

**Limited Arbitrage is Necessary and Sufficient  
for the Existence of an Equilibrium**

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# Limited Arbitrage is Necessary and Sufficient for the Existence of an Equilibrium

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## Abstract

In 1991, in [1], [2] and in [8], I introduced the concept of global cone and used it to define a condition on endowments and preferences, 'limited arbitrage', which I showed to be necessary and sufficient for the existence of a competitive equilibrium [8]. In response to a comment [19], I show here that the authors misunderstood my results by focussing on brief announcements which cover other areas, social choice [6] and algebraic topology [3], rather than on the publication which contains my proofs on equilibrium [8]. The comment's example is irrelevant to my results in [8] because it starts from different conditions. Limited arbitrage is always necessary and sufficient for the existence of a competitive equilibrium, with or without short sales, with the global cones as I defined them, and exactly as proved in [8].

## 1 Introduction

Limited arbitrage is a unifying concept for resource allocation. Defined on the traders' endowments and preferences it is simultaneously necessary and sufficient for the existence of a competitive equilibrium,<sup>1</sup> for the nonemptiness of the core<sup>2</sup> and for the existence of satisfactory social choice rules.<sup>3</sup> Limited arbitrage extends to a topological invariant for competitive markets which contains exact information on the equilibrium, social choice and the core of all subeconomies, and predicts a failure of 'effective demand,' [12]. In strictly regular economies, limited arbitrage is necessary and sufficient for the uniqueness of equilibrium [13]. For a complete presentation of my results on existence of an equilibrium the reader is referred to another paper in this issue [12].

This paper responds to a recent comment [19] on my work on market equilibrium in [3], [6] and [8]. The three authors misread my papers. I correct the comment's errors and show that its example is consistent with my results: limited arbitrage is always necessary and sufficient for the existence of a competitive equilibrium, with or without short sales, with the global cones which I introduced and exactly as proved in [8]. A summary of the response is as follows:

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<sup>1</sup>[2], [7], [9], [3], [6] and [8].

<sup>2</sup>[5], [7], [9], [17].

<sup>3</sup>[1], [7],[6], [9].

- The comment consists of a single example of a “mixed economy”, while none of my three publications which the comment addresses—[3], [6] or [8]—claim to cover mixed economies. The comment is therefore irrelevant to the publications on which it comments.
- The comment concentrates on my publications [3] and [6], which are brief announcements on equilibrium and cover other areas. The comment therefore concentrates on the wrong references,
- Details and proofs of my results on equilibrium were published in [8]. These proofs in [8] are correct exactly as given. There is no inconsistency between the comment’s example and any of my results, because they start from different conditions,
- My global cone  $G_h$ , introduced in [1], [2] and in [8], is a well-defined concept which differs from the recession cones that are used elsewhere in the literature on no-arbitrage,
- Based on global cones I defined limited arbitrage, a concept which bounds trades and utility exactly as needed for the existence of a competitive equilibrium. The concepts of global cones and limited arbitrage are always the same throughout my work; the notation is adapted to the context,
- Limited arbitrage is necessary and sufficient for bounded and attainable gains from trade [8],
- Limited arbitrage is necessary and sufficient for the compactness of the Pareto frontier,
- All the above results in [8] hold in economies with or without short sales, and whether traders’ indifferences contain half lines or not.

Furthermore

- Rather than a counterexample, the example given in the comment [19] is a special case of my results for mixed economies in [9], reported in this issue [12]. The three authors cite [9] as Chichilnisky (1995a) but, for reasons which I leave the reader to surmise, fail to inform the reader of the consistency of their example and the results of my paper [9].

It seems that the three authors have misread my papers.

How does limited arbitrage work? Limited arbitrage is strictly weaker than other conditions used to ensure the existence of an equilibrium, which is why it can be necessary and well as sufficient. Other conditions bound the feasible and individually rational trades. Defined by using global cones, limited arbitrage is unique in that it only bounds the utility levels which are achieved by such trades but not the trades themselves. The crucial insight is the equivalence of limited arbitrage and the compactness of the Pareto frontier in utility space;<sup>4</sup> this controls the existence of an equilibrium, of the core and of satisfactory social choice rules.

