A Comparative Study of Korean Abacus Users' Perceptions and Explanations of Use: Including a Perspective on Stigler's Mental Abacus

Soomi Kim

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ABSTRACT

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Soomi Kim

The purpose of this study was to determine the prevalence of using a "mental abacus" by adults whose mathematics education in Korea included extensive use of the actual abacus as both a teaching and computational aid. One hundred and sixty-nine Korean adults between the ages of 25 to 65 who had abacus training and its uses for a minimum of one year participated in the study. The study had two phases: a quantitative phase and a qualitative phase. The quantitative phase focused on the participants' perceptions of their training and use of the abacus as well as an assessment of their basic arithmetic competencies. This served as a context for a more in-depth analysis of their perceptions of, and thinking about, the use of the abacus in arithmetic operations obtained in the qualitative phase. All participants were asked and then answered a total of 6 questions regarding basic background information about their abacus training as well as their current use of the abacus for arithmetic computations in order to examine the extent of Korean abacus uses. The questionnaires included an assessment of participants' arithmetic computation skills. Among them, 59 adults were selected and interviewed to explore the extent of the "mental abacus" influence on their qualitative thoughts and tasks. From this research, it was expected that the study would provide information concerning the power of Stigler's mental abacus in mathematics and how it relates to Korean adults' daily life.

Apparently, although computation tools such as calculators and computers are widely available and convenient to use, the abacus is still used as one of the arithmetic tools by Korean adults. Considering the fact that the Korean national standard mathematics education curriculum has not included abacus training, although some commercial educational institutions included it,

the rate of learning the abacus and the period and frequency of its use tell us that abacus skill could affect the basic mathematics competency of Koreans. The data show that Korean adults who have been educated in abacus use provide self-reported evidence that they have the competency of mental computation and the ability to develop a mental abacus image depending on their period of frequency of abacus use. Further evidence indicates that most Korean abacus users who participated in the study report self-confident and accurate perceptions of their ability and arithmetic accuracy in doing basic arithmetic computations. Moreover, they are more confident and accurate in addition test problems than subtraction, multiplication, and division from the assessment results. It is concluded that mental abacus image occurrence may be associated with mental computation among some Korean adults who had learned to use the abacus in the past. Many of the Korean adult participants in this study who trained on the abacus can automatically conjure the mental abacus image and employ the skill during mental computation. The ease with which the abacus mental representation is activated during mental calculation is related to how frequently or intensively the adults practiced or exercised its use.

Among further findings, about the positive aspects of “mental abacus” use, most of the Korean adults in the study expressed opinions that there were positive influences of having learned the abacus, not only increased mathematics competency but also an additional “reward” in greater competency in other academic subjects and activities. This study reveals that intensive training with an abacus and the continuous use of an abacus can promote mental visualization and manipulation of the abacus during arithmetic computations, and result in a sense of positive effects from “mental abacus” use among those who have had sufficient opportunities to use the abacus.

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(1 Corinthians 15:10) But by the grace of God I am what I am, and His grace toward me did not prove vain; but I labored even more than all of them, yet not I, but the grace of God with me.

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DEDICATION

To Rev. Abraham Yoon-Sik Park, D.Min., D.D. and my beloved family

Chapter I

INTRODUCTION

Need for the Study

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) states that increased attention should be given to mental computation, the use of concrete materials and "number sense". Mental computation is thought to promote flexible thinking skills and number sense (Reys & Barger, 1994; Reys, 1985). In the National Council of Teachers of Mathematics President's message bulletin, Seeley (2005) emphasizes the important role of mental mathematics and calls for investing in mental mathematics proficiency because mental computation competency improves student problem solving skills. Reys and Barger (1994) claim that work needs to be done to help teachers plan curriculum in the area of mental computation. In their study, students learned to use knowledge of the abacus to promote mental computation strategies. Use of this technique as part of the curriculum has had success in elementary mathematics classrooms in Asia, and has a direct effect on students' spatial and concentration skills (Stigler, Chalip, & Miller, 1986; Hatano, Amaiwa, & Shimizu, 1987). Stigler, Chalip and Miller (1986) also assert that building the necessary skills to use the abacus encourages flexible thinking. Much research from Asia (Kojima, 1954; Hatano & Osawa, 1983; Hatano, Miyake, & Binks, 1977; Hatano, Amaiwa, & Shimizu, 1987; Ezaki, 1980; Shibata, 1994; Hanakawa *et al*., 2003) involving the use of the abacus and abacus-like configurations in the classroom shows that abacus skills have a direct effect on mental computation skills, spatial ability and general mathematics achievement.

After studying the effects of abacus training on Chinese children's mental calculations, Stigler (1984) explained that skilled abacus operators report visualizing a "mental image" of the abacus, and performing mental calculations by manipulating the beads on such "mental abacus". Stigler hypothesizes that the mental abacus is used by the brain as a virtual tool for performing mental arithmetic. Physical abacus users push beads up and down as they operate the abacus, while mental abacus calculators manipulate the imaginary abacus in their minds. Through extensive abacus training it appears that people can visualize the abacus image and exploit the abacus skills for mental calculation. The speed of this mental calculation is much greater than that of physical abacus calculation. Research suggests that the visual image processing employed by mental abacus experts can, with intensive practice, help early learners. Positive long-term effects have been reported (Hanakawa *et al*., 2003; Chen *et al*., 2006; Wu *et al*., 2009; Hu *et al.*, 2011). Stigler's results indicate that abacus training has both quantitative and qualitative effects on children's mental calculation skills. A subsequent study, "*Consequences of Skill: The Case of Abacus Training in Taiwan*" (Stigler, Chalip, & Miller, 1986), concludes that abacus skill training and its mental representations as well as transfer of abacus skills to other tasks demonstrate the multiple ways such training contributes to cognitive development.

Purpose of the Study

The purpose of this study was to determine the prevalence of using a "mental abacus" by adults whose mathematics education in Korea included extensive use of the actual abacus as both a teaching and computational aid. The research questions answered by the study are as follows:

1. What relationship exists if any between abacus training and reported use of the abacus:

a. For each of the age groups?

b. Specifically for the correlations among the reported preferences for use of arithmetic operation tools?

c. In relation to correlations among mental abacus use, mental computation confidence level, and confidence of accuracy in use of abacus?

