Redistribution and Non-Consumption Smoothing in an Open Economy

by

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This paper presents a model where income distribution and redistributive fiscal policy interact to affect the budget deficit and the pattern of net borrowing of a country. According to the standard representative agent paradigm, a small open economy should smooth consumption by borrowing from (lending to) the rest of the world when its income increases (declines) over time. The simple model of this paper delivers exactly the same predictions in the absence of income dispersion. When income distribution is not degenerate, however, the same model gives rise to a surprising wealth of results. In particular, poor economies with high inequality may exhibit completely counterintuitive patterns of fiscal policy and external borrowing. The country's production path declines over time, because the more mobile agents leave the country to escape taxation; yet, the country might end up having a budget deficit and borrowing from abroad, thereby reinforcing rather than smoothing the asymmetry in consumption between the two periods. An important feature of this outcome is that it is backed by both the poor and the rich, who gain from the fiscal system at the expense of the middle class.

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1 Introduction.

The standard representative agent, neoclassical model implies that a country should smooth consumption by borrowing from (lending to) the rest of the world when its production increases (declines) over time (see e.g. Sachs (1982) and Svensson and Razin (1983)). While the neoclassical model seems to fit well the behavior of industrialized countries (see for instance Ahmed (1986)), it has difficulty explaining certain patterns of behavior in the recent history of developing countries. An important example is the failure of several Latin American and African countries during the '80’s to adjust their fiscal policies, and the resulting current account imbalances, in response to what was widely perceived as permanent negative shocks.

The neoclassical model is inherently unable to account for these phenomena because a representative, forward-looking agent would always internalize all the costs of not smoothing consumption. Seemingly irrational patterns of borrowing and lending, however, can be rationalized as the outcome of an interaction between different groups over the distribution and redistribution of resources. Indeed, many observers have documented the role of an unequal distribution of income or political power in hampering the adjustment to external shocks in many developing countries (see, among others, Berg and Sachs (1988) and Tornell and Lane (1994)). At a more theoretical level, several recent contributions (including Alesina and Drazen (1991), Laban and Sturzenegger (1992) and Velasco (1992)) have modeled delays in stabilization as the outcome of power struggles between two groups with different interests and characteristics. The common elements to all these models is that some policies that are inefficient in an aggregate sense are kept in place because the two parties cannot reach an agreement on how to replace them.

The present paper too shows that the interaction of a non-degenerate income distribution and a redistributive fiscal policy in a small open economy can lead to a very different dynamic behavior from that of a representative agent economy. Unlike the papers cited above, the basic structure of the model comprises three, rather than two, groups, who interact through a repeated voting process. This basic framework can give rise to a surprising wealth of possibilities, depending on the average income of the economy and the degree of inequality of its distribution of income.

The model has two periods, one factor supplied inelastically, one good produced with a linear production function, a given world interest rate, and a fiscal system that redistributes tax revenues from rich to poor individuals. The only complication to this minimal set of assumptions is that any agent can escape taxation in the second period by paying a fixed cost. In other words, in the long run the tax base is elastic to taxation.
Initially, only the government can borrow or lend: the government budget surplus is thus equal to the current account surplus. I then show that income dispersion does not matter in rich economies: at any degree of inequality, they mimick exactly the behavior of a representative agent economy. Poor economies are much more sensitive to the pattern of income distribution, and may exhibit completely counterintuitive patterns of fiscal policy and external borrowing. At high levels of inequality, the richer agents will leave the country to escape taxation. Hence, the country has a declining income over time, and yet it might end up running a budget deficit and borrowing from abroad, thereby reinforcing rather than smoothing the asymmetry in consumption between the two periods. An important feature of this outcome is that it is backed by both the poor and the rich, who use the fiscal system at the expense of the middle class.

Thus, the model fits the stylized facts described above in that rich economies always follow the neoclassical paradigm, regardless of how income is distributed, while poorer economies with high income dispersion can exhibit completely different, and sometimes very extreme, behaviors. In particular, the phenomenon of delays in stabilization can arise as a special case in conditions of high inequality. Furthermore, this outcome has many features in common with the stylized facts of “populist experiences”: high budget deficits caused by high redistributive expenditures and supported (for different reasons) by both the poor and the rich, followed by sudden and anticipated reversals characterized by the collapse of government expenditure and the need to repay the external debt (see e.g. Dornbusch and Edwards (1990)).

Of course, letting private agents lend and borrow privately implies that each agent can undo the effects of official borrowing and smooth consumption perfectly. Precisely for this reason, the behavior of fiscal policy in poor, unequal countries becomes even more extreme, with different majorities initially backing the highest possible budget deficit and external borrowing, and as inequality increases further, the highest possible budget surplus and external lending. Furthermore, the case of private borrowing and lending has the interesting implication that poor, unequal economies exhibit two-way flows of resources with the rest of the world, with private flows going in opposite direction to official flows. This occurs even though private agents and the government face exactly the same world interest rate.

It is important to emphasize that the dynamics of the model are driven by the interplay of income distribution and the redistributive fiscal system. In fact, when private agents cannot borrow or lend the current account is entirely a reflection of the government budget. Thus, the model can also be interpreted as a study of the relationship between income distribution and budget deficits when fiscal policy is primarily redistributive, as it is indeed the case in most industrialized and developing countries.
The paper is organized as follows. The next section introduces the model. In order to isolate the role of income distribution, section 3 studies the case of the representative agent version of the economy. Section 4 discusses some general features of the case of income dispersion and introduces some restrictions on the parametrization of the model in order to allow a clearer exposition of this case. Section 5 derives the equilibrium in a poor economy, while section 6 does the same for the case of a rich economy. Section 7 analyse the effects of shocks to productivity and income distribution, and of opening up private credit markets, respectively. Section 9 discusses the main assumptions and possible extensions of the model, and its relationship with some related literature. In sections 3 to 8, the emphasis is on the main intuition, rather than on algebra. Complete formal proofs of all the propositions of the model can be found in the appendices.

2 The model.

1. Technology and factor endowments. A single factor, labor, can produce a single non-storable good using a constant returns to scale production function: \( y = \theta n \), where \( n \) is the input of labor and \( \theta \) - a strictly positive, deterministic parameter - represents a technological shift factor. The good can be traded freely, and its world price is normalized to 1. Each individual supplies his endowment of labor inelastically. Thus, the income of an agent with labor endowment \( n \) is \( \theta n \). The economy is inhabited by a total mass 1 of individuals. The total - and average - endowment of labor of the economy is normalized to 1. As a consequence, the total and average income of the economy is \( \theta \).

The economy lasts for two periods. Any individual can move abroad at the beginning of the second period. To use the foreign technology and earn income abroad, the individual must pay the fixed cost \( d \) in the second period. Hence, \( d \) can be interpreted as the loss of income that an individual must suffer in order to use the foreign technology. The production function abroad is \( y = n \). Thus, a value of \( \theta \) greater than 1 means that the home country is richer than the rest of the world, and conversely if \( \theta \) is less than 1. Income abroad is not taxed. In summary, an agent with labor endowment \( n \) who moves abroad earns \( n - d \), as opposed to the pre-tax income \( \theta n \) he would be earning at home.

2. The distribution of endowments. The total endowment of labor in the home country is distributed among three types of agents, A, B and C, with per capita endowments \( n_A \), \( n_B \) and \( n_C \) respectively, where \( n_A \leq n_B \leq n_C \). The mass of type \( i \) agents is \( p_i \), where obviously \( p_A + p_B + p_C = 1 \). In addition, I assume that:
(i) \( p_i < .5 \);
(ii) \( p_A \leq p_B \)
According to condition (i), a group cannot impose its proposal without the support of at least another group. Hence, this assumption rules out trivial equilibria of the voting process. Condition (ii) implies that, when agents C are not present in the economy, a policy cannot be adopted if it does not have the support of the middle class, group B.

3. Preferences. Individuals value consumption according to the utility function:

$$U = \frac{C_{1i}^{1-\phi}}{1-\phi} + \beta \frac{C_{2i}^{1-\phi}}{1-\phi} \quad i = A, B, C$$

where the first subscript refers to the time period and the second to the individual's type. The elasticity of intertemporal substitution is constant and equal to the inverse of \( \phi \): individuals with a lower \( \phi \) are more willing to substitute consumption between the two periods. In the limit, as \( \phi \) tends to 0, the utility function becomes close to linear and the individual is almost indifferent to when he consumes his lifetime income. Most of the results of the paper require an elasticity of intertemporal substitution greater than 1, i.e. \( \phi < 1 \). If this were not the case, the utility of an agent that consumes 0 in a given period would be minus infinity in that period. Consequently, that agent would be indifferent to any pattern of consumption that involves zero consumption in at least one period. To avoid these situations, I assume that \( \phi < 1 \). Since nothing substantial depends on the discount factor \( \beta \), from now on I will assume that it is equal to 1.

4. Fiscal policy. Fiscal policy consists of the simplest possible redistributive system: a proportional tax on income whose proceeds are redistributed lump-sum to all individuals in the economy. Thus, if \( \tau_j \) is the tax rate in period \( j \) and all agents are present in the economy, the economy's total income, \( \theta \), is the tax base and \( \tau_j \theta \) represents total tax revenues. Since tax revenues are redistributed lump-sum, \( \tau_j \theta \) is also the per capita subsidy.

Initially I assume that private agents cannot borrow or lend. They can however shift consumption between periods through the government, which can borrow from and lend to the rest of the world at the given world interest rate. For simplicity, the world interest rate is equal to the rate of time preference, \( \beta \). Hence, borrowing or lending is not motivated by differences between the marginal rate of transformation for the country as a whole and the marginal rate of substitution when consumption is the same in the two periods.

