Cointegration, Aggregate Consumption, and the Demand for Imports: A Structural Econometric Investigation

by

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June 1991, Revised March 1992

Discussion Paper Series No. 614
This paper was completed during my stay as a Visiting Scholar at the Federal Reserve Bank of New York. I would like to thank Richard Davis, Akbar Akhtar, Charles Pigott, Bruce Kasman, Susan Hickok, Juann Hung and seminar participants at the Federal Reserve Bank of New York; Mike Gavin, Ricardo Caballero, Jordi Gali and seminar participants at Columbia University; Peter Hooper, William Helkie, Jaime Marquez, Dale Henderson and seminar participants at the Federal Reserve Board of Governors; Bill Branson, Ken Rogoff, and seminar participants at the 1991 NBER Summer Institute; the editor and two referees of this journal; and seminar participants at Princeton, Chicago, Wisconsin, the IMF, Virginia, New York University, and UC Davis for their comments and suggestions. All remaining confusions are my doing.
COINTEGRATION, AGGREGATE CONSUMPTION, AND THE DEMAND FOR IMPORTS:
A STRUCTURAL ECONOMETRIC INVESTIGATION

Abstract

This paper uses a two-good version of the rational expectations permanent income model to derive a structural import demand equation for non-durable consumer goods. Under the identification restriction that taste shocks are stationary, the model is shown to imply that log imports, log domestic goods, and the log relative price of imports are co-integrated. The rational expectations permanent income hypothesis in conjunction with assumption of addilog (Houthakker (1960)) preferences implies that the log of the demand for domestic goods is the correct "activity" variable on the right-hand-side of the import demand equation. This is because consumption of domestic goods is a noisy proxy for the unobservable utility index of permanent income, the marginal utility of wealth.

Using the econometric approach suggested by Phillips-Loretan (1990), we estimate the cointegrating vector and use these estimates to recover estimates of the utility parameters of the representative household. Given these utility parameters, we calculate expressions for the price and expenditure elasticities of import demand. The price elasticity of import demand is estimated to average -0.95 during the sample. The elasticity of import demand with respect to an increase in real spending is estimated to average 2.20. These estimates fall within the range reported in studies by Helkie and Hooper (1986), Cline (1989), and the many studies surveyed by Goldstein and Kahn (1985). The similarity between the OLS and Phillips-Loretan estimates of the parameters suggests that the simultaneous equation bias is not large.

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COINTEGRATION, AGGREGATE CONSUMPTION, AND THE DEMAND FOR IMPORTS: A STRUCTURAL ECONOMETRIC INVESTIGATION

Richard H. Clarida

1. Introduction:

Employing a two-good version of the rational expectations permanent income model, this paper derives a structural econometric equation that can be used to estimate the parameters of the demand for imported consumer goods. With strongly separable, addilog (Houthakker (1960)) preferences, the log of the demand for imported goods is shown to be linear in the log of the relative price of imports, the log of the demand for domestically produced varieties, and the log of an unobservable shock to tastes. The rational expectations permanent income hypothesis in conjunction with the addilog preference structure implies that the log of the demand for domestic goods is the correct "activity" variable on the right-hand-side of the import demand equation. This is because consumption of domestic goods is a noisy proxy for the unobservable utility index of permanent income, the marginal utility of wealth.

The permanent income hypothesis implies that the demand for domestic non-durable goods and the demand for foreign non-durable goods share a common stochastic trend (Stock and Watson (1988)) and that this trend may be identified with the marginal utility of wealth. According to the theory, log imports, log domestic goods, and the log relative price of imports will be cointegrated if the equilibrium relative price of imports contains an independent stochastic supply trend. If these three variables are cointegrated, the import demand equation's structural parameters -- the elasticities of marginal utility with respect to foreign goods consumption, \( \eta \) and home goods consumption, \( \alpha \) -- are exactly identified by the cointegrating vector.
The data decisively reject the null hypothesis that imports, the relative price of imports, and the consumption of home goods are not cointegrated. To correct for simultaneous equations bias, we employ the non-linear least squares technique recently proposed by Phillips and Loretan (1990) to estimate the parameters of the structural import demand equation.

The results of the empirical work may be summarized as follows. The price elasticity of import demand is estimated to average -0.95 during our sample. Given the precision of the estimate, it is not possible to reject the null hypothesis of a unitary price elasticity, thus putting our estimate in the range of earlier empirical studies (Goldstein and Kahn (1985); Helkie and Hooper (1986); Cline (1989)). The elasticity of import demand with respect to an increase in real spending is estimated to average 2.15 during our sample, roughly the same as reported by Helkie and Hooper (1986), somewhat smaller than reported by Cline (1989), and somewhat larger than the average of the many studies surveyed recently by Goldstein and Kahn (1985). In the context of our theoretical specification, the Marshallian price elasticity of import demand is not constant but in fact converges to -1 as the share of total spending that falls on imports rises, while the elasticity of import demand with respect to an increase in real spending is not constant but in fact declines over time as the share of spending that falls on imports rises. An advantage of our utility-based, cointegration approach is that, by recovering consistent estimates of the utility parameters via Phillips-Loretan non-linear least squares, we are able to estimate the income elasticity of import demand without having to specify a time series model for actual income (as is the case in Sheffrin and Woo (1990)).

