Optimal Exchange Rate Policy Under Collateral Constraints and Wage Rigidity

Pablo Ottonello

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Columbia University
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Pablo Ottonello∗

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Abstract

This paper conducts a quantitative study of the optimal exchange-rate policy in a small open economy that faces the “credit access–unemployment” trade-off: In the presence of nominal wage rigidity, exchange-rate depreciation reduces unemployment; in the presence of collateral constraints linking external debt to the value of income, exchange-rate depreciation tightens the collateral constraint and leads to higher consumption adjustments. It is shown that the optimal policy during financial crises generally features large currency depreciation, since welfare costs related to higher unemployment and lower consumption typically outweigh welfare costs associated with intertemporal misallocation of consumption. The optimal policy also implies a lower currency depreciation than that necessary to achieve full employment, which is consistent with a managed-floating exchange-rate policy, frequently observed during financial crises in emerging market economies. Sudden stops (or large current-account adjustments) are part of the endogenous response to large negative shocks under the optimal exchange-rate policy.

JEL Classification: E52, F41

Keywords: Exchange-rate policy, financial crises, debt deflation, sudden stops, downward wage rigidity, unemployment

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1 Introduction

During an external crises, exchange-rate policy in emerging market economies (EMs) seem to leave policymakers between a rock and a hard place: Preventing currency depreciation could bring more unemployment, but if liabilities are denominated in foreign currency, currency depreciation could increase debt in terms of domestic income, leading to financial destabilization, and compromising credit access. The potential conflict for exchange-rate policy between these two welfare concerns, credit access and unemployment, is often a central element of the policy debate, as was observed during the East Asian and Latin American crises in the late 1990s (Fischer, 1998; Calvo, 2001; Stiglitz, 2002) and during the peripheral European crises that started in 2008 (see, for example, Krugman, 2010; Feldstein, 2011).

This paper conducts a quantitative analysis of the optimal exchange-rate policy when facing this “credit access–unemployment” trade-off. It constructs an environment that provides a theoretical justification for this trade-off, combining two frictions that have been largely studied in the literature: a downward nominal wage rigidity (as in Schmitt-Grohe and Uribe, 2011), and a financial friction by which external borrowing is denominated in the international unit of account and limited by the value of collateral in the form of tradable and nontradable income (as in Mendoza, 2002). In this framework, credit-access and unemployment are two conflicting factors affecting welfare: Devaluations are associated with a welfare gain – by decreasing the real value of wages they reduce involuntary unemployment – but are also associated with a welfare cost – by increasing the value of external debt in terms of domestic income, they tighten the collateral constraint and can trigger an endogenous “sudden stop.”

The main finding, in calibrated versions of the model, is that two features characterize the optimal exchange-rate policy during financial crises (defined as episodes of binding credit constraints). First, the optimal allocation generally implies a large real exchange rate depreciation (between a 17 and 40 percent fall on average in the relative price of non-tradables), which is achieved by allowing for nominal currency depreciation. The reason is that, the welfare costs related to higher unemployment and lower consumption are typically higher than the welfare costs related to intertemporal misallocation of consumption. Second, optimal currency depreciation is generally lower than that associated with full employment. Thus, the optimal policy is consistent with a managed-floating exchange-rate policy, frequently observed in EMs during financial crises (see, for example, Calvo and Reinhart, 2002). Moreover, the real exchange-rate depreciation and current account adjustment under the optimal exchange rate policy during episodes of binding collateral constraints is in line with the dynamics observed in the data during sudden stops. The

\[1\] Calvo (1998) labeled sudden stops episodes of large and abrupt reversals in external credit flows that characterize EMs. For a review of the Fisherian debt-deflation approach to sudden stops, including the form of collateral constraint used in this paper, see Korinek and Mendoza (2013).
paper shows that the nature of the shocks and the structural characteristics of the economy are key determinants for the optimal degree of “fear of floating” during financial crises: Higher external interest rates, a larger intertemporal or intratemporal elasticity of substitution, or a large mobility of labor across sectors, call for a smaller unemployment; a more elastic labor supply, or a higher share of income that can be used as collateral, call for more contained currency depreciation.

Welfare under the optimal exchange-rate policy is compared to that under a full-employment and a fixed exchange-rate regimes. The full-employment exchange-rate regime is costly in terms of welfare for making consumption adjust more than what is optimal during periods of binding collateral constraints. The fixed exchange-rate regime is costly for inducing an inefficient adjustment to negative shocks with involuntary unemployment, in periods of both nonbinding and binding collateral constraints. This different nature of welfare costs generally makes the fixed exchange-rate regime more costly, in terms of welfare, than the full-employment exchange-rate regime. The welfare cost of the full-employment and fixed exchange-rate regimes, with respect to the optimal exchange-rate policy, are larger in regions of the state-space where the collateral constraint binds, with an average welfare cost during periods of binding collateral constraints of 0.06 percent and 1.8 percent of consumption per period, respectively.

This is the first paper that conducts a quantitative study of nominal exchange-rate policy under a collateral constraint by which debt is limited by the value of collateral in the form of tradable and nontradable income. Introduced in Mendoza (2002), this form of financial friction has been widely used to capture the main stylized facts about sudden stops in EMs. This form of collateral constraint causes endogenous sudden stops through Fisher’s (1933) debt-deflation mechanism: Binding constraints lead to deleveraging, which leads to a fall in the price of nontradables, which further tightens the collateral constraint. Previous studies using this form of financial friction have considered real models, in which the policy instrument during periods of binding collateral constraints is a tax (subsidy) on nontradable or tradable goods (see, for example, Benigno et al., 2012a). The present paper expands this literature by considering a nominal model and a monetary instrument, which present the policymakers a different trade-off: While subsidizing nontradable goods leads simultaneously to increased employment and higher prices of nontradable goods (relaxing the credit constraint), currency depreciation leads to an increase in employment and a decrease in the price of nontradable goods (tightening the credit constraint).\(^2\)

\(^2\)See, for example, Mendoza (2005), Durdu, Mendoza, and Terrones (2009), Korinek (2011), Bianchi (2011), Benigno et al. (2011,2012a,b,c).

\(^3\)In particular, it can be shown that in the model economy presented in Section 2, using taxes on nontradable or tradable consumption together with the capital-control tax, a Ramsey planner can achieve an allocation characterized by full employment and nonbinding collateral constraint in all states. See Benigno et al. (2012b) for a similar result in an economy without wage rigidity.
The paper is related to the large body of literature that studies nominal exchange-rate policy in small open economies during financial crises. A key difference with respect to this literature is the form of financial friction studied in the present paper, which in turn leads to different policy implications. For instance, in a large subset of this literature, borrowing access is linked to asset prices: Cespedes, Chang, and Velasco (2004), Devereux, Lane, and Xu (2006), Curdia (2007), and Gertler, Gilchrist, and Natalucci (2007) study economies featuring the financial accelerator mechanism (Bernanke, Gertler, and Gilchrist, 1999). In a recent related paper, Fornaro (2013) study an economy with a collateral constraint that limits external debt to a fraction of the market value of asset holdings (as in Bianchi and Mendoza, 2011). In these frameworks, currency depreciations have a positive effect on output, that leads to higher asset prices and improved credit access. As a consequence, contrary to the present paper’s result, in these papers flexible exchange rates lead to more financial stability during crises than fixed exchange rates. In the present setup, borrowing access is linked to goods prices: Debt denominated in a foreign currency is limited by the market value of tradable and nontradable income. The combination of a nontradable sector and liability dollarization creates a currency mismatch that makes currency depreciation financially destabilizing (in line with the traditional “original sin” argument; see Eichengreen, Hausmann, and Panizza, 2005; Calvo, 1999).

The hypothesis of currency depreciations being financially destabilizing has been previously formalized, for instance, in Aghion, Bacchetta, and Banerjee (2001), and in Bragion, Christiano, and Roldos (2009) using credit constraints on firms. In these papers, however, currency depreciations are financially destabilizing because they cause output contraction. In the present paper, currency depreciations are not contractionary (they reduce unemployment), but the associated currency mismatch reduces the value of income, leading to a large consumption adjustment under binding credit constraints and entailing a welfare cost. For these reasons, the form of financial friction considered in this paper gives rise to a trade-off between credit access and unemployment that has not been formally studied in the literature of exchange-rate policy in small open economies during financial crises.

The policy choice under the trade-off studied in this paper has engendered a long-standing and still lively policy debate. Keynes, for instance, was first actively opposed to the return of Britain to the gold standard after World War I, arguing that it would be associated with high unemployment (Keynes, 1925). However, when the Great Depression started, Keynes recommended against devaluation, claiming that now the costs in terms of debt revaluation and financial destabilization would outweigh the benefits (Irwin, 2011). In the same line, Diaz-Alejandro (1965), analyzing Argentina’s exchange-rate policy in the 1950s, highlighted the possibility that devaluations would lead to negative wealth effects and adjustment in consumption from income distribution and balance-sheet effects. This policy debate was triggered again by the crisis in peripheral Europe that started
in 2008, in which there are, simultaneously, high unemployment and high debt levels denominated in euros. Moreover, the empirical literature suggests that both sides of the debate are supported by evidence. Cross-country regressions for EMs tend to show both that fixing the exchange rate during financial crisis episodes is associated with larger output contractions (see, for example, Ortiz et al., 2009), and that currency mismatch plays a key role in determining the access to international credit markets (see, for example, Calvo, Izquierdo, and Mejia, 2008).

Finally, it is worth noting that while most of the above-mentioned literature on nominal exchange rate policy in small open economies during financial crises compare different (possibly nonoptimal) exchange-rate regimes, the present paper derives the fully optimal exchange-rate policy. The paper shows that the optimal allocation is a nonmonotonic function of the states; therefore, considering the optimal policy, instead of comparing exchange-rate regimes, is relevant.

The rest of the paper is organized as follows. Section 2 presents the model economy. Section 3 defines three possible exchange-rate regimes in this setup (optimal, full-employment, and fixed exchange-rate policies) and provides analytical results describing the exchange-rate policy trade-off that emerges in this economy. Section 4 presents the quantitative analysis comparing the aggregate dynamics and welfare under the three exchange-rate regimes. Section 5 examines the sensitivity of results to different calibrations and changes in the baseline model’s assumptions. Section 6 concludes.

2 The Model Economy

This section describes the model economy used to conduct exchange-rate policy analysis. It extends the two-sector (tradable and nontradable), dynamic, stochastic, small open economy model with a downward nominal wage rigidity from Schmitt-Grohe and Uribe (2011), to include a collateral constraint in the form of tradable and nontradable income. The economy only has access to a one-period, non-state-contingent debt instrument, denominated in units of tradable goods, capturing liability dollarization. The model then features a nominal rigidity and two financial frictions that will interact to determine the exchange-rate policy trade-off.

Tradables are endowed to the economy, and their price is determined by the law of one price. Nontradables are produced by the economy, and their price is determined by domestic demand and supply. Fluctuations in the small open economy are driven by

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4Optimal monetary policy has been largely studied in open economies with complete asset markets, and in open economies in which the financial friction is that financial markets are incomplete; see, for example, Clarida, Gali, and Gertler (2001), Schmitt-Grohe and Uribe (2001), Devereux and Engel (2003), Corsetti and Pesenti (2005), Gali and Monacelli (2005), De Paoli (2009), Corsetti, Dedola, and Leduc (2010), Schmitt-Grohe and Uribe (2011). The present paper constitutes a contribution in this direction for a small open economy in which financial frictions include an imperfect access to credit markets, with the presence of occasionally binding collateral constraints.
exogenous shocks to the value of the tradable endowment (which can be interpreted as shocks to terms of trade or to productivity in the tradable sector) and to the interest rate on external debt, two sources of business-cycle fluctuations that have been widely studied in EMs (Mendoza, 1995; Neumeyer and Perri, 2005; Uribe and Yue, 2006).

2.1 Households

Households’ preferences over consumption are described by the expected utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t),$$

where $$c_t$$ denotes consumption in period $$t$$; the function $$U(\cdot)$$ is assumed to be continuous, twice differentiable, strictly increasing, and concave; the subjective discount factor $$\beta \in (0, 1)$$, and $$E_t$$ denotes expectation conditional on the information set available at time $$t$$.

The consumption good is assumed to be a composite of tradable and nontradable goods, with a CES aggregation technology:

$$c_t = A(c_T^t, c_N^t) = \left[a \left(\frac{c_T^t}{c_N^t}\right)^{1-\xi} + (1-a) \left(\frac{c_N^t}{c_T^t}\right)^{1-\xi}\right]^\frac{1}{1-\xi},$$

where $$c_T^t$$ denotes tradable consumption and $$c_N^t$$ denotes nontradable consumption.

Each period, households receive a stochastic endowment ($$y_T^t$$) and profits from the ownership of firms producing nontradable goods ($$\Pi_t$$). They inelastically supply $$h_t$$ hours of work to the labor market. (Section 5.3 relaxes these assumptions studying production in the tradable sector and an elastic labor supply.) Due to the presence of the wage rigidity (discussed in detail in the next sections), households will only be able to sell $$h_t \leq \bar{h}$$ hours in the labor market. The level of actual hours worked ($$h_t$$) is determined by firms and is taken as given by the households.

