The Effect of the Establishment of Reinforcement Value for Math on Rate of Learning for Pre-Kindergarten Students

Emmy N. Maurilus

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ABSTRACT

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The objective of Experiment I was to determine whether establishing conditioned reinforcement for engaging in math for pre-kindergarten students was possible using the three conditioning procedures outlined in previous research for conditioning book stimuli. The purpose of Experiment II was to determine whether this change in preference for engaging in math had an effect on 6 pre-kindergarten participants’ rate of learning math. In Experiment I a counterbalanced pre- and post-intervention ABAB/BABA functional analysis and a delayed multiple probe across dyads design, was used to measure the indirect and direct reinforcement value of math for each participant. Indirect measures referred to a functional analysis where the participants’ rate of responding to a performance task during a 1-min session when Play-Doh® was delivered as a reinforcer was compared to their rate of responding when math was delivered as a reinforcement operation. Direct measures referred to the number of 5-s intervals (out of 60) each participant engaged in math when given math worksheets and Play-Doh®. The individualized reinforcement intervention consisted of a sequence of conditioning procedures until a defined successful outcome resulted. First learn units were delivered, then stimulus-stimulus pairing, and then observational conditioning-by-denial. Learn unit instruction resulted in the establishment of conditioned reinforcement for the first dyad, while the stimulus-stimulus pairing procedure was necessary for the remaining dyad. The purpose of Experiment II was to test if establishing conditioned reinforcement for math would change rate of learning. The dependent variable was each participant’s rate of learning as measured by the number of learn
units required to meet mastery criterion for 4 units of the Multiple Exemplar Functional Math (MEF-Math) curriculum. The dependent variable, rate of learning, was tested using a multiple probe design. The independent variable was the establishment of conditioned reinforcement for math using individualized reinforcement procedures as detailed in Experiment I. The intervention also consisted of a multiple probe design on testing the effect of the individualized reinforcement procedures on establishing conditioned reinforcement. Three participants required learn units, 2 participants required the stimulus-stimulus pairing procedure, and 1 participant required observational conditioning-by-denial to establish conditioned reinforcement for math. Results showed an educationally significant acceleration of learning following the establishment of conditioned reinforcement for math across all 6 participants. Results are discussed in terms of the significance of early math instruction.
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DEDICATION

I dedicate this dissertation to my mother and father who, by the grace of God, without hesitation left the only home they knew to provide better opportunities for their three “rocks”. Your sacrifice never mentioned, is also never forgotten. Your venture into the unknown produced a woman who will never give up on her dreams and who will inspire others to do the same. You are my heroes and I will work to finish what you started.
CHAPTER I
INTRODUCTION AND REVIEW OF LITERATURE

Introduction

Conditioned reinforcement for math, or math enjoyment, may be critical for students. If so, this should be established as early as pre-kindergarten, as conditioning the components of math as reinforcement arguably leads to more complex mathematical operations. The purpose of Experiments I and II was to determine if the establishment of conditioned reinforcement for math influenced students’ rate of learning math. In Experiment I, I tested the effect of an individualized reinforcement intervention on the establishment of reinforcement value for math. Reinforcement value referred to the participants’ interest or level of math enjoyment as measured by the number of intervals the participant selected math during a 5-min observation. The individualized reinforcement intervention consisted of a sequence of procedures. First learn units were delivered, then stimulus-stimulus pairing, and then observational conditioning-by-denial, until a defined successful outcome resulted. This sequence of conditioning procedures was found in research on conditioning book stimuli where Buttigieg (2015) determined that different preschoolers required different conditioning procedures to establish conditioned reinforcement for the observation of books.

Overall, this study seeks to determine whether students’ preference or interest in math, can be changed. Prior research on reading (Buttigieg, 2015; Tsai & Greer, 2006) has suggested that the establishment of conditioned reinforcement results in an accelerated rate of acquisition of sight words. Studies on generalized visual match-to-sample suggested that conditioned reinforcement for two-dimensional stimuli results in an accelerated rate of learning (Du, Broto, & Greer, 2015; Delgado, Greer, Speckman, & Goswami, 2009; Greer & Han, 2015). Studies on
conditioned reinforcement for voices and faces have suggested an accelerated rate of acquisition for listener programs (Greer, Pistoljevic, Cahill, & Du, 2011; Maffei, Singer-Dudek, & Keohane, 2014). The results of the above studies suggest that conditioned reinforcement value may have the same effect on math. Given that the United states is ranked at only number 36 (out of 65 developed countries) in the world in math performance (“Pisa Tests: Top 40,” 2015), I sought to test whether: 1) establishing conditioned reinforcement for math activities so that students would prefer math and engage in mathematic activities in free play settings was indeed possible, 2) whether different students required different procedures to establish conditioned reinforcement for math, and 3) whether the establishment of conditioned reinforcement for math would result in a change in the students’ rate of learning. The review of literature will review conditioned reinforcement for math from the education perspective (math enjoyment or interest) as well as from the behavior selection perspective.

**Review of Literature**

Research has shown that children are indeed able to learn math at an early age (Ginsburg, Lee, & Boyd, 2008). According to cognitive psychologists, from birth to five-years children acquire what is called *everyday mathematics* even without direct instruction. Everyday mathematics includes complex yet, “informal ideas of more and less, taking away, shape, size, location, pattern and position” (Ginsburg et al., 2008, p. 3). In addition to these everyday mathematics skills, between the ages of 3- and 4- years, students learn to subitize, compare quantities, and understand part-whole relations (Charlesworth, 2005). Notwithstanding these findings, some parents and educators criticize direct math instruction and do not believe it is developmentally appropriate. “However, much of the mathematical thinking that some people say *cannot be done* until age seven, can be learned by children— most children — in a high-
quality environment” (Clements, 2015). Students’ *everyday mathematics* as well as any additional math skills students acquire before the onset of kindergarten are critical indicators of their future success in both reading and math (Engel, Claessens, & Finch, 2013). Despite these discoveries, compared to reading instruction, math is underemphasized in the early grades (Engel et al., 2013).

The National Council of Teachers of Mathematics (2002) recommended math instruction begin in pre-kindergarten. However, the majority of research on early mathematical skills is primarily concerned with kindergarten. Despite the recommendation of the National Council and the recent rise in investment in early childhood education, teachers have reported discomfort in instructing young children in more advanced mathematics (Ginsburg et al., 2008). According to Ginsburg and colleagues (2008), early childhood teachers, “are poorly trained to teach the subject, are afraid of it, feel it is not important to teach, and typically teach it badly or not at all” (p. 3). Engel and colleagues (2013) found that teachers continued to *teach* basic math concepts that kindergarteners previously mastered in an attempt to avoid teaching more advanced concepts.

While it is clear that it is developmentally appropriate for children to learn mathematics in preschool, the controversy lies in the content that should be taught (to expand on students’ *everyday mathematics*) and whether or not instruction should be teacher-led or play-based. Gray (2012) believed that the rise in children’s mental disorders is due to a lack of free play and exploration in early education. On the other hand, Ginsburg and colleagues (2008) assert that, “although essential for children’s intellectual development generally and for mathematics learning in particular, play is not enough. It does not usually help children to mathematize—to interpret their experiences in explicitly mathematical form and understand the relations between
the two” (Ginsburg et al., 2008, p. 7). Ginsburg and colleagues assert that early math instruction should include play, projects, teachable moments, and intentional teaching guided by curriculum.

Math Enjoyment from the Perspective of Educational Research

Math attitudes. While early math development is a predictor of future success in more complex mathematical operations (Classens & Engel, 2013), it is evident that as early as the preschool level some students perform well in math while others do not. Lyons and Beilock (2011) propose math anxiety as a variable in this fact. Lyons and Beilock used functional magnetic resonance imaging (fMRI) to distinguish between neural activity while participants were completing math work and neural activity when students anticipated completing math work. The results suggest that, “education interventions emphasizing control of negative emotional responses to math stimuli will be most effective in revealing a population of mathematically competent individuals, who might otherwise go undiscovered” (Lyons & Beilock, 2011, p. 2103).

In addition to math anxiety, previous research has found that academic interest is a factor in learning, “the presence of interest positively influences learners’ attention, strategy use, and goal setting. With interest, learners are better able to self-regulate and persist to complete tasks even when they are challenging” (Renninger, Nieswandt, & Hidi, 2015, p 2). In a longitudinal study, Fisher, Dobbs-Oates, Doctoroff, and Arnold (2012) measured students’ math abilities, language and cognition, levels of interest, time playing a math game, and the teacher’s report of the child’s level of interest. Results indicate a positive correlation between math enjoyment and math performance in preschoolers. Fisher et al., (2012) built upon the research of Ma (1997) who:

proposed a bidirectional paradigm between ability and attitudes specifically for math development, with attitudes and achievement strengthening one another over time. . .
heightened interest, increases time spent pursuing math activities, which improves ability. While framed positively, individuals with low interest or ability could experience a negative cycle. (p. 674)

Behavior selectionists would refer interest as reinforcement strength. According to Renninger and colleagues (2015), interest has four phases: triggered situational, maintained situational, emerging individual, and well-developed individual. In the triggered situational phase, an individual may attend to stimuli for a short period of time. The individual enters the maintained situational phase when s/he reengages with the stimuli with some prompting. The later phases of interest development occur on the independent level, which means the individual engages with the stimuli without prompting and demonstrates curiosity by posing questions about the stimuli. The individual enters the well-developed interest development stage only when s/he, “can persevere through frustration and challenge order to meet goals” (p. 4). This is synonymous to what is called “resistance to extinction” in behavior analytic research. Resistance to extinction refers to the strength of the reinforcer or the “time elapsed or number of trials until performance meets some extinction criterion” (Catania, 2013, p. 461). For example, resistance to extinction could be the number of math problems above grade level a student attempts before stopping or giving up. In this case, students who have math as a conditioned reinforcer would complete more problems than those who do not have math as a conditioned reinforcer. See Table 1 for a comparison of the behavior selection perspective and educational research in terms of the phases of interest.
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<td>Triggered Situational</td>
<td>Brief changes in cognitive processes</td>
<td>Child briefly observes the object</td>
<td>Observing Response</td>
<td>Each instance of the child looking at, smelling, touching, listening to, or tasting the stimulus</td>
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<td>Maintained Situational</td>
<td>Extensive focused attention</td>
<td>Child has positive feelings about the object but still requires prompting to engage</td>
<td>Conditioning Procedures</td>
<td>Number of correct responses to 5-s test trials, Number of correct responses to learn units, Number of mands for the target stimulus</td>
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<td>Emerging Individual</td>
<td>Increased tendency to engage</td>
<td>Child independently engages with object</td>
<td>Conditioned Reinforcer</td>
<td>80% of 5-s intervals during a 5-min free play observation</td>
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<td>Well-Developed Individual</td>
<td>Lasting tendency to reengage</td>
<td>Child is curious about the object, asks, questions about it, and persists through frustration</td>
<td>Resistance to Extinction</td>
<td>The duration of time the child manipulates the stimulus before stopping</td>
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Students with low interest in math are proposed to have low self-confidence in their math ability as supported by Pinxten, DeFraine, Van Den Noortgate, and Van Damme’s (2014) study that found a positive relation between math competency beliefs in addition to math enjoyment and actual math competency. The previous research however was conducted with students in first grade. Math enjoyment, math competency beliefs, and math anxiety all influence students’ math performance (Fisher et al., 2012; Lyons & Beilock, 2012; Ma, 1997; Pinxten, 2014).

The gender gap. According to Renninger and colleagues (2015), while the gender gap in math and science performance has been reduced, the gap between perceived competence persists. This means that the absence of girls in STEM can no longer be attributed to a lack of competence (Renninger et al., 2015). Despite their equal performance to males, females perceive themselves as less competent in math and science. Males demonstrate a more positive attitude towards math and are attuned to its function and importance in later success (Renninger et al., 2015, p. 35). “Girls’ fears and anxieties toward mathematics may even prevent the math knowledge they do possess to be used to solve problems. . . their gender-biased expectations may lead to lower achievement and lower math self-concept” (Tichenor, Welsh, Corcoran, Piechura, & Heins, 2016, p. 93). In addition to a gender gap, the results of Cheema and Galluzzo’s (2013) experiment indicated a race and socioeconomic status gap in the areas of math anxiety and math self-efficacy. They assert that once race and socioeconomic status (predictors of math achievement) are controlled for, the gender gap no longer exists.

