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Simple Analytics of the Government Expenditure Multiplier†

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This paper explains the key factors that determine the output multiplier of government purchases in New Keynesian models, through a series of simple examples that can be solved analytically. Sticky prices or wages allow for larger multipliers than in a neoclassical model, though the size of the multiplier depends crucially on the monetary policy response. A multiplier well in excess of one is possible when monetary policy is constrained by the zero lower bound, and in this case welfare increases if government purchases expand to partially fill the output gap that arises from the inability to lower interest rates. (*JEL* E12, E23, E32, E62, H20, H50)

The recent worldwide economic crisis has brought renewed attention to the question of the usefulness of government spending as a way of stimulating aggregate economic activity and employment during a slump. Interest in fiscal stimulus as an option has been greatly increased by the fact that in many countries, by the end of 2008, the short-term nominal interest rate used as the main operating target for monetary policy had reached zero—or at any rate, some very low value regarded as an effective lower bound by the central bank in question—so that further interest rate cuts were no longer available to stave off spiraling unemployment and fears of economic collapse. Increases in government spending were at least a dimension on which it was possible for governments to do more—but how effective should this be expected to be as a remedy?

Much public discussion of this issue has been based on old-fashioned models (both Keynesian and anti-Keynesian) that take little account of the role of intertemporal optimization and expectations in the determination of aggregate economic activity. This paper, instead, reviews the implications for this question of the kind of New Keynesian dynamic stochastic general equilibrium (DSGE) models that are now commonly used in monetary policy analysis. It focuses on one specific question of current interest: the determinants of the size of the effect on aggregate output of an increase in government purchases, or what has been known since John Maynard Keynes (1936) as the government expenditure "multiplier."

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I discuss this issue in the context of a series of models that are each simple enough for the effects to be computed analytically, so that the consequences of parameter variation for the quantitative results will be completely clear. It is hoped that the economic mechanisms behind the various results will be fairly transparent as well. I also restrict my attention to policy experiments that are defined in such a way that the time path of the increase in output has the same shape as the time path of the increase in government purchases, so that there is a clear meaning to the calculation of a "multiplier" (though more generally this need not be the case). These models are too simple to be taken seriously as the basis for quantitative estimates of the effects of some actually contemplated policy change; nonetheless, I believe that the mechanisms displayed in these simple examples explain many of the numerical results obtained by a variety of recent authors in the context of empirical New Keynesian DSGE models,[1] and the simpler analysis here may be of pedagogical value.

I begin by reviewing in Section I the neoclassical benchmark under which intertemporal optimization should result in a multiplier less than one. Section II then shows that in simple New Keynesian models, if monetary policy maintains a constant real interest rate, the multiplier is instead equal to one. Section III shows that under more realistic assumptions about monetary policy under normal circumstances, the multiplier will be less than one, because real interest rates will increase; but Section IV shows that when the zero lower bound is a binding constraint on monetary policy, the multiplier is instead greater than one, because fiscal expansion should cause the real interest rate to fall. Section V considers the welfare effects of government purchases in these various cases, while Section VI briefly discusses the consequences of allowing for tax distortions. Section VII summarizes the paper's conclusions.

I. A Neoclassical Benchmark

I shall begin by reviewing the argument that government purchases necessarily crowd out private expenditure (at least to some extent), according to a neoclassical general-equilibrium model in which wages and prices are both assumed to be perfectly flexible. This provides a useful benchmark, relative to which I shall wish to discuss the consequences of allowing for wage or price rigidity. I shall confine my analysis here to a relatively special case of the neoclassical model, first analyzed by Robert J. Barro and Robert G. King (1984), though the result that the multiplier for government purchases is less than one does not require such special assumptions.[2]

[1] See, for example, comments below on the studies of Lawrence Christiano, Martin Eichenbaum, and Sergio Rebelo (2009); John F. Cogan et al. (2010); Thorsten Drautzburg and Harald Uhlig (2010); Christopher J. Erceg and Jesper Lindé (2010).

[2] More general expositions of the neoclassical theory include Barro (1989); S. Rao Aiyagari, Christiano, and Eichenbaum (1992); and Marianne Baxter and King (1993).

A. *A Competitive Economy*

Consider an economy made up of a large number of identical, infinite-lived households, each of which seeks to maximize

$$\sum_{t=0}^{\infty} \beta^t [u(C_t) - v(H_t)], \tag{1}$$

where C_t is the quantity consumed in period t of the economy's single produced good, H_t is hours of labor supplied in period t, the period utility functions satisfy $u' > 0, u'' < 0, v' > 0, v'' > 0$, and the discount factor satisfies $0 < \beta < 1$. The good is produced using a production technology yielding output

$$Y_t = f(H_t), \tag{2}$$

where $f' > 0, f'' < 0$. This output is consumed either by households or by the government, so that in equilibrium

$$Y_t = C_t + G_t \tag{3}$$

each period. I shall begin by considering the perfect foresight equilibrium of a purely deterministic economy; the alternative fiscal policies considered will correspond to alternative deterministic sequences for the path of government purchases $\{G_t\}$. I shall also simplify (until Section VI) by assuming that government purchases are financed through lump-sum taxation; a change in the path of government purchases is assumed to imply a change in the path of tax collections so as to maintain intertemporal government solvency. (The exact timing of the path of tax collections is irrelevant in the case of lump-sum taxes, in accordance with the standard argument for "Ricardian equivalence.")

One of the requirements for competitive equilibrium in this model is that in any period

$$\frac{v'(H_t)}{u'(C_t)} = \frac{W_t}{P_t}. \tag{4}$$

This is a requirement for optimal labor supply by the representative household, where W_t is the nominal wage in period t, and P_t is the price of the good. Another requirement is that

$$f'(H_t) = \frac{W_t}{P_t}. \tag{5}$$

This is a requirement for profit-maximizing labor demand by the representative firm. In order for these conditions to simultaneously be true, one must have $v'/u' = f'$ at each point in time.

Using (2) to substitute for H_t and (3) to substitute for C_t in this relation, one obtains an equilibrium condition

$$u'(Y_t - G_t) = \tilde{v}'(Y_t) \tag{6}$$

in which Y_t is the only endogenous variable. Here, $\tilde{v}(Y) \equiv v(f^{-1}(Y))$ is the disutility to the representative household of supplying a quantity of output Y, so that $\tilde{v}' = v'/f'$. (Note that our previous assumptions imply that $\tilde{v}' > 0, \tilde{v}'' > 0$.) This is also obviously the first-order condition for the planning problem of choosing Y_t to maximize utility, given preferences, technology, and the level of government purchases. Thus, this equilibrium condition reflects the familiar result that competitive equilibrium maximizes the welfare of the representative household (in the case that there *is* a representative household).

Condition (6) can be solved for equilibrium output Y_t as a function of G_t. Differentiation of the function implicitly defined by (6) yields a formula for the "multiplier,"

$$\frac{dY}{dG} = \frac{\eta_u}{\eta_u + \eta_v}, \tag{7}$$

where $\eta_u > 0$ is the negative of the elasticity of u' and $\eta_v > 0$ is the elasticity of $\tilde{v}'$ with respect to increases in Y.[3] It follows that the multiplier is positive, but necessarily less than one. This means that private expenditure (here, entirely modeled as nondurable consumer expenditure) is necessarily crowded out, at least partially, by government purchases. In the case that the degree of intertemporal substitutability of private expenditure is high (so that η_u is small), while the marginal cost of employing additional resources in production is sharply rising (so that η_v is large), the multiplier may be only a small fraction of one.

B. *Monopolistic Competition*

The mere existence of some degree of market power in either product or labor markets does not much change this result. Suppose, for example, that instead of a single good there is a large number of differentiated goods, each with a single monopoly producer; and, as in the familiar Dixit-Stiglitz model of monopolistic competition, let us suppose that the representative household's preferences are again of the form (1), but that C_t is now a constant-elasticity-of-substitution aggregate of the household's purchases of each of the differentiated goods,

$$C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}}\, di\right]^{\frac{\theta}{\theta-1}}, \tag{8}$$

where $c_t(i)$ is the quantity purchased of good i, and $\theta > 1$ is the elasticity of substitution among differentiated goods. Let us suppose for simplicity that each good is produced using a common production function of the form (2), with a single homogeneous labor input used in producing all goods. In this model, each producer will face a downward-sloping demand curve for its product, with elasticity θ; profit maximization will then require not production to the point where marginal cost is equal to the price

[3] That is, $\eta_u \equiv -\bar{Y}u''/u'$, $\eta_v \equiv \bar{Y}\tilde{v}''/\tilde{v}'$.

for which it sells its good, but only to the point at which the price of good i is equal to μ times marginal cost, where the desired markup factor is given by

$$\mu \equiv \frac{\theta}{\theta - 1} > 1. \tag{9}$$

Hence, condition (5) must be replaced by the requirement that $p_t(i) = \mu W_t / f'(h_t(i))$ for each good i.

Let us consider a monopolistically competitive equilibrium, in which each firm chooses its price optimally, taking as given the wage and the demand curve that it faces. (I continue to assume perfectly flexible prices, and a competitive labor market or some other form of efficient labor contracting.) Since each firm faces the same wage and a demand curve of the same form, in equilibrium each firm chooses the same price, hires the same amount of labor, and produces the same quantity. It follows that we must also have

$$P_t = \mu W_t / f'(H_t), \tag{10}$$

where P_t is the common price of all goods (and also the price of the composite good), and H_t is the common quantity of labor hired by each firm (and also the aggregate hours worked). It also follows that aggregate output Y_t (in units of the composite good) and aggregate hours worked H_t must again satisfy (2). Optimal labor supply by the representative household also continues to require that (4) hold, where P_t is now the price of the composite good.

Relations (2), (4), and (10) allow us to derive a simple generalization of equation (6),

$$u'(Y_t - G_t) = \mu \tilde{v}'(Y_t), \tag{11}$$

which again suffices to determine equilibrium output as a function of the current level of government purchases. While the equilibrium level of output is no longer efficient, the multiplier is still given by (7), regardless of the value of μ. A similar conclusion is obtained in the case of a constant markup of wages relative to households' marginal rate of substitution: aggregate output is again determined by (11), where μ is now an "efficiency wedge" that depends on the degree of market power in both product and labor markets, and so the multiplier calculation remains the same.[4]

A different result can be obtained, however, if the size of the efficiency wedge is endogenous. One of the most obvious sources of such endogeneity is delay in the adjustment of wages or prices to changing market conditions.[5] If prices are not immediately adjusted in full proportion to the increase in marginal cost resulting

[4] The same result is obtained in the case of a constant rate of taxation or subsidization of labor income, firms' payrolls, consumption spending, or firms' revenues. The tax distortions simply change the size of the efficiency wedge μ in equation (11).

[5] Another possible source of endogeneity is cyclical variation in desired markups due to implicit collusion, as in the model of Julio J. Rotemberg and Woodford (1992). In that model, a temporary increase in government purchases reduces the ability of oligopolistic producers to maintain collusion; the resulting decline in markups increases equilibrium output more than would occur in a perfectly competitive model.

from an increase in government purchases, the right-hand side of (10) will increase more than does the left-hand side; as a consequence, the right-hand side of (11) will increase more than does the left-hand side of that expression. This implies an increase in Y_t greater than the one implied by (11). One can similarly show that if wages are not immediately adjusted in full proportion to the increase in the marginal rate of substitution between leisure and consumption, the right-hand side of (11) will increase more than does the left-hand side, again implying a larger multiplier than the one given in (7).

