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Optimal Menu of Menus with Self-Control Preferences*

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Abstract

This paper studies how a seller should design its price schedule when consumers' preferences are subject to temptation. As in Gul and Pesendorfer (2001), consumers exercise costly self-control to some degree and foresee their impulsive behavior and self-control. Since consumers may pay a premium for an option set that is less tempting, the seller may offer multiple small menus. Building on the standard model of adverse selection and second-degree price discrimination, we characterize the optimal menu of menus for the seller. In particular, we show that if consumers are tempted by goods of higher quality, the seller can achieve perfect discrimination: consumers' choices appear as if the seller can observe consumers' preferences directly. To achieve this, the seller "decorates" menus by adding items that are never chosen but are tempting to consumers.

JEL Classification: D11, D42, D82, L12, L15, M31
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1 Introduction

Impulse purchasing is widespread among consumers. A marketing study finds that between 27% and 62% of department stores sales fall into impulse purchasing (Bellenger, Robertson, and Hirschman, 1978). If consumers' behavior exhibits impulse purchasing and temptation, sellers may try to take advantage of this behavior, perhaps by offering a large selection of tempting products. There is a complication, however, since consumers usually recognize their impulsive behavior and may be able to control it to some degree, or may try to stay away from undesirable and tempting opportunities. Behavior of this kind with temptation and self-control can be analyzed with a class of utility functions introduced by Gul and Pesendorfer (2001). Using their utility-function formulation, the present paper extends the standard model of adverse selection and studies a seller's optimal strategy against consumers who exhibit temptation and self-control.

Standard economics assumes that a consumer evaluates a choice set based only on the most preferred element in the set, since a consumer cares only about what he will choose from the set. This implies that if a set X contains another set Y, then a consumer likes X at least as well as Y: there is no disutility of having more options. Consumers who are subject to temptation, however, may dislike a larger choice set since it may contain options that are tempting and undesirable. For example, suppose that there are two options s (salad) and b (burger) and that the consumer prefers s to b but b is tempting to him (whatever this means). The consumer may then prefer not to have b in the choice set. The reason is that, confronted with the choice set $\{s, b\}$, the consumer may succumb to temptation and choose a burger, and even if he resists temptation and chooses a salad, he may have to incur psychological costs in the process. Anticipating these possibilities, the consumer may prefer the singleton choice set $\{s\}$. The set $\{s\}$ is desirable for the consumer since it gives no room for temptation and allows him to commit to the ex ante desirable choice.

To formulate choice behavior of this kind, Gul and Pesendorfer (2001) introduced the following class of preferences: the consumer prefers a choice set X to another set Y if and only if $W(X) > W(Y)$ where W is defined by

$$W(X) \equiv \max_{x \in X} \Big[U(x) + V(x) \Big] - \max_{x \in X} V(x).$$

The interpretation is that U is the utility function of the *untempted* part of the consumer while V is the utility function of the *tempted* part. The first maximization identifies what the consumer actually chooses as a compromise between the preferences of these selves. The second maximization identifies the *most tempting* alternative. To better understand this formulation, we can rearrange it as

$$W(X) = U(\hat{x}) - \Big[\max_{x \in X} V(x) - V(\hat{x}) \Big] \tag{1}$$

where $\hat{x}$ denotes what the consumer chooses, i.e., a maximizer of $U(x) + V(x)$. The term $\max_{x \in X} V(x) - V(\hat{x})$ denotes the forgone utility of the tempted self: the tempted self wants to choose a maximizer of $V(x)$ but ends up with $\hat{x}$ after self-control. The forgone utility can be interpreted as the cost of self-control. With this interpretation, (1) says that the overall utility $W(X)$ is equal to the untempted self's utility from the chosen item minus the cost of self-control.

This paper considers consumers who have preferences of the type described above and studies a seller's optimal supply decision. We use the classic model of nonlinear pricing and second-degree price discrimination by Mussa and Rosen (1978) and Maskin and Riley (1984) and consider a monopolist selling goods that are indexed by a single-dimensional quality level $q \in \mathbb{R}$. The seller does not observe consumers' preferences directly and therefore can set prices only via indirect price discrimination schemes that rely on consumers' self-selection.

In the standard nonlinear pricing problem, the seller chooses a set of goods $Q \subseteq \mathbb{R}$ to sell and a price function $p\colon Q \to \mathbb{R}$ that specifies the price $p(q)$ for each quality level. The choice of Q and p determines the choice set for consumers, which is a menu of quality-price pairs given by $M = \{(q, p(q)) : q \in Q\}$. Given the menu, each consumer chooses the most preferred pair $(q, p(q))$ in the menu. Anticipating consumers' choices, the seller chooses a menu that maximizes the expected profits.

The present paper considers the same profit-maximization problem when consumers' preferences exhibit temptation and self-control as described above, extending the problem by allowing the seller to offer multiple menus. With standard consumer preferences, the number of menus that the seller offers is immaterial. However, if consumers may prefer smaller, less tempting menus, the seller may profit from offering multiple menus, i.e., a menu of menus. For example, the seller may open multiple retail stores (possibly with different brand names) with smaller and specialized selections and let consumers choose which store to visit. By offering multiple stores, the seller can make the selection in each store less tempting and hence more appealing to consumers with self-control costs.

Another example is weight-loss programs (e.g., Weight Watchers), which specify the aimed level of weight loss, the number of weekly visits, food discounts, as well as the penalty fee for weight gain. In this example, q denotes the realized level of weight loss and $p(q)$ specifies the total fee for each realized weight loss. By offering multiple plans or fee schedules, the firm may be able to better discriminate participants.

A feature of the extended problem is that consumers may not choose a menu if it contains tempting options and the ex-ante overall utility from the menu is low, or if another menu offered by the seller yields a higher ex-ante utility. Therefore, the seller's problem has to deal with the new conditions of individual rationality and incentive compatibility that pertain to the choice of a menu, as well as the usual conditions of individual rationality and incentive compatibility that pertain to the choice of a quality-price pair within the chosen menu.

We study a simple case in which there are only two types of consumers: high and low. As usual, we assume that the high-type consumers have a higher marginal valuation for additional quality than the low-type consumers. However, this statement is ambiguous in our problem since a consumer has multiple states of mind and his marginal willingness to pay depends on which state he is in. Specifically, each consumer has three possible states of mind: (i) *untempted state*, which is captured by the utility function U and in which the consumer is not tempted at all; (ii) *tempted state*, which is captured by the utility function V and in which the consumer is tempted and succumbing to temptation; and (iii) *self-controlled state*, which is captured by $U + V$ and in which the consumer is tempted but exercising self-control. We assume that a high-type consumer in a given state of mind has a higher marginal valuation for additional quality than a low-type consumer in the same state of mind. That is, a change of the state does not reverse the relative positions of types.

There is a number of possibilities for the heterogeneity of consumers since each consumer is characterized by a pair of utility functions. One important way to classify these possibilities is to look at the relation between the two utility functions for a given consumer, which pertains to the direction in which the consumer is tempted. We say that a consumer is tempted *upwards* if his marginal valuation for additional quality is higher when he is tempted than when he is not, which means that he is tempted toward goods of higher quality. Conversely, a consumer is tempted *downwards* if his marginal valuation for additional quality is lower under temptation. While upward temptation may be easier to imagine, downward temptation is not unusual. For example, in the case of weight-loss programs, downward temptation means that consumers are tempted to lose less weight. Even in the case of shopping, consumers may become more frugal when they make decisions and pay.[1]

Our first result says that if the high-type consumers are tempted upwards, then regardless of the direction of the low-type consumers' temptation, the seller can obtain the same level of profits as in the case where he can observe each consumer's preferences directly. That is, consumers' choices under the seller's optimal scheme appear as if perfect discrimination is feasible. To achieve perfect discrimination, the seller offers two menus and "decorates" the one intended for the low type, adding a quality-price pair that is irrelevant for the low type but is ex post tempting and ex ante undesirable for the high type. By adding such an item, the seller can lower the high type's ex-ante utility from the menu intended for the low type. We can show that there is a way to decorate the menu so that the high type has no incentive to choose it. Then, although the added item is not chosen by any consumer, it completely eliminates the high type's

[1]Using survey techniques, Ameriks, Caplin, Leahy, and Tyler (2003) find evidence of heterogeneity in the direction of temptation in a two-period saving problem among TIAA-CREF participants: 20% of participants are tempted to consume less in the first period.

incentive to mimic the low type and therefore enables the seller to extract the full surplus from the high-type as well as the low-type consumers.

On the other hand, the perfect-discrimination result does not hold if the high-type consumers are tempted downwards. If the high type is tempted downwards, tempting the high type requires adding an item of relatively lower quality, but adding such an item also changes the actual choice of the low type, and therefore extracting full surplus from both types is not possible.

When the high-type consumers are tempted downwards, the seller's optimal strategy depends on the degree of the high type's temptation. We say that the high type's temptation is *strong* if the high type in the tempted state has a lower marginal valuation for additional quality than the low type in the self-controlled state. That is, when the high type is fully tempted and the low type is self-controlled, the relative positions of the types are reversed. If this reversal does not occur, i.e., if the high type's temptation is *weak*, then we show that offering a single menu is never optimal for the seller. Under weak temptation, the item offered to the low type is tempting to the high type. Therefore, by offering a separate menu for each type, the seller can reduce the high type's self-control cost and weaken its incentive-compatibility condition. On the other hand, if the reversal occurs, i.e., if the high type's temptation is strong, then the seller gains nothing from offering multiple menus. We show that the optimal scheme in this case coincides with the solution to the standard nonlinear pricing problem where consumers are assumed to be always in the self-controlled state, i.e., their preferences are always $U + V$.

In our basic model, the seller is not allowed to charge *entry fees*, fees that are charged even to consumers who end up buying nothing or using no service (e.g., annual membership fees, fixed monthly fees, etc). In Section 4, we extend the model to allow for entry fees. The perfect-discrimination result continues to hold without any change. On the other hand, the optimal scheme when the high-type consumers have downward temptation is affected considerably by the availability of entry fees. In particular, when the high type's temptation is strong, while offering one menu is optimal without entry fees, it is never optimal if entry fees can be charged. This also implies that the seller strictly prefers to charge entry fees. The seller may also profit from decorating the menus with items that are never chosen by consumers.

