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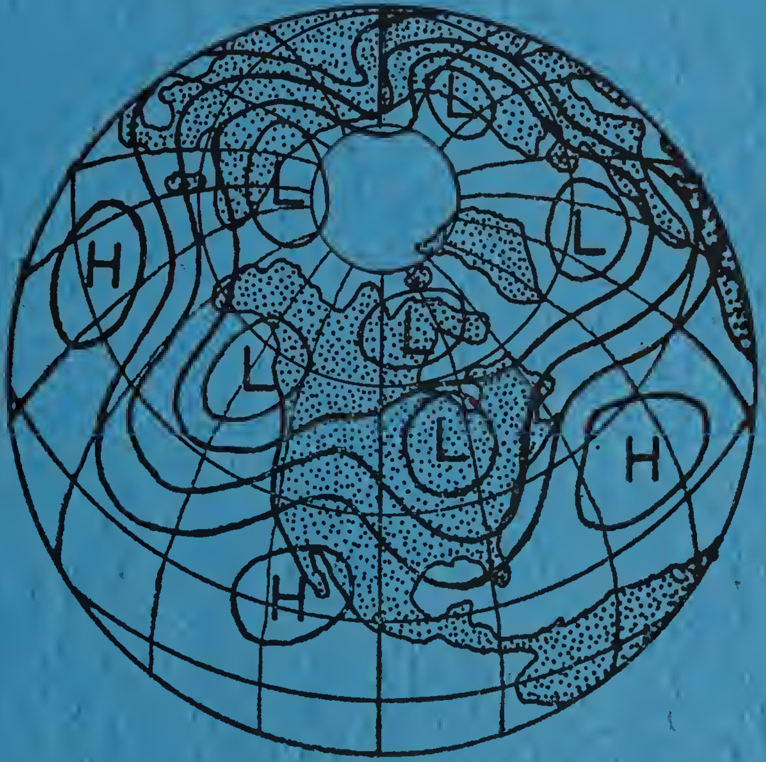
DYNAMICS OF HURRICANE MOTION

by

RICHARD L. PFEFFER



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ABSTRACT

Hurricane motion is discussed from the standpoint of the balance of angular momentum in each one of an array of cylindrical volumes located in different positions relative to the moving vortex. It is shown that the initial increase of angular momentum about a local vertical axis, which must take place when a hurricane moves through a region, is brought about by the transport of such angular momentum into the volume from the surroundings. This transport can be accomplished by either of two processes --- vertical circulations or horizontal eddy exchanges --- or by some combination of both.

The results of the present investigation reveal that horizontal exchange processes account almost entirely for the angular momentum transports associated with the motion of the mature hurricane. The asymmetries found in the distribution of the angular momentum transports around the hurricane cannot be explained in terms of simple concepts such as the "steering" of a circular vortex by a uniform current, but must be the reflection of a more complex mechanism of hurricane motion. It is suggested that any theory of hurricane motion, in order to be considered valid, should be able to account for the observed asymmetries in the field of angular momentum transports around the hurricane.

1. INTRODUCTION

When a rotating wind system such as a hurricane moves through a region, the relative angular momentum of the air about a local vertical axis increases, perhaps sporadically, until the center of the vortex is approximately at its nearest position to the axis, and then decreases, as shown schematically in fig. 1. Although simple processes such as the "steering" of a circularly symmetrical vortex by a uniform basic current could account for such changes, these processes are not the only ones that could bring about this sequence of events. Indeed, if they were, the forecasting of hurricane motion would be a simple task.

One can study the motion of the hurricane from a more general point of view by making use of an equation derived by Starr (1953) for the time rate of change of local angular momentum -- that is, the angular momentum of the air about an arbitrarily located vertical axis which is fixed with respect to the rotating earth. Conservation of angular momentum is a fundamental principle which has been used with considerable success as a basis for studying the general circulation of the earth's atmosphere (see, for example, Jeffrey's, 1926, Starr, 1948, Starr and White, 1954, and references therein). The equation for local angular momentum has been used by Starr (1953), Lorenz (1953), Pfeffer and Saltzman (1955), Saltzman (1955) and Pfeffer (1958 a, b) to study dynamical processes which take place in extratropical and tropical circulations.