Hence, the key to obtaining a larger multiplier is an endogenous decline in the labor-efficiency wedge.[6] In a model with sticky prices or wages, however, the degree to which the efficiency wedge changes depends on the degree to which aggregate demand differs from what it was expected to be when prices and wages were set. Equilibrium output is thus no longer determined solely by supply-side considerations; we must, instead, consider the effects of government purchases on aggregate demand.

II. A New Keynesian Benchmark

What is the size of the government expenditure multiplier if prices or wages are sticky—as many empirical DSGE models posit, in order to account for the observed effects of monetary policy on real activity? The answer does not depend solely on the assumed structure of the economy. If prices or wages are sticky, monetary policy affects real activity, and so the consequences of an increase in government purchases depend on the monetary policy response. One might suppose that the question of interest should be the effects of government purchases "leaving monetary policy unchanged;" but one must take care to specify just what is assumed to be unchanged. It is not the same thing to assume that the path of the money supply is unchanged as to assume that the path of interest rates is unchanged, or that the central bank's inflation target is unchanged, or that the central bank continues to adhere to a "Taylor rule," to list only a few of the possibilities.

I shall first consider, as a useful benchmark, a policy experiment in which it is assumed that the central bank *maintains an unchanged path for the real interest rate,* regardless of the path of government purchases. This case corresponds, essentially, to the standard "multiplier" calculation in undergraduate textbooks, where the question asked is how much the "IS curve" shifts to the right—that is, how much output would be increased if the real interest rate were not to change. Here, I wish to consider a similar question, but in a dynamic model, it is necessary to define the hypothetical policy in terms of the entire forward *path* of the real interest rate. The answer to this question provides a useful benchmark for two reasons. The first is that it is simple to calculate; but the second is that the answer is the same under a wide range of alternative assumptions about the nature of price or wage stickiness.

Again, I consider a purely deterministic economy, and let the path of government purchases be given by a sequence $\{G_t\}$, such that $G_t \to \overline{G}$ for large t; the long-run

[6] Robert E. Hall (2009) says that the key is a decline in the price markup; but this is not the only possibility, as is discussed further at the end of Section II.

level of government purchases $\bar{G}$ is held constant while considering alternative possible assumptions about near-term government purchases. Thus, I shall consider only the consequences of *temporary* variations in the level of government purchases. I shall furthermore assume that monetary policy brings about a zero rate of inflation in the long run. (That is, the inflation rate $\{\pi_t\}$ is also a deterministic sequence, such that $\pi_t \to 0$ for large t.) Under quite weak assumptions about the nature of wage and price adjustment, these assumptions about monetary and fiscal policy in the long run imply that the economy converges asymptotically to a steady state in which government purchases equal $\bar{G}$ each period, inflation is equal to zero, and output is equal to some constant level $\bar{Y}$.[7]

Given preferences (1), optimization by households requires that, in equilibrium,

$$\frac{u'(C_t)}{\beta u'(C_{t+1})} = e^{r_t} \qquad (12)$$

each period, where r_t is the (continuously compounded) real rate of return between t and $t+1$. It follows from (12) that in the long-run steady state, $r_t = \bar{r} \equiv -\log\beta > 0$ each period. Since I wish to consider a monetary policy that maintains a constant real rate of interest, regardless of the temporary variation in government purchases, it is necessary to assume that monetary policy maintains $r_t = \bar{r}$ for all t; this is the only constant real interest rate consistent with the assumption of asymptotic convergence to a long-run steady state.

We may suppose that the central bank chooses an operating target for the nominal interest rate i_t according to a Taylor rule of the form

$$i_t = \bar{i}_t + \phi_\pi \pi_t + \phi_y \log\left(Y_t/\bar{Y}\right), \qquad (13)$$

where the response coefficients ϕ_π, ϕ_y are chosen so as to imply a determinate equilibrium under this policy,[8] and where the sequence $\{\bar{i}_t\}$ is chosen so that $\bar{i}_t \to \bar{r}$ for large t (the requirement for asymptotic convergence to the zero-inflation steady state) and so that the equilibrium determined by this monetary policy involves $r_t = \bar{r}$ each period. However, there is no need to assume that the equilibrium is implemented in this way; all that matters for the analysis here is that a monetary policy can be specified that implements the equilibrium in which the real interest rate is constant.

Let us set aside for the moment the question whether such an equilibrium exists (and what sort of monetary policy implements it), and consider what such an equilibrium must be like if it exists. If $r_t = \bar{r}$ for all t, it follows from (12) that $C_t = C_{t+1}$ for all t. Thus, the representative household must be planning a constant level of consumption over the indefinite future, at whatever level is consistent with its intertemporal budget constraint. Convergence to the steady state referred to above

[7] Under many reasonable assumptions about wage and price adjustment, the steady-state level of output $\bar{Y}$ will be the same as in the model with flexible wages and prices, namely, the solution to (11) when $G_t = \bar{G}$.

[8] See Woodford (2003, Proposition 4.3) for the conditions required in the case of the Calvo model of price adjustment described in Section III. In general, the precise conditions for determinacy of equilibrium will depend on the details of wage and price adjustment.

implies that $C_t \to \overline{C} \equiv \overline{Y} - \overline{G}$ for large t; hence, equilibrium must involve $C_t = \overline{C}$ for all t.[9] It then follows from (3) that

$$Y_t = \overline{C} + G_t \tag{14}$$

for all t. Hence, in this case, we find once again that equilibrium output depends only on the level of government purchases in the current period—so that the effects of a given size increase in government purchases are the same regardless of how persistent the increase is expected to be[10]—but now the multiplier (dY_t/dG_t) is equal to one. There is no crowding out of private expenditure by government purchases, though no stimulus of additional private expenditure, either.[11]

An interesting feature of this simple result is that it is quite independent of any very specific assumption about the dynamics of wage and price adjustment. Under the particular assumption about monetary policy made here, the effect on aggregate output depends purely on the demand side of the model. The supply side of the model matters only in solving for the implied path of inflation, wages, and employment, and for the monetary policy required to achieve the hypothesized path of real interest rates. I have, however, made one crucial assumption about the supply side. I have supposed that it is *possible* for monetary policy to maintain $r_t = \overline{r}$ at all times, regardless of the chosen short-run path of government purchases. This assumption is violated by the model with fully flexible wages and prices. However, under many specifications of sticky prices or wages (or both), it is possible for monetary policy to affect real interest rates, and a path for monetary policy can be chosen under which $r_t = \overline{r}$ will hold, in the case of any path for government purchases satisfying certain bounds.

Essentially, it is simply necessary to use the model of wage and price adjustment implied by such a model to determine the paths of wages and prices implied by the dynamics of consumption and output solved for above. Assuming that a solution exists, the implied path for inflation, and hence for inflation expectations, will then yield the required path of the nominal interest rate. (Adjoining a money-demand equation to the model would then allow one to determine the required path of the money supply as well.) In the next section, I present the equations of a particular familiar model of price adjustment (the model with flexible wages and Calvo-style staggered adjustment of prices), and show how it is possible to determine the monetary policy required to keep the real interest rate constant in that model. But it should be evident that the conclusion that *some* monetary policy would be consistent with a constant real rate is in no way dependent on the special details of the Calvo model

[9] This is the point at which it matters to the argument that I consider only paths for government purchases such that $C_t \to \overline{C}$. In the case of a change in the long-run level of government purchases, the long-run steady-state value $\overline{C}$ would also change.

[10] This statement is subject to the proviso, of course, that the long-run level of government purchases, $\overline{G}$, is not changed. If the short-run increase in G_t actually implies that government purchases will have to be *reduced* in the long run, then consumption will *increase*, and the multiplier will be greater than one, as concluded by Giancarlo Corsetti, André Meier, and Gernot J. Müller (2009).

[11] It is possible, instead, to obtain an increase in private expenditure, and hence a multiplier greater than one, if household preferences are nonseparable between consumption and leisure, as discussed by Tommaso Monacelli and Roberto Perotti (2010) and Florin Ovidiu Bilbiie (2009).

of price adjustment; it is equally true in many other models of the dynamics of price adjustment, in models with sticky wages instead of (or in addition to) sticky prices, in models with "sticky information" instead of sticky prices, and so on.

It may seem surprising that the multiplier in this baseline case is independent of the degree of flexibility of prices and wages. There thus appears to be a discontinuity in the case of *complete* flexibility (and *full* information), where the multiplier is given by (7). The explanation is that the derivation of (14) requires that it be possible for monetary policy to maintain a constant real interest rate despite an increase in government purchases; and while such a policy is technically possible, according to the model of price adjustment presented in Section IIIA, for any positive degree of price stickiness, as the degree of price stickiness becomes small, the required degree of inflation becomes extreme. Hence, it becomes implausible to believe that a central bank will actually maintain a constant real interest rate (even if this is technically feasible) in the case of sufficiently flexible (even though not perfectly flexible) prices. For this reason, the relevance of the New Keynesian benchmark does depend on the existence of a sufficient degree of stickiness of prices, wages, or information (or more than one of these).

It is also noteworthy that, in this benchmark case, the predicted multiplier is independent of the degree to which resource utilization is slack; in the derivation of (14), the costs of supplying a given level of output do not figure at all. But once again, supply costs do generally matter for the rate of inflation associated with a given size of government purchases under the assumed monetary policy; more steeply increasing marginal costs as output increases will lead to larger price increases. Again, this means that it is much more plausible to imagine a central bank holding real interest rates constant in response to an increase in government purchases when there is a great deal of excess capacity (so that marginal cost increases little with increased output), rather than when capacity utilization is high (so that marginal cost is steeply increasing). And if capacity constraints are severe enough, it may actually be infeasible to maintain a constant real interest rate under any monetary policy, because no amount of monetary stimulus can induce the increase in supply required in order for the current goods not to be expensive relative to future goods (or indexed bonds).

The simple case considered in this section suffices to establish that New Keynesian models can easily deliver multipliers higher than the one predicted by the neoclassical model. This makes them easier to reconcile with empirical evidence. For example, Hall's (2009) review of the empirical evidence concludes that "GDP rises by roughly the amount of an increase in government purchases" under normal circumstances,[12] which is to say that the multiplier is roughly one. While this is too large an effect to be consistent with neoclassical theory, at least in standard models, it is easily consistent with a simple New Keynesian model, to the extent that monetary policy has in fact maintained a relatively constant real interest rate in response to fiscal shocks.[13] (The response of the real interest rate to fiscal shocks is seldom

[12] He notes that the multiplier may be substantially larger when monetary policy is constrained by the zero bound. This special case is discussed in Section IV.

[13] Under some familiar hypotheses about monetary policy, such as the Taylor rule, the New Keynesian model would predict a smaller multiplier, as is discussed in Section III. However, authors such as John B. Taylor (1999) and Richard Clarida, Jordi Galí, and Mark Gertler (2000) argue that US monetary policy in the 1960s and 1970s

considered in the literature that Hall (2009) reviews; this is a topic that deserves further attention.)