2. To what extent do Korean adults report that the physical abacus was an important tool:

   a. With respect to their percepts of their experiences and uses of it?

   b. With respect to their responses to interview questions intended to elicit their reflective and analytical thinking about their use of the abacus?

3. To what extent do Korean adults report use of a "mental abacus" when performing mental calculations:

   a. With respect to their responses of their duration of learning and use of it?

   b. With respect to their response to interview questions intended to draw out if mental abacus image use is associated to duration of learning or the way that the physical abacus is utilized?

4. With respect to arithmetic competencies:

   a. Do Koreans who learned to use the physical abacus have arithmetic competency?

   b. To what extent do Korean abacus learners perform accurate arithmetic computations? If so, which arithmetic operation, if any, is performed the best?

   c. Is there any influence on computation assessment results by current preference for computation tool?

5. To what extent do Korean adults report that mental abacus use influences their thoughts and tasks when responding to interviews?
6. What is the status of Stigler's claims for the mental abacus in view of the reported experience of Korean adults in this study in response to interviews?

Procedures of the Study

The study required the identification of a sample of Korean-educated adults between the ages of 25 to 65. One hundred sixty-nine subjects participated in the study. The subjects were asked to complete an interactive questionnaire prepared by the investigator. The questionnaire included arithmetic activities to be performed by the subjects. The questions themselves were designed in clusters to encourage introspection and metacognition. An example of a set of questions from the questionnaire follows:

Multiply 279 by 9 in your head.

1) Did you use a memorized procedure to compute the product?

2) Would you have preferred to find the products:

(a) With pen and paper?

(b) With an abacus?

(c) With a calculator?

3) Which of the above aids did you think about as you computed the product?

It was anticipated that the interactive questionnaire would consist of approximately 10 items. A sub-sample of the 59 subjects who reported extensive reliance upon a "mental abacus" was selected for further study via interviews. The investigator developed an interview protocol that explored the origins of participants' abacus and implications of this reliance upon quantitative and qualitative tasks in their everyday and/or professional life.

Chapter II

BACKGROUND FOR THE STUDY

Theory of Cognitive Development

Cognitive theory is a learning theory of psychology that attempts to explain human behavior by analyzing the human thought process. It is concerned with the development of a human being's mental process. It also studies how mental processes influence human behavior development and how people understand and interact with the world through their learning (Piaget, 1973).

Piaget's theory of cognitive development is a theory of child intellectual development. Through his observation, Piaget developed a universal theory of intellectual development including four stages: the sensori-motor stage, the pre-operational stage, the concrete operational stage, and the formal operational stage. Development, in his view, is based on biological maturation. This is the foundation of research in cognitive process and ability. In Piaget's view, early cognitive development involves process based upon actions and later progresses into changes in mental operations. Piaget considered only qualitative changes in how children think as they gradually progress through these four stages and did not view children's intellectual development as a quantitative process. He proposed a discrete stage of development, marked by qualitative differences, rather than a gradual increase in the number and complexity of behaviors, concepts, and ideas. To Piaget, cognitive development was a progressive reorganization of mental processes as a result of biological maturation and environmental experience.

Piaget's theory is concerned with children, rather than all learners, and only focuses on development without addressing the learning of information or specific behaviors. In contrast, Vygotsky asserted the fundamental role of social interaction in the development of cognition, known as Social Development Theory (Vygotsky, 1978). He insisted that community plays a central role in the procedure of “making meaning”. Unlike Piaget’s notion that children’s development must necessarily precede their learning, Vygotsky (1978) argued that “learning is a necessary and universal aspect of the process of developing culturally organized, specifically human psychological function” (p. 80). In his theory, individual development cannot be understood without reference to the social and cultural context within which it is embedded. He strongly believed that higher mental processes in the individual have their origin in social processes.

Like Piaget, Vygotsky claimed that infants are born with the basic materials and abilities for intellectual development (Vygotsky, 1978). He refers to Elementary Mental Functions: Attention, Sensation, Perception, and Memory. Through interaction within the socio-cultural environment, these are developed into more sophisticated and effective mental processes and strategies that he refers to as Higher Mental Functions. He emphasizes the importance of cultural determinants in types of memory strategies that people develop. Also, he refers to tools of intellectual adaptations, which allow children to use basic mental functions more effectively and adaptively, and these are culturally determined (e.g., mnemonics and mind maps). Therefore, according to Vygotsky’s theory, intellectual adaptation to the culture in which a person develops has an effect on cognitive functions.

The zone of proximal development is another important principle of Vygotsky’s work in social cognition theory. He sees the zone of proximal development as the area where the most careful instruction or guidance should be given. This will allow the children to develop skills they will then use on their own in developing higher mental function.

One of the approaches to studying the development of cognition is to investigate a repertoire of tools and skills which vary by culture. According to Stigler, Chalip and Miller (1986), culture provides the repertoire of tools and leads the navigation of users with powerful cognitive routines and subjects for thought. They questioned how the acquisition of specific skills affects the development of general cognitive abilities by examining in Asian children both abacus skill training and the significance of mastering the abacus. They held that general cognitive abilities develop as a result of the acquisition of specific skills (Thorndike & Woodworth, 1901; Vygotsky, 1962, 1978; Ferguson, 1953, 1956, 1959). They also explored the nature and cognitive consequences of a particular culturally transmitted tool; namely, abacus calculation as practiced in Taiwan. In Taiwan, children study the abacus as part of the fourth- and fifth-grade mathematics curriculum. Among these children, some children develop what has been called a “mental abacus” (Hatano, Miyake, and Binks, 1977; Stigler, 1984) after special intensive after-school training programs. Using this method, they are capable of extremely rapid and accurate mental calculations. With this example of a culturally specific skill program, they studied several questions:

1. What does obtaining a skill such as mental abacus calculation mean?
2. How is this the skill acquired and represented mentally?
3. Which factors distinguish intermediates from experts?
4. How is this kind of mathematical knowledge related to other aspects of mathematics that children are exposed to in school?

In their view, cognitive development is best described as the gradual accumulation of a repertoire of cognitive tools and routines that may then transfer to other domains.