There is a growing number of papers that study optimal strategies against agents who have non-standard preferences. O'Donoghue and Rabin (1999), Gilpatric (2001), and DellaVigna and Malmendier (2004) study optimal contracts when agents have (quasi-) hyperbolic discounting. Eliaz and Spiegler (2004) derive the optimal contract when the principal knows that consumers' preferences change in the second period but consumers themselves believe that the change may not occur. Esteban, Miyagawa, and Shum (2003) consider the same problem as the present paper but examine the case where the seller offers a *single* menu and there is an infinite number of types. Esteban and Miyagawa (2004) extend the model to oligopoly and characterize Bertrand–Nash equilibria when firms compete by offering a menu of menus.[2]

There is also an empirical literature that tests for preference reversals with pricing data. Wertenbroch (1998) finds evidence that consumers tend to forgo quantity discounts for goods that have delayed negative effects (e.g., cigarettes). Della Vigna and Malmendier (2002) find evidence of time inconsistent behavior in consumers' enrollment decisions in health clubs. Miravete (2003) looks for evidence of irrational behavior in consumers' choices of calling plans and finds that their behavior is actually consistent with rationality and learning. Oster and Morton (2004) find evidence that magazines that have payoff in the future (e.g., intellectual magazines) are sold at a higher price.

2 Model

We consider a monopolist that sells a collection of goods (or services). The goods are indexed by $q \in \mathbb{R}_+$, which represents the quality (or quantity) of the good. The good with quality $q = 0$ is the equivalent of nothing. Each consumer is interested in consuming at most one unit of one good. An *offer* from the monopolist is defined as a pair $(q, t) \in \mathbb{R}_+^2$, which means that the monopolist offers one unit of good q for a price of t.[3] A set of offers $M \subseteq \mathbb{R}_+^2$, such that $(0, 0) \in M$, is referred to as a *menu*. The restriction $(0, 0) \in M$ comes from the assumption that those consumers who do not buy any good can avoid payments; the assumption will be relaxed in Section 4. To simplify exposition, we often specify a menu M without noting that it includes $(0, 0)$: by writing $M = \{(q, t), (q', t'), \ldots\}$, we mean $M = \{(0, 0), (q, t), (q', t'), \ldots\}$.

2.1 Consumers' Preferences

Consumers have preferences over menus and their preferences are indexed by a number $\gamma \in \mathbb{R}_+$. Let $n(\gamma) \in [0, 1]$ denote the proportion of consumers whose preferences are of type γ, so that $\sum_\gamma n(\gamma) = 1$. Let Γ denote the support of $n(\cdot)$ and assume that Γ is finite.

Using the utility representation by Gul and Pesendorfer (2001), we assume that the utility function of a type γ consumer is given by

$$W_\gamma(M) \equiv \sup_{(q,t)\in M} \big[U_\gamma(q, t) + V_\gamma(q, t)\big] - \sup_{(q,t)\in M} V_\gamma(q, t), \tag{2}$$

where U_γ and V_γ are functions from $\mathbb{R}_+^2$ to $\mathbb{R}$. Although sup is used in (2) to accommodate all menus, we will focus on menus such that at least the maximization problem

[2] Gul–Pesendorfer preferences have been applied to a variety of models: e.g., Krusell, Kuruşcu, and Smith (2000) to a neoclassical growth model; Krusell, Kuruşcu, and Smith (2002) and DeJong and Ripoll (2003) to an asset-pricing problem; and Miao (2004) to an optimal stopping problem.

[3] We write $(q', t') \gg (q, t)$ if $q' > q$ and $t' > t$.

associated with $U_\gamma + V_\gamma$ has a maximum.

Functions U_γ and V_γ are the utility functions of the two different selves of the consumer γ. Function U_γ represents the preferences of the untempted (or committed) self, while V_γ represents the preferences of the tempted self. What the consumer actually chooses is an offer $(\hat{q}, \hat{t}) \in M$ that maximizes $U_\gamma + V_\gamma$ (if a maximum exists), which is considered as a compromise between the preferred alternatives of the two selves. On the other hand, the second maximization problem identifies the most tempting alternatives in the menu.

To see why we have the second maximization problem, we can rearrange (2) to obtain

$$W_\gamma(M) = U_\gamma(\hat{q}, \hat{t}) - \Big[\sup_{(q,t) \in M} V_\gamma(q,t) - V_\gamma(\hat{q}, \hat{t}) \Big],$$

where $(\hat{q}, \hat{t})$ denotes a maximizer of $U_\gamma + V_\gamma$. Then, the second term on the right-hand side measures the utility that the tempted self loses from self-control: the tempted self would like to maximize V_γ but ends up with $(\hat{q}, \hat{t})$ after self-control. The forgone utility can be thought of quantifying the disutility from self-control and will be called the *self-control cost*. With this definition, the overall utility from a menu M for type γ is given by $U_\gamma(\hat{q}, \hat{t})$ minus the self-control cost.

While the first maximization in (2) attaches equal weights to the two utility functions, this is without loss of generality since different weights can be accommodated by changing the scales of the utility functions.

We assume that U_γ and V_γ are continuous, strictly increasing in q, strictly decreasing in t, quasi-concave, and satisfy $U_\gamma(0,0) = V_\gamma(0,0) = 0$.

We now introduce binary relations ($\succsim$, $\succ$, $\sim$) defined over utility functions. Given two utility functions U and $\hat{U}$, we write $U \succsim \hat{U}$ if at any point $(q,t) \in \mathbb{R}^2_+$, the indifference curve of U is at least as steep as that of $\hat{U}$ when we measure the first (resp. second) argument on the horizontal (resp. vertical) axis. Formally, $U \succsim \hat{U}$ if and only if for all $(q,t), (q',t') \in \mathbb{R}^2_+$ such that $q' > q$, we have

$$\begin{aligned} &\hat{U}(q',t') \geq \hat{U}(q,t) \text{ implies } U(q',t') \geq U(q,t), \text{ and} \\ &\hat{U}(q',t') > \hat{U}(q,t) \text{ implies } U(q',t') > U(q,t). \end{aligned}$$

If $U \succsim \hat{U}$ and $U \succsim \hat{U}$, then the two functions are ordinally equivalent in the sense that they induce the same indifference map. This is denoted as $U \sim \hat{U}$.

We also write $U \succ \hat{U}$ if the indifference curve of U is strictly steeper than that of $\hat{U}$ at any point $(q,t) \in \mathbb{R}^2_+$. Formally, $U \succ \hat{U}$ if and only if for all $(q,t), (q',t') \in \mathbb{R}^2_+$ such that $q' > q$, we have

$$\hat{U}(q',t') \geq \hat{U}(q,t) \text{ implies } U(q',t') > U(q,t).$$

We assume the following on U_γ and V_γ.

A1. For all $\gamma, \gamma' \in \Gamma$, if $\gamma' > \gamma$, then $U_{\gamma'} \succ U_\gamma$, $V_{\gamma'} \succ V_\gamma$, and $(U_{\gamma'}+V_{\gamma'}) \succ (U_\gamma+V_\gamma)$.

A2. For all $\gamma \in \Gamma$, either $V_\gamma \succ U_\gamma$ or $V_\gamma \prec U_\gamma$.

A3. For any pair of utility functions $f, g \in \{U_\gamma, V_\gamma, U_\gamma + V_\gamma : \gamma \in \Gamma\}$ such that $f \prec g$ and any pair of offers $x, y \in \mathbb{R}^2_+$ such that $f(x) > f(y)$, there exists an offer $z \in \mathbb{R}^2_+$ such that

$$f(z) = f(y),$$
$$g(z) = g(x).$$

A1 is a single-crossing property saying that the indifference curves of U_γ, V_γ, and $U_\gamma+V_\gamma$ are steeper (in the strict sense) for higher types. Given a menu, this assumption implies that the most preferred quality level is (weakly) larger for higher types.

A2 says that each consumer is tempted in one direction or the other. For γ such that $V_\gamma \succ U_\gamma$ (which implies $V_\gamma \succ U_\gamma + V_\gamma \succ U_\gamma$), the consumer's marginal willingness to pay for additional quality is higher when he is tempted than when he is not. This means that the consumer is tempted towards goods of higher q, and hence we say that the consumer is tempted *upwards*. On other hand, if $V_\gamma \prec U_\gamma$ (which implies $V_\gamma \prec U_\gamma + V_\gamma \prec U_\gamma$), then the consumer's marginal willingness to pay for additional q is lower when she is tempted. Thus, the consumer is tempted *downwards*.

A3 appears complex but it suffices that for any two utility functions f and g such that either $f \prec g$ or $f \succ g$, any indifference curve of f crosses any indifference curve of g somewhere, provided that these curves are first extended from $\mathbb{R}^2_+$ to $\mathbb{R}^2$ (that is, the curves may hit the axes before they cross). The assumption simply rules out the case where the indifference curves get closer and closer asymptotically but never cross.

We sometimes consider the case where preferences exhibit no income effect. Given that a consumer is characterized by multiple preference relations, there are a few ways to assume quasi-linearity. The simplest way is to assume that the preference relations associated with U, V, and $U+V$ are all quasi-linear (in the ordinal sense). An example is the following:

Example. There exists a pair of functions $u, v \colon \mathbb{R}_+ \times \Gamma \to \mathbb{R}$ such that

$$U_\gamma(q,t) = u(q,\gamma) - t, \tag{3}$$
$$V_\gamma(q,t) = \beta_\gamma\big[v(q,\gamma) - t\big], \tag{4}$$

where $\beta_\gamma > 0$ is the weight attached to the tempted utility.

2.2 Computation of Ex-Ante Utility

Here is a convenient way to compute the (ex-ante) utility $W_\gamma(M)$. Given a menu M and a consumer type γ, let $y \in \mathbb{R}^2_+$ be an offer solving the following two equations:

$$U_\gamma(y) + V_\gamma(y) = \sup_{x \in M} \Big[U_\gamma(x) + V_\gamma(x) \Big],$$
$$V_\gamma(y) = \sup_{x \in M} V_\gamma(x).$$

That is, y is the intersection of the highest attainable indifference curves of $U_\gamma + V_\gamma$ and V_γ. The offer y itself does not necessarily belong to M but is well-defined if the menu is bounded. The offer y is useful since $U_\gamma(y)$ gives the consumer's ex-ante utility for the menu M:

$$W_\gamma(M) = U_\gamma(y) + V_\gamma(y) - V_\gamma(y) = U_\gamma(y).$$

2.3 Monopolist's Problem

The monopolist can offer any finite number of menus. Given a number of menus S, let $(M_s)_{s=1}^S$ denote the list of menus offered by the firm.

