The Effects of a Grouping by Tens Manipulative on Children’s Strategy Use, Base Ten Understanding and Mathematical Knowledge

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ABSTRACT

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Manipulatives have the potential to be powerful tools in helping children improve their number sense, develop advanced mathematical strategies, and build an understanding of the base ten number system. Physical manipulatives used in classrooms, however, are often not designed to promote efficient strategy use, such as counting on, and typically do not encourage children to perceive higher-order units in multi-digit numbers. The aim of this study was to closely examine the affordances of a novel grouping by tens virtual manipulative. Seventy-nine first grade students were randomly assigned to one of two math software comparison groups or a reading software control group. In the math comparison groups, children received scaffolding and feedback while playing a computerized enumeration game that required them to use the novel grouping by tens manipulative. Children in the Transformation group used a manipulative that transformed from a unitized to a continuous model, while children in the Unitized group used a manipulative that remained discrete. Researchers recorded children’s strategy use and accuracy when determining how many objects appeared on the screen, and the data were examined microgenetically. Children’s counting on abilities, base ten understanding, and number sense were tested at posttest to examine group differences. The results showed that using the transforming manipulative significantly improved children’s ability to count on at posttest. The math software also improved girls’ base ten understanding at posttest. Children who used the math software in both conditions improved in their advanced strategy use and accuracy over
time. These findings suggest that virtual manipulatives have the potential to improve children’s strategy use and base ten understanding in ways that physical manipulatives may not. Suggestions for future research are discussed.
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DEDICATION

This work is dedicated to my daughter, Georgia Elle Krasner. Georgia, you have taught me more about growth and development than I could have learned from any text. Thank you for inspiring me. I love you always.
Chapter 1

INTRODUCTION

Addition and place value are two essential aspects of children’s mathematical development. The Common Core State Standards highlight these concepts as two of the four critical instructional areas for educators to focus on with students in Grade 1 (Common Core State Standards Initiative, 2010, p.13). In order to ensure children’s deep learning of addition and place value, educators must present these concepts in meaningful contexts that build on children’s informal everyday mathematics knowledge (Ginsburg, 2006). Enumeration is one such context that is uniquely suited to bridge the gap between children’s informal mathematics experiences and children’s formal learning of number and operations. Counting objects is a familiar task to children and has been shown to promote children’s understanding of cardinality, offer children an intuitive method for solving addition problems, and serve as a potentially useful context for learning about the underlying base ten structure (Ginsburg, 1989; Houlihan & Ginsburg, 1981; Zur & Gelman, 2004).

Building upon children’s informal knowledge of enumeration, however, will not guarantee that children gain insight into operations and place value. Rather, enumeration activities must be structured in ways that encourage these new types of mathematical thinking. One way to design powerful enumeration activities is by incorporating manipulatives. For instance, manipulatives that segment objects to be counted into distinct subsets can help children build an intuitive understanding of addition. Tools that group objects into tens and ones may also help children gain a rudimentary understanding of the base ten system. While there are many types of concrete and virtual manipulatives that have been designed for either promoting addition or base ten concepts, few tools have been developed that foster both types of
understanding, especially in the context of an enumeration task. This paper underscores the need for such manipulatives and argues that key features of these manipulatives must be empirically tested to determine whether they do, in fact, improve children’s mathematical understanding and advance their strategy use.

*Enumeration and Number Sense*

Enumeration is a fundamental part of young children’s everyday mathematics experiences. Because of children’s familiarity with counting objects, enumeration tasks are excellent entry points for teaching children more formal mathematics. As many researchers have shown, the task of enumeration is substantially more complex than saying the counting words and may require the understanding of several underlying principles. Gelman (2000) proposes five key principles that underlie counting and enumeration, including the “one-one principle,” which states that each object to be counted receives one and only one counting word, the “stable ordering principle,” referring to the importance of the counting sequence, and the “cardinal principle,” meaning that the last number stated when enumerating refers to the entire set of counted objects. The “abstraction principle” states that one can count any type of entities and the “order-irrelevance principles” expresses the idea that items can be counted in any order (Gelman, 2000). While some argue that these principles do not need to be understood in order for children to enumerate (Briars & Siegler, 1984, Fuson, 1987; Siegler, 1991), researchers agree that repeated experiences with counting objects aids in the development of children’s understanding of number and cardinality (Baroody, 1993; Briars & Siegler, 1984; Fuson, 1987).

*Addition Strategies*
Children’s initial understanding of simple addition problems often emerges from experiences with enumeration. Ginsburg (1989) notes that young children often approach addition problems as tasks of enumeration involving two sets. When using concrete manipulatives to add, children typically determine the union of the two sets by counting all objects, starting at 1. As children become more familiar with single-digit addition problems, they frequently are able to invent new, more efficient strategies, such as counting on, grouping, performing mental calculations and recalling facts from long-term memory (Baroody, 1987; Baroody & Gannon, 1984; Ginsburg, 1989). In their work with preschool children, Zur and Gelman (2004) stress the importance of understanding counting in the context of arithmetic tasks, noting that the principles of counting and arithmetic are “mutually constraining” (p.122). Therefore, it is crucial to give children enumeration tasks that are designed to be precursors to early addition problems, for instance by presenting to children problems that visually separate the set to be counted into distinct subsets.

Children use a variety of different strategies to solve addition problems, including using everyday manipulatives, such as fingers, and carefully designed math manipulatives, such as unifix cubes, to count the sets in different ways. These strategies have been extensively studied over the last few decades (Ashcraft, 1982; Carpenter & Moser, 1984; Fuson, 1992; Siegler, 1987). Studies examining children’s arithmetic show a general progression over the course of children’s mathematical development from counting-based strategies to retrieval-based strategies (Ashcraft, 1982; Carpenter & Moser, 1984; Fuson, 1982; Siegler, 1987). Carpenter and Moser (1984) delineate three basic levels of children’s strategy use when solving single-digit addition problems. The earliest strategy, often referred to as “counting all,” involves directly modeling a problem with objects or fingers and counting the union of sets (Carpenter & Moser, 1984).
Children who count all often fail to understand that the cardinal value of the first addend is also the starting point in the count sequence of the combined sets; therefore, they recount the first addend, an unnecessary and inefficient behavior (Secada, Fuson & Hall, 1983). In his study of kindergartener’s addition strategies, Baroody (1987) found that many children persist in using the counting all strategy with concrete objects throughout the kindergarten year. However, extensive practice with this strategy often leads children to invent more advanced and efficient strategies (Groen & Resnick, 1977).

“Counting on” is a more advanced strategy than counting all and can be used with or without manipulatives. This strategy requires a more sophisticated understanding of the count sequence, for children must be able to begin the count sequence from any number word, what Fuson (1982) refers to as the “breakable chain level.” Children also must know that counting begins with the cardinal value of the second addend, referred to as the “cardinal-to-count transition” (Fuson, 1982). Thus, for the problem \( m + n \), children who count on either begin counting at \( m \) and continue the count sequence for \( n \) more counts or begin counting at \( m + 1 \) and continue counting \( n - 1 \) more counts (Secada, Fuson, & Hall, 1983). Groen and Resnick (1977) describe a more advanced and faster variation of the count on strategy, known as the \( min \) strategy, which children in their study invented on their own after being taught how to count on. This strategy involves counting on from the larger addend, rather than the second addend, and is considered more advanced than counting on because it minimizes the amount of counting children must do to solve the problem. Supporting children’s counting on is an important instructional aim as studies show that this strategy may increase the rate at which children learn addition (Fuson, 1987; Fuson & Fuson, 1992).
Finally, Carpenter and Moser (1984) describe a third level of strategy use, fact retrieval, which entails recalling specific number combinations that are stored in long-term memory. Fact retrieval often refers to children’s use of rote recall to answer known number facts. However, fact retrieval may also involve using “derived facts” to solve given problems. When a child is unable to recall a given math fact, he or she may use another known math fact as a back-up strategy to help them determine the answer. Utilizing “derived facts” requires that children apply known number facts, such as ties or doubles, to help solve related, unknown problems that cannot be quickly recalled (Carpenter & Moser, 1984; Houlihan & Ginsburg, 1981; Siegler, 1987; Steinberg, 1985). For instance, children who fail to remember that 6+7=13 may use the known fact of 6+6=12 to help them find the answer. Siegler (1987) also found that decomposing problems into smaller known number facts, for instance breaking 5+2 into 2+3+2, is also a frequently used back-up strategy by 1st and 2nd graders when solving single-digit addition problems.

Despite the general developmental trends from counting-based strategies to retrieval-based strategies, children’s use of addition strategies, like children’s strategy use more generally, is not a clear, linear progression of acquiring more advanced strategies and abandoning less efficient ones (Carpenter & Moser, 1984; Siegler, 1987; Siegler & Jenkins, 1989). In their longitudinal study of 1st through 3rd graders strategy use, Carpenter and Moser (1984) found that changes in children’s strategy use occur gradually; even after learning advanced methods for adding and subtracting, children continued to employ less efficient strategies, such as counting all. They also found that children tended to use count all and count on strategies concurrently in the same session (Carpenter & Moser, 1984).
Siegler (1987) also found a high degree of variability in strategies used by a single child within one session. For instance, he found that 99% of the kindergarten, first and second graders in his study reported using 2 or more strategies and 62% reported using 3 or more different strategies. In another study, Siegler and Shrager (1984) found that children often use as many as 4 different strategies when faced with solving a single digit addition problem. How children determine what strategy to use depends on specific conditions of a problem, problem difficulty, and the total set of available strategies that they know (Siegler, 1987; Siegler & Shrager, 1984). Siegler (1987) stresses the important role that accuracy plays in shaping children’s strategy use, noting that children chose more efficient, faster strategies only when they are likely to yield accurate solutions. These same children regress to using less efficient strategies when they anticipate that the more efficient strategies will produce incorrect answers.

Finally, studies have also highlighted gender differences in the strategies children use to solve addition problems. Carr and Jessup (1997) examined the strategies of 58 first grade students when solving addition and subtraction problems. They found significant differences between the strategies that boys and girls used to solve problems: boys were more likely to use recall, while girls were more likely to use observable counting strategies. However, the researchers did not distinguish between advanced and non-advanced counting strategies when assessing children’s strategies. Fennema et al. (1998) also investigated the role that gender plays in children’s strategy use. They examined the accuracy and strategy use of 82 first grade students over the course of 3 years. Their results showed that while there were no significant gender differences in terms of accuracy scores, girls were more likely to use counting-based strategies when solving problems, whereas boys were more likely to use derived facts or other
invented strategies. More studies are needed to replicate these findings and to determine whether these gender differences emerge when examining advanced versus non-advanced strategy use.

**Base Ten Understanding**

Gaining insight into the underlying base ten structure of the number system is another important learning outcome that may emerge in the context of enumeration tasks. While enumeration in its basic form of iterating by ones does not promote base ten understanding, educators can structure large number enumeration tasks to highlight the efficiency of grouping by tens. Partitioning large sets into subsets of tens and ones may help children see the embedded nature of numbers and may consequently impact their place value understanding (Cowan, 2003; Nunes & Bryant, 1996). For instance, problems that highlight a subset of 10 objects from a collection of 14 objects may help children notice the powerful role that the number 10 plays in our number system. Empirical studies should be designed to examine whether frequent exposure to grouping by tens can be an effective way to build rudimentary base ten concepts in young children.

Understanding the base ten system is a pivotal mathematical goal as it is foundational for more advanced mathematical understanding, for instance multi-digit written calculation (Baroody, 1990; Hiebert & Wearne 1992; Ho & Cheng, 1997). However, despite the importance of the base ten system and the call by the National Council of Teachers of Mathematics (NCTM) to teach it to pre-K through 2nd grade students, researchers continue to disagree on when base ten concepts should be first introduced in formal schooling (Baroody, 1990; Fuson & Briars, 1990; NCTM, 2000). Fuson and Briars (1990) argue that multi-unit concepts should not be introduced until 2nd grade because younger children struggle to understand ten-unit and hundred-unit items
composed of ones; therefore, they argue, instruction should focus on building young children’s unitary conceptual structures.