Conditioned Reinforcement – The Behavior Selection Perspective

From a behavior selection perspective, math enjoyment or positive attitudes towards math is referred to as conditioned reinforcement for math or reinforcement value and preference for engaging in math activities where the control of the activities reinforce the related behaviors.
Hence, according to Verbal Behavior Development Theory (Greer & Ross, 2008) doing math would be considered a learned or conditioned reinforcer. Math anxiety can otherwise be defined as a lack of conditioned reinforcement for math and/or math functioning as aversive stimuli (i.e., negative reinforcement or punishment). Reinforcement is, “the response-produced presentation of positive reinforcers or termination of negative reinforcers, or the increase or maintenance of responding resulting from this operation” (Catania, 2013, p. 460). By extension, stimulus control for engaging in an activity is the reinforcer for said activity. Conditioned positive reinforcement occurs when a previously neutral or aversive stimulus acquires reinforcing properties. This can occur through the following types of conditioning: operant, classical, and observational conditioning (Greer & Singer-Dudek, 2008; Greer, Singer-Dudek, & Gautreaux, 2006; Pavlov, 1927; Skinner, 1953). Operant conditioning focuses on the role of consequences on behavior as well as the automaticity of reinforcement (Skinner, 1953). Classical or respondent conditioning (Pavlov, 1927) calls for the stimulus-stimulus pairing procedure in which an unpreferred or neutral stimulus is paired with a reinforcing conditioned stimulus that results in that same neutral stimulus becoming a conditioned reinforcer. Observational conditioning refers to a change in existing behaviors as a result of observing the delivery of the stimulus to others while being repeatedly denied access to it (Greer & Singer-Dudek, 2008; Singer-Dudek, Oblak, & Greer, 2011).

**Operant conditioning.** Skinner argued that the story of life did not begin with a big bang, but instead the “moment when a molecule came into existence which had the power to reproduce itself. It was then that selection by consequences made its appearance as a causal mode” (Skinner, 1981, p. 501). Skinner considered reproduction as the very first consequence and suggested that it was that consequence that led to the evolution of cells and organisms.
According to Skinner, reproduction was made possible through, “the evolution of two processes through which individual organisms acquired behavior appropriate to novel environments” (Skinner, 1981, p. 501). The processes Skinner referred to were operant conditioning, “responses strengthened by events which immediately followed them” and respondent conditioning, “responses prepared in advance by natural selection under the control of new stimuli” (Skinner, 1981, p. 501).

Operant conditioning occurs at an observable speed. Skinner viewed operant conditioning as a second kind of selection by consequences (Skinner, 1981). It can be seen as natural selection in progress. Skinner’s hypothesis can currently be seen in the field of epigenetics. Robinson and Barron’s (2017) article on epigenetics and the evolution of instincts discuss the “experience-dependent transgenerational” behavior change. This behavior change refers to an organism’s reaction to a stimulus, changing a protein in its brain and thus transmitting this behavior change to its offspring. Operant conditioning and natural selection are redundant when selecting consequences are the same. According to Skinner, selection by consequences was discovered late in the history of science and was challenged because operant conditioning “provides a controversial account of the “voluntary” behavior traditionally attributed to a creative mind” (Skinner, 1981, p. 502). Skinner cites the following as other reasons selection by consequences are not understood: a prior act of creation, purpose or intention, certain essences, and certain definitions of good and value (Skinner, 1981).

**Respondent conditioning.** Pavlovian second-order conditioning or respondent conditioning occurs when a neutral stimulus acquires value due to its pairing with a primary reinforcer and becomes a reinforcer itself.

That is, a stimulus paired with a Pavlovian reinforcer acquires not only the ability to elicit the responses appropriate to the Pavlovian reinforcer but its reinforcing value as well.
The implication is that variables that govern Pavlovian conditioning also determine when initially neutral stimuli will become conditioned reinforcers (Williams, 1994, p. 261).

In the 1950s conditioned reinforcement was used as an explanatory idea across several topics in psychology including but not limited to: personality theory, psychopathology, human cognition, and social behavior (Williams, 1994). While Hullian behaviorists subscribed to the idea of conditioned reinforcement as an explanatory idea, Skinner, Keller, and Schoenfeld believed that conditioned reinforcement played, “an essential explanatory role for extrapolating the concept of reinforcement to human behavior in real-world situations” (Williams, 1994, p. 263).

Kelleher and Gollub (1962) conducted an extensive review of conditioned reinforcement. They outlined the role of chains of stimuli and responses in primary and secondary reinforcers, analyzed the conditions necessary for the stimuli in said chains to indeed become conditioned reinforcers, and summarized experiments in which conditioned reinforcers were used to elicit responding during extinction. Kelleher and Gollub (1962) asserted that the process of conditioning an operant results in the establishment of a chain. “In a respondent chain, a response follows an eliciting stimulus and produces the eliciting stimulus for another response. In an operant chain, responses occur in the presence of a discriminative stimuli for other responses” (Kelleher & Gollub, 1962, p. 544).

In terms of respondent or classical conditioning, Skinner emphasized that an individual could only imitate or model a behavior that was already in repertoire for the organism that modeled it. Respondent conditioning is different in that the individual is solely under control of the environment s/he is exposed to. Skinner proposed the following as a simplified explanation of respondent conditioning: the sounding of a bell followed by food elicited salivation after several pairings. The natural progression should have been salivation to the mere appearance of
food however the contingencies established a stronger response to taste. Saliva was a “weak reflex arising from natural selection and as a conditioned reflex” (Skinner, 1984, p. 218).

Respondent conditioning is often described as an increase in the strength of reflexes; this is evident because conditioned reflexes do not survive unless they are followed by unconditioned responses (Skinner, 1998). The key difference between operant and respondent conditioning is the relation between the stimuli, response, and reinforcement. Unlike respondent conditioning, during operant conditioning, one must wait for behavior to appear in order to reinforce it (unless it is prompted). Because of this, Skinner specified the importance of shaping behavior through successive approximations (Skinner, 1998).

Greer, Dorow, Wachhaus and White (1974) were the first to use conjugate duration, as opposed to frequency, to measure reinforcement value and observing responses. Experimenters tested the effect of adult approvals and disapprovals on music preference. There were 110 fifth grade participants in this experiment. Participants had access to a specific genre of music contingent upon holding down a micro switch. However, if the participant continuously held the switch for longer than two minutes, the participants’ selection was interrupted, thus requiring him/her to search through other micro switch keys to find the selection. Results indicated that participants selected music that was taught during high approval conditions. This study was one of the first studies to use duration as opposed to frequency to measure reinforcement value.

Sundberg, Michael, Partington, and Sundberg (1996) used Pavlov’s stimulus-stimulus pairing procedure when they conditioned vocal sounds to function as reinforcers for preschool students. They paired a target sound, word, or phrase with an already established form of reinforcement (i.e., tickling). The results indicated that all participants emitted the target sounds as a function of the pairing procedure.
**Observational conditioning-by-denial.** Observational learning is defined as “learning based on observing the responding of another organism, and / or its consequences” (Catania, 2013, p. 452). The capability to learn from observing the instruction of others is fundamental to children’s success (Greer & Ross, 2008). Observational learning consists of observational acquisition, performance, and conditioning. The distinction between observational acquisition and performance is that observational acquisition refers to new operants as opposed to operants in repertoire. Observational conditioning refers to changes in existing stimuli from non-reinforcing to reinforcing as a function of observation by denial (Greer & Singer-Dudek, 2008). Greer, Singer-Dudek, and Gautreaux (2006) suggest that observational learning should be distinguished as “changes in behavior resulting from the observation of the contingencies received by others,” (Greer, et al., 2006, p. 496). These changes in behavior can lead to the acquisition of conditioned reinforcers.

Greer and Singer- Dudek (2008) found that stimuli may become conditioned reinforcers through observing others receive the stimuli while simultaneously being denied access. Six students between the ages of three and five years participated in the study. Prior to the intervention, each participant’s responding was compared on (1) performance tasks in which the child received either a preferred food item, a disc, or string for correct responses, and (2) the acquisition of new skills where the disc or string was the consequence for correct responses. Experimenters conducted a reversal design with alternating phases of food and discs or strings provided as consequences for correct responses to a performance task. Pre-observational intervention data demonstrated that discs or string did not function as reinforcers for correct responses compared to edibles. The intervention consisted of denying the participant access to the stimulus being conditioned. The participant was seated side-by-side with a peer. Between
them was an opaque divider so that the participant could only view the plastic cup on the peer’s table. The peer was given plastic discs in the cup following correct responses to the performance task while the participant was denied discs and string regardless of correct or incorrect responses. After the peer observation intervention, the performance and acquisition tasks used in the pre-intervention phase were repeated. Results indicated that the discs and strings acquired reinforcing properties for correct responding for both performance and acquisition tasks for all participants.

Zrinzo and Greer (2013) isolated the role of adults and the view of the peer’s face during the observational procedure. There were three male participants, who functioned at early reader and writer levels of verbal behavior, in this study. The participants were four years old and were classified as preschoolers with a disability. The participants were selected because the neutral stimuli did not function to reinforce the probe tasks. The dependent variable was the number of correct and incorrect responses per min to pre- and post-intervention performance and learning tasks when either edibles or a metal washer (previously neutral stimulus) were the consequent stimuli delivered. The second dependent variable was the learn units-to-criterion prior to and following the observational intervention when the delivery of a washer was the consequence for correct responding. The independent variable was observational conditioning.

Unlike the initial study, the neutral stimulus in Zrinzo and Greer’s (2013) study was delivered through a metal chute in order to eliminate the role of adults and the view of peers’ faces in the establishment of socially learned conditioned reinforcement. Following the observational intervention, the participants responded similarly when washers or edibles were earned. Not only were these results consistent with the previous experiments, but the results also demonstrated the continued effectiveness of the observational intervention when the presence of
adults and the view of confederates’ faces were eliminated. A functional relation was shown and reinforcement effects remained six to ten weeks following the intervention. The results of this study strengthened the internal validity and functional relation between the observational conditioning procedure and the establishment of conditioned reinforcement for previously neutral stimuli in that it controlled for the possibility of confounding variables such as the presence of the experimenter as well as reactivity. Similar observational procedures have been used to condition socially and educationally significant reinforcers such as adult praise (Greer, Singer-Dudek, Longano, & Zrinzo, 2008), books (Buttigieg, 2015; Singer-Dudek, Oblak, & Greer, 2011), math (O’Rourke, 2006), and writing (Lee, 2016).

**Conditioned Reinforcement—Fall from Grace**

Conditioned reinforcement fell out of favor in the basic science of behavior during the 1960s. Williams (1994) proposed that this decrease in attention was due to criticisms of, “its validity by major behavior theorists and in part because its explanatory function in a variety of different conditioning procedures has become uncertain” (Williams, 1994, p. 261). These behavior theorists questioned whether conditioned reinforcers were in fact reinforcing in value. They also suggested “the effects of conditioned reinforcement contingencies, although often potent, occur for reasons other than the process of reinforcement” (Williams, 1994, p. 262). Due to these damaging opinions, very little about the concept of conditioned reinforcement was included in introductory textbooks as it had become unclear how to depict the concept (Williams, 1994). Although conditioned reinforcement fell out of favor of the basic science, researchers such as Edmund Fantino (Fantino & Romanowich, 2007), Ben Williams (Williams, 1994), Robert Rescorla (Rescorla & Wagner, 1972) as well as verbal behavior development theorists continued to conduct research on conditioned reinforcement. Verbal Behavior Development
Theory (VBDT) research suggests that this principle of behavior is educationally significant and critical in the acquisition of new higher-order operants with children from preschool through middle school.

**Verbal Behavior Development Theory (VBDT)**

A program of research in Verbal Behavior Development (Greer & Ross, 2008), influenced by Skinner’s Verbal Behavior theory, proposes a trajectory of fundamental cusps and capabilities an individual must achieve to be deemed truly verbal. The developmental and educational implications of conditioned reinforcement are detailed in a series of VBDT studies conducted with preschool aged children just as in the present study.

**Preverbal cusps.** Conditioned reinforcement procedures have been effective in inducing the following preverbal cusps: conditioned reinforcement for voices and faces (Greer, Pistoljevic, Cahill, & Du, 2011; Maffei, Singer-Dudek, & Keohane, 2014), two-dimensional and three-dimensional stimuli (Du, Broto, & Greer, 2015; Greer & Han, 2015; Pereira-Delgado, Greer, Speckman, & Goswami, 2009), and books (Buttigieg, 2015). Maffei and colleagues (2014) identified a face conditioning protocol to establish conditioned reinforcement for the observation of voices and faces in students who do not attend to faces. The intervention consisted of delivering reinforcement, such as singing or “motherese” (a sing song voice mothers typically use with their infants), contingent upon the participant observing the face of the experimenter. A functional relationship was shown between the protocol and students’ observing responses. Following the intervention, the students required fewer learn units to meet criterion on listener programs (Greer et al., 2011).

Using a delayed pre- and post- intervention probe design, Greer, Pistoljevic, Cahill, and Du (2011) tested the effects of a voice conditioning procedure on students’ numbers of learn
units to meet an objective on listener programs, number of intervals in which the participants listened to an adult voice reading a story, number of occurrences of stereotypy in the story setting, and observing responses. Voice conditioning included a stimulus-stimulus pairing of voices and preferred stimuli. The results demonstrated that after voice conditioning, the participants required fewer learn units-to-criterion. Moreover, observing responses increased and stereotypy decreased following voice conditioning in two participants.

