International Emission Permits: 
Equity and Efficiency

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International Emission Permits: Equity and Efficiency

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ABSTRACT. Not all possible distributions of a given total of emission permits are compatible with the attainment of first-best Pareto efficiency. In fact, of the infinitely many ways of distributing a given total of permits between a fixed number of parties, only a finite number can lead to efficiency. We can therefore evaluate permit allocations not only in terms of their equity, but also in terms of their efficiency. If there are no other redistributive instruments in the policy environment, the traditional orthogonality of equity and efficiency does not hold here. This has important implications for arguments about the initial international distribution of entitlements to produce carbon dioxide.

Key words: Carbon dioxide, environment, global warming, emissions, emission permits, tradeable permits, public goods.

JEL classification: Q2, H4.

CONTENTS

1. EQUITY, EFFICIENCY AND CO₂ ABATEMENT

The atmospheric concentration of carbon dioxide has become a matter of widespread concern in the last decade. It is generally recognized that it has the capacity to change the global climate in ways which are potentially very harmful but are presently not forecastable (for a review see Chichilnisky and Heal [3]). Consequently countries at the 1992 “Earth Summit” in Rio de Janeiro agreed to cut back CO₂ emissions to their

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1990 levels by the end of the century. This policy could easily cost several percent of GNP (Weyant [21]). In conformity with the conclusions of the “Earth Summit”, the US administration has recently made a very tentative move in the direction of policies to reduce $CO_2$ emissions. In this it is lagging some way behind other industrial countries, most of which have already made more definitive moves.

The adoption of an ambitious target for $CO_2$ emission has naturally brought in its wake a focus on the policy the instruments available for achieving this goal. This comes at the same time as an increasing awareness of the economic burden of environmental protection, and the two together have produced a particular interest in market-based policy instruments. There are two frameworks from which such instruments emerge: the Pigouvian framework of corrective taxes and subsidies, as developed by Meade [15], and the Coasian tradition of property rights [7], which has evolved to tradeable permits. The OECD has recently studied in detail the possibility of a global carbon tax (see Coppel [8] and Chichilnisky [2]), and the principles behind this have already been subject to detailed examination (see Hartwick [13] for an interesting perspective). Tradeable permits are already in use in the USA for regulating a variety of emissions: $SO_2$, lead and various water-borne wastes. The introduction of a global permit market for $CO_2$ is now firmly on the international agenda (Chichilnisky and Heal [6] and Grubb [12]). Tradeable permits have many economic attractions as instruments for controlling atmospheric emissions (see for example Dales [9] or any standard environmental text), and there is now a growing body of literature on their practical application (Noll [17], Stavins [19], Stavins and Hahn [20]).

This paper is about the implementation of a global market for $CO_2$ emission permits. We point out that such a market has an important characteristic which had not previously been noted, a characteristic which has quite significant economic and political implications. The point which we highlight is an unexpected link between equity and efficiency: the initial distribution of property rights or emission permits determines whether or not a global $CO_2$ permit market will operate efficiently. Prior to now, it has been generally assumed that the manner in which emission permits are initially distributed will not affect the efficiency of the market—though it will of course affect the distribution of income resulting from the operation of the market. This is the original Coase position [7]: that whatever the initial distribution of permits, the market can bring about a Pareto efficient allocation of resources. In fact a stronger claim is sometimes made: that the equilibrium allocation of resources is not affected by the initial distribution of permits. Clearly the conditions for this stronger claim to be true are very restrictive indeed—a total absence of income effects (see Milgrom and Roberts, chapter 2 [16]).

We show below that the manner in which emission rights are initially distributed determines the possibility of the market attaining a Pareto efficient outcome. There are many ways (uncountably many, in fact) of distributing a given total of emission
rights between participants: in general, only a finite number of these distributions will be compatible with the efficient operation of the market. In this case equity and efficiency are not orthogonal, as in the first and second theorems of welfare economics. How does this happen?