In contrast, Baroody (1990) contends that multi-unit concepts should be taught when two-digit numbers are introduced, which occurs within the kindergarten and first grade years. He notes that teaching aspects of the base ten system when introducing teen numbers, for instance, may help children gain a stronger foundation for understanding grouping, trading up and larger written numbers (Baroody, 1990). In pilot work, I found evidence that second-semester kindergarteners who grouped by tens when enumerating displayed a developing awareness that two-digit numbers are composed of tens and ones. While young children may not be ready to understand much of the complexity of place value, especially in its written form, my findings suggest that a grouping by tens manipulative has the potential to transform young children’s approach to enumeration problems and improve their strategy use and base ten understanding.

There are several obstacles that must be overcome if young children are to better understand the base ten structure of numbers when enumerating. The biggest conceptual hurdle is shifting from perceiving enumeration as a counting by ones task to one that involves counting multi-units and leftover single units. This task is especially challenging, as Fuson and Briars (1990) note that English-speaking children, in particular, tend to conceive of multi-digit numbers as “concatenated single-digit numbers” (p. 181), rather than numbers composed of tens and ones. This assertion is substantiated by results from Miura et al.’s (1993) study that found that children from non-Asian language groups tend to represent multi-digit numbers as collections of singular units, rather than as composed of tens and ones.
An important reason why children, especially in non-Asian countries, may struggle with base ten understanding is because of difficulties posed by language. Research shows that children in English-speaking countries struggle to understand place value even when they can enumerate and count to high numbers, especially compared to children in Asian countries (Fuson, 1990; Fuson & Kwon, 1992; Ginsburg, 1989; Kamii, 1986; Kouba, et al., 1988; Miura et al, 1993). Irregularities in the English counting words, in particular the teen numbers (11-19), are believed to contribute to this lack of understanding, for the English number naming system fails to clearly distinguish between tens and ones, a feature typical of many Asian languages (Fuson, 1990; Fuson & Kwon, 1992; Miller & Stigler, 1987; Miura et al., 1993). For example, in both the formal and informal Korean number systems, 11, 12 and 13 are referred to as “ten one,” “ten two,” and “ten three,” names that explicitly divide each teen number into tens and ones. In order to help English-speaking children overcome linguistic obstacles posed by the English counting words, children need sufficient practice conceptualizing numbers as composites of tens and ones.

*Manipulatives*

For over a century, educators have argued for the use of manipulatives when teaching young children mathematics. Theorists such as Piaget (1952), Dienes (1960), Bruner (1966), and Montessori (1964), all advocated the benefits of teaching with physical objects, primarily because they were believed to offer children a bridge to understand complex, abstract ideas. Consequently, a great deal of research over the past few decades has been conducted on the use of manipulatives, with many studies finding positive effects on learning (Sowell, 1989).

There are several theories that attempt to explain why manipulatives make learning
difficult mathematical concepts easier for children. Researchers stress that manipulatives may help children “see” mathematical ideas, which would otherwise be invisible, by offering a physical representation of important mathematical relationships, often expressed through spatial relationships (Mix, 2010). Hoyles and Noss’s (2009) notion of situated abstraction underscores the importance of actively engaging with semiotic tools in order to contextualize and better understand abstractions. They note that it is through manipulating tools, making connections between different types of models, symbols, and concepts, and receiving immediate feedback that learners gain deep understanding of abstract concepts.

McNeil and Jarvin (2007) believe that manipulatives may offer children additional resources for learning and may help children draw upon their real-world experiences. Having additional resources may enable children to achieve goals they otherwise would not be able to achieve. For instance, solving addition problems with countable objects might help children be more accurate in their counting. Children may carefully count out each addend, combine them in an organized line, and touch each object as they say the count sequence. Manipulatives can also help children connect formal mathematics with their everyday mathematical problem solving. In fact, researchers have found that young children who are unable to solve traditional addition and subtraction problems are often capable of solving these problems when they are posed in the context of a story that children can act out with concrete objects (Ginsburg, 1989).

Mix (2010) delineates four mechanisms through which manipulatives may aid children’s learning: by generating actions, acting as conceptual metaphors, offloading intelligence, and focusing attention. Mix’s notion of “generating action,” or providing children with something to act upon, is supported by research on embodied cognition and physically distributed learning, which highlight the iterative, co-evolving relationship between actions, thinking, and memory.
Several studies underscore the role that interacting with manipulatives plays on learning in different academic domains. An example of this is a study conducted by Glenberg et al. (2004) on the effects of using toy manipulatives to act out story events. Second graders in their study who used toys or imagined using toys to enact what they read showed significantly better inferencing skills and memory of the stories compared to children who were not instructed with manipulatives.

Martin & Schwartz (2005) studied the ways that actions impact conceptual understanding in the realm of mathematics. They found that fourth grade students who were given the opportunity to move around pie pieces and tiles gained better understanding of fraction concepts than those who were only able to look at these pieces. Importantly, they note that the process of moving the pieces around itself, and not an initial understanding of fractions, led children to gain insight into the underlying concepts (Martin & Schwartz, 2005). Taken together, these studies suggest that action, real or imagined, is effective in encouraging children’s conceptual learning and thinking. Because manipulatives provide material for children to act upon mathematical ideas, they are potentially useful tools to support mathematics learning.

Mix (2010) distinguishes between two types of conceptual metaphors, which both may be responsible for why children learn from manipulatives. *Grounding metaphors* emerge, often spontaneously, from direct experience with objects, for instance, when one learns about the concept of containment or subsets through playing with various types of containers and objects (Mix, 2010). Working with base ten manipulatives may offer children a new grounding metaphor for the compositional nature of numbers (i.e. 10 ones blocks are the same size as 1 tens rod). This metaphor could have important effects on children’s ability to learn about place value.
Linking metaphors emerge through formal instruction and are used to bridge different domains together. For instance, teachers may help children see that proportional base ten blocks offer insight into the relationship between numbers and space.

Finally, Mix (2010) argues that manipulatives may promote learning because of their ability to offload intelligence and focus attention. The supposition that manipulatives offload intelligence is based in literature on situated cognition, which stresses the important function of the environment in cognitive tasks (Clark, 1997; Greeno, 1989; Thelen & Smith, 1994). Manipulatives can ease the cognitive demands of a task for children, by providing scaffolds and making goals more attainable. For example, a base ten 100’s block allows children to perceive “100” without requiring children to count out 100 unit blocks, an extremely laborious, error-prone task. Mix (2010) also describes how manipulatives may help focus students’ attention because they isolate particular mathematical relationships. For example, the mathematical relationship between ones, tens and hundreds is highlighted by base ten blocks through the proportionate sizes of the blocks. To ensure that manipulatives focus children’s attention on underlying mathematical relationships, these tools should have few extraneous features, which could distract children from the learning goals (Uttal et al., 2009).

Despite the potential for manipulatives to promote conceptual understanding and advance strategy use, manipulatives do not assure early mathematics learning. Research shows that while manipulatives can be useful for making mathematical concepts and relationships explicit, they do not in and of themselves guarantee that students will draw meaningful connections to deeper ideas or transfer their knowledge to new contexts (Baroody, 1989; Clements & McMillen, 1996; McNeil & Jarvin, 2007). On the contrary, empirical studies have highlighted the ineffective and even deleterious effects that manipulatives can have on children’s learning (McNeil, Uttal, Jarvin
Because of the ambiguous results surrounding the effectiveness of manipulatives, researchers have begun to refocus their efforts on investigating which circumstances are best for using manipulatives as learning tools, rather than simply advocating for or against these tools (Kamii, Lewis, & Kirkland, 2001; McNeil & Uttal, 2009; McNeil & Jarvin, 2007; Mix, 2010; Sarama & Clements, 2009; Sowell, 1989). In addition to identifying particular contexts that are most suitable for the use of manipulatives, researchers must also examine which features of different manipulatives are effective at helping children understand particular ideas. Researchers must empirically study the learning gains that arise from features of a given manipulative and differentiate between features that improve accuracy, advance strategy use, and promote conceptual understanding. Research is also needed to uncover whether certain tools help children move from inefficient strategies to more advanced strategies and whether the positive effects of these manipulatives persist after they are no longer available for children to use.

Instruction appears to be an essential aspect of effective use of manipulatives to enhance mathematics learning. Sarama and Clements (2009) stress the importance of having educators guide children to actively reflect on the underlying mathematics, noting that children must be lead to analyze their actions with manipulatives through appropriate scaffolding. Providing this type of guidance requires that educators understand the mathematics embodied by the manipulatives, the ways in which children frequently misunderstand or misuse the tools, and the best methods for helping children move towards meaningful understanding. Fuson and Briars (1990) also advocate for the use of instruction when using manipulatives. In their work on place value, they developed carefully crafted teaching lessons using base ten manipulatives. Their results showed that second grade students performed better on multi-digit addition and
subtraction after learning from instruction with manipulatives than older students exposed to traditional curricula. While their study failed to isolate what aspect of the lesson plans were responsible for place value learning, their results offer evidence for the educational value of including instruction when teaching with manipulatives.

Offering children sufficient time to work with a given manipulative before introducing a new material or representation is also a crucial component of using manipulatives effectively. Mix (2010) notes that children must be given enough time to work with a particular manipulative to enable them to build stable responses to it. Once their responses are solidified, educators can then vary the situation by either changing the manipulative in some way or introducing an alternative representation of the same underlying mathematical idea. This enables children to experience different representations of an idea without being overloaded by having to simultaneously analyze and integrate multiple representations (Mix, 2010). These ideas suggest that while manipulatives can be a valuable way to help children perceive and make connections between mathematical ideas, they need to be introduced in careful ways and accompanied by scaffolds that help children build meaning.

Finally, manipulatives do not need to be physical in order to be effective or “concrete” (Sarama & Clements, 2009). Researchers have begun to outline potential benefits and advantages of virtual manipulatives in certain contexts over physical objects (Hoyles & Noss, 2009; Mix, 2010; Sarama & Clements, 2009). For instance, Sarama and Clements (2009) describe the precision and flexibility of virtual manipulatives, which enable students to more accurately extend and resize shapes and show geometric transformations, as well as the dynamic nature of virtual manipulatives, which can model mathematical actions. Hoyles & Noss (2009) note that being able to accurately “sketch” on the computer and reflect on these sketches enables
students to make conjectures, which otherwise would likely not be made with imprecise
drawings. Computerized and touch-screen manipulatives also enable developers to dynamically
link the virtual tools to mathematical symbols and build in immediate feedback for users (Hoyles
& Noss, 2009; Mix, 2010; Sarama & Clements, 2009).

Improving Strategy Use and Base Ten Understanding

Manipulatives offer a potentially powerful way to improve children’s number sense, advance their strategy use, and help them understand the base ten number system. However, these tools must be carefully designed to take into account specific learning principles and must be used appropriately, with guidance and feedback. While certain manipulatives may be effective for particular mathematical activities, they may be ineffective for others. This is particularly true for manipulatives typically used in elementary classrooms to model addition and base ten concepts. Few materials are specifically designed to foster advanced strategy use and help children conceive of higher order units. In fact, the primary manipulatives used for modeling addition in classrooms are small sets of objects, such as chips, cubes, or toys. While these individual objects nicely model counting all, they do not encourage advanced strategies, like counting on, because they do not discourage children from counting over the first addend.

Few studies have examined the role that manipulatives play in shaping children’s addition strategy use. For instance, while Carpenter and Moser (1984) found that some children capable of counting on reverted back to counting all in the presence of manipulatives, they did not see this pattern with all children. It is essential to understand if and how manipulatives impact students’ strategy use precisely because manipulatives are so prevalent in early childhood classrooms and teachers rely so heavily upon these objects when teaching single-digit addition
and base ten concepts. More research is needed to examine the features of different
manipulatives that appear to promote advanced strategy use. Additionally, researchers must
investigate whether any positive impacts of manipulatives persist after these tools are removed
from children’s use.