The paper ends with some concluding remarks.
2. The Model:

We begin by deriving the demand for non-durables foreign goods, $F_t$, from a standard rational expectations permanent income model. Letting $P_t$ denote the price of imports in terms of domestic goods, $H_t$ the consumption of domestic non-durable goods, $A_t$ assets, $y_t$ labor income, and $r_t$ the real interest rate, the representative household selects $(H_t, F_t, A_{t+1})$ $t = 0, \ldots, T$ so as to:

\begin{align*}
\text{(1) } & \quad \max_{H_t, F_t, A_{t+1}} \sum_{t=0}^{T} (1 + \delta)^{-t} u(H_t; F_t) \\
\text{s.t.} & \\
\text{(2) } & \quad H_t + P_t F_t + A_{t+1} - (1 + r_t) A_t + y_t; \\
\text{(3) } & \quad A_t \geq 0.
\end{align*}

Assuming an interior solution, the first-order conditions are given by:

\begin{align*}
\text{(4a) } & \quad u_H = \lambda_t; \\
\text{(4b) } & \quad u_F = \lambda_t P_t; \\
\text{(5) } & \quad \lambda_t = (1 + \delta)^{-1} E_t (\lambda_{t+1} (1 + r_{t+1}));
\end{align*}

where $\lambda_t$ is the Lagrange multiplier on the accumulation constraint (2).

We shall assume that $u$ is an addilog (Houthakker (1960)) utility function:

\begin{align*}
\text{(6) } & \quad u(H_t, F_t) = D_t H_t^{1-\alpha} (1 - \alpha)^{-1} + B_t F_t^{1-\eta} (1 - \eta)^{-1};
\end{align*}

where $B_t$ and $D_t$ are random, trend stationary shocks to preferences. Using (6), (4a) and (4b) are easily solved for the optimal consumption of domestic and
foreign goods as a function of $\lambda_t$ and $P_t$:

\begin{align*}
(7a) \quad H_t &= \lambda_t^{-1/\alpha}D_t^{1/\alpha}; \\
(7b) \quad F_t &= \lambda_t^{-1/\eta}P_t^{-1/\eta}B_t^{1/\eta}.
\end{align*}

Letting lower case letters denote logs, we see that:

\begin{equation}
(8) \quad f_t = b_t/\eta - (l/\eta)p_t - (l/\eta)\log \lambda_t.
\end{equation}

Along the optimal path, the log of the demand for imported consumer goods is linear in the log of the relative price of imports and the log of the marginal utility of wealth, the utility index of permanent income implied by the permanent income hypothesis.

If, given the assumption of addilog preferences, we had data on $\log \lambda_t$, this utility index of permanent income would be the proper "activity" variable to include on the right-hand-side of our import demand equation. Such data is not available. However, using the fact that:

\begin{equation}
(9) \quad H_t^{\alpha/\eta} = \lambda_t^{-1/\eta}D_t^{1/\eta};
\end{equation}

we may express the demand for imported consumer goods as:

\begin{equation}
(10) \quad f_t = \gamma_t - (l/\eta)p_t + (\alpha/\eta)h_t + e_t;
\end{equation}

where $\gamma_t = (b_0 + b_1t - d_0 - d_1t)/\eta$ is the difference between the linearly deterministic components of the log shocks to preferences divided by $\eta$ and

\begin{equation}
(11) \quad e_t = (b_t - b_0 - b_1t)/\eta - (d_t - d_0 - d_1t)/\eta.
\end{equation}

Thus, if the model is true, log consumption of domestically produced goods may be used as a noisy proxy for the unobserved marginal utility of wealth.
A well known property of the standard permanent income model with a constant real interest rate is that the marginal utility of consumption follows a martingale (Hall (1978)). Allowing for stationary shocks to the real interest rate, it follows from (5) that if the variance in forecasting \( \lambda_t(l + r_t) \) is small, \( \log \lambda_t \) is well approximated by the following unit root process:

\[
(12) \quad \log\lambda_t = (\delta - r_t) + \log\lambda_{t-1} + [\lambda_t(l + r_t) - E_{t-1} \lambda_t(l + r_t)]/\lambda_{t-1}(1 + \delta).
\]

Taking logs of both sides of (7a) and using (8) we obtain:

\[
(13) \quad f_t = b_t/\eta - (1/\eta)p_t - (1/\eta)\log\lambda_t;
\]

\[
(14) \quad h_t = d_t/\alpha - (1/\alpha)\log\lambda_t.
\]

Thus, the permanent income hypothesis implies that the log consumption of foreign goods and the log consumption of home goods share a common stochastic trend, and that this trend can be identified with the marginal utility of wealth, \( \log\lambda_t \).