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<sup>4</sup>The Pareto frontier is the set of undominated and individually rational utility values. The equivalence between the compactness of the Pareto frontier and limited arbitrage was pointed out first and established in a number of papers since 1984, [14], [18], [2], [7], [8], [9] and was used there to prove existence of an equilibrium in markets with or without short sales, with finite or infinite dimensions, and whether traders’ indifferences contain half line or not. Observe that the compactness of the Pareto frontier is irrelevant for Hart-type economies as studied by Page [20] and others, because such economies are incomplete: their equilibria are typically inefficient and are not contained in the Pareto frontier.

## 2 Mixed Economies

In this section I correct two errors in the comment: one consists of misreading my conditions on preferences, and the second one in misreading my definition of global cones. These errors invalidate the comment's claims.

1. The comment [19] consists of one simple example of a "mixed economy" with short trades, two traders and two goods. Mixed economies were not covered by my results. Trader one has a utility  $u_1(x_1, x_2) = x_1 + x_2 + 2 - \sqrt{(x_1 - x_2)^2 + 4}$ , and trader two's is  $u_2(x_1, x_2) = x_1$ .<sup>5</sup> Trader one's indifferences are bounded below, and the second trader's are not; trader two has indifferences with half lines and the other does not. This combination of preferences is a 'mixed' economy, a type of economy which I never claimed to cover in my work in [8] which the comment addresses. My results in [8] dealt only with homogenous economies: the statement of my Theorem 1 in [8] does not mention whether mixed economies were included, but it is clear to anyone who read my proofs that I only covered homogenous economies as I considered the two cases quite separately—indifferences bounded below and those which are not bounded below—see p. 103, Section 7.0.1, lines 1-5 of [8].<sup>6</sup> Since the comment's example is **not** a homogeneous economy, it is not a counterexample to the results in the publications which it addresses, all of which pertain and refer to the proofs in [8]. Therefore the comment's example is irrelevant to the publications which it addresses.

2. The three authors made another error, stating of their example "Thus limited arbitrage is satisfied"<sup>7</sup>. However, their economy does **not** have limited arbitrage. The details are as follows. By my definition in [8] the first trader in the example of [19] Section 3 has as global cone  $G_1 = \{x_1, x_2 : x_1 \geq 0 \text{ and } x_2 \geq 0\}$  because this trader's indifferences contain no half lines: this is case (b) of [8], p. 85 (4). However the three authors got this wrong, stating that my global cone  $G_h$  is the set they denote  $A_1$  in the statement of their Theorem 3 [19]. My definition in [8] p. 85, (4) states clearly that when preferences contain no half lines (case (b)) the global cone  $G_h$  is the **closure** of the set  $A_h$ , i.e. it is the set they denote  $I_1$  in Theorem 3 of [19]. The three authors confused  $I_1$  with  $A_1$ . This is an error which invalidates the rest of the statements in the comment.

The second trader in [19] has half-lines in the indifferences (case (a) of [8] p. 85, (4)), and by my definition his/her global cone is the open half space  $G_2 = \{x_1, x_2 \in R^2 : x_1 > 0\}$ , as the authors themselves point out in Theorem 3 of Section 3.2. It is now immediate to observe that, contrary to what the comment states, limited arbitrage is **not** satisfied in their example in Section 3, Theorem 3 of [19]. The market cones are respectively  $D_1 = \{(x_1, x_2) : x_1 > 0 \text{ and } x_2 > 0\}$  and the  $D_2 = \{(x_1, x_2) = x_1 = 0\}$ . Since  $D_1 \cap D_2 = \emptyset$ , limited arbitrage is **not** satisfied. It is not surprising, therefore, that their economy has **no** equilibrium, as the comment acknowledges. This is just a confirmation of my results.<sup>8</sup>

<sup>5</sup>See [19] Section 3, Figures 1(a), 1(b), 2(a) and 2(b), and subsection 3.2.