To help children understand fundamental aspects of the base ten system, many physical
manipulatives, such as Unifix cubes, Montessori beads, base ten blocks, bundled sticks and
Cuisenaire rods, have been developed and are frequently used in elementary classrooms. Fuson
(1990) refers to these types of manipulatives as a “physical collectible multiunits,” stressing that
in order to learn from these tools children must focus on the cardinal value of each bundle, 10-
stick or 100 block, and relate the individual units within each collection to the overall multiunit
collection. Thus, children must be able to conceive of a 10-stick as a new single multiunit, rather
than simply a collection of 10 individual units (Fuson, 1990). A main drawback of both Unifix
cubes and bundled sticks is that they do not help children perceive the collection as a multi-unit
because they do not transform into a continuous higher-order unit (Sharma, 1993). The primary
perceptual characteristic of the manipulatives, therefore, remains the units, not the powers of
tens. Cuisenaire rods, on the other hand, have a different size and color rod for each of the
numbers 1-10, and are continuous, proportional blocks that emphasize the multi-unit. However,
Cuisenaire rods do not represent the individual units that make up each rod, and thus fail to
demonstrate the relationship between the higher-order unit and the ones units.

Base ten blocks and Montessori beads attempt to combine the continuous and unitized
representation into one manipulative (Dienes, 1960). Base ten and Montessori “10-blocks” are
proportional, continuous rods; however, unitized ridges and beads highlight the individual units
of each 10-block. While the base ten blocks and Montessori beads come closest to highlighting
the higher-order relationship between ones and tens and have been shown to be effective at promoting base ten awareness in conjunction with instruction, they do not fully eliminate the concatenated view of the 10-block (Fuson & Briars, 1990; Hiebert & Wearne, 1992). The individual units are always visible on each of the multiunit blocks, which may discourage children from seeing a 10-block as 1 ten rather than 10 ones. Thus, there is clearly a need for a base ten manipulative that is capable of shifting from a unitized to a continuous representation of a ten.

*Grouping by Tens Virtual Manipulative*

In our work developing the MathemAntics software, we have designed a virtual manipulative that groups objects into a stack of ten boxes and visually highlights both the higher-order unit and the individualized units within the stack. As a virtual tool, this grouping mechanism transforms a unitized 10-bar that contains ten individual objects into a continuous bar that does not display units or objects. I believe this affordance of shifting from the unitized model of a 10-box to a continuous model is uniquely suited to improve children’s understanding of the higher-order unit and to promote more advanced strategy use, for instance counting on. However, research is needed to determine whether the unitized/continuous 10-box manipulative improves children’s accuracy, advances their strategy use, and promotes understanding of base ten concepts during enumeration tasks. Research is needed to examine whether this affordance does in fact promote advanced strategy use and encourage children to conceive of larger numbers as composed of tens and ones.
Study Goals

The aim of this study was to closely examine the affordances of a grouping by tens virtual manipulative. I investigated whether having the manipulative transform from a unitized to continuous model impacts children’s strategy use and understanding of base ten concepts. To measure the transformation affordance, I compared one group that uses the transformation model to another that uses a unitized model. Because the transformation condition enables children to see the 10-box as both ten individual units and as one tens unit, I hypothesized that children in this condition would improve in their strategy use, base ten understanding, and ultimately accuracy as compared to children in the unitized condition.

An additional study goal was to evaluate the effectiveness of our software more generally. To measure learning gains from the software, I had a third group that controls for the effects of ordinary classroom learning and everyday experience. My overall design involved three groups (Transformation, Unitized, Control) to assess the effectiveness of the transform affordance and the overall power of the mathematics software.

Research Questions

1) Does the Transformation condition improve children’s ability to count on at posttest more than does the Unitized or Control condition?

2) Does the Transformation condition improve children’s base 10 understanding at posttest more than does the Unitized or Control condition?

3) Do the Transformation and Unitized conditions improve children’s overall number sense at posttest more than does the Control condition?
4) Does the Transformation condition use more advanced strategies with more accuracy than does the Unitized Condition during the computer intervention sessions?
Chapter 2

METHODS

Design

This study was conducted using a new mathematics software program, MathemAntics, which is currently being developed by researchers at Teachers College, Columbia University. First grade students played either The Boxes Game, a MathemAntics activity where children solve a series of enumeration problems, or Reader Rabbit, a widely-used reading software developed to promote literacy skills. A central feature of The Boxes Game is the 10-box tool, an electronic manipulative designed to help children organize randomly arranged sets of objects into vertical stacks of ten objects.

This study used a pre/posttest between subjects design with condition as the main factor. The study also looked microgenetically at the math software intervention sessions. Because there is some evidence of gender differences in strategy use by early elementary school students, I also examined gender differences at posttest and during the intervention sessions. Both the Transformation and Unitized groups played the MathemAntics software, while the Control group played the reading software. In the Transformation condition, the 10-box tool transformed from a unitized stack of 10-boxes, with each box containing an object, into a continuous box with no objects visible. In the Unitized condition, the 10-box did not transform; rather, it remained unitized and continued to display the objects to be counted. The Unitized condition more closely resembles traditional base ten manipulatives, as it does not encourage children to move beyond perceiving the multiunit as a concatenation of single units.

All study children were assessed on several pretest/posttest measures to better understand their enumerating abilities, strategy use, base ten knowledge, and overall mathematics abilities.
before and after the software interventions. During the following weeks, children met with researchers for 5 study sessions, where they played either the reading or math software. The 5\textsuperscript{th} session resembled the previous MathemAntics sessions but required children to enumerate larger sets with no modeling or feedback. Immediately after the fifth session, children were assessed on a Near Transfer measure using physical base ten blocks.

I examined children’s accuracy (correct/incorrect) and strategy use (See Coding section and Appendix E) microgenetically over the course of the study sessions. Additionally, researchers recorded field notes about children’s behavior and strategy use during the study sessions. All researchers were trained in taking detailed field notes and I reviewed the notes weekly.

Participants

Participants were 79 first grade students (37 boys, 42 girls). Students were recruited from two public schools and one charter school in Harlem, New York. Subjects were recruited from the same schools that we worked with on the MathemAntics project. Two of the three schools in the study serve children from low socioeconomic status (SES) backgrounds. The third school serves children from mixed SES backgrounds. Exact socioeconomic information for individual children was not collected. Children’s ages ranged from 5.7 to 7.2 years of age (M= 6.3, SD= .316). Participants were through fliers sent home by their teachers and parent meetings that researchers attended. In the fliers, parents were asked to check off whether they consent to their children being videotaped. Two of the 79 students did not complete the computer intervention trials and posttesting because they were absent too many times.
The 10-Box tool

The “10-box” tool is a visual representation designed to aid in enumeration by grouping objects into stacks of 10 boxes and making the cardinal value of the objects in the stacked boxes explicit. The tool is activated upon clicking on a small icon at the bottom of the screen. After the stacked boxes appear, animals on the screen automatically move one at a time into the boxes, until the stack of 10 boxes is filled. In the Unitized condition, the stack of boxes remains unitized, showing ten animals and displaying the numeral “10” above the box (See example of an activated unitized 10-box below).

Figure 1: Screen shot of enumeration activity with the unitized manipulative

In the Transformation condition, the animals fill the stacked 10-box in the same way; however, when the entire 10-box is filled with animals, the box becomes continuous, hiding the animals, and the number “10” appears on the box (See activated continuous 10-box below).

Figure 2: Screen shot of enumeration activity with the transforming manipulative
In both conditions, if there are more animals than there are stacked boxes, “left over” animals remain scattered across the screen.

Children were allowed to press the 10-box icon numerous times, as long as 10 animals remained on the field to fill the box. Therefore, if a child pressed the 10-box 3 times, 3 sets of stacked boxes appeared on the left side of the screen and 30 animals would enter the boxes. While this study did not specifically ask children to solve addition problems, the boxes tool visually partitions objects to be counted into distinct subsets; therefore, children had to combine objects in the subsets, which is essentially an addition problem.

Procedure

All children worked individually with researchers who were blind to the study objectives. Children in the Control group met with researchers for the same number of sessions as children in the math conditions and for approximately the same length of time. The only difference between the two math conditions was whether the 10-boxes remained unitized or transformed into continuous boxes.

Children were pretested individually in a quiet space at each school by trained researchers, who were blind to condition; children who had parental permission were videotaped during pretesting and posttesting. Pretesting occurred during one session that lasted approximately 30 minutes. The 3 pretest measures (MClass, Count On, Base Ten) were counterbalanced to control for order effects. Researchers were trained to code children’s strategies for the Count On and Base Ten measures using videotapes from pilot work. Six out of eight researchers scored greater than 90% agreement with me when coding pilot children’s strategy use. For the trials where there total agreement was not achieved, researchers discussed
the trials and came to a consensus about which strategies were used to solve the problems. The two researchers that did not reach 90% agreement with me worked only with children who were videotaped. I later coded the videotapes that were recorded by these two researchers.

Sessions occurred two times per week for two and a half weeks. During each session, all children in the math conditions were presented with the same instructions, demonstrations, number of objects to count, and feedback. At the onset of the first session, children in both math conditions were given two practice trials in order to gain familiarity with the software. Children were required to use at least one 10-box per trial before submitting their answers. The software provided strategy-based and accuracy feedback after the child entered his or her response for the first 3 problems; strategy-based feedback modeled the count on strategy. For the remaining trials, only accuracy feedback was given.

During sessions 1-4, children in the math groups watched instructional videos that modeled using the 10-box tool with advanced strategies (See Appendix A). Before the first trial, researchers reminded children that they could press the 10-box tool as many times as needed. Children were required to press the 10-box tool at least one time prior to responding on the number line. Researchers worked individually with children, setting up the software and recording children’s responses and strategy use during each trial. At the end of each trial, children were asked how they figured out their answer. There were 9 trials in each of the 4 sessions with sets of animals ranging from 11-15 and 20-24 animals.

For the 5th computer session, children in the math groups were asked to enumerate sets of 30-34 animals with no modeling or feedback for 5 trials. This session had fewer trials than the other sessions because the set sizes were large. Children in the Unitized group saw boxes that remained discrete, while children in the Transformation groups saw boxes that transformed into
continuous boxes. Children in the reading group played the reading software for a comparable amount of time.

Five researchers, who did not collect pretest/posttest data, collected session data. Researchers coded observable strategies children used during the intervention sessions (specifically, states number fact, count all, count on, count by 10’s, points, fingers, no visible strategy-quick, no visible strategy-delay, and other). Prior to the start of the trials, all session data collectors were trained in the coding system using videos of non-study children playing the activity and percent agreement with me was calculated. Three out of the five researchers scored greater than 90% agreement with an expert when coding videotapes of children from a pilot study. Researchers discussed any trials where there was not total agreement and came to a consensus about which strategy the child used to solve the problem. The two researchers that did not reach 90% agreement worked only with children who were videotaped. I later coded the videotapes recorded by these two researchers.

Immediately after the 5th computer session, all study children were given a near transfer test with base ten blocks, which examined children’s ability to determine how many tens and ones blocks there are altogether (See Measures). The Near Transfer measure was administered by the same researchers who collected session data, since the coding scheme was identical to that used during the computer sessions. This measure took approximately 7 minutes to complete. The following week, children were posttested. Researchers administered 3 post-test measures, which they had also taken at pretest, including a standardized mathematics assessment and a counting on and base ten measure to assess far transfer (See Measures section). The researchers who collected pretest data also collected posttest data and worked individually with children for approximately 30 minutes. Researchers were blind to condition.
Standardized Measures

*mClass: Math* test (Lee et al., 2007): *mClass: Math* is a standardized, electronic mathematics assessment designed to assess early number skills and understandings for kindergarten through third grade students (Lee et al., 2007). Children were administered six first grade level Curriculum Based Measures (CBM) of counting, missing number, next number, number identification, number facts and quantity discrimination. Each CBM took approximately 1 minute to administer.

Pre and Posttests:

**Count On Measure (See Appendix B):** I used a task adapted from Secada, Fuson and Hall’s (1983) study with first grade students to assess children’s ability to count on. This task measures children’s ability to count on an array of dots from a given numeral.