While the theory implies that the log consumption of home goods, \( h_t \) and foreign goods, \( f_t \) share a common stochastic trend, these two variables are not necessarily cointegrated (Granger and Engle (1987)). In fact, as is revealed by equation (10),

\[
(10) \quad f_t = \gamma_t - (1/\eta)p_t + (\alpha/\eta)h_t + \epsilon_t;
\]

if the equilibrium relative price of imports contains a stochastic supply trend that is independent of \( \log \lambda_t \), the model implies that \( f_t \) and \( h_t \) are not cointegrated. Rather, the model implies that \( f_t \), \( h_t \), and \( p_t \) are cointegrated so long as the preference shocks are trend stationary. Furthermore, by the results of Stock and Watson (1988), the existence of two stochastic trends among
three non-stationary variables implies that there exists a unique (at least up to a scale factor) cointegrating vector. In the context of our model, if two stochastic trends are found to be present in the data, these trends can be identified with the log marginal utility of wealth \( \log \lambda_t \) and a shock to the supply schedule for imported goods. The unique co-integrating vector is 
\[
[1, 1/\eta, -\alpha/\eta]'
\]
and is defined by equation (10).

It follows that, in a co-integrating regression of \( f_t \) on \( p_t \) and \( h_t \), the utility parameters \( \eta \) and \( \alpha \) - the elasticities of marginal utility with respect to foreign and home goods - are just identified. In Section 4, after presenting estimates of \( \alpha \) and \( \eta \), we shall use (7) and these estimates to obtain estimates of the Marshallian price elasticity of import demand holding constant real expenditure \( C = H + PF \), \( \epsilon_{f,P;C} \), as well as of the elasticity of import demand with respect to a change in real spending, \( \epsilon_{f,C;P} \) holding constant import prices.

3. The Data

The NIPA accounts provide quarterly, seasonally adjusted nominal and 1982 dollar data on non-durable consumer goods imports, \( M_t \), beginning with 1967:1. The NIPA accounts do not provide data on the spending on or consumption of domestically produced consumer goods, but of course do provide quarterly, seasonally adjusted nominal and 1982 dollar data on non-durables consumption.

Our measurement of \( H_t \) is defined as:

\[
H'_t = (E_t - P_{ft}M_t) / P_{ht}
\]

where \( E_t \) is the NIPA definition of quarter \( t \) consumption of non-durable goods valued in current dollars, \( P_{ft} \) is the NIPA deflator for non-durable consumer goods imports, and \( P_{ht} \) is the producer price index for non-durable consumer
goods. A constant, or even random but stationary mark-up of the unobservable deflator for home goods over the ppi for home goods could be incorporated without changing the thrust of the argument. It follows that:

(16) \( H' = H_t + P_t(F_t - M_t) \);

where \( P_t = P_{Ft}/P_{Ht} \), \( H_t \) is the 1982 dollar value of quarter \( t \) consumption of domestic non-durable goods, \( H'_t \) is the 1982 dollar value of measured quarter \( t \) consumption of domestic goods, and \( F_t \) is the 1982 dollar value of quarter \( t \) consumption of imported non-durable goods.

By using data on imports of foreign consumer goods instead of data on consumption of imported goods, we introduce measurement error. Letting the measurement errors \( z_t \) and \( u_t \) be defined by:

(17) \( m_t = f_t + z_t \);

(18) \( h' = h_t + u_t \);

we substitute for \( f_t \) and \( h_t \) in (10) to obtain the equation to be estimated:

(19) \( m_t = \gamma_t - (1/\eta)p_t + (\alpha/\eta)h'_t + v_t \);

where:

(20) \( v_t = e_t + z_t - (\alpha/\eta)u_t \).

The stationarity of preference shocks \( e_t \) is assumed. In the NBER working paper version of this paper, I examine the conditions under which we would expect the measurement errors \( z_t \) and \( u_t \) to be stationary. If measurement errors are stationary, the model implies that \( m_t \), \( p_t \), and \( h'_t \) are cointegrated and that the parameters of interest, \( \alpha \) and \( \eta \), can be recovered from the co-integrating vector defined by equation (19), \([1, 1/\eta, -\alpha/\eta]'\).
4. Testing for Unit Roots and Common Trends

We begin by reporting the results obtained from a Dickey-Fuller (1979) test of the hypothesis that each of the series \( m_t, p_t, \) and \( h'_t \) possesses a unit root. The alternative hypothesis is that these series are stationary about a deterministic trend. The Dickey-Fuller test is just a t-test that the coefficient \( \beta \) is equal to zero in the following regression:

\[
\Delta x_t = \mu_0 + \mu_1 t + \beta x_{t-1} + \rho_1 \Delta x_{t-1} + \ldots + \rho_p \Delta x_{t-p} + \epsilon_t.
\]

The results of these tests are reported in Table 1 and are easily summarized. We cannot reject at even the 10% level the null hypothesis of a unit root in any of the three variables \( m_t, p_t, \) and \( h'_t \). With no strong evidence against the null hypothesis of a unit root in \( m_t, p_t, \) or \( h'_t \), we turn next to an investigation of the number of stochastic trends that are present among the three variables in our system.

