationship, the DA method only focuses on the unit and does not show this relation. Thus, literature that advocates the use of PR in solving science problems for its conceptual advantage in its method (Canagaratna, 1993; Cook & Cook, 2005).

Though many educators have assumed that successful problem solving implies a good understanding of the content, "the ability to solve problem is different from solving with understanding" (DeMeo, 2008, p. 101). The purpose of solving numerical problems is not only getting the correct answer. It expects the students to apply the conceptual knowledge that they have obtained in an appropriate way to solve the problems. Unfortunately, the DA method does not require explicit recognition of conceptual knowledge. Some instructors regard this as an advantage for the academically weak students, especially in mathematics, because they can obtain a correct answer just by looking at the units (Canagaratna, 1993).

**Table 1**

*Examples of Dimensional Analysis and Proportional Reasoning method*

| Questions: How many kilograms are equal to 125 grams given that 1000 g=1 kg? | |
|---|---|
| Dimensional analysis (DA) method | Proportional reasoning (PR) method |
| Step 1. x kg = (125 g)($\frac{1 \text{ kg}}{1000g}$)<br><br>Step 2. x kg = 0.125 kg | Step 1. $\frac{\text{x kg}}{125 \text{ g}} = \frac{1 \text{ kg}}{1000 \text{ g}}$<br>Step 2. Cross multiple the two ratios and isolated x by multiplying 125 g on both side of the equation<br>(x kg)(1000 g) = (1 kg)(125g)<br>x kg = $\frac{(125 \text{ g})(1 \text{ kg})}{1000g}$<br>Step 3. x g = 0.125 kg |

Within the PR method, the most used algorithmic method for solving proportion problems is cross-multiplication. Once the two ratios (or fraction) are set to be equal, students automatically cross-multiply the two equated ratios (fractions) without understanding the relationship between the two. In other words, the denominator of one fraction and the numerator of another fraction are multiplied; and it equates the other fraction. Therefore, even the PR method needs attention in understanding the conceptual knowledge that underlies stoichiometry problems.

**Teaching Stoichiometry Problem-solving**

The thinking of teachers when solving stoichiometry problems is shaped by an adult understanding of the problem. Thus, an algorithmic approach that they have mastered often does not resemble the way the students approach the problem. To enhance a student's understanding of proportional reasoning, it is critical to analyze and understand student's mathematical thinking in relation to that of adults (Kastberg et al., 2012; Lobato & Ellis, 2010; Lobato et al., 2011).

Stoichiometry is a difficult topic not only for secondary school students to understand, but also for teachers to teach (Furio, Azcona, & Guisasola, 2002). Especially, teaching stoichiometry calculation is a difficult task (Schmidt, 1990) because it is an abstract concept that requires an understanding of qualitative aspects of chemical reactions. Consequently, teachers' understanding of the difficulties students encounters when solving stoichiometric problems is essential for designing and utilizing appropriate instructions (BouJaoude & Barakat, 2000). Likewise, teachers should have sufficient knowledge and strategies in order to effectively support students' development of proportional reasoning (Kastberg et al., 2012).

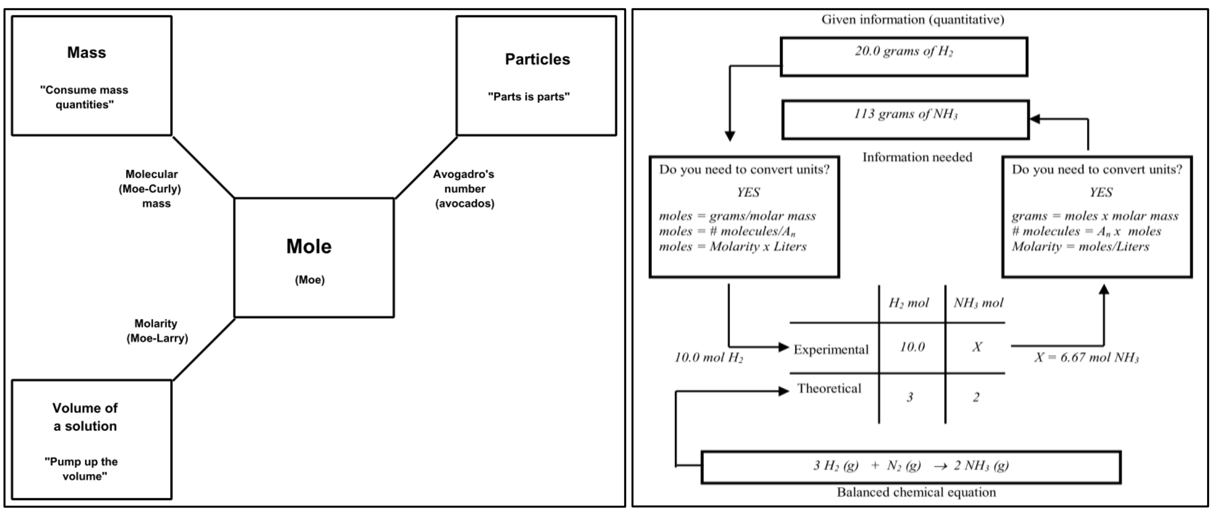
**Instructions and Strategies for Teaching Stoichiometry**

Generally, three instructional approaches have been introduced to overcome a blind manipulation of symbols without understanding the concepts involved: (a) an approach that emphasizes the qualitative aspect of the concepts involved in the mathematical equation; (b) an approach that emphasizes model development; and (c) an approach which uses physical materials (de Berg, 1992). For example, BouJaoude and Barakat (2000) suggested two key aspects of teaching problem-solving. The first is to make sure that students clearly understand the concepts before they start solving problems. This allows students to understand the content conceptually before going into a quantitative perspective. The second aspect requires students to think rather than to simply memorize and use algorithms. As some students place too much emphasis on the use of algorithms, even when it is not necessary, this allows students to develop their own mental model and make a decision on when it is necessary to use a formula (Dahsah & Coll, 2007).

Other teaching instructions have been suggested for improving students' understanding of stoichiometry. For example, Krieger (1997) suggested the creation of a flow chart based on The Three Stooges (Figure 1). Wagner (2001) also proposed each student's creation of a Mole Ratio Flow Chart (Figure 1) that emphasizes proportional relationship and showed the efficacy of the chart in helping students to solve stoichiometry problems. Flow charts are also suggested for visualizing how amounts of substances are represented (Ault, 2001) and for stoichiometry in chemical engineering (Felder, 1990). Witzel (2002) recommended teaching stoichiometry using LEGO blocks. Haim, Cortón, Kocmur, and Galagovsky (2003) suggest visualizing stoichiometry by the creation of hamburgers. Finally, DeMeo (2008) proposed using a graphic organizer, such as a decision map, for solving stoichiometry because it is believed to integrate conceptual and procedural knowledge.

**Figure 1**

*The Three Stooges (Left) and Mole Ratio Flow Chart (Right)*




## Teachers' Knowledge and Belief

The *Framework* (NRC, 2012) stated that "well-designed instruction is needed if students are to assign meaning to the types of ratios and proportional relationships they encounter in science" (p. 91). Moreover, it was found that student performance on proportional reasoning was improved when teachers intervened students' performance by asking questions or feedback (Farrell & Farmer, 1985). As such, teachers' role in the classroom is critical in developing students' proportional reasoning. The role of teachers in teaching proportional reasoning is supported by a body of research that explored the effect of science teachers' knowledge and beliefs on all aspects of their teaching (Carlsen, 1991; Smith & Neale, 1989).

**Teachers' Beliefs**

Teachers' beliefs are known to be a critical factor for predicting their professional practices (Jones & Carter, 2006). According to Keys and Bryan (2001), every aspect of teaching is influenced by the complex web of attitudes and beliefs that teachers hold, such as knowledge acquisition and interpretation and choices of assessment. Therefore, to understand the teacher's practice of teaching, it is necessary to explore what beliefs science teachers hold. Cronin-Jones (1991) suggested four major categories of beliefs that influenced the curriculum implementation process: "(a) beliefs about how students learn, (b) beliefs about a teacher's role in the classroom, (c) beliefs regarding the ability levels of students in a particular age group, and (d) beliefs about the relative importance of content" (p. 246). These four categories were employed in this study to reveal what beliefs chemistry teachers hold about teaching proportional reasoning in their stoichiometry instructions.

**Teachers' Knowledge**

In the 1980s, Lee Shulman, an educational psychologist, proposed three domains of teacher knowledge: (a) content knowledge, (b) pedagogical content knowledge (PCK), and (c) pedagogical knowledge. Among these domains, PCK has drawn the most attention by educators. Shulman (1986) defined PCK as "a particular form of content knowledge that embodies the aspects of content most germane to its teachability" (p. 9). It is a unique knowledge for the profession of teachers that includes knowledge of how particular subject matter topics, problems, and issues can be presented to a diverse level of learners.

Shulman's (1986) study has been incorporated by other researchers into a model of teacher knowledge containing four domains: subject matter knowledge, general pedagogical knowledge, PCK, and knowledge of the context (Grossman, 1990). This model also delineates Shulman's conception of PCK into four more finely grained components—(a) knowledge for teaching subject matter, (b) knowledge of students' understanding, (c) curricular knowledge, and (d) knowledge of instructional strategies. The first component of PCK, the purpose of teaching subject matter, includes teachers' overarching conceptions, knowledge, and beliefs regarding the goals for teaching their subject matter. The second component, knowledge of students' understanding, involves an awareness of what students already know about a topic and what they are likely to find challenging. The third component of PCK, curricular knowledge, includes information about available curricular materials and knowledge about how particular content is developed in the curriculum prior to, during, and after a given grade level. Finally, the fourth component, knowledge of instructional strategies, addresses the set of explanations, illustrations, representation, metaphors, and activities that teachers have found to be effective for teaching particular topics (Grossman, 1990).

Building upon the work of Grossman (1990) and Tamir (1988), Magnusson, Krajcik, and Borko (1999) defined PCK as a "teacher's understanding of how to help students understand specific subject matter" (p. 96). They conceptualize five components of PCK for teaching science: (a) orientations toward science teaching, (b) knowledge and beliefs about science curriculum, (c) knowledge and beliefs about students' understanding of specific science topics, (d) knowledge and beliefs about assessment in science, and (e) knowledge and beliefs about instructional strategies for teaching science. Because PCK is "integral to effective science teaching" (p. 96), it is necessary to understand the PCK that teachers possess in order to explore the effectiveness of teaching. Thus, these five components are employed in this study to explore chemistry teachers' PCK in teaching proportional reasoning in stoichiometry.

**Teachers' Knowledge for Solving Proportion Problems**

According to Lenton and Stevens (2000), the child's development of process skills and conceptual thinking depends on the teacher's experience in overcoming the difficulties that the students face when learning the same material. This reflects the importance of teachers' experiences becoming proportional reasoners themselves with proper content knowledge to facilitate students' development of proportional reasoning (Thompson & Bush, 2003). At the same time, teachers should know how to develop students' proportional reasoning appropriately. For example, as proportional reasoning is not simply an ability to set up proportions and solve by cross-multiplication (Lobato & Lester, 2010), teachers should not only know how to support and assure that the students' have conceptually understood the concept.

Unfortunately, it was found that neither mathematics teachers nor science teachers have sufficient knowledge on proportional reasoning, as well as knowledge for teaching it (Kastberg et al., 2012; Lenton & Stevens, 2000; Lobato et al., 2011). Lenton and Stevens (2000) found that science teachers have their own conceptual difficulties with proportional reasoning. This also aligns with Lobato et al. (2011) study that "many elementary and middle grades [mathematics] teachers and prospective teachers lack a deep understanding of proportional reasoning and rely too heavily on rote procedures such as the cross-multiplication algorithm" (p. 3). Moreover, Watson, Callingham, and Donne (2008) found that teachers were challenged when asked to craft instructional interventions and develop productive lines of questioning to support students' proportional reasoning. This may be due to the fact that proportional reasoning is rarely explicit in the K-12 curriculum. As many teachers only have secondary experience of doing proportional reasoning, they may not have possessed sufficient knowledge, especially pedagogical content knowledge, to solve and teach proportional reasoning (Kastberg et al., 2012; Lobato et al., 2011).

## Summary

While reviewing the theoretical discussions of proportional reasoning, it was realized that a great deal of research on proportional reasoning was mostly conducted in the 1970s and 1980s. Though proportional reasoning is considered an important concept in mathematics and science learning, there are not many studies in recent years compare to the 1970s. Moreover, though proportional reasoning is an essential concept in both mathematics and science learning, most studies were conducted in the field of mathematics education compared to science education. However, more studies in the field of science education are necessary as the concepts of ratio and proportion are prevalent in all science disciplines (Hilton & Hilton, 2016; NRC, 2012; Taylor & Jones, 2009). Moreover, as the *Framework* (NRC, 2012) pointed out, the use of ratio in science is different from a ratio of numbers describing fractions of a pie and thus different from how mathematicians use these compared to science and stoichiometry specifically. In science, it must be understood in the context of different types of quantities, often discussed in a small amount. As Hilton and Hilton (2016) pointed out, learning science has great potential for improving students' proportional reasoning ability because it not only helps students to understand the science concepts that involve mathematics, but it may also extend and challenge students' mathematical understanding of these concepts (NRC, 2012). Thus, science education research is needed to understand the concept of proportional reasoning. This study addresses this concept in chemistry education.

Overall, the literature on proportional reasoning and stoichiometry mostly examined students' understanding of them. There are fewer studies that examine how teachers teach and perceive teaching these concepts. In mathematics education, some studies explored mathematics teachers' understanding and their practice of teaching proportional reasoning. However, when it gets to science teachers, the number significantly drops. Considering the demand to incorporate mathematical thinking and the concept of proportion in the related science concepts from the *Framework* (NRC, 2012) and NGSS (NGSS Lead States, 2013), more investigation is needed regarding science teachers' practice on delivering them, difficulties they face, and the supports they need in incorporating proportional reasoning in their classroom. Moreover, in the topic of stoichiometry, there are few studies that investigate teachers' understanding of stoichiometry, problem-solving strategies in learning stoichiometry, and difficulties teaching stoichiometry problems. Thus, it is a field that needs attention as teachers' teaching practice is an important factor in students' learning of stoichiometry.

Both conceptual knowledge and procedural knowledge are required for solving stoichiometry problems. Many concepts for conceptual knowledge, such as mole, molar mass, balancing equation, algebraic skills, and interpreting a word problem into procedural steps (Wagner, 2001), are required for solving those problems. However, literature mostly focused on improving students' understanding of scientific conceptual knowledge, such as the concept of the mole. Again, there are few studies that explored the understanding of ratio and proportion in science education, which is also required knowledge for solving stoichiometry problems (Padilla & Garritz, 2011). Therefore, this study is expected to reveal a new perspective on stoichiometry instruction.

Finally, related to science teachers' knowledge and beliefs, though there are many studies that measure teachers' epistemological beliefs on science, little is known about their beliefs on teaching specific concepts related to stoichiometry. Especially, after the *Framework* (NRC, 2012) and NGSS (NGSS Lead States, 2013) suggested integrating all three dimensions—crosscutting concepts, science and engineering practices, and disciplinary core ideas—teachers' perceptions of teaching them are needed. Therefore, this study explores in-service chemistry teachers' understanding of three-dimensional learning, especially the concept of proportion and its understanding when teaching stoichiometry, and their perceptions on teaching it.

# Chapter III: Methodology

Proportional reasoning is a common scientific practice that scientists engage in (NGSS, 2013). For example, in planning the reaction of nitrogen and hydrogen to make ammonia, a chemist prepares the exact amount of nitrogen and hydrogen (reactants) based on the chemical reaction to get the needed amount of the product (ammonia). An area that studies the quantitative relationship between two entities in chemistry is called stoichiometry. As the quantitative relation is based on a chemical reaction, which is a statement of proportionality of the entities (reactants), manipulating the proportionality is necessary for understanding stoichiometry. Therefore, proportional reasoning becomes a central scientific and mathematical practice in learning stoichiometry. Students' engagement in such practices is crucial not only to understand the concept of stoichiometry but also to understand the work of science (NRC, 2012).

Though proportional reasoning is closely associated with solving stoichiometry problems, there is a lack of certainty with respect to how chemistry teachers understand proportional reasoning and how they incorporate it into their teaching of stoichiometry. Thus, this study explored chemistry teachers' knowledge on proportional reasoning and their belief in integrating it when teaching stoichiometry. The following research questions were addressed:

1. What beliefs does a sample of chemistry teachers hold about teaching proportional reasoning and stoichiometry?
2. What problem-solving strategies were used by a sample of chemistry teachers when solving proportional reasoning and stoichiometry problems?
3. What pedagogical content knowledge (PCK) do chemistry teachers bring when asked about how they would teach stoichiometry focusing on proportional reasoning?

4. What characteristics do three of the chemistry teachers show based on the evidence of Likert scale survey, proportion problems, and interview?

## Research Approach

This study used a mixed method research approach by combining the elements of qualitative and quantitative research approaches (Niaz, 2008; Onwuegbuzie & Leech, 2005). Earlier studies on proportional reasoning often employed paper-pencil assessment to measure students' understanding and their problem-solving strategies. Their responses were often categorized either as correct and incorrect. However, limitations in this assessment can "overestimate the ability since a correct answer can be generated from non-proportional reasoning" (Tourniaire & Pulos, 1985, p. 183); therefore, a qualitative research methodology (e.g., interview) was also incorporated. Therefore, a mixed-method design is appropriate (Niaz, 2008; Onwuegbuzie & Leech, 2005).

Among the five approaches for qualitative research (Creswell, 2013), multiple case study was employed because this study "entails looking at a specific example (e.g., each chemistry teacher's understanding and teaching of proportional reasoning) in a specific context (e.g., teaching stoichiometry) to shed light on a larger more generalized issue (e.g., chemistry teachers' teaching of proportional reasoning practice in science context)" (DeMeo, 2008, p. 34). Therefore, this study collected both quantitative data (e.g., tests and surveys) and qualitative data (e.g., interviews) to explore chemistry teacher's belief and knowledge on proportional reasoning and on their teaching of stoichiometry.

### Participants and Settings

Crouch and McKenzie (2006) suggested around 10-15 participants for qualitative research methods. Therefore, ten chemistry teachers participated in this study. When recruiting the participants, an invitation email with a flyer was sent out to teacher education institutes, professional development organizations, and teacher organizations located in the New York City (NYC) metropolitan area. As a result, about 15 responded back, and ten teachers from this group fully participated in the study. Only teachers with at least three years of teaching chemistry in the U.S. were considered because K-12 teachers, in general, get tenure around their third or fourth year of teaching as a professional marker. Also, according to van Driel, Verloop, and de Vos (1998), around this time, teachers build up their own PCK based on their teaching experiences.

The 10 participants had a diverse background in terms of gender, race/ethnicity, years of teaching, major, and highest degree achieved (four males and six females; seven White, two African-American, and one other ethnicity; six with Master's degree and four with Ph.D. degree). While there were six participants from different schools in different districts, there were four teachers (Jessica, Jeffry, Stella, and Lena) who were from the same school. Each participant's demographic information is summarized in Table 2.

A short questionnaire was given to explore (a) demographic information (e.g., gender and years of teaching chemistry), (b) contextual information (e.g., school information, a sequence of the school's science curriculum, requirements for the students for taking chemistry), and (c) perceptual information (e.g., teachers' belief on prerequisite class for taking chemistry). All three sets of information played a critical role in understanding the based line of participants' teaching practices.

The demographic information assured that the teachers meet the criteria for participating in the study (at least three years of teaching) and the diversity of the participants. As a result, the range of teachers' years of chemistry teaching was from three years to around 15 years. In this study, two teachers were from a private school, and the rest of the eight teachers were teaching at public schools. School type was important information in understanding teacher's chemistry curriculum knowledge. In public schools, the curriculum is usually given with a specific learning goal and guide pacing, which allows less flexibility in modifying curriculum than private school teachers. Thus, this information was helpful for understanding the teacher's chemistry curriculum knowledge, which is one category of the PCK framework.

**Table 2**

*Participants Demographic Information*

| | Gender | Years of teaching chemistry | College major | Highest degree |
|---|---|---|---|---|
| Jake | M | 4 years | Chemistry | Master's |
| Annie* | F | 6 years | Chemical Engineering | PhD (Pursuing) |
| Tina | F | 13 years | Chemistry | PhD |
| John | M | 13 years | Biology | PhD |
| Levy | F | 6 years | Biology | Master's |
| Jack | M | 3 years | Chemistry | Master's |
| Jessica | F | 8 years | Chemistry | Master's |
| Jeffry | M | >10 years | Chemistry (Minor: Math) | PhD (Pursuing) |
| Stella | F | 5 years | Chemistry | Master's |
| Lena | F | 14 years | Chemical Engineering | Master's |

*Note.* * Does not have a teaching certificate

Student's grade, level of chemistry class (e.g., honors or general chemistry class), and a prerequisite for taking chemistry were also valuable as it informs the teacher's knowledge about students. All the teachers were teaching chemistry to $10^{th}$ or $11^{th}$ grade students. Each school had a different prerequisite for taking a chemistry class. Some schools had mathematics class, and the other schools had a science class as a prerequisite. For example, Annie's school placed Algebra 2 /Trigonometry class as a prerequisite while living environment class was a prerequisite for Levy's school. The level of chemistry class that the teachers taught varied. About half of the teachers (6 out of 10) taught both two levels of classes: lower level (e.g., general level) and higher level (e.g., honors) chemistry. These were evitable pieces of evidence in understanding teacher's instructional knowledge as their expectation and preparation for the class varies based on students' level and ability. Therefore, the contextual information collected through this questionnaire was significant for understanding the baseline of teacher's teaching and pedagogical content knowledge. Each participants' contextual information, particularly related to school demographics, is summarized in Table 3.

**Table 3**

*Participants' School Contextual Information*

| | School Type/Location | Level of Chemistry Class Currently Teaching | Students' Grade in Chemistry Class | Prerequisite for Taking Chemistry Class | Number of students in one class |
|---|---|---|---|---|---|
| Jake | Public/ City | General | 10 & 11 | N/A | 26 - 30 |
| Annie | Private/ Rural | General & Honors | 10 & 11 | Algebra 2 /Trigonometry | 16 - 20 |
| Tina | Public/ City | General & Honors | 10 | Algebra 1 | 16 - 20 |
| John | Private/ City | Modified*, General, & AP | 10 | N/A | 6 - 20 |
| Levy | Public/ City | General | 10 & 11 | Living Environment | 21 - 25 |
| Jack | Public/ City | General | 11 | N/A | 21 - 25 |
| Jessica**** | Public/ Suburban | College-prep** & Honors | 10 | Physics | 21 - 25 |
| Jeffry**** | Public/ Suburban | General & AP | 10 | Physics | 21 - 25 |
| Stella**** | Public/ Rural | General & Honors | 10 | Physics | 27 max |
| Lena**** | Public/ Suburban | General & Honors | 10 | Epstein test*** & Physics | 20 - 26 |

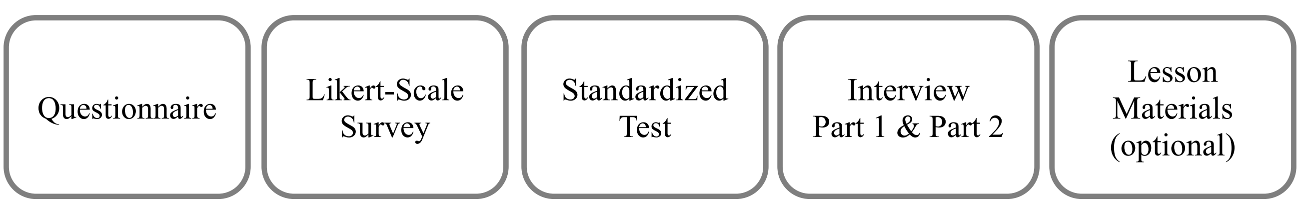
*Note.* * Lower-level chemistry than the general class
** Between general and honors class
*** A set of mathematics tests used for assessing students' mathematics ability
****All from the same school

## Data Collection

As this study employed a mixed method design, both quantitative (e.g., Likert-scale surveys and standardized tests) and qualitative data (e.g., interviews and lesson materials) were collected at each teacher's school classroom. On the day of the interview, as a first step, a questionnaire was given to collect the participants' demographic, contextual information of their teaching and school. Then, a Likert-scale survey was given to explore the teacher's belief in incorporating proportional reasoning during stoichiometry instruction. After the survey, they solved a standardized test, which included four proportion problems and one stoichiometry problem. Finally, an interview was conducted after solving the standardized test of problems. The first part of the interview was a think-a-loud interview where participants explained their problem-solving strategies for the five problems. Then, the interview transitioned to a stoichiometry teaching perspective. By adopting the Content Representation (CoRe) instrument developed by Loughran, Mulhall, and Berry (2004), the teacher's PCK for teaching proportional reasoning in stoichiometry instructions was probed. At the end of the interview, teachers were asked to share their lesson materials or lesson plan on stoichiometry. Figure 2 summarized the data collection process on the day of the interview. The following section describes how each piece of evidence was developed and collected in detail.

**Figure 2**

*Order of Data Collection Process*



## Quantitative Data Collection

### *Likert Survey*

After the questionnaire, the 5-point Likert scale survey was given to explore the teachers' beliefs and perceptions towards teaching stoichiometry with a focus on proportional reasoning practice (Appendix C). The survey items were developed by the researcher based on four major categories that are found to be most influential on teachers' curriculum implementation process: (a) beliefs about how students learn, (b) beliefs about a teacher's role and confidence, (c) beliefs regarding the ability levels of students in a particular age group, and (d) beliefs about the relative importance of content (Cronin-Jones, 1991). For each category of belief, five items were developed, except for the category of belief on the teacher's role and confidence in integrating proportional reasoning in stoichiometry instructions. This category was divided into two parts—belief on teachers' role and belief on teachers' confidence—and each part had five items. In general, the items within one category targeted teacher belief on proportional reasoning, stoichiometry, and integrating the two, respectively. For example, when developing items for the last category of belief—teacher's beliefs about the importance of content—one survey item asked whether the teacher believed proportional reasoning was important for students to learn, the other item focused on whether the teacher believed stoichiometry was an important topic for students to learn, and the next item asked whether the teacher believed integrating proportional reasoning in their stoichiometry instruction was important. Finally, the last item was added to examine whether the teacher believed they needed more support to involve mathematics practices in their chemistry instruction. As a result, a total of 26 items were developed for the Likert-scale survey.

### *Standardized Test*

After the survey, teachers solved a set of standard problems designed to explore their problem-solving strategies for proportional reasoning and stoichiometry problems. This test included five problems in total-- four proportion problems and one stoichiometry problem (Appendix A). The four proportion problems were intended to unpack participants' conceptual knowledge and procedural knowledge for solving proportion problems. The one stoichiometry problem, which was also a proportional reasoning problem, was included to understand how the teachers taught these problems. For all five problems, teachers were asked to solve each problem in two different ways in order to prevent teachers from solving the problem without thinking. As these problems were adopted from the studies that measured students' proportional reasoning ability, the problems may be too easy that they are likely to solve the problems without deeply thinking about the underlying concept. In addition, multiple ways of solving the proportional reasoning problem is an indicator of having a concrete understanding of the concepts (DeMeo, 2008).

Stoichiometry problem (Appendix A: Problem 1) was derived from a well-known chemical reaction, Haber–Bosch chemical reaction. This chemical reaction was selected not only because it is one of the most famous reactions in chemistry but also because the coefficients in the chemical equation are not as simple as a 1: 1 ratio or 1: 2 ratio (DeMeo, 2008). There are three commonly used stoichiometry problems; (1) mole-to-mole problem, which has one proportional relationship, (2) mole-to-mass problem, which has two proportional relationships, and (3) mass-to-mass problem, which has three proportional relationships. In this study, the mass-to-mass problem was selected because it requires multiple steps to solve. For example, the stoichiometry problem used in this study involves three different proportional relationships: (a) between grams and moles of product ($NH_3$), (b) between moles of product ($NH_3$) and reactant ($H_2$), and (c) between grams and moles of reactant ($H_2$). In order to solve this problem, all these three relationships must be visited. More importantly, between the two different methods that they used for solving the stoichiometry problem, teachers were asked to choose one method that they prefer to use in front of their students. They also choose one method that they preferred to use when solving it by themselves, as an adult. This question was intended to capture teachers' awareness of the gap between their approach for solving the problems and that of the students' as the literature found that teachers' algorithmic approach often does not resemble the way in which students approach problems (Kastberg et al., 2012; Lobato & Ellis, 2010; Lobato et al., 2011).

The four proportion problems were carefully selected from previous studies on proportional reasoning. As a result, the only missing-value types with the context of rate and mixture tasks were selected because those are the most commonly seen types and contexts of proportion problems in science education. The first proportional reasoning problem was a plastic jar problem (Appendix A: Problem 2), which was adopted from the Group Assessment of Logical Thinking (GALT). This assessment is a popular tool for measuring formal reasoning ability in science education. As proportional reasoning is considered to elucidate formal reasoning ability, the proportional reasoning problem in this assessment was adopted. The second problem was a density problem (Appendix A: Problem 3), and it was included because it is the most critical concept in science that all students learn prior to taking chemistry and because it is the most relevant proportional reasoning context of learning stoichiometry. The orange juice problem (Appendix A: Problem 5) is the most well-known proportional reasoning task, which originated from Noelting's (1980) study and modified by Kouba and Wearne (2000). Finally, the non-proportion problem (Appendix A: Problem 4), adopted from Cramer, Post, and Currier's

(1993) study, was included because identifying proportional relationship is the most critical ability in proportional reasoning. Thus, this non-proportion problem was added to reveal teachers' competency in proportional reasoning ability.

## Qualitative Data Collection

### *Interviews*

Before this study, a pilot study was conducted. From the pilot study, minor changes were made to the interview questions. For example, two sub-interview questions were added, specifically related to teaching instruction. A question on how teachers transition from the Smore's analogy to the chemistry context was added not only because the analogy was commonly referred for teaching stoichiometry but also because literature found that students' performance of proportional reasoning depended on the familiarity of the context of the problem. In addition, in the pilot study, I noticed that teachers stated their different instructions for different levels of chemistry classes. Their stoichiometry instruction varied based on students' understanding of chemistry and their mathematics skills. Thus, how their instruction differed between the two levels of chemistry class was added. The main interview questions can be found in Appendix B. The interviews were conducted individually with the teachers at their school.

This qualitative data collection via interview was conducted for two reasons. First, it is generally known that paper-pencil assessments inform limited insight into one's understanding of a specific concept. Thus, interviewing chemistry teachers was inevitable in order to deeply understand their understanding of proportional reasoning. Secondly, as Loughran et al. (2004) found, the teacher's PCK is hard to capture because it is a multifaceted and internal construct. Therefore, teachers should articulate their PCK using their own language. The interview was conducted in two parts.

**Interview Part 1.** The first part of the interview focused on the five problems that the teachers solved. It was mostly a think-a-loud interview where participants were asked to explain their two methods of solving the five standardized test problems. In addition, they explained how they think the two methods were different. Finally, they were asked to choose one method they prefer to use as an adult. These questions were expected to provide deeper information on teachers' knowledge and competency in proportional reasoning.

**Interview Part 2.** The second part of the interview focused on exploring participants' PCK for delivering proportional reasoning during stoichiometry instruction. When examining each teacher's PCK, modified Content Representations (CoRe) interview questions, developed by Loughran, Mulhall, and Berry (2004) were used. CoRe questions were developed to explore and gain insights of the expert science teacher's PCK. It "sets out and discusses science teachers' understanding of particular aspects of PCK (e.g., an overview of the main ideas; knowledge of alternative conceptions; insightful ways of testing for understanding; known points of confusion; effective sequencing; and important approaches to the framing of ideas)" (Loughran et al., 2004, p. 376). In the original CoRe questions, there are eight questions. These eight questions were modified by the researcher to target the PCK categories suggested by Magnusson, Krajcik, and Borko (1999): (a) orientations toward science teaching, (b) knowledge and beliefs about science curriculum, (c) knowledge and beliefs about students' understanding of specific science topics, (d) knowledge and beliefs about assessment in science, and (e) knowledge and beliefs about instructional strategies for teaching science. Based on this modification and the piolet study, ten interview questions were used as a final version.

### *Lesson Plan and Materials*

To further understand how the teachers teach and support the proportional reasoning practices in stoichiometry lessons, they were asked to submit lesson materials, such as a lesson plan, practice packet, or tests, at the end of the interview. Except for Jessica and Jeffry, the other eight participants shared their lesson materials. Through these materials, the researcher was able to have a concrete view of stoichiometry instruction.

## Data Analysis

Data analysis was done after the data was collected from all the ten teachers.

### Quantitative Data Analysis

Regarding the quantitative data sources, a Likert-scale survey and standardized test problems were given to the participants. Both data sources were analyzed by the percentage of responses to each item.

#### *Likert Survey*

The Likert scale survey, which revealed the chemistry teachers' beliefs in teaching proportional reasoning when teaching stoichiometry, was analyzed by the percentage of responses made to each of the items from strongly disagree to strongly agree using Microsoft Excel. This allowed for a better display of the distribution of responses to each item and also to satisfy the statistical requirements for ordinal data (Sullivan & Artino, 2013).

#### *Standardized Test*

Ten teachers solved five problems in two different ways, and each strategy was considered as if twenty teachers solved the problem once. Thus, the total number percentage of responses was 20 (N=20).

The five problems of proportional reasoning and stoichiometry were analyzed using the framework suggested by DeMeo (2008) and Cramer and Post (1993). The problem-solving strategies used for solving the five problems were first sorted into two categories—either the dimensional analysis method or the proportional reasoning method. Within the dimensional analysis strategy, it was sorted into three sub-strategies: (a) Dimensional analysis (DA), (b) Dimensional analysis T-chart equation (DA-T chart), and (c) Dimensional analysis one chain equation (DA-Chain) strategies. The details and example of each strategies are summarized in Table 4.

**Table 4**

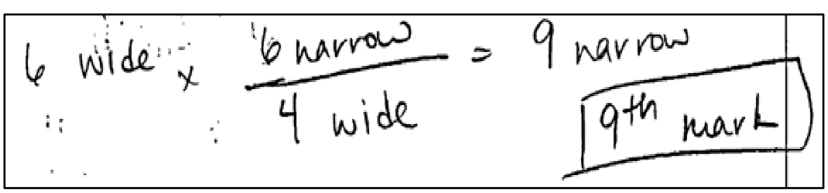
*Dimensional Analysis Codes for Problem-solving Strategies*

| | Strategy | Characteristics |
|---|---|---|
| Dimensional analysis (DA) | Dimensional analysis (DA) | • DA equation for one single proportional relationship |
| | T-chart equation (DA-T chart) | • DA equation in a format of T-chart for multiple proportional relationships |
| | One chain equation (DA-Chain) | • One single equation for multiple proportional relationships |

Dimensional analysis (DA) code was used when the one-dimensional analysis equation was set for a single proportional relationship (Figure 3). It is usually written with a whole number in the front multiplied by the fraction. This is usually how proportion problems are taught in mathematics class.

**Figure 3**

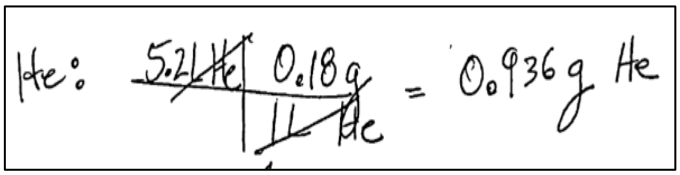
*Annie's Example of Dimensional Analysis (DA) Strategy for Solving Proportion Problems*




Dimensional Analysis T-chart equation (DA-T chart) was added because this is a unique way of writing a dimensional analysis equation that science teachers often use to minimize the confusion of multiplying and dividing the fractions (Figure 4). In other words, in order to prevent students from getting confused with calculating the numbers, whether to multiply or divide the numbers, chemistry teachers often use T chart equations. In this way, students clearly understand that the top numbers in the T chart are for multiplying, and the bottom numbers are for dividing.

**Figure 4**

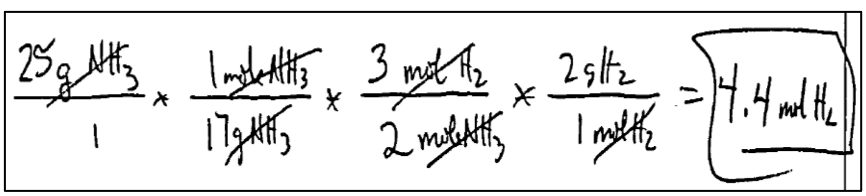
*Tina's Example of Dimensional Analysis T-chart Equation Strategy for Solving Proportion Problems*




Finally, Dimensional Analysis One chain equation (DA-Chain) strategy is a chain equation of DA strategy, which is frequently used when there are more than two steps of changing units (Figure 5).

**Figure 5**

*Jeffery's Example of Dimensional Analysis One Chain Equation Strategy for Solving Proportion Problems*




Cramer and Post (1993) suggested four strategies for solving proportion problems: (a) unit-rate, (b) factor-of-change, (c) fractions, and (d) cross-product algorithm. This study adapted these strategies for analyzing the four proportion problems (jar, density, stoichiometry, and orange juice problem). As mentioned in Chapter 1, there are some differences in understanding of proportional reasoning in science as compared to mathematics because the extent of understanding the proportion required for learning science may differ from that of mathematics education. Thus, the four strategies for solving proportion problems have been adapted and modified to reflect the use of proportion in science education. The details and examples of each code are described in Table 5.

