articles

An American Precursor of Non-tonal Theory: Ernst Bacon (1898–1990)

by Severine Neff

Ernst Bacon was one of the very first American theorists to investigate aspects of extended tonality and atonality. At the age of nineteen, Bacon made his sole contribution to music theory: a long article, "Our Musical Idiom," which appeared in the Chicago journal *The Monist.*[1] His intention was to classify non-tonal scales and harmonies "in a logical order" and "to develop a system of nomenclature describing any possible combination of tones." "Our Musical Idiom" is one of the earliest American contributions to the non-tonal theoretical literature; it predates such European works as Josef Hauer's *Vom Wesen des Musikalischen* and Herbert Eimert's *Atonale Lehrbuch.*[2]

Ernst Bacon was born in Chicago in 1898 and died in Orinda, California, in 1990. His earliest teachers in Chicago were the composers Arne Oldberg, Thörvald Otterström, and the pianist-critic-conductor Glenn Dillard Gunn. Gunn and Otterström had been students of Bernhard Ziehn (1845–1912), the first major German theorist to emigrate to America. Oldberg had been instead a pupil of the organist Wilhelm Middelschulte, a disciple of Ziehn. Bacon subsequently studied composition with Karl Weigl[3] in Vienna and worked extensively with Ernst Bloch in San Francisco from 1919 to 1921. In 1945, Bacon became Professor of Music at Syracuse University. He composed over 300 songs and was the author of an aesthetic treatise, *Words on Music,* and a technical manual, *Notes on the Piano;*[4] among his students were Carlisle Floyd and Donald Martino.[5]

When Bacon published "Our Musical Idiom," American theorists were writing pedagogical treatises solely in the tonal tradition. Bacon's article instead appeared in a journal concerned with the philosophy of science.[6] His musical audience consisted almost exclusively of his teachers Gunn, Otterström, and Oldberg, and other Chicago musicians in the circle of Bernhard Ziehn. Ziehn's own theoretical works treat the harmonic language of Wagner, Bruckner, Richard Strauss, and Debussy.

I shall describe first the background for Bacon's work, focusing specifically on Ziehn's concepts of order permutation, "plurisignificance" *(Mehrdeutigkeit),* and symmetrical inversion. Second, I shall examine "Our Musical Idiom" and its compositional implications for some of Bacon's works.

Ziehn's theories

In 1954, Ziehn's disciple Julius Gold wrote that his teacher's principal achievement was "the formulation and substantiation of an extremely inclusive theory of chromatic harmony at a time when chromaticism was still a highly experimental matter."[7] In view of the current understanding of theoretical rigor, Ziehn's thoughts on chromaticism seem less an "inclusive theory" than merely a classification system of possible chromatic chord events. His major works, including the *Harmonie- und Modulationslehre* of 1887, were all written in Chicago.[8]

Ziehn was a mathematician as well as a music theorist. When he came to Chicago, he taught both music and mathematics at the German Lutheran School. Though Ziehn's books contain no mathematical formulas, Julius Gold, Otterström, and others attest that Ziehn used standard formulas of probability to calculate structures and classifications of chromatic chords, scales, progressions, and canons, such as are found in late nineteenth- and early twentieth-century music.[9]

Using the formula

$$\frac{n!}{(n_1!)\,(n_2!)}$$

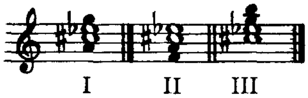
one can calculate the number of order permutations in which different intervals can occur. To determine the number of possible order permutations of two different major thirds and a minor third, n is assigned the value 3, the total number of intervals; n_1 is assigned a value of 2, which indicates the two major thirds; and n_2 is assigned the value 1, which indicates the minor third. The calculation yields the result 3, indicating that there are three possible permutations of this interval grouping.[10]

Ziehn selected combinations of major, minor, or diminished thirds and then calculated possible permutations of these thirds to form chromatic seventh chords for his classification system of chromatic chord events. The three chords that could be generated from re-orderings of two major thirds and one diminished third are shown in example 1. The values of n, n_1, and n_2 are the same as those above, again yielding the result 3. Chord "I" contains the ordering (from the bass) major third, diminished third, major third; chord "II," major third, major third, diminished third; and chord "III," diminished third, major third, major third. Example 2 shows six chromatic seventh chords generated in analogous fashion from two diminished thirds, one major, and one minor third. In this case,

$$\frac{n!}{(n_1!)\,(n_2!)\,(n_3!)}$$

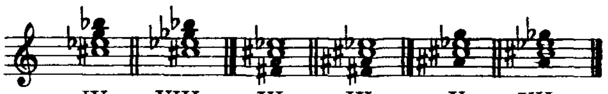
for n=4, n_1=2, n_2=1, and n_3=1, there exist six possible orderings of the intervals. Together, examples 1 and 2 demonstrate the nine chromatic seventh chords that in derivation are unique to Ziehn's system of chromatic harmony.[11]

Example 1. Ziehn's representations of chords. Reproduced from *Manual of Harmony,* 21.



I II III

Example 2. *Manual of Harmony,* 21.



IV VIII VI IX V VII

All nine seventh chords contain C♯ and E♭. The repetition of these pitches illustrates a second major theoretical concern for Ziehn, which he called "plurisignificance": the structural and functional reinterpretation of a single interval or pitch in different harmonies. Example 3 shows the plurisignificance of the pitch D.

Example 3. The plurisignificance of the pitch D. *Manual of Harmony,* 6.

The tone as part of diatonic Thirds.

A tone can be considered as lower or upper tone of a Third, and belongs, consequently, to two large and two small Thirds.

The tone as part of diatonic triads.