Hall (2009) argues that while New Keynesian models can explain the possibility of a multiplier on the order of one, they can do so only under the hypothesis of countercyclical movement in the markup of prices relative to marginal cost, and he questions the realism of the latter assumption, citing evidence such as the findings of Christopher J. Nekarda and Valerie A. Ramey (2010). Nekarda and Ramey (2010) find that increases in government purchases have little effect on their measure of the markup (the ratio of average labor productivity to the real wage). However, New Keynesian models do not necessarily imply that this measure of the markup must decline in response to an increase in government purchases; the real wage may remain constant, or even fall, if wages are sticky, while average labor productivity may remain constant, or even increase, in the presence of overhead labor or procyclical effort (to cite only two familiar hypotheses). Yet, hypotheses of these types, which are consistent with the Nekarda-Ramey findings, are also consistent with the reasoning given above; under the hypothesis of a central bank that maintains the path of real interest rates fixed despite the increase in government purchases, the multiplier will equal one. Hence, Hall's (2009) critique of the basic mechanism that allows New Keynesian models to predict multipliers of this size seems to be misplaced.

III. Alternative Degrees of Monetary Accommodation

The result obtained in the previous section applies only under one specific assumption about monetary policy, namely, that the path of the real interest rate will remain fixed despite the temporary increase in government purchases. Under alternative assumptions about the degree of monetary accommodation of the fiscal stimulus, the size of the increase in output will be different. Indeed, under some assumptions about monetary policy, the output response predicted by the New Keynesian model may be even smaller than in the neoclassical model. Hence, an empirical finding of a multiplier less than one, under the monetary policy that has been followed historically, does not necessarily disconfirm the validity of the New Keynesian model.

In order to illustrate this point by computing multipliers associated with alternative monetary policies, it is necessary to adopt a specific model of wage and price adjustment. The calculations in this section and the one that follows are based on a particular, very familiar New Keynesian model, in which wages are flexible and prices adjust according to the Calvo model of staggered price adjustment.

A. *Inflation Dynamics and Aggregate Supply: A Simple Model*

Let us assume Dixit-Stiglitz monopolistic competition, as discussed in Section I, but now let us suppose that each differentiated good i is produced using a constant-returns-to-scale technology of the form

was considerably more "passive" than the Taylor rule would prescribe, allowing the real interest rate to fall in response to increases in inflation, and it is possible that the fiscal multipliers found in the empirical literature mainly reflect responses from such periods.

$$y_t(i) = k_t(i) f\big(h_t(i)/k_t(i)\big), \tag{15}$$

where $k_t(i)$ is the quantity of capital goods used in production by firm i, $h_t(i)$ are the hours of labor hired by the firm, and $f(\cdot)$ is the same increasing, concave function as before. I shall assume for simplicity that the total supply of capital goods is exogenously given (and can be normalized to equal one), but that capital goods are allocated to firms each period through a competitive rental market. This assumption implies that each firm will have a common marginal cost of production, a homogeneous degree 1 function of the two competitive factor prices that is independent of the firm's chosen scale of production.

Cost-minimization will imply that each firm chooses the same labor/capital ratio, regardless of its scale of production, and in equilibrium this common labor/capital ratio will equal H_t, the aggregate labor supply (recalling that aggregate capital is equal to one). Hence, the common nominal marginal cost of production S_t in any period will equal

$$S_t = W_t/f'(H_t). \tag{16}$$

If we assume flexible wages and a competitive labor market, (4) must again hold in equilibrium; substituting this for W_t in (16) yields

$$S_t = P_t \frac{\tilde{v}'\big(f(H_t)\big)}{u'(Y_t - G_t)}. \tag{17}$$

Note that in the case that each firm's price is a fixed markup μ over marginal cost (as would follow from Dixit-Stiglitz monopolistic competition with flexible prices), condition (17) together with (2) would imply that output must satisfy (11), as concluded previously.

In the Calvo model of staggered price adjustment, it is assumed that fraction $1 - \alpha$ of all firms reconsider their prices in any given period, while the others continue to charge the same price as in the previous period. (The probability that any firm will reconsider its price in any period is assumed to be independent of the time since it last reconsidered its price, and of how high or low its current price may be.) To a log-linear approximation,[14] the optimal price p_t^* chosen by each firm that reconsiders its price in period t will be given by[15]

$$\log p_t^* = \log \mu + \sum_{j=0}^{\infty} (1 - \alpha\beta)\alpha^j \beta^j E_t[\log S_{t+j}]. \tag{18}$$

[14] Here, I log-linearize around the zero-inflation steady state, which under the assumed monetary policy is the equilibrium in the case that government purchases equal $\bar{G}$ each period. Hence, the approximation is valid if in all periods G_t remains close enough to $\bar{G}$. Further details of the calculation sketched here are presented in Woodford (2003, chap. 3).

[15] Here, I write the condition in the more general form that applies in the case of a stochastic environment, as preparation for further applications below.

(This is just a weighted geometric average of the prices $p^f_{t+j} = \mu S_{t+j}$ that a profit-maximizing flexible-price firm would choose in each of the future periods $t + j$.)

Since in each period, a fraction $(1 - \alpha)\alpha^j$ of all firms chose their current price j periods earlier (for each $j \geq 0$), in a similar log-linear approximation the price index evolves according to a law of motion

$$\log P_t = \alpha \log P_{t-1} + (1 - \alpha)\log p^*_t. \tag{19}$$

Condition (19) together with (18) allow one to show that

$$\log(p^*_t/P_t) = (1 - \alpha\beta)\sum_{j=0}^{\infty} \beta^j E_t[\log\mu + \log S_{t+j} - \log P_{t+j}]. \tag{20}$$

Thus, a firm that reconsiders its price will choose a high relative price, to the extent that a weighted geometric average of the profit-maximizing relative prices $\mu S_{t+j}/P_{t+j}$ in the various future periods $t + j$ is high. In the case of fully flexible prices, P_t must equal p^*_t each period, in which case (20) requires that $P_t = \mu S_t$ each period, leading again to (11). But with sticky prices, it is possible for P_t to differ from μS_t (and hence for Y_t to violate equation (11)); this simply requires that firms that reconsider their prices choose a price different from the general level of prices ($p^*_t \neq P_t$), resulting in inflation or deflation ($P_t \neq P_{t-1}$) in accordance with (19).

A similar log-linear approximation to (17) takes the form[16]

$$\log(S_t/P_t) = -\log\mu + \eta_v \hat{Y}_t + \eta_u(\hat{Y}_t - \hat{G}_t), \tag{21}$$

where the elasticities $\eta_v, \eta_u > 0$ are defined as in (7), and the deviations from steady state are defined as $\hat{Y}_t \equiv \log(Y_t/\bar{Y}), \hat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$.[17] Hence, an increase in $\hat{Y}_t$ greater than the one implied by the flexible-price multiplier (7) requires that real marginal cost S_t/P_t increases. Substituting this into (20), we obtain

$$\log(p^*_t/P_t) = (1 - \alpha\beta)(\eta_u + \eta_v)\sum_{j=0}^{\infty} \beta^j E_t[\hat{Y}_{t+j} - \Gamma\hat{G}_{t+j}], \tag{22}$$

where $\Gamma < 1$ is the flexible-price multiplier defined in (7). Then, since (19) implies that the inflation rate is given by

$$\pi_t \equiv \log(P_t/P_{t-1}) = \frac{1 - \alpha}{\alpha}\log(p^*_t/P_t), \tag{23}$$

[16] Note that because the steady state around which the approximation is computed involves the same level of production of each good, log-linearization of (15) and integration over i implies that, to this order of approximation, the aggregate quantities Y_t and H_t satisfy (2). This allows an expression to be derived for real marginal cost as a function of $\hat{Y}_t$ and $\hat{G}_t$ only.

[17] The latter definition is chosen so that $\hat{G}_t$ is defined even if $\bar{G} = 0$, and so that $\hat{G}_t$ and $\hat{Y}_t$ are in comparable units (i.e., percentages of steady-state output).

we obtain

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t[\hat{Y}_{t+j} - \Gamma \hat{G}_{t+j}], \tag{24}$$

where $\kappa \equiv (1 - \alpha)(1 - \alpha\beta)(\eta_u + \eta_v)/\alpha > 0$.

We can now answer the question whether it is possible for monetary policy to maintain a constant real interest rate in the case of an arbitrary path $\{G_t\}$ for government purchases, at least in the case that G_t remains always close enough to $\bar{G}$ for the log-linear approximation to be accurate. For an arbitrary path $\{G_t\}$, the solution for the path of output $\{Y_t\}$ is given by (14). Substituting this into (24), one obtains a solution for the path of the inflation rate as well.[18] It is then straightforward to solve for the equilibrium path of the nominal interest rate, and for the path $\{\bar{i}_t\}$ of intercepts for the central bank reaction function (13). One thus obtains a policy that implements the equilibrium conjectured in Section II.

B. *A Strict Inflation Target*

As an example of another simple hypothesis about monetary policy, suppose that the central bank maintains a strict inflation target, regardless of the path of government purchases. (For conformity with the assumption made above about the long-run steady state, suppose that the inflation target is zero.) In the case of the Calvo model of price adjustment, (23) implies that maintaining a zero inflation rate each period requires that $p_t^* = P_t$ each period. It then follows from (20) that this requires that $\mu S_t = P_t$ each period.[19] If we assume flexible wages (or efficient labor contracting), (17) implies that this will hold if and only if Y_t satisfies (11) each period. Hence, under this policy, aggregate output Y_t will be the same function of G_t as in the case of flexible prices, and the multiplier will be given by (7).

Again, this result does not depend on the precise details of the Calvo model of price adjustment. In a wide range of specifications with sticky prices (or prices set on the basis of sticky information), a sufficient (and often necessary) condition for zero inflation each period is maintenance of aggregate conditions under which the marginal cost of production satisfies $S_t = P_{t-1}/\mu$ each period. For if this condition holds, then under the assumption that each firm that reconsiders its price at any date chooses $p_t^* = P_{t-1}$, not only will all prices remain constant over time, but each firm will find that marginal revenue equals marginal cost each period, so that no firm would expect to increase profits by deviating from this pricing strategy. But such a policy thus ensures that each firm's price is equal to μS_t each period, so that the equilibrium is the same as if all prices were fully flexible and set on the basis of full information. Hence, the multiplier will be given by (7), just as in the neoclassical model.

[18] Note that for any bounded sequence $\{\hat{G}_t\}$, the infinite sum is well defined.

[19] One can show that this is true in the exact model, and not merely in the log-linear approximation used in (20).

C. *Monetary Accommodation under a Taylor Rule*

A less extreme hypothesis would assume that policy is not tightened so much in response to a fiscal expansion as to prevent any increase in prices, but that real interest rates do rise in response to any increase in prices that occurs, rather than being held constant regardless of the consequences for inflation. For example, suppose that interest rates are set in accordance with a Taylor rule of the form

$$i_t = \overline{r} + \phi_\pi \pi_t + \phi_y(\hat{Y}_t - \Gamma\hat{G}_t), \tag{25}$$

where i_t is a short-term riskless nominal rate (the central bank's policy instrument), $\overline{r}$ is the value of this rate in a steady state with zero inflation (so that the policy rule is consistent with that steady state), and the response coefficients satisfy $\phi_\pi > 1$, $\phi_y > 0$, as proposed by Taylor (1993). Here, $\hat{Y}_t - \Gamma\hat{G}_t$ corresponds to one interpretation of the "output gap," namely, the number of percentage points by which aggregate output exceeds the flexible-price equilibrium level.