Let an *assignment function* for a given list of menus $(M_s)_{s=1}^S$ be a function $\alpha = (s, (q, t)) \colon \Gamma \to \{0, 1, \ldots, S\} \times \mathbb{R}^2_+$ that specifies the menu and the offer that the monopolist expects a given consumer type to choose. The first component $s(\gamma) \in \{0, 1, \ldots, S\}$ specifies the menu that consumers of type γ are expected to choose. If $s(\gamma) = 0$, consumers γ are not expected to choose any menu and we define $M_0 \equiv \{(0, 0)\}$. The second and third components $(q(\gamma), t(\gamma))$ specify the offer that consumers γ are expected to accept. Since these consumers choose menu $s(\gamma)$, the offer $(q(\gamma), t(\gamma))$ has to be included in the menu: for all $\gamma \in \Gamma$,

$$(q(\gamma), t(\gamma)) \in M_{s(\gamma)}. \tag{5}$$

We often denote the pair $(q(\gamma), t(\gamma))$ as $x(\gamma)$, so we write an assignment function as $\alpha(\gamma) = (s(\gamma), x(\gamma))$.

Let $C(q)$ denote the per-consumer cost of producing good q. We assume that C is differentiable, strictly increasing, convex, and satisfies $C(0) = 0$. For a given offer $(q, t) \in \mathbb{R}^2_+$, let $\pi(q, t)$ denote the per-consumer profits generated by the offer: $\pi(q, t) \equiv t - C(q)$. To simplify exposition, we assume that for any type γ and any utility function $f_\gamma \in \{U_\gamma, V_\gamma, U_\gamma + V_\gamma\}$, there exists a unique offer $x \gg 0$ that maximizes $\pi(x)$ subject to $f_\gamma(x) \geq 0$.

The monopolist's problem is to choose a list of menus $(M_s)_{s=1}^S$ and an associated assignment function $\alpha = (s, (q, t))$ that maximize expected profits

$$\sum_{\gamma \in \Gamma} n(\gamma)\big[t(\gamma) - C(q(\gamma))\big] \tag{6}$$

subject to the following pair of incentive-compatibility conditions:

Ex-Ante Incentive Compatibility. For each type $\gamma \in \Gamma$, choosing the menu $s(\gamma)$ is at least as good as choosing any other menu in terms of ex-ante utility:

$$W_\gamma(M_{s(\gamma)}) \geq W_\gamma(M_s) \quad \text{for all } s \in \{0, 1, \ldots, S\}. \tag{ex-ante IC}$$

Note that the right-hand side includes the option of $s = 0$. By definition, $M_0 \equiv \{(0,0)\}$ represents the option of not choosing any real menu: e.g., not entering any store or not participating in any weight-loss plan. Thus ex-ante IC implies the following ex-ante condition of individual rationality:

$$W_\gamma(M_{s(\gamma)}) \geq 0. \tag{ex-ante IR}$$

Ex-Post Incentive Compatibility. For each type $\gamma \in \Gamma$, the offer $x(\gamma) \equiv (q(\gamma), t(\gamma))$, which is contained in the menu $M_{s(\gamma)}$ by (5), is at least as good as any other offer in the menu in terms of ex-post utility:

$$U_\gamma(x(\gamma)) + V_\gamma(x(\gamma)) \geq U_\gamma(x) + V_\gamma(x) \quad \text{for all } x \in M_{s(\gamma)}. \tag{ex-post IC}$$

Since we assume that each menu contains $(0,0)$ and we normalize $U_\gamma(0,0) = V_\gamma(0,0) = 0$, ex-post IC implies the following ex-post version of individual rationality:

$$U_\gamma(x(\gamma)) + V_\gamma(x(\gamma)) \geq 0. \tag{ex-post IR}$$

That is, choosing the assigned offer $x(\gamma)$ is at least as good as not choosing any offer.[4]

A list $\sigma = ((M_s)_{s=1}^S, \alpha)$ that satisfies both of the IC conditions is called a *feasible schedule*. If it solves the maximization problem, we call it an *optimal schedule*.

Note that the profit in (6) depends only on the offers that the consumers actually choose, i.e., $(q(\gamma), t(\gamma))$. This comes from our assumption that adding offers to a menu is costless if they are not chosen by any consumer. Including such offers in a menu is immaterial in the standard nonlinear pricing problem, but may be advantageous for the seller in the present problem since such offers may tempt consumers and affect their ex-ante utilities.[5]

[4]Note that the ex-ante conditions pertain to the assignment of menus while the ex-post conditions pertain to the assignment of offers. Therefore, ex-ante IR does not imply ex-post IR since ex-ante IR itself does not place any restriction on $x(\gamma)$.

[5]Another implicit assumption in the above formulation is that the seller can add menus costlessly. Introducing a small cost of creating menus (e.g., setup costs for retail stores) does not affect our

To see the difference between our profit-maximization problem and the standard one in the nonlinear pricing literature, suppose, for the moment, that the ex-ante utility function for each consumer γ is given by

$$W_\gamma(M) = \sup_{x \in M} \big[U_\gamma(x) + V_\gamma(x)\big].$$

With this utility function, the utility for a menu depends only on the most preferred offer in the menu. Therefore, the optimal schedule is identical to the standard optimal tariff with consumers' preferences given by $U_\gamma + V_\gamma$. This reduced problem is a useful benchmark for our problem and will be referred to as the *standard problem with utility functions* $U_\gamma + V_\gamma$.

Another useful benchmark is the case when there is no temptation at all: $V_\gamma = U_\gamma$ (or $V_\gamma \sim U_\gamma$). In this case, $W_\gamma(M) = \sup_{x \in M} U_\gamma(x)$ and the seller's problem reduces to the *standard problem with utility functions* U_γ.

2.4 Perfect Discrimination

It is also useful to consider the case where the seller can observe each consumer's type directly and offer a menu for each type separately. A *perfect discrimination offer* for a given type γ is an offer x^*_γ that maximizes the per-consumer profit $\pi(x)$ subject to

$$U_\gamma(x) + V_\gamma(x) \geq 0, \tag{7}$$

$$U_\gamma(x) + V_\gamma(x) - \max\{0, V_\gamma(x)\} \geq 0. \tag{8}$$

That is, x^*_γ is the most profitable offer that satisfies ex-post IR and such that the associated singleton menu $\{x^*_\gamma\}$ satisfies ex-ante IR. Since (8) implies (7), the binding constraint is (8):

$$U_\gamma(x^*_\gamma) + V_\gamma(x^*_\gamma) - \max\{0, V_\gamma(x^*_\gamma)\} = 0. \tag{9}$$

If the consumer is tempted upwards (i.e., $V_\gamma \succ U_\gamma + V_\gamma$), then $V_\gamma(x^*_\gamma) > 0$ and therefore (9) implies $0 = U_\gamma(x^*_\gamma) + V_\gamma(x^*_\gamma) - V_\gamma(x^*_\gamma) = U_\gamma(x^*_\gamma)$. Thus x^*_γ is the offer at which the iso-profit curve is tangent with the curve of $U_\gamma = 0$.

If the consumer is tempted downwards, on the other hand, an offer where the ex-post IR condition (7) binds also satisfies the ex-ante IR condition (8) since the consumer is tempted by $(0,0)$: $V_\gamma(0,0) > V_\gamma(x^*_\gamma)$. Thus, the ex-post IR condition binds at x^*_γ and therefore x^*_γ is the offer where the iso-profit curve is tangent with the curve of $U_\gamma + V_\gamma = 0$.

To summarize, at the perfect discrimination offer x^*_γ, the iso-profit curve is tangent

qualitative results.

with the curve of $F_\gamma = 0$ where F_γ is defined by

$$F_\gamma \equiv \begin{cases} U_\gamma & \text{if } U_\gamma \prec U_\gamma + V_\gamma, \\ U_\gamma + V_\gamma & \text{if } U_\gamma \succ U_\gamma + V_\gamma. \end{cases} \tag{10}$$

Thus, what is tangent with the iso-profit curve is the "flatter" curve between $U_\gamma = 0$ and $U_\gamma + V_\gamma = 0$. Under our assumption, the offer x^*_γ is unique for each type. The following lemma will be useful.

Lemma 1. *Suppose that an offer x maximizes $U_\gamma + V_\gamma$ in a menu M. Then M satisfies ex-ante IR for γ only if $F_\gamma(x) \geq 0$. If x is the only non-trivial offer in the menu, i.e., $M = \{x\}$, then M satisfies ex-ante IR for γ if and only if $F_\gamma(x) \geq 0$.*

Proof. Since $\sup_{y \in M} V_\gamma(y) \geq \max\{0, V_\gamma(x)\}$,

$$\begin{aligned} W_\gamma(M) &= U_\gamma(x) + V_\gamma(x) - \sup_{y \in M} V_\gamma(y) \\ &\leq \min\{U_\gamma(x), U_\gamma(x) + V_\gamma(x)\}. \end{aligned} \tag{11}$$

The second line is non-negative if and only if $F_\gamma(x) \geq 0$. If $M = \{x\}$, then $\sup_{y \in M} V_\gamma(y) = \max\{0, V_\gamma(x)\}$ and hence the inequality in (11) holds with equality. Q.E.D.

3 Optimal Menu of Menus

We now characterize the optimal schedule for the seller when there are only two types of consumers: γ_L and γ_H such that $\gamma_L < \gamma_H$. Let n_L and n_H denote the fractions of consumers of types γ_L and γ_H, respectively ($n_L + n_H = 1$).

The following lemma is useful and general.

Lemma 2. *For consumers with downward temptation, any menu satisfies ex-ante IR.*

Proof. Let γ be any type who is tempted downwards. Let M be any menu and x be any offer in M such that $V_\gamma(x) \geq 0$. Since the consumer is tempted downwards, $V_\gamma(x) \geq 0$ implies $U_\gamma(x) \geq 0$ and hence

$$U_\gamma(x) + V_\gamma(x) \geq V_\gamma(x),$$

which implies

$$\sup_{y \in M} \left[U_\gamma(y) + V_\gamma(y)\right] \geq V_\gamma(x).$$

Since this holds for all $x \in M$ such that $V_\gamma(x) \geq 0$, we have

$$\sup_{y \in M} \left[U_\gamma(y) + V_\gamma(y)\right] \geq \sup_{y \in M} V_\gamma(y),$$

which means $W_\gamma(M) \geq 0$. Q.E.D.

