

*A STATISTICAL MECHANICS OF SOME  
INTERCONNECTION NETWORKS.*

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ABSTRACT

Despite intensive research on distributed processor interconnection architectures, relatively little work has been done on the performance analysis of such systems. The reason for this, besides the complexity of the behavior of such systems, is that Queueing Theory cannot easily handle systems consisting of many tightly interacting components. An alternate approach, based upon statistical mechanics, is used. We analyze interconnection structures such as crossbar, linear array, binary tree and ring.



## INTRODUCTION

The objective of this paper is to investigate the possibility of using methods, borrowed from statistical mechanics as a possible alternative to the traditional Queueing Theory approach, to analyze the performance of multiprocessor interconnection networks.

Multiprocessor interconnection networks provide communication between a set of distributed processing modules. In high performance multiprocessor systems, a high capacity interconnection network is required if inter-module communication is not to be a system bottleneck. Recent developments towards supercomputers has spurred increased interest in the design and analysis of multiprocessor interconnection networks [Siegel79, Thurber82].

Typically, the interconnection networks do not provide a dedicated communication path between any two agents. Therefore, an established path may block other paths. For example, in a crossbar switch connecting processors to memories, two processors cannot simultaneously access the same memory module. The description of possible states and blocking between them often presents challenging combinatorial problems. Relatively little analytic results are known for the crossbar, tandem, banyan, delta, shuffle-exchange, loop and other interconnection networks. [Bhandakar73, Bhuyan83, Diaz81]

Queueing Theory approach encounters a number of difficulties when it comes to the analysis of these systems. Firstly, it requires a very detailed study of the evolution of each component to the steady state even when the only questions of interest are some global averages. Secondly, and most importantly, Queueing Theory approach can offer very little when it comes to analyze systems consisting of many tightly-interacting components. The approach only works if a large-scale system can be decomposed into many independent "easy-to-analyze" subsystems.

An alternative to Queueing Theory is the application of ideas from Statistical Mechanics. [Benes63, Ferdinand70, Yemini82]. Such approach can provide us with tools for handling tightly-interacting components. In contrast to the Markovian Analysis of Queueing Theory, the "Statistical Mechanics" approach does not require detailed tracking of the evolution of each component to steady-state, but can derive all important system measurements directly using a partition function, derived from an interaction potential.

## THE MODEL

Consider a resource shared by a set of distributed agents. An agent becomes active according to exponential interarrival law with rate  $\lambda$  and once active, (that is, once it has acquired the resource) it uses the resource for a period distributed

exponentially with rate  $\mu$ . Assume further that agents may interfere with each other and that two interfering agents may not acquire simultaneous access to the resource. We call such a system an Interference System. In an interconnection network, for example, the contending agents are transmissions accessing the shared communication switch.

Clearly, most multiprocessor systems do not have an exponentially distributed service times. However, techniques such as memory interleaving, read/write access mechanisms and cache memories suggest that the exponential assumption may be as good as the assumption of the fixed service time. [Bhandarkar73]. In the above model, requests that are not accepted, are rejected altogether. In real systems, the rejected requests must be resubmitted. Simulation studies [Bhandarkar73, Diaz81, Strecker70] show however that this turns out to be a very good approximation. For the crossbar switch, for example, the analytic results are within five percent of the simulation results [Strecker70].

An interference system can be represented by an interference graph  $G = \langle N, E \rangle$  where the set of nodes  $N$  represents the agents and the and an edge between two agents represents a mutually exclusive interference between them. With the above assumptions, the evolution of the network is that of a spatial birth-death process over the interference graph. Let  $\pi(A)$  represent the equilibrium probability of the set  $A \in V$  being active while  $V-A$  being idle. Then it is clear that the process is time-reversible and the equilibrium probability distribution  $\pi(\cdot)$  satisfies the following detailed balance equation:

$$\lambda\pi(A) = \mu\pi(A \cup x) \quad \dots(1)$$

for  $x$  not in  $cl(A)$ , both sides being zero otherwise. [Here  $cl(A)$  denotes the closure of  $A$ , that is the set of vertices in  $A$  and those neighboring to vertices in  $A$ .]

