OCEAN ENGINEERING DIVISION - HENRY KRUMB
SCHOOL OF MINES Columbia University
New York, N. Y. 10027

TECHNICAL REPORT No. 1

CHANGES OF DEEP HORIZONTAL CURRENTS IN THE OCEAN

by
T. E. Pochapsky and F. D. Malone

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This work was supported by the Office of Naval Research under Contract N00014-67-A-0108-011AA, Project No. NR 083-243.

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ABSTRACT

The separations and velocities of separation of neutrally buoyant floats in pairs have been investigated in detail. Observations at three locations where the inertial periods are 18, 24, and 57 hours indicate strong relative movements around circles at those periods. Just within the Caribbean Sea, however, the motion is in a semidiurnal ellipse.

Spectrums for the velocities of separation vary above the inertial frequency so that the energy density falls approximately as f^{-3}. This decrease is consistent with the trend in the absolute velocities obtained at WHOI Mooring Site "D" (UNESCO) at frequencies well above the inertial period. Correlations exist between the motions of nearby pairs of deep floats so that the motions are not entirely random. The possible effect of these on the spectra has yet to be investigated.

INTRODUCTION

The oceans have neither a uniform density or current structure. Regions of fairly uniform density tend to be layered, onion-wise, but the density fluctuates even within these layers. The current changes with both position and time and has a magnitude which is far from ignorable in even the deepest waters. The layered structure is also continually wiggled in all directions by internal waves. Acoustic waves travelling in the horizontal direction tend to be trapped or even disallowed in those layers while they are also reflected in unpredictable ways by the continuous shifts of horizontal tilt. Variations in the density produce "patches" which scatter or diffract sound. These attributes of the oceanic structure not only affect sound propagation but they also produce and are associated with unusual diffusive transport characteristics. Contaminants, for example, do not spread from sources in readily predictable fashions. No suitable theory exists to explain these characteristics nor have enough experiments been done to get a parametric description. The present work is part of an experimental program designed to find out significant features and to determine the magnitudes of relevant quantities. Because the problem is basically one in the field of hydrodynamics, much reliance is placed on hydrodynamic theory on the assumption that the general acoustic or diffusive characteristics can be derived.

This presentation deals with some results obtained by using instrumented neutrally buoyant floats. The capabilities of these floats have been continuously developed over a period of a number of years by the senior author and experimental results have been regularly reported in the literature. Full use was not made of all the data obtained in past experiments nor was any general comparison between the results of the various experiments attempted. A general summary is not the intent of this report. It is more a presentation of the results of further analyses of data relating to the separation between pairs of floats. Nevertheless, that analysis utilizes results obtained in all past pertinent experiments. A familiarity with that past work is assumed and generalizations based on that work are made on occasion.

The following section presents the results. To minimize distraction, many of the experimental details are presented and elaborated in the succeeding Appendix. References to past publications of the senior author will utilize the initials TEP.

REVIEW OF EXPERIMENTAL RESULTS

When a neutrally buoyant float is put in the ocean, it sinks slowly until it reaches a level where the density of the float matches that of the surrounding water. The initial velocity is near 1/2 knot when the float is adjusted for deep hovering. If a second identically trimmed float is released a minute or so later, it follows a trajectory which is displaced horizontally from that of the first float by the distance that the ship has drifted between drops. That distance in the experiments to be discussed is measured in tens of meters. At a given time the delayed float will also be 10 or more meters shallower until after the floats settle at the same equilibrium depth. When the spatial current structure is stationary in time, both trajectories should be identically shaped and hovering floats should remain separated by the same horizontal distance as existed at the surface.

Float experiments performed over a number of years at various locations have revealed a number of features in common which will be reviewed before presenting experimental details in the Appendix. Further details will be found in past publications by the senior author listed in the bibliography (TEP, 1963 through 1970).

Two floats placed in the deep sea a few minutes apart tend to remain the same distance apart during sinking. Although relative velocities of the order of 1 cm $\sec^{-1}$ occur, these fluctuate in direction so as to tend to nullify total relative displacements while sinking. More generally, the relatively small changes in separation imply a uniform rate of sinking and a current structure which is approximately stationary for the short time interval between drops. Stationarity does not exclude the possible existence of intense gradients of the current.

Separations and relative velocities between floats in pairs at different locations in the deep sea are illustrated in Figure 1, 2 and 3. Pertinent parameters are presented in Table I. Solid circles on the separation curves mark hours after insertion in the water. Equilibrium depth is reached at time t = 0 in Figure 3. Further details are presented in the Appendix.

Floats cannot be adjusted precisely enough to hover at identical depths. Final differences amount to at least tens of meters at depths of thousands of meters. After equilibrium depths are reached, fluctuations continue to take place in the relative velocities but occur more slowly so that the separation between floats tends to change dramatically with time. Experiments at different locations have shown this separation to increase initially to a distance of approximately a kilometer but subsequently it diminishes so as to approach the initially small value. The relative velocity associated with this motion is a few cm sec^{-1}. After a time at depth comparable to the inertia period, that throbbing motion of the separation is repeated. Additionally, there is often a slow drifting apart of the floats as well as more rapid fluctuations in the rate of separation. Each hovering experiment was conducted over a time interval of a few inertia periods. A typical result reported in the past (TEP, 1966) is shown in Figure 4 for the separation between a common "master" float and the various individual floats numbered 35, 47, 48 and 49.

Such experiments have shown that the shear currents in deep water are larger than generally anticipated, but the values remain below a cm sec^{-1} in a vertical distance of 10 meters, or the gradients remain below 10^{-3} sec^{-1}. When weakly stratified water of Vaisala period 3 hrs is subjected to such gradients, the associated Richardson number is

$$R_i = \frac{\omega_v^2}{\left(\frac{du}{dz}\right)^2} = \left(\frac{6\times10^{-4}}{10^{-3}}\right)^2 = 0.36$$

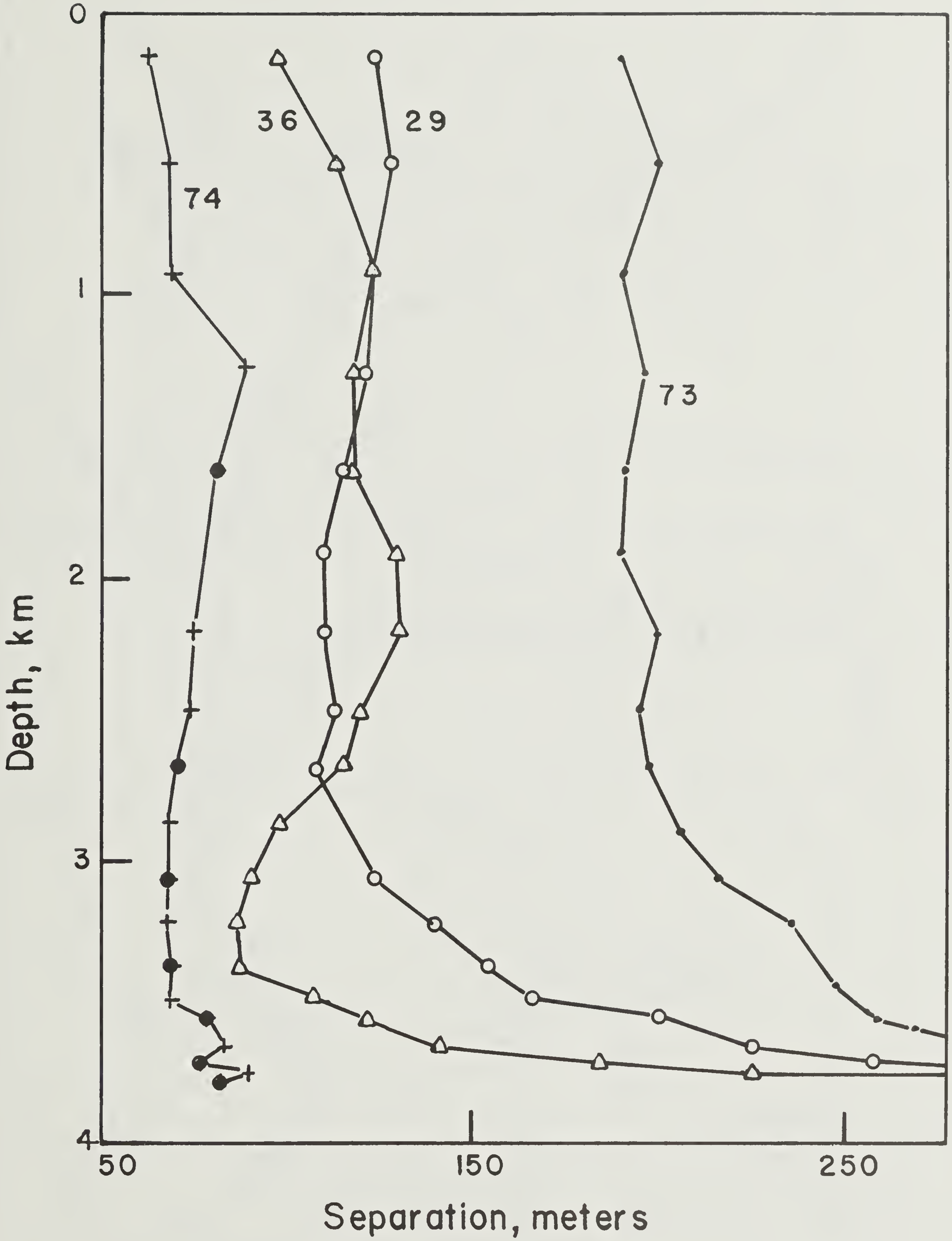

Figure 1. Radial separation between a master float and floats 29, 36, 73 and 74 during sinking at 12°N, 27°W.

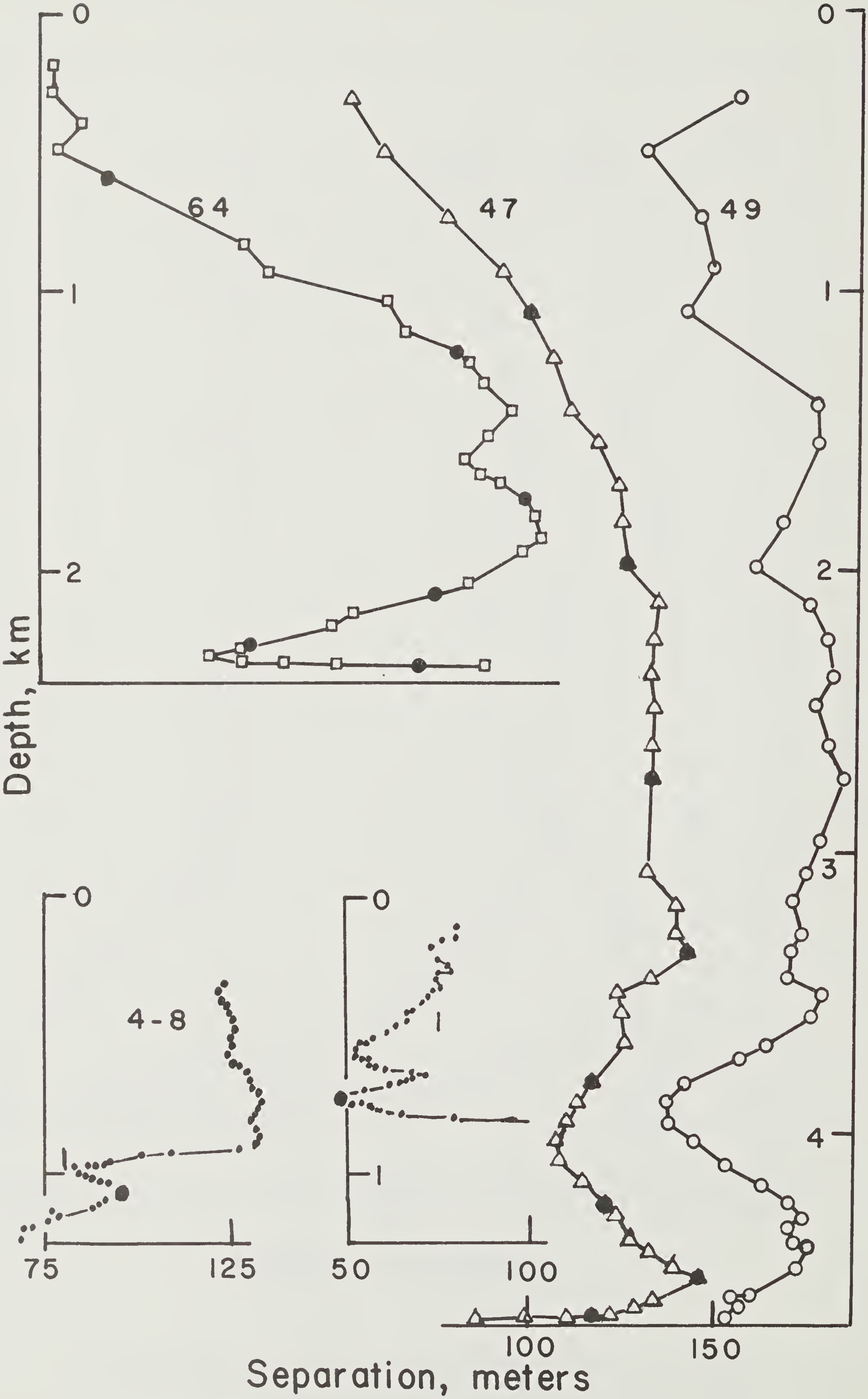

Fig. 2. Radial separation between sinking floats in pairs at various locations noted in Table I.