In order to determine the equilibrium implications of a policy rule of this kind, it is useful also to log-linearize equilibrium relation (12), yielding[20]

$$\hat{Y}_t - \hat{G}_t = E_t[\hat{Y}_{t+1} - \hat{G}_{t+1}] - \sigma(i_t - E_t\pi_{t+1} - \overline{r}), \tag{26}$$

where $\sigma \equiv \eta_u^{-1} > 0$ measures the intertemporal elasticity of substitution of private expenditure.[21] If we consider deterministic paths for government purchases of the simple form $\hat{G}_t = \hat{G}_0\rho^t$ for some $0 \leq \rho < 1$, then the future path of government purchases looking forward from any date t is a time-invariant function of the level of $\hat{G}_t$ at that date. Conjecturing a solution of the form

$$\hat{Y}_t = \gamma_y \hat{G}_t, \tag{27}$$

$$\pi_t = \gamma_\pi \hat{G}_t, \tag{28}$$

$$i_t = \overline{r} + \gamma_i \hat{G}_t, \tag{29}$$

for some coefficients $\gamma_y, \gamma_\pi, \gamma_i$, we can substitute these equations into (24), (25), and (26), and solve for the values of the coefficients for which all three equilibrium conditions are satisfied each period.

There is easily a unique solution of this form, in which

$$\gamma_y = \frac{1 - \rho + \psi\Gamma}{1 - \rho + \psi}, \tag{30}$$

[20] Again, I write the log-linear approximation for the more general stochastic form of this equilibrium condition, as this will be used in the next section.

[21] Here, i_t is a continuously compounded nominal rate — that is, $i_t \equiv -\log Q_t$, where Q_t is the nominal price of a bond that pays one unit of currency with certainty in period $t+1$ — and $\overline{r} \equiv -\log\beta$ is the corresponding continuously compounded rate of time preference.

where

$$\psi \equiv \sigma\left[\phi_y + \frac{\kappa}{1 - \beta\rho}(\phi_\pi - \rho)\right] > 0.$$

It follows from (27) that in this case the multiplier is simply the coefficient γ_y. One observes from (30) that under this policy, $\Gamma < \gamma_y < 1$. Thus, the multiplier is necessarily higher than in the flexible-price model (or under the strict inflation targeting policy), but smaller than under the constant-real-interest rate policy. It is higher than under strict inflation targeting, because under the Taylor rule, inflation is allowed to rise somewhat in response to fiscal stimulus; but lower than under the constant-real-interest-rate policy, because the real interest rate is increased in response to the increases in inflation and in the output gap. Note also that for a policy rule of this form, the size of the multiplier depends on the degree of stickiness of prices (through the dependence of ψ upon the value of κ); the more flexible are prices (i.e., the smaller the value of α), the larger is κ and hence ψ, and the smaller is the multiplier.

A still more realistic assumption about monetary policy might be to assume a Taylor rule of the form (13), but with a constant intercept. (I shall assume $\bar{i}_t = \bar{r}$, for consistency with the zero-inflation steady state.) In this case, the central bank is assumed to respond to deviations of aggregate output from its average (or trend) level, rather than to departures from the flexible-price equilibrium level. (In fact, most central banks use measures of potential output that do not assume that potential should depend on the level of government purchases, as in specification (25).) In this case, we again obtain a solution of the form (27)–(29), but with different constant coefficients; the multiplier is now given by

$$(31) \qquad \gamma_y = \frac{1 - \rho + (\psi - \sigma\phi_y)\Gamma}{1 - \rho + \psi}.$$

The multiplier is necessarily smaller under this kind of Taylor rule, since (for any $\phi_y > 0$) the degree to which monetary policy is tightened in response to expansionary fiscal policy is necessarily greater. In fact, in the case of any large enough value of ϕ_y, the multiplier under this kind of Taylor rule is even smaller than the one predicted by the neoclassical model. In such a case, price stickiness results in even less output increase than would occur with flexible prices, because the central bank's reaction function raises real interest rates more than would occur with flexible prices (and more than is required to maintain zero inflation). Hence, while larger multipliers are possible according to a New Keynesian model, they are predicted to occur only in the case of a sufficient degree of monetary accommodation of the increase in real activity; and, in general, this will also require the central bank to accommodate an increase in the rate of inflation.

IV. Fiscal Stimulus at the Zero Interest Rate Lower Bound

One case in which it is especially plausible to suppose that the central bank will not tighten policy in response to an increase in government purchases is when monetary policy is constrained by the zero lower bound on the short-term nominal interest rate. This is a case in which it is plausible to assume not merely that the real interest rate does not rise in response to fiscal stimulus, but that the *nominal* rate does not rise; this will actually be associated with a *decrease* in the real rate of interest, to the extent that the fiscal stimulus is associated with increased inflation expectations. Hence, government purchases should have an especially strong effect on aggregate output when the central bank's policy rate is at the zero lower bound.[22] This is also a case of particular interest, since calls for fiscal stimulus become more urgent when it is no longer possible to achieve as much stimulus to aggregate demand as would be desired through interest-rate cuts alone.

In practice, the zero lower bound is most likely to become a binding constraint on monetary policy when financial intermediation is severely disrupted, as during the Great Depression or the recent financial crisis.[23] A simple extension of the model proposed above allows us to see how this can occur. Suppose that the interest rate that is relevant in condition (12) for the intertemporal allocation of expenditure is not the same as the central bank's policy rate, and furthermore that the spread between the two interest rates varies over time, owing to changes in the efficiency of financial intermediation.[24] If we let i_t denote the policy rate, and $i_t + \Delta_t$ the interest rate that is relevant for the intertemporal allocation of expenditure, then (26) takes the more general form

$$\hat{Y}_t - \hat{G}_t = E_t[\hat{Y}_{t+1} - \hat{G}_{t+1}] - \sigma(i_t - E_t\pi_{t+1} - r_t^{net}), \tag{32}$$

where $r_t^{net} \equiv -\log\beta - \Delta_t$ is the real *policy* rate required to maintain a constant path for private expenditure (at the steady-state level). If the spread Δ_t becomes large enough, for a period of time, as a result of a disturbance to the financial sector, then the value of r_t^{net} may temporarily be negative. In such a case the zero lower bound on i_t will make (32) incompatible, for example, with achievement of the steady state with zero inflation and government purchases equal to $\overline{G}$ in all periods.

A. *A Two-State Example*

As a simple example (based on Gauti B. Eggertsson 2009), suppose that under normal conditions, $r_t^{net} = \overline{r} > 0$, but that as a result of a financial disturbance at date 0, credit spreads increase, and r_t^{net} falls to a value $r_L < 0$. Suppose that each period thereafter, there is a probability $0 < \mu < 1$ that the elevated credit spreads persist in period t, and that r_t^{net} continues to equal r_L, if credit spreads were elevated in period $t - 1$; but with probability $1 - \mu$ credit spreads return to their normal level, and $r_t^{net} = \overline{r}$. Once credit spreads return to normal, they remain at the normal

[22] In fact, it matters only that the policy rate be at a level that the central bank is unwilling to go below; this "effective lower bound" need not be zero.

[23] See Christiano (2004) for a quantitative analysis of the conditions under which the zero bound would be a binding constraint even in the absence of financial frictions.

[24] Vasco Cúrdia and Woodford (2009) present a complete general equilibrium model with credit frictions in which the policy rate is lower than the rate of interest that enters the equilibrium relation that generalizes (26), and describe a number of sources of variation in the spread between the two rates.

level thereafter. (This exogenous evolution of the credit spread is assumed to be unaffected by either monetary or fiscal policy choices.)

Suppose, furthermore, that monetary policy is described by a Taylor rule, except that the interest rate target is set to zero if the linear rule would call for a negative rate; specifically, let us suppose that

$$i_t = \max\{\overline{r} + \phi_\pi \pi_t + \phi_y \hat{Y}_t, 0\}, \tag{33}$$

so that the rule would be consistent with the zero-inflation steady state, if r_t^{net} were to equal $\overline{r}$ at all times. (We shall again suppose that $\phi_\pi > 1$, $\phi_y > 0$, as prescribed by Taylor.) Finally, let us consider fiscal policies under which government purchases are equal to some level G_L for all $0 \leq t < T$, where T is the random date at which credit spreads return to their normal level, and equal to $\overline{G}$ for all $t \geq T$. The question we wish to consider is the effect of choosing a higher level of government purchases G_L during the crisis, taking as given the value of $\overline{G}$ (the level of government purchases during normal times) and the monetary policy rule (33).

Since there is no further uncertainty from date T onward, and the equilibrium conditions (24), (32), and (33) are all purely forward-looking, it is natural to suppose that the equilibrium from date T onward should be the zero-inflation steady state; hence, the equilibrium values will be $\pi_t = \hat{Y} = 0$, $i_t = \overline{r} > 0$ for all $t \geq T$.[25] Given this solution for the equilibrium from date T onward, we wish to determine the equilibrium evolution prior to date T. Equilibrium conditions (24), (32), and (33) can be "solved forward" to obtain a unique bounded solution if and only if the model parameters satisfy

$$\kappa\sigma\mu < (1 - \mu)(1 - \beta\mu). \tag{34}$$

Note that this condition holds for all $0 \leq \mu < \overline{\mu}$, where the upper bound $\overline{\mu} < 1$ depends on the model parameters (β, κ, σ). Here, I consider only the case in which (34) is satisfied, which is to say, in which it is not expected that the crisis is likely to persist for too many years. Then, since at each date $t < T$, the probability distribution of future evolutions of fundamentals (the joint evolution of $\{r_t^{net}, \hat{G}_t\}$) is the same, the unique bounded solution obtained by "solving forward" is one in which $\pi_t = \pi_L, \hat{Y}_t = \hat{Y}_L, i_t = i_L$ for each $t < T$, for certain constant values $(\pi_L, \hat{Y}_L, i_L)$.

These constant values can be obtained by observing that (24) requires that

$$\pi_L = \frac{\kappa}{1 - \beta\mu}(\hat{Y}_L - \Gamma\hat{G}_L), \tag{35}$$

and that (32) requires that

$$(1 - \mu)\big(\hat{Y}_L - \hat{G}_L\big) = \sigma(-i_L + \mu\pi_L + r_L). \tag{36}$$

[25] One can show that this is a locally determinate rational-expectations equilibrium for dates $t \geq T$, under the policies assumed; that is, it is the only solution in which inflation and output remain within certain bounded intervals.

Using (35) to substitute for π_L in (36), one obtains an equation that can be solved to yield

$$\hat{Y}_L = \vartheta_r(r_L - i_L) + \vartheta_G \hat{G}_L, \tag{37}$$

where

$$\vartheta_r \equiv \frac{\sigma(1 - \beta\mu)}{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu} > 0, \tag{38}$$

$$\vartheta_G \equiv \frac{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu\Gamma}{(1 - \mu)(1 - \beta\mu) - \kappa\sigma\mu} > 1.$$

(Here, the indicated bounds follow from (34) and the fact that $\Gamma < 1$.)

One can then substitute (37) and the associated solution for the inflation rate into (33) and solve the resulting equation for i_L. The solution lies on the branch of (33), where $i_L = 0$ for values of $\hat{G}_L$ near zero if and only if

$$\overline{r} + \left(\frac{\kappa}{1 - \beta\mu}\phi_\pi + \phi_y\right)\vartheta_r\, r_L < 0. \tag{39}$$

This is the case of interest here; assuming that r_L is negative enough for (39) to hold, the zero lower bound will bind in the case that government purchases remain at their normal (steady-state) level.[26] In fact, it will bind in the case of any $\hat{G}_L < \hat{G}^{crit}$, where

$$\hat{G}^{crit} \equiv \frac{\left(\frac{\kappa}{1 - \beta\mu}\phi_\pi + \phi_y\right)\vartheta_r(-r_L) - \overline{r}}{\frac{\kappa}{1 - \beta\mu}\phi_\pi(\vartheta_G - \Gamma) + \phi_y\vartheta_G} > 0.$$

For any level of government purchases below this critical level, equilibrium output will be given by

$$\hat{Y}_L = \vartheta_r r_L + \vartheta_G \hat{G}_L \tag{40}$$

for all $t < T$, and the inflation rate will equal the value π_L given by (35).