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1Note, however, that empirical estimates of the elasticity of intertemporal substitution tend to reject a value above unity.
As long as private agents cannot lend or borrow, the government budget balance is equal to the current account balance. I denote the budget and current account deficits in period 1 by $X$. A positive value of $X$ indicates that in period 1 the country is running budget and current account deficits and borrows $X$ from abroad, while a negative value of $X$ indicates that in the same period the country lends the amount $-X$ to the rest of the world by running budget and current account surpluses. Any amount the government borrows in period 1 is added to the tax revenues, $\tau_1 \theta$, and distributed lump-sum among all individuals. Conversely, any amount the government lends in period 1 is subtracted from the tax revenues that can be redistributed. Hence, the consumption of an individual of type $i$ in period 1 is:

$$C_{1i} = (1 - \tau_1) \theta n_i + \tau_1 \theta + X \quad (2)$$

In period 2 the government, and the country through it, must repay the amount it borrowed in period 1 if it was a net borrower or, in case it was a net lender in period 1, it receives the amount it lent to the rest of the world. If all agents are present in the economy, so that the income of the economy is $\theta$, the consumption of an agent of type $i$ in period 2 is therefore

$$C_{2i} = (1 - \tau_2) \theta n_i + \tau_2 \theta - X \quad (3)$$

Expressions (2) and (3) make clear that the tax rate that maximizes the consumption of an individual of type $i$ depends only on the value of $n_i$ as compared to 1: if the individual has above-average endowment, his optimal tax rate is 0; if he has below-average endowment, his optimal tax rate is $1^2$.

Note also that, as in many models that analyse the internal politics of debt repayment, such as Alesina and Drazen (1991), Alesina and Tabellini (1990) and Velasco (1992), by assumption the government cannot default on the debt. However, by moving abroad in period 2, an individual can avoid contributing to the repayment of the debt incurred by the government in period 1.

5. The political system. In the first period, all agents vote on the tax rate $\tau_1$ and on the budget and current account deficit, $X$. Because in the second period the amount received from or transferred to the rest of the world is just the opposite than in the first period, in the second period only the tax rate $\tau_2$ remains to be decided by majority voting.

A "policy" in period 1 is a vector whose elements are a value of $\tau_1$ and a value of $X$.

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2Obviously, an individual with exactly the average endowment of labor is indifferent between any tax rate. As a convention, I will assume that in this case he prefers a tax rate of 0. This can be justified, for instance, if there are infinitesimal fixed costs in setting up a tax system.
In period 2, a policy is just a value of \( \tau_2 \). In each period, the "proposal" by an individual of type \( i \) is the policy that maximizes his utility.

The proposal that beats the other two in pairwise comparison is adopted. In the first period the issue space is therefore bidimensional; as it is well known, in this case the existence of a winning proposal is not guaranteed in general. However, since there are only three distinct groups of individuals, the number of proposals that can be voted on is finite. As shown below, this allows to identify stable majorities even in a bidimensional issue space. Note also that, in period 1, each proposal wins or loses as a whole: in other words, when the proposals by agents \( A \) and \( B \) are compared pairwise, it is not possible to vote for, say, the tax rate proposed by agents \( A \) and the value of \( X \) proposed by agents \( B \).

3 The benchmark: the representative agent economy.

An analysis of the representative agent version of the economy described so far helps isolate the specific role of income distribution in this model. The representative agent version of the model is obtained as a special case of the setup of section 2, namely by assuming that there is no dispersion in the endowments of labor, so that \( n^a = n^b = n^c = 1 \).

Intuitively, when all agents have the same income, the fiscal system cannot transfer resources across agents. The only reason to have an unbalanced government budget or current account would be to transfer resources across the two periods. In fact, with non-distortionary taxation, the representative agent can effectively borrow and lend at the world interest rate through the government. However, as long as the economy is rich enough that an agent earns more at home than abroad, the two periods look exactly the same, and the utility of the representative agents is maximized at a balanced government budget and current account.

The following proposition describes formally the behavior of the representative agent economy:

**Proposition 1.**
(i) For all values of \( \theta \), in the representative agent version of the model the government budget and the current account are always balanced.
(ii) For \( \theta \) sufficiently large - specifically, for \( \theta \geq 1 - d \), production and consumption are the same in the two periods, i.e. the economy exhibits perfect consumption smoothing.
Proof.

(i) Note first that each individual is indifferent to the tax rate, for when all agents have the same income, at any tax rate they pay in taxes what they receive in transfers. Thus, from now on the analysis can focus on the only remaining policy variable, the budget deficit and current account $X$. It is easy to show that borrowing a positive amount $X > 0$ is dominated by a strategy involving a balanced current account, $X = 0$. Suppose that the government borrows $X > 0$ in period 1. It is easy to see that there are only two possible outcomes in period 2: either all agents stay, or all agents leave. Suppose first that all agents stay in period 2: their consumption is $\theta + X$ in period 1 and $\theta - X$ in period 2. This is clearly dominated by a strategy involving a balanced current account $X = 0$, which gives the same lifetime income but a perfectly smooth consumption path of $\theta$ in each period. Now suppose that all agents leave in period 2, in which case they can escape repaying the debt. But then, the government will find it impossible to borrow in period 1. Similarly, it is easy to show that $X = 0$ dominates $X < 0$, i.e. any strategy that involves lending a non-zero amount in period 1. This proves the first part of the proposition.

(ii) As long as $\theta \geq 1 - d$, all agents stay in period 2, and by part (i) of this proposition they produce and consume $\theta$ in each period. Thus, for $\theta$ "sufficiently large", the production and consumption paths of the representative agent are also perfectly flat over time.

The basic message of this proposition is that income dispersion is the crucial assumption of the model. Without income dispersion, the assumption that private agents cannot borrow or lend privately is irrelevant, because they can borrow and lend through the government at the world interest rate. It is therefore not surprising that this economy delivers exactly the same predictions as the neoclassical model.

Note also that temporary and permanent shocks will induce the familiar patterns of borrowing and lending.\textsuperscript{3} For instance, an anticipated future negative shock will induce a government budget and current account surplus, while a permanent negative shock will be absorbed in the same proportion in each period and will not induce any imbalance in the government budget or in the current account.

\textsuperscript{3}This is strictly true as long as these shocks are not so large as to induce all agents to move abroad in period 2.
4 The case of income dispersion: introduction.

Before solving formally the case of positive income dispersion, it is useful to discuss and simplify its structure, with the goal of simplifying the exposition without sacrificing the main insights. As it is, the model has many free parameters, which would make the exposition particularly cumbersome. In particular, given the per capita income of the economy $\theta$, there are four free parameters that characterize the distribution of income: two $n_i$'s and two $p_i$'s. A given change in, say, the Gini coefficient can arise from movements in the endowments $n_i$ of the various groups or from movements in their sizes $p_i$, or both. Hence, very little is lost in terms of the analysis of the role of income distribution if one fixes the sizes $p_i$'s of the various groups at some specific value. On the other hand, this assumption simplifies the exposition and the notation substantially. Thus, from now on I will assume the following values for the sizes of the various groups:

$$p_A = p_B = \frac{2}{5}, \quad p_C = \frac{1}{5}.$$  

Still, even after fixing the sizes $p_i$'s, any movement in the endowment of one group can be accompanied by very different patterns of movements in the endowments of the other two groups, leading to a situation where practically anything could happen. The solution of the model would have to consider all possible cases, and would turn out to be lengthy and tedious. Hence, in order to parametrize the distribution of income in a compact way, I assume that the low income class is unproductive: $n_A = 0$. Pinning down the value of $n_A$ has the important implication that given the sizes $p_i$'s, the endowments of the other two groups are monotonically and negatively related: $n_C = [1 - p_B n_B]/p_C$. It is then straightforward to show that frequently used measures of inequality like the Gini coefficient are completely characterized by the value of $n_C$ or, equivalently, $n_B$. Moreover, the two Lorenz curves corresponding to two different values of $n_C$ do not intersect: an increase in $n_C$ is therefore associated unambiguously with an increase in inequality. Note also that a mean-preserving spread in the distribution of income is necessarily associated with an increase in $n_C$. Thus, in this model the terms "increase in inequality" and "high inequality" will be synonymous with "increase in $n_C$ (decrease in $n_B$)" and "high value of $n_C$ (low value of $n_B$)".

Because the values of $n_B$ or $n_C$ completely characterize the distribution of income, it is useful to provide a few reference points. When inequality is at a minimum, $n_B = n_C$ and both are equal to $5/3$ \[5/3 = (1 - p_A n_A)/(p_B + p_C),\] after substituting the specific values assumed above for all the parameters in this expression. At the other extreme, when inequality is at a maximum, $n_B = 0$ and $n_C = 5$ \[5 = (1 - p_A n_A - p_B n_B)/p_C,\] again after substituting in the specific values assumed above and $n_B = 0$. Also, of particular importance in the analysis of the model is whether $n_B$ is greater or smaller than the
average endowment 1, since in the former case both agents B and C favor the lowest possible tax rate, while in the latter case both groups A and B favor the highest possible tax rate, 1. For future reference, \( n_B > 1 \) implies \( n_C \leq 3 \).

There is also a second, more substantive reason for fixing \( n_A \) at 0, besides a clearer exposition. This assumption captures the existence of consistent segments of the population that are outside the production process and whose consumption is therefore closely tied to the extent of redistribution in the economy. These segments of the population therefore represent a powerful constituency whose only goal is to maximize redistribution. Group A in this model captures exactly this notion.

Because the focus of the paper is on the role of the distribution of income rather than factor mobility per se, the exposition and notation of the model can be further simplified without sacrificing much in terms of insights by restricting also the cost of moving abroad, \( d \). For convenience, I assume that \( d \) is equal to the average endowment of labor of the home country, 1. Note that this assumption implies that \( \theta \) is always "sufficiently large" in the terminology of Proposition 1, i.e. that \( \theta > 1 - d = 0 \), so that a representative agent economy would always exhibit perfectly flat production and consumption paths over time.

Another important issue concerns the possible existence of multiple equilibria. For some values of the debt and of the other parameters, in this model there might be more than one equilibrium. To illustrate the nature of this issue, suppose the country borrows \( D \) in period 1, and in period 2 the tax rate is just what is needed to repay the debt. If all agents B and C stay in period 2, this tax rate is defined by \( \tau_2 \theta = D \). Agents C’s consumption in this case would be \( n_C(\theta - D) \). If all agents C leave while all agents B stay, the tax rate will be such that \( \tau_2 {\theta} n_B = D \). An agent C that deviates and decides to stay would therefore consume \( n_C(\theta - D) < n_C - 1 < n_C(\theta - D) \). Hence, for a range of values of \( D \) there would be three Nash equilibria, one with all agents C staying, one with all leaving, and one with some agents C staying. Since the focus of this paper is not on multiple equilibria, I assume that in these cases the first, Pareto-superior equilibrium is chosen. The same assumption applies also to agents B.