A tone can be fundamental tone, Third or Fifth of a triad, and can, therefore, belong to three triads of every kind.

The tone as part of diatonic Seventh-chords.

A tone can be fundamental tone, Third, Fifth or Seventh of a Seventh-chord, and can, therefore, belong to four Seventh-chords of every kind.

The tone as part of large and small Ninth-chords.

Ziehn further considers the numerical consequences of plurisignificance. Example 3 indicates that a single pitch can belong to twelve diatonic triads, twenty-eight diatonic seventh chords, and ten diatonic ninth chords. By applying probability formulas to the numerical consequences of plurisignificance, Ziehn could exhaustively calculate possible chord progressions. For instance, in G major, Ziehn finds that the pitch G belongs to five chords, the pitch A to seven (example 4a).[12] Applying a probability formula, one can determine that the possible orders of these harmonies in two-chord progressions number 792.

$$\frac{12!}{(5!)(7!)} = \frac{4790016}{6048} = 792$$

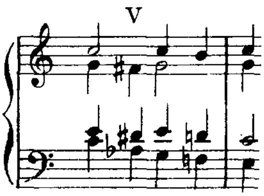
Example 4a. *Manual of Harmony,* 10–11.

Ziehn lists fifty-six—only "a few" of them (example 4b).

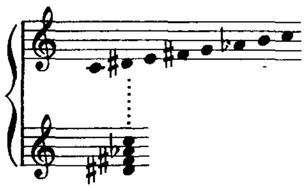
Example 4b. *Manual of Harmony,* 11.

Many of Ziehn's mathematically calculated progressions are tonally ambiguous, sometimes extending to the very limits of the tonal system. Others interject highly chromatic chords into diatonic areas; Ziehn believes that these generate possible chromatic variants of the diatonic scale (examples 5a and 5b). Ziehn, however, never pursued the generation of potential chromatic scales within the octave with his customary mathematical rigor. Thus, Ziehn's list of eight-note scales hardly names all possible ones.[13] It was Ernst Bacon who continued Ziehn's work in this area.

Example 5a. Chord progression in C major including the chromatic seventh chord called "V." *Manual of Harmony,* 23.



Example 5b. A chromatically altered scale including the chromatic seventh called "V." *Manual of Harmony,* 22.



The third major concern of Ziehn's work is his operation of symmetrical inversion around the axis D-A♭. He called this a "keyboard" inversion because D and A♭ are symmetrical with respect to the distribution of black and white keys on the piano.[14] Example 6a illustrates the operation of symmetrical inversion. The soprano pitch, B♭ in the first progression, becomes F♯ in the bass of the symmetrical inversion, the alto E♯ becomes a tenor C♭, and so on. Ziehn applied this operation of symmetrical inversion to scales and suspension figures as well (example 6b and 6c). These symmetrical inversions often produced exotic cadence forms and tonally ambiguous material that he hoped would interest contemporary composers.[15]

Example 6a. Symmetrical inversion around the pitch axis D-A♭ *Manual of Harmony,* 28.

Example 6b. Symmetrical inversion of the whole-tone scale. *Manual of Harmony,* 28.

Example 6c. Symmetrical inversion of suspension figures. *Manual of Harmony,* 28.

designates the lower second

Ziehn was not a professional composer, but he did write short piano pieces to celebrate the birthdays of his young students. Each of the eight surviving pieces is filled with applications of this theories.[16] "Albumblatt," a piece in ternary form dedicated to his "best" young pupil Helen Rudolph, has a theme using the letters of her name (figure 1).

The composer himself gave the following description of the work:

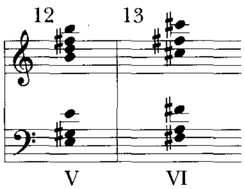
> The musical letters of the name "*He*l*e*n Ru*d*olp*h*" [B–E–E–D–B] structure the theme.
> Measures 3–7: The theme in its original form and in contrary motion; both forms also retrograded.

The next tones heard build a major ninth chord.
Measure 8: The ninth chord arpeggiated. Measure 9: The thematic figure repeated in a different way.
First Half of Measure 10: The ninth chord with passages in contrary motion; Second Half: Modulation.
Measure 11: The theme in the upper voice. The remaining material needs no explanation.[17]

These analytical notes confirm his conscious deployment of theoretical ideas throughout the piece. Moreover, the relation of its first half to its second illustrates symmetrical inversion. Ziehn also uses the enharmonic "plurisignificance"[18] of A♭ and G♯ to circle back from m. 22 to m. 1 (example 7). The final cadence, from V to VI, illustrates Ziehn's concept of "irregular cadence": a final cadence to a triad other than tonic (example 8).

Example 7. The enharmonic plurisignificance of A♭.

