The Taylor Rule and Optimal Monetary Policy

By Michael Woodford*

John B. Taylor (1993) has proposed that U.S. monetary policy in recent years can be described by an interest-rate feedback rule of the form

\[ i_t = 0.04 + 1.5(\pi_t - 0.02) + 0.5(y_t - \tilde{y}_t) \]

where \( i_t \) denotes the Fed’s operating target for the federal funds rate, \( \pi_t \) is the inflation rate (measured by the GDP deflator), \( y_t \) is the log of real GDP, and \( \tilde{y}_t \) is the log of potential output (identified empirically with a linear trend). The rule has since been subject to considerable attention, both as an account of actual policy in the United States and elsewhere, and as a prescription for desirable policy. Taylor argues for the rule’s normative significance both on the basis of simulations and on the ground that it describes U.S. policy in a period in which monetary policy is widely judged to have been unusually successful (Taylor, 1999), suggesting that the rule is worth adopting as a principle of behavior.

Here I wish to consider to what extent this prescription resembles the sort of policy that economic theory would recommend. I consider the question in the context of a simple, but widely used, optimizing model of the monetary transmission mechanism, which allows one to reach clear conclusions about economic welfare. The model is highly stylized but incorporates important features of more realistic models and allows me to make several points that are of more general validity. Out of concern for the robustness of the conclusions reached, the analysis here addresses only broad, qualitative features of the Taylor rule and attempts to identify features of a desirable policy rule that are likely to hold under a variety of model specifications.

I. The Taylor Principle and Determinacy

A first question about the Taylor rule is whether commitment to an interest-rate rule of this kind, incorporating no target path for any monetary aggregate, can serve to determine an equilibrium price level at all. It is sometimes argued that interest-rate rules as such are undesirable, as they lead to indeterminacy of the rational-expectations equilibrium price level. But this familiar result assumes a rule that specifies an exogenous path for the short-term nominal interest rate; determinacy is instead possible in the case of feedback from an endogenous state variable such as the price level. In fact, many simple optimizing models imply that the Taylor rule incorporates feedback of a sort that suffices to ensure determinacy, owing to the dependence of the funds-rate operating target upon recent inflation and output-gap measures.

Here I consider the question in the context of the “neo-Wicksellian” model derived in Woodford (2000). This reduces to a pair of log-linear relations, an intertemporal “IS” equation of the form

\[ y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + g_t \]

and an expectations-augmented “AS” equation of the form

\[ \pi_t = \kappa(y_t - y^*_t) + \beta E_t \pi_{t+1}. \]

Here \( g_t \) and \( y^*_t \) are composite exogenous disturbances, and the coefficients satisfy \( \sigma, \kappa > 0, 0 < \beta < 1. \)

Let monetary policy be specified by an interest-rate rule of the form

\[ i_t = i^*_t + \phi_i(\pi_t - \tilde{\pi}) + \phi_y(y_t - y^*_t - \tilde{x}), \]

where \( i^*_t \) is any exogenous stochastic process for the intercept, and \( \tilde{\pi} \) and \( \tilde{x} \) are constant “target”

---

* Department of Economics, Princeton University, Princeton, NJ 08544-1021. I thank Jim Bullard, Julio Rotemberg, John Taylor, and John Williams for helpful comments, Argia Sbordone for discussion and for providing the figures, and the NSF for research support.
values for the inflation rate and the output gap, respectively. Then using (4) to eliminate $i_t$ in (2), the system in (2) and (3) can be written in the form

\[ (5) \quad E_t z_{t+1} = A z_t + e_t \]

where $z_t = [\pi_t, y_t]$, and $e_t$ is a vector of exogenous terms. System (5) has a unique stationary solution (assuming stationary disturbance processes) if and only if both eigenvalues of the matrix $A$ lie outside the unit circle. If we restrict attention to policy rules with $\phi_\pi, \phi_y \geq 0$, this condition holds if and only if

\[ (6) \quad \phi_\pi + \frac{1 - \beta}{\kappa} \phi_y > 1. \]

The determinacy condition (6) has a simple interpretation. A feedback rule satisfies the Taylor principle if it implies that, in the event of a sustained increase in the inflation rate by $k$ percent, the nominal interest rate will eventually be raised by more than $k$ percent. (Taylor [1999] stresses this as a criterion for sound monetary policy.) In the context of the model sketched above, each percentage point of permanent increase in the inflation rate implies an increase in the long-run average output gap of $(1 - \beta)/\kappa$ percent; thus a rule of the form represented by (4) conforms to the Taylor principle if and only if the coefficients $\phi_\pi$ and $\phi_y$ satisfy (6). In particular, the coefficient values in (1) necessarily satisfy the criterion, regardless of the size of $\beta$ and $\kappa$. Thus the kind of feedback prescribed in the Taylor rule suffices to determine an equilibrium price level. Woodford (2000) shows that the Taylor principle continues to be necessary and sufficient for determinacy when the family of rules is extended to allow for interest-rate inertia of the kind characteristic of estimated Federal Reserve Board reaction functions.