Stock and Watson (1988) demonstrate that any system of \( m \) I(1) variables has a common trends representation, and that in a system comprised of \( m \) I(1) variables being driven by \( n \leq m \) common trends, the number of linearly independent co-integrating vectors must equal \( m - n \). It follows immediately from Stock and Watson's result that if there exists one common trend among \( m \) variables, then all \( m(m-1)/2 \) possible pairs of these variables must be co-integrated. Of course, if there exists \( n = m - 1 \) common trends among \( m \) variables, the co-integrating vector is unique up to scale.

We recall from Table 1 that the hypothesis of a unit root in the relative price of imports cannot be rejected. Consider the hypothesis that the relative price of imports and \( \log \lambda_t \), the utility index of permanent income, do not share a common stochastic trend, as would be the case if the relative price of imports
is driven in part by a stochastic supply shock trend. Following Granger and Engle (1987) we test the null hypothesis that \( p_t \) and \( h'_t \) are not cointegrated by running the regression:

\[
(22) \quad p_t = \mu_0 + \beta h'_t + \epsilon_{pht}.
\]

We then regress changes in the estimated residuals, \( \Delta \epsilon_{pht} \), on one lagged level of the residual and lagged changes:

\[
(23) \quad \Delta \epsilon_{pht} = \delta_0 \epsilon_{pht-1} + \rho_1 \Delta \epsilon_{pht-1} + \ldots + \rho_p \Delta \epsilon_{pht-p} + \epsilon_{pht}.
\]

The test is just a t-test on the coefficient \( \delta_0 \); the appropriate critical values are those reported in Engle and Yoo (1987) since the co-integrating regression has a constant term. We also run the test allowing for the alternative that \( p_t \) and \( h'_t \) are stationary about a deterministic trend, obtaining critical values from Phillips and Ouliaris (1989). As can be seen from the results in Table 2, \( p_t \) and \( h'_t \) do not appear to be cointegrated according to the Granger-Engle test: the t-ratios fall well below the level that would be required to reject the null of no-cointegration at even the 10 percent level.

We recall that if \( p_t \) is driven in part by a stochastic supply trend, we should not expect \( m_t \) and \( h'_t \) to be cointegrated. Table 2 also reports the results of tests that \( m_t \) and \( h'_t \) are not cointegrated, again both excluding as well as allowing for the presence of a time trend. As can be seen from Table 2, \( m_t \) and \( h'_t \) do not appear to be cointegrated according to the Granger-Engle test. For completeness, Table 2 also reports the results of tests that \( m_t \) and \( p_t \) share a common trend. Again, these variables do not appear to be cointegrated.
These findings are consistent with the prediction of the model that two common stochastic trends, one identified with the log marginal utility of wealth \( \log \lambda_t \) and the other identified with supply shocks to the relative price of imports \( p_t \), are driving the non-stationary components of the system's three variables, \( m_t \), \( p_t \), and \( h'_t \). If in fact there are two common trends present among \([m_t, p_t, h'_t]\), these three variables will be cointegrated, and the cointegrating vector will be unique - up to a multiplicative scale factor. It follows that the parameters of interest, \( \alpha \) and \( \eta \), can be recovered from the unique cointegrating vector defined by equation (19), \([1, 1/\eta, -\alpha/\eta]'\). In light of the results reported in Table 2, a rejection of the null of no cointegration among \( m_t \), \( p_t \), and \( h'_t \) is evidence in favor of the model.

Granger and Engle (1987) suggest estimating \([1, 1/\eta, -\alpha/\eta]'\) directly from the first-stage OLS regression:

\[
(24) \quad m_t = \mu_0 + \mu_t t + \beta_1 p_t + \beta_2 h'_t + \epsilon_{mph't}'.
\]

If it is found that, in the Dickey-Fuller regression:

\[
(25) \quad \Delta \epsilon_{mph't} = \delta_1 \epsilon_{mph't-1} + \rho_1 \Delta \epsilon_{mph't-1} + \ldots + \rho_p \Delta \epsilon_{mph't-p} + \zeta_t;
\]

\( \delta_1 \) is significantly negative, the OLS estimates of \([1, 1/\eta, -\alpha/\eta]'\) given by \([1, -\beta_1, \beta_2]'\) are consistent, despite the fact that \( \nu_t \) is correlated with \( p_t \) and \( h'_t \) and is also likely to be serially correlated.

Recent research, as summarized in the survey of Campbell and Perron (1991), has documented that, with the samples sizes available for macroeconomic time series research, the OLS estimate of the co-integrating vector can be severely biased. Furthermore, it is not possible to test hypotheses about the parameters of the co-integrating vector when these are estimated by OLS (Campbell
and Perron (1991), p. 56). Fortunately, both Stock and Watson (1989) and Phillips and Loretan (1990) have discovered tractable methods for obtaining asymptotically FIML estimates of the co-integrating vector. For this reason, we will rely on the co-integrating regression primarily for its estimates of $\epsilon_{mph,t}$ and $\Delta \epsilon_{mph,t}$ that are needed to test the null of no co-integration among $m_t$, $p_t$, $h'_t$.