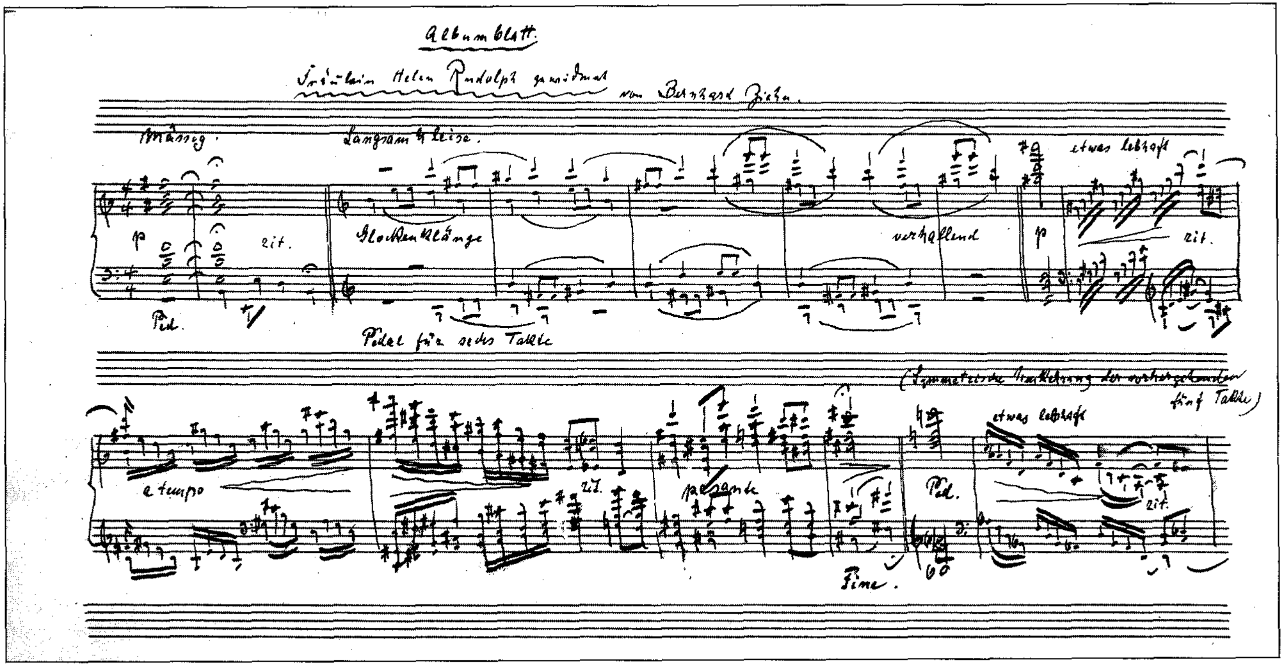
Example 8. The "irregular" cadence.



Bacon's Theories

Ernst Bacon studied Ziehn's *Manual of Harmony* in detail with Glenn Dillard Gunn.[19] In his preface to Bacon's "Our Musical Idiom," Gunn loosely connects Ziehn's work to Bacon's:

> "Harmony is that which sounds together," wrote Bernhard Ziehn twenty-five years ago. But the average theoretician comprehends only those simultaneously produced sounds which may be arranged in a series of superimposed thirds. . . . the world is still seeking a general system which will include all possible harmonies in logical order. . . . The system has now been evolved by Mr. Ernst Lecher Bacon.[20]


Albumblatt.
Mässig.
Langsam u. leise.
Glockenklänge
verhallend
etwas lebhaft
rit.
Ped.
Pedal für sechs Takte
a tempo
pesante
Fine.

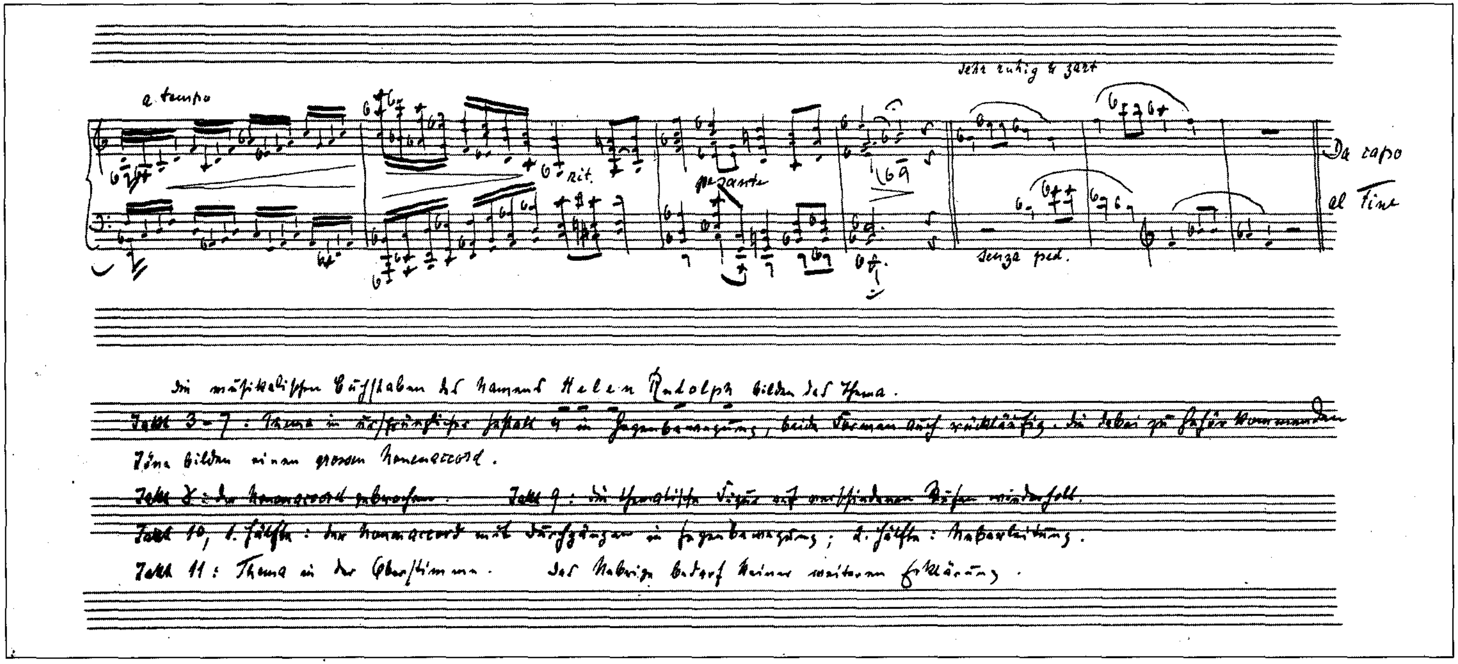


Figure 1. "Albumblatt." Holograph manuscript (Helen Rudolph Heller Collection, Newberry Library). Reproduced with permission.