A corollary of this lemma is that if all consumers are tempted downwards, the ex-ante IR condition can be ignored completely. An interesting implication is that if the optimal schedule consists of a single menu, the only effective condition is ex-post IC (which includes ex-post IR) and therefore the optimal schedule coincides with that in the standard problem with utility functions $U_\gamma + V_\gamma$.

3.1 When High Type is Tempted Upwards

To characterize the optimal schedule, we first consider the case when the high-type consumers are tempted upwards: $V_H \succ U_H + V_H$. The next proposition states that if the high-type consumers are tempted upwards, the monopolist can obtain the level of profits associated with perfect discrimination: it is as if the seller can observe consumers' types directly. To achieve perfect discrimination, the monopolist offers two menus and "decorates" the one assigned to the low type by adding an offer that is irrelevant for the low type but is tempting and ex ante undesirable for the high type. By adding such an offer, the seller can deter the high type from choosing the menu. This effectively eliminates the incentive-compatibility condition for the high-type consumers and enables the seller to extract the full surplus from them.

Proposition 1. *If the high-type consumers are tempted upwards, then the optimal schedule assigns the perfect discrimination offer to each type and generates the same level of profits as perfect discrimination.*

Proof. Let x_L^* and x_H^* denote the perfect discrimination offers for L and H, respectively. See Figure 1. Since the high type is tempted upwards, $F_H = U_H$. Let y be the intersection of the $U_H = 0$ curve and the indifference curve of $U_H + V_H$ through x_L^*:

$$\begin{aligned} U_H(y) + V_H(y) &= U_H(x_L^*) + V_H(x_L^*), \\ U_H(y) &= 0. \end{aligned} \tag{12}$$

The offer y is well defined by Assumption A3.

If $V_L(y) \leq V_L(x_L^*)$ (which occurs if $V_L \prec U_H + V_H$, which is the case if the low type is tempted downwards), then let $z = y$. Otherwise, let z be the intersection of the

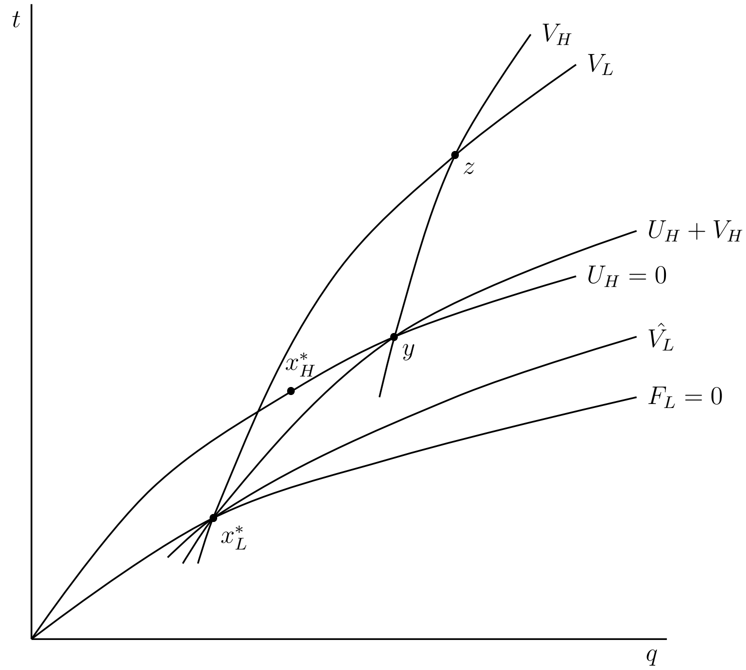

Figure 1: Optimal schedule in Proposition 1

indifference curve of V_L through x_L^* and the indifference curve of V_H through y:

$$\begin{aligned} V_L(z) &= V_L(x_L^*), \\ V_H(z) &= V_H(y). \end{aligned} \tag{13}$$

We claim that an optimal schedule is to offer $M_L = \{x_L^*, z\}$ to L and $M_H = \{x_H^*\}$ to H, assigning x_L^* and x_H^* to L and H, respectively. To see that this schedule is feasible, we compute each type's ex-ante utility and ex-post optimal choice for each menu.

If the high type chooses M_H: By the definition of x_H^* (see (9)), the high type's ex-ante utility from $\{x_H^*\}$ is zero. Since $U_H(x_H^*) = 0$ and the high-type consumers are tempted upwards, we have $U_H(x_H^*)+V_H(x_H^*) > 0$. Thus, upon choosing M_H, high-type consumers will indeed choose x_H^* over $(0, 0)$.

If the high type chooses M_L: Upon choosing M_L, high-type consumers are willing to choose x_L^*. Indeed, (12) implies that they are indifferent between x_L^* and y. In the case of $z \neq y$, since $z \gg y$ and $V_H \succ U_H+V_H$, we have $U_H(y)+V_H(y) = U_H(x_L^*)+V_H(x_L^*) \geq U_H(z) + V_H(z)$. On the other hand, the high-type consumers are tempted by z in the menu: since x_L^* and y are indifferent for $U_H + V_H$, $y \gg x_L^*$, and $V_H \succ U_H + V_H$, we have $V_H(y) > V_H(x_L^*)$, which together with (13) implies $V_H(z) > V_H(x_L^*)$. Putting these facts together, we obtain that the high-type consumers' ex-ante utility from M_L

is

$$U_H(x_L^*) + V_H(x_L^*) - V_H(z) = U_H(y) = 0.$$

Thus, ex ante, the high-type consumers are indifferent between M_L, M_H, and M_0.

If the low type chooses M_H: Since $U_L(x_H^*) < 0 = U_L(0,0)$ and the self-control costs are non-negative, the low type's ex-ante utility from M_H is at most 0.

If the low type chooses M_L: By the previous argument, the high-type consumers like x_L^* at least as well as z in terms of $U_H + V_H$. Since $U_L + V_L \prec U_H + V_H$ and $z \gg x_L^*$, the low-type consumers prefer x_L^* to z in terms of $U_L + V_L$. Since x_L^* satisfies ex-post IR by (7), the low-type consumers will indeed choose x_L^* upon facing the menu M_L. We also show that the low type's ex-ante utility from M_L is zero. Indeed, since $V_L(x_L^*) \geq V_L(z)$, i.e., x_L^* is at least as tempting as z, the offer z does not affect the low type's ex-ante utility at all. Then M_L is equivalent to the singleton menu $\{x_L^*\}$ for the low type, and $\{x_L^*\}$ gives zero ex-ante utility to the low type by the definition of x_L^*. Thus, the low type has a (weak) incentive to choose M_L over M_H and M_0. Q.E.D.

Therefore, if the high-type consumers are tempted upwards, then regardless of the direction of the low-type consumers' temptation, the seller can achieve the same level of profits and purchasing pattern as in the case of perfect discrimination.[6] The seller decorates the menu intended for the low type with the offer z. The offer z is irrelevant for the low type, but it is tempting to the high-type consumers and increases their self-control costs up to the point where their ex-ante utility from the menu is zero. Then the high type does not have any incentive to mimic the low type, and therefore the menu for the high type needs to satisfy ex-ante IR only.[7]

For this perfect-discrimination result, it is immaterial how strongly the high-type consumers are tempted upwards. What matters is $V_H \succ U_H$, i.e., the marginal valuation for additional quality is higher for V_H than U_H. The magnitude of the difference may be arbitrarily small as long as the crossing condition A3 is satisfied. In addition, since the condition $V_H \succ U_H$ depends only on the ordinal preferences associated with these utility functions, the *scales* of these functions are not important. In particular, the result holds even if the scale of V_H is arbitrarily small compared to that of U_H. That is, the result remains true even if we scale down V_H by multiplying it by a small positive number $\varepsilon > 0$.

[6]For some cases, perfect discrimination can be achieved with a single menu. Indeed, if the low-type consumers are tempted downwards and $U_H(x_H^*) + V_H(x_H^*) \geq U_H(x_L^*) + V_H(x_L^*)$, then an optimal schedule offers a single menu $M = \{x_L^*, x_H^*\}$.

[7]The perfect-discrimination result also holds if self-control is costless, that is, consumers' ex-ante utility from a menu is given by $U_\gamma(\hat{x}_\gamma)$ where $\hat{x}_\gamma$ is a maximizer of $U_\gamma + V_\gamma$. Thus the consumer anticipates his temptation and self-control (i.e., the fact that he will maximize $U_\gamma + V_\gamma$) but does not incur psychological costs of self-control. With this utility formulation, perfect discrimination can be achieved by the schedule constructed above with $z \equiv y$.

The result implies that the optimal schedule and the maximum attainable profits for the seller may change discontinuously as consumers' preferences change. This can be seen by considering the case where consumers have no temptation problem: $V_\gamma = U_\gamma$ for all consumers. With no temptation, the seller's optimal strategy is to offer the optimal menu $M^s = \{x_L^s, x_H^s\}$ in the standard nonlinear pricing problem where utility functions are given by U_γ. By comparing these offers with perfect discrimination offers, we can easily see that $\pi(x_L^s) \leq \pi(x_L^*)$ and $\pi(x_H^s) \leq \pi(x_H^*)$, and at least one of the inequalities holds strictly under our assumptions. Thus

$$n_L \pi(x_L^s) + n_H \pi(x_H^s) < n_L \pi(x_L^*) + n_H \pi(x_H^*). \tag{14}$$

Then, under no temptation, the optimal menu generates strictly less profits than perfect discrimination.[8] However, according to Proposition 1, once the high type's temptation preferences V_H change slightly to exhibit upward temptation, the seller can achieve perfect discrimination and obtain the profit level equal to the right-hand side of (14).

3.2 When High Type is Tempted Downwards

We now characterize the optimal schedule when the high-type consumers are tempted downwards (i.e., $V_H \prec U_H + V_H \prec U_H$). As mentioned in the introduction, downward temptation is not unusual. In some contexts, downward temptation is actually the norm. We characterize the optimal schedule by proving a series of lemmas.

The first lemma shows that at any optimal schedule, the offer chosen by a high-type consumer is at least as profitable for the seller as the one chosen by a low-type consumer. The lemma also gives a lower bound for the maximum attainable profit for the seller.