5. Cointegration, Consumption, and the Demand for Imports: Empirical Results

The results of the Granger-Engle test of the null hypothesis that $m_t$, $p_t$, $h'_t$ are not cointegrated are presented in the top panel Table 3. The critical values are those reported in Phillips and Ouliaris (1989) since both a constant and a linear time trend are included in (24), the cointegrating regression. It is seen that the estimated value of $\delta_1$ is -0.4119 with a standard error of 0.0863 and a t ratio of -4.774. Under the null hypothesis that $\Delta \epsilon_{mph,t}$ is a random walk, the estimated $\delta_1$ is significant at the 1% level using the Phillips-Ouliaris critical values.

In light of the results reported in Table 2, we conclude that the data are consistent with the prediction of the model that two stochastic trends and thus one co-integrating vector describe the data. The OLS estimate of the co-integrating vector is $[1, 0.96, -2.33]$. This implies an OLS estimate of $\eta$, minus the elasticity of marginal utility with respect to the consumption of foreign goods, of $\eta^{ols} = 1.04$ and an OLS estimate of $\alpha$, minus the elasticity of marginal utility with respect to the consumption of home goods, of $\alpha^{ols} = 2.37$.

As discussed above, if $v_t$ is correlated with the regressors $p_t$ and $h'_t$, OLS estimates of the co-integrating vector can be biased in small samples. We would expect the structural preference shock, $b_t$ to be positively correlated with $p_t$. 

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That is, a transitory rise in consumption of foreign goods brought about by a jump in $b_t$ would be positively correlated with $p_t$ and thus negatively correlated with $-p_t$. We would also expect the structural preference shock, $d_t$ to be positively correlated with $h_t$. It follows that $e_t = (b_t - d_t)/\eta - \gamma_t/\eta$ is likely to be negatively correlated with the regressors in equation (24).

Phillips and Loretan (1990) propose a parametric procedure for estimating the co-integrating vector in an equation in which the variables are in fact known to be co-integrated. The Phillips and Loretan approach tackles the simultaneity problem by including lagged and lead values of the change in the regressors. The approach deals with the autocorrelation in the residuals by including lagged values of the stationary deviation from the co-integrating relationship. Phillips and Loretan prove that the estimates of the co-integrating vector obtained from this approach are asymptotically FIML. They also show that the likelihood ratio test can be used to test hypotheses about the parameters of the co-integrating vector.

Let $y_t$ denote the vector $[1, t, p_t, h'_t]'$ and let $\beta$ denote the vector $[\mu_0, \mu_1, \beta_1, \beta_2]'$. The Phillips-Loretan equation is given by:

\begin{equation}
(26) \quad m_t = \beta'y_t + \rho(m_{t-1} - \beta'y_{t-1}) + \sum_{j=\tau_1}^{\tau_2} \varphi_j \Delta p_{t-j} + \sum_{j=\tau_1}^{\tau_2} \psi_j \Delta h'_{t-j} + \epsilon_{mt}.
\end{equation}

The $\beta$ vector and $\rho$ are estimated by non-linear least squares. The implied estimates of $\beta$ along with standard errors are reported in Table 4.
As shown in Table 4, the NLS estimate is quite similar to the OLS estimate of the co-integrating vector. The NLS estimate of the co-integrating vector is [1, 0.94, -2.21]. This implies a NLS estimate of \( \eta \), minus the elasticity of marginal utility with respect to the consumption of foreign goods, of \( \eta_{nls} = 1.05 \) and a NLS estimate of \( \alpha \), minus the elasticity of marginal utility with respect to the consumption of home goods, of \( \alpha_{nls} = 2.27 \).

We now use these NLS estimates of \( \eta \) and \( \alpha \) to construct estimates of the familiar Marshallian price elasticity and the expenditure elasticity of the demand for imports. If total real expenditure \( C = H + PF \) is to remain constant in the face of an increase in the relative price of foreign goods, (7) can be used to show that:

\[
(\eta - 1)(1 - s)d\log P/\eta = [s/\alpha + (1 - s)/\eta]d\log \lambda;
\]

where \( s \) is the share of spending that falls on domestic goods. Substituting for \( \log \lambda \) in (13), we obtain the expression for the Marshallian price elasticity:

\[
(28) \quad \epsilon_{f,p;C} = -(1/\eta)[1 - (1 - \eta)(1 - s)/((\eta s/\alpha) + (1 - s))].
\]

Since our estimate of \( \eta \), \( \eta_{nls} = 1.06 \) exceeds 1, the estimated Marshallian elasticity must, in absolute value, exceed \( 1/\eta_{nls} = 0.94 \). In our sample \( (1 - s) \), the share of total non-durables spending that falls on imports, rises from 0.01 in 1967 to 0.04 in 1990. Using our estimate of \( \alpha_{nls} = 2.27 \), we determine that, in our sample, the Marshallian price elasticity of the demand for imports falls in the following range:

\[
(29) \quad 0.94 \leq \epsilon_{f,p;C} \leq 0.95.
\]
We now derive an expression for the elasticity of import demand with respect to an increase in real expenditure, holding constant the relative price of imports. From (13) and (14), we see that the source of such a permanent rise in real spending must be a permanent decline in the marginal utility of wealth. Using (7) it is straightforward to show that:

\[ \frac{d \log C}{d \log \lambda} = -(s/a + (1 - s)/\eta) \frac{d \log \lambda}{d \log C}. \]