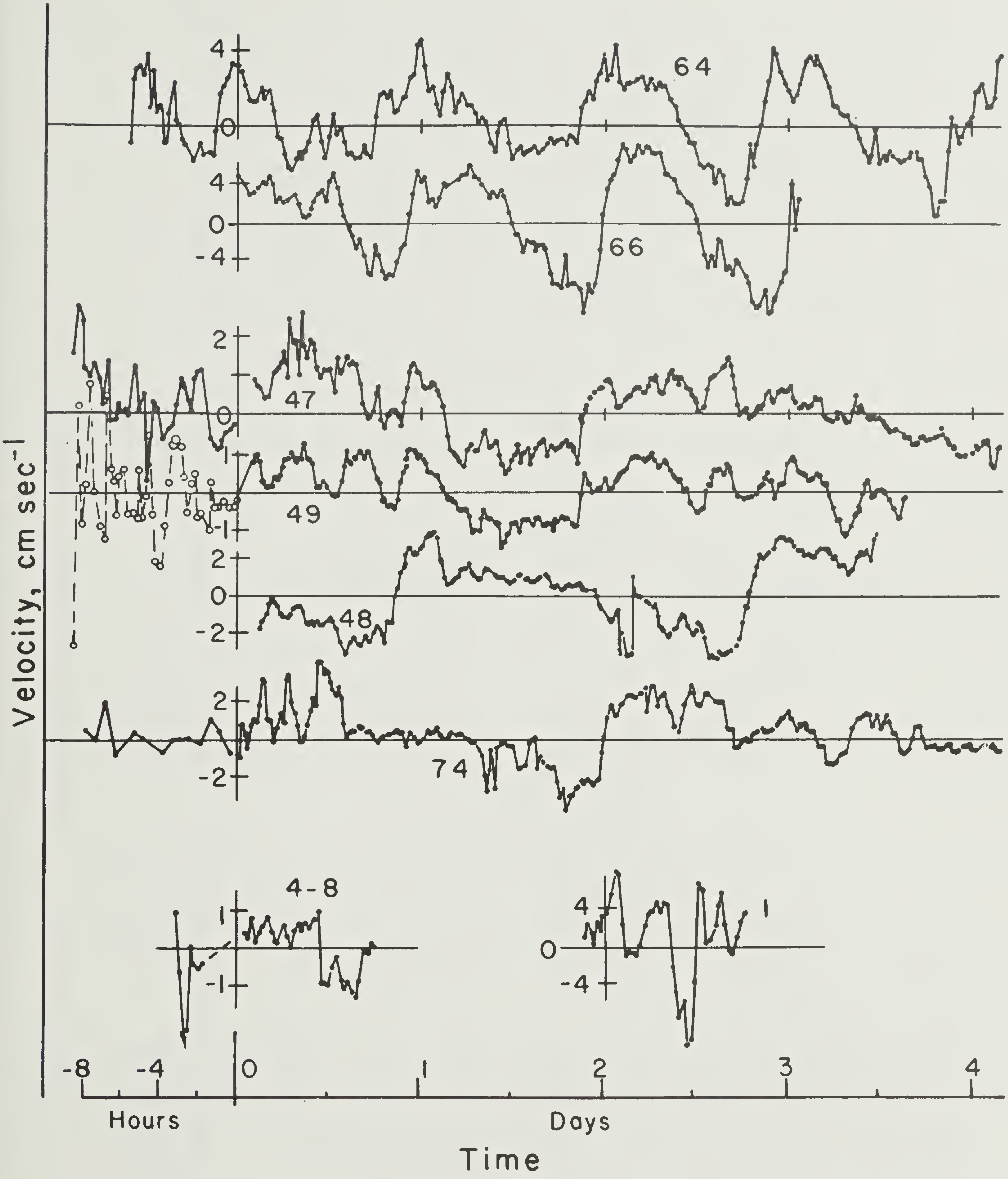

Fig. 3. Velocities between floats in pairs at various locations during sinking and after reaching equilibrium depth at t = 0.

Table I. Experimental Parameters

Float pair slave number	Location	Inertial period, hrs	Depth, meters	Depth difference, meters to master	True mean velocity of master float, cm sec^{-1}	Mean radial relative velocity between pair cm sec^{-1}
1	18° 16′ N, 64° 23′ W	38.2	840	(100)	(3)	0.
73	41° 43′ N, 65° 24′ W	18.0	274	- 46	-	2.22
64	27° 56′ N, 55° 22′ W	25.6	2265	75	2	0.33
66	"	"	2480	- 140	"	0.23
74	12° N, 27° W	57.6	3880	- 80	2	0.28
47	"	"	4530	150	1	0.11
48	"	"	4430	250	"	0.42
49	"	"	4530	150	"	0.11

	Orbit diameter, km	Orbital speed, cm sec^{-1}	Sample interval, minutes	Number of data points in analysis	Stability period, f_v^{-1}, hrs	Float vertical amplitude above f_v	Separation above f_v, m, rms	Separation velocity above f_v cm sec^{-1} rms
1	0.38/0.65	3.7	2	527	0.44	(2)	0.8	0.7
73	2.2	10.7	5	498	0.50	2	3.2	1.7
64	0.82	2.8	10	623	1.92	2.5	3.9	0.63
66	2.23/2.53	8.2	"	443	"	"	5.4	1.24
74	1.0	1.5	20	317	3.10	0.2	4.5	0.45
47	0.53	.33	"	310	3.40	"	2.2	0.20
48	1.64	3.2	"	242	"	"	4.4	0.35
49	0.38	.17	"	256	"	"	2.1	0.13

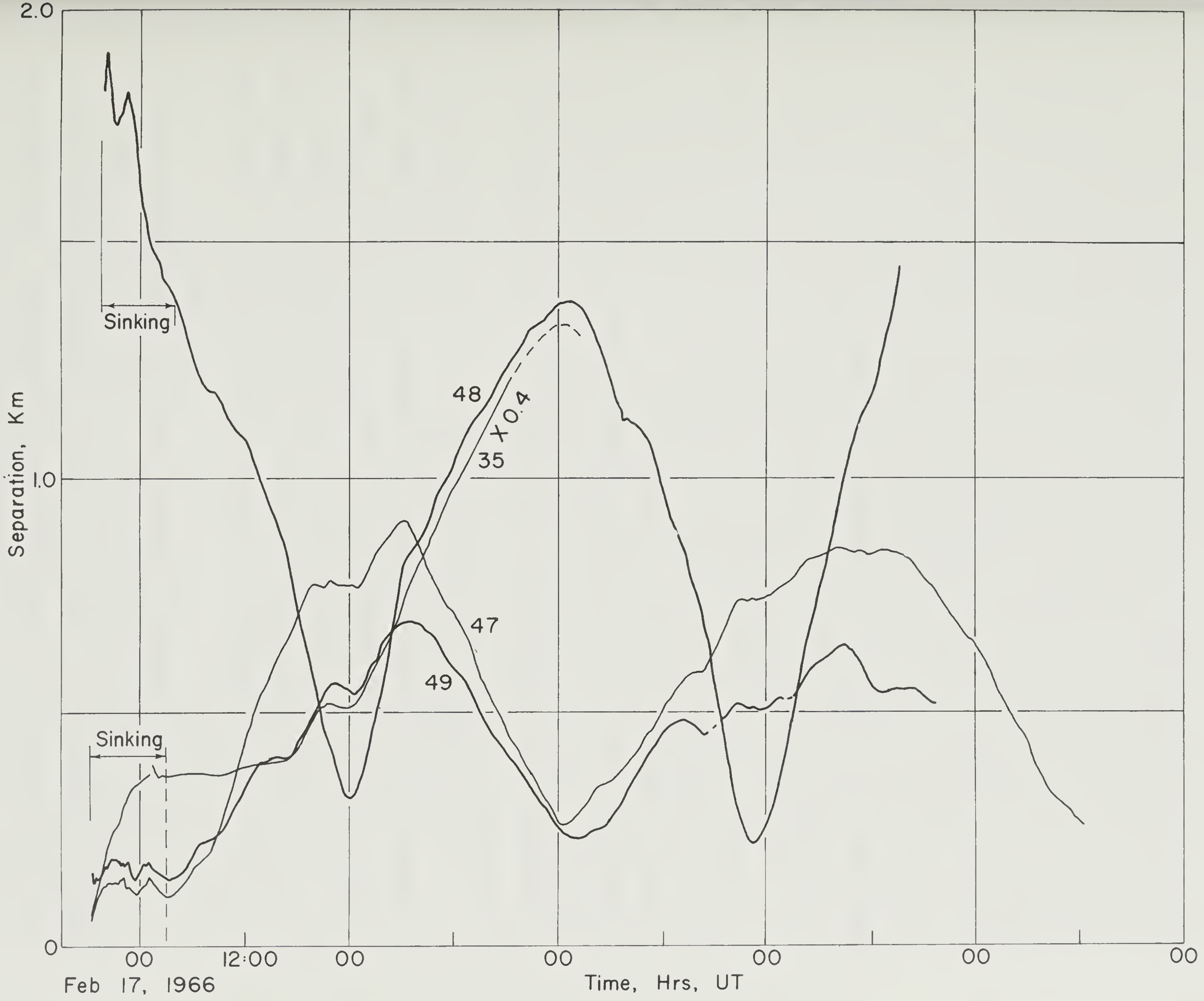

Fig. 4. Separation between floats of a cluster at 12°N, 27°W, 4500m.

where ω_v is the angular stability frequency and du/dz is the vertical gradient of the current. This value of the Richardson number is a substantial underestimate and can be approached only in regions of unusual temporary activity. Our experiments have given no evidence of values of R_i less than unity and so for any predicted presence of instabilities.

Because floats which differ in depth by less than 100 meters can separate a distance of the order of a kilometer, it might be surmized that floats a kilometer apart in depth would have a throbbing separation measured in tens of kilometers. Otherwise expressed, some individual floats would be expected to have a large horizontal movement relative to the earth. Such has not been observed and, surprisingly, the fluctuations in absolute position compare in magnitude with the changes in separation between floats of a pair which differ only slightly in depth. Consequently, horizontal velocity fluctuations must alternate in some fashion with depth. A sinusoidal variation in the vertical direction with a "wavelength" of approximately 400m would be one way of explaining such results.

