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Abstract

In a winner-take-all duopoly market for systems in which firms invest to improve their products, a monopoly supplier of an essential system component may have an incentive to advantage itself by technological tying; that is, by designing the component to work better in its own system. If the vertically integrated firm is prevented from technologically tying, then there is a pure strategy equilibrium in which the more efficient firm invests and serves the entire market. However other equilibria may exist, including a pure strategy equilibrium in which the less efficient firm invests and captures the market and mixed strategy equilibria in which each firm captures the market with positive probability. In contrast, if the vertically integrated firm is able to degrade the quality of its rival's system with a technological tie, and if the wholesale price of the essential component is insufficiently remunerative, then there is a unique equilibrium outcome in which the supplier of the essential component invests alone and forecloses a more efficient rival with an actual, or merely threatened, technological tie. A comparison of these equilibria for the two game forms demonstrates that a prohibition of technological tying can either increase or decrease social welfare depending on equilibrium selection.

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1 Introduction

During the "browser war" between Microsoft and Netscape, Microsoft and its defenders argued that its Internet Explorer browser gained market acceptance because it was a superior product (see e.g., Liebowitz and Margolis, 1999). Microsoft's critics responded that Internet Explorer benefited from Microsoft's exclusionary practices associated with the distribution and use of its ubiquitous Windows operating system. In this paper we examine strategic competition in markets for systems that combine one or more component inputs to produce a final output, when of the components is controlled by a monopoly. Examples of such markets can be found in telecommunications, electricity service, and other industries in addition to information technologies. In these markets a single supplier often controls an essential component of the system such as an operating system, a local telephone exchange network, or an electricity transmission grid. We examine competition under the assumption that the firm that supplies the system with the lowest quality-adjusted price wins the entire market. When system suppliers have unfettered access to the essential component, a firm can prevail by investing to improve the quality of its system. We show that even on this level playing firm, the firm that is the most efficient supplier of systems need not emerge as the market leader. We also consider the incentives of the monopolist to tip the scales of competition by granting itself superior technological access to the essential component. This could be accomplished by designing the essential component to work better with its own system, thereby degrading the performance of rival systems relative to its own. Such a "technological tie" can give the monopoly component supplier a greater incentive to innovate and is an additional reason why the market structure *ex post* need not reflect the most capable supplier of systems *ex ante*.[2]

The monopolist confronts a trade-off in considering the merits of a technological tie. On the one hand, by limiting rivals' access to an essential component, the monopolist profits by curtailing competition in the market for systems. On the other hand, the technological tie reduces the monopolist's ability to extract rents from more efficient rivals through sales of the essential component. If rivals have a superior ability to innovate, or produce systems that appeal to a large number of consumers, then by providing access the monopolist profits from sales of the component. The technological tying trade-off depends on the price of the component. A high component price encourages the monopolist to provide efficient access to the essential component, while a low price encourages the monopolist to limit access with a technological tie. Therefore technological tying is likely to be an attractive business strategy for the monopolist only if sales of the upstream component are insufficiently remunerative.

The traditional "Chicago School" emphasizes that there is no incentive for technological tying (other than for efficiency reasons) if a monopoly profit for the tying product can be extracted by charging a profit-maximizing price.[3] There are reasons, however, why a

[2] A technological tie refers to the physical integration of a product with another product, in a manner that makes it costly for rivals to sell similar integrated products. A technological tie may also be accomplished by designing an interface or withholding technical information to impede the interoperability of a complementary product. See Lessig (2000) for a review of the case law on tying and its applicability to U.S. v. Microsoft.

[3] Bowman (1957) and Bork (1978), among others, maintain that the owner of an essential input that is used in fixed proportions with another competitively supplied good has no incentive to bundle the input and the complementary good or to tie purchase of the complementary good to the essential input. The

monopolist may have limited flexibility to charge rivals a profit-maximizing price for an essential component. One possibility is that the component has other uses that consumers value differently. The monopolist may opt for a mixed bundling strategy, selling both systems and the component on a stand-alone basis, while imposing a technological tie to prevent arbitrage between the two offerings.[4] Alternatively, regulation (including antitrust scrutiny) can constrain the price that the monopolist can charge for the essential input.

In light of these considerations our analysis focuses on tying incentives conditional on the price of the tying product. Consistent with the traditional view, we find that the ability to technological tie does not affect market outcomes when the price of the tying good is sufficiently high. Otherwise, a technological tie, or even the threat of a technological tie, can significantly impact market structure, prices, and innovation. In some cases, these impacts have negative welfare consequences; in other cases, the ability to impose a technological tie can actually increase social welfare. The ambiguity is due to multiple equilibria, as we discuss shortly.

Economides (1998) shows that a price-regulated upstream monopolist participating in a downstream Cournot (quantity-setting) oligopoly has an incentive for non-price discrimination.[5] Our analysis develops this theme by analyzing the incentives for and consequences of technological tying for product improvement in a downstream systems market where firms compete on quality and price.[6] In this paper, we consider the case of homogeneous consumer preferences over vertically differentiated products. We defer consideration of heterogeneous consumers and horizontal product differentiation for future research.

More specifically, we study markets for systems as duopoly games. Each of M consumers demands a system comprised of two components. Firm 1 offers a system comprised of one unit of component A and one unit of its version of component B. Firm 2 purchases component A from Firm 1 at a predetermined price w,[7] and offers consumers a system

argument is that there is a single monopoly profit, which the owner of the essential input can capture by charging a monopoly price.

[4] For example, suppose some consumers are willing to pay q_i for computer systems consisting of an operating system and application software, but other consumers would pay no more than w for a computer with e-mail and Internet browsing capability and have no demand for other applications. If there are enough consumers of the second type, Firm 1 would maximize profits by selling component A separately at price w, provided that it can prevent rivals or consumers from purchasing the operating system at the lower price and combining it with applications.

[5] In line with Chicago School reasoning, no incentive for non-price discrimination exists if the input monopolist is a less efficient supplier of systems in the downstream market and the upstream price is profit-maximizing. See Sibley and Weisman (1998), Bergman (2000), and Economides (2000).

[6] The new vertical foreclosure literature, which includes papers by Salop and Scheffman (1983), Krattenmaker and Salop (1986), Ordover, Saloner and Salop (1990), Riordan and Salop (1995), Hart and Tirole (1990), and Bolton and Whinston (1991) and Rey and Tirole (1997), identifies incentives for a firm that operates in both upstream and downstream markets to use price and exclusionary contracts to influence downstream competition and to “raise rivals’ costs”. Our analysis can be interpreted as an exploration of technological tying as a raising rivals’ costs strategy.

[7] We assume that Firm 1 is commited to the wholesale price w. Without such a commitment, Firm 2 would not invest, because of the ability of Firm 1 to hold up Firm 2 by raising the wholesale price after Firm 2 has invested in product improvement. Farrell and Katz (2000) reach the same conclusion for their price leadership model in which the integrated firm prices the essential component after observing the realized qualities.

that consists of component A and its version of component B.[8] In our basic "product improvement game," given the wholesale price of component A, rival firms invest in quality improvements of component B, and subsequently compete on the price of systems. In the companion "technological tying game" there is also an intermediate stage in which Firm 1 can act to degrade the quality of Firm 2's system.

Given that consumers have homogeneous preferences, the market has a winner-take-all character and multiple equilibria of the product improvement game are possible. There always exists an efficient pure strategy equilibrium in which Firm 2 improves its product optimally and captures the entire market. If the quality advantage of Firm 2 is sufficiently small, then there also exists an inefficient pure strategy equilibrium in which Firm 1 invests in product improvement and captures the market. In these equilibria, consumers enjoy a positive surplus as long as the losing firm imposes some competitive price pressure on the winner. There also can exist mixed strategy equilibria of the product improvement game in which each of the firms invests with positive probabilities. Possible mixed strategy equilibrium outcomes include one or the other firm investing alone, duplicative investments, and a complete failure to invest.

The technological tying game, in contrast, has a unique equilibrium outcome if the wholesale price of the essential component is not too large. Firm 1 forecloses competition with an actual or perhaps merely threatened technological tie,[9] improves its product efficiently, and sets a monopoly price that fully extracts consumer surplus. In this case, technological tying distorts market structure and reduces consumer welfare by eliminating competition from Firm 2. Surprisingly, technological tying can improve social welfare compared to a mixed strategy equilibrium of the product improvement game, even though consumers are worse off. The technological tying game also has a unique equilibrium outcome if the wholesale price is sufficiently close to the monopoly level. Firm 1 prefers the role of supplier to Firm 2 and declines to compete for the retail market. For an intermediate range of wholesale prices, the technological tying game can have multiple equilibria in which both firms invest with some probability, but Firm 1 forecloses its rival when both firms invest in product improvement.

Farrell and Katz (2000) also study the incentives for product innovation and technological tying in a market for vertically differentiated systems. In their model, a monopoly supplies an essential component and competes with others to supply a complementary component to consumers who assemble a system. By "overinvesting" in product R&D for the complementary component, the integrated firm squeezes the rents of rival suppliers and is able to charge consumers more for the essential component.[10] Moreover, the integrated

[8] While we assume that Firms 1 and 2 supply systems comprised of component A and firm-specific versions of component B, our analysis would be unchanged if consumers were to purchase component A from Firm 1 and combine A with component B from Firm 1 or Firm 2. In this case component A can be an operating system (e.g. Microsoft Windows), and component B can be an application program, such as Microsoft Word or Wordperfect.

[9] When a merely threatened technological tie does the job, Firm 2 is foreclosed by a price squeeze, meaning that the wholesale price is prohibitive compared to Firm 2's equilibrium quality. Firm 2 declines to invest in product quality because it rationally believes that Firm 1 would foreclose a competitive product with a technological tie.

[10] See Bolton and Whinston (1993) and Kranton and Minehart (2002) for related models of strategic

firm has no incentive to disadvantage rivals with a technological tie. These results depend on their assumptions that the monopolist prices the essential component after observing the qualities and prices of the competitively-supplied component. In contrast, we assume that the wholesale price of the essential component is determined prior to systems market competition. The difference is crucial for considering the integrated firm's incentives for technological tying.

Choi and Stefanadis (2001) analyze a model of a systems market that is similar in structure to Farrell and Katz (2000). In their model an incumbent owns two complementary components and firms invest to lower the cost of one or both of the components, which they may combine with a component owned by the incumbent. They compare investment incentives with and without tying and show that tying strengthens the incumbent's investment incentives. Choi, Lee and Stefanadis (2002) explore a similar model with discrete investment. In contrast, our analysis focuses on the incentive for technological tying by an integrated monopolist when a competitor requires access to an essential component of the system at a predetermined price.

Section 2 describes the structure of the market for systems and the technology for product improvement. This section introduces the assumptions that Firm 2 is the higher quality supplier of systems when neither firm invests and that, by investing, Firm 1 can leapfrog Firm 2's quality advantage. These assumptions frame the policy issue by defining an environment in which Firm 1 can use its control over access to the essential component to influence investment incentives and thereby distort market outcomes. Section 3 introduces the product improvement and technological tying games, identifies their pure strategy equilibria, and examines the welfare properties of the pure strategy equilibrium outcomes. Section 4 considers possible mixed strategy equilibria of these games and Section 5 concludes.

2 Vertical Product Differentiation

Systems are differentiated in quality, which is partly exogenous and partly endogenous. Each of M identical consumers demands a single system. A consumer's willingness-to-pay for a system consisting of components A and B from firm i is

$$q_i = \gamma_i + q(r_i), \tag{1}$$

where γ_i is an exogenous quality parameter specific to systems sold by Firm i and the endogenous variable r_i is Firm i's investment in R&D to improve the quality of its system (or, equivalently, the quality of its component B). For analytical convenience, we assume that there are no additional variable costs of producing systems.

It is convenient to reinterpret (1) as Firm i choosing a level of quality improvement

$$z_i = q_i - \gamma_i$$

overinvestment.

by incurring an R&D cost

$$r_i = r(z_i).$$

We maintain several additional assumptions.

A1: The symmetric R&D cost function $r(z)$ is increasing, strictly convex, twice differentiable, and satisfies $r(0) = r'(0) = 0$.

The first assumption implies that there is a unique z^M that maximizes the net benefits from quality improvement $zM - r(z)$ and is the solution to $r'(z^M) = M$. Thus z^M is the efficient level of quality improvement for a firm selling to the entire market and

$$\pi^M \equiv z^M M - r(z^M) > 0.$$

A2.: $\Gamma \equiv \gamma_2 - \gamma_1 > 0$.

The second assumption implies that Firm 2 is the more efficient supplier of systems for any level of investment in quality improvement.[11]

A3: $\pi^M > \Gamma M$.