**Table 5**

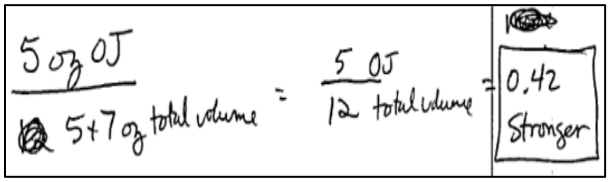
*Proportional Reasoning Codes for Problem-solving Strategies*

| | Strategy | Characteristics |
|---|---|---|
| Proportional Reasoning (PR) (Cramer & Post, 1993) | Unit-rate (PR-UR) | • Incorporates the concept of the unit-rate with two different entities in a pair (e.g., 20 miles/minutes)<br>• Used ‘x amount per y amount,’ ‘x amount for every y amount,’ and ‘assuming there are same amount/speed of x’ wording |
| | Factor-of-change (PR-FC) | • Focus on the multiplicative relationship between the two given numbers<br>• Focus on the proportional relation of given numbers<br>• Does not set up an equation<br>• Used ‘times as many as’ or ‘times as’ wording |
| | Fractions (PR-F) | • Set up an equation of two equivalent fractions<br>• No units used for setting up two equivalent fractions<br>• Apply fraction rule (multiply both in numerator and denominator) |
| | Cross-product algorithm (PR-CP) | • Proportional relationship described with units<br>• Using an arrow to show cross-multiplication for calculating the two-equivalent fraction<br>• When two equivalent fractions were in a non-integer multiplicative relationship |
| Other | Qualitative | • Does not involve the use of mathematics<br>• Verbally describes the steps/logistics/manipulates for solving the problem<br>• Compare quantity by less/more |

According to Cramer and Post (1993), the unit-rate (PR-UR) strategy incorporates two different entities in a pair of quantities, such as 20 minutes/4 miles (Figure 6). Students are more familiar with this strategy by a phrase such as 'how many for one’ (Cramer & Post, 1993, p. 406). Thus, whenever teachers used the concept of the unit, it was coded as a unit-rate (PR-UR) strategy; ‘x amount per y amount,’ ‘x amount for every y amount,’ and ‘assuming there is same amount/speed of x’ are example phrases for capturing the teachers’ use of this strategy in chemistry. Before moving on to the next strategy, I would like to point out that the unit-rate strategy should not be confused with a unit-rate method, which is just another name for dimensional analysis method often used in the literature.

**Figure 6**

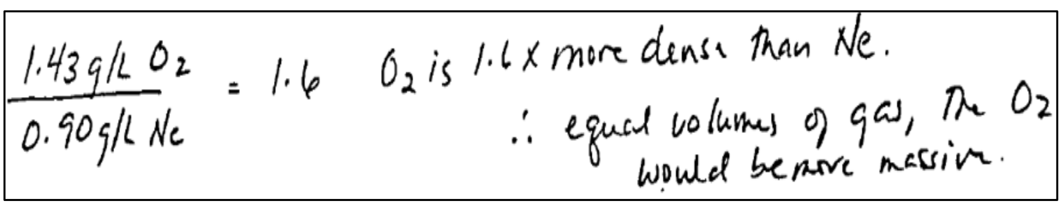
*Jeffery's Example of Unit-rate Strategy for Solving Proportion Problems*




A second strategy—factor-of-change (PR-FC)—involves changing or scaling a given ratio by multiplying a factor to both aspects of the ratio (Figure 7). When teachers solved problems only focusing on the multiplicative relationship between the two given numbers, without setting up an equation, it was considered as factor-of-change (PR-FC) strategy. This was indicated when teachers used words such as *'times as many as'* or *'times as.'*

**Figure 7**

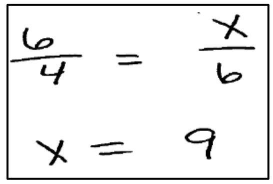
*Jessica's Example of Factor-of-change Strategy for Solving Proportion Problems*




The third strategy, Fraction (PR-F) strategy, is similar to the unit-rate (PR-UR) strategy. However, when the units are dropped from unit-rate (PR-UR) strategy and when the two ratios were calculated by applying the fraction rule – multiply both numerator and denominator by a factor (Cramer and Post, 1993) (Figure 8). In addition, when teachers set up a two-equivalent fraction without units and indication of cross-multiplying, such as arrows, it was considered as a fraction (PR-F) strategy.

**Figure 8**

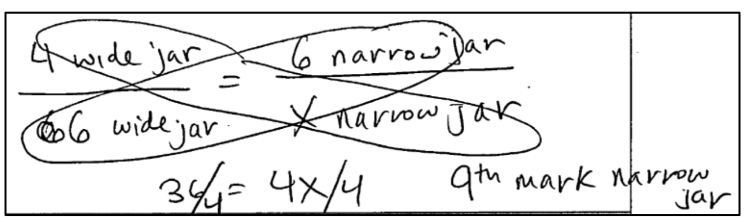
*Stella's Example of Fraction Strategy for Solving Proportion Problems*




Finally, cross-product algorithm (PR-CP) strategy is often used in the secondary school mathematics and science class when solving for a missing value problem; here, the solver sets up two equivalent ratios, cross multiplies the two ratios, and then determines the unknown value by dividing the coefficient in the x term (Figure 9). When a teacher draws an arrow to indicate that they did cross multiplying, or when the two equivalent fractions were set up with non-integers or were not in a simple integer relationship, it was considered as the cross-product algorithm (PR-CP) strategy.

**Figure 9**

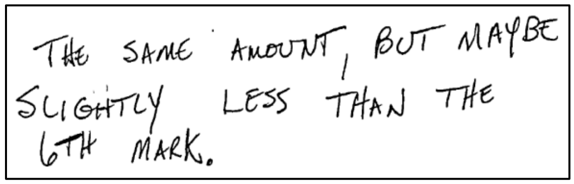
*Tina's Example of Cross-product Algorithm Strategy for Solving Proportion Problems*




In addition, the responses that did not involve the use of mathematics were coded as a qualitative strategy (Figure 10). In other words, qualitative responses included only and explanation of the process of solving the problem or description of how teachers would present to students. These five problem-solving strategies informed chemistry teachers' content knowledge and procedural knowledge for solving proportion problems.

**Figure 10**

*Tina's Example of Qualitative Strategy for Solving Proportion Problems*




## Qualitative Data Analysis

### *Interview*

After the data collection was done from the ten teachers, all the interview responses were transcribed by the researcher. The first part of the interview, which focused on explaining the two methods for solving the five problems, supplemented the problem-solving methods. This part of the interview responses allowed the researcher to understand the underlying thinking process in each step of problem-solving. Also, the method that a teacher preferred to use between the two problem-solving methods and the reason for it was organized in a table.

For the second part of the interview, the four forms of case study research analysis suggested by Stake (1995) were used to analyze the transcripts: (a) direct interpretation, (b) category, (c) cross-case synthesis, and (d) naturalistic generalization. These forms are expected to add coherence when analyzing the data. As the first cycle of analysis, each participant's interview transcripts were read employing an in vivo process of analysis for direct interpretation (Saldaña, 2009). Then, the codes derived from the first cycle of in vivo analysis were categorized using the five components of PCK from Magnusson et al. (1999). For cross-case synthesis, the excerpts were compared and contrasted across the participants for each category of PCK as an inductive analysis. Through this synthesis, a pattern or theme was established for the five categories of PCK and was organized as a table. More details of the coding process are detailed in Chapters IV and V.

***Lesson materials***

This set of data sources were used to supplement the interview responses regarding the teachers' instruction on teaching stoichiometry. The lesson materials, such as test and lesson plan, clarified how teachers teach and assess students' understanding of stoichiometry and the underlying concept of proportion. For example, when John just explained how he assesses students at the end of his stoichiometry unit, it was unclear what problems he actually included in the test and what was his intention and goal for each problem. His test sheet became concrete evidence of his explanation. As a result, the researcher was able to clearly understand John's attention for including a specific type of problem. Table 6 summarizes the research questions, the data collection methods, and data analysis process used in this study.

**Table 6**

*Summary of the Research Questions, Data Collection, and Data Analysis*

| Research Question | Data Collection | Data Analysis |
|---|---|---|
| 1. What beliefs does a sample of chemistry teachers hold about teaching proportional reasoning and stoichiometry? | • Likert scale survey (Cronin-Jones, 1991) | • Proportionality of the response to each item |
| 2. What problem-solving strategies were used by a sample of chemistry teachers when solving proportional reasoning and stoichiometry problems? | • Standardized Test (Cramer et al., 1993; Kouba & Wearne, 2000; Noelting, 1980; Roadrangka, 1985)<br>• Think-aloud interview | • Categorize the solving method and strategies (Cramer & Post, 1993; DeMeo, 2008) |
| 3. What pedagogical content knowledge (PCK) do chemistry teachers bring when asked about how they would teach stoichiometry focusing on proportional reasoning? | • Semi-structured interview using modified CoRe questions (Loughran et al., 2004)<br>• Lesson materials | • Open-coded for the interview responses<br>• Code using PCK categories by Magnusson et al. (1999) |
| 4. What characteristics do three of the chemistry teachers show based on the evidence of Likert scale survey, proportion problems, and interview? | • Each participant's Likert scale survey, standardized test, and semi-structured interview | • Holistic analysis of data for each participant |

## Reliability and Validation

This research is limited to the scope of chemistry teachers in the northeast United States. Therefore, this research, at this stage, has a limitation in generalization and would oblige much more replication for external validity. Creswell (2013) suggested eight strategies to increase the validity, or trustworthiness, of the study and to engage in at least two strategies to ensure the validity of the study. This study used triangulation and peer review to meet the trustworthiness of the study. For triangulation, I used three different types of data that were collected to understand teachers' knowledge and beliefs in delivering proportional reasoning in the context of teaching stoichiometry. By checking the codes used for analyzing problem-solving strategies and teachers' PCK with a colleague, a peer debriefing strategy was performed.

## Role of the Researcher

Stake (1995) suggested several roles of the researcher in a case study. Among many roles, as the researcher in this study, I behaved as a colleague teacher, advocate, and interpreter. As a former chemistry teacher in high school, I observed students struggling to learn stoichiometry and heard various lessons taught by teaches and learned of the different ways that teachers support their chemistry learning and teaching. Thus, I expected teachers to see me as a colleague who can easily share science teaching strategies and as an advocate of their teaching practice in chemistry. Moreover, I anticipated myself to be an interpreter between the field of teaching and educational research. In other words, while teachers share their teaching strategies with me (in my role as a researcher), I hoped the teachers might also see me (in my roles as a chemistry teachers) as understanding their role as chemistry teaches, their process, limitations, and supports we needed for teaching proportional reasoning, teaching stoichiometry, and helping students to gain a better understanding of science.

# Chapter IV: Findings

The purpose of this study was to explore chemistry teachers' knowledge of proportional reasoning and their beliefs about integrating it in teaching stoichiometry. This chapter presents the key findings obtained from four sources of data: (a) Likert survey results, (b) five problems solved by teachers, (c) semi-structured interviews with teachers who completed the problems, and (d) lesson materials or plans that eight teachers submitted (Jessica and Jeffry did not agree with sharing the materials due to the copyright issue). The findings are presented for each research question sequentially; starting from the chemistry teacher's belief on teaching stoichiometry, the strategies that teachers employed for the five problems, and the pedagogical content knowledge (PCK) that the teachers have for teaching stoichiometry. Finally, the four sources of evidence will be revisited for the three selected teachers as a case study to holistically understand their teaching of stoichiometry and its relations to proportional reasoning. Chapter IV is the first chapter of Findings that addresses research questions 1, 2, and 3. Chapter V is the second chapter of Findings that addresses research question 4, which gives wholistic data for three teachers presented as cases.

## Research Question 1.

## What beliefs does a sample of chemistry teachers hold about teaching proportional reasoning and stoichiometry?

As described in Chapter 3, to understand teachers' beliefs about teaching stoichiometry with a focus on proportional reasoning practice, a five-point Likert scale survey was given to teachers. The results of the survey for each item are presented as percentages to better display the distribution of responses to each item and also to satisfy the statistical requirements for ordinal data (Sullivan & Artino, 2013). Because the objective was to assess the beliefs of a sample of ten teachers, the results are not intended to generalize to a larger population of chemistry teachers.

However, as some coherency was noticed for most of the survey items, teachers' beliefs are reported initially for the entire group. A detailed description of each participant's beliefs about teaching proportional reasoning and its implication for stoichiometry teaching are presented in chapter 6.

In the following section, the evidence is reported on participants' beliefs based on the four major categories of the belief that are known to affect teaching the most: "(a) beliefs about how students learn, (b) beliefs about a teacher's role in the classroom, (c) beliefs regarding the ability levels of students in a particular age group, and (d) beliefs about the relative importance of content" (Cronin-Jones, 1991, p. 246).

**Belief 1. Beliefs about How Students Learn Proportional Reasoning and Stoichiometry**

Table 7 shows a summary of the Likert survey results for teachers' beliefs about how students learn proportional reasoning and stoichiometry. Survey items 1, 2, 9, 20, and 25 pertained to this Belief. The ten teachers showed some consensus belief over this category. For example, in item 1, teachers all either agreed or strongly agreed that proportional reasoning could be adopted in daily life. At the same time, they believed (item 25) that students' proportional reasoning ability can be improved through teachers' support. These two items imply that teachers believe that proportional reasoning not only can be adopted in their daily lives but also can be developed through school education.

All the participants responded either agreed or strongly agreed to item 9, which asked whether emphasizing a proportional relationship between two substances in stoichiometry can improve students' proportional reasoning ability. Moreover, 90% chose either agreed or strongly agreed that proportional reasoning can be improved through solving stoichiometry problems (item 20). Such agreement to these two survey items indicates that teachers believe that students' proportional reasoning ability and their stoichiometry problem-solving can benefit each other.

However, teachers had a diverse opinion for item 2, which addressed whether successfully solving stoichiometry problems implies that students have a conceptual understanding of stoichiometry. Fifty percent chose either strongly disagree or disagree with the statement, whereas 40% chose strongly agree or agree. One teacher (10%) remained neutral. Though problem-solving is a common way of assessing students' understanding of stoichiometry concepts, such a diverse opinion may indicate that teachers believe that problem solving is not the only way of assessing the students' understanding of stoichiometry, rather agreeing for employing other forms of assessment to assure the students' understanding. Thus, in the latter section of this chapter (Research Questions 3 and 4), the researcher will examine what other types of assessments are used by the participants.

**Table 7**

*Teachers' Beliefs on How Students' Learn Proportional Reasoning and Stoichiometry*

| | Strongly disagree | Disagree | Neutral | Agree | Strongly agree |
|---|---|---|---|---|---|
| 1. Proportional reasoning can be fully adopted just by daily life/experiences (i.e., tip, interest, recipes, etc.) | | | | 50% | 50% |
| 2. Successfully solving stoichiometry problems implies that students have conceptually understood the concept of stoichiometry | 20% | 30% | 10% | 10% | 30% |
| 9. Emphasizing the proportional relationship between the reactants/products in stoichiometry problems helps high school students to improve their proportional reasoning ability | | | | 30% | 70% |
| 20. Student's proportional reasoning ability can be improved through solving stoichiometry problems | | | 10% | 40% | 50% |
| 25. Students' proportional reasoning ability can be advanced to a higher level through teachers' support | | | | 10% | 90% |

*Note*. N = 10

**Belief 2. Beliefs about a Teacher's Role and Confidence in Integrating Proportional Reasoning in Stoichiometry Instructions**

Survey items 4, 5, 6, 7, 8, 12, 15, 18, 24 reveals the beliefs about a teachers' role and confidence in the classroom (Table 8). More specifically, Items 4, 5, 7, and 12 are related to the teacher's role in students' learning of stoichiometry and improving their proportional reasoning ability, while the remaining items are related to the teachers' confidence in teaching and implementing proportional reasoning in their stoichiometry instructions.

A finding of 100% of positive responses (either Strongly agree or Agree) to item 5 and 7, and 90% of negative responses (either Strongly disagree or Disagree) to item 4, indicates a general belief among the participants that not only the mathematics teachers but also they are in a responsibility to improve students' proportional reasoning ability.

Overall, teachers showed confidence in teaching proportional reasoning and stoichiometry. For example, 90% responded that they could teach students to fully understand the concept of proportion through any type of problems that involve a proportional relationship (item 15), whereas 10% showed less confidence for teaching proportional reasoning practice by disagreeing with this item. In addition, all the teachers (100%) indicated that their students understand the concept of stoichiometry after their teaching (item 24). They also showed confidence in their role of presenting the proportional relationship in their chemistry instruction and integrating it in their stoichiometry instructions. For example, every teacher in this study indicated that they often emphasize the proportional relationship that underlies stoichiometry problems (item 8). Such agreement was also evident in item 18: confident in integrating mathematical thinking practice, particularly the concept of proportion, in my stoichiometry instructions.

**Table 8**

*Teachers' Belief on Their Role and Confidence in Teaching Proportional Reasoning and Implementing It to Stoichiometry Instructions*

| | Strongly disagree | Disagree | Neutral | Agree | Strongly agree |
|---|---|---|---|---|---|
| 4. Improving students' proportional reasoning ability is more of mathematics teachers' responsibility than that of chemistry teachers | 30% | 60% | | 10% | |
| 5. Chemistry teachers are in a critical position in improving students' proportional reasoning ability | | | | 40% | 60% |
| 6. I present different solution methods for solving stoichiometry problem to students | 10% | | | 50% | 40% |
| 7. Chemistry teachers should improve students' proportional reasoning ability while teaching stoichiometry problems | | | | 40% | 60% |
| 8. During the stoichiometry instructions, I often emphasize the proportional relationship that underlies in stoichiometry problems | | | | 20% | 80% |
| 12. Chemistry teacher should present multiple solution methods for solving stoichiometry problems | | | 10% | 30% | 60% |
| 15. I can teach students to fully understand the concept of proportion through any types of problems that involve proportional relationship | | 10% | | 70% | 20% |
| 18. I am confident in integrating mathematical thinking practice, particularly the concept of proportion, in my stoichiometry instructions | | | | 60% | 40% |
| 24. Most of my students conceptually understand the concept of stoichiometry after my instructions | | | | 80% | 20% |

*Note*. N = 10

For the statement addressing whether chemistry teachers should present multiple solution methods for solving stoichiometry (item 12), 90% responded positively, and 10% were neutral. Moreover, 90% responded that they present different solution methods during their stoichiometry instructions, while 10% do not (item 6). These two items indicate that the teachers generally understand that it is their responsibility to present multiple solution methods to their students when solving stoichiometry problems. However, though teachers have indicated this belief in presenting multiple problem-solving methods, teachers expressed difficulty coming up with the second problem-solving method for the five problems that they solved for this study. Therefore, this item is revisited later in the section on Research question 4, where there is an inconsistency between their belief in using multiple strategies and their ability to use multiple strategies.

**Belief 3. Beliefs Regarding the Ability Levels of Students in Learning Proportional Reasoning and Stoichiometry**

Among the major four categories of the belief that Cronin-Jones (1991) suggested, beliefs regarding the ability levels of students for learning proportional reasoning and stoichiometry showed the most diverse opinions among the teachers. Table 9. summarizes the results for this category of belief. For example, among the five items—10, 13, 16, 17, and 21—the consensus was evident only for the two items 13 and 16. For item 16, all teachers believed that it was too much for high school students to learn proportional reasoning ability while learning stoichiometry. Also, 70% agreed that high school students could not solve stoichiometry problems without the concept of proportion and problem-solving skills (item 13). Thus, teachers generally agreed that stoichiometry is a hard topic for students as well as that the concept of proportion as an underlying concept or skill for solving and learning stoichiometry.

Teachers also had a different belief about the difficulty of the stoichiometry concept. Forty percent responded that stoichiometry is hard for high school students because it requires many concepts, whereas 60% disagreed (item 21). Finally, 40% claimed that students' lack of mathematics knowledge limited the teachers from teaching the concept more deeply (item 17), while the other 40% believed the opposite way.

**Table 9**

*Teachers' Belief on Ability Levels of Students in Learning Proportional Reasoning and Implementing It to Stoichiometry Instructions*

| | Strongly disagree | Disagree | Neutral | Agree | Strongly agree |
|---|---|---|---|---|---|
| 10. All high school students should have mastered proportional reasoning ability in their middle school years | 10% | 20% | 10% | 30% | 30% |
| 13. Without fully understanding the concept of proportion and knowing how to solve proportional relationship problems, high school students cannot understand and solve stoichiometry problems | 10% | 10% | 10% | 20% | 50% |
| 16. High school students cannot improve their proportional reasoning ability while learning stoichiometry because it is too much to learn at once | 60% | 40% | | | |
| 17. Students' lack of mathematics knowledge or ability to solve stoichiometry problems limits me from teaching the concept more deeply | 20% | 20% | 20% | 30% | 10% |
| 21. Stoichiometry is a hard concept for all high school students to understand because it requires many concepts | 10% | 30% | | 60% | |

*Note*. N = 10

**Belief 4. Beliefs about the Relative Importance of Proportional Reasoning and Stoichiometry**

Beliefs about the relative importance of proportional reasoning and its relation to teaching stoichiometry were explored through survey items 3, 11, 14, 19, 22, 23 (Table 10). Overall, the teachers mostly understood the importance of proportional reasoning, stoichiometry, and the relation between the two. For example, item 3 asked whether they think proportional reasoning is important in learning other science concepts. Ninety percent agreed, while 10% disagreed. The same pattern was found for item 11. 90% agreed that proportional reasoning for becoming a science- and mathematics-literate citizen while 10% responded neutral.

**Table 10**

*Teachers' Belief on the Relative Importance of Proportional Reasoning and Its Implementation to Stoichiometry Instructions*

| | Strongly disagree | Disagree | Neutral | Agree | Strongly agree |
|---|---|---|---|---|---|
| 3. Proportional reasoning is an important ability in understanding and learning other science concepts | | 10% | | 10% | 80% |
| 11. Proportional reasoning is an important ability for a student to become a science and mathematics literate citizen | | | 10% | 30% | 60% |
| 14. Engaging students to mathematically think about the concept of proportion in science class helps to understand the work of scientists | | | | 70% | 30% |
| 19. Proportional reasoning is a common mathematical practice that scientists engage in | | | 10% | 10% | 80% |
| 22. Solving stoichiometry problems helps students to understand the work of scientists | | 10% | | 60% | 30% |
| 23. It is important to emphasize the proportional relationships that underlie in stoichiometry to understand the concept of stoichiometry | | | | 40% | 60% |
| 26. Chemistry teachers need more support (e.g., knowledge, time, and etc.) to involve the mathematical thinking practice in stoichiometry instructions | | 10% | 20% | 10% | 60% |

*Note*. N = 10

Items 14, 19, and 22 asked about teachers' belief in integrating the proportional reasoning skills and solving stoichiometry problems for the sake of understanding the work of scientists. 90% of the chemistry teachers understood that proportional reasoning is a common mathematical practice that scientists engage in (tem19), whereas 10% were neutral. In addition, everyone either strongly agreed or agreed that students' engagement with proportional reasoning in science class and solving stoichiometry problems helps students to understand the work of scientists (items 14 & 22). Most importantly, teachers agreed that emphasizing the proportional relationships in stoichiometry helps to understand the concept of stoichiometry (item 23).

Finally, the last item 26 was added to understand whether teachers believe they need support in implementing mathematical thinking practices in their teaching because it is one of the practices from the NGSS. Seventy percent believed they needed more support, while 30% were either neutral or disagreed for more support.

**Research Question 2.**

**What problem-solving strategies were used by a sample of chemistry teachers when solving proportional reasoning and stoichiometry problems?**

Teachers' knowledge of proportional reasoning was revealed by solving the five problems (Appendix A): three proportion problems (jar, density, and orange juice problem), one non-proportion problem, and one stoichiometry problem (Haber–Bosch process problem). The strategies that teachers employed for solving the five problems were coded using the framework suggested by DeMeo (2008) and Cramer and Post (1993).

**Stoichiometry Problem**

Table 11 summarizes the strategies used by teachers for the stoichiometry problem. In this stoichiometry problem, three different proportional relationships are involved: (a) proportional relationship between grams and moles of product ($NH_3$), (b) proportional relationship between moles of product ($NH_3$) and reactant ($H_2$), and (c) proportional relationship between grams and moles of reactant ($H_2$). Thus, each proportional relationship was considered as one problem, because other proportion problems (jar, density, and orange juice problems) engage only one proportional relationship. When converting the grams of product ($NH_3$) to moles, 95% used dimensional analysis, and the remaining 5% used a cross-product (PR-CP) strategy. For the second proportional relationship, involving molar ratio between the product ($NH_3$) and reactant ($H_2$), 70% used dimensional analysis (DA) methods, 15% used the cross-product (PR-CP) strategy; and for the remainder, they used factor-of-change (PR-FC) and fractions (PR-F), 10% and 5%, respectively. Finally, when changing the amount of reactant ($H_2$) from moles to grams, 80% used the dimensional analysis (DA) method, and 20% used unit-rate (PR-UR). Moreover, 45% used dimensional analysis chain equations (DA-Chain) (i.e., one long equation for solving the problem). Both dimensional analysis chain equation (DA-Chain) and T-chart (DA-T chart) equation were used by 10%.

**Table 11**

*Problem-solving Strategies for Stoichiometry Problem*

| Strategy | Stoichiometry Problem | | |
|---|---|---|---|
| | Gram to Mole | Mole to Mole | Mole to Gram |
| DA-Chain (& DA-T chart) | | 45% (10%)* | |
| DA | 50% | 25% | 35% |
| PR-UR | 0% | 0% | 20% |
| PR-FC | 0% | 10% | 0% |
| PR-F | 0% | 5% | 0% |
| PR-CP | 5% | 15% | 0% |

*Note.* Dimensional analysis (DA); Dimensional analysis T-chart equation (DA-T-chart); Dimensional analysis one chain equation (DA-Chain); Unit-rate (PR-UR); Factor-of-change (PR-FC); Fractions (PR-F); Cross-product algorithm (PR-CP)
N=20
*Two participants used both DA-Chain and DA-T chart strategy

As mentioned earlier, only for stoichiometry problem, teachers were asked to choose the problem-solving strategy that they, as an adult, preferred to use and the one they prefer to present to the students. Six out of ten teachers had a mismatch of these strategies for their own reasons. Overall, most teachers preferred step-by-step process no matter what strategy that they used for each proportional relationship (Jake, John, Jack, Jessica, Jeffry, Stella, and Lena) while Annie, Tina, and Levy preferred using one long equation, either Dimensional Analysis T-chart (DA-T-chart) or Chain Equation (DA-Chain). For example, teachers advocated for the step-by-step process because each step targets one concept at a time. However, Levy believed that the step-by-step process is more abstract and conceptual that students have to really understand what is going on for each step (Levy, Interview data). Therefore, teachers focused on breaking down the proportional relationship so that each equation or step represents one proportional relationship or concept rather than the strategies itself for teaching the stoichiometry problem.

## Proportion Problems

### *Jar problem*

Unlike the other two proportion problems, teachers employed diverse strategies for this jar problem. Dimensional analysis (DA), unit-rate (PR-UR), fraction (PR-F), and cross-product (PR-CP) strategies were used. Twenty percent used dimensional analysis (DA), unit-rate (PR-UR), and cross-product (PR-CP) strategies and 25% used a fraction (PR-F) strategy, while the remaining answered qualitatively. Overall, when all the sub-strategies for the proportional reasoning methods—unit-rate (PR-UR), fraction (PR-F), and cross-product (PR-CP) strategies—were added up, it was noticed that teachers preferred using proportional reasoning strategies rather than dimensional analysis for this problem. Surprisingly, 15% of the strategies were incorrectly solved.

### *Density problem*

The respondents adopted dimensional analysis (DA), T-chart (DA-T chart), unit-rate (PR-UR), factor-of-change (PR-FC), and cross-product (PR-CP) strategies for solving the density problem. A majority of the population (60%) employed the dimensional analysis (DA) strategy; surprisingly, all the participants used this strategy as their first method. Though it is a small portion (5%), the T-chart strategy also was adopted. For this density problem, unlike the jar problem, teachers preferred using dimensional analysis (65%) rather than the proportional reasoning method (30%). Within the proportion problems, cross-product (PR-CP) strategies were used the most (15%) followed by 10% for factor-of-change (PR-FC) and 5% for unit-rate (PR-UR) strategy. The remaining 5% solved the problem qualitatively (explained their logical thinking process verbally without involving numbers); however, this solution ended up as a wrong answer.

***Orange juice problem***

The unit-rate (PR-UR), factor-of-change (PR-FC), and cross-product algorithm (PR-CP) strategy were used for this orange juice problem. Eighty-five percent used the unit-rate (PR-UR) strategy for the orange juice problem. 7 out of 10 teachers used this strategy for both methods. 4 out of 10 teachers simply changed the unit from per solution to per solvent, vice versa. This may result from the fact that the participants were chemistry teachers who teach all kinds of concentrations – molarity, molality, and percent. During the interview, half of the participants incorporated the concept of molarity, molality, solvent, and solution. Ten percent used a cross-product (PR-CP) strategy by assuming that the two orange juices have the same concentration. Again, though this is the same proportional reasoning problem as the other ones (jar and density problems), no one used the dimensional analysis strategy for this orange juice problem. Table 12 summarizes the strategies used by teachers for the three proportion problems: jar, density, and orange juice problem.

**Table 12**

*Teachers' Problem-Solving Strategies for Proportion Problems*

| Strategy | Jar problem (15%)* | Density problem (10%)* | Orange Juice problem (10%)* |
|---|---|---|---|
| DA-T chart | - | 5% | - |
| DA | 20% | 60% | 0% |
| PR-UR | 20% | 5% | 85% |
| PR-FC | 0% | 10% | 5% |
| PR-F | 25% | 0% | 0% |
| PR-CP | 20% | 15% | 10% |
| Other (Qualitative) | 15% | 5% | 0% |

*Note.* Dimensional analysis (DA); Dimensional analysis T-chart equation (DA-T-chart); Unit-rate (PR-UR); Factor-of-change (PR-FC); Fractions (PR-F); Cross-product algorithm (PR-CP)
N=20; * Incorrect answers

**Non-proportion Problem**

As mentioned in Chapter 3, identifying proportional situations (Cramer et al., 1993; Lamon, 1995) is one of the abilities proportional reasoners should acquire. Thus, this non-proportion problem, which is a non-proportional relationship problem, was added to the problem set. This question asked the participants to find the distance that Matt ran when Sophie was running ahead (2 miles) at the same speed as him.

Referring to Table 13, every teacher (except Tina) correctly identified the non-proportional situation of this non-proportion problem. Tina was the only one who did not notice that this was a non-proportional situation. She used the cross-product (PR-CP) strategy for both methods. Since this problem was in the middle of five proportional situations, a number of teachers (3 out of 10, 30%) admitted that they did not catch that the rate was the same, at first. However, because each problem was solved in two different ways, they eventually noticed that this problem was different from the other four and that it was non-proportional. Therefore, it is hard to see that Tina has simply made a mistake; rather, it implies that she has a limited understanding of the proportional relationship. Also, Lena's second method for solving this non-proportion problem was considered incomplete not only because the answer was incorrect but also because it was impossible to understand one's thinking process by looking at her work.

**Table 13**

*Teachers' Problem-solving for Non-proportion Problem*

| Correct | Incorrect | |
|---|---|---|
| | Proportional reasoning | Incomplete |
| 85% | 10% | 5% |

*Note*. N=20

**Overall Findings for Problem-solving Strategies**

Though all three proportion problems involve the same concept (proportion), the strategies that teachers preferred were different. While dimensional analysis (DA) methods were popularly used over the proportional reasoning method for the density problem, by contrast, the jar and orange juice problems were solved popularly via the proportional reasoning method. Such a trend may imply that teachers have standardized ways of solving specific types of problems. Or, the context of the problem may affect their problem-solving strategies. For example, teachers may tend to use unit-rate strategy for all types of concentration problems because it takes the shortest time to solve it; or alternatively, to use dimensional analysis for all the density problems because the majority learn to solve in that way. Therefore, to understand this trend (i.e., teachers' preference for the use of the specific problem-solving strategy for a specific context of the problem), further studies on what factors affect such preferences are needed.

Another interesting aspect that was noticed is the trend toward using dimensional analysis as related to the structure of the numbers. For example, among the three proportion problems, the number for the jar and the orange juice problems was given in integers, rather than a more precise decimal notation. In a stoichiometry problem, the molar ratio between the product ($NH_3$) and reactant ($H_2$) is expressed in integers by the coefficients in the Harbor-Bosh process. Most of the teachers preferred using the proportional reasoning method for these three problems. This may indicate that the structure of the numbers affects the teacher's choice in problem-solving strategies. It is commonly known that students tend to do better with integer numbers than decimal numbers (Karplus et al., 1983; Riehl & Steinthorsdottir, 2017; Tourniaire & Pulos, 1985). However, there are few studies that examine this from a teacher's perspective. In other words, whether the number format influences their teaching and/or their problem-solving strategies is unresolved. Thus, teachers' choice of problem-solving strategies and their relationship to the number format needs to be studied further.

## Research Question 3.

## What pedagogical content knowledge (PCK) do chemistry teachers bring when asked about how they would teach stoichiometry focusing on proportional reasoning?

For this research question, the five categories of PCK from Magnusson, Krajcik, and Borko (1999) were used to frame teachers' PCK: (a) orientations toward science teaching, (b) knowledge and beliefs about science curriculum, (c) knowledge and beliefs about students' understanding of specific science topics, (d) knowledge and beliefs about assessment in science, and (e) knowledge and beliefs about instructional strategies for teaching science.

### Orientation Towards Science Teaching

According to Magnusson et al. (1999), orientation towards the science teaching category refers to "teachers' knowledge and beliefs about the purposes and goals for teaching science at a particular grade level" (p. 97). In this study, the teachers' orientations of teaching stoichiometry were analyzed using sub-categories of this category of PCK, which were identified in the literature (Magnusson et al., 1999): process, academic rigor, didactic, conceptual change, activity-driven, discovery, project-based science, inquiry, and guided inquiry. These sub-categories were used as Magnusson et al. (1999) defined them namely:

- Process: Teacher introduces students to the thinking processes employed by scientists to acquire new knowledge.
- Academic Rigor: Students are challenged with difficult problems and activities.
- Didactic: Teacher presents information, generally through lecture or discussion.
- Conceptual Charge: Facilitate the development of scientific knowledge by confronting students with contexts to explain that challenge their naive conceptions.
- Activity-driven: Students participate in “hands-on” activities used for verification or discovery.
- Discovery: Provide opportunities for students on their own to discover targeted science concepts. Student-centered.
- Project-based Science: Teacher and student activity centers around a “driving” question that organizes concepts and principles and drives activities within a topic of study. Involve students in investigating solutions to authentic problems.
- Inquiry: Teacher supports students in defining and investigating problems, drawing conclusions, and assessing the validity of knowledge from their conclusions.
- Guided Inquiry: Constitute a community of learners whose members share responsibility for understanding the physical world, particularly with respect to using the tools of science.

When analyzing each participant's orientation of teaching stoichiometry, the categories appeared more than once because s stoichiometry unit typically runs for one to two weeks (Table 14). Thus, more than one orientations were found to be used from teachers' instructions during the two weeks.

**Table 14**

*Teachers' Knowledge of Orientation Towards Teaching Stoichiometry*

| Orientation | Number of participants | Participants |
|---|---|---|
| Didactic | 5 | John, Tina, Lena, Jack, & Jake |
| Discovery | 4 | Annie, Jessica, Lena, & Levy |
| Activity-driven | 4 | Jeffry, Stella, Jack, & Jake |
| Process | 3 | John, Tina, & Lena |
| Project-based | 1 | Levy |

Five out of 10 teachers (John, Tina, Lena, Jack, & Jake) oriented their lessons toward a didactic approach. In other words, they directly delivered a step-by-step process of solving the stoichiometry problems during their lesson. For example, Jack provided a guideline that specifically elaborates on the steps of calculations for solving stoichiometry problems:

> How to Complete Stoichiometric Calculations:
> 1. Ensure the reaction is balanced. Perform a quick check to see if the element inventory is balanced on both product and reactant side.
> 2. Identify the compounds discussed in the question and determine the molar ratio between them.
> 3. Depending on what is given, perform necessary gram-molar conversions. You may need to perform multiple (Jack, lesson material, April 18, 2018).

Four teachers (Annie, Jessica, Lena, & Levy) gave discovery-oriented lessons. They all incorporated a lab, such as copper and silver nitrate lab or magnesium strip lab, and they used the quantitative data as evidence for students to discover and construct their understanding of the quantitative relationship among the reagents in the chemical reaction. These teachers employed those kinds of discovery-oriented labs because it entailed all the stoichiometry concepts, from the ratio among the reagents to the concept of limiting reagent. Levy's lab material (Figure 11) specifically guides students to use the quantitative data that they have collected to construct their understanding of the quantitative relationship of the reagents.

**Figure 11**

*Levy's Lab Material for Flint Crisis Activity*

**Procedure:**

1. In Data Tables 1 and 2, fill in the blanks in the first column with the substance that your team will be testing, then determine the ratio that each combination represents.
2. Label your well plate as row A and B, and for each 1-5.
3. Place the correct amount of drops of each substance in the appropriate well, and then compare the five wells to determine which produced the most precipitate.

**Data Table 1. Ionic Compound Possibility 1**

| **Well Plate Row A** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| Ratio of $AgNO_3$ to _______________ | 1:3 | | | | |
| Drops of $AgNO_3$ | 6 | 8 | 12 | 16 | 18 |
| Drops of _______________ | 18 | 16 | 12 | 8 | 6 |

Some teachers (4 out of 10; Jeffry, Stella, Jack, & Jake) organized their lessons to be activity-based by using hands-on activities and labs. In general, they implemented these activities to increase the students' engagement in learning stoichiometry. In other words, they did not report any other meaning for including activities in their lesson other than students having fun and being motivated. For example, Jack made a card sort activity (Figure 12) to increase students' engagement in solving stoichiometry problems.

**Figure 12**

*Jack's Card Sorting Activity for Stoichiometry Lesson (Front and Back)*

| | |
|---|---|
| **START**<br>What is the molar ratio between $CH_4$ and $H_2O$? | How many grams of $O_2$ are needed to completely combust 256g of $CH_4$? |
| If 72g of $CH_4$ combust, how many moles of $CO_2$ are produced? | If 64g of $CH_4$ are combusted, how many moles of $H_2O$ will be produced? |

| | |
|---|---|
| ***352 g*** | ***1024 g*** |
| ***2 moles*** | ***308 g*** |

> Something I've tried this year has worked really well, where instead of just giving them a practice worksheet, I've made like a card sort activity, where it's basically I'll take a sheet of paper and split it into 10 boxes or so, and they have to solve for an equation on the first side of the page, flip it over and that will give them an answer to look at for the next question. So, as they rotate that, they can self-check because they see if they finished it and if everything ended up in the right spot. So, it's kind of like solving a little puzzle. So, it kind of just increases the engagement in terms of them just doing practice problems. Like in the past, I've given them a worksheet and be like, you have a half-hour to get this stuff and this is just another way for them to think about it. It gets them more engaged into it. They're having more fun with it and they're working with their group members to make it happen (Jack, interview, April 18, 2018).