The stimulus-stimulus pairing procedure has also been effective in inducing the acquisition of conditioned reinforcement for observing and manipulating stimuli (Longano & Greer, 2006). The dependent variable of Longano and Greer’s (2006) study was the number of 5-s whole intervals the participant emitted appropriate play and the number of partial intervals of stereotypy or passivity. Appropriate play consisted of looking at books, playing on the computer and manipulating toys. The independent variable was the stimulus-stimulus pairing procedure. The stimulus-stimulus pairing procedure resulted in increases in the number of intervals in which the student emitted the target behavior and a decrease in stereotypy and passivity. The second experiment was additive to the first in that the same intervention was used to increase two additional participants’ completion of seat work. Following the stimulus-stimulus pairing procedure, the number of intervals the participants emitted the target behavior increased.

The use of stimulus-stimulus pairings to condition books and toys was important because it decreased the amount of stereotypy and passivity emitted (Nuzzolo-Gomez, Leonard, Ortiz, Rivera, & Greer, 2002). Nuzzolo and colleagues conditioned books in an effort to teach students to prefer books or toys and reduce stereotypy and passivity. After implementing the book conditioning procedures, the student showed a significant increase in looking at books and a decrease in passivity and stereotypy. The second experiment used the stimulus-stimulus pairing
procedure to condition toy play. The results show that a decrease in stereotypy followed by an increase in appropriate toy play. Toys became reinforcers for the students, meaning instead of emitting stereotypy during free play, the participants manipulated toys following the intervention (Nuzzolo-Gomez et al., 2002).

Conditioned reinforcement for two-dimensional and three-dimensional stimuli is another crucial cusp (Du, Broto, & Greer, 2015; Delgado, Greer, Speckman, & Goswami, 2009; Greer & Han, 2015). The establishment of conditioned reinforcement for two-dimensional print stimuli has had numerous outcomes. Greer and Han (2015) tested the effect of conditioned reinforcement for the observation of print stimuli on generalized match-to-sample responses. The results indicated that conditioning the observation of two-dimensional stimuli resulted in the emergence of a generalized match-to-sample repertoire, enhanced discrimination training, and a decrease in students’ learn units-to-criterion (Greer & Han, 2015).

Tsai and Greer (2006) sought to determine whether conditioned reinforcement for the observation of books would accelerate the rate at which students acquire textual responses to sight words. The results supported the hypothesis in that all students required fewer learn units-to-criterion for textually responding to sight words following the establishment of conditioned reinforcement for the observation of books (Tsai & Greer, 2006).

**Conditioning stimuli for learning academics.**

The establishment of conditioned reinforcement for academic stimuli has resulted in either a faster rate of learning or a new way of learning (Buttigieg, 2015; Lee, 2016; O’Rourke, 2006; Tsai & Greer, 2006). For example, conditioned reinforcement for books, math, and writing have been established via observation. Singer-Dudek, Oblak, and Greer (2011) conducted a study to test the effectiveness of an observational intervention in establishing books as
conditioned reinforcers using a delayed multiple baseline design. The participants were three preschool students with mild language and developmental delays. The participants were selected because although they had a variety of reinforcers, books did not function as a conditioned reinforcer for observing. The dependent variables of this experiment included the percentage of 5-s intervals the participant looked at books in the free play area, the number of correct responses to three learning tasks, and the number of correct responses to a maintenance task before and after the observational intervention.

During the observational intervention, the participant and confederate were seated next to each other with an opaque divider between them. While both students were presented with the same task, only the confederate was given books for 5-s following each response. The divider was removed when the confederate was looking at books so that the participant could observe this. The intervention continued for eight sessions. Following the intervention, books served as conditioned reinforcers for participants and the rate of responding for maintenance and acquisition responses increased for all three participants. A functional relationship was shown (Dudek et al., 2011). The above procedure was successful in conditioning praise as a reinforcer as well (Greer, Dudek, Longano, & Zrinzo, 2008).

Tsai and Greer (2006) found that all four preschool participants required fewer learn units-to-criterion for textually responding to sight words following the establishment of conditioned reinforcement for the observation of books (Tsai & Greer, 2006). Buttigieg’s (2015) study on the effect of conditioned reinforcement on students’ rate of acquisition of novel textual responses was additive to Tsai and Greer’s (2006) study in that Buttigieg’s (2015) study sought to condition books through either learn unit instruction, stimulus-stimulus pairings, or observational conditioning-by-denial. If the participant demonstrated to have conditioned
reinforcement for books following sight word learn unit instruction, then the participant was said to have acquired conditioned reinforcement for the observation of books through operant conditioning, as learn unit instruction is an operant teaching procedure. Buttigieg (2015) found that following the establishment of conditioned reinforcement for the observation of books students required fewer learn units-to-criterion on novel sight words. The establishment of conditioned reinforcement for the observation of books is crucial in that it not only exposes students to a variety of word and picture relationships, but it also promotes reading readiness (Buttigieg, 2015). A functional relationship was shown between the establishment of conditioned reinforcement for looking at books and a decrease in learn units-to-criterion for the acquisition of novel sight words (Buttigieg, 2015). This cusp can be established through several procedures; stimulus-stimulus pairing procedure (Nuzzolo et al., 2002), observational conditioning procedure (Singer-Dudek, Oblak, & Greer, 2011), and textual operant discrimination training (learn unit instruction) (Buttigieg, 2015).

In an unpublished dissertation, O'Rourke (2006) conducted two experiments to test the effects of an observational conditioning intervention on conditioned reinforcement for math. The participants were between six and eight years old and were selected because math was aversive to them. Experiment 1 was a replication of Greer and Singer-Dudek’s (2008) study to determine if previously non-preferred tokens would acquire reinforcing properties through an observational intervention for learning and performance tasks. Results of Experiment 1 indicated that tokens did in fact function as conditioned reinforcement for performance and learning due to the observational intervention.

In her second experiment, O'Rourke, (2006) tested the observational conditioning-by-denial intervention on academic performance in math. Four students from second and third grade
were selected as participants in this ABABA reversal design. A delayed multiple baseline across participants for the three learning tasks tested for changes in reinforcing properties for acquiring new math repertoires. The pre- and post-intervention assessment measured learn units-to-criterion to assess the rate of acquisition of math skills. Results showed that math tasks (e.g. addition and subtraction, time, and word problem worksheets) functioned as reinforcers for performance and acquisition as a result of the observational intervention. According to O'Rourke, (2006), the intervention did “establish a shift in preference of activities, changing a previously non-preferred activity into a preferred activity,” (O'Rourke, 2006, p. 80).

In a recent unpublished doctoral dissertation, Lee (2016) extended observational conditioning to writing. She conducted two experiments to determine whether an observational condition would be effective in establishing conditioned reinforcement for opportunities to emit writing responses. Lee measured two dimensions of conditioned reinforcement that she referred to as direct and indirect reinforcement. Indirect reinforcement referred to the use of math as a reinforcement operation for another behavior unrelated to writing and direct reinforcement referred to the number of intervals participants responded to a writing prompt. There were three dependent variables: the number of letters of the alphabet written when a green ticket (signaling writing as a consequence) and a red ticket (signaling free-play as a consequence) was given following a one-minute session (indirect reinforcement measure), the participants’ rate of learning novel chemical element symbols when writing prompts were delivered as a consequence for correct responding (indirect reinforcement measure), and the number of 5-s whole intervals in 5-min probe sessions participants wrote in response to a writing prompt (direct reinforcement measure). The social observational condition consisted of the participant being denied access to functional writing tasks while observing two confederates completing a writing task. The writing
task required both confederates to interact with one another. The participant was seated between the confederates and a divider was placed so that the participant could not view the confederates. All of the students were given three performance task directions followed by the removal of the dividers. The confederates were provided reinforcement for the completion of the task in the form of taking turns writing a story. The participant was denied access to the writing task as a consequence for the completion of the performance task. Data were collected on the participants’ number of mands emitted for writing. Intervention sessions continued until there was a steady state of mands across sessions or when the extinction of performance responses occurred. Post intervention data demonstrated an increase in direct and indirect reinforcement value of writing. Lee’s (2016) intervention was based off of the observational procedure in Greer and Dudek (2008), in which being denied the target stimuli resulted in the establishment of conditioned reinforcement for said stimuli.

**Rationale and Educational Significance**

Claessens and Engel suggest that, “early mathematics knowledge and skills are the most important predictors not only for later math achievement, but also for achievement in other content areas” (2013, p. 1). Despite the math education crisis in America, procedures for establishing an interest in math, or math readiness, in pre-kindergarten students have not been established nor has an empirical definition for conditioned reinforcement for math been produced. Establishing conditioned reinforcement has been found effective in inducing cusps and increasing students’ rate of learning. The purpose of this study was to extend these results with math using the proposed sequence of conditioning detailed in Buttigieg’s (2015) study; learn units, stimulus-stimulus pairing, and observation.
Research Questions

The research questions for Experiment I were as follows: (1) Can conditioned reinforcement for math and preference for math over Play-Doh® be established in students as young as four years old?; and (2) Do the procedures for establishing conditioned reinforcement for math differ for students?
CHAPTER II

EXPERIMENT I

Methods

Participants

Four preschool students, between four- and five-years old, participated in this experiment. These students were selected from a Comprehensive Application of Behavior Analysis to Schooling Accelerated Independent Learner (CABAS® AIL) Pre-Kindergarten classroom that contained 16 students, one teacher and two teacher’s assistants. These students were chosen to participate in this study because they did not have conditioned reinforcement for math in repertoire. Participants I, K, T, and L functioned at speaker and listener levels of verbal behavior. At the onset of the study all four participants could count intraverbally to 20 and had the following prerequisite cusps and capabilities in repertoire: teacher presence resulted in instructional control, adult praise functioned as reinforcement for learning and performance, generalized match to sample, conditioned reinforcement for observing two-dimensional stimuli, and conditioned reinforcement for the observation of books. Tables 2 and 3 contain a full description of each participant.
The Brigance Inventory of Early Development -II is a norm referenced test for students between Pre-Kindergarten and First grade. Students physical, language, and cognitive skills are assessed and compared to that of other students. Scores less than 70 are considered very poor, 70-79 are considered poor, 80-89 are below average, 90-110 are average, 111-120 are above average, 121-130 are considered superior and scores over 130 are considered very superior.

Note. The Brigance Inventory of Early Development -II is a norm referenced test for students between Pre-Kindergarten and First grade. Students physical, language, and cognitive skills are assessed and compared to that of other students. Scores less than 70 are considered very poor, 70-79 are considered poor, 80-89 are below average, 90-110 are average, 111-120 are above average, 121-130 are considered superior and scores over 130 are considered very superior.  

^aAge at the onset of the study

### Table 2

**Description of Participants**

<table>
<thead>
<tr>
<th>Participant</th>
<th>I</th>
<th>K</th>
<th>T</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age^a</td>
<td>4.9 years</td>
<td>4.6 years</td>
<td>4.6 years</td>
<td>4.4 years</td>
</tr>
<tr>
<td>Gender</td>
<td>Male</td>
<td>Female</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>Grade level</td>
<td>Pre-K</td>
<td>Pre-K</td>
<td>Pre-K</td>
<td>Pre-K</td>
</tr>
<tr>
<td>Free/Reduced Lunch</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Tuition</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Relevant Cusps and Capabilities</td>
<td>Listener, Listener Literacy, Teacher Presence, Conditioned R+ for 2D, Generalized Matching (2D/3D)</td>
<td>Listener Literacy, Teacher Presence, Conditioned R+ for 2D, Generalized Matching (2D/3D)</td>
<td>Listener Literacy, Teacher Presence, Conditioned R+ for 2D, Generalized Matching (2D/3D)</td>
<td>Listener Literacy, Teacher Presence, Conditioned R+ for 2D, Generalized Matching (2D/3D)</td>
</tr>
<tr>
<td>Level of VB</td>
<td>Listener/Speaker</td>
<td>Listener/Speaker</td>
<td>Listener/Speaker</td>
<td>Listener/Speaker</td>
</tr>
</tbody>
</table>
Table 3

Summary of Pre-Reader Cusps and Capabilities for Participants

<table>
<thead>
<tr>
<th>Pre-Reader Capabilities</th>
<th>Participant I</th>
<th>Participant K</th>
<th>Participant T</th>
<th>Participant L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book stimuli conditioned reinforcement for observing</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Self-talk in fantasy play</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Say-do in speaker-as-own listener function</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Bidirectional Naming (BiN)</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Unilateral Naming (UiN)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Transformation of establishing operations</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Independent mands</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Echoic-to-tact</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Echoic to mand</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Auditory matching</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Listener literacy</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Generalized imitation</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Match 2D and 3D objects</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Conditioned reinforcement for 2D print stimuli</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Conditioned reinforcement for 3D objects/visual stimuli on desktop</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Setting

Experimental sessions took place in the participants’ classroom at a horseshoe table with the necessary materials detailed below. All other students in the classroom were participating in small group reading and language instruction during probe and intervention sessions. See Figure 1 for the direct reinforcement probe setting. During observational conditioning sessions, the participant sat next to the confederate with a foam divider between them such that they could not see each other, but could see the experimenter delivering math worksheets to the confederate.
Figure 1. Direct Reinforcement Measure Setting. During direct reinforcement, pre- and post-intervention probes, participants were given Play-Doh® and math worksheets (see above).
Materials

Several materials were used in this study. Materials used were based on the math skills each participant had in repertoire at the onset of the study. Three 85.05g Play-Doh® containers and three Play-Doh® tools were used during the functional analysis and direct reinforcement sessions. Play-Doh® was selected because all participants demonstrated a preference for Play-Doh® as evidenced by their frequency of selecting Play-Doh® from the menu of the classroom token economy.