The third assumption implies that Firm 1 can profitably leapfrog Firm 2's initial quality advantage by investing efficiently in product improvement; i.e., $(\gamma_1 + z^M)M - r(z^M) > \gamma_2 M$. Although this assumption is not necessary for some of our results, it describes an environment in which investment effects can dominate firm-specific efficiencies, which is the focus of our analysis.

A4: $w < \bar{w}$, where

$$\bar{w} \equiv \gamma_2 + \pi^M/M.$$

The fourth assumption simplifies our analysis by restricting attention to exogenous wholesale prices of component A below the monopoly level, $\bar{w}$. The maximum social surplus in this market is $\gamma_2 M + \pi^M$, which corresponds to investment of z^M by Firm 2.[12] This surplus is fully extracted by Firm 1 with a wholesale price equal to $\bar{w}$. Firm 1 is content to supply Firm 2 as long as the wholesale discount below $\bar{w}$ does not exceed Firm 2's initial efficiency advantage, Γ. We make extensive use of $\bar{w}$ below. For reference, note that $\bar{w} > \gamma_1$ and $\bar{w} - \Gamma > \gamma_2$.

We compare the subgame perfect Nash equilibria of two different game forms. In the "product improvement game," competition proceeds in two stages. In the first stage, the

[11] This assumption serves to emphasize the potential social costs of strategic conduct by a vertically integrated supplier. The analysis for $\Gamma \leq 0$ is similar; see footnote 14, below.

[12] More formally, let xM be the allocation of consumers to Firm 1 and $(1-x)M$ the allocation to Firm 2, and let z_1 and z_2 be the firms' investments in quality improvement. The social planner chooses (x, z_1, z_2) to maximize

$$W(x, z_1, z_2) = M\left[x(\gamma_1 + z_1) + (1-x)(\gamma_2 + z_2)\right] - r(z_1) - r(z_2).$$

Clearly $W(0, 0, z^{\mathsf{M}}) > W(1, z^{\mathsf{M}}, 0)$. Furthermore, the convexity of $r(z)$ implies that $W(0, 0, z^{\mathsf{m}}) > W(x, z_1, z_2)$ for $x \in (0, 1)$. Therefore, the welfare optimum has $x = 0$, $z_1 = 0$, and $z_2 = z^{\mathsf{M}}$.

firms simultaneously and independently choose quality improvements z_i at cost $r(z_i)$. In the second stage, the firms simultaneously and independently set prices $P_i \geq w$ after observing each other's quality. The "technological tying game" amends the product improvement game by allowing the upstream monopolist to degrade the quality of its rival's systems. In the first stage, the firms choose costly quality improvements z_i as in the product improvement game. In the second stage, Firm 1 can degrade Firm 2's quality by an amount $\delta \geq 0$. In the final stage, the firms set prices.

The price subgame is the same in both game forms. Consumers observe prices and qualities and choose the product that offers the greatest net utility. Consumers have identical preferences, so Firm i makes sales to all M consumers if $q_i - P_i > \max(q_j - P_j, 0)$. When both products offer the same net utility, consumers are assumed to choose the higher quality product, and if both firms also have the same quality, then consumers are assumed to choose Firm 1.

Equilibrium prices and sales in the market for systems depend on the level of w and product qualities. In an equilibrium of the price subgame, only one firm sells to the entire market. If $q_1 > q_2$ and $q_2 \geq w$, then Firm 2 sets price $P_2 = w$ and Firm 1 wins the market at price $P_1 = q_1 - q_2 + w$. A similar result holds for Firm 2 if $q_2 > q_1$ and $q_1 \geq w$, because w is an opportunity cost of sales for Firm 1 and a direct cost for Firm 2. If $q_2 < w$, then Firm 1 wins the market at price $P_1 = q_1$ even if $q_1 < q_2$.

Summarizing, Firm 1 sells to all M customers at a price $P_1 = q_1 - \max(q_2 - w, 0)$ if $q_1 \geq q_2$ or if $w \geq q_2$, and Firm 2 sells to all M customers at a price $P_2 = q_2 - \max(q_1 - w, 0)$ if $q_2 > q_1$ and $w < q_2$. The losing firm serves as a competitive check when the quality of its system is above its opportunity cost.[13]

We begin by analyzing the pure strategy equilibria of the product improvement and technological tying games. The pure strategy equilibria of the product improvement game illustrate the ability and incentives of a firm that controls an essential input to improve its system in order to gain a competitive advantage over downstream rivals. The upstream monopolist may win the market with a better product even when it would be more efficient for a downstream rival to engage in product improvement. Thus, an *ex post* measure of product superiority can be a misleading indicator of market performance.

Technological tying by an upstream monopolist is another source of market failure. A technological tie can effectively foreclose downstream rivals from participating in the market for systems. When technological tying is feasible and the price of the upstream good is insufficiently remunerative, the upstream monopolist has an incentive to foreclose rivals and to substitute its own innovative efforts.

Pure strategy equilibria are natural outcomes in a winner-take-all market when firms are able to coordinate their investment decisions. In contrast, mixed strategy equilibria describe industry conduct when firms are uncertain about the investment decisions of their rivals. After characterizing and comparing pure strategy equilibria, we consider mixed strategy

[13] Let xM be Firm 1's sales of systems, with $x \in [0, 1]$. Firm 2's sales are $(1 - x)M$. If w is the price of component A, then the respective profits of the two firms are $\pi_1 = (P_1 x + w(1 - x))M - r(z_1) = (P_1 - w)xM + wM - r(z_1)$ and $\pi_2 = (P_2 - w)(1 - x)M - r(z_2)$. The wholesale price of component A (w), is an opportunity cost of system sales for Firm 1 as well as a direct marginal cost for Firm 2.

equilibria of both the product improvement and the technological tying games. We show that total expected welfare can be lower when firms play mixed strategies compared to a product improvement game in which firms play pure strategies. This is a consequence of both the low and redundant investment levels that can occur when firms play mixed strategies. As in the case of pure strategy equilibria, technological tying is potentially costly because it facilitates product improvement and market dominance by a less efficient firm. Nonetheless, technological tying is also potentially beneficial because it avoids the inefficiencies from low and redundant investments that can occur in mixed strategy equilibria of the product improvement game.

3 Pure strategies

3.1 The Product Improvement Game

We begin with the product improvement game. In stage 1, the firms invest in quality improvement anticipating the Bertrand-Nash equilibrium of the price subgame discussed above. It is immediate that both firms cannot make positive investments in a pure strategy equilibrium. One of the firms will capture the entire market, leaving the other firm better off not investing.

Proposition 1 *There does not exist a pure strategy equilibrium of the product improvement game in which both firms make positive investments. Either* $z_1 = 0$ *or* $z_2 = 0$ *in equilibrium.*

Given our maintained assumptions, there always exists an equilibrium in which Firm 2 captures the entire market. This equilibrium is efficient because Firm 2 has an exogenous quality advantage.

Proposition 2 *There exists a pure strategy equilibrium of the product improvement game in which Firm 2 invests efficiently* ($z_2 = z^M$) *and Firm 1 does not invest in quality improvement* ($z_1 = 0$). *Firm 2 sets a price equal to* $P_2 = \gamma_2 + z^M - \max(\gamma_1 - w, 0)$, *sells systems to all* M *customers, and earns* $\pi_2 = [\bar{w} - w - \max(\gamma_1 - w, 0)]M \geq 0$. *Firm 1 sets price* $P_1 = w$, *sells* M *units of component A and no systems, and earns* $\pi_1 = wM$.

Proof. Suppose Firm 1 deviates from the assumed equilibrium by investing $z_1 > 0$. This cannot be profitable unless Firm 1 can win the market from Firm 2, which requires $z_1 \geq z^M + \Gamma$. The best deviation for Firm 1 maximizes $\pi_1 = \quad z_1 - z^M - \Gamma + w \quad M - r(z_1)$ subject to this constraint. Convexity of $r(z)$ implies that the constraint binds and Firm 1's maximum deviation profit is $\pi_1 = wM - r(z^M + \Gamma) < wM$. Thus Firm 1 earns less profit by deviating from $z_1 = 0$. Given that Firm 1 chooses $z_1 = 0$, the profit-maximizing investment for Firm 2 is $z_2 = z^M$. Firm 2's profit is $\pi_2 = [\bar{w} - w - \max(\gamma_1 - w, 0)]M$. ■

In the efficient equilibrium, Firm 1 is effectively foreclosed from competing with Firm 2 if $w > \gamma_1$. Firm 2's profit in this case is the maximum surplus $\bar{w}M$, less its payments to Firm 1 for component A. Firm 2's profit is independent of w for $w \leq \gamma_1$. In this range, Firm 1 exercises a competitive constraint on Firm 2's pricing equal to $\gamma_1 - w$, so Firm 2's net profit is $[\bar{w} - \gamma_1]M$.

Firm 1 does not profit by investing in quality improvement if it expects Firm 2 to do so. However, there may exist an alternative equilibrium in which only Firm 1 invests in product improvement. By assumption, Firm 1 can profitably leapfrog Firm 2's initial quality advantage by investing efficiently. Moreover, if $r(z^M) \geq \Gamma M$, then Firm 2 cannot profitably leapfrog Firm 1's post-investment product quality.

Proposition 3 *There exists a pure strategy equilibrium of the product improvement game in which Firm 1 invests* $z_1 = z^M$ *and Firm 2 does not invest if and only if* $r(z^M) \geq \Gamma M$. *Firm 1 sets price* $P_1 = \gamma_1 + z^M - \max(\gamma_2 - w, 0)$, *sells systems to all* M *customers, and earns* $\pi_1 = [\bar{w} - \Gamma - \max(\gamma_2 - w, 0)]M \geq 0$. *Firm 2 sets price* $P_2 = w$, *sells no systems, and earns* $\pi_2 = 0$.

Proof. If $z_1 = z^M$ and $z_2 = 0$, Firm 1 will make all sales at a price equal to $P_1 = \gamma_1 + z^M - \max(\gamma_2 - w, 0)$. If $w \leq \gamma_2$, then $P_1 = z^M - \Gamma + w$ and Firm 1 earns a profit equal to $\pi_1 = \pi^M - \Gamma M + wM \geq wM$ given the "leapfrogging" assumption A3. Furthermore, Firm 1 has no incentive to deviate and earn $\pi_1 = wM$ by choosing $z_1 = 0$, and has no incentive to choose any other level of quality improvement. If it is profitable for Firm 2 to deviate, then Firm 2 would choose z^M and earn $\pi_2 = \Gamma M - r(z^M)$. Therefore, Firm 2 has no incentive to deviate if $\Gamma M - r(z^M) \leq 0$. If $w > \gamma_2$, then $P_1 = \gamma_1 + z^M$ and Firm 1 earns an equilibrium profit equal to $\pi_1 = [\bar{w} - \Gamma]M$. If Firm 1 were to deviate and choose $z_1 = 0$, then Firm 1 would earn only $\gamma_1 M$ (because Firm 2 is foreclosed by $w > \gamma_2$). Thus Firm 1 has no incentive to deviate. Firm 2 has no incentive to deviate by the same reasoning as before. ■

In the equilibrium in which only Firm 1 invests, Firm 2 is foreclosed even though it is able to produce systems more efficiently *ex ante*. Such foreclosure can occur for any price of component A for suitable parameter values. By improving its system, Firm 1 endogenously becomes such a formidable competitor that Firm 2 cannot effectively compete. For illustration, Figure 1 shows the payoffs to Firm 1 in each of the possible pure strategy equilibria as a function of w.[14]

The pure strategy equilibria of the product improvement game show that one cannot rely on *ex post* market structure to infer which firm is the more efficient supplier. Product superiority is endogenous and in our model either firm can invest to become the market leader. In particular, the firm that has a quality disadvantage can overcome its disadvantage by investing in quality. Having done so, the more efficient firm cannot profitably invest to win the market even though it can supply a better product than its rival. Thus the "wrong" firm can emerge as the market leader. This indeterminacy exists even when the owner of component A does not engage in technological tying to impede access by its rival. The next section explores equilibrium outcomes when such strategies are feasible.

[14] We have assumed that $\Gamma > 0$. If $\Gamma \leq 0$, then an equilibrium exists in which the more efficient Firm 1 invests z^{M} for any $0 \leq w \leq \bar{w} - \Gamma$. There is a second equilibrium in which the less efficient Firm 2 invests z^{M} if $0 \leq w \leq \bar{w}$ and $r(z^{\mathsf{M}}) > -\Gamma M$. The component price would foreclose Firm 2 if $\bar{w} < w \leq \bar{w} - \Gamma$. Firm 1 would leapfrog Firm 2's post-investment quality if $r(z^{\mathsf{M}}) \leq -\Gamma M$.