Lemma 3. *Suppose that the high-type consumers are tempted downwards. If x_H and x_L are the offers chosen by γ_H and γ_L at an optimal schedule, respectively, then $\pi(x_H) > \pi(x_L)$ and $n_H \pi(x_H) + n_L \pi(x_L) > \pi(x_L^*)$.*

Proof. To prove the first inequality (which is not trivial since x_L and x_H might belong to different menus), suppose that the inequality does not hold. Then $\pi(x_H) \leq \pi(x_L) \leq \pi(x_L^*)$, where x_L^* is the perfect discrimination offer for L. See Figure 2. Consider the schedule σ that offers a single menu $M = \{x_L^*, \hat{x}_H\}$, where $\hat{x}_H$ maximizes $\pi(\cdot)$ subject to

$$U_H(\hat{x}_H) + V_H(\hat{x}_H) = U_H(x_L^*) + V_H(x_L^*). \tag{15}$$

[8] On the other hand, if there is temptation, the seller's profit in the standard problem with utility functions $U_\gamma + V_\gamma$ may be higher or lower than the profit from perfect discrimination. The reason is that while x_H^* has to be on the $U_H = 0$ curve, the standard problem with $U_\gamma + V_\gamma$ may assign an offer above $U_H = 0$ to the high type.

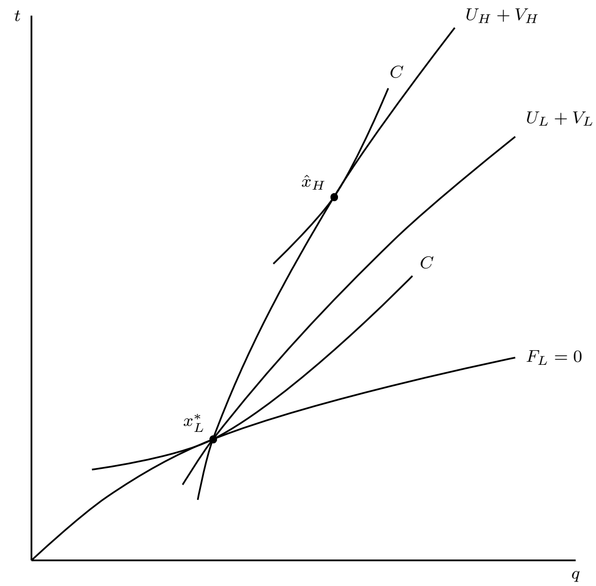

Figure 2: Proof of Lemma 3. The curves labeled C are iso-profit curves and vertical translations of the cost curve.

The assignment function is such that all consumers choose the menu M (over the trivial menu $M_0 = \{(0,0)\}$) and x_L^* and $\hat{x}_H$ are chosen by the low-type consumers and the high-type consumers, respectively. We first show that this schedule is feasible.

Since $F_L \precsim U_L + V_L \prec U_H + V_H$ and the indifference curve of F_L is tangent with the iso-profit curve at x_L^*, we have $\hat{x}_H \gg x_L^*$, $F_L(\hat{x}_H) < 0$, and $\pi(\hat{x}_H) > \pi(x_L^*)$.

Given the menu M, we first consider the choice and utility of the low-type consumers. Since $F_L(x_L^*) = 0$ and $F_L \precsim U_L + V_L$, we have $U_L(x_L^*) + V_L(x_L^*) \geq 0$. On the other hand, (15), $\hat{x}_H \gg x_L^*$, and $U_H + V_H \succ U_L + V_L$ imply that $U_L(x_L^*) + V_L(x_L^*) \geq U_L(\hat{x}_H) + V_L(\hat{x}_H)$. Therefore, x_L^* is an ex-post optimal choice for the low-type consumers once they choose the menu M.

To show that M satisfies ex-ante IR for the low type, suppose that the low type is tempted upwards (otherwise, ex-ante IR follows from Lemma 2). Since the high-type consumers are tempted downwards,

$$U_H + V_H \succ V_H \succ V_L.$$

This together with (15) and $\hat{x}_H \gg x_L^*$ implies that $V_L(x_L^*) \geq V_L(\hat{x}_H)$, i.e., x_L^* is at least as tempting as $\hat{x}_H$. Thus, the low type's ex-ante utility from M is 0 by (9).

For the high-type consumers, since they are tempted downwards, any menu satisfies ex-ante IR. It is evident that $\hat{x}_H$ satisfies ex-post IC.

We have shown that offering the menu $M = \{x_L^*, \hat{x}_H\}$ satisfies all the incentive constraints. Since $\pi(\hat{x}_H) > \pi(x_L^*) \geq \pi(x_L) \geq \pi(x_H)$, the menu generates more profits than the optimal schedule, a desired contradiction.

Lastly, the argument above also shows that the monopolist can always guarantee itself profits of $n_L\pi(x_L^*) + n_H\pi(\hat{x}_H) > \pi(x_L^*)$. Q.E.D.

The next lemma shows that at any optimal schedule, the low-type consumers derive a higher utility in terms of F_L from the offer assigned to them than from the offer assigned to the high type (even if these offers belong to different menus).

Lemma 4. *Assume that the high-type consumers are tempted downwards and consider any optimal schedule. If x_L and x_H are the offers assigned to L and H, respectively, then $F_L(x_L) > F_L(x_H)$, where F_L is defined by* (10).

Proof. Suppose otherwise. Let M_L denote the menu assigned to the low type. Since M_L satisfies ex-ante IR for the low type, Lemma 1 implies $F_L(x_L) \geq 0$. Since x_L^* is the most profitable offer satisfying $F_L(x) \geq 0$ and we have $F_L(x_H) \geq F_L(x_L) \geq 0$, it follows that $\pi(x_L^*) \geq \pi(x_H)$. But Lemma 3 implies $\pi(x_H) > \pi(x_L^*)$, a contradiction. Q.E.D.

To obtain sharper characterizations, we further classify the high-type consumers on the basis of the *degree* of their downward temptation, which can be measured by comparing V_H and $U_L + V_L$.

Definition. The high type's downward temptation is *weak* if $V_H \succ U_L + V_L$ and *strong* if $V_H \prec U_L + V_L$.

Since the high type is tempted downwards, $U_L + V_L \prec U_H + V_H \prec U_H$ and $V_H \prec U_H + V_H$. Thus if V_H is sufficiently similar to U_H in terms of the marginal rate of substitution, then temptation is weak. It is worth noting that the distinction of weak vs. strong temptation depends only on ordinal preferences induced by the utility functions and is independent of the scales of these functions. That is, the high type's temptation may be strong even when the scale of the function V_H is arbitrarily small and therefore self-control is not very costly. In other words, weak vs. strong temptation pertains to how much temptation can possibly affect preferences (i.e., V_H) and not how much it actually does (i.e., $U_H + V_H$). This notion of weak vs. strong temptation turns out to be useful for characterizing optimal schedules.

In what follows, we consider each case in turn, starting with the case of weak temptation.

3.2.1 When High Type's Temptation is Weak

The following lemma shows that the seller gains nothing from decorating the menu intended for the low type.

Lemma 5. *Suppose that the high type's downward temptation is weak, and let M_L and x_L denote the menu and offer assigned to the low type at an optimal schedule. If the high type is assigned to a different menu, then replacing M_L with $\{x_L\}$ yields another optimal schedule.*

Proof. Since offers in $M_L \setminus \{x_L\}$ may be tempting to the low type, $W_L(M_L) \leq W_L(\{x_L\})$. Therefore, it suffices to show $W_H(M_L) \geq W_H(\{x_L\})$, i.e., decorating the menu for the low type does not decrease the high type's ex-ante utility from the menu. To prove this, let $\varepsilon > 0$. Then there exists an offer $z \in M_L$ such that

$$\begin{aligned} V_H(z) &> \sup_{y \in M_L} V_H(y) - \varepsilon, \\ V_H(z) &\geq V_H(x_L). \end{aligned} \tag{16}$$

Thus the high type's ex-ante utility from M_L is

$$W_H(M_L) \geq U_H(z) + V_H(z) - \sup_{y \in M_L} V_H(y) > U_H(z) - \varepsilon.$$

Note that z has to satisfy $U_L(z) + V_L(z) \leq U_L(x_L) + V_L(x_L)$ (by ex-post IC) as well as (16) and that $V_H \succ U_L + V_L$ (weak temptation). Thus if $z \neq x_L$, then $z \gg x_L$ and hence $U_H(z) > U_H(x_L)$. Therefore, whether $z = x_L$ or not, $W_H(M_L) > U_H(x_L) - \varepsilon$. Since $\varepsilon > 0$ is arbitrary, $W_H(M_L) \geq U_H(x_L) = W_H(\{x_L\})$. Q.E.D.

The next lemma shows that if the high type's downward temptation is weak, then at any optimal schedule, $F_L = 0$ holds.

Lemma 6. *Assume the high type's downward temptation is weak. If x_L is the offer assigned to the low type in an optimal schedule, then $F_L(x_L) = 0$. Thus $U_L(x_L) = 0$ if the low-type consumers are tempted upwards and $U_L(x_L) + V_L(x_L) = 0$ if they are tempted downwards.*

Proof. Let M_L and M_H be the menus assigned to the low type and the high type, respectively (possibly $M_L = M_H$). Let $x_H \in M_H$ be the offer assigned to the high type. Since M_L satisfies ex-ante IR for the low type, $F_L(x_L) \geq 0$. Suppose, by way of contradiction, that $F_L(x_L) > 0$. By Lemma 4, $F_L(x_L) > F_L(x_H)$. Hence, there exists a small number $\varepsilon > 0$ such that

$$F_L(x_L + (0, \varepsilon)) > \max\{0, F_L(x_H)\}. \tag{17}$$

Let $x'_L \equiv x_L + (0, \varepsilon)$. Consider a schedule σ' that offers $M'_L \equiv \{x'_L\}$ and $M'_H \equiv \{x_H\}$. We show that this generates more profits.

Consider the low-type consumers first. By the construction of x'_L (i.e., (17)), x'_L satisfies ex-post IR and $M'_L = \{x'_L\}$ satisfies ex-ante IR for the low type. Thus, if the low-type consumers indeed choose M'_L, then they will choose x'_L (over $(0,0)$) and this increases the seller's profits since $\pi(x'_L) > \pi(x_L)$. However, the low-type consumers may prefer M'_H. If they prefer and choose M'_H, they will choose x_H, which also increases the seller's profits since $\pi(x_H) > \pi(x_L)$ by Lemma 3.