Bacon himself acknowledges Ziehn's penchant for listing "new" harmonies, such as the nine chromatic seventh chords, as a source of inspiration for "Our Musical Idiom." His mathematical operations for generating scales and pitch collections are identical to those used by Ziehn in his *Manual of Harmony*.[21] His interest in exotic scales and in the invariance of pitch collections also has its roots in Ziehn's work.

Bacon derives the list of all possible forms of chromatic scales within the octave as follows. Using the same probability formula used by Ziehn, he determines the number of all possible order permutations of a group of intervals—none larger than a major third—that can occur within the octave:

$$\frac{6!}{(3!)(3!)} = \frac{720}{36} = 20$$

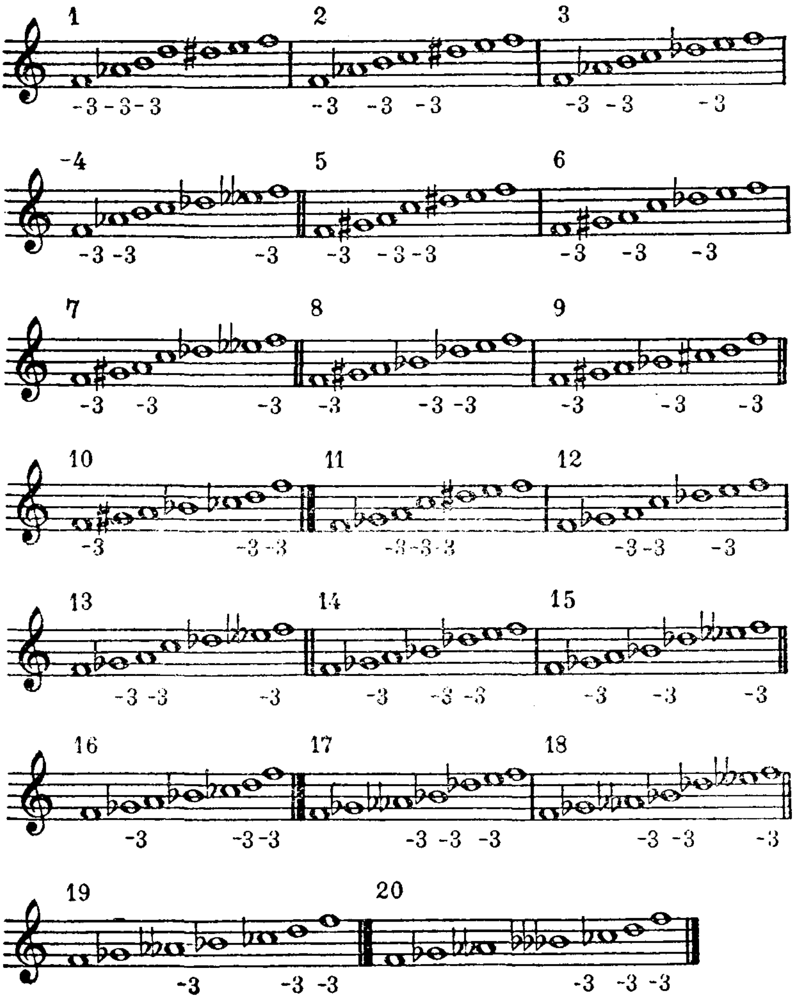
(The major third is a limit beyond which Bacon believes an interval will not sound scalar.) For example, three minor thirds and three minor seconds yield twenty possible scales (example 9). Bacon represents the scales in a pitch order which aims to keep initial pitches and intervals the same. In all, Bacon calculates 1,490 types of six-, seven-, eight-, and nine-note scales within the octave.[22] In figure 2, the columns titled "combinations" show the number of intervals of each kind used for the generation of each scale type. Each row shows the number of scales generated by particular interval combinations. For instance, the number of scales having ten semitones and one whole tone is eleven (see the second row on the left half of figure 2).

Bacon singles out from among the 1,490 scales those that have inversionally symmetrical structures, which he terms "equipartite." In these equipartite scales, "the same [ordered] pattern of intervals is repeated an integral number of times within equal divisions of the octave"—what George Perle now terms "interval cycles."[23] For instance, the scale pictured in example 10 is an equipartite one: it is made up of two successive 0134 tetrachords. Bacon defines two types of equipartite scales: "tripartite," whose interval patterns repeat three times within the octave, and "bipartite," whose patterns repeat two times. Figures 3a and 3b show Bacon's tabulations of possible equipartite scales.

A second purpose of "Our Musical Idiom" is to list and name what Ziehn called "all possible harmonies": what we now generally call all "nontonal collections." For Bacon, the problem of listing all possible harmonies is that of finding the number of cyclically unrelated permutations of a given collection. The familiar harmonic inversions of a C-major triad constitute a cyclic permutation. The intervallic content of a major triad, however, can also be ordered major third–perfect fourth–minor third to pro-

Example 9. Twenty possible scales generated from three minor thirds and three minor seconds. Reproduced from "Our Musical Idiom," 564.

The symbol (—3) will be written below respective minor thirds.



duce the minor triad whose harmonic inversion forms yet another cyclic collection. For contemporary atonal theorists, the cyclic permutations of the major and minor triads are equivalent; for Bacon they are not.[24]

Bacon uses two formulas to calculate the non-cyclically related permutations of interval collections from which he generates his complete list of

Figure 2. The 1,490 types of scales. "Our Musical Idiom," 566.