3.2 The Technological Tying Game

Firm 1 can avoid competition from Firm 2 by foreclosing Firm 2's access to component A, which we assume is available only from Firm 1. It conceivably might do this by contractually conditioning the purchase of A on the purchase of its component B, by selling a system consisting of components A and B and refraining from selling A separately (a pure bundling strategy), by charging a price for component A that is so high that Firm 2 cannot compete (or, equivalently, refusing to sell component A), or by designing component A so that a system performs worse when used with Firm 2's component B. This last strategy is a technological tie. A technological tie lowers the quality of a system made with component B from Firm 2 by an amount δ. That is, $q_2 = \gamma_2 + z_2 - \delta$ and we assume that q_1 is unchanged by the tie at $\gamma_1 + z_1$. A technological tie forecloses competition if δ is sufficiently large.

Foreclosure strategies that are based on a contractual tie, pure system sales, or refusals to deal in the upstream product may be ineffective if there is a separate demand for component A that the upstream monopolist wishes to serve. In contrast, a technological tie that obstructs the ability of Firm 2 to offer a competitive system product, or makes it expensive for consumers to assemble a system using component B from Firm 2, does not limit the ability of the upstream monopolist to pursue a mixed bundling strategy in which the firm both sells systems and makes separate sales of component A in a different market. For simplicity, we assume that technological tying is costless for Firm 1 (other than the indirect cost of lost revenues from sales of component A to Firm 2) and consider only Firm 1's incentives to engage in this activity. Foreclosure is clearly inefficient because it eliminates competition from a more efficient producer. Nonetheless, Firm 1 may profit by foreclosing production by Firm 2 under some circumstances.

Consider the following three-stage "technological tying game," which amends the basic product improvement game studied in the previous subsection. In stage one, the firms choose costly quality improvements z_i as before. In stage two, Firm 1 is able to impose a technological tie that degrades Firm 2's quality by an amount $\delta \geq 0$. In stage three, the firms set prices $P_i \geq w$ as before.

With vertical product differentiation, Firm 1 may profit by degrading Firm 2's quality only if it wins the system competition; i.e., only if $q_1 > q_2 - \delta$. Thus, it is sufficient to focus on technological tying strategies that foreclose Firm 2 from the market. The following proposition establishes the existence of a unique pure strategy equilibrium outcome of the technological tying game for sufficiently low values of the component price. When $w < \bar{w} - \Gamma$, Firm 1 invests z^M and Firm 2 is foreclosed from the systems market by either an actual or threatened technological tie. For very low values of w ($w < \gamma_2$), Firm 1 would foreclose Firm 2 with a technological tie. For intermediate values of w, Firm 1 has no need to impose a technological tie in equilibrium because Firm 2 poses no competitive threat unless it invests and Firm 2 is deterred from investing by the credible threat of a technological tie if it were to leapfrog Firm 1. Although the threat of a technological tie is critical to the equilibrium outcome, whether or not Firm 1 actually imposes a technological tie in equilibrium in this case has no effect on profits or welfare.

Proposition 4 *In the technological tying game:*

(i) There exists an equilibrium in which Firm 1 invests z^M, Firm 2 does not invest, and Firm 1 forecloses Firm 2 with a technological tie. In this equilibrium, Firm 1 sets $P_1 = \gamma_1 + z^M$, sells systems to the entire market, and earns $\pi_1 = [\bar{w} - \Gamma]M$. Firm 2 sets $P_2 = w$ and earns $\pi_2 = 0$.

(ii) If and only if $\gamma_2 \leq w < \gamma_1 + z^M$, there exists an equilibrium in which Firm 1 invests z^M and Firm 2 does not invest. Firm 1 does not impose a technological tie, sets $P_1 = \gamma_1 + z^M$, sells systems to the entire market, and earns $\pi_1 = [\bar{w} - \Gamma]M$. Firm 2 sets $P_2 = w$ and earns $\pi_2 = 0$.

(iii) If and only if $w \geq \bar{w} - \Gamma$, there exists an equilibrium in which Firm 2 invests z^M and Firm 1 does not invest. Firm 1 does not impose a technological tie. Firm 2 sets $P_2 = \gamma_2 + z^M$, sells systems to the entire market, and earns $\pi_2 = [\bar{w} - w]M$. Firm 1 sets $P_1 = w$ and earns $\pi_1 = wM$.

(iv) There are no other pure strategy equilibria.

Proof. Given the winner-take-all nature of the market, $0 < z_i < z^M$ is never a best response in pure strategies. Therefore, without loss of generality, we can restrict attention to $z_i \in \{0, z^M\}$. For any value of w, there is an equilibrium in which Firm 1 invests, Firm 2 does not invest, and Firm 1 imposes a technological tie. The tie makes it unprofitable for Firm 2 to invest, and if Firm 2 does not invest, then it is costless for Firm 1 to impose the tie. The most that Firm 2 would pay for the component is $\gamma_2 M$. By investing and foreclosing Firm 2 with a technological tie, Firm 1 sets $P_1 = \gamma_1 + z^M$, sells systems to the entire market, and earns $\pi_1 = [\bar{w} - \Gamma]M$. By Assumption A3, $\bar{w} - \Gamma > \gamma_2$, so foreclosure is profitable.

If $w < \bar{w} - \Gamma$, investing z^M and foreclosing Firm 2 with a technological tie is a dominant strategy for Firm 1. This strategy is clearly subgame perfect because, with a tie, Firm 1 earns $(\gamma_1 + z^M)M$ at the second stage, which exceeds wM when $w < \bar{w} - \Gamma$. Firm 2's best response to this strategy is to invest zero. Any lesser investment by Firm 1 would yield a lower profit. Any positive investment by Firm 2 that threatened Firm 1's profit would be undone by the technological tie.

If $w \geq \gamma_2$, Firm 1 has no need to tie when Firm 2 does not invest because Firm 2 is foreclosed by the high component price. Furthermore, if $w < \gamma_1 + z^M$, Firm 1 would impose a tie even if Firm 2 invests. Thus, if $\gamma_2 \leq w < \gamma_1 + z^M$, there exists an equilibrium in which Firm 1 invests z^M and Firm 2 does not invest. Firm 1 does not impose a technological tie, sets $P_1 = \gamma_1 + z^M$, sells systems to the entire market, and earns $\pi_1 = [\bar{w} - \Gamma]M$. Firm 2 sets $P_2 = w$ and earns $\pi_2 = 0$.

If $w \geq \bar{w} - \Gamma$, then there is also an equilibrium in which Firm 2 invests, Firm 1 does not invest, and Firm 1 does not impose a technological tie. In the second stage, Firm 1 would earn $\gamma_1 M$ if it imposes a tie, which is less than it would earn by selling the component to Firm 2. In this equilibrium Firm 2 sets $P_2 = \gamma_2 + z^M$, sells systems to the entire market, and earns $\pi_2 = [\bar{w} - w]M$. Firm 1 sets $P_1 = w$ and earns $\pi_1 = wM$. ■

Summarizing, for any value of w, there is an equilibrium in which Firm 1 invests, Firm 2 does not invest, and Firm 1 imposes a tie. This is the unique equilibrium if $w < \gamma_2$. For $\gamma_2 \leq w < \gamma_1 + z^M$, there is an additional equilibrium in which Firm 1 invests z^M, Firm 2 does not invest, and Firm 1 does not impose a technological tie. For $w \geq \bar{w} - \Gamma$, a third equilibrium exists in which Firm 2 invests z^M, Firm 1 does not invest and does not impose a technological tie. This last equilibrium Pareto-dominates the equilibrium in which Firm 1 invests and is consistent with the standard Chicago School result that a firm does not benefit from a technological tie if it can charge the monopoly price for an essential upstream input.[15]

The mere threat of a technological tie is sufficient to foreclose Firm 2 when $\gamma_2 \leq w < \gamma_1 + z^M$. In equilibrium, Firm 2 responds to the threat by declining to invest. Firm 1 invests in an efficient level of R&D and provides a superior system to Firm 2. In this case, no actual anticompetitive behavior is ever observed. Yet market structure is distorted and the more efficient supplier of systems is effectively foreclosed by the ability of the vertically integrated firm to impose a technological tie and its incentive to do so off the equilibrium path.

Firm 1 does not benefit from tying when $w \geq \bar{w} - \Gamma$. In this case, the ability to impose a technological tie is a trap that the tying firm would prefer to avoid. Nonetheless there is an equilibrium in which the less efficient Firm 1 invests and imposes a tie. Thus, when $w \geq \bar{w} - \Gamma$, Firm 1 has no profitable use for a technological tie, either threatened or actual, and would be better off if it could relinquish the ability to impose a tie.[16]

3.3 Welfare

We now consider the welfare implications of the pure strategy equilibria of the product improvement and the technological tying games. Social welfare is obviously at a maximum in the efficient pure strategy equilibrium of the product improvement game. Nonetheless, consumers (weakly) prefer the inefficient pure strategy equilibrium to the efficient one. With Bertrand competition, the equilibrium price is the quality level of the investing firm less the margin between quality and cost for the rival firm, provided this margin is positive. This margin determines consumer surplus. The margin is (weakly) larger for Firm 2 because Firm 2's quality level exceeds Firm 1's when neither firm invests. Firm 2 is a greater competitive threat to Firm 1 in the inefficient equilibrium than Firm 1 is to Firm 2 in the efficient equilibrium. Consumers benefit directly from the greater competitive threat of Firm 2 in the inefficient equilibrium.

Proposition 5 *Consumer surplus is weakly higher in the inefficient pure strategy equilibrium of the product improvement game than in the efficient equilibrium, and strictly higher when $w < \gamma_2$.*

[15] We have assumed that $\Gamma > 0$. If $\Gamma \leq 0$, Firm 1's dominant strategy is to foreclose Firm 2 and invest z^{M}. This strategy extracts the maximum attainable surplus, $\pi^{\mathsf{M}} = (\bar{w} - \Gamma)M$. Foreclosure can be achieved with a technological tie or a sufficiently high component price.

[16] AT&T's voluntary divestiture of Western Electric was explained in part by the costs of being both a supplier to downstream firms and a competitor of those firms. Our analysis confirms that the ability to distort downstream competition can indeed be liability that a monopoly supplier of an essential component would want to relinquish.

Proof. The surplus that each consumer enjoys from the purchase of system i is $CS_i = q_i - P_i$. The equilibrium price of system i when $q_i > q_j$ is $P_i = q_i - \max(q_j - w; 0)$. Therefore, consumer surplus when Firm i wins the market is $CS_i = \max(q_j - w, 0)$. It follows that $CS_1 \geq CS_2$, with a strict inequality when $\gamma_2 > w$. ■

The ability of Firm 1 to impose a technological tie obviously threatens social welfare when there is a unique equilibrium of the product improvement game. If $r(z^M) < \Gamma M$, efficient investment by Firm 2 is the unique pure strategy equilibrium market structure. However, equilibria with tying exist for all values of w and Firm 1 has a strictly positive incentive to impose a technological tie unless the wholesale price for the component input is sufficiently remunerative. When $r(z^M) < \Gamma M$, technological tying destroys the possibility of an efficient market structure.

Corollary 1 If $r(z^M) < \Gamma M$, then technological tying reduces social welfare relative to the pure strategy equilibrium of the product improvement game.

If $r(z^M) \geq \Gamma M$, there are multiple pure strategy equilibria of the product improvement game. Technological tying does not improve total welfare in this case and can reduce welfare by preventing an efficient pure strategy equilibrium.

Corollary 2 If $r(z^M) \geq \Gamma M$, then technological tying weakly reduces social welfare relative to pure strategy equilibria of the product improvement game. If the efficient equilibrium is focal, then technological tying strictly reduces social welfare. Otherwise, the ability of Firm 1 to technologically tie is irrelevant for market structure and social welfare (but not for consumer welfare).

Technological tying is never in the interest of consumers. It is evident from Proposition 4 that consumer surplus is zero for all values of w in the technological tying game. Tying eliminates Firm 2 as a potential competitor when Firm 1 invests, and the high component price eliminates Firm 1 as a potential competitor when Firm 2 invests and wins the market. Absent any effective competition, consumer surplus is fully extracted.