Substituting for \( \log \lambda \) and differentiating with respect to \( \log C \), we obtain:

\[ \epsilon_{f,C;p} = \frac{a}{\eta} \left[ \frac{1}{s + \left( \frac{a}{\eta} \right)(1 - s)} \right]. \]

Thus, since \( a^{\text{nls}} \) exceeds \( \eta^{\text{nls}} \), the elasticity of import demand with respect to a rise in real expenditure is bounded above by 2.21, the NLS estimate of \( \beta_z \). Using the fact that \( (1 - s) \) rises from 0.01 to 0.04 in our sample, we obtain:

\[ 2.11 \leq \epsilon_{f,C;p} \leq 2.18. \]

These elasticity estimates are firmly in the range of those reported in the many studies surveyed by Goldstein and Kahn (1985), and those reported by Helkie and Hooper (1986) and Cline (1989). However, it should be pointed out that the Marshallian price elasticity and the expenditure elasticity are not constant if, as is the case in our sample, the share of spending that falls on imports is changing over time. It is easily verified that, as the share of spending on imports, \( (1 - s) \), rises over time, the permanent expenditure elasticity must decline over time from 2.21 to 1.00, while the Marshallian price elasticity must rise - in absolute value - over time from -0.94 to -1.00.4 Marquez (1991) has recently emphasized the importance of allowing for time varying elasticities in empirical trade models.
One message of this paper is that, at least for non-durable consumer goods, it is possible to interpret the traditional import demand equation as a co-integrating regression. The striking similarity between the OLS and Phillips-Loretan estimates suggests that the simultaneous equation bias is not large.

A second message of this paper is that the permanent income theory, along with the empirically testable restriction that the log relative price of imports and the log marginal utility of wealth are not cointegrated, predicts that the co-integrating vector for $[f_t, p_t, h_t]$ is unique, and that estimates of this vector can be used to identify the parameters of the household utility function. An expenditure elasticity in excess of unity is consistent with the theory when the concavity of the sub-utility function for home goods exceeds the concavity of the sub-utility function for foreign goods. Our estimate is that the elasticity of the marginal utility of home goods consumption, $\alpha$, is a bit more than twice the elasticity of the marginal utility of foreign goods consumption.

6. Concluding Remarks

This paper has employed a rational expectations permanent income model to derive a structural econometric specification of the demand for imported consumer goods. With strongly separable, addilog preferences, the log of the demand for foreign goods is shown to be linear in the log of the relative price of imports, the log of the demand for domestic goods, and the log of an unobservable shock to tastes. The rational expectations permanent income hypothesis in conjunction with the addilog preference structure implies that the log of the demand for domestic goods is the correct "activity" variable on the right-hand-side of the import demand equation. This is because consumption of domestic goods is a noisy proxy for the unobservable utility index of permanent income, the marginal utility of wealth.$^5$
The model implies that log consumer goods imports, the log price of imports, and log consumption of domestically produced varieties are cointegrated, and that the cointegrating vector is unique. Using the approach of Granger and Engle (1987) we were able to decisively reject the null hypothesis that imports, the relative price of imports, and the consumption of home goods are not cointegrated.

The estimation technique proposed by Phillips and Loretan (1990) was employed to estimate the parameters of the structural import demand equation. The long-run price elasticity of import demand was estimated to average -0.95. The elasticity of import demand with respect to a permanent increase in real spending was estimated to average 2.15, roughly the same as reported by Helkie and Hooper (1986), somewhat smaller than reported by Cline (1989), and somewhat larger than the average of the many studies surveyed recently by Goldstein and Kahn (1985). In the context of the optimization problem of the representative household, the Marshallian price elasticity of import demand is not constant but in fact converges to -1 as the share of total spending that falls on imports rises, while the elasticity of import demand with respect to a permanent increase in real spending converges to 1 as the share of spending that falls on imports rises. An advantage of our utility-based, co-integration approach is that, by recovering consistent estimates of the utility parameters via Phillips-Loretan non-linear least squares, we are able to estimate the permanent income elasticity of import demand without having to specify a proxy for permanent income or having to estimate a time series model for actual income.
1. The addilog utility function has been estimated in a number of previous studies of consumer demand and intertemporal substitution, including Deaton (1974), Miron (1986), Ball (1990), and Ceglowski (1991). I will discuss below the recent contributions of Ogaki (1988; 1990) and Ogaki and Park (1989).

2. The assumption that $d_t$ and $b_t$ are trend stationary is actually stronger than required to proceed with the cointegration approach. All that is needed is that $b_t - d_t$ be trend stationary. We allow for a deterministic trend in the cointegrating relationship for two reasons. First, for comparability with the vast empirical literature devoted to estimating ad hoc import demand equations. Second, to capture the influence of what are certain to be omitted variables such as improvements in product quality and the accumulation of knowledge about the characteristics of imported varieties of consumer goods. A non-linear trend would probably be preferable on theoretical grounds, but the bulk of the available research on cointegration has focused on cointegrating relationships about a deterministic linear trend.