Consider now the high-type consumers. Since $M'_H \supseteq M_H$ and offers in $M'_H \setminus M_H$ may be tempting to the high type, $W_H(M'_H) \geq W_H(M_H)$. By the argument in the proof of Lemma 5, $W_H(M_L) \geq U_H(x_L)$. Thus

$$W_H(M'_H) \geq W_H(M_H) \geq W_H(M_L) \geq U_H(x_L) > U_H(x'_L) = W_H(M'_L).$$

This shows that the high type's ex-ante optimal menu is M'_H. Once they choose M'_H, they choose x_H. Therefore, the schedule σ' generates the same profits from the high-type consumers as the optimal schedule. Since σ' generates strictly more profits from the low-type consumers, we obtained a desired contradiction. Q.E.D.

We now show that offering a single menu is not an optimal strategy if the seller sells to both consumer types. The optimal strategy is to tailor a menu for each type.

Proposition 2. *If the high type's downward temptation is weak, then any optimal schedule that sells some good $q > 0$ to low-type consumers offers a separate menu for each type of consumers.*

Proof. Suppose, by way of contradiction, that there exists an optimal schedule that offers a single menu M and such that the low-type consumers choose an offer (q, t) such that $q > 0$. Let x_L and x_H denote the offers that each type chooses under the optimal schedule. By Lemma 6, $F_L(x_L) = 0$. Profit maximization within one menu implies that the ex-post IC binds for the high type: $U_H(x_H) + V_H(x_H) = U_H(x_L) + V_H(x_L)$. Since the cost function is differentiable, $x_H \gg x_L$. This and $V_H \prec U_H + V_H$ imply $V_H(x_L) > V_H(x_H)$. Since $x_L \gg 0$ and $V_H \succ U_L + V_L \succsim F_L$, we have $V_H(x_L) > 0$ and hence $V_H(x_L) > \max\{0, V_H(x_H)\}$. This implies that the high type prefers a singleton menu $\{x_H\}$ to another singleton menu $\{x_L\}$:

$$\begin{aligned} W_H(\{x_H\}) &= U_H(x_H) + V_H(x_H) - \max\{0, V_H(x_H)\} \\ &= U_H(x_L) + V_H(x_L) - \max\{0, V_H(x_H)\} \\ &> U_H(x_L) = W_H(\{x_L\}). \end{aligned}$$

Now, consider an alternative schedule that offers $M_L \equiv \{x_L\}$ and $M_H \equiv \{x_H + (0, \varepsilon)\}$ where $\varepsilon > 0$. If ε is sufficiently small, the high-type consumers prefer M_H to M_L, and once they choose M_H, they will choose $x_H + (0, \varepsilon)$ over $(0,0)$. Since $F_L(x_H + (0, \varepsilon)) < 0$, it is easy to verify that the low-type consumers get a zero ex-ante utility from M_L and M_H, having a weak incentive to choose M_L and x_L. Since $\varepsilon > 0$, this schedule generates more profits than the optimal one, a desired contradiction. Q.E.D.

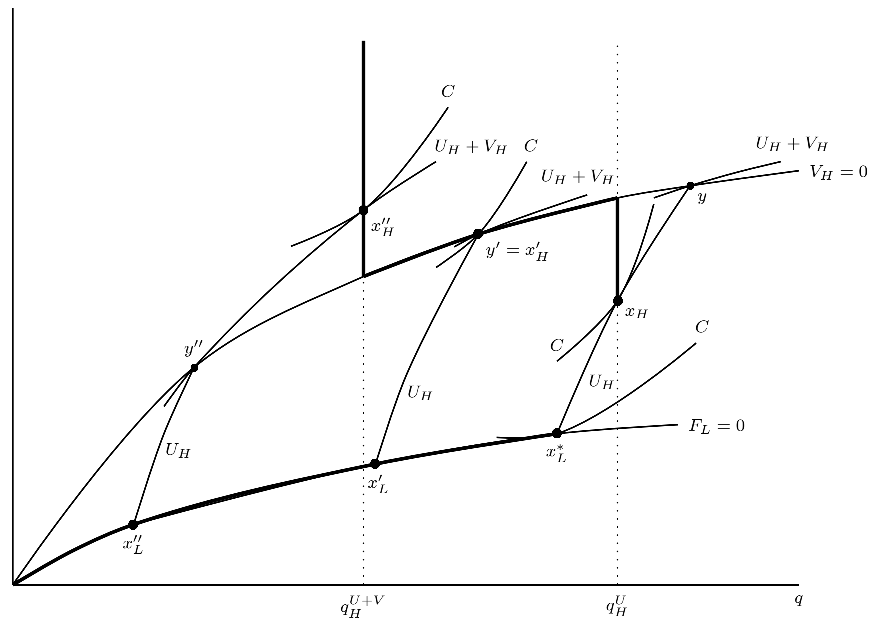

Figure 3: Optimal schedule when high type has weak downward temptation

The intuition behind Proposition 2 is simple. If only one menu is offered, the high-type consumers are tempted by the offer intended for the low type. Thus, the high-type consumers are willing to pay a premium if there is a separate menu that does not contain the tempting offer. For example, in the case of weight-loss programs, if a single plan serves dieters with different levels of eagerness, a reasonable weight loss for a group of dieters may work as a tempting outcome for more eager dieters and make them suffer more from self-control. Thus these eager dieters are willing to pay a premium for a plan that is targeted for their aimed level of weight loss.

We can now describe the optimal schedule concretely. See Figure 3. By Lemmas 5 and 6, we can assume that the menu given to the low type is of the form $\{x_L\}$ such that $F_L(x_L) = 0$. One option for the seller is to offer the perfect discrimination offer x_L^* to the low type. With this choice, the set of offers that can be assigned to the high type is given by the kinked curve that follows the indifference curve of U_H from x_L^* to y and then follows the indifference curve of $U_H + V_H$ to the right. This kinked curve is the set of offers x such that $\{x\}$ gives the same ex-ante utility as $\{x_L^*\}$ for the high type. Let x_H denote the offer that is most profitable on the kinked curve. To consider the interesting case, suppose, as in the figure, that the offer x_H is on the indifference curve of U_H.

We now decrease the quality level offered to the low type along the $F_L = 0$ curve. Then, the kinked curve shifts upwards and therefore the most profitable offer that can be given to the high type moves up (straight if preferences are quasi-linear). However, it eventually hits the $V_H = 0$ curve. At this point, the offer for the high type is at the kink and remains so for a while as we continue moving the offer given to the low type. For example, if x'_L is given to the low type, the most profitable offer that can be given to the high type is y'. As we continue, the offer for the high type eventually moves to an indifference curve of $U_H + V_H$, as depicted by x''_H. The pair of thick lines in the figure show the paths of the offers. The seller's problem is then to identify an offer $\hat{x}_L$ between the origin and x^*_L on the $F_L = 0$ curve to maximize the profits $n_L\pi(\hat{x}_L) + n_H\pi(\hat{x}_H)$ where $\hat{x}_H$ is the associated offer for the high type. If the solution is such that $\hat{x}_L \neq 0$, then the optimal schedule is to offer the singleton menus $\{\hat{x}_L\}$ and $\{\hat{x}_H\}$. If $\hat{x}_L = 0$, the optimal schedule is to offer $\{\hat{x}_H\}$ only.

It is worth noting that the standard result of "no distortion at the top" does not necessarily hold here. Indeed, if the optimal schedule offers a pair of menus like $\{x'_L\}$ and $\{x'_H\}$ in the figure, then at the offer x'_H, the iso-profit curve is not tangent with the indifference curve of either U_H or $U_H + V_H$.

To see the benefit of offering multiple menus, suppose that the seller is restricted to offer one menu. Then, by ex-post IC, the offer to the high type needs to be on the indifference curve of $U_H + V_H$ that passes through the offer to the low type. Since the high type is tempted downwards, the indifference curves of $U_H + V_H$ are flatter than those of U_H. Therefore, one can see that, by offering two menus, the seller can expand the set of offers that can be assigned to the high type, for any given offer to the low type on $F_L = 0$, if the low type is assigned a non-trivial good $q > 0$.

It is also interesting to compare the optimal schedule described above and that when there is no temptation, i.e., $V_\gamma = U_\gamma$. To make the comparison meaningful, consider the same set of functions U_γ and vary V_γ. Recall that, in the case when the high type is tempted upwards, the presence of temptation makes perfect discrimination possible and increases profits. In the current case, however, the presence of temptation actually lowers profits, at least weakly and sometimes strictly. Indeed, without any temptation, the optimal schedule offers a menu $\{x_L, x_H\}$ such that $F_L(x_L) = U_L(x_L) = 0$ and $U_H(x_H) = U_H(x_L)$. A critical difference from the case with temptation is that, as we move x_L to the left on the $F_L = 0$ curve, the associated x_H moves up without hitting the constraint of $V_H = 0$ since $V_H = U_H$. Facing fewer constraints, the seller can earn at least as much as in the presence of temptation. Recall that when there is temptation, the constraint $V_H = 0$ matters since an offer x_H above the curve generates a positive self-control cost for the high-type consumers, which lowers their ex-ante utility and limits the amount of profits the seller can extract from them. Without temptation, this effect disappears since there is no self-control cost.

3.2.2 When High Type's Temptation is Strong

We now consider the case in which the high-type consumers have preferences with *strong* downward temptation: $V_H \prec U_L + V_L \prec U_H + V_H$. Note that since $V_L \prec V_H$, we have $V_L \prec V_H \prec U_L + V_L$ and hence the low-type consumers are also tempted downwards.

Since all consumers are tempted downwards, if the optimal schedule offers only one menu, then as we mentioned earlier, the menu is simply the optimal menu in the standard problem with utility functions $U_\gamma + V_\gamma$. We show that, indeed, offering a single menu is optimal for the seller provided that preferences are quasi-linear. The key is the following result.