Corollary 3 *Consumer welfare is weakly lower in the technological tying game than in the product improvement game. If $w < \gamma_2$, then consumer surplus is strictly lower in the technological tying game relative to the inefficient pure strategy equilibrium of the product improvement game. If $w < \gamma_1$, then consumer surplus also is strictly lower relative to the efficient equilibrium of the product improvement game.*

An important caveat is in order. The above welfare analysis is premised on the assumption that the exogenous wholesale price is the same in the product improvement game and the corresponding technological tying game. The assumption seems most reasonable when the wholesale price is determined by regulation. It seems less reasonable if it is a market price. A possibility that we alluded to earlier is that technological tying may allow the upstream monopolist to price discriminate between different uses for the component. The non-discriminatory profit-maximizing price that the upstream monopolist would charge without technological tying is a compromise between the profit-maximizing prices for each possible use. Technological tying would allow the firm effectively to set a high price for the component when it is used as part of a system, and a low price for other uses. In this case, a prohibition against technological tying could cause the wholesale price to increase, depending on the price elasticities of demand for other uses. This possibility tempers the case against technological tying.

4 Mixed strategies

4.1 The Product Improvement Game

The product improvement game lacks pure strategy equilibria in which both firms invest (Proposition 1). Moreover, firms' preferences over alternative equilibria disagree if the wholesale price of the component is low. When $w < \bar{w} - \Gamma$, Firm 1 strictly prefers the inefficient pure strategy equilibrium in which it invests and forecloses Firm 2. Firm 2, of course, prefers the efficient equilibrium in which it wins the market. There would be a compelling case to restrict our analysis to the pure strategy equilibria of the games if the firms could coordinate their R&D investments. This could come about because, for example, one of the firms gets to move first in its choice of R&D expenditure. However, the characteristics of R&D investment do not suggest that such coordination would be easy. Firms often invest with limited information about their rivals' investments and there is often no natural first mover in R&D. Absent coordination on a pure strategy equilibrium, mixed strategy equilibria in which both firms invest with some probability are plausible. The possibilities of wasteful investments in product improvement, deficient product improvement, or no investments at all, are all realistic outcomes when firms are unsure of each other's incentives and must form beliefs about what the other will do.[17]

In general, there exist multiple mixed strategy equilibria. The Appendix shows that in any mixed strategy equilibrium, at least one firm's strategy must have at least one discrete component.[18] To keep things simple we only consider mixed strategy equilibria in which each firm randomizes over a binary support. That is, Firm 1 randomizes between z_1^L and z_1^H with $z_1^H > z_1^L$ and Firm 2 randomizes between z_2^L and $z_2^H > z_2^L$. While equilibrium mixed strategies involving three or more discrete levels of investment may exist, our focus on binary mixed strategy equilibria is sufficient to demonstrate that welfare conclusions depend on equilibrium selection and that consumers can be better off when the firms play mixed strategies.

There are two possible binary mixed strategy equilibria to consider, depending on which firm wins the market when both firms invest at high levels. The first candidate mixed strategy equilibrium features $z_2^H \geq z_1^H - \Gamma$. The more efficient Firm 2 is sure to win the market by investing high and does so efficiently by choosing $z_2^H = z^M$. This being the case, Firm 1 only has an incentive to invest if it wins when Firm 2 invests low, i.e. $z_1^H \geq z_2^L + \Gamma$.

[17] Mixed strategy equilibria can be interpreted in terms of each player's beliefs about the actions of others. Alternatively, it is possible to "purify" mixed strategy equilibria along the lines of Harsanyi (1973) as the limit of a Perfect Bayesian Equilibrium of a corresponding game of incomplete information in which the two firms are unsure of each other's incentive for product improvement. Thus the mixed strategy equilibrium has a realistic interpretation as rational conduct in a strategically uncertain market environment. Following a different approach, Cheng and Zhu (1995) show that if agents have quadratic utility, mixed strategy equilibria exist with unique best-reply probabilities for each agent.

[18] This assumes $\Gamma > 0$. A symmetric continuous mixed strategy equilibrium may exist if $\Gamma = 0$.

Firm 2 also has an incentive for a lower level of investment that wins the market when Firm 1 invests low. As there are no other scenarios for profitable investment, Firm 1's low level of investment is zero.

Proposition 6 *For Γ sufficiently small, a mixed strategy equilibrium exists in which Firm 2 randomizes between z^M and z_2^L and Firm 1 randomizes between z_1^H and 0, with $z^M \geq z_1^H - \Gamma > z_2^L > 0$ if and only if:*

$$w \leq \gamma_2 + z_2^L \tag{2}$$

$$r'(z_1^H)(z_1^H - z_2^L - \Gamma) = r(z_1^H) \tag{3}$$

$$r'(z_2^L)(z_2^L + \Gamma) - r(z_2^L) = \pi^M - (z_1^H - \Gamma)M + r'(z_2^L)z_1^H. \tag{4}$$

Firm 1 invests z_1^H with probability

$$\alpha = 1 - \frac{r'(z_2^L)}{M} \tag{5}$$

and Firm 2 invests z^M with probability

$$\beta = 1 - \frac{r'(z_1^H)}{M}. \tag{6}$$

The equilibrium exists if $\Gamma M < r(z^M)$.

The proof is in the Appendix. Firm 1 must be indifferent between not investing and investing z_1^H given Firm 2's strategy. Furthermore, z_1^H must be locally optimal. Together, these conditions imply equations (3) and (6). Similarly, Firm 2 must be indifferent between investing z^M and z_2^L given Firm 1's strategy, and z_2^L must be locally optimal. These conditions imply equations (4) and (5). Moreover, w must not be so large as to foreclose either z_1^H or z_2^L. Since $z_1^H + \gamma_1 > z_2^L + \gamma_2$, the binding condition is $w \leq z_2^L + \gamma_2$. The mixed strategy equilibrium in Proposition 6 exists only if Γ is sufficiently small; otherwise equations (3)-(4) do not have an appropriate solution.

As an example, consider the quadratic case: $r(z) = \frac{1}{2}kz^2$ and define $m \equiv \frac{M}{k}$. Then

$$z_1^H = \frac{2}{3}m(1 + \frac{\Gamma}{m}); \quad z_2^L = \frac{1}{3}m(1 - 2\frac{\Gamma}{m})$$

and

$$\alpha = \frac{2}{3}(1 + \frac{\Gamma}{m}); \quad \beta = \frac{1}{3}(1 - 2\frac{\Gamma}{m}).$$

The mixed strategy equilibrium exists if $w \leq \gamma_2 + z_2^L$ and $\Gamma < \frac{1}{2}m.$

There may exist other mixed strategy equilibria. For example, under some conditions there is a mixed strategy equilibrium in which Firm 1 randomizes between z^M and z_1^L and Firm 2 randomizes between z_2^H and 0, with $z^M \geq z_2^H + \Gamma > z_1^L \geq 0$. These two equilibria have similar properties when Γ is small. For the quadratic case,

$$z_1^L = \frac{1}{3}m(1 + 2\frac{\Gamma}{m}); \quad z_2^H = \frac{2}{3}m(1 - \frac{\Gamma}{m})$$

and

$$\alpha = \frac{1}{3}(1 + 2\frac{\Gamma}{m}); \quad \beta = \frac{2}{3}(1 - \frac{\Gamma}{m}).$$

The Appendix shows that this equilibrium exists if $w \leq \gamma_1 + z_1^L$ and $\Gamma < \frac{1}{4}m.$

4.2 The Technological Tying Game

Mixed strategy equilibria also can exist for the technological tying game, but only for high values of w. In the technological tying game, if w is sufficiently small, Firm 1's optimal strategy is to foreclose Firm 2. It does this by imposing a technological tie in the second stage of the tying game if Firm 2 is not already foreclosed by a price squeeze; i.e. a prohibitive component price.

Proposition 7 *Investing z^M and imposing a technological tie is a dominant strategy for Firm 1 when $w < \bar{w} - \Gamma$.*

Proof. *Firm 1's profit if it imposes a technological tie and forecloses Firm 2 is*

$$\pi_1^{tie} = \max_{z_1}\{(\gamma_1 + z_1)M - r(z_1)\} = (\bar{w} - \Gamma)M.$$

If Firm 1 does not foreclose, its profit is either

$$\pi_1^{no\ tie} = wM$$

if it sells the component to Firm 2 or

$$\begin{aligned} \pi_1^{no\ tie} &= \max_{z_1}\{[\gamma_1 + z_1 - \max(\gamma_2 + z_2 - w, 0)]M - r(z_1)\} \\ &\leq (\bar{w} - \Gamma)M. \end{aligned}$$

if it sells systems. A sufficient condition for $\pi_1^{tie} \geq \pi_1^{no\ tie}$ is $w < \bar{w} - \Gamma$. Hence, tying is a dominant strategy for Firm 1 if $w < \bar{w} - \Gamma$. ■

The component price would foreclose Firm 2 if $w > \gamma_2 + z_2$. In that case, tying is only a weakly dominant strategy. Tying is a strictly dominant strategy if $w < \bar{w} - \Gamma$ and $w \leq \gamma_2 + z_2$.

Because tying is a dominant strategy for Firm 1 when $w < \bar{w} - \Gamma$, mixed strategy equilibria of the tying game can exist only for higher values of w. Furthermore, in the limit as $\Gamma \to 0$, tying is a dominant strategy for any $w < \bar{w}$. Consequently, mixed strategy equilibria do not exist for the tying game for Γ sufficiently small.

Corollary 4 Only pure strategy equilibria of the technological tying game exist when Γ is sufficiently small.

Surprisingly, a mixed strategy equilibrium of the technological tying game exists under some conditions when $w \geq \overline{w} - \Gamma$. In this case, Firm 1 and Firm 2 both prefer the pure strategy equilibrium in which Firm 2 captures the market. In the mixed strategy equilibrium, however, the firms fail to coordinate their beliefs on this outcome. Firm 1 invests in product improvement with positive probability out of a concern that Firm 2 might fail to improve its product. And having invested, Firm 1's finds it in its own self interest to impose a technological tie even when Firm 2 does invest. Firm 2, for its part, becomes hesitant to invest out of fear that Firm 1 will also invest and impose a technological tie.

Proposition 8 *In the technological tying game, there exists a mixed strategy equilibrium in which Firm 1 randomizes between* z^M *and* z_1^L *and Firm 2 randomizes between* z_2^H *and* 0 *with* $z^M > z_2^H + \Gamma > z_1^L$ *if and only if*[19]

$$\bar{w} - \Gamma \leq w < \gamma_1 + z^M - \frac{r(z^M - \Gamma)}{r'(z^M - \Gamma)},$$

$$r'(z_1^L)(w - (\gamma_1 + z_1^L)) + r(z_1^L) = M(w - \gamma_1) - \pi^M,$$
$$r'(z_2^H)(z_2^H + \gamma_2 - w) - r(z_2^H) = 0,$$

and $\Gamma < z^M$. *Firm 1 invests* z^M *with probability*

$$\alpha = 1 - \frac{r'(z_2^L)}{M}$$

and Firm 2 invests z_2^H *with probability*

$$\beta = 1 - \frac{r'(z_1^H)}{M}.$$

Firm 1 imposes a technological tie whenever $\gamma_1 + z_1 > w$.

Proof. See Appendix. ■

Using these results, direct calculation proves the following special case.

Corollary 5 Suppose $r(z) = \frac{1}{2}kz^2$ and define $m \equiv \frac{M}{k}$. A binary mixed strategy of the technological tying game exists in which Firm 1 randomizes over z^M and $z_1^L = 2[w-(\bar{w}-\Gamma)]$ and Firm 2 randomizes over $z_2^H = 2(w - \gamma_2)$ and 0 if and only if

$$\bar{w} - \Gamma < w < \bar{w} - \frac{1}{2}\Gamma.$$

We focus the rest of our analysis of mixed strategies on the binary equilibria described in Propositions 6 and 8 for the product improvement game and the technological tying game, respectively. We implicitly assume that other mixed strategy equilibria are not selected. This approach is sufficient to demonstrate the implications of equilibrium selection for evaluating technological tying.

4.3 Welfare

How do consumers fare if the firms play a mixed strategy equilibrium? The next proposition demonstrates that consumers do better in the mixed strategy equilibrium of the product improvement game than in the efficient pure strategy equilibrium. They may do better or worse than in the inefficient pure strategy equilibrium (which yields $max(\gamma_2 - w, 0)$), depending on parameters.

[19] The first condition requires $\bar{w} - \Gamma \leq \gamma_1 + z^M - \frac{r(z^M - \Gamma)}{r'(z^M - \Gamma)}$, or $\frac{r(z^M - \Gamma)}{r'(z^M - \Gamma)} \leq \frac{r(z^M)}{r'(z^M)}$, which is satisfied for convex $r(\cdot)$.

Proposition 9 *Expected consumer surplus is higher in the mixed strategy equilibrium of the product improvement game (of Proposition 6) than in the efficient pure strategy equilibrium.*

Proof. There are two cases to consider.