Another argument against interest-rate rules with a venerable history asserts that targeting a nominal interest rate allows for unstable inflation dynamics when inflation expectations extrapolate recent inflation experience. The basic idea, which originates in Knut Wicksell’s description of the “cumulative process,” is that an increase in expected inflation, for whatever reason, leads to a lower perceived real interest rate, which stimulates demand. This generates higher inflation, increasing expected inflation still further and driving inflation higher in a self-fulfilling spiral. But once again, the classic analysis implicitly assumes an exogenous target path for the nominal interest rate. The sort of feedback from inflation and the output gap called for by the Taylor rule is in fact of the sort needed to damp such an inflationary spiral.

James Bullard and Kaushik Mitra (2000) consider the stability of rational-expectations equilibrium under a form of adaptive learning dynamics in the model sketched above, again in the case of a policy rule of form (4). They find that condition (6) is also necessary and sufficient for “expectational stability” of the equilibrium (i.e., for convergence of the learning dynamics to rational expectations). Thus they confirm the Wicksellian instability result in the case of feedback from inflation or the output gap that is too weak; but this is not a problem in the case of a rule that conforms to the Taylor principle. Taylor’s emphasis upon raising interest rates sufficiently vigorously in response to increases in inflation is again justified.

II. Inflation and Output-Gap Stabilization Goals

Even granting that the Taylor rule involves feedback of a kind that should tend to exclude instability due purely to self-fulfilling expectations, one must consider whether the equilibrium determined by such a policy is a desirable one. The dependence of the funds-rate target upon the recent behavior of inflation and of the output gap is prescribed in order to damp fluctuations in those variables, and Woodford (2000) shows that in the simple model described above it has this effect. But are inflation and output-gap stabilization in fact sensible proximate goals for monetary policy?

Woodford (1999a) argues that both inflation and output-gap stabilization are sensible goals of monetary policy, as long as the “output gap” is correctly understood. In fact, the paper shows that in the context of the simple optimizing model behind equations (2) and (3), it is possible to motivate a quadratic loss function as a second-order Taylor-series approximation to the expected utility of the economy’s representative household,
equal to the expected discounted sum of period losses for certain coefficients \( \lambda > 0 \) and \( x^* > 0 \):

\[
L_t = \pi_t^2 + \lambda(y_t - y_t^* - x^*)^2.
\]

Here \( y_t^* \) is the same exogenously varying natural rate of output as in (3). This is defined as the equilibrium level of output that would obtain in the event of perfectly flexible prices; in general, this will not grow with a smooth trend, as a result of real disturbances of many kinds.

There is a simple intuition for the two stabilization objectives in (7). To the degree of approximation discussed in Woodford (1999a), the efficient level of output \( y_t^* \) (the same for all goods, in the presence of purely aggregate shocks) varies in response to real disturbances in exactly the same proportion as does the flexible-price equilibrium level \( y_t^* \), the two differ at all times by the constant factor \( x^* > 0 \).

The average squared deviation of the log output of each good from the efficient level can then be decomposed into the squared deviation of the average log output \( y_t \) from the efficient level and the variance of the log output level across individual goods. This latter output dispersion term is in turn proportional to the dispersion of prices across goods due to imperfect synchronization of price changes, which in the case of a particular model of staggered price-setting is proportional to the square of the inflation rate. This last result [and hence the exact form (7)] is somewhat special. But the connection between price dispersion and instability of the general level of prices holds more generally, so that a goal of inflation stabilization may be justified on more general grounds.

We thus find that the stabilization goals implicit in the Taylor rule have a sound theoretical basis, subject to two important qualifications. The first is that Taylor’s classic formulation of the rule seeks to stabilize inflation around a target rate of 2 percent per annum. Instead, the welfare-theoretic loss function (7) implies that the target rate of inflation should be zero, as this is the rate that minimizes relative-price distortions associated with imperfect synchronization of price changes. Taking account of additional frictions may modify this conclusion, but in general this will also justify the introduction of additional stabilization goals as well (Woodford, 1999a).