For the first 3 trials, children were shown a card with an array of dots on it. The researcher placed an index card with a numeral above the array card that tells the cardinal value of the array and explained the relationship of the two cards to the child. The researcher then flipped the array card over to hide the dots. A second array card was placed to the right of the 1st array card and a second numeral card with the cardinal value of the second set was placed above the 2nd array card. The child was asked to determine how many dots there are on both array cards all together.

The final 3 trials had the same procedure, except the researcher did not flip over the first array card. Rather, the researcher provided a hint to the child that it is not necessary to count the dots on the first array card. The final 3 trials tested children’s ability to count on in the face of
visual stimuli that promote counting all. I recorded children’s accuracy on each trial and whether or not the child counted on (See Appendix E for coding scheme).

Because children often count in their minds and with their fingers, we also coded children’s pointing behaviors. When children pointed to individual dots on the second addend card and not the first, this was classified as counting on. When children pointed to individual dots on both of the addend cards, whether or not dots were visible on the first addend card, this behavior was classified as counting all. Pilot data showed that children who were not ready or capable of counting on often counted imaginary dots on the hidden first addend card, despite the likelihood of being inaccurate. The Count On measure had a reasonably high internal consistency using the Kuder-Richardson formula 20, $\alpha(20) = .77$.

**Base Ten measure (See Appendix C):** This measure was adapted from a task developed by Miura and Okamoto (1989). The original task was designed to assess children’s cognitive representation of number; however, this task also captures rudimentary base ten understanding and has been shown to correlate with place-value understanding (Miura & Okamoto, 1989). Because first grade students in the United States performed very poorly on this task (Miura & Okamoto, 1989; Miura et al., 1993), I simplified and shortened it. The researcher counted 10 unit blocks and demonstrated the equivalence between these unit blocks and one tens block. The child was then asked to read aloud a numeral written on a card and to show the number using the blocks. There were 2 demonstrations with numbers less than 10. Three numerals were then presented: 11, 13, and 28. Children were asked to make these numbers with the blocks. After each trial, the researcher asked the child to show another way to make the number using the blocks.
Children’s responses were categorized as correct or incorrect. Each correct, novel construction was given a score of 1, with a total possible score ranging from 0 to 6. Each correct construction was coded as a 1-to-1 representation (28 units blocks for 28), a Canonical Base 10 representation (2 tens and 8 units for 28), or a Non-canonical base 10 representation (1 ten and 18 units for 28) (See Miura et al, 1993 and Ross, 1986). Children were given a score from 0-3 for the total number of correct, canonical base ten constructions that were made across the 2 trials for the 3 problems. This measure had a moderate internal consistency using the Kuder-Richardson formula 20, $\alpha(20) = .65$.

Near Transfer Measure (Post-test only; See Appendix D): This measure was designed to assess children’s ability to combine tens and ones using base ten blocks. Children solved the problems by counting on, counting by 10’s, counting by 10’s and adding ones, counting by 10’s and counting on, or using base ten knowledge and subitizing skills to automatically recognize the number of blocks. Researchers showed children a 10-block and 10 units blocks and explained the equivalence between the 10-block and 10 unit blocks. Researchers then showed children a combination of 10’s blocks and unit blocks and asked children to determine how many there were altogether. Children were asked to enumerate sets of 15, 12, 32, 13, 21, and 23 blocks. Children were given a total accuracy score from 0 to 6 and researchers coded children’s strategy use using the same set of codes that were used during the study sessions (See Appendix E and Coding section below).
Coding

In order to determine children’s strategy use during study sessions and the Near Transfer task, I used codes that I developed during my pilot study (See Appendix E). These codes were developed based on the pilot data in conjunction with guidance from the literature. There are several strategies that I coded, namely, states number fact, count all, count on, count by 10’s, points, fingers, no visible strategy-quick, no visible strategies-delay, and other. States number fact was coded whenever a child stated a math fact, such as “10+2” or “10 and 10 is 20 and 3 is 23,” when working out an answer. Using number facts to solve the problem is a very sophisticated strategy, as noted previously, as it demonstrates the use of retrieval or derived facts to solve the problem. Counting all was coded when children counted each object on the screen, starting at 1. Counting on was coded when children began their count with the cardinal value of the first set or the cardinal value of the first set plus 1, for instance, when children began counting animals outside of the 10-box as “11, 12, 13…” As has been described above, counting on is a more efficient and sophisticated strategy than counting all. Researchers also marked from which number children began counting on. Counting by 10’s was coded when a child audibly counted by 10’s. Counting by 10’s was generally used in conjunction with count on, for instance when children counted 3 10-boxes and a few remaining animals on the field, such as, “…10, 20, 30, 31, 32.” Counting by 10’s is an efficient strategy that shows a developing understanding of base ten concepts.

Children’s pointing was coded in two different ways to note when children pointed to each individual animal on the screen, whether it was hidden by the box or not, or whether they pointed only to individual animals outside of the box. When children pointed to each individual animal, this implied children’s use of the count all strategy as children were likely counting in
their minds as they touch each object. In contrast, when children only pointed to animals outside of the box or boxes, this suggested the count on strategy for children did not individually attend to the objects in the box/boxes. After all data was collected and entered, I recoded when children pointed to all the animals as counting all and recoded when children pointed only to animals outside of the box as counting on. Researchers also coded whether children used their fingers to count at any point during a trial.

No visible strategies-quick was coded when the child responded immediately (less than 3 seconds) after either seeing the problem or after pressing the boxes tool. This strategy suggests that children may be quickly using arithmetic or recalling a known number fact to help them solve the problem. For instance, upon seeing a 10-box and 2 leftover animals, a child may respond quickly because she sees 2 and knows that 10 and 2 is 12. No visible strategy-delay was coded when children made no observable behaviors, but responded more than 3 seconds after seeing the problem or pressing the 10-box tool. This category encompasses a range of possible strategies, including silent counting, using arithmetic, recalling known number facts, or using a derived strategy. While it is not possible to determine precisely which strategy children use when they make no observable behaviors, no visible strategies is generally considered to be more advanced than simply counting all in the literature, as it requires children to use more cognitive resources and often involves retrieval of known facts (Siegler & Robinson, 1982).

After all data was collected, I classified strategies as either Advanced or Not Advanced (See Table 1). In order to ensure that children who used no visible strategy-correct were not merely guessing randomly, I examined all instances where children used this strategy and answered incorrectly. When children’s answers were within two of the correct answer (nearly accurate) when using no visible-quick, they were considered to be using an advanced strategy.
When children were off by 10, for instance responding 13 instead of 23, or were off by 11, responding 13 instead of 24, they were also considered to be using an advanced strategy and not randomly guessing. This type of error suggests that children used knowledge of base ten concepts to help them determine the answer. There were only 8 instances of random guessing when using the no visible-quick across all 5 sessions. In these instances, children’s use of the no visible-quick strategy was coded as Not Advanced.

<table>
<thead>
<tr>
<th>Advanced</th>
<th>Not Advanced</th>
</tr>
</thead>
<tbody>
<tr>
<td>Count On</td>
<td>Count all</td>
</tr>
<tr>
<td>Count by 10's</td>
<td>No visible strategy-delay</td>
</tr>
<tr>
<td>No visible strategy-quick</td>
<td>Other</td>
</tr>
<tr>
<td>Multiple strategies</td>
<td></td>
</tr>
</tbody>
</table>

*Table 1: Classification of advanced strategies on Near Transfer measure and computer intervention trials*

I also computed whether children primarily used advanced strategies during each session. When children’s used an advanced strategy for 6 or more trials during Sessions 1,2,3, and 4, they were considered to primarily use advanced strategies. Because Session 5 only had 5 trials, children were considered to primarily use advanced strategies if they used them on 3 or more trials.
Chapter 3

RESULTS

Pretest Analyses

In order to ensure equivalent groups at pretest, I ran a Multivariate Analysis of Covariance (MANCOVA) with the four MClass CBM pretest scores (Missing Number, Next Number, Number Identification, Quantity Discrimination) and two Base Ten pretest scores (Accuracy, Strategy use) as the dependent variables (DVs). I did not include the Counting or Number Facts CBM scores, as these CBM scores were dropped from all analyses. The counting CBM was not included because of technical recording problems on the devices used to score the data. The number facts CBM was not included because most children scored extremely low on this task and were discontinued from completing the task. Preliminary analyses detected no statistically significant group differences at pretest, Wilks’ Lambda=.809, F(12,98)=.915, p=.536.

I also conducted two Generalized Linear Mixed Methods analyses to assess group differences at pretest on the Count On measure. The first GLM examined the effects of condition on pretest accuracy scores (Y/N) and the second examined the effects of condition on children’s use of the count on strategy at pretest (Y/N). Mixed method GLM analyses were used for the Count On pretest scores because these were the methods I used for analyzing posttest data on this measure. There were no significant group differences detected on either of the pretest scores, F(2,461)=1.698, p=.184 for accuracy and F(2,453)=.290, p=.748 for strategy use. However, the interaction between gender and condition approached significance for accuracy scores, F(2,461)=.067. More specifically, girls in the Unitized group performed significantly
better than girls in the Control group, adjusted p=.044. I consider this finding in the results that follow.

Question 1: Does the Transformation condition improve children’s ability to count on at posttest more than does the Unitized or Control condition?

Thirty-four children were able to solve at least 1 of the hidden addend Count on problems at pretest. At posttest, 47 children were able to solve at least 1 of the hidden addend Count on problems. Overall, children made significant improvements in their ability to count on accurately from pretest to posttest, F(1,74)=6.988, p=.010.

<table>
<thead>
<tr>
<th></th>
<th>Transformation (n=26)</th>
<th>Unitized (n=25)</th>
<th>Control (n=26)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>SD</td>
<td>M</td>
</tr>
<tr>
<td>Pretest</td>
<td>2.93</td>
<td>2.017</td>
<td>2.6</td>
</tr>
<tr>
<td>Posttest</td>
<td>4.08</td>
<td>2.038</td>
<td>3.04</td>
</tr>
</tbody>
</table>

Table 2: Means and standard deviations on the Count On measure by condition and time

To determine whether condition affected children’s ability to accurately count on at post-test, I ran a Mixed Effects Generalized Linear Model with student as a random factor, item as a within-subjects, fixed factor, and gender, school, and condition as between-subjects, fixed factors on Count On accuracy scores (Y/N). I ran a full-factorial model and stepwise removed non-significant interactions. My final model included 2-way interactions between gender and
condition and between item and condition. Results of the Omnibus F tests showed main effects for condition, $F(2,439)=4.955$, $p=.007$, and item, $F(5,439)=4.051$, $p=.001$.

As figure 3 shows, the Transformation group was 82% likely to solve an item correctly, while the Unitized group was 56% likely and the Control group 45% likely to solve a problem correctly.

![Figure 3: Estimated accuracy means and standard errors on the Count On posttest by condition](image)

I conducted pairwise contrasts using the Bonferroni correction to examine the specific differences between each of the conditions. As was expected, the transforming manipulative had a significantly greater positive impact on children’s ability to count on accurately at posttest than did the unitized manipulative, adjusted $p=.039$. Children who used the transforming manipulative were also significantly more likely to count on accurately at posttest than were

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1 I also ran a GLM analysis after removing the 4 children who scored at ceiling on the Count On measure for both accuracy (score of 6) and advanced strategy use. Gender by condition was the only interaction that remained in the model in predicting accuracy. I found the same overall results: condition significantly predicted accuracy scores, $F(2,425)=4.200$, $p=.016$ and item significantly predicted accuracy scores, $F(5,425)=3.955$, $p=.002$. 

those in the Control group, adjusted p=.003. Children who used the unitized manipulative were not more likely to count on accurately at posttest than were children in the Control group, adjusted p=.421. These findings show that having the manipulative transform from a unitized to a continuous model positively influenced children’s counting on abilities.