Functional analysis of reinforcement value of math. Participants I, K and L’s performance task material was a laminated 21.59 cm by 27.94 cm Microsoft Word® document with an 8 by 10 cell table. Each cell of the table contained a letter corresponding to one of the following previously mastered letters in a randomized fashion: m, a, s, t, d. Participant T’s performance task was a laminated 21.59 cm by 27.94 cm with a 6 by 7 table that contained one previously mastered stimulus in each cell. Multiple versions of each performance task were created to account for practice effects. See Figure 2 for an example of each participants’ performance task. Also, during functional analyses the participants earned one match-quantity-to-quantity or quantity-to-Arabic number math problem for each correct response to the performance task. Each problem was created using Microsoft Word® and printed on a 10.16 cm by 12.7 cm piece of paper. See Figure 3 for an example of each problem.
Figure 2. Performance Tasks for Functional Analyses. Above is the page of tacts used for Participant T’s performance task. Below Participant T’s performance task is the page of letter sounds used for Participant I, K, and L’s performance task. Several versions of these sheets were made to control for practice effects.
Figure 3. Individual Math Problems for Functional Analyses. Above is an example of the math problems given as a response to correct responses to the performance task during indirect pre- and post- math conditioning probes. Participant I, K, and L received the match Arabic number-to-quantity sheet, while Participant T received the match quantity-to-quantity sheet. Various versions of these sheets were created using Arabic numbers 1-10 and quantities 1-10.
Direct reinforcement measure. Two of the aforementioned 85.05 g Play-Doh® containers were used in direct reinforcement probes. Additionally, a stack of 20 match quantity to Arabic number worksheets were used during these measures for Participant I, K, and L. Participant T’s stack consisted of match quantity-to-quantity worksheets. Each worksheet in the stack was different. See Figure 4 for examples of the worksheets used for each participant.
Figure 4. Direct Reinforcement Probe Math Materials. Match quantity-to-quantity and number-to-quantity worksheets such as the above were used during direct reinforcement probes for Participant I, K, and L. Participant T’s was only given match quantity-to-quantity worksheets.
Learn Units. A computer was used to display Microsoft PowerPoint® slides during learn unit instruction. Each objective contained 20 PowerPoint® slides with one target number on each slide. See Table 3 for the list of numbers in each objective for all four participants.

Table 4

Learn Unit Instruction Objectives

<table>
<thead>
<tr>
<th>Objective</th>
<th>Participant I</th>
<th>Participant K</th>
<th>Participant T</th>
<th>Participant L</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11, 12, 13, 14, 15</td>
<td>11, 12, 13, 14, 15</td>
<td>1, 2, 3, 4, 5</td>
<td>10, 11, 12, 13, 14</td>
</tr>
<tr>
<td>2</td>
<td>20, 30, 40, 50, 60</td>
<td>20, 30, 40, 50, 60</td>
<td>6, 7, 8, 9</td>
<td>15, 16, 17, 18, 19</td>
</tr>
<tr>
<td>3</td>
<td>76, 54, 92, 33, 28</td>
<td>76, 54, 92, 33, 28</td>
<td>12, 13, 15, 17</td>
<td>20, 30, 40, 50, 60</td>
</tr>
</tbody>
</table>

Note: Equal opportunities to respond to each operant were included in each 20 learn unit session.

Stimulus-stimulus pairing and observational conditioning-by-denial. Multiple 21.59 cm by 27.94 cm “More or Less” worksheets were used during stimulus-stimulus pairing and observational conditioning-by-denial sessions. See Figure 5 for an example of these worksheets. Uppercase and lowercase letter flashcards and privacy dividers were also used for observational conditioning.
Figure 5. Stimulus-Stimulus Pairing and Observational Conditioning-by-denial Worksheet. Multiple exemplars of the above worksheets were used to conduct stimulus-stimulus pairing trials and observational conditioning-by-denial trials in an attempt to establish conditioned reinforcement for math.
Dependent Variable

The dependent variable was the reinforcement value of math; this was measured both directly and indirectly. The reinforcing value of math was measured indirectly using a functional analysis comparing the participants’ rate of correct and incorrect responding to a performance task when alternating phases of either a math problem or a piece of Play-Doh® was delivered as a consequence for each correct response. The direct measure of conditioned reinforcement for math was the number of 5-s intervals in a 5-min observation the participant emitted math observing responses or engaged in math when presented with both Play-Doh® and math materials. A math observing response consisted of engaging in math related activities including looking at and manipulating the worksheet in a mathematical manner e.g., counting, pointing, picking up the marker and writing a number or drawing a line within 3 s.

Individualized Reinforcement Intervention – Independent Variable

The independent variable was an individualized reinforcement intervention. This intervention consisted of three conditioning procedures: learn units, stimulus-stimulus pairing, and observational conditioning. The procedures were implemented in the above sequence until conditioned reinforcement for math was established. Learn unit instruction consisted of participants receiving reinforcement contingent upon correct responses to math objectives, the stimulus-stimulus pairing procedure consisted of a 5- and 10-s pair test, and the observational intervention consisted of the participant being denied access to math stimuli while observing a peer gaining access to said stimuli.

Data Collection

Functional analysis or indirect measures were collected by counting the number of letter sounds or tacts the participant emitted and calculated as the number of correct and incorrect
responses per min. These data were recorded using pen and paper. The mean number of correct responses per phase was calculated by totaling the number of correct responses per phase and dividing by the number of sessions in that phase.

Direct reinforcement measures were collected using the plus (+) and minus (-) system. A plus was recorded for each 5-s whole interval the participant emitted a math observing response, while a minus (-) was recorded for each 5-s interval the participant played with Play-Doh®. These data were displayed visually out of 60 intervals following each session.

Learn unit data were collected using the “plus” and “minus” system as well and reported as the number of correct responses to learn units per session. Stimulus-stimulus pairing data were collected using the plus (+) and minus (-) system and reported as the number of correct responses to whole interval 5-s or 10-s pair-test trials. Observational conditioning data were collected using tallies and reported as the number of vocal and nonvocal attempts to access math. Additionally, observational conditioning data were collected using the plus (+) and minus (-) system and reported as the number of correct responses to trials.

**Design**

The following two designs were used in this study: a counterbalanced pre- and post-intervention ABAB/BABA functional analysis and a delayed multiple probe across dyads design.

**Functional analysis.** A functional analysis was used to determine the indirect reinforcement value of math. The order of each participants’ phases was determined by coin flip. An ABAB design was used with Participant I and Participant L and a BABA design was used with Participants K and T. The A phase represented Play-Doh® reinforcement (a conditioned reinforcer) and the B phase represented math problems. Following the
establishment of conditioned reinforcement for math, post-intervention functional analyses were conducted in the same manner.

**Delayed multiple probe across dyads.** A delayed multiple probe across dyads design was used for direct reinforcement assessments. The first dyad, Participant I and Participant K, entered the intervention following initial direct conditioned reinforcement probes. Following mastery criterion on learn unit instruction, post-direct conditioned reinforcement probes were conducted for the first dyad and initial-probes of direct reinforcement for math were conducted for the second dyad; Participant T and L. Once the dyad entered the experiment, the sequence consisted of learn unit instruction, stimulus-stimulus pairing, and observational conditioning until the participant met criterion on the direct reinforcement assessment. See Figure 6 for a visual representation of the sequence of the experiment.
Figure 6. Sequence of the experiment. Dyads entered the experiment following post-probes of direct conditioned reinforcement for math for the preceding dyad. Once entered the study, experimenters followed the sequence above. The design follows the multiple probe logic as detailed above.
Procedures

**Pre- and post-conditioned reinforcement functional analyses.** Before entering the study, a pre-conditioned reinforcement for math functional analysis was conducted using 1 min sessions. Students were presented with a performance task sheet either textually responding to letter sounds or emitting mastered tacts (see Figure 2) and were given the respective consequence for each phase. During the math phase, participants were given a single math problem for each correct response, while in the Play-Doh® phase participants were given a piece of Play-Doh® for each correct response (see Figure 3). Play-Doh® was used as it was a conditioned reinforcer for all participants. The antecedent delivered to the participants was, “I will give you one of these (either Play-Doh® or math) for every answer you say, you have one min”. Incorrect responses were consequated after the 1-min session in which the experimenter emitted the correct response and gave the student the opportunity to independently emit the correct response. Following each Play-Doh® phase participants were given 5-min to play with Play-Doh® while in the math phase the participants were required to complete the math problems they earned. Sessions for each phase continued until steady state was achieved. Following the establishment of conditioned reinforcement for math, post-conditioned reinforcement for math functional analyses were conducted in the same manner.

**Pre- and post-direct conditioned reinforcement for math probes.** Prior to entering the intervention direct probes were conducted. Experimenter placed a stack of 20 math worksheets, a marker, two Play-Doh® containers, and three Play-Doh® tools on the table. The experimenter called over the participant and delivered the antecedent “These are for you, you have 5-min.” while moving her arms in a circular motion around all the materials. The participants were given 5-min with the material while the experimenter collected the data using a
preconstructed data sheet. Experimenters did not attend to the participant during this time. If the student sought attention experimenters stated, “We can talk in a little bit.” Five direct probes were conducted for each participant. Criterion for direct math probes was set at 144/300 cumulative intervals. Following mastery criterion on direct math probes, post- conditioned reinforcement for math functional analyses were conducted.

**Learn unit instruction.** Operant conditioning was conducted with learn unit instruction (Albers & Greer, 1991). The fidelity of this instruction is evidenced by the experimenters’ number of errorless Teacher Performance Rate Accuracy, or TPRAs (Ross, Singer-Dudek, Greer, 2005). The experimenter was trained in delivering learn units, thus controlling for the quality of instruction. Objectives were created based on the textual responses to numbers the participants did not have in repertoire. See Table 3 for the objectives used for each participant. Each session of conditioning consisted of 20 learn units with equal opportunity for each operant. Learn unit instruction was as follows: the experimenter presented the target number to the participant, the participant emitted a response, and a consequence was delivered. If the student emitted a correct response, vocal praise and playful physical (tickles, high-fives, etc.) contact was delivered. It is important to note that behavior-specific praise was used, e.g., “Wow you’re such a mathematician,” “Awesome job learning your numbers,” “You’re doing a great job with your math work.” If the student emitted an incorrect response, the experimenter emitted the correct response and re-presented the antecedent in order to provide the participant the opportunity to independently emit the correct response. Criterion was set at 90% accuracy (18/20) across two consecutive sessions or 100% (20/20) accuracy in one session. Instructional decisions were made using the Decision Tree Protocol (Greer, 2001) and instructional tactics were used as necessary, such as: increase in learn units for operants of difficulty, and multiple
exemplar instruction across listener and speaker response topographies (Greer, Stolfi, & Pistoljevic, 2007). Once criterion was met on all three objectives, post-probes of direct conditioned reinforcement for math were conducted.

**Stimulus-stimulus pairing.** If learn unit instruction was not successful in establishing conditioned reinforcement for math then the stimulus-stimulus pairing procedure was used. The stimulus-stimulus pairing procedure consisted of a block of 20 pair and test trials, in which the test trial did not occur unless there was a successful pair trial. The pair trial was 5 s, and consisted of the participant observing or completing the More/Less worksheet while the experimenter provided math-specific praise. In order to continue to the 5-s test trial, the participant was required to view/complete the math materials for the entirety of the 5-s interval, if this did not occur another pair was conducted. During the test trials, a correct response was defined as the participant viewing/completing the math materials for 5-s without the attention of the experimenter. If the student did not view the math material for 5-s, a pair trial was conducted. Data were recorded on test trials. Each session consisted of 20 test trials. Criterion was set at 18/20 or 90% accuracy across two consecutive sessions. After achieving criterion on the 5-s pair and test trials a 10-s pair test was conducted. Once criterion for both the 5-s and 10-s pair-test was met, a post-probe of direct conditioned reinforcement for math was conducted. If conditioned reinforcement for math was not established following the 5-s and 10-s pair-test, a 15-s pair-test was conducted before continuing observational conditioning-by-denial.

**Observational conditioning-by-denial.** During observational conditioning, had it occurred, the participant sat next to the confederate with a foam divider separating the two, such that the participant could see the experimenter delivering math worksheets to the confederate contingent upon correct responses. The participant and the confederate were each given their
own field of three letter flashcards. The experimenter emitted the antecedent, “match__ with ___” to both the participant and confederate. A correct response was defined as the participant placing the card on top of the matching card. The confederate was given 5 s to complete math worksheets contingent upon correct responding, while the participant was denied access to the worksheets as well as all other forms of reinforcement. The partition was removed when the confederate was doing math so that the participant could observe the confederate. Data were collected on the number of mands (e.g., “I want that!” “Gimmie!” or “What about me!”) and nonvocal attempts to obtain the math worksheets as well as the number of correct responses in each session. Each session consisted of 10 match trials for a total of five sessions.