The key to understanding this result, is to note that the atmospheric concentration of CO$_2$ is a global public good. Carbon dioxide mixes thoroughly in the atmosphere: the concentration is therefore fairly uniform over the globe. It is, however, a privately produced public good: it is produced by every individual who runs a car or a heating furnace, and by every firm operating transportation or burning fuel in any other way. Carbon dioxide emission, the production of a public good, is in fact a by-product of the consumption and production of private goods.

These points were made in Chichilnisky [2] and Chichilnisky and Heal [4], where this simple observation was shown to have other far-reaching consequences. In particular, these papers establish that the equalization of marginal abatement costs across countries is not sufficient for Pareto efficiency, and is also not necessary in the sense that Pareto efficient allocations may have different marginal costs. In this paper we show that this line of argument, when developed further, has the implication already mentioned, namely that efficiency and distribution cannot be separated.

We also investigate the extent to which a Lindahl equilibrium, rather than a Walrasian equilibrium, is the appropriate concept when seeking efficiency in permit markets. There is a simple reason why this might be so: a Lindahl equilibrium is the only market equilibrium that we know which leads to efficiency with public goods (see Foley [11]). And a permit market is in fact a market which determines the production of public goods. We might therefore expect that efficiency would require the key feature of a Lindahl equilibrium, which is a multiplicity of prices. In a Lindahl equilibrium, each producer of a public good is paid for her production by each consumer, and the per unit payment typically varies from consumer to consumer. In section 5 we establish conditions that are sufficient for uniform prices to achieve efficiency in a permit market.

2. Efficiency and International Emissions

Following the models in Chichilnisky [2] and Chichilnisky and Heal [4], we consider a world economy with $I$ countries, $I \geq 2$, indexed by $i = 1, \ldots, I$. Each country has a utility function $u_i$ which depends on its consumption of a vector of private goods $c_i = (c_{i,1}, c_{i,2}, \ldots, c_{i,m})$ where $m$ is the number of private goods, and also on the quality of the world's atmosphere, $a$, which is a public good. Formally, $u_i(c_i, a)$ measures welfare, where $u_i : \mathbb{R}^{m+1} \rightarrow \mathbb{R}$ is a continuous, strictly concave and increasing function. It is assumed to be twice continuously differentiable. The quality of the atmosphere, $a$, is measured by for example the reciprocal or the negative of its concentration of CO$_2$. The concentration of CO$_2$ is "produced" by emissions of carbon, which are positively
associated with the levels of production of private goods. Let $y_i$ be a vector giving the production levels of the $m$ private goods in country $i$.

$$a = \sum_{i=1}^{I} a_i, \quad a_i = \Phi_i(y_i), \text{ for each } i = 1, \ldots, I, \text{ and } \frac{\partial \Phi_i}{\partial y_{i,l}} < 0 \forall i.$$  \hspace{1cm} (1)

$a$ is a measure of atmospheric quality overall, and $a_i$ is an index of the abatement carried out by country $i$. The “production functions” or “abatement functions” $\Phi_i$ are continuous, continuously differentiable and strictly concave, and show the level of abatement or quality of the atmosphere decreasing with the output of consumption$^1$. An allocation of consumption and abatement across all countries is a vector $(c_1, a_1, \ldots, c_I, a_I) \in \mathbb{R}^{(m+1)I}$.

Feasibility in this case is defined by constraint (1) and by the condition that the total consumption of each private good worldwide be equal to the total production, i.e.,

$$\sum_{i=1}^{I} c_i = \sum_{i=1}^{I} y_i$$  \hspace{1cm} (2)

Constraint (2) allows private goods to be transferred freely between countries, i.e., it allows unrestricted lump sum international redistributions. This is a rather strong and unrealistic assumption, which gives a full first-best solution. It is not of course equivalent to modeling free trade between countries as no balance of payments condition is imposed (see Chichilnisky and Heal [5]). Free trade would be modeled by the constraint

$$\left(\sum_{i=1}^{I} c_i - \sum_{i=1}^{I} y_i\right) p = 0$$  \hspace{1cm} (3)

where $p \in \mathbb{R}^m$ is a world price vector. This condition requires the value of the difference between consumption and production to be zero at world prices, which implies that the value of goods which are imported, and for which consumption exceeds production, equals the value of goods which are exported and for which production therefore exceeds consumption.