Lemma 7. *Suppose that the high type's downward temptation is strong and the preference relations associated with $U_\gamma + V_\gamma$ and V_γ are quasi-linear for each type. Then if x_L is the offer assigned to the low type under an optimal schedule, $U_L(x_L) + V_L(x_L) = 0$.*

Proof. Fix an optimal schedule σ. Let M_L and M_H (possibly $M_L = M_H$) denote the menus that each type chooses and let $x_L \in M_L$ and $x_H \in M_H$ denote the offers that each type chooses. See Figure 4. (For simplicity, x_H is not shown since its location is immaterial for the proof.) Suppose, by way of contradiction, that the ex-post IR is not binding for the low type: $U_L(x_L) + V_L(x_L) > 0$. Then there exists $t' > 0$ such that the offer defined by $x'_L \equiv x_L + (0, t')$ satisfies $U_L(x'_L) + V_L(x'_L) = 0$. We consider a schedule σ' that offers the following set of menus:

$$\begin{aligned} \{x'_L\} \text{ and } \{x_H\} \quad & \text{if } \pi(x_H) > \pi(x'_L), \\ \{x'_L\} \quad & \text{if } \pi(x_H) \leq \pi(x'_L). \end{aligned}$$

We claim that offering this set of menus generates more profits than the optimal schedule, which is a desired contradiction. Since x'_L is more profitable than x_L, it suffices to show that the low type chooses x'_L and the high type chooses x_H in the first case and x'_L in the second. Since x'_L satisfies ex-post IR for both types and ex-ante IR is vacuous, what remains to be proved is that the high type likes the menu $\{x_H\}$ at least as well as $\{x'_L\}$: $W_H(\{x_H\}) \geq W_H(\{x'_L\})$.

First, $W_H(\{x'_L\})$ is given by

$$\begin{aligned} W_H(\{x'_L\}) &= U_H(x'_L) + V_H(x'_L) - \max\{0, V_H(x'_L)\} \\ &= U_H(x'_L) + V_H(x'_L) \end{aligned} \tag{18}$$

since $V_H \prec U_L + V_L$. On the other hand, since offers in $M_H \setminus \{x_H\}$ may be tempting for the high type,

$$W_H(\{x_H\}) \geq W_H(M_H). \tag{19}$$

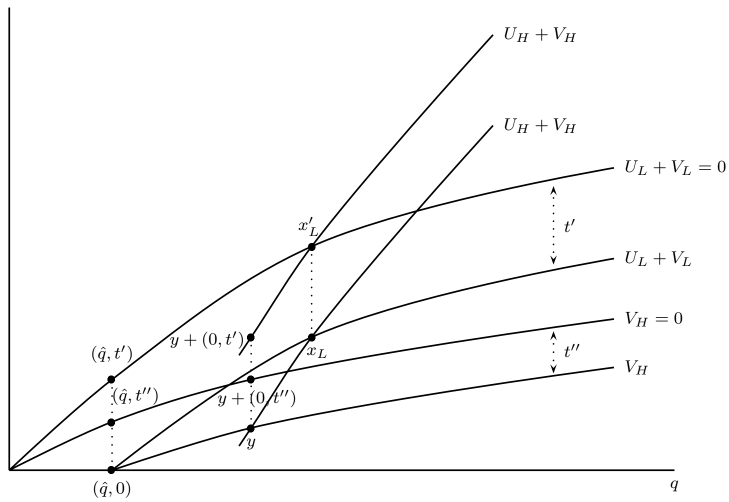

Figure 4: Proof of Lemma 7

Since the initial schedule satisfies ex-ante IC,

$$W_H(M_H) \geq W_H(M_L). \tag{20}$$

The ex-post IC for the low type in the initial schedule implies that none of the offers in M_L is below the indifference curve of $U_L + V_L$ that passes through x_L. Since $V_H \prec U_L + V_L$, no offer in M_L is more tempting for the high type than the offer $(\hat{q}, 0)$ defined by

$$U_L(\hat{q}, 0) + V_L(\hat{q}, 0) = U_L(x_L) + V_L(x_L). \tag{21}$$

That is,

$$V_H(\hat{q}, 0) \geq V_H(x) \qquad \text{for all } x \in M_L.$$

Since the right-hand side of (21) is strictly positive, $\hat{q}$ is well-defined and $\hat{q} > 0$. By quasi-linearity, $U_L(\hat{q}, t') + V_L(\hat{q}, t') = 0$. Let $t'' > 0$ be defined by $V_H(\hat{q}, t'') = 0$. Since $V_H \prec U_L + V_L$, we have $t'' < t'$.

Let y be an offer such that

$$V_H(y) = V_H(\hat{q}, 0), \tag{22}$$

$$U_H(y) + V_H(y) = U_H(x_L) + V_H(x_L). \tag{23}$$

Then

$$\begin{aligned}
W_H(M_L) &\geq U_H(x_L) + V_H(x_L) - V_H(\hat{q}, 0) \\
&= U_H(y) + V_H(y) - V_H(y) \\
&= U_H(y) + V_H(\hat{q}, t'') && \text{since } V_H(\hat{q}, t'') = 0 \\
&= U_H(y) + V_H(y + (0, t'')) && \text{by (22)} \\
&> U_H(y + (0, t')) + V_H(y + (0, t')) && \text{by } t' > t'' > 0 \\
&= U_H(x'_L) + V_H(x'_L) && \text{by (23).}
\end{aligned}$$

This, together with (18)–(20), implies $W_H(\{x_H\}) > W_H(\{x'_L\})$, as desired. Q.E.D.

An implication of Lemma 7 is that the seller gains nothing by decorating the menu assigned to the low type. To see this, consider an optimal schedule and let x_L and M_L denote the offer and the menu assigned to the low-type consumers. Then since the low type's ex-post IR binds at x_L and the high type's temptation is strong (i.e., $V_H \prec U_L + V_L$), the most tempting offer in M_L for the high type is $(0, 0)$, i.e, $\sup_{x \in M_L} V_H(x) = 0$, regardless of other offers that may be present in the menu. The seller cannot change this by decorating the menu, without violating the low type's ex-post IC. This implies

$$W_H(M_L) \geq U_H(x_L) + V_H(x_L) = W_H(\{x_L\}).$$

Thus, decorating the menu for the low type cannot lower the high type's ex-ante utility from the menu and therefore the conclusion of Lemma 5 extends to the current case.

The difference from the previous case is that the seller also gains nothing from offering multiple menus. To see this, note that

$$U_H(x_L) + V_H(x_L) \leq W_H(M_L) \leq W_H(M_H) \leq U_H(x_H) + V_H(x_H),$$

where M_H and x_H denote the menu and the offer assigned to the high type. The inequalities imply that at any optimal schedule, x_H and x_L satisfy the ex-post IC for the high type even though x_H and x_L may belong to different menus. This means that offering multiple menus does not enlarge the set of offers that can be given to the high type and therefore it suffices to offer a single menu. Thus

Proposition 3. *If the high type's downward temptation is strong and the preference relations associated with $U_\gamma + V_\gamma$ and V_γ are quasi-linear for each type, then an optimal schedule is a solution to the standard problem with utility functions $U_\gamma + V_\gamma$.*

The comparison with the case with no temptation is relatively transparent. When there is no temptation, the optimal schedule is identical to the one given in Proposition 3 with $V_\gamma = U_\gamma$. As we change V_γ to introduce downward temptation, the optimal schedule changes and the seller's profits decrease. However, the profits change continuously and do not exhibit the discontinuous jump that we saw in the case when the high type is tempted upwards.

4 Entry Fee

We have so far assumed that any menu contains $(0,0)$, which means that consumers who do not buy any good do not have to pay. This assumption plays an important role in the above analysis since consumers with downward temptation may be tempted by the option $(0,0)$ and the firm cannot remove this option from the menus. The assumption that $(0,0)$ is included in any menu is certainly reasonable if menus represent retail stores or restaurants. On the other hand, for weight-loss programs, gym clubs, cell-phone plans, and other services, firms often charge initial fees independently of service usage (e.g., fixed monthly fees), which correspond to the price of choosing $q = 0$. Once a consumer chooses a menu with such a fee, $(0,0)$ as the final consumption is not available. In this section, we extend our analysis to the case where the seller can charge initial fees.

The model is the same as before except that menus are not required to include $(0,0)$. With this change, ex-post IR does not follow from ex-post IC and hence need not be imposed. On the other hand, the change does not affect ex-ante IR, since consumers continue to have the option of not choosing any menu.

The perfect discrimination offer x^*_γ is then the most profitable offer subject to the constraint that $\{x^*_\gamma\}$ satisfies ex-ante IR. Since the menu $\{x^*_\gamma\}$ does not contain the option $(0,0)$ (i.e., the menu is truly a singleton), it does not induce any temptation for the consumer. Hence, for each type, x^*_γ is simply the offer that maximizes $\pi(x)$ subject to $U_\gamma(x) = 0$.

High Type with Upward Temptation. The perfect discrimination result (Proposition 1) continues to hold without any change. The construction of the optimal schedule (and the decoration) is also identical. The proof is therefore omitted.

Proposition 4. *If the high-type consumers are tempted upwards, the optimal schedule assigns the perfect discrimination offer to each type and generates the same level of profits as perfect discrimination.*

High Type with Weak Downward Temptation. For the case where the high-type consumers are tempted downwards and their temptation is weak, Proposition 2 continues to hold: the optimal schedule that does not exclude the low-type consumers offers multiple menus. However, the optimal menu of menus may differ from that without entry fees. Specifically, the optimal schedule can be characterized as follows. By the argument in the proof of Lemma 5, $W_H(M_L) \geq U_H(x_L)$ for any optimal schedule. By ex-ante IC, $W_H(M_H) \geq W_H(M_L)$. Since the self-control cost is non-negative,

$U_H(x_H) \geq W_H(M_H)$. Putting these inequalities together, we get

$$U_H(x_H) \geq U_H(x_L). \tag{24}$$

For the low type, non-negative self-control cost and ex-ante IR imply

$$U_L(x_L) \geq W_L(M_L) \geq 0. \tag{25}$$

Inequalities (24) and (25) show that the menu $\{x_H, x_L\}$ satisfies the pair of binding constraints in the standard nonlinear pricing problem with utility functions U_γ. Thus, the standard problem with utility functions U_γ has fewer constraints. Let $\{x_H, x_L\}$ denote the optimal menu in the standard problem (possibly $x_L = 0$). We claim that offering a pair of singleton menus $\{x_H\}$ and $\{x_L\}$ is optimal in our problem. Indeed, since these menus are truly singletons and do not induce any temptation, consumers evaluate them by U_γ. Thus

$$\begin{aligned} W_H(\{x_H\}) &= U_H(x_H) = U_H(x_L) = W_H(\{x_L\}), \\ W_L(\{x_L\}) &= U_L(x_L) = 0 \geq U_L(x_H) = W_L(\{x_H\}). \end{aligned}$$

This shows that the pair of singleton menus satisfies ex-ante IC and ex-ante IR. Since the pair (x_L, x_H) maximizes profits subject to (24) and (25), there is no schedule that generates more profits. Thus

Proposition 5. *If the high-type consumers are tempted downwards and their temptation is weak, an optimal schedule is to offer $\{x_L\}$ and $\{x_H\}$ where (x_L, x_H) is a solution to the standard problem with utility functions U_γ.*

High Type with Strong Downward Temptation. In the case where the high-type consumers are tempted downwards strongly, the feasibility of entry fees changes the character of the optimal schedule considerably: the optimal schedule differs from the one in Proposition 3 in many respects, as described below.