Case 1: $w \leq \gamma_1$. Consumer surplus (for each of the M consumers) in the efficient pure strategy equilibrium is equal to $\gamma_1 - w$ with probability one. Expected consumer surplus in the mixed strategy equilibrium is

$$CS_{mixed} = \gamma_1 - w + \alpha\,(1-\beta)\,(z_2^L + \Gamma) + \alpha\beta z_1^H > \gamma_1 - w.$$

Case 2: $\gamma_1 < w \leq \gamma_2 + z_2^L$.[20] Consumers get zero surplus in the efficient pure strategy equilibrium. Expected consumer surplus in the mixed strategy equilibrium is

$$\begin{aligned} CS_{mixed} &= \alpha\,(1-\beta)\,(z_2^L + \Gamma) + \alpha\beta z_1^H + \alpha(\gamma_1 - w) \\ &\geq \alpha\beta(z_1^H - z_2^L - \Gamma) > 0. \end{aligned}$$

■

Corollary 6 Expected consumer surplus is a decreasing function of the wholesale price w.

This last result follows from the fact that competitive pressure is greater when the wholesale price is less.

Social welfare (measured by social surplus) obviously is highest in the efficient pure strategy equilibrium. But how do the mixed strategy and the inefficient pure strategy equilibria of the product improvement game compare from a social welfare perspective? The answer is not obvious.

Total welfare in the mixed strategy equilibrium of the product improvement game is equal to

$$\begin{aligned} W_{mixed} &= \alpha\,(1-\beta)\Big[\gamma_1 M + \pi(z_1^H) - r(z_2^L)\Big] \\ &\quad + (1-\alpha)\,\beta\Big[\gamma_2 M + \pi^M\Big] \\ &\quad + \alpha\beta\Big[\gamma_2 M + \pi^M - r(z_1^H)\Big] \\ &\quad + (1-\alpha)\,(1-\beta)\,[\gamma_2 M + \pi(z_2^L)]. \end{aligned}$$

where $\pi(z) \equiv zM - r(z)$. Note that $\pi(z) < \pi^M$ if $z < z^M$. The level of welfare in the efficient pure strategy equilibrium is $W_{pure_+} = \gamma_2 M + \pi^M$. Welfare is lower in the mixed strategy equilibrium compared to the efficient pure strategy equilibrium for three reasons. First, market structure is distorted because the less efficient Firm 1 sometimes wins the market in the mixed strategy equilibrium. Second, there is wasteful investment in product improvement when both firms invest. Third, product improvement is deficient when neither firm invests at the efficient level z^M.

[20] Recall that the mixed strategy equilibrium of the product improvement game does not exist for $w > \gamma_2 + z_2^L$ (Proposition 6).

The comparison with the inefficient pure strategy equilibrium is less clear on first inspection. The level of welfare in the inefficient pure strategy equilibrium is $W_{pure_-} = \gamma_1 M + \pi^M$. Therefore, the difference in welfare between the efficient pure strategy equilibrium and the mixed strategy equilibrium is

$$\begin{aligned} W_{mixed} - W_{pure_-} &= \alpha\,(1-\beta)\left[\pi(z_1^H) - \pi^M - r(z_2^L)\right] \\ &\quad + (1-\alpha)\,\beta[\Gamma M] \\ &\quad + \alpha\beta\left[\Gamma M - r(z_1^H)\right] \\ &\quad + (1-\alpha)\,(1-\beta)\left[\Gamma M - \pi^M + \pi(z_2^L)\right]. \end{aligned} \tag{7}$$

The first term is negative, because Firm 1 has deficient investment incentives in the mixed strategy equilibrium and there is redundant investment by Firm 2. The second term is positive because the mixed strategy equilibrium sometimes selects an efficient market structure. The third term is ambiguous because, even though Firm 2 is more efficient, Firm 1's investment in product improvement is wasteful. The fourth term is also ambiguous because $\Gamma M < \pi^M$ from assumption A3 (Firm 1 can profitably leapfrog Firm 2) and $\pi(z_2) \geq 0$. Thus, on the one hand, the mixed strategy equilibrium sometimes beneficially achieves a more efficient market structure. On the other hand, a pure strategy equilibrium eliminates wasteful investment and improves investment incentives. Thus it appears that welfare may be higher or lower in the mixed strategy equilibrium, depending on the strength of these various effects.

It is, however, possible to resolve this ambiguity when Firm 2's efficiency advantage is small.

Proposition 10 *Expected welfare is lower in the mixed strategy equilibrium than in an inefficient pure strategy equilibrium of the product improvement game if Γ is sufficiently small.*

Proof. >From equation (7), as $\Gamma \to 0$,

$$\begin{aligned} W_{mixed} - W_{pure_-} &\to \alpha(1-\beta)[\pi(z_1^H) - \pi^M - r(z_2^L)] \\ &\quad -\alpha\beta r(z_1^H) + (1-\alpha)(1-\beta)[-\pi^M + \pi(z_2^L)]. \end{aligned} \tag{8}$$

Each term on the right-hand-side of (8) is negative. ■

The result that expected welfare is lower in the mixed strategy equilibrium than in the pure strategy equilibria of the product improvement game as $\Gamma \to 0$ is intuitive. The mixed strategy equilibria incur unnecessary costs when both firms invest and, in the limit as $\Gamma \to 0$, provide no benefit when Firm 2 invests instead of Firm 1. Figure 2 compares expected total welfare in the mixed and pure strategy equilibria when the R&D function has the form $r(z) = kz^2$ for different values of Γ. Note that the mixed strategy equilibrium yields slightly greater expected welfare than the inefficient pure strategy for some values of Γ.

A welfare analysis of technological tying is more complicated when there exists a mixed strategy equilibrium of the product improvement game. When $w < \bar{w} - \Gamma$, technological tying accomplishes the same market structure as the inefficient pure strategy equilibrium. As observed in Proposition 10, expected social welfare in the mixed strategy equilibrium is lower than social welfare in an inefficient pure strategy equilibrium when Γ is sufficiently small. Therefore, technological tying can increase social welfare when a mixed strategy exists and is focal.

Corollary 7 *If $w < \bar{w} - \Gamma$ and Γ is sufficiently small, technological tying increases social welfare relative to the mixed strategy equilibrium of the product improvement game.*

Allowing for mixed strategy equilibria does not change the conclusion that technological tying is not in the interest of consumers. Consumers earn no surplus in the mixed strategy equilibria of the tying game because tying eliminates Firm 2 as a potential competitor when Firm 1 wins the market and the high component price eliminates Firm 1 as a potential competitor when Firm 2 wins the market. However, mixed strategy equilibria of the tying game exist only for $w \geq \bar{w} - \Gamma > \gamma_2$ and consumers earn no surplus in the pure strategy equilibria of the product improvement game when $w \geq \gamma_2$. Thus, consumers are no worse off in a mixed strategy tying equilibrium than they are in the pure strategy equilibria of the product improvement game, but they are better off compared to a mixed strategy equilibrium of the product improvement game. Thus, we can generalize Corollary 3.

Corollary 8 *Consumer welfare is weakly lower in pure and mixed strategy equilibria of the technological tying game compared to pure and mixed strategy equilibria of the product improvement game.*

5 Conclusions

We have examined the causes and consequences of technological tying in a winner-take-all model of a market for systems. In this model, a vertically integrated upstream monopolist supplies an essential component to a more efficient independent competitor in the downstream systems market. The two firms compete on the price and quality for sales to consumers with homogeneous preferences over these vertically differentiated products. If the wholesale price of the essential component is insufficiently remunerative, then the upstream monopolist has an incentive to foreclose rival systems, either by selling only systems, contractually tying components, or designing an essential component so that it works better with its own systems. The equilibrium market structure is inefficient in this case. A technological tie, or even in some cases the mere threat of technological tie, can reduce social welfare by distorting market structure.

The ambiguity regarding the welfare effects of technological tying has to do with the nature of equilibrium when technological tying is infeasible for the vertically-integrated firm, e.g. due to antitrust enforcement. In some cases, a mixed strategy equilibrium can emerge when technological tying is infeasible. If a mixed strategy equilibrium exists and is focal, then the prevention of technological tying reduces social welfare under some conditions. If instead an efficient pure strategy equilibrium is focal, then preventing technological tying increases social welfare. If the inefficient pure strategy equilibrium is focal, then technological tying is irrelevant for social welfare.

If the wholesale price of the essential component is sufficiently near the monopoly price, then the upstream monopolist and independent downstream firm both prefer a pure strategy equilibrium in which the upstream monopolist supplies the component efficiently and the independent firm wins the downstream market. Surprisingly, in this case there can also exist a mixed strategy equilibrium in which the vertically integrated firm invests with positive probability to improve its product and forecloses the more efficient downstream firm with a technological tie. This unfortunate coordination failure would be prevented by a ban on technological tying.

The simple vertical differentiation model does not admit pure strategy equilibria in which both firms invest in product improvement. We plan in future work to allow for systems that are both vertically and horizontally product differentiated, so that some consumers prefer the system sold by Firm 1 and others prefer Firm 2's system, even when each has the same (vertical) quality and is sold at the same price. In this richer model, both firms may have an incentive to improve their products in a pure strategy equilibrium, and the ability of Firm 1 to technological tie might reduce Firm 2's market share short of complete foreclosure. The welfare effects of technological tying are more subtle in this case.

Some brief lessons from our analysis are relevant for local telecommunications markets. Competitive local exchange carriers (CLECs) have complained about a lack of cooperation from incumbents (ILECs) who are required by law to provide interconnection services. CLECs also seek lower wholesale prices for these services. While the enforcement of interconnection requirements is in the hands of the Federal Communications Commission, the wholesale prices of interconnection services typically are set by state regulators in arbitration proceedings. A perennial policy proposal is for "structural separation" that would force an ILEC to form a separate wholesale services company operating separately from its retail services company. The idea is that structural separation would prevent technological tying by making interconnection requirements more transparent and easier to enforce. Our analysis suggests two points about the structural separations policy debate. First, structural separation that prevents technological tying may fail to improve social welfare if participants in the local services market coordinate on product improvement strategies that form a mixed strategy or inefficient pure strategy equilibrium. Second, a higher wholesale price may improve an ILEC's incentive to cooperate in the provision of interconnection services and avoid unwanted equilibria in the product improvement game played by the ILECs.[21]

[21] Our model assumes that retail prices are not regulated. In local telecommunications market, local exchange service is regulated by state authorities, although "vertical services" (e.g. voice mail) are not regulated. See Riordan [2003] for a discussion of incentives for product improvement with retail price regulation.

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Appendix

A.1 Proof that in a mixed strategy equilibrium, at least one firm's strategy must have a discrete component

Let S_i denote the support of Firm i's mixed strategy, and define $\overline{z}_i = \inf S_i$. We prove by contradiction that either $\overline{z}_1$ is a discrete element of S_1, or $\overline{z}_2$ is a discrete element of S_2. Let S_i denote the support and Ψ_i the c.d.f. of Firm i's mixed strategy for $i = 1, 2$. The assumption (A1) that $r(z)$ is convex increasing requires that the S_i are bounded. Define the maximal investment levels $\bar{z}_i = \sup S_i$. Obviously we must have $\Psi_i(\bar{z}_i) = 1$. Given quality choices the (expected) profit of Firm 1 is

$$\pi_1(z_1) = M\left[w + \int_0^{z_1-\Gamma} (z_1 - \Gamma - \max\{z_2, w - \gamma_2\})\, d\Psi_2(z_2)\right] - r(z_1),$$

and of Firm 2 is

$$\pi_2(z_2) = M \int_0^{z_2+\Gamma} (z_2 + \Gamma - \max\{z_1, w - \gamma_1\})\, d\Psi_1(z_1) - r(z_2).$$

Suppose that both firms' strategies are continuous over a region that includes the upper bounds of S_1 and S_2. Thus $[z', \bar{z}_1] \subseteq S_1$ and $[z'', \bar{z}_2] \subseteq S_2$ for $z' < \bar{z}_1$ and $z'' < \bar{z}_2$. Firm 1's indifference condition, $\pi_1(z_1) = \pi_1(\bar{z}_1)$ for all $z_1 \in S_1$, requires $\Psi_2(\bar{z}_2) = \frac{r'(\bar{z}_2+\Gamma)}{M}$ for $z_2 \in [z' - \Gamma,\ \bar{z}_1 - \Gamma) \subset S_2$. Therefore, $\bar{z}_1 - \Gamma \leq \bar{z}_2$. Similarly, Firm 2's indifference condition, $\pi_2(z_2) = \pi_2(\bar{z}_2)$ for all $z_2 \in S_2$, requires $\Psi_1(\bar{z}_1) = \frac{r'(\bar{z}_1-\Gamma)}{M}$ for $z_1 \in [z'' + \Gamma,$ $\bar{z}_2 + \Gamma) \subset S_1$. Therefore, $\bar{z}_1 - \Gamma = \bar{z}_2$. Furthermore, $\Psi_1(\bar{z}_1) = \Psi_1(\bar{z}_2 + \Gamma) \geq \frac{r'(\bar{z}_2)}{M}$ and $\Psi_2(\bar{z}_2) = \Psi_2(\bar{z}_1 - \Gamma) \geq \frac{r'(\bar{z}_1)}{M}$.