In this equilibrium, there will be both deflation and a negative output gap (output below its level with flexible wages and prices), for as long as credit spreads remain elevated, in the case of any level of government purchases $G_L \leq G^{crit}$.[27] The

[26] Note that if, as in Eggertsson and Woodford (2003), it is assumed that the central bank pursues a strict zero inflation target as long as this is consistent with the zero lower bound, then the zero lower bound necessarily binds at dates $t < T$ if $\hat{G}_L = 0$, as long as $r_L < 0$. The values computed here for the multipliers dY_L/dr_L and dY_L/dG_L are the same under that simpler hypothesis.

[27] As illustrated in Figure 1, output may nonetheless exceed its steady-state level; for the parameter values assumed in the figure, Y_L exceeds $\overline{Y}$ (so that $\hat{Y}_L > 0$) for values of G_L near G^{crit}, though the output gap remains negative, because the increased government purchases increase the "natural" level of output.

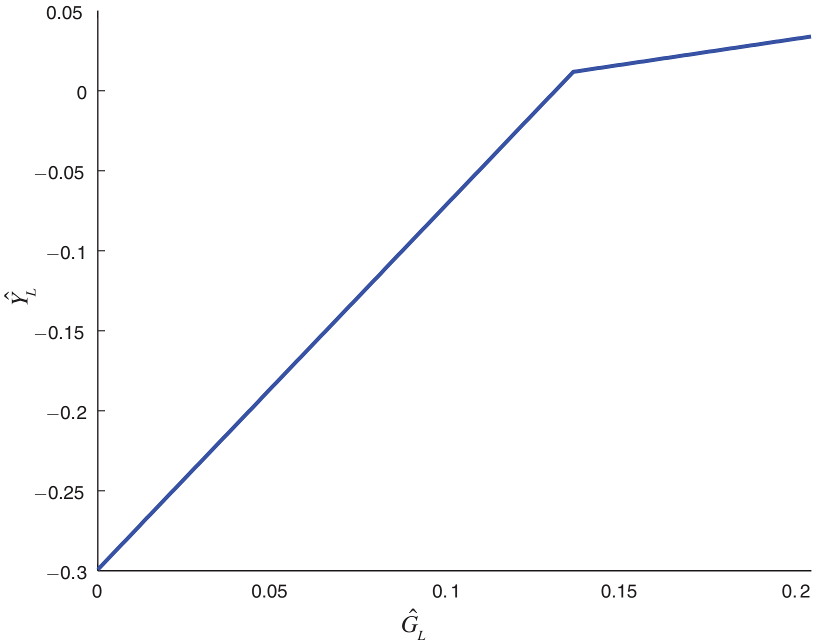

Figure 1. Output as a Function of the Level of Government Purchases during the Period in which Credit Spreads Remain Elevated

deflation and economic contraction can be quite severe, for even a modestly negative value of r_L, in the case that μ is large; in fact, ϑ_r (the multiplier dY/dr plotted in Figure 2) becomes unboundedly large as μ approaches $\overline{\mu}$. Under such circumstances, it can be highly desirable to stimulate aggregate demand by increasing the level of government purchases.

For levels of government purchases up to G^{crit}, (40) implies that each additional dollar spent by the government increases gross domestic product (GDP) by ϑ_G dollars.[28] Increases in government purchases beyond that level result in even higher levels of GDP, though the increase per dollar of additional government purchases is smaller, as shown in Figure 1, owing to the central bank's increase in interest rates in accordance with the Taylor rule. (Figure 1 plots $\hat{Y}_L$ as a function of $\hat{G}_L$, for the numerical parameter values proposed by Eggertsson (2009).[29] Under these parameter values,

[28] Note that this multiplier is calculated using approximations to the model structural equations that have been log-linearized around the zero-inflation steady state, as in Eggertsson (2009) and Christiano, Eichenbaum, and Rebelo (2009). However, the case considered here is necessarily some distance from that steady state, so the derivatives used need not yield a correct multiplier. (The multiplier computed here is correct only in the case that r_L is a sufficiently *small* negative quantity, so that π_L and $\hat{Y}_L$ remain close to zero when $\hat{G}_L = 0$.) R. Anton Braun and Yuichiro Waki (2010) find that log-linearization around the zero-inflation steady state can substantially exaggerate the size of the multiplier under realistic parameter values; but they still conclude on the basis of their nonlinear analysis that the multiplier is well above one.

[29] Eggertsson (2009) chooses parameter values to fit the size of the contraction experienced by the US economy during the Great Depression. According to his modal parameter estimates (for a quarterly model), $\beta = 0.997$, $\kappa = 0.00859$, $\sigma = 0.862$, and $\Gamma = 0.425$. The shock required to account for the size of the contraction during the Great Depression is one under which $r_L = -0.010$ (minus 4 percent per annum) and $\mu = 0.903$ (an expected mean duration a little over ten quarters); the response coefficients for monetary policy are assumed to be $\phi_\pi = 1.5$,

G^{crit} is reached when government purchases exceed their steady-state value by 13.6 percent of steady-state GDP.[30]) For values $G_L > G^{crit}$, the multiplier is no longer ϑ_G, but instead the coefficient γ_y defined in (31), where the persistence parameter ρ is now replaced by μ.[31]

It follows from (38) that the multiplier $dY_L/dG_L = \vartheta_G$ for government purchases up to the level $\hat{G}^{crit}$ is necessarily greater than one (for any $\mu > 0$). The reason is that, given that the nominal interest rate remains at zero in periods $t < T$, an increase in G_L, which increases π_L, accordingly increases expected inflation (given some positive probability of elevated credit spreads continuing for another period), and so *lowers* the *real* rate of interest.[32] Hence, monetary policy is even more accommodative than is assumed in the benchmark analysis in Section II, and the increase in aggregate output is correspondingly higher.

The degree to which the multiplier exceeds one in this case can, in principle, be quite considerable. In fact, for any given values of the other parameters, the multiplier, while the policy rate remains at the zero bound, can be unboundedly large, for a sufficiently large value of the persistence parameter μ. Figure 2 plots the multiplier as a function of μ, holding the other model parameters fixed at the values used by Eggertsson (2009). The figure illustrates something that can be observed from (38) to hold quite generally; the multiplier is monotonically increasing in μ, and increases without bound as μ approaches $\bar{\mu}$. The figure also indicates that the multiplier is in general not too much greater than 1, except if μ is fairly large. However, it is important to note that the case in which μ is large (in particular, a large fraction of $\bar{\mu}$) is precisely the case in which the multiplier dY_L/dr_L is also large, which is to say, the case in which a moderate increase in the size of credit spreads can cause a severe output collapse.[33]

Thus, increased government purchases when interest rates are at the zero bound should be a powerful means through which to stave off economic crisis precisely in those cases in which the constraint of the zero lower bound would otherwise be most crippling—namely, those cases in which there is insufficient confidence that the disruption of credit markets will be short-lived. For example, in Eggertsson's (2009)

$\phi_y = 0.25$. (The justification of these parameter values is discussed in greater detail in Matthew Denes and Eggertsson 2009.) Note that because I use a simpler model of the labor market in the current exposition, κ is not the same function of underlying parameters in (24) above as in Eggertsson's (2009) paper. Here, I parameterize the model so that the value of κ is the same as in Eggertsson's (2009) paper, meaning that implicitly the value of α is larger than the value assumed by Eggertsson (2009). The difference in the values assumed for α has no consequences for the multiplier calculations discussed here.

[30] In drawing Figure 1, I have also assumed that the credit spread is zero in the "normal" state, so that $\bar{r} = -\log\beta$. Allowing for a small positive credit spread in this state would raise the value of G^{crit}.

[31] Under Eggertsson's (2009) parameter values, this quantity is equal only to 0.3. (Note that this is a case in which, when the central bank is not constrained by the zero bound, the multiplier under a Taylor rule that responds to detrended output is actually lower than the neoclassical benchmark; for under Eggertsson's (2009) parameter values, $\Gamma = 0.4$.) Under the alternative hypothesis that the central bank implements a strict zero inflation target, except when prevented by the zero bound, the multiplier above the critical level of government purchases is equal to Γ. If, instead, the central bank follows a Taylor rule of the form (25), the multiplier beyond the critical level of government purchases is given by (30).

[32] Note that the increase in expected inflation referred to here is actually a reduction in the expected rate of *deflation*. For all levels of government purchases below G^{crit}, the output gap remains negative (output remains below the flexible-price equilibrium level), and it is expected to be nonpositive in all future periods as well, so that a negative rate of inflation is implied by (24).

[33] See Denes and Eggertsson (2009) for further discussion of this point.

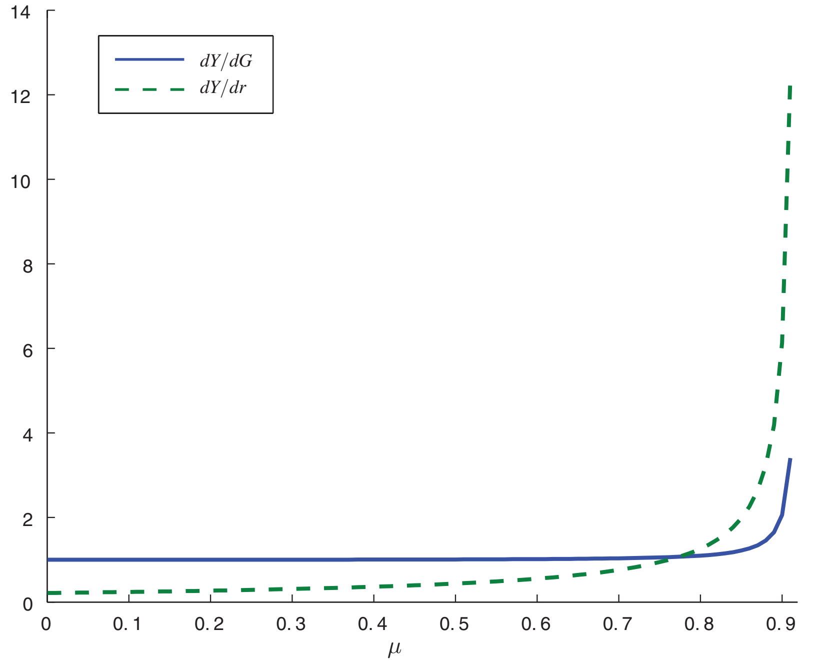

Figure 2. Derivatives of Y_L with Respect to the Values of r_L and G_L, for Alternate Degrees of Persistence μ of the Financial Disturbance

numerical example, a contraction of the size experienced during the Great Depression occurs as a result of a disturbance with a persistence coefficient of $\mu = 0.903$. In the case of this kind of disturbance, his parameter values imply a multiplier of 2.3. Christiano, Eichenbaum, and Rebelo (2009) similarly find that a multiplier above two is possible at the zero lower bound, in the context of a more complex New Keynesian model that is estimated to match a large number of features of postwar US data.