These findings of a large motion near the inertia frequency and of the possibility of a large vertical "mode number" are consistent with an internal wave model suggested by W. Munk and N. Phillips (1968) and used by them in conjunction with current meter measurements. Those measurements by Webster (1968), however, suggest a degree of intermittency and incoherence of the wave field which makes application of the theory difficult.

The radial separation between floats in a pair changes with time as though one of the floats moves largely in a circle relative to the other. Suitable orbiting times were found to be the same as the local inertial periods of 57, 24 and 18 hrs at three locations. Just within the Caribbean, however, the relative motion followed an ellipse. Drawings illustrating such relative movements are presented in Figures 5 through 11. Figure 9 is reproduced from an earlier report and has its own coding (TEP, 1966). Experimental parameters can be found in Table 1. One float is assumed to orbit the circular path every inertial period. The first orbiting is denoted by triangles, the second by circles and the third by crosses. The second float, labelled M, is assumed to move at a constant speed relative to the circle along paths as illustrated. Both floats of a pair may have some horizontal separation at the beginning, t = 0. The extension or contraction of the hypothetical separation needed to obtain the observed separation is marked at regular time intervals. In many cases, there is impressively little difference between the observed and anticipated separations. Other frequencies, however, contribute significantly and the results obtained at the location off Bermuda, Figure 9, suggest that the circle radius changes appreciably during successive inertial periods. The first two minima in the separations at Cape Verde were approximately 48 hrs apart but when a small constant drift was added in a suitable direction to the M float it was possible to obtain consistency with a 57 hr inertial period. This drift correction has a strong "fudge-factor" influence when only a few inertial rotations are involved. Nevertheless, the procedure cannot bring into accordance motions that are excessively at variance with that hypothesized. As an example, an attempt was made to fit a linear rather than circular sinusoidal relative motion of 20 hr period along with a steady drift of 3m hr^{-1} to a circle of the same period. The results, Figure 12, are not particularly good.

The spectrums for the separation velocities at four locations are presented in Figure 13. Within the confidence limits they can be characterised as decreasing with some power of the frequency, f, except for a tendency to remain steady or to rise above the Vaisala frequency

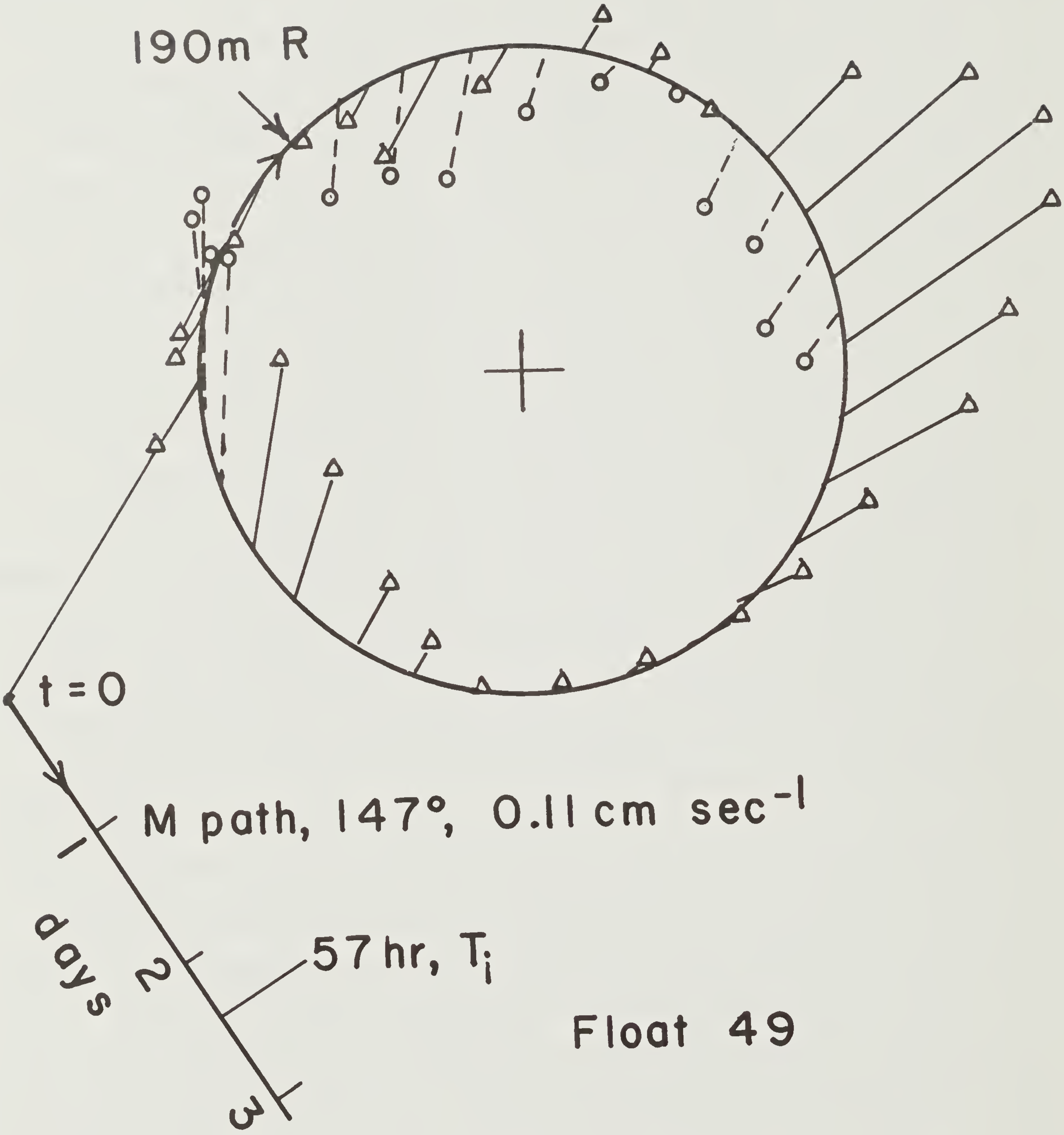

Fig. 5. Relative movements in float pair at 12°N, 27°W, 4500m.

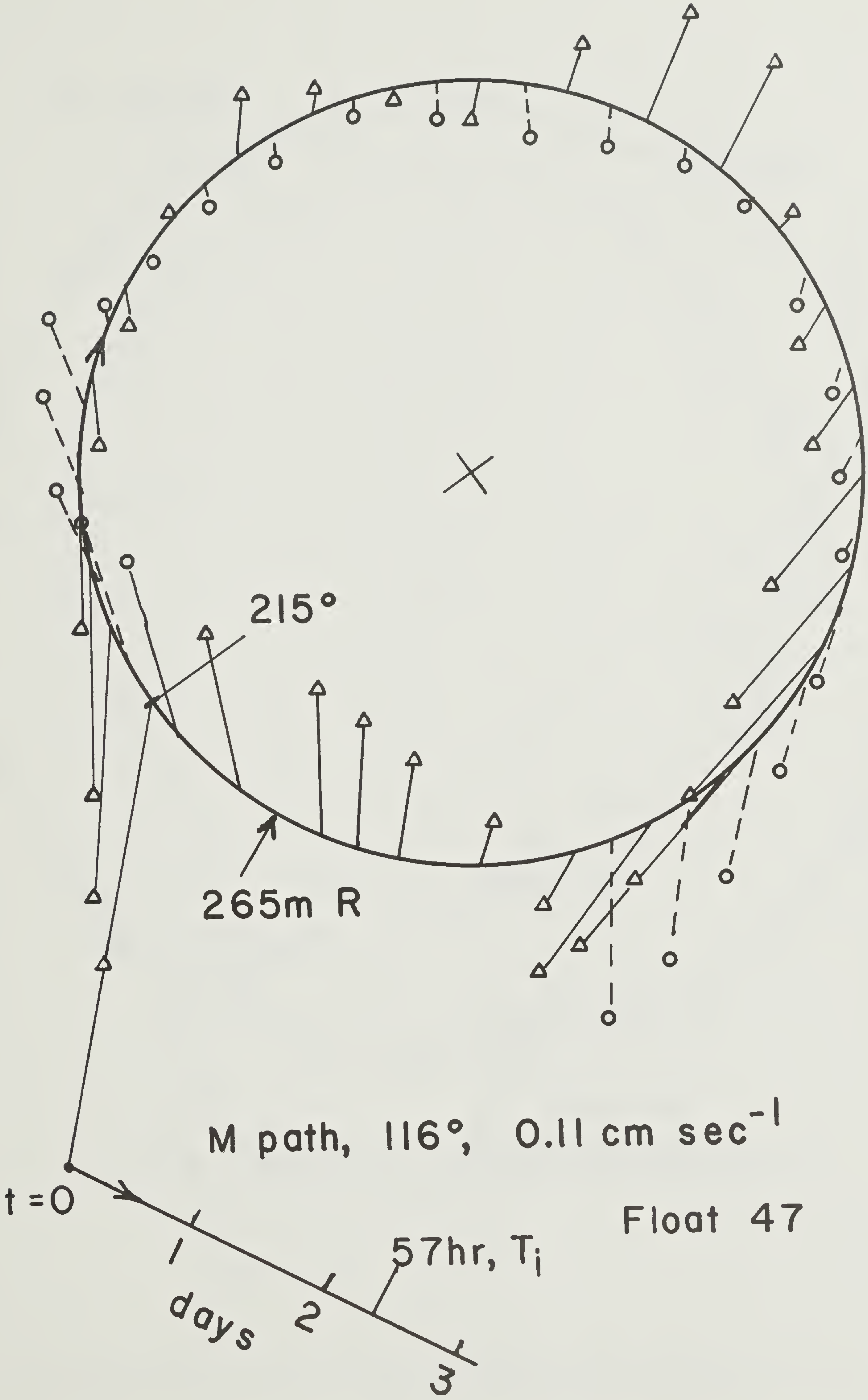

Fig. 6. Relative movements in float pair at 12°N, 27°W, 4500m.