While Lena and Jessica used the copper and silver nitrate lab for students to discover the quantitative relationship, Stella, who works at the same school, did not consider this lab more than a typical lab she does for every unit. She considered this lab as one of the chemical reactions that she could use to explain balancing equations and the concept of limiting reagent. Depending on the purpose of the lab, the same lab could be used and delivered differently to students. Therefore, unlike Lena and Jessica, Stella’s orientation in using the copper and silver nitrate lab was an activity-driven lesson.

> We use the copper and silver nitrate lab [to make the transition from the daily life analogy (e.g. S’mores) to scientific practice (e.g. microscopic level)]. Um, so I have them look at the equation for copper and silver nitrate, and then we actually do a lab where they have a piece of copper and they have a sample of silver nitrate and then they have to make their solution of silver nitrate and they have to collect their silver at the end. And then, workout limiting reactant problems based off of that (Stella, interview, April 20, 2018).

To familiarize students with the dimensional analysis method, Jeffry also used a card activity to help his students to learn about ratios, which is a concept needed to learn stoichiometry:

> We do some other things with just some, long chains of, you know, we have a problem that you want to find out how many turtles are on the island and they give you a thousand different things to kind of work through of, you know, there's many hands to this many eyes on the island. We've taken that as well to a different level because it used to be a very mathematical. They just sit and work the math problem out and now we've given them little cards with different ratios on them that are a part of the problem and

> they have to kind of align the cards up to see if they can solve it and develop a kind of a methodology for how they're gonna get to turtles from whatever started with and I can't remember now. So, we do some of those things to put a little bit more hands-on with that (Jeffry, interview, April 20, 2018).

Three teachers (3 out of 10; Tina, John, & Lena) also provided process-oriented lessons for teaching stoichiometry. For example, Tina strongly believed stoichiometry problems have a formal process or system that should be followed to solve them. This process or system that she refers to was the dimensional analysis method. Because she believed the dimensional analysis method is important in chemistry, especially for preparing entry into college, she constantly referred to the dimensional analysis method for learning stoichiometry. In addition, she believed that students typically do not learn this method in Algebra class; as a result, they have to adopt this method from the stoichiometry unit:

> This new lesson plan will show how dimensional analysis is modeled and used by students for the instruction of mass-mass and theoretical yield chemistry problems…… They [students] do need a formal process or system that they can use to solve just about any type of stoichiometric problem. And with the dimensional analysis method, you know, you see how the units are canceling out based on Algebra and you're left with numbers and the numbers that are side by side and multiplying and then underneath the numbers underneath those are used to divide. So, um, yeah, they, they can see that with this type of method. So, it's definitely useful. And then, they can use to manipulate numbers so they have to go from mass to moles. They can easily reverse that direction. They could do that (Tina, interview, March 20, 2018).

Levy was the only participant who employed project-based lessons for learning stoichiometry. She incorporated the Flint water crisis as a real-life project for the stoichiometry unit. She used this project as a driving force for covering all the stoichiometry concepts, from molar ratio to limiting reagent:

> Just like in this problem [Haber-Bosch problem], I'm giving them now a real-life problem. If they have a sample and this is where connecting it to the Flint crisis. So, if they have a sample of contaminated water, they need to know "OK, there's a treatment you can add, it will precipitate out the contaminant, but you don't know how much. How can you prove to someone that you've really gotten it all out?" I need to be able to use math as a proof to say "OK, I know this is exactly what I need to add to clean out that

> contaminant." Like [the] ultimate purpose [is] you need to be able to show someone that I really know this is what I made. This is what I had to add (Levy, interview, March 23, 2018).

Overall, the instructional approaches used by the teachers included didactic, discovery, activity-driven, process, and project-oriented lessons. However, other subcategories for orientations toward teaching science that Magnusson et al. (1999) identified—inquiry, guided inquiry, conceptual change, and academic rigor—were not used by the participants in this study. This may imply that inquiry-based learning is hard to implement for the stoichiometry unit. This could be noticed from the case of Jack. Though his school was an inquiry-based learning school that uses the 5E lesson plan (Engagement, Exploration, Explanation, Elaboration, and Evaluation), his stoichiometry lesson was either didactic or activity driven. More interestingly, didactic and discovery, the very opposite orientations, were most frequently used for teaching stoichiometry by the participants. This finding may be explained speculatively as a result of teachers' effort to make stoichiometry lessons more engaging and student-centered. At the same time, they had to incorporate didactic lessons to effectively deliver the problem-solving steps within the given time.

## Knowledge and Beliefs about Science Curriculum

Science curricular knowledge is explained by Magnusson et al. (1999) as knowing the general learning goals of the curriculum as well as the instructional activities and materials to meet those goals. This component of PCK consists of two categories: knowledge of goals and objectives, and specific curricular programs. These two categories were used as a framework for analyzing teachers' knowledge about the science curriculum.

### *Finding 1: Goals and Objectives of Stoichiometry Lessons*

In terms of the goals and objectives of stoichiometry lessons, six teachers out of ten listed mathematics concepts or skills (including proportion, unit conversion, mathematical thinking, dimensional analysis, and equation as a concept) as a goal of stoichiometry lessons (Table 15).

**Table 15**

*Teachers' Goals and Objectives of Stoichiometry Lessons*

| Rank | Goals and objectives |
|---|---|
| 1 (60%) | • Mathematics skill<br>  o Proportion (ratio); Unit conversion; Mathematical thinking; Dimensional analysis<br>• Each equation represents a concept |
| 2 (40%) | • Practical use of stoichiometry |
| 3 (30%) | • Mass conservation<br>• Problem-solving skills<br>• Thinking different methods for problem-solving |
| 4 (20% | • Molar ratio<br>• Law of definite proportion |
| 5 (10%) | • Conceptual understanding of chemical reactions<br>• Transformation of bonding; Rearrangement of atoms<br>• Accounting method for particles (atoms and molecules)<br>• Concept of mole<br>• Difference between gram and mole; Thinking backward & different ways (from product to reactant) |

Depending on the mathematical problem-solving methods that teachers use for solving stoichiometry problems, the mathematics skills that teachers emphasized differed. For example, because Tina indicated focusing on dimensional analysis for solving stoichiometry problems, she listed the dimensional analysis method as a goal for stoichiometry lessons.

> They're learning the vocabulary term, dimensional analysis. Number two, the learning of systematic process for problem-solving because they'd already know that the scientific method is used and a lot in that's a systematic process. But, in this case, we're dealing with numbers. And so this is one way that scientists use. And they use this method for analyzing units and converting from one unit to another. And then, I also go into explanations about the term dimensional analysis (Tina, personal communication, March 20, 2018).

The practical use of stoichiometry was included as an objective of stoichiometry lessons by four of the ten teachers. For example, Jessica emphasized students' usage of the concepts learned during stoichiometry lessons in their future careers in science or other fields.

> What I really want them to learn is … to be able to go to a lab and use it. … I mean we talked about industry, if you want to know how much coke you need to make, you need to know how much sugar you need to buy. If you're going to produce x number of cans of coke because, otherwise, you're not running an efficient business. So, I try to get them to make those links (Jessica, interview, April 20, 2018).

Two goals—the law of conservation of mass and problem-solving skills—were among the third most common topics that teachers referred to as goals for stoichiometry instruction; four out of 10 teachers noted these goals. The following is Annie's and Levy's response regarding the goal for stoichiometry lessons—Law of mass conservation and problem-solving skills, respectively.

> Learning goals would be around, I never taught to the standards, so I never had to write [a lesson plan]. So, getting to big ideas around conservation of matter, getting to think more broadly or more specifically about a molar ratio and understanding how molar ratios work in chemical equations [are the goals I have for students] (Annie, interview, March 14, 2018).

> I think one piece is that this [problem-solving] is a tool that they can use. So, I want to offer this as a tool for students to use when they're doing their designed work in chemistry. So, like this is a way of managing information and like processing and converting information. It's also a thinking skill (Levy, interview, March 23, 2018).

***Finding 2: Concepts within the Chemistry Curriculum***

Teachers mentioned the chemistry concepts that connect with the stoichiometry concept (Table 16). This implies that teachers tend to consider stoichiometry as being a concept that is taught before or after other key concepts in the chemistry curriculum.

For instance, teachers listed conceptual understanding of chemical reactions (transformation of bonding and rearrangement of atoms) as an accounting method for learning the particle-nature of matter, e.g., the concept of mole (the difference between gram and mole). Interestingly, Stella pointed out students' difficulty in thinking from product to reactant, which is backward thinking in terms of the arrow in a chemical equation. Thus, she believes stoichiometry problems, which are finding the amount of product from a given amount of reactant, can be a brain-stretching activity for overcoming students' misunderstanding of a chemical equation.

> It is just another way of stretching their brain or it's a reasoning kind of problem. It's a problem that makes you almost think backwards because you're given the solution and you're looking for part of the original question if you think of reactants and products that way. Because a lot of students think from left to right, they think reactants and products are starting and finish. They aren't sure what to do when they're given the finish and asked to look for the start (Stella, interview, April 20, 2018).

**Table 16**

*Teachers' Knowledge of Connecting Stoichiometry with Other Concepts within the Chemistry Curriculum*

| | |
|---|---|
| Before the stoichiometry lesson | 1. [40%] Chemical equation (balancing, coefficient, & subscript)<br>2. [30%] Chemical reactions (classifying & predicting chemical reactions); Mole & molar mass; & Empirical formula, naming, and bonding<br>3. [20%] Atomic theory (atom & molecule) & Unit conversion (molar ratio, mole-to-gram)<br>4. [10%] Law of conservation of mass & Periodic table |
| After the stoichiometry lesson | 1. [40%] Gas law<br>2. [30%] Solution chemistry (concentration)<br>3. [20%] Limiting reagent; Energy and mass transfer; Equilibrium; & Acid and base<br>4. [10%] Atomic model development & No other related unit or concept with stoichiometry |

Understanding of what chemical equations represent, which includes balancing equations and understanding the meaning of coefficients and subscripts, was most frequently referred to as prior knowledge by teachers; that is, they believed learning these concepts reinforced students' learning of stoichiometry. The concept of mole and molar mass and understanding chemicals (e.g., empirical/molecular formula, naming of a compound) and its reactions (e.g., types of bonding and classifying and predicting chemical reactions) were needed in learning stoichiometry. The following response from Levy illustrates how students need to know related concepts to be able to understand stoichiometry.

> I think this [stoichiometry] really helps them to understand chemical transformation in a new way. So, we introduce that topic of the difference between physical reactions and chemical reactions and nuclear reactions earlier on. And then, we've learned about bonding and this really pushes for them to understand how bonding can be transformed during reactions. And then, also how does the quantities that we've talked about in an individual molecule help me to predict what's going to form. So, it's kind of chemistry in action. So, we've learned sort of the foundational pieces of this [stoichiometry]. And, this is what everything is made out of. And then, how do we manipulate it to be able to do something with it (Levy, interview, March 23, 2018).

Gas laws were most frequently referred to as a concept learned later that connects with stoichiometry in the chemistry curriculum (4 out of 10 teachers). For example, Tina believed that the gas law chemistry unit synchronizes with the stoichiometry chemistry unit due to the calculations and conversions involved in the laws. Solution chemistry, which includes different types of concentrations (e.g., molarity), was noticed by the teachers as a chemistry unit that connects with stoichiometry for its calculation similarity (3 out of 10 teachers).

> I would go into probably acids and bases [after stoichiometry]. For gas laws, I probably would do that after acids and bases just because acids and bases is a little bit quicker. The students will learn that faster…… But I'm only introducing acids and bases right after this because the timing and trying to get through the curriculum and thinking about what will be on the final exam. But I would think more in harmony with gas laws because you're still dealing with calculations and sometimes conversions are needed (Tina, interview, March 20, 2018).

Limiting reagent, energy and mass transfer, acid/base, and equilibrium concepts followed next in order of preference by 2 out of 10 teachers. Because limiting reagent is usually the last concept within the stoichiometry unit, many teachers referred to this concept as “the concept that follows-from and most typically connect with the stoichiometry concepts.” However, in this research study, the concept of limiting reagent was considered as a separate concept from stoichiometry, because it does not emphasize the quantity relationship. A limiting reagent is a reagent that limits the chemical reaction due to its insufficient amount to sustain a reaction, even though other reactants are still present. Thus, this does not directly involve quantity relationships among the reagents. The following is Annie's reasoning for putting limiting reagent after stoichiometry.

> Next would probably be limiting reactants. Again, we're thinking about conservation of mass and if we're using an evidence to support this, we would want to look at 'OK, so we know that there are these molar ratios that exists. What if we start with lots of x reactant and very little of y reactant, how does that impact the amount of x y that we might produce? So, that idea of limiting reactant sort of flows naturally from doing the molar ratios (Annie, interview, March 14, 2018).

While Tina taught the acid/base unit immediately after the stoichiometry unit due to the time limitation for the state exam, Levy taught the acid/base unit in conjunction with the stoichiometry unit because the concept is similar to the stoichiometry unit.

> I usually do a mini case study on acids and bases [after stoichiometry] because it's a similar idea and it builds on that. So, the titration idea kind of can build off of what they've seen already and like how many hydrogen ions, how many hydroxide ions can I pick up, and how does that help me to learn more about concentration. So, that comes next (Levy, interview, March 23, 2018).

Stella thought the stoichiometry unit connects well with an equilibrium unit due to the instructions that she uses—modeling instruction. The BCA (Before-Change-After) chart that she uses to represent the chemical reaction in stoichiometry has a similar format and conceptual organization that follows logically with the organization of the ICE chart that she uses in the equilibrium unit. She believes this made it easier for her to teach it, as well as for her students to overcome their fear of learning equilibrium more quickly.

> Moving forward. We definitely do circle back. When I took that chemistry course in the summertime from the AMTA one [or] two years ago, I hadn't seen ICE (Initial-Change-Equilibrium) charts in a while, like I hadn't taught equilibrium in a while. But, since we had done BCA charts, they're so similar that I instantly made the connection and it took the fear right out of it. So, I think that for this similar thing would happen to the students that you have that reinforcement all the time. That it's not new. You've seen this before. It takes the fear out of it and gives them more confidence (Stella, interview, April 20, 2018).

### *Finding 3: Concepts Outside of Chemistry Curriculum*

Only three teachers out of 10 choose to talk about outside curriculum concepts that connect with the stoichiometry concept (Table 17). This implies that teachers tend to consider stoichiometry as being within the chemistry curriculum and not a topic to be taught outside the chemistry curriculum.

**Table 17**

*Teachers' Knowledge of Connecting Stoichiometry with Other Concepts Outside the Chemistry Curriculum*

| | |
|---|---|
| Before the chemistry curriculum | • Physics: Includes many proportional relationships<br>• Mathematics (Algebra): Setting up a proportion |
| After the chemistry curriculum | • Chemical Engineering (pollution, industry process): Energy and heat transfer<br>• Biology (respiration, photosynthesis, biochemical pathways, and limiting factor in growth in population)<br>• Pre-calculus<br>• College success (specifically dimensional analysis)<br>• Anything outside of science related to proportional reasoning |

More interestingly, the teacher's view of topics that connect outside the curriculum depended on their background. For example, because John had a background in biology, he knew that the concept of respiration, photosynthesis, biochemical pathways, and limiting factors in population growth could be connected with the stoichiometry unit, such as the concept of molar ratio among the reagents and limiting reagent. Also, Annie's background in chemical engineering led her to design stoichiometry instructions around the concept of heat and mass transfer as she believed those were the core concepts that connect with chemical engineering.

> In my own learning experience, having been a chemical engineer for undergrad, ideas around heat and mass transfer really start from this point. So, that's something that, when I started teaching, chem[istry] played really saliently into my planning of curricula. It was those ideas around if we're thinking more broadly about actual problems that our society is encountering, let's take pollution and pollution based on industrial processes. A

lot of what chemical engineers do is take those ideas about heat and mass transfer and say like, ‘OK, well, if we're creating all of this $CO_2$, how can we capture it and get it to a point where it's not $CO_2$ anymore, but it's a safe waste product or how can we repurpose it into something that's useful?’ And it's using those ideas of, again, heat and mass transfer to do that and you have to know how to do this [stoichiometry] in order to be able to solve those more complex problems (Annie, interview, March 14, 2018).

Physics and mathematics curricula, especially Algebra, were referred to as curriculum topics that connect with stoichiometry. For example, Jessica believed that her students could recall the concept of proportion from physics class during stoichiometry lessons and can experience how it is constantly applied in science. In addition, Tina believed that students’ understanding of proportion learned in Algebra class would be reinforced during stoichiometry.

> The stoichiometry lessons will help what they learn before because they are definitely recalling the prior knowledge: just setting up proportions and Algebra a process with canceling out similar units or like terms when you have a ratio (Tina, interview, March 20, 2018).

To clarify, Tina’s intension for mentioning ‘setting up a proportion’ refers to setting up a dimensional analysis equation correctly. In other words, putting the factors in the correct position: putting the eliminating factor both on the numerator and denominator of the dimensional analysis equation. She interchangeably used proportion with dimensional analysis because her notion of solving proportion problems was using dimensional analysis methods.

***Finding 4: Curricular Programs for Stoichiometry Instructions***

Four out of 10 participants (Jeffry, Stella, Jack, & Jake) cited specific curricular programs or materials that they use for stoichiometry instructions. Jake and Jack used curricula programs and materials to provide practice problems for students. For example, in his lesson plan, Jake indicated using POGIL (Process Oriented Guided Inquiry Learning; a student-centered instructional approach that generally follows a learning cycle paradigm). He used the curriculum program to reinforce students' understanding of the molar ratio after teaching the stoichiometry lab. Jack specified that the stoichiometry practice problems that he provides to students were attained from the Problem-Attic website, which is a database for all kinds of exams (e.g., Regents and TIMSS). Jeffry, Jessica, and Stella, who are from the same school, indicated that they use modeling instruction from the American Modeling Teachers Association (AMTA) for stoichiometry instructions.

> We have modeling instruction. So, we follow the model instructional strategies, which really puts you into the cycle of building and understanding this [proportional relationship in stoichiometry] (Jeffry, interview, April 20, 2018).

**Knowledge and Beliefs about Students Understanding of Specific Science Topics**

This component of pedagogical content knowledge refers to teachers' knowledge about students. It includes knowledge and beliefs about students' prerequisite knowledge or skills as well as the areas of student's difficulties for learning specific scientific science content (Magnusson et al., 1999). These two categories and the subcategories were used to unpack teachers' knowledge of student understanding of stoichiometry concepts. Teachers identified five different areas where students have difficulties in learning stoichiometry: mathematics, chemistry, problem-solving skills, connecting mathematics and science, and motivation. Table 18 summarizes teachers' knowledge of students' difficulties in learning stoichiometry. Teachers identified that students have difficulty learning stoichiometry because of their lack of knowledge in mathematics, stoichiometry, problem-solving skills, and motivation as well as their compartmented view of the world (mathematics and science).

> Misconceptions... knowing where to place the numbers in this dimensional analysis chart. So, a lot of times they don't recognize that the units have to cancel out. So, they place numbers with their units in incorrect places. So, I have to point out how they're doing that and how it's wrong (Tina, interview, March 20, 2018).

**Table 18**

*Teachers' Knowledge of Students' Difficulties in Learning Stoichiometry Concept*

| | Students' difficulties for learning stoichiometry concepts | Support for overcoming the difficulties |
|---|---|---|
| Mathematics | • Understanding proportion (50%)<br>○ Setting up a proportion: Matching units on top and bottom of the proportion<br>○ Understanding the proportional relationship in macro and micro-scale<br>• Placing over 1 for every whole number | • Define where the units go in the proportion<br>• Representation of proportion: Pizza<br>• Technology: using iPad app for sharing students' work |
| | • Using dimensional analysis (20%)<br>• Placing factors in the correct position (top and bottom of the equation) | • Activities on unit conversations<br>• Visual cues<br>• More practice problems |
| | • Fear of mathematics (10%)<br>• Confused for calculating (20%)<br>○ Order of operation<br>• Using calculator | |
| Stoichiometry | • Chemical reaction (50%)<br>○ Conservation of mass/atom<br>○ Balancing equation<br>○ Meaning of coefficient proportionality of mole (not mass)<br>○ Meaning of arrow: ability to think from product to reactant<br>• Mole and molar mass (40%)<br>• Writing/Naming chemical formula (40%): Diatomic<br>• Limiting reagent (10%) | Some difficulties only exist in the beginning |
| Problem-solving skills | • Labeling units (40%)<br>• Not reading the problems carefully (10%)<br>• Solving problems without conceptual understanding (20%)<br>• Remembering/Skipping a step for multi-step problems (60%)<br>• Showing work (20%) | • Using different visual cues for multiple steps problems<br>• Mandatory showing work |
| Compartment | • Compartmentalization of mathematics and science subjects (10%)<br>• Mathematics as a tool/language of science (10%) | |
| Motivation | • Reason for studying stoichiometry (20%) | • Providing practical examples:<br>○ Counting nail from weight<br>○ Multiplying recipe<br>○ Manufacturing: coke production |

To support students to overcome this difficulty of placing factors, Annie allotted a lesson for practicing dimensional analysis with made-up units before solving stoichiometry problems.

> I did a whole activity around just dimensional analysis before using any chemicals or any moles or anything like that where they are looking to convert from. And I did this with my regular students, too. Like flip flaps to zonks. It's just made up units essentially. I would give them the conversion factors above and they would have to sort of figure out how to solve that problem that way. So, they built dimensional analysis through that (Annie, interview, March 14).

Three out of 10 teachers (Jake, Jack, Jeffry) who use proportional reasoning strategy for solving stoichiometry problems referred to a similar difficulty that they found in teaching stoichiometry. The following statement from Jack illustrates this difficulty with fractions.

> I don't know if it's necessarily a difficulty, but it's something that the kids do mess up a lot with is getting the fractions upside down, dividing during multiplication. So, that's just something that they mix up with a lot (Jack, interview, April 18, 2018).

Related to the proportional reasoning strategy, Levy indicated that students struggle to understand proportional relationships at both the macroscopic and microscopic levels. In chemistry education, shifting among macroscopic, microscopic, and symbolic levels of chemicals is an important skill for a good understanding of chemistry (Johnstone, 2000). Thus, applying and transferring proportional understanding in those three levels is a necessary skill in learning chemistry.

> I think that the proportional relationships between macroscopic and microscopic that's a real challenge. So, I think it requires a lot of time. It's something that I try to do and I think by having a real world application of it that they're working towards helps to support this because they're not just solving a problem (Levy, interview, March 23, 2018).

Levy also indicated that students' fear and resistance over mathematics hinders them from learning the stoichiometry.

> This math is also not geometry or Trig. This is like conceptual math. And there's a lot of fear still around it and a lot of resistance to trying things out or pushing themselves. I do see some students like shutting down when they see math (Levy, interview, March 23, 2018).

Though it was just two teachers (Lena and Levy), it was interesting that they referred to students' segmentation between mathematics and science subjects as a reason for having difficulty in learning stoichiometry. For example, Lena denoted the compartmentalization of mathematics and science subjects. In other words, students have difficulty transferring the concept of proportion from mathematics class to chemistry class. Moreover, Levy believed that students have difficulty seeing mathematics as a tool or language of science. She attributed to the difficulty in learning stoichiometry.

> I think for a lot of students who have struggled in math … have success earlier in my chemistry class. When they see math and suddenly they're like 'uh! I'm not good at math but I'm good at chemistry but I'm not good at math'. And it's hard for them to recognize math as a tool and this is the language of science (Levy, interview, March 23, 2018).

Moreover, teachers (two out of 10) pointed out students' lack of calculating skills as one of the difficulties in teaching stoichiometry. Stella and Jack indicated that students are often confused with the order of operation, even when using a calculator.

> I do see students who just flat out [and] struggle with how to solve it. As far as when do you multiply, I guess order of operation, how you multiply things. And then, when do you divide and how you use your calculator. There's so many students who don't know how to use their calculator (Stella, interview, April 20, 2018).

***Finding 2: Students' Loose Understanding of the Chemical Reaction***

In the area of chemistry, 50% of teachers highlighted that students' have difficulty learning stoichiometry due to their loose understanding of the meanings of a chemical reaction or chemical equation. This included visualizing chemical reactions; balancing chemical equations; understanding the meaning of coefficients, subscripts, and arrow in the chemical equations; and conservation of mass and atoms.

> There are just kids who have trouble seeing it and visualizing it [chemical reaction]. They maybe don't have a really strong understanding of how molecules come together. So, they forget things like hydrogen is a diatomic molecule. Little things like that that impede

> finding a correct solution even if they're stoichiometry understanding are there (Annie, interview, March 14).

Moreover, students have difficulty understanding the meaning of the coefficients in the chemical reactions. In other words, students often are confused that the proportional relationship is present in the masses of the reactants and products rather than a mole or number of molecules.

> [Students have misconceptions] that it's a mass relationship, not a particle relationship. That's definitely a big one. It's hard to overcome for a lot of students (Stella, interview, April 20, 2018)

As Johnstone (2000) indicated, students have a hard time understanding the symbolic level of chemical reactions. Stella and Jessica noticed this struggle and illustrated because the arrow in the chemical reaction often goes from left to right, students have trouble thinking about the chemical reaction backward, from product to reactant.

> Because a lot of students think from left to right, they think reactants and products are starting and finish and they aren't sure what to do when they're given the finish and asked to look for the start (Stella, interview, April 20, 2018).

***Finding 3: Students' Lack of Problem-solving Skills***

Six out of ten teachers stated that a lack of problem-solving skills as a reason for students' difficulty for learning stoichiometry. Four teachers out of 10 (Jack, John, Stella, and Jeffry) pointed out missing units on the numbers as a reason for the unsuccessful solving of stoichiometry problems.

> They don't like to fully label … the little proportions, whatnot. So, they will leave out the one mole of $NH_3$, those have numbers on top of the variable and not know what represents what and they'll get frustrated when they get it wrong. So, I tell them that if they have it labeled, it's foolproof as long as you match with everything you need to but then often times they just are careless in not doing that (Jack, interview, April 18, 2018).

Teachers (six out of 10) illustrated students' common mistake on multiple-step problems, such as the gram-to-grams type of stoichiometry problems, that need three steps of converting units. As Jeffry stated in the following, students often forget or miss a step. Tina believed that students' age level, more specifically their memory capacity, makes it hard for them to process problems that have several steps. However, John believed it is because they solve problems mechanically without thinking about the conceptual meaning of each step. As found in research question 1, this reflects that teachers have different beliefs about students' abilities in learning science concepts.

> They will sometimes not finish a problem. I'd mentioned some before the flipping of the ratios in the numbers, but they will at times go through that process and forget to do a mole-to-mole conversion. So, they'll just somehow jump to moles of ammonia to directly grams and moles of hydrogen. And they don't use the relationship from the chemical reaction to do that. And I don't know why they necessarily do that because it's not consistent (Jeffry, interview, April 20, 2018).

According to Tina, missing steps is a challenge not only for students but also for teachers because it is difficult to help them remember all the steps. She implied that using cues, such as a chart or song, can help students remember the steps of solving stoichiometry problems.

> Definitely, [students have difficulty] just learning the process to convert from mass to mass. A number of steps that are involved and the memory for students in this age level, it's just sometimes I guess weak so to speak. So, that's been my biggest struggle and helping them to remember each of the steps. I tried to use different ques like in this location of the chart, the given number goes here and then here you have your mole ratio and then this section is your wanted. So, try to set up your numbers in this particular way. That sort of helps (Tina, interview, March 20, 2018).

***Finding 4. Students' Motivation***

Motivation was also one of the difficulties that teachers identified for learning stoichiometry (2 out of 10 participants). John and Tina indicated the difficulty of involving students in learning stoichiometry. John solved this problem by providing practical examples, such as buying nails in grams. Tina also used different cues to help students remember multiple-step in solving stoichiometry problems as well as to make it interesting for her students. She strongly believed that without their interest and open attitude to learning, they will not learn anything.

> That's the thing because ultimately as a learner, if you're not interested in learning something, you're going to resist, and you won't learn it because you've thought that you don't want to learn it. You don't like it. But, if you are open minded, I think that helps with learning more of the content (Tina, interview, March 20, 2018).

***Finding 5. Varication in Learning***

As Stella stated below, knowing the variation in individual differences among the students is a challenging job for teachers. However, it seemed all participants recognize and differentiate their instruction and assessment based on student variation.

In terms of required knowledge and skill for learning stoichiometry, seven out of 10 teachers listed mathematics skills – Algebra, unit, dimensional analysis, cross multiplication, and proportionality.

> I assume that they have a certain baseline math ability. At this point, I assume that they can do dimensional analysis because we've been doing that all year [in physics class]. I'm assuming that they're going to be good about doing units, and I'm assuming correct use of significant figures because we've taught that. And I assume that they can do sort of cross multiplication stuff. But I don't like them to do that. I don't want to see that so much because I feel like that's not the thinking that is involved. And I want them to think about (Jessica, interview, April 20, 2018).

Teachers commented that students' self-motivation and independence of learning are requirements for learning stoichiometry. Depending on students' attitude for learning, teachers differentiate their instruction toward guided instruction or collaborative instruction.

> Identify where in the lesson modifications are placed: B10 (Honors) will have more independence with completing calculations, B13 (General) will likely need more guidance (Jack, lesson plan, April 24, 2018).

Most of the teachers provided more practice problems to general students or students who struggle to solve the stoichiometry problem. However, unlike other teachers, Tina gave more practice problems to honors level class students than to general level students so that they could self-guide their learning by solving harder problems.

> Just making sure that the honors class gets enough practice, so they'll get more problems,

> especially for the homework assignments. And then, we might go more in-depth, just some of the different types of problems that are encountered in stoichiometry (Tina, interview, March 20, 2018).

Table 19 summarizes teachers' knowledge of the requirements students need for learning stoichiometry.

**Table 19**

*Teachers' Knowledge of Students' Prerequisite Knowledge or Skills for Learning Stoichiometry*

- Mathematics skill (70%)
  - Algebra; Unit; Dimensional analysis; Cross multiplication; Proportionality
- Abstract thinking
- Critical thinking
  - Chemistry: Significant figures
- Independence
- Self-motivation

*Note*. * Description of variation is based on general level students

**Variation in Instructions.** Participants differentiated their teaching of stoichiometry in the aspect of their instructional practices, assessment, and curriculum (Table 20). Significant differentiation was found in each teacher's instructional practices when teaching honors and general level classes. In general, they provided additional support and scaffolding in terms of time, practice, and guidance to general level students than to honors-level students. John, Annie, Tina, and Lena indicated that they give more time and practice problems to the students for their own processing time.

> I think that the difference between the two or at least the way that I conceptualized it was to provide a little bit more time for my regular students and have a little bit more in class practice so that I could attend to whatever issues and problems and misconceptions that they might have had. Or, even if it's not misconceptions, just like process issues, where they understand that they're using a molar ratio and they understand why they're using molar ratio, but their use of fractions is not great. Dimensional analysis just really messed with them. So, just providing more space for them to be able to flush out some of those issues with me there instead of forcing them to learn it too fast (Annie, interview, March 14, 2018).

**Table 20**

*Teachers' Knowledge of Variation for Teaching Stoichiometry*

| Variation between honors chemistry class and general chemistry class* | | |
|---|---|---|
| Curriculum | Instruction | Assessment |
| • Limiting reagent<br>• Solution: Concentration | • Needs more practice problems and time<br>• More guidance for solving problems<br>• More scaffolding with examples and analogies<br>• Discrete learning and assessing<br>• Solve only similar types of problems<br>• Practice DA method with non-chemistry context problems | • Limit number of things assessing<br>• Only ask problems seen in class<br>• Short answers & No explaining problems<br>• Types of problems<br>o Simple mathematics<br>o Give a balanced equation<br>o Give a simple chemical equation<br>o Short passage |

*Note*. * Description of variation is based on general level students

Teachers (Tina, Jessica, John, and Jack) expected honors students to be more independent in their learning. In other words, teachers involved more collaborative learning with their peers and allowed them to lead the class than the teacher.

> I do give more homework assignments, and then I like to have the students take more of the lead in my honors class. My general chemistry class, there's more time taking to just explain things and to have them finish assignments. But, my honors class, it's more freedom for me as a teacher and having students take the lead in showing and helping one another and in some cases teaching the class (Tina, interview, March 20, 2018).

Jeffry considers different mathematics background between the honors and general level students. Based on this difference, he encouraged general level students to utilize more of the conceptual tools (i.e., mini whiteboards and magnetic chips for representing the atoms/molecules) that they have used earlier in the chemistry class.

> I definitely consider their math background when I'm going through this [stoichiometry] because I know that some of them are going to come to me with really an initial negative feeling about the math. I think with that in mind with the level of students that I'm dealing with, we definitely went through the conceptual model and a little bit more detail

> and with little bit more scaffolds to help them through that. And, there was sometimes where we encouraged them [that] there's the mini-whiteboards that you've used. So, definitely wanted them to go back more to those tools in that. That's at that kind of general chemistry level class. I think at an honors level class, they've somewhat kind of moved beyond that. Once they've got it, they don't need that in the same way. So, I think when we were planning it out, we definitely understood that they were going to need some of those scaffolds, so kinda help their thinking along (Jeffry, interview, April 20, 2018).

Jessica also used a variation in instructional practice between the honors and general students based on students' cognitive differences. Because she knows that students in general classes tend to forget the earlier chemistry topics easily, she designed discrete learning experiences for them. In other words, she would not ask for, or link to, topics that they had learned earlier with stoichiometry.

> My honors kids, I feel like if I've taught it to them earlier in the year, I should still be able to pull on that even though it wasn't overtly reviewed for that unit. I expect them to hold on to that. My College Prep [lower level chemistry class] kids don't tend to hold on to things. They don't tend to see the connection from one thing to the next. So, I feel like it's more discrete learning, which isn't necessarily as in-depth of learning (Jessica, interview, April 20, 2018).

To get students familiar with the dimensional analysis strategy for solving stoichiometry problems, Stella solved more of non-chemistry context unit conversion problems with the general level students than honors students.

> Absolutely, so a lot of it is in the supports that we give. For example, with dimensional analysis, I probably did a lot of or I did do a lot of pre-teaching with non-chemistry related materials. So, I started off with inches and feet and seconds and years, just doing conversions that students are used to doing in their head with units that they're very comfortable with. And then, applying it to chemistry and units that they have no prior experience with. Whereas on honors, there's less of that pre-teaching and more into the chemistry unit, you know, grams and moles. So, a lot less practice I guess before getting right into the chemistry (Stella, interview, April 20, 2018).

**Variation in Assessments.** Four out of ten teachers (Jessica, Lena, Tina, and Jack) used variations of assessment between honors and general level classes. Jessica limited the number of items or topics that she assessed in one problem for general level students.

> I feel like you need to scaffold them a little bit more, you need to give them a little bit more support at the base and you need to limit how many things you're assessing at a time (Jessica, interview, April 20, 2018).

Also, Jessica and Lena indicated that they give only short answer questions to the general level students, while honors-level students were asked to explain the chemical reactions.

> I might ask the same question in terms of a short answer question from my CPA and my honors, but I would expect my honors to be able to go deeper in the explanation than my CP. And it might be worth more points on an honors test than it is on a CP test for the same reason (Jessica, interview, April 20, 2018).

Jessica also described the difference in the context of the problems. For example, she would usually give general level students one-step process problems or problems that have simple chemical equations because it does not coincide with the purpose of learning the stoichiometry.

Tina, Lena, and Jack also indicated that they only give simple mathematics problems and only short passage problems to their general level class.

> With college prep CP classes, which are the lower level of the two, I don't push them with the stoichiometry. I wouldn't ask them "How do I put this?" The problems aren't as complicated. I'd give them easier reaction equations. I still do the particle diagrams with them. I don't do as much of the mass-to-mass problems. I try to keep it more mass-to-moles and moles-to-moles, but not go mass-to-mass. I still expect them to be able to tell me what's limiting in a very simple equation. But I wouldn't give them a more complex equation. Honors, I expect them to balance their own chemical reaction equation. CP, I would give them the balanced reaction equation because I'm trying to assess can they use it at this point not they can balance it. So, if they screw up the balancing, then obviously they screw up everything else. So, I just want to know "OK, if I give that to you, can you do this?" So, that's one of the big ways I differentiate between them (Jessica, interview, April 20, 2018).

**Curriculum Differentiation.** Finally, three out of 10 (Jessica, John, and Lena) indicated that they differentiate the curriculum for the two levels of chemistry classes. John and Lena said they did not teach the concept of limiting reagent to their general level students. This may be due to a number of steps in limiting reagent stoichiometry problems. Jessica differentiated the two levels of classes by not introducing stoichiometry problems with solution concepts (i.e., molarity and molality) to general level chemistry class.

> I do stoichiometry with solutions with my honors. I don't do stoichiometry with solutions and molarity concentrations with my CP. That's another differentiation (Jessica, interview, April 20, 2018).

To sum up this category of knowledge and beliefs about students' understanding of specific science topics, teacher's had difficulty teaching stoichiometry for serval reasons: (a) students' difficulty placing variables in the mathematics equation, (b) students' loose understanding of the chemical reaction, which is a prior knowledge that students should have adopted in the earlier years of the chemistry curriculum, (c) students' lack of problem-solving skills, and (d) lack of students' motivation to learn stoichiometry. In addition, teachers differentiated the two levels of chemistry class (general and honors) in terms of instruction, assessment, and curriculum based on students' ability level.