Evidence on the effects of defense spending during the 1930s suggest that substantial multipliers of this kind may indeed be possible during circumstances like those of the Great Depression. For example, Miguel Almunia et al. (2010) estimate panel vector autoregressions (VARs) using data from 27 countries for the period 1925–1939, and look at the response to innovations in defense purchases, taken to represent exogenous changes in government purchases. Depending on the specification used, they find a multiplier during the year of the innovation of either 2.5 (their figure 14) or 2.1 (their figure 19). Robert J. Gordon and Robert Krenn (2010) similarly find a multiplier greater than one for the effects of innovations in government purchases on US real GDP during the military buildup between 1940:I and 1941:IV. It is arguable that these relatively high multipliers for defense purchases during the Great Depression, relative to those found by studies of the effects of defense purchases at other times (e.g., those summarized in Hall 2009), reflect a greater degree of monetary accommodation under Great Depression circumstances than has been typical of other military buildups.[34]

[34] In fact, the VAR results of Almunia et al. (2010) show central-bank discount rates being reduced, rather than increased, in response to a positive innovation in defense purchases.

B. *Importance of the Duration of Fiscal Stimulus*

Cogan et al. (2010), instead, find that a leading empirical New Keynesian model of the US economy predicts small multiplier effects of increased government purchases during a situation in which the zero lower bound is assumed to bind. For example, when Cogan et al. (2010) consider the effect of a permanent increase in government purchases of 1 percent of GDP, they find an increase in GDP of only 1.0 percent in the first quarter, which falls to only 0.6 percent by the end of the second year (the period over which they assume that the federal funds rate remains at zero), and to only 0.4 percent after four years. In the case of an assumed path of government purchases intended to mimic projected expenditure under the February 2009 US federal stimulus package, their model implies an increase in GDP substantially smaller than the increase in government purchases in all quarters, and hence a particularly modest increase in output during the first year of their simulation.

What accounts for the difference with the large multiplier obtained at the zero bound by Eggertsson (2009)? While the empirical model used by Cogan et al. (2010) is substantially more complex, this is probably not the most important difference in their analysis.[35] The crucial difference is that the calculations above assume an increase in government purchases that lasts precisely as long as credit spreads are elevated, and hence precisely as long as the zero lower bound is a binding constraint, following which period $G_t = \overline{G}$ again each period. Cogan et al. (2010), instead, consider increases in government purchases that are initiated at a time when interest rates are zero, but that extend much longer than the period over which the interest rate is assumed to remain at zero.

In our simple model as well, the increase in output is predicted to be much smaller if a substantial part of the increased government purchases are expected to occur after the zero lower bound ceases to bind. For, as explained above, once interest rates are determined by a Taylor rule, a higher level of government purchases should crowd out private spending (raising the marginal utility of private expenditure), and may well cause lower inflation as well.[36] But the expectation of a higher marginal utility of expenditure and of lower inflation in the event that credit spreads normalize in the following period both act as disincentives to private expenditure while the nominal interest rate remains at zero. Hence, while there is a positive effect on output during the crisis of increased government purchases at dates $t < T$, an anticipation of increased government purchases at dates $t \geq T$ has a *negative* effect on output prior to date T.

A simple calculation can illustrate this. Suppose that, instead of the two-state Markov chain considered above, there are three states: after the "crisis" state (in which $r_t^{net} = r_L$ and $\hat{G}_t = \hat{G}_L$) ends, there is a probability $0 < \lambda < 1$ each period that government purchases will remain at their elevated level ($\hat{G}_t = \hat{G}_L$), even

[35] The empirical model considered by Christiano, Eichenbaum, and Rebelo (2009) has a structure very similar to the one used by Cogan et al. (2010), yet Christiano, Eichenbaum, and Rebelo (2009) obtain multipliers well in excess of one for a policy experiment similar to the one analyzed above.

[36] Both things occur in the case of the Eggertsson (2009) parameter values explained in footnote 29.

though $r_t^{net} = \overline{r}$, though with probability $1 - \lambda$ each period the economy returns to the "normal" state (in which $r_t^{net} = \overline{r}$ and $G_t = \overline{G}$) and remains there forever. If we let $(\pi_S, \hat{Y}_S, i_S)$ be the constant values for $(\pi_t, \hat{Y}_t, i_t)$ in the transitional state (i.e., for all $T \geq t < T'$, where T' is the random date at which government purchases return to their "normal" level), then the value of $E_t \hat{Y}_{t+1}$ during the "crisis" period is not $\mu \hat{Y}_L$, but $\mu \hat{Y}_L + (1 - \mu)\lambda \hat{Y}_S$, and similarly for expected future government purchases and expected future inflation. We can repeat the previous derivation, obtaining instead of (40) the more general form

$$\hat{Y}_L = \vartheta_r r_L + \vartheta_G \hat{G}_L + \vartheta_\pi \pi_S + \vartheta_C (\hat{Y}_S - \hat{G}_L), \tag{41}$$

where

$$\vartheta_\pi \equiv (1 - \mu)\lambda \vartheta_r > 0, \qquad \vartheta_C \equiv \sigma^{-1} \vartheta_\pi > 0.$$

The fact that $\vartheta_\pi, \vartheta_C > 0$ indicates that an expectation of either lower private expenditure or lower inflation in the transitional state will lower output during the crisis.

Using the same reasoning as in the previous section, one can show that the levels of output and inflation during the transitional state, when the interest rate is determined by the Taylor rule but government purchases remain high, are given by $\hat{Y}_S = \gamma_y \hat{G}_L, \pi_S = \gamma_\pi \hat{G}_L$, where γ_y is the coefficient defined in (31) (but with the persistence coefficient ρ equal to λ) and γ_π is the corresponding inflation coefficient. One thus obtains a multiplier

$$\frac{dY_L}{dG_L} = \vartheta_G + \vartheta_\pi \gamma_\pi + \vartheta_C(\gamma_y - 1) \tag{42}$$

for government purchases below the critical level that causes the zero bound to no longer bind even in the crisis state. Since $\gamma_Y < 1$ as explained earlier, the contribution of the final term is necessarily negative. In the case that either of the response coefficients (ϕ_π, ϕ_y) is sufficiently large, the Taylor rule will not allow a large increase in inflation during the transitional phase, and one obtains a multiplier smaller than ϑ_G when $\lambda > 0$.

Figure 3 plots the value of the multiplier (42) as a function of λ, in the case that the other parameters take the values proposed by Eggertsson (2009). When $\lambda = 0$, the multiplier is nearly 2.3, as reported by Eggertsson, but it steadily falls as λ is increased. For values of λ equal to 0.8 or higher (an expected duration of the fiscal stimulus for four quarters or more after the end of the financial disturbance), the multiplier falls below one. For values of λ equal to 0.91 or higher (an expected duration of ten quarters or more), the multiplier is negative. In particular, in the case of a permanent increase in the level of government purchases (the case $\lambda = 1$), as in the first case considered by Cogan et al. (2010), the multiplier is strongly negative (nearly -5!). Hence a finding that a long-lasting fiscal stimulus is predicted to increase output only modestly, as in the simulations of Cogan et al. (2010), does not mean that a better-targeted fiscal stimulus cannot be much more effective.

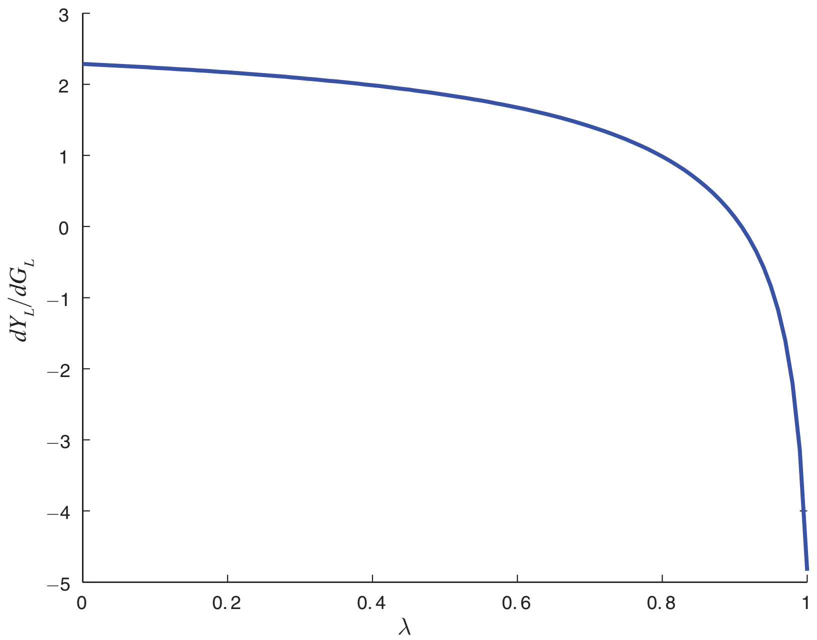

Figure 3. Derivative of Y_L with Respect to G_L, for Alternative Degrees of Persistence λ of Fiscal Stimulus after the End of the Financial Disturbance

Nor is it the case that to be effective, the government spending must occur immediately. In the model considered here, an increase in government purchases during a period in which the interest rate is zero, which is expected to last for *the current quarter only,* so that there is no change in expected future government purchases, has a multiplier of exactly one. (This is because with no change in expected future fiscal policy, there is no change in expected future output or inflation. This means no change in expected real interest rates in future periods, and, as long as the temporary increase in G_t remains within the range that implies a current nominal interest rate of zero, no change in the current real interest rate either. Hence, the benchmark analysis in Section II applies.) It follows that when Eggertsson (2009) obtains a multiplier of 2.3, 1.0 of this is due to the increase in government purchases during the current quarter, while the other 1.3 is the effect of higher anticipated government purchases in the future.

Hence, even if there were no increase in government purchases in the current quarter at all, an expectation of higher government purchases in all future quarters prior to date T would increase output *immediately* by an amount that is 1.3 times as large as the promised future increase in the level of government purchases. Of course, an even longer delay would attenuate the effects on output at the time of the announcement to an even greater extent. Still, New Keynesian models certainly do not imply that a delayed fiscal stimulus will serve no purpose—as long as the eventual increase in government spending is *contingent on the continued existence of the financial disruption* that justifies the emergency measures. The kind of stimulus package that is ineffective, or even counter-productive, is one under which a large part of the increased government purchases are expected to occur in a postcrisis environment in which monetary policy is not expected to accommodate an increase in aggregate demand.[37]

V. Government Purchases and Welfare

Thus far, I have simply considered the extent to which it is possible for an increase in government spending to increase aggregate output and employment, taking for granted (as in much popular discussion) that an increase in output would be desirable, at least under circumstances where output would otherwise be below its trend path. But it is reasonable to ask whether our models imply not only that increased government purchases will increase GDP, but that they will increase economic welfare as well. This does not follow trivially from the existence of a positive multiplier (or even a multiplier greater than one); one must consider the value of the use to which the resources consumed by the government would otherwise be put.