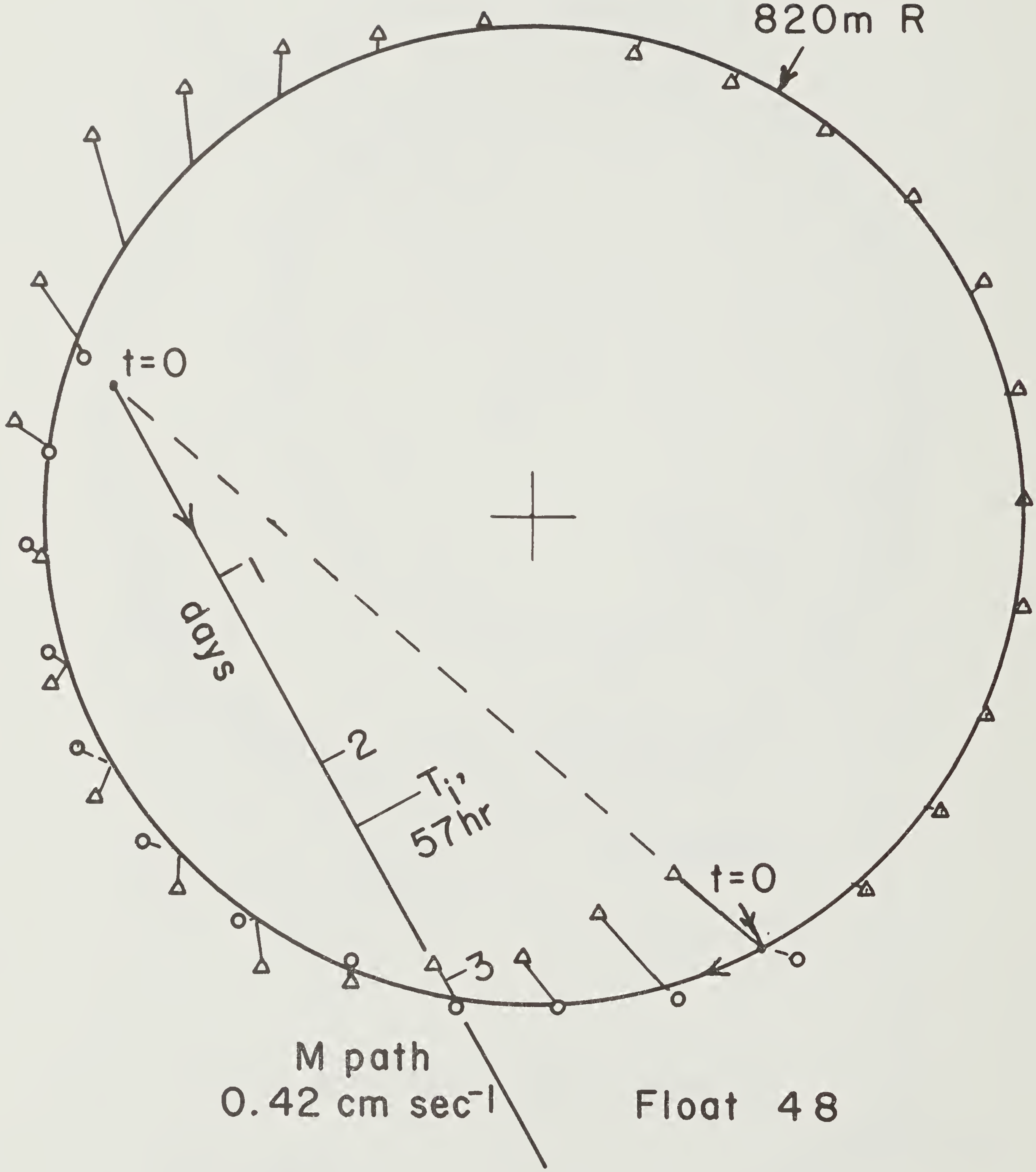

Fig. 7. Relative movements in float pair at 12°N, 27°W, 4500m.

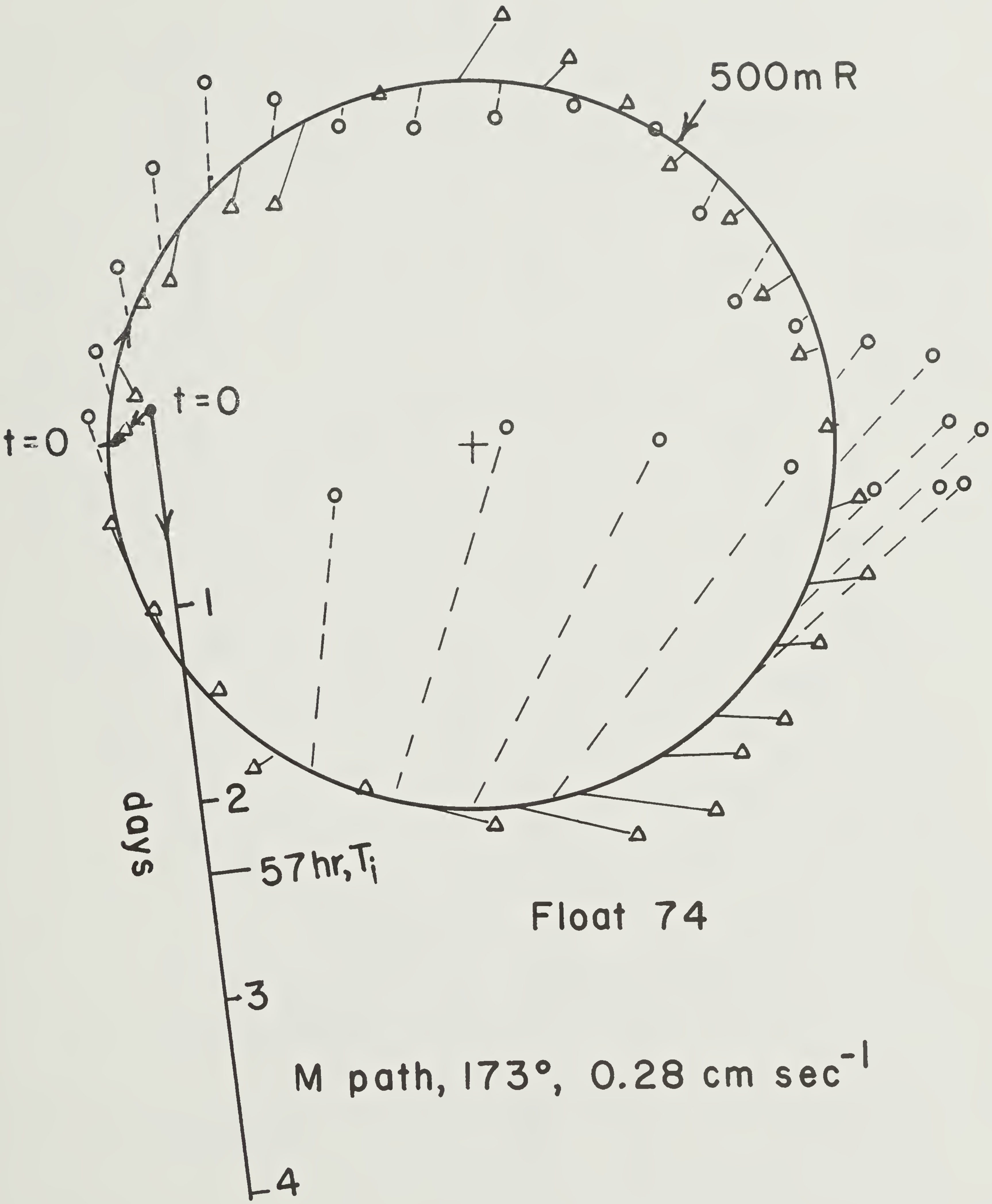

Fig. 8. Relative movements in float pair at 12° N, 27° W, 3800m.

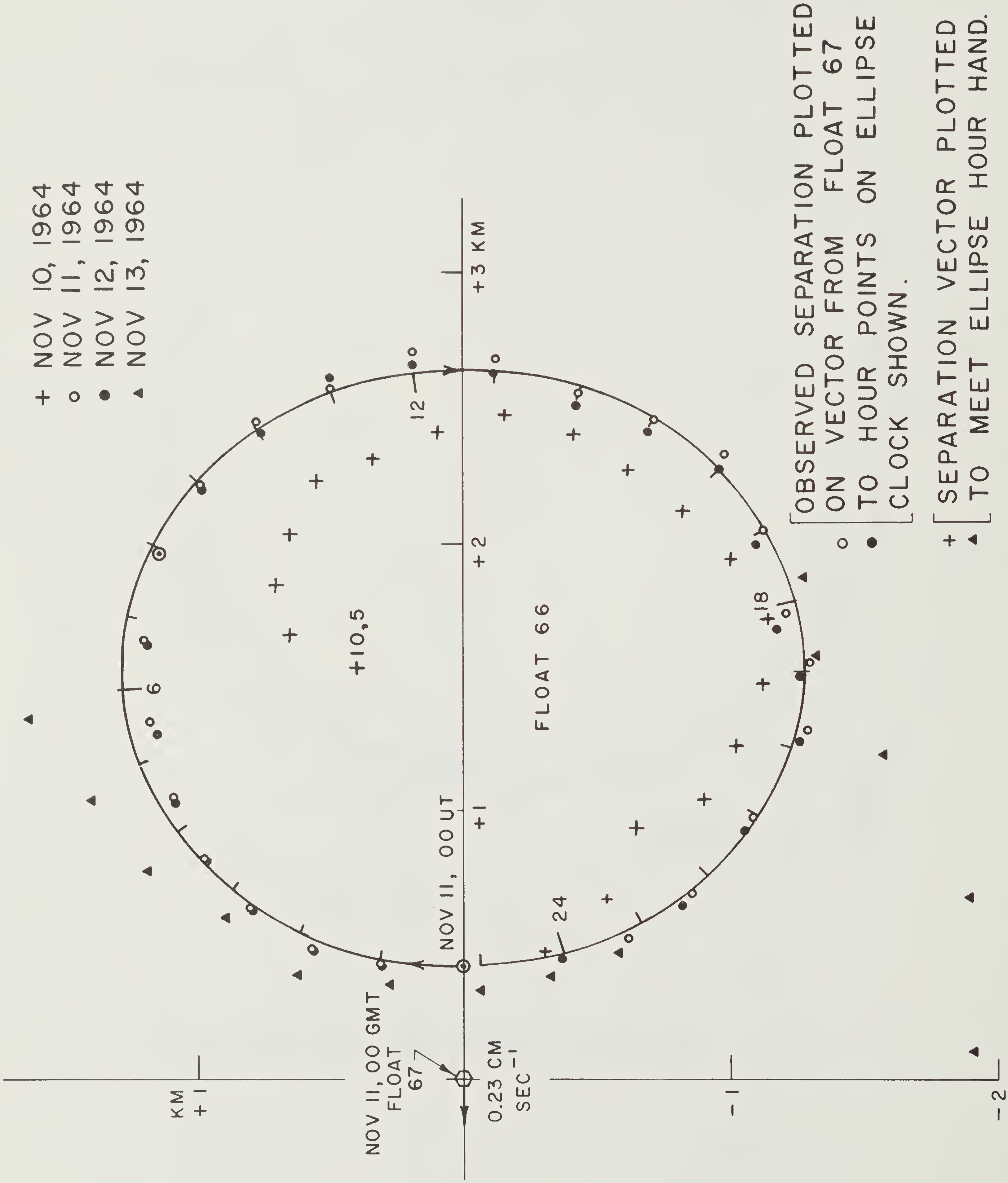

Fig. 9. Relative movements in float pair at 28°N, 55°W, 2500m.