**Knowledge and Beliefs about Instructional Strategies for Teaching Science**

Teachers' instructional knowledge is composed of two categories: knowledge of subject-specific strategies and knowledge of topic-specific strategies. While subject-specific strategies refer to teaching science as opposed to other subjects, topic-specific strategies focus on teaching specific topics within a science subject. Because this study focuses on teaching a specific topic, or stoichiometry, only topic-specific strategies were examined (Table 21). This strategy includes not only knowing about representations and activities—illustrations, models, analogies, problems, demonstrations, simulations, investigations, experiments—for teaching a specific topic but also knowing the relative strengths and weaknesses of the different approaches.

**Table 21**

*Teachers' Knowledge and Beliefs about Instructional Strategies for Teaching Stoichiometry*

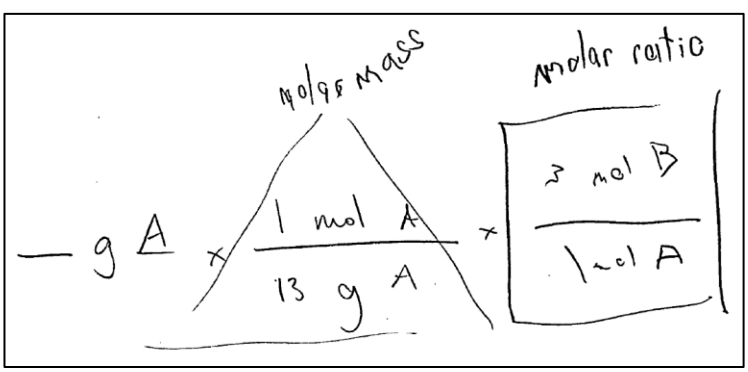
| Representation | Activity |
|---|---|
| • Visual cue for equation (10%)<br>• Analogy (30%): Pizza, S'mores, cooking<br>• Particle model/diagram (20%)<br>• BCA chart (30%)<br>• CER chart (10%)<br>• Example: Environmental context (10%)<br>• Example: counting by weighing (10%)<br>• Song (10%) | • Activity (50%)<br>  o S'mores, Peanut butter, Chocolate chip, Sandwiches<br>  o Crayon drawing activity<br>• Lab (60%)<br>  o Copper and silver nitrate lab<br>  o Combustion of magnesium lab<br>  o Vinegar and baking soda lab<br>• Discussion (40%): Whiteboarding session, iPad app<br>• Online math activity (30%)<br>• Demo: Balloon race and mole-to-particle (20%)<br>• Card sorting activity (10%)<br>• POGIL activity (10%)<br>• Simulation: PhET (10%) |

***Finding 1: Representations for Stoichiometry Instruction***

Participants used various representations—such as visual cues, models, charts, and analogies—for different reasons. Annie indicated that she uses visual cues as shown in Figure 13 to help students figure out what factor goes where when using dimensional analysis method. She also talked about how she used the dimensional analysis with her students by explaining Figure 13.

**Figure 13**

*Annie's Representation of Equation to Help Students Set Up Dimensional Analysis Method*



> We would, a lot of that was in visual cues in terms of like how I was working on the board. I would put like different shapes around different pieces. If I were doing that problem right, I would start with x grams of whatever, and then, uh, one mole of, we'll call this A one mole of A is 13 grams of A, right. And so I would do something like this and maybe put like a triangle around this and say the triangle gives us the molar mass except I wouldn't write it upside down because my upside down handwriting has gotten really bad even though it used to be so good. And then, I would give three moles of B for every one mole of A. And put a rectangle around that and say, 'Here's the molar ratio.' And so start this off this way with those visual cues that usually helped them. I would do it in color too, to again just highlight and be consistent and make it bright and attend to those things (Annie, interview, March 14, 2018).

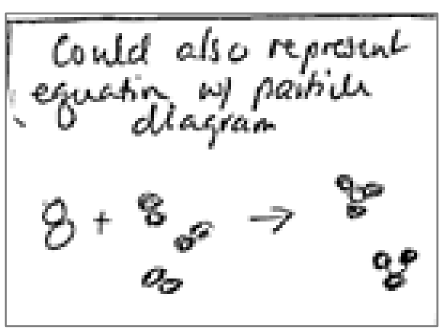
An analogy, especially cooking recipes, was commonly used to represent the concept of stoichiometry. Jack referred to pizza to remind students of the ratio and proportion that they had learned in Algebra class and to help them arrange the factors in an equation.

> Another way that kind of helps them is I'll usually refer to pizza. So, they know that one pie is equal to eight slices. So, if I have two pies, how many slices would that be? That really triggers this quick math. Like, “Oh! 16, four, five, 16 slices. How many times would that be?" And they can figure out how to rearrange [the equation] based on that (Jack, interview, April 18, 2018).

Teachers employed a particle model or particle diagram to promote a conceptual understanding of stoichiometry. They used this diagram to encourage students to think more reflectively, rather than relying on memorization and rote application of equations for solving stoichiometry problems. The following expert and figure 14 are evidence for the particle diagram that she used for teaching stoichiometry.

**Figure 14**

*Jessica’s Particle Diagram Used to Represent Chemical Reaction*



> We're doing these other pictures. These other things with particle diagrams and using that to represent reaction mixtures, not actual numbers because you can't see them. You can see these now, but it's a ratio and that's what they need to recognize (Jessica, interview, April 20, 2018).

Jessica, Lena, and Stella indicated that they use the BCA (Before-Change-After) chart (Figure 15) to represent a quantitative relationship and its chemical process in stoichiometry.

> We introduced the BCA chart in the beginning because I want them to recognize that you have this relationship between the reactant used and the product performed. Once they get familiar with it, I don't have them complete an entire chart, and if they can recognize that proportional reasoning from the balanced chemical equation, then I tell them to go for it. But, some of them do find the chart useful. I never assess that they have to do the chart. I just give it as an optional tool, and I start with that (Jessica, interview, April 20, 2018).

**Figure 15**

*BCA Chart for Stoichiometry Instruction*

$$2\,H_2S + 3\,O_2 \rightarrow 2\,SO_2 + 2\,H_2O$$

Before:

Change

After

Jessica also favored the CER (Claims-Evidence-Reasoning) chart because it allows students to practice using the evidence that they have with their reasoning.

> And then, at the end, I had them write something called a CER: claims, evidence, reasoning. So, I made them give me a claim. I actually give them the claim. I told them the claim is that the product form is copper-two-nitrate. They have to give me the evidence and they had to give me the reasoning to support that and if you can give me the reasoning then you're making that bridge between a claim and the evidence and that to me is important. And we do a lot of claim, evidence, reasoning (Jessica, interview, April 20, 2018).

John used unique representations to motivate students to learn stoichiometry topics and to recognize the importance of converting gram-to-mole. A detailed description of these representations is unfolded in the next section—Research question 4.

Finally, Tina made a song that summarizes the conceptual and procedural aspects of solving stoichiometry problems. Her intention was to help students memorize the new vocabulary in stoichiometry, such as dimensional analysis, and to reduce the fear that students have over mathematics involved in solving stoichiometry problems.

> Well, this year I took it upon myself to make a song to introduce some of the vocabulary terms and for them to see what they will eventually get in the end master. So, that was one way I bridged into this…… The song is my way of connecting with the students psychologically. The students when they first see this kind of math, they're intimidated. They reluctant to write anything. … And, I wanted to get in touch with the students and help them to see that this is something that they should not be afraid of and let's just sing a little bit about what we're going to learn. And also, they're learning some of the vocabulary terms that are going to be frequently used throughout the lessons for these next two to three weeks. So, it's a two-fold purpose for the song (Tina, interview, March 20, 2018).

***Finding 2: Activities for Stoichiometry Instruction***

In terms of the activities for stoichiometry instruction, cooking recipes were mostly used (five out of 10 teachers). Among the different recipes, peanut butter and bread sandwiches, chocolate chip cookies, and S'mores were the most frequently used by the chemistry teachers. They favored recipes for different reasons. Annie preferred recipes as a stoichiometry activity because it relates to students' daily lives. Lena believed that such a connection delivers a reason to learn stoichiometry. Some believed it helps students to transfer their conceptual understanding of stoichiometry from macroscopic to microscopic level. Most of all, teachers (three out of 10) favored the *S'mores* activity because it can be extended to limiting reagent concept.

> All of my metaphors for stoichiometry are around cooking pretty much cause that's the easiest one. But I used to do it with chocolate chip cookies a lot. I also used S'morse and they loved that because they would get to play with them and eat them. And especially with when we were getting into limiting reactants, S'morse were really very good because I could give different groups, different numbers of marshmallows, different numbers of pieces of chocolate, or different numbers of Graham crackers, and we could experiment that way and then they could eat the sugary foods at the end. So, that was primarily it, just thinking about like what kind of food do you like to make and trying to connect it to their lives somehow so that you can see more of a macroscopic view of

> what's going on and then take that to understand what's happening on a microscopic level (Annie, interview, March 14, 2018).

Teachers usually employed a cooking recipe activity as an introduction of the stoichiometry unit. Jeffry began his lesson by using the online activity of making sandwiches so that students can familiarize themselves with the vocabularies involved in stoichiometry, such as limiting reagent and excess reagent.

> We start out the stoichiometry with some activities online. … And they look at, you know, when you do this, you could actually have leftover bread and leftover cheese, which is a reality. I mean, it is not a big stretch when you make a sandwich, usually put more bread back in the bag and cheese back to the refrigerator. So, looking at those relationships, we spend a lot of time talking about what's an excess. And then, trying to see those pieces as the concepts that they are before we start naming them as limiting reactant or, you know, something other than that. So, we definitely have kind of adhere more to the concept before name in this particular unit (Jeffry, interview, April 20, 2018).

In addition to cooking recipe activity, teachers used a crayon/chalk drawing activity as their introductory lesson. John and Tina used this activity to practice the related calculation in stoichiometry: unit conversion from mass to mole.

> And then, today I had them do a simple lab where they took a piece of chalk and then they drew pictures, diagrams. And then from that, they measured the amount of mass that was used for drawing the pictures. And they took the mass and they converted to moles. Very simple one step t-chart process. And from there, I'll get them to add on to this t-chart and have them do more practice with the dimensional analysis thing. Going through the full stoichiometric process of this. This part of our lesson, I will call this as a pre-stoichiometric process. It's the introduction (Tina, interview, March 20, 2018).

A lab experiment with actual chemicals, such as copper and silver nitrate, was popularly used (six out of 10 teachers). Stella, Jessica, and Lena preferred this lab because it can lead the entire unit of stoichiometry from the ratio to the concept of limiting reagent. Annie saw the copper and silver nitrate lab beneficial because it gives a high yield and allows collecting a full set of data/evidence of the proportional relationship between the two substances.

There are supplemental activities that can be really valuable in terms of helping kids

> who can't get to the ratio really easily or the idea of molar ratio really easily. The thing that really has been most beneficial to them, understanding conceptually what's happening is to use evidence. So, we'll do labs that have generally speaking really high yields so that they can construct this idea from the actual quantitative evidence that they come up with. In the past, things like a copper and silver nitrate reaction is usually pretty good and gives pretty decent yield. I know some people use the magnesium strips, so burning magnesium for it because it gives that bright light and kids like that. But I've found that kids don't always get really good yields on that and so it's harder for them to get the idea from that. … So, we really were constructing a full set of data as opposed to just looking at the one reaction that they did or something like that. So, in that way we emphasized more of 'OK, so what are the ratios that we're getting? What are the proportions that are needed to do this and how does that work?' (Annie, interview, March 14, 2018).

Four out of 10 teachers employed a small group activity to promote students' discussion for solving stoichiometry problems. When facilitating the discussion, teachers used different tools. For example, Jake used an iPad app that allows students to share their problem-solving strategies. Tina, Jeffry, and Jessica used the whiteboard. From this discussion, teachers expected students to learn from each other.

> I definitely have had whiteboard sessions where students are asked to work in a group and talk about what they're learning and how to set up a problem. So, if students give them a problem, they would discuss it in the collaborative group setting. And then, draw out the concept on a whiteboard so that it's expressed visually, but then also explain it in writing on their board. That way people can understand how they're thinking and compare it to other group members and their responses and that a lot of times reveals any misconceptions that the students might have and any point of their solving the problem because then we can also come together as a whole class, compare our whiteboards and then talk about it as a class. How is our board different? How is this person thinking differently or group thinking differently than this group? (Tina, interview, March 20, 2018)

Annie, Jeffry, and Levy indicated that they do a separate activity or lab to practice the mathematics required for solving stoichiometry problems. Depending on teachers' problem-solving methods, the focus of the activity varied. When Annie focused on a dimensional analysis method for solving stoichiometry problems, she allotted a lesson on practicing dimensional analysis within a non-chemistry context. Because Levy believed that it is important to understand the proportional relationship in a chemical reaction, she did a lab to practice converting the whole numbers into ratio.

> I do a lab where they just test out different ratios and before they can test them out they have to say, "OK, how many drops of this and how many drops would that and what ratio does that give me?" And just being able to convert the [whole] number into a ratio number, like simplifying that, that's a significant. That's a math skill that they need (Levy, interview, March 23, 2018).

As mentioned in an earlier section, the orientation of teaching stoichiometry, Jack employed a card sorting activity to increase students' interest and engagement in learning stoichiometry. Jake used various activities such as a POGIL activity, simulation from the PhET digital learning app, and a demo in his stoichiometry instruction. These activities are addressed in detail in the next section for Research question 4.

To sum up this category of knowledge, patterns of the teaching of stoichiometry unit were noticed among the teachers. First, they tended to teach stoichiometry from the macroscopic scale to the microscopic scale. Most teachers employed concrete activities, such as S'mores or POGIL activity, at the beginning of their lesson and moved on to the chemistry content. In general, teachers organized their lessons from solving one or two examples of stoichiometry problems as a whole class, moving on to small group discussion sessions, and then having individual practice.

**Knowledge and Beliefs about Assessment in Science**

A sample of chemistry teachers' knowledge about assessment was categorized into two: methods and dimensions of science learning (Magnusson et al., 1999). The first category—methods of assessing science learning—is about knowing the multiple ways of assessing students' learning of a science concept. Dimensions of science learning assessment refer to teachers' knowledge of the aspects of students' learning that are important to be assessed. When analyzing the dimension of assessment, teachers' reasons for using a specific assessment were used as evidence.

***Finding 1. Formative Assessment***

In terms of formative assessment, teachers employed several methods to keep track of students' conceptual and procedural understanding of stoichiometry (Table 22). They use quizzes, exit slips, do now problems, practice problems, and discourse methods. Teachers expected students to consider the formative assessment as a self-assessing tool or resource for their own understanding of stoichiometry.

**Table 22**

*Teachers' Knowledge and Beliefs about Formative Assessment in Science*

| | Methods | | Dimensions |
|---|---|---|---|
| | Methods | Types of problems | |
| Formative assessment | • Quiz/Test<br>• Exit slip/Do now<br>• Practice packet/worksheet<br>• Homework<br>• Video<br>• Student responsive system<br>• Discourse methods<br>• (=public discourse)<br>• Lab report: conclusion | • Traditional stoichiometry problems | • For self-assessing<br>• Checking conceptual knowledge of stoichiometry: Molar ratio, molar conversion, proportional reasoning from macro to micro-scale |
| | | • Decontextualized problems | • Measuring procedural knowledge<br>• Familiarizing procedural steps |
| | | • Incorrect set up of equation problem | • Practice how to set up a proportion |
| | | • Conservation of mass problems | • Measure the understanding of big idea of stoichiometry |
| | | • Low stake practice problems | • Tracking the development of solving stoichiometry problems |
| | | • Lab report: conclusion questions | • Scientific investigation: How/what they do in the lab impacts the result<br>• Applying/understanding proportional reasoning<br>• Mole relationship/ratio |

> I think I'm giving them a lot of practice with the scaffolds. We did a lot of whiteboarding, so there was a lot of public articulation of it. I think that's a really key piece when we were talking discourse in the classroom: to have the kids talking to their peers and sharing their ideas of doing that. I wish I was a little bit more efficient at it in my classroom and practice. But, I think discourse methods are really helpful in those formative assessment pieces. I don't really feel like I need to give them a bunch of little

> quizzes to tell me they don't know it. You can get that from when they were trying to explain their work (Jeffry, interview, April 20, 2018).

Tina included traditional stoichiometry problems in her formative assessment because the end of the unit exam, which is provided by the Board of Education, involves these types of problems. Thus, to prepare students for that exam, she included those types of problems in the practice packet.

> The worksheet assignments are definitely used [to assess students learning]. A lot of that is used in this part of chemistry and that's because the test like the final exam. Those types of problems are very like they're wordy and require the students to think in terms of the textbook. So, basically what's on the problems that are in the textbook. They mirror what's going to be on the final exam (Tina, interview, March 20, 2018).

Annie gave traditional stoichiometry problems in her formative assessment to allow students to self-assess their learning of stoichiometry. She expected that decontextualized, or non-chemistry context, problems will assist students from familiarizing and adopting the procedure of solving stoichiometry problems. In addition, because Annie knows that stoichiometry falls under the big idea of conservation of mass, she denoted bringing in problems that underlie this big idea.

> I ask them questions about the conservation of matter as well. And then, give them some really good decontextualized problems that asks them to do this [stoichiometry] kind of thing and some very contextual problems that this is self-assessing. I was looking for how you're conserving mass throughout and what does this look like in this particular setting and how can we address this in this new problem (Annie, interview, March 14, 2018).

Participants included traditional stoichiometry problems (e.g., finding an amount of product from a given amount of reactant) in both formative and summative assessments. Jack’s intention for including this type of problem in the formative assessment was to allow students to practice applying the concepts in solving stoichiometry problems: applying molar ratio, molar conversion, and proportions from macro- to a microscopic level.

> By just giving them the practice questions, so they are converting from grams to moles and going and taking the equation and breaking it down into that point. So, by applying the molar ratios and the molar conversions, they're setting up their proportions to go from atomic to macroscopic, or back and forth (Jack, interview, April 18, 2018).

***Finding 2. Assessing Students' Conceptual Understanding of Stoichiometry***

In order to assess students' conceptual understanding and to avoid students relying on algorithmic techniques for solving the stoichiometry problems, teachers often used particle diagram drawing problems or explaining problems (Table 23). Teachers assessed students' conceptual understanding of chemical reaction and the proportional relationship in stoichiometry by drawing a particle diagram of molecules in the chemical reaction. They believed that such a drawing reveals more about students' understanding of stoichiometry than the traditional stoichiometry problems.

**Table 23**

*Teachers' Knowledge and Beliefs about Summative Assessment in Science*

| | Methods | | Dimensions |
|---|---|---|---|
| | Methods | Types of problems | |
| Summative assessment | Final exam/test | Traditional stoichiometry problems | • Goal accomplishment of stoichiometry lesson |
| | | Drawing particle diagram problems | • Understanding how molecules interact<br>• Understanding the ratio between the two substances |
| | | Explaining problems | • Capture the reasoning behind each step of the problem-solving<br>• Assess the conceptual understanding of stoichiometry<br>• Stay away from rote memorization or application of equation |
| | | High-level problems (multiple steps) | • Assessing mathematical thinking |
| | | Numerical/Equation set up problems | • Easy to fix and catch mistake/misconception |
| | | Real word example problems | • Measure proportional reasoning understanding<br>• No need for the equation for solving proportion problems<br>• Measuring problem-solving skills and critical thinking skills |

> I think that in terms of assessing that, there are certainly give them problems to see if they can do those problems. I really like having kids draw. I think that that form of assessment does a lot more to show their understanding than cranking out some numbers (Annie, interview, March 14, 2018).

To examine students' conceptual understanding of stoichiometry, teachers also employed explaining problems. In other words, students justify each step of the calculation and their answers to the problem (Stella and Levy). John even specifically asked students to explain what the numbers in the chemical reaction represent.

> Sometimes we ask them to draw pictures. Sometimes we ask them to do the calculation. And then, we ask them to describe in words: how they did the calculation? And why? What was the meaning behind the calculation? How they made the decisions? For example, what is the limiting reactant? Why did you choose that as the limiting reactant? (Stella, interview, April 20, 2018)

As Stella indicated, several different types of problems were included in one assessment. Specifically, Jeffry believed that such multiple representations reveal students' conceptual understanding of chemical reactions and a deeper understanding of the proportional relationship between the two chemicals.

> I think giving them multiple representations that they have to give and was not unlike our first piece where you said, "Do this in two different ways." So, we're going to give them chemical reactions to analyze and to think about. But we're going to ask them to do it from the particle diagram perspective, from the mathematical perspective, from lots of different ways that you can approach. This is to make sure that it's not just about the number and that they deeply understand how these things are interacting and why the number ends up being in this ratio (Jeffry, interview, April 20, 2018).

In order to reduce the pressure of making calculation mistakes for solving stoichiometry problems, teachers employed equation set-up problems. In other words, instead of asking students to find the final answer, the equation is only asked to be provided. This is often used in major exams, such as the end of a state test or SAT chemistry test. Jack preferred this type of problem because it is easier for him to notice and fix students' mistakes while solving problems.

> Another thing is a lot of times, the questions I ask don't involve solving for the exact answer will just like on the Regents it says like show a numerical setup. So, something like that I can easily fix for them or show them how to fix it (Jack, interview, April 18, 2018).

As an indicator of students' conceptual understanding of stoichiometry, Jessica, Lena, Tina, Jack, and Levy indicated that they ask conceptually challenging problems in the post-lab questions or quiz.

> I do dig pretty deep in terms of conclusion questions like "what could make your mole ratio be off?", where suppose that your copper silver mole ratio was OK, but your percent yield and your silver (Ag) [and] silver nitrate ($AgNO_3$) ratio is okay, it's one to one, but then your percent yield is really low. Like how could that happen? Like I want them to be able to understand how what they do in the lab impacts what they take out this data (Jessica, interview, April 20, 2018).

Finally, teachers used both low-level and high-level problems when assessing students' understanding of stoichiometry problems. Low-stake problems were used more often for formative assessments to track students' step-by-step understanding for solving stoichiometry problems.

> Along the way [of assessment], the practice problems, I am sure of tracking the development of that [stoichiometry concept]. So, I gave them a lot of practice problems and after the first few I gave him the answers with it. I said, "look this is not a test, this is for you to learn it, so I'm going to give it to try each one and check the answer. If you didn't get it right, go back and figure out what the mistake was and check the worksheet". So, I just checked off. Did you do the first? We weren't able to do all the problems in the first one. You get them all correct the first time. OK. How about the second one? So, if I see they're gaining until the point where they can do the third one without any mistakes, then I know they're ready for a test. Anyone who is having trouble on the way to intervene at that point, so I'm trying to give them lots of low stakes practice (John, interview, March 22, 2018).

Then the high-level problems were used in summative assessments to assure that students are thinking mathematically rather than just relying on memorization.

> If I give them more higher-level problems. So, things that involve more than one step, like for them to go from grams to moles is a simple thing. That's something that they can memorize. They know how to do that. It's even on the reference table that they get. Like converting, like going from that into applying to a chemical equation, you're adding

> more than one step, so that's assessing their mathematical thinking (Jack, interview, April 18, 2018).

To sum up this category of chemistry teachers' knowledge about assessment, the teachers employed formative assessment and summative assessments. All ten teachers used the final exam as a summative assessment for the stoichiometry unit. Moreover, whether it is formative or summative, all the assessments heavily depended on solving problems for this unit. This is reasonable as the key feature of the stoichiometry unit is solving numerical problems. Thus, when analyzing teachers' assessments, each type of problem was attended with its purpose to understand not only the assessment methods that teachers use but also the dimension of each assessment as suggested by Magnusson et al. (1999).

# Chapter V: Findings from Three Cases

In the previous chapter, the findings addressed the first three research questions. In Chapter 5, it is the second chapter of Findings that addresses research question 4. This chapter of findings gives wholistic data for three of the chemistry teachers that are presented as cases.

## Research Question 4.

## What characteristics do three of the chemistry teachers show based on the evidence of Likert scale survey, proportion problems, and interview?

In this section, Likert scale survey evidence, problem-solving strategies, and interview responses will be revisited for three teachers (Jake, John, and Lena) to capture their unique characteristics for teaching stoichiometry and the application of the proportional reasoning concept in their instruction. Jake was selected as he showed the typical characteristics of teaching stoichiometry and proportional reasoning as a new teacher, while John had a unique insight on teaching stoichiometry and the mathematics practices involved compared to the other eight teachers. For example, he was the only teacher who did not think it was his role to make a connection with the mathematics class (John, Interview data). Finally, Lena's data were closely examined because she showed the most active support for students' understanding and development of proportional reasoning in her chemistry teaching.

### Case of Jake

Jake is a male chemistry teacher in his 30's. He teaches at a public school located in New York City. Jake majored in chemistry while in college and the highest degree he earned is a Master's in Education. At the time of the study, he was teaching general level chemistry and living environment (biology). Also, it was his fourth year of teaching chemistry to 10th and 11th grade students, who all had taken Algebra 1. Jake's school did not have pre-requisites for taking the chemistry class. In response to the Likert survey, he indicated that the science department and mathematics department sometimes collaborate and that he also sometimes consults with mathematics teachers. Jake participated in professional development opportunities that focused on chemistry content, stoichiometry, and the NGSS, but not on mathematical thinking.

***Belief***

In the dimension of how students learn, Jake believed that students' proportional reasoning ability could be developed through both their daily life and schooling. He also believed that proportional reasoning ability and stoichiometry instruction could benefit from each other. In other words, proportional reasoning can be improved through stoichiometry instruction, and vice versa. Jake believed that students' successful stoichiometry problem solving indicates their understanding of the concept. These beliefs indicate that Jake values diverse ways of high school students' learning of proportional reasoning and stoichiometry.

Regarding the teacher's role, he strongly believed that it is both mathematics and chemistry teachers' responsibility to teach proportional reasoning and that chemistry teachers should work to improve students' proportional reasoning ability. Also, he believed that chemistry teachers should present multiple solution methods for solving stoichiometry problems. However, when Jake explained the difference between the two methods that he solved, he did not seem confident in presenting multiple solutions. He did not think the two methods that he suggested for each of the five problems were different.

Jake expressed his confidence in teaching proportional reasoning. For example, he was confident in teaching proportional reasoning with any type of problem and integrating proportional reasoning in his stoichiometry instruction. Jake strongly agreed to all the items that asked for his opinion on whether he feels responsible for teaching both concepts (Appendix D:

items 5, 7, 8, and 12). However, he only agreed to all the confidence related items (items 6, 15, 18, and 24). Based on this tendency, Jake strongly believes that it is his role to teach the two concepts, but it seems he is not quite confident in teaching them.

In the aspect of students' ability to learn proportional reasoning and stoichiometry, Jake thinks proportional reasoning should be mastered during the student's middle school year (item 10). He agreed that a student's lack of understanding of proportion hinders them from learning stoichiometry (item 13). He also expressed that student's insufficient mathematical knowledge limited him from teaching deeply about the concept of stoichiometry (item 17). However, Jake believes that proportional reasoning ability can be improved while learning stoichiometry concepts and does not think this is too demanding for students.

Finally, Jake understood the importance of student's learning of proportional reasoning and stoichiometry. For example, he strongly agreed that proportional reasoning is important in terms of learning other science concepts (item 3), being a mathematics and science-literate citizen (item 11), and understanding the work of scientists (item 14). Also, he understood that proportional reasoning is commonly used by scientists (item 19).

***Problem-solving strategies***

**Stoichiometry problem.** Jake used a dimension analysis method for converting the grams of ammonia to moles of ammonia. Then, a cross-product was used for molar ratio relation between ammonia and hydrogen. Finally, the dimensional analysis method was again used to find out the grams of hydrogen from the previous step (Figure 16). There was no difference between the two methods, except that the proportional relationship between the moles of two substances was arranged differently.

> For the first one [problem], um, with the equation, with the Harbor-Bosch process, I was just trying to use the ratio method. So, I was looking at the ratio of ammonia to

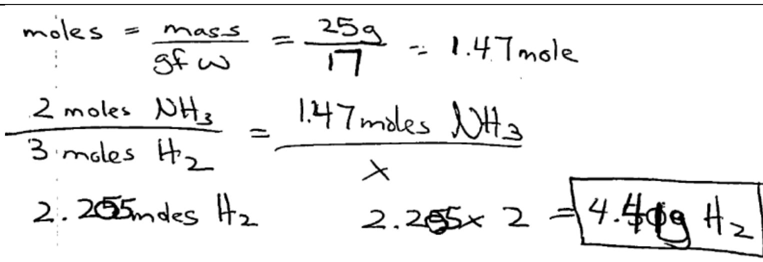
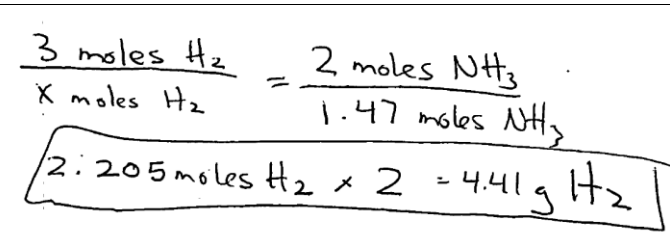
hydrogen, just trying to set up some proportions with that. Um, given the mass that was provided for the ammonia, so I converted the mass to moles [of ammonia], um, and figured out how much moles of the hydrogen you would need to, um, to have and then convert, um, that, those moles to mass [of hydrogen]. And I was trying to think about, um, different, uh, different methods. So, the only thing that I tried to figure out to do differently is, um, the, the, the proportions and just in terms of how you set up the proportions might be different (Jake, interview, March 13, 2018).

Also, Jake preferred the first method not only for himself but also for teaching purposes. He explained his process.

> So, when I did the first one [method], I did it from moles of ammonia to moles of hydrogen. And then in the second one [method] I set it up, have mole of the hydrogen [in the chemical equation] to how much moles of hydrogen that you would need. Um, and then, on the other side [of the equation], the moles of the ammonia, um, that was provided in the formula to the moles of ammonia that you had given the 25 grams, that was the, the only slight difference in terms of how I would set it [the two fractions] up (Jake, interview, March 13, 2018).

**Figure 16**

*Jake's Problem-solving Strategies for Solving the Stoichiometry Problem*

| | |
|---|---|
| Method 1.<br>Dimensional analysis (DA)<br>Fraction (PR-F)<br>Unit-rate (PR-UR)<br><br>[Preferred method] |  |
| Method 2.<br>Dimensional analysis (DA)<br>Fraction (PR-F)<br>Unit-rate (PR-UR) |  |

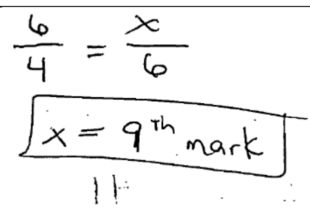
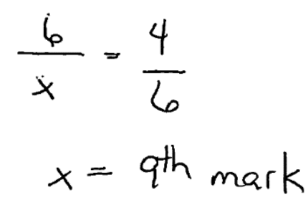
For the three proportion problems (jar, density, and orange juice, Appendix A), Jake employed diverse methods. However, while explaining his work for each problem, he constantly showed a lack of confidence in defining the differences in his two methods.

**Jar problem.** Jake used the fraction method (PR-F) for both methods (Figure 17) and expressed his concern over defining the two methods as different methods. Though Jake did not see these two as different methods, he preferred to use the first method.

> Once again, with the second problem, I was just thinking about the proportions. So, um, it stated that when the, the water level was at the sixth mark in the narrowed jar that it was at the fourth mark in the wider jar. So, I just set up those proportions, um, like six over four to x, the x mark and the narrow jar to the six mark in the wider jar to solve those proportions. Um, and then for the second method, like, I was just trying to think of a different way of solving it. Um, but I just essentially just set up the proportions slightly differently (Jake, interview, March 13, 2018).

**Figure 17**

*Jake's Problem-solving Strategies for Solving the Jar Problem*

| | |
|---|---|
|  |  |
| Method 1. Fraction (PR-F)<br>[Preferred method] | Method 2. Fraction (PR-F) |

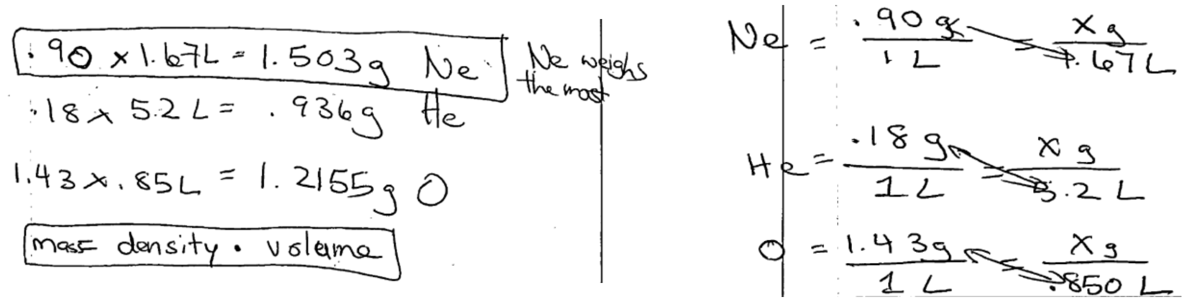
Moreover, he voluntarily came up with a third method of showing the proportional relationship visually, but he was still skeptical if this third method was different from the other two. The following elaborates his thought for this method.

> And then, it just got me to thinking about, well, if I was to do it visually, like I know that it's in a ratio of three to two, maybe if I had a third method of just showing a visual that, um, for every three, that the water level is going up in the narrow jar that is going up two levels in the wider jars. So, cause I don't know that if it's essentially that different in terms of the problem-solving methods since I'm still using proportions. But I guess it could be considered a different method (Jake, interview, March 13, 2018).

**Density problem.** This was the only problem that Jake actually solved in two different ways. Dimensional analysis was used for the first method, while the cross-product method was used for the second one (Figure 18).

**Figure 18**

*Jake's Problem-solving Strategies for Solving the Density Problem*



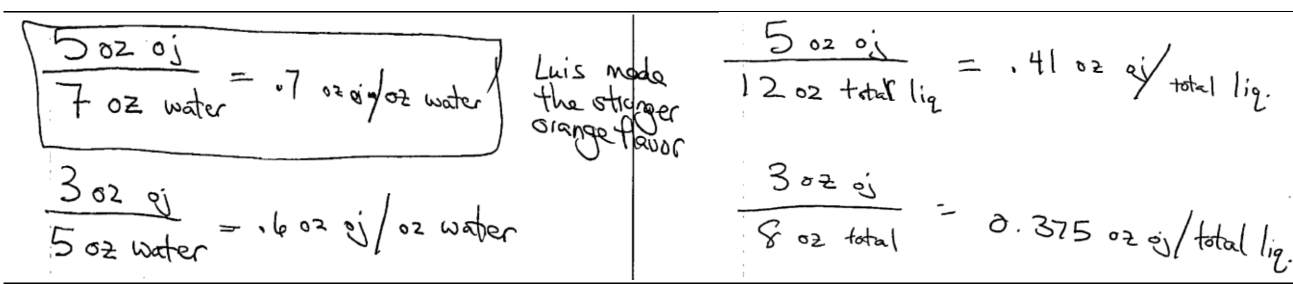

| Method 1. Dimensional analysis (DA) [Preferred method] | Method 2. Cross-product algorithm (PR-CP) |
|---|---|

So once again, I don't know how different they are, but essentially what I did was… since I wanted to figure out the mass, the mass is equal to the density times the volume. So, I just multiplied all of the densities that were provided by the volume. And the only thing was that the volume of density was given in the milliliters. So, I just converted that to liters since the density was provided in grams per liters. And then, in the second method, I just set up that proportion, so if like the Neon, it's 0.09g/L. So, I just set that as the proportions equal to x grams of Neon and over 1.76L of the Neon. And, I essentially did that with the other gases that were provided as the second method (Jake, interview, March 13, 2018).

**Orange juice problem.** Jake and the other teachers used the unit-rate (PR-UR) method for the orange juice problem. However, each method refers to a different concept of concentration, i.e., solute over solution and solute over solvent, respectively (Figure 19).

**Figure 19**

*Jake's Problem-solving Strategies for Solving the Orange Juice Problem*




| Method 1. Unit-rate (PR-UR) [Preferred method] | Method 2. Unit-rate (PR-UR) |
|---|---|

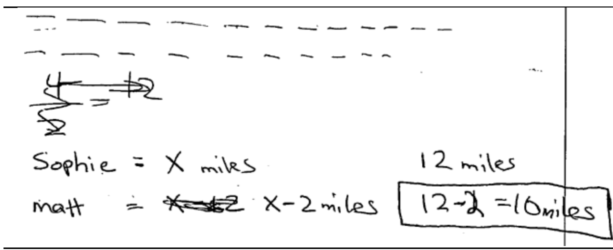
I just looked at the ratio of the juice concentrate to the water as the first method. And that the second method, I'm probably not the most ideal, but I just looked at the juice concentrate to the total amount of liquid (Jake, interview, March 13, 2018).

**Non-proportion problem.** For this non-proportion problem, Jake solved algebraically for both methods (Figure 20) by setting up an equation for the constant distance difference (two miles) between the two runners, Matt and Sophie.

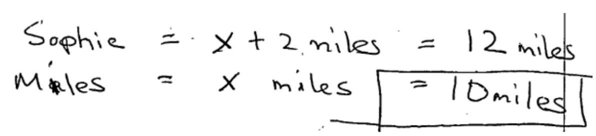
> I think the fourth problem [non-proportion problem] I was overthinking. So, Sophie essentially was running two miles more than, um, than Matt, and they were both running at the same speed, so they would essentially be at that same gap. So, the first method, I was like, well, if Sophie is running x miles than Matt is running two miles less than, um, than Sophie. And then, the second method, um, slight variation. If Matt is running x miles, then Sophie is running two more than that. So, that was my second (Jake, interview, March 13, 2018).

**Figure 20**

*Jake's Problem-solving Strategies for Solving the Non-proportion Problem*




Method 1
[Preferred method]




Method 2

Then, Jake was asked to elaborate on what made him hesitate about this problem. Based on the following excerpt, it seems he had a hard time coming up with two different equations.

> Well, I guess essentially when I'm looking at the problem-solving method, I was just trying to figure out... I should probably show some sort of algebraic relationships. So, I guess I was just trying to figure out, like, what algebraic relationship, you know, should I set up (Jake, interview, March 13, 2018).