To solve equation (1), we define a set of nodes in the interference graph to be independent if no two nodes in such a set are neighbors. Thus, independent sets of nodes represent possible concurrent transmissions. Let  $J$  denote the set of independent subsets of  $G$ . Let us define  $\alpha_G^i$  to be the number of distinct independent subsets of  $G$  having  $i$  nodes. It is easy to verify that:

Theorem 1:

The equilibrium probability that solves equation (1) is given by

$$\pi(A) = \begin{cases} \rho^{|A|} / Z_G, & A \in J \\ 0, & \text{otherwise} \end{cases} \quad \dots(2)$$

where

$$\rho = \lambda/\mu$$

and  $Z_G$ , the "partition" function of the system is given by:

$$Z_G = \sum \rho^{|A|} = \sum_{i=0}^N \alpha_G^i \rho^i$$

Let us now note an analogy to statistical mechanics: The equilibrium behavior of a large mechanical system is completely described by its partition function

$$Z = \sum_i \exp(-\beta \epsilon_i)$$

where the summation is over all microstates of the system (enumerated by  $i$ );  $\epsilon_i$  is the energy of  $i$ -th microstate and  $\beta=1/kT$  where  $T$  is the absolute temperature and  $k$  is the Boltzmann constant. The equilibrium probability that the system is in state  $i$  is given by the Gibbs distribution:

$$\pi_i = \frac{\exp(-\beta \epsilon_i)}{Z}$$

All thermodynamic functions describing the system (energy, entropy, pressure) are obtained in terms of the partition function and its derivatives.

One can easily see the analogy to statistical mechanics. A microstate of an interference system is described by an independent set of nodes in the interference graph. The cardinality of an independent set corresponds to the energy of a microstate. The global energy thus corresponds to the average number of concurrent transmissions processed by the interconnection network, that is the thruput of the system. The probability of non-zero energy corresponds to the utilization of the system. Pressure  $P$  can be shown to correspond to the average rate of blocking experienced by transmissions. A detailed elaboration of this analogy can be found in [Yemini82]. In this paper we will show how to calculate the partition function, thruput, utilization and average blocking probability of some interesting interconnection structures.

If we let  $T = -1/k \ln \rho$ , we can rewrite the partition function as:

$$Z = \sum_{A \in J} e^{-|A|/kT}$$

in complete analogy to the statistical mechanics model. Note that  $\rho = 0$  corresponds to  $T=0$  and  $\rho = 1$  corresponds to  $T=\infty$ , so traffic increase is associated with raising the "temperature".

Formally, the thruput can be defined in terms of the "logarithmic derivative" of the partition function as follows:

$$E = \sum_{i=0}^n i \alpha^i \rho^i = \rho \frac{d \ln Z}{d \rho}$$

The utilization of the system (the probability of at least one transmission) is then

$$U = 1 - \frac{1}{Z}$$

It can be shown [Yemini82] that the pressure of the system corresponds to the average probability of blocking and can be computed as follows:

$$P = \frac{\lambda d}{Z} \frac{dZ}{dn}$$

where  $d$  denotes the average degree of a node in the interference graph, that is, the average number of edges incident on that node. Of course, the above formula is meaningful only for sufficiently large interference graphs.

### APPLICATIONS

Example 1 (A non-blocking interconnection network). Consider a system capable of producing  $n$  transmissions which do not interfere with each other. Its interference graph is the graph with no edges.