The second qualification is that the “output gap” that one should seek to stabilize is the gap between actual output and the natural rate of output defined above. This contrasts with the assumption made in Taylor’s (1993) comparison between the proposed rule and actual U.S. policy, where the output gap is assumed to be measured by output relative to a deterministic trend. In theory, a wide variety of real shocks should affect the growth rate of potential output in the relevant sense; as shown in Woodford (2000), these include technology shocks, changes in attitudes toward labor supply, variations in government purchases, variation in households’ impatience to consume, and variation in the productivity of currently available investment opportunities, and there is no reason to assume that all of these factors follow smooth trends. As a result, the output-gap measure that is relevant for welfare may be quite different from simple detrended output.

One source of evidence that this is so comes from a comparison of a detrended output series with the behavior of real unit labor costs. In the model that underlies both (3) and (7), the output gap \( y_t - y_t^* \) appears because the average ratio of marginal supply cost to price is an increasing function of it; this cost/price ratio determines both the incentive to raise prices in (3) and the deadweight losses in (7). A measure of real marginal cost is thus an appropriate proxy for the relevant output gap. But in quarterly U.S. data, variations in real unit labor cost are negatively correlated with detrended real GDP (Fig. 1). Moreover, Argia M. Sbordone (1998) shows that equation (3) gives a very poor account of U.S. inflation when detrended real GDP is used as the gap measure but explains much of the medium-frequency variation when real unit labor costs are used instead (see also Jordi Gali and Mark Gertler, 1999.).

In each panel of Figure 2, a small, unrestricted vector autoregression (VAR) is used to forecast the future evolution of the gap proxy, and then (3) is “solved forward” to obtain a predicted quarterly inflation series. The assumed value of \( \beta \) is 0.99; in panel (b), the elasticity \( \kappa \) is chosen to minimize the mean-square prediction error, while in panel (a) an arbitrary positive value is assumed (as predicted inflation is negatively correlated with actual inflation in any event). The dramatic improve-
ment in fit in panel (b) suggests that real unit labor cost is a much better measure of the true output gap, at least for purposes of explaining inflation variation. But Figure 1 indicates that the use of such an alternative measure would matter greatly for practical implementation of the Taylor rule.

III. Optimal Responses to Real Disturbances

Supposing now that a central bank responds to appropriate measures of the economy’s departure from its stabilization goals, I turn to a subtler question. Is the contemporaneous feedback prescribed in (1) sufficient to ensure an optimal response of policy to real disturbances? The answer is that in general, a rule this simple (one that avoids any direct response to other information about the real disturbances, and that incorporates only contemporaneous feedback from the goal variables) must be suboptimal.

As a simple illustration of this, suppose again that all real disturbances affect \( y_t^n \) and \( y_t^e \) equally. Then it is optimal to completely stabilize both inflation (at zero) and the output gap (at the level consistent with zero inflation). However, this is not possible with a rule of form (4) where the intercept is a constant, for if in equilibrium inflation and the output gap are both constant, such a rule would prescribe a constant interest rate. Instead, in the optimal equilibrium the interest rate must satisfy \( i_t = r^n_t \), where

\[
(8) \quad r^n_t = \sigma^{-1}[g_t + E_t(y_{t+1}^n - y_t^n)]
\]

is the Wicksellian natural rate of interest (i.e., the equilibrium real rate under flexible prices). In our simple model, \( r^n_t \) is an exogenous process (independent of monetary policy), but it should vary in response to a wide range of real disturbances.

A policy rule of form (4) is consistent with the optimal equilibrium, however, if it satisfies two requirements. First, \( \phi_p \) and \( \phi_y \) must satisfy (6), in order to ensure determinacy. Second, the rule must include a time-varying intercept \( i_t^* = r^n_t \), for consistency with a stable inflation rate and output gap. Such a variable intercept is actually in the spirit of Taylor’s prescription, which describes the intercept as incorporating “the central bank’s estimate of the equilibrium real rate of interest” (Taylor, 1999 p. 325). But for his empirical illustration, Taylor assumes this to be a constant (2 percent), while in reality there may be substantial variation in the natural
rate. Failure to adjust the intercept to track variation in the natural rate of interest will result in fluctuations in inflation and the output gap, just as in Wicksell’s analysis (Woodford, 2000).

Of course, in a more realistic analysis, the optimal equilibrium is unlikely to involve complete stabilization of inflation and output. For example, while many real disturbances should affect \( y_n \) and \( y_e \) equally, others may not (Woodford, 1999a; Marc P. Giannoni, 2000), in which case it is no longer possible to fully stabilize both inflation and the welfare-relevant gap \( y - y^*_e \). Alternatively, it may be desirable to accept some variability of inflation and the output gap for the sake of less variable nominal interest rates. In these cases, the optimal equilibrium will involve some fluctuations in inflation and the output gap in response to real disturbances; but contemporaneous feedback from the goal variables is still generally insufficient to bring about optimal interest-rate responses, for when the private sector is forward-looking, optimal policy almost always involves a commitment to some later response to current shocks, which then implies that policy must be history-dependent at that later date.