It must also be that $\bar{z}_1 = \bar{z}_2 + \Gamma = z^M$. Clearly, $\bar{z}_1 \geq z^M$; otherwise, Firm 1 could more profitably choose $z_1 = z^M$. But then $\Psi_2(\bar{z}_2) \geq \frac{r'(\bar{z}_1)}{M} \geq \frac{r'(z^{\mathrm{M}})}{M} = 1$ implies $\bar{z}_1 = z^M$. Finally $\bar{z}_2 = z^M - \Gamma$ implies that Firm 2 could increase its profit by choosing $z_2 = z^M$. This contradicts the definition of equilibrium. Therefore, either $\bar{z}_1$ is a discrete element of S_1 or $\bar{z}_2$ is a discrete element of S_2.

A.2 Proof of Proposition 6: Mixed strategy equilibrium of the product improvement game.

In the assumed equilibrium, Firm 1 randomizes between z_1^H and 0 and Firm 2 randomizes between z^M and z_2^L, with $z^M \geq z_1^H - \Gamma > z_2^L > 0$.

Necessity: In the candidate mixed strategy equilibrium, Firm 1 must be indifferent between investing z_1^H and investing zero. This requires

$$(1-\beta)[\gamma_1 + z_1^H - \max(\gamma_2 + z_2^L - w, 0)]M + \beta w M - r(z_1^H) = wM$$

Local optimality requires

$$(1-\beta)M = r'(z_1^H).$$

If $w > z_2^L + \gamma_2$, then optimality and indifference for Firm 1 require

$$r'(z_1^H)[z_1^H + \gamma_1] = r(z_1^H),$$

which is impossible because $r(\cdot)$ is strictly convex. Given $w \leq z_2^L + \gamma_2$, indifference and local optimality require

$$r'(z_1^H)(z_1^H - z_2^L - \Gamma) = r(z_1^H).$$

Similarly, Firm 2 must be indifferent between investing z^M and z_2^L. This requires

$$\begin{aligned} & (1-\alpha)[\gamma_2 + z^M - \max(\gamma_1 - w, 0) - w]M + \alpha[\gamma_2 + z^M \\ & - \max(\gamma_1 + z_1^H - w, 0) - w]M - r(z^M) \\ = \; & (1-\alpha)[\gamma_2 + z_2^L - \max(\gamma_1 - w, 0) - w]M - r(z_2^L). \end{aligned}$$

Local optimality requires

$$(1-\alpha)M = r'(z_2^L).$$

If $w \leq z_2^L + \gamma_2 < z_1^H + \gamma_1$, indifference and local optimality for Firm 2 requires

$$r'(z_2^L)(z_2^L + \Gamma - z_1^H) = r(z_2^L) + (z^M - z_1^H)M + (\Gamma M - r(z^M)).$$

Therefore equations (2)-(6) in Proposition 6 are necessary for the mixed strategy equilibrium. Note that the left-hand side of the equation above is negative and the first two terms on the right are positive. Therefore, $\Gamma M < r(z^M)$ is also necessary.

Sufficiency: Firm 1 would not deviate from the assumed equilibrium by investing $0 < z_1 < z_2^L + \Gamma$, because it would have no sales. For $z_2^L + \Gamma \leq z_1 < z^M + \Gamma$, Firm 1 wins the market with probability $1-\beta$ and, by convexity, $z_1 \neq z_1^H$ is not profitable in this interval. If Firm 1 invests $z_1 \geq z^M + \Gamma$, it earns no more than $r'(z_1^H)(z^M - z_2^L) - r(z^M + \Gamma) + wM$. This is less than $r'(z_1^H)(z_1^H - z_2^L - \Gamma) - r(z_1^H) + wM$, its payoff when it invests z_1^H. Hence Firm 1 would not deviate from the assumed equilibrium. Firm 2 could deviate by not investing. This is unprofitable if $(1-\alpha)(z_2^L + \Gamma)M - r(z_2^L) > (1-\alpha)\Gamma M$, or if $r'(z_2^L)z_2^L > r(z_2^L)$, which is true by convexity. Finally, Equation (3) directly implies $z_1^H > z_2^L + \Gamma$, while equation (4) and the convexity of $r(\cdot)$ imply $z^M > z_1^H - \Gamma$. Therefore equations (2)-(6) are sufficient for the mixed strategy equilibrium.

Existence: Assume $w \leq \gamma_2 + z_2^L$. Equation (3) implicitly defines

$$z_2 = \frac{1}{r'(z_1)}[z_1 r'(z_1) - r(z_1)] - \Gamma \equiv \omega(z_1)$$

with the properties $\omega(0) \to -\Gamma$, $\omega'(z_1) = \frac{r''(z_1)r(z_1)}{r'(z_1)^2} \geq 0$ for $z_1 > 0$, $\omega(z^M) = \frac{\pi^{\mathsf{M}}}{M} - \Gamma$, and $\lim_{\Gamma \to 0} \omega(z_1) > 0$ for $z_1 \in (0, z^M]$. Equation (4) implicitly defines

$$z_1 = \frac{\pi^M - [r'(z_2)z_2 - r(z_2)]}{M - r'(z_2)} + \Gamma \equiv \varphi(z_2)$$

with the properties $\varphi(0) = \frac{\pi^{\mathsf{M}}}{M} + \Gamma$, $\varphi'(z_2) = \frac{r''(z_2)[\pi^{\mathsf{M}} - (z_2 M - r(z_2))]}{[M - r'(z_2)]^2} > 0$, and $\varphi(z_2) \to z^M + \Gamma$ as $z_2 \to z^M$. Now define $\hat{z}_1$ by $\omega(\hat{z}_1) = 0$ and $\tilde{z}_2$ by $\varphi(\tilde{z}_2) = z^M$. Sufficient conditions for a solution $(z_1^H, z_2^L) \in (0, z^M)^2$ are $\hat{z}_1 < \frac{\pi^{\mathsf{M}}}{M} + \Gamma$ and $\tilde{z}_2 > \frac{\pi^{\mathsf{M}}}{M} - \Gamma$. (See Figure 3, which shows a fixed point of $\omega(z_1)$ and $\varphi(z_2)$). By continuity, if Γ is sufficiently small, then $\omega(\frac{\pi^{\mathsf{M}}}{M} + \Gamma) > 0$. Furthermore, $\omega(\Gamma) < 0$. These inequalities imply $\hat{z}_1 < \frac{\pi^{\mathsf{M}}}{M} + \Gamma$. Note that $\tilde{z}_2 \to z^M$ as $\Gamma \to 0$. Thus $\tilde{z}_2 > \frac{\pi^{\mathsf{M}}}{M} - \Gamma$ if Γ is sufficiently small.

A.3 Mixed strategy equilibrium of the product improvement game with quadratic R&D costs.

Assume R&D costs are quadratic, $r(z) = \frac{1}{2}kz^2$, and let $m \equiv M/k$. In the assumed equilibrium, Firm 1 randomizes between z^M and z_1^L and Firm 2 randomizes between z_2^H and zero, with $z^M > z_2^H + \Gamma > z_1^L \geq 0$. Furthermore, equilibrium requires $z_1^L > \Gamma$ if $w < \gamma_2$. A mixed strategy equilibrium cannot exist if $w \geq \gamma_2 + z_2^H$, because then Firm 2 would not invest. Therefore, without loss of generality we assume $w < \gamma_2 + z_2^H$. There are two cases to consider, corresponding to $w \leq \gamma_1 + z_1^L$ and $w > \gamma_1 + z_1^L$.

Case (i): $w \leq \gamma_1 + z_1^L$.

Necessity: In the candidate mixed strategy equilibrium, Firm 2 must be indifferent between investing z_2^H and investing zero. The indifference condition for Firm 2 is

$$\begin{aligned} \pi_2(z_2^H) &= (1-\alpha)(\gamma_2 + z_2^H - \max(\gamma_1 + z_1^L - w, 0) - w)M - r(z_2^H) \\ &= (1-\alpha)(z_2^H - z_1^L + \Gamma)M - r(z_2^H) = 0. \end{aligned}$$

Local optimality of z_2^H requires:

$$r'(z_2^H) = (1-\alpha)M. \tag{9}$$

Equation (9) gives for quadratic R&D costs

$$z_2^H = 2(z_1^L - \Gamma). \tag{10}$$

Similarly, assuming $z_1^L > \Gamma$, the indifference condition for Firm 1 is

$$\begin{aligned} \pi_1(z_1^L) &= (1-\beta)(\gamma_1 + z_1^L - \max(\gamma_2 - w, 0))M + \beta w M - r(z_1^L) = \\ \pi_1(z^M) &= (1-\beta)(\gamma_1 + z^M - \max(\gamma_2 - w, 0))M + \beta(\gamma_1 + z^M - \max(\gamma_2 + z_2^H - w, 0))M - r(z^M) \end{aligned}$$

or

$$(1-\beta)z_1^L M - r(z_1^L) = \pi^M - \beta(z_2^H + \Gamma)M.$$

The local optimality condition is

$$r'(z_1^L) = (1-\beta)M. \tag{11}$$

Equation (11) gives for quadratic R&D costs

$$z_2^H = \frac{1}{2}(m + z_1^L) - \Gamma. \tag{12}$$

Equations (10) and (12) imply

$$\begin{aligned} z_1^L &= \frac{1}{3}(m+2\Gamma) \\ z_2^H &= \frac{2}{3}(m-\Gamma). \end{aligned}$$

Equilibrium also requires $m > z_2^H + \Gamma$, or $m > \Gamma$. This condition also implies $z_2^H + \Gamma > z_1^L > \Gamma$. Note that $w \le \gamma_1 + z_1^L$ requires $w < \gamma_1 + \frac{1}{3}(m+2\Gamma)$. The investment probabilities (α, β) follow directly from the local optimality conditions (9) and (11).

Sufficiency: If Firm 2 deviates from the assumed equilibrium and invests z^M, it would earn $(1-\alpha)(z^M + \Gamma - z_1^L)M + \alpha\Gamma M - r(z^M)$, which cannot dominate Firm 2's profit at $z_2 = 0$. This requires

$$r(z^M) > r'(z_2^H)(z^M - z_1^L) + M\Gamma$$

or, for the case of quadratic R&D costs, $\Gamma < \frac{1}{4}m$.

Firm 1 could deviate from the assumed equilibrium by investing $z_1 < z_1^L$, which also must be unprofitable. If $z_1 \ge \Gamma$, this requires $\pi_1(z) = (1-\beta)(\gamma_1 + z - \max(\gamma_2 - w, 0))M + \beta w M - r(z) \le (1-\beta)(\gamma_1 + z_1^L - \max(\gamma_2 - w, 0))M + \beta w M - r(z_1^L)$ for $z < z_1^L$. This is satisfied because$(1-\beta)zM - r(z) > 0$ is increasing in z for $z < z_1^L$. If $z_1 < \Gamma$, then Firm 1 would make no sales and have no incentive to invest unless Firm 2 is foreclosed by w. The assumption that $w \le \gamma_1 + z_1^L$ implies that $w < \gamma_2$. Hence the corresponding condition is $wM \le (1-\beta)(z_1^L - \Gamma + w)M + \beta w M - r(z_1^L)$ for $z < z_1^L$. For quadratic R&D costs, this is satisfied if $\Gamma < \frac{1}{4}m$.

Finally, the firms have no incentives to deviate from prescribed strategies. By construction, Firm 2 is indifferent between investments z_2^H and 0, and Firm 1 is indifferent between z^M and z_1^L. And we have already argued that Firm 1 has no incentive to deviate to 0 and Firm 2 has no incentive to deviate to z^M. Other possible deviations are unprofitable by the concavity of profit functions over relevant ranges. Hence, we conclude that the assumed equilibrium exists in case (i) if $\Gamma < \frac{1}{4}m$ and $w < \gamma_1 + \frac{1}{3}(m+2\Gamma)$.

Case (ii): $w > \gamma_1 + z_1^L$.