<sup>6</sup>I quote from [8], p.103, Section 7.0.1, lines 1-5: "When all indifferences are bounded below the proof is identical to the case where  $X = R_+^N$  a case where the result is known (cf. Arrow and Hahn (1)). Therefore we need only consider the case where  $X = R^N$  and where the indifferences of the traders are not all bounded below". Observe that by Assumption 2 on p. 84 of [8] if one indifference surface of a trader is bounded below, then all of his/her indifference surfaces are bounded below. Therefore the two exclusive cases considered in [8] are: either all indifference surfaces of the traders bounded below, or none are. It is clear that the example in Section 3 of the comment [19] does not satisfy my conditions because it mixes the two types of preferences.

<sup>7</sup>On the last line of Section 3.1 of [19].

<sup>8</sup>Since there is no equilibrium and limited arbitrage fails, this example is clearly consistent with my results on mixed economies in Theorem 2 of [9], which the authors acknowledge they have, and they cite

### 3 Markets without short sales

In this section I correct two other errors in the comment, about my results for the classic Arrow Debreu economy without short sales, in [19] Section 5, last paragraph. These errors invalidate the comments' claims.

1. The three authors claim that my example in Figure 4B of [8], p. 88, has a competitive equilibrium but does not satisfy limited arbitrage. This is incorrect. This example does **not** have a competitive equilibrium. This example is in fact the classic example of nonexistence of a competitive equilibrium with boundary endowments, first proposed by Kenneth Arrow and reported in his 1970 book with Frank Hahn. It has been known for over 25 years that the economy in this example has no competitive equilibrium. All this is in accordance with my Theorem 1 of [8]. All this confirms my results in [8].

The details are as follows. Figure 4B in [8] is a two person economy with trading space  $X = R_+^2$ . Trader one owns the second good,  $\Omega_1 = (0, x_2)$ ,<sup>9</sup> and trader two owns both goods,  $\Omega_2$  is in the interior of the positive orthant.<sup>10</sup> Trader one has a preference which is strictly increasing in both goods.<sup>11</sup> For trader one the global cone is  $G_1(\Omega_1) = R_+^2 = \{(x_1, x_2) : x_1 \geq 0 \text{ and } x_2 \geq 0\}$  i.e. the whole positive orthant<sup>12</sup>  $R_+^2$  and by definition  $D_1 = \{(x_1, x_2) : x_1 > 0 \text{ and } x_2 > 0\}$ . Trader two is indifferent in the second good as shown in Figure 4B of [8]. Furthermore since this trader has an interior endowment<sup>13</sup>  $\Omega_2$ , the only price in the set<sup>14</sup>  $S(E)$  is the vector  $v = (1, 0)$  as indicated in Figure 4B.<sup>15</sup> Since  $\langle v, \Omega_1 \rangle = 0$ , by definition  $S(E) \subset N$ . Therefore by definition  $D_1^+ = D_1 \cap S(E) = \emptyset$ .<sup>16</sup> It follows that limited arbitrage is not satisfied in Figure 4B, because  $D_1^+ = \emptyset \Rightarrow D_1^+ \cap D_2^+ = \emptyset$ . As Arrow pointed out a long time ago, this economy has no competitive equilibrium: the only possible supporting price is  $v$ , at which excess demand of trader one is not well defined. The comment went wrong by stating that in my example “one trader likes only one good and is endowed with the other good” see last paragraph of Section 5 of [19]. Neither of my two traders has the characteristic described by the comment: trader one only owns the second good and prefers both, and trader two owns both goods and prefers only the first. As detailed above, this example fails to have limited arbitrage since  $D_1^+ \cap D_2^+ = \emptyset$ . Since there is no equilibrium and limited arbitrage fails, this example confirms my Theorem 2 above [9].