Stigler, *et al*. (1986) stated that application to problem-solving situations is not a part of abacus training. However, many recent mental abacus studies have reported the various effects of mental abacus training in mathematics education in Japan where the abacus training is a part of the mathematics curriculum, and in China where the value of the training program was recently reexamined. The abacus training helped children develop number sense, mental arithmetic and mathematical applications such as problem-solving as well as rapid and accurate calculation. Even though children who intensively receive and practice abacus training in Asia do so primarily for competition, both on a national and international level, these children apparently have developed more speed and accuracy in calculations. They also have developed self-confidence in mathematics by achieving basic arithmetic skills in mental computation. Eventually, the acquisition of abacus skills develops children's mathematics knowledge in cognition and helps their learning.

Mental Computation

Importance

Mental computation is a term regarding the act of performing calculations without the aid of external record-keeping procedures. Mental abacus usage is a way of managing and carrying out computations throughout the thinking process. The purpose of mental arithmetic is to develop meaningful strategies in each student to aid in the problem-solving process. Also, it helps children to learn number sense and comparisons of numbers, rewards a creative thought process, grows the skill of basic calculation, and suggests various alternative algorithms to solve problems and formulate mathematical techniques.

Reys and Yang (1998) describe number sense as "a person's general understanding of number and operations. It…includes…[using] this understanding in flexible ways to make mathematical judgments, and to develop useful strategies for handling numbers and operations…[T]he characteristics typically associated with number sense include using multiple representations of number, recognizing the relative and absolute magnitudes of numbers, selecting and using benchmarks, decomposing and recomposing numbers, understanding the relative effects of operations with numbers, and flexibly and appropriately performing mental computation and estimation" (p. 225-226). Many aspects of this definition can be extended beyond "number sense" to "operation sense", "symbol sense", "graph sense" and others. In all instances, students should be able to operate mentally, estimate, be flexible, transform, use multiple representations and choose strategies that are based on the particular question at hand. Every curriculum strand should have mental mathematics objectives.

The first step in building success in mental mathematics is to specify objectives clearly and to provide sample items. Adequate time should be taken early in the school term to discuss those objectives seen as being prerequisites for a course, and time should be allotted during the term to introduce new objectives as they are added. The objectives should be appropriately developed. The second step is to offer frequent, in-class practice.

According to Reys *et al*. (1995), mental calculation is recognized as important, valuable and useful in everyday living, and promotes and monitors higher-level mathematical thinking. There are several reasons for its importance. First, mental computation is a universally valued skill. It is frequently used in real-world problem-solving and provides essential prerequisites for doing computational estimation. Second, mental computation provides opportunities to engage in mathematical thinking, contributes to number sense and develops valued processes associated with problem solving (Sowder, 1992). According to Thompson (1999a), based on earlier literature, there are four reasons for teaching mental calculation:

1. Most calculations in adult life are done mentally.

2. Mental work develops insight into the number system ("number sense").

3. Mental work develops problem-solving skills.

4. Mental work promotes success in later written calculations.

The benefits of developing mental computation and related thinking strategies have been discussed (Cobb & Merkel, 1989; Josephina, 1960; McIntosh, 1990; Reys, 1985; Reys & Barger, 1994; Sowder, 1990). Reys (1985) stated that the major advantage of mental computation lies in helping students develop flexibility in dealing with numbers. When students engage in mental computation, they develop an understanding of the number system and its properties, and they often engage in the invention of alternative algorithms (Heirdsfield, 1996). Everyday mathematics problems are primarily solved by employing mental computation or computational estimation (Maier, 1977; Clarke & Kelly, 1989). Much research suggests that curriculum developers should pay attention to the role of mental computation as a valid computational method as well as the contribution it makes to mathematical thinking as a whole (Australian Education Council, 1991; Beberman, 1985; Cobb & Merkel, 1989; Cockcroft, 1982; Jones, 1989; Reys, 1985; Reys & Bargen, 1991; Sowder, 1990). According to Heirdsfield (1996), research results suggest that students are capable of formulating their own legitimate strategies for mental computation and computational estimation. Given the importance of mental computation and computational estimation in the context of number sense, it is imperative that methods which enhance and build upon natural skills be developed. To do this, it is essential that students regularly verbalize strategies in class and group discussion, and that they be able to choose from a variety of procedures. It is also important to recognize that, while particular strategies are meaningful to one student, teaching alternative and prescriptive strategies may cause confusion. Individual students may have developed idiosyncratic strategies that were meaningful to them, and the imposition of alternative strategies may not be effective for these students. Less emphasis on algorithms and more emphasis on developing students' legitimate, spontaneous strategies may result in better understanding. Schools effecting curriculum changes should recognize that students construct mathematical knowledge. Thus, teachers should allow students to participate in their own learning and to construct their own knowledge. Another major benefit of mental mathematics is that it facilitates learning many important structural topics such as mathematical properties. It also facilitates the understanding of inverse operations (Rubenstein, 2001).

The significance of mental mathematics is also highlighted in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), in other recent reports on mathematics education in the United States (Mathematical Sciences Education Board, 1990), and in similar reports from abroad (Australian Education Council, 1991; Cockcroft, 1982; Ministry of Education, 1989). Mental mathematics is also significant because number sense matures with experience (Reys & Barger, 1994). The teaching of mental computation is advocated in at least three countries (United States, Australia and Japan), but only in Japan is there consistent teaching of mental computation strategies and precedence given in the early years to mental computation over written computation (Australian Education Council, 1991; NCTM, 1989; Shibata, 1994; Reys & Barger, 1994).

In the school context, these purposes are translated into short-term curriculum objectives focused on skills, in particular the ability to calculate accurately and reasonably quickly over a range of demands deemed be appropriate to the age of the pupil. By the late primary years, this usually includes mental calculations with two-and three-digit numbers. Since expectations about skills are readily encapsulated in tests of mental calculation, the purpose of teaching mental calculation can become identified with achieving high levels of performance on such tests. Thompson (2001, p. 75) suggests that the phrase "mental calculation" is used in official documentation in England and Wales to stress the importance of the "used strategies" aspect of mental work. Thompson (1999b) suggests that mental calculation "strategies" are: "… about the application of known or quickly calculated number facts in combination with specific properties of the number system to find the solution of a calculation whose answer is not known" (p. 2).