Households have access to a one-period, non-state-contingent bond denominated in units of tradable goods that can be traded internationally paying an exogenous and stochastic gross interest rate $$R_t$$. The model therefore assumes full liability dollarization. It is assumed that the vector of exogenous states, $$s_t^X \equiv [y_T^t, R_t]$$, follows a first-order Markov process. Debt acquired in period $$t$$ is taxed at rate $$\tau_d^t$$. Households’ sequential budget constraint is therefore given by

$$\frac{d_{t+1}^d}{R_t} \left(1 - \tau_d^t\right) = d_t + c_T^t + p_t c_N^t - (y_T^t + w_t h_t + \Pi_t) - T_t,$$

where $$d_{t+1}$$ denotes the level of debt assumed in period $$t$$ and due in period $$t+1$$, $$p_t \equiv \frac{p_T^t}{p_T^t}$$ denotes the relative price of nontradables in terms of tradables, $$w_t$$ denotes the wage rate in terms of tradable goods, and $$T_t$$ denotes a lump sum transfer in period $$t$$. It is assumed that households face a collateral constraint by which external debt cannot exceed a fraction $$\kappa$$ of income:

$$d_{t+1} \leq \kappa (y_T^t + w_t h_t + \Pi_t),$$
where \( \kappa > 0 \). This form of collateral constraint, introduced in Mendoza (2002), has been used extensively in the literature on small open economies to capture the effect of currency mismatch on external credit-market access: While collateral includes income from both tradable and nontradable sectors, external debt is fully denominated in units of tradables. The credit-market frictions from which this constraint arises are not modeled here explicitly, but this form of collateral constraint can be seen as describing an environment in which lenders manage default risk by imposing a debt limit linked to households’ current income, as is typically the case of lending criteria in mortgage or consumer credit markets.  

Empirical evidence suggests that current income is a significant determinant of credit market access (Jappelli, 1990).

In addition, households are assumed to face a no-Ponzi game constraint of the form

\[
d_{t+1} \leq d^N, \tag{5}
\]

where \( d^N \) denotes the natural debt limit. As in Aiyagari (1994), this is defined as the maximum value of external debt that the household can repay almost surely starting from that period, assuming that its tradable consumption is zero forever. Formally, denoting \( y^T_t \) as the minimum possible level of tradable endowment and \( \bar{R} \) as the maximum possible level of external interest rate, the natural debt limit is defined as

\[
d^N \equiv \frac{\bar{R}}{R - 1} y^T_t. \tag{5}
\]

Since the collateral value in the credit limit (4) depends on relative prices which can be affected by policy variables, constraint (5) is imposed in addition to (4) is in order to prevent Ponzi schemes induced by the policymaker (Mendoza, 2005; Benigno et al., 2012b).

The household problem is to choose state-contingent plans for \( c_t, c^T_t, c^N_t, \) and \( d_{t+1} \) that maximize the expected utility (1) subject to the consumption aggregation technology (2), the sequential budget constraint (3), the collateral constraint (4), and the no-Ponzi game constraint (5), for a given initial debt level, \( d_0 \); for the given sequence of prices, \( w_t \) and \( p_t \); for the given sequence of hours worked, \( h_t \), profits, \( \Pi_t \), stochastic tradable endowment, \( y^T_t \), and interest rate, \( R_t \); and for the given sequence of policies, \( \tau^d_t \) and \( T_t \).

Denoting by \( \lambda_t \) the Lagrange multiplier associated with the budget constraint (3) and by \( \mu_t \) the Lagrange multiplier associated with the collateral constraint (4), the optimality conditions (provided \( d_{t+1} < d^N \)) are (2), (3), and (4), with the first-order conditions

\[
\begin{align*}
\lambda_t R_t^{-1} \left(1 - \frac{\tau^d_t}{d_t} \right) &= \beta \mathbb{E}_t \lambda_{t+1} + \mu_t, \tag{6} \\
U_{c^T} \left(c^T_t, c^N_t \right) &= \lambda_t, \tag{7} \\
\left( \frac{1}{a} \right) \left( \frac{c^T_t}{c^N_t} \right)^{\frac{1}{2}} &= \mathcal{P} \left(c^T_t, c^N_t \right) = p_t. \tag{8}
\end{align*}
\]

and the complementary slackness conditions

\[
\mu_t \geq 0, \mu_t \left( \kappa \left(y^T_t + w_t h_t + \Pi_t \right) - d_{t+1} \right) = 0. \tag{9}
\]

\(^{5}\)Korinek (2011) shows that this form of the collateral constraint can be rationalized as a renegotiation-proof form of debt contract in an imperfect credit market in which households can renegotiate external debt and lenders can extract at most a fraction of borrowers’ current income if debt is renegotiated.
2.2 Firms

Each period, operating in competitive labor and product markets, firms hire labor to produce the nontradable good, \( y_t^N \). Profits each period are given by

\[
\Pi_t = p_t F(h_t) - w_t h_t,
\]

where the production function, \( F(\cdot) \), is assumed to be increasing and concave.

The firms’ problem is to choose \( h_t \) to maximize profits given prices \( p_t \) and \( w_t \). The first-order condition of this problem is

\[
p_t F'(h_t) = w_t. \tag{10}
\]

This condition implicitly defines the firms’ demand for labor.

2.3 The Labor Market

Nominal wages (\( W_t \)) are assumed to be downwardly rigid as in Schmitt-Grohe and Uribe (2011):\(^6\)

\[
W_t \geq \gamma W_{t-1},
\]

for \( \gamma > 0 \).

It is assumed that the law of one price holds for tradable goods, implying that \( P_t^T = E_t P_t^{T*} \), where \( E_t \) is the nominal exchange rate and \( P_t^{T*} \) is the foreign currency price of tradable goods. Assuming that \( P_t^{T*} \) is constant and normalized to one, wages in terms of tradable goods (\( w_t \)) can be expressed as

\[
w_t = \frac{W_t}{E_t}.
\]

From this, the wage rigidity can be expressed as

\[
w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \tag{11}
\]

where \( \epsilon_t \) is the gross depreciation rate of the nominal exchange rate: \( \epsilon_t \equiv \frac{E_t}{E_{t-1}} - 1 \). However, actual hours worked cannot exceed the inelastically supplied level of hours:

\[
h_t \leq \overline{h}. \tag{12}
\]

When the nominal wage rigidity binds, the labor market can exhibit involuntary unemployment, given by \( \overline{h} - h_t \). This implies a slackness condition must hold at all dates and states:

\[
\left( w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) (\overline{h} - h_t) = 0. \tag{13}
\]

\(^6\)The assumption of an asymmetric nominal wage rigidity is consistent with empirical evidence using microeconomic data (e.g., Gottschalk, 2005; Barattieri, Basu and Gottschalk, 2010; Daly, Hobijn, and Lucking, 2012).
This condition means that when the nominal wage rigidity is not binding, the labor market must exhibit full employment, and if it exhibits unemployment, it must be the case that the nominal wage rigidity is binding.

### 2.4 The Government

The government determines the exchange-rate depreciation, \( \epsilon_t \), and imposes a proportional tax (subsidy) on debt \( \tau_t^d \), which is rebated lump sum to households \( (T_t) \), to balance its budget each period:

\[
\frac{d_{t+1}^T \tau_t^d}{R_t} = T_t. \tag{14}
\]

Section 3 defines different exchange-rate regimes and how the capital-control tax is determined.

### 2.5 General Equilibrium Dynamics

The market for nontradable goods clears at all times:

\[
c_t^N = F(h_t). \tag{15}
\]

Combining the equilibrium price equation, (8), with condition (15), the firms’ optimality condition, (10), can be expressed as

\[
w_t = \left(\frac{1 - a}{a}\right) \left(\frac{c_t^T}{a}\right)^\frac{1}{\xi} F(h_t)^{-\frac{1}{\xi}} F'(h_t). \tag{16}
\]

Combining condition (15) with households’ budget constraint, (3), the definition of firms’ profits, and the government’s budget constraint, (14), the resource constraint of the economy becomes

\[
\frac{d_t^T}{R_t} = d_t + c_t^T - y_t^T. \tag{17}
\]

Using the definition of firms’ profits, the equilibrium price equation, (8), and the market clearing condition for nontradables, (15), the collateral constraint, (4), can be reexpressed as

\[
d_{t+1} \leq \kappa \left( y_t^T + \left(\frac{1 - a}{a}\right) \left(\frac{c_t^T}{a}\right)^\frac{1}{\xi} F(h_t)^{-\frac{1}{\xi}} F'(h_t) \right) \equiv d(h_t, c_t^T, y_t^T). \tag{18}
\]

The general equilibrium dynamics are then given by stochastic processes \( \{c_t^N, c_t^T, h_t, p_t, w_t, d_{t+1}, \lambda_t, \mu_t, T_t\}_{t=0}^\infty \) satisfying the set of equations (GE):
given an exchange-rate policy \( \{ \epsilon_t \}_{t=0}^{\infty} \), a capital-control tax policy \( \{ \tau^d_t \}_{t=0}^{\infty} \), initial conditions \( w_{-1} \) and \( d_0 \), and exogenous stochastic processes \( \{ y^T_t, R_t \}_{t=0}^{\infty} \).

3 Exchange-Rate Regimes: Definitions and Analytical Results

This section formally defines the optimal exchange-rate policy, and discusses the trade-off between credit access and unemployment that exchange-rate policy can face in the model economy presented in the previous section. Analytical results relating credit access and unemployment are established, providing a framework for understanding the quantitative characterization of the optimal exchange-rate policy to be presented in the next section. Two additional exchange-rate regimes are also defined in this section – full-employment and fixed exchange-rate policy – to provide standard benchmarks for the study of the optimal exchange-rate policy.

3.1 Definition of Exchange-Rate Regimes

This section defines three possible exchange-rate regimes: optimal, full-employment, and fixed exchange-rate policy. Exchange-rate regimes are defined conditional on an optimal capital-control tax policy \( \{ \tau^d_t \}_{t=0}^{\infty} \). The reason for using this capital-control tax is twofold.

First, previous literature has shown that both the credit constraint and the downward wage rigidity considered in this paper embody a pecuniary externality that may induce inefficient external borrowing (Bianchi, 2011; Beningo et al., 2012a; and Schmitt-Grohe and Uribe, 2013).\(^7\) The optimal capital-control tax policy eliminates any borrowing inef-

\(^7\)Inefficient borrowing arises when the social costs of borrowing differ from the private costs of borrowing. Bianchi (2011) shows that in an endowment economy, the collateral constraint in the form of tradable
ficiency, and allows for a comparison across exchange-rate regimes isolating the effect of exchange-rate policy from this distortion.

Second, without the optimal capital-control tax, the set of restrictions for the optimal policy includes a forward-looking constraint (namely, the household’s intertemporal borrowing decision (6)). As shown in Kydland and Prescott (1977), Bellman’s (1957) principle of optimality fails in this context, and standard dynamic programming techniques cannot be applied. Using an optimal capital-control tax technically simplifies the problem, allowing for the use of standard dynamic programming techniques. Nevertheless, Section 5.4 studies the sensitivity of the optimal exchange-rate policy to the assumption of optimal capital-control taxes by restricting the Ramsey planner’s set of available instruments to the nominal exchange rate. In this context, time-invariant optimal policies under commitment are obtained using the recursive saddle-point method developed in Marcet and Marimon (2011).

3.1.1 Optimal Exchange-Rate Policy

**Definition 1** The optimal exchange-rate policy with optimal capital-control taxes is the set of processes \( \{ \epsilon_t, \tau^d_t \} \) that maximize households’ expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (GE).

To characterize the allocation under the optimal exchange-rate policy with optimal capital-control taxes, I set up the Ramsey problem dropping constraints (6)–(9), (11), and (13)–(16). Appendix A shows that any \( \{ d_{t+1}, c^T_t, h_t \} \) that satisfy (5), (12), (17), and (18) also satisfy (GE). The Ramsey problem is then to maximize (1) with respect to \( \{ d_{t+1}, c^T_t, h_t \} \), subject to (5), (12), (17), and (18). The dynamics under the optimal exchange-rate policy with optimal capital-control taxes can be thus expressed with the Bellman equation,

\[
V^{OP}(s^X, d) = \max_{d', c^T, h} \left[ U \left( A \left( c^T, F(h) \right) \right) + \beta E_{s^X} V^{OP} \left( s^{X'}, d' \right) \right]
\]

s.t. \[
\frac{d'}{R} = d + c^T - y^T,
\]
\[
d' \leq \kappa \left( y^T + \left( \frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{\frac{\xi-1}{\xi}} \right),
\]
\[
d' \leq d^N,
\]
\[
h \leq \bar{h},
\]

and nontradable income induces overborrowing, in the sense that the social costs of borrowing exceed the private costs of borrowing; in this setup, the constrained social planner borrows less than the competitive equilibrium. Beningo et al. (2011, 2012a) define overborrowing (underborrowing) as a situation in which a constrained social planner would take on less (more) debt than decentralized agents; in this sense, the authors find that whether an economy with this form of collateral constraint features overborrowing or underborrowing depends on the structure of the economy (e.g., endowment or production), and on the calibration. Schmitt-Grohe and Uribe (2013) show that the downward wage rigidity, combined with a fixed exchange-rate policy, induces overborrowing.
where time subscripts for variables dated in period $t$ have been dropped, and a prime is used to indicate variables dated in period $t+1$; $V^{OP} (s^X, d)$ denotes the value function for households under optimal exchange-rate and capital-control tax policies. This formulation will be used in the quantitative analysis.

### 3.1.2 Full-Employment Exchange-Rate Policy

For this regime, consider an exchange-rate policy aimed at maintaining full employment at all states and dates: Under the full-employment policy,

$$ h_t = \overline{h}, \forall t. $$

**Definition 2** The full-employment exchange-rate policy with optimal capital-control taxes is the set of processes $\{\epsilon_t, \tau^d_t\}$ that maximize households’ expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (GE), and the full-employment constraint (20).