The number of independent sets of with  $i$  vertices is  $\alpha_N^i = \binom{n}{i}$  and so the partition function is given by

$$Z_n = 1 + \binom{n}{1} \rho + \dots + \binom{n}{n} \rho^n = (1 + \rho)^n$$

The energy (thruput) of the complete interconnection network is given by

$$E_n = \frac{n \rho}{(1 + \rho)}$$

The thruput per node is then

$$\frac{E_n}{n} = \frac{\rho}{(1 + \rho)}$$

Utilization of such system is

$$U_n = 1 - \frac{1}{(1 + \rho)^n}$$

Since the degree of each node in the interference graph is 0,  $P = 0$  which reflects the fact that no blocking is experienced by concurrent transmissions.

For  $\rho \mapsto 0$  we have  $\frac{E_n}{n} \mapsto 0$  and  $U_n \mapsto 0$ .

For  $\rho \mapsto 1$  we have  $\frac{E_n}{n} \mapsto \frac{1}{2}$  and  $U_n \mapsto 1 - \frac{1}{2^n}$ . So, the utilization approaches 1 very fast, as one expects: the system is non-blocking.

Example 2 (Complete interference model) Suppose we have  $n$  processors connected to a carrier-sense bus with negligible propagation delay. Any two concurrent transmissions interfere with each other. The interference graph is  $K_n$ . Clearly,  $\alpha_N^1 = 1$  and  $\alpha_N^i = 0$  for  $i > 1$ .

The partition function of the bus, is thus given by

$$Z_n = 1 + n\rho$$

The energy (thruput) of the bus is given by

$$E_n = \frac{n\rho}{(1 + n\rho)}$$

The energy per node is then

$$E_n/n = \frac{\rho}{(1 + n\rho)}$$

The utilization of such a bus is

$$U_n = 1 - \frac{1}{1 + n\rho}$$

The pressure (average blocking probability) is

$$P = \frac{\lambda(n-1)\rho}{1 + n\rho}$$

For  $\rho \mapsto 0$  we have  $\frac{E_n}{n} \mapsto 0$ ,  $U_n \mapsto 0$  and  $P \mapsto 0$ .

For  $\rho \rightarrow 1$  we have  $\frac{E_n}{n} \rightarrow \frac{1}{1+\rho}$ ,  $U_n \rightarrow 1 - \frac{1}{1+\rho}$  and  $P_n \rightarrow \infty$ . This is what one expects - in heavy traffic each processor will be involved in a transmission approximately  $\frac{1}{n}$  of the time, and since any two concurrent transmissions interfere with each other, the average rate of blocking approaches infinity.

**Example 3** Consider  $n$  processors arranged in a linear array. Suppose that any processor can be involved in a transmission with one of its neighbors, but not with both. To calculate the partition function  $Z_n$  we will derive a recursive relations for  $\alpha_n^i$ .

Suppose one more processor is added to the tandem. Let us examine a configuration involving  $i$  transmissions. There are clearly two cases to consider:

Case 1: the  $(n+1)$ -st processor is not transmitting. There are  $\alpha_n^i$  such configurations.

Case 2: the  $(n+1)$ -st processor is involved in a transmission. There are  $\alpha_{n-1}^i$  such configurations.

$$\text{Hence, } \alpha_{n+1}^i = \alpha_{n-1}^i + \alpha_n^i.$$

The above relation implies the following recursive relation

$$Z_{n+1} = Z_n + \rho Z_{n-1} \text{ with } Z_0 = Z_1 = 1$$

The solution is readily seen to be

$$Z_n = \frac{1}{\sqrt{1+4\rho}} \left[ \left( \frac{1+\sqrt{1+4\rho}}{2} \right)^{n+1} - \left( \frac{1-\sqrt{1+4\rho}}{2} \right)^{n+1} \right]$$

The energy (average number of transmissions) is

$$E_n = \frac{-2\rho}{1+4\rho} + \frac{2\rho(n+1)}{\sqrt{1+4\rho}(1+\sqrt{1+4\rho})} \left[ \frac{1 + \left( \frac{1-\sqrt{1+4\rho}}{1+\sqrt{1+4\rho}} \right)^n}{1 - \left( \frac{1-\sqrt{1+4\rho}}{1+\sqrt{1+4\rho}} \right)^{n+1}} \right]$$