An allocation is called feasible if it satisfies the constraints (1) and (4). A feasible allocation $(c_1^*, a_1^*, \ldots, c_I^*, a_I^*)$ is Pareto efficient if there is no other feasible solution at which every country’s utility is at least as high, and one’s utility is strictly higher, than at $(c_1^*, a_1^*, \ldots, c_I^*, a_I^*)$. A Pareto efficient allocation can be characterized as a solution to the problem of maximizing the utility of a designated country, subject to the other

$^1$We can suppose that the functions $\Phi_i$ embody information about countries’ initial endowments of goods. By assuming strict concavity, we are ignoring the problem of the fundamental non-convexity associated with externalities (Starrett [18]).
countries all reaching prescribed utility levels. The solutions of this problem as the prescribed utility levels vary over all feasible values describe the utility possibility frontier. The characterization of Pareto efficiency is formalized and solved in the Appendix: a solution has to satisfy the following conditions:

\[
\frac{\partial u_i}{\partial c_{i,l}} = \lambda_k \frac{\partial u_k}{\partial c_{i,l}} \quad \forall l = 1, \ldots, m \text{ and } \forall k \neq i.
\]

(4)

where country \(i\) is the designated country whose utility is being maximized, and \(\lambda_k\) is a Lagrange multiplier associated with the constraint that country \(k\) reach a specified welfare level, and

\[
\frac{\partial \Phi_i}{\partial y_{i,l}} = \frac{\partial u_i}{\partial c_{i,l}} \quad \forall l, \text{ and for } k \neq i, \quad \frac{\partial \Phi_k}{\partial y_{k,l}} = -\frac{\lambda_k \partial u_k}{\partial c_{k,l}} \quad \forall l
\]

(5)

Note that the marginal cost of abatement in country \(i\) in terms of good \(l\) is just the reciprocal of the marginal productivity with respect to \(l\) of the function \(\Phi_i\):

\[
MC_{i,l}(a_i) = -\frac{1}{\frac{\partial \Phi_i}{\partial y_{i,l}}}
\]

(6)

Chichilnisky and Heal [4] established the following proposition in the case of one private good. The extension to the present case is immediate.

Proposition 1. At a Pareto efficient allocation \((c_i^*, a_i^*, \ldots, c_i^*, a_i^*)\), in each country the marginal cost of abatement \(MC_i(a_i^*)\) in terms of private good \(l\) is inversely proportional to the marginal valuation of the private good \(l, \lambda_i \partial u_i / \partial c_{i,l}\). In particular, the marginal costs will be equal across countries if and only if the marginal valuations of the private goods are equal, i.e., for each good \(l, \lambda_i \partial u_i / \partial c_{i,l}\) is independent of \(i\).

It follows that with constraint (2), marginal costs will always be equalized, as private goods can always be shifted between countries via lump sum redistributions to equate their marginal valuations. However, if each country is required to consume what it produces, or to trade internationally subject to a balance of trade constraint, this will not be true (see Chichilnisky and Heal [5]).

3. INTERNATIONAL EMISSION MARKETS

So far we have characterized first best Pareto efficient allocations, i.e., allocation which are Pareto efficient in a framework in which lump sum redistribution are possible. Next we introduce an international market for tradeable permits, and investigate the efficiency of the equilibria in this market. Now we model a policy-relevant situation and assume that the initial distribution of emission permits is the only policy variable.
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that can address distributional issues, and in particular that unrestricted lump sum redistributions of private goods are not possible. We assume that each country is given an initial endowment of permits to emit $E_i$ units of $CO_2$, where $\sum_i E_i = E^*$, the desired level of total emissions. They can trade these as price takers in a market in which there is a single price $p_e$ for a permit to emit one unit. We shall remark below that the assumption of a single price for the emission permits could be a restrictive one: given the that the resource allocation problem involves public goods, there is a presumption in favor of a more complex pricing system.