To characterize the optimal allocation under the full-employment policy, I follow the same strategy as for the optimal exchange-rate policy and drop constraints (6)–(9) and (11)–(16). Appendix A shows that any $\{d_{t+1}, c^T_t, h_t\}$ that satisfy (5), (17), (18), and (20), also satisfy (GE) and (20). Therefore, the dynamics under the full-employment exchange-rate policy with optimal capital-control taxes can be expressed with the Bellman equation,

$$ V^{FE} (s^X, d) = \max_{d', c^T} \left[ U (A (c^T, F (\overline{h}))) + \beta \mathbb{E}_{s^X} V^{FE} (s^{X'}, d') \right] $$

s.t. $d' = d + c^T - y^T,$

$$ d' \leq \kappa \left( y^T + \left( \frac{1-a}{a} \right) \left( c^T \right)^\frac{\xi}{\xi-1} F (\overline{h}) \right)^{\frac{\xi-1}{\xi}}, $$

$$ d' \leq d^N, $$

where $V^{FE} (s^X, d)$ denotes the value function for households under the full-employment exchange-rate policy with optimal capital-control taxes.

### 3.1.3 Fixed Exchange-Rate Policy

Finally, consider a policy aimed at keeping the exchange rate fixed at all states and dates: Under the fixed exchange-rate policy or currency peg,

$$ \epsilon_t = 1, \forall t. $$

**Definition 3** The fixed exchange-rate policy with optimal capital-control taxes is the set of processes $\{\epsilon_t, \tau^d_t\}$ that maximize households’ expected lifetime utility (1) subject to the set of equations describing the general equilibrium dynamics (GE), and currency peg constraint (22).
To characterize the allocation under the currency peg with optimal capital-control taxes, I follow a similar strategy to that of the optimal exchange-rate policy and drop constraints (6)–(9) and (14)–(15). Appendix A shows that any \( \{d_{t+1}, c^T_t, h_t, w_t, \epsilon_t\} \) that satisfy (5), (11)–(13), (16)–(18), and (22), also satisfy (GE) and (22). Thus, the dynamics under the currency peg with optimal capital-control tax policy can be expressed with the Bellman equation,

\[
V^{\text{PEG}}(s^X, d, w_{-1}) = \max_{d', c^T, h, w} \left[ U \left( A \left( c^T, F(h) \right) \right) + \beta \mathbb{E}_{s^X} V^{\text{PEG}}(s^{X'}, d', w) \right] \tag{23}
\]

\[\text{s.t. } \frac{d'}{R} = d + c^T - y^T, \]

\[d' \leq \kappa \left( y^T + \left( \frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{\frac{\xi-1}{\xi}} \right), \]

\[d' \leq d^N, \]

\[w \geq \gamma w_{-1}, \]

\[h \leq \bar{h}, \]

\[(w - \gamma w_{-1}) (\bar{h} - h) = 0, \]

\[w = \left( \frac{1-a}{a} \right) (c^T)^{\frac{1}{\xi}} F(h)^{-\frac{1}{\xi}} F'(h), \]

where \( V^{\text{PEG}}(s^X, d, w_{-1}) \) denotes the value function for households under the currency peg and optimal capital-control taxes and the subscript \(-1\) is used to indicate variables dated in period \( t-1 \).

### 3.2 Optimal Exchange-Rate Policy, Unemployment and Credit Limit: Analytical Results

This section studies the relationship between unemployment and the credit limit under the optimal exchange-rate policy. Although, given the complexity of the model, a numerical solution is required for a full characterization, some analytical results can be obtained to show the trade-off involved in exchange-rate policy. These results will be relevant to understanding the next section’s numerical solution for the dynamics of the economy under the optimal exchange-rate policy. Proposition 1 characterizes the allocation under the optimal exchange-rate policy defined in the previous section.

**Proposition 1.** Under the optimal exchange-rate policy with optimal capital-control taxes (Definition 1) the following conditions hold at all dates and states:

- If \( \xi < 1 \), \( (\bar{h} - h_t) (\bar{d}(h_t, c^T_t, y^T_t) - d_{t+1}) = 0 \).
- If \( \xi \geq 1 \), \( h_t = \bar{h} \).

**Proof.** See Appendix B. \(\blacksquare\)

Two conclusions follow from this result. First, the allocation under the optimal exchange-rate policy and capital-control taxes features no unemployment under no binding collateral.
constraints. Given that the capital-control tax eliminates any inefficient borrowing, eliminating unemployment when the credit constraint does not bind leads to a welfare gain (a higher consumption of nontradables), without any welfare cost.\footnote{Note that while the presence of incomplete financial markets leads to inefficient consumption fluctuations relative to an economy with complete asset markets, eliminating unemployment does not lead to \textit{more} inefficient consumption fluctuations.}

Second, if the intratemporal elasticity of substitution is less than one, a slackness condition is established between unemployment and the collateral constraint under the optimal exchange-rate policy: If the collateral constraint is not binding, the labor market must exhibit full employment, and if there is unemployment, the collateral constraint must be binding. As discussed at the end of this section, empirical evidence from EMs provides wide support for the intratemporal elasticity of substitution being less than one. To understand the role of the intratemporal elasticity of substitution and the interaction between unemployment and the collateral constraint under the optimal exchange-rate policy, a discussion is in order regarding the trade-off facing exchange-rate policy in this economy.

### 3.2.1 The Credit-Access–Unemployment Trade-off

Parallel to the traditional inflation–unemployment trade-off in the New Keynesian literature, the exchange-rate policy in this economy may face a “credit-access–unemployment” trade-off. Under binding nominal downward wage rigidity, a depreciation of the nominal exchange rate decreases real wages and, thus, helps reduce unemployment. But it is also associated with a real exchange-rate depreciation, which decreases the value of nontradable output in tradable units. Recall that the collateral in this economy is given by the value, in tradable units, of tradable and nontradable output. Accordingly, if the price effect (real exchange-rate depreciation) dominates the quantity effect (employment increase), an exchange-rate depreciation can decrease the collateral value and tighten the credit limit. The price effect dominates the quantity effect if the intratemporal elasticity of substitution between tradables and nontradables is less than one ($\xi < 1$). As discussed in the next section, this assumption is widely supported by empirical evidence from EMs.

Under this assumption, the following proposition can be established:

**Proposition 2.** If $\xi < 1$, given an initial state $(s^X_t, d_t)$, for any debt level $d^*_{t+1}$ with associated tradable consumption $c^T_{t^*} = (d^*_{t+1}R_t^{-1} - d_t + y^T_t) > 0$ for which $d^*_{t+1} > \bar{d}(h^*_t, c^T_{t^*}, y^T_t)$, there exists a level of employment $h^*_t \in (0, \bar{h})$ for which $d^*_{t+1} = \bar{d}(h^*_t, c^T_{t^*}, y^T_t)$.

**Proof.** See Appendix B. $\blacksquare$

This result shows that for any debt level that does not satisfy the credit limit under full employment, there exists a level of employment below full employment for which the real exchange rate is sufficiently appreciated to ensure the credit limit is satisfied for
that debt level. This result stems from the fact that if the intratemporal elasticity of substitution is less than one \((\xi < 1)\), the collateral constraint is decreasing in the level of employment.\footnote{If the intratemporal elasticity of substitution is greater than or equal to one \((\xi \geq 1)\), the credit access-unemployment trade-off vanishes, as implied by Proposition 1. In particular, if the intratemporal elasticity of substitution is equal to one \((\xi = 1)\), employment does not influence the collateral constraint. If the intratemporal elasticity of substitution is greater than one \((\xi > 1)\), the credit-access-unemployment trade-off overturns, and a decrease in unemployment also helps relax the collateral constraint.} This provides a theoretical justification for the existence of the exchange-rate policy debate, typically observed during financial crises in EMs, that weighs the two policy objectives. The optimal choice under this trade-off can be characterized using the first-order conditions of the optimal policy problem (19):

\section*{Remark 1} If \(\xi < 1\), in an allocation under the optimal exchange-rate policy with optimal capital-control taxes (Definition 1) in which, at time \(t, h_t < \bar{h}\), the following conditions hold:

\begin{equation}
U_cA_N(c_t^T,F(h_t)) = \phi^\mu_t \left( \frac{1-\xi}{\xi} \right) \kappa P(c_t^T,F(h_t)),
\end{equation}

\begin{equation}
\phi^\mu_t = \left( \frac{\phi^F_t}{\bar{R}_t} - \beta E_t \phi^F_{t+1} \right),
\end{equation}

where \(\phi^\mu_t\), and \(\phi^F_t\) denote the nonnegative multipliers associated with the collateral constraint (18), and the resource constraint (17), respectively, in the Ramsey problem of optimal exchange-rate policy with optimal capital-control taxes.

\textbf{Proof.} See Appendix B.

Equation (24) shows that in any optimal allocation in which there is unemployment, the Ramsey planner equates the marginal benefit of increasing employment, given by the marginal utility of nontradable consumption, to its marginal cost in terms of tightening the collateral constraint. Equation (25) shows that the shadow price of relaxing the credit constraint for the Ramsey planner, \(\phi^\mu_t\), is the wedge between the current shadow value of wealth for the Ramsey planner and the expected value of reallocating wealth to the next period. This shows a relevant aspect of the trade-off involved in exchange-rate policy: While the costs of exchange-rate depreciations are associated with intertemporal misallocation of consumption, their benefits are related to a higher level of consumption.

\subsection*{3.2.2 Empirical Evidence on the Intratemporal Elasticity of Substitution}

As shown in Propositions 1 and 2, a tension exists between credit access and unemployment only if the elasticity of substitution between tradable and nontradable goods is less than one \((\xi < 1)\). If this is the case, tradable and nontradable goods are gross complements, and the price effect (real exchange-rate depreciation) associated with increasing employment dominates the quantity effect (employment increase). As a result,
exchange-rate depreciation can decrease the collateral value and make the credit limit tighter.

There is wide support from the empirical literature for the intratemporal elasticity of substitution being less than one. In a sample of developed and emerging market economies, Stockman and Tesar (1995) estimate a value of the elasticity of substitution of 0.44. Separating the samples of developed and emerging economies, Mendoza (1995) finds values of the elasticity of 0.74 and 0.43, respectively. In studies for EMs, Gonzalez-Rozada et al. (2004) found estimates in the range between 0.4 and 0.48 for Argentina and Lorenzo, Aboal, and Osimani (2005), found estimates in a range between 0.46 and 0.75 for Uruguay.10

Moreover, following this empirical literature, the studies referenced in the present paper that calibrate a two-sector, small open economy model generally use a parameter value of the elasticity of substitution in the range between 0.44 and 0.83.

4 Quantitative Analysis

The objective of this section is to quantitatively characterize the aggregate dynamics of the model economy under the optimal exchange-rate policy and to compare its performance, in terms of welfare, to that under the full-employment and fixed exchange-rate policies, both during periods of financial crises and under regular business-cycle fluctuations.

4.1 Calibration and Computation

To characterize the aggregate dynamics under the different exchange-rate regimes, calibrated versions of the functional equations (19), (21) and (23) are solved numerically. Due to the presence of occasionally binding constraints, I resort to the method of value-function iteration over a discretized state-space to compute the numerical solutions.

As mentioned in Section 2, the consumption aggregator is assumed to be a CES aggregator. I also assume a CRRA period utility function and an isoelastic form for the production function:

\[
U(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},
\]

\[
F(h) = h^{\alpha N}.
\]  

(26)

The model is calibrated at the annual frequency, to match Argentinean data. Argentina is used as a benchmark to conduct this exercise as an EM country whose exchange-rate regimes and financial crises have been widely studied, particularly in the two branches of the literature this paper combines.

10 Ostry and Reinhart (1992) found evidence inconclusive in this respect with estimates between 0.66 and 1.44, depending on the EM region and the instrumental variable considered. For a survey on the methodologies used to estimate the elasticity of substitution between tradable and nontradable goods, see Akinci (2011).
Table 1. Baseline Parameter Values

<table>
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<th>Parameter</th>
<th>Value</th>
<th>Description</th>
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<tr>
<td>$\sigma$</td>
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<td>Inverse of intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.44</td>
<td>Intratemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\alpha$</td>
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<td>Share of tradables</td>
</tr>
<tr>
<td>$\beta$</td>
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<td>Annual subjective discount factor</td>
</tr>
<tr>
<td>$\kappa$</td>
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<td>Share of income used as collateral</td>
</tr>
<tr>
<td>$\alpha^N$</td>
<td>0.75</td>
<td>Labor share in nontradable sector</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.96</td>
<td>Degree of downward nominal rigidity</td>
</tr>
</tbody>
</table>

All parameter values used in the baseline calibration are shown in Table 1. The inverse of the intertemporal elasticity of substitution is set to $\sigma = 2$, a standard value in the business-cycle literature for small open economies (see, for example, Mendoza 1991). The intratemporal elasticity of substitution is set to $\xi = 0.44$, using the estimates of Gonzalez-Rozada et al. (2004) for Argentina (see Section 3.2 for a review of the literature on this parameter). I set $\alpha^N = 0.75$, following the evidence in Uribe (1997) on the labor share in the nontradable sector in Argentina, and $\gamma = 0.96$, following the evidence in Schmitt-Grohe and Uribe (2011) on downward nominal wage rigidity. The mean level of tradable output and the labor endowment $(\bar{h})$ are normalized to one.

The parameters $\{\beta, a, \kappa\}$ are used to match three key moments in the ergodic distributions of the model under the optimal exchange-rate policy to the ones observed in historical Argentinean data (for the period 1975–2011). The three data moments considered are typically targeted in the related literature (following Bianchi, 2011): an average level of external debt-to-GDP ratio of 21 percent, a share of tradable output in GDP of 32.9 percent, and a frequency of sudden stops of 5.5 percent. A sudden stop in the model is defined as a period in which the economy exhibits a change in the current account larger than one standard deviation, following Eichengreen, Gupta and Mody (2006), from which the frequency of sudden stops is obtained for a sample of EMs.\(^{11}\) (See the Data appendix for further details on data sources, and on the construction of the series). The parameter values obtained from this calibration are $\beta = 0.8$, $a = 0.295$ and $\kappa = 0.263$. Section 5.1 studies the sensitivity of the optimal policy to this calibration.