According to Threlfall (2002), in order to achieve flexibility in mental calculation, solutions to problems would be better approached as specific examples of how particular numbers can be dealt with: how numbers can be taken apart and put together, rounded and adjusted. Teaching needs to focus on the number knowledge and understanding that is drawn on when calculating. Thompson (1999a) lists four "attributes" that in his view assist in the development of flexible mental calculation:

1. Good knowledge of number facts (appropriate to the student's age)
2. Clear understandings of what can be done legitimately with numbers, such as when the order can be changed and when it cannot, how numbers can be split and dealt with as parts, reverse operations, and the properties of zero
3. Well developed skills (appropriate to the student's age) such as counting skills, and automated mental calculations at a lower level
4. Positive attitudes, such as the confidence to "have a go at it" and not be put off by immediate lack of success

Threlfall (2002) asserts that teaching towards flexible mental calculation must include extensive development of factual knowledge about numbers, so that children will notice and observe a range of different things about the numbers when faced with a calculation, and are then better placed to develop an "easy" solution to it.

Emphasis on Mental Computation in Basic Mathematical Competency

Many researchers claim that practicing mental computation fits in with the *Curriculum & Evaluation Standards for School Mathematics* (NCTM, 1989) due to its importance to the grade school mathematics curriculum. For example, "In grades K-4, the mathematics curriculum should develop whole number computations so that students can use a variety of mental computation and estimation techniques" (p. 38). Teaching different techniques for mental computation helps students develop other types of thinking skills. According to Reys and Barger (1994), mental computation is increasingly thought to be a vehicle for promoting thinking, conjecturing and generalizing processes based on conceptual understanding. Sowder (1994) states "Individuals who are considered to be successful at mental computation and computational estimation are usually characterized as flexible, self-confident, tolerant of errors in estimates and disposed toward making sense of mathematics being undertaken and seeking reasonableness in results" (p. 140).

The *Curriculum & Evaluation Standards for School Mathematics* (NCTM, 1989) states that increased attention should be given to mental computation, the use of concrete materials and "number sense". Mental computation is thought to promote flexible thinking skills and number sense (Reys & Barger, 1994; Reys, 1985). In the NCTM President's message bulletin, Seeley (2005) emphasizes the important role of mental mathematics and calls for investing in mental mathematics proficiency because mental computation competency influences students' ability in solving problems. Reys and Barger (1994) claim that work needs to be done to help teachers plan curricula in the area of mental computation. In their study, students learned to use knowledge about the abacus to promote mental computation strategies. This technique was part of the curriculum, has had success in elementary mathematics classrooms in Asia, and has a direct effect on students' spatial and concentration skills (Stigler, Chalip, & Miller, 1986; Hatano, Amaiwa & Shimizu, 1987). Stigler, Chalip and Miller (1986) also assert that building the skills necessary to use the abacus encourages flexible thinking. Much research from Asia concerning the abacus and abacus-like configurations in the classroom shows that abacus skills have a direct effect on (i) mental computation skills, (ii) spatial ability and (iii) general mathematics achievement. Reys and his colleagues (1995) assessed Japanese students in grades 2, 4, 6 and 8 to determine whether Japanese students apply procedures they have been taught, or whether they formulate and apply their own thinking strategies for efficient mental computation. They concluded that the students develop a wide range of appropriate strategies for mental computation (when defined as self-developed strategies based on conceptual knowledge), and that such strategies should be a central focus of a computation curriculum.

Developing Tools for Mental Computation in the Curriculum

Interest in establishing mental computation in the mathematics curriculum is not a new phenomenon. In fact, emphasis on mental computation has a long sporadic history within the school mathematics curriculum in the United States (Reys & Barger, 1994). Recent initiatives in curricular reform and in the alignment of learning theory and instruction have heightened interest in mental computation. In the area of curricular reform, the availability of calculators has drawn attention to the need for focusing on a wider range of computation tools, including increased attention to mental computation and estimation as well as a decreased emphasis on traditional written algorithms. Research into children's learning processes has focused attention on constructivism, learner autonomy and the use of invented thinking strategies as methods for developing basic facts and computational procedures.

In addition to recognizing mental computation as an important skill, there is renewed interest in using it as a vehicle for promoting thinking, conjecturing, and generalizing. The contemporary view of mental computation contrasts sharply with current methods of developing pen and paper computation procedures in the United States. Today there is a call for mental computation (or thinking) strategies to originate from understanding and active problem solving rather than memorized rules or standard procedures. Although the benefits of a curriculum whose values encourage the development of mental computation are generally agreed upon, instructional treatments vary widely.

Abacus

History of the Abacus

The abacus, also called the counting frame, was one of the earliest and most effective calculating devices. Historically, the abacus was widely used and dates back to Mesopotamia, ancient Egypt, the Middle East, ancient Rome, ancient China, and ancient South and Central America. According to the Mathematical Association, a brief history of the abacus is as follows:

> The principles of abacus arithmetic were first developed in the Middle East over 5000 years ago by the Sumerian civilization. This civilization was probably the first to develop the subject of mathematics and their sexagesimal number system which used base 60 and

that is still used to measure time. In its earliest form, the abacus was probably a sand table with pebbles being used as counters. From this form, it evolved to its modern design with beads moving on rods. This version dates from the Greek and Roman civilizations. The abacus in its various forms continued to be used in Western Europe until the Middle Ages (Maxwell, 1981, p. 2-3).

The earliest abaci were made using sand and employing stones as the counters. From this ancient form, Rome developed the abacus board, which had long grooves and beads instead of pebbles. The Chinese abacus, known as the *suanpan*, was first described in a book written by Xu Yue of the Eastern Han Destiny, namely *Supplementary Notes on the Art of Figures* written about the year 190 A.D. The exact design of such abacus is not known. It is different from the Romans' version. Due to the similarity between the Roman abacus and the Chinese abacus, people suggest that one must have inspired the other; however, there is no direct connection that can be demonstrated. The Chinese abacus usually has more than seven rods. It has two beads on each section of rod in the upper deck and five wooden beads on each in the bottom deck for both decimal and hexadecimal computation. The beads are counted by moving them up or down toward the beam. The modern *suanpan* has 5 (4 bottom + 1 upper) beads, colored beads to indicate position and a clear-all button. *Suanpan* arithmetic was still being taught in school in the People's Republic of China into the 1990s. However, when calculators became available, school children's willingness to learn the use of the *suanpan* decreased dramatically. Still, many parents continue to send their children to private tutors or schools as well as government-sponsored after-school activities to learn bead arithmetic as a learning aid and a stepping stone to faster and more accurate mental arithmetic, or as a matter of cultural preservation. Speed competitions are still held (Martzloff, 2006).