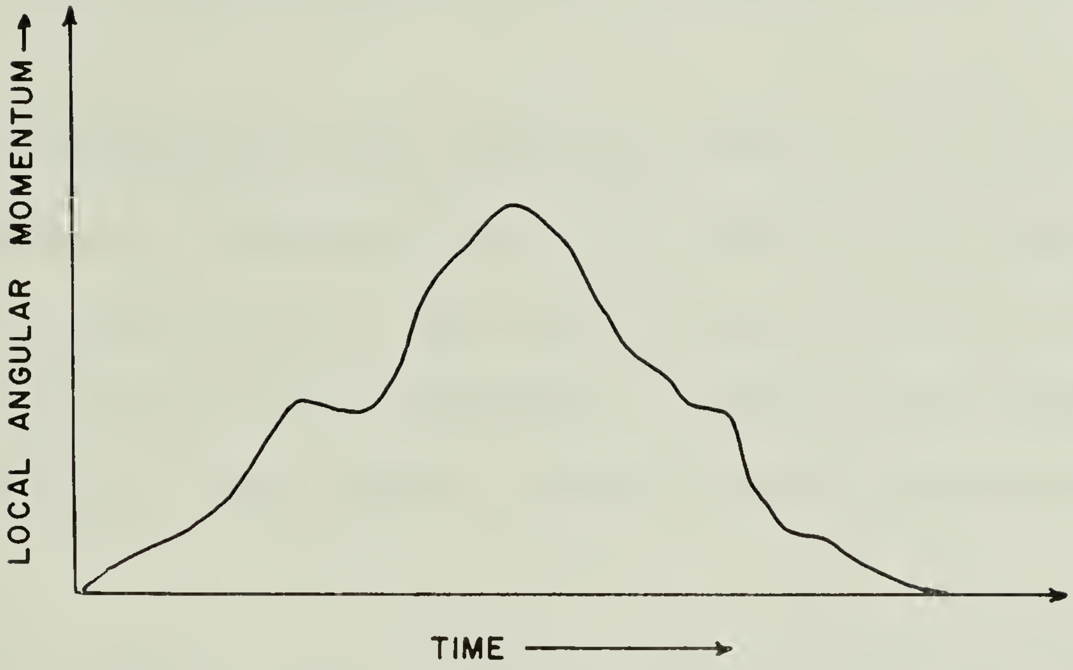


Fig. 1 Schematic picture showing the time variation of the relative angular momentum about a local vertical axis during the passage of a hurricane.

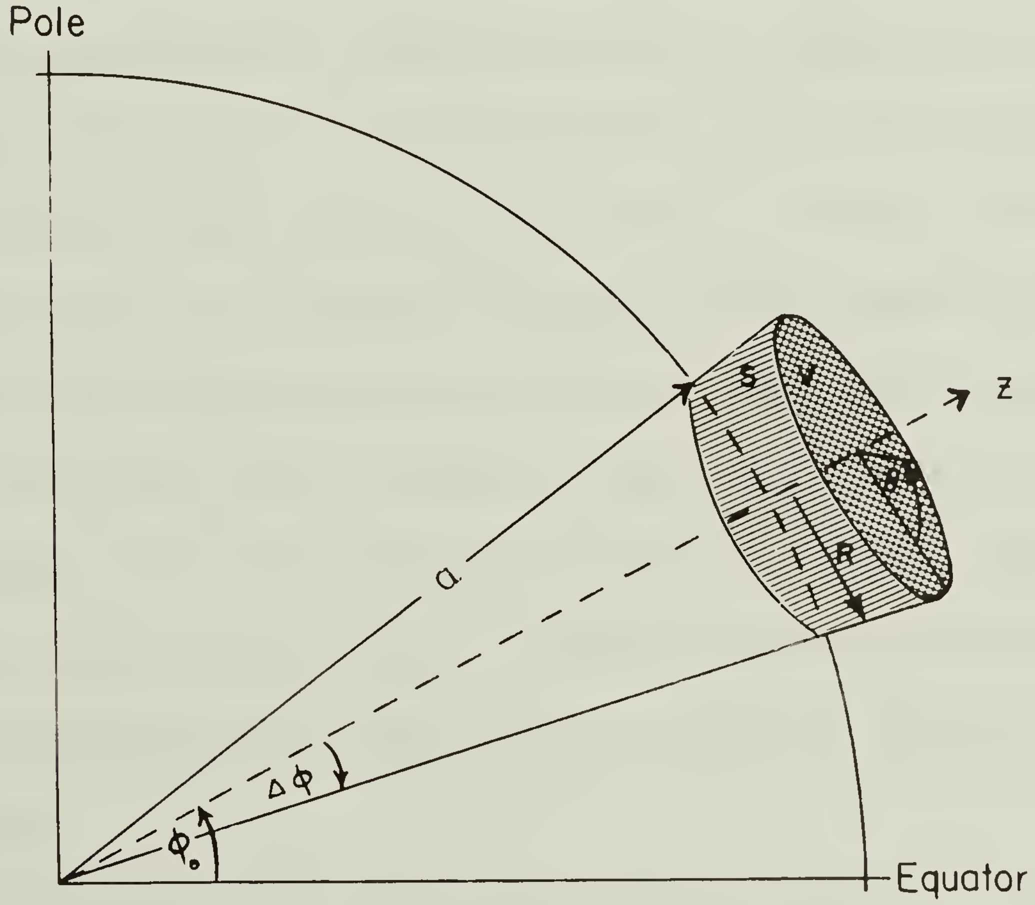


Fig. 2 Schematic picture of the approximately cylindrical volume, V (shaded region). ϕ_o is the latitude of the axis, $\Delta\phi$ is the angle between the axis and the wall, a is the radius of the earth and R is the radius of the cylinder.

II THE EQUATION FOR LOCAL ANGULAR MOMENTUM

Adopting a cylindrical polar coordinate system (R, θ, Z) and considering the approximately cylindrical volume (V) shown by the shaded region in fig. 2 (really, a frustrum of a cone), which extends from the surface of the earth to the upper limit of the atmosphere, we may write the equation for the time rate of change of local angular momentum without sensible error in the form,

$$\frac{\partial}{\partial t}\int_V \rho R u \, dV = R\int_S \rho u v \, dS - \int_V \frac{\partial P}{\partial \theta} dV + \int_V \rho f R v \, dV + \int_V R D \, dV \qquad (1)$$

where u and v are the tangential (positive counterclockwise) and radial (positive inward) components of the wind velocity, respectively, ρ is density, P is pressure, f is the Coriolis parameter, D is the tangential component of the friction force per unit mass, and t is time. This equation states that the time rate of change of relative local angular momentum (left hand integral) can be brought about by a flux of such momentum across the boundaries of the volume (first integral on the right) or by torques due to pressure gradients, the Coriolis force or viscosity (second, third and fourth integrals on the right, respectively). For the purpose of this discussion we shall consider volumes in which there are no orographic features so that the pressure integral,

$$\int_V \frac{\partial P}{\partial \theta} dV \equiv \int_Z \int_R R\left[\oint \frac{\partial P}{\partial \theta} d\theta\right] dR \, dZ,$$

vanishes identically.