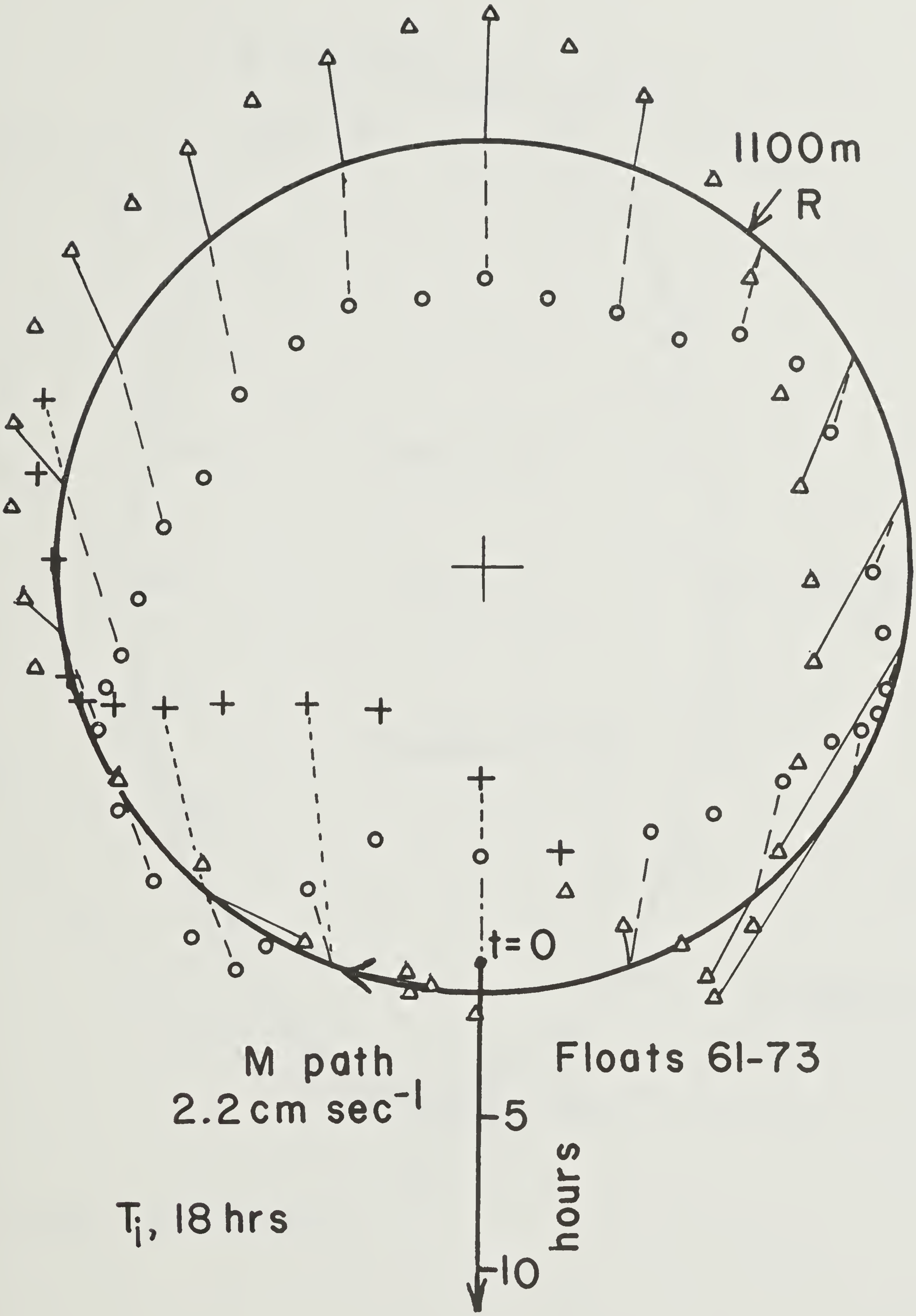

Fig. 10. Relative movements in float pair at 42°N, 65°W, 274m.

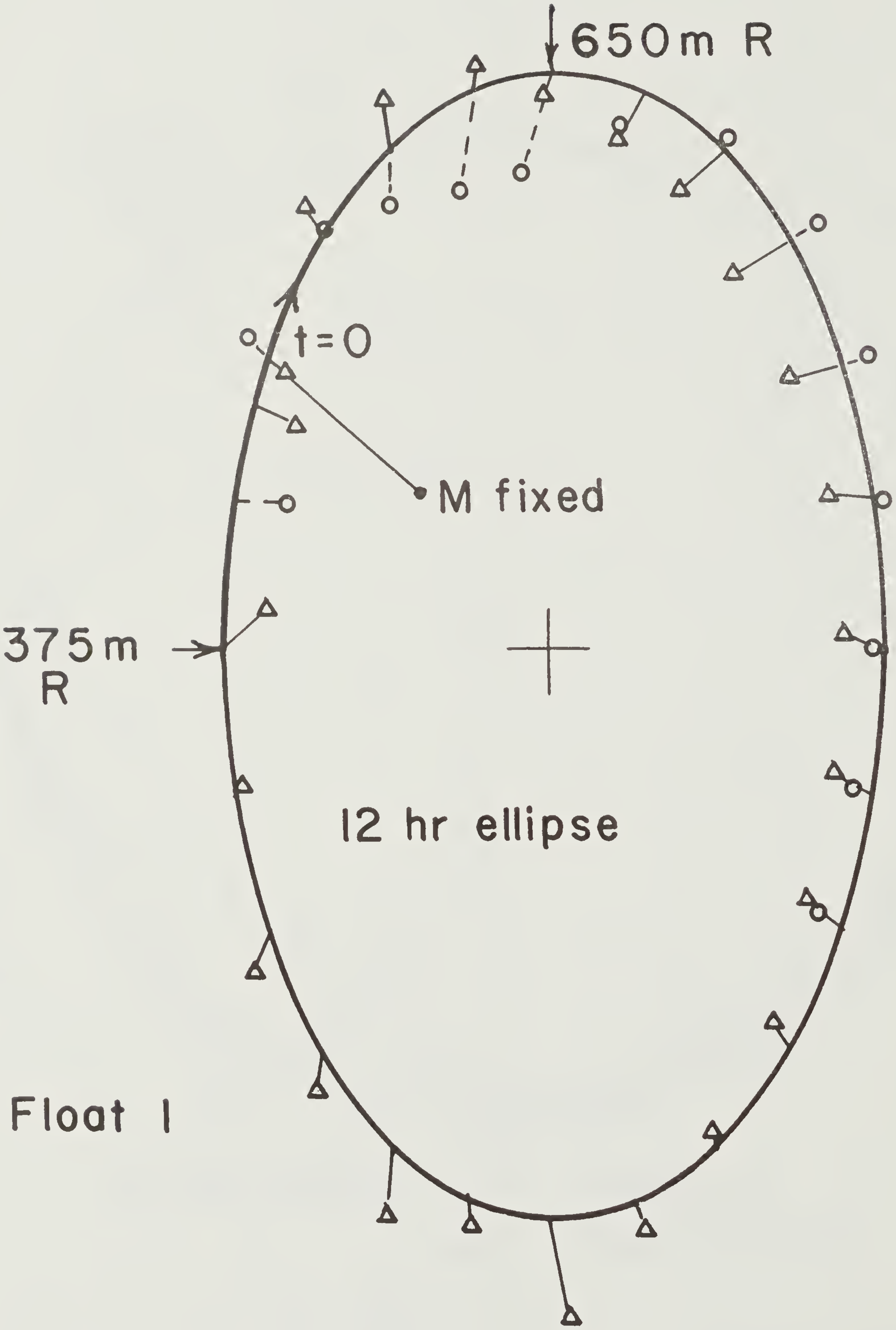

Fig. 11. Relative movements in float pair at 18°N, 64°W, 840m.

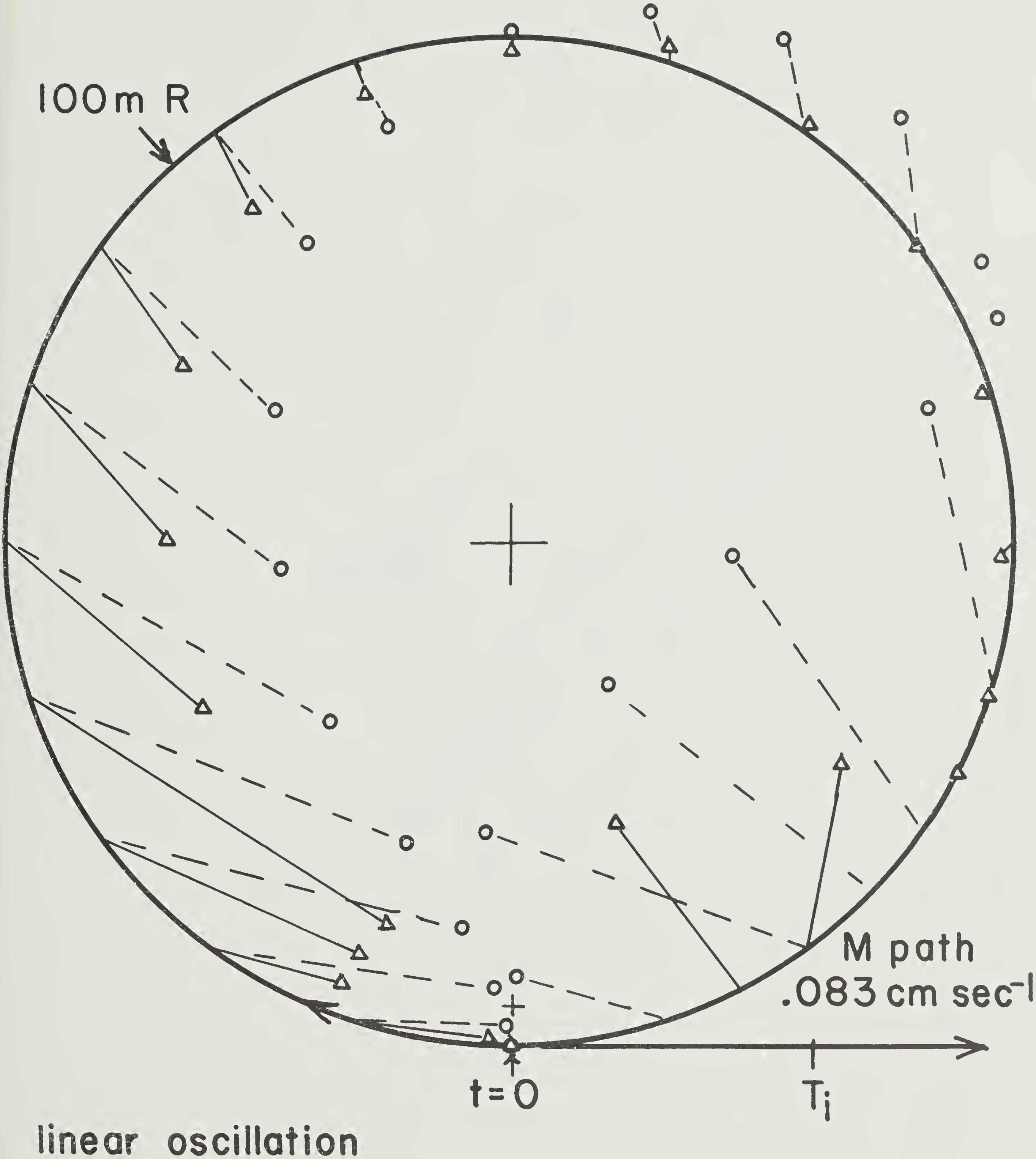

Fig. 12. Relative movements in hypothetical float pair. Linear oscillation compared to an assumed circular motion.