Necessity: The assumed equilibrium cannot exist if $\gamma_1 + z_1^L < w < \gamma_2$, because Firm 2 would win the market when Firm 1 invests z_1^L, so Firm 1 would not invest. Suppose instead that $w \ge \gamma_2$. Then Firm 2 is foreclosed if it does not invest, even if $0 \le z_1^L \le \Gamma$. The indifference condition for Firm 2 is

$$\begin{aligned} \pi_2(z_2^H) &= (1-\alpha)(\gamma_2 + z_2^H - \max(\gamma_1 + z_1^L - w, 0) - w)M - r(z_2^H) \\ &= (1-\alpha)(\gamma_2 + z_2^H - w)M - r(z_2^H) = 0. \end{aligned}$$

Using local optimality of z_2^H (equation (9)) and assuming quadratic costs gives

$$z_2^H = 2(w - \gamma_2). \tag{13}$$

The indifference condition for Firm 1 is $\pi_1(z_1^L) = \pi_1(z^M)$,or

$$\begin{aligned} & (1-\beta)(\gamma_1 + z_1^L)M + \beta w M - r(z_1^L) \\ = \; & (1-\beta)(\gamma_1 + z^M)M + \beta(z^M - \Gamma - z_2^H + w)M - r(z^M). \end{aligned}$$

Using the local optimality condition (11) and assuming quadratic R&D costs gives

$$z_1^L = 2(z_2^H + \Gamma) - m,$$

and substituting (13) gives

$$z_1^L = 2(2w - (\gamma_1 + \gamma_2)) - m. \tag{14}$$

The assumption that $w > \gamma_1 + z_1^L$ along with equation (14), $z_1^L \geq 0$, and $w \geq \gamma_2$ imply that

$$\max[\gamma_2, \gamma_1 + \frac{1}{4}(m + 2\Gamma)] \leq w < \gamma_1 + \frac{1}{3}(m + 2\Gamma). \tag{15}$$

This in turn implies that a necessary condition for equilibrium is $m > \Gamma$. This also implies $w < \gamma_1 + z^M$.

Sufficiency: As in case (i), Firm 1 would not profit by deviating to 0 and has no incentive to invest at levels other than z_1^L or z^M. If Firm 2 deviates by investing z^M, it would earn

$$\begin{aligned}\pi_2(z^M) &= (1-\alpha)(\gamma_2 + z^M - w)M + \alpha(\gamma_2 + z^M - \max(\gamma_1 + z^M - w, 0) - w)M - r(z^M) \\ &= \pi^M - (1-\alpha)(w - \gamma_2)M - \alpha(z^M - \Gamma)M,\end{aligned}$$

which has to be less than the zero payoff when it does not invest. Substituting the local optimality condition (9) and assuming quadratic R&D costs, this requires

$$\pi^M - r'(z_2^H)(w - \gamma_2) - (M - r'(z_2^H))(z^M - \Gamma) \leq 0.$$

For the case of quadratic R&D, using (13), sufficiency requires

$$(w - \gamma_2)^2 - (w - \gamma_2)(m - \Gamma) + \frac{1}{4}m(m - 2\Gamma) \geq 0.$$

This, in turn, implies either $w \leq \gamma_1 + \frac{1}{2}m$ or $w \geq \gamma_2 + \frac{1}{2}m$. However, the necessary conditions for an equilibrium require $\max[\gamma_2, \gamma_1 + \frac{1}{4}(m + 2\Gamma)] < w < \gamma_1 + \frac{1}{3}(m + 2\Gamma)$. Notice that $w < \gamma_1 + \frac{1}{3}(m + 2\Gamma)$ contradicts $w \geq \gamma_2 + \frac{1}{2}m$. Therefore, an equilibrium exists in this case (ii) only if

$$\max[\gamma_2, \gamma_1 + \frac{1}{4}(m + 2\Gamma)] \leq w \leq \min(\gamma_1 + \frac{1}{2}m, \gamma_1 + \frac{1}{3}(m + 2\Gamma)).$$

No equilibrium exists when $w \geq \gamma_2$ if $m < 2\Gamma$. Furthermore, $\gamma_2 \geq \gamma_1 + \frac{1}{4}(m + 2\Gamma)$ when $m \geq 2\Gamma$. Thus equilibrium requires

$$\gamma_2 \leq w \leq \min(\gamma_1 + \frac{1}{2}m, \gamma_1 + \frac{1}{3}(m + 2\Gamma))$$

and $m \geq 2\Gamma$. As $\Gamma \to 0$, this condition becomes

$$\gamma_2 \leq w \leq \gamma_1 + \frac{m}{3}.$$

The equilibrium exists for an intermediate range of w if Γ is sufficiently small.

A.4 Proof of Proposition 8: Mixed strategy equilibrium of the technological tying game.

Consider a candidate mixed strategy equilibrium with $z_1^H = z^M > z_2^H + \Gamma > z_1^L > z_2^L = 0$. Assume $\gamma_1 + z^M > w \geq \bar{w} - \Gamma \geq \gamma_2$. A mixed strategy equilibrium of the tying game does not exist if $w < \bar{w} - \Gamma$ or if $w > \gamma_1 + z^M$. In the former case, Firm 1 will always foreclose. In the latter case, Firm 1 would always prefer to sell the component. Define $\pi(z) = r'(z)z - r(z)$. Suppose Firm 1 invests $z_1^H = z^M$. The component price would foreclose Firm 2 if it does not invest because $w \geq \gamma_2$. If it does invest, Firm 1 would impose a tie because $\gamma_1 + z^M > w$. Thus, when Firm 1 invests z^M, it earns

$$\pi_1(z^M) = \gamma_1 M + \pi^M.$$

Suppose Firm 1 invests z_1^L. As before, the component price forecloses Firm 2 if it does not invest. If Firm 2 invests, then Firm 1 would not foreclose if $\gamma_1 + z_1^L < w$. Then

$$\pi_1(z_1^L) = \beta w M + (1-\beta)(\gamma_1 + z_1^L)M - r(z_1^L).$$

A mixed strategy equilibrium requires

$$r'(z_1^L) = (1-\beta)M$$

and

$$(M - r'(z_1^L))(w - \gamma_1) = \pi^M - \pi(z_1^L).$$

Define

$$\psi(z) = (M - r'(z))(w - \gamma_1) - \pi^M + \pi(z).$$

A necessary condition for a mixed strategy equilibrium is $\psi(z_1^L) = 0$. At $z = 0$, $\psi(0) = M(w - \gamma_1) - \pi^M \geq 0$ if and only if $w \geq \bar{w} - \Gamma$, and $\psi(z^M) = 0$. Note that

$$\psi'(z) = (z - (w - \gamma_1))r''(z)$$

and $\psi'(z)$ has the same sign as $(z - w + \gamma_1)$. Furthermore, $\psi'(z^M) > 0$ if and only if $w < \gamma_1 + z^M$ and $\psi'(0) < 0$ if and only if $w > \gamma_1$. Therefore, if $\bar{w} - \Gamma \leq w < \gamma_1 + z^M$, then there exists $z^M > z_1^L \geq 0$ such that $\psi(z_1^L) = 0$. (See Figure 4.) Moreover, as necessary, $z_1^L < w - \gamma_1$.

Firm 2 earns a profit only if it invests $z_2 = z_2^H$ and if $z_1 = z_1^L$ and $w \leq \gamma_2 + z_2^H$. Then

$$\pi_2(z_2^H) = (1-\alpha)[\gamma_2 + z_2^H - w]M - r(z_2^H)$$

Local optimality requires $r'(z_2^H) = (1-\alpha)M$. Thus we can write

$$\pi_2(z_2^H) = \pi(z_2^H) + r'(z_2^H)(\gamma_2 - w).$$

Existence of a mixed strategy equilibrium requires $\pi_2(z_2^H) = 0$, or $\pi(z_2^H) + r'(z_2^H)(\gamma_2 - w) = 0$. Define

$$\phi(z) = \pi(z) + r'(z)(\gamma_2 - w).$$

Note that

$$\phi'(z) = (z - (w - \gamma_2))r''(z)$$

and $\phi'(z)$ has the same sign as $(z - w + \gamma_2)$. Therefore, if $\phi(0) \leq 0$ and $\phi(z^M - \Gamma) > 0$, then there exists a $z_2^H < z^M - \Gamma$ such that $\phi(z_2^H) = 0$ and $z_2^H > w - \gamma_2$. (See Figure 5.) Now $\phi(0) \leq 0$ if $w > \gamma_2$. Assume $z^M > \Gamma$. Then $\phi(z^M - \Gamma) = r'(z^M - \Gamma)(z^M + \gamma_1 - w) - r(z^M - \Gamma) \geq 0$ if and only if $w \leq \gamma_1 + z^M - \frac{r(z^{\mathsf{M}} - \Gamma)}{r'(z^{\mathsf{M}} - \Gamma)}$. Furthermore, $z_2^H > w - \gamma_2$, hence $z_2^H + \Gamma > w - \gamma_1 > z_1^L$.

Summarizing, Proposition 8 provides a statement of the necessary and sufficient conditions for existence of a mixed strategy equilibrium of the tying game with $z_1^H = z^M > z_2^H + \Gamma > z_1^L > z_2^L = 0$. As an example, consider the quadratic case: $r(z) = \frac{1}{2}kz^2$ and let $m \equiv M/k$. Then $\psi(z_1^L) = 0$ implies

$$z_1^L = 2(w - \gamma_1) - m,$$

and $\phi(z_2^H) = 0$ implies

$$z_2^H = 2(w - \gamma_2).$$

A mixed strategy equilibrium exists with $z_1^H = z^M > z_2^H + \Gamma > z_1^L > z_2^L = 0$ if

$$\bar{w} - \Gamma \leq w < \bar{w} - \frac{1}{2}\Gamma,$$

and $\Gamma < m$.

Figure 1. Firm 1 profit in pure strategy equilibria of the product improvement game