As discussed by Starr (1953) the last two integrals in (1) act to oppose the initial increase of angular momentum in the volume. The Coriolis integral also acts to oppose the subsequent decrease of angular momentum. It follows that the initial increase of angular momentum must take place as a result of a horizontal transport of such momentum into the volume from the surrounding atmosphere and that the subsequent decrease is accomplished by some combination of flux processes and friction. This does not, however, imply that the earth's rotation (represented in equation (1) by the Coriolis integral) has a negligible effect on the rate of change of the relative angular momentum within the volume. Although the integral, $\int_V \rho f R v \, dV$, taken over the entire volume of the cylinder extending from the surface of the earth to the upper limit of the atmosphere, is of the wrong sign to account for the direction of the angular momentum changes, the integrand takes on large positive and negative values on a point-for-point basis. At individual points inward motion generates cyclonic angular momentum and outward motion generates anticyclonic angular momentum. The immediate effect of the Coriolis integral, therefore, is to redistribute angular momentum within the volume. The accumulation of angular momentum in the volume then takes place as a result of the inward transport of those fluid elements which have acquired cyclonic angular momentum and the outward transport of those which have acquired anticyclonic angular momentum. The net transport of angular momentum is represented by the flux integral, $R\int_S \rho u v \, ds$.

In order to investigate the way in which the motions of the atmosphere are organized in the hurricane to bring about the required transports of local angular momentum, we may make use of the identity,

$$q \equiv \overline{[q]} + [q]'' + q' \qquad (2)$$

where q may be any quantity and the brackets, bar and primes are defined as follows:

$$[q] \equiv \frac{1}{2\pi}\oint q\,d\theta \; ; \qquad q' \equiv q - [q]$$

$$\bar{q} \equiv \frac{1}{P_0}\int_0^{P_0} q\,dP \; ; \qquad q'' \equiv q - \bar{q}$$

Using (2) we may write the integral representing the horizontal transport of angular momentum into the volume in the form,

$$R\int_S \rho u v\,ds \simeq \frac{2\pi R^2 P_0}{g}\left\{\overline{[u]''[v]''} + \overline{[u'v']}\right\}. \qquad (3)$$

Here we have neglected a term which depends upon $\overline{[v]}$, or the net mass transport into the volume.

The first term on the right, which depends upon the covariance of $[u]$ and $[v]$ on the boundary, measures the angular momentum transport brought about by vertical circulations. This term gives a positive contribution when there is a net mass inflow at those levels at which the circulation on the boundary is large and positive, and a net mass outflow at those levels at which it is negative or small positive. Vertical circulations will be of importance if, for example, hurricane motion takes place as a result of a regeneration of the vertical motion field by heating in a new region ahead of the existing position of the vortex; for example, by a warm tongue of ocean water.

The last term, which depends upon the covariance of $[u]$ and $[v]$ along the boundary at each level, measures the transport of angular momentum due to horizontal exchange processes and is positive when, at individual levels, the angular momentum is greater at inflow points than it is at outflow points. This term, which we shall call the "horizontal-eddy transport" of angular momentum, will be

of importance if the motion of the hurricane is a form of wave motion, whether the wave is an asymmetrical disturbance such as an easterly wave or a symmetrical disturbance such as a circular vortex imbedded in a uniform current.

III OBSERVATIONAL STUDY

If a dense network of simultaneous wind observations in individual hurricanes were available, it would be a simple matter to evaluate the terms in (3) and thereby to determine the role of each of the transport processes. Since present-day data coverage, particularly at upper levels in the hurricane, is inadequate for such purposes, it was decided to make use of composite wind charts which have been prepared by E. Jordan (1952) for altitudes 4,000, 7,000, 10,000, 18,000, 30,000, 40,000 and 45,000 ft., and by L. Hughes (1952) for 1,000 ft., using wind reports from a number of mature hurricanes. The wind charts for elevations above 1,000 ft., many of which were unpublished, were kindly loaned to the author several years ago by Mrs. Jordan. Since these data are in the nature of ensemble averages of the velocity field taken over a large number of hurricanes, they cannot be expected to contain all of the information that might be present on synoptic charts prepared from simultaneous wind observations. In particular, features which may be common to all hurricanes, but which do not have a preferred location within the hurricane, tend to be suppressed by averaging. Such limitations in the data should be kept in mind when evaluating the results of the present study. A further limitation is that the present observations cover only the region between 2° and 6° latitude from the hurricane eye and, therefore, do not give us information about

the structure of the inner vortex. The data do, however, provide good definition of the large-scale circulation surrounding this vortex. As we shall see, this circulation is an integral part of the hurricane system which plays an important role in the motion of the hurricane.