The Japanese abacus is called the *soroban*, and was adapted from the Chinese abacus through Korea around 1600. Like the present-day Chinese *suanpan*, the *soroban* from long ago had two beads above the beam and five below. In Japan, the form and operational methods of the abacus have undergone some improvements. Toward the close of the 19th century, it was simplified by reducing the two beads above the beam to one. Finally, around 1920 to 1930 it acquired its present form which looks more elegant, is much smaller and sometimes has more columns. But those with fewer columns (13 or 15) are most often used. The *soroban* is taught in primary schools as a part of lessons in mathematics so that the decimal number system can be perceived visually. People who become proficient in using the *soroban* almost automatically become adept at mental calculation. As a part of *soroban* instruction, intermediate students are asked to do calculations mentally by visualizing the *soroban* and working out problems without trying to figure out the answer beforehand.

The Korean abacus is called *jupan*, and was imported from the Chinese around 1592. Until 1900, the Chinese abacus had (5 bottom + 2 upper) beads on each rod and their calculation method was used only by merchants. After the Cho-Seon Abacus Association was established in 1920, the Japanese simplified abacus was introduced to Korea and it became the current Korean abacus. From 1932, the current Korean abacus with (4 + 1) beads was accepted. The *soroban* is still in use today. It is the most widely used abacus (Stigler, 1984). After 1945, abacus education was adopted only in the curriculum of Commercial High School. In 1960, the Korea Department of Education established a program in standard abacus education (Level 1~7) where students achieved higher levels as their abacus skills progressed. Each level was achieved by passing a test under time constraints. The abacus level tests were regularly administered. Moreover, the abacus was taught as a gifted education program in elementary and middle school. Specialized programs for after-school abacus training were also operated and widely available throughout Korean society. In order to promote the abacus-training program, the Korean government issued a license and certificate to the gifted program abacus trainers. Therefore, many students became involved in abacus competitions that tested speed and accuracy in computations. However, this fervent trend of abacus education and its uses has decreased since the introduction of calculators and computers. Currently, the importance of developing mental arithmetic skills for young students through abacus training in Korea is a much discussed topic.

The Configuration of the Korean Abacus – *Jupan*

The Korean abacus, the *jupan* with (4 + 1) beads on each rod, has the same configuration as the Japanese abacus, the *soroban*. A crosswise bar divides the *jupan* into two parts. The upper part is called "heaven" and consists of five-value beads. The lower part is called "earth" and consists of one-value beads. The zero-position of all abaci is when the upper beads are moved up and the lower beads are moved down. The beads represent a number only if one or more beads are moved toward to the beam.

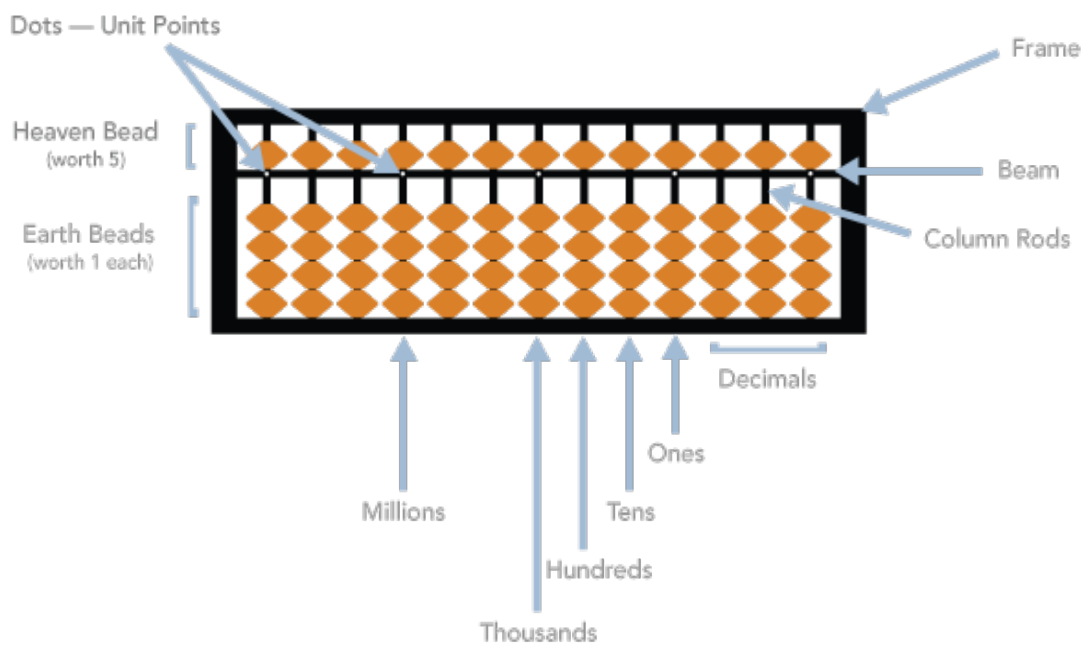

Figure 1. Korean abacus *Jupan*

In principle, the rods (starting from the right side and continuing to the left side) represent the "ones", the "tens", the "hundreds" and so on. The user has to decide at which rod the "ones" should be. If there are numbers with values less than one, it is convenient to put the column with "ones" just in the middle of the abacus. That way, numbers can grow or decrease on both sides. But if there are only numbers with values from one and higher the user can start just at the right side of the frame developing the calculation to the left side.