If the number of units of $CO_2$ emitted exceeds the number of permits a country has, it has to buy the difference in the permit market: otherwise, it can sell excess permits and use the proceeds to buy private goods at prices $p_i$. A country therefore maximizes its utility $u_i(a_i, a)$ subject to the following budget constraint:

$$\sum_i c_{i,i} p_i = \sum_i y_{i,i} p_i + p_e \{ E_i + a_i \}$$  \hspace{1cm} (7)

In (7) the level of abatement $a_i$ enters with a positive sign as $p_e$ multiplies the difference between the endowment of permits to emit and the actual level of emissions, $e_i$. Clearly emissions and abatement are negatively related and we shall take it for simplicity that $e_i = -a_i$. This budget constraint requires that for each country the value of consumption equal the value of production plus the net revenue from the sale of permits. Note that (7) can be rewritten as

$$\left( \sum_i c_{i,i} - \sum_i y_{i,i} \right) p_i = p_e \{ E_i + a_i \}$$  \hspace{1cm} (8)

Here the left hand side is the difference between the value of domestic consumption and production, i.e., the balance of trade. A surplus of consumption over production (i.e., a position of net imports) is funded by the revenue generated by sales of permits in international markets. Conversely, a net purchase of permits in international markets has to be matched by a surplus of production over consumption and hence a net export position. This interpretation of the budget constraint, and a comparison of the balance of trade condition (8) with the actual budget constraint (3), makes it clear that controlling the initial endowments of emission rights, net of actual emissions, act as a substitute for lump sum transfers by allowing countries to avoid the need to balance budgets internationally. This point will be important later in the argument.

Each country seeks to maximize its utility $u_i(c_i, a)$ subject to the budget constraint (7) and to the production relations given in (1). We shall assume that in so doing it supposes the total level of emissions to be fixed at $E^*$, the desired total level. This in effect implies the existence of a credible inter governmental agency which sets and implements global emission targets: an alternative, which we do not explore
here, would be to look for a Nash equilibrium in countries’ abatement levels. In this case, each country would observe the emissions of each other and then choose its optimal emission level on the assumption that these levels are fixed. (For a similar development, see Dasgupta and Heal [10], chapter 3).

In the case of a fixed total level of emissions \( E^* \), each country chooses consumption levels and abatement or emission levels to satisfy

\[
\frac{\partial u_i}{\partial c_{i,t}} = p_i \frac{\partial u_k}{\partial c_{i,t}} \quad (9)
\]

and

\[
\frac{\partial \Phi_i}{\partial y_{i,t}} = p_i p_e \quad (10)
\]

These are standard conditions: (9) just requires that marginal rates of substitution between goods be equated to their price ratios, and (10) requires tangency between the production possibility frontier and an iso-profit hyperplane. The latter implies in particular that for given prices, levels of production (and therefore also of emission) are determined independently of the utility function. (Of course, in equilibrium the prices will depend on preferences.)

How do the first order conditions (9) and (10) chosen by the country compare with the conditions (4) and (5) which describe Pareto efficient allocations? Clearly (10) is the same as (5) provided that

\[
\sum_k \lambda_k \frac{\partial u_k}{\partial c_{i,t}} = \frac{p_i}{p_e} \quad (ii)
\]

This condition can only hold if \( \lambda_k \) and \( \frac{\partial u_k}{\partial c_{i,t}} \) are independent of \( i \) and \( k \). (Note that without the assumption of a single price \( p_e \) for permits, there would be no such restriction.) Condition (4) required for Pareto efficiency automatically implies this. However, condition (9) from the countries’ optimization problems does not. So utility maximization subject to the budget constraint (7) does not lead to the conditions needed for efficiency. There is an additional requirement represented by (4), namely that \( \frac{\partial u_i}{\partial c_{i,t}} = \lambda_k \frac{\partial u_k}{\partial c_{i,t}} \forall k, \forall k \neq i \). This condition would of course be satisfied if there were policy instruments available to redistribute resources without restriction across countries—if for example lump sum redistributions were possible. In the absence of such instruments, what is required to ensure that (4) is met and efficiency attained in the permit market?