### ***Pedagogical Content Knowledge***

Jake's stoichiometry instruction was mostly didactic and activity-driven. For example, his lesson plan (85 minutes block schedule time) and lesson materials for the stoichiometry were based on PowerPoint and guided notes that he prepared for the students. Half of the lesson (30 minutes of independent practice time) was focused on doing a stoichiometry computer lab, working on a set of molar ratio problems from POGIL (Process Oriented Guided Inquiry Learning; a student-centered instructional approach that generally follows a learning cycle paradigm), and watching video clips from Georgia Public Radio website. In addition, he shared two other resources that he uses: (a) PhET Interactive Simulations, which is popular for its simulations for a number of science and mathematics concepts, and (b) BrainPOP, well-known for short animated videos in K-12 major subjects (i.e., social studies, English, mathematics, health, and arts). All these resources indicate that Jake has great knowledge of curriculum programs and materials to support his chemistry teaching.

Three categories were noticed for Jack's knowledge on science goals and objectives: (a) understanding the related concept, (b) acquiring the related skills, and (c) understanding the work of scientists. In the interview and lesson plan, he indicated that the goal of this lesson was to understand the concept of mole and the law of mass conservation. This goal was reflected in Jack's guided notes for students. He specifically required students to write the definition of the law of mass conservation.

> CONNECTION to ACHIEVEMENT GOAL. Students will be able to apply the principle of the law of conservation to chemical reactions. They will be able to make to appropriate calculations to determine the amount (mass /moles) of the reactants & the products (Jake, lesson plan, March 13, 2018).

Jack also hoped that the students, even for those who will not go into chemistry, will perform and understand the importance of mathematical thinking, especially related to proportion and ratio.

> In terms of the chemical sense, just getting them to understand that the primary counting principle in chemistry is in moles, um, to get them to drive that point across. In a more general sense, you know, so even if students never go into chemistry or anything like

> that, but just understanding the importance of proportions and ratios and being able to establish like, um, you know, just like how to think mathematically in, you know, in that respect (Jake, interview, March 13, 2018).

In addition, as an implicit goal, Jake hoped that students would adopt that the mole is how chemistry accounts for invisible particles or substances.

> I intend them to just learn that in the world of chemistry, the way we [chemistry related people] account for, um, substances, molecules, um, uh, atoms. The way I kind of expressed it to them [is] just in terms of the budget keeping that, um, this is the manner in which we account for everything. Um, you know, we talk about the law of conservation of energy and mass and all of that type of thing. So, in order to keep track of everything, this is how we keep track of these things that you can't see (Jake, interview, March 13, 2018).

In terms of students' understanding, he indicated that students have a hard time setting up proportions correctly. In other words, students often flip the relationship in the two equivalent fractions.

> Um, setting up the actual proportions. So, understanding that you know if you are setting up your proportions of moles of hydrogen to ammonia that, you know, it has to be in that sequence. So, sometimes the students will flip the items. Um, then the other thing I guess would be, um, sometimes not realizing that. So, for example, 25 grams of ammonia that was provided, sometimes those numbers make their way into the proportion. Um, so understanding that once again, you set up your proportion then those items or that unit of substance has to be consistent (Jake, interview, March 13, 2018).

Jake thinks that the abstract and unfamiliar context is a real barrier for students to think mathematically about the stoichiometry concept.

> I think the challenge is that, one, you have many students who have difficulty with setting up proportions and ratios and then compounded by the fact that you're asking them to set up these proportions in just totally abstract scenarios. So, I think that's probably just like the biggest challenge. Like I think it's easier to understand, you know, setting up these proportions if you took it, talking about everyday things, like baking or cooking, you know, things like that where it makes it a little bit more sense. Um, but then you're asking them to do something that's challenging in a context that's challenging.

The following piece reveals the component that Jake considers in terms of student variation when planning a stoichiometry lesson.

> I look to see, um, which students have a little bit of stronger versus weaker math skills. So, some students, they don't even have to really take a look at the example of, you know, like, the *S'mores* or something relatable. They [can] think mathematically and they can set up those relationships. Whereas some students struggle to think mathematically. So, I guess I just look at, you know, what their ability level in math is (Jake, interview, March 13, 2018).

According to this quotation, he considers the role of a student's mathematic ability for planning stoichiometry lessons, as this variation affects students' acquisition of the concept.

As mentioned earlier, because Jake believes that stoichiometry involves very abstract concepts, he starts his stoichiometry lesson with a concrete example and enlarges it to include a more abstract context.

> I try to always start off with something a little bit more relatable and concrete and then go into some of the more abstract, taking a look at the chemical formulas. Well, a lot of this is very abstract. I mean they're not dealing with nitrogen or hydrogen on a regular basis. So, you know, um, if we could get something that's a little bit more concrete and tangible and then hopefully extend that to um, you know, something along, you know, chemical reaction than I think I'm, hopefully they can transfer that knowledge to situation like this (Jake, interview, March 13, 2018).

As a result, he starts his stoichiometry lesson with activities that are more familiar to students, such as the *S'mores* recipe activity.

> I would essentially, well typically like I'll just start off with something, um, more common to them. So, like if they bake or cook something like that… Um, the other thing that they really enjoy is, like, it's pretty popular, um, *S'mores* lab, like if you wanted to make a *S'mores* and given this amount of material, Graham crackers and chocolate bars, things like that. And if you wanted to make x amount of *S'mores* enough for the entire class, then how many of each of the individual items that you would need (Jake, interview, March 13, 2018).

In terms of an assessment, Jake relied on understanding students' conceptual understanding not only through solving stoichiometry problems but also through conceptual problems. For example, a question that requires an explanation of incorrect equation set up. He checks students' understanding of the stoichiometry through a number of practice problems, assignments, and exit slips.

> At the end of a lesson, I would typically like give them once again like some sort of exit slip or something like that. And a common one would be, once again, like, to give, um, a problem that's set up incorrectly and then for them to explain, you know, like why this is set up [like this]. So, then that'll give me an idea of whether they understand how to, how to set up, um, the, uh, the problems. Um, um, and then just sort of practice problems, homework assignments, quizzes, um, give me an indication of if they understand how to set up these proportions (Jake, interview, March 13, 2018).

In the lesson plan, Jack also indicates that he will use student response systems to answer sample Regents exam questions on stoichiometry as a formative assessment of the lesson.

***Summary of Jake's Case***

According to Schneider and Plasman's (2011) category of science teacher experience, Jake was fairly a new teacher (four years of teaching chemistry). Jake's belief that students' successful problem-solving indicates their understanding of the stoichiometry concept aligns with his assessing strategy (item 2). This belief was reflected in his lesson materials, such as practice worksheets, exit slips, and homework, as the number of traditional stoichiometry problems included in these assignments. Moreover, he assured students' proficiency by successfulness of solving those problems.

Though Jake showed his confidence in presenting multiple solution methods for teaching stoichiometry problems (item 6) and understood his role for showing them (item 12), the two methods that he provided for each problem (except the density problem) were actually the same strategy of solving the problems. Also, no evidence either in his lesson plan or interview was found that he was introducing multiple solutions.

There was no relationship or connection among the five categories of PCK in Jake's case. In other words, the five categories of PCK played independently. Though he incorporated many interesting instructional strategies and assessments in his stoichiometry instructions, it was hard to reveal the reasons for using those materials. Considering that he is fairly a new teacher, it is speculated that he might be at the stage where he collected all the good lesson materials that are known to be effective in teaching stoichiometry instructions. Table 24 summarizes Jake's PCK.

**Table 24**

*Summary of Jake's PCK in Teaching Stoichiometry*

| | |
|---|---|
| Orientation to teaching science | Didactic & Activity-driven |
| Knowledge of science curriculum | Goals & objectives<br>• Understand the concept of mole and law of mass conservation<br>• Understand and able to solve ratio and proportion<br>• Understand how chemists account for particles that are invisible<br>Curriculum programs<br>• Brain pop, PhET, Georgia Public Radio, and POGIL |
| Knowledge of students' understanding | Requirements<br>• Math skills (related to proportion and ratio) & Balancing equation<br>Students' difficulty<br>• Setting up two equivalent fractions correctly<br>• Abstract scenarios of applying the proportion |
| Knowledge of instructional strategies | Subject-specific strategies: Using technology (ex. iPad apps)<br>Topic-specific strategies: S'mores activity |
| Knowledge of assessment | Dimension of science learning: Conceptual understanding<br>Methods of assessing: Exit slips, practice problems, quizzes, student response system |

## Case of John

John is a male chemistry teacher in a private school located in New York City. He was a Biology major in college, and earned a Master's and Ph.D. in Agronomy. John taught chemistry for thirteen years. At the time of the interview, he was teaching general level, modified (lower level), and AP chemistry to $10^{th}$ grade students. He had about six to twenty students in one class. John's school does not have a prerequisite for taking chemistry classes. Also, John indicated that he rarely consulted with mathematics teachers. However, his science department sometimes collaborated with the mathematics department. John rarely participated in any professional development experiences, except the one that focused on improving students' mathematical thinking. Also, because he works at a private school, he was not very familiar with any of the standards or curriculum that we often encounter in public schools.

***Belief***

John generally agreed that proportional reasoning could be adopted both by daily life (item 1) and by school education (item 25). He also believed that the proportional relationship in stoichiometry and solving stoichiometry problems improve student's proportional reasoning ability (items 9 and 20). Moreover, John agreed that successfully solving stoichiometry problems indicate that the students have understood the concept of stoichiometry (item 2). This belief was reflected in his instructional methods and assessments for stoichiometry, where both were heavily focused on solving problems.

Though John considered teaching proportional reasoning as one of his responsibilities as a chemistry teacher (items 5, 7, and 8), he believed that it is more of a mathematics teacher's responsibility to teach it (item 4). In addition, he was skeptical about presenting multiple solutions methods for stoichiometry problems. For example, John stated that he does not show different methods of solving stoichiometry problems (item 6). Also, he did not express his opinion for item 12, which asked if chemistry teachers should present multiple solution methods for stoichiometry problems. Moreover, he was very confident about teaching and integrating proportional reasoning during stoichiometry instruction. He strongly agreed to all the items (15, 18, and 24) regarding the confidence of teaching the two.

In terms of students' ability, John strongly believed that proportional reasoning should be mastered during the middle school years (item 10). However, he still believed that students could learn stoichiometry without proper adoption of proportional reasoning ability. John disagreed with item 13 that asked if he assumes that high school students cannot learn stoichiometry if they don't have the proper knowledge and ability to solve proportion problems (item 13). In addition, he indicated that he personally did not have limitations in teaching stoichiometry due to students' lack of mathematics knowledge (item 17).

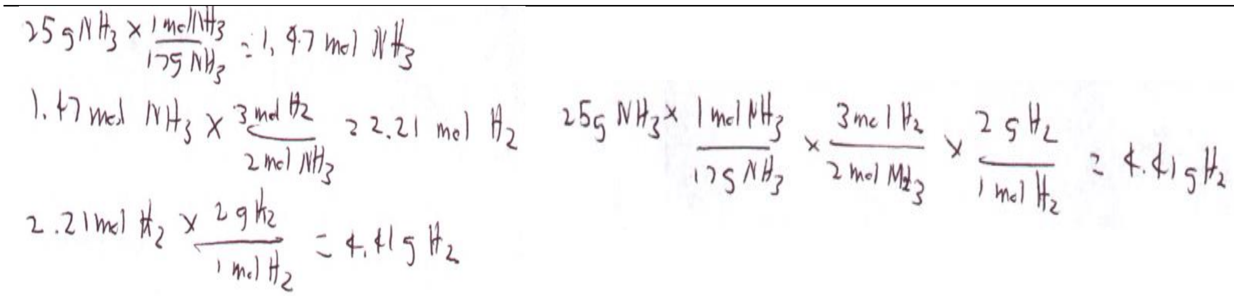
Overall, John was skeptical about developing proportional reasoning ability while learning science concepts (item 3) and being a science and mathematics literate citizen (item 11). Though he was not sure whether proportional reasoning is a common mathematical practice that scientists engage in (item 19), he believed that it would help students to understand the work that scientists do in lab (item 14). Finally, John was not sure if chemistry teachers need more support in integrating mathematical practices in their stoichiometry instruction (item 26).

***Problem-solving strategies***

**Stoichiometry problem.** Figure 21 and the following interview excerpt show the two methods that John used for solving the stoichiometry problem.

**Figure 21**

*John's Problem-solving Strategies for Solving the Stoichiometry Problem*



| Method 1. 3 steps of dimensional analysis (DA-DA-DA) [Preferred method] | Method 2. Dimensional analysis one chain equation (DA-chain) |
|---|---|

> So, the calculation itself is really the same. The difference is that the first method was the way I teach it, which is in three steps. And the second [method] is one a chain calculation. And I mentioned the students that you can do this if you want, but [it is] a little more intricate (John, interview, March 22).

John gave a concrete reason for preferring the first method.

> I preferred [the first method] for teaching because I think breaking it up into steps like that is easier for students. Each step is a concept that the first is converting a mass to number of moles. Second is using proportionality of the two substances from the chemical equation to determine how many moles of one is really how many moles of the other. And then finally it's just a conversion of a number of moles back into mass. So, each one has a simple concept they've learned that that is being done in each step (John, interview, March 22).

John preferred the first method because students can identify what they are doing for each step, and it corresponds to what they have learned earlier in his class. In terms of catching errors, he thought the chain calculation was harder than the step-by-step one because students actually have to cancel out all seven units used in this problem in order to confirm their answer.

> If you do it in a chain calculation, you do all the math all at once. Um, it's a little bit more intricate checking for cancellation of the units that it's here is correct. But, I think the most difficult part of that for teaching is… (pause)… that the student can say in words what they're accomplishing each of the three steps in the first method and they can give it a name. Whereas here [second method], I think I mentioned this earlier, you have to kind of conceive of what you've accomplished in the first multiplication. And then, decide what you wanna do with that result in the second and the third. And you're essentially doing mentally what was done on paper first. So, it requires student to be more familiar with the process and it's more intricate to have a chain calculation where there is more potential for error. It may not be immediately obvious, I think until you cancel out all the units, which they often don't bother to do to catch the error. And then if you have three factors here that you have six items in there that any one of which could well actually seven and in one of which could be wrong. So, it's more difficult to correct, to catch an error and correct it. So, from a teaching point of view, I think the first is more accessible (John, interview, March 22).

Moreover, John also pointed out that the second method (dimensional analysis chain calculation) would be harder for learners who are not familiar with the context of this problem. This aligns with the literature (Akatugba & Wallace, 2009; Saunders & Jesunathadas, 1988) that context affects learners' performance of proportional reasoning ability.

> Um, I think for them, if they are just learning it to think about adding on…(pause)… additional, um, terms and multiplication…(pause)… different additional factors and different multiplication it is something that results that they haven't seen yet. I think it's a more complex way to think about it [second method] for someone who's not used to solving these (John, interview, March 22).

For the three proportion problems, John tended to use proportional reasoning strategies. However, for the stoichiometry problem, he used dimensional analysis strategies for all three proportional relationships in the problem. Most of all, he was very skeptical about solving these problems in two different ways. For every problem, he disapproved the idea of solving in two different ways. This aligns with his belief that it is not his responsibility, as a chemistry teacher, to provide multiple solutions for stoichiometry problems. The following is John's overall thought for the five problems:

> The [five] problems themselves were easy enough. Mostly pretty familiar. One of them was a little different type. But, um, coming up with two methods, was a little tricky because I'm used to doing it a certain way and it had to kind of... it is kind of artificial to come up with a second one that was just sort of a variation on the first, you know, it wasn't really fundamentally different (John, interview, March 22, 2018).

**Jar problem.** Figure 22 and the following quotation from his interview shows the two strategies that John used to solve the jar problem and his view on them.

> Since I didn't have actual concrete units for this, I had to… There was no alternative way to set this up as a portion, um, which I did. So, in the first method, um, I define the ratio of, um, it's essentially the ratio of the cross-sectional area. The two jars based on what marks the water came up to and then used that as a multiplier to convert from one jar to the other. Um, and the other [second] method, um, I set it up as a proportion. It has just as, um, you know, six here is four to there, as x is 6. And this is, this is the way that my students used to doing it from their math classes. Um, so it, it's essentially, you know, mathematically equivalent. It's just a matter of um, two different approaches to the same facts, that same answer. So, um, I would probably use the first method (John, interview, March 22).

**Figure 22**

*John's Problem-solving Strategies for Solving the Jar Problem*

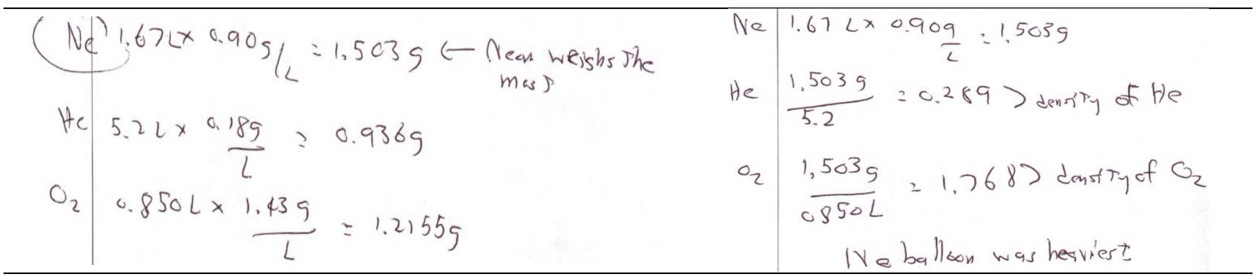
| | |
|---|---|
| $6/4 = 1.5$<br>$1.5 \times 6 = 9$ | $\frac{6}{4} = \frac{x}{6}$<br>$x = \frac{6 \cdot 6}{4} = 9$ |
| Method 1. Unit-rate (PR-UR)<br>[Preferred method] | Method 2. Cross product (PR-CP) |

For the first method, John used the unit-rate strategy by finding the multiplier. He knew that the second strategy (cross-product) is what students learn in mathematics class. John expressed that these two methods were mathematically equivalent; but in approach, they are different. In addition, unlike other participants who doubtlessly used ‘wide jar mark’ and ‘narrow jar mark’ as a unit, John did not see the marks as a unit.

**Density problem.** For the density problem, John used dimensional analysis and unit-rate strategy (Figure 23). Again, he showed his disapproval for coming up with a second method. The following illustrates his two problem-solving methods and his thoughts about the second method.

**Figure 23**

*John’s Problem-solving Strategies for Solving the Density Problem*




| Method 1. Dimensional analysis (DA) [Preferred method] | Method 2. Unit-rate (PR-UR) |
|---|---|

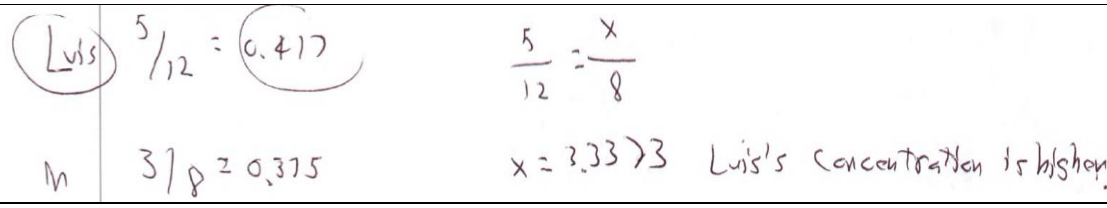
> The first method was just to calculate the mass of each of the three balloons and then just see which one was the greatest. And then, I had to come up with a method that wasn't sort of the obvious one to me. So, um, so what I did here was I calculate the mass of one of the arbitrarily, one of them. And it turned out to be the one that was the heaviest. I had already found that, but just one of them. And then, I took the other two and I essentially used that mass of the neon in this case and said ‘well, what would the density have to be of the other two to come up to the, to the same mass?’ And then compare that to the actual density and the table…. So, to me this is a little bit more cumbersome and I basically did this because you asked for two methods, but it, I mean, I wouldn't have thought of doing it this way initially (John, interview, March 22).

**Orange juice problem.** For the orange juice problem, John employed the unit-rate and cross-product strategies (Figure 24), where both are one of the proportional reasoning strategies that could be used. For the second method, he used a similar strategy with the density problem:

he assumed that it involved making the same concentration of juice and compared it with the given amount of concentration.

**Figure 24**

*John's Problem-solving Strategies for Solving the Orange Juice Problem*




| Method 1. Unit-rate (PR-UR) [Preferred method] | Method 2. Cross product (PR-CP) |
|---|---|

> Initially, I just determined the concentration of the concentrate in the mixture as a fraction of the total and saw that one was greater than the other. I just compared them. And then, the second method, what's set it up as a proportion and just determine what would the amount [be], if this is a mass or weight or volume here, but the number of ounces of concentrate for Martin would have to come to the same concentration as Luis and so that, that was actually greater than the amount he actually applied. So, sort of similar to what I did in, um, with the gas and the balloons is determine use proportion to turn what would have to be to be equal and saw that it was in fact greater than the actual amount. And therefore, that second one must be less concentrated (John, interview, March 22, 2018).

John also added an interesting comment for choosing his preferred method. He chose the first method (unit-rate) for himself as well as for teaching, because he thought it was more straight forward and less likely to make an error. However, he pointed out students' tendency to use the cross-product strategy (setting up two equivalent fractions) because it is what they learn in mathematics class, but at the same time, he identified their common mistake for this method that students were setting up the proportion incorrectly by flipping the relationship.

> I would prefer the first, I think it's a little more straight forward. And my students would choose the opposite because they use this for math, the same things of proportions, even if it involved some steps to isolate their unknown from the other numbers. To me it's more cumbersome and more prone to error… I would teach them the first and they would then use the second. And they would probably get it wrong because they would set up the proportion wrong (John, interview, March 22, 2018).

**Non-proportion problem.** John used equations for both methods for the non-proportion problem (Figure 25). He was the only participant who specifically identified this problem differently from the other four problems, and that it is a non-proportional relationship problem. In addition, he also admitted that it took him a moment to realize that it was a different problem rather than a proportional one.

> This took me a moment to catch that the speeds were the same. I should have read more carefully. Then, it was really just a different problem rather than a proportionality like the others. So, then that was a simple process. Basically, see the difference in their distances at the outset and start at the same speed. The difference should stay the same (John, interview, March 22, 2018).

**Figure 25**

*John's Problem-solving Strategies for Solving the Non-proportion Problem*

| | |
|---|---|
| $4-2=2$<br>$12-2=10$ | $x = 12-(4-2)=10$ |
| Method 1<br>[Preferred method] | Method 2 |

Once again, John also expressed that coming up with a second method was trivial for solving this problem.

> So, then I had to come up with another way to do this and that was, it all seems so trivial. It's hard to come up with it. So, basically, I just sort of set the whole thing up as, as one calculation. In other words, rather than finding the difference and then applying it. I just sort of said, well, what, what would that distance he ran have to be equal to when set up as her difference minus the differences in their initial mileages. So that, um, it came out to the same... And so, it's the same, uh, again, the same calculation but sort of done in, combined in one step that way. Just more of an Algebra approach, I guess you'd say (John, interview, March 22, 2018).

### *Pedagogical Content Knowledge*

The following table 25 summarizes John's PCK in teaching stoichiometry.

**Table 25**

*Summary of John's PCK in Teaching Stoichiometry*

| Orientation to teaching science | Didactic & Process |
|---|---|
| Knowledge of science curriculum | Goals & objectives<br>• Application and use of dimensional analysis<br>• Understand that every equation represents a concept<br>• Reinforce the concept of atom/molecule and gas law<br>• Relate stoichiometry to biology concepts<br>Curriculum programs: None |
| Knowledge of students' understanding of science | Requirements<br>• Prepare stoichiometry lesson based on students' mathematics and abstract thinking level: more time, concrete examples/analogies, and practice problems are needed for general level students<br>Students' difficulty<br>• Difficulty calculating without putting one under the whole number<br>• Motivation to learn stoichiometry<br>• Understanding that every molecule has a different molar mass<br>• Understanding of the concept and the use of units<br>• Considering stoichiometry problems as a mechanically routine mathematics problem (lack of conceptual understanding) |
| Knowledge of instructional strategies | Subject-specific strategies<br>• Discussion organization: class → group → individual<br>Topic-specific strategies:<br>• Nail and bag analogy<br>• Environmental issue example for promoting practical motivation<br>• Crayon activity |
| Knowledge of assessment of scientific literacy | Dimension of science learning<br>• Emphasized conceptual understanding<br>Methods of assessing<br>• Solving stoichiometry problems |

John's stoichiometry lesson was oriented toward didactic and process teaching. In his PowerPoint presentation for a stoichiometry lesson (Figure 26), he directly taught the steps of solving problems. Also, he indicated that for his lower-level chemistry class, he used the audio recording to help students follow the steps for practice problems.

**Figure 26**

*John's PowerPoint for Teaching to Solve Stoichiometry*

| How much $CO_2$ is produced by burning 100 g of propane? | **Strategy:** g propane ➔ mol propane ➔ mol $CO_2$ ➔ g $CO_2$ |
|---|---|

> I had several different frameworks to coach them through it. Um, we have a PowerPoint that introduces it. I have recorded audio narration for the PowerPoint that I made and I don't use that much nowadays as I don't have a modified course… I used to use it for that and they [students] could study on their own and listen to it over and over and it walks them through. Um, and then we have some sample ones [stoichiometry problems] that we work [together]. I give them those as a work model for the others. So, that's, uh, those are the main resources that I use (John, interview, March 22, 2018).

Regarding the science curriculum knowledge, John identified skill-wise and concept-wise goals for his stoichiometry lessons. First, he hoped that the students would adopt the dimensional analysis (or factor label) method for practical applications. John was one of the teachers who constantly considered the situation of the students who will not be pursuing science after high school. Thus, he believed that they should benefit from stoichiometry lessons by adopting the calculation skills (dimensional analysis) that can be applied in other areas.

> I pointed out to them that, well, let's say several things. First of all, I, there are some practical application. Very few of them will go on to be research scientists or even work in applied areas where they're going to need these. However, this kind of factor label or dimensional analysis calculation is a useful one for all kinds of things. It didn't need even necessarily in science, but in many, many different areas. So, that's a useful skill … so the calculation skills are useful in terms of using factor label to set up and calculate, multiple calculations (John, interview, March 22, 2018).

Moreover, as an implicit goal, John wanted students to understand that a mathematical equation is a language for expressing a concept. In other words, students should understand that every step in solving a problem represents a concept.

> So, mathematics is the language for expressing a concept. So, the concept here [stoichiometry problem] has to do with the relationship in the first step, for example, has some of the relationship between a mass and moles and the second step do with the proportionality of things that chemical equation is telling you, proportions of reactants

> and products, and it's expressed in mathematical terms because it allows you to determine quantitatively how much of something you're talking about. Um, so that's an underlying goal in terms of the practical day to day (John, interview, March 22, 2018).

Within the chemistry curriculum, John referred to his view that the stoichiometry unit reinforces the concept of atoms, molecules, and mole, because chemicals react by atoms or molecules.

> I think in terms of the goals of the course, I want them to be able to perform the calculations, and to also reinforces the concept, um, of atoms and molecules because we're going through moles, you have to have these three-step calculations. Why can't we just go from grams of reactants to grams of products because they don't really react in gram by gram and they're reacting atom by atom or molecule by molecule. And so, um, it emphasizes [that] the different molecules are different substance, in this case, [so] their masses are different. Um, so that's another piece of understanding, you know, how chemistry works. Substances are made up of particles with different properties including different masses… Another is that, um, we're gonna use this [stoichiometry] later on in the course if they, if that, depending on level of the class, for example, if I'm going to teach about a limiting reagent, questions are going to need to know stoichiometry for that.

John believed that teaching the gas law could be a unit, where the concept of stoichiometry can be reinforced later in the chemistry curriculum. However, he implied that this is not what he actually does in his class. Gas law is just a possible unit where John thinks the concept of stoichiometry could be revisited.

> Um, so this [stoichiometry] is sort of like a foundation. So, if you had to deal with a more complicated situation like a, they haven't learned the gas laws yet, so if you could. I don't usually do this, but I could, for example, give a problem that involves a chemical reaction in the gas phase where they'd have to use both gas laws and stoichiometry. That would, could be like a nice kind of summative assessment later they've gone to final exam or something. Um, so it would come up in that way, he would be using these things over again, it might be an example, beyond that (John, interview, March 22, 2018).

John strongly believed that stoichiometry is more useful in other science subjects, especially in biology, more than within the chemistry curriculum. For example, he constantly referred to biology units (i.e., respiration and photosynthesis of a plant and ecology) as a unit where stoichiometry can be useful.

> I think they're going to encounter the [stoichiometry] concepts more in, um, in biology…… I mean all these stoichiometric ideas come out in biochemical pathways and in ecology too. And that [was] the idea of the law… the minimum and limiting factors in the growth of the population. So, those are other places where it gets applied (John, interview, March 22, 2018).

> It also come up in biology to some extent depending on the level of the class also, but, um, when we look at things like, uh, say the respiration and photosynthesis and the stoichiometry is significant there (John, interview, March 22, 2018).

As a requirement, John believed that there should not be a pre-requisite for taking chemistry class, but he did refer that it could limit the nature of the curriculum and the level of his teaching chemistry: “That [prerequisite] may limit the level or nature of the curriculum. Accepting that allows us to teach chemistry to students who are less capable in mathematics” (John, written response to the survey, March 22, 2018).

When John was asked what student factors he considers when preparing stoichiometry lessons, he referred to students’ mathematics and abstract thinking ability. Based on these factors, he adjusted his lessons on stoichiometry.

> I think where they are in math and abstract thinking. I think [it] is a big factor in how fast we go and how I approach it and how much I have to rely on physical analogies, um, that, that can help a lot in those cases. And it also, that'll also affect, um, well that should be taken account of the leveling. It isn't always, but, um, whether I'm going to include more advanced topics like limiting reagents, those would all be factors (John, interview, March 22, 2018).

John differentiated between the lower- and honors-level chemistry classes by adjusting the scope of the concepts that he introduces, the speed of presenting the concepts, and the use of concrete examples or analogies (e.g., recipes).

> The higher-level classes get the limiting reagent problems as well as lower-level classes do not. Um, the way we're set up currently, we basically have two levels now: a regular and honors. I teach the regular, someone else teaches the honors, so we really are able to work independently, we can do whatever we want, and I just decided not to bother limiting reagent this year in the regular class. I thought it wasn't really necessary and we need the time for other things. Um, I would not do that in the modified [regular] class. I did it in my accelerated [honors] chem. I did do it. The other thing is in a lower-level

class, it just hasn't been so true this year, but I think for the modified [regular] class, I would introduce the concept more slowly with more concrete examples and analogies that we might use (John, interview, March 22, 2018).

Um, recipe that I did, um, recipes, cooking recipes, you know, halving or doubling a cooking recipe. Um, it's a little bit different there because they're mixing volume and mass, so it's not quite the same thing. But I would use more of those things, those analogies (John, interview, March 22, 2018).

While teaching stoichiometry, John noticed that at the beginning of the stoichiometry unit, students' have difficulty understanding that every substance (molecules) have different molar masses and that the chemicals react in atoms or molecules.

A couple of things, one is students who forget that different substances have different molar masses and [the other is] they want to go right from grams to grams. So that, that's usually in the beginning. It's a typical beginning area that we usually get over pretty well (John, interview, March 22, 2018).

In general, John noticed that students solve stoichiometry problems like a mechanical routine rather than truly understanding the meaning of each step. For example, instead of understanding why they convert from the moles to grams (or vice versa), students will just mechanically convert it without thinking about the meaning of it.

They see this [stoichiometry problem] as a mechanic… just as a mechanical routine. They don't see the conceptual meaning. They'll make mistakes that lead into that. For example, … I probably only use the coefficients in the second [step: converting from moles of ammonia to moles of hydrogen] of the three steps, but they just put them [coefficients] into the first or the third. Now there is a gram to moles, moles to grams conversion because they're thinking 'well, it says three moles, so I have to find the mass of three moles.' So, it's a matter of just kind of not sorting out the conceptual meaning of the different parts of the process. Those are some common mistakes (John, interview, March 22, 2018).

John also knew that students would either not label units or, even they do, use it meaninglessly.

So, I keep telling you got to put your units on everything because it helps you make sure you're doing this correctly. Because if the units don't cancel out, right then, you know, there's a mistake. You can go back and find it. Um, and invariably, students who don't put units on, they make a mistake and they go on. They don't see there was a mistake and

> they just go on to the next one. … Sometimes, they'll put them in because I told them to, but they don't really understand why they're doing it and they'll make a mistake and not see it or they'll put the wrong units in because they don't really understand there. Just as a formality, you know, they don't really take it seriously. So, those are kind of things that come up along the way (John, interview, March 22, 2018).

In addition, John found students' odd patterns of putting a number one under the whole number when multiplying with fractions. He did not understand the reason for doing this, though he knew that it is what mathematics teachers teach them to do.

> There's some eccentricities, I don't know, I am never able to explain to me why they do this, but for some reason when you have these kinds of proportion set up, they want to put a one under everything. For example, they learn in math that you can multiply fractions, but you don't multiply a fraction by a whole number, which to me makes no sense … you can multiply the numerator by a whole number. That's essentially the same thing. So, they want to put one under everything. Since I've stressed putting units on every number one, I don't know what units to put on that one. I said, 'I don't know what you need that one for. You're multiplying by one. What are you doing here? It doesn't do anything?' But they have this idea that it has to be parallel like that. And the math teachers encourage that. So, I don't think it adds anything. So, that's one thing that comes up sometimes (John, interview, March 22, 2018).

Finally, John pointed out students' difficulty in finding the reason for learning stoichiometry. For those students, he tried to provide practical examples, such as running a business of construction or multiplying a recipe, where the proportional concept in stoichiometry could be used to reduce the waste and save money.

> Another thing is some students' questions… it seems like very complex and arcane to some of them and they want to know why they have to do it. And I try to give practical examples and if depending on who the students are, anything from what we just talked about, like you know, nails for a construction project or multiplying a recipe to some of the more business-oriented kids. I talk to them about manufacturing and you want to order as much of the materials that you need, but no more. That would be a waste of money. So, you have to buy them in certain proportions, and you need to know what the proportions are. Um, so when that seems to help sometimes, depending on how resistant they feel (John, interview, March 22, 2018).

John's knowledge of instructional strategies was revealed through the representations and activities that he used for his stoichiometry instruction. He used different examples and activities compared to other teachers. While the *S'mores* is a common introductory activity for stoichiometry, John started his lesson by presenting an example of combusting propane within an environmental point of view to interest students. The following excerpt also shows John's didactic orientation of presenting the process of solving combustion problems.

> I explain this as a conceptual process; 'OK. So, we know the equation of telling us the ratio of moles, not massive, so we have to convert our grams into moles, then it meets to use the mole ratio to go from one to the other and finally the molar mass of the other substance to go back to, uh, to grams'. So, um, that's kind of how I do it with them (John, interview, March 22, 2018).

John borrowed the nuts and bolts example to represent the idea of converting grams to moles, or the idea of weighing to counting.

> Physical objects, for example, um, there are a lot of activities like this where you know, you the idea of, of counting by weighing. I did this verbally with this class I talked about, if you're going to build a house and you're going to buy nails by the pound, you don't know what's going to count. How many are there in pounds? So, it's the same idea basically. Um, so we might actually do something like that. I've done that with that in the past where I had sets of nuts and bolts and say 'OK, if we want to have two washers and a nut, let's say on each bolt to be in certain proportions and there's a lot of them, so we'll just weigh them and then we could develop the concept' that I've done like that (John, interview, March 22, 2018).

John used a crayon activity to motivate students and to get them familiar with calculating the power of 10. He believed that this is a fun activity for high school students as it links their personal identity and chemistry.

> Just for fun, but I have them, um, weigh a piece of paper and then they write their name and fill it up with crayon drawing and they weigh it again. And then, they give them the molar mass of average molar mass of the wax and they calculate how many atoms are in their name. So, I keep that one, just they can get familiar with operations, with powers of 10. And, also, um, I think it's nice connecting with it in terms of a lot of the issues of this age. It's their name. It's about personal identity but it's also chemistry (John, interview, March 22, 2018).

Regarding the methods for assessing students' understanding of stoichiometry, most of the time, John employed solving problems. He used actual stoichiometry problems not only that asks for students to solve the full process (mass-to-mass), but also ones that ask for part of the process (moles-to-mass). John also included problems that focus on the conceptual meaning of the chemical equation. The following comment and Figure 27 illustrate how John assesses students' understanding of stoichiometry.

> So, the assessments that are used are partly solving problems like these [stoichiometry problems given for this study]. They actually solve stoichiometry problems. I tell them to show their work, so I can catch any mistakes, but also some questions probing just parts of this [stoichiometry problem]. For example, on a stoichiometry test, you might have some tests just about, um, mass-to-mole conversions. So, the parts of the process and also some more abstract ones like about the conceptual meaning of these parts. So, I might say, for example, in this chemical equation, point out there's a number that's a coefficient. What is the meaning of that coefficient? What does it mean here? (John, interview, March 22, 2018).

In order to avoid students' rote memorization of solving stoichiometry problems, John used problems that involve contexts that the students have never seen (i.e., bonus questions) or that can't be answered by memorization (i.e., Figure 27 - Problem 4). John personally favored problems like bonus questions because this not only deepens the conceptual understanding of stoichiometry but also allows students to retain the concept longer.

> These [bonus questions] are kind of more of the conceptual ones. You know, my feeling is that in chemistry and science, in general, more rigorous to me doesn't just mean more facts, more information. But I'm more interested in broadening or deepening the conceptual understanding, which is something they may actually remember. If you have a student [who] memorized, you know, solubility series, they might remember for tests, but the next day it's gone. I mean, what's the point? What have you accomplished? But, if they understand the concepts of why some things are more soluble than others, that's more [meaningful]. That might actually make some sense, has some meaning, and can stay with them, and they actually use it at some point. See why calcium compounds are less soluble than sodium compounds. I mean, you know, that's, that would be a, something they could explain so that was what I was getting at here (John, interview, March 22, 2018).