For "large"  $n$  the term in brackets may be ignored to obtain the following asymptotic expression the energy per node in a linear array

$$\frac{E_n}{n} = \frac{2\rho}{\sqrt{1+4\rho}(1+\sqrt{1+4\rho})}$$

The utilization of the array is



$$U_n = 1 - \frac{\sqrt{1+4\rho}}{\left[ \left( \frac{1+\sqrt{1+4\rho}}{2} \right)^{n+1} - \left( \frac{1-\sqrt{1+4\rho}}{2} \right)^{n+1} \right]}$$

The pressure is easily computed to be

$$P = 2\lambda \log \left( \frac{1 + \sqrt{1+4\rho}}{2} \right)$$

Pressure is thus nearly linear in  $\lambda$ . For  $\rho \mapsto 1$  the thrupt per node  $\frac{E_n}{n} \mapsto \frac{1}{2} - \frac{1}{2\sqrt{5}}$ . The term  $\frac{1}{2}$  is the thrupt per node if no interference existed; the amount  $\frac{1}{2\sqrt{5}}$  is lost due to interference. The average rate of blocking  $P \mapsto 2\lambda \log \left( \frac{1+\sqrt{5}}{2} \right)$ , while utilization  $U_n \mapsto 1 - \frac{\sqrt{5}}{\left( \frac{1+\sqrt{5}}{2} \right)^{n+1}}$

**Example 4** Consider  $n$  processors, arranged in a linear array as before, but now assume circuit switching, so that once a path is established between nodes A and B which passes thru node C, this node C cannot be involved in any transmission. An example of such a network is the MP/C architecture, suggested by Arden and Ginosar [Arden81, Ginosar81]. To calculate  $\alpha_n^i$  note that the choice of  $2i$  processors uniquely determines  $i$  transmissions.

Therefore,  $\alpha_n^i = \binom{n}{2i}$  The partition function is

$$Z_n = \frac{1}{2} [(1 + \sqrt{\rho})^n + (1 - \sqrt{\rho})^n]$$

So, the energy is

$$E_n = \frac{n\sqrt{\rho}}{4} \left[ \frac{(1 + \sqrt{\rho})^{n-1} - (1 - \sqrt{\rho})^{n-1}}{(1 + \sqrt{\rho})^n + (1 - \sqrt{\rho})^n} \right]$$

$$E_n = \frac{n\sqrt{\rho}}{4(1 + \sqrt{\rho})} \left[ \frac{1 - \left( \frac{1 - \sqrt{\rho}}{1 + \sqrt{\rho}} \right)^{n-1}}{1 + \left( \frac{1 - \sqrt{\rho}}{1 + \sqrt{\rho}} \right)^n} \right]$$

For very "large"  $n$ , the term in the bracket can be ignored. Thus, asymptotically, the average number of transmissions in a linear array per node is

$$E_n \mapsto \frac{\sqrt{\rho}}{4(1 + \sqrt{\rho})}$$

Utilization of such linear array of n processors is

$$U_n = 1 - \frac{2}{(1 + \sqrt{\rho})^n + (1 - \sqrt{\rho})^n}$$

Asymptotically, the utilization is

$$U_n \mapsto 1 - \frac{2}{(1 + \sqrt{\rho})^n}$$

For  $\rho \mapsto 1$  the thruput per node  $\frac{E_n}{n} \mapsto \frac{1}{8} = \frac{1}{2} - \frac{3}{8}$  Here  $\frac{1}{2}$  represents the thruput per node without an interference, the amount  $\frac{3}{8}$  is lost due to interference. The utilization  $U_n \mapsto 1 - \frac{1}{2^{n-1}}$ .