It is assumed that the two exogenous driving forces, the tradable endowment and the interest rate, follow a first-order VAR of the form

$$
\begin{bmatrix}
\ln \left( \frac{y^T_t}{y^T_{t-1}} \right) \\
\ln \left( \frac{R_t}{R_{t-1}} \right)
\end{bmatrix} = \Phi
\begin{bmatrix}
\ln \left( \frac{y^T_t}{y^T_{t-1}} \right) \\
\ln \left( \frac{R_{t-1}}{R_{t-2}} \right)
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon^y_t \\
\varepsilon^R_t
\end{bmatrix},
$$

(27)

where $\begin{bmatrix}\varepsilon^y_t \varepsilon^R_t\end{bmatrix}' \sim \text{i.i.d. } N(\theta, \Omega)$ and $R$ denotes the mean interest rate level.

\(^{11}\)The frequency for EMs is similar to the frequency in Argentina during this period, and to other empirical estimates, such as Calvo et al. (2008).
The parameters of this stochastic process are estimated using Argentinean data since 1983. Tradable endowment is measured with the cyclical component of value added in agriculture and manufacturing. Interest rates on external debt are measured as the sum of EMBI spreads and the Treasury-bill rate. (Section 5.2 studies an economy with interest rate shocks calibrated to those of the risk-free rate). Since the data on EMBI spreads for Argentina is available since 1994, the series were extended back to 1983, using the Neumeyer and Perri (2005) dataset, which uses a measure similar to the one considered here. The interest rate series is then deflated with a measure of expected dollar inflation. (See the Data appendix for further details on data sources, and on the construction of tradable endowment and interest rates.) The years 2002–2005, in which Argentina defaulted and was excluded from international markets (Cruces and Trebesch, 2013), are not included in the estimation. The following OLS estimates are obtained.

\[ \hat{\Phi} = \begin{bmatrix} 0.42 & -0.28 \\ 0.32 & 0.93 \end{bmatrix}, \quad \hat{\Omega} = \begin{bmatrix} 0.002 & -0.001 \\ -0.001 & 0.001 \end{bmatrix}, \quad \hat{R} = 1.113. \]

This process is approximated with a Markov chain, setting a grid of 15 equally spaced points for both \( \ln(y_T^t) \) and \( \ln(R_t) \), yielding 225 exogenous states. To estimate the transition-probability matrix, I use the method proposed by Terry and Knoteck (2011) extending Tauchen (1986).\(^{12}\)

Finally, to approximate the aggregate dynamics of the economy under the optimal and the full-employment policies, I discretize the endogenous state space \((d_t)\) using 1,001 equally spaced points. To approximate the dynamics under a currency peg, I use 251 equally spaced points for debt \((d_t)\) and 250 equally spaced points for the log of previous period wage \((w_{t-1})\). The next sections present the results of the quantitative analysis.

4.2 Policy Functions

This section analyzes the policy functions under the optimal exchange-rate policy and compares them to those under the two benchmark exchange-rate regimes: full-employment and fixed exchange rate.

Figure 1 shows decision rules for the nominal devaluation rate, the real exchange rate, unemployment, and next-period debt as a function of the state variables: current debt, tradable endowment, and the external interest rate. In each panel, only one state variable varies (on the horizontal axis), and the remaining state variables are fixed at their unconditional means (under the optimal policy, if the state is endogenous). In each panel, a shaded region depicts the state-space in which the collateral constraint binds under the optimal policy. The panels on the right do not have a shaded region since varying the

\(^{12}\)I am grateful to Stephen J. Terry and Edward S. Knotek II for sharing the code for the Markov-chain approximations of vector autoregressions, which were used in this paper to estimate the transition-probability matrix of the stochastic process.
interest rate – while keeping the rest of the states fixed at their respective means – is not sufficient to make the collateral constraint bind.

The decision rules for the nominal devaluation rate and real exchange rate under the optimal policy are nonmonotonic, in sharp contrast with the decision rules under the full-employment or fixed exchange-rate policies. The change of the sign in the slope under the optimal policy occurs at the point at which a higher initial level of debt or a lower tradable endowment entails a binding credit constraint. In the region of nonbinding collateral constraint, the decision rules of optimal and full-employment policies coincide, as implied by Proposition 1. In this region, currency depreciation is increasing in the initial debt level and the interest rate, and decreasing in the level of tradable endowment. In the region of binding collateral constraint, while currency depreciation in the full-employment policy continues to be increasing in the initial debt level and decreasing in the level of tradable endowment, currency depreciation under the optimal exchange-rate policy becomes decreasing in the initial debt level and increasing in the level of tradable endowment. Positive unemployment emerges under the optimal exchange-rate policy in the region of binding collateral constraint, increasing in the initial level of debt and decreasing in the level of tradable endowment.

The decision rule for next-period debt under the optimal policy is monotonic, as it would be without an endogenous collateral constraint. Again in sharp contrast, under the full-employment policy, the decision rule for next-period debt is nonmonotonic, with a change in the slope at the point at which a higher initial level of debt or a lower tradable endowment implies a binding credit constraint (for a similar result in an endowment economy, see Bianchi, 2011). Hence, consistent with Proposition 2, the optimal policy restores the monotonicity in the policy functions of debt by making the decision rule of the real exchange rate nonmonotonic. In other words, the optimal choice under the credit-access–unemployment trade-off implies no corner solution: the optimal policy is willing to choose unemployment in the region of binding collateral constraint to allow for a higher next-period debt.

The decision rules for the currency peg show that this regime, in contrast to the optimal and full-employment policies, implies positive unemployment in the state-space regions in which the collateral constraint does not bind. In these regions, the fixed exchange-rate regime makes the downward rate rigidity binding. Consistent with this, the decision rule of the real exchange rate under the currency peg displays less sensitivity than that of the other two exchange-rate regimes.

4.3 Optimal Exchange-Rate Policy during Financial Crises

Under no binding collateral constraints, the optimal exchange-rate policy always consists of depreciating the nominal exchange rate in response to negative shocks to achieve full employment, as implied by Proposition 1. This section characterizes the optimal
Figure 1. Policy Functions. Note: The real exchange rate is expressed in log deviations from its sample mean. Devaluation rate, unemployment rate and next-period debt are expressed in levels. In each panel, only one state variable varies (on the horizontal axis), and the remaining state variables are fixed at their unconditional means (under the optimal policy, if the state is endogenous). Shaded regions denote regions of the state-space in which the collateral constraint binds under the optimal policy.
exchange-rate policy under periods of binding collateral constraints, or financial crises, and compares the dynamics of the economy under the different exchange-rate regimes.

To do this, the calibrated version of the model is simulated for 2 million quarters, identifying periods in which the collateral constraint is binding under the optimal exchange-rate policy. The beginning of a financial crisis episode \( t = 0 \) is defined as the first period in which the collateral constraint binds. The responses of the variables, during all episodes of financial crisis, are then averaged.

Figure 2 depicts the average external shock during a financial-crisis episode. In the two years that precede such an episode, tradable endowment contracts and interest rates increase. At the crisis trough \( t = 0 \), tradable output is 10 percent below its mean, and the annual interest rate is 16 percent, 4 percentage points above its mean. In the three years following the trough, both tradable output and the interest rate recover their precrisis levels.

![Figure 2. Financial Crises – Exogenous Variables.](image-url)

The average responses of the nominal exchange rate and endogenous variables under the different exchange-rate regimes are shown in Figure 3. Optimal and full-employment exchange-rate policies display striking similarities and offer a sharp contrast to the response under a currency peg. Even under binding collateral constraints \( t = 0 \), the optimal exchange-rate policy does not fix but it substantially depreciates the nominal exchange rate, 52 percent on average. This depreciation is less than that under the full-employment policy (71 percent). As a result, some involuntary unemployment emerges under binding collateral constraints (1.6 percent on average at the crisis trough). How-
ever, unemployment under the optimal exchange-rate policy is significantly lower than that observed under the currency peg (6.2 percent on average at the crisis trough).

![Figure 3: Financial Crises – Endogenous Variables. Note: Real exchange rate, real wage, and external debt expressed in log deviations from their sample means. Current account expressed in deviations of its sample mean. Devaluation rate and unemployment rate expressed in levels.](image)

In periods of binding collateral constraints, the large real-exchange-rate depreciation under the optimal exchange-rate policy (the relative price of nontradable goods being 39 percent below its mean at the crisis trough), implies a large adjustment of external debt and tradable consumption. Under the optimal policy, the contraction of tradable
consumption is much larger than the contraction in nontradable consumption: at the crisis trough, tradable consumption is 20.8 percent below its mean, while nontradable consumption is only 1.2 percent below its mean. The intuition for this result is that while the benefits of reducing unemployment are related to higher nontradable consumption (by market clearing of nontradables) its costs are related to intertemporal miscallocation of consumption. In this sense, sudden stops (understood as large current-account adjustments), are in fact part of the endogenous response to large negative external shocks under the optimal exchange-rate policy to prevent greater unemployment. This is again in sharp contrast to the behavior under the currency peg, where, for the same exogenous shock, external debt continues increasing and the current-account deficit expands; at the crisis trough, tradable consumption is 7.6 percent below its mean and nontradable consumption 4.2 percent below its mean.

The large, but still contained, optimal currency depreciation during periods of financial crises is consistent, for instance, with the typical behavior observed in EMs during the global financial turbulence of 2008 (see Figure 4). During this episode, EMs considerably depreciate the exchange rate (24 percent on average), but also contain the depreciation, as can be observed in the fall in international reserves. Calvo (2013) shows that this pattern of large nominal depreciation (more than 20 percent on average) with simultaneous exchange-rate intervention is the typical policy observed in EMs during periods of sudden stops since 1980.

4.4 Means and Volatilities by Exchange-Rate Regime

Table 2 shows that the differences between the optimal and full-employment policies during periods of binding collateral constraint (analyzed in Sections 4.2 and 4.3) translate, on the one hand, into a lower volatility of tradable consumption and total consumption, and, on the other hand, into a higher average unemployment rate. This reflects the fact that, under binding collateral constraints, the optimal policy allows for lowering nontradable consumption to improve intertemporal allocation of consumption. The differences in first and second moments between optimal and full-employment policies are slight since the unconditional probability of binding collateral constraints is low (1.5 percent).

The currency peg displays a larger difference in terms of the average unemployment rate with respect to the optimal exchange-rate regime. The reason is that, as previously analyzed, currency pegs also display unemployment when the collateral constraint does not bind but the wage rigidity does. This response of currency pegs to negative shocks also results in a higher volatility of nontradable consumption and total consumption with respect to the other two regimes.
4.5 Welfare and Exchange-Rate Regimes

This section compares welfare under the different exchange-rate regimes. The welfare costs of an exchange-rate regime $i$ with respect to an exchange-rate regime $j$ are computed as the percentage increase in the consumption stream under exchange-rate regime $i$ that will make the representative household indifferent between that consumption stream and the one under the exchange-rate regime $j$. Formally, the compensation rate under the regime $i$ with respect to regime $j$, $\lambda^{ij}$, in a state $s_t$ is implicitly defined by

$$E \left\{ \sum_{k=0}^{\infty} \beta^k U \left( c_{t+k}^i \left( 1 + \lambda^{ij} (s_t) \right) \right) \mid s_t \right\} = E \left\{ \sum_{k=0}^{\infty} \beta^k U \left( c_{t+k}^j \right) \mid s_t \right\},$$

where $i, j \in \{OP, FE, PEG\}$. 

Figure 4. Exchange-Rate Policy in Emerging Market Economies – Lehman Episode.

Note: Nominal exchange rate and international reserves figures were computed as the simple average for the countries included in the EMBI, except countries with no separate legal tender (Ecuador, El Salvador, and Panama). The countries included in the sample are Algeria, Argentina, Belarus, Belize, Brazil, Bulgaria, Chile, China, Colombia, Cote d’Ivoire, Croatia, Czech Republic, Dominican Republic, Egypt, Gabon, Georgia, Hungary, Indonesia, Jamaica, Jordan, Kazakhstan, Korea, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Namibia, Nigeria, Pakistan, Peru, Philippines, Poland, Romania, Russia, Senegal, South Africa, Sri Lanka, Thailand, Trinidad and Tobago, Tunisia, Turkey, Ukraine, Uruguay, Venezuela, and Vietnam. Data sources: See Data appendix.
Table 2. Means and Volatilities by Exchange Rate Regime

<table>
<thead>
<tr>
<th></th>
<th>OP</th>
<th>FE</th>
<th>CP</th>
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</thead>
<tbody>
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<td><strong>Means</strong></td>
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<tr>
<td>( \mu(c) )</td>
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<td>0.98</td>
<td>0.98</td>
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<td>0.94</td>
<td>0.95</td>
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<td>1.00</td>
<td>0.99</td>
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<tr>
<td>( \mu(p) )</td>
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<td>2.09</td>
<td>2.16</td>
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<tr>
<td>( \mu(u) )</td>
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<tr>
<td>( \mu(d) )</td>
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<td>0.64</td>
<td>0.52</td>
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<tr>
<td><strong>Volatilities</strong></td>
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<td>0.04</td>
<td>0.12</td>
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</table>

Note: OP, FE and CP denote optimal exchange-rate policy, full-employment exchange-rate policy, and currency peg, respectively, as defined in Section 3. The variables \( c, c^T, c^N, p, u \), and \( d \), denote, respectively, consumption, tradable consumption, nontradable consumption, relative price of nontradables, unemployment rate, and external debt. Volatilities and mean unemployment expressed in percent. Moments computed using parameters from Table 1.