**Figure 27**

*John's Test Problems for the Stoichiometry Unit*

**3.** In the following chemical equation, what do the numbers 4 and 5 on the lefthand side mean?

$$4\ P + 5\ O_2 \rightarrow 2\ P_2O_5$$

**4.** Complete the following sentence to describe the equation in question 3:

For every four ____________________ of phosphorus (P) that are consumed in this reaction, ______________________________ of $P_2O_5$ are formed.

5-7. Suppose you carried out the reaction in question 3 with more oxygen than you could possibly need. How many moles of $P_2O_5$ would be formed from …

**5.** ...4 moles of P? ____________________________________________

**6.** ...2 moles of P? ____________________________________________

**7.** ...8 moles of P? ____________________________________________

12-14. Convert the following masses to moles.

**12.** 34 g Ne ____________________

**Bonus Question (5 extra-credit points)**

An inventor tries to sell you a new kind of balance that he claims can tell you how many moles of a substance you have put on it, instead of how many grams. "You just put anything on it," he says, "and it tells you how many moles it is."

Is this possible? If you think that it is, explain how it could work. If you think that it is not possible, explain why not.

### *Summary of John's Case*

John was one of the teachers who had unique characteristics for teaching stoichiometry and proportional relationship. For example, though John emphasized the importance of understanding and using units, he often did not label units for solving the five problems. Interestingly, he only labeled units for density and stoichiometry problems. This seems to be a habit of himself that evolved during his teaching of chemistry. Moreover, John was careless about the problem-solving methods and equations that students learn in mathematics class. He even believed that what students learn in math class are more vulnerable to making errors. For example, though John knew mathematics teachers often teach the use of a cross-product strategy for solving proportion problems, he believed that strategy makes students make mistakes.

While other teachers advocated for improving every student's conceptual learning of stoichiometry, John was skeptical about this idea because he believed this is not necessary for all students, especially for those who are not planning to pursue science. Overall, John strongly believed that chemistry is a hard subject because it requires students to understand the unseen world. This aligns with his belief that stoichiometry is a hard concept for high school students (item 21). Regarding John's PCK, he harmonized the five categories of PCK with the goal of conceptual understanding of stoichiometry. Especially, his knowledge of the goals and objectives, students' understanding and difficulty, and assessment reconciled. He emphasized students' understanding that every mathematical equation represents a concept because he knows that students mechanically solve stoichiometry problems without a conceptual understanding of the stoichiometry. He also included conceptual problems that cannot be solved simply by plugging in the numbers in his test so that he can assure that they have conceptually understood the stoichiometry concept as well as the mathematical concept that underly in stoichiometry problems.

**Case of Lena**

Lena is a female chemistry teacher in a public school located in a suburban area in New Jersey. She majored in chemical engineering in her college and earned a Master's degree in education. At the time of the interview, it was her 14$^{th}$ year of teaching chemistry and she was teaching general level and honors level chemistry to 10$^{th}$ grade students. She had about 20-26 students in one class. As a prerequisite to taking chemistry courses, students must pass an

Epstein test (an assessment for measuring student basic mathematics skills) and physics class. Lena thought these prerequisites are necessary because “otherwise, students struggle beyond [their] capabilities” (Lena, interview, April 20, 2018). Considering her long experience as a chemistry teacher, Lena participated in many different professional development programs. She participated in a chemistry modeling program, district-wide NGSS program, and another modeling course specifically related to stoichiometry. However, she did not take any professional development regarding improving students’ mathematics thinking. Lena expressed her desire to take in the future because she believed mathematics skills is a critical component in learning chemistry. According to Lena’s survey responses, the science department sometimes collaborated and consulted with the mathematics department.

***Belief***

In terms of students’ learning of proportional reasoning, Lena strongly believed that it could be adopted in daily life experiences and, at the same time, be advanced through school education (item 1 and 25). She also believed that a proportional relationship in the stoichiometry concept and solving those problems could improve students’ proportional reasoning ability (item 9 and 20). However, she clearly differentiated understanding of the concept with solving problems correctly. For example, she disagreed with the statement that successfully solving stoichiometry problems implies that students have conceptually understood the concept of stoichiometry (item 2). In addition, when answering for item 13 (Without fully understanding the concept of proportion and knowing how to solve proportional relationship problems, high school students cannot understand and solve stoichiometry problems), which intentionally grouped problems solving and understanding the concept, she specifically expressed that “they [students] might be able to solve and not understand (Lena, interview, April 20, 2018).”. Interestingly, though she disagreed that successfully solving stoichiometry problems imply their conceptual understanding of stoichiometry (item 6), her class materials (i.e., worksheets and practice problems) heavily relied on solving stoichiometry problems.

In the survey, Lena showed her confidence in teaching proportional reasoning and stoichiometry (item 15, 18, and 24). She strongly believed that it was not only mathematics teachers' obligations, but also her responsibility, as a chemistry teacher, to improve students' proportional reasoning ability (items 4, 5, and 7).

> I think it's everybody's job. Um, I mean maybe it's because it's my math slash chemistry background of being a chemical engineer, but I think both have to work together, um, to learn that because kids are not, not living in a chemistry world, they're not living in the math world, they're living in 'the' world and being able to hear it from both perspectives. I think even just learning the lingo from isn't just lingo. I mean common vocabulary. I think we have to work together, and I don't know that we are, we're not there yet, unfortunately (Lena, interview, April 20, 2018).

However, Lena thought proportional reasoning should be mastered during middle school years (item 10). She also expressed that it would be easier to teach chemistry if the students have adopted the concept of proportional reasoning before coming to her class.

> So, hopefully by the time they come to us [chemistry class], between them having had a little bit of that chemistry in 8$^{th}$ grade and they've had proportional reasoning in physics [prerequisite course for taking chemistry]. By the time they come to us, they [should] really get the value of proportionality, so it makes our job a little easier, which was not the case (Lena, interview, April 20, 2018).

Moreover, she indicated that she presents different solution methods for solving stoichiometry problems (item 6) and that chemistry teachers should present multiple solutions (item 12). However, she was very passive about presenting multiple solution methods. In other words, Lena relied on students' different solution methods rather than herself presenting multiple methods. Thus, when solving the five problems in this study, she expressed her difficulty of coming up with two different methods by herself, because it was an unfamiliar context for her.

> Some of these [problems] were easier for me because there are typically problems that I've given to students. So, I've already had seen that they've solved them in different ways and some I haven't really been exposed to. So, I had some difficulty in coming up with a different way other than the way that my mindset would be. So, it really depends if I've seen it before. For example, the stoichiometry problem, it's obvious to me there's more than one way to do it because I know how I was taught [which] was much more algorithmically. Whereas the way that I teach now as much more proportionality wise. So, it was easy for me to come up with different ways (Lena, interview, April 20, 2018).

In general, Lena believed that students could improve their proportional reasoning ability while learning stoichiometry concepts (item 16). She disagreed that stoichiometry is a hard topic for high school students (item 21). Lena indicated that she had difficulty teaching the stoichiometry concept more deeply due to students' lack of mathematics knowledge (item 17).

Regarding the importance of proportional reasoning and stoichiometry, Lena agreed that proportional reasoning is important for understanding later science concepts (item 3) and being a science and mathematics literate citizen (item 11). She also believed that proportional reasoning is a common mathematical practice that scientist engages in (item 19) and that students' engagement in such practice while learning science can help students to understand the work of scientists (item 14). The following shows how she relates stoichiometry to real-world examples so that students can understand the work of scientists:

> I have a couple of engineering degree and one of the things that I really liked about being a chemical engineer was this idea that you could calculate exactly how much stuff you need to make the right amount of stuff without wasting anything. And I tell the students about some of the stories of that, you know, like if you're a plant manager, you may go to your engineer and say 'OK, we need to make x amount of whatever it is by next week, what do we need'? and how, you know, you need to know the rate of this and how much of this material you need and, you know, your job depends on making sure that you get the right yield and that you're not wasting anything (Lena, interview, April 20, 2018).

Finally, she strongly believed that chemistry teachers need more support to integrate mathematics thinking practice in stoichiometry instructions (item 26).

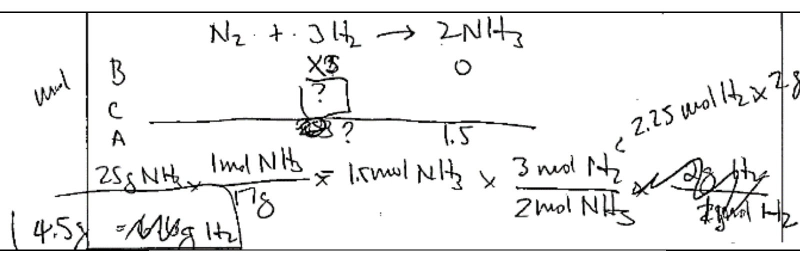
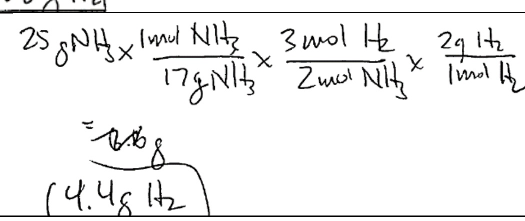
### ***Problem-solving strategies***

As soon as Lena started to explain about her problem-solving strategies, she linked it to her students. She voluntarily mentioned which strategy will be better for improving students' understanding of proportional reasoning and what method she would prefer to use for students. In addition, Lena strongly advocated for presenting the proportional relationship of the variables for the three proportion problems rather than solving them mechanically. However, she did not have a limited understanding of dimensional analysis strategy and the proportional reasoning strategies.

**Stoichiometry problem.** For the first method, Lena mixed strategies of dimension analysis and unit-rate (Figure 28). She used dimension analysis for converting the grams of ammonia to moles of ammonia, and for shifting the moles of ammonia to moles of hydrogen. Finally, she used the unit-rate strategy to change the moles of hydrogen to grams of hydrogen. These strategies were used under the name of the BCA (Before-Change-After) chart, which is commonly used in modeling instruction. She extremely favored this chart because she believed it represents the proportional relationship in a way that students can clearly understand.

**Figure 28**

*Lena's Problem-solving Strategies for Solving the Stoichiometry Problem*

| | |
|---|---|
| Method 1.<br>DA<br>DA<br>PR-UR<br>[Preferred method] |  |
| Method 2.<br>DA-Chain |  |

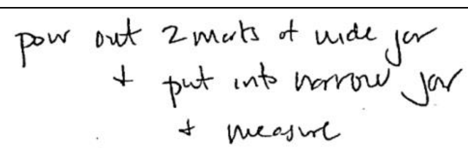
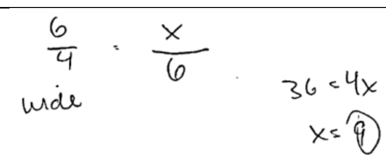
Lena used dimensional analysis chain strategy for the second problem. During the interview, she referred that this is a strategy of how she was taught and used to teach in her earlier years of teaching stoichiometry. Thus, as an adult, she preferred using the second method, but strongly advocated for using the first method for teaching.

**Jar problem.** Lena solved jar problems verbally described the process of getting the answer (which did not lead to answer) and using a cross-product (PR-CP) strategy (Figure 29). The following excerpt shows her struggle to come up with the second method of solving this problem. Although she personally prefers to use the cross-product (PR-CP) strategy for solving this problem, she expects students to verbally explain the problem first as it visualizes the proportional relationship of the two jars and, later, solve it in a mathematical way.

> It was very interesting to me because I had mentioned to you if can I pour out from the jars. And then, when [the researcher said no] that kind of limited me. I kinda got stuck for second and trying to figure it out until I realized 'well, wait a minute, if I could look at the whole, again, proportionality, then it became easy to solve the problem.' Um, so you could mathematically figure it out, but you could also solve it by manipulation of the information, which I think sometimes I'm getting the students more to try to do that. So, if I gave them this kind of problem, I probably would give them actual jars because I think visual aids really help them and I think they would still get to the end and then ask them to 'OK, well how could you do with math, mathematically?' And I almost liked the idea of teaching them to look at it in both ways, um, because it gets to the proportionality piece I think a lot faster (Lena, interview, April 20, 2018).

**Figure 29**

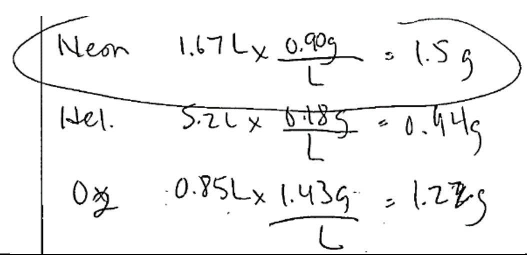
*Lena's Problem-solving Strategies for Solving the Jar Problem*

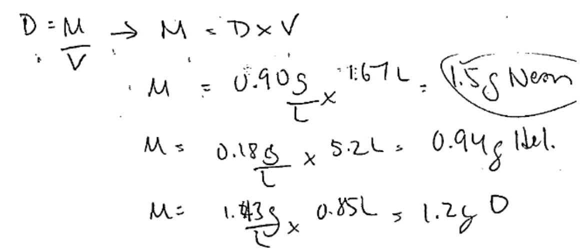
| | |
|---|---|
|  |  |
| Method 1: Qualitative [Incorrect] | Method 2: Cross-product [Preferred method] |

**Density problem.** For this density problem, Lena employed dimensional analysis strategies for the two methods (Figure 30). However, even though the two equations look exactly the same for the two methods, she believed that the first method focuses on the proportional relationship more than the second method, because the thinking process of the first method does not rely on the equation (D = $\frac{m}{V}$).

**Figure 30**

*Lena's Problem-solving Strategies for Solving the Density Problem*



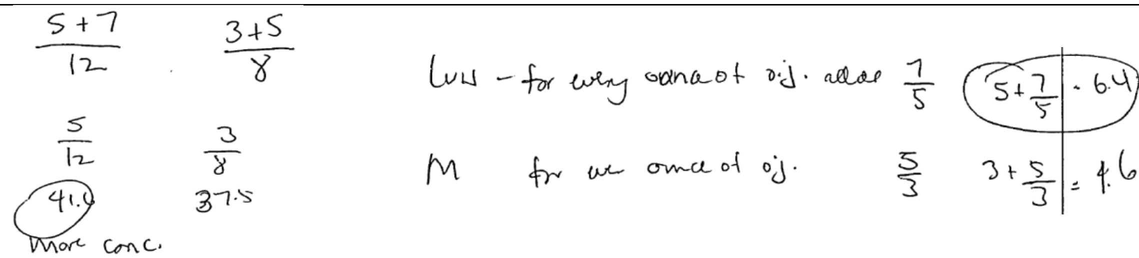

| Method 1: DA [Preferred method] | Method 2: DA |
|---|---|

> When I first started using the modeling approach, equations are not used at all [for teaching to solve density]. So, they really have to understand what density is in terms of relationship… So, it was easy for me to just show both ways to do the problem because they know how the old D is equal to M over V would be solved. But I really appreciate the using density as a factor or relationship between mass and volume. Make so much more sense to me now. And, um, that there really isn't this necessity to have an equation that you have to memorize if you understand what, what does it mean to be grams over liters? Um, so I prefer the proportional relationship way versus the other way now (Lena, interview, April 20, 2018).

**Orange juice problem.** For both methods, Lena used the unit-rate strategy for solving organge juice problem (Figure 31). Her answer was correct for the first method, but the second method was incorrect. She added the amount of orange juice (5 oz) and the concentration of orange juice ($\frac{7\ water}{5\ orange\ jucie}$), which cannot be added as the unit of the two amounts are different.

**Figure 31**

*Lena's Problem-solving Strategies for Solving the Orange Juice Problem*



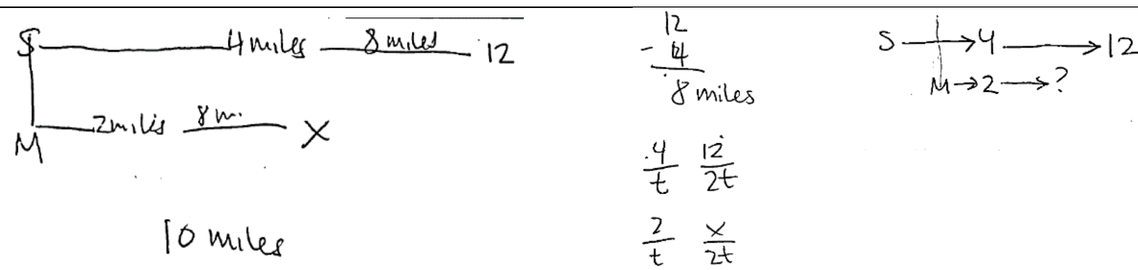

| Method 1: PR-UR<br>[Preferred method] | Method 2: PR-UR<br>[Incorrect] |
|---|---|

**Non-proportion problem.** For this non-proportion problem, Lena first visually represented the situation of the problem (Figure 32). She indicated that her anxiety over physics immediately led her to draw a picture of the context.

> Problem four kind of gave me a hard time because to be honest with you, a physics and I never really got along. So, when I see problems of so and so left the train, you know, I immediately get an anxiety attack, um, and I immediately go to drawing a picture (Lena, interview, April 20, 2018).

**Figure 32**

*Lena's Problem-solving Strategies for Solving the Non-proportion Problem*




| Method 1<br>[Preferred method] | Method 2 |
|---|---|

Most of all, though her strategy for the first method and the answer were correct, Lena was not able to identify that this problem was a non-proportion problem. While she explained her solutions, she constantly referred to this problem as a proportion problem.

> And, I think that goes back to how I solved problems way back when, so I was doing it much more of a proportionality thing, but then I also was trying to figure out how they would do it with an, with an equation. So, but again, I struggled with it because it's not something that, um, I'm used to solving. But, I think I would go back to the proportional, which I don't know that I would've done that before I started teaching this way (Lena, interview, April 20, 2018).

***Pedagogical Content Knowledge***

Lena's whole unit of stoichiometry lessons was discovery-oriented, while her delivery of the proportional relationship in the stoichiometry concept was didactic. She used a virtual *S'mores* activity to familiarize students with the concept of proportional relationship in chemical equations. Then, Lena derived all the related concepts in stoichiometry, such as percent yield and limiting reagent, from one lab: copper and silver nitrate reaction. The following comment illustrates how the lab connects and brings all the concepts that need to be understood.

> Before we talk about a [chemical] reaction equation where one copper goes with every whatever, the other thing is they realize that 'wait a minute, it's also problem solving and that we don't know which copper nitrate that is actually gonna form.' So, they realize that 'well, there's different mole relationships that we could end up getting.' So, we have them predict, 'OK, it could be a two to one relationship or it could be one to one relationship. What would we need? What kind of information would we need to get there?' So, the lab itself really is very powerful. Because, one – it's a fun lab, you know, lots of visuals, they get to see the silver forming and the copper nitrate forming. Um, but they're collecting the data and they get pretty good results to see that it's a two to one ratio. From there, they get to figure out what limited them. Um, what kind of yield they should get. And, we do percent yield. So, we basically teach the whole stoichiometry content with one lab (Lena, interview, April 20, 2018).

However, when representing the process of actual reaction using BCA (Before-Change-After) chart, Lena was very didactic and process-oriented. She directly introduced what to write, how to format the chart, and the things to do for each step of BCA. For example, in the worksheet, she specifically instructed to write down the given information in the Before line.

Regarding the goals and objectives of the stoichiometry lessons, which is one category of knowledge of the science curriculum (Magnusson, Krajcik, & Borko, 1999), Lena expected the students to acquire the ability to scale up the proportional relationship from micro-level to macro-level. In other words, she expects students to transfer their understanding of the proportional relationship in the *S'mores* activity (macro-level) to chemical reactions (micro-level).

> I want them to learn that there are these [proportional] relationships at a particle level that you can scale up to a much larger level. It's really the same thing in this whole idea that we talk about. Moles are not this, like, obscure and stoichiometry is not this obscure idea that the chemist use that. It's basically just scaling up (Lena, interview, April 20, 2018).

Lena also specified that the stoichiometry unit sums up a lot of concepts that students have learned in earlier classes, such as moles, balancing chemical equations, and classifying reactions.

> OK, well it reinforces, um, like I said, they at this point, I would have had a whole unit about moles and they've had a whole unit on balancing and classifying reactions and predicting reactions. Um, so now this sort of ties those two things together, and from there we actually start talking about solution chemistry. … So, they appreciate that because we do a lot of spiraling back to things we've done earlier in the year, but now they know more information. So, they get excited about that because they're like, oh yeah, now we can do that lab and we, we, if we wanted to, you know, scale it up, what would you do? So, we do a lot of that (Lena, interview, April 20, 2018).

However, before her school changed the curriculum, the lessons on gas laws, which are also based on the proportional or inversely proportional relationship among the variables, she taught the concepts after the stoichiometry unit. However, with their new curriculum, her school does not teach gas laws anymore; as a result, there is no unit or concept that relates to stoichiometry. Thus, it became an exclusive unit in their new curriculum.

> Well, unfortunately after stoichiometry, we sort of go into nuclear chemistry, so we don't really get to go beyond. Um, it used to be, we had a unit where we did applications of stoichiometry and that's where the solution part [comes in]. I pulled that back into that unit, but we don't get to any of the gases, which is what I would like to get into that. … So, it sort of ends the unit of bringing the moles and reactions. Stoichiometry is just sort of its own little entity, so we don't get to really go back to it unfortunately (Lena, interview, April 20, 2018).

Regarding the knowledge of student understanding, Lena pointed out three difficulties that her students experience while learning stoichiometry: (a) misunderstanding of coefficients, (b) lack of foundation from earlier concepts in chemistry, and (c) showing the work for solving problems. One of the misunderstandings that students easily adopt is that coefficients in the chemical reaction represent the mass relationship between the substances. Lena also stated this in her interview when we talked about the misconceptions that students have in learning stoichiometry.

> When I'm teaching of stoichiometry, common misconceptions are kids who still want to think that mass got some kind of proportionality to the equations. You know, that one of the test questions that we give them is an equation and we use the one that's on the first [stoichiometry] problem. So, $N_2 + 3H_2$ goes to $2NH_3$. So, we might have 'which of the following is not appropriate for this would be? One gram of nitrogen plus three grams of grams of hydrogen gives two grams of ammonia'. I will still have kids who will do that because there's still relating those coefficients to mass when it really isn't. So, I think that's something that picks that up, that they have to realize that that's [chemical equation] proportional to particles and moles, but not to mass (Lena, interview, April 20, 2018).

Secondly, Lena pointed out that students' lack of foundation of earlier chemistry concepts, such as balancing equations, causes the struggle to understand stoichiometry. The chemistry curriculum is unique from other subject areas in that if the students fail to understand one unit or one concept within the curriculum, the domino effect occurs, meaning that catching up the later concepts becomes harder. Lena also stated that the foundation that students built during the earlier time of the year in chemistry class contributes to the variation among the students.

> Another issue is a student who may be still struggling with balancing equations and even writing formulas. And it's still there. You keep telling him, look, you know, it's not gonna go away just because we finished. OK, we finished balancing equation. I don't have to worry about that anymore. No, it's still there. I'll show the students that 'hey look, if you had figured that out to units before, you wouldn't be struggling with this, but you're still struggling with this because you don't have the foundation'. I still have kids who will say 'can you give me more practice with naming? Can you give me more practice with balancing?' because they recognize that without that they're still messing things up or struggling with it. So, that's the difference between the kids who are doing well in those

> or not is usually because their foundation isn't there still (Lena, interview, April 20, 2018).

Because stoichiometry lessons depend heavily on problem-solving, students often have difficulty solving problems rather than the chemistry concept itself. In order to exactly understand what the difficulty the students are having for solving the problem, whether they are having trouble setting up an equation or they are simply making calculation mistake, teachers often require students to write their work. In other words, students are required to write every step of solving the problem. However, as Lena stated below, students often do not recognize the importance of showing their work for solving the problem, which makes it hard for teachers to provide feedback.

> I still have a lot of kids who don't want to show work. And that frustrates me because then they have an answer and I tell him I can't give you feedback on what part of the problem that you're struggling with that wrong answer. Is it because you didn't balance the equation? Is it because you plugged into your calculator wrong? I mean how am I supposed to know? Is it proportionality that you're struggling with or was it just you can't put a number in calculator correctly? So, I think that's a big struggle is getting them to recognize the importance of putting work down (Lena, interview, April 20, 2018).

Lena indicated that students' mathematical ability is the main reason for being assigned either into a general level or honors level class. Because she observed that general level students become passive when they see mathematics involved in stoichiometry, she tries to simplify the math so that they could be more successful in solving the stoichiometry problems.

> And a lot of them [students] are very reticent when it comes to math in general [level class]. The reason why they're tracked into general is because their math skills are really, really low. So, I have kids who have no idea how to find a percent of something and now you're asking them to do something that 'what do you mean I'm want to convert from here to there?' but I try to alleviate some of that with making the types of math and we're doing fairly simple so it makes them feel more successful (Lena, interview, April 20, 2018).

Lena believed that students' proportional reasoning skill comes up to the surface in the stoichiometry unit than any other units in the chemistry curriculum. Students are suddenly challenged in learning stoichiometry due to their low understanding of proportionality.

Moreover, even within the honors class, Lena expressed her difficulty in giving differentiated lessons to meet diverse student's needs.

> I think the difficulties I have is because we do not have prerequisites in our classes and in an honors-level class, I may have students who have very poor math skills. When we get to a stoichiometry unit, all of a sudden, they're having a very difficult time because they don't have the proportional reasoning skills. So, when you got a big difference in your class it becomes difficult. So, I try to give those students who really get it more complicated problems and the differentiation really becomes hard at this level. So, there's a lot more. It gives a lot more independent type of work or I'll give them a whole packet where some kids will need to finish the whole packet to practice and really spend time on. And another group, you know, three problems and they got this. So, there is a lot of differentiation that really comes up in stoichiometry, more so than maybe some of the other topics that we cover in honors (Lena, interview, April 20, 2018).

Nonetheless, as a result of the variation in students learning, Lena sequenced the content differences between her general level and honors-level students. She also did not go deep in stoichiometry with her general level students. For example, she focused on real-world examples and did not introduce the limiting reagent concept. Lena also differentiated the two classes in terms of assessing their understanding of stoichiometry. She only gave problems that general students are familiar with. In other words, she only gave a test on the type of problems that they have seen previously in class. In addition, unlike honors level students, general level students are given less of the explaining kind of problems.

> Like I said I don't give them something on a quiz that they haven't already seen at some point. So, it's the critical thinking. They don't have to explain necessarily as much as an honors kid would have to explain through a problem (Lena, interview, April 20, 2018).

Similar to the ways the other teachers differentiated general and honor level classes, Lena gave more practice problems and time to general students and more independence and self-motivated studying time to honors-level students.

> I do a lot more practice and a lot more chunking of how I assess them. Like, you're going to learn about significant figures and here's the quiz we learned about scientific notation. Here's a quiz. Factor labeling, here's a quiz. Whereas the honors level, it's here, here it is. Here's some practice worksheets. We're not going to go over everything. Here's

> the key. You can check it in your own words. There's a lot more independence and self-motivation that I'm expecting at the honors level. Whereas I do a lot of having to handhold a lot more, um, with the general kids (Lena, interview, April 20, 2018).

According to Magnusson, Krajcik, and Borko's (1999), there are two categories of instructional strategies: subject-specific strategy and topic-specific strategy. In terms of subject-specific strategy, Lena emphasized the scaling up of the understanding from micro-level, which Lena referred to as particle bases, to macro-level of chemistry.

> We start with them talking about it on a particle basis. So, one Graham cracker, let's say two chocolate bars and one marshmallow, and then I lead them into 'well, what if we had a dozen of each one? Does that change things in it anyway? No. Well, what have you had a moles-worth of them? No, it doesn't'. We put up an equation. They've seen many, many times. The Haber process and 'OK, what if I had one nitrogen, three hydrogens, how many ammonia molecules would we make?' And we do the same thing 'what if I had a dozen of each of those?' And we say 'well then, OK, what about moles?' So, now we can tie what you've learned about reactions and balanced equations at the particle level, which is what we had talked about before to now looking at it from a scaled up to moles, which we can measure molar quantities and materials in the lab. So, that's how we get them into scaling it up (Lena, interview, April 20, 2018).

Lena indicated that her stoichiometry instruction has shifted from teaching algorithmic to proportional reasoning. BCA charts, which is a popular method that is used in modeling instructions, was used to represent the chemical reaction process. She believed that the BCA chart highlights not only the proportional relationship in stoichiometry but also helps students to understand that the same relationship is applied to micro- and macro-level of substances.

> We start off with and using the BCA charts, which we've taught them with the Graham crackers. And we now say 'OK, well let's use it on a particle basis'. So, we can talk about dozens of Graham crackers, and then we can talk about moles of Graham crackers. Well, then we could do the same thing with copper (Lena, interview, April 20, 2018).

Lena employed two activities for teaching stoichiometry: a virtual *S'mores* activity and copper and silver nitrate lab. She would first do the virtual *S'mores* activity so that students can find a reason for learning stoichiometry. Then, once students get the general idea of the proportional reasoning between the materials used in virtual *S'mores* activity.

> I do have a lot of students who will say 'well, why do I need to know stoichiometry' and I actually start stoichiometry with my students by giving them real world examples of where they can use it so that, you know, for those kids who maybe don't even care about chemistry or science, at least it's a tool for them if they ever have any other kind of problems. We actually talk about making virtual *S'mores*. So, each group gets a different amount of Graham Crackers, chocolate bars, and marshmallows, and they're not real. And they have to solve how many can they make? And, how did you come up with your way of doing it? And so many of them do exactly what we would teach them, the proportionality. … You have to always do the same thing with the equations (Lena, interview, April 20, 2018).

Lena goes into chemistry learning by doing a copper and silver nitrate lab. As mentioned earlier, Lena uses this lab to lead the whole unit of stoichiometry. She purposely leaves the reaction to be incomplete so that students can predict the chemical formula of the product (copper nitrate), whether it is a one-to-one ratio ($CuNO_3$) or a one-to-two ratio ($Cu(NO_3)_2$) of copper and nitrate.

> The next thing is we do a lab, we use copper and silver nitrate. And, they actually get to mass the copper. They mass the silver nitrate and make a solution from that because we haven't talked to them yet about molarity. So, we want them to just really focus on mass-to-mass relationships and we purposely don't allow the reaction to completion. So, they have excess of both reactants (Lena, interview, April 20, 2018).

In terms of methods of assessing science learning, a number of problems were given as test items. As a formative assessment, she used a whiteboarding method. In this approach, the students will solve problems using a whiteboard and present it to the class. During this process, Lena even purposefully left the incorrect answers so students can learn from each other.

> We do a lot of whiteboarding, which means that, um, the kids have access to these. There are about two- by three-foot whiteboards and if I've given them homework, they've assigned different stoichiometry problems. They have to present to the class. And when I say present, they have to explain why they did each of the steps. Um, sometimes I actual[ly] have students purposely make wrong mistakes on the boards and get the kids to see if they can pick up those mistakes. … So, there's a lot of students helping other students, which I love because sometimes they realize they can't explain until I'd had to try to explain it to somebody else (Lena, interview, April 20, 2018).

Lena focused on assessing two types of dimensions: conceptual understanding and practical reasoning. To confirm the extent of students' conceptual understanding, Lena employed a lot of explaining by using whiteboards.

> We do a lot of explaining in class. Like ‘can you explain to me how did you know that was limiting?’ In fact, on our test, not only do they have to tell me what was the limiting [reagent], but how did you know that was a limiting one? Explain it in words. And some of the kids were like ‘well, I just know.’ ‘Well, no. You have to be able to tell me what. How did you know that?’ ‘Well, it just worked out!’ ‘No, there has to be a reason.’ So, kids struggle with that, but then they realized that if they can explain it to their classmates, they have a better understanding of it… So, I would say whiteboarding really has become a very powerful tool (Lena, interview, April 20, 2018).

The following shows Lena’s method of assessing students’ practical reasoning skills. She took out the numbers from the problem and gave a scenario of changing the amount of reactants and products. Lena believed problems like this that do not involve numbers allow the students to focus on the proportional relationships between the two entities (e.g., reactants and products) in stoichiometry.

***Summary of Lena’s Case***

Lena strongly advocated for developing students’ proportional reasoning ability. She not only realized students’ lack of ability but also stressed it is time for educators to work together to solve this problem. However, though Lena noticed the importance of proportional reasoning ability in learning chemistry and the students’ daily lives, she did not have a strong understanding of proportional reasoning. For instance, though she insisted that her problem-solving method for the five problems was focused on proportionality, the actual method she used was dimensional analysis. Also, she did not identify the non-proportion problem from among the proportional ones. In addition, the real-world problem that she suggested as an outside stoichiometry context was not a proportional reasoning problem. While Jake and John did not reveal a strong connection among the five categories of PCK, Lens’s knowledge of instructional strategies was mostly informed from the knowledge of students’ understanding and knowledge of the science curriculum. Because she believed the goal of the stoichiometry unit was raising the students' ability to scale up/down from micro- to macro-level of chemistry understanding, she employed S'more activity, which represents a macro-level understanding of stoichiometry, at the beginning of her stoichiometry lessons. Again, because she knew that the students have difficulty understanding the coefficients in the chemical reaction, she favored using the BCA chart as it emphasizes the meaning and relationship of the coefficients.

**Table 26**

*Summary of Lena's PCK in teaching stoichiometry*

| | |
|---|---|
| Orientation to teaching science | Discovery, Didactic, and Process |
| Knowledge of science curriculum | Goals & objectives:<br>• Ability to scale up from micro-level to macro-level<br>• Recall and sum up concepts learned earlier in the curriculum<br>Curriculum programs: None |
| Knowledge of students' understanding | Requirements:<br>• Mathematical skill; especially proportional reasoning skill<br>• More practice and time for general students; More independence and self-motivation is expected for honors-level<br>Students' difficulty<br>• Understanding the meaning of coefficients in the chemical equation<br>• Successfully following chemistry curriculum to building the foundation for understanding stoichiometry<br>• Recognizing the importance of showing their thinking process for solving a problem |
| Knowledge of instructional strategies | Subject-specific strategies:<br>• Activity: Lab & Virtual S'mores activity<br>• Representation: BCA chart<br>Topic-specific strategies: Scaling up from particle level to macroscopic level |
| Knowledge of assessment | Dimension of science learning: Practical reasoning & Conceptual understanding<br>Methods of assessing: Problem-solving |

## Summary

In this chapter, the findings from cases of three of the chemistry teachers' beliefs, problem-solving strategies for proportion problems (including stoichiometry problem), and their

PCK were shared. Among the four categories of belief, the ability of students in learning proportional reasoning in stoichiometry instruction had the most varied belief among the three case teachers. In terms of problem-solving, the strategies they used for solving proportion problems depended on the context of the problems. Moreover, even though teachers showed their confidence in teaching multiple-problems and solving methods during their stoichiometry instructions, they had a hard time thinking of the second method for solving the problems. The interviews allowed for probing of the teachers' PCK and the interviews revealed their intertwined understanding of incorporating proportional reasoning into teaching stoichiometry. In other words, they had a great deal of knowledge of incorporating the two concepts (proportional reasoning and the conceptual understanding of stoichiometry) into their stoichiometry lessons. Finally, the three cases revealed a strong connection between knowledge of students' learning and their instructional knowledge as chemistry teachers among the five components of PCK. The assessment knowledge was independent from other components.

# Chapter VI: Discussion, Implications, and Conclusions

The focus of this study was to look at chemistry teachers' knowledge and belief in teaching proportional reasoning in the context of stoichiometry. The study addressed the following research questions:

1. What beliefs does a sample of chemistry teachers hold about teaching proportional reasoning and stoichiometry?
2. What problem-solving strategies were used by a sample of chemistry teachers when solving proportional reasoning and stoichiometry problems?
3. What pedagogical content knowledge (PCK) do chemistry teachers bring when asked about how they would teach stoichiometry focusing on proportional reasoning?
4. What characteristics do three of the chemistry teachers show based on the evidence of Likert scale survey, proportion problems, and interview?

A mixed method research design was used to accomplish the goal of this study. More specifically, problem-solving, surveys, interviews, and lesson materials were collected to gain an insight into chemistry teachers' knowledge and belief in teaching proportional reasoning in stoichiometry. The results of this study may not be generalized beyond the ten in-service teachers who participated. However, it is believed to have implications that go beyond the participants in this study. In the following section, a discussion of each research question is presented, followed by implications and suggestions for educational research in science education.

## Research Question 1.

## What beliefs does a sample of chemistry teachers hold about teaching proportional reasoning and stoichiometry?

Chemistry teachers' belief over teaching proportional reasoning during stoichiometry instruction was explored mainly using the Likert scale survey. The four major categories of beliefs that influenced teaching suggested by Cronin-Jones (1991) were probed in this study: "(a) beliefs about how students learn, (b) beliefs about a teacher's role in the classroom, (c) beliefs regarding the ability levels of students in a particular age group, and (d) beliefs about the relative importance of content" (p. 246). Participant teachers generally understood that incorporating proportional reasoning practice in the stoichiometry instruction benefits from learning both concepts. Among these four categories, teachers had the most varied belief regarding the ability of students to learn proportional reasoning during stoichiometry instructions. For example, about half of the teachers (6 out of 10) believed that all high school students should have mastered proportional reasoning ability in their middle school years (item 10). The response to this item catches the attention not only because proportional reasoning is generally known to develop in early adolescent years (Piaget & Inhelder, 1958; Tourniaire & Pulos, 1985) but also because teachers knew that it is taught in middle school years (Common Core State Standards Initiative, 2012). Moreover, as Lena's case has revealed, some teacher's instructional knowledge is mainly informed by their knowledge of student understanding. Thus, teachers' belief and knowledge are significant factors in their teaching practices (Cronin-Jones, 1991; Fang, 1996; Jones & Carter, 2006; Mansour, 2009). Unfortunately, the relationship between teachers' beliefs and knowledge and their teaching practices is perplexing. However, the findings of this study (Lena's case and teachers' diverse belief regarding the ability of students in learning proportional reasoning during stoichiometry instructions) may suggest the first step for exploring the relationship between teachers' beliefs and knowledge and their teaching practices—examining the relationship between teacher's instructional knowledge and knowledge of student understanding. Therefore, as other researchers (e.g., Mansour, 2009) are arguing for continuous research on examining teachers' beliefs and how they affect their teaching practice, here I am urging more studies on how science teachers' beliefs and knowledge on mathematical thinking practice influence their science teaching.