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4As shown below, this occurs for instance when \( n_C > 1 \) and \( n_B > 1 \), so that both groups vote for the minimum tax rate required to repay the debt.

5In addition, note that the second equilibrium is unstable.

6Appendix A explores more formally the implications of this assumption.
5 Equilibrium policies in a poor economy.

The problem solved by all agents in an economy with positive income dispersion is fundamentally different from that of the representative agent economy: since taxation is proportional but redistribution is lump-sum, any reallocation of consumption between the two periods through the government budget also implies some redistribution of lifetime income across the three groups. It is this property of the government budget that can induce patterns of fiscal policy and external borrowing which are inconsistent with the logic of the representative agent model. Moreover, whether and to what extent this can happen depends critically on the level of income per capita and on the degree of inequality.

I begin the analysis of the effects of income dispersion with the case of a poor economy. I define a poor economy as an economy where \( \theta < 1/2 \). One important implication of this inequality is that, because productivity at home is so low, an agent with a sufficiently large endowment will leave in period 2. Thus, there is a specific sense in which poor economies are more vulnerable to inequality in the distribution of income. In fact, the key result of this section is that a poor economy with very low inequality will exhibit perfect consumption smoothing and balanced current account and government budget; however, as inequality increases the economy displays a surprisingly different behavior from that of a representative agent economy, and can exhibit highly counterintuitive patterns of the consumption path and the current account. In fact, at high levels of inequality, a majority consisting of the two groups at the extreme ends of the distribution of income, A and C, support a policy involving borrowing the maximum possible amount in period 1, even though production is already higher in period 1 than in period 2\(^7\). As inequality increases further, a majority composed of groups A and B support a policy that smooths aggregate consumption partially, involving a current account surplus.

I provide here the main intuition for this result, leaving a formal proof to Appendix B. At the heart of the result are the different incentives of agents C to move abroad at different levels of inequality\(^8\). Recall that an agent of type C will move abroad in period

\(^7\)To avoid a tedious list of all possible cases, I also assume here the more interesting case of \( \theta > 2/5 \). This ensures that, when \( n_B \geq 1 \), the income of agents B if they stay and there is no debt to repay, \( \theta n_B \), is not less than what they could earn abroad, \( n_B - 1 \). Thus, agents B stay as long as the debt is not too high. If instead \( \theta n_B < n_B - 1 \), agents B would leave even at \( X = 0 \), and the country cannot borrow any amount in period 1 since no agent with positive endowment would be left to repay it in period 2. The resulting equilibrium would be trivial to analyse. If \( n_B < 1 \), agents B cannot leave and the equilibrium would be similar to that derived in this section. The case \( \theta < 2/5 \) can be analysed easily along the lines of this section, but it does not add any important insight to the main intuition.

\(^8\)Agents B could also move abroad in period 2, when their endowment of labor is above the cost of moving, 1. However, it is intuitive that, if agents B move, agents C also move (see Appendix A), and
2 whenever the differential between consumption at home and abroad is negative. If all agents are present in the economy, so that the average income is \( \theta \), this differential is 
\[
(1 - \tau_2) \theta n_C + (\tau_2 \theta - X) - (n_C - 1),
\]
where the first two terms represent consumption at home (see expression (3) and its explanation) and the third represents consumption abroad. Hence, there are three determinants of the consumption differential in period 2. First, the pre-tax income differential \( \theta n_C - (n_C - 1) \): at low levels of \( n_C \), this differential is positive; as \( n_C \) increases, the incidence of the moving cost in total income abroad falls, and past some value of \( n_C \) this differential becomes negative, and increasing in absolute value. Second, the tax rate: since agents C have above-average income, their post-tax income at home is a decreasing function of the tax rate. Third, the current account: if the country borrowed a positive debt \( X > 0 \) in period 1, the debt repayment reduces consumption in period 2 for an agent C who decides to stay.

When inequality is in a region around its minimum value, \( n_C = 5/3 \), all three components of the consumption differential in period 2 work in favor of agents C staying. At low level of inequality, the pre-tax income differential is positive; in addition, because agents B too have above-average endowment, if agents C stay in period 2 both groups B and C will vote for the lowest possible tax rate. The overall utility of both groups B and C is then maximized when \( \tau_1 = \tau_2 = 0 \) and \( X = 0 \). Under this policy, both groups smooth consumption perfectly and maximize their lifetime income, since as we have seen above at \( \tau = 0 \) both groups earn more at home than abroad.\(^9\)

When \( \tau_2 = 0 \), \( X = 0 \), the consumption of agents C in each period is \( \theta n_C \), their pre-tax income, and the consumption differential in period 2 is just \( \theta n_C - (n_C - 1) \). As inequality increases, and so does \( n_C \), this differential, although still positive, becomes very small. Therefore, after some point \( n_C^1 \), the policy that maximizes agents C’s utility changes drastically. Rather than smoothing consumption perfectly, agents C can leave the country in period 2, and maximize consumption in period 1 by having the government borrow the maximum possible amount \( X = D_{\text{max}} \).\(^{10}\) In fact, once they leave the country in period 2, any increase in period 1 consumption obtained by borrowing from abroad comes at no cost to them in terms of period 2 consumption. Relative to the previous policy, this causes a fall in period 2’s consumption from \( \theta n_C \) to \( n_C - 1 \). However, this is

\( \text{the country cannot borrow any amount}. \) The maximum debt that the country can incur, therefore, is the debt that leaves agents B indifferent between staying and moving. As argued in the previous footnote, for \( \theta > 2/5 \) this debt is positive. Therefore, in this intuitive exposition of the solution one can concentrate on the decision to leave of agents C only.

\(^9\)Because the consumption differential is decreasing in \( n_C \), if it is positive for agents C, \textit{a fortiori} it is positive for agents B.

\(^{10}\)As shown in Appendix A, \( D_{\text{max}} \) is that value of borrowing that leaves agents B indifferent between staying in period 2 and repaying the debt, or leaving and consuming \( \theta n_C - 1 \).
more than compensated by the increased period 1 consumption from $\theta n_C$ to $\theta n_C + D_{\text{max}}$. Thus, past a certain value $n^1_C$, agents C propose $\tau_1 = 0$ and $X = D_{\text{max}}$, and then leave the country.

At still higher levels of $n_C$, the pre-tax income differential $\theta n_C - (n_C - 1)$ becomes negative even at $X = 0$. Hence, even if there is no debt to be repaid and $\tau_2 = 0$, agents C certainly leave in period 2. A fortiori, then agents C will propose $\tau_1 = 0$ and $X = D_{\text{max}}$ in period 1.

Importantly, whenever agents C leave the country in period 2, agents A too propose borrowing the highest possible amount $D_{\text{max}}$. The fundamental reason is that, once agents C have moved abroad, there is no redistribution of labor income in period 2, as the only group left with positive endowment, group B, clearly opposes any redistribution to agents A. Hence, the tax rate in period 2 will be just enough to repay the debt contracted in period 1. But then, any borrowing in period 1 increases agents A’s consumption at no cost in terms of their period 2’s consumption. Clearly, agents A too propose borrowing the highest possible amount in period 1, $X = D_{\text{max}}$. Thus, in an interval to the right of $n^1_C$, a majority composed of the two groups at the extremes of the distribution of income votes for $X = D_{\text{max}}$. However, the two groups diverge on the preferred tax rate: agents C prefer $\tau_1 = 0$, agents A $\tau_1 = 1$. As long as $n_B \geq 1$, the former prevails, because agents B prefer C’s proposal to A’s proposal: both have the same level of debt $D_{\text{max}}$, but at least the former has the tax rate that agents B prefer, $\tau_1 = 0$.

When inequality increases further, so that $n_B$ falls below 1, a fortiori agents C will leave in period 2 and vote for $X = D_{\text{max}}$ in period 1. In fact, at these levels of inequality, the pre-tax income differential is already negative; in addition, if they stayed, the tax rate in period 2 would be 1, as both groups A and B now would have below-average endowments. The logic of the model, however, remains the same: it is still true that a majority composed of groups A and C vote for $X = D_{\text{max}}$ in period 1. The only difference is that now the tax rate in period 1 will be 1 rather than 0, since agents B now prefer A’s proposal (involving $X = D_{\text{max}}$ and $\tau_1 = 1$) to C’s proposal (also involving $X = D_{\text{max}}$ but $\tau_1 = 0$).

When inequality increases still further, past a certain value $n^2_C$, the position of agents A changes. At these levels of inequality, $n_B$ is very small and therefore $D_{\text{max}}$ is also very small. When $D_{\text{max}}$ is small, it does not pay for agents A to give up consumption smoothing in order to consume everything in period 1. Rather, they prefer a policy involving perfect consumption smoothing, which implies lending some of the tax revenues

\footnote{In fact, when $n_B < 1$ agents B cannot move and therefore $D_{\text{max}}$ is equal to the aggregate income of agents B, $\frac{1}{2} \theta n_B$, which is decreasing in $n_C$.}
collected in period 1 (recall that the equilibrium tax rate is 1 in period 1). As agents B too would like to lend, a policy with $\tau_1 = 1$ and $X < 0$ prevails in equilibrium. Thus, at very high levels of inequality the country lends a positive amount, and the policy has the support of the two groups A and B.

The following proposition summarizes the results of this section:

**Proposition 2.**
(i) At very low levels of inequality (for $n_C \geq n_C^1$, with $n_C^1 < 3$), the average income and consumption of a poor economy are the same in each period. As the distribution of income becomes more unequal (for $n_C \in (n_C^1, n_C^2)$, with $n_C^2 > 3$), the average income declines over time, yet the economy runs budget and current account deficits in period 1. At still higher levels of inequality (for $n_C \geq n_C^2$), the economy runs budget and current account surpluses in period 1.
(ii) Except at very low and very high levels of inequality, the winning proposal involves high budget and current account deficits, which are supported by the two groups at the opposite extremes of the distribution of income, A and C.