Although composite wind charts cannot furnish us with a time sequence of events as the hurricane passes through a fixed volume, they do provide an instantaneous picture of the flow from which we can obtain useful information. In the present investigation, it was decided to use these data to compute the angular momentum flux into each one of an array of cylindrical volumes located in different positions relative to the moving hurricane. The computations were made in an effort to determine (a) the distribution of the angular momentum transport into volumes surrounding the hurricane, (b) the relative magnitudes of the two processes represented by the terms in equation (3), and (c) the scale of the processes involved in the motion of the hurricane; in particular, whether the motion of the hurricane is associated with large-scale transports of angular momentum, or whether it is independent of such transports? Although the size of the cylinder to be used is dictated in a rough way by the scale of the hurricane circulation, the exact choice is somewhat arbitrary. For the purposes of the present study it was decided to make the computations for four different cylinder sizes, namely, $\mathcal{R} = 2^\circ, 3^\circ, 4^\circ$ and 5° latitude. In a number of cases a portion of the cylinder wall passed through the region within 2° or beyond 6° latitude from the eye of the hurricane where there were no data. Since only a small portion of the cylinder wall was involved in all but a few instances, it is felt that the main conclusions of the present study are not affected by this shortcoming in the data.

The numbers in fig. 3 give the total transport of angular momentum into each one of an array of cylindrical volumes with radii 3° latitude. The positions of the numbers denote the locations of the axes of the respective cylinders. The hurricane eye is designated by the symbol,  , and the direction of motion of the hurricane is to the left, as indicated by the arrow. For convenience the array of numbers is contoured. It is seen that the motion of the hurricane is along the line connecting the centers of maximum positive and negative angular momentum transport, the sense being from negative toward positive values. The maximum positive transport is found about 1° to 2° latitude ahead of the eye and the maximum negative transport is found 3° behind the eye. There is also a positive transport of angular momentum into the cylinder centered at the eye. This is required to balance the frictional drain of angular momentum which is brought about by the interaction between the hurricane circulation and the earth's surface.

In fig. 4 a similar calculation is presented for an array of cylinders with radii 4° latitude. In this case, the maximum positive transport of local angular momentum is found 2° latitude ahead of the eye and the maximum negative transport is found 4° latitude behind it. Once again there is a positive transport of angular momentum into the cylinder centered at the eye of the hurricane.

Calculations using larger and smaller cylinders confirm that, as the cylinder size is increased, the centers of positive and negative angular momentum transport move farther apart along the line of hurricane motion, with the maximum positive transport always located closer to the eye of the hurricane than the maximum negative transport. In every case there is a positive transport of angular momentum into the cylinder centered at the eye.