This study mainly employed a Likert scale to measure teachers' beliefs because it is one of the most popular tools for investigating them. Also, because the main focus of this study was exploring in-service teachers' stoichiometry instruction with a focus on proportional reasoning practice and because the belief was explored as a lens for understanding their instructions, it did not go more in-depth in measuring the beliefs. However, many educators are incorporating reflective practices (reflecting their experience and skills, including writing a journal) for probing in-depth understanding of teachers' beliefs because personal experience is a critical factor in forming their beliefs (Akerson, Pongsanon, Park Rogers, Carter, & Galindo, 2017; Clarà, Kelly, Mauri, & Danaher, 2017; Min, Akerson, & Aydeniz, 2019). For example, Min et al. (2019) incorporated collaborative oral reflection to explore preservice teachers' beliefs about effective science teaching. Some argued that reflection is vital for teachers' growth because it helps them develop their critical thinking skills, develop practical theories and identities, and their pedagogical content knowledge (Akerson et al., 2017). Thus, reflection on practice is recommended for future studies to deeply assess and develop science teachers' belief in teaching proportional reasoning practices in teaching stoichiometry.

The most interesting finding regarding teachers' belief is the mismatch with their knowledge. Most of the participant teachers indicated in the survey (items 6 and 12) that they believed it is their role to present multiple solutions for stoichiometry problems. However, teachers expressed the difficulty coming up with the second way of solving the proportion problems, including the stoichiometry problem. Moreover, while teachers believed that they solved the problem in two different ways, the two were found to be the same. This finding indicates that even though they have a firm belief, they may not teach as they believe due to their lack of knowledge. Thus, in order to realize their beliefs, knowledge has to be the foundation.

**Research Question 2.**

**What problem-solving strategies were used by a sample of chemistry teachers' when solving proportional reasoning and stoichiometry problems?**

This research question was mainly investigated by the five problems that teachers solved. The problem-solving strategies and teachers' explanations of the strategies have revealed several interesting points. First, as the literature has indicated (Akatugba & Wallace, 2009; Saunders & Jesunathadas, 1988), a problem-solving strategy depended on the context of the problem. For example, Saunders and Jesunathadas (1988) found that students scored significantly higher when the proportional reasoning tasks were directly related to everyday, real-life experiences compared to unfamiliar problems, such as science. In this study, most of the teachers employed dimensional analysis strategy for density problems while unit-rate strategy was popularly used for solving the orange juice problem. Secondly, it is known that teachers heavily rely on an algorithmic approach (Arican, 2019a; Kartal & Kartal, 2019; Ramful & Narod, 2014). However, the participants in this study used various strategies for solving proportional relationship problems. For example, for the jar problem, there was fairly an even distribution on strategies that the teachers used. Finally, chemistry teachers in this study did show their limited understanding of proportional reasoning. This finding aligns with literature that continually reports limited understanding of proportional relationships among college-level students and even pre-service mathematics teachers (Arican, 2019b; Weiland, Orrill, Brown, &

Nagar, 2019). Tina, who had more than ten years of teaching chemistry, incorrectly solved proportion problems. Also, Jessica showed her limited understanding of proportion by listing age difference as an example of daily life proportional relationship during the interview. Therefore, the findings regarding teachers' knowledge often aligned with other literature in that teachers can also have limited knowledge of proportional reasoning.

To genuinely measure the proficiency in proportional reasoning, two types of proportional relationships should be examined: directly proportional relationships ($y = k \cdot x$) and inversely proportional relationships ($y \cdot x = k$). This study specifically focused on the direct proportional relationship as the underlying relationship in stoichiometry only includes direct proportion. However, inversely proportional reasoning is also a prevalent mathematical concept in science. Boyle's Law (the volume of an ideal gas is inversely proportional to the pressure of the gas) is a typical example of an inversely proportional relationship in the chemistry curriculum. Therefore, as a future study, the science teacher's conceptual understanding of the two types of proportional relationships and how they teach those two needs to be explored, respectively.

According to DeMeo (2008), chemistry teachers also use mass ratio for solving stoichiometry problems. However, there were no teachers who used this relationship in this study. Therefore, a separate case study for teachers who use mass ratio for solving and teaching stoichiometry should be examined. Again, it is recommended to closely examine the mass ratio strategy with the teacher's belief, knowledge, and teaching practice.

**Research Question 3.**

**What pedagogical content knowledge (PCK) do chemistry teachers bring when asked about how they would teach stoichiometry focusing on proportional reasoning?**

The teachers' PCK was explored using the modified CoRe interview questions by Loughran, Mulhall, and Berry (2008) and the PCK framework suggested by Magnusson, Krajcik, and Borko's (1999). Teachers in this study significantly considered the mathematics component specific to the proportion in their teaching of stoichiometry. Adopting related mathematics skills was one of the main goals of stoichiometry lessons. Teachers listed Algebra as a prerequisite skill/knowledge for learning stoichiometry. They also expressed that students lack of mathematics skill or knowledge as one of the factors that makes them difficulty to teach stoichiometry as well as students to learn stoichiometry. Finally, mathematics knowledge and skill occupied one dimension of assessment for stoichiometry unit. All these finding regarding mathematics indicates that chemistry teachers are fully aware of the importance and involvement of mathematics in learning stoichiometry.

In this study, and as Kartal and Kartal (2019) found, the teachers indicated that they solve and teach stoichiometry problems as they were taught in their prior learning experiences. For example, Annie stated that she taught the dimensional analysis strategy for solving stoichiometry problems because that was how she learned stoichiometry. She used this strategy for a while until she noticed that students were solving the stoichiometry problems without catching the underlying chemistry concept and that they took most of the time figuring out how to use this method. Thus, after years of teaching, she employed evidence-based teaching because she believed it was better to convey both proportional relationship and conceptual understanding of chemistry. Lena, who has more than ten years of teaching chemistry, also stated a similar transition that she took from teaching algorithmic to proportional reasoning focused instruction. The transition that Annie and Lena took for their instruction may imply that there is a hypothetical trajectory or learning progression in teachers' PCK. This aligns with Schneider and

Plasman's (2011) study that reviewed the literature and found a learning progression for each component of PCK. While the authors summarized the progression based on the career stage (years of teaching), no correlation was noticed between the transition and the years of teaching chemistry in this study. For example, Jeffry taught chemistry for over ten years, as Lena did. However, at the time of this study, he was in the process of transitioning from teacher-centered instruction (algorithmic and dimensional analysis focused instruction) to student-centered instruction (discovery-oriented). After years of struggling to teach stoichiometry, with his colleague's suggestion he took professional development from the American Modeling Teachers Association and adopted a modeling instruction in his stoichiometry lessons. As such, the instructional transition occurs not only based on the years of teaching experiences but also based on the colleague's suggestion and the teacher's need. However, considering that teaching is a career-long endeavor, it is strongly suggested to examine the development of teachers' PCK for different topics.

After Shulman (1986) introduced the term pedagogical content knowledge, researchers proposed various frameworks as they reconceptualized PCK or based on the content area (Ball et al., 2008; Grossman, 1990; Magnusson et al., 1999; Park & Chen, 2012). In 2013, a group of researchers suggested the 'PCK Summit Consensus Model' and met some level of agreement regarding the various conceptualizations of PCK (Gess-Newsome, 2015). This model includes teacher knowledge for teaching as a profession, how this knowledge comes into the classroom, and how that relates to students' outcomes. Furthermore, as technology plays a role in the daily lives and teachers' teaching practices, Koehler and Mishra (2009) suggested the teachers' technological pedagogical content knowledge (TPACK) framework that builds on Shulman's PCK to include technology knowledge. Among various framework, this study employed

Magnusson, Krajcik, & Borko's (1999) framework not only because it is the most commonly used framework in science education for teacher’s PCK but also because this study was an initial exploration of the chemistry teachers’ professional knowledge on teaching proportional reasoning in the context of stoichiometry. In other word, as the focus of the study was on exploring the teacher’s knowledge for each component of PCK rather than the interaction among the components of knowledge or its reflection on teaching practice. However, other models, especially the ‘PCK Summit Consensus Model’, is suggested to be used for examining teachers’ PCK on proportional reasoning practice because the next stage of this research is understanding how the knowledge comes into the classroom and how it affects students’ outcome. As the field of chemistry education is in need for studies regarding mathematics practices, chemistry teachers’ PCK on integrating mathematical practices, or proportional reasoning in teaching science concepts and its impact on teaching should be continued by using other PCK frameworks.

**Research Question 4.**

**What characteristics does each of the participant teachers show based on the evidence of Likert scale survey, standardized test, and interview?**

Three cases were carefully examined to understand the relationship among the components of a teacher’s belief and knowledge. Each participant’s problem-solving strategies, survey, interview, and lesson materials were holistically assessed. While Jake showed no relationship among the five categories of PCK, John reconciled the five categories of PCK in reaching a goal of conceptual understanding of stoichiometry, as well as the mathematical concept that underlies in each equation. Through Lena’s case, it was found that a teacher’s knowledge of instructional strategies was mainly informed by the knowledge of students, especially regarding the difficulties that students have in learning stoichiometry. This finding contradicts the other literature. According to Magnusson et al.'s (1999) framework and work from Abell (2008), teachers' orientation to teaching is an overarching component that shapes the other four components of PCK. This contradiction might be due to the depth of investigating the knowledge of orientation and the interaction among the components. As mentioned earlier, because the focus of the study was exploring in-service chemistry teachers' professional knowledge on teaching proportional reasoning in stoichiometry, the interaction among the components was not deeply explored. This might have limited the finding of genuine interactions. To deeply understand the multifaceted and internal construct of teachers' knowledge for teaching, more studies on the interactions among the components is needed. Similarly, Ekiz-Kiran and Boz (2020) suggested more studies on in-service teachers PCK and the interaction among the components as most interaction studies were conducted with pre-service teachers.

## Significance and Implications

Most proportional reasoning studies were conducted in the field of mathematics education compared to that in science education (Taylor & Jones, 2009). Thus, this study has its meaning in that it explored the chemistry teacher's knowledge and belief in teaching proportional reasoning. This study, as well as in the literature, found that teachers had limited understanding of proportional reasoning. Furthermore, students are known to have difficulty learning and transferring proportional reasoning ability form one context to another, say from mathematics learning to chemistry learning. Because we cannot expect different results by doing the same things, various solutions should be tried out. Here, I suggest two possible solutions: collaboration between mathematics and science education and teacher education.

**Collaboration between Mathematics and Science Education**

As one solution to this problem, the two disciplines are suggested to work closely together (Becker & Towns, 2012; Cooke & Canelas, 2019; Phelps, 2019). However, teachers indicated in the survey that they never or seldomly consult or collaborate with mathematics teachers, except Lena's experience of collaborating with mathematics teachers. More specifically, when she requested mathematics teachers to teach linear equations in the context of density, her students conceptually understood density better that year. This simple case reveals that collaboration between the two fields helps students learn chemistry better.

According to Phelps (2019), students learn mathematics in a very different context than in their chemistry class. For example, while students can add a negative number and positive numbers in mathematics class, they had a hard time doing the same calculation in chemistry class because they were not able to relate the negative numbers as the electrons and positive numbers as protons. Therefore, studies on how mathematics and chemistry instructors can support students as they reinterpret mathematics in chemistry learning are needed. In addition, because students come into chemistry class with personal experiences and sometimes negative experiences (i.e., mathematics anxiety), Becker and Towns (2012) suggested that science educators pay attention to the resources students bring from mathematics class to chemistry class. Therefore, chemistry teachers and mathematics teachers should work closely together to understand what resources students bring in to the chemistry class as well as how they are expected to apply what they learned in a mathematics class in another context. Moreover, students are expected to benefit from learning both subject areas by seeing "some commonality and enjoy the differences in approaches used within the subjects" (Lenton & Stevens, 2000, p.188).

I propose the collaboration between mathematics and science education departments in two phases. On a larger scale, the mathematics department and science department within the school districts or schools are suggested to collaborate to develop, for example, an integrated curriculum or projects that incorporate the common concept delivered in the two subject areas. In a narrower sense, an individual science teacher is encouraged to consult or communicate with a mathematics teacher so that students can transfer the same concept in different contexts.

When relating mathematics and chemistry performance, Cooke and Canelas (2019) suggested to work on answering the following two questions continuously: “(a) how well do we understand the nature of this relationship? and (b) what barriers exist to facilitating the transition of students in the application of math concepts to chemistry problems?” (p.120). Prior to collaborating, it is critical to address these two questions because there are some differences in understanding of proportional reasoning in science as compared to mathematics, as mentioned in chapter 1. For example, the concept of ratio used in science, such as density (ratio of mass to volume), is quite different from a ratio of numbers describing fractions of a pie (NRC, 2012). As such, the extent of understanding the proportion required for learning science is different from that of mathematics education. Therefore, as the first step of collaboration to improve students understanding of proportion, I suggest mathematics and science educators work collaboratively to make a consensus on how the use of ratio and proportion in science is different from mathematics and, furthermore, how the approach to teach proportion in mathematics is different from that of chemistry class (Phelps, 2019). Such consensus will add to the coherence of K-12 education.

**Teacher Education**

It is well acknowledged that information about teacher's knowledge and beliefs is an essential prerequisite for establishing an effective science teacher education (De Jong, Veal, & Van Driel, 2002). According to Ekiz-Kiran and Boz (2020), "knowing more about PCK components and their interactions helps researchers to identify methods for developing more qualified teachers" (p. 98). Thus, understanding in-service teachers' PCK is a prerequisite for preparing and training the pre-service teachers (Abell, 2008). This study revealed in-service teachers' PCK on teaching stoichiometry with a specific focus on proportional reasoning practice. Therefore, this study is expected to shed light on the teacher education programs for preparing pre-service science teachers, especially for chemistry and physics teachers, with sufficient knowledge of integrating mathematics practices in their science instruction.

This study proposed a hypothetical trajectory of PCK for teaching stoichiometry. For example, in earlier years of teaching, chemistry teachers taught stoichiometry algorithmic heavy because that was how they were taught. As they gain more experience with students, their teaching transitioned to promoting conceptual understanding. From this finding, it can be inferred that preservice teachers (PSTs) may not know the importance of, and how to promote, conceptual understanding of mathematics components that underly in science because they do not have experience with students yet. Therefore, I propose teacher education programs to emphasize the mathematical components that underlie in science concepts so that PSTs are prepared to interrelate science and mathematics (Kartal & Kartal, 2019).

This study revealed that in-service chemistry teachers have limited knowledge on proportional reasoning. In addition, even though they understood the importance and their role in teaching multiple-solution methods for stoichiometry problems, they had a hard time solving it in two different ways. Therefore, professional development that enhances not only science teachers' understanding of proportional reasoning but also their teaching of multiple-solution methods is needed. For example, during the professional development, teachers should be encouraged to share their problem-solving strategies for solving problems that involve mathematics as well as the advantages and disadvantages of each strategy for teaching. From this sharing, teachers' knowledge of proportional reasoning and presenting multiple ways of solving the problem is expected to improve.

## Summary

In this chapter, I presented the discussions, implications, and future studies for incorporating mathematics, especially proportional reasoning, in teaching chemistry. What I found was that (a) chemistry teachers had various belief on students' level of ability to learn proportional reasoning in stoichiometry lesson; (b) teachers had limited knowledge on proportion reasoning; (c) there may be a hypothetical trajectory in the development of PCK for teaching stoichiometry.; and (d) each teacher had unique relationship and development among the five components of PCK. The results of this study have implications for teacher education as well as science education regarding collaboration with mathematics education. Finally, I offered some recommendations for future studies on how science teachers' beliefs and knowledge of mathematical thinking practice influence their science teaching.

## References

Abell, S. K. (2008). Twenty Years Later: Does pedagogical content knowledge remain a useful idea? *International Journal of Science Education*, *30*(10), 1405–1416.

Akatugba, A. H., & Wallace, J. (1999). Mathematical Dimensions of Students' Use of Proportional Reasoning In High School Physics. *School Science Mathematics*, *99*(1), 31–41.

Akatugba, A. H., & Wallace, J. (2009). An Integrative Perspective on Students' Proportional Reasoning in High School Physics in a West African Context. *International Journal of Science Education*, *31*(11), 1473–1493.

Akerson, V. L., Pongsanon, K., Park Rogers, M. A., Carter, I., & Galindo, E. (2017). Exploring the Use of Lesson Study to Develop Elementary Preservice Teachers' Pedagogical Content Knowledge for Teaching Nature of Science. *International Journal of Science and Mathematics Education*, *15*(2), 293–312.

Al-wattban, M. S. (2001). *Proportional reasoning and working memory capacity among saudi adolescents: A neo -piagetian investigation*.

Arican, M. (2019a). Facilitating Mathematics Teachers ' Understanding of Directly and Inversely Proportional Relationships using Hands-on and Real-World Problems.

Arican, M. (2019b). Preservice Mathematics Teachers' Understanding of and Abilities to Differentiate Proportional Relationships from Nonproportional Relationships. *International Journal of Science and Mathematics Education*, *17*(7), 1423–1443.

Ault, A. (2001). How to Say How Much: Amounts and Stoichiometry. *Journal of Chemical Education*, *78*(10), 1347.

Ball, D. L., Thames, M. H., Phelps, G., Loewenberg Ball, D., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher*

*Education*, *59*(5), 389–407.

Becker, N., & Towns, M. (2012). Students' understanding of mathematical expressions in physical chemistry contexts: An analysis using Sherin's symbolic forms. *Chemistry Education Research and Practice*, *13*(3), 209–220.

Bird, L. (2010). Logical Reasoning Ability and Student Performance in General Chemistry, *87*(5), 541–546.

Bodner, G. M., & Herron, D. J. (2002). Problem-Solving in Chemistry. In J. H. Gilbert, J. K., de Jong, O., Justi, R., Treagust, D. F., & van Driel (Ed.), *Chemical education: towards research-based practice* (pp. 235–266). Dordrecht, The Netherlands: Kluwer Academic Publishers.

BouJaoude, S., & Barakat, H. (2000). Secondary school students' difficulties with stoichiometry. *School Science Review*, *81*(296), 91–98.

BouJaoude, Saouma, & Barakat, H. (2003). Students' problem solving strategies in stoichiometry and their relationship to conceptual understanding & learning approaches. *Electronic Journal of Science Education*, *7*(3).

Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology*, *44*(5), 1478–1490.

Brown, R. E., Nagar, G. G., Abington, P. S. U., Orrill, C. H., Weiland, T., & Burke, J. (2016). Coherency of a teacher's proportional reasoning knowledge in and out of the classroom. In *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 450–457).

Canagaratna, S. G. (1993). Is Dimensional Analysis the Best We Have to Offer? *Journal of Chemical Education*, *70*(1), 40–43.

Carlsen, W. S. (1991). Subject-matter knowledge and science teaching: A pragmatic perspective. *Advances in Research on Teaching*, *2*, 115–143.

Chandran, S., Treagust, D. F., & Tobin, K. (1987). The role of cognitive factors in chemistry achievement. *Journal of Research in Science Teaching*, *24*(2), 145–160.

Clarà, M., Kelly, N., Mauri, T., & Danaher, P. A. (2017). Can massive communities of teachers facilitate collaborative reflection? Fractal design as a possible answer. *Asia-Pacific Journal of Teacher Education*, *45*(1), 86–98.

Common Core State Standards Initiative. (2012). *Common Core State Standards for Mathematics*. *Common Core State Standards Initiative*.

Cook, E., & Cook, R. L. (2005). Cross-Proportions: A Conceptual Method for Developing Quantitative Problem-Solving Skills. *Journal of Chemical Education*, *82*(8), 1187.

Cooke, B. P., & Canelas, D. A. (2019). Transition of Mathematics Skills into Introductory Chemistry Problem Solving. In *It's Just Math: Research on Students' Understanding of Chemistry and Mathematics* (Vol. 1316, pp. 119–133). American Chemical Society.

Cracolice, M. S., Deming, J. C., & Ehlert, B. (2008). Concept Learning versus Problem Solving: A Cognitive Difference. *Journal of Chemical Education*, *85*(6), 873–878.

Cramer, K., & Post, T. (1993). Proportional Reasoning. *The Mathematics Teacher*, *86*(5), 404–407.

Cramer, K., Post, T., & Currier, S. (1993). Learning and Teaching Ratio and Proportion: Research Implications. In D. Owens (Ed.), *Research Ideas For the Classroom* (pp. 159–178). NY: Macmillan Publishing Company.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches*. Sage publications.

Cronin-Jones, L. L. (1991). Science teacher beliefs and their influence on curriculum implementation: Two case studies. *Journal of Research in Science Teaching*, *28*(3), 235–250.

Crouch, M., & McKenzie, H. (2006). The logic of small samples in interview-based qualitative research. *Social Science Information*, *45*(4), 483–499.

Dahsah, C., & Coll, R. K. (2007). Thai grade 10 and 11 students' understanding of stoichiometry and related concepts. *International Journal of Science and Mathematics Education*, *6*(3), 573–600.

de Berg, K. C. (1992). Mathematics in science: The role of the history of science in communicating the significance of mathematical formalism in science. *Science and Education*, *1*(1), 77–87.

De Jong, O., Veal, W. R., & Van Driel, J. H. (2002). Exploring chemistry teachers' knowledge base. In *Chemical Education: Towards Research-based Practice* (pp. 369–390). The Netherlands, Kluwer Academic Publishers.

DeLorenzo, R. (1994). Expanded Dimensional Analysis: A Blending of English and Math. *Journal of Chemical Education*, *71*(9), 789–791.

DeMeo, S. (2008). *Multiple Solution Methods for Teaching Science in the Classroom - Improving Quantitative Problem Solving using Dimensional Analysis and Proportional Reasoning*. Universal-Publishers.

Deters, K. M. (2003). View from My Classroom What Are We Teaching in High School Chemistry ? *Journal of Chemical Education*, *83*(10), 1492–1498.

Ekiz-Kiran, B., & Boz, Y. (2020). Interactions between the science teaching orientations and components of pedagogical content knowledge of in-service chemistry teachers. *Chem.*

*Educ. Res. Pract*, *21*(95), 95.

Fang, Z. (1996). A review of research on teacher beliefs and practices. *Educational Research*, *38*(1), 47–65.

Farrell, M. A., & Farmer, W. A. (1985). Adolescents' performance on a sequence of proportional reasoning tasks. *Journal of Research in Science Teaching*, *22*(6), 503–518.

Felder, R. M. (1990). Stoichiometry Without Tears. *Chemical Engineering Education*, *24*(4), 188–196.

Freudenthal, H. (1978). *Weeding and sowing: Preface to a science of mathematical education*. Springer Science & Business Media.

Furio, C., Azcona, R., & Guisasola, J. (2002). The Learning and Teaching of the Concepts "Amount of Substance" and "Mole": a Review of the Literature. *Chemistry Education Research and Practice*, *3*(3), 277–292.

Gabel, D. L., Sherwood, R. D., & Enochs, L. (1984). Problem-solving skills of high school chemistry students. *Journal of Research in Science Teaching*, *21*(2), 221–233.

Gess-Newsome, Julie. (2015). A model of teacher professional knowledge and skill including PCK: Results of the thinking from the PCK Summit. In *Re-examining Pedagogical Content Knowledge in Science Education*.

Glazar, S. A., & Devetak, I. (2002). Secondary School Students Knowledge of Stoichiometry. *Acta Chim. Slov.*, *49*, 43–53.

Grossman, P. (1990). *The making of a teacher: Teacher knowledge and teacher education*. New York, NY: Teachers College Press.

Guckin, A. M., & Morrison, D. (1991). Math* Logo: A project to develop proportional reasoning in college freshmen. *School Science and Mathematics*, *91*(2), 77–81.

Gulacar, O., Overton, T. L., Bowman, C. R., & Fynewever, H. (2013). A novel code system for revealing sources of students' difficulties with stoichiometry. *Chemistry Education Research and Practice*, *14*(4), 507–515.

Gulacar, Ozcan, Overton, T. L., & Bowman, C. R. (2013). A Closer Look at the Relationships between College Students' Cognitive Abilities and Problem Solving in Stoichiometry. *Eurasian Journal of Physics and Chemistry Education*, *5*(2), 144–163.

Haim, L., Cortón, E., Kocmur, S., & Galagovsky, L. (2003). Learning Stoichiometry with Hamburger Sandwiches. *Journal of Chemical Education*, *80*(9), 1021–1022.

Hart, K M. (1988). Ratio and proportion. In *Number Concepts and Operations in the Middle Grades* (pp. 198–219).

Hart, Kathleen M. (1984). *Ratio: Children's strategies and errors*. Windsor, England: NFER-Nelson.

Herron, J. D. (1975). Piaget for chemists: Explaining what "good" students cannot understand. *Journal of Chemical Education*, *52*(3), 146–150.

Heyworth, R. M. (1999). Procedural and conceptual knowledge of expert and novice students for the solving of a basic problem in chemistry. *International Journal of Science Education*, *21*(2), 195–211.

Hilton, A., & Hilton, G. (2016). Proportional Reasoning: An Essential Component of Scientific Understanding. *Teaching Science*, *62*(4), 32–42.

Hilton, A., Hilton, G., Dole, S., & Goos, M. (2016). Promoting middle school students proportional reasoning skills through an ongoing professional development programme for teachers. *Educational Studies in Mathematics*, *92*(2), 193–219.

Hwang, B. T., & Liu, Y. S. (1994). A Study of Proportional Reasoning and Self-Regulation

Instruction on Students' Conceptual Change in Conceptions of Solution. In *1994 Annual NARST Meeting*. Anaheim, CA.

Jensen, W. B. (2003). The Origin of Stoichiometry Problems. *Journal of Chemical Education*, *80*(11), 1248.

Johnson, M. A., & Lawson, A. E. (1998). What are the relative effects of reasoning ability and prior knowledge on biology achievement in expository and inquiry classes? *Journal of Research in Science Teaching*, *35*(1), 89–103.

Johnstone, A. H. (2000). Teaching of Chemistry - Logical or Psychological? *Chemistry Education Research and Practice in Europe*, *1*(1), 9–15.

Jones, M. G., & Carter, G. (2006). Science Teacher Attitudes and Beliefs. In S. K. Abell & N. G. Lederman (Eds.), *Handbook of research on science education* (pp. 1067–1104).

Karplus, R., Pulos, S., & Stage, E. K. (1983). Early Adolescents Proportional Reasoning on Rate Problems. *Educational Studies in Mathematics*, *14*(3), 219–233.

Kartal, T., & Kartal, B. (2019). Examining Strategies Used by Pre-service Science Teachers in Stoichiometry Problems in Terms of Proportional Reasoning, (1), 910–944.

Kastberg, S. E., D'Ambrosio, B., & Lynch-Davis, K. (2012). Understanding proportional reasoning for teaching. *The Australian Mathematics Teacher*, *68*(3), 32–40.

Keys, C. W., & Bryan, L. A. (2001). Co-constructing inquiry-based science with teachers: Essential research for lasting reform. *Journal of Research in Science Teaching*, *38*(6), 631–645.

Koehler, M. J., & Mishra, P. (2009). What is Technological Pedagogical Content Knowledge (TPACK)? *Contemporary Issues in Technology and Teacher Education*, *9*(1), 60–70.

Kouba, V. L., & Wearne, D. (2000). Whole number properties and operations. In *Results from the* *seventh mathematics assessment of the national assessment of educational progress* (pp. 141–161). National Council of Teachers of Mathematics.

Krieger, C. R. (1997). Stoogiometry : A Cognitive Approach to Teaching Stoichiometry. *Journal of Chemical Education*, *74*(3), 306–309.

Lamon, S. (2012). *Teaching Fractions and Ratios for Understanding*. *Ratio and Proportion*.

Lamon, S. J. (1993). Ratio and proportion: Connecting content and children's thinking. *Journal for Research in Mathematics Education*, *24*(1), 41–61.

Lamon, Susan J. (1995). Ratio and proportion: Elementary didactical phenomenology. In *Providing a foundation for teaching mathematics in the middle grades* (pp. 167–198).

Lamon, Susan J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In *The second handbook of research on mathematics teaching and learning* (pp. 629–668).

Langrall, C., & Swafford, J. (2000). Three Balloons for Two Dollars: Developing Proportional Reasoning. *Mathematics Teaching in the Middle School*, *6*(4), 254–261.

Lenton, G., & Stevens, B. (2000). Numeracy in science: Understanding the misunderstandings. In *Issues in Science Teaching* (pp. 175–190).

Lobato, J., & Ellis, A. B. (2010). *Developing Essential Understandings of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics: Grades 6-8*.

Lobato, J., & Lester Jr, F. K. (2010). *Teaching and Learning Mathematics: Translating Research for Secondary School Teachers*. National Council of Teachers of Mathematics.

Lobato, J., Orrill, C. H., Druken, B., & Jacobson, E. (2011). Middle School Teachers' Knowledge of Proportional Reasoning for Teaching. In *Annual Meeting of the American Educational Research Association (AERA)* (pp. 1–14). New Orleans, LA.

Loughran, J. J., Mulhall, P., & Berry, A. (2008). Exploring pedagogical content knowledge in science teacher education. *International Journal of Science Education*, *30*, 1301–1320.

Loughran, J., Mulhall, P., & Berry, A. (2004). In Search of Pedagogical Content Knowledge in Science: Developing Ways of Articulating and Documenting Professional Practice. *Journal of Research in Science Teaching*, *41*(4), 370–391.

Luke, C. (1990). *Young adolescents'performance on proportional reasoning tasks*. Harvard University, Cambridge, MA.

Magnusson, S., Krajcik, J., & Borko, H. (1999). Nature, sources, and development of pedagogical content knowledge for science teaching. In J. Gess-Newsome & N. G. Lederman (Eds.), *PCK and Science Education* (pp. 95–132). Netherlands.: Kluwer Academic Publishers.

Mansour, N. (2009). Science Teachers' Beliefs and Practices: Issues, Implications and Research Agenda. *International Journal of Environmental & Science Education*, *4*(1), 25–48.

McBride, J. W., & Chiappetta, E. L. (1978). The Relationship Between the Proportional Reasoning Ability of Ninth Graders and Their Achievement of Selected Math and Science Concepts. In *51st Annual meeting of the National Association for Research in Science Teaching*. Toronto, Canada.

Min, M., Akerson, V., & Aydeniz, F. (2019). Exploring Preservice Teachers' Beliefs about Effective Science Teaching through Their Collaborative Oral Reflections. *Journal of Science Teacher Education*, *00*(00), 1–19.

Mitchell, A., & Lawson, A. E. (1988). Predicting genetics achievement in nonmajors college biology. *Journal of Research in Science Teaching*, *25*(1), 23–37.

Nakhleh, M. B., & Mitchell, R. C. (1993). Concept learning versus problem solving: There is a difference. *Journal of Chemical Education*, *70*(3), 190.

NGSS Lead States. (2013). *Next Generation Science Standards: For States, By States*. Washington, DC: The National Academies Press.

Niaz, M. (2008). A rationale for mixed methods (integrative) research programmes in education. *Journal of Philosophy of Education*, *42*(2), 287–305.

Noelting, G. (1980). The development of proportional reasoning and the ratio concept Part 1.Differentiation of stages. *Educational Studies in Mathematics*, *11*(2), 217–253.

NRC. (2012). *A Framework for K-12 Science Education: Practices, Crosscutting Concepts, and Core Ideas*. Washington, DC: The National Academies Press.

Nurrenbern, S. C., & Pickering, M. (1987). Concept learning versus problem solving: Is there a difference? *Journal of Chemical Education*, *64*(6), 508.

Onwuegbuzie, A., & Leech, N. (2005). On becoming a pragmatic researcher: The importance of combining quantitative and qualitative research methodologies. *International Journal of Social Research Methodology: Theory and Practice*, *8*(5), 375–387.

Padilla, K., & Garritz, A. (2011). The Pedagogical Content Knowledge of University Chemistry Professors Teaching Stoichiometry. In *Annual Meeting of National Association of Research in Science Teaching*.

Park, S., & Chen, Y. C. (2012). Mapping out the integration of the components of pedagogical content knowledge (PCK): Examples from high school biology classrooms. *Journal of Research in Science Teaching*, *49*(7), 922–941.

Phelps, A. J. (2019). But You Didn't Give Me the Formula!" and Other Math Challenges in the Context of a Chemistry Course. *ACS Symposium Series*, *1316*, 105–118. chapter.

Piaget, J., & Inhelder, B. (1958). *The growth of logical thinking from childhood to adolescence:*

*An essay on the construction of formal operational structures*.

Ramful, A., & Narod, F. B. (2014). Proportional reasoning in the learning of chemistry: Levels of complexity. *Mathematics Education Research Journal*, *26*(1), 25–46.

Riehl, S. M., & Steinthorsdottir, O. B. (2017). Investigations in Mathematics Learning Missing-value proportion problems: The effects of number structure characteristics. *Investigations in Mathematics Learning*, *11*(1), 1–13.

Roadrangka, V. (1985). *The construction and validation of the group assessment of logical thinking (GALT)*.

Saldaña, J. (2009). *The Coding Manual for Qualitative Researchers*.

Sanger, M. J., Campbell, E., Felker, J., & Spencer, C. (2007). Concept Learning versus Problem Solving: Does Particle Motion Have an Effect? *Journal of Chemical Education*, *84*(5), 875–879.

Saunders, W. L., & Jesunathadas, J. (1988). The effect of task content upon proportional reasoning. *Journal of Research in Science Teaching*, *25*(1), 59–67.

Sawrey, B. A. (1990). Concept learning versus problem solving: Revisited. *Journal of Chemical Education*, *67*(3), 253.

Schliemann, A. D., & Carraher, D. W. (1992). Proportional reasoning in and out of school. In *Context and Cognition: Ways of Learning and Knowing* (pp. 47–73).

Schmidt, H. J. (1990). Secondary school students' strategies in stoichiometry. *International Journal of Science Education*, *12*(4), 457–471.

Schneider, R. M., & Plasman, K. (2011). Science Teacher Learning Progressions: A Review of Science Teachers' Pedagogical Content Knowledge Development. *Review of Educational Research*, *81*(4), 530–565.

Seminara, L. (1996). *An exploration of the relationship between conceptual knowledge, sex, attitude and problem-solving in chemistry*. Teachers College, Columbia University.

Shulman, L. (1986). Those who understand: knowdge growth in teaching. *Educational Researcher*, *15*(2), 4–14.

Singer-Freeman, K. E., & Goswami, U. (2001). Does half a pizza equal half a box of chocolates?: Proportional matching in an analogy task. *Cognitive Development*, *16*(3), 811–829.

Smith, D. C., & Neale, D. C. (1989). The construction of subject matter knowledge in primary science teaching. *Teaching and Teacher Education*, *5*(1), 1–20.

Stake, R. (1995). *The Art of Case Study Research*. *Thousand Oaks, CA: Sage*.

Steinthorsdottir, O. B. (2003). *Making meaning of proportion: A study of girls in two Icelandic classrooms*. University of Wisconsin-Madison.

Sullivan, G. M., & Artino, A. R. (2013). Analyzing and Interpreting Data From Likert-Type Scales. *Journal of Graduate Medical Education*, *5*(4), 541–542.

Tang, H., Kirk, J., & Pienta, N. J. (2014). Investigating the effect of complexity factors in stoichiometry problems using logistic regression and eye tracking. *Journal of Chemical Education*, *91*(7), 969–975.

Taylor, A., & Jones, G. (2009). Proportional Reasoning Ability and Concepts of Scale: Surface area to volume relationships in science. *International Journal of Science Education*, *31*(9), 1231–1247.

Thompson, S., & Bush, S. (2003). Improving Middle School Teachers' Reasoning about Proportional Reasoning. *Mathematics Teaching in the Middle School*, *8*(8), 398–403.

Tourniaire, F. (1984). *Proportional reasoning in grades three, four, and five*. University of

California, Berkeley.

Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics*, *16*(2), 181–204.

van Driel, J. H., Verloop, N., & de Vos, W. (1998). Developing science teachers' pedagogical content knowledge. *Journal of Research in Science Teaching*, *35*(6), 673–695.

Wagner, E. P. (2001). A Study Comparing the Efficacy of a Mole Ratio Flow Chart to Dimensional Analysis for Teaching Reaction Stoichiometry. *School Science & Mathematics*, *101*(1), 10–22.

Watson, J., Callingham, R., & Donne, J. (2008). Proportional Reasoning : Student Knowledge and Teachers ' Pedagogical Content Knowledge, 563–571.

Weiland, T., Orrill, C. H., Brown, R. E., & Nagar, G. G. (2019). Mathematics teachers' ability to identify situations appropriate for proportional reasoning. *Research in Mathematics Education*, *0*(0), 1–18.

Witzel, J. E. (2002). Lego Stoichiometry. *Journal of Chemical Education*, *79*(3), 352A-352B.

Wolfer, A. J. (2000). *Introductory College Chemistry Students' Understanding of Stoichiometry: Connections Between Conceptual and Computational Understandings and Instruction*. Oregon State University.

**Appendix A. Standardized Test**

Problem 1.

The Haber–Bosch process, which was developed in the first half of the 20th century, is an artificial nitrogen fixation process and is the main industrial procedure for the production of ammonia today. It is named after its inventors, the German chemists Fritz Haber and Carl Bosch, who were later awarded Nobel prizes in 1918 and 1931, respectively, for their work in overcoming the chemical and engineering problems of large-scale, continuous-flow, high-pressure technology. The process converts atmospheric nitrogen ($N_2$, Molar mass = 28 g/mole) to ammonia ($NH_3$, Molar mass = 17g/mole) by a reaction with hydrogen ($H_2$, Molar mass = 2 g/mole) using a metal catalyst under high temperatures and pressure

$$N_2\ (g) + 3H_2\ (g) \rightarrow 2NH_3\ (g)$$

If you want to produce 25 grams of ammonia gas ($NH_3$), how many grams of hydrogen gas ($H_2$) would you need? (Please provide TWO different solution methods).