3. In prior, but independent work Ogaki (1988;1990) and Ogaki and Park (1989) also exploit the fact that if the equilibrium consumption paths of different goods are each $I(1)$, the assumption of addilog preferences - and stationary preference shocks - implies a cointegration restriction across the consumption of different goods and the relative prices of these goods. These authors show that cointegration methods can be used to estimate the parameters of the addilog utility function, and apply their approach to estimating the "long-run intertemporal elasticity of substitution" and the "Engle's Law" relationship in US data. Ogaki and Park (1989) also explore the conditions under which the addilog utility function - which is not homothetic - can be aggregated across heterogenous consumers.
4. Given the point estimates I/η = 0.94 and α/η = 2.21, we obtain the following relationship between s, the share of spending falling on domestic goods, and the relevant elasticities:

<table>
<thead>
<tr>
<th>Elasticities</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1/η</td>
<td>0.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α/η</td>
<td>2.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>0.99</td>
<td>0.96</td>
<td>0.9</td>
<td>0.8</td>
<td>0.67</td>
<td>0</td>
</tr>
<tr>
<td>ε(IF,p;C)</td>
<td>0.94</td>
<td>0.95</td>
<td>0.95</td>
<td>0.96</td>
<td>0.97</td>
<td>1.00</td>
</tr>
<tr>
<td>ε(IF,C;p)</td>
<td>2.18</td>
<td>2.11</td>
<td>1.97</td>
<td>1.78</td>
<td>1.58</td>
<td>1.00</td>
</tr>
</tbody>
</table>

5. Ogaki (1990) and Ogaki and Park (1989) point out that it is possible to recover estimates of the addilog utility parameters from the cointegrating vector if each period utility is given by:

\[(6') \quad v(u(H_t, F_t)) = v(D_t H_t^{1-\alpha}(1 - \alpha)^{-1} + B_t F_t^{1-\eta}(1 - \eta)^{-1});\]

where \(v\) is a concave transformation of the addilog utility function. In this case, \(h_t\) is no longer the simple noisy proxy for \(\log\lambda_t\) that it is in the absence of said concave transformation. For example, if \(v(u) = (1 - \sigma)^{-1}u^{1-\sigma}\), we have

\[(14') \quad h_t = d_t/\alpha - (1/\alpha)(\log\lambda_t + \sigma\log\eta_t);\]

and

\[(13') \quad f_t = b_t/\eta - (1/\eta)p_t - 1/\eta(\log\lambda_t + \sigma\log\eta_t);\]

\(h_t\) remains a noisy proxy for the correct "activity" variable on the right-hand-side of the import demand equation (13'). \((\log\lambda_t + \sigma\log\eta_t)\). Substituting (14') into (13') we see that the cointegrating equation is unchanged:

\[(10') \quad f_t = \gamma_t - (1/\eta)p_t + (\alpha/\eta)h_t + \epsilon_t.\]
This means that if utility is given by (6'), the cointegration approach discussed in Ogaki (199) and Ogaki and Park (1989) and derived independently here can be used to estimate the addilog parameters \( \eta \) and \( \alpha \); it cannot be used to recover the parameter \( \sigma \). While \( h_t \) remains the correct "activity variable", it can only be identified the marginal utility of wealth under the assumption \( \sigma = 0 \).
REFERENCES


TABLE 1

Testing for Unit Roots

The Dickey-Fuller Regression: \( \Delta x_t = \mu_0 + \mu_1 t + \delta x_{t-1} + \rho \Delta x_{t-1} + \epsilon_{xt} \).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated ( \beta )</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_t )</td>
<td>-0.0958</td>
<td>-2.080</td>
</tr>
<tr>
<td>( p_t )</td>
<td>-0.0555</td>
<td>-1.461</td>
</tr>
<tr>
<td>( h_t )</td>
<td>-0.0417</td>
<td>-1.860</td>
</tr>
</tbody>
</table>

The Fuller (1976) critical values from Table 8.5.2 are:
- -3.12 at the 10 percent level;
- -3.41 at the 5 percent level;
- -3.96 at the 1 percent level.

The sample is 1968:2 through 1990:2. Variables are as defined in the text. All three equations were re-estimated with four, three, and two lags of \( \Delta x_t \), and the lag length for calculating the t-test was chosen as recommended by Campbell and Perron (1991). Using this approach, the null hypothesis of a unit root in \( h_t, m_t, \) or \( p_t \) was never rejected at even the 10 percent level.
TABLE 2

Testing for A Common Trend

The Co-integrating Regression: \( x_t = \mu_0 + \mu_1 t + \beta y_t + \epsilon_{xyt} \).

The Dickey-Fuller Regression: \( \Delta \epsilon_{xyt} = \delta_1 \epsilon_{xyt-1} + \rho \Delta \epsilon_{xyt-1} + \epsilon_{xyt} \).

(Augmented)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimated ( \delta_1 )</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>([m_t, h_t])</td>
<td>-0.1508</td>
<td>-2.2500</td>
</tr>
<tr>
<td>([m_t, p_t])</td>
<td>-0.1240</td>
<td>-2.5230</td>
</tr>
<tr>
<td>([p_t, h_t])</td>
<td>-0.0543</td>
<td>-1.4414</td>
</tr>
</tbody>
</table>

The Phillips-Ouliaris (1989) asymptotic critical values from Table IIc are:

- -3.51 at the 10 percent level;
- -3.80 at the 5 percent level;
- -4.36 at the 1 percent level.