**Proof.**
See Appendix B.

Thus, when inequality is high, although not extreme - for $n_C \in (n_C^1, n_C^2)$ - the country borrows the largest possible amount it can repay. This occurs despite the fact that the average income is higher in period 1 than in period 2. Hence, to an outside observer the economy exhibits a perverse pattern of budget deficits and external borrowing: the average income declines over time, and consumption declines even faster. This is true even if one considers only the average income of the two groups of agents that remain in the country in both periods, A and B: their disposable income is higher in period 1 than in period 2, yet the pattern of borrowing amplifies this asymmetry rather than smoothing it.

One puzzling aspect of these experiences is the fact that this drastic reversal of policies can be easily anticipated, and seems therefore inconsistent with any rational, forward-looking behavior on the part of private individuals and policymakers alike. In the existing literature on the topic (e.g. Alesina and Drazen (1991), Laban and Sturzenegger (1992), Velasco (1992)), delays in stabilization result from the failure of the two groups to agree on the distribution of the costs and on the features of a stabilization. As time passes, the costs of the inefficient policy become so large that eventually stabilization takes place. The models differ in exactly how and why the stabilization
takes place. In Alesina and Drazen, each group is imperfectly informed on the costs of the stabilization to the other group; after engaging in a "war of attrition" with the other, eventually one group concedes and bears most of the costs of the stabilization. In Velasco, after the inefficient policy has been in place for some time, an equilibrium in which all groups agree to the new policy becomes sustainable. In Laban and Sturzenegger, one group is increasingly affected by the inefficient policy and eventually becomes willing to agree to a stabilization with uncertain results.

In this model, similar outcomes stem from the interplay of factor mobility and redistributive fiscal policy in the presence of high inequality and perfectly rational and forward-looking agents. A distinctive feature of this model is that its dynamics is generated by the interaction of three, rather than two, groups. This framework captures two important features of many episodes of delays in stabilization. First, difficulties in stabilizing an economy seem to be correlated with high inequality in the distribution of income (see Berg and Sachs (1988) for some evidence on the correlation between external borrowing and inequality). Second, the initial fiscal expansion is often backed by both the trade unions and the associations of industrialists, in the latter case because the high resulting level of demand leads to high profits (see Dornbusch and Edwards (1990)). Both features appear, albeit highly stylized, in this model: when the distribution of income is unequal, the two groups at the extremes of the distribution of income, A and C, support an expansionary fiscal policy financed by high budget and current account deficits, at the expense of the middle group, B.

Note also that the equilibrium policy involving maximum borrowing is the preferred outcome of agents C only for \( n_C \in (n_C^1, 3] \). For \( n_C \in (3, n_C^2) \), the equilibrium policy implies \( \tau_1 = 1 \), which agents C dislike. However, this policy is still preferred to agents B’s proposal, since it allows agents C a higher consumption in period 1 while still enabling them to escape all the costs of the stabilization.

6 Equilibrium policies in a rich economy.

In contrast to poor economies, rich economies exhibit perfectly flat output and consumption paths over time regardless of how income is distributed. Thus, to an outside observer these economies appear isomorphic to the representative agent economy of section 3. The basic intuition is simple. Consider a rich economy, with \( \theta > 4 \). In this economy, if there is no debt to be repaid, the richest possible individual of type C, with \( n_C = 5 \), still consumes more at home when the tax rate is 1 and he only gets the average income \( \theta \), than abroad, where he gets \( n_C - 1 \). Thus, in this economy, agents C never leave the country if \( X = 0 \), whatever tax policy is implemented. A fortiori, this is true for
agents B, who have a lower endowment and therefore benefit less from moving abroad. A majority of agents can therefore maximize their lifetime income by implementing in each period the tax rate they prefer \((\tau = 0\), supported by groups B and C if \(n_B \geq 1\), or \(\tau = 1\), supported by groups A and B if \(n_B < 1\)) and smooth consumption at the same time by having a balanced current account.

The following proposition develops this intuition formally:

**Proposition 3.**
At any level of inequality, the aggregate income and consumption paths of a rich economy are identical to those of a representative agent economy with the same average income: in particular, the budget and the current account are always balanced.

**Proof.**
Consider first the case of low inequality, where \(n_C \leq 3\), or, equivalently, \(n_B \geq 1\). Since both agents B and C have above average endowments, both groups dislike taxation and redistribution, and propose the lowest possible tax rate in each period. In addition, both groups also propose a perfectly balanced government budget and current account. Intuitively, at \(\tau_1 = 0, X = 0\) and \(\tau_2 = 0\), disposable income and consumption are the same in the two periods and lifetime income is maximized. In fact, because both groups have above-average endowments, there is no linear redistributive scheme that increases their lifetime disposable income.

Now consider the case of high inequality, where \(n_C > 3\), or, equivalently, \(n_B < 1\). If agents C are present in period 2, both groups A and B have below average endowments, and therefore both propose \(\tau_2 = 1\). And indeed, as shown above, if there is no debt to be repaid agents C are present in period 2, even when \(\tau_2 = 1\). It is then clear that at \(\tau_1 = 1, X = 0\) and \(\tau_2 = 1\), agents A and B can maximize their lifetime income and achieve perfect consumption smoothing.

Hence, in a rich economy a majority of agents always favor a balanced budget and current account. The same majority also favors a policy of no redistribution when inequality is low, and the largest possible redistribution when inequality is high. To an outside observer, a rich economy with income inequality is indistinguishable from an economy with no income dispersion, and from the representative agent, neoclassical model.

At levels of income between a poor and a rich economy, i.e. for \(\theta \in (1/2, 4)\), the logic of the model is unchanged. However, under some configurations of the distribution of income some complications arise regarding the definition of an equilibrium and the existence of a non-cycling majority. I illustrate the nature of these problems and some
possible solutions in Appendix D.

7 The effects of shocks.

The differences between rich and poor economies, and between equal and unequal societies, extend to their responses to exogenous shocks. Two types of shocks in this model generate interesting implications: shocks to productivity and to the distribution of endowments.

As an example of the effects of productivity shocks, consider a permanent negative shock, i.e. a permanent fall in $\theta$. Both an economy with no income dispersion at any level of income or a rich economy with any degree of dispersion will respond exactly as in the neoclassical model, by reducing the consumption in both periods by the same amount. Thus, the government budget and the current account will remain perfectly balanced and there will be no distributional effects.

The response of a poor economy can be remarkably different, however. As an example, suppose inequality is low, so that $n_C$ is slightly below $n_C^1$, as defined in section 5. In other words, initially both groups B and C vote for $\tau_1 = 0$, $X = 0$, and the government budget and the current account are perfectly balanced. As shown formally in Appendix E, a permanent fall in $\theta$ causes a fall in $n_C^1$, i.e. an enlargement of the interval over which agents C propose $X = D_{\text{max}}$ rather than $X = 0$. The intuition is simple: when $\theta$ falls, the consumption of agents C when $X = 0$, $\theta n_C$, falls proportionally in each period. When $X = D_{\text{max}}$, however, their consumption falls only in the first period, because in the second period they move abroad, where they consume $n_C - 1$ independent of $\theta$; hence, this policy becomes more attractive at any level of $n_C$. After the fall in $\theta$, then, agents C might propose $\tau_1 = 0$ and $X = D_{\text{max}}$, while before they proposed $\tau_1 = 0$, $X = 0$. As agents A too favor the highest possible debt, the economy might respond to a permanent shock by switching from perfectly balanced budget and current accounts with no redistribution to a policy of the highest possible external indebtedness. Since the debt is redistributed to agents A, this shock has also important distributional effects.

Now consider shocks to the distribution of endowments, such as an increase in the Gini coefficient. As explained in section 4, in this model any change in inequality must correspond to a movement of $n_C$ in one direction with a corresponding movement of $n_B$ in the opposite direction. In particular, a mean-preserving spread or an increase in the Gini coefficient are all equivalent to an increase in $n_C$.

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Footnote: Appendix E shows that the fall in $\theta$ makes it less attractive for agents A to vote for $D_{\text{max}}$ rather than for agents B's proposal. However, it might still be true that at the new value of $n_C^1$ agents A prefer $D_{\text{max}}$. 

16
Once again, the government budget and the current account of a rich economy are immune from an increase in inequality: both remain perfectly balanced, so that aggregate production and consumption too remain perfectly flat over time. At most, the tax rate changes from 0 to 1 in both periods, if the increase in inequality drives $n_B$, the income of the median group, below the average income. In a poor economy, however, an increase in inequality can have a dramatic effect on the current account. For instance, now an increase in inequality that causes $n_C$ to rise above $n_{C}^1$, again shifts the equilibrium policy from a balanced current account to the maximum possible external indebtedness.\textsuperscript{13}

In summary, in this model \textit{permanent} productivity or income distribution shocks might not be absorbed equally over time, contrary to the predictions of the neoclassical model. Rather, they can induce responses in aggregate production and consumption that have a highly unbalanced temporal profile.

8 Private borrowing and lending.

So far, private agents were prevented from borrowing and lending. Removing this assumption means that individuals can now propose the value of $X$ that maximizes their lifetime \textit{income}, using the private credit markets to smooth \textit{consumption} by allocating the lifetime income evenly between the two periods.

Thus, allowing for private borrowing and lending has two important effects in this model. First, it encourages even more extreme patterns of fiscal policy and external borrowing or lending by the government, since individuals are not prevented from proposing extreme values of $X$ by the need to smooth consumption. Second, it gives rise to a simultaneous two-way flow of resources, with private flows going in opposite direction to official flows, exactly because individuals use the private credit markets to smooth the path of consumption relative to income.

However, these two results obtain only in poor economies with high inequality. As shown in sections 5 and 6, in rich economies or in poor economies with low inequality, in the absence of private credit markets the equilibrium tax rate is the same in both periods and the current account is always balanced. Hence, a majority of agents already maximize lifetime income \textit{and} smooth consumption at this policy. As a consequence, there is no need to compensate with private borrowing or lending when private credit markets are opened.