**Interobserver agreement (IOA)**

Interobserver agreement (IOA) for measures were conducted using point-to-point interobserver agreement by dividing the number of agreements by the number of agreements and disagreements then multiplying by 100 (Cooper, Heron, & Heward, 2007). For direct measures, there was IOA for 100% of sessions with 100% agreement. For indirect measures, there was IOA for 49% of sessions with 100% agreement. There was 100% IOA for 53% of learn unit instruction sessions and 96% IOA for 60% of stimulus-stimulus pairing sessions.

**Results**

**Functional Analysis Results**

The results of the functional analysis demonstrate that it is indeed possible to enhance the reinforcement value of math for children as young as 4-years. Figure 7 shows the functional analysis of reinforcement value for math for all four participants.

**Participants I and K.** There was a clear difference in level of responding in Participant I’s third and fourth phase. Extinction effects became apparent in her second math phase.
Following the establishment of conditioned reinforcement for math, her level of responding was similar across Play-Doh® and math phases. The results of a two month follow up probe demonstrated the maintenance of intervention effects; Participant I emitted 100 and 99 correct textual responses in Play-Doh® and math phases respectively.

Extinction effects were apparent in each of Participant K’s math phases prior to the establishment of conditioned reinforcement for math. Following the establishment of conditioned reinforcement for math Participant K’s number of correct responses to a performance task were an overall descending trend. In a two month follow up probe Participant K emitted 40 correct responses during the math phase and 43 correct responses during the Play-Doh® phase indicating the maintenance of treatment effects.

**Participants T and L.** The second dyad consisted of Participant T and Participant L. There was a clear difference in the level of responding during Play-Doh® and math phases prior to the establishment of conditioned reinforcement for math. Both participants’ responding went into extinction in their second math phase. Following the establishment of conditioned reinforcement for math both participants level of responding was similar between math and Play-Doh® phases. Participant T’s two week follow up probe indicates the maintenance of conditioned reinforcement for math with 60 correct responses in the math phase and 46 correct responses in the Play-Doh® phase. In the two-week follow up probe, Participant L emitted 100 correct responses in both the math and Play-Doh® phase. Figure 7 shows the rate of correct responding to performance tasks for all four participants.
Figure 7. Functional Analyses of Reinforcement Value of Math. This figure shows the number of correct responses to a performance task when phases of either Play-Doh® or math were delivered as a consequence prior to and following the establishment of conditioned reinforcement for math. Participant I and K’s functional analyses were conducted concurrently, Participant T and L’s were conducted concurrently following the establishment of conditioned reinforcement for math for the previous dyad. P and M were used as abbreviations for Play-Doh® and math phases.
Direct Reinforcement Results

Not only do the direct reinforcement results demonstrate that it is possible to establish conditioned reinforcement for math for children as young as 4-years old, but these results also demonstrate that different students require different procedures to enhance the reinforcement value of math. Figure 8 shows the number of 5-s whole intervals out of 60 that each participant emitted math observing responses prior to and following each conditioned reinforcement for math procedure. In initial probes of direct reinforcement for math, Participant I emitted 0, 5, 0, 0, and 0 correct responses in each respective probe session. Following learn unit instruction she emitted 37, 29, 60, 60, and 39 correct responses. During two-month follow up probes she emitted 52 correct responses. Participant K emitted 0 correct responses across all five initial probes of direct reinforcement for math. Following learn unit instruction, he emitted 34, 27, 34, 60 and 45 correct responses. In a two-month follow-up probe Participant K emitted 60 out of 60 correct responses.

Prior to learn unit instruction, Participant T emitted 26, 0, 0, 0, and 0 correct responses. Following learn unit instruction he emitted 0 correct responses as well, however, following the stimulus-stimulus pairing procedure Participant T emitted 27, 38, 23, 20, and 27 correct responses in each respective probe session. In a two week follow up probe session he emitted 60 out of 60 correct responses. Participant L emitted 0 correct responses prior to and following learn unit instruction. Following the stimulus-stimulus pairing procedure, Participant L emitted 19, 39, 28, 60, and 52 correct responses during probe sessions. Similar to Participant T, Participant L emitted 60 out of 60 correct responses during two-week follow-up probes.
Figure 8. Direct Reinforcement for Math Pre- and Post-Probes. This figure shows the number of 5-sintervals each participant emitted math observing responses or played with Play-Doh® before and following each conditioning procedure.
**Cumulative direct reinforcement results.** The cumulative number of 5-s intervals each participant emitted math observing responses prior to and following each conditioning intervention is depicted in Figure 9. Overall, Participant I emitted 5 correct responses prior to the establishment of conditioned reinforcement for math and 225 correct responses following learn unit instruction. Participant K emitted 0 correct responses during pre-probes of direct reinforcement for math and 200 correct responses in post-probes of direct conditioned reinforcement for math. Participant T emitted 25 correct responses prior to learn unit instruction, 0 correct responses following learn unit instruction and 165 correct responses following the stimulus-stimulus pairing procedure. Participant L emitted 0 correct responses both in the pre-conditioned reinforcement for math probe and following learn unit instruction, but emitted 198 correct responses following the stimulus-stimulus pairing procedure.
Figure 9. Direct Reinforcement for Math Pre- and Post-Probes. This figure shows the total number of 5-sintervals each participant emitted math observing responses or played with Play-Doh® before conditioned reinforcement for math was established and following the establishment of conditioned reinforcement for math.
Individualized Reinforcement Intervention Results

Learn unit instruction. Figure 10 shows the intervention data for Participant I, K, T, and L. During learn unit instruction, Participant I required 160, 100, and 40 learn units to meet criterion for each respective objective while Participant K required 300, 40, and 40 learn units to achieve mastery criterion for each respective objective. Participant T required 180 learn unit to meet criterion (LUC) for his first math objective and 470 LUC on his second math objective. He required 240 LUC for his final math objective. Participant L required 60, 80, and 40 LUC on her respective math objectives during learn unit instruction. See Figure 11 for a summary of these results.
Figure 10. Learn Unit Instruction. This figure shows the number of correct responses to learn units for Participant I, K, T, and L. Increased opportunity to respond (I.O.), interspersal of known items, and multiple exemplar instruction (MEI) across listener and speaker responses were used as instructional tactics.
Figure 11. Summary of Learn Units to Criterion. This figure shows the number of learn units required for each participant to meet criterion on each objective during learn unit instruction. See Table 3 for a list of each participants’ objectives.
**Stimulus-stimulus pairing procedure.** Both Participant T and Participant L required the stimulus-stimulus pairing procedure to establish conditioned reinforcement for math. Both participants required 60 train-test trials to meet criterion in the 5-s pair-test phase and 40 learn units-to-criterion during the 10-s pair-test phase. See Figure 12 for a graphical representation.
Figure 12. Stimulus-Stimulus Pairing Intervention Participant T and Participant L. This figure shows the number of correct responses to learn units for Participants T and L during 5-s and 10-s pair/test trials.
Discussion and Rationale for Experiment II

Results show that conditioned reinforcement for math can indeed be established for students as young as 4 years old. Additionally, different participants required different conditioning procedures to establish reinforcement for math; learn unit instruction was effective in establishing conditioned reinforcement for math for Participants I and K while a stimulus-stimulus pairing was necessary for Participants T and L. Should Participants T and L not have acquired conditioned reinforcement for math following the stimulus-stimulus pairing, an observational conditioning-by-denial procedure would have been conducted. Across all four participants the Crayola® marker included in direct reinforcement probes, to complete the math worksheets, were instead used as rolling pins for the Play-Doh®. Following conditioned reinforcement for math, all participants used the marker for its intended function.

Participant I’s functional analysis data were consistent with her direct reinforcement data in that following conditioned reinforcement for math, she emitted similar correct responses to letter sounds per min in Play-Doh® and math phases indicating math functioned as a prosthetic reinforcer. Anecdotally, Participant I requested math work to do at home with her family following conditioned reinforcement for math. Participant K’s functional analysis data demonstrated an overall descending trend following conditioned reinforcement for math. The performance task became aversive to him as evidenced by his decrease in correct responding across Play-Doh® and math phases; however, extinction effects were not shown in post-conditioned reinforcement for math results as it was in the pre-conditioned reinforcement for math functional analysis results. It is important to note that Participant K requested to do small group math instruction as opposed to the performance task on three occasions during post-conditioned reinforcement for math probes. While pre- and post- conditioned reinforcement for
math rate of learning measures were not conducted, the number of learn units necessary to achieve criterion (LUC) for math objectives decreased across each objective for both Participants I and K. Anecdotally both Participants I and K began to look for math in the environment after math was conditioned. For example, Participant I counted the spots on each half of the back of a lady bug in a book and stated, “Look three and three makes six. He’s got six spots!” Like Zrinzo and Greer’s (2013) intervention, effects remained more than 6 weeks after the experiment concluded. Six months after the establishment of conditioned reinforcement, Participant I’s parents report that she “has expressed a new love for math.”

Both Participants T and L required the stimulus-stimulus pairing procedure to establish conditioned reinforcement for math. Extinction effects were evident in Participant T’s pre-conditioned reinforcement for math functional analysis. Similar to Participant K, during these measures Participant T did not emit incorrect responses, but simply insisted he couldn’t see the pictures and spent the majority of the 1-min session flipping the page back and forth and pretending to zoom in to the pictures in an attempt to avoid receiving additional math problems. As a rule-governed student, Participant T emitted 26 correct response in his first direct probe session. In his remaining four sessions, he told the experimenter, “I’m going to do Play-Doh®. Okay?” The experimenter reminded him that he could do, “whatever he wants” with his 5 min.

Reactivity may have been in effect for Participant L as well as an establishing operation to complete the entire page during her first math phase in her functional analysis. During Play-Doh® phases, she simply completed the task and retrieved her Play-Doh®. During math phases, she looked toward the experimenter for attention stating, “look I finished the whole page.” Additionally, when she skipped a line or decreased her rate of responding she looked to see if there were any changes on the experimenter’s face. The responses in her first math phase may be
a result of the instructional control of the teacher in that work was given to her and so Participant L completed it. This occurred until her second math phase in which she asked the experimenter, “Do I have to say all the letters?” to which the experimenter responded, “it’s up to you” followed by the performance task antecedent. Of all the participants, Participant L required the fewest learn units to achieve mastery of each objective during learn unit instruction. The low number of exposures to math instruction may have had an impact on learn unit instruction not succeeding in establishing conditioned reinforcement for math. While Participants T and L’s learn units to criterion during learn unit instruction decreased in each successive phase, learn unit instruction was not effective in establishing conditioned reinforcement for math for these participants.

A potential limitation of this experiment is that learn unit instruction sets were not counterbalanced. However, they were based on the Arabic numbers students did not have in repertoire and were sequenced such that the students abstracted the rules for textual responding to numbers 1-100 (i.e. stating the tens column followed by the single digit). Additionally, a low number of sessions with IOA during indirect and intervention sessions as well as a lack of two-month follow-up probes for Participant T and L were a possible limitation in this study. In terms of the indirect measures (or functional analysis) of reinforcement value, it took one phase of each consequence for students to contact the relationship between their responding and the amount of Play-Doh® or math given, thus creating false positives for the reinforcement value of math during each participants’ first math phase. It can be determined that these were false positives as anecdotally participants complained during these phases stating, “I don’t want all of these,” “Why can’t I earn Play-Doh®?” “I don’t want to do this.” and “Why do I have to earn that?”

The results of the study demonstrate that conditioned reinforcement can indeed be established in students as young as four years old and the reinforcement procedure to achieve
this is different for different students. These results are consistent with Buttigieg’s (2015) study which examined the effects of the establishment of conditioned reinforcement for the observation of books, using individualized reinforcement interventions, on students’ rate of acquisition of novel textual responses. While the current study was not a replication of Buttigieg’s study, both studies used individualized reinforcement procedures. The participants in both studies required different procedures to establish conditioned reinforcement. Future research should examine the relation between the participants and which conditioning procedure was effective for said student. The purpose of Experiment II was to determine if conditioned reinforcement for math would result in an accelerated rate of learning, just as in previous studies on conditioned reinforcement (Buttigieg, 2015; Delgado, Greer, Speckman, & Goswami, 2009; Du, Broto, & Greer, 2015; Greer & Han, 2015; Greer, Pistoljevic, Cahill, & Du, 2011; Maffei, Singer-Dudek, & Keohane, 2014; Tsai & Greer, 2006) using the individualized reinforcement procedures from Experiment I.
CHAPTER III
EXPERIMENT II

Experiment II was designed to test the effects of conditioned reinforcement for math on rate of learning. In Experiment I, the dependent variable was the conditioning of reinforcement and the individualized conditioning procedures leading to that were the independent variable. Embedded within Experiment II was a replication of Experiment I as the individualized reinforcement intervention was the independent variable and rate of learning as the dependent variable. As a result, all procedures relating to conditioning were the same. The differences include: the dependent variable, the math objectives, and the participants.