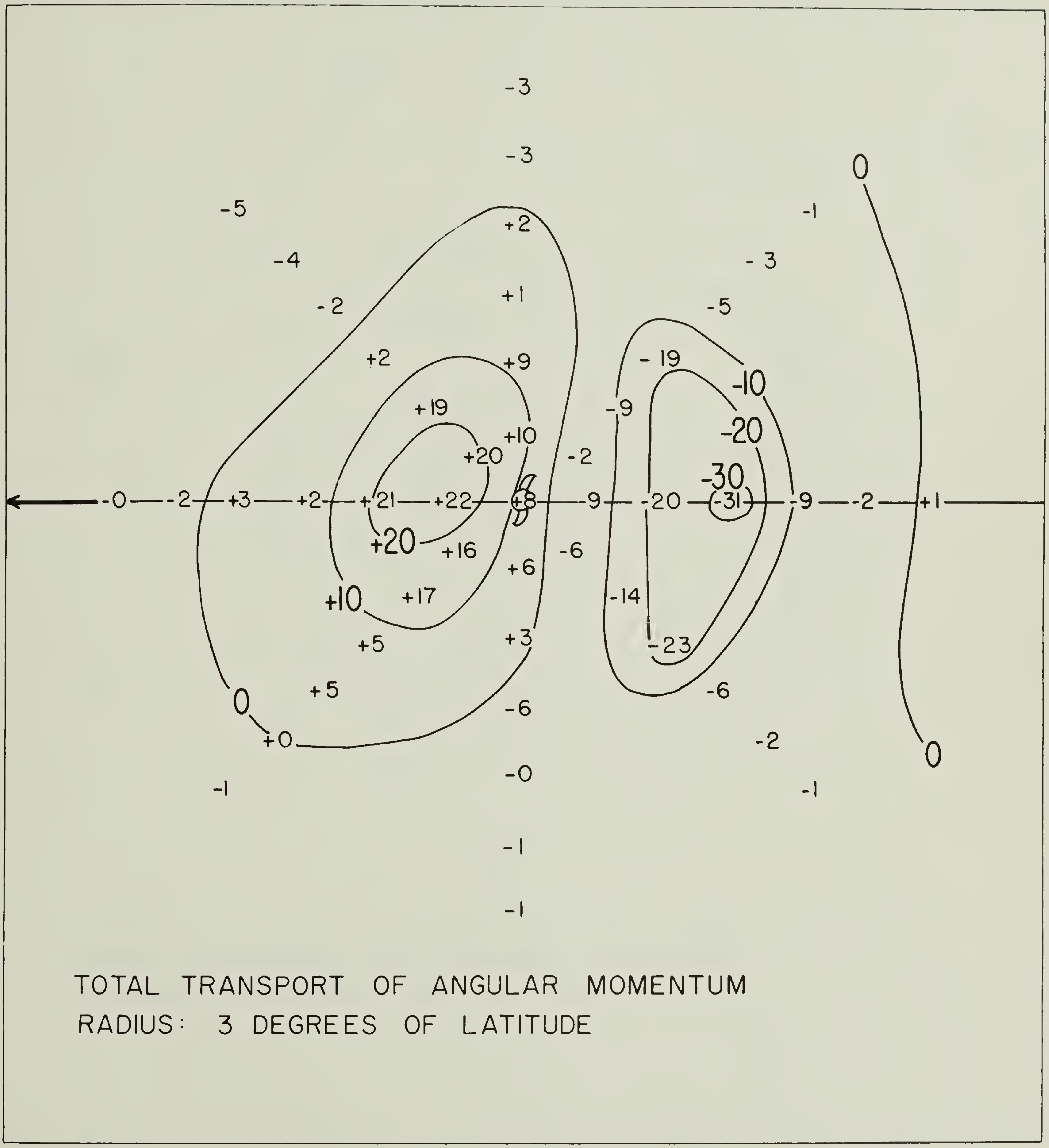


Fig. 3 Total transport of angular momentum into each one of an array of cylindrical volumes with radii 3° latitude. The positions of the numbers denote the locations of the axes of the respective cylinders. The hurricane eye is designated by the symbol, 𝒮 , and the direction of motion of the hurricane is to the left, as indicated by the arrow.

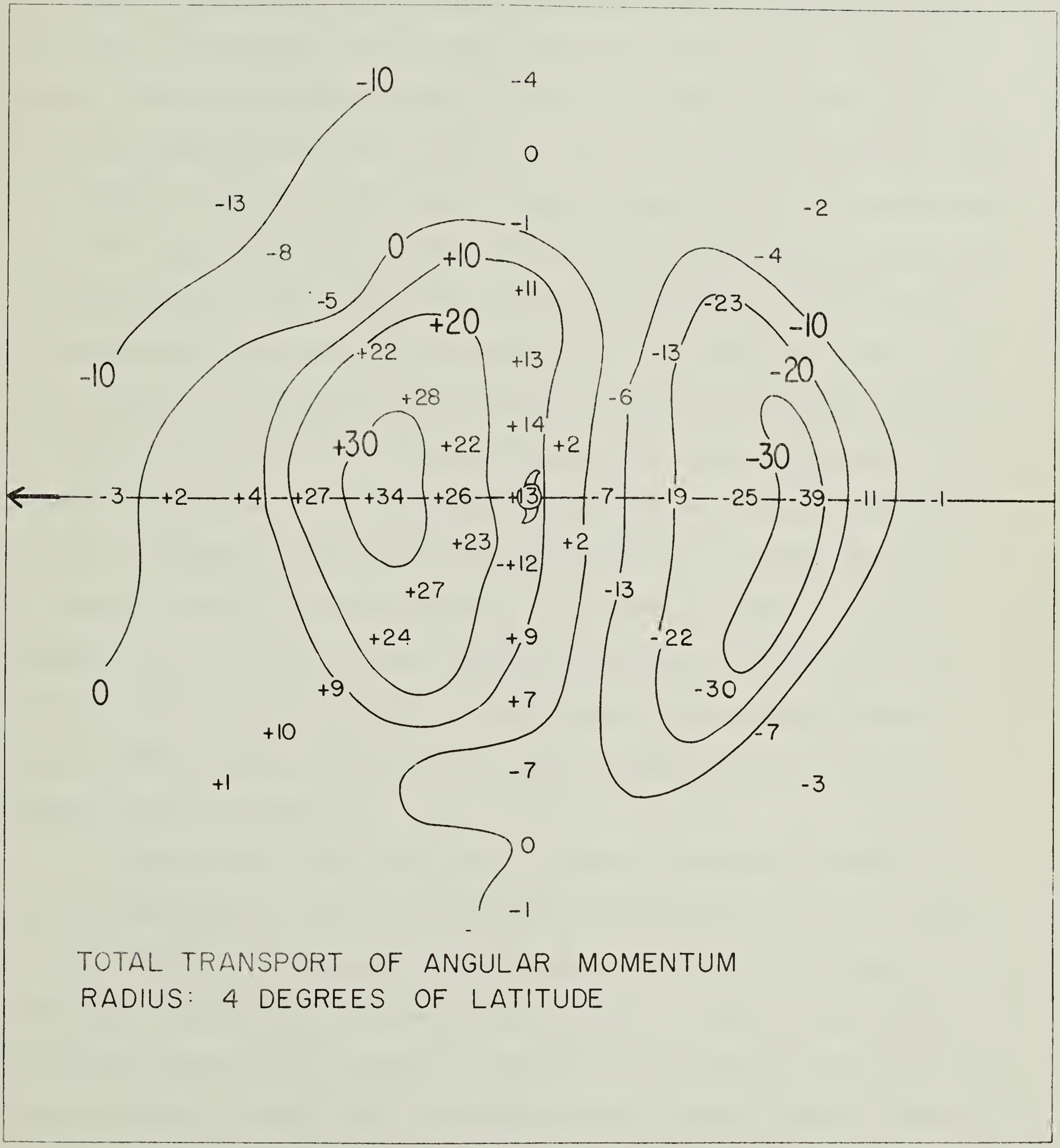


Fig. 4 Total transport of angular momentum into each one of an array of cylindrical volumes with radii 4° latitude.