By contrast, opening up private credit markets in poor economies with high inequality

\textsuperscript{13}Note that, contrary to the case of a productivity shock, now $n_C^1$ is not affected by the shock.
has rather drastic consequences on the equilibrium policies. I illustrate the main intuition by considering the case of a poor economy with high inequality, with \( n_C \) in a region to the right of \( n_C^1 \), as defined in Section 5.

First, the range of values of \( n_C \) over which agents C move abroad and propose the maximum debt becomes larger. In fact, in the absence of private credit markets, agents C move abroad and vote for \( X = D_{\text{max}} \) whenever this policy gives a higher \textit{utility} than the alternative policy involving staying and \( X = 0 \). This is exactly the criterion that defines the cut-off value \( n_C^1 \) in Proposition 2. With private credit markets, agents C vote for the first policy, \( X = D_{\text{max}} \), whenever it gives a higher \textit{lifetime income}. This obviously occurs on a wider region of values of \( n_C \), including a region to the left of \( n_C^1 \), between some value \( \tilde{n}_C \) and \( n_C^1 \). In fact, at \( n_C^1 \) the lifetime income of agents C was already higher under the first policy, but lifetime utility was the same because the first policy implied a more unbalanced income and therefore consumption path. With private credit markets, however, consumption is delinked from income, and all that matters is maximizing lifetime income. As the policy of borrowing \( D_{\text{max}} \) still maximizes agents A’s lifetime income, the region where a majority composed of groups A and C propose \( X = D_{\text{max}} \) becomes larger, and includes the interval between \( \tilde{n}_C \) and \( n_C^1 \). Hence, opening up private credit markets leads to more extreme fiscal policies and official borrowing.

As in section 5, since \( n_B \geq 1 \) when \( n_C \) is close to \( n_C^1 \), the winning tax rate in period 1 is \( \tau_1 = 0 \). Also, because agents C leave, in period 2 the winning tax rate is just enough to repay the debt, without any redistribution to agents A. Therefore, under the winning policy \( \tau_1 = 0 \) and \( X = D_{\text{max}} \), the income path of all agents is highly unbalanced. The post-tax income of agents A is \( D_{\text{max}} \) in period 1 and 0 in period 2. As a consequence, when private credit markets are opened, agents A will use them to smooth consumption by lending privately. The same reasoning applies to agents B, who must repay the debt in period 2. By contrast, agents C have a higher income in period 2, and therefore they borrow privately in period 1. Thus, the existence of private credit markets leads to a two-way flow of resources, with private flows going in opposite direction to official flows.

Similarly, at even higher levels of inequality (for \( n_C \) in an interval \([\tilde{n}_C, 5]\), with \( \tilde{n}_C > n_C^1 \)), the government runs the highest possible budget surplus and lends abroad all tax revenues. The reason is that, at these levels of inequality, agents C leave anyway in period 2. By running a budget surplus and lending abroad, agents A and B effectively transfer the tax revenues collected in period 1 to period 2, where they do not have to share them with agents C. Clearly, the lifetime income of agents A and B is maximized when \( \tau_1 = 1 \) and all period 1 national income \( \theta \) is transferred to period 2. To balance their consumption paths, all individuals then borrow privately.

It is also important to emphasize that in this model the government and all the
private agents face the same world interest rate. As a consequence, there would be no reason for resources to flow in both directions in a representative agent economy. Here, the result arises from the combination of a positive dispersion in the distribution of income and a fiscal system that redistributes resources across different groups.

9 Discussion and conclusions.

This paper has presented a model where income distribution and redistributive fiscal policy interact to affect the budget deficit and the pattern of net borrowing of a country. Income distribution is irrelevant at high levels of income, but becomes a major determinant of the shape of the aggregate consumption path at low levels of income. Furthermore, while a rich economy behaves according to the traditional paradigm, a poor economy with an unequal income distribution might follow a consumption path which is essentially the opposite of that posited by standard theories.

It is interesting to compare these conclusions with those of the representative agent model of Dornbusch (1983). There, the presence of a non-traded good creates a wedge between the world real interest rate and the marginal rate of transformation facing the representative agent whenever the price of the non-traded good is changing over time. In the present model too the underlying driving force is a wedge between the world interest rate and the marginal rate of transformation perceived by the agents of the economy. However, here the difference is not due to the changing price of the composite non-traded good. Rather, the existence of a redistributive system that allocates the costs of debt repayment asymmetrically across income groups means that the low income and high income groups might face a very low marginal rate of transformation, which can be even 0 in the limit. This of course might create a strong constituency in favor of a high budget deficit and external borrowing.

It is also interesting to compare the rationale for budget deficits in this model to that of Tabellini and Alesina (1990) and Alesina and Tabellini (1990). There, the median voter in period 1 is uncertain about the future median voter’s identity and preferences over a public good. By running a budget deficit, a risk-averse median voter in period 1 can therefore constrain the future median voter to use future tax revenues to repay the debt, rather than to spend on a public good that he might dislike. In these contributions as well as in the model of my paper the driving force behind the existence of budget deficits is that the future looks different from the present from the viewpoint of the current majority. The two explanations differ in what causes this asymmetry between the two periods. In Tabellini and Alesina (1990) and Alesina and Tabellini (1990), the cause is a random shock to preferences. In my model, it is the interplay between factor
mobility and redistribution.

In a similar vein, Alesina and Tabellini (1989) present an explanation for the simultaneous existence of private and official flows which differs from that developed in this paper. In their model, two classes, capitalists and workers, alternate in office with given probabilities. When capitalists are in office, they might decide to borrow from abroad in order to constrain the choice of the future policymakers. At the same time, they might export some capital as a form of insurance against future changes in policy. In my model, private capital flows are not due to uncertainty about the future, but to the desire to smooth consumption in the presence of large imbalances in the government budget and the current account.

Several assumptions of the model deserve further discussion. First, the model assumes that an individual can escape taxation by moving abroad. In the real world, the phenomenon of actual migration of individuals with all their human capital in response to taxation is relatively rare, and of limited macroeconomic significance. However, the assumption that individuals can move abroad can be easily reinterpreted as capturing the possibility of escaping high tax rates by exiting the formal sector, which can be done at a cost. This reinterpretation would require very little changes in the structure of the model. In particular, one only needs to impose the additional, reasonable condition that an individual that operates illegally in the underground economy and therefore does not pay taxes is not entitled to any redistribution.

The model can also accommodate, in a more stylized manner, other important phenomena that are highly influenced by fiscal policy. One could reinterpret the endowment of labor of the rich and mobile agents as capital. Although migration of physical capital in response to fiscal conditions is also rare, large drops in investment rates are frequently observed in response to macroeconomic imbalances and mismanagement. The act of moving capital abroad in the second period captures in a stylized way the effects of letting the capital stock depreciate at home while investing resources abroad. The rapid decadence of the manufacturing sector during prolonged periods of overvaluation of the real exchange rate in Latin America is an example of this phenomenon.

A prolonged fall in investment has at least two effects on the disposable income of the immobile factors that combine with capital in the production function: it reduces their pre-tax income, and it reduces the tax base and therefore the resources available for redistribution. Because it has only one non-accumulable factor, the present model can rigorously capture only the second type of effect. To capture the first effect, it would be necessary not only to add a second factor, capital, but also a second intertemporal problem, how much capital to accumulate. This problem would have to be solved together with the problem of whether to move capital or not in the second period. It is
easy to see that the model would quickly become very difficult to handle.

Furthermore, the advantage of having only one factor with a linear production function is that the reward to that factor per unit provided is constant regardless of the total employment of the factor in the economy. Consequently, changes in income distribution are easy to track. If the economy produces two goods, factor rewards would still be constant because of the factor price equalization theorem, and movements in income distribution would still be fairly easy to trace. However, once enough capital has moved abroad in response to taxation, the economy would specialize in the production of one good, and factor rewards would become endogenous. The model would again become extremely difficult to handle.

Fortunately, however, these complications are not necessary. In the present model, the tax base is mobile in the long run. When part of the tax base escapes taxation, because of the linearity of production factor rewards do not change; however, the second effect of a fall in investment and the capital stock, namely the reduction in tax revenues for redistribution, is still present. This is enough to generate the dynamics the model focuses on. The first effect, namely the drop in the reward to the immobile factor, would make the model more realistic, but the underlying logic would be the same.

The act of moving abroad in this model could also be thought of as a proxy for capital flights, which are typically associated with mismanagement of fiscal policy. Strictly speaking, capital flights are difficult to capture in this model for two reasons. First, there is no room for financial instruments that can be moved quickly into a different denomination; second, in the first part of the model I assume that private individuals cannot borrow and lend, which would be difficult to reconcile with the existence of capital flights. However, the important feature of capital flights from the point of view of the logic of this model is that they are typically associated with a decline in investment and therefore in the standards of living. Thus, the model seems to be able to capture the events typically associated with capital flights.

A second important assumption concerns the treatment of debt. In this model, the only role of the government is to redistribute income. As in all real, dynamic models, there are two ways to finance government expenditure: taxation and debt. Hence, in this model government debt accomplishes two tasks at once: it redistributes income across individuals with different lifetime incomes, and it shifts consumption from one period to the other. In principle, the redistributive and intertemporal allocative function of the government budget could be separated. The former can be accomplished by taxing labor proportionally and redistributing the proceeds lump-sum. The latter can be accomplished by distributing the debt proportionally to each individual’s income, and taxing individuals proportionally to repay it. However, this would make government debt
equivalent to forcing individuals to borrow privately a given proportion of their income, which seems highly unrealistic.

Note that, when voluntary private borrowing and lending is allowed, as in the second part of the paper, there is still a role for government borrowing and lending: the reason is precisely that government borrowing and lending is a way to finance redistribution in a given period, over and above the tax revenues collected in that period. This illustrates the importance of considering the redistributive role of government in models of open economies, and not just its allocative role, i.e. purchases of goods and services.