Method

Participants

Six preschool students were selected from the same setting as in Experiment I. Participants L and M functioned at early reader/writer levels of verbal behavior, while Participants C, N, G, and J functioned at speaker and listener levels of verbal behavior. The following participants were classified as a preschooler with a disability: L, N, C, and J. See Tables 5 and 6 for a full description of each participant.
Table 5

*Description of Participants*

<table>
<thead>
<tr>
<th>Participant</th>
<th>L</th>
<th>N</th>
<th>C</th>
<th>M</th>
<th>J</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age*</td>
<td>5 years</td>
<td>5.3 years</td>
<td>4.8 years</td>
<td>4.7 years</td>
<td>4.4</td>
<td>5.3</td>
</tr>
<tr>
<td>Gender</td>
<td>Male</td>
<td>Male</td>
<td>Male</td>
<td>Female</td>
<td>Male</td>
<td>Female</td>
</tr>
<tr>
<td>Preschooler w/disability</td>
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<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Free/Reduced Lunch</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Tuition</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>ELL</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Level of VB</td>
<td>Listener / Speaker</td>
<td>Listener/ Speaker Early Reader/ Writer</td>
<td>Listener/ Speaker Early Reader/ Writer</td>
<td>Listener/ Speaker</td>
<td>Listener/ Speaker</td>
<td>Listener/ Speaker</td>
</tr>
</tbody>
</table>

*Note.* The Brigance Inventory of Early Development -II is a norm referenced test for students between Pre-Kindergarten and First grade. Students physical, language, and cognitive skills are assessed and compared to that of other students. Scores less than 70 are considered very poor, 70-79 are considered poor, 80-89 are below average, 90-110 are average, 111-120 are above average, 121-130 are considered superior and scores over 130 are considered very superior.

*Age at the onset of the study*
Materials

The direct reinforcement probe, stimulus-stimulus pairing, and observational conditioning materials were the same as in Experiment I. The only differences in materials were those used for learn unit instruction and rate of learning measures. The pilot version of the Weber and Greer (in progress, 2017) Multiple Exemplar Functional Math Curriculum (MEF-Math) was used for these measures. Aligned to Common Core Standards, this curriculum contains units that include four math objectives rotated across match, point to, tact, and

Table 6

Summary of Pre-Reader Cusps and Capabilities for Participants

<table>
<thead>
<tr>
<th>Cusp or Capability</th>
<th>Participant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L</td>
</tr>
<tr>
<td>Book stimuli conditioned reinforcement for observing</td>
<td>X</td>
</tr>
<tr>
<td>Self-talk in fantasy play</td>
<td>X</td>
</tr>
<tr>
<td>Say-do in speaker-as-own listener function</td>
<td>X</td>
</tr>
<tr>
<td>Bidirectional Naming (BiN)</td>
<td>X</td>
</tr>
<tr>
<td>Unilateral Naming (UiN)</td>
<td>X</td>
</tr>
<tr>
<td>Transformation of establishing operations</td>
<td>X</td>
</tr>
<tr>
<td>Independent mands</td>
<td>X</td>
</tr>
<tr>
<td>Echoic-to-tact</td>
<td>X</td>
</tr>
<tr>
<td>Echoic to mand</td>
<td>X</td>
</tr>
<tr>
<td>Auditory matching</td>
<td>X</td>
</tr>
<tr>
<td>Listener literacy</td>
<td>X</td>
</tr>
<tr>
<td>Generalized imitation</td>
<td>X</td>
</tr>
<tr>
<td>Match 2D and 3D objects</td>
<td>X</td>
</tr>
<tr>
<td>Conditioned reinforcement for 2D print stimuli</td>
<td>X</td>
</tr>
<tr>
<td>Conditioned reinforcement for 3D objects/visual stimuli</td>
<td>X</td>
</tr>
</tbody>
</table>
intraverbal response topographies. This rotation of response topographies is referred to as multiple exemplar instruction, or MEI (Greer, Stolfi, Chavez-Brown, & Rivera-Valdes, 2005; Greer, Stolfi, Pistoljevic, 2007; Greer & Yuan, 2008). The curriculum includes a teacher script, student answer book, and data sheet for each unit. The teacher script outlined the instructional demonstration learn units for each objective in the unit as well as the antecedent and correct procedure for each learn unit. The student answer book contained the printed material necessary for the participant to respond to each learn unit. Multiple versions of each answer booklet were created to control for memorization. See Appendix A, B, and C for a sample of these materials. See Table 7 for a list of each participants’ units and Table 8 for a list of the objectives in each unit.

Table 7

*Pre- and Post-Intervention Math Units*

<table>
<thead>
<tr>
<th></th>
<th>Pre-Intervention Units</th>
<th>Post-Intervention Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant L</td>
<td>Unit 7, 8, 9AB, and 10</td>
<td>Unit 14, 15, 16, 17</td>
</tr>
<tr>
<td>Participant N</td>
<td>Unit 3, 4, 5, 6</td>
<td>Unit 7, 8, 9A, 9B</td>
</tr>
<tr>
<td>Participant C</td>
<td>Unit 2, 3, 4, 5,</td>
<td>Unit 6, 7, 8, 9A</td>
</tr>
<tr>
<td>Participant M</td>
<td>Unit 3, 4, 5, 6</td>
<td>Unit 7, 8, 9A, 9B</td>
</tr>
<tr>
<td>Participant J</td>
<td>Unit 3, 4, 5, 6</td>
<td>Unit 7, 8, 9A, 9B</td>
</tr>
<tr>
<td>Participant G</td>
<td>Unit 3, 4, 5, 6</td>
<td>Unit 7, 8, 9A, 9B</td>
</tr>
</tbody>
</table>
Table 8

**Unit Objectives**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Objectives</th>
</tr>
</thead>
</table>
| Unit 2 | Given a written number, student will produce the corresponding quantity of objects (6-10).  
Given a number of shapes in a field of 3 (6-10), student will point to the correct number of shapes (2D or 3D).  
Given a written number in a field of 3 (1-5), student will point to the correct number.  
Given a group of objects and the antecedent, “how many?” student will count the objects and respond with the amount of objects in the group (6-10). |
| Unit 3 | Given a group of 2D shapes or pictures and three exemplars of numbers on a page, student will match the quantity with the written Arabic number (numbers 1-5)  
Given two groups of lines, students will count the lines to get the correct total number (numbers 1-5)  
Given a group of 3D objects and an Arabic number, student will give a peer or teacher the corresponding number of objects (antecedent: give me _____) (numbers 1-5)  
Given 3-5 numbers, student will identify the missing number within a set of numbers (numbers 1-5) |
| Unit 4 | Given a group of objects and three exemplars of numbers on a page, student will match the quantity with the written Arabic number  
Given two groups of lines, students will count the lines to get the correct total number  
Given a group of 3D objects and an Arabic number, student will give a peer or teacher the corresponding number of objects (antecedent: give me _____) (Numbers 6-10)  
Given 3-5 numbers, student will identify the missing number |
| Unit 5 | Given 3D objects (1-5), student will demonstrate one more or one fewer by adding or removing objects.  
Given the frame _____ is more/less than _____ student will place the numbers in the corresponding spaces.  
Given a number and a set of objects (1-5), student will identify one more and one less than the given number.  
Given two groups of objects student will count each group and compare the groups (which has more and which has fewer). |
Unit 6
Given 3D objects (6-10), student will demonstrate one more or one fewer by adding or removing objects.
Given the frame _____ is more/less than _____ student will place the numbers in the corresponding spaces (numbers 6-10).
Given a number and a set of objects (6-10), student will identify one more and one less than the given number.
Given two groups of objects student will count each group and compare the groups (which has more and which has fewer) (numbers 6-10).

Unit 7
Given a number, student will identify the set of 3 numbers that come before or after the target number (numbers 11-20)
Given an addition, subtraction, or equals sign student will tact the symbol
Given an addition or subtraction problem, student will point to the addition sign or subtraction sign within an equation.
Given an addition or subtraction sign, student will produce a model of what addition, subtraction, and equal sign means.

Unit 8
Given a number and the quantity, student will decompose the number into two groups (1-10).
Given a number and a number line, student will take away or add 1 (1-20)
Given a vocal word problem, student will identify the number sentence
Given an equation, student will identify the corresponding number sentence.

Unit 9A
Given a number, student will decompose the number into two parts using manipulatives.
Given an addition equation, with a missing number, student will identify the missing number.
Given a vocal addition word problem with corresponding pictures, student will identify the equation and produce the solution of the problem
Given an equation, student will produce the corresponding picture (by drawing the picture)

Unit 9B
Given a number, student will decompose the number into two parts.
Given a subtraction equation, with a missing number, student will identify the missing number.
Given a vocal subtraction word problem with corresponding pictures, student will identify the equation and produce the solution of the problem
Given a subtraction equation, student will produce the corresponding picture (by drawing the picture)

**Unit 9AB**
- Given a number, student will decompose the number into two parts.
- Given an addition or subtraction equation, with a missing number, student will identify the missing number.
- Given a vocal addition or subtraction word problem with corresponding pictures, student will identify the equation and produce the solution of the problem.
- Given an addition or subtraction equation, student will produce the corresponding picture (by drawing the picture)

**Unit 10**
- Given pictures only, student will produce equation
- Given an equation (with addition, subtraction, or equals sign), student will produce the solution
- Given one-step addition word problems, student will use manipulatives to find the solution of the problem.
- Given one step subtraction word problems, student will use manipulatives to find the solution of the problem.

**Unit 14**
- Given an addition equation, student will identify (select) another equation that results in the same solution.
- Given an addition equation, student will produce an equation that results in the same solution.
- Given an addition equation, student will identify whether the statement is true or false.
- Given an addition equation with an unknown number, student will produce the unknown number to make the equation true

**Unit 15**
- Given a subtraction equation, student will identify (select) another equation that results in the same solution.
- Given a subtraction equation, student will produce an equation that results in the same solution.
- Given a subtraction equation, student will identify whether the statement is true or false.
- Given a subtraction equation with an unknown number, student will produce the unknown number to make the equation true

**Unit 16**
- Given a two-digit number, student will identify the ones and tens place of the number
- Given a two-digit number, student will identify the number of ten(s) and number of one(s) in a number (numbers 11-19)
- Given two numbers, student will identify which number is bigger and which number is smaller
Note: An earlier version of the curriculum, in which unit 9A and 9B were combined, was used for Participant L. The previous version of this unit is referred to as Unit 9AB.

Dependent Variable

The dependent variable was the rate of learning, (i.e., number of learn units required to meet criterion) for four units of the MEF-Math curriculum. Instruction consisted of state of the science teaching as behavior analysis. That is, instruction had all of the components of the learn unit and correction, thus controlling for instructional conditions prior to the intervention and following the intervention. Each unit included 4 objectives rotated across match, point, tact, and intraverbal responses for a total of 20 learn units per session.

Individualized Reinforcement Intervention

The individualized reinforcement intervention was the same as that in Experiment I. The only difference was the materials and objectives used for learn unit instruction. Instead of textually responding to Arabic numbers, the MEF-Math curriculum was used for learn unit instruction. All the components for the learn unit were scripted within the curriculum.
Data Collection

Rate of learning and learn unit instruction measures were collected by totaling the number of learn units delivered to each participant for four units of the MEF-Math curriculum. Learn unit data for each objective in the curriculum were collected using the plus (+) and (-) system and graphed as the number of correct responses to learn units. Direct reinforcement, learn unit instruction, stimulus-stimulus pairing, and observational conditioning-by-denial data were collected in the same manner as in Experiment I.

Design

A multiple probe across dyads design with a nested multiple probe across dyads design was used in this experiment. The dependent variable, rate of learning, was tested using a multiple probe design; the intervention nested within that was also a multiple probe design on testing the effect of the individualized reinforcement procedure on establishing conditioned reinforcement. All participants’ initial probes of direct reinforcement for math were conducted at the onset of the study. Following initial conditioned reinforcement probes, all participants began the MEF-Math curriculum which was used to collect rate of learning and learn unit instruction data. Following mastery of four units of MEF-Math, conditioned reinforcement probes were conducted to determine whether learn unit instruction was effective in establishing conditioned reinforcement for math. If learn unit instruction was indeed successful, then post-intervention rate of learning measures were conducted. If learn unit instruction was not successful then the intervention continued with the stimulus-stimulus pairing procedure. This sequence continued with observational conditioning if the stimulus-stimulus pairing procedure was not effective. Dyads remained in the pre-intervention rate of learning phase until the preceding dyad entered post-intervention rate of learning phase. See Figure 13 for the design sequence flow chart.
Figure 13. Sequence of the experiment. Dyads remained in the pre-intervention rate of learning phase until the preceding dyad entered post-intervention rate of learning phase. Once entered the study, experimenters followed the sequence above.
Procedures

Pre- and post-probes of direct conditioned reinforcement for math. Conditioned reinforcement probes were conducted for all participants at the onset of the study. These probes were identical to the direct reinforcement probes detailed in Experiment I. Unlike Experiment I, indirect reinforcement measures were not conducted. Conditioned reinforcement probes were conducted prior to and following each conditioning procedure to determine if the specific procedure was effective in establishing conditioned reinforcement for math. Initial conditioned reinforcement probes were conducted for all participants simultaneously.