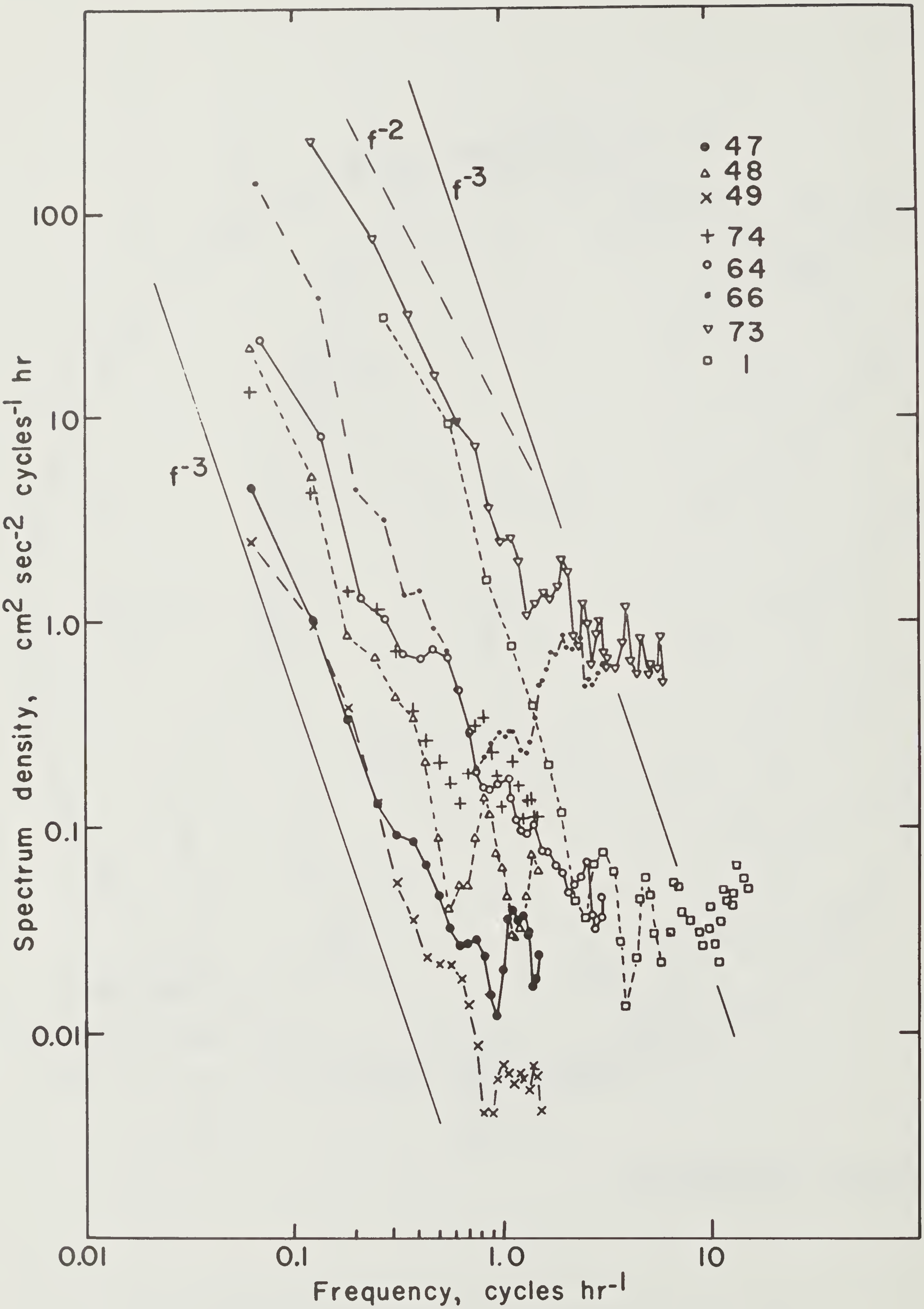

Fig. 13. Spectrums for velocities between floats in pairs at various locations.

at the Bermuda and Gulf of Maine locations. Within a cluster, the frequency dependence changes from a slope near f^{-2} to one closer to f^{-3} as either the depth difference or the maximum separation increases in a given cluster. The relatively shallow Virgin Gorda results at 18°N, 64°W have an f^{-3} dependence.

Only the rise in the Virgin Gorda spectrum at the highest frequencies can be attributed to contributions from random least count instrumental noise. Otherwise, additions can come from vertical movements of the floats and from errors resulting from gaps in the data. Analyses of short sections of particularly "clean" data over time intervals when the floats were close together or far apart suggested that the first of these factors was important. Nevertheless, the gaps in the data which resulted when ship movements were necessary and which were filled by interpolation probably were responsible for most of the energy at the highest frequencies. The extent of the horizontal motions above the Vaisala frequency is tabulated in Table I as determined from these spectra. This work shows the practicality of using floats to investigate both the horizontal and vertical motions at frequencies near and above the Vaisala frequency in the deep ocean in order to determine features of vertical transport. Less contamination at the highest frequencies is easily obtained.

The few spectrum points at the lowest frequencies have values which can be attributed to a sharp spectrum line at the inertia frequency which has an amplitude determined from the relative inertia circle described earlier and which is passed through the "hanning" filter of the spectrum analysis. When separations to this hypothetical circle or ellipse are removed from the separations actually observed, the resulting velocity spectrum for the new values of separation differs from the original chiefly in that the spectrum stops rising at the lowest frequencies - insofar as such a statement can apply to the one or two points involved. Densities at the higher frequencies were practically unaffected.

The ratio of energy densities a few hundred meters below the surface near the Gulf of Maine and those 4-1/2 km down in quiet ocean is a factor of somewhat over 100. Velocities then differ by a factor of a little over 10. The "95 percent confidence limits" for a particular estimate lie in the region from 0.6 to almost twice the value of the esimate.

Spectrums for absolute currents determined by means of Savonius rotors have been published by Webster (1963, 1968; see also UNESCO Tech Paper). Some of these, obtained from his published graphs and smoothed somewhat, are compared with some of the results for relative currents in Figure 14. The long duration of Webster's individual experiments allows a higher resolution and yields spectrum densities at lower frequencies than were obtained with the floats. The extent of the general agreement between the two different approaches is surprising although there are some important differences. The absolute energy densities at a depth near 500m for the WHOI Mooring Site D, 39°20'N, 70°W, plotted as W4, compare with those for relative values obtained with floats at depths of 2265m and 2480m at 28°N, 55°W. The spectrum slopes are also similar.

Savonius rotors have not been calibrated for their response at high frequencies and there is some question as to the extent of the Doppler shifting of energy to the higher frequencies by the low frequency flow. The comparisons in Figure 14 suggest that the rotors may indeed yield accurate results at those frequencies. There are differences, however, and a convincing comparison could best be made by operating floats

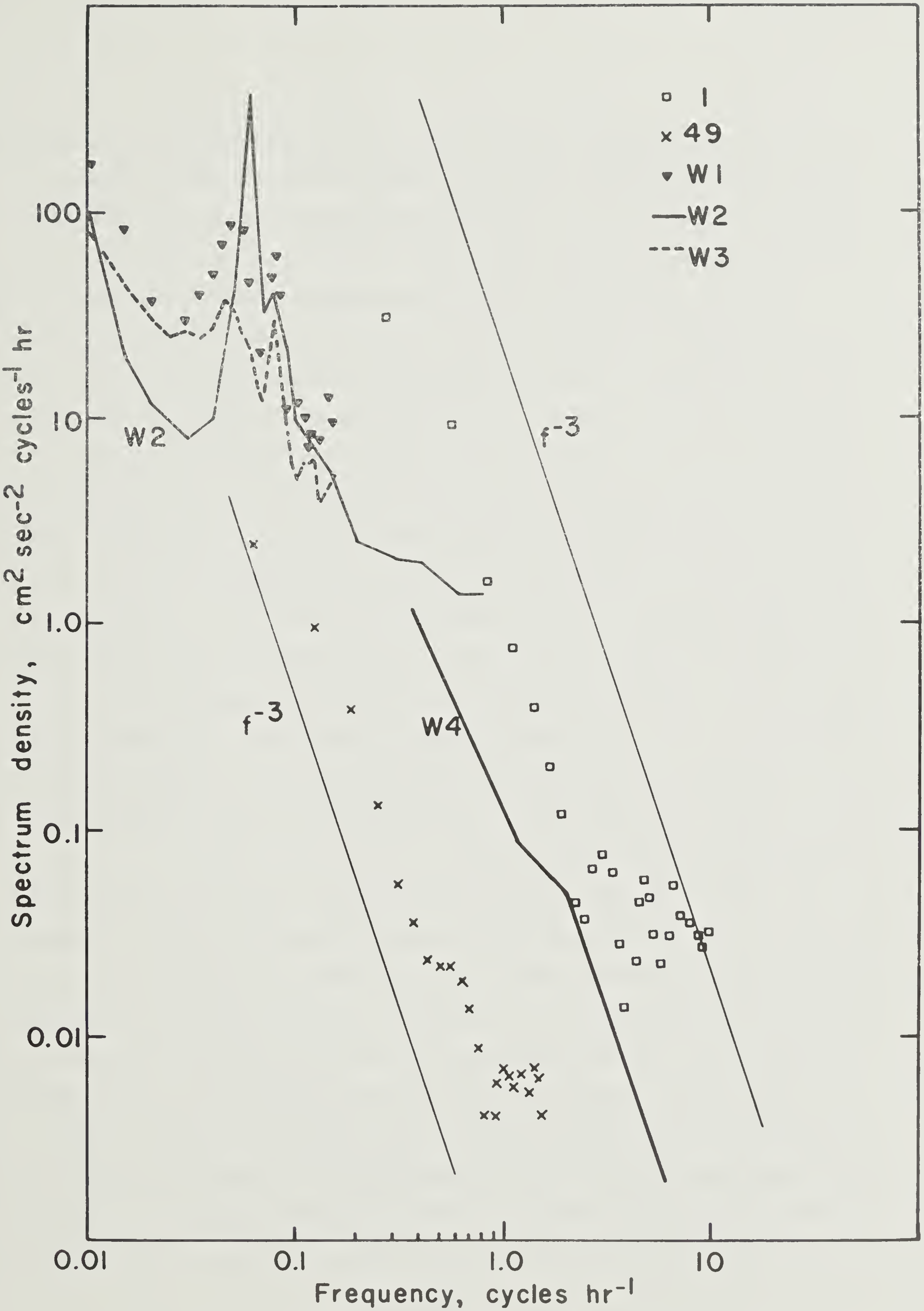

Fig. 14. Spectrums obtained at current meters compared to those obtained with float pairs, W1, 39°30'N, 70°W, 500m (Webster, 1963); W2, 38°30'N, 70°W, 3300m (Webster, 1968); W3, 34°N, 66°W, 3000m (Webster 1963); W4, 39°20'N, 70°W, 480m (UNESCO).

and rotors in a combined experiment. At these high frequencies, motions between floats are probably uncorrelated and the spectrum densities represent twice the values for the unidirectional motion of a single float. They should then be comparable to the total energy as are Webster's.

It is pertinent to estimate the wavelengths expected for the internal waves which probably dominate the motions at the lower frequencies. A non-rotating ocean of depth D and constant Vaisala frequency N can propagate waves of frequency $n = N \cos\theta$ where θ is the angle of the propagation vector to the horizontal. The upper and lower boundaries of the ocean limit allowable vertical wave numbers to m/D and so horizontal wave numbers are measured by $\pi m/(D\tan\theta)$ where m is the vertical mode number. At the frequency N/2 that wave length is 14 km for m = 1 in a 4 km deep ocean. This length varies as $n/\sqrt{N^2 - n^2}$ so that a strong correlation is expected in the horizontal low frequency motions of floats separated by a distance of 1 km. An analysis of propagation in a more realistic ocean has been made by Ekart (1960, p 197). His diagram of frequency versus horizontal wave number shows, for example, that a mode number of a little over 10 is required for a wave of 4 hr period to have a wavelength as short as 2 km.

When the mode number is not too large, floats are separated by fractional wavelengths and will have relative velocities that are corresponding fractions of the maximum water velocity amplitude for a given wavelength. In a frequency range not too close to either the stability or inertial periods, the observed approximate n^{-3} slope for the relative velocities then implies an n^{-5} dependence for the total velocity, a result completely at variance with the Savonius rotor data. It is then likely that large mode numbers are involved along with a large number of randomly distributed sources so that wave lengths exceed the average float separations only in the low frequency part of the spectrum. The experimental evidence favors such an explanation in that the far floats of a given cluster have the steepest slopes.

The internal wave field in the ocean is usually considered to be made up of a pair of waves at each frequency. One wave is inclined upward and the other downward so as to form a vertical standing wave pattern which advances in the horizontal direction. Phase differences between floats within a pair would increase continuously with an increase in the average horizontal separation. Changes in the vertical direction would take place in steps of π. Two pairs with a common master float and near the same depth would have separations which are almost in phase when the horizontal wavelengths are appreciably larger than the pair separation. The velocity of separation would be larger for those pairs whose floats are farther apart. As the frequency is increased and the stability frequency approached, the horizontal wavelength decreases and should approach zero length. Near and above the stability frequency the phasing should start to vary erratically and there would be little in common between the motions in each pair. At low frequencies, depending on the initial configurations, a high correlation and a constant phase difference would be expected between pairs. This accord would be lost at frequencies above, say, $N/\sqrt{2}$.