	Combinations				Permutations			Combinations				Permutations	
	Minor Seconds	Major Seconds	Minor Thirds	Major Thirds	Calculations	Permutations		Minor Seconds	Major Seconds	Minor Thirds	Major Thirds	Calculations	Permutations
1	12				P12!/12!	1	18	3		3		6!/3! 3!	20
2	10	1			11!/10!	11	19	2	5			7!/2! 5!	21
3	9		1		10!/9!	10	20	2	3		1	6!/2! 3!	60
4	8	2			10!/2! 8!	45	21	2	2	2		6!/2! 2! 2!	90
5	8			1	9!/8!	9	22	2	1		2	5!/2! 2!	30
6	7	1	1		9!/7!	72	23	2		2	1	5!/2! 2!	30
7	6	3			9!/3! 6!	84	24	1	4	1		6!/4!	30
8	6	1		1	8!/6!	56	25	1	2	1	1	5!/2!	60
9	6		2		8!/2! 6!	28	26	1	1	3		5!/3!	20
10	5	2	1		8!/2! 5!	168	27	1		1	2	4!/2!	12
11	5		1	1	7!/5!	42	28		6			6!/6!	1
12	4	4			8!/4! 4!	70	29		4		1	5!/4!	5
13	4	2		1	7!/2! 4!	105	30		3	2		5!/2! 3!	10
14	4	1	2		7!/2! 4!	105	31		2		2	4!/2! 2!	6
15	4			2	6!/2! 4!	15	32		1	2	1	4!/2!	12
16	3	3	1		7!/3! 3!	140	33			4		4!/4!	1
17	3	1	1	1	6!/3!	120	34				3	3!/3!	1
												Total	1490

Example 10. An equipartite scale formed of (0134) tetrachords.

non-tonal harmonies. In the first formula,

$$H = (n - 1)!$$

H is the number of harmonies and n is the number of non-repeating intervals. A minor second, a major second, and a perfect fourth generate

Figure 3a. The possible bipartite scales identified by Bacon. "Our Musical Idiom," 567.

	Combinations				Permutations	
No.	1 Minor Second	2 Major Second	3 Minor Third	4 Major Third	Calculations	No. of Perm.
1	6				P=6!/6!	1
2	4	1			P=5!/4!	5
3	3		1		P=4!/3!	4
4	2	2			P=4!/2! 2!	6
5	2			1	P=3!/2!	3
6	1	1	1		P=3!	6
7		3			P=3!/3!	1
8		1		1	P=2!	2
9			2		P=2!/2!	1
					Total	29

Figure 3b. The possible tripartite scales identified by Bacon. "Our Musical Idiom," 567.

	Combinations				Permutations	
No.	1 Minor Second	2 Major Second	3 Minor Third	4 Major Third	Calculations	No. of Perm.
1	4				P=4!/4!	1
2	2	1			P=3!/2!	3
3	1		1		P=2!	2
4		2			P=2!/2!	1
5				1	P=1!	1
					Total	8

$(3 - 1)!$, or two possible, non-cyclically related harmonies; these two have interval orderings as follows: minor second, major second, perfect fourth; and minor second, perfect fourth, major second, respectively. The second formula,

$$H = \frac{(n-1)!}{n_1!n_2!n_3! \ldots}$$

applies to harmonies contatining intervals that repeat in combination with one that occurs uniquely. The variable n is the total number of intervals in the harmony, n_1 is the number of repetitions of one repeating interval, n_2

is that of a second repeating interval, and so on. Thus, for example, there is only one harmony having the following seven intervals: one major second and six minor seconds, i.e., $(7-1)!/6! = 1$. In the case of harmonies in which all intervals occur more than once, Bacon chooses to calculate his results by trial and error.

Characteristically, Bacon uses his formulas and trial experiments to create tables (eleven in all) that show the number of harmonies derived from each intervallic combination (Figures 4a and 4b present two of these tables). He concludes that there are 350 possible harmonies. These harmonies correspond to Allen Forte's list of atonal sets along with the inversional equivalents of asymmetrical sets.[25]

Figure 4a. Harmonies generated from uniquely occuring intervals. "Our Musical Idiom," 587.

	Various Intervals											Calculations of Harmonies	(No. of Harm.) H
	1	2	3	4	5	6	7	8	9	10	11		
1	1										1	$H=(n-1)!=(2-1)!$	1
2		1								1		"	1
3			1						1			"	1
4				1				1				"	1
5					1		1					"	1
6						2						"	1
												Total	6

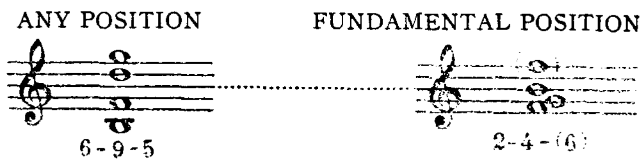
Because he does not acknowledge "pitch class," Bacon chooses to represent all 350 harmonies in pitch notation. He voices each harmony in what he calls "fundamental position": within the smallest possible range, the smallest interval occupies the lowest position, the next smallest the next lowest, and so on. Such voicing is akin to the "normal order" of sets (example 11).[26] Bacon names his harmonies in integer notation according to their interval content, 1 representing a minor second, 2, a major second, etc. Example 12 shows Bacon's list of the twelve trichords and their inversional equivalents.

In 1924, seven years after the publication of "Our Musical Idiom," Bacon wrote in Otto Luening's copy of the article: "This is pretty puerile. At present the opinions expressed herein do not interest me—only the sub-

Figure 4b. Harmonies generated from intervals, of which one must occur uniquely. "Our Musical Idiom," 588.