Under the assumed form of period utility function, it follows that

\[
\lambda^{i,j}(s_i) = \left[ \frac{V^j(s_i) (1 - \sigma) + (1 - \beta)^{-1}}{V^i(s_i) (1 - \sigma) + (1 - \beta)^{-1}} \right]^{\frac{1}{1+\sigma}} - 1.
\]

Since welfare costs are state dependent, Figure 5 begins by showing the welfare costs of the full-employment and fixed exchange-rate policies, with respect to the optimal exchange-rate policy, as functions of the states, and the welfare cost of the fixed exchange-rate policy, with respect to the full-employment exchange-rate policy, as a function of the states. As in Figure 1, in each panel only one state variable varies (on the horizontal axis), and the remaining state variables are fixed at their unconditional means (under the optimal policy, if the state is endogenous). In each panel, a shaded region show where in the state space the collateral constraint binds under the optimal policy. As in Figure 1, the panels on the right do not have a shaded region since varying the interest rate – while keeping the rest of the states fixed at their respective means– is not sufficient to make the collateral constraint bind.

The welfare costs of the full-employment policy with respect to the optimal policy are increasing in the initial debt level, and decreasing in the level of tradable endowment and
Welfare Costs of Full-Employment Policy with respect to Optimal Policy

Welfare Costs of Currency Peg with respect to Optimal Policy

Welfare Costs of Currency Peg with respect to Full-Employment Policy

Figure 5. Welfare Costs by State. Note: the welfare costs of an exchange-rate regime $i$ with respect to an exchange-rate regime $j$ are defined as the percentage increase in the consumption stream under exchange-rate regime $i$ that will make the representative household indifferent between that consumption stream and that under the exchange-rate regime $j$ in a given state. In each panel, only one state variable varies (on the horizontal axis); the remaining state variables are fixed at their unconditional means (under the optimal policy, if the state is endogenous). Shaded regions denote regions of the state-space in which the collateral constraint binds under the optimal policy.

The welfare costs of the full-employment policy are significantly higher in the regions of the state space in which the collateral constraint binds. The higher welfare costs of the full-employment policy in this region stem from the fact that in these states, the decision rules from the optimal policy differ from those of the full-employment policy, implying a looser credit limit, as shown in the study of the policy functions in Section 4.2. This suggests that the welfare costs of the full-employment policy are decreasing in the interest rate because a higher interest rate leads to a reduction in the shadow value from relaxing the constraint.

The welfare costs of the currency peg with respect to the optimal and full-employment policies are nonmonotonic. In the region of nonbinding collateral constraint, welfare costs are decreasing in the level of endowment, and for high levels of debt or interest rates, welfare costs are increasing in the initial debt level and the interest rate. The intuition is that in the region of nonbinding collateral constraint, while the optimal policy
Table 3. Welfare Costs by Exchange-Rate Regime

<table>
<thead>
<tr>
<th>Welfare Costs of:</th>
<th>Full-Employment Policy</th>
<th>Currency Peg</th>
<th>Currency Peg</th>
</tr>
</thead>
<tbody>
<tr>
<td>with respect to:</td>
<td>Optimal Policy</td>
<td>Optimal Policy</td>
<td>Full-Employment Policy</td>
</tr>
<tr>
<td>Mean</td>
<td>0.006</td>
<td>0.576</td>
<td>0.568</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.023</td>
<td>0.324</td>
<td>0.325</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.7</td>
<td>5.065</td>
<td>5.063</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.001</td>
<td>0.22</td>
<td>−1.71</td>
</tr>
</tbody>
</table>

Note: welfare costs expressed in percent. The welfare costs of an exchange-rate regime $i$ with respect to an exchange-rate regime $j$ are defined as the percentage increase in the consumption stream under exchange-rate regime $i$ that will make the representative household indifferent between that consumption stream and the one under the exchange-rate regime $j$ in a given state.

maintains full-employment (Proposition 1), the currency peg displays a positive level of unemployment, which is increasing in the initial debt level and the interest rate, and decreasing in the tradable endowment (see Section 4.2). In the regions where the collateral constraint binds, the welfare costs of the unemployment generated by the currency peg are increasing in the initial debt level and decreasing in the level of tradable endowment. The intuition is that, in this region, the optimal policy displays a positive unemployment, which, as shown in Section 4.2, is increasing in the debt level and decreasing in tradable endowment.

Table 3 shows the moment of the distribution of welfare costs and indicates that the average welfare costs of the full-employment policy with respect to the optimal policy (0.006 percent) are significantly lower than the welfare costs of the currency peg with respect to the optimal policy (0.58 percent).\(^{13}\)

Finally, Figure 6 shows welfare costs during financial crisis episodes (as defined in Section 4.3). It can be observed that financial crises are periods in which the welfare costs of both the full-employment policy and the currency peg increase. The size of the increase in the welfare costs of the full-employment policy are, again, much smaller than the increase in the welfare costs of currency pegs: At the crisis trough the welfare costs of the full-employment and currency peg policies, with respect to the optimal exchange-rate policy, are 0.06 percent and 1.83 percent, respectively. As a consequence, the welfare costs of the currency peg with respect to the full-employment exchange-rate regime also rise during financial crises, reaching 1.77 percent at the crisis trough, meaning that currency pegs are particularly costly in terms of welfare during periods of binding collateral constraints.

\(^{13}\)Formally, the mean of the welfare costs of an exchange-rate regime $i$ with respect to an exchange-rate regime $j$, denoted $\bar{\lambda}^{i,j}$ is given by

$$\bar{\lambda}^{i,j} = \sum_{s_t} \pi^t(s_t) \lambda^{i,j}(s_t)$$

where $\pi^t(s_t)$ denotes the unconditional probability of state $s_t$ under exchange-rate regime $i$.  

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4.6 Data and Model Predictions

This subsection compares the data with the predictions of the model, during both financial crises and regular business cycles. Although the structure of the model is relatively simple, several features of the data are in line with the predictions of the model, as in previous literature using similar model structures. The predictions of the model are compared with data from Argentina, the economy for which the model economy was calibrated. Figure 7 illustrates the fact that Argentina, as did most EMs, alternated between different exchange-rate regimes during the period of study (Ilzetzki, Reinhart, and Rogoff, 2010). For this reason, the predictions of the three exchange-rate regimes are relevant to a comparison of the data.

Figure 8 shows that, in most dimensions, the dynamics of the average sudden stop episode in the data is within the predictions of the model during a financial crisis episode (as defined in Section 4.3).\footnote{Nominal exchange rates and external debt were excluded from the comparison due to hyperinflation episodes and default episodes that occurred in some of these periods (for hyperinflation episodes, see Sargent, Williams, and Zha, 2009; for default episodes, see Cruces and Trebesch, 2013).} The average episode in the data is constructed using three
sudden-stop episodes observed in Argentina in 1982, 1989, and 2001 (Eichengreen, Gupta and Mody, 2006; Calvo, Izquierdo and Talvi, 2006). The current-account reversal, the real-exchange-rate depreciation, and the contraction in real wages observed in the average sudden-stop episode in the data are similar in magnitude to the predictions of the model under the optimal exchange-rate policy. The increase of unemployment in the data is also within the predictions of the model, between the unemployment predicted by the currency peg and that predicted under the optimal policy. This can be related to the fact that, as Figure 7 indicates, sudden-stop episodes are periods of transition between exchange-rate regimes. A dimension in which the quantitative behavior of the average sudden-stop episode is not in line with the model is in nontradable-output and consumption. Although the model predicts a significant contraction in these two variables, the contraction observed in the data is larger. Since the behavior of unemployment in the data is in line with the predictions of the model, a key factor driving the larger fall in out-
Figure 8. Financial Crises – Model and Data. Note: Real exchange rate, real wage, real GDP and consumption expressed in log deviations from their sample means in the model, and from a log quadratic trend in the data. Current-account-to-GDP ratio expressed in deviations from its sample mean in the model and from a quadratic trend in the data. Unemployment rate expressed in levels in the model, and in deviation from $t-2$ values in the data. See Section 4.3 for the definition of a financial crisis episode. Data sources: see Data appendix.

Put in the data is the contraction in measured total factor productivity (TFP), typically observed during sudden stop episodes (see Calvo et al., 2006). In particular, the average sudden stop episode in the data displays a contraction in measured TFP of 7 percent.\footnote{Data on measured TFP for Argentina obtained from Aravena and Fuentes (2013). I am grateful to}
However, to maintain a simple structure, the model does not feature TFP shocks in the nontradable sector, or an endogenous propagation mechanism that shows up as measured TFP (e.g., capacity utilization, as in Gertler et al., 2007, or imported intermediate inputs as in Mendoza and Yue, 2012).

Table 4 shows that, in several dimensions, the second moments observed in the Argentinean data are in line with the second moments predicted by the model. The observed standard deviations of the unemployment rate, the real wage and the real exchange rate are in line with those predicted by the model. The data is closer to the predictions of the model under a currency peg, which can be related to the fact that, as illustrated in Figure 7, in more than 60 percent of the periods since 1980, Argentina was under either a peg or a narrow crawling peg. The model also captures that consumption volatility exceeds output volatility, which is a key feature of EM business cycles (Aguiar and Gopinath, 2007; Uribe and Schmitt-Grohe, 2014). The ratio of standard volatility of consumption to standard volatility of output observed in the data is 1.1, close to the 1.2 predicted under a currency peg. Despite the fact that the volatility of unemployment in the data is similar to the volatility predicted by the model, the volatility of output in the data is significantly higher than the volatility of output predicted by the model. Similar to what was discussed for sudden-stop episodes in the previous paragraph, the higher volatility of output in the data, with respect to the model, can be explained by measured TFP: If the variations in the Solow residual observed in the data are extracted from output, the volatility of output in Argentina in the period of study decreases to 1.9 percent, which is in line with the predictions of the model.

Another relevant dimension observed in the data for Argentina, and in general for EMs, which is qualitatively captured by the model is the countercyclical trade balance (Aguiar and Gopinath, 2007; Uribe and Schmitt-Grohe, 2014). However, the correlation of the trade balance with output observed in the data is less in absolute value than the one predicted by the model. Since the correlation with net factor income from abroad is strongly procyclical both in the data and in the model, the model predicts a procyclical current account. As discussed in Benigno et al. (2012a), an element that could make the model predict a more countercyclical trade balance and current account is the presence of investment (see, for example, Backus, Kehoe, and Kydland, 1993). For the rest of the variables, the correlations with output observed in the data are in line with those predicted by the model. Finally, the autocorrelations observed in the data are also consistent with those predicted by the model. For output, consumption, unemployment, trade balance and the current account, the autocorrelation coefficient observed in the data are higher than those predicted by the model, while the correlation coefficient of the real wage and the real exchange rate observed in the data are within the model’s predictions.

Claudio Aravena for sharing the data on measured TFP.
### Table 4. Second Moments – Data and Model

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>OP</td>
</tr>
<tr>
<td><strong>Volatilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(Y)$</td>
<td>8.2</td>
<td>1.6</td>
</tr>
<tr>
<td>$\sigma(C)/\sigma(Y)$</td>
<td>1.1</td>
<td>1.8</td>
</tr>
<tr>
<td>$\sigma(u)$</td>
<td>3.2</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma(p)$</td>
<td>19.2</td>
<td>40.7</td>
</tr>
<tr>
<td>$\sigma(w)$</td>
<td>23.3</td>
<td>30.5</td>
</tr>
<tr>
<td>$\sigma(TB/Y)$</td>
<td>2.8</td>
<td>2.0</td>
</tr>
<tr>
<td>$\sigma(CA/Y)$</td>
<td>2.8</td>
<td>1.4</td>
</tr>
<tr>
<td><strong>Correlations with output</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(C,Y)$</td>
<td>98.6</td>
<td>75.2</td>
</tr>
<tr>
<td>$\rho(u,Y)$</td>
<td>-23.0</td>
<td>-28.5</td>
</tr>
<tr>
<td>$\rho(p,Y)$</td>
<td>76.3</td>
<td>77.8</td>
</tr>
<tr>
<td>$\rho(w,Y)$</td>
<td>78.5</td>
<td>77.7</td>
</tr>
<tr>
<td>$\rho(TB/Y,Y)$</td>
<td>-95.5</td>
<td>-29.6</td>
</tr>
<tr>
<td>$\rho(CA/Y,Y)$</td>
<td>-88.3</td>
<td>7.9</td>
</tr>
<tr>
<td><strong>Autocorrelations</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho(Y_t,Y_{t-1})$</td>
<td>74.9</td>
<td>55.3</td>
</tr>
<tr>
<td>$\rho(C_t,C_{t-1})$</td>
<td>74.9</td>
<td>36.0</td>
</tr>
<tr>
<td>$\rho(u_t,u_{t-1})$</td>
<td>75.3</td>
<td>8.7</td>
</tr>
<tr>
<td>$\rho(p_t,p_{t-1})$</td>
<td>68.1</td>
<td>43.3</td>
</tr>
<tr>
<td>$\rho(w_t,w_{t-1})$</td>
<td>62.3</td>
<td>43.3</td>
</tr>
<tr>
<td>$\rho(TB_t/Y_t,TB_{t-1}/Y_{t-1})$</td>
<td>70.2</td>
<td>10.3</td>
</tr>
<tr>
<td>$\rho(CA_t/Y_t,CA_{t-1}/Y_{t-1})$</td>
<td>56.5</td>
<td>-12.9</td>
</tr>
</tbody>
</table>

**Note:** Values expressed in percent. OP, FE and CP denote optimal exchange-rate policy, full-employment exchange-rate policy, and currency peg, respectively, as defined in Section 3. The variables $Y$, $C$, $TB$, and $CA$ denote, respectively, output, consumption, trade balance, and current account, expressed in real terms; the variables $u$, $p$, and $w$ denote, respectively, unemployment rate, relative price of nontradables, wages in units of tradables. Moments in the data were computed for the period 1980–2011. Variables in the data were quadratically detrended. Moments predicted by the model were computed using parameters from Table 1. Data sources: see Data appendix.
5 Sensitivity Analysis and Extensions

This section studies how the characterization of the optimal exchange-rate policy during financial crises is affected by alternative parametrizations of the model, alternative shocks, and alternative modeling assumptions.