Any analysis of two independent series of random data will show some correlation between them. Furthermore, any analysis of two series of oceanographic data taken at completely different times would probably show strong correlations caused by the tides and presumably by inertia currents. In our averages, there is a 95% confidence that correlations above 0.45 are significantly a result of non-randomness, but a value of 0.64 must be exceeded because pairs have a float in common. Phase angles for higher correlations achieve the same confidence in the range of angles of $\pi/10$, or less, depending on the extent of the correlation. A demonstration of the correlation and phase angles between independent series of data obtained at sea with floats is given in the lower half of Figure 15. These results apply to the three unreal pairs formed by three floats in very deep water and a common single float at the same station but 600m shallower and observed 6 days later. The results are as might be expected for random conditions.

Corresponding results for floats forming pairs within the same cluster are shown at the top of Figure 15 and in Figure 16. In the former case, significant correlations exist only at the lower frequencies for the two pairs inserted together. In the third pair, float 48 was dropped in later and it shows no significant velocity correlation with the other two. The two pairs in the location off Bermuda, Figure 16, show some significant correlation even though float 66 carried out excursions over a range from 1/2 to 2-1/2 km from its mate.

Below the stability frequency, there tends to be no phase difference between pairs in a cluster. The out of phase condition for float 48 at the lowest frequency is as expected from the results shown in Figure 4. This isolated result may be related to the circular paths of inertial motions. Phasing becomes erratic only at periods shorter than 1.3 hrs and so at times shorter than the calculated stability periods, Table I, suggest should be the case.

The small correlations and close phasings present are at variance with the independence assumed earlier on the basis of the Savonius meter data. They suggest that the relative motions may not have the same spectra as absolute velocities obtained at a fixed point. With sufficient correlation, the latter would fall faster than f^{-3} in a range between, but excluding, the Vaisala and inertial frequencies. Further measurements are needed to show whether the existing correlations are large enough to have an important influence on the spectral slope.

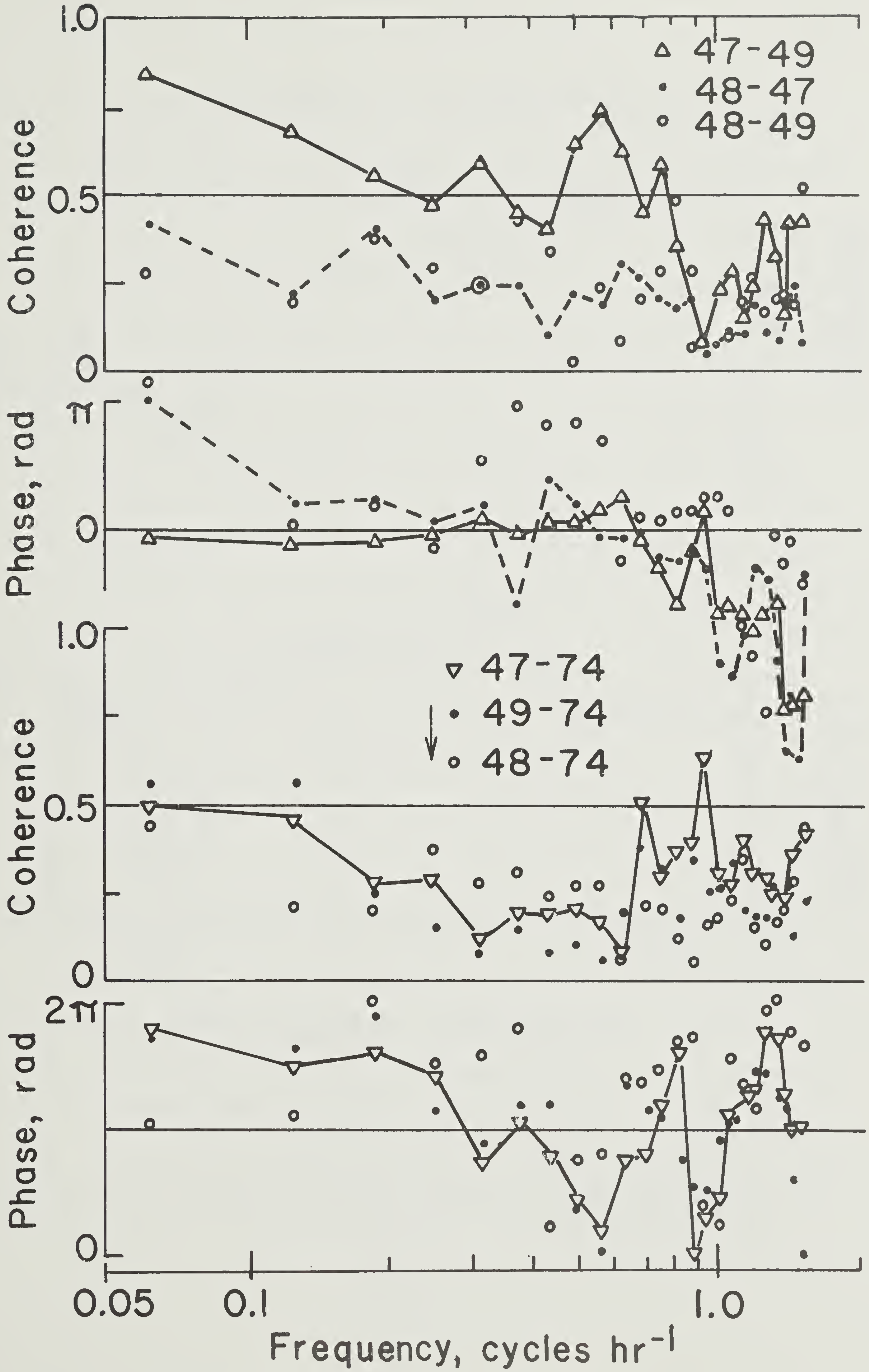

Fig. 15. Coherence and phase differences between pairs of floats, 12°N, 27°W.

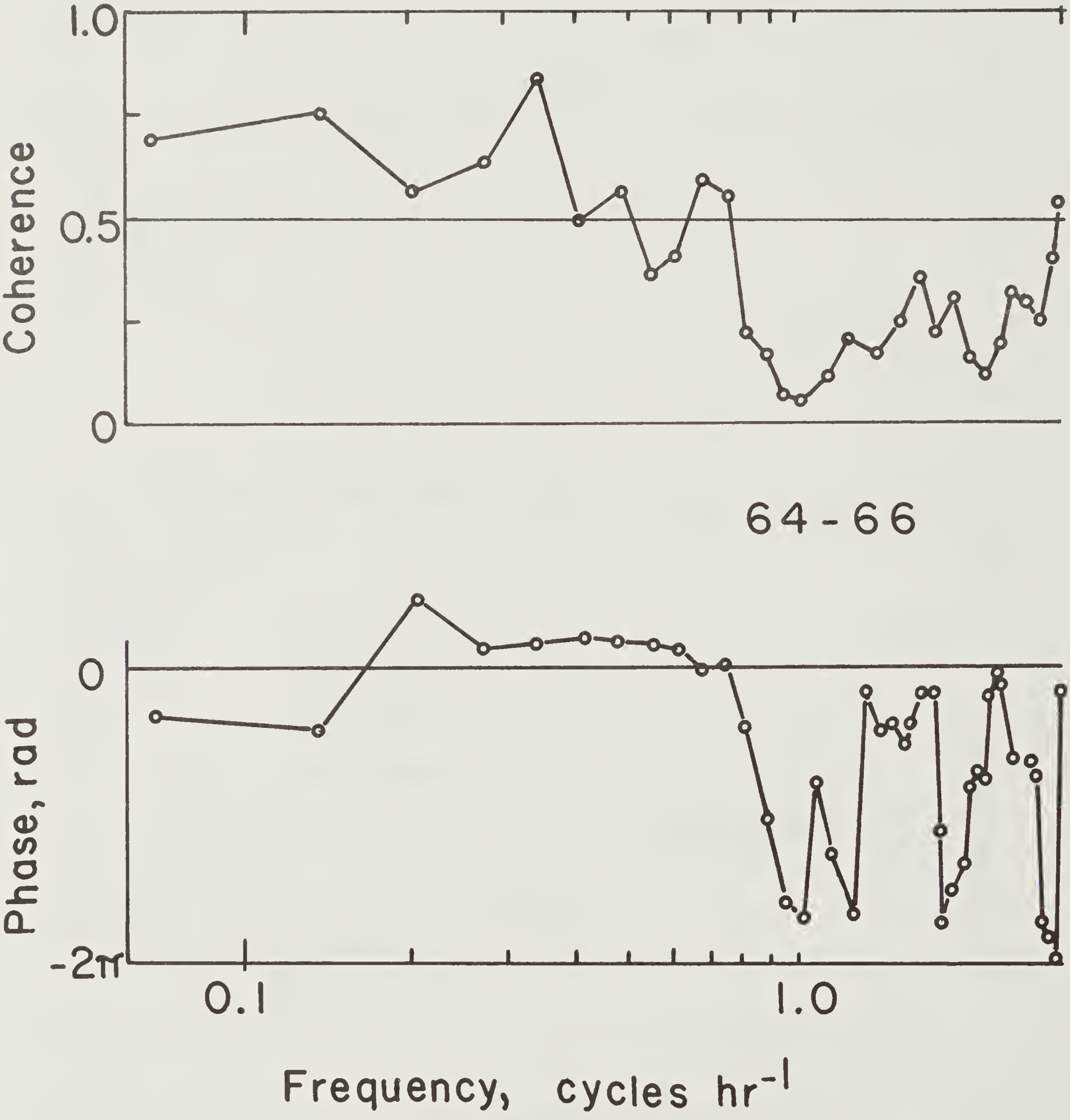

Fig. 16. Coherence and phase differences between pairs of floats, 28°N, 55°W, 2500m.

DISCUSSION AND FUTURE WORK

Particle motions in internal waves are normal to the propagation vector. The ratio of horizontal to vertical displacements is

$$\tan\theta = \sqrt{\frac{N^2 - n^2}{n^2}}$$

where θ is the angle of the propagation vector with the horizontal. For n not too close to N, that ratio varies as n^{-1} so that corresponding to a horizontal velocity changing as n^{-3} is a vertical velocity spectrum changing as n^{-1} and a vertical displacement spectrum as n^{-3}. The latter is predicted by Phillips (1966) for internal waves in either a weak velocity shear field or a field that is limited by shear instability. The observed persistence of such an approximate relationship in the relative movements of floats and extending near and above the stability frequency, however, is surprising.

Measurements made at hovering floats are Lagrangian insofar as the floats duplicate water movements. That duplication is far from exact (TEP, 1963) although measurements do pertain more to a particular parcel of water than to a sequence of particles, each with its own history, which flows past a fixed current meter. Observations at the floats are expected to be steadier in time and any flow common to the two floats is automatically filtered from measurements of separations. There are other attractive features but mention should be made of some important disadvantages: 1) measurements cannot be interpreted as definitely Lagrangian or related easily to an Eulerian coordinate system, 2) radial rather than vector separations are determined, and 3) cost and weight factors, including those involved in recovering floats, have necessitated pressure measuring inaccuracies which prevent determining differences in depth to better than tens of meters.

Uncertain non-linear viscous forces between the floats and water determine the extent to which floats move vertically so as to follow water motions. Unfortunately, those forces dominate the response between the inertia and float resonance frequencies and so over the frequency range of primary physical interest. When temperature changes are measured as well as pressures, it is possible to approximate the extent of water motion at the lowest frequencies. Spectra of vertical water displacements based on temperature and pressure spectra are, however, of poor quality. It is found that the amplitude spectra increase sharply as the frequency decreases and that total water amplitudes are measured in tens of meters regardless of whether the measurements are made in shallow or very deep water. Further details can be found in publications by the senior author listed in the bibliography.

Much of the difficulty in giving meaning to measurements obtained with floats stem from the large horizontal water amplitudes and the associated current shears. If floats remained a reasonably fixed distance apart, a series of measurements could establish various correlations as a function of separation distances. One would hope that floats at identical depths could maintain such almost constant separations but measurements so far suggest that dispersive motions would prevent that (see for example TEP, 1963, Figure 6). Such dispersion takes place because uncorrelated high frequency motions carry floats into different layers of a shear flow which varies with time. Water itself will not behave in this way unless it "tumbles" in its motion.