	1	2	3	4	5	6	7	8	9	10	Calculations of Harmonies	H
1	2									1	$H=(3-1)!/2!$	1
2	1	1							1		$(3-1)!$	2
3	1		1					1			"	2
4	1			1			1				"	2
5	1				1	1					"	2
6		2						1			$2!/2!$	1
7		1	1				1				$2!$	2
8		1		1		1					"	2
9		1			2						$2!/2!$	1
10			2			1					"	1
11			1	1	1						$2!$	2
12				3							By trial	1
											Total	19

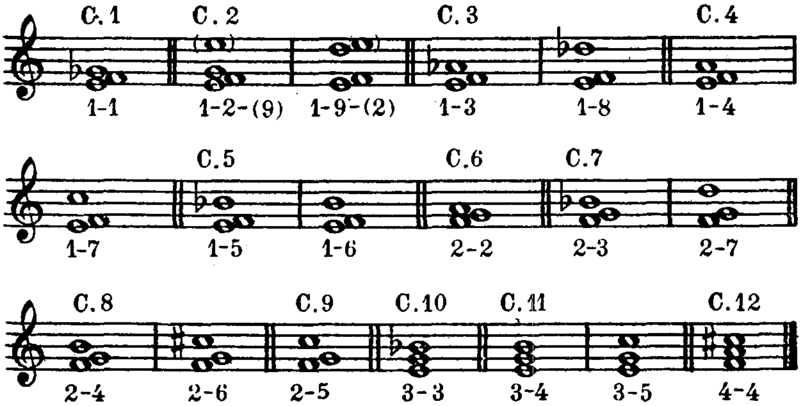
Example 11. "Our Musical Idiom," 592.



stantiated facts are of worth."[27] In a 1978 essay accompanying the song collection *Tributaries,* he further explained these thoughts:

> Growing up in an age of science, I, like many others felt the urge to experiment before settling down to seriously writing music. I tabulated and examined the three hundred fifty harmonies possible in our tuning system, invented new scales latent within the chromatic ladder, discovered new sounds through keyboard symmetry, [and] worked out new progressions in juxtaposing familiar chords. These

Example 12. "Our Musical Idiom," 594.



were all exercises in permutations and combination, a practice by no means new, that may have led to the belief that music and mathematics are closely related which I have never seen substantiated, apart from acoustical science.

A little of this experimentation found its way into my writing, but less than might have been expected. Instead, I took to studying the classical song literature.[28]

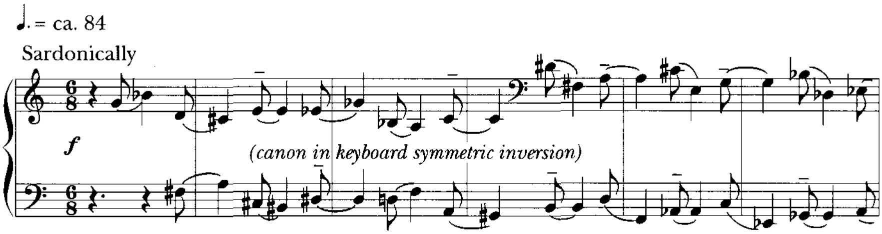
"I gave up Ziehn for Schubert."[29] Despite his remarks, Bacon did not completely "give up Ziehn." The songs "The Spider" and "No More Milk," "Water" (from the song cycle *Tributaries*), the *Riolama* Concerto for Piano and Orchestra, and *The Words of Lao-Tzu* for Narrator and String Quartet written in the 1950s, 1960s, and 1970s use equipartite scales and Ziehn's operation of symmetrical inversion to generate pitch material. Bacon himself points out in *Notes on the Piano* that sections of the *Riolama* Concerto are built from pitch material symmetrical around the D–A♭ axis (examples 13a and 13b).[30] Bacon again points out instances of D-symmetrical material on the autograph score of "No More Milk," a song dedicated to the composer

Example 13a. Alternating symmetrical chords in the *Riolama* Concerto. Transcribed from *Notes on the Piano*, 49.

Example 13b. Symmetrical chords used simultaneously in the *Riolama* Concerto. Transcribed from *Notes on the Piano*, 49.

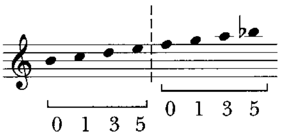
Otto Luening, a friend with whom he often discussed Ziehn's theories (example 14).[31]

Example 14. "No More Milk" (mm. 1–6).

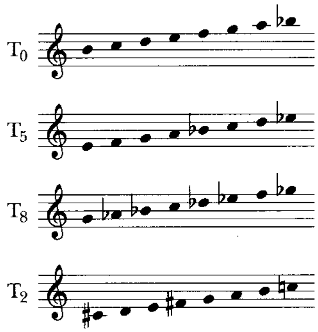


The fourth movement of *The Words of Lao-Tzu* illustrates Bacon's use of exotic scalar material.[32] The work contains five sections, each corresponding to a line of text. The identical pitch-class content of the first, third, and fifth sections of the piece articulates order permutations of a scale formed of two symmetrically related tetrachords (example 15a). For instance, in the first section, the first violin line favors an ordering of the scale beginning on B; the second violin and cello, orderings on A and F (example 15b, p. 23). In the second phrase of the work, the first violin line consists of permutations and transpositions of the basic scale on E, G, B, and C♯ (examples 15c and 15d).

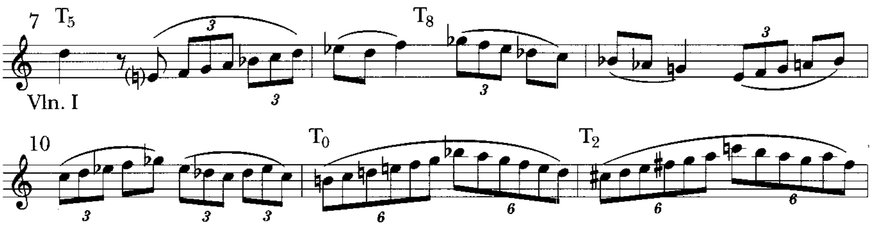
Example 15a. The basic scale of *The Words of Lao-Tzu,* movement iv.