A third important assumption of the model is that each individual proposes the policy that maximizes his utility and votes sincerely. As it is often the case, allowing for strategic voting could introduce significant complications: the outcome would depend on the allowable strategies and on the definition of equilibrium one adopts. However, in this particular model strategic voting is unlikely to change the results significantly. First, it is clear that every time the same proposal maximizes the utility of at least two groups, allowing for strategic voting would have no effect on the outcome, as neither group would have any incentive to make a different proposal. Second, if the three proposals are all different, a stable winner under sincere voting exists whenever a proposal is a "median in all directions", i.e. its tax rate is intermediate between the tax rates of the other two proposals and the same is true for its external borrowing. In this case, the median in all directions is the most reasonable and intuitive winner under sincere voting, since the other two proposals are at the opposite extremes on both dimensions. It is unlikely that strategic voting can reshuffle proposals as to put together these two "extreme" proposals. In most of the cases explicitly analysed in the paper, there is a median in all directions; consequently, the results presented here appear to be quite robust to strategic voting. However, for those configurations that do not have a non-cycling majority under sincere voting, it is difficult to predict what effects strategic voting would have on the existence and the characteristics of the equilibrium.

Note however that sincere voting does not mean that voters are irrational or myopic. In fact, in this model voters are perfectly forward-looking, as they take into account the effects of their first period proposal, if it prevails, on the economic and political equilibria of the economy in the second period.

Finally, an important feature of the model presented in this paper is that it allows both for repeated voting and a bidimensional issue space. In general, the median voter result does not apply in this case. However, by allowing only the preferred policies of each agent as admissible proposals, a stable majority can be identified in this model. Furthermore, in this setup the interesting possibility arises that the winning policy is the proposal of one of the two groups at the extreme ends of the distribution of income.
Appendix A.

This Appendix shows how the maximum possible debt in period 1, $D_{\text{max}}$, varies with $n_C$. For expository purposes, I consider two cases separately, first when $n_B \geq 1$ and then when $n_B < 1$. In both cases, I consider the case of a poor economy, $\theta \leq 1/2$.

$n_C \in \left[\frac{5}{3}, 3\right]$.

Suppose the country borrows the amount $D$ in period 1. Because $n_B \geq 1$ and $n_C > 1$, in period 2 both groups B and C vote for the minimum tax rate required to repay $D$. There are three cases:

(i) If all agents stay, the tax base is $\theta$ and the tax rate is $D/\theta$. Therefore, all agents stay if and only if:

$$n_C(\theta - D) \geq n_C - 1, \quad n_B(\theta - D) \geq n_B - 1 \quad (A.1)$$

(ii) When $\theta < 1$, if inequality (A.1) is satisfied for agents C, it is also satisfied for agents B. Therefore, outside case (i), necessarily $n_C(\theta - D) < n_C - 1$ or, in other words, $D$ is larger than

$$D_{\text{max}}^C \equiv \max \left\{0, \ (\theta - 1 + \frac{1}{n_C}) \right\} \quad (A.2)$$

and all agents C leave. Once agents C leave, the maximum debt that the country can incur is then limited by the fact that it must be repaid by agents B only. Hence, once agents C have left, $D$ must be such that

$$\theta n_B - \frac{5}{2} D \geq n_B - 1 \quad (A.3)$$

This implicitly defines the maximum possible debt $D_{\text{max}}^B$ that can be incurred after agents C leave as

$$D_{\text{max}}^B \equiv \max \left\{0, \ \theta - 3/5 + \frac{1}{5} n_C(1 - \theta) \right\} \quad (A.4)$$

Hence, there are two possibilities outside case (i). In the first case, $D_{\text{max}}^C > D_{\text{max}}^B$ and the maximum possible debt is $D_{\text{max}}^C$, so that agents C stay. In fact, if the country tried to borrow more in period 1, agents C would leave, but then agents B would leave too, and there would be nobody left to repay the debt.

(iii) Conversely, if $D_{\text{max}}^B > D_{\text{max}}^C$, the country can borrow up to $D_{\text{max}}^B$, as agents B stay even after agents C have left.

\[14\text{Recall that, when both all agents C staying and all leaving is a Nash equilibrium, by assumption the former, Pareto-superior equilibrium prevails.}\]
Therefore, one can define $D_{\text{max}}$ as:

$$
D_{\text{max}} \equiv \max \left\{ D_{\text{max}}^B, D_{\text{max}}^C \right\}
$$

(A.5)

Note that $D_{\text{max}}^B$ is increasing and $D_{\text{max}}^C$ decreasing in $n_C$. It is then easy to show that there is a value of $n_C, n'_C$, such that $D_{\text{max}} = D_{\text{max}}^C$ for $n_C \in \left[\frac{5}{3}, n'_C\right]$ while $D_{\text{max}} = D_{\text{max}}^B$ for $n_C \in \left[n'_C, 3\right]$. For future reference, note that $n'_C < \frac{1}{(1 - \theta)}$.

$n_C \in (3, 5)$.

When $n_B < 1$, agents B cannot move abroad. Also, now $D_{\text{max}}^C = 0$, as agents C certainly leave even for $D = 0$. Therefore, the maximum debt once agents C have left is the aggregate income of agents B:

$$
D_{\text{max}} = D_{\text{max}}^B = \frac{2}{3} \theta n_B
$$

(A.6)

Note that now $D_{\text{max}}$ is always decreasing in $n_C$.

**Appendix B.**

This appendix proves Proposition 2. Because all agents are rational and forward-looking, the equilibrium is determined by backward induction. First, the equilibrium in period 2 is determined as a function of the policy adopted in period 1. Then, in period 1 each individual proposes the policy that maximizes his utility, taking into account the effects of the policy on period 2's outcome. The equilibrium policy in period 1 is then determined by finding the winning proposal. I determine each group's proposal in turn, starting from group C. As in Appendix A, it is useful to consider first the case of $n_B \geq 1$, and then $n_B < 1$.

1. $n_C \in \left[\frac{5}{3}, 3\right]$

Agents C.

Because both groups B and C have above-average endowment, both propose the lowest possible tax rate in period 2. In particular, this means that, in equilibrium, $\tau_2 = 0$ if $X = 0$. In period 1, clearly agents C always propose the lowest possible $\tau_1$, which is positive only if the country is a net lender in period 1.

Consider initially values of $n_C$ such that $\theta n_C \geq n_C - 1$, i.e. $n_C \in \left[\frac{5}{3}, \frac{1}{1 - \theta}\right]$. Only two policies can conceivably be optimal for agents C. First, they can stay in period 2 and
smooth consumption perfectly at $\tau_1 = 0, X = 0$, thus consuming $\theta n_C$ in each period. This policy clearly dominates all others such that agents C stay in period 2, i.e. any policy with $X \leq 0$ or $0 < X \leq D^C_{\text{max}}$. Alternatively, agents C can leave the country in period 2, and maximize period 1 consumption by borrowing the maximum possible amount $D_{\text{max}}$. As shown in Appendix A, this policy is only feasible when $D^C_{\text{max}} < D^B_{\text{max}}$, i.e. for $n_C$ greater than some value $n'_C$.

Hence, for $n_C \in \left[\frac{5}{3}, n'_C\right]$, the optimal policy for agents C is certainly $\tau_1 = 0, X = 0$. On an interval $(n'_C, n''_C)$, with $n''_C \geq n'_C$, both policies are feasible, but the first gives a higher lifetime income. At $n_C = n''_C$, the two policies give the same lifetime income, but the first is still preferred because it allows perfect consumption smoothing. On the other hand, at $n_C = \frac{1}{1-\theta}$, the second policy gives a higher lifetime income and utility: it ensures the same period 2 consumption $n_C - 1$ as the first policy, but a higher period 1 consumption. It is then easy to show that there exists a value $n''_C$, between $n''_C$ and $\frac{1}{1-\theta}$, such that the policy $\tau_1 = 0, X = 0$ maximizes utility for $n_C \in \left[\frac{5}{3}, n''_C\right]$, while $\tau_1 = 0, X = D_{\text{max}}$ dominates for $n_C \in (n''_C, \frac{1}{1-\theta}]$.

For higher values of $n_C$, such that $\theta n_C \in \left(\frac{1}{1-\theta}, 3\right]$, the second policy is clearly optimal over the whole interval. In fact, agents C leave under both policies; given this, they can maximize consumption in period 1 and overall utility by borrowing $D_{\text{max}}$.

In summary, agents C propose $\tau_1 = 0, X = 0$ for $n_C \in \left[\frac{5}{3}, n''_C\right]$, and $\tau_1 = 0, X = D_{\text{max}}$ for $n_C \in (n''_C, 3]$, with $n''_C < \frac{1}{1-\theta}$.

**Agents B.**

It is easy to see that agents B maximize their lifetime utility at $\tau_1 = 0, X = 0$.

**Agents A.**

In period 1, agents A always propose $\tau_1 = 1$ regardless of their proposal on $X$. Similarly to the case of agents C, there are only two values of $X$ that can conceivably be optimal for agents A. First, they can smooth consumption optimally by lending an amount $L_A^*$. Under this policy, consumption is $\theta - L_A^*$ in period 1 and $kL_A^*$ in period 2, where $k$ depends on the mass of agents C who are present in period 2. Appendix C shows how $L_A^*$ and $k$ vary with $n_C$. Second, since in period 2 no labor income is redistributed, any debt incurred in period 1 does not entail any cost to agents A in terms of period 2’s consumption. Thus, the maximum possible debt, $X = D_{\text{max}}$, dominates any other $X > 0$. Under this second policy, consumption is $\theta + D_{\text{max}}$ in period 1 and 0 in period 2.

Thus, the only two candidates as proposals for agents A are $\tau_1 = 1, X = -L_A^*$ and $\tau_1 = 1, X = D_{\text{max}}$. An exhaustive determination of the optimal policy for all values of
$n_C$ in the interval $[\frac{5}{3}, 3]$ would be rather long and tedious. However, the intuition for
the logic of the model can easily be obtained by considering what happens at the two
extremes of the interval.