**Example 5** Consider n processors arranged in a ring. Two transmissions between 2 different pairs of nodes interfere if they share at least one node in common. To calculate  $\alpha_n^i$  observe that once we fix 2i nodes for i transmissions there are only two possibilities for a pair of adjacent nodes among those chosen: either they are the end nodes of the same transmission or of two different transmissions.

$$\text{Hence, } \alpha_n^i = 2 \binom{n}{2i}$$

Therefore,

$$Z_n = [(1 + \sqrt{\rho})^n + (1 - \sqrt{\rho})^n]$$

The energy (average number of transmissions) of the ring is

$$E_n = \frac{n\sqrt{\rho}}{2} \left[ \frac{(1 + \sqrt{\rho})^{n-1} - (1 - \sqrt{\rho})^{n-1}}{(1 + \sqrt{\rho})^n + (1 - \sqrt{\rho})^n} \right] \mapsto \frac{n\sqrt{\rho}}{2(1 + \sqrt{\rho})}$$

The utilization of the ring is

$$U_n = 1 - \frac{1}{[(1 + \sqrt{\rho})^n + (1 - \sqrt{\rho})^n]}$$

For "large"  $n$ , the utilization

$$U_n \mapsto 1 - \frac{1}{(1 + \sqrt{\rho})^n}$$

For  $\rho \mapsto 1$  the energy per node converges to  $\frac{1}{4}$ . In the absence of interference the thruput per node would have been  $\frac{1}{2}$  in heavy traffic, the amount  $\frac{1}{2}$  is lost due to interference.

To calculate the pressure, observe that the cardinality of the interference graph for the ring is  $n^2 - n + 2$  and that a vertex representing a transmission involving  $k$  consecutive nodes in the ring has degree  $n^2 - n + 1 - \binom{n-k}{2}$ . Thus, the average degree  $d = \frac{(n-2)(5n^2-2n+3)}{n^2-n+2}$ . The average rate of blocking is then  $\frac{5}{2}\lambda \log(1 + \sqrt{\rho})$ .

Example 6 Consider a full binary tree interconnection network with  $n$  processors at the leaves. Such a network switch has been suggested as a multi-processor communication mechanism [Shaw82] and as a local area network architecture [Yemini82]. Let  $n = 2^{k-1}$  be the number of leaves. ( $k$  is the depth of the tree)

Two processors can be involved in a transmission through their least common father on the tree. Two transmissions paths that cross, interfere with each other. It is clear from this that to a configuration with  $i$  transmissions there corresponds a choice of  $2i$  leaves, and conversely, a choice of  $2i$  leaves uniquely determines the possible  $i$  transmissions. Therefore  $\alpha_n^i = \binom{n}{2i}$ . Hence, the partition function is given by

$$Z_n = \frac{1}{2}[(1 + \sqrt{\rho})^n + (1 - \sqrt{\rho})^n]$$

This is identical with the partition function for the linear array in example 4. Therefore, the thruput and utilization of the binary tree interconnection network are the same as that of a linear array. This proves Ginosar-Arden's argument that an MP/C bus is as good as a binary tree. [Arden82, Ginosar82].

Comparing this with the results of the previous example, one can see that the utilization of each node in a ring interconnection network is twice as that of a binary tree. Moreover, the ring is a 2-connected graph whereas any single node failure is fatal to the binary tree interconnection. Therefore, the ring network is more reliable. In terms of the number of nodes to connect  $N$  processors, both require the same ( $N+1$  for the ring,  $N$  for the binary tree). In this respect they are equivalent. On the other hand, the longest interconnection may require a maximum of  $2\log_2 N$  nodes in a tree and  $\frac{N}{2}$  for the ring. This advantage of a binary tree interconnection network has been utilized in a number of computer systems. [Shaw82, Yemini82, Stolfo81].