Example 15c. Transposition of the basic scale of *The Words of Lao-Tzu* (mm. 7–12, violin I).



Example 15d. *The Words of Lao-Tzu* (mm. 7–12).



These pieces typify Bacon's employment of scalar material from "Our Musical Idiom" and Ziehn's operation of symmetrical inversion. Their pitch structures show that Bacon read the compositional potential of "Our Musical Idiom" in pitch-centered fashion, ignoring strictly atonal applications. He explained:

> Atonality, with its cousin, the tone row, never had any attraction for me. Appropriate to its originator for reasons remote from the American experience, it seemed a precarious foundation for what has become an international school. Its constructions could not hide their

Example 15b. *The Words of Lao-Tzu* (mm. 1–6).

> artificiality. It appeared to offer more denial than affirmation, imposed limitations far to be beyond promised liberations. What was there to be liberated from? Tonality is no more a tyranny than sentence structure.[33]

Such compositional conservatism contrasts sharply with the theoretical radicalism of "Our Musical Idiom," which places American music theory in a unique historical perspective. Five years before the debut of twelve-tone music and over thirty years before the proliferation of mathematically based theories of non-tonal music, Bacon was working on order permutation, invariance, and symmetrical inversion of non-tonal collections. Moreover, "Our Musical Idiom" is the only work using theoretical principles and methodologies of a radical nineteenth-century work, Ziehn's *Harmonie- und Modulationslehre.* Ziehn and Bacon's works are thus among the first to wed the progressive sides of the German and American music-theoretical traditions.

NOTES

A version of this article was presented at the 1985 national meeting of the Society for Music Theory (Vancouver, British Columbia).

1 *The Monist. A Quarterly Magazine Devoted to the Philosophy of Science* 27 (1917):560–607.

2 Josef Hauer, *Vom Wesen des Musikalischen* (Leipzig, 1920; reprint, Berlin-Lichterfelde, 1966); Herbert Eimert, *Atonale Lehrbuch* (Leipzig, 1924).

3 Karl Weigl was the brother of the theorist Bruno Weigl, an admirer of Ziehn's work.

4 Ernst Bacon, *Words on Music* (Syracuse, New York, 1960); *Notes on the Piano* (Syracuse, New York, 1963). For further biographical information, see *Dictionary of Contemporary Music,* ed. John Vinton (New York, 1974) s.v. "Ernst Bacon"; Philip Lieson Miller, "Ernst Bacon," *The New Grove Dictionary of American Music* (London, 1986), 1:109–10.

5 Donald Martino dedicated his *Seven Pious Pieces* (Boston, 1974) to Bacon.

6 Bacon's article was the first to treat music in this journal.

7 Julius Gold began studying with Bernhard Ziehn in Chicago in 1909. An ardent disciple, he taught Ziehn's theories both in San Francisco and in Los Angeles to the critic Winthrop Sargeant, to Isaac Stern, and to Meredith Willson (composer of *The Music Man*). For further biographical data on Gold, see Charlotte Serber, "Music as Science Championed by Julius Gold," *Musical America,* 25 October 1935. The quotation is drawn from an autograph annotation on some harmonic progressions in the Julius Gold Collection, Library of Congress (Washington, D.C.). Also concerning Ziehn and Gold, see Julius Gold, "Bernhard Ziehn's Contributions to the Science of Music," *Musical Courier,* 1 July 1914, 21–22.

Ferruccio Busoni met Bernhard Ziehn in Chicago in 1910 and was an ardent admirer of his work. Ziehn gave Busoni his solution to the unfinished fugue in Bach's *Der Kunst der Fuge,* which Busoni incorporated in his keyboard work, the *Fantasia contrappuntistica.* In appreciation, the composer wrote an essay "The Gothics of Chicago," praising Ziehn and his protégé the organist Wilhelm Middelschulte. The essay is reprinted in Hans Moser, *Bernhard Ziehn: die deutsch-amerikanische Musiktheoretiker* (Bayreuth, 1950). Busoni also mentions the compositional potential of Ziehn's theories in his *Essence of Music and Other Papers,* trans. Rosamund Ley (London, 1957), 47.

[8] Ziehn's works are: *System der Übungen für Klavierspieler, ein Lehrgang für den ersten Unterricht* (Hamburg: Verlag Hugo Pohle, 1881); *Harmonie- und Modulationslehre* (Berlin: Verlag Chrs. Friedrich, 1887, 1888, 1910); *Manual of Harmony* (Milwaukee: Wm. A. Kaun, 1907); *Five- and Six-Part Harmonies and How to Use Them* (Milwaukee: Wm. A. Kaun, 1911); *Canonical Studies: A New Technique in Composition* (Milwaukee: Wm. A. Kaun, 1912; reprint, *Canonical Studies,* ed. Ronald Stevenson, New York: Crescendo Press, 1976); Ziehn's essays are to be found in *Jahrbuch der deutsch-amerikanische historischen Gesellschaft von Illinois,* ed. Julius Goebel, vol. 26–27 (Chicago, 1927). Some of Ziehn's musical compositions are reprinted in *Bernhard Ziehn: The Doric Hymns of Mesomedes* (Chicago, 1979). His last work, a second volume of the *Manual of Harmony,* was destroyed in a fire at the office of his son Dr. Robert Sebastian Ziehn. (This is attested in a 1968 letter from Ziehn's son to Donald Krummel, formerly of the Newberry Library, which is preserved in the Helen Rudolph Heller Collection. For biographical information, see Moser, *Bernhard Ziehn,* 7–8; and Philip Lieson Miller, "Ernst Bacon," *The New Grove Dictionary of American Music* (London, 1986), 1:109–10.