In particular, in the model sketched above, it is optimal for the nominal interest rate to be adjusted only gradually in response to new information about the natural rate of interest (Woodford, 1999b; Giannoni, 2000). This is because (2) implies that aggregate demand is as much affected by expected future short real rates of interest as by current short rates. Thus a predictable policy of gradual interest-rate adjustment allows substantial effects on aggregate demand without requiring large swings in short-term interest rates. The advantages of interest-rate inertia in a generalized Taylor rule have also been shown through numerical analysis in the context of more complex econometric models that nonetheless incorporate realistic degrees of forward-looking private-sector behavior (e.g., John C. Williams, 1999).

IV. Conclusions

The Taylor rule incorporates several features of an optimal monetary policy, from the standpoint of at least one simple class of optimizing models. The response that it prescribes to fluctuations in inflation or the output gap tends to stabilize those variables, and stabilization of both variables is an appropriate goal, at least when the output gap is properly defined. Furthermore, the prescribed response to these variables counteracts dynamics that could otherwise generate instability due to self-fulfilling expectations.

At the same time, the original formulation of the rule may be improved upon. The measure of the output gap suggested in Taylor’s (1993) empirical discussion may be quite different from the theoretically correct measure, as the natural rate of output should be affected by a wide variety of real disturbances. The empirical discussion also assumes a constant intercept, but a desirable rule is likely to require that the intercept be adjusted in response to fluctuations in the Wicksellian natural rate of interest, and this too should vary in response to a variety of real disturbances. Finally, the classic formulation assumes that interest rates should be set on the basis of current measures of the target variables alone, but an optimal rule will generally involve a commitment to history-dependent behavior; in particular, more gradual adjustment of the level of interest rates has important advantages. These considerations call for further research to improve measurement of the natural rates of output and of interest, and to analyze the consequences of inertial rules in the context of more detailed models.

REFERENCES


Taylor, John B. “Discretion versus Policy Rules


5. Sarah Zubairy. 2013. INTEREST RATE RULES AND EQUILIBRIUM STABILITY UNDER DEEP HABITS. *Macroeconomic Dynamics* 1-18. [CrossRef]


27. JOHN DUFFY, WEI XIAO. 2011. Investment and Monetary Policy: Learning and Determinacy of Equilibrium. *Journal of Money, Credit and Banking* **43**:5, 959-992. [CrossRef]


32. Kevin Lee, Kalvinder K. Shields. 2011. Decision-making in hard times: What is a recession, why do we care and how do we know when we are in one?. *The North American Journal of Economics and Finance* **22**:1, 43-60. [CrossRef]


38. Philip Arestis, Alexander Mihailov. 2010. CLASSIFYING MONETARY ECONOMICS: FIELDS AND METHODS FROM PAST TO FUTURE. *Journal of Economic Surveys* no-no. [CrossRef]


43. John B. Taylor, John C. Williams. Simple and Robust Rules for Monetary Policy 3, 829-859. [CrossRef]


45. ICHIRO MUTO. 2009. ESTIMATING A NEW KEYNESIAN PHILLIPS CURVE WITH A CORRECTED MEASURE OF REAL MARGINAL COST: EVIDENCE IN JAPAN. Economic Inquiry 47:4, 667-684. [CrossRef]

46. PETER TILLMANN. 2009. Optimal Monetary Policy with an Uncertain Cost Channel. Journal of Money, Credit and Banking 41:5, 885-906. [CrossRef]


50. GISLE JAMES NATVIK. 2009. Government Spending and the Taylor Principle. Journal of Money, Credit and Banking 41:1, 57-77. [CrossRef]


52. T MOLODTSOVA, A NIKOLSKORZHEVSKYY, D PAPELL. 2008. Taylor rules with real-time data: A tale of two countries and one exchange rate#. Journal of Monetary Economics 55, S63-S79. [CrossRef]


55. Silvia Sgherri. 2008. Explicit and implicit targets in open economies. Applied Economics 40:8, 969-980. [CrossRef]


57. WEI XIAO. 2008. INCREASING RETURNS AND THE DESIGN OF INTEREST RATE RULES. Macroeconomic Dynamics 12:01. [CrossRef]


67. EFELEM CASTELNUOVO. 2007. TAYLOR RULES AND INTEREST RATE SMOOTHING IN THE EURO AREA. *The Manchester School* 75:1, 1–16. [CrossRef]

68. M BRUCKNER, A SCHABERT. 2006. Can money matter for interest rate policy?#. *Journal of Economic Dynamics and Control* 30:12, 2823–2857. [CrossRef]