5.1 Parameter values

This section shows that the main conclusions regarding the characterization of the optimal exchange-rate policy during financial crises are robust to alternative parameter values. In particular, Figure 9 shows the average values of nominal exchange-rate depreciation, real exchange rate, unemployment, and tradable consumption under the optimal exchange-rate policy during periods in which the collateral constraint binds for alternative parameter values. The focus is on financial crises since, as shown in Proposition 1, periods of nonbinding collateral constraint are always characterized by full employment under the optimal exchange-rate policy, independent of parameter values.

I begin by studying alternative values for the intratemporal elasticity of substitution, considering values in the range used in the literature, between $\xi = 0.4$ and $\xi = 0.83$ (see Section 3.2 for a survey). The value of this parameter used in the baseline calibration is $\xi = 0.44$, following the estimates of Gonzalez-Rozada et al. (2004) for Argentina. As explained in Section 3.2, this parameter determines the extent to which exchange-rate depreciations decrease collateral values. If $\xi \geq 1$, there is no negative effect of a currency depreciation on collateral values, and the optimal policy is always to achieve full employment. As expected from this, Figure 9 indicates that the higher the value of the intratemporal elasticity of substitution, the lower the unemployment rate under the optimal exchange-rate policy. In this sense, the conclusions obtained in the baseline calibration are conservative with respect to this parameter value: A higher intratemporal elasticity of substitution would imply an optimal policy even closer to full employment.

I then study alternative values for $\kappa$, the parameter that governs the collateral constraint. To the best of my knowledge, there is no empirical estimate available of this parameter. In the baseline calibration this parameter was set to $\kappa = 0.263$ to match the probability of sudden stops. I now consider alternative values for the collateral parameter ranging from $\kappa = 0.2$ (average debt-to-GDP ratio in Argentinean data) to $\kappa = 0.645$ (maximum debt-to-GDP ratio in Argentinean data). Results in Figure 9 indicate that, in this range of parameter values, the higher the collateral parameter, the lower the depreciation rate under the optimal exchange-rate policy, and the higher resulting unemployment and tradable consumption. The difference is nontrivial: for instance, with a value of $\kappa = 0.645$ the average depreciation rate in a period of financial crisis is 10.3 percent and the resulting unemployment rate, 5.3 percent (which compares to 1.6 percent in the baseline scenario). The intuition for this result is that, the higher the collateral parameter, the higher the effect that containing real exchange-rate depreciation has on collateral values, and thus
the higher the benefits of containing depreciation. However, even in this case, it can be 
observed that the optimal policy features a large real exchange-rate depreciation during 
financial crises (a 24 percent fall in the relative price of nontradable goods), implying that 
it is optimal to contract tradable consumption more than nontradable consumption and 
employment.

In the third place, I study the sensitivity, of the optimal policy during financial crises, 
with respect to the intertemporal elasticity of substitution. In the baseline calibration, the 
inverse of the intertemporal elasticity of substitution was set to $\sigma = 2$, a standard value in 
the business-cycle literature. Since, as studied in Section 3.2, the benefits of credit-market 
access are related to the intertemporal allocation of consumption, this is a key parameter 
in the optimal exchange-rate policy. The range of values of the inverse of the intertemporal 
elasticity of substitution consider is from $\sigma = 2$ to $\sigma = 5$, the value estimated in Reinhart 
and Vegh (1995) for Argentina. Results for this range of parameter values are shown 
in Figure 9. As expected, a lower intertemporal elasticity of substitution is associated 
with a lower optimal exchange-rate depreciation during financial crises. Quantitatively, 
the conclusions are similar to those obtained in the baseline calibration. For instance, 
for $\sigma = 5$ the optimal policy in financial crises is a large nominal and real exchange-rate
depreciation of 35 percent.

Finally, it worth mentioning that $\gamma$, the degree of wage rigidity, does not affect the allocation under the optimal exchange-rate policy, except for the value of the optimal nominal depreciation rate, $\epsilon_t$. This can be seen from the fact that $\gamma$ does not enter in the formulation of the Bellman equation (19) that describes the dynamics of $\{d_{t+1}, c^T_t, h_t\}$ under the optimal exchange-rate policy. It follows that the real exchange-rate depreciation, under the optimal exchange-rate policy, does not depend on $\gamma$ either. The only variable under the optimal exchange-rate policy that is affected by $\gamma$ is the nominal exchange-rate depreciation: the lower the $\gamma$, the lower the average nominal exchange-rate depreciation required to implement the optimal allocation during a financial crisis episode, as illustrated in Figure 9.

In summary, the main findings regarding the optimal exchange-rate policy during financial crises in previous sections are robust with respect to alternative values of structural parameters: The optimal exchange-rate policy implies, on average, large real exchange-rate depreciations during financial crises; this is achieved by depreciating the nominal exchange rate, and implies a relatively small increase in the unemployment rate as compared to the decline in tradable consumption.

### 5.2 Stochastic structure

The baseline quantitative analysis (Section 4) includes interest rate shocks, and uses the contractual interest rate to calibrate these shocks. This section considers instead three alternative stochastic structures: i) an economy with interest-rate shocks calibrated to those of the risk-free rate; ii) an economy with no interest-rate shocks; and iii) an economy with no interest rate shocks but shocks to the parameter $\kappa$, that governs the collateral constraint. These sensitivity analyses are important because, in the presence of credit constraints, calibrating the model using the EM’s contractual interest rate (as in the baseline calibration), which incorporates default risk, could lead to overestimating the costs of borrowing during crises. It is shown that the optimal policy under these alternative stochastic structures still features large nominal and real exchange-rate depreciations during financial crises (between 20 and 25 percent). However, since higher interest rates lead to a lower shadow value from relaxing the credit constraint, the optimal currency depreciation under these stochastic structures is half of that under the baseline stochastic structure.

The first case considers an economy with interest-rate shocks calibrated to those of the risk-free rate. As in the baseline calibration, it is assumed that the two exogenous driving forces – tradable endowment and interest rate – follow a first-order VAR of the form in (27), with the risk-free rate taking the place of the country’s interest rate. The risk-free rate is measured by a US real interest rate (Treasury-bill rate, deflated with a measure of expected dollar inflation, constructed as detailed in the Data appendix). Following
Uribe and Yue (2006), I assume that the risk-free rate follows a univariate process (i.e., $\Phi_{21} = 0$). The parameters of the stochastic process are estimated for the same period and using the same methodology as in the baseline calibration. The parameters $\{\beta, a, \kappa\}$ are used, as in the baseline calibration, to match the average level of external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data. All the rest of the parameters are set as in the baseline calibration (Table 1).

The second case considers an economy with no interest-rate shocks. In this economy, tradable endowment shocks are the only source of uncertainty and are assumed to follow the same stochastic process as in the baseline calibration. The fixed gross interest rate is set to 1.018, the average US real interest rate from 1980 to 2011. Again, as in the baseline calibration, the parameters $\{\beta, a, \kappa\}$ are used to match the average level of external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data, with all the rest of the parameters set as in the baseline calibration (Table 1).

Finally, the third case considers an economy with no interest-rate shocks, but instead with shocks to the parameter $\kappa$, that governs the collateral constraint. These types of shocks have been used in the literature to capture sudden stops driven by shocks to foreign investors’ confidence in EMs (see, for example, Benigno and Fornaro, 2012; Bianchi, Hatchondo and Martinez, 2013). Formally, the collateral constraint (18) is now replaced with

$$dt+1 \leq \kappa_t \left( y_t^T + \left( \frac{1-a}{a} \right) (c_t^T)^{\frac{1}{2}} F(h_t)^{\frac{1}{2}+1} \right),$$

(28)

where $\kappa_t$ follows a first-order Markov process. As is standard in this literature, we assume for simplicity that $\kappa_t \in \{\kappa^L, \kappa^H\}$, with $\kappa^L < \kappa^H$. The value of $\kappa^H$ is set to an arbitrarily high value such that the collateral constraint never binds in period $t$ if $\kappa_t = \kappa^H$. Similar to the baseline calibration, the parameters $\{\beta, a, \kappa^L\}$ are used to match the average external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data. Following Benigno and Fornaro (2012), the probability of entering a low-collateral-constraint state, denoted $\pi_{\kappa}^L$, is set to 0.1 (Jeanne and Ranciere, 2011), and the probability of exiting a low-collateral-constraint state, denoted $\pi_{\kappa}^H$, is set to 0.5 (Alfaro and Kanczuk, 2009). The other parameters are set as in the baseline calibration (Table 1).

Figure 10 displays the optimal nominal exchange rate and endogenous variables for the average financial crisis episode (as defined in section 4.3), under the baseline and the three alternative stochastic structures. The optimal nominal and real exchange-rate depreciation during the average financial crisis episode, under the three alternative stochastic structures, is between 20 and 25 percent. Thus, independently of the assumption on the behavior of interest rates during financial crisis episodes, it is optimal to allow for large currency depreciation, and to induce a relatively small increase in the unemployment rate compared to the adjustment of tradable consumption. However, the fact that,
under the alternative stochastic structures, currency depreciation is half that under the baseline structure suggests that the behavior of interest rates during a financial crisis is key to determining the optimal degree of “fear of floating.” In particular, when a financial crisis occurs with no significant increase in interest rates, optimal policy calls for a more contained depreciation, a smaller current-account adjustment and a larger increase
in unemployment than when the episode occurs with a sizable increase of interest rates.

5.3 Model structure

This section studies the characterization of optimal exchange-rate policy after relaxing two assumptions of the baseline model: endowment in the tradable sector and inelastic labor supply.

5.3.1 Production in the Tradable Sector

This subsection relaxes the baseline model’s assumption of tradable endowment and instead considers production in the tradable sector. This a relevant modification to study since, as shown in Benigno et al. (2012a), labor reallocation is an important mechanism in dealing with financial crises in the presence of a collateral constraint like the one studied in this paper. To facilitate the reallocation mechanism it will be assumed – in sharp contrast to the baseline model – that labor is perfectly mobile across sectors.

As in the nontradable sector, production in the tradable sector is now assumed to be conducted by firms that operate in competitive labor and product markets, each period hiring labor to produce the tradable good, \( y_T \), and using an isoelastic production function. Profits each period, expressed in units of tradable goods, are given by

\[
\Pi_T = Z_T^T (h_T^T)^\alpha - w_t h_T^T,
\]

where \( h_T^T \) denotes labor employed in the tradable sector, \( Z_T^T \) denotes productivity in the tradable sector, assumed to be exogenous and stochastic.

The firms’ problem is to choose \( h_T^T \) to maximize profits given prices \( w_t \) and productivity \( Z_T^T \). The first-order condition of this problem is

\[
Z_t^T \alpha (h_T^T)^{\alpha - 1} = w_t. \tag{29}
\]

Total hours worked is now given by the sum of hours worked in the tradable sector and hours worked in the nontradable sector (denoted \( h_t^N \)):

\[
h_t = h_T^T + h_t^N. \tag{30}
\]

The rest of the equilibrium conditions are the same as in the baseline economy. To calibrate the model, the labor share in the tradable sector is set to \( \alpha_T = 0.5 \), following evidence in Uribe (1997). The parameters \( \{ \beta, a, \kappa \} \) are used, as in the baseline calibration, to match the average level of external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data. All the rest of the parameters are set as in the baseline calibration (Table 1). Given the lack of historical sectoral data on measured TFP for Argentina, and to facilitate comparison with the baseline calibration, it is assumed that the stochastic process is the same as in (27), with \( Z_t^T \) taking the place of \( y_t^T \).
Figure 11 shows that the optimal exchange-rate policy in the economy with production in the tradable sector features less unemployment than in the baseline economy. This is because, in this modified setup, currency depreciations have a larger benefit than in the baseline economy, to reallocate resources and increase production in the tradable sector. This increase in tradable production, in turn, reduces the tightening in the collateral constraint and the required current-account adjustment. Therefore, in equilibrium, even if currency depreciation is smaller than in the baseline economy, unemployment is also smaller than that observed in the baseline economy. These results indicate that the degree of labor mobility across sectors is a key characteristic of the economy for determining the optimal degree of “fear of floating,” and also the severity of financial crises in terms of unemployment and current-account adjustment.

5.3.2 Endogenous Labor Supply

In this section, the assumption of inelastic labor supply is relaxed. I assume, instead of (1), that households’ preferences are given by the expected utility function

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (U(c_t) - v(h_t)),$$

(31)

where the function $v(\cdot)$ is assumed to be continuous, twice differentiable, strictly increasing and convex.

The first-order condition with respect to hours worked is given by

$$v'(h^*_t) = w_t \lambda_t,$$

(32)

where $h^*_t$ denotes the number of hours supplied by households to the labor market.

Actual hours worked cannot exceed labor supply, meaning that labor-market conditions (12) and (13) are replaced respectively by

$$h_t \leq h^*_t,$$

(33)

$$\left( w_t - \gamma \frac{w_t - 1}{\epsilon_t} \right) (h^*_t - h_t) = 0.$$

(34)

The rest of the equilibrium conditions are the same as in the baseline economy.