4. EQUITY AND EFFICIENCY IN PERMIT MARKETS

Condition (4) requires that for each good, its marginal social valuation be equal for every country. This is clearly a condition on the distribution of income or wealth.
Let us look in more detail at the determinants of the terms $\frac{\partial u_i}{\partial c_i}$. As $u_i = u_i(c_i, E^*)$, where $E^*$ is fixed, the derivatives of $u_i$ with respect to consumption can depend only on consumption levels. These in turn depend via the budget constraint (7) on prices $p_i$, production levels $y_{ij}$, abatement levels $a_i$ and initial endowments of emission rights $E_i$. Once prices are given, production and abatement levels are fully determined via (10). In the absence of policy instruments which can effect unrestricted redistributions across countries, the only variables then available for ensuring that marginal social valuations of consumption are equalized across countries are therefore the initial allocations of permits, and only those initial permit allocations which ensure that (4) is satisfied will lead to Pareto efficient allocations. We formalize this below, and show that very few initial allocations satisfy this condition.

4.1. An example—one private good and two countries. Before giving a general treatment of these results, we give a diagrammatic analysis of the case of a single private good with two identical countries. Figure 1 shows the abatement-production frontier and the preferences over combinations of public and private goods for each country. We suppose a total emission level $E^*$ to have been chosen: this is of course assumed to be a level associated with a Pareto efficient allocation on the utility possibility frontier. Then the total abatement level of the two countries must sum to $-E^*$: as they are identical, each must produce a level of abatement of $-E^*/2$. Each country’s production of the private good is now determined to be the level that corresponds to an abatement level of $-E^*/2$, and the relative price of the public and private good is therefore determined to be the slope of the frontier at this point. Each country’s consumption of abatement is $A^* = -E^*$: its consumption of the private good is determined by maximizing utility subject to the equation

$$c_i = y_i + p_e(E_i + a_i)$$

where $c_i$ and $y_i$ are country $i$'s consumption and production of the single private good, and $p_e$ is the relative price of the emission permits. Here $y_i$, $p_e$ and $a_i$ are all fully determined from the total level of emissions $E^*$. Hence only $E_i$, the initial endowment of permits, is available to control $c_i$. This variable has therefore to be used to ensure that marginal valuations of the private good satisfy the condition (4) needed for Pareto efficiency.

Figure 1 illustrates how this can be done. If both countries are given endowments of permits equal to their levels of emission, neither will buy or sell private goods for permits, and each will consumne the same amount of the private good, namely the amount that they produce. They will consume levels of the private good given by the horizontal coordinate of the production point in figure 1. Their
Figure 1: two identical countries producing one private good and emissions. Each produces abatement of $A^*/2$, where $A^*$ is the total level of emissions. This determines the output of private goods $Q^*$ and the price ratio, as shown. If each country is given $A^*/2$ permits, each will consume $Q^*$ of the private good and have the same marginal utility for this good. A country with permits in excess of $A^*/2$ will sell permits and buy the private good arriving at point $a$ on the line tangent to the production point: the other country will be at the symmetric point $a'$. $b$ and $b'$ form a similar symmetric pair. At the pairs $(a,a')$ and $(b,b')$ the marginal utilities of consumption differ. The country at $b$ consumes at point $b^*$, where the level of abatement is $A^*$ and the consumption level is production plus imports of private goods. Similarly the country at $b'$ consumes at $b''$. 
consumption of the public good abatement will be the sum of the production levels of both countries, which is $A^*$. Hence each country's consumption vector has a vertical coordinate equal to $A^*$ and a horizontal coordinate equal to its consumption of the private good, which in general is production (the same for both countries) plus imports
from the sale of permits or minus exports to pay for the purchase of permits.