Figs. 5a and 5b show the contributions of horizontal exchange processes and vertical circulations, respectively, to the total transport of angular momentum into the cylinders with radii 3° latitude. Figs. 6a and 6b give the results of similar calculations for the cylinders with radii 4° latitude. In both of these cases, and in the case of larger and smaller cylinders (not shown here), the angular momentum transports due to horizontal exchange processes are of greater magnitude than those due to vertical circulations, and the pattern of the horizontal-eddy transports correlates well with that of the total transports. Figs. 7a and 7b show how the streamline pattern in the middle troposphere is organized to accomplish the required transports of angular momentum. In the cylinder located ahead of the hurricane, cyclonic motion about the axis of the cylinder is present at both inflow and outflow points; but because the velocity is greater at inflow points the result is a net inward transport of angular momentum. In the cylinder located behind the hurricane, cyclonic motion about the axis of the cylinder is found at outflow points and anticyclonic motion is found at inflow points, thus leading to a net outward transport of angular momentum from this volume.

As noted earlier, the "steering" of a circular vortex by a uniform current would result in positive and negative eddy transports of angular momentum into cylinders located ahead of and behind the center of the vortex, respectively. However, this phenomenon cannot account for positive eddy fluxes of angular momentum into cylinders of various sizes centered at the eye of the hurricane, or for the fact that the maximum negative fluxes of angular momentum are found in cylinders which lie farther from the eye of the hurricane than the ones in which the maximum positive fluxes are found. Such asymmetries must be the reflection of a more complex mechanism of hurricane motion which future