First, notice that when $n_C \in [\frac{5}{3}, n_C^1]$, the proposal of agents A is irrelevant, as both
groups B and C vote for $\tau_1 = 0, X = 0$. At the opposite extreme of the interval under
consideration, in a neighborhood of $n_C = 3$, $D_{\text{max}}$ is equal to $D_{\text{max}}^{B}$ and large. Agents A’s
lifetime income is then larger under the second policy, $\tau_1 = 1$ and $X = D_{\text{max}}$, although
the first one, $\tau_1 = 1$ and $X = -L_A^*$, allows a better smoothing of consumption. Therefore,
for a sufficiently high elasticity of intertemporal substitution, agents A propose the
second policy.

For intermediate values of $n_C$, there might be problems with the existence of an equi-
librium if agents A, B and C propose $X = -L_A^*$, $X = 0$ and $X = D_{\text{max}}$, respectively.
In fact, denoting by $\Omega_i$ group i’s proposal in period 1, $\Omega_C$ beats $\Omega_B$, as agents A get 0
in both periods under the latter; $\Omega_B$ beats $\Omega_A$, as agents C clearly prefer the former;
and $\Omega_A$ might (but need not) beat $\Omega_C$. However, a stable majority certainly exists on
the whole interval $[\frac{5}{3}, 3]$ if agents A propose $X = D_{\text{max}}$ whenever agents C do, i.e. for
$n_C \in (n_C^1, 3]$, since in this case $\Omega_C$ always prevails in pairwise comparison. Agents A
will indeed propose $D_{\text{max}}$ for $n_C \in (n_C^1, 3]$ if the elasticity of substitution is high enough,
for in this case the consumption smoothing motive is not very important and the policy
that maximizes lifetime income gives a higher utility. Under this condition, it is now
relatively easy to establish the equilibrium policies as functions of $n_C$.

Equilibrium policies.
For $n_C \in [\frac{5}{3}, n_C^1]$, the proposals are:
$\Omega_C$: $\tau_1 = 0, X = 0$
$\Omega_B$: $\tau_1 = 0, X = 0$
$\Omega_A$: $\tau_1 = 1, X = D_{\text{max}}$ or $X = -L_A^*$
and obviously $\tau_1 = 0, X = 0$ is adopted, regardless of group A’s proposal. For
$n_C \in (n_C^1, 3]$, the proposals are:
$\Omega_C$: $\tau_1 = 0, X = D_{\text{max}}$
$\Omega_B$: $\tau_1 = 0, X = 0$
$\Omega_A$: $\tau_1 = 1, X = D_{\text{max}}$
and $\Omega_C$ defeats in pairwise comparison the other two proposals. In fact, agents A vote
for $\Omega_C$ against $\Omega_B$ because, under the latter, their consumption is 0 in both periods.
Agents B vote for $\Omega_C$ against $\Omega_A$ because both proposals imply the same period 2 con-
sumption, but consumption in period 1 is higher under the former.
The important property of a poor economy with high inequality is that, for virtually all values of $n_C$, all agents C always leave in period 2, whatever tax and debt policies are adopted in period 1. In fact, even if $\tau_2 = 0$, the country lends all its income in period 1, and all agents C leave in period 2, the disposable income of the marginal agent C that stays in period 2 would be $\theta n_C + \frac{5}{4} \theta$, which is certainly less than $n_C - 1$ for all $n_C \in (\frac{26}{8}, 5]$. In the interval $n_C \in (3, \frac{26}{8}]$, if the country lends an amount $X < 0$ large enough in absolute value and $\theta$ is large enough, some agents C might stay. In fact, Appendix D shows that in this case the only Nash equilibrium in period 2 is such that a mass $\mu_1 < 1/5$ of agents C stay, where $\mu_1$ is defined by $\theta n_C + X/(4/5 + \mu_1) = n_C - 1$, so that agents C are indifferent between staying or moving. This is a Nash equilibrium because at $\mu_1$ $n_B$ is certainly higher than the average income of the economy, implying that indeed $\tau_2 = 0$. Thus, even in this case, there is never any redistribution of labor income in period 2, and as before the consumption of agents A is 0 unless the country is a net lender in period 1. It is then relatively straightforward to determine the proposals of the various types of agents.

Agents C.

As shown above, agents C always get $n_C - 1$ in period 2. In fact, when $n_C \in (\frac{26}{8}, 5]$, all agents C leave regardless of the policy adopted in period 1. When $n_C \in [3, \frac{26}{8})$ and a mass $\mu_1 < 1/5$ stays (which as shown above can occur only if the country lends a large amount in period 1) in equilibrium agents C consume $n_C - 1$ in both countries. In both cases, it is obvious that utility is maximized when period 1 consumption is maximized. This occurs at $\tau_1 = 0$, $X = D_{\text{max}}$.\(^\text{15}\)

Agents B.

Since $n_B < 1$, agents B certainly propose $\tau_1 = 1$. Under this proposal, at $X = 0$ agents B consume $\theta$ in period 1 and $\theta n_B < \theta$ in period 2. Any borrowing $X > 0$ would therefore make consumption even more unbalanced and would decrease lifetime income. In fact, at these levels of productivity, all agents C leave the country for any $X > 0$,\(^\text{16}\) and all the burden of the debt repayment would be on agents B. Therefore, $X > 0$ cannot be optimal for agents B, and they always propose $\tau_1 = 1$, $X = -L_B^*$.\(^\text{15}\)

\(^\text{15}\)Note that, when $n_C > 3$, $D_{\text{max}} = D_{\text{max}}^B$ always, since $D_{\text{max}}^B = \frac{2}{3} \theta n_B$ and $D_{\text{max}}^C = 0$.

\(^\text{16}\)Formally, the statement is true because, at these levels of productivity and inequality, $\theta n_C < n_C - 1$ $\forall \theta \leq 1/2$, $\forall n_C \geq 3$. 

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Agents A.
As usual, regardless of their proposal on \( X \) agents A always propose \( \tau_1 = 1 \). Given this, the problem they face is similar to the one they solve when \( n_C \leq 3 \): they can smooth consumption by lending an amount \( L_A^* \), or they can maximize period 1 consumption by borrowing \( D_{\text{max}} \). The main difference with the case of \( n_C \leq 3 \) is that now agents B cannot move, and therefore \( D_{\text{max}} = \frac{2}{5} \theta n_B \) is a decreasing function of \( n_C \). Consequently, for a sufficiently high elasticity of intertemporal substitution, the second policy maximizes utility at low values of \( n_C \), in an interval \((3, n_C^2)\).

For \( n_C \in [n_C^2, 5] \), however, \( D_{\text{max}} \) is too low, and it does not pay agents A to give up smoothing consumption in order to consume \( \theta + D_{\text{max}} \) in period 1 and 0 in period 2. Consequently, the first policy dominates. It is easy to see that \( L_A^* \), is always larger than \( L_B^* \). In fact, for any \( X \), the consumption of the two types of agents is the same in period 1, \( \theta - X \), but it is higher for agents B in period 2; in addition, at \( \tau_1 = 1 \) the marginal rate of transformation when lending is the same for both types of agents.

Thus, agents A propose \( \tau_1 = 1, X = D_{\text{max}} \) for \( n_C \in (3, n_C^2) \), and \( \tau_1 = 1, X = -L_A^* \) for \( n_C \in [n_C^2, 5] \).

Equilibrium policies.
To summarize, when \( n_C \in (3, n_C^2) \), the proposals are as follows:
\[
\begin{align*}
\Omega_C: & \quad \tau_1 = 0, X = D_{\text{max}} \\
\Omega_B: & \quad \tau_1 = 1, X = -L_B^* \\
\Omega_A: & \quad \tau_1 = 1, X = D_{\text{max}}
\end{align*}
\]
and \( \Omega_A \) defeats in pairwise comparison both \( \Omega_B \) and \( \Omega_C \). In fact, agents C vote for \( \Omega_A \) over \( \Omega_B \) since they are taxed at the maximum rate under both \( \Omega_A \) and \( \Omega_B \), but at least obtain \( D_{\text{max}} \) under \( \Omega_A \). Also, agents B prefer \( \Omega_A \) to \( \Omega_C \), because their consumption in period 2 is the same under the two proposals, but consumption in period 1 is higher under \( \Omega_A \).

When \( n_C \in [n_C^2, 5] \), the proposals that can be voted on are:
\[
\begin{align*}
\Omega_C: & \quad \tau_1 = 0, X = D_{\text{max}} \\
\Omega_B: & \quad \tau_1 = 1, X = -L_B^* \\
\Omega_A: & \quad \tau_1 = 1, X = -L_A^*
\end{align*}
\]
It is then straightforward to show that \( \Omega_B \) defeats the other two proposals. In fact, agents C prefer \( \Omega_B \) to \( \Omega_A \) because the tax rate is the same but net lending is lower under the former. Also, agents A prefer \( \Omega_B \) to \( \Omega_C \) because they now dislike borrowing and there is no redistribution of labor income under \( \Omega_C \).
Appendix C.

This appendix shows how the optimal net lending of agents A, \( X = -L_A^* \), varies with \( n_C \) in a poor economy where \( n_C \in [\frac{5}{3}, 3] \). Let \( L > 0 \) denote the amount the country lends in period 1. For \( \theta \in [2/5, 1/2] \), all agents B stay for any \( L \), since \( \theta n_B > n_B - 1 \) \( \forall n_B \). The issue is therefore the behavior of agents C. Recalling that \( r_2 = 0 \) always, there are several possibilities, depending on the value of \( n_C \).

(i) Suppose first \( \theta n_C \geq n_C - 1 \), i.e. \( n_C \leq \frac{1}{(1-\theta)} \). Clearly, all agents C stay for any \( L \). Under agents A's proposal \( \tau_1 = 1 \), the intertemporal rate of transformation for agents A is 1, and they can maximize utility by smoothing consumption perfectly. Hence, for \( n_C \in [\frac{5}{3}, \frac{1}{(1-\theta)}] \), perfect consumption smoothing is achieved at \( L_1 = \frac{1}{2} \theta \).