2. Another example refers to Figure 7 in page 92 of [8]. This figure is the same as Figure 4B above, and as stated in its legend, it has no competitive equilibrium. The comment [19] states that “limited arbitrage is satisfied in this case (Figure 7)”, see last paragraph of [19]. However, as was shown in Example I above, the comment is wrong: Figure 7 in page 92 does **not** satisfy limited arbitrage. This is an example where there is no competitive equilibrium and limited arbitrage fails, and which is consistent with my Theorem 1 in [8].

as Chichilnisky (1995a), see also this issue [12].

<sup>9</sup>See [8] page 89, lines 3 and 10-11.

<sup>10</sup>See lines 1-2 and 10-11, p. 89 of [8].

<sup>11</sup>There is an obvious switch in the indices 1 and 2 here but in any case the argument is clear.

<sup>12</sup>As stated in lines 12-15 p. 89 of [8].

<sup>13</sup>As stated in lines 1-2 and 10-11 p. 89 of [8].

<sup>14</sup>The definition of  $S(E)$  had a well known typographical error missing the expression “ $\langle v, x_h - \Omega_h \rangle = 0$ ” in [8], but this was corrected in the revised version of [9] which the three authors acknowledge they have and cite as Chichilnisky (1995a).

<sup>15</sup>And as stated in lines 13-15 p. 89 of [8].

<sup>16</sup>As stated in page 89, line 13 and in my Figure 4B of [8].

## 4 Global Cones

In this section I correct errors in the comment about my global cones. The commentators misread my definitions of global cones. Below I show that the global cone which I introduced is a well-defined concept which never changes. The notation is adapted to fit the context. On the basis of these global cones I defined limited arbitrage, and showed that it is exactly what is needed for the existence of an equilibrium, the core and social choice.

1. In Section 5, under the title “Impact of Changes in Chichilnisky [8]”, the comment states: “Below we argue that global cones in Chichilnisky [8] are not well defined”. However the comment [19] misread my definition of global cones and this error invalidates the rest of the statements in the comment.

My global cone  $G_h$  introduced in 1991 [1], [2] and [8] is different from all other cones used in the no-arbitrage literature which is based, instead, on the recession cones used by Rockafeller in 1970, see e.g. [20]. In its more general form reported in [8] my global cone  $G_h$  is the set of directions along which utility never ceases to increase. This is quite different from a recession cone, which is in this case the set of directions of non-decreasing utility. For example, for a constant function on  $R^N$  the recession cone is the whole of  $R^N$ , while my global cone is empty.<sup>17</sup> A detailed exposition of my global cones is in [8] which is the only publication among the three discussed in the comment to give details and proofs on limited arbitrage and equilibrium. Therefore I refer the reader to [8], p. 84 where I define global cones. In Assumption 2, lines 3-5 of [8] p. 84 I consider explicitly two cases: “ $\forall h$  : (a) the directions of the gradients of each indifference surface which is not bounded below define a closed set, or (b) indifferences contain no half lines”. Preferences in case (a) always contain half lines in the indifferences, while those in case (b) never do. It is clear that whenever preferences have indifferences without half lines, we are in case (b). Because the two cases encompass different types of preferences, it can be expected that the global cone will have slightly different realizations in the two cases and indeed they do. In [8] p. 84 (4) I state: “in case (a), the global cone is  $A_h(\Omega_h)$  and in case (b), its closure  $\bar{A}_h(\Omega_h)$ ”. In all cases, (a) and (b), my global cone is always the set of directions along which utility never ceases to increase.<sup>18</sup> This global cone is well defined and never changes. It appears that the commentators misread my papers.

2. The comment mentions that cases (a) and (b) in Assumption 2 of [8] may overlap, and that my global cones may therefore be ill-defined.<sup>19</sup> This is irrelevant. The only possible overlap is a trivial case where short sales do not matter, when indifferences are bounded below *and* contain no half lines, but in this case *any* definition of the global cone ( $A_h$  or  $\bar{A}_h$ ) works, because all my conditions and results hold true trivially. In this case limited arbitrage is always trivially satisfied (with either cone  $A_h$  or  $\bar{A}_h$ ), an equilibrium always exists, and the Pareto frontier is always compact because indifferences are bounded below. There is no ambiguity and no problem with my definition of global cones.