Consider the case in which both countries have an initial allocation of permits equal to their production of CO₂. As they both neither import nor export the private good and so consume and produce the same amounts, and also consume the same amount of the public good (by definition), their marginal valuations of the private goods must be the same. Suppose now that condition (4) requires for efficiency that \( \frac{\partial u_1}{\partial c_{1,l}} = \lambda_2 \frac{\partial u_2}{\partial c_{2,l}} \) where 1 and 2 are the two countries, \( l \) denotes the single private good, and \( \lambda_2 < 1 \). Then to satisfy (4) country two's consumption of the private good has to be decreased and country one's increased from their common production level. This can be achieved by giving country one an endowment of permits in excess of its emission, and country two an endowment less than its emission. Country one then increase its consumption of the private good by selling its spare permits and using the proceeds to buy the private good, whereas two is forced to sell the private good to buy permits. One's marginal utility of the private good will be less than two's, and the ratio will decrease continuously from unity as one's initial endowment of permits is raised above the emission level corresponding to its production of the private good (and two's is correspondingly reduced, as the total must be constant at \( E^* \)).

Consider the straight line through the countries' production points tangent to the production frontier, as shown in figure 1. Each country produces a mix of abatement and private good given by the point of tangency and then trades private goods for emission permits along the line tangent to the production frontier. If it has more permits than needed (i.e., more than \( E^*/2 \)) it will add consumption of the private good by selling permits and buying the private good along the tangency line, whose slope is the relative price of permits and the private good. As its move along this line, its consumption of abatement remains constant: it is selling surplus permits, not abatement. However, its consumption of the private good changes. The other country will be symmetrically placed on this line relative to the production point. In this way we can reach an allocation at which all markets will clear, total emissions will be \( E^* \), and condition (4) needed for efficiency will be satisfied. We can do this by picking the permit allocations and therefore consumption levels of the private good correctly. As the ratio of the countries' marginal utilities changes continuously with their initial allocations of permits, there will generally be at most a finite number of initial allocations at which the efficiency conditions hold. In fact, in this simple example, one would expect that there would be just one initial distribution of permits which will lead to efficiency. This argument establishes the following result:

**Proposition 2.** Let \( E^* \) be the level of total emissions at a Pareto efficient allocation of resources in the economy described in section 2 with a single private good and two identical countries. Then only a finite number of ways of allocating the total emission \( E^* \) among the countries as initial endowments will lead to market equilibria which
are Pareto efficient.

The diagrammatic analysis illustrating proposition 1 can in fact be pushed further, as in figure 2. As figure 1 shows, each possible distribution of the total emission permits \( E^* \) between the two countries leads them to a pair of levels of consumption of the private good given by the horizontal coordinates of pairs of points such as (a,a') or (b,b') which are symmetrically placed on the line which is tangent to the production frontier at the production point. These pairs of points in turn give rise to consumption vectors for the public and private and private goods together represented by points such as b" and b* in figure 1. From figure 1 we can ascertain the utility levels of these points. Suppose we plot the utility levels arising from all such possible distributions of the total \( E^* \) permits: what does this set of points look like?

We know that few points will be Pareto efficient, so that this must form a curve largely inside the utility possibility frontier, touching this frontier at at most a finite number of points (see the following proposition for a complete formal proof). In fact in the present two-country fully symmetric case it is easy to see that once we have an allocation of permits that satisfies (4), departures from this allocation increase the difference from equality of the two sides in (4), so that the efficient allocation is unique. Figure 2 therefore illustrates the set of utility vectors associated with different allocations of the total of \( E^* \) permits, and also shows the overall utility possibility frontier. Each point on the frontier corresponds to a different total emission level and hence to a different total number of permits, and for each point on the frontier there is one way of allocating the corresponding total of permits which is efficient and gives the utility vector on the utility possibility frontier. Lin [14] solves analytically for the curves in figure 2 for specific utility and production functions.
Overall utility possibility frontier, given unrestricted lump-sum redistributions.

Utility vectors at the equilibria attainable by redistributing a total number of emission permits equal to total emissions at point A. Only one way of distributing this total leads to a Pareto efficient allocation at equilibrium. There is a similar curve corresponding to each point on the utility possibility frontier.

Figure 2: utility levels associated with different allocations of a fixed total of permits
4.2. The general case. The above result in fact holds for the general case, but the argument is less intuitive. Formally, we establish the following proposition:

**Proposition 3.** Let $E^*$ be the level of total emissions at a Pareto efficient allocation of resources in the economy described in section 2. Assume countries maximize utility subject to the budget constraint (7) given by the ability to trade emission permits. Assume furthermore that a regularity condition defined in the Appendix is satisfied. Then only a finite number of ways of allocating the total emission $E^*$ among countries as initial endowments will lead to market equilibria which are Pareto efficient.