To calibrate the model, we assume the functional form

$$v(h^*) = -\varphi \frac{(\overline{h} - h_t)^{1-\theta} - 1}{1 - \theta},$$

(35)

where $\varphi > 0$ and $\theta > 0$. The values of $\overline{h}$ and $\varphi$, are set to 3 and 1.5 to match an average level of hours worked at unity (to preserve the size of the nontradable sector), assuming that households at full employment spend one third of their time working. The value of $\theta$ is set to 1.6 which corresponds to an elasticity of labor supply of 1.25, following the estimates in Mendoza (2010). The parameters $\{\beta, a, \kappa\}$ are used, as in the
baseline calibration, to match the average level of external debt-to-GDP ratio, the share of tradable output in GDP, and the frequency of sudden stops observed in the data. The other parameters are set as in the baseline calibration (Table 1).

Figure 11 shows that the optimal exchange-rate policy in an economy with an endogenous labor supply features significantly more unemployment during financial crises than...
in the baseline economy, showing that labor elasticity is also a relevant characteristic of the economy for determining the optimal degree of “fear of floating.” The contraction in the level of employment is greater than in the baseline economy since, in this setup, decreasing employment has a benefit not only in terms of relaxing the credit constraint, but also in terms of increasing leisure. However, most of this increase in unemployment is driven by an increase in the labor supply: Unlike the Ramsey planner, agents do not incorporate the effect that increasing employment has on tightening the collateral constraint, and increase labor supply during financial crises, given the large contraction in consumption that occurs during these episodes. While currency depreciation is smaller than in the baseline economy, the optimal allocation is still characterized by a relatively large real exchange-rate depreciation during financial crises (a 21 percent fall in the relative price of nontradable goods), implying a relatively small decrease in nontradable consumption rate compared to that of tradable consumption.

5.4 No Capital-Control Taxes

So far, the paper has characterized the optimal exchange-rate policy as conditional on the use of an optimal capital-control tax. As discussed in Section 3, this optimal capital-control tax eliminates any inefficient borrowing that might stem from the collateral constraint or from the downward wage rigidity. This section studies the sensitivity of the optimal policy to this assumption by extending the analysis to the case where the only instrument available to the Ramsey planner is the nominal exchange rate.

The Ramsey planner is assumed to have access to a commitment technology. Since the household’s intertemporal optimality condition (6) is part of the set of restrictions, the Ramsey problem cannot, as in Section 3, be expressed with a standard recursive formulation.\(^{16}\) However, the method developed in Marcet and Marimon (2011) can be applied to reformulate the nonrecursive problem with forward-looking variables as a recursive saddle-point problem.\(^{17}\) This approach implies the inclusion of the Lagrange multipliers associated with forward-looking constraints (in this case, equation (6)) as costate variables. As shown in Appendix D, using this method the dynamics under the optimal exchange-rate policy (without capital-control taxes) can be expressed with the recursive

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\(^{16}\)This problem does not arise when the Ramsey planner has access to an optimal capital-control tax since, as shown in Appendix A, the capital-control tax can always be picked so that the optimal allocation satisfies the intertemporal optimality condition (6).

\(^{17}\)For related applications of the Marcet and Marimon (2011) method, see Adam and Billi (2005), Monacelli (2008), and Svenson (2010).
saddle-point problem:

\[
W^{OP} \left( s^X, d, \tilde{\phi}^D \right) = \min_{\tilde{\phi}^{D'}} \max_{d', c^T, h, \mu} \left[ H \left( c^T, h, \mu, \tilde{\phi}^D, \tilde{\phi}^{D'} \right) + \beta \mathbb{E}_{s'X} W^{OP} \left( s^{X'}, d', \tilde{\phi}^{D'} \right) \right]
\]

(36)

s.t. \[
d' = \frac{d + c^T - y^T}{R},
\]

\[
d' \leq \kappa \left( y^T + \left( \frac{1 - a}{a} \right) \left( c^T \right)^\frac{1}{2} F (h)^\frac{1}{2-H} \right),
\]

\[
d' \leq d^N,
\]

\[
h \leq h,
\]

\[
\mu \geq 0,
\]

\[
\mu \left( \kappa \left( y^T + \left( \frac{1 - a}{a} \right) \left( c^T \right)^\frac{1}{2} F (h)^\frac{1}{2-H} \right) - d' \right) = 0,
\]

with

\[
H \left( c^T, h, \mu, \tilde{\phi}^D, \tilde{\phi}^{D'} \right) \equiv U \left( A \left( c^T, F (h) \right) \right) -
- U_c \left( c^T, F (h) \right) A_T \left( c^T, F (h) \right) \left( \frac{\tilde{\phi}^{D'} R - \tilde{\phi}^{D}}{\tilde{\phi}^{D}} \right) + \tilde{\phi}^{D'} \mu,
\]

where the costate variable \( \tilde{\phi}^D \) denotes the Lagrange multiplier of the household’s intertemporal optimality condition (6) chosen in the previous period by the Ramsey planner and can be interpreted as the value to the planner of promises that must be honored from past commitments. To approximate the dynamics under the optimal exchange-rate policy, the functional equation (36) is solved numerically. The parameters used are the same as in Table 1.

Figure 12 shows that the dynamics during financial crisis episodes under the optimal exchange-rate policy without capital-control taxes are similar to those under the optimal exchange-rate policy with optimal capital-control taxes. The main difference is that, in the case without capital-control taxes, the optimal allocation displays a higher nominal and real exchange-rate depreciation and, as a result, a smaller unemployment and a larger current-account adjustment. Therefore, without capital-control taxes, the optimal policy commits to make credit-access tighter during financial crisis episodes, increasing the private costs of borrowing.

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18The presence of additional states and controls in the case of optimal policy without an optimal capital-control tax makes the numerical approximation computationally more demanding. For this reason, the approximation uses a grid of five equally spaced points for both \( \ln (y^T) \) and \( \ln (\frac{R}{P}) \), 51 equally spaced points for \( (d_t) \), and 21 equally spaced points for \( (\tilde{\phi}^D_t) \). For comparison purposes (in this section only) these grids were also used to solve numerically for the optimal exchange rate with optimal capital-control taxes.
Figure 12. Financial Crises – Optimal Policy with No Capital-Control Taxes.

Note: Real exchange rate, real wage, and external debt expressed in log deviations from their sample means. Current account expressed in deviations of its sample mean. Devaluation rate and unemployment rate expressed in levels. See Section 4.3 for the definition of a financial crisis episode.

6 Conclusions

This paper conducts a quantitative study of the optimal exchange-rate policy facing a trade-off between credit access and unemployment, which captures a central discussion of the policy debate typically observed during financial crises in EMs. In the presence of downward nominal wage rigidity, allowing for nominal exchange-rate depreciations can
help attenuate unemployment. In the presence of liability dollarization and collateral constraints linked to tradable and nontradable income, fighting real exchange-rate depreciation alleviates the consumption adjustment.

The main finding is that the optimal exchange-rate policy during financial crises is consistent with managed-floating exchange-rate regimes, widely used by EMs in periods of sudden stops: It is optimal to allow for large currency depreciation (between a 17 and 40 percent average fall in the relative price of nontradable goods), but also to contain currency depreciation with respect to full-employment levels. The bias of the optimal policy towards large currency depreciation is related to the fact that the welfare costs from unemployment and lower consumption typically outweigh those of intertemporal misallocation of consumption.

The findings of the paper suggest that simple policy recipes will, in general, not be optimal during EMs’ financial crises. For instance, both full-employment and fixed-exchange-rate regimes entail relatively large welfare costs compared to the optimal exchange-rate policy during financial crisis episodes – 0.06 percent and 1.8 percent of consumption per period, respectively. Moreover, the paper shows that the optimal degree of “fear of floating” during financial crises depends on the nature of the shocks, and structural characteristics of the economy. This means that what is optimal during a financial crisis episode in a given economy might not be optimal in another economy or in an episode involving a different combination of shocks.

A novel finding is that sudden stops, understood as large current-account adjustments, are generally part of the endogenous response to large negative shocks under the optimal exchange-rate policy. In other words, while exchange-rate policy could prevent sudden stops by resisting real exchange-rate depreciation, the associated unemployment costs make this policy suboptimal.

In future research, several extensions related to the present paper’s framework could be considered. First, the paper abstracts from capital accumulation. Including capital accumulation would be computationally demanding, but would enrich the study of the trade-off faced by the policymaker. Second, the paper studies the optimal exchange-rate policy when the policymaker has access to a commitment technology. An interesting area of future research is the optimal time-consistent exchange-rate policy in a framework in which the policymaker does not have commitment. Third, the paper assumes that all debt is denominated in a foreign currency. A relevant extension would be a study of the interaction between exchange rate policy and optimal currency composition of external debt under the framework of this paper. These extensions are planned for future research.
7 Appendices

7.1 Appendix A: Omitted Constraints in Ramsey Problems

In Section 3.1, to characterize the allocation under the different exchange-rate regimes, I follow the strategy of setting up the Ramsey problem dropping constraints; this Appendix shows that the omitted constraints are satisfied.

7.1.1 Optimal Exchange-Rate Policy

This section shows that any \( \{d_{t+1}, c_t^T, h_t\} \) that satisfy (5), (12), (17), and (18) also satisfy (GE). Pick \( c_t^N = F(h_t) \) to satisfy (15). Pick \( p_t = \left( \frac{1-a}{\gamma} \right) \left( \frac{c_t^T}{c_t^N} \right)^{\frac{1}{\gamma}} \) to satisfy (8). Pick \( \mu_t = 0 \). This makes (9) hold. Pick \( \lambda_t = U_c(c_t^T, c_t^N) A_T(c_t^T, c_t^N) \) to satisfy (7). Choose \( \tau_t^d = 1 - R_t \beta \frac{E_t \lambda_t + 1 + \mu}{\lambda_t} \) to satisfy (6). Choose \( T_t \) to satisfy (14) as: \( T_t = \tau_t^d d_{t+1} R_t^{-1} \).

Pick \( w_t = \left( \frac{1-a}{\gamma} \right) \left( c_t^T \right)^{\frac{1}{\gamma}} F'(h_t) \) to satisfy (16). Given \( w_{t-1} \), pick \( \epsilon_t \) to satisfy (11) with equality: \( \epsilon_t = \gamma \frac{w_{t-1}}{w_t} \). Finally, since (11) holds with equality, (13) always holds: \( \left( w_t - \gamma \frac{w_{t-1}}{w_t} \right) (h_t - h_t) = 0 \).

7.1.2 Full-Employment Exchange-Rate Policy

This section shows that any \( \{d_{t+1}, c_t^T, h_t\} \) that satisfy (5), (17), (18), and (20), also satisfy (GE) and (20). Pick \( c_t^N = F(h_t) \) to satisfy (15). Pick \( p_t = \left( \frac{1-a}{\gamma} \right) \left( \frac{c_t^T}{c_t^N} \right)^{\frac{1}{\gamma}} \) to satisfy (8). Pick \( \mu_t = 0 \). This makes (9) hold. Pick \( \lambda_t = U_c(c_t^T, c_t^N) A_T(c_t^T, c_t^N) \) to satisfy (7). Choose \( \tau_t^d = 1 - R_t \beta \frac{E_t \lambda_t + 1 + \mu}{\lambda_t} \) to satisfy (6). Choose \( T_t \) to satisfy (14) as: \( T_t = \tau_t^d d_{t+1} R_t^{-1} \).

Pick \( w_t = \left( \frac{1-a}{\gamma} \right) \left( c_t^T \right)^{\frac{1}{\gamma}} F'(h_t) \) to satisfy (16). Given \( w_{t-1} \), pick \( \epsilon_t \) to satisfy (11) with equality: \( \epsilon_t = \gamma \frac{w_{t-1}}{w_t} \). Finally, by (20) and (12), (13) always holds: \( \left( w_t - \gamma \frac{w_{t-1}}{w_t} \right) (h_t - h_t) = 0 \).

7.1.3 Fixed Exchange-Rate Policy

This section shows that any \( \{d_{t+1}, c_t^T, h_t, w_t, \epsilon_t\} \) that satisfy (5), (11)–(13), (16)–(18), and (22), also satisfy (GE) and (22). Pick \( c_t^N = F(h_t) \) to satisfy (15). Pick \( p_t = \left( \frac{1-a}{\gamma} \right) \left( \frac{c_t^T}{c_t^N} \right)^{\frac{1}{\gamma}} \) to satisfy (8). Pick \( \mu_t = 0 \). This makes (9) holds. Pick \( \lambda_t = U_c(c_t^T, c_t^N) A_T(c_t^T, c_t^N) \) to satisfy (7). Choose \( \tau_t^d = 1 - R_t \beta \frac{E_t \lambda_t + 1 + \mu}{\lambda_t} \) to satisfy (6). Choose \( T_t \) to satisfy (14) as: \( T_t = \tau_t^d d_{t+1} R_t^{-1} \).

7.2 Appendix B: Proofs

7.2.1 Proof of Proposition 1

The Ramsey problem of optimal exchange-rate policy under an optimal capital-control tax is to maximize (1) with respect to \( \{d_{t+1}, c_t^T, h_t\} \), subject to (5), (12), (17) and (18).
The Lagrangean of the Ramsey problem is then given by
\[
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ U \left( A \left( c_t^T, F(h_t) \right) \right) + \phi_F^t \left[ \frac{d_{t+1} - d_t + y_t^T - c_t^T}{R_t} \right] + \phi_t^c \left[ \kappa \left( y_t^T + \left( \frac{1-a}{a} \right) \left( c_t^T \right)^{\frac{1}{\xi}} F(h_t)^{1-\frac{1}{\xi}} \right) - d_{t+1} \right] + \phi_t^w \left[ d^N - d_{t+1} \right] + \phi_t^\eta \left[ R_t - h_t \right] \right\},
\]
where \( \phi_F^t, \phi_t^c, \phi_t^w, \) and \( \phi_t^\eta \) are Lagrange multipliers.