[9] Otterström recounts the following story: "The week after [I gave Ziehn] a canon in thirteen parts on the same theme. . . . [Ziehn told me:] the other day I was on the train and felt the necessity of fooling around with something. I figured out that this canon in thirteen parts can be exchanged 6227020800 times without resulting in two positions that are alike" (Goebel, ed., *Jahrbuch,* Bd. 26, 22). 13! equals 6,083,020,800, a number similar to Ziehn's; notice it shares the opening "6" and the "20800" pattern at the end.

Otterström's treatise *A Theory of Modulation* (Chicago, 1935) relies heavily on permutation of chords and progressions through various formulas of probability and number theory. The mathematical formulas are explained in the appendix to that work. Julius Gold's papers contain a sheet on which exponential operations are used to calculate the number of variants of a progression (Julius Gold Collection, Library of Congress).

Otto Luening, a pupil of Ziehn's disciple Wilhelm Middelschulte, recounts in his *The Autobiography of Otto Luening: The Odyssey of an American Composer* (New York, 1980): Middelschulte "worked out my weekly assignments in fifteen different ways to show me *the* way. It annoyed me that he was a better student than I was. I thought I had matched him when I arrived at 126 solutions to a harmonic progression that interested him. He was pleased, but he said, 'I'm sure if we searched a little further we could find other possibilities'." Luening says Middelschulte never showed him how he calculated those "other possibilities" (interview with the author, 15 April 1985).

[10] See K. B. Henderson, Z. Usiskin, and W. Zaring, *Precalculus Mathematics* (New Jersey, 1971), 546–47. The variable n may equal any integer.

[11] Ziehn's roman numerals do not represent functions but are his preferred nomenclature. The chords are grouped first according to the nature of their inherent sevenths: roman numerals I–III contain the minor or "small" seventh; IV–IX the diminished seventh. IV–IX are further grouped in the following way: "The last six chords have the sound of diatonic seventh chords; the numbering, therefore, is arranged in the corresponding manner: IV and V according to the dominant, VI and VII according to the small, and VIII and IX according to the small seventh-chord" (*Manual of Harmony,* 21). The actual order of the example—IV, VIII, VI, IX, V, VII—shows, first, how adjacent chords are chromatic variants of each other (e.g., C♯-E♭-G-B♭ and C♯-E♭-G♭-B♭); and, second, how the intervallic content of the chords, taken as a whole, is symmetric to the axis C♯-E♭ (e.g., B♭-G in chord IV paired with F♯-A in IX):

B♭ – G – C♯ – E♭ – A – F♯

Symmetrical inversion is one of Ziehn's general theoretical concerns (see p. 9f).

[12] G obviously can also belong to G dominant seventh, E minor seventh, and C major seventh chords.

[13] See *Manual of Harmony*, 61.

[14] Ziehn's explanation of the sources of his theory is in an essay on symmetrical inversion (Goebel, ed., *Jahrbuch*).

[15] The subtitle of Ziehn's *Canonical Studies*, which extensively explains symmetrical inversion, is "A New Technique in Composition" (see note 8).

[16] Ziehn's pieces are in the Helen Rudolph Heller Collection of the Newberry Library.

[17] Translated by the author.

[18] In a letter to Eugene Luening (Otto Luening's father) dated 1909 (now in the Otto Luening Collection of the New York Public Library at Lincoln Center), Ziehn points out that he is working theoretically with more than symmetrical inversion around D. Notice that "Albumblatt" is doing the same thing compositionally.

[19] Glenn Dillard Gunn was a champion of new music as a pianist, a conductor of the American Symphony Orchestra from 1915 to 1917, and a critic for the Chicago *Tribune*. Certain of his papers are now in the Glenn Dillard Gunn Collection, Library of Congress. See my article, "Glenn Dillard Gunn," *The New Grove Dictionary of American Music*, 1:1332.

[20] Bacon, "Our Musical Idiom," 2.

[21] Bacon confirmed this in a telephone interview with the author (22 April 1985).

[22] Bacon also acknowledges his debt to Busoni's idea of 113 scales. See Bacon, "Our Musical Idiom," 3.

[23] George Perle, *Twelve-tone Tonality* (Berkeley, Calif.: University of California Press, 1977), 18.

[24] Allen Forte, *The Structure of Atonal Music* (New Haven, Conn.: Yale University Press, 1973), 7–11.

[25] Forte, *The Structure of Atonal Music*, appendix 1.

[26] Forte, *The Structure of Atonal Music*, 3–5; John Rahn, *Basic Atonal Theory* (New York, 1981), 31–39.

[27] This copy is in the Otto Luening Papers at the New York Public Library at Lincoln Center.

[28] See "Afterword" to *Tributaries* (Ernst Bacon, 1978).

[29] Bacon, interview with the author, March 1986.

[30] See Ernst Bacon, *Notes*, 48–49.

[31] See *Fifty-Three Songs*. Otto Luening confirmed in an interview with the author (15 April 1985) that he and Bacon discussed the theories of Ziehn. Examples of symmetrical inversion also occur in Bacon's song "A Spider" (found in *Fifty-Three Songs*). See also *Spirits and Places for Organ* (Cincinnati, Ohio, 1974).

[32] The score to *The Words of Lao-Tzu* is housed at the American Music Center, New York.

[33] "Afterword" to *Tributaries*.