Rate of learning and learn unit instruction. Prior to and following the establishment of conditioned reinforcement for math, participants’ rate of learning math was measured using the MEF-Math curriculum. Sessions were conducted individually with each participant. The curriculum included scripted antecedents and corrections. The experimenter delivered the antecedent and provided reinforcement for correct responses in the form of approvals, high-fives, and tickles. If the participant emitted an incorrect response, the correction procedure (detailed in the teacher script) was delivered in which the experimenter emitted the correct response and represented the antecedent, giving the participant the opportunity to independently emit the correct response. There were 20 learn units in each session. Criterion was set at 90% (18/20) across two consecutive sessions or 100% (20/20) in one session. Once the participant mastered four units of the STEM curriculum, a post-direct conditioned reinforcement probe was conducted to determine whether learn unit instruction was effective in establishing conditioned reinforcement for math. If so, post-intervention rate of learning measurement began, if not then the experimenter began the stimulus-stimulus pairing procedure as detailed in Experiment I.
**Interobserver Agreement**

Point-by-point interobserver agreement (IOA) was collected for both the dependent and independent variables to ensure fidelity. Point-by-point IOA was calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100%. For rate of learning/learn unit sessions there was 98% IOA on 40% of learn unit sessions. There was 99% IOA on 100% of direct reinforcement probes sessions, 100% IOA on 100% of stimulus-stimulus pairing sessions, and 100% IOA on 50% of observational conditioning sessions.

**Results**

Figure 14 shows the pre- and post- intervention rate of learning data for each participant. Prior to and following the establishment of conditioned reinforcement for math, Participant L’s learn units to criterion or LUC was 160 and 100 respectively. Participant N required 200 and 160 learn units to meet criterion respectively prior to and following the establishment of conditioned reinforcement for math. Prior to the establishment of conditioned reinforcement for math Participant C’s LUC was 320 compared to 200 following conditioned reinforcement for math. Participant M’s number of learn units required to meet criterion was 340 prior to the intervention and 160 following the intervention. Prior to and following the intervention Participant J and G’s LUC were as follows: 440 to 260 and 480 to 240 respectively. Figure 15 shows each participant’s cumulative rate of learning prior to and following the establishment of conditioned reinforcement for math.
Figure 14. Rate of Learning. This figure shows the number of learn units required to meet criterion prior to and following the establishment of conditioned reinforcement for math.
Figure 15. Cumulative Rate of Learning. This figure shows each participants’ cumulative rate of learning prior to and following the establishment of conditioned reinforcement for math.
Figure 16 shows the pre- and post-probes of conditioned reinforcement for math.

Participant L required only learn unit instruction to establish conditioned reinforcement for math. Prior to the establishment of conditioned reinforcement for math, Participant L selected math during direct probe sessions for 60/300 intervals. Following learn unit instruction, he selected math for 300/300 intervals. Participant N required the stimulus-stimulus pairing procedure to establish conditioned reinforcement for math as he selected math for 7/300 intervals during initial direct probe sessions and 0/300 following learn unit instruction. After meeting criterion on the stimulus-stimulus pairing procedure, Participant N selected math for 181/300 intervals. Like Participant L, Participant C and J only required learn unit instruction to establish conditioned reinforcement for math. Prior to learn unit instruction Participants C and J selected math for 0/300 intervals, following learn unit instruction Participants C and J selected math for 195/300 and 284/300 intervals respectively. Participant M required observational conditioning to establish conditioned reinforcement for math. During initial direct probes, post-learn unit, and post-stimulus-stimulus pairing probes she selected math for 0/300 intervals. Following observational conditioning, she selected math for 300/300 intervals. Like Participant N, Participant G required the stimulus-stimulus pairing procedure to establish conditioned reinforcement for math. In pre-probes of direct reinforcement for math, Participant G selected math for 3/300 intervals, following learn unit instruction she selected math for 50/300 intervals, and after the stimulus-stimulus pairing procedure she selected math for 190/300 intervals.
Figure 16. Duration of selection of Math in Free Operant Play. This figure shows the number of 5-s intervals each participant emitted math observing responses as opposed to play prior to and following each conditioning procedure.
Figure 1 shows the participants’ number of correct responses during each session of learn unit instruction prior to and following the establishment of conditioned reinforcement for math. Participant L required 40, 60, 40, and 20 learn units to meet criterion on his pre-conditioned reinforcer unit instruction for math units and 20, 40, 20, and 20 learn units on his post-conditioned reinforcement for math units respectively. In each respective pre-conditioned reinforcement for math unit Participant N required 60, 20, 60, and 60 learn units to meet criterion and 20, 20, 60, and 40 learn units during post-conditioned reinforcement rate of learning measures. Participant C required 60, 40, 40, and 180 learn units to master Units 2-5 respectively compared to following the establishment of conditioned reinforcement where he required 40, 40, 60, and 60 learn units to master Units 6-9A. Participant M required 60, 60, 60, and 160 learn units to master Units 3-6 and 40, 60, 40, and 20 learn units to master Unit 7-9B respectively. Participant J required 140, 120, 100, and 80 learn units to master his units prior to the establishment of conditioned reinforcement for math and 40, 60, 80, and 80 learn units to master his units following conditioned reinforcement for math. Participant G required 100, 140, 120, and 120 learn units to master Units 3-6 respectively compared to 60, 40, 80, and 60 to master Units 7-9B respectively following the establishment of conditioned reinforcement for math.
Figure 17. Rate of learning/ Learn Unit Instruction. This figure shows the number of correct responses for each session of learn unit instruction for all six participants.
Learn unit instruction did not suffice in establishing conditioned reinforcement for math for Participants M, N and G, thus they continued with the stimulus-stimulus pairing procedure. Participants M and G required 40 pair-test trials to meet criterion on the 5-s pairing and 60 pair-test trials to meet criterion on the 10-s pairing. Participant N required 40 learn units for both the 5-s pairing and for the 10-s pairing.

Observational conditioning was conducted with Participant M as learn unit instruction and the stimulus-stimulus pairing procedure were ineffective in establishing conditioned reinforcement for math for this participant. During observational conditioning she did not emit any mands for math but her correct responding to the performance task were low. Participant M emitted 3, 1, 1, 3, and 2 correct responses out of 10 during observational conditioning-by-denial-sessions.
Discussion

The results of Experiment II strengthened the findings of Experiment I in that the individualized reinforcement intervention was effective in establishing conditioned reinforcement for math for all six participants. Experiment II was additive to Experiment I such that the findings of Experiment II indicate that establishing conditioned reinforcement for math results in an accelerated rate of learning math.

Learn unit instruction alone was effective in establishing conditioned reinforcement for math for Participants L, C, and J. These participants simply needed to access reinforcement for math and learn the function of math for math itself to function as a conditioned reinforcer. Due to Participant L’s rule governed behavior, he selected math for 60/60 intervals in his first probe of direct reinforcement for math. Prior to beginning the next probe, Participant L asked the experimenter, “I can do whatever I want?” Participant L was reassured that he could choose either the Play-Doh® or math without consequence. Following the intervention Participants L, C, and J’s learn units to criterion decreased by almost half, meaning they acquired new math objectives twice as fast as they did prior to having conditioned reinforcement for math in repertoire.

Unlike Participants L, C, and J, learn unit instruction did not suffice in establishing conditioned reinforcement for math for Participants N and G. These participants required both learn units and the stimulus-stimulus pairing procedure to establish conditioned reinforcement; once established their rate of learning significantly accelerated just as with Participants L, C, and J. Participant M required observational conditioning to establish conditioned reinforcement for math. Even after the stimulus-stimulus pairing procedure, Participant M did not attend to the math material and opted to play with Play-Doh® for the entirety of direct probes. During
observational conditioning she emitted a high number of incorrect responses to a performance task when she was denied access to math. While she did not emit any mands for math, she did state that, “this is just hard for me.” The observational conditioning procedure was effective in establishing conditioned reinforcement for math for Participant M, which led to an accelerated rate of learning math.

A possible limitation of this experiment is that the math units were not counterbalanced. However, like in Experiment I, probes were conducted to determine which units the participants did not have in repertoire in order to ensure the objectives were indeed novel. Though the math units were not counterbalanced, the MEF-Math curriculum increases in difficulty meaning that the participants learned faster following the establishment of conditioned reinforcement for math despite an increase in the difficulty (number of steps required to complete the problem) of the math objectives.

The next chapter will discuss the educational significance and implications of the results of both Experiment I and II.
CHAPTER IV

GENERAL DISCUSSION

Experiments I and II determined that the individualized reinforcement intervention could establish math as a conditioned reinforcer for preschoolers, and once established, Experiment 2 showed that the establishment of conditioned reinforcement resulted in accelerated rates of learning math. The purpose of Experiment I was to determine if conditioned reinforcement for math using the sequence of conditioning procedures outlined in Buttigieg (2015) could be accomplished. When reinforcement was established in Experiment I, Experiment II was conducted to test for replication of these results while also investigating the effect of the establishment of conditioned reinforcement for math on pre-kindergarten students’ rate of learning math.

Experiment I showed that the individualized reinforcement intervention was an effective means of establishing conditioned reinforcement for math or preference for math over Play-Doh® for four pre-kindergarten participants. Buttigieg’s (2015) study was the first to use all three conditioning procedures in an attempt to establish conditioned reinforcement for the observation of books. Like in Buttigieg’s (2015) study, the participants of Experiment I and II required different conditioning procedures in order to enhance the reinforcement value of math.

Experiment II was an extension of Experiment I. After determining that conditioned reinforcement for math could be established using the procedures outlined in Buttigieg (2015), the purpose of Experiment II was to determine whether enhancing the reinforcement value of math would increase participants’ rate of learning. The results of Experiment II were consistent with prior studies that showed that establishing or enhancing reinforcement value reduces students learn units to criterion (Buttigieg, 2015; Delgado, Greer, Speckman, & Goswami, 2009;
Major Findings and Implications

Experiments I and II were the first studies to establish conditioned reinforcement for math in pre-kindergarten children. The results of these studies demonstrate that even though students, without instruction, have everyday math skills and can learn complex math, does not necessarily mean that they have interest in or conditioned reinforcement for math. The establishment of conditioned reinforcement for math may be the missing piece of the math crisis in America.

Experiment I Questions: Can conditioned reinforcement for math be established at the age of four? Do different students require different procedures? The results of Experiment I demonstrate that conditioned reinforcement for math can be established in students as young as 4.4 years old and that different students require different procedures to increase or establish reinforcement value for math. As previously mentioned, according to Ginsburg and colleagues (2008), from birth to five-years children acquire everyday mathematics even without direct instruction. This leaves the responsibility of building upon the everyday mathematics of each child on caretakers and teachers. Students’ math skills at the onset of kindergarten are critical indicators of their future success in both reading and math (Engel, Claessens, & Finch, 2013). The individualized reinforcement intervention would allow teachers to create or increase an interest in math in their students, thus allowing for more complex math instruction.

In Experiment I, there was no need to conduct observational conditioning-by-denial as two participants acquired conditioned reinforcement for math following learn unit instruction and the remaining dyad required the stimulus-stimulus pairing procedure. The participants who
only required learn units simply needed to contact reinforcement for engaging in math activities, thus allowing the stimulus control to shift from the approvals delivered during instruction to math itself. The second dyad required the stimulus-stimulus pairing procedure because learn units alone did not shift the stimulus control of reinforcement to the actual stimuli embedded in the process of math. In the case of these participants, the positive reinforcement they contacted following correct responses to learn units was paired with math, but not enough for the math itself to become reinforcing. Increasing the duration of engaging in math using the stimulus-stimulus pairing procedure allowed the stimulus control to shift to the actual math.

The results of Experiment I also expand upon the results of Lee (2016) with the use of direct and indirect measures of reinforcement. Lee defined direct reinforcement as, “an individual emitting a behavior and accessing reinforcement intrinsic to the stimulus” and indirect reinforcement as a conditioned reinforcer functioning to reinforce other behaviors as a prosthetic reinforcer (p. 99). The indirect measures for Participants L, T, and I provide evidence that following the establishment of conditioned reinforcement for math, math itself functioned as a prosthetic reinforcer or educational reinforcer for responding to a performance task. Even though Participant K’s indirect measures showed an overall descending trend, his direct probes clearly reveal that math became a conditioned reinforcer as a function of the individualized reinforcement intervention. Thus, math may only function as a direct reinforcer for Participant K as opposed to a direct and indirect reinforcer like the remaining participants in Experiment I. These results are consistent with the view that there are two subcategories of conditioned reinforcers. Future studies should investigate how these different dimensions of reinforcement are established.
Experiment II Question: Does conditioned reinforcement for math result in an accelerated rate of learning? Experiment II replicated Experiment I in that the individualized reinforcement intervention was used to establish conditioned reinforcement for math. Experiment II expanded the results of Experiment I in that participants’ rate of learning prior to and following the establishment of conditioned reinforcement was measured. The establishment of conditioned reinforcement for math using the individualized reinforcement intervention resulted in an educationally significant accelerated rate of learning math in pre-kindergarten students.