Example 7 Consider an  $m \times m$  crossbar with "non-overlap" restriction, that is once we choose  $i$  inputs and  $i$  outputs, there is a unique way to connect them. (Each of the  $m^2$  switches can only have at most one transmission going thru it) We can choose  $i$  inputs in  $\binom{m}{i}$  ways and  $i$  outputs in  $\binom{m}{i}$  ways. Therefore,

$$\alpha_m^i = \binom{m}{i}^2$$

Hence

$$Z_m = \sum_{i=0}^m \binom{m}{i}^2 \rho^i$$

To calculate  $Z_m$  we note the following identities

$$\begin{cases} i^2 \binom{m}{i}^2 = m^2 \binom{m-1}{i-1}^2, \\ (m-i)^2 \binom{m}{i}^2 = m^2 \binom{m-1}{i}^2, \\ i \binom{m}{i}^2 = i \binom{m-1}{i}^2 + (2m-i) \binom{m-1}{i-1}^2, \end{cases}$$

From this we can immediately verify the following

$$\begin{cases} Z'_m + \rho Z''_m = m^2 Z_{m-1}, \\ m^2 Z_m - (2m-1)\rho Z'_m + \rho^2 Z''_m = m^2 Z_{m-1}, \\ Z_m = (2m-1)Z_{m-1} + (1-\rho)Z'_{m-1}, \end{cases}$$

Therefore,

$$mZ_m = (2m-1)(1+\rho)Z_{m-1} - (m-1)(1-\rho)^2 Z_{m-2}$$

Comparing this with the recurrence relation of the Legendre polynomials, namely

$$mP_m(x) = (2m-1)xP_{m-1}(x) - (m-1)P_{m-2}(x)$$

we have the following formula:

$$Z_m = (1-\rho)^m P_m\left(\frac{1+\rho}{1-\rho}\right)$$

where  $P_m$  denotes the Legendre polynomial of degree  $m$ .

The energy of the "restricted" crossbar is

$$E_m = \rho \frac{d \log Z_m}{d \rho} = \frac{-m\rho}{1-\rho} + \rho \frac{P'_m\left(\frac{1+\rho}{1-\rho}\right)}{P_m\left(\frac{1+\rho}{1-\rho}\right)} \quad \dots (*)$$

From the recurrence relation for the Legendre polynomials:

$$(x^2 - 1)P'_m(x) = mxP_m(x) - mP_{m-1}(x)$$

the following expression for the average energy of the system is obtained

$$E_m = \frac{m}{2} \left[ 1 - \frac{P_{m-1}\left(\frac{1+\rho}{1-\rho}\right)}{P_m\left(\frac{1+\rho}{1-\rho}\right)} \right]$$

Let's compute the asymptotic behavior of the energy. When  $\rho \mapsto 1$  we have,  $\frac{1+\rho}{1-\rho} \mapsto \infty$ . It is easy to show that if  $f_m(x)$  is a polynomial of degree  $m$ , such that  $x \mapsto \infty$  we have  $\frac{f'_m}{f_m} \approx \frac{m}{x}$

Therefore, for  $\rho \mapsto 1$ , from equation (\*) above for the number in the system we obtain

$$E_m \approx \frac{-m\rho}{1-\rho} + \frac{2m\rho}{(1+\rho)(1-\rho)} = \frac{m\rho}{1+\rho}$$

Thus, in heavy traffic the thruput per input for the crossbar is  $\frac{1}{2}$ . This means that as multiprocessor systems with such an interconnection networks grow, one is not faced with diminishing returns in heavy traffic, no matter how many modules are there - one can expect half of them to be active.

The utilization is

$$U_n = 1 - \frac{1}{(1-\rho)^m P_m\left(\frac{1+\rho}{1-\rho}\right)}$$

From the above examples one can see some of the difficulties in evaluating the partition function and interpreting the results. So far, we have not been able

to analyze a class of delta networks (omega, shuffle-exchange, banyan) because the recursive expressions for these networks seem to be not easily amenable to closed form expressions. We have been able, however, to analyze the crossbar, and we do this in:

**Example 8** Consider now a network capable of realizing any permutation of inputs to outputs such as the crossbar or a full banyan network. (we assume that an input can be connected to just one output). For such networks we have  $\alpha_n^i = \binom{n}{i}^2 i!$ .