The proof of this proposition is given in the Appendix. It is worth noting that the assumption of strict concavity, and the regularity assumption, are needed for this result. Otherwise, one can construct counterexamples. For example, with quasi-linear preferences of the form $u_i(a) + \alpha_i c_i, m \alpha_i > 0$, there may be infinitely many allocations of permits that will lead to efficient outcomes.
Intuitively, why will only some initial allocations lead to efficiency? In the context of public goods, an equilibrium must satisfy a more demanding set of conditions than a competitive equilibrium with only private goods. In addition to the first order condition for utility maximization, and to the equality of supply and demand for each good (both of which are common to the two problems), there is another requirement. This is that every agent (country in the present case) should have the same demand for the public good. The need for this of course follows immediately from the definition of a public good. For a given number of goods and agents, there are more equations to be satisfied at an equilibrium and consequently one would expect equilibria to be more difficult to obtain.

It is worth noting that although the dependence of efficiency on distribution runs quite counter to the thrust of the first and second welfare theorems, there are parallels in the literature. For example, in economies with increasing returns to scale, there are some allocations of a given total of initial endowments which are compatible with attainment of efficiency at a marginal cost pricing equilibrium and some that are not—see Brown and Heal [1]. The orthogonality of efficiency and distribution may therefore be limited to "classical" economic environments free from increasing returns and public goods or externalities.

5. COUNTRY-SPECIFIC PERMIT PRICES

We have remarked several times that the use of a uniform permit price, to be paid by every country, is restrictive. In this section we investigate the gains from having a permit price specific to each country. The intuition suggesting that there may be gains in this case comes from two sources.

One is the structure of Lindahl equilibria (for a definition of Lindahl equilibria, see Foley [11] or Dasgupta and Heal [10]): as already remarked, at a Lindahl equilibrium each producer of a public good is paid by every consumer for each unit produced, and in principle all consumers may pay different prices. In the present context, the analog would be the following. Any country thinking of producing one more unit of emissions would have to purchase from every other country the right to emit that extra unit. Only when it has been sold that right by each affected country is it entitled to emit. It would therefore have to buy an emission permit from each affected country, with possibly a different price ruling in each bilateral trade. This would give as many prices as there are in a Lindahl equilibrium.

An alternative way of reaching the same intuition is to think of markets for externalities, as described by Meade [15] in his famous bees and apples example (see [10] for an exposition relevant to the present model). In this context, each pairwise externality is a separate commodity, separately priced. There are therefore as many prices as there are pairs of interacting producers and consumers of externalities.

If each country faces a different price for emission permits, the budget constraint
(7) becomes instead
\[ \sum_i c_{i,l} p_i = \sum_i y_{i,l} p_i + p_{i,e} \{ E_i + a_i \} \] (12)
where \( p_{i,e} \) is the price of an emission permit to country \( i \). Instead of (10), each country's first order condition in production now becomes
\[ \frac{\partial \Phi_i}{\partial y_{i,l}} = -\frac{p_i}{p_{i,e}} \] (13)
and in place of (11) the condition for permit markets to attain efficiency is
\[ \frac{p_i}{p_{k,e}} = \frac{\lambda_k \frac{\partial u_k}{\partial c_k,l}}{\sum_k \lambda_k \frac{\partial u_k}{\partial a}} \forall k \] (14)
This condition is satisfied if \( \lambda_k \frac{\partial u_k}{\partial c_k,l} \) is the same for all \( k \). In fact this is always implied by the necessary condition (4) for Pareto efficiency, at least for the version of the resource allocation problem set out in section 2 above. If however this resource allocation problem were altered so that lump sum redistributions of private goods between countries were no longer possible, i.e., if (2) were modified to require domestic consumption to equal production, or alternatively to require the value of any difference to be zero (trade balance), then the equalization of marginal social valuations of the private goods would no longer be a condition needed for efficiency. We show in the Appendix that if international lump sum redistributions are ruled out and domestic consumption is required to equal domestic production, then condition (4) is no longer necessary for Pareto efficiency and only condition (5) is required. This point is discussed further in Chichilnisky and Heal [4] and [5]. In these case there is a real efficiency gain to having permit prices that are country-specific, for without them it would not be possible to attain a Pareto efficient allocation.