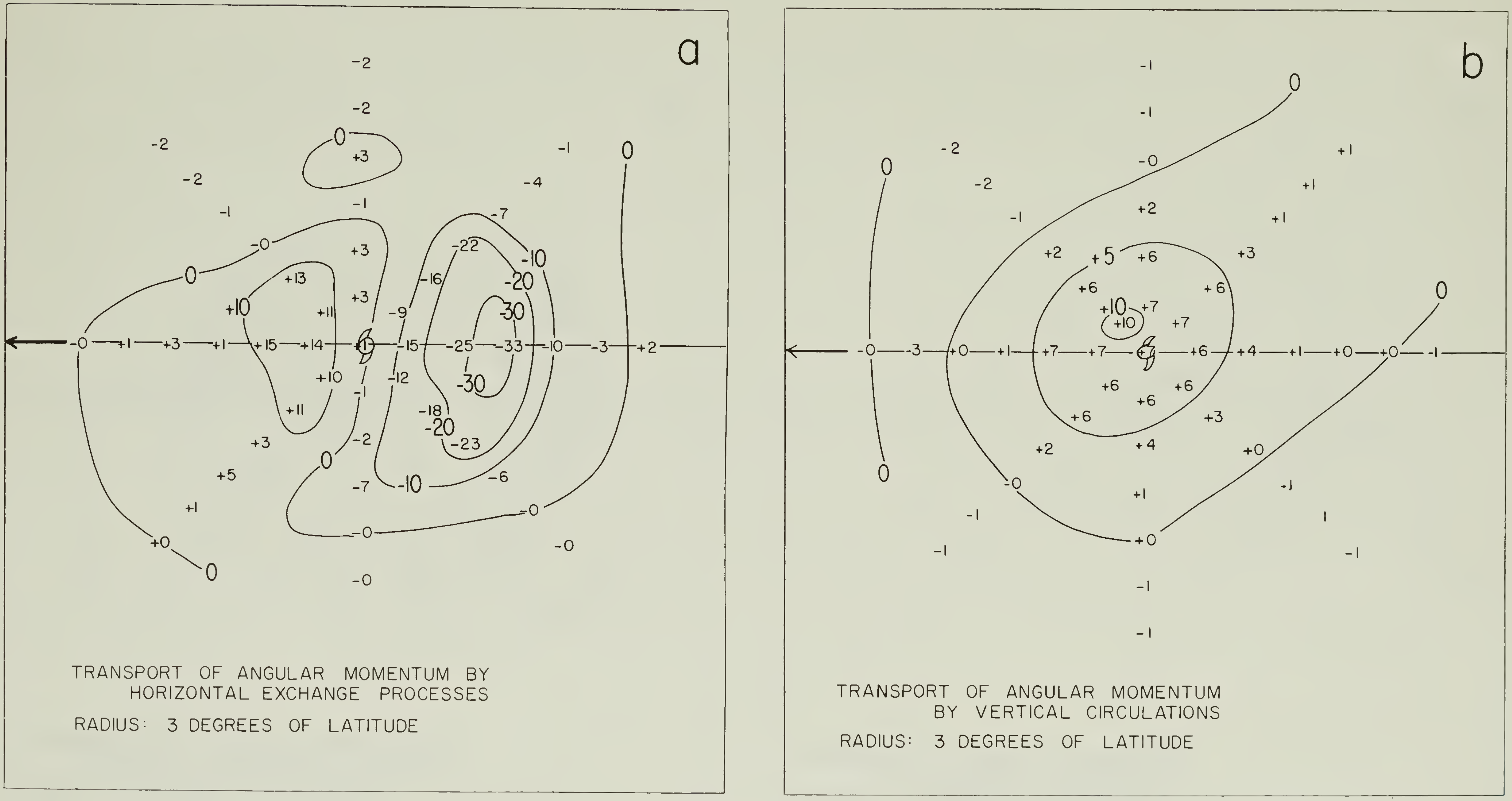
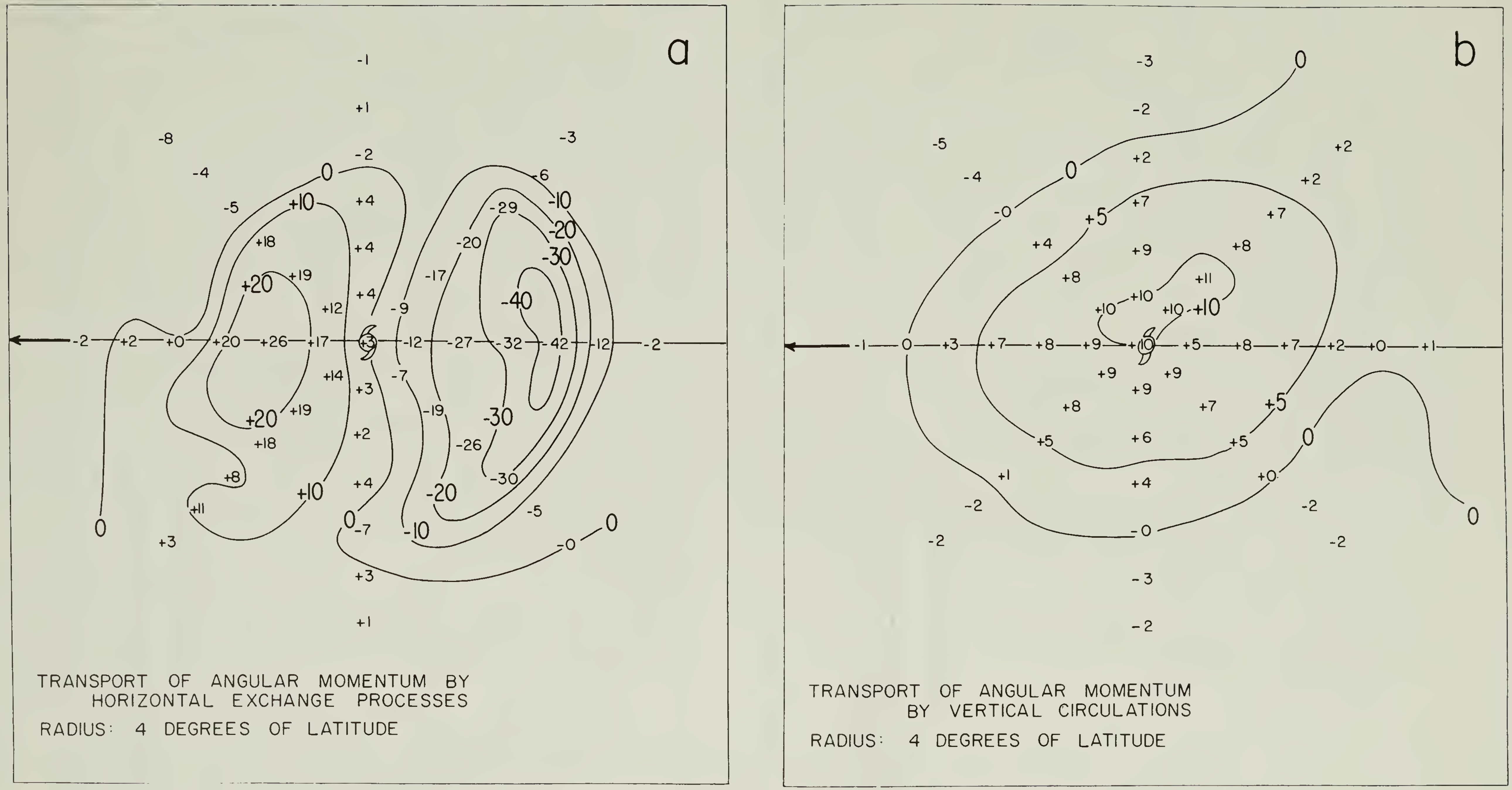


Fig. 5 Transport of angular momentum into each one of the array of cylindrical volumes with radii 3^{o} latitude

a. by horizontal exchange processes.

b. by vertical circulations.



TRANSPORT OF ANGULAR MOMENTUM BY HORIZONTAL EXCHANGE PROCESSES

RADIUS: 4 DEGREES OF LATITUDE

TRANSPORT OF ANGULAR MOMENTUM BY VERTICAL CIRCULATIONS

RADIUS: 4 DEGREES OF LATITUDE

Fig. 6 Transport of angular momentum into each one of the array of cylindrical volumes with radii 4° latitude
a. by horizontal exchange processes.
b. by vertical circulations.