(ii) Suppose now that \( \theta n_C < n_C - 1 \), and that \( L \) is such that all agents C have left. For this to happen, it must be the case that even when \( L \) has to be shared with agents A and B only, an individual of type C is better off abroad: \( \theta n_C + \frac{5}{4} L < n_C - 1 \). In this case, the marginal rate of transformation in consumption for agents A is larger than 1, as any unit of consumption in period 1 can be converted into \( \frac{5}{4} \) units of consumption in period 2. It is easy to show that in this case agents A maximize utility by lending \( L = L_2 \), where \( L_2 = \frac{1}{k} \theta 1 \), and \( k = 1 + \frac{4}{5} - 1 \). Note that \( k \in [1, 2] \), and \( k = 1 \) when \( \phi = 0 \), i.e. when utility is linear. Hence, \( L_2 \geq L_1 \). When the country is a net lender, ceteris paribus agents A are better off if agents C leave in period 2, as they share \( L \) in period 2 with agents B only. Therefore, if \( \theta n_C + \frac{5}{4} L_2 < n_C - 1 \), i.e. for \( n_C \in \left(\frac{(1+\frac{5}{4})}{(1-\theta)}\right), 3 \), \( -L_2 \) dominates any other \( X < 0 \).

(iii) Now assume that \( n_C \) is such that \( \theta n_C + L_1 \geq n_C - 1 \), i.e. \( n_C \in \left(\frac{1}{1-\theta}, \frac{(1+\frac{5}{4})}{(1-\theta)}\right) \). Because now \( \theta n_C + \frac{5}{4} L_2 > n_C - 1 \), at \( X = -L_2 \) a mass \( \mu_1 > 0 \) of agents C stay such that \( \theta n_C + \frac{L_2}{4/5+\mu_1} = n_C - 1 \). Let \( L_3 \) be the value of \( L \) that makes agents C indifferent between staying and moving when no agents C are present: \( \theta n_C + \frac{L_3}{4/5} = n_C - 1 \), i.e. \( L_3 = \frac{3}{5}(n_C(\theta - 1) - 1) \). For any \( L \in [L_3, \frac{5}{4} L_3] \), period 2's consumption of agents A is constant at \( \frac{L}{4/5+\mu_1} = n_C(1-\theta) - 1 \). Hence, in this interval of values of \( L \), any increase in \( L \) reduces period 1 consumption, but does not increase period 2 consumption, as it only attracts more agents C from abroad. It is then clear that \( L_3 \) dominates any \( L \in [L_3, \frac{5}{4} L_3] \). Thus, either \( L_A^* = L_1 \) or \( L_A^* = L_3 \). When \( n_C = \frac{1}{1-\theta} \), \( L_3 = 0 \) and \( L_1 \) clearly dominates. At the other extreme of the interval considered here, \( n_C = \frac{(1+\frac{5}{4})}{(1-\theta)} \), \( L_3 = \frac{4}{5} L_1 \) and \( L_3 \) clearly dominates. Thus, there exists an \( n_C^4 \) such that \( L_A^* = L_1 \) for \( n_C \in [\frac{4}{5}, n_C^4] \) and \( L_A^* = L_3 \) for \( n_C \in [n_C^4, \frac{(1+\frac{5}{4})}{(1-\theta)}] \).

\(^{17}\)Note that, for very low values of \( \phi \), \( (1 + \frac{5}{4} \phi)/(1 - \theta) \) could be larger than 3, in which case the interval considered here would be empty.
(iv) Finally, consider values of $n_C$ such that $\theta n_C + L_1 < n_C - d$ and $\theta n_C + \frac{5}{4} L_2 > n_C - 1$, i.e. $n_C \in \left( \frac{\theta}{\theta - 1}, \frac{\theta}{\theta - 1} \right)$. Now neither $L_1$ nor $L_2$ can be optimal, and it is easy to see that $L_A = L_3$.

To summarize:

- $n_C \in [\frac{\theta}{\theta - 1}, n_C^4)$: $L_A^* = \frac{\theta}{2}$;
- $n_C \in [n_C^4, \left( \frac{1 + \frac{5}{4}}{\theta - 1} \right)]$: $L_A^* = \frac{4}{5} (n_C - 1 - \theta)$;
- $n_C \in \left( \frac{1 + \frac{5}{4}}{\theta - 1}, 3 \right]$: $L_A^* = \frac{\theta}{4}$.

### Appendix D.

This appendix illustrates the nature of some problems that can arise for intermediate levels of income. Consider an economy with $\theta$ less than, but close to, 1, and with $n_B < 1$. Therefore, if all agents $C$ stay, the tax rate in period 2 is 1. Now suppose in period 1 the country lends the amount $X = -L$, which is such that, at $r_2 = 1$, agents $C$ certainly leave. In other words, even if all agents $C$ leave, an agent $C$ who stays would be able to consume

$$\theta + \frac{5}{4} L < n_C - 1 \quad (D.1)$$

However, $L$ is such that, if the tax rate where 0 and the marginal agent $C$ were able to retain all his labor income, $\theta n_C + \frac{5}{4} L > n_C - 1$. It is easy to see that all agents $C$ staying or all agents $C$ leaving are not Nash equilibria. Now let $\mu_1$ be the mass of agents $C$ staying such that at $r_2 = 0$ agents $C$ are indifferent between staying and leaving:

$$\theta n_C + \frac{L}{\frac{4}{5} + \mu_1} = n_C - 1 \quad (D.2)$$

Also, define $\mu_2$ as the mass of agents $C$ staying such that the average income of the economy is equal to $n_B$: $\mu_2 n_C + \frac{2}{5} n_B = n_B$. It is clear that in any Nash equilibrium the mass of agents $C$ staying must be no greater than $\mu_2$. Otherwise, the income of the median voter would be below the average and $r_2 = 1$. But at this tax rate, from (D.1) all agents $C$ would leave. Given $\tau_2 = 0$, a Nash equilibrium with a positive mass of agents $C$ staying can occur only if this mass is exactly $\mu_1$, so that the marginal agent is indifferent between staying and leaving. Thus, a Nash equilibrium requires $\mu_1 \leq \mu_2$, so that at $\mu_1 \tau_2$ is indeed 0. However, at high values of $\theta$ and $L$, $\mu_1 > \mu_2$, and it is easy to see that neither $\mu_1$, nor $\mu_2$, nor any other mass of agents $C$ leaving is a Nash equilibrium.

The problem, of course, is the discontinuity in agents $C$'s income that occurs when the mass of agents $C$ staying is $\mu_2$. Furthermore, it is easy to see that in this setup there
is no Nash equilibrium in mixed strategies, either. Notice that, when $\theta < 1/2$, $\mu_1 < \mu_2$ and therefore $\mu_1$ was a Nash equilibrium.$^{18}$

There are several possibilities to address the problem of the non-existence of a Nash equilibrium for certain levels of $L$ when $\theta \in (1/2,4)$. Suppose the economy is composed of a large, but finite, number of agents. One could assume that, before taxes are voted on in period 2, agents C have to decide, sequentially, whether they want to stay or not. The order is decided randomly, and agents C cannot reconsider their decision. Abstracting from integer problems, it is easy to see that initially, all agents C will decide to stay, until a proportion $\mu_2$ of them has been asked to commit to moving or staying. After this, all other agents C will decide to leave. An alternative, although less satisfactory, solution does not require assuming a finite number of agents. In the problem described above, the cooperative solution for agents C is such that exactly a mass $\mu_2$ of agents stay. With some form of cooperation among agents C, $\mu_2$ is therefore the equilibrium.

Appendix E.

This appendix shows that, when $\theta$ increases, $n_C^1$ increases while $n_C^2$ falls. Recall that $n_C^1$ is defined implicitly by

$$H \equiv (\theta n_C + D_{max}^B)^{1-\phi} + (n_C - 1)^{1-\phi} - 2(\theta n_C)^{1-\phi} = 0 \quad (E.1)$$

Let $M \equiv \theta n_C + D_{max}^B$, $N \equiv n_C - 1$ and $Q \equiv \theta n_C$. Also, let $\hat{K}_z$ indicate $\frac{\partial K}{\partial z}$, with $K = M, N, Q$ and $z = \theta, n_C$. Then one can write, ignoring multiplicative constants:

$$\frac{\partial H}{\partial \theta} = M^{1-\phi}\hat{M}_\theta - 2Q^{1-\phi}\hat{Q}_\theta$$

where $\hat{Q}_\theta = 1$. Using (E.1), (E.2) can be rewritten as

$$\frac{\partial H}{\partial \theta} = 2Q^{1-\theta}(\hat{M}_\theta - \hat{Q}_\theta) - \hat{M}_\theta N^{1-\theta} \quad (E.3)$$

The r.h.s. of (E.3) is negative for $\phi < 1$, since $Q/N < 3/2$, while $\hat{M}_\theta/(\hat{M}_\theta - \hat{Q}_\theta) > 3$.

Using a similar method, one can write

$$\frac{\partial H}{\partial n_C} = (M^{1-\phi} - 2Q^{1-\phi})(1 - \hat{N}_{n_C}) \quad (E.4)$$

$^{18}$To show that $\mu_1 \leq \mu_2$, from (D.2) notice that $\mu_1$ is maximum at $X = X_{max} = \theta$ and $\theta = \theta_{max} = 1/2$, while $\mu_2$ is independent of both $X$ and $\theta$. Let $\mu_{max}$ be this maximum value of $\mu_1$. It is easy to see that $\mu_{max} = \mu_2$ at $n_C = 3$; moreover, both $\mu_{max}$ and $\mu_2$ are decreasing functions of $n_C$, but the former falls faster as $n_C$ increases. Therefore, $\mu_{max} \leq \mu_2$. 

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which is positive. Hence, \( d^{n_C^1}_{\theta} = \frac{\partial H}{\partial \theta} / \frac{\partial H}{\partial n_C} > 0 \).

Now consider how \( n_C^2 \) is affected by changes in \( \theta \). \( n_C^2 \) is defined by

\[
T \equiv (\theta + D_{\text{max}}^B)^{1-\phi} - 2 \left( \frac{\theta}{2} \right)^{1-\phi} = 0
\]  \hspace{1cm} (E.5)

Using a procedure similar to that followed above, one can easily show that \( \frac{\partial T}{\partial \theta} > 0 \) and \( \frac{\partial T}{\partial n_C} > 0 \), and therefore \( d^{n_C^2}_{\theta} < 0 \).
References.


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