First, since ex-post IR does not have to be satisfied, the seller can offer x_L such that $U_L(x_L) = 0$. That is, the seller can offer the optimal single menu for the standard problem where utility functions are U_L for the low type and $U_H + V_H$ for the high type. Since U_L is steeper than $U_L + V_L$, this menu generates more profits than the one in Proposition 3.

Second, the seller can earn even more profits by offering multiple menus, as in Proposition 5. Indeed, if the seller divides the menu into two separate menus $\{x_L\}$ and $\{x_H\}$, ex-post IC becomes vacuous (since menus are singletons) and ex-ante IC for the high type requires $U_H(x_H) \geq U_H(x_L)$, which allows for more profits than ex-post IC for the high type in the single-menu schedule, i.e., $U_H(x_H) + V_H(x_H) \geq U_H(x_L) + V_H(x_L)$. Thus, the schedule in the previous paragraph is dominated by the

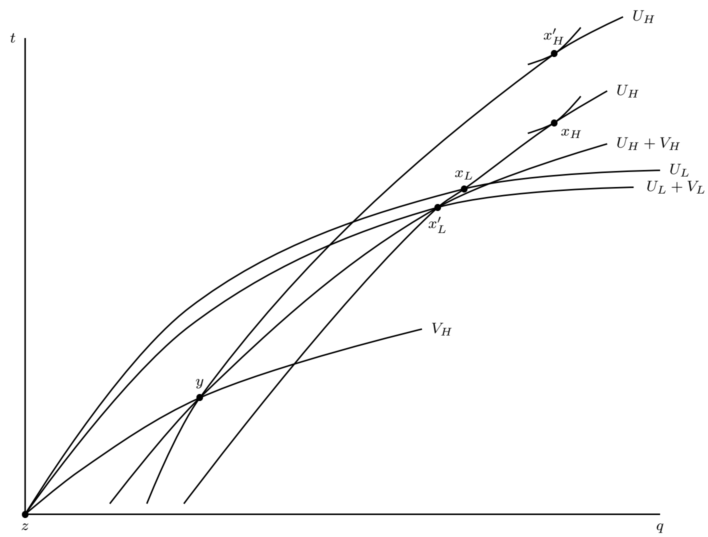

Figure 5: Decoration is profitable

schedule in Proposition 5.

Finally, the seller may be able to do even better by decorating the menu intended for the low type and thereby weakening the ex-ante IC condition for the high type. To see this, let $\{x_L\}$ and $\{x_H\}$ be the pair of menus offered by the schedule in Proposition 5. Suppose further that the scale of the function V_L is small and therefore $U_L + V_L$ and U_L induce similar indifference curves. That is, the low type's temptation is not intense and has little influence on the actual choice. For the example (4), this means that the weight β_L is small. The seller can then offer a pair of menus $\{x'_L, (0,0)\}$ and $\{x'_H\}$ depicted in Figure 5, where $(0,0)$ is a decoration that is added in the menu intended for the low type to tempt the high type. The high type's ex-ante utility from the decorated menu is then

$$W_H(\{x'_L,(0,0)\}) = U_H(x'_L) + V_H(x'_L) - V_H(0,0) = U_H(y) < U_H(x_L) = W_H(\{x_L\}).$$

The strict inequality holds as long as x'_L is sufficiently close to x_L, which is guaranteed if the indifference curve of $U_L + V_L$ is sufficiently close to that of U_L. The inequality implies that the decoration makes the menu intended for the low type less desirable for the high type and hence the seller can extract more from the high type. The offer x'_H is located strictly above x_H and the difference between them is bounded away from zero as x'_L approaches to x_L. Thus, if the indifference curves of $U_L + V_L$ and U_L are sufficiently close to each other, the decorated schedule generates strictly larger profits.

The optimal schedule can be identified as follows. See Figure 6. Pick an offer z such that $U_L(z) = 0$ and another offer x_L that is indifferent to z for $U_L + V_L$ and such that

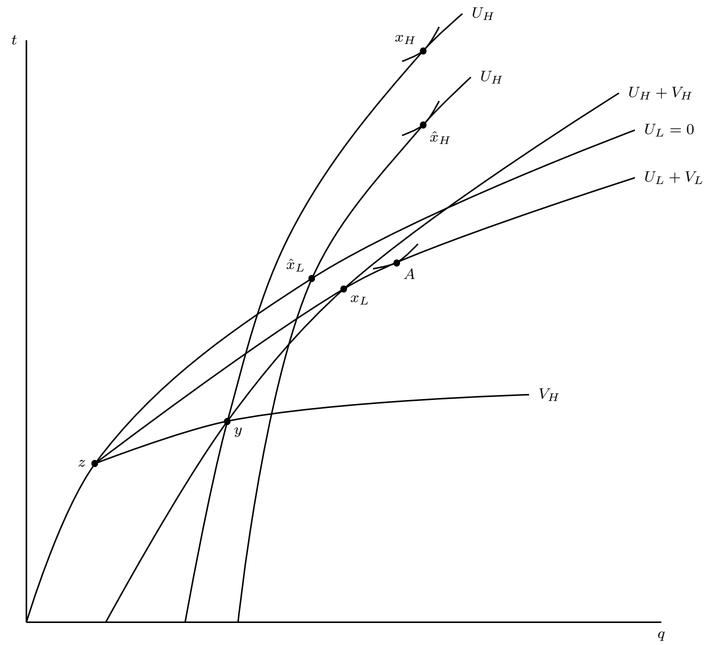

Figure 6: Optimal schedule with strong downward temptation

$x_L \geq z$ (so that $U_L(x_L) \geq 0$). Given a choice of z and x_L, compute the high type's ex-ante utility from the menu $\{x_L, z\}$, which is equal to $U_H(y)$ in the figure. Then, among the offers x_H such that $U_H(x_H) = U_H(y)$, choose one that maximizes the per-consumer profit $\pi(x_H)$. Then $n_H\pi(x_H) + n_L\pi(x_L)$ gives the maximum level of profits $\Pi(x_L, z)$ that the seller can earn given the choice $\{x_L, z\}$. To look for the optimum, first vary x_L between z and A on the $U_L + V_L$ indifference curve to maximize $\Pi(x_L, z)$. This gives an optimal offer $x_L(z)$ as a function of z. Finally, choose z between $(0, 0)$ and $\hat{x}_L$ to maximize $\Pi(x_L(z), z)$, where $\hat{x}_L$ is the offer given to the low type in the schedule of Proposition 5. If the chosen z equals $\hat{x}_L$, the optimal schedule coincides with that in Proposition 5.

The optimal schedule can be characterized analytically for the class of quasi-linear specifications given by (3) and (4) with differentiable functions. The seller's problem is to choose three offers (q_L, t_L), (q_H, t_H), and (q_z, t_z) to maximize the expected profits

$$n_L(t_L - C(q_L)) + n_H(t_H - C(q_H))$$

subject to

$$
\begin{aligned}
u_H(q_H) - t_H = u_H(q_L) - t_L + \beta_H\big[v_H(q_L) - t_L\big] - \beta_H\big[v_H(q_z) - t_z\big], \\
u_L(q_L) - t_L + \beta_L\big[v_L(q_L) - t_L\big] = u_L(q_z) - t_z + \beta_L\big[v_L(q_z) - t_z\big], \\
u_L(q_z) - t_z \geq 0, \\
u_L(q_L) - t_L \geq 0. \qquad (26)
\end{aligned}
$$

The first-order conditions are

$$
\begin{gathered}
u'_H(q_H) = C'(q_H), \\
u'_L(q_L) - n_H u'_H(q_L) - n_L C'(q_L) + J(q_L) = 0, \\
J(q_z)q_z = 0, \quad J(q_z) \geq 0, \quad q_z \geq 0
\end{gathered}
$$

where $J(\cdot)$ is defined by

$$
\begin{aligned}
J(q) &\equiv n_H\beta_H\Big[\frac{u'_L(q) + \beta_L v'_L(q)}{1+\beta_L} - v'_H(q)\Big] + \frac{\beta_L(1-\eta_L)}{1+\beta_L}\big[v'_L(q) - u'_L(q)\big] \qquad (27) \\
&= n_H\beta_H\big[v'_L(q) - v'_H(q)\big] + \frac{\beta_L(1-\eta_L) - n_H\beta_H}{1+\beta_L}\big[v'_L(q) - u'_L(q)\big] \qquad (28)
\end{aligned}
$$

where $\eta_L \geq 0$ is the Lagrange multiplier for (26).

In (28), the first term is negative, and the term $[v'_L(q) - u'_L(q)]$ is also negative. Since $J(q_z) \geq 0$, we have $\beta_L - n_H\beta_H < \beta_L\eta_L$. This inequality has the following implication. If the optimal schedule involves decoration, then $U_L(x_L) > 0$ and so $\eta_L = 0$, which is possible only if

$$
n_H\beta_H > \beta_L.
$$

That is, if this inequality is not satisfied, the decoration does not increase profits and therefore the schedule in Proposition 5 is optimal.

5 Conclusion Remark

We have shown that a monopolist can separate consumers better and raise more profits by offering multiple menus, adding items that are never chosen by consumers, and charging entry fees. On the other hand, clearly our model captures only a fraction of real-life maximization problems of sellers. A particularly important difference from reality comes from our assumption that consumers have complete information about the menus offered by the seller and their own preferences. In reality, we often have to visit a store to find out what are offered and what we are looking for. This gives an advantage to stores with large selections and makes advertisements important. Extending the model in this direction therefore appears to be an interesting topic for research.

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