Because the Count On measure was composed of two different problem types (the first addend dot array was hidden for the first 3 trials, but visible for the final 3 trials), I also examined the effects of item on children’s accuracy scores. Students were significantly more likely to be accurate when addend arrays were visible than when the first addend array was hidden (See Figure 4). More specifically, children were more accurate on item 4 than on items 1, adjusted p=.049, 2, adjusted p=.023, or 3, adjusted p=.013. Children were also significantly more likely to be correct on item 5 than on items 2, adjusted p=.049, or 3, adjusted p=.030. This finding highlights the higher difficulty level of the first 3 trials, which require children to use the more advanced count on strategy. Using the count all strategy on the first 3 trials was an ineffective strategy choice, resulting in low accuracy levels.

![Figure 4: Estimated accuracy means and standard errors on the Count On posttest by item](image-url)
No significant interaction was detected between item and condition on Count On accuracy scores, F(10,439)=.857, p=.573. This was surprising as I expected the Transformation group to score significantly higher than the other two groups on the hidden addend items in particular, as these items required counting on to be accurate. However, because there was a main effect for condition, it appears that the transforming manipulative positively impacted children’s accuracy on both problem types, those with hidden and visible addends, compared to the unitizing manipulative and the reading software.

\textit{Does condition affect observed strategy use at post-test?}

In addition to analyzing accuracy scores on the Count On task, I also examined whether condition impacts observed strategy use at posttest. I conducted another GLM analysis using the same set of factors from the previous analysis, with use of the count on strategy (Y/N) as the dependent variable.\textsuperscript{2} Interactions between gender and condition as well as condition and index remained in the model. The analysis revealed a significant interaction between condition and gender in predicting children’s use of the count on strategy, F(2,451)=5.474, p=.004. Boys in the Transformation condition were significantly more likely to count on than were boys in the Unitized condition, adjusted p=.020, or than boys in the Control condition, adjusted p=.016. Boys in the Transformation group were 57% likely to use the count on strategy, while boys in the Unitized group were 6% likely to use the count on strategy and those in the Control group were 3% likely to count on. A similar trend was not found amongst female participants. Rather, girls

\textsuperscript{2} I also conducted this analysis after removing the four children who scored at ceiling on the Count On task. Condition by gender and item by gender remained in the final model. Condition by gender remained significant, F(2,432)=4.998, p=.007. Gender by index was also significant, F(5,432)=2.437, p=.034; however, there were no significant pairwise differences between items based on gender or between gender based on items.
in the Unitized condition were 48% likely to use Count On compared to 13% of girls in the Transformation group and 11% of girls in the Control group. While pairwise differences between girls in the different conditions were not significant, it was unexpected that girls in the Transformation condition would not improve in their strategy use at posttest more than girls in the other two conditions.

Boys were significantly more likely to count on than were girls after using the transforming manipulative, adjusted $p=.033$, while girls were significantly more likely than boys to count on after using the unitized manipulative, adjusted $p=.021$. I did not anticipate that girls would benefit more from the unitized manipulative than from the transforming manipulative. There are several possible reasons for these results, which will be discussed further in the Discussion. However, it is possible that group differences among girls at pretest in their accuracy scores on the Count On measure may have contributed to girls’ greater use of the count on strategy at posttest.

Given the use of the count on strategy, does conditions impact accuracy?

In order to determine whether the transforming manipulative improved children’s accuracy levels when using the count on strategy more than did the unitized manipulative, I examined accuracy scores given children’s use of the count on strategy. I ran a Generalized Linear Mixed Effects Model with subject as a random factor and condition and item as fixed factors, controlling for gender and school. The interactions between gender and condition,
gender and school, condition and use of the count on strategy, and item and use of count on strategy remained in the model.³

There was not a statistically significant interaction detected between condition and use of the count on strategy, \( F(2,139)=1.642, p=.197 \). However, a clear trend emerged: given the use of the count on strategy, the Transformation group was more likely to be accurate than the other two groups. Given use of the count on strategy, the Transformation group was 96% likely to be accurate, whereas the Unitized group was 80% likely to be accurate and the Control group was 72% likely to be accurate. Using the Bonferroni correction, there were significant pairwise differences between the Transformation and Unitized groups, adjusted \( p=.009 \) and between the Transformation and Control groups, adjusted \( p=.017 \). Using the transforming manipulative helped children become more accurate in their use of the count on strategy at posttest than did using the unitized manipulative. The transforming manipulative also helped children become more accurate at counting on than did the reading software.

³ I also conducted a similar GLM with the 4 children who scored at ceiling removed from the analysis. No interactions remained in the model. My results were similar when rerunning the analysis: Condition approached significance, \( F(2,426)=2.869, p=.058 \). Use of the count on strategy was significant in predicting accuracy scores, \( F(1,426)=15.680, p<.001 \), as was item, \( F(5,426)=4.562, p<.001 \).
Figure 5: Estimated accuracy means and standard errors by condition given the use of the count on strategy.

The transforming manipulative not only helped children learn to count on in a far transfer context, but also to use this strategy accurately. The high-level of accuracy of all students who used the count on strategy attests to the benefits of using this highly efficient strategy. As was expected, the use of the count on strategy predicted accuracy scores, F(1,139)= 41.524, p<.001. Those who used the count on strategy were 86% likely to be correct, whereas those who did not count on were 50% likely to be correct, p<.001.

Question 2: Does the Transformation condition improve children’s base 10 understanding at posttest more than does the Unitized or Control condition?

This study also sought to determine whether having the grouping by tens manipulative transform from a unitized to a continuous model would improve children’s base ten
understanding compared to the unitized model or the reading software. Two post-test measures were used to assess children’s base ten knowledge: the Base Ten measure and the Near Transfer measure. The Base Ten measure asked children to construct 3 multi-digit numbers in two different ways using base 10 blocks. Children were given two scores on this measure: an accuracy score for each correct, unique construction and a score for the number of correct, canonical constructions children made across the 3 problems. The Near Transfer task, which was administered at posttest only, asked children to enumerate using base ten blocks; children received an accuracy score (Y/N) as well as a strategy score (Advanced/Not Advanced). The Near Transfer task was designed to capture children’s base ten knowledge in addition to children’s strategy use because of its reliance on base ten blocks. For instance, many children were able to quickly recognize that a 10’s rod and 2 units blocks were 12 without having to use a counting strategy; this was coded as an advanced strategy (See Table 1 for Classification of Advanced strategies).

*Does condition affect performance on the Base Ten measure?*

To examine how condition impacted children’s base ten understanding at posttest, I conducted a mixed MANCOVA with time (pre/post) as a within-subjects factor and condition as a between-subjects factor, controlling for gender and school. The dependent variables were total accuracy score on the Base 10 measure (0-6) and the number of correct, canonical constructions children (0-3). I initially ran a full-factorial model and then removed all non-significant interactions between condition, gender and school. My final model included the interaction between gender and condition and all within-subjects interactions. For significant tests, I
conducted Tukey’s Honestly Significant Difference (HSD) post-hoc, pairwise comparisons to examine group differences.\textsuperscript{4}

Overall, children improved on the Base 10 measure from pretest to posttest, Wilks’ Lambda = .684, \( F(2,66)=15.253, p<.001 \). Children made significant improvements in their accuracy scores over time, \( F(1,67)=30.189, p<.001 \); total accuracy scores were 1.113 points higher at posttest (\( M=4.032, \text{SE}=.211 \)) than at pretest (\( M=2.920, \text{SE}=.201 \)), \( p<.001 \).

Additionally, children made significantly more canonical constructions at posttest (\( M=1.421, \text{SE}=.159 \)) than at pretest (\( M=1.021, \text{SE}=.158 \)), \( F(1,67)=5.835, p=.018 \).

<table>
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<th>Sum of Squares</th>
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<th>Mean Square</th>
<th>F</th>
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<td>3.793</td>
<td>1.178</td>
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</tr>
<tr>
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<td>3.22</td>
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<td><strong>Within Subjects</strong></td>
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<td>0</td>
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<td>2.202</td>
<td>0.032</td>
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<tr>
<td>Error(Time)</td>
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<td>1.029</td>
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</tr>
</tbody>
</table>

* Adj. p < 0.05; ** Adj. p<.001

\textit{Table 3:} Base Ten Accuracy ANOVA Summary

\textsuperscript{4}I also conducted this analysis after removing from the data the five children who scored at ceiling on the Base Ten measure (score of 6 for accuracy and 3 for number of canonical constructions). My results were similar: Time remained significant, Wilks’ Lambda=.657, \( F(2,62)=16.208, p<.001 \), and gender by condition remained significant, Wilks’ Lambda=.770, \( F(4,124)=4.329, p=.003 \).
In terms of group differences on the Base Ten measure, condition and gender interacted to predict Base 10 accuracy scores, $F(2,67)=3.849$, $p=.026$, and the number of correct canonical constructions children made, $F(2,67)=6.390$, $p=.003$. Girls who used the transforming manipulative were more likely to make unique constructions than were girls in the Control group, although this finding approached significance, adjusted $p=.091$. Surprisingly, however, the same pattern was not true for boys. In fact, boys in the control group performed very similarly to boys in the other two groups. Boys in the Control group made significantly more unique constructions than did girls in the Control group, adjusted $p=.032$, which was not true for the other two conditions. This finding was also not anticipated.

* Adj. p < 0.05; ** Adj. p<.001

Table 4: Base Ten Number of Canonical Constructions ANOVA Summary

<table>
<thead>
<tr>
<th>Item</th>
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<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>Partial Eta Squared</th>
</tr>
</thead>
<tbody>
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<td><strong>Between Subjects</strong></td>
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<td></td>
<td></td>
<td></td>
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<td>9.611</td>
<td>5.257**</td>
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<td>0.687</td>
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</table>
In terms of the number of canonical base ten constructions children made, girls in the Unitized condition made more canonical constructions than did girls in the Control group, which approached significance, adjusted $p=.097$. A similar finding was not true for boys. Boys in the Control group made significantly more canonical constructions than did girls in the Control group, adjusted $p=.004$, which did not occur in the other two conditions and was unexpected.
In summary, girls appeared to learn base ten concepts from using both the transforming and unitized manipulatives, but not from working with the reading software. Boys, on the other hand, improved in base ten concepts in all three conditions. Boys’ relatively high performance in the control group was very unexpected and suggests that despite not using the mathematics software, they learned base ten concepts during the course of the study.

While there are many possible explanations for these findings, some of which will be discussed in greater detail in the Discussion section, I will also address a few possibilities here. After looking more closely at children’s pretest scores on the Base Ten measure by condition and gender, I found initial group differences which help to explain these unanticipated findings. A very similar gender by condition pattern emerged after plotting the pretest data, suggesting that differences at posttest were highly related to initial condition by gender differences. While there were no significant pairwise comparisons at pretest, boys in the Control group had higher accuracy scores (M=3.38, SE=.417) than did girls (M=2.08, SE=.336) and higher number of canonical constructions (M=1.54, SE=.332) than did girls in the Control group (M=.33, SE=.256). While this does not explain why boys in the Control group appeared to learn base ten concepts over the course of the study, it does suggest that boys’ higher base ten performance within the Control group was likely due to initial gender differences within this condition.

Does condition affect performance on the Near Transfer task?

To evaluate group differences in performance on the Near Transfer task, I ran a GLM model with subject as a random factor, condition as a between-subjects, fixed factor and item as a within-subjects, fixed factor. The analysis controlled for school and gender and the DV was children’s accuracy scores (Y/N). Gender by condition remained in the model. For significant
tests, I conducted pairwise contrasts using the Bonferroni correction. The results showed that condition and gender interacted to predict children’s accuracy scores on the Near Transfer task, F(2,443)=3.170, p=.043. More specifically, girls in both software conditions enumerated base 10 blocks more accurately than did girls who used the reading software. Girls in the Transformation condition were more accurate than were girls in the Control condition, adjusted p=.001, and girls in the Unitized condition were more accurate than were girls in the Control condition, adjusted p=.002. A similar effect was not seen for boys. Additionally, boys in the Control group were significantly more accurate than were girls in the Control group, adjusted p=.001, which was unexpected. This finding, however, aligns with the gender comparison within the Control group that was found on the Base Ten measure. Boys in the Control group showed greater knowledge of base ten concepts at pretest when compared to girls in the Control group, which likely impacted their learning and abilities at posttest.