A. *Fiscal Stabilization in the Neoclassical Model*

In the case of the neoclassical model, it is evident that if government purchases are of no intrinsic value ("paying people to dig holes and then fill them again"), the optimal level of government purchases must be zero, for any government purchases crowd out private expenditure *and* increase the disutility of working. But, of course, some kinds of government spending do benefit the public; we can represent this by making the utility of the representative household depend on G_t, the level of public goods provision. The calculations above are unaffected by this hypothesis, as long as we suppose that utility is additively separable in public goods (the tacit assumption earlier).[38] Let us suppose, then, that the utility of the representative household is given by

$$\sum_{t=0}^{\infty} \beta^t [u(C_t) + g(G_t) - v(H_t)], \tag{43}$$

where $g' > 0, g'' \leq 0$. (Of course, the value of public projects does not depend solely on the amount that is spent on them. But it is an obvious principle of optimal fiscal policy that the projects financed should be those that yield the greatest additional utility per dollar spent; the function $g(G)$ accordingly indicates the utility obtained in this case.)

Given that for any path $\{G_t\}$ of government purchases, the competitive equilibrium will maximize the utility of the representative household, it is easily seen that the optimal path of government purchases will be the one that satisfies the first-order condition

$$g'(G_t) = u'(Y_t - G_t) \tag{44}$$

[37] This is illustrated not only by the simulations of Cogan et al. (2010), but also by those of Erceg and Lindé (2010) for the case of a "gradual increase in government purchases" that continues beyond the point at which the zero bound ceases to bind.

[38] For extension of the neoclassical theory to the case in which public goods are at least partially substitutes for private expenditure, see, e.g., Baxter and King (1993).

each period. This condition has a simple interpretation: government purchases should be undertaken if and only if they have a marginal utility as high as that associated with additional private expenditure—i.e., if they satisfy the conventional (microeconomic) cost-benefit criterion. One way of stating this criterion is to say that government purchases should be chosen so as to maximize $u(Y_t - G_t) + g(G_t)$, taking as given the quantity of aggregate expenditure Y_t. (I shall call this the criterion of *efficient composition of expenditure.*) Plainly, this is not a criterion that requires one, in choosing whether to undertake a particular public project, to think about the consequences of government spending for aggregate demand.

B. *Fiscal Stabilization when Monetary Policy Is Optimal*

There is greater scope for fiscal stabilization policy in the case that prices or wages are sticky (or based on older information than that available to the government). If a recession is a time when output is below the full-information flexible-wage/price level, owing to stickiness of one sort or another, this implies a misallocation of resources, and a potential justification for fiscal stimulus to "fill the output gap." If an increase in government purchases G_t is associated with an increase in output Y_t that period (abstracting, for the moment, from changes in the allocation of resources in any other periods), utility will be increased if the relative size of the two changes satisfies the condition

$$(u' - \tilde{v}')\frac{dY}{dG} + (g' - u') > 0. \tag{45}$$

In the neoclassical case, equilibrium condition (6) implies that the first term in (45) is necessarily zero, so that increased government purchases increase welfare only to the extent that g' exceeds u'. But if during a recession, $u' > \tilde{v}'$, the condition can be satisfied even when u' exceeds g' to some extent; this will be more likely the greater the extent to which u' exceeds $\tilde{v}'$ (i.e., the more negative the output gap), and the greater the multiplier effects of government purchases.

Yet it is important to remember that in New Keynesian models, both the size of the output gap and the size of the multiplier will depend on monetary policy; and while there might well be significant opportunities for fiscal stabilization policy under the assumption that prices, wages or information are sticky *and that monetary policy is inept,* the most obvious solution in such a case is to increase the accuracy of *monetary* stabilization policy. Indeed, given that effective monetary stabilization policy should prevent large variations in the ratio of u' to $\tilde{v}'$ (by stabilizing the output gap), it is not obvious that the novel considerations mentioned in the previous paragraph should be of great quantitative significance when monetary policy is used optimally.

A case that is especially simple to analyze is that in which we suppose that there exists a constant employment or output subsidy, of precisely the magnitude necessary to offset the distortion owing to the market power of monopolistically competitive producers.[39] In this case, the factor $\mu > 1$ in (11) is canceled, and the equilibrium

[39] For example, it suffices that there be a subsidy equal to fraction τ of a firm's payroll, where $\tau = 1 - \mu^{-1} > 0$, and $\mu > 1$ is the markup factor in (11).

with (full-information) flexible prices and wages is efficient, despite the assumption of monopolistic competition. Now, suppose that prices are sticky (or set on the basis of sticky information), while wages are flexible (or there is efficient contracting in the labor market). A monetary policy that maintains price stability at all times achieves the (full-information) flexible-price equilibrium allocation, regardless of the path of government purchases, as discussed in Section IIIA; hence, this policy maximizes expected utility, given the path of government purchases.[40] Thus, one may conclude that, regardless of the path of government purchases, an optimal monetary policy achieves the allocation of resources predicted by the neoclassical model.[41] But then the condition for optimality of the level of government purchases is again simply (44), which is to say, the principle of efficient composition of expenditure.

It is not simply a matter of there being two instruments that can each, in principle, address the problem of an insufficient level of aggregate nominal expenditure, given the existing level of prices or wages, so that it does not matter which instrument is used for the job. Rather, to the extent that the problem can be solved using monetary policy, it is *costless to do so,* since monetary policy has no *other* aims to fulfill; whereas, while government spending can also be used to ameliorate the problem, this has a cost, since it requires the diversion of real resources to alternative uses. Whenever government purchases are used for aggregate demand management, there is a tension between this goal and the choice of government purchases so as to maintain an optimal composition of expenditure. Since there is no equally important conflict in the case of the use of monetary policy for aggregate demand management, monetary policy should be used to the extent possible; and this should largely allow decisions about government purchases to be made from the standpoint of the optimal composition of expenditure.

C. *Fiscal Stabilization at the Zero Lower Bound*

There is, however, one case in which a much stronger argument can be made for the usefulness of variations in government spending for stabilization purposes. This is when a financial disturbance makes it impossible for monetary policy to maintain price stability and a zero output gap at all times, as the required path for the policy rate would violate the zero lower bound. Under such circumstances, substantial distortions due to deflation and a large negative output gap can exist in equilibrium, even with a central bank that maintains a strict zero inflation target whenever this is consistent a non-negative interest rate. It can then be desirable to use government purchases to “fill the output gap,” at least partially, even at the price of distorting, to some extent, the composition of expenditure in the economy.

As an example, let us consider the welfare effects of fiscal stimulus in the two-state example of Section IVA. Suppose that the central bank maintains a strict zero inflation target whenever this is possible, and a nominal interest rate of zero

[40] For a more formal presentation of this argument, see Woodford (2003, chap. 6, sec. 3.1).

[41] This result depends on an assumption that the zero lower bound on interest rates does not prevent monetary policy from achieving its inflation target at some points in time. The importance of this caveat is made clear in the following section.

whenever deflation is unavoidable;[42] and let us consider only fiscal policies under which G_t is equal to some constant G_L for all $t < T$, and equal to $\bar{G}$ for all $t \geq T$, where $\bar{G}$ is the optimal level of government purchases under "normal" conditions, that is, the value that satisfies (44) when $Y_t = \bar{Y}$. The analysis is simplified if we again assume the existence of a subsidy such that the flexible-price equilibrium allocation would be optimal. In this case, the steady state with $Y_t = \bar{Y}$ and $G_t = \bar{G}$ represents an optimal allocation of resources, and the assumed monetary policy would be optimal in the event that credit spreads were to remain always modest in size, so that the zero bound were never a binding constraint. I wish to consider the welfare effects of increasing G_L above the normal level $\bar{G}$, and the way in which the optimal choice of G_L depends on the size and expected duration of the financial disturbance.

One can show that a quadratic approximation to the expected value of (43)[43] varies inversely with

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_y (\hat{Y}_t - \Gamma \hat{G}_t)^2 + \lambda_g \hat{G}_t^2\right], \tag{46}$$

where

$$\lambda_y \equiv \frac{\kappa}{\theta} > 0, \qquad \lambda_g \equiv \left[\frac{\eta_g}{\eta_u} + 1 - \Gamma\right] \Gamma \, \lambda_y > 0,$$

and $\eta_g \geq 0$ is (the negative of) the elasticity of g' with respect to G, a measure of the degree to which there are diminishing returns to additional government expenditure. Here, the final two terms inside the square brackets represent a quadratic approximation to $u(Y_t - G_t) + g(G_t) - \tilde{v}(Y_t)$, which would be the period contribution to utility if the prices of all goods were the same, as would occur with flexible prices or in an environment with complete price stability. The additional π_t^2 term represents the additional welfare loss owing to an inefficient composition of the economy's aggregate product as a result of price dispersion.

If the zero bound were never a binding constraint on monetary policy, the only constraint on feasible paths for the inflation rate and the output gap $\hat{Y}_t - \Gamma \hat{G}_t$ would be (24), regardless of the path of $\{\hat{G}_t\}$. Hence, optimal monetary policy would maintain a zero inflation rate and output gap at all times, reducing each of the first two terms inside the square brackets in (46) to their minimum possible values each period. The optimal path of government purchases would then be chosen simply to minimize the remaining term, by setting $\hat{G}_t = 0$ each period. (This would achieve an optimal composition of expenditure, as it would result in $Y_t = \bar{Y}, G_t = \bar{G}$ each period.)

In the case considered here, however, the zero lower bound on interest rates precludes this first-best outcome. Under a policy in the family proposed above, the

[42] This corresponds to a limiting case of the policy considered in Section IVA, in which ϕ_π is made unboundedly large.

[43] See Woodford (2003, chap. 6, sec. 2) for the derivation.

equilibrium is of the kind characterized in Section IVA. In any equilibrium of this kind, the objective (46) takes the value

$$\frac{1}{1 - \beta\mu}\left[\pi_L^2 + \lambda_y(\hat{Y}_L - \Gamma\hat{G}_L)^2 + \lambda_g\hat{G}_L^2\right]. \tag{47}$$

The optimal policy within this family is therefore obtained by minimizing (47) with respect to $\hat{G}_L$, taking into account the dependence of $(\pi_L, \hat{Y}_L)$ on $\hat{G}_L$ implied by (35) and (40). The first-order conditions for the minimization of this quadratic objective subject to the two linear constraints can be uniquely solved for a linear solution,

$$\hat{G}_L = -\frac{\xi(\vartheta_G - \Gamma)\vartheta_r}{\xi(\vartheta_G - \Gamma)^2 + \lambda_g} r_L > 0, \tag{48}$$

where

$$\xi \equiv \left(\frac{\kappa}{1 - \beta\mu}\right)^2 + \lambda_y > 0.$$

(This solution for the optimal value of $\hat{G}_L$ is necessarily positive, because $\vartheta_G > \Gamma$ and $r_L < 0$.)

Figure 4 plots the optimal value of $\hat{G}_L/|r_L|$ defined by (48), for alternative values of μ, assuming the values for the model parameters $\beta, \kappa, \sigma, \Gamma$ and θ proposed by Eggertsson (2009).[44] For a given financial disturbance parameterized by (r_L, μ), the optimal size of the increase in government purchases can be determined from the figure by observing the optimal ratio for that value of μ, and then multiplying by the value of $|r_L|$. Thus, a value of two on the vertical axis means that if r_L is equal to -4 percent per annum, it would be optimal to increase government purchases by an amount equal to 8 percent of GDP.