The importance of conditioning stimuli as reinforcers for observing and the effect of this conditioning on learning has been well documented in a series of verbal developmental studies (Buttigieg, 2015; Delgado, Greer, Speckman, & Goswami, 2009; Du, Broto & Greer, 2015; Greer & Han, 2015; Greer, Pistoljevic, Cahill, & Du, 2011; Maffei, Singer-Dudek, & Keohane; Tsai & Greer, 2006). Experiment II adds to this body of research. Like the participants in the present study, the above studies were conducted with preschool aged children. Each of the above studies used the stimulus-stimulus pairing procedure to establish conditioned reinforcement for: voices and faces (Greer et al., 2011; Maffei et al., 2014), two-dimensional and three-dimensional stimuli (Du et al., 2015, Greer & Han, 2015; Pereira-Delgado et al., 2009), and books (Buttigieg, 2015; Tsai & Greer, 2006). All of these participants acquired a verbal developmental cusp (Rosales-Ruiz & Baer, 1996) following the intervention. This means that the participants were now able to contact the environment in ways they previously could not. The results of these studies demonstrate that acquiring certain reinforcers also results in the acquisition of new cusps (Greer & Du, 2015). In addition to acquiring a verbal developmental cusp, conditioning stimuli as reinforcers for observing function to accelerate students’ rate of learning for listener programs,
visual match to sample programs, and sight words respectively. The results of this study are consistent with the above studies in that conditioning stimuli as reinforcers, in this case math, led to an increase in participants’ rate of learning math.

The results of this study extended those of O’Rourke’s (2006) math conditioning study conducted with second and third grade students. While O’Rourke was successful in establishing conditioned reinforcement for math as a function of observational conditioning-by-denial, the participants’ rate of learning math only slightly improved for 3 out of 4 of the participants. The slight effects on rate of learning may be related to the discrepancy between the type of math that was conditioned as compared to the math learning used for the dependent variable. The objectives used to test for changes in rate of learning were within the geometry domain while the math that was conditioned was within the numeracy domain. For this reason, units 11, 12, and 13 (geometry units) of the MEF-Math curriculum were omitted from Participant L’s rate of learning measures. All conditioning materials within Experiment I and math tasks in Experiment II were conducted with objectives within the numeracy domain in order to make accurate comparisons about the changes within participants’ rate of learning following the establishment of conditioned reinforcement for math.

Early math instruction greatly impacts students’ later math development. Research has shown that children are able to learn complex math (Ginsburg et al., 2008). Increasing students’ rate of learning will maximize instructional time and allow for high quality instruction on more complex mathematical skills. The participants in Experiment II were pre-kindergarten students who by the end of the study were either at the start or well into kindergarten math while still in the middle of their pre-kindergarten school year. The results of Experiment II not only show that students’ rate of learning increases as a function of conditioned reinforcement for math, but also
that students are indeed able to acquire more complex math operations without sacrificing the students’ play time. In fact, many of the participants in Experiment I and II consider their math groups as play time following the establishment of conditioned reinforcement for math.

**Bridging the Gap- Low SES Students**

Math achievement at the onset of kindergarten is the strongest predictor of future performance in math and reading. This means that at the pre-kindergarten level there may already be gaps between students who come from low socioeconomic homes and those that do not. “For these children especially, the long-term success of their learning and development requires high-quality experience during their early ‘years of promise’” (Clements & Sarama, 2014, p. 2). According to the National Math Panel (2008), the math gap between low-income and middle-income students increasingly expands throughout their school-aged years. The results of Experiment II demonstrate that students’ rate of learning accelerates by 1.5 to 2 times following the establishment of conditioned reinforcement for math. These low SES students who are deemed “at risk” would benefit from conditioned reinforcement for math as it would increase their rate of learning thus bridging the gap between themselves and their middle-class peers.

While a relation between SES and the presence or absence of conditioned reinforcement for math was not investigated in Experiments I and II, the Brigance scores of the participants in both experiments varied across the physical, language, and cognitive domains. The variance in Brigance scores supports the notion that conditioned reinforcement for math can be established and would be beneficial for all students.

**Limitations**

The study is not without limitations. The textual responses in Experiment I and the units of MEF-Math in Experiment II were not counterbalanced, making it difficult to account for the
difficulty of the sets or units. That is, the possible differences in difficulty were not controlled for. The units used prior to the establishment of conditioned reinforcement for math consisted of matching numbers to quantities, counting, and other basic foundational math skills. The MEF-Math units used for post-rate of learning measures consisted of arguably more complex math, including addition and subtraction, word problems, and finding the missing numbers within equations. From the students’ point of view, basic math skills may be just as difficult as addition and subtraction, however, addition and subtraction are indeed higher-level math skills that require multiple steps to complete compared to basic math that usually only requires one step. This means that following the establishment of conditioned reinforcement for math all six participants in Experiment II acquired more complex math objectives at a faster rate.

Another limitation was the rule-governed behavior of the participants in Experiment I as evidenced by their first phase of math during indirect probes of conditioned reinforcement. The participants in Experiment I have an instructional history of following directions and teacher presence functioning as instructional control. Therefore, when the performance task was presented to Participants I, T, and L they completed it without regard to whether they were receiving Play-Doh® or math as a consequence. By the second math phase, the participants responded directly to the contingency between their correct responses and how much Play-Doh® or math they received. Once they contacted the contingency, extinction effects were shown. This stimulus control was also apparent for Participant T and Participant L’s first direct reinforcement probe. Once the students were certain that they would not receive any consequence following direct probes, they very clearly selected Play-Doh® for the remaining four direct probes.

In addition to the math sets/units and the rule-governed behavior of the students, the lack of two-month follow-up probes for the participants in Experiment II is another limitation in this
study. Follow-up probes would determine whether or not math remained a conditioned reinforcer after the intervention and could demonstrate whether or not the participants continued to acquire math objectives at an accelerated rate. However, anecdotally, the participants of Experiment II continue to request math instruction and tact math principles within the environment.

**Future Research**

The results of Experiment I and II demonstrate that it is possible to establish conditioned reinforcement for math for children as young as 4.4 years old and different conditioning procedures are necessary for different students. Across both experiments, five participants required learn units, four required the stimulus-stimulus pairing procedure, and one participant required observational conditioning-by-denial. The results of Experiments I, II, and Buttigieg (2015) suggest that observational conditioning-by-denial would be most effective for the students who have more cusps and capabilities. This may be due to observational stimulus control. Students who do not have observational stimulus control may not attend to the confederate’s access to the target stimulus. Compared to learn unit instruction or the stimulus-stimulus pairing procedure, observational conditioning-by-denial is faster to implement. The unaddressed matter is whether the five participants who only required learn units or the four participants who required the stimulus-stimulus pairing in the present study would have acquired conditioned reinforcement for math faster if observational conditioning-by-denial was implemented first. Another possibility would be to begin with learn unit instruction, since that alone is successful for some, followed by observational conditioning-by-denial, and then the stimulus-stimulus pairing procedure if necessary. In order to maximize instructional time, future research should investigate which procedure is most appropriate for which kids and whether
there is a relation between the students’ rate of acquisition or other cusps and the procedure that was effective in establishing reinforcement value for math.

Establishing conditioned reinforcement for math may also have collateral effects on students’ reading objectives. Previous research has shown that children’s math skills predict their reading and math ability later in life (Claessens & Engel, 2013). Future research should examine the effects of the establishment of conditioned reinforcement for math using the individualized reinforcement intervention on students’ rate of learning both math and reading.

Conclusion

The results of Experiment I showed that the individualized reinforcement intervention, which includes the delivery of learn units, stimulus-stimulus pairing, and observational conditioning-by-denial was effective in establishing conditioned reinforcement and preference for math over Play-Doh®. Experiment II replicated and extended upon Experiment I. Following the establishment of conditioned reinforcement for math, all six participants’ rate of learning significantly accelerated. Not only did math become a conditioned reinforcer, but participants were learning math 1.5-2 times as fast. There is still much that is unknown about the effects of establishing conditioned reinforcement for math – are there collateral effects on reading rate of learning and what is the relation between students’ cusps and capabilities and the specific procedure that is effective in establishing conditioned reinforcement for math? However, the present study offers the individualized reinforcement intervention as an effective means to establish conditioned reinforcement for math while also accelerating learning.
References


Appendix A

MEF-Math Curriculum teacher script sample

STEM MATH UNIT 4

Scripted Objective

Given a group of objects and three exemplars of numbers on a page, student will match the quantity with the written number.

Given two groups of lines, students will count the lines to get the correct total number.

Given a group of 3D objects and an Arabic number, student will give a peer or teacher the corresponding number of objects (antecedent: give me ____ antecedent should have a set number of objects on the table)

Given 3-5 numbers, student will identify the missing number.

Materials:

- Number cards: 7 & 9 (Index cards)
- STEM MATH UNIT 4 Version 1 and 2

Instructional Demonstration Learn Units

Before presenting the lesson, provide the following models:

**Instructional Demonstration 1:**
1. Put out 7 3D objects with a number 7 card.
2. Count each object while pointing to the object (1, 2, 3, 4, 5, 6, 7).
3. Point to the card, and say “7” There are a total of 7 _____ on the table.

**Instructional Demonstration 2:**
1. Put out 9 3D objects with a number 9 card.
2. Count each object while pointing to the object (1, 2, 3, 4, 5, 6, 7, 8, 9).
3. Point to the card, and say “9” There are a total of 9 _____ on the table.

CONSEQUENCES FOR LEARN UNITS. Each learn unit presentation receives a consequence in the form of reinforcement or a correction.

CORRECT RESPONSES: Correct Response: Provide vocal reinforcement (i.e., great job) and record a plus on the data sheet

INCORRECT RESPONSES:
Incorrect responses differ for the type of learn unit presented. Corrections should be presented no more than 3 times before moving onto the next learn unit.

<table>
<thead>
<tr>
<th>Type A: Match</th>
<th>Type B: Quantity of Objects</th>
<th>Type C: Count</th>
<th>Type D: Missing Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>With the LU card on the table: Teacher gives student an opportunity to first read each number. Reinforcement is provided if student correctly reads the number. Corrections are provided if student submits incorrect response. Corrections consist of modeling correct number and giving student an opportunity to read the number correctly. Once student reads each number, teacher says: Match the same</td>
<td>With the LU card on the table and a pile of objects, teacher says: Give me ____ (objects)</td>
<td>With the LU card on the table: Teacher models counting of one colored line. Student is given the instruction to count from where the teacher left off. Teacher says: How many in all? If student submits incorrect response follow the correction procedure: 1. Count the target number of objects within a pile 2. Re-present antecedent: “Give me ____ objects” Student counts the objects requested and gives it to a teacher or peer</td>
<td>With LU card on the table: Teacher counts the numbers (but skips the missing number). Teacher gives instruction: “Point to the missing number.” If student submits incorrect response: 1. Count the numbers (with the missing number) while pointing to each number 2. Re-present the instruction: “Point to the missing number” Student points to the correct missing number</td>
</tr>
<tr>
<td>If student submits incorrect response follow the correction procedure: 1. Match by pointing to the shape and corresponding number while saying the number 2. Re-present antecedent: “match the same” Student matches by pointing to the shape and corresponding number</td>
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<tr>
<td>Stem Math - Version 1</td>
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<td><strong>Learn Unit</strong></td>
<td><strong>Teacher Instructions</strong></td>
<td><strong>Student Stimuli</strong></td>
<td><strong>Correct Student Response</strong></td>
</tr>
<tr>
<td>1</td>
<td>MATCH</td>
<td>1. While student is attending, present student with LU 1&lt;br&gt;2. Point to &quot;9&quot; (No vocal instruction- student says: 9)&lt;br&gt;3. Point to &quot;7&quot; (No vocal instruction- student says: 7)&lt;br&gt;4. Point to &quot;6&quot; (No vocal instruction- student says: 6)&lt;br&gt;5. Match the same</td>
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<tr>
<td>2</td>
<td>COUNT</td>
<td>1. While student is attending, present LU 2 card&lt;br&gt;2. <em>I am going to count the red lines. 1, 2, 3, 4 (While pointing to the line)</em>&lt;br&gt;3. There are 4 red lines. Now we can count all of the lines. Start from 4.&lt;br&gt;4. 4 (While pointing to the last red line).&lt;br&gt;5. How many lines in all?</td>
<td></td>
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<tr>
<td>3</td>
<td>MISSING NUMBER</td>
<td>1. While student is attending, present LU 3 card&lt;br&gt;2. (Point to each number) 5, 6, 7, point to the line: 9&lt;br&gt;3. Point to the missing number</td>
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Appendix B

MEF-Math Curriculum student answer book sample
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### Appendix C

**MEF-Math Curriculum data sheet sample**

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**Correct/Total**

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