Fingering technique is important when using the abacus. The technique is simple: the thumb is used to add beads (move them upward) in the lower part of the abacus, and the forefinger to subtract beads (move them down). In the upper part of the abacus, the forefinger is used to both add and subtract beads. To add one, the top bead of the lower section is slid toward the crossbar (moving it upward). To add five, the bead in the top section is moved toward the crossbar, downward. Unlike adding with pen and paper, where calculation is typically started on the right side and progresses to the left, in abacus usage the learner starts on the left and moves to the right. The abacus can be used to perform the basic operations (addition, subtraction, multiplication and division) as well as extraction of square and cube roots.

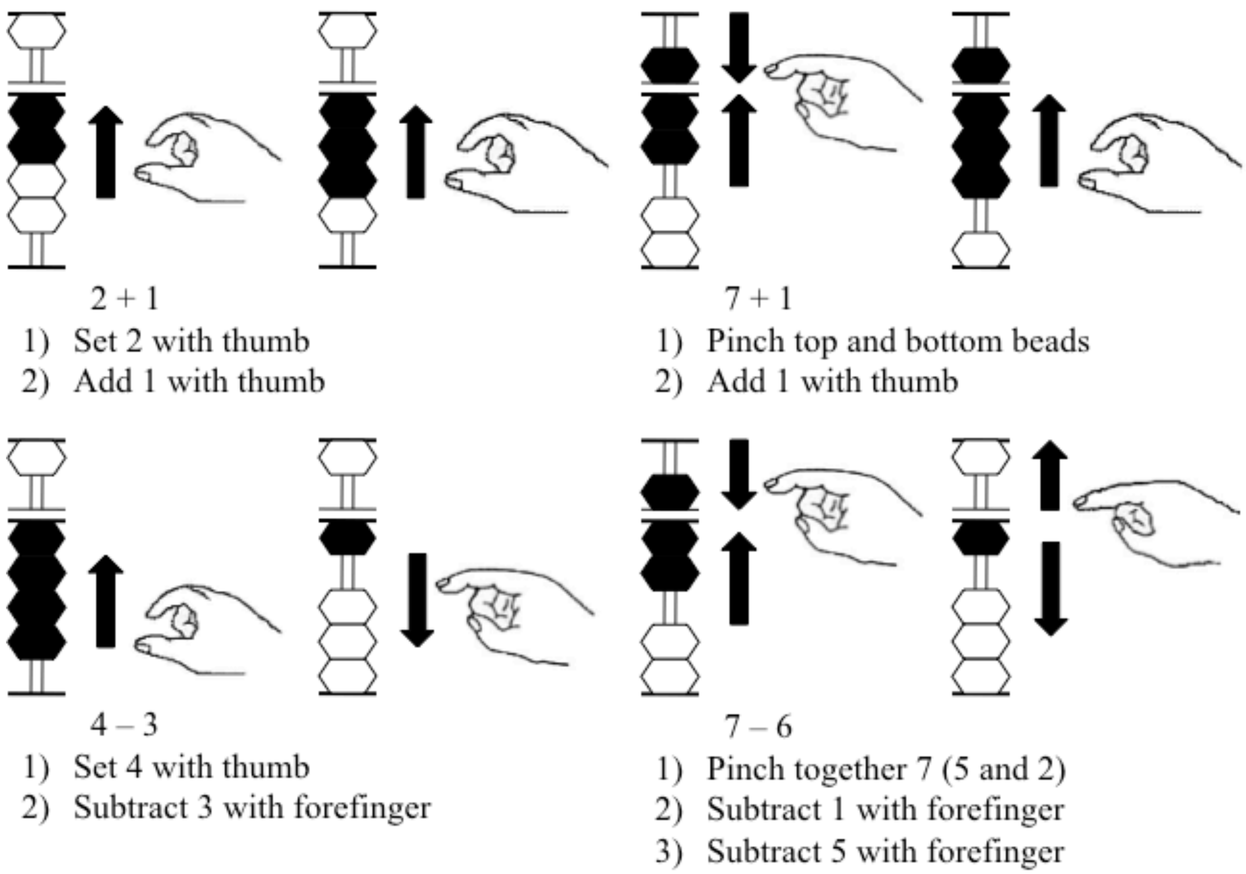

Figure 2. Calculation by abacus

Since the advent of computers, the abacus has started to shift from being used purely as a calculating device to being a useful tool in general mathematics education. The *jupan* represents numbers exactly as they are written. Each column can contain zero; i.e., the numbers run from 0 to 9 not from 1 to 10. Practicing with the *jupan* improves the understanding of basic ideas such as place value and number bonds. Furthermore, *jupan* beads help to create concrete images of numbers, with which one can perform mental calculations. As there are only five beads per column, the *jupan* reduces the amount of information the user's mind has to retain. The visual representation of numbers by patterns of beads is easy to remember and helps with mental calculation.

According to Stigler (1984), he chose to work with the *soroban*, the Japanese abacus, because it best represents our number system. In particular, one can only write the numbers 0 - 9 in any particular column on the *soroban*. This is the same as in the base ten number system. It is important to note that the abacus was not developed as an educational tool, but was developed as a way to indicate numbers and to compute quickly and accurately. This is significant because many manipulations of the abacus in the classroom are introduced as novelties or are used just to introduce a topic. The abacus can be used as a tool for discussing concepts in mathematics, but it also can be used as a serious computational tool for finding answers to problems. Although the abacus has been replaced by computers and calculators, it is still widely used in elementary classrooms for mathematics education in Japan and China (Hatano & Osawa 1983).

The Abacus as a Means to Develop Skills

Abacus training is useful for developing number representations for children. An abacus enables students to see concepts concretely and bridges the gap between concrete and mathematical symbolism (Metallo, 1988). It has also been found to help students see the relationship between numbers and abstract concepts (Tang, 1993). For example, in the early grades there is much emphasis on rote and sequential counting (Sharma, 1987). Most children while counting sequentially do not attend to the change that takes place in the representations of numbers when moving from 9 to 10 to 11. These changes are important for children to discover, since the movement involves an understanding of place value. In manipulating the beads of an abacus, children usually are able to appreciate the importance of that movement.