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<sup>17</sup>The global cone is also different from the cone of directions where utility is strictly increasing: it is easy to show an increasing function on  $R_+^N$  which is locally satiated at 0, in which the latter cone is empty while the global cone is the positive orthant.

<sup>18</sup>My global cone  $G_h$  is also identical to that I used in [9], in another paper in this issue [12] and in [13], papers which also contain details of the cones and proofs on equilibrium. In all cases my global cone is the same.

<sup>19</sup>In paragraph 1 of the subsection on “impacts”, Section 5.

## 5 Gains from Trade

In this section I correct another error in the comment. The comment states that my concept of gains from trade is unrelated to equilibrium, when in fact they are very closely related.

1. [19] states “Chichilnisky’s notion of gains from trade has no relevance for the existence of an equilibrium.”<sup>20</sup> However, as shown in Proposition 2 page 90 of [8] and mentioned in Proposition 1 of [6] p. 428,<sup>21</sup> gains from trade are closely related to limited arbitrage, and therefore from Theorem 1 of [8], closely related to the existence of a competitive equilibrium. Limited arbitrage is always necessary and sufficient for attainable and bounded gains from trade, in case (a) and (b).<sup>22</sup> As stated in [8] the connection between limited arbitrage and bounded gains from trade is very close.

## 6 No Half-Lines

In this section I correct another error in the comment, about my results for economies where preferences have no half lines. The comment implies that I do not cover this case, when in fact my work covers preferences without half lines in many publications, starting in 1984.

1. [19] states<sup>23</sup> that “One of the main objectives of Chichilnisky [6] and [3] appears to have been to obtain necessary and sufficient conditions for existence of equilibrium in terms of conditions limiting arbitrage for economic models in which agent’s indifference surfaces are allowed to contain half lines”. This statement contains two errors. The main purpose of my papers [6] and [3] was *not* to prove a necessary and sufficient condition for the existence of an equilibrium: far from this, [3] contains only results in algebraic topology and [6] contains only results in social choice.<sup>24</sup> Of the three papers, the only one to contain details and proofs on equilibrium and limited arbitrage is [8], in Theorem 1 p. 94, and this Theorem 1 of [8] is correct exactly as stated and proved.

3. The second error in the statement quoted above is to imply that I focus on the case of indifferences with half lines (a). This is incorrect: although the case without half lines (b) is very simple I cover this case as well.<sup>25</sup> Indeed my work on arbitrage and equilibrium in preferences **without half lines** goes back to 1984 [14] and 1993 [18]: these two papers contain the first results on no-arbitrage, the compactness of the Pareto frontier and the existence of a competitive equilibrium in economies with or without short sales, with finite and infinite dimensions, and include preferences without half lines.<sup>26</sup>

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<sup>20</sup>See [19], page 1, first paragraph.

<sup>21</sup>And in [12].

<sup>22</sup>In case (a) of [8] limited arbitrage is also necessary and sufficient for bounded gains from trade.

This is also stated in [6], and in Corollary 1 of [12].

<sup>23</sup>In para. 5 of its Introduction.

<sup>24</sup>Both of these papers refer the reader to [8] for details and proofs on equilibrium, see [6] p. 430, line 19, and [3] p. 195, lines 8-9.

<sup>25</sup>Theorem 1 and Lemma 2 of [8] cover preferences with half lines in their indifferences as well as those without, see p. 84 of [8], Assumption 2, lines 3-4, where case (a) is with half lines and case (b) without half lines. See also the definition of global cones for cases (a) and (b) in p. 85, section 2.2 of [8]; Lemma 2 p. 96 and 103 of [8] on the compactness of the Pareto frontier covers also cases (a) and (b), as does Theorem 1 on limited arbitrage being necessary and sufficient for the existence of a competitive equilibrium, p. 94 of [8].

<sup>26</sup>See e.g. Lemmas 4 and 5 and Theorem 1 of [18]

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