Problem 2.
There are two plastic jars, a wide one and a narrow one.

Each has equally space marks along the side. Dany pours the same amount of water into each jar. The water level comes up to 4th mark in the wide jar and to the 6th mark in the narrow jar. Dany pours more water into the wide jar and the water level comes up to the 6th mark. How high would the water level be when the same amount of extra water is poured into the narrow jar? (Please provide TWO different solution methods).

Problem 3.

| Gases (at 25°C) | Density (g/L) |
|---|---|
| Neon | 0.90 |
| Helium | 0.18 |
| Oxygen | 1.43 |

There are three balloons filled with each gas. A balloon with neon has a volume of 1.67 L. A balloon with helium has a volume of 5.2 L. Finally, a balloon with oxygen has a volume of 850 ml. Which balloon weighs the most? (Please provide TWO different solution methods).

Problem 4.
Sophie and Matt ran a marathon at the same speed. Sophie started before Matt, so when Sophie ran 4 miles, Matt ran 2 miles from the starting line. How far had Matt run when Sophie had run 12 miles? (Please provide TWO different solution methods).

Problem 5.
Luis mixed 5 ounces of orange juice concentrate with 7 ounces of water to make orange juice. Martin mixed 3 ounces of the same orange juice concentrate with 5 ounces of water. Who made the stronger orange juice? (Please provide TWO different solution methods).

## Appendix B. Interview Questions

1. For each of the problems, please briefly explain:
   A. How you solved for each method.
   B. How do you think the two methods are different?
   C. Which method do you prefer to use? Why?
   D. [For stoichiometry problem] As a chemistry teacher, which method do you prefer to present in your class or do you prefer another method? Why?

2. Why do you think it is important for students to know how to solve such a stoichiometry problem?

3. What do you intend the students to learn while solving such a stoichiometry problem?

4. Can you briefly describe how you teach the unit of stoichiometry to students?
   A. How do you teach differently between honors and general chemistry classes?
   B. How will your stoichiometry instruction help students to reinforce what they have learned before and what they will learn later?

5. What difficulties do you have when you teach these stoichiometry problems?

6. What student factors do you consider when planning to teach a stoichiometry unit?
   A. What are the common misconceptions/mistakes that students make when solving stoichiometry problems?
   B. How you implement them in your teaching?

7. How do you assess students understanding of stoichiometry?
   A. How would you ensure that the students have understood the concept of stoichiometry?

## Appendix C. Survey result by participants

| | Jake | Annie | Tina | John | Levy | Jack | Jessica | Jeffry | Stella | Lena |
|---|---|---|---|---|---|---|---|---|---|---|
| **Belief 1. How students learn** | | | | | | | | | | |
| 1. Proportional reasoning can be fully adopted just by daily life/experiences (i.e., tip, interest, recipes, etc.) | SA | SA | A | A | A | A | SA | A | SA | SA |
| 2. Successfully solving stoichiometry problems implies that students have conceptually understood the concept of stoichiometry | SA | SD | N | A | SA | SA | D | D | SD | D |
| 9. Emphasizing the proportional relationship between the reactants/products in stoichiometry problems helps high school students to improve their proportional reasoning ability | SA | SA | A | SA | SA | SA | A | SA | A | SA |
| 20. Student's proportional reasoning ability can be improved through solving stoichiometry problems | SA | N | A | A | SA | SA | SA | A | A | SA |
| 25. Students' proportional reasoning ability can be advanced to a higher level through teachers' support | SA | SA | SA | SA | SA | SA | SA | SA | A | SA |
| **Belief 2. Teachers role and confidence** | | | | | | | | | | |
| 4. Improving students' proportional reasoning ability is more of mathematics teachers' responsibility than that of chemistry teachers | SD | D | D | A | D | D | D | SD | SD | D |
| 5. Chemistry teachers are in critical position in improving students' proportional reasoning ability | SA | A | SA | A | SA | A | SA | SA | SA | A |
| 6. I present different solution methods for solving stoichiometry problem to students | A | SA | SA | SD | A | A | SA | SA | A | A |
| 7. Chemistry teachers should improve students' proportional reasoning ability while teaching stoichiometry problems | SA | SA | SA | A | A | SA | SA | SA | A | A |
| 8. During the stoichiometry instructions, I often emphasize the proportional relationship that underlies in stoichiometry problems | SA | SA | SA | SA | SA | A | SA | SA | A | SA |

| | Jake | Annie | Tina | John | Levy | Jack | Jessica | Jeffry | Stella | Lena |
|---|---|---|---|---|---|---|---|---|---|---|
| 12. Chemistry teacher should present multiple solution methods for solving stoichiometry problems | SA | SA | A | N | SA | SA | SA | SA | SA | A |
| 15. I can teach students to fully understand the concept of proportion through any types of problems that involve proportional relationship | A | A | A | SA | A | A | A | A | D | SA |
| 18. I am confident in integrating mathematical thinking practice, particularly the concept of proportion, in my stoichiometry instructions | A | SA | SA | SA | A | A | SA | A | A | A |
| 24. Most of my students conceptually understand the concept of stoichiometry after my instructions | A | A | A | SA | A | A | SA | A | A | A |
| **Belief 3. Ability levels of students in a particular age group** | | | | | | | | | | |
| 10. All high school students should have mastered proportional reasoning ability in their middle school years | SA | N | SA | A | A | D | SA | SD | D | A |
| 13. Without fully understanding the concept of proportion and knowing how to solve proportional relationship problems, high school students cannot understand and solve stoichiometry problems | SA | SA | D | SA | SA | N | SA | A | SD | A |
| 16. High school students cannot improve their proportional reasoning ability while learning stoichiometry because it is too much to learn at once | D | SD | SD | SD | D | D | SD | SD | D | SD |
| 17. Students' lack of mathematics knowledge or ability to solve stoichiometry problems limits me from teaching the concept more deeply | A | SD | D | A | N | N | D | SD | A | SA |
| 21. Stoichiometry is a hard concept for all high school students to understand because it requires many concepts | A | A | D | A | A | A | SD | A | D | D |
| **Belief 4. Relative importance of content** | | | | | | | | | | |
| 3. Proportional reasoning is an important ability in understanding and learning other science concepts | SA | A | D | SA | SA | SA | SA | SA | SA | SA |

| | Jake | Annie | Tina | John | Levy | Jack | Jessica | Jeffry | Stella | Lena |
|---|---|---|---|---|---|---|---|---|---|---|
| 11. Proportional reasoning is an important ability for student to become a science and mathematics literate citizen | SA | SA | N | SA | SA | A | SA | SA | A | A |
| 14. Engaging students to mathematically think about the concept of proportion in science class helps to understand the work of scientists | SA | A | A | A | SA | A | SA | A | A | A |
| 19. Proportional reasoning is a common mathematical practice that scientists engage in | SA | SA | N | SA | SA | SA | SA | A | SA | SA |
| 22. Solving stoichiometry problems helps students to understand the work of scientists | A | SA | A | A | SA | A | SA | A | D | A |
| 23. It is important to emphasize the proportional relationships that underlie in stoichiometry to understand the concept of stoichiometry | A | SA | A | SA | A | A | SA | SA | SA | SA |
| 26. Chemistry teachers need more support (e.g., knowledge, time, and etc.) to involve the mathematical thinking practice in stoichiometry instructions | SA | SA | SA | N | SA | SA | N | A | D | SA |

SA – Strongly agreed; A –Agreed; N – Neutral; D –Disagree; SD – Strongly agreed

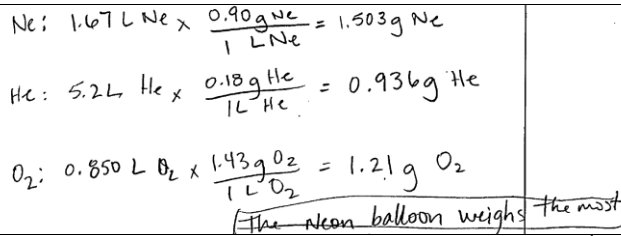
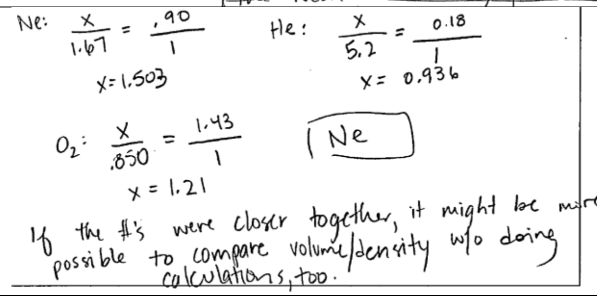
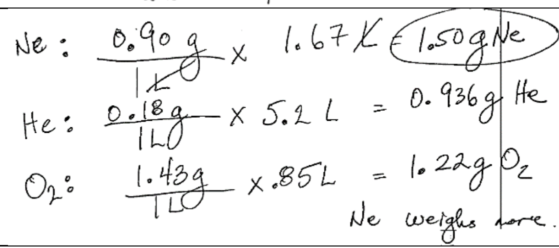
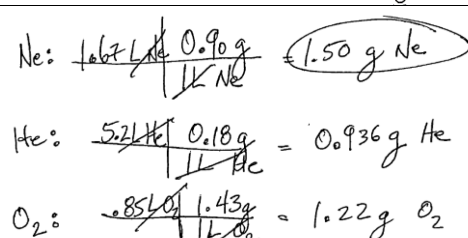
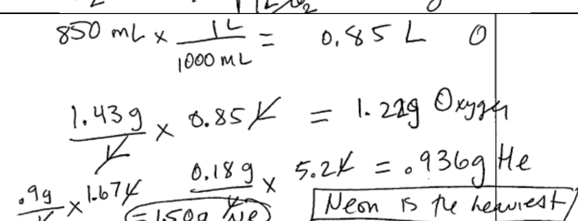
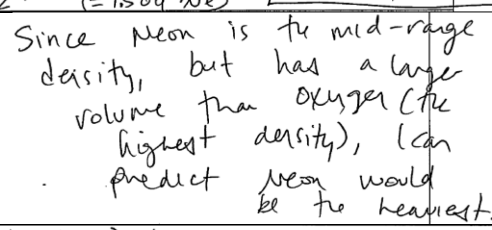
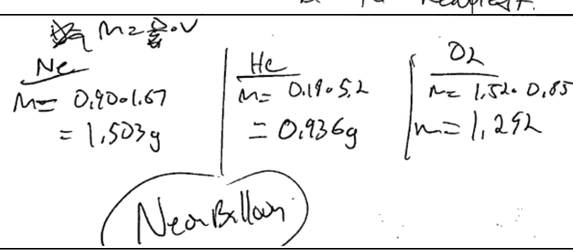
## Appendix D. Participants' problem-solving strategies for jar problem

| | | |
|---|---|---|
| Annie's work | **Method 1.<br>DA** | 4 wide = 6 narrow<br>6 wide × 6 narrow / 4 wide = 9 narrow<br>9th mark |
| | Method 2.<br>PR-F | 4 wide / 6 wide = 6 narrow / x narrow<br>x = 9 narrow<br>9th mark |
| Tina's work | **Method 1.*<br>Qualitative** | THE SAME AMOUNT, BUT MAYBE SLIGHTLY LESS THAN THE 6TH MARK. |
| | Method 2.*<br>Qualitative | THE WATER LEVEL WOULD RISE BACK TO THE 6TH MARK. |
| Levy's work | **Method 1.<br>PR-CP** | 4 wide jar / 6 wide jar = 6 narrow jar / X narrow jar<br>36/4 = 4X/4  9th mark narrow jar |
| | Method 2.<br>PR-UR | 4/6 = 0.66<br>the wide jar is always 2/3 of the narrow jar<br>6 is 2/3 of X<br>1/3 = 3  2/3 = 6  3/3 = 9 |
| Jack's work | Method 1.<br>DA | 4w = 6n  w = 6n/4<br>6w = x·n  6(6n/4) = x·n  x = 9, it would reach the 9th line<br>36/4 · 9n = x·n |
| | **Method 2.<br>PR-UR** | If poured 60mL ...<br>narrow = 60mL = 10/Line<br>wide > 60mL = 15/Line → 30 more to 6th Line<br>90mL = 90/10 = 9 Lines |

| | | |
|---|---|---|
| Jessica's work | Method 1.<br>PR-F | $6A = 4B$<br>$? = 6B$<br>$\frac{6A}{4B} = \frac{xA}{6B}$<br>$x = 9$<br>The level would be up to 9 in the narrow jar |
| | **Method 2.<br>PR-UR** | $4/6 = 2/3$<br>$6/10 = 3/5$<br>$\frac{2}{3}B = \frac{6}{10}A = \frac{3}{5}A$<br>1 notch B = 2/3 # A<br>so if 6B = 2/3 # notches on A<br>Then need 9 notches on A |
| Jeffery's work | Method 1.<br>PR-CP | 4 marks wide = 6 marks narrow<br>$\frac{4 \text{ wide}}{6 \text{ narrow}} = \frac{2 \text{ wide}}{x \text{ narrow}} \rightarrow \frac{4x}{4} = \frac{12}{4}$<br>Narrow would be 6 marks + 3 marks [9 marks] $x = 3$ |
| | **Method 2.<br>DA** | 4 marks wide = 6 marks narrow<br>$\frac{6 \text{ wide marks}}{} \times \frac{6 \text{ narrow marks}}{4 \text{ wide marks}} = \frac{36 \text{ narrow marks}}{4}$<br>9 narrow marks |
| Stella's work | **Method 1.<br>DA** | $6w \times \frac{6N}{4w} = 9N$<br>The water would reach the 9th mark in the narrow jar |
| | Method 2.<br>PR-F | $\frac{6}{4} = \frac{x}{6}$<br>$x = 9$ |

*Note.* Dimensional analysis (DA); Dimensional analysis T-chart equation (DA-T-chart); Dimensional analysis one chain equation (DA-Chain); Unit-rate (PR-UR); Factor-of-change (PR-FC); Fractions (PR-F); Cross-product algorithm (PR-CP)
* Incorrect answer
**Bold**: Preferred method

## Appendix E. Participants' problem-solving strategies for density problem

| | | |
|---|---|---|
| Annie's work | **Method 1. DA** |  |
| | Method 2. PR-CP |  |
| Tina's work | Method 1. DA |  |
| | Method 2. DA-T Chart |  |
| Levy's work | Method 1. DA |  |
| | Method 2.* Qual |  |
| Jack's work | Method 1. DA |  |

| | | |
|---|---|---|
| | Method 2.<br>PR-FC | $\frac{0.90}{0.18} = 5$ $\quad \frac{5.2}{1.67} = 3.11$<br>3.11 < 5, so Neon will be heavier than He<br>$\frac{0.90}{1.43} = 0.629$ $\quad \frac{0.85}{1.67} = 0.5$<br>0.629 > 0.5, so Ne is heavier than $O_2$ |
| Jessica's work | Method 1.<br>DA | $\left(\frac{0.90g}{1L}\right)(1.67L\ Ne) = 1.5g\ Ne$<br>$\left(\frac{.18g}{1L}\right)(5.2L\ He) = 0.94g\ He$<br>$\left(\frac{1.43g}{L}\right)(.850L) = 1.2g\ O_2$<br>The balloon w/ Ne weighs the most. (Has the most mass) |
| | Method 2.<br>PR-FC | $\frac{1.43 g/L\ O_2}{0.90 g/L\ Ne} = 1.6$ $O_2$ is 1.6 x more dense than Ne. ∴ equal volumes of gas, the $O_2$ would be more massive.<br>850 mL → 0.850 L<br>$\frac{1.67 L\ Ne}{0.850 L\ O_2}$ = approx 2 x the volume so Ne will be more massive over all b/c $O_2$ is only 1.6 x more massive if comparing equal # particles g:g |
| Jeffery's work | Method 1.<br>DA | $\frac{1.67 L\ Ne}{1} \times \frac{0.90 g\ Ne}{1 L\ Ne} = 1.5 g\ Ne$ (weighs the most)<br>$\frac{5.2 L\ He}{1} \times \frac{0.18 g\ He}{1 L\ He} = 0.94 g\ He$<br>$\frac{0.850 L\ O_2}{1} \times \frac{1.43 g\ O_2}{1 L\ O_2} = 1.22 g\ O_2$ ($O_2$ balloon weighs the most) |
| | Method 2.<br>PR-CP | Ne weighs most<br>$\frac{0.90 g\ Ne}{1 L\ Ne} = \frac{x\ g\ Ne}{1.67 L\ Ne}$ $\quad x = 0.90(1.67)$ — 1.5 g Ne<br>$\frac{0.18 g\ He}{1 L\ He} = \frac{y\ g\ He}{5.2 L\ He}$ $\quad y = 0.18(5.2)$ — 0.94 g He<br>$\frac{1.43 g\ O_2}{1 L\ O_2} = \frac{z\ g\ O_2}{0.850 L\ O_2}$ $\quad z = 1.43(0.850)$ — 1.22 g $O_2$ |
| Stella's work | Method 1.<br>DA | $1.67L\ Ne \times \frac{.90g}{1L} = 1.5g\ Ne$ Neon weighs the most<br>$5.2L\ He \times \frac{.18g}{1L} = 0.94g\ He$<br>$.850L\ O_2 \times \frac{1.43g}{1L} = 1.2g\ O_2$ |

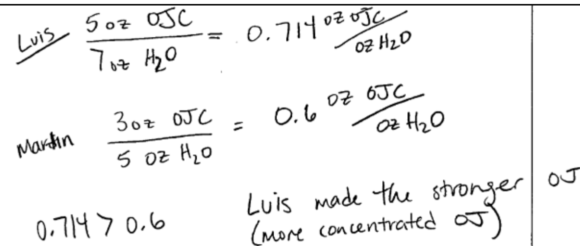
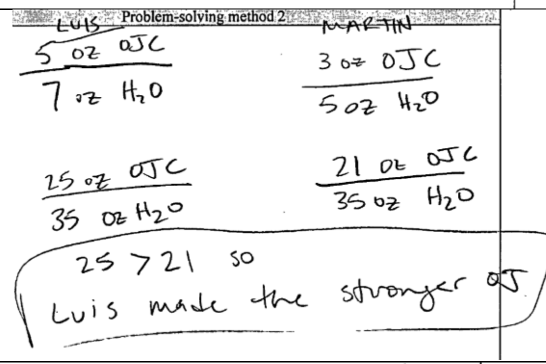
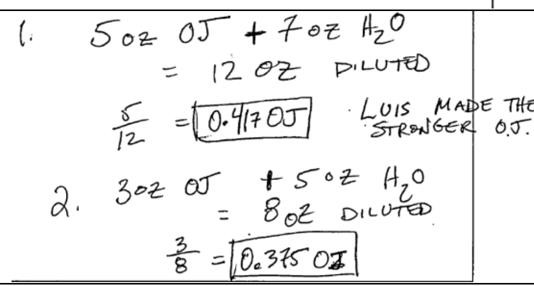
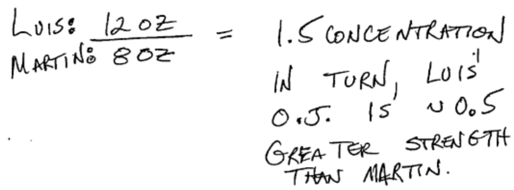
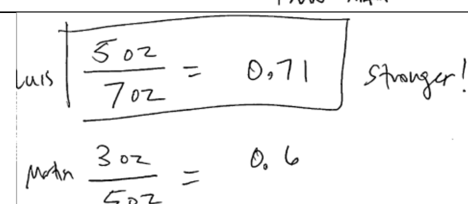
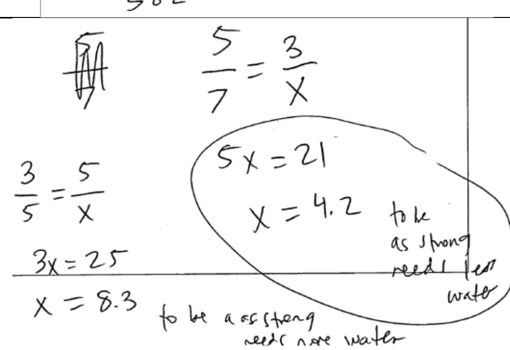
| | | |
|---|---|---|
| | Method 2.*<br>DA | $D = \frac{m}{V}$ $m = \frac{D}{V}$<br>$Ne: m = \frac{.90 g/L}{1.67 L} = 1.5 g\ Ne$<br>$He: m = \frac{.18 g/L}{5.2 L} = .94 g\ He$<br>$O_2: m = \frac{1.43 g}{.850 L} = 1.2 g\ O_2$ |

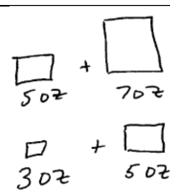
*Note.* Dimensional analysis (DA); Dimensional analysis T-chart equation (DA-T-chart); Dimensional analysis one chain equation (DA-Chain); Unit-rate (PR-UR); Factor-of-change (PR-FC); Fractions (PR-F); Cross-product algorithm (PR-CP)

* Incorrect answer

**Bold**: Preferred method

## Appendix F. Participants' problem-solving strategies for orange juice problem

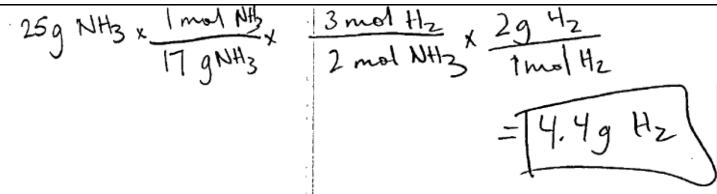
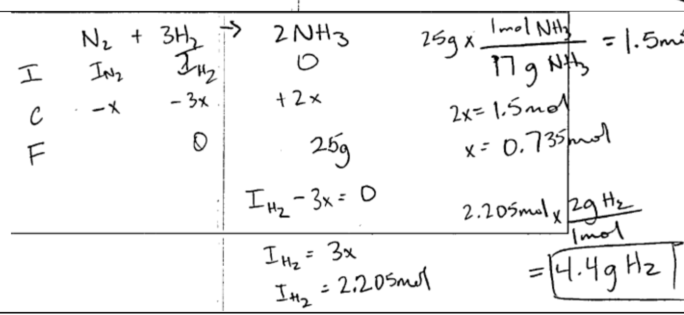
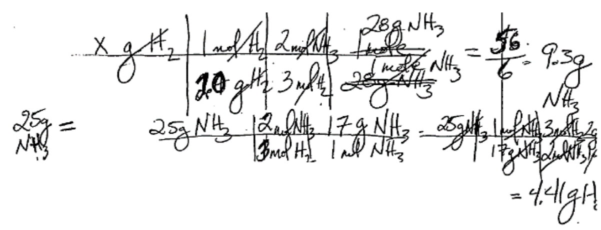
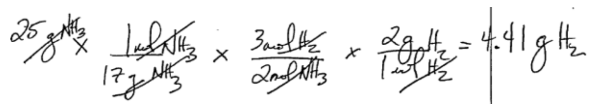
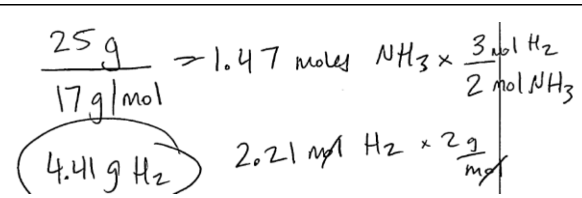
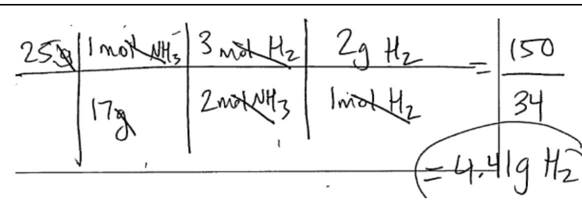
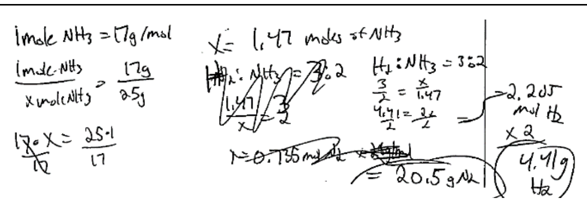
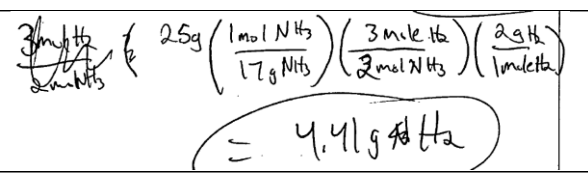
| | | |
|---|---|---|
| Annie's work | **Method 1. PR-UR** |  |
| | Method 2. PR-UR |  |
| Tina's work | **Method 1. PR-UR** |  |
| | Method 2.* PR-FC |  |
| Levy's work | **Method 1. PR-UR** |  |
| | Method 2. PR-CP |  |

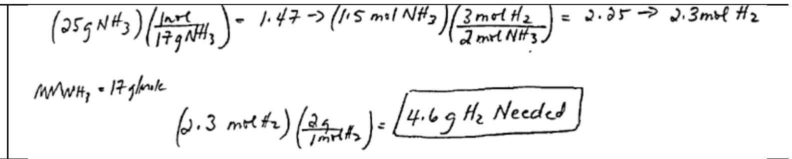
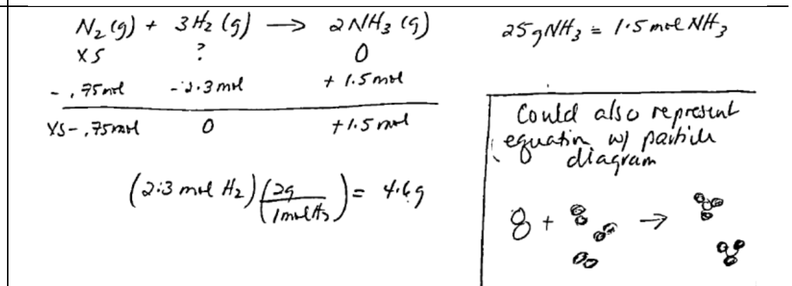
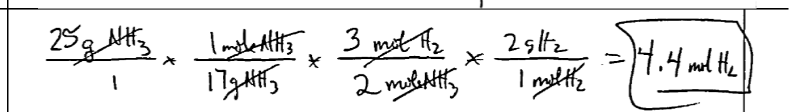
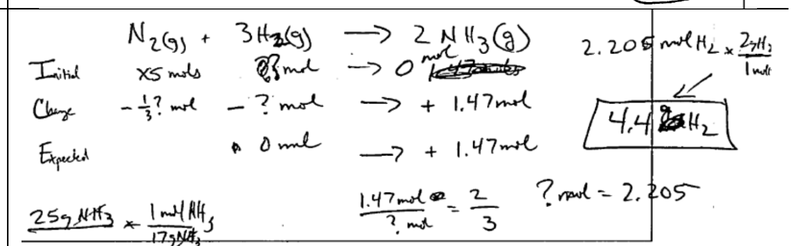
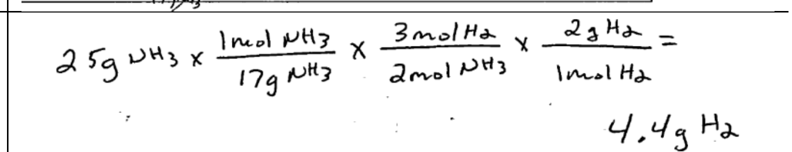
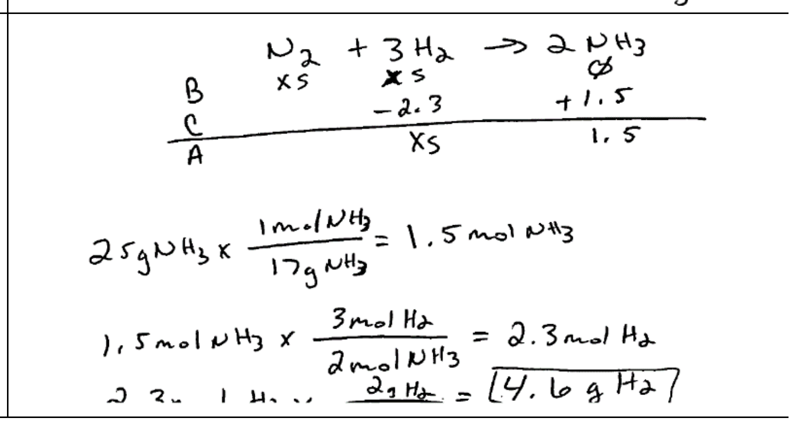
| | | |
|---|---|---|
| Jack's work | **Method 1. PR-UR** | $\frac{5\text{ oz conc.}}{7\text{ oz. water}}$ = 0.714 conc/wat<br>$\frac{3\text{ oz conc.}}{5\text{ oz wat}}$ = 0.6 conc/wat<br>Luis |
| | Method 2. PR-UR | $\frac{5}{5+7}$ = 0.4167 Luis<br>$\frac{3}{5+7}$ = 0.375 |
| Jessica's work | **Method 1. PR-UR** | Luis – 5 oz oj + 7 oz $H_2O$ 5 oz/12 total assume volume conserved<br>Martin 3 oz oj + 5 oz $H_2O$ 3 oz/8 TOTAL<br>Luis 0.42<br>Martin 0.38 |
| | Method 2. PR-UR | not sure this helps. <br>$\frac{5}{12} = \frac{x}{100}$ 42% oj<br>$\frac{3}{8} = \frac{x}{100}$ 38% oj |
| Jeffery's work | **Method 1. PR-UR** | Luis ↓ $\frac{5\text{ oz OJ}}{5+7\text{ oz total volume}} = \frac{5\text{ OJ}}{12\text{ total volume}}$ = 0.42 Stronger<br>Martin $\frac{3\text{ oz OJ}}{3+5\text{ total volume}}$ $\frac{3\text{ OJ}}{8\text{ total volume}}$ = 0.38 |
| | Method 2. PR-UR | To find how much concentrate to water ratio each had<br>$\frac{5\text{ oz OJ}}{7\text{ oz water}}$ = 0.71 Luise has 0.71 oz of concentrate for every ounce of water. Stronger<br>$\frac{3\text{ oz OJ}}{5\text{ oz water}}$ = 0.60 Martin has 0.60 oz of conc for every ounce of water |
| Stella's work | **Method 1. PR-UR** | <br>5 oz + 7 oz = $\frac{5\text{ oz}}{12\text{ oz}}$ = 0.4<br>3 oz + 5 oz = $\frac{3}{8\text{ oz}}$ = 0.375 = 0.4<br>According to sig figs they are the same concentration |

| | | |
|---|---|---|
| | Method 2.<br>PR-UR | $\frac{5}{7} = .7$ more concentrated would be Luis's mixture<br>$\frac{3}{5} = .6$ |

*Note.* Dimensional analysis (DA); Dimensional analysis T-chart equation (DA-T-chart); Dimensional analysis one chain equation (DA-Chain); Unit-rate (PR-UR); Factor-of-change (PR-FC); Fractions (PR-F); Cross-product algorithm (PR-CP)
* Incorrect method
**Bold**: Preferred answer

## Appendix G. Participants' problem-solving strategies for stoichiometry problem

| | | |
|---|---|---|
| Annie's work | **Method 1. DA-Chain** |  |
| | Method 2. DA PR-FC DA |  |
| Tina's work | **Method 1. DA-Chain & DA-T Chart** |  |
| | Method 2. DA-Chain |  |
| Levy's work | Method 1. DA DA DA |  |
| | **Method 2. DA-Chain & DA-T Chart** |  |
| Jack's work | **Method 1. PR-CP PR-F PR-UR** |  |
| | Method 2. DA-Chain |  |

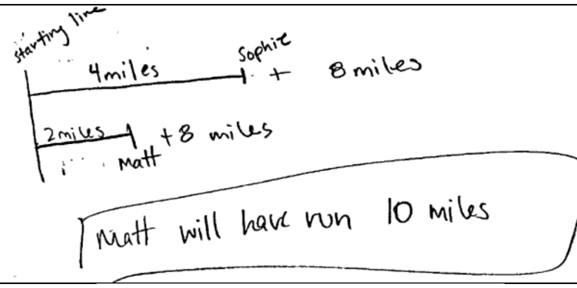
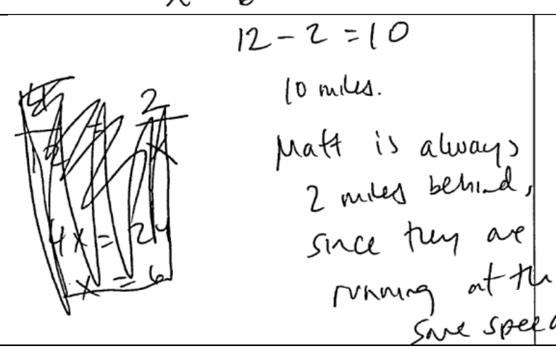
| | | |
|---|---|---|
| Jessica's work | Method 1.<br>DA<br>DA<br>DA |  |
| | **Method 2.**<br>**DA**<br>**PR-FC**<br>**DA** |  |
| Jeffery's work | Method 1.<br>DA-Chain |  |
| | **Method 2.**<br>**DA**<br>**PR-CP**<br>**DA** |  |
| Stella's work | Method 1.<br>DA-Chain |  |
| | **Method 2.**<br>**DA**<br>**DA**<br>**DA** |  |

*Note.* Dimensional analysis (DA); Dimensional analysis T-chart equation (DA-T-chart); Dimensional analysis one chain equation (DA-Chain); Unit-rate (PR-UR); Factor-of-change (PR-FC); Fractions (PR-F); Cross-product algorithm (PR-CP)

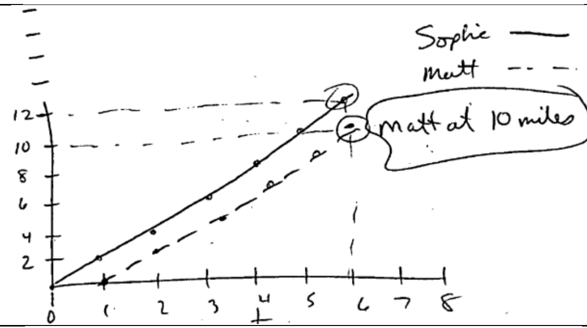
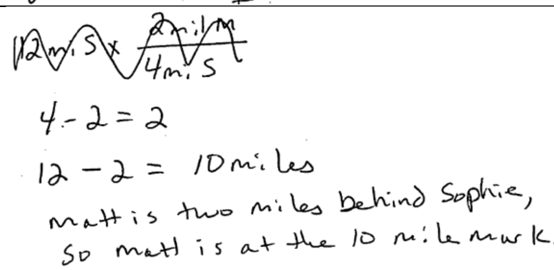
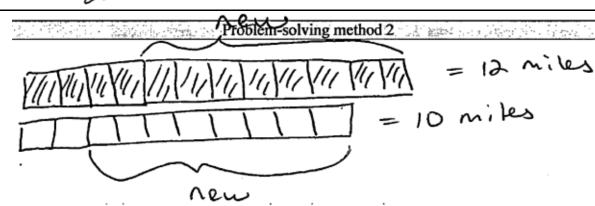
* Incorrect answer

**Bold**: Preferred method for teaching

## Appendix H. Participants' problem-solving strategies for non-proportion problem

| | | |
|---|---|---|
| Annie's work | Method 1. |  |
| | Method 2. | $S_d = M_d + 2$ miles<br>12 miles $= M_d + 2$ miles<br>$M_d = 10$ miles |
| Tina's work | Method 1.* | $\frac{4 \text{ miles Sophie}}{2 \text{ miles Matt}} = \frac{12 \text{ miles Sophie}}{X \text{ Matt}} =$<br>$\frac{4x}{4} = \frac{24}{4}$ mile<br>$x = 6$ miles |
| | Method 2.* | 4 miles Sophie \| X miles Matt<br>12 miles Sophies \| 2 mile Matt<br>$\frac{4x}{4} = \frac{24}{4}$<br>$x = 6$ |
| Levy's work | Method 1. |  |

| | | |
|---|---|---|
| | Method 2. | assume 6 miles / hour<br>Sophie ran for 2 hours<br>$12 \text{ miles} \times \frac{hr}{6 \text{ miles}} = 2 \text{ hours}$<br>$2/3$ hour → $\frac{1}{3}$ hour<br>Matt has only run for $1\,2/3$ hours<br>$1.66 \text{ hrs} \times \frac{6 \text{ miles}}{1 \text{ hr}} = 10$ miles! |
| Jack's work | Method 1. | $4 - 2 = 2$<br>$12 - x = 2$<br>$x = 10$ miles |
| | Method 2. | Say they ran at 8 mph<br>$\frac{1}{2}$ hr = Sophie ran 4 miles<br>Matt started 0.25 hr later<br>12 miles at 8 mph<br>= 1.5 hrs<br>$1.5 - 0.25 = 1.25 \times 8 = 10$ miles |
| Jessica's work | Method 1. | $t_0$ $\frac{4 \text{ mile}}{t}$ → 12 miles is $3t$.<br>$V_s = V_m$<br>$d_s$ = 4 miles<br>$d_m$ = 2 miles<br>$V_m = V_s = \frac{d}{t}$ $V_s = \frac{4}{t_0 + x} = \frac{2}{}$<br>2 miles $= \frac{1}{2} t_1$<br>$3t_1 - \frac{1}{2} t_1$<br>$2\frac{1}{2} t_1$<br>$(2.5)(4)$ = 10 miles |
| | Method 2. | 0 —4 mile ($t_1$)— 8 — 12<br>12 miles $= 3t_1$<br>will always be 2 miles behind.<br>10 miles |

| | | |
|---|---|---|
| Jeffery's work | Method 1. Verbal explanation | Sophie is 2 miles ahead so if the speeds are the same then Matt will be two miles behind Sophie always so he is at the 10 mile mark. |
| | Method 2. |  |
| Stella's work | Method 1. |  |
| | Method 2. |  |

*Note.* Dimensional analysis (DA); Dimensional analysis T-chart equation (DA-T-chart); Dimensional analysis one chain equation (DA-Chain); Unit-rate (PR-UR); Factor-of-change (PR-FC); Fractions (PR-F); Cross-product algorithm (PR-CP)
* Incorrect method