Despite the difficulties of interpreting results when using instrumented neutrally buoyant floats, the measurements of temperatures, pressures and separations coupled with physical insight have revealed otherwise unobserved features of oceanic movements, as illustrated by the results presented in this report. Further measurements using both floats and anchored current meters are needed. The former are useful for measuring uncorrelated motions of short period while the latter are better suited for determining the characteristics of slow internal wave motions. Some important problems that can be handled uniquely by floats are:

1) Measurements of temperatures, pressures and separations in float pairs at frequencies near and above the Vaisala frequency. These will help to establish the characteristics of the turbulent region where accelerating forces dominate gravity and which probably determine the method of vertical transport.

2) Detailed measurements of the absolute trajectories of hovering floats. Such experiments will investigate the implications of the present work that nearly circular absolute motions at the inertial frequency will not be much larger than observed in the separations between floats. They will resolve movements in two directions in the horizontal plane and describe the displacements in long internal waves. The mean flows in deep water at the locations discussed in this report have been small enough for floats to move only a few miles each day and communication has been maintained between a central master float and its satellite slaves. Such slaves can be anchored near the sea floor over a large area to form a listening array and the absolute trajectory of the master determined to within a fraction of a meter on the basis of communication between it and the array. Experiments made with the present floats in 3000 fathoms of water showed promise in that direction.

3) Detailed measurements of vertical profiles of the horizontal velocity structure and the variation of those profiles with time. These studies will investigate the vertical mode structure and the possible existence of discontinuities in the change of horizontal velocity with depth. Float experiments have, so far, given no evidence of large

discontinuities. A float sinking slowly to the sea floor and returning to the surface will communicate with the bottom anchored array mentioned in the previous paragraph. Data obtained in this way will serve to establish the absolute position and temperature at the sinking float. Repeated drops will be made in order to investigate changes in these profiles with time. Preparations are practically complete for these experiments. Time permitting, the experiments noted in the previous paragraph will be done with this equipment.

APPENDIX

An example of the small changes in separation which take place during the sinking of a cluster of floats is illustrated in Figure 1. These results were obtained when five floats were dropped together so as to ultimately hover at a depth near 3700m in the Cape Verde Basin. The separations are between pairs formed by a specific "master" float and the other floats designated by the numbers 29, 36, 73 and 74. Floats 73 and 74 were placed in the water a few minutes before and after the master float was inserted; all three were dropped from the stern. Floats 29 and 36 were also inserted before and after the master but from the bow approximately 100m forward. Initial depth differences with the master were approximately 50m except for float 73 which was 100m deeper. Temperature-time profiles during descent showed less than 50m variations in the initial depth differences. Heavy circles on the curve for float 74 mark hourly intervals after insertion. Other details are recorded in Table I.

These profiles show that the changes in separation within pairs are measured in tens of meters during the first five hours. The increasing rate of separation with depth near the equilibrium level reflects the slowed sinking rate rather than any increased relative horizontal velocity between floats.

Separation results for a second drop of two pairs at the Cape Verde location are shown in Figure 2. Also shown are the results for some other locations. The latter drops are shallower but sampling of the data took place more often. The nature of the curves indicates that the sampling rate is sufficient to avoid important aliasing of the variations of separation with depth.

In the deeper drops, floats sank at a rate of 200m in approximately 1/2 hour. At an observed typical separation velocity of 1 cm sec^{-1}, separations would change 20 meters in that time and alternations of ± 10 meters would take place over a vertical "wavelength" of 400m. Such variations are indicated by the curves but there are no coherent wavelengths near 400m between floats. Although one float was common in all four of the pairs illustrated in Figure 1, there was little similarity in results.

The velocities of separation during these drops are shown in Figure 3. Also included are the velocities present during subsequent hovering. Equilibrium depth was attained at approximately the time indicated by the heavy vertical line. Sinking took place during the times shown on the left where the time scale is expanded by a factor of 2-1/2. The rapidity with which velocity changes take place during sinking as compared with those during hovering is clearly illustrated. On the other hand their magnitudes do not differ impressively.

The results in Figure 3 show that a broad spectrum of frequencies contributes to the velocity during hovering. The kinetic energy associated with the low, near inertial, frequencies is large compared with that at the upper end of the spectrum. It is not clustered sharply enough to produce a dominating sinusoidal curve having the inertial period for the velocity in the low frequency region.

Separation amplitudes which depend on the ratio of velocity to frequency have been found almost predominantly large near the inertia period. It is reasonable to assume that such horizontal motion is almost circular and the shapes of many of the separation curves are such as to conform with that assumption, Figure 4. The small differences between some of the separations observed and those expected in such hypothetical circular motions is often surprising. When such comparisons are pursued, it is found that the amplitude or radius of an inertia circle can change appreciably from cycle to cycle and that the time between repeated closest approaches can differ from the actual inertia period by 10% or so, Figure 9. Both these factors imply that a spectrum representing the motions over many inertial cycles would be broader than implied by the present measurements which are carried out over only a few periods. Each of the experiments described here is too short in duration to give meaningful results for the spectrum of random motions at the lowest frequencies. On the other hand, the inertia period forms a limit for both internal and Rossby waves so that it is by no means a characterless random number.

The weak higher frequency components of the separations should probably depend on uncorrelated motions between floats of a pair. For purposes of analysis, it would be helpful to remove as much of the low frequency energy as possible in order to avoid leakage of that energy into the higher frequencies as a consequence of leakage through the filters used in the spectrum analyses. A steady drift was removed from the data as was the contribution from motion in an inertia circle. The appropriate parameters for this two dimensional circular or elliptical inertial motion and the associated drift were found by seeking as little difference between the separations given by a particular model of relative orbital movement and the observed separations as possible. A graphical procedure was used to determine those parameters and so the final parameters are not necessarily the best possible. The eye was thought a more trustworthy arbiter than the computer for this purpose.

An analytical expression for the hypothetical separation is obtained as follows. Two floats are assumed to traverse independent horizontal inertia circles each with its own phase, center of rotation and radius. Both have the same horizontal coordinates initially and the centers of the circles are assumed to move apart with a constant velocity $\vec{v}$. They may be in different planes. The separation at time t as determined from the relative velocity is

$$\hat{i}\,[v_x t + R_s \cos\alpha_s(t) - R_M \cos\alpha_M(t)] + \hat{j}\,[v_y t + R_s \sin\alpha_s(t) - R_M \sin\alpha_M(t)] + \vec{C}$$

where v_x and v_y are the components of relative velocity, R the radius, $\alpha = \theta + \omega t$ is the phase angle and $\vec{C}$ is a vector such that the horizontal separation is zero and the vertical separation equals the difference in depth at t = 0. Subscripts refer to the master or slave float. When θ_M is chosen to be zero at t = 0, and there is no difference in depth,

$$\vec{C} = -(\vec{R}_S - \vec{R}_M) \text{ at } t = 0 \text{ with } \vec{R}_M \text{ along the x-axis,}$$

The subsequent separation can be written as the sum of three terms: 1) a constant $\vec{C}$, 2) $-\vec{C}e^{i\omega t}$, a vector equal to $-\vec{C}$ at t = 0 and which rotates with angular velocity ω, and 3) $\vec{v}t$, the relative motion of centers.

When v = 0, the separation between floats at the same depth is

$$\left| 2C \sin\frac{\omega t}{2} \right|$$

This dependence on time is the same as that of a full wave rectified sine wave having one half the inertia frequency. That appearance is characteristic of the separation versus time plots. A spectrum analysis of such a separation would suggest erroneously the presence of high frequencies at a single float, particularly at times when the floats are near one another.

In the absence of a relative steady velocity and strong motions at higher frequencies floats should return to nearly zero separation after each inertia period if inertia motion dominated. When there is a relative velocity, depending on its direction, the time of return can be advanced or retarded. Observations that show a time of closest approach which differs from an inertia period can be brought into accord by altering the relative velocity. Because only a few periods of rotation are followed during a given experiment, this procedure conceals a fudge factor.

Comparisons of the observed and expected inertia circle separations are made in Figures 5 to 11. In many instances the similarity is impressive. At the least, they show the extent of the important contributions of displacements having nearly the inertia period. The dimensions of the appropriate circles of motion in a pair are given on these drawings. It is of interest to note the results obtained when linear rather than circular relative displacements take place. These are illustrated in Figure 12 which shows the result of trying to fit a linear motion of 100m amplitude and 20 hr period which has a steady drift of 3m hr^{-1} to a circle.

The results for the stations off the Gulf of Maine and Bermuda and in the Cape Verde Basin are consistent with orbital motions at the inertial times 18, 24 and 57 hrs, respectively, used for the circles. In the Caribbean side of the Anegada Passage the separations clearly fit a tidal ellipse having a 12 hr period. At the latter location the inertia period is 38 hours but data was obtained for only 18 hours. A strong inertia current during that time would probably appear to be an almost steady drift but no drift at all was found. The results for float 48 at Cape Verde are of particular interest because that float was dropped into the cluster later than the other floats. The large phase difference can be explained on the basis of a large rather than zero initial displacement from the master; the closeness of the approaches is probably a result of luck in picking a drop position.

Spectrums for both the separation distances and radial velocities for a pair were obtained by means of a variety of analytic approaches. The rapid drop with frequency of the separation spectrum densities makes it more convenient to report results in terms of speeds rather than distances. Spectra were obtained for the following conditions: 1) separaration spectra directly from separation data, 2) separation and velocity spectra from data "prewhitened" to form a velocity input, 3) complete spectra from data "prewhitened" to form acceleration input and 4) repeats of these procedures for input data consisting of the differences, S - I; i.e., between measured separations, S, and those expected for ideal inertia circles, I, as discussed earlier. The least count of the recording equipment was ±0.7m except at Cape Verde where it was ±0.07m.

Within the confidence limits, 1) prewhitening to form velocity input data was found necessary and adequate, 2) except at the lowest frequency of each analysis, there was little difference which data, S or S - I, were used. The latter result was encouraging because it shows that motion in a circle of itself introduces relatively little energy at high frequencies as a result of the dependence of the separation on the absolute value of the term $\sin(\omega t/2)$. Nevertheless, the meaning of such spectra is not yet clear for the following reasons: 1) At the highest frequencies, motions at each float are uncorrelated but they are not necessarily isotropic in a horizontal plane and could be a function of the distance between floats. Fluctuations along a line between floats may not be typical of average horizontal motions. 2) At the lowest frequencies, internal waves probably dominate the motions and so movements are correlated, as in Figures 15 and 16. Differential responses between floats of a pair will depend on phase differences and so be frequency dependent even in the unlikely event that the response at an individual float is independent of frequency. 3) Vertical motions may contribute to the separation, especially when floats are near one another. 4) All these factors can be present in the mid-frequency range.

Such considerations discouraged performing spectrum analyses in the past but consistencies in preliminary analyses suggested the need for the thorough analysis reported here. The initial pessimism was apparently unwarranted.

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13. ABSTRACT

The separations and velocities of separation between pairs of neutrally buoyant floats in pairs have been investigated in detail. Observations at three locations where the inertial periods are 18, 24 and 57 hours indicate strong relative movements around circles at those periods. Just within the Caribbean Sea, however, the motion is in a semidiurnal ellipse.

Spectrums for the velocities of separation vary above the inertial frequency so that the energy density falls approximately as f^{-3}. This decrease is consistent with the trend in the absolute velocities obtained at WHOI Mooring Site "D" (UNESCO) at frequencies well above the inertail period. Correlations exist between the motions of nearby pairs of deep floats so that the motions are not entirely random. The possible effect of these on the spectra has yet to be investigated.

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