I also examined whether condition affected children’s advanced strategy use on the Near Transfer task. Using the same GLM approach that I used on the Near Transfer accuracy scores, I reran the GLM with Advanced Strategy Use (Y/N) as the dependent variable, with condition by index as the only interaction included in the model. Condition approached significance in predicting children’s advanced strategy use on the Near Transfer measure, F(2,427)=2.483, p=.085. Children in the Transformation condition had a 62% probability of using an advanced strategy, compared to children in the Unitized condition, who had a 46% likelihood of using an advanced strategy, and children in the Control condition, who had a 30% likelihood of using an advanced strategy. Children who used the transforming manipulative were significantly more likely to use an advanced strategy when enumerating with base 10 blocks than were the Controls, adjusted p=.048.
The finding that children in the Transformation condition were more likely to use advanced strategies than were children in the Control group captures both base ten understanding as well as strategy use more generally. Advanced strategies included no visible-quick, a strategy that requires base ten knowledge, as well as count on, which does not necessarily require base ten knowledge. That children who used the transforming manipulative were significantly more likely to use an advanced strategy when enumerating base ten blocks than were children who used the reading software provides evidence for the power of this transforming manipulative in helping children learn both advanced strategies and rudimentary base ten concepts.

Question 3: Do the Transformation and Unitized conditions improve children’s overall number sense at posttest more than does the Control condition?

Because the MathemAntics software offers children structured practice in enumerating sets, it was anticipated that children’s overall number sense would improve after using the math software. In order to assess how condition impacted overall number sense, I ran a full factorial MANCOVA with condition as a between-subjects factor, controlling for gender, school, and MClass CBM pretest scores, on posttest MClass CMB scores (Missing Number, Next Number, Number Identification, and Quantity Discrimination). I stepwise removed non-significant interactions, leaving condition by gender in the final model.

The results showed that the interaction between condition and gender approached significance, Wilks’ Lambda=.785, F(8,112)=1.797, p=.085. Follow-up univariate analyses revealed that condition by gender approached significance for Next Number, F(2,59)=3.134, p=.051, and Number Identification, F(2,59)=2.670, p=.078 (See Table 5). Despite these results,
no significant pairwise differences were uncovered. Thus, while it appears that boys who used the transforming manipulative improved in their number sense more than did those who used the unitized manipulative, an effect which was not seen for girls, these results were not in fact significant and should be interpreted with caution.

<table>
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<tr>
<th>Source</th>
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</table>
Table 5: MClass:Math ANOVA Summary

**Question 4:** Does the Transformation condition use more advanced strategies with more accuracy than does the Unitized Condition during the computer intervention sessions?

In order to better understand how the transforming manipulative differentially impacted children’s learning while playing the math software compared to the unitized manipulative, I examined children’s accuracy and strategy use during the five MathemAntics computer sessions. During the first four sessions, children were asked to enumerate sets ranging from 11-15 and 21-24 animals, which were presented in random order over the course of 9 trials. At the onset of each of the first four sessions, children watched a brief video modeling an advanced strategy; they also received feedback on their answers. During the fifth session, children received no modeling or feedback and were asked to enumerate larger sets, ranging from 30-35 animals for 5
trials. I conducted a Generalized Linear Mixed Effects Model with subject as a random factor, condition as a between-subjects, fixed factor, and session, trial, and set size as within-subjects, fixed factors. I controlled for gender and school differences, beginning with a full factorial model and stepwise removing non-significant interactions. For significance tests, I conducted pairwise contrasts using the Bonferroni correction.

To determine whether condition impacted accuracy scores during the computer sessions, I ran the GLM with accuracy (Y/N) as the DV. Condition by session remained in the model. My results showed that the interaction between condition and session approached significance, F(4,2003)=2.160, p=.071. Children who used the transforming manipulative became significantly more accurate when using the manipulative to enumerate in Sessions 3 and 4 as compared to in Session 1, adjusted p<.001 and p=.002 respectively. This was not true for children who used the unitized manipulative (See Figure 8).

![Figure 8](image-url)  
*Figure 8: Estimated accuracy means and standard errors on computer trials by condition and session*
Because the interaction between condition and session only approached significance, I also examined the main effect for session, which was highly significant, $F(3,2003)=8.070$, $p<.001$. Overall, children were more accurate in Session 3 than in Session 1, adjusted $p=.001$, and in Session 4 than in Session 1, adjusted $p=.003$. Importantly, children’s accuracy levels remained consistently high after Session 3, when controlling for set size; children had approximately an 80% probability of being correct during Sessions 3-5 (See Figure 9). That children were able to maintain a high level of accuracy throughout the three weeks of computer sessions suggests that the software may help children solidify their understanding and use of newly acquired strategies.

![Figure 9: Estimated accuracy means and standard errors on computer trials by session](image)

As was anticipated, set size also predicted accuracy scores, $F(12,2003)=11.560$, $p<.001$. Children were significantly more accurate when enumerating smaller than larger set sizes on the math software (See Figure 10).
To determine whether condition affected advanced strategy use during the computer intervention sessions, I ran a GLM with the same set of factors that I included in the previous GLM predicting accuracy; however, in this analysis the new dependent variable was Advanced Strategy Use (Y/N) (For a list of advanced strategies, see Table 1). After stepwise removing non-significant interactions, condition by session and session by trial remained in the model. My results revealed a significant interaction between condition and session in predicting children’s use of advanced strategies, $F(4,1988)=4.794$, $p<.001$. During session 1, children who used the transforming manipulative were significantly more likely to use an advanced strategy than were children who used the unitized manipulative, adjusted $p=.017$. This was not true for the other sessions (See Figure 11). Additionally, children in both conditions were more likely to use advanced strategies after playing the math software for more than 1 session.

Figure 10: Estimated accuracy means and standard errors intervals on computer trials by set size
Session 1:
T > U, p=.017

Unitized:
S5 > S1, p=.002
S4 > S1, p<.001
S4 > S2, p=.029
S3 > S1, p<.001
S2 > S1, p<.001

Transformation:
S4 > S1, p=.004
S3 > S1, p=.003
S2 > S1, p=.006

Figure 11: Estimated mean probabilities and standard errors of using an advanced strategy by condition and session

There was also a significant interaction between session and trial, F(16,1988)=1.682, p=.043, which was very difficult to interpret. Children did better on trials 1 and 2 in later sessions than in earlier sessions. This effect was not true for all trials or sessions.

Finally, I also investigated how condition impacts accuracy scores during the computer intervention sessions, given the use of an advanced strategy. In addition to the same set of factors used in the GLM analysis on computer session accuracy scores, I reran the GLM with advanced strategy use (Y/N) as a within-subjects, fixed factor in predicting accuracy scores. The interaction between condition and advanced strategy use remained in the model.

The results showed a significant interaction between condition and advanced strategy use in predicting accuracy scores, F(1,1992)=14.165, p<.001. Children who used the transforming manipulative were significantly more likely to be accurate when using an advanced strategy (79%) than when not using an advanced strategy (46%), adjusted p<.001. This was not true for those in Unitized condition. For children who used the unitized manipulative, there was very
little difference in accuracy depending on whether they used an advanced or not advanced strategy. Children who used the unitized manipulative were 77% likely to be correct when using an advanced strategy and 73% likely to be correct when not using an advanced strategy. Thus, the transforming manipulative not only encouraged children to use more advanced strategies earlier than did the unitized manipulative, but it also helped children to be accurate when using advanced strategies.

Children who did not use advanced strategies were significantly more likely to be accurate in the Unitized condition than in the Transformation condition, adjusted p=.018. This finding makes sense considering that the transforming manipulative hides the animals within the box, making it difficult for children to be accurate when using less sophisticated strategies, such as counting all. The unitized manipulative, however, leaves all animals are visible, making it more likely that children will be accurate when using less advanced strategies.
Chapter 4

DISCUSSION

Overview of findings

In this study, I examined the affordances of a grouping by tens manipulative on children’s mathematics abilities. The main findings revealed that using the transforming manipulative over the course of 5 study sessions improved children’s abilities to count on in a far transfer context at posttest. This finding expands upon the research on children’s strategy use as it highlights specific features of a novel manipulative that encourage the adoption and employment of advanced addition strategies. This study also extends the body of literature focused on the use of manipulatives with children. Few studies isolate particular features or affordances of manipulatives that contribute to children’s strategy advancement and base ten understanding. My findings demonstrate the value of closely examining the features of manipulatives to better understand how they impact different types of learning.

Another major finding of this study was that girls in both math software conditions made significant learning gains in the realm of base ten understanding compared to girls who used the reading software. Girls learned from using both the unitized and transforming manipulatives when enumerating objects. This suggests that rich, scaffolded experiences with a virtual grouping by tens manipulative have the potential to improve children’s understanding of base ten concepts. Because boys in the Control group also appeared to learn during the course of the study, it is difficult to determine whether learning occurred for boys due to the software. Possible explanations for these gender differences will be discussed in greater detail below.

This research also showed that children who used the math software became more likely to enumerate accurately and use advanced strategies after playing the activity for more than one
session. These learning gains were maintained throughout the remainder of the following four sessions and persisted after the virtual manipulatives were no longer available for children to use. This finding offers insight into the ways in which virtual manipulatives should be used with children to promote learning and suggests that repeated exposure to mathematics software can build and solidify strategic and conceptual learning.

This study builds upon Zur and Gelman’s (2004) findings regarding the effectiveness of hiding objects in order to advance children’s thinking. Zur and Gelman (2004) “arithmetic-counting” task encouraged preschoolers to predict and check solutions in hidden addend and minuend problems. They found that children as young as 3 and 4 years old were able to make reasonable predictions about addition and subtraction solutions after enumerating a set, observing a teacher cover the set, and then watching additional items either being added to or subtracted from the hidden set. My study extends these findings, as I also demonstrated the usefulness of designing hidden addend tasks within the context of enumeration activities. While Zur and Gelman (2004) found that hiding was an effective paradigm for promoting arithmetic thinking, my results showed this paradigm to be a powerful method for advancing children’s strategy use and base ten understanding. This dissertation goes beyond Zur and Gelman’s (2004) work as it outlines possible mechanisms through which the transforming and hiding actions may encourage learning.
<table>
<thead>
<tr>
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<td>Accuracy</td>
<td>Condition*</td>
<td>T&gt;U*, T&gt;C*</td>
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<td>Condition by Gender*</td>
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<td>Particular Strategy</td>
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<td>Base Ten</td>
<td>Accuracy</td>
<td>Time*</td>
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<td>Condition by Session**</td>
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<td>Condition by Advanced Strategy**</td>
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<td>Particular Strategy</td>
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<td>T: Advanced&gt;No Advanced strategy**</td>
</tr>
</tbody>
</table>

* Adj. p < 0.05; ** Adj. p<.001; *** Adj. p <.1

Note: M=Males, F= Females, T=Transformation Condition, U=Unitized Condition, C=Control Condition, Q=Question, S=Session, CO=Use of Count On

**Table 6: Summary of Main Findings**

**Counting On Abilities**

My results confirmed the hypothesis that using the transforming manipulative would positively affect children’s ability to count on more than would the unitized manipulative or the
reading software. There are numerous possible explanations for why the transforming affordance improved children’s accuracy scores on the Count On measure at posttest. One such explanation is that the move from a discrete to a continuous model shifted children’s attention from the individual items within the set of 10 animals to the cardinal value of the set. As Mix (2010) has argued, focusing attention is a key mechanism through which well-designed manipulatives may promote learning. Despite the cardinal value label attached to the 10-boxes in both the Unitized and Transformation conditions, only the transforming manipulative sought to actively shift children’s visual attention to the cardinality of the box, by hiding the contents of the box. Focusing attention is a likely mechanism through which this novel manipulative improved learning.