The partition function for the network is then

$$Z_n = \sum_{i=0}^n \binom{n}{i}^2 i! \rho^i$$

From the definition of Laguerre polynomials of degree n, namely

$$L_n(x) = \sum_{k=0}^n (-1)^k \binom{n}{n-k} \frac{x^k}{k!}$$

we have

$$n!(-x)^n L_n\left(-\frac{1}{x}\right) = \sum_{k=0}^n \binom{n}{n-k} \frac{n!}{k!} \left(-\frac{1}{x}\right)^{n-k}$$

Therefore, we obtain

$$Z_n = (-1)^n n! \rho^n L_n\left(-\frac{1}{\rho}\right)$$

The energy of the network is

$$E_n = \rho \frac{Z'_n}{Z_n} = n + \rho \frac{L'_n\left(-\frac{1}{\rho}\right)}{L_n\left(-\frac{1}{\rho}\right)}$$

From the recurrence relation of the Laguerre polynomials, namely

$$xL'_n(x) = nL_n(x) - n^2L_{n-1}(x)$$

we obtain the following formula for the energy  $E_n$

$$E_n = n(1 - \rho^4) + n^2 \rho^4 \frac{L_{n-1}(-\frac{1}{\rho})}{L_n(-\frac{1}{\rho})}$$

The utilization of the network:

$$U_n = 1 - \frac{1}{(-1)^n n! \rho^n L_n(-\frac{1}{\rho})}$$

### CONCLUDING REMARKS

This paper presents some preliminary analytic results for the analysis of some complex interconnection structures, based upon ideas and tools analogous to those in statistical mechanics. Such an approach gives an attractive alternative to Queueing Theory, offering a relatively easy way to derive important quantities of interest without going into "fine-grained" analysis of the evolution of each component. Future work will extend the applications to other models of interconnection networks and distributed systems.

### REFERENCES

- B.W. Arden, R. Ginosar, "MP/C: A Multiprocessor/Computer Architecture" Dept. of Computer Science, Princeton University, 1981.
- V.E. Benes, "A 'Thermodynamic' Theory of Traffic in Connecting Networks" Bell System Technical Journal, XLII(3), pp.567-607, 1963
- D. Bhandarkar, "A Survey of Techniques for Analyzing Memory Interference in Multiprocessor Computer Systems", Technical Report, CMU, April 1973.
- L.N. Bhuyan, "An Interference Analysis of Interconnection Networks", IEEE Transactions on Computers, 1983.
- D.M. Diaz, "Packet Communication in Delta and Related Networks", Ph.D. Thesis, Rice University, 1981.
- A.E. Ferdinand, "A Statistical Mechanics Approach to Systems Analysis" IBM Journal of Research and Development, September 1970.
- L. Kleinrock, "Queueing Theory". John Wiley, 1976.

C. J. Preston, "Gibbs States on Countable Sets", Cambridge University Press, 1974.

D. Ruelle, "Statistical Mechanics: Rigorous Results", Benjamin Inc., 1969.

D. Shaw, "The Non-Von Supercomputer", Technical Report, Dept. of Computer Science, Columbia University, 1982.

H. S. Siegel, "A Model of SIMD Machines and a Comparison of Various Interconnection Structures", IEEE Transactions on Computers, December 1979.

S. Stolfo, D. Miranker, "DADO: A Parallel Processor for Expert Systems", Proceeding of the 1984 International Conference on Parallel Processing.

W. D. Strecker, "Analysis of the Instruction Execution Rate in Certain Computer Structures", Ph.D. Thesis, CMU, 1970.

K. J. Thurber, "Distributed Processor Communication Architecture", Lexington Books, 1982.

Y. Yemini, "A Statistical Mechanics of Distributed Resource Sharing Mechanisms", Dept. of Computer Science, Columbia University.

Y. Yemini, "Tinkernet: Or Is There Life Between LANs and PBXs?" Proceeding of the 1983 International Conference on Communications.