The optimal value is plotted under two different assumptions about the degree of diminishing returns to additional government expenditure. In case A, it is assumed that utility is linear in government purchases ($\eta_g = 0$); this provides an upper bound for the degree to which it can be cost-effective to increase government purchases. In case B, it is instead assumed that $\eta_g = 4\eta_u$; this corresponds to the case in which the marginal utility of government purchases decreases at the same rate (per percentage point increase in spending) as the marginal utility of private purchases, and private expenditure is four times as large as government purchases in the steady state. In this case, because of the diminishing returns to additional government purchases, the optimal increase in government spending is less for any given financial disturbance. For purposes of comparison, the solid line in Figure 4 also plots the level of government purchases that would be required to fully eliminate the output gap (i.e., keep output at the flexible-price equilibrium level) and prevent any decline in inflation as a result of the financial disturbance. (This line also indicates the critical level

[44] In addition to the parameter values reported in footnote 29, it is now also assumed that $\theta = 12.77$.

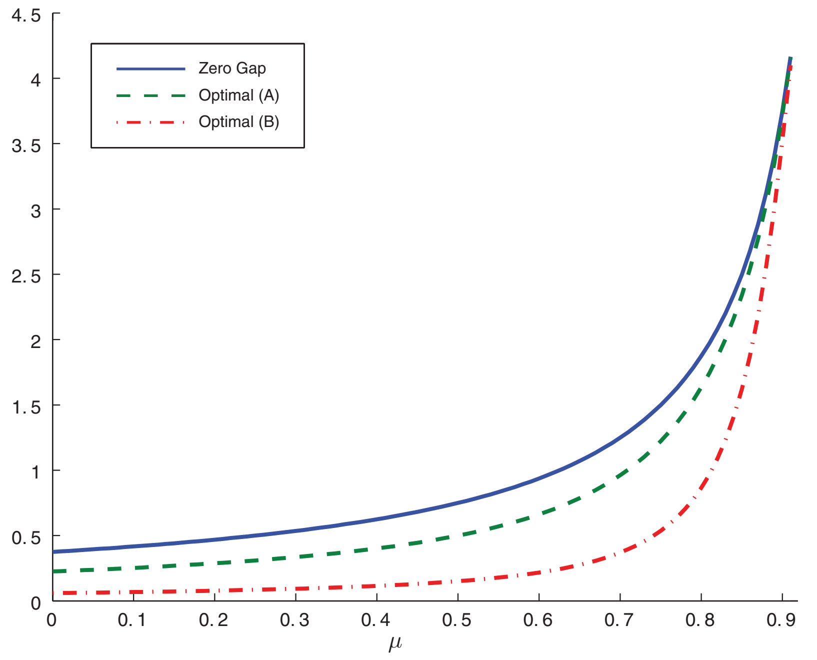

FIGURE 4. THE OPTIMAL VALUE OF $\hat{G}_L/|r_L|$, FOR ALTERNATE VALUES OF μ, UNDER TWO DIFFERENT ASSUMPTIONS ABOUT THE SIZE OF η_g

Notes: Case A: $\eta_G = 0$. Case B: $\eta_G = 4\eta_u$. The solid line shows the value of $\hat{G}_L/|r_L|$ required to maintain a zero output gap.

of government purchases at which the zero lower bound ceases to bind, given the central bank's assumed policy.)

The figure shows that it is optimal to use discretionary (state-dependent) government purchases to partially offset the decline in output and inflation that would otherwise occur as a result of the financial disturbance. It should be noted, however, that it is not optimal to *fully* stabilize inflation and the output gap, despite the feasibility of doing so, because of the inefficient composition of expenditure that this would involve. In the case that the financial disturbance is not too persistent ($\mu = 0.5$ or less), the optimal increase in government purchases is only a small fraction of the increase that would be required to eliminate the output gap, if we assume diminishing returns to additional public expenditure similar to those that exist for private expenditure. (The optimal fiscal stimulus would be even smaller if one were to assume even more sharply diminishing returns to public expenditure, or if one were to take into account the distortions involved in raising government revenues.) At the same time, the optimal size of fiscal stimulus can be quite substantial, and a large fraction of the size required for full stabilization of both inflation and the output gap, in the case that μ is large. In this case—when there is believed to be a substantial probability that the financial disruption will persist for years, and when a serious depression could result in the absence of fiscal stimulus—welfare is maximized by an aggressive increase in government purchases, of nearly the size required to fully stabilize inflation and the output gap.

VI. Consequences of Distortionary Taxation

The analyses above have for simplicity assumed lump-sum taxation. This is clearly unrealistic, but because there is no necessary connection between a path of government purchases and the path of distorting taxes (of various types) used to finance it, a full analysis of the complications raised by taking into account tax distortions is not possible here. If increased government purchases are financed by an increase in a proportional tax on wage income or on consumption purchases (for example), the increased tax wedge will increase the real marginal cost of supplying a given level of output (assuming flexible wages). In a neoclassical model (where real marginal cost can never differ from one in equilibrium), the increased tax distortion will lower equilibrium output, and may even negate the increase in equilibrium output that would occur with lump-sum taxation for the reason explained in Section I.

For example, in the case of financing entirely through a proportional tax τ_t on sales revenues, condition (6) becomes, instead,

$$(1 - \tau_t)u'(Y_t - G_t) = \tilde{v}'(Y_t), \tag{49}$$

from which it follows that Y_t is a decreasing function of τ_t, for a given level of G_t. If $u(C) = \log C$ and a balanced budget is maintained each period (so that $\tau_t Y_t = G_t$), equation (49) reduces to

$$Y_t^{-1} = \tilde{v}'(Y_t),$$

and it is easily seen that the solution for Y_t is independent of G_t (so that the multiplier is zero). If the intertemporal elasticity of substitution of private expenditure is even smaller, the multiplier will even be negative. Thus, it might seem a serious omission to discuss the plausibility of a substantial government expenditure multiplier without taking into account the effects of distorting taxes.

But here, again, the stickiness of prices and/or wages and the nature of the assumed monetary policy response make an important difference. In the benchmark case considered in Section II, where monetary policy is assumed to maintain a constant path for the real interest rate, taking account of tax distortions would not change the conclusion that the government expenditure multiplier is equal to one, as long as the change in fiscal policy is assumed not to change the *long-run* level of tax distortions (which would matter for the determination of $\bar{Y}$ and hence of $\bar{C}$). If a temporary increase in government purchases requires a corresponding temporary increase in the tax rate applied to wage income, the increase in real marginal cost will imply that the monetary policy required to keep the real interest rate constant will be *even more inflationary* than in the case of lump-sum taxation.

For example, in the case of Calvo pricing, and again assuming a proportional sales tax and a balanced budget each period as in the example above, inflation will again be determined by (24), where Γ will now be the balanced-budget neoclassical multiplier implied by (49). Under the constant-real-interest-rate policy, the multiplier will still equal one, so that $\hat{Y}_t = \hat{G}_t$ each period; but since the tax distortions reduce the size of Γ (for given preferences, technology, and steady-state level of government purchases), the implied increase in $\hat{Y}_t - \Gamma \hat{G}_t$ will be greater than in the case of lump-sum taxation, and so the implied increase in inflation will be greater. (The multipliers implied by the hypothesis of a strict inflation target, or by monetary policy following a Taylor rule, will instead be lower in the case of the distorting tax, just as in the neoclassical model.)

In the case of an increase in government purchases while monetary policy is constrained by the zero lower bound, the multiplier would actually be *increased* if we assume that the increased government purchases are financed by a balanced-budget increase in the tax rate on wage income. The reason is that the increase in the tax wedge makes the policy even more inflationary, for the reason just explained. But an increase in expected inflation during the period while the nominal interest rate is constrained to equal zero will mean that real interest rates fall even more than in the analysis in Section IV, resulting in an even greater increase in output.[45]

Thus, the main conclusions of the simple analysis above have not been exaggerated by abstracting from the effects of tax distortions. Even if the increase in government purchases must be financed entirely by an increase in a wage income tax, it remains the case that sticky prices and/or wages make multipliers greater than or equal to one possible; that a monetary policy that maintains the real interest rate constant is sufficient to ensure a multiplier of one; and that a multiplier greater than one (under certain circumstances, substantially greater) should be expected in the case of an increase in government purchases while monetary policy is constrained by the zero lower bound—though in the last case, an additional proviso is now required, that the increase in the wage income tax must also occur while interest rates remain at zero. Taking account of tax distortions further underlines the importance of the expected duration of "fiscal stimulus" in response to an economic crisis, already emphasized in the analysis above. To the extent that tax distortions (such as an increased tax on wage income) are expected to continue to be higher even after the zero lower bound ceases to be a binding constraint, then—assuming that monetary policy is subsequently determined by a strict inflation target or by a Taylor rule, as above—this fact will further reduce expected output after credit spreads normalize, further increase the expected marginal utility of income at that time, and thus give households a motive to save more during the crisis. Policy expectations of this kind can therefore be highly counterproductive, as Drautzburg and Uhlig (2010) find in the context of a more complex, empirical New Keynesian model.[46]

[45] See Eggertsson (2009) for a detailed analysis of the expansionary effects of certain kinds of tax increases when monetary policy is constrained by the zero lower bound.

[46] See, also, the discussion of the consequences of delayed financing through labor income taxation in Erceg and Lindé (2010).

VII. Conclusion

We may summarize our conclusions as follows. Under circumstances like those of the Great Depression—that is, when a disturbance to the financial sector results in insufficient aggregate demand even with the central bank's policy rate at the lower bound of zero, and when there is feared to be a substantial probability of the constraint continuing to bind for years to come—standard models of the kind widely used in analyses of monetary stabilization policy imply that the government expenditure multiplier should be larger than one, and may be well above one. Moreover, in the case of the kind of (purely forward-looking) monetary policy assumed in the analysis above, we have found that not only is there a large effect on output of an increase in government expenditure under Depression-like circumstances, but up to a certain point an increase in government purchases will increase welfare as well. In the case of a sufficiently persistent disturbance (the case in which the zero bound can lead to a serious output collapse), the optimal increase in government purchases can be nearly as large as the increase that would be required to completely eliminate the "output gap," i.e., to raise output to its flexible-price equilibrium level (which will itself be higher due to the temporary increase in government purchases). Hence, a case can be made for quite an aggressive increase in government purchases under such circumstances, even taking account of the increased tax distortions required in order to finance the increase in government purchases.

Nonetheless, under less extreme circumstances, the case for using variations in government purchases for stabilization purposes is much weaker. Even when the zero lower bound is a binding constraint, if the disturbance that causes it to bind is not expected to be too persistent, then even though the multiplier for increased purchases while the constraint still binds will be at least slightly greater than one, it need not be much greater than one; and the optimal increase in government purchases is probably only a small fraction of what would be required to "fill the output gap." When monetary policy is not constrained by the zero lower bound, there is a good case for leaving output-gap stabilization largely to monetary policy, and basing decisions about government purchases primarily, if not entirely, on the principle of efficient composition of aggregate expenditure.

And, finally, even when the zero lower bound is a temporarily binding constraint on monetary policy, the case just made for fiscal stimulus while the constraint binds applies only to the case in which the increased government purchases will be terminated as soon as the constraint ceases to bind, and in which the tax increases required to finance them also occur while the constraint binds. Either an increase in government purchases that continues after monetary policy ceases to be constrained, or tax increases thereafter that may be required to pay off debt issued during the crisis, is likely—to the extent such a change in future fiscal policy is correctly forecasted, and intertemporal expenditure decisions are forward-looking—to significantly reduce the stimulative effects of increased government purchases during the crisis, and a fortiori to reduce the net welfare gains from the policy. Hence, while a case for aggressive fiscal stimulus can be made under certain circumstances, such a policy must be designed with care if it is to have the desired effect. And, as is now widely understood in the context of monetary stabilization policy, careful signalling about the likely direction of future policy is likely to be as important as current actions.

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