Another explanation for why the transforming manipulative positively affected children’s strategy use as compared to the unitized manipulative may involve the streamlined design of the 10-box after it transforms into a continuous box. Research shows that decreasing extraneous, potentially distracting features can aid in focusing children’s attention on the important mathematical concepts (Uttal, O’Doherty, Newland, Hand & DeLoache, 2009). After the 10-box transforms from a unitized to a continuous model, the animals within the box become hidden, which decreases the visual imagery within the box and lowers the number of potentially distracting elements on the screen. Consequently, the continuous box may have provided a simpler, less distracting model for children, which directed their attention to the cardinal value of the 10-box.

Inhibitory control may also play a role in explaining the benefits of the transforming 10-box over the unitized 10-box. In order to count on while using the unitized manipulative, children must resist the urge to count the animals they see within the unitized 10-box, a task
which may require substantial inhibitory control for many children. Several studies describe children’s tendencies to regress to less efficient strategy use even after acquiring a novel, more advanced strategy (Siegler, 1987; Siegler & Shrager, 1984). One possible reason why children may revert to less efficient strategy may relate to contextual, problem-based features, which increase the need for children to inhibit their prepotent responses. While this study offers evidence for one context that appears to promote advanced strategy use, more research is needed to examine the role of executive functioning in shaping children’s strategy use.

**Gender by Condition Effects**

Several possible explanations help account for why there was a gender by condition interaction in predicting children’s observed strategy use on the Count On measure. Boys in the Transformation group were significantly more likely to display the count on strategy at posttest than were boys in the other two conditions; however, girls were more likely to display counting on behaviors after using the unitized manipulative rather than the reading software. Boys’ improvement after using the transforming manipulative aligns with my earlier finding that this manipulative significantly improved children’s ability to accurately count on at posttest. Why, then, did similar results not occur for girls in their observed use of the count on strategy?

There are several possible reasons why girls appeared to benefit more from the unitized manipulative as compared to the transforming manipulative or reading software. One possibility is that the unitized manipulative more closely resembles the types of physical manipulatives typically used in elementary classrooms for counting and addition activities. It is possible that girls benefitted from this similarity; however, it is unclear why this similarity would positively impact girls and not boys. Girls also may have been better able to inhibit the inclination to count.
all of the animals in the unitized 10-box, whereas boys may have been more likely to regress to counting all in the face of this visual stimuli, despite their knowledge of the count on strategy. Extensive evidence shows that girls tend to have better inhibitory control than do boys (See Else-Quest, Hyde, Goldsmith & Van Hulle, 2006 for a meta-analysis).

Girls may have had more difficulty taking the risk of using a newly acquired strategy with the transforming 10-box. To count on from the continuous 10-box, one must either have a strong grasp of cardinality or one must take the intellectual risk of counting on from 10, even though one may not fully grasp that there are 10 animals hidden by the continuous box. There is ample evidence that boys are more likely to engage in risk-taking behaviors than girls (Forman & Kochanska, 2001; Lahey et al., 2006; Serbin, 1990). Even in the realm of strategy use, there is evidence that boys are more likely than girls to take the risk of using novel, invented algorithms when solving addition and subtraction problems (Fennema et al., 1998). In fact, Villalobos (2009) argues that differences in gender performance in mathematics are due to the socialization of girls to follow the rules and of boys to take intellectual risks and break rules.

Finally, while there were no significant gender by condition pretest differences on children’s observed strategy use on the Count On measure, girls in the Unitized group were more accurate on the Count On measure as compared to girls in the Control group at pretest. Because accuracy and observed strategy use scores both help explain children’s counting on abilities, it is possible that pretest abilities impacted girls’ increased use of the count on strategy in the Unitized group as compared to the Control group.

Base Ten Understanding
Girls who used the transforming manipulative made more accurate, unique constructions of numbers than did girls in the Control group; girls who used the unitized manipulative made more base ten canonical constructions of numbers than did girls in the Control group. In addition to these findings, girls in both math software conditions also used significantly more advanced strategies on the Near Transfer task than did girls in the Control group. There were no significant pairwise differences between groups for boys. In fact, the only significant finding with respect to boys was that boys in the Control group showed greater base ten understanding than did girls in the Control group. These results were not expected.

It was anticipated that the act of transforming from a unitized to a continuous manipulative would highlight the role of the tens in multi-digit numbers and help children move past seeing the set as merely a collection of ones. However, this study found that for girls, both the unitized and transforming manipulative improved base ten knowledge. It appears that the experience of using a grouping by tens manipulative, whether it was unitized or transforming, helped girls conceive of multi-digit numbers as being composed of tens and ones, rather than just ones. Girls may have been able to learn from both of the grouping by tens manipulatives because they were able to overlook the individual items within the unitized 10-box and focus their attention on the grouping mechanism of the 10-box.

Perhaps the more perplexing question that arises from this study concerns the improvement that boys in the Control group made in their base ten understanding. While this finding was unexpected, there are plausible explanations that help to explain these results. Perhaps the most likely explanation for this finding is that children’s initial abilities at pretest contributed significantly to their posttest scores. As was previously described, while there were no significant effects of condition at pretest on the Base Ten measures, the same pattern of
performance scores between conditions and genders that existed at posttest also existed at pretest. Thus, despite the fact that overall all children made significant learning gains in base ten understanding over the course of the study, initial condition by gender differences likely explain posttest differences in base ten understanding.

Another important discovery that may explain my findings is that during the middle of the study, I learned that one of the schools was teaching place value concepts using base ten blocks. I did not anticipate any classroom instruction of place value concepts, as this is not part of typical early first grade math curricula. However, upon discovering that teaching was occurring in some classrooms, it became clear that children in my study might have improved in their base ten understanding because of classroom instruction. More specifically, boys in the Control group likely improved due to classroom learning; however, because girls in the Control group did not improve on the Base Ten measure, it appears that girls benefited from the math software, either in conjunction with classroom instruction or without such instruction, and not from classroom instruction alone.

Another possible explanation for Control boys’ improvement on the Base Ten measure is testing effects. Because of the pre/posttest design, it is possible that boys learned from the pretest and consequently performed better at posttest. It is also possible that working closely with a researcher using software led the boys to perform better at posttest. Follow-up studies need to investigate this finding to determine the causes of this gender by condition interaction.

Number Sense

My third research question sought to determine whether condition improves children’s overall number sense at posttest. I predicted that children in both math software conditions
would improve in their number sense compared to children in the Control condition, as the math software provides rich, scaffolded mathematical experiences, including making children submit their responses on an interactive number line. I did not anticipate significant performance differences between the Transformation and Unitized conditions, for both conditions exposed children to repeated enumeration activities and required interaction with the number line.

My results showed that an interaction between gender and condition approached significance for the Next Number and Number Identification CBM posttest measures. The general trend was that boys did better than girls in the Transformation condition, girls did better than boys in the Unitized condition and both genders scored approximately the same in the Control condition. However, there were no significant pairwise contrasts, making these findings extremely difficult to interpret. Overall, children in the math software conditions did not appear to expand their number sense more than did children in the Control condition. It is possible that children did not have enough time working with the math software to make substantial improvements in their number sense. Additionally, there was no explicit instruction in the math software that targeted children’s number sense. It is possible that with additional instruction, children would have made more substantial gains using the math software.

**Microgenetic Findings**

Looking more closely at children’s performance during the math computer sessions reveals that using the transforming manipulative propelled children to begin using advanced strategies earlier than did using the unitized manipulative. Children were significantly more likely to use advanced strategies during the first computer session if they were using the transforming manipulative rather than the unitized manipulative. This finding suggests that the
transforming manipulative encouraged children to approach the enumerating task in a different way than did the unitized manipulative. This finding should be examined further as it highlights a context that might expedite the employment of efficient strategies. Because children were also more likely to be accurate when using an advanced strategy than when not using an advanced strategy, this suggests that the software did not just help children use efficient strategies in careless ways; also, the advanced strategy resulted in more accurate answers.

Importantly, after playing the math software for more than one session, children in both conditions became equally likely to use advanced strategies. Children in both conditions became approximately 80% likely to use an advanced strategy by the third and fourth intervention sessions. This finding attests to the power of the grouping by tens tool; regardless of the grouping by tens manipulative they used, children showed learning gains from working with the math software. However, because there were significant group differences in performance on the Count On measure at posttest, it appears that the transforming manipulative may have solidified children’s acquisition of the count on strategy more than did the unitized manipulative and may have helped children better apply this strategic knowledge in far-transfer contexts.

Limitations

Despite my attempt to carefully design this study, there are certain limitations to my research that must be addressed. The ambiguous results on the Base Ten measure suggest that initial differences between conditions for boys likely impacted their posttest performances more than did the intervention sessions. This suggests that the math software did not make substantial improvements to boys’ base ten knowledge. Also, the study did not find strong evidence that the
transforming manipulative positively impacted base ten knowledge as compared to the unitized manipulative for boys or girls.

One reason for these results may be that there was not a strong enough focus in the math software on base ten concepts. The short instructional videos that were shown at the onset of the first four math computer sessions primarily modeled advanced strategy use, such as counting on, and not base ten concepts. Additionally, all feedback involved either modeling the count on strategy or providing accuracy statements; there was no feedback that directly related to base ten understanding. The study results would likely have been strengthened had the math software explicitly modeled base ten concepts and had feedback more directly related to the number of tens and ones on the screen. Making small changes to the layout of the software, such as more clearly separating the 10-boxes from the single animals and labeling the side of the screen with the 10-boxes as “Tens” and the field with animals as “Ones,” might have also bolstered children’s learning of base ten concepts. Finally, the use of the 10-box tool required children to click on the tool icon, but did not require children to move the objects into the box themselves. It is possible that children would have benefitted from dragging the individual objects into the box, rather than having the computer automatically carry out this action, as this gesture would more closely matched the real-life action of filling up a box with 10 items.

Another limitation of the study is that some classrooms taught place value concepts during the study. The improvement of boys in the Control group made my results extremely difficult to interpret and made it impossible to attribute base ten learning gains to the math software. I should have initially determined which math topics teachers planed to cover during the first half of the year in order to exclude classrooms where place value would be taught. In addition, I should have made my Base Ten and Near Transfer measures more challenging. It is
possible that with harder measures, children in the math software groups would have shown more learning gains than those who had the reading software and some base ten classroom instruction.

Finally, choosing the Reader Rabbit reading software may not have been the best choice of software for the Control group. While my intention was to offer children in the Control group a similar experience of working one-on-one with a researcher while using computerized software, I should have selected either another math software, which focused on topics other than addition and base ten concepts, or chosen reading software that incorporated mathematical ideas into its activities. The reading software used in this study did not address mathematical topics and was therefore not the best option to use with the Control group.

Implications for future research

This study demonstrated the learning benefits that result from children’s use of a novel, grouping by tens virtual manipulative. This study has significant implications for research on children’s mathematical learning. One important implication of this study concerns the value of empirically testing different features of a manipulative to determine the effects on children’s learning. More studies are needed that closely investigate which features of given manipulatives promote learning. Studies should also attempt to uncover the mechanisms through which manipulatives advance children’s thinking and strategy use.

In order to better understand the mechanisms underlying the transforming manipulative, I should examine the features of this manipulative at an even more fine-grained level in future experiments. For instance, I was not able to assess whether it was the act of seeing the manipulative transform that impacted children’s learning or whether it was the hidden nature of
the objects in the continuous 10-boxes that impacted learning. An important follow up study should clarify whether the act of transforming from unitized to continuous is what prompts strategy change or whether using a continuous model that does not transform would also advance the strategies children employ.

This study displayed many positive learning effects that arise from using virtual manipulatives with children. A useful follow up study would examine whether the virtual nature of the manipulatives impacted learning. Future studies should compare how the virtual transforming manipulative compares with a similar physical manipulative. While there are few physical manipulatives designed to both teach base ten concepts and advance strategy use, future studies could examine a range of different physical manipulatives in relation to the MathemAntics virtual transforming manipulative. One potential physical manipulative to compare with the transforming manipulative is the Japanese Association of Mathematics Instruction (AMI) base 10 tile (Ginbayashi, 1984). The AMI base ten tile is a rod of 10 tiles that is unitized on one side and continuous on the other. Closely comparing this tool with the transforming manipulative could shed insight into the processes and mechanisms that promote learning.