The sample is 1968:2 through 1990:2. The data are defined in the text.

The Co-integrating Regression: \( x_t = \mu_0 + \beta y_t + \epsilon_{xyt} \).

The Dickey-Fuller Regression: \( \Delta \epsilon_{xyt} = \delta_0 \epsilon_{xyt-1} + \rho \Delta \epsilon_{xyt-1} + \epsilon_{xyt} \).

<table>
<thead>
<tr>
<th>Variables</th>
<th>Estimated ( \delta_0 )</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>([m_t, h_t])</td>
<td>-0.0382</td>
<td>-1.2857</td>
</tr>
<tr>
<td>([m_t, p_t])</td>
<td>-0.0492</td>
<td>-1.5287</td>
</tr>
<tr>
<td>([p_t, h_t])</td>
<td>-0.0488</td>
<td>-1.6545</td>
</tr>
</tbody>
</table>

The Engle-Yoo (1987) critical values from Table 2 for a sample of 100 are:

- -3.03 at the 10 percent level;
- -3.37 at the 5 percent level;
- -4.07 at the 1 percent level.

The sample is 1968:2 through 1990:2. All six equations were re-estimated with four, three, and two lags of \( \Delta \epsilon_{xyt} \), and the lag length for calculating the t-test was chosen as recommended by Campbell and Perron (1991). Using this approach, the null hypothesis of no co-integration among any pair of \([m_t, h_t, p_t]\) was never rejected at even the 10 percent level.
TABLE 3

Testing for Cointegration

The Co-integrating Regression:  
\[ m_t = \mu_0 + \mu_1 t + \beta_1 p_t + \beta_2 h'_t + \epsilon_{mph,t}. \]

The Dickey-Fuller Regression:  
\[ \Delta \epsilon_{mph,t} = \delta_1 \epsilon_{mph,t-1} + \xi_t \]

<table>
<thead>
<tr>
<th>Estimated ( \delta_1 )</th>
<th>t-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.4119</td>
<td>-4.7740*</td>
</tr>
</tbody>
</table>

The Phillips-Ouliaris (1989) critical values from Table IIc are:
- -3.84 at the 10 percent level;
- -4.16 at the 5 percent level;
- -4.65 at the 1 percent level*.

The augmented Dickey-Fuller regression:
\[ \Delta \epsilon_{mph,t} = \delta_1 \epsilon_{mph,t-1} + \rho_1 \Delta \epsilon_{mph,t-1} + \ldots + \rho_4 \Delta \epsilon_{mph,t-4} + \xi_t \]

was also estimated and the lag length used to calculate the t-statistic for \( \delta_1 \) was chosen as recommended by Campbell and Perron (1991). As none of the \( \rho_j \) was significant, the t-test for the significance of \( \delta_1 \) is based on the simple Dickey-Fuller regression.

The OLS estimates of the parameters are:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0 )</td>
<td>-6.4105</td>
</tr>
<tr>
<td></td>
<td>(0.1661)</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>0.0170</td>
</tr>
<tr>
<td></td>
<td>(0.0004)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.9577</td>
</tr>
<tr>
<td></td>
<td>(0.0684)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>2.3258</td>
</tr>
<tr>
<td></td>
<td>(0.1386)</td>
</tr>
</tbody>
</table>

The \( R^2 \) is 0.979892. The Durbin-Watson statistic is 0.8107. The sample is 1967:2 through 1990:2. Variables defined in text.
TABLE 4

Estimation of the Parameters
Phillips and Loretan(1990) Non-Linear Least Squares

Phillips-Loretan equation with \( \beta = [\mu_0, \mu_1, \beta_1, \beta_2]' \):

\[
m_t = \beta'y_t + \rho(m_{t-1} - \beta'y_{t-1}) + \sum_{j=-1}^{j=1} \varphi_j \Delta p_{t-j} + \sum_{j=-1}^{j=1} \psi_j \Delta h'_{t-j} + \epsilon_{mt}.
\]

The non-linear least squares estimates of \( \theta \) are:

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0 )</td>
<td>-6.2096</td>
</tr>
<tr>
<td></td>
<td>(0.3289)</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>0.0164</td>
</tr>
<tr>
<td></td>
<td>(0.0008)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.9404</td>
</tr>
<tr>
<td></td>
<td>(0.1366)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>2.2062</td>
</tr>
<tr>
<td></td>
<td>(0.2721)</td>
</tr>
</tbody>
</table>

The implied elasticities are:

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Estimated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon_{i,p;C} )</td>
<td>-0.95</td>
</tr>
<tr>
<td>( \varepsilon_{i,C;p} )</td>
<td>2.15</td>
</tr>
</tbody>
</table>

The elasticities are derived in the text, equations (36) and (39). The Phillips-Loretan equation was estimated with up to \( r = 3 \) leads and lags and with up to 2 lags of the equilibrium error with no significant difference in the results.