6. APPENDIX

Characterization of Pareto efficiency.

A Pareto efficient allocation solves the following problem:
\[
\max u_i (c_i, a) \text{ subject to } u_k (c_k, a) = N_k \\
\sum_i y_{i,l} = \sum_i c_{i,l} \forall l \\
a_i = \Phi_i (y_i) \text{ and } \sum_i a_i = a
\]
The second line of this problem allows unrestricted international lump sum redistribution: world-wide consumption has to equal world-wide production. To solve this problem write out the Lagrangian
\[
\mathcal{L} = u_i \left( c_i, \sum_i \Phi_i (y_i) \right) + \lambda_k \left( u_k \left( c_k, \sum_i \Phi_i (y_i) \right) - N_k \right) + \theta \left( \sum_i y_{i,l} - \sum_i c_{i,l} \right)
\]
where \( a \) has been replaced by \( \sum_i \Phi_i(y_i) \). Differentiating with respect to the components of \( c_i \) and \( y_i \) gives the first order conditions for efficiency used in the text. Without the possibility of international lump sum redistributions, the problem and the necessary condition are different. For example, suppose that in each country consumption is required to equal production. Then the second line of the problem above is dropped, and the vector \( y_i \) in the third line replaced by \( c_i \). In this case the necessary conditions for Pareto efficiency are just (5): conditions (4) are no longer required.

**Proof of Proposition 3.**

The conditions (4) required for Pareto efficiency, \( \frac{\partial u_i}{\partial c_{i,l}} = \lambda_k \frac{\partial u_k}{\partial c_{k,l}} \) \( \forall i, \forall k \neq i \), constitute a system of \((I - 1)m\) equations. Rewrite them as

\[
\frac{\partial u_i}{\partial c_{i,l}} - \lambda_k \frac{\partial u_k}{\partial c_{k,l}} = 0
\]

Efficiency now requires that we locate a zero of a system of \((I - 1)m\) non-linear equations given by (15). Note that the independent arguments of the functions in (15) are \( E_i, i = 1, \ldots, I \) and \( p_l, l = 1, \ldots, m \) and \( e \). For once the prices of all goods are chosen, the production levels of private goods and of abatement are determined by equation (10) giving first order conditions in production. And these levels, together with prices and endowments of permits, determine consumption levels through the budget constraint (7) and the first order conditions on consumption (9). Now, as both the \( E_i \) and the prices are non-negative and sum to a fixed number, the left hand side of system (15) is a function, call it \( \Omega \), defined on \( \mathbb{R}^{(I-1)m} \). This function also takes values in \( \mathbb{R}^{(I-1)m} \): \( \Omega : \mathbb{R}^{(I-1)m} \rightarrow \mathbb{R}^{(I-1)m} \), \( \Omega(x) = \frac{\partial u_i(x)}{\partial c_{i,l}} - \lambda_k \frac{\partial u_k(x)}{\partial c_{k,l}} \) where \( x \in \mathbb{R}^{(I-1)m} \). Proposition 3 uses the following *regularity condition*, which essentially states that the first order conditions for efficiency in equation (4) change smoothly as prices and permit allocations change:

*Regularity condition*: the matrix of first partial derivatives of the function \( \Omega \) has full rank.

Note that \( \Omega \) is defined on a compact set in \( \mathbb{R}^{(I-1)m} \). It therefore follows that if the rank of the matrix of first partial derivatives of \( \Omega \) is maximal, there will be at most a finite number of points \( (E_1, \ldots, E_I, p_1, \ldots, p_m, p_e) \in \mathbb{R}^{(I-1)m} \) at which \( \Omega = 0 \) and the conditions needed for efficiency are satisfied. This is implied by the regularity condition, so that the proposition is proven.

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