Children’s accuracy and strategy use improved after one study session and children maintained their learning gains over the following four sessions. An important question that remains is whether the length of the computer intervention impacted children’s ability to solidify their learning gains. Future research should determine whether providing children with one computer intervention session, as opposed to multiple sessions, would have the same effect on children’s strategy use at posttest as did five intervention sessions. In future studies, children should also be administered an additional post-test, several weeks after the study concludes, to
determine whether children retain learning gains and whether retention is based on the number of sessions children used the computerized activity. Another follow up study should examine whether boys in the Control group made improvements in this study because of repeated testing. Using a posttest only design to replicate my findings would rule out the possibility of testing effects.

This dissertation also alluded to the possible role that executive function plays in impacting how and why children select a given strategy. More specifically, the findings in this study suggest that inhibitory control may impact children’s decision of whether to use counting all versus counting on. More research is needed to examine how executive function shapes children’s strategy use and to determine whether inhibitory control may also explain the gender differences in strategy use and base ten understanding that were uncovered by this study.

Implications for Educators and Software Designers

There are several important implications from this study that could shape how educators approach teaching and learning. My results showed that using math software with modeling and feedback made a significant impact on the thinking of first grade students. Children were able to learn sophisticated addition strategies after one computer session and were able to retain their learning gains after five intervention sessions. My findings underscore the positive effects that software can have on children’s learning, especially in the realm of strategy use, when the software is carefully designed based on the principles of cognitive psychology. Teachers should make substantial efforts to seek out appropriate, research-based learning software and incorporate this software into their instruction and classroom activities.
Another crucial finding was that the majority of the first grade children I worked with were capable of learning rudimentary base ten concepts. Several studies demonstrate the poor math performance of elementary school children in the United States, particularly by children from low SES backgrounds (Fuson, 1990; Fuson & Kwon, 1992; Ginsburg, 1989; Kamii, 1986; Kouba, et al., 1988; Miura et al., 1993). Researchers often stress the difficulty of learning base ten concepts for young children and conclude that these concepts should not be introduced until later in the elementary grades (Fuson & Briars, 1990). My findings, however, show that first grade children from low and mixed SES backgrounds were capable of learning base ten concepts when taught within the context of software using rich virtual manipulatives, scaffolding, and substantive feedback. These results suggest that children as young as first grade, including those from low SES backgrounds, should be introduced to base ten concepts in school.

This study also has important implications for software designers. As my results show, all virtual manipulatives are not created equally. Making small changes to a virtual manipulative, such as having the tool transform from a discrete to a continuous model, can have large consequences on children’s learning. It is essential that designers develop their software based on theoretically sound principles and then empirically test various aspects of their designs to ensure that they foster the learning objectives that they were designed to promote.
REFERENCES


APPENDIX A: Scripts for instructional videos

Sessions 1:

Both conditions:
“I can use the 10-box tool to help figure out how many animals there are altogether. I will click on the 10 box tool. Here is 10. I will keep 10 in my head and count on. 10-STOP, 11, 12, 13,14,15. 15 animals altogether.”

Session 2:

Both conditions:
“That’s a lot of animals. I can use the 10-box tool to help figure out how many animals there are altogether. (Press 10-box) That’s 10. There are still a lot of animals on the field. Here is another 10 (press 10 box again). 10, 20-STOP, 21,22, 23,24,25,26. 26 animals altogether.”

Session 3:

Both conditions:
“That’s a lot of animals. I can use the 10-box tool to help figure out how many animals there are altogether. (Press 10-box) That’s 10. There are still a lot of animals on the field. Here is another 10 (press 10 box again). 10, 20-STOP, 21,22, 23,24,25,26,27. 27 animals altogether.”

Session 4:

Both conditions:
“I can use the 10-box tool to help figure out how many animals there are altogether. (Press 10-box) That’s 10. I can see there are 2 animals left on the screen without even counting. I know that 10 and 2 is 12 so there are 12 animals altogether.”
APPENDIX B: Script for Count On measure

Trials 1-3: (FLIP) “We are going to play a counting game.”

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<tr>
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<th>1st addend</th>
<th>2nd addend</th>
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<tr>
<td>2)</td>
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</tr>
<tr>
<td>3)</td>
<td>18</td>
<td>8</td>
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Lay out LONGER (1st) array card and 1st numeral card directly above array card.
- “This card [Point to numeral card] tells you how many dots there are here [Gesture across array card].
- “But now I’m going to hide these dots from you [Turn array card over, facing down to hide array of dots].

Lay out SHORTER (2nd) array card to child’s right of 1st array card and 2nd numeral card above it.
- “This card [Point to 2nd numeral card] tells you how many dots there are here (gesture across 2nd array card).
- “How many dots are there on both of these cards all together (in all)?”
- After 2nd and subsequent trials, after child responds, ask, “How did you figure that out?”
  o If child answers, “I counted,” ask:
    • “Show me the first thing you counted” AND
    • “Tell me the first word you said.”

Trials 4-6: (NO FLIP)

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</tr>
<tr>
<td>6)</td>
<td>15</td>
<td>9</td>
<td>turquoise</td>
</tr>
</tbody>
</table>

Lay out 1st array card and 1st numeral card directly above array card.
- “This card (point to 1st number card) tells you how many dots there are here (gesturing across dots on 1st array card).

Lay out 2nd array card to immediate right of 1st array card and place 2nd numeral card above it.
- “This card (point to 2st number card) tells you how many dots there are here (gesture across dots on 2nd array card).
- “See, this card (indicating 1st number card) tells you how many dots there are here (gesture to 1st array card) so you don’t have to count them, but you can if you need to.
- How many dots are there on both of these cards altogether (in all)? Remember, you don’t have to count these (gesture to 1st array card) over again.
- “How did you figure that out?”
Scaffolds:
- If child is idle for 5 seconds, say, “How many dots are there altogether (in all)?”
- If child answers, “I counted,” ask:
  - “Show me the first thing you counted” AND
  - “Tell me the first word you said.”

After final trial: “Great job. Thanks for playing the counting game with me.”
APPENDIX C: Script for Base Ten measure

Intro: Place 10 units blocks and 1 tens block in front of child. After you finish the intro, keep the 10 ones and 1 tens block in front of you as a model. This model should be out of child’s reach during task.

- “These blocks can be used for counting and to show numbers.” [Count out the 10 units blocks] “1,2,3,4,5,6,7,8,9,10. There are 10 ones blocks here [Point to the 10 units blocks]. They are the same as 1 of these tens block [Point to the 1 tens block]. Do you see how they are the same? In this game, you will read the number on the card and make that number using the blocks. Let’s practice 2 together.”

Practice #1: Place all blocks within child’s reach. Show card with “2” on it.

- “Read the number on this card and make that number using the blocks. Let me know when you are done.”
  - If correct: “Yes, the card says 2 and you made 2 with the blocks.”
  - If incorrect: “Nice try. The card says 2.” [Show the child 2 units blocks]. “This is 2.”

Practice #2: Clear all blocks (except for the model). Show card with “7” on it.

- “Read the number on this card and make that number using the blocks. Let me know when you are done.”
  - If correct: “Yes, the card says 7 and you made 7 with the blocks.”
  - If incorrect: “Nice try. The card says 7. [Show the child 7 using 7 units blocks]. “This is 7.”

Trials:
“Now it is your turn to play. Remember that 10 of these ones blocks [Point to 10 units blocks in the model] are the same as 1 tens block [Point to 1 tens block in the model].”

1) “Read the number on this card and show the number using the blocks. Let me know when you are done.” [Show card 11]
   - When finished, move blocks away from child. “Now, show me another way to make 11 using the blocks.”

2) “Read the number on this card and show the number using the blocks. Let me know when you are done.” [Show card 13]
   - When finished, move blocks away from child. “Now, show me another way to make 13 using the blocks.”

3) “Read the number on this card and show the number using the blocks. Let me know when you are done.” [Show card 28]
   - When finished, move blocks away from child. “Now, show me another way to make 28 using the blocks.”

Scaffolds:
- If child is idle for 5 seconds, say, “Read the number on this card and make that number using the blocks. Let me know when you are done.”
• If child needs help reading # or reads incorrectly, do not read # for them. Say, “Just do your best to read the number.” If they can’t read # after this prompt, note this on your sheet and tell them the #.
• If child tries to draw out number with the blocks, say, “Don’t draw the number. Show the number by putting out that many blocks.” If child continues drawing, allow them to and record their answer and strategy.

After final trial: “Great job. Thanks for playing the block game with me.”
APPENDIX D: *Script for Near Transfer measure*

**Materials:** 5 tens blocks, 20 units blocks. After you finish the intro, keep the 10 ones and 1 tens block in front of you as a model. This model should be out of child’s reach during task.

**Intro:**
“These blocks can be used for counting and to show numbers.” [Count out the 10 units blocks] 1,2,3,4,5,6,7,8,9,10. There are 10 ones blocks here [Point to the 10 units blocks]. They are the same as 1 of these tens block [Point to the 1 tens block]. Do you see how they are the same? I am going to show you some blocks and your job is to figure out how many blocks there are altogether. Are you ready?”

**Practice #1:** [Put out 5 blocks, randomly arranged]: “Let’s practice one together. How many blocks are there?”
   If correct: “Yes, there are 5 blocks altogether.”
   If incorrect: “No, that’s not quite right. There are 5 blocks altogether.”

“Now it’s your turn. Are you ready?”

**Trials:**
1) 15 (Put out 1 ten, 5 ones)
   “How many are there altogether? Show me how you know that.”
2) 12 (Put out 1 ten, 2 ones)
   “How many are there altogether?” Show me how you know that.”
3) 32 (Put out 3 ten, 2 ones)
   “How many are there altogether?” Show me how you know that.”
4) 13 (Put out 1 tens and 3 ones)
   “How many are there altogether?” Show me how you know that.”
5) 21 (Put out 2 tens, 1 ones)
   “How many are there altogether?” Show me how you know that.”
5) 23 (Put out 2 tens, 3 ones)
   “How many are there altogether?” Show me how you know that.”

**Scaffolds:**
- If child is idle for a long time, say, “How many blocks are there in all?”
- If child answers, “I counted,” to “Show me how you know that,” ask:
  - “Show me the first thing you counted” AND “Tell me the first word you said.”

**After final trial:**
“Great job. Thanks for playing this block game with me.”
APPENDIX E: *Strategy coding recoding sheet for Count On and Near Transfer measures and session data*

**Coding scheme for Count On measure:**

<table>
<thead>
<tr>
<th>Observed (Check all that apply)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Child’s answer:</strong></td>
</tr>
<tr>
<td>___ No visible strategy</td>
</tr>
<tr>
<td>□ Quick</td>
</tr>
<tr>
<td>□ Delay</td>
</tr>
<tr>
<td>□ Stares mostly at 2nd dot card</td>
</tr>
<tr>
<td>Points (1x1):</td>
</tr>
<tr>
<td>□ both addends</td>
</tr>
<tr>
<td>□ 2nd addend only</td>
</tr>
</tbody>
</table>

**Other:**

**Coding scheme for Near Transfer measure and session data:**

<table>
<thead>
<tr>
<th>HM</th>
<th>Observed (Check all that apply)</th>
<th>Counts aloud (Check all that apply)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Child’s answer:</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>___ States number fact</td>
<td>___ Counts aloud</td>
</tr>
<tr>
<td></td>
<td>___ Fingers</td>
<td>Points:</td>
</tr>
<tr>
<td></td>
<td>___ No visible strategy</td>
<td>□ 1x1 all</td>
</tr>
<tr>
<td></td>
<td>□ Quick</td>
<td>□ 1x1 out of box</td>
</tr>
<tr>
<td></td>
<td>□ Delay</td>
<td>(points to units only)</td>
</tr>
</tbody>
</table>

**Other:**