According to Spitzer (1942), there are absolutely significant characteristics related to the representation of quantities on the abacus, as follows:

- The markers (beads) can be used to represent various concrete objects and to aid the children in securing an understanding of these efficient abstract uses of number.
- The value of a number depends on its position—consider how easily position can be explained on an abacus.
- Closely associated with the ideas discussed in the preceding paragraphs, the abacus can be used to illustrate (e.g, the idea of a placeholder or the function of zero).
- The function of the number system illustrated by the abacus is the idea of collection (p. 450-451).

To develop abacus skills, the Japanese government set the training as part of the elementary school curriculum. The Mathematical Association of America (1975) reported:

> Indeed, in Japan, where abacus lessons are mandatory for third and fourth grades of the primary school, a skilled student can add and subtract faster than by pressing the buttons on a calculator, the light wooden beads are skillfully flicked to give the answer. To gain high skill in the use of the abacus, Japanese children attend private schools and take nationally organized examinations. Even though elementary schools in Japan have a mandatory course just for the abacus, it is not good enough for making abacus experts (p. 18).

Stigler *et al*. (1986) also reported: “Some students who pursue additional training attend after-school classes as private lessons. Children who pursue this additional abacus training use their skill primarily for competition, both on a national and international” (p. 448).

Stigler’s Mental Abacus

Abacus Skills and Mathematics Competency

In general, Asian students perform better on international standardized tests in mathematics than students in the United States (Beaton, 1996). D’Ailly (1992) cites several possible reasons for this difference, one of which is the use of the abacus in the early grades. Chang (1984) states, “One of the unique features in the Chinese elementary school is the development of the skills needed to use the abacus” (p. 16). In Japan, the *soroban* plays a vital role in early mathematics education. Metallo (1988) argues that the *soroban* is the main reason why elementary or secondary or high school children in Japan consistently score higher on the standardized math achievement test.

According to Vygotsky (1978), general cognitive abilities develop as a result of the acquisition of specific skills. In other words, cognitive development is the gradual accumulation of a repertoire of cognitive tools and routines that may then transfer to other domains. The effect of learning the *soroban* is in the area of mental computation. When students learn to compute using the *soroban* in Japan, mental computation is part of the curriculum. According to Stigler, Chalip and Miller (1986), “In Japan and Taiwan, for example, there are efforts to train children in mental computation, both in school and in extracurricular programs. An emphasis on speed of mental calculation is included in the regular school curriculum beginning in the first grade” (p.

452). Students practice and perform mental computations by visualizing the beads of the *soroban* in their minds. Stigler, Chalip and Miller (1986) report:

> From the American point of view, the goings-on at Donyan Buxiban (an elementary school abacus program in Taiwan) are quite impressive. Children go there after school and usually sit on long benches in large rooms filled with students. A typical mental abacus calculation exercise begins when the teacher, standing at the front of the room, raises his hand, whereupon the room falls silent in anticipation. The teacher then reads aloud a list of 20 three-digit numbers as fast as he can, so fast in fact that the numbers are almost unintelligible. After the last number is read, every hand in the room shoots up, and the teacher calls on one child to report the sum. Usually the child's answer is right (p. 454).

Stigler (1984) compared the differences of mental computation skills between fifth graders in Taiwan and adults in the United States. The fifth graders in the study had abacus skills, and they apparently used a mental image of the abacus to perform mental computations. The American subjects were college students who were recruited for the study because, the same grades of American as Taiwanese were unable to perform mental computation. In every case, the Taiwanese students were faster than the Americans. Stigler (1984) concludes his study by stating, "perhaps the most powerful tools a culture can provide to the mental development of a child will come in the form of specialized mental representations that are passed down through education" (p. 175). In a later study, Stigler, Chalip and Miller (1986) studied 714 fifth graders at Dongyuan Elementary School in Taipei, Taiwan. The research was narrowed to four factors that might affect learning of abacus skills. Of these, only the amount of time spent practicing with the abacus had a direct effect. A follow-up study conducted a year later at the same school investigated how abacus skills affect other cognitive domains. Stigler, Chalip and Miller (1986) found "Abacus skill enhances mental calculation, which in turn, transfers to component mathematical sub-skills of numeration and calculation. These sub-skills equally affect mathematics achievement. The transfer from abacus skill to numeration has a further indirect impact on mathematics achievement through numeration's effect on mathematical applications. In the case of the abacus, at least the improvement of sub-skills seems to be a consequence of learning an effective mental analog, the mental abacus" (p. 474).

Mental Abacus

In addition to using the physical device, mental abacus users are trained to visualize an abacus and to move imaginary beads on this abacus in order to perform arithmetic calculations. Many users appear to move these imaginary beads using their hands, and thus move their hands in the air as they perform calculations, suggesting that motor representation somehow interfaces with the number representation created by the mental abacus. This mental abacus is commonly used for calculations such as addition and subtraction, but with practice users can also learn routines to perform multiplication and division or even calculation of square and cube roots. Because of its incredible speed and accuracy, the mental abacus compares favorably to other methods of computation, including electronic calculators (Kojima, 1954) and alternative systems of mental arithmetic.

Hatano and Osawa (1983) authored a pioneering work concerning the memory function of three *soroban* grand champions. They demonstrated that

(i) While the champions could recall a larger number of digits (13 to 16), they could not recall between five and nine alphabet letters and fruit names;

(ii) Their digit memory was disrupted more by concurrent visual-spatial tasks such as matching of figures than by verbal tasks such as answering questions;

(iii) They could not recall the digits they stored at all after they were told to erase them. From these results, they proposed that *soroban* grand masters have a specific ability to represent visually and spatially a long enough for calculation number of digits;

Hatta (1985) and Hatta and Ikeda (1988) examined the hemisphere functioning of *soroban* experts with a time-sharing task devised by Kinsbourne and Cook (1971), where both sequential finger-tapping and mental arithmetic tasks were given simultaneously. They demonstrated that *soroban* experts showed a larger reduction in performance when tapping with their left hand than when using their right hand in both auditory and visually administered mental calculation problems, whereas control subjects showed the reverse tendency. From these findings they suggest that the right hemisphere engages more in mental calculation than the left hemisphere of *soroban* experts, while the position is reversed in non-expert subjects. Hatta *et al.* (1989) found *soroban* experts have a superior ability to memorize digits compared with ordinary people. Their experiments showed that *soroban* experts use a mental *soroban* to improve digit memory retention, but ordinary subjects who have negligible *soroban* learning experiences do not employ such an imagery-based strategy.