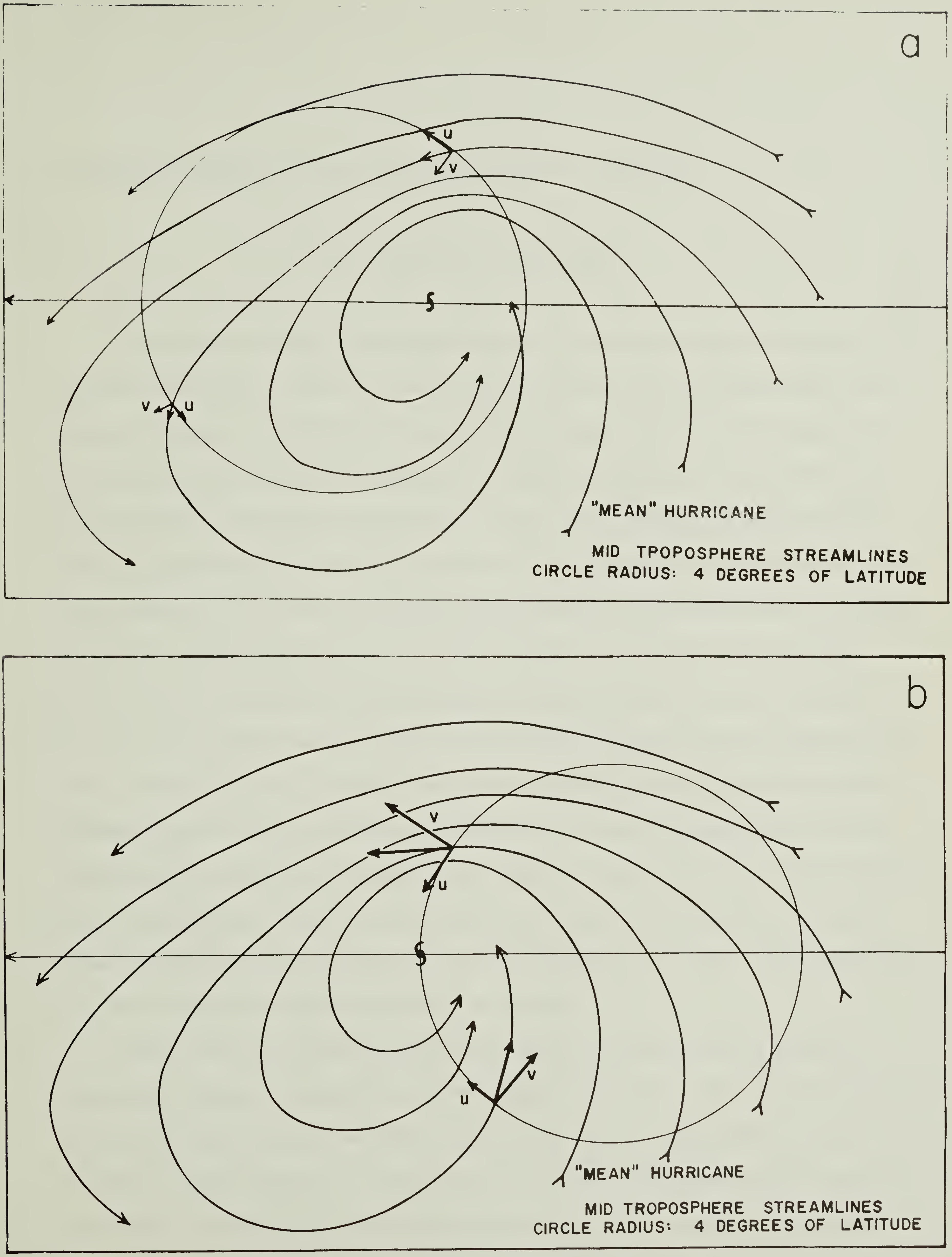


Fig. 7 Streamline pattern in the middle troposphere showing how the motions of the atmosphere are organized to transport angular momentum
a. into a cylinder located in advance of the eye.
b. into a cylinder located to the rear of the eye.

theoretical treatments of the problem will have to explain.

IV SUMMARY AND CONCLUSIONS

Transports of local angular momentum into cylindrical volumes located in different positions relative to the moving hurricane can be computed with reasonable accuracy, even when the data within a radius of 2° latitude from the hurricane eye are missing. Computations for various cylinder sizes show that the transports are accomplished mainly by horizontal exchange processes and that the instantaneous direction of motion of the mature hurricane is along the line connecting the centers of positive and negative horizontal-eddy transports of angular momentum (the sense being from negative toward positive values). The cylinders in which the maximum positive transports are found lie closer to the eye of the hurricane than those in which the maximum negative transports are found. There is also a positive transport of angular momentum by horizontal exchange processes into cylinders centered at the eye, the magnitude of which increases with increasing cylinder size within the range of sizes considered in this investigation. The observed asymmetries cannot be accounted for by the "steering" of a circular vortex in a uniform current, but must be the reflection of a more complex mechanism of hurricane motion.

From time to time theoretical models will be proposed which attempt to explain the mechanism of hurricane development and motion. It will be necessary to find criteria by which to judge the validity of the conclusions reached on the basis of such theories. Regardless of any other considerations, it may be stated that if the model satifies the principles of conservation of mass, momentum and energy <u>in the same way that the atmosphere does</u>, then it consitutes

a valid explanation of the behavior of the atmosphere. The burden of proof thus rests on studies in which an effort is made to determine, quantitatively, the physical processes which take place in the atmosphere. The present investigation is an attempt in this direction. Owing to the limitations of the data, it must be regarded merely as a crude first look at the mechanism of hurricane motion. If the results of this study are confirmed by future computations using more accurate and more complete data, they will serve as one test of the validity of proposed theories of hurricane motion. Any theory of hurricane motion, in order to be considered valid, should be able to account for the observed asymmetries in the field of angular momentum transports around the hurricane.

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