The optimality conditions associated with this problem (provided \( d_{t+1} < d^N \)) are (12), (17), (18), the first-order conditions
\[
\frac{\phi_F^t}{R_t} = \beta \mathbb{E}_t \phi_F^{t+1} + \phi_t^c,
\]
\[
\phi_t^c = U_c A_T \left( c_t^T, F(h_t) \right) + \phi_t^c \kappa \left( \frac{1}{\xi} \right) \left( \frac{1-a}{a} \right) \left( \frac{c_t^T}{F(h_t)} \right)^{\frac{1}{\xi}-1},
\]
\[
\phi_t^w = F'(h_t) \left[ U_c A_N \left( c_t^T, F(h_t) \right) + \phi_t^\eta \left( \frac{\xi-1}{\xi} \right) \kappa \left( \frac{1-a}{a} \right) \left( \frac{c_t^T}{F(h_t)} \right)^{\frac{1}{\xi}} \right],
\]
and the complementary slackness conditions
\[
\phi_t^c \geq 0; \ \phi_t^c \left[ \kappa \left( y_t^T + \left( \frac{1-a}{a} \right) \left( c_t^T \right)^{\frac{1}{\xi}} F(h_t)^{1-\frac{1}{\xi}} \right) - d_{t+1} \right] = 0,
\]
\[
\phi_t^w \geq 0; \ \phi_t^w \left[ R_t - h_t \right] = 0.
\]

First, consider the case with \( \xi < 1 \). Assume, contrary to the statement of the proposition, that under the optimal exchange-rate policy, at some date \( t, h_t < \overline{h} \) and \( d_{t+1} < \overline{d}(h_t, c_t^T, y_t^T) \). By (41) it follows that \( \phi_t^w = 0 \). By (39), and since \( c_t^T > 0, h_t > 0 \) \( F'(h_t) > 0, U_c A_N \left( c_t^T, F(h_t) \right) > 0 \), and \( \left( \frac{\xi-1}{\xi} \right) < 0 \), this implies that \( \phi_t^c > 0 \), which contradicts (40), which requires \( \phi_t^c \left( \overline{d}(h_t, c_t^T, y_t^T) - d_{t+1} \right) = 0 \).

Second, consider the case with \( \xi \geq 1 \). Assume, contrary to the statement of the proposition, that under the optimal exchange-rate policy, at some date \( t, h_t < \overline{h} \). By (39), and since \( c_t^T > 0, h_t > 0 \) \( F'(h_t) > 0, U_c A_N \left( c_t^T, F(h_t) \right) > 0 \), \( \left( \frac{\xi-1}{\xi} \right) \geq 0 \) and \( \phi_t^c \geq 0 \), this implies that \( \phi_t^w > 0 \), which contradicts (41), which requires \( \phi_t^w \left( \overline{h} - h_t \right) = 0 \).

### 7.2.2 Proof of Proposition 2

Given the initial state \( (s_t^X, d_t) \) and a debt level \( d_{t+1}^* \) with associated tradable consumption \( c_t^T = (d_{t+1}^* R_t^{-1} - d_t + y_t^T) \), pick \( h_t^* \) such that \( d_{t+1}^* = \overline{d}(h_t^*, c_t^T^*, y_t^T) \). This implies setting \( h_t^* \) as:
\[
h_t^* = F^{-1} \left( \left( \frac{1-a}{a} \right) \left( d_{t+1}^* R_t^{-1} - d_t + y_t^T \right)^\frac{1}{\xi} \left( d_{t+1}^* \kappa \left( \frac{\xi}{\xi-1} \right) \left( d_{t+1}^* - y_t^T \right)^{\frac{\xi}{\xi-1}} \right)^\frac{1}{\xi} \right).
\]

By assumption \( d_{t+1}^* > \overline{d}(h_t^*, c_t^T^*, y_t^T) \). Since \( \xi < 1 \), it follows that
\[
\overline{h} > F^{-1} \left( \left( \frac{1-a}{a} \right) \left( d_{t+1}^* R_t^{-1} - d_t + y_t^T \right)^\frac{1}{\xi} \left( d_{t+1}^* \kappa \left( \frac{\xi}{\xi-1} \right) \left( d_{t+1}^* - y_t^T \right)^{\frac{\xi}{\xi-1}} \right)^\frac{1}{\xi} \right),
\]
and thus \( h_t^* < \overline{h} \).
Finally, since \((d_{t+1}^* R_t^{-1} - d_t + y_t^T)^\frac{1}{2} > 0\), and \(F(\bar{h}) > 0\), by the assumption that \(d_{t+1}^* > \bar{d}(\bar{h}, c_t^*, y_t^T)\), it also follows that \(d_{t+1}^* > \kappa y_t^T\), and thus \(h_t^* > 0\).

### 7.2.3 Proof of Remark 1

As shown in Section 7.2.1, the optimality conditions associated with the problem of optimal exchange-rate policy with capital-control taxes (Definition 1) are (12), (17), (18), (37), (38) (39), (40) and (41). In an allocation in which at time \(t\), \(h_t < \bar{h}\), by (41) it follows that \(\phi_t^W = 0\). Replacing in (39), (24) is obtained.

### 7.3 Appendix C: Data Appendix

1. Sectoral data, Argentina: Constructed using data on value added from agriculture, manufacturing, and services from the WDI dataset. The tradable sector was defined as the sum of agriculture and manufacturing sectors. The nontradable sector was defined as services. Data on relative prices was constructed using current and constant value added on each sector.

2. External debt, Argentina: Measured using net foreign assets, obtained from Lane and Milesi-Ferreti (2007) dataset.

3. National accounts, Argentina: Output, consumption, and net exports, obtained from WEO dataset.

4. Interest rates: For Argentina, since 1994, the country interest rate on external debt was measured as the sum of country EMBI spreads and the US Treasury-Bill rate, obtained, respectively, from Datastream and the Federal Reserve of Saint Louis datasets. The series were extended back to 1983 using Neumeyer and Perri (2005) dataset, which uses a measure similar to the one considered here. The risk-free rate was measured with the Treasury-Bill rate. The interest rate series is then deflated with a measure of expected dollar inflation. In particular, \(R_t\) is measured as \(R_t = (1 + i_t) E_t \left(\frac{1}{1 + \pi_{t+1}^*}\right)\), where \(i_t\) denotes the interest rate on Argentinean external debt in US dollars, and \(\pi_{t+1}^*\) denotes US CPI. \(E_t \left(\frac{1}{1 + \pi_{t+1}^*}\right)\) is obtained as the one-step-ahead forecast of an estimated AR(1). US CPI data was obtained from the Federal Reserve of Saint Louis dataset. For EMs, spreads (Figure 4) were measured with the EMBI, obtained from Datastream.

5. Unemployment rate, Argentina: Obtained from INDEC. Since 2003, excludes government social plan “Jefas y Jefes “.


7. Balance of payments, Argentina: Current account and factor services obtained from IFS dataset.

9. Nominal exchange rates and international reserves, EMs: Obtained from IFS dataset.

7.4 Appendix D: Recursive Formulation of the Optimal Exchange-Rate Policy without Capital-Control Taxes

This section shows how to obtain a recursive formulation of the problem of optimal exchange-rate policy without capital-control taxes, using the recursive saddle-point method developed in Marcet and Marimon (2011). Without capital-control taxes, the general equilibrium dynamics are given by stochastic processes \( \{c^N_t, c^T_t, h_t, p_t, d_{t+1}, \lambda_t, \mu_t\}_{t=0}^{\infty} \) satisfying the set of equations (GE'): \( \{ (5)-(9),(11)-(13),(15)-(18) \} \) given an exchange-rate policy \( \{ \epsilon_t\}_{t=0}^{\infty} \), initial conditions \( w^{-1} \) and \( d_0 \), and exogenous stochastic processes \( \{ y^T_t, R_t\}_{t=0}^{\infty} \).

To characterize the allocation under the optimal exchange-rate policy, without optimal capital-control taxes, and under commitment, I begin by setting up the Ramsey problem and dropping constraints (8), (11), (13), and (15)–(16). Any \( \{d_{t+1}, c^T_t, h_t, \lambda_t, \mu_t\} \) that satisfy (5)–(7), (9), (12), (17), and (18), also satisfy (GE'). To see this, pick \( c^N_t = F(h_t) \) to satisfy (15). Pick \( p_t = \left(\frac{1-a}{a}\right) \left(\frac{c^T_t}{c^N_t}\right)^{\frac{1}{2}} F(h_t)^{-\frac{3}{2}} F'(h_t) \) to satisfy (8). Pick \( w_t = \left(\frac{1-a}{a}\right) \left(\frac{c^T_t}{c^N_t}\right)^{\frac{1}{2}} F(h_t)^{-\frac{1}{2}} F'(h_t) \) to satisfy (16). Given \( w_{t-1} \), pick \( \epsilon_t \) to satisfy (11) with equality: \( \epsilon_t = \gamma \frac{w_t-1}{w_t} \). Finally, since (11) holds with equality, (13) always holds: \( (w_t - \gamma \frac{w_t-1}{w_t}) \left(\bar{h} - h_t\right) = 0 \). The Ramsey problem is then given by

\[
\max_{\{d_{t+1}, c^T_t, h_t, \lambda_t, \mu_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U \left( A \left(c^T_t, F(h_t)\right)\right) \tag{42}
\]

s.t. \( d_{t+1} \leq d^N \),

\[
\lambda_t R_t^{-1} = \beta \mathbb{E}_t \lambda_{t+1} + \mu_t,
\]

\[
U_c \left(c^T_t, c^N_t\right) A_T \left(c^T_t, c^N_t\right) = \lambda_t,
\]

\[
\mu_t \geq 0,
\]

\[
\mu_t \left(\kappa \left(y^T_t + p_t F(h_t)\right) - d_{t+1}\right) = 0,
\]

\[
h_t \leq \bar{h},
\]

\[
d_{t+1} \leq \kappa \left(y^T_t + \left(\frac{1-a}{a}\right) \left(\frac{c^T_t}{c^N_t}\right)^{\frac{1}{2}} F(h_t)^{-\frac{1}{2}} F'(h_t)\right).
\]

To obtain the recursive formulation using the method of Marcet and Marimon (2011), the following steps are followed (as in Adam and Billi, 2005). First, set the Lagrangean of problem (42), denoting \( \phi^D_t \) the Lagrange multiplier associated with the forward looking constraint (6). Since equation (6) is forward-looking, some terms in the Lagrangean involve period \( t \) Lagrange multipliers multiplied by period \( t+1 \) controls (e.g., \( \phi^D_t \beta_t \lambda_{t+1} \)). Second, in these terms, relabel the Lagrange multipliers multiplied by period \( t+1 \) controls.
as \( \bar{\phi}_{t+1} \). This relabeling defines a “transition” equation: \( \phi_t^D = \bar{\phi}_{t+1} \). Third, define the period objective function,

\[
H \left( c_t^T, h_t, \mu_t, \bar{\phi}_{t+1}^D, \phi_t^D \right) \equiv U \left( A \left( c_t^T, F \left( h_t \right) \right) \right) - U_c \left( c_t^T, F \left( h_t \right) \right) A_T \left( c_t^T, F \left( h_t \right) \right) \left( \frac{\phi_t^D}{R_t} - \bar{\phi}_{t+1}^D \right) + \phi_t^D \mu_t.
\]

(43)

Fourth, using the equivalence results shown in Marcet and Marimon (2011), re-express the problem (42) as an infinite-horizon saddle-point problem:

\[
\min_{\phi_t^D} \max_{d_t+1, c_t^T, h_t, \mu_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t H \left( c_t^T, h_t, \mu_t, \bar{\phi}_{t+1}^D, \phi_t^D \right)
\]

s.t.

\[
\bar{\phi}_{t+1} = \phi_t^D,
\]

\[
d_{t+1} \leq d^N,
\]

\[
\mu_t \geq 0,
\]

\[
\mu_t \left( \kappa \left( y_t^T + p_t F \left( h_t \right) \right) - d_{t+1} \right) = 0,
\]

\[
h_t \leq \bar{n},
\]

\[
d_{t+1} R_t^{-1} = d_t + c_t^T - y_t^T,
\]

\[
d_{t+1} \leq \kappa \left( y_t^T + \left( \frac{1 - a}{a} \right) \left( c_t^T \right)^\frac{1}{2} F \left( h_t \right) \right),
\]

\[
\bar{\phi}_0 = 0.
\]

Finally, rewrite the infinite-horizon saddle-point problem (44) in recursive form:

\[
W^{OP} \left( s^X, d, \bar{\phi}^D \right) = \min_{\phi_t^D} \max_{d', c_t^T, h_t, \mu_t} \left[ H \left( c_t^T, h_t, \mu_t, \bar{\phi}_{t+1}^D, \phi_t^D \right) + \beta \mathbb{E}_{s^X} W^{OP} \left( s^X', d', \bar{\phi}_{t+1}^D \right) \right]
\]

s.t.

\[
d' = d + c^T - y^T,
\]

\[
d' \leq \kappa \left( y^T + \left( \frac{1 - a}{a} \right) \left( c^T \right)^\frac{1}{2} F \left( h \right) \right),
\]

\[
d' \leq d^N,
\]

\[
h \leq \bar{n},
\]

\[
\mu \geq 0,
\]

\[
\mu \left( \kappa \left( y^T + \left( \frac{1 - a}{a} \right) \left( c^T \right)^\frac{1}{2} F \left( h \right) \right) - d' \right) = 0,
\]

where time subscripts for variables dated in period \( t \) have been dropped, and a prime is used to indicate variables dated in period \( t + 1 \).
References


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