Along with astounding research on abacus experts with a “mental abacus” in the aspects of cognitive representation (Hatano, Miyake, & Binks, 1977), Stigler (1984) investigated the human intelligence and the role of culture in mathematics learning. He was interested in studies of cultural differences in mathematics in human’s own learning and mental representation of mathematics. Stigler focused on the context of abacus learning and the transformation from a physical abacus to a mental abacus image in order to perform accurate arithmetic operations with great speed, which is achieved from a vast amount of practices and efforts in China, Taiwan, and Japan. Stigler assumed that mental abacus acquisition is necessarily abstract and free of content in domain-specific, culture-specific knowledge representation in thought processes. Stigler claims that culture-specific representations of number do not merely influence the development of mathematical knowledge, but in fact remain part of that knowledge. His hypothesis is that the abacus maintains its abacus-like qualities when one internalizes it as a tool for thought. Stigler questioned, "What evidence could we find that would reveal effects of the abacus per se, above and beyond the simple effects of practice? What aspects of the abacus were preserved when incorporated into the cognitive system?"

Stigler (1984) studied abacus skill focusing on the "mental abacus" and also studied the consequences of mental abacus skills with his colleagues (Stigler, Chalip, & Miller, 1986). He was interested in the phenomenon of mental abacus calculation because it was a case in which a culturally specific activity employing a culturally specific technology appeared to have both a quantitative and qualitative effect on the cognitive processes of individuals. He wanted to document the calculating speed and accuracy of experts using a mental abacus. He also wanted to understand the nature of the mental abacus, how it is represented mentally and how it functions.

Stigler (1984) showed that calculation times increased with the increase of addends in *soroban* novice children, but remained constant in *soroban*-expert children. This occurred when subjects were asked to answer questions about intermediate state calculations that are unique to *soroban* calculation, and the amount of time required to answer was a linear function of the position of the intermediate state. From these findings, he proposed that skilled *soroban* operators can visualize a mental image of the *soroban* which is analogous to the actual one.

Furthermore, once people have mastered the way in which the mental abacus is employed to improve number processing, they can use it thereafter without special training. Hatta *et al.* (1989) suggested that *soroban* experts utilize imagery to conduct calculation tasks, that they use their right hemispheres to engage in mathematical activity as a result of a substantial amount of practice, and that their superior competence compared with ordinary people arises from this.

The Korean Mental Abacus

What are the nature and cognitive consequences of use of the *jupan* as a tool for mathematics education in terms of enhancing mental computation; a controversy has arisen as to whether the *jupan* is an effective tool for developing mathematics competency. However, only a few studies of this tool are currently available (Lim, 2006; Kim & Gang, 2006; Sung, 2008; Shin, 2012). The present study was aimed at the prevalence of Korean adults' abacus usage and its effects, and the extent of its usage and relevance for mental calculation as expressed by those who had already been trained in use of the abacus. In addition, this study examined the relationship between Stigler's mental abacus and Koreans' abacus expertise and the effectiveness of the mental abacus as a tool for mathematics education.

Chapter III

METHODOLOGY

This study used a combined quantitative and qualitative design. In the first section of this chapter, the quantitative methods used in the study are described. In this study, a survey was used to examine the uses and effects of the abacus and mental abacus skill training on Korean adults' mental computation. The effects on mental computation were measured by examining results of arithmetic computation tests, each arithmetic computations test contains two questions. The following topics are addressed in this section of Chapter III:

(i) Population and selection of sample,

(ii) Data collection procedure

(iii) Data analysis, and

(iv) Limitations of the study

The second section of this chapter discusses the qualitative methods used, and the third section discusses the limitations of the study.

The six research questions for this study are as follows:

1. What relationship exists if any between abacus training and reported use of the abacus:
    a. For each of the age groups?
    b. Specifically for the correlations among the reported preferences for use of arithmetic operation tools?
    c. In relation to correlations among mental abacus use, mental computation confidence level, and confidence of accuracy in use of abacus?

2. To what extent do Korean adults report that the physical abacus was an important tool:
    a. With respect to their percepts of their experiences and uses of it?
    b. With respect to their responses to interview questions intended to elicit their reflective and analytical thinking about their use of the abacus?
3. To what extent do Korean adults report use of a "mental abacus" when performing mental calculations:
    a. With respect to their responses of their duration of learning and use of it?
    b. With respect to their response to interview questions intended to draw out if mental abacus image use is associated to duration of learning or the way that the physical abacus is utilized?
4. With respect to arithmetic competencies:
    a. Do Koreans who learned to use the physical abacus have arithmetic competency?
    b. To what extent do Korean abacus learners perform accurate arithmetic computations? If so, which arithmetic operation, if any, is performed the best?
    c. Is there any influence on computation assessment results by current preference for computation tool?
5. To what extent do Korean adults report that mental abacus use influences their thoughts and tasks when responding to interviews?
6. What is the status of Stigler's claims for the mental abacus in view of the reported experience of Korean adults in this study in response to interviews?

Quantitative Phase

Research Questions

The research questions addressed in the quantitative phase as listed in the preceding section were: 1a, b, c; 2a; 3a; and 4a, b, c.

Population and Selection of Sample

The quantitative data collection was conducted in banks, universities, offices and private institutions of abacus education in South Korea. A flyer was used to invite participants. The flyer (Appendix A) was distributed in major banks, universities and offices in major cities including Seoul, Incheon and Pusan. With the cooperation of the Korean National Abacus Mathematics Education Association (KNAMEA), the flyer was exhibited throughout South Korea by the use of posters and the Internet. Because the focus of this study was to assess the prevalence of use of the abacus and its effect among Korean adults who had been educated in the use of the physical abacus, the locations where appropriate participants could be found were limited. The study specifically focused on adults aged from 25 to 65 who had abacus training at least one year. Each of the participants answered a questionnaire prepared by the researcher to obtain basic information (Appendix B). The respondents replied to the questionnaire in a common meeting room with guidance from the researcher or by responding individually using forms sent to them. In the latter case, the researcher sent questionnaires to the survey participants in electronic form by email, or in paper