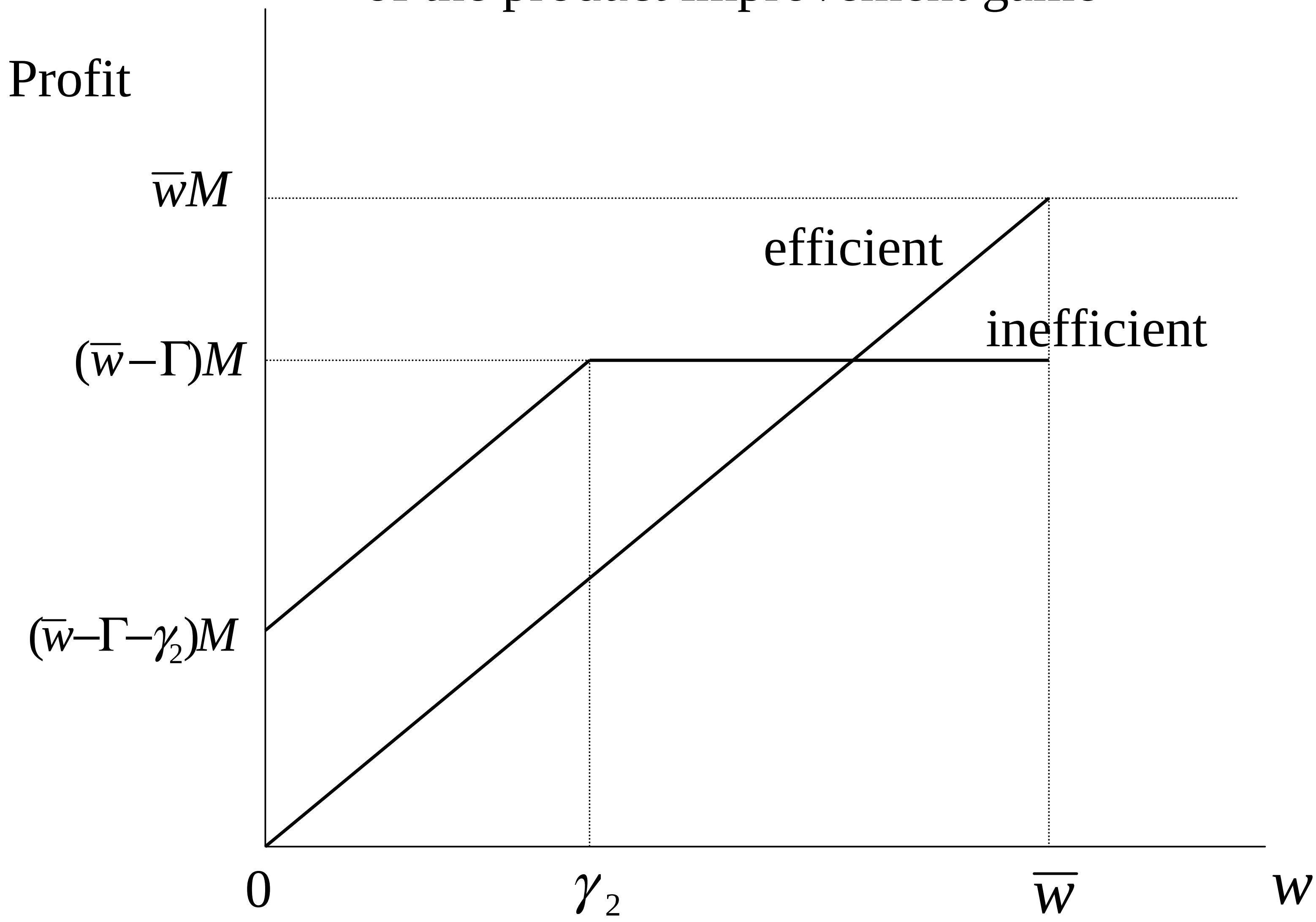

Figure 2. Expected welfare with pure and mixed strategy equilibria

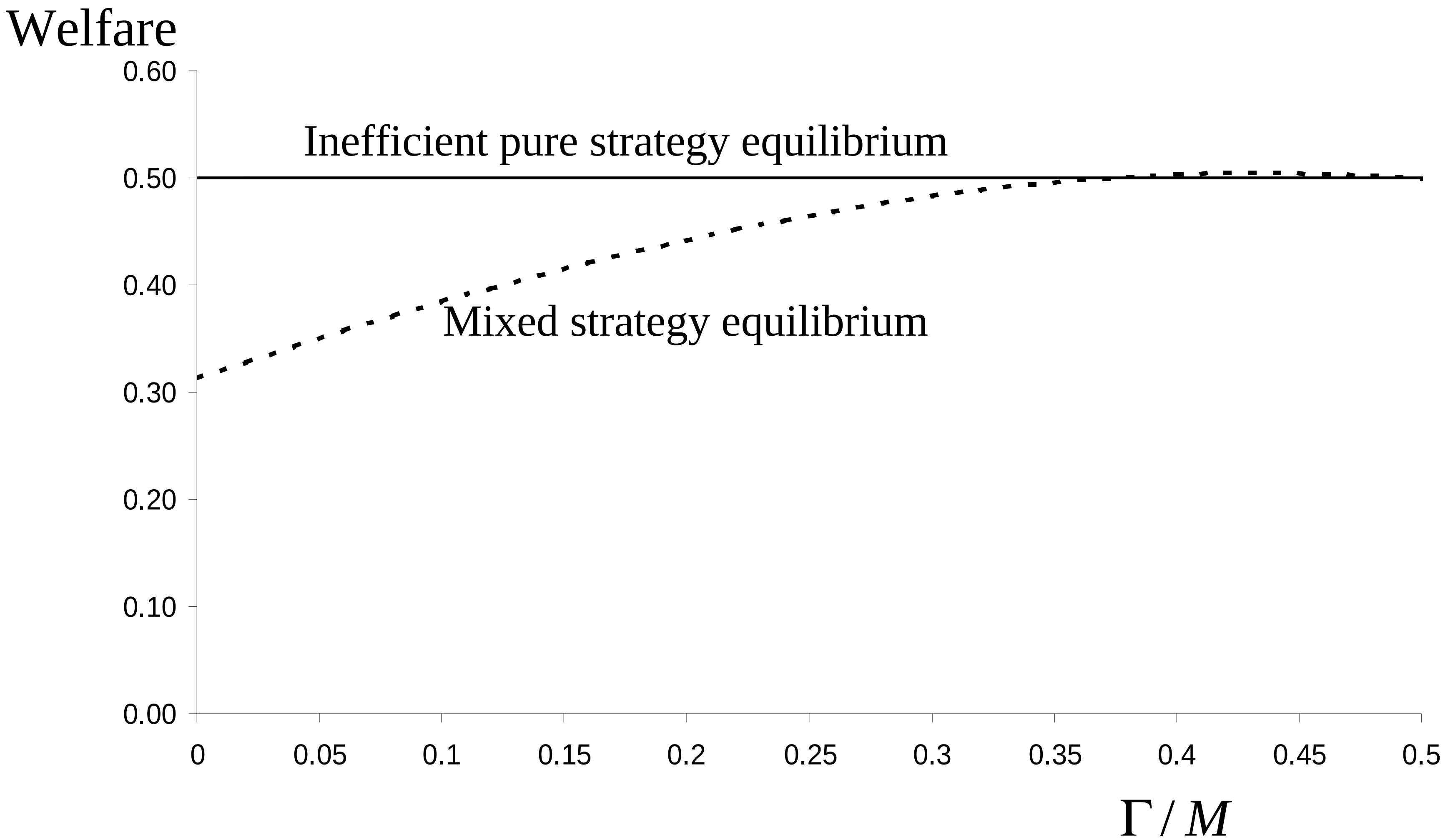

Figure 3. Illustration of fixed point for z_1^H.

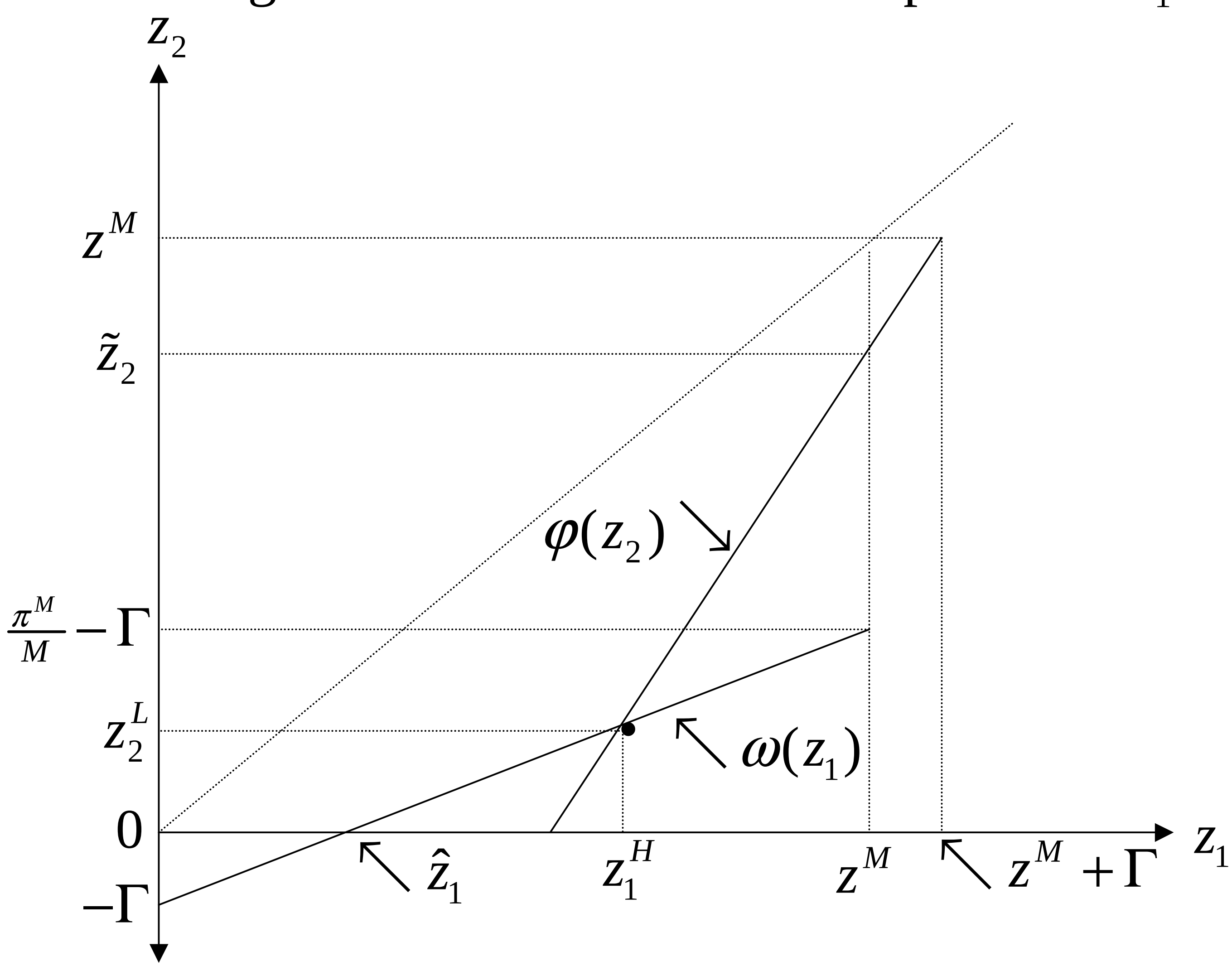

Figure 4. Demonstration of $z_1^L < w - \gamma_1$.

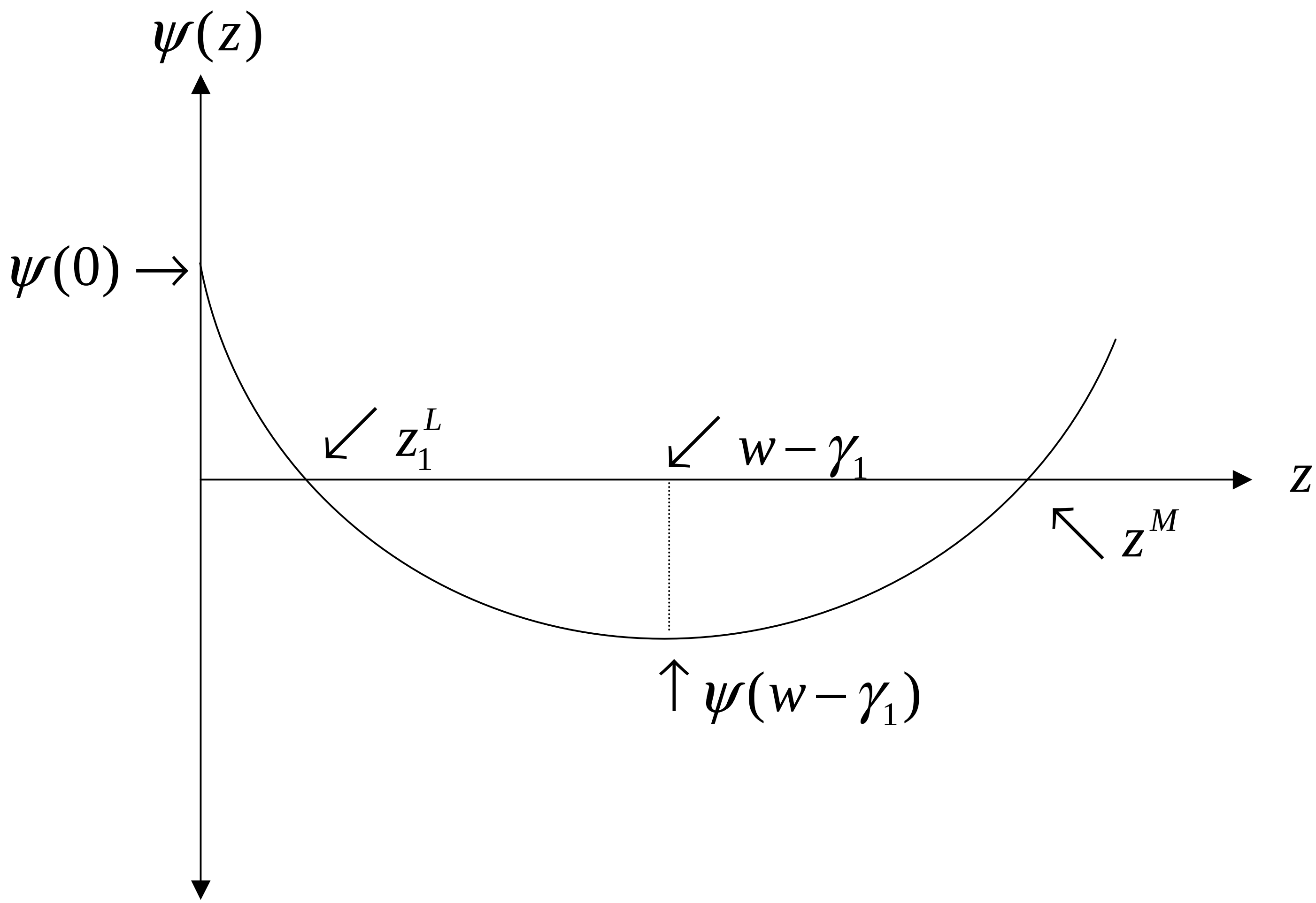

Figure 5. Demonstration of $z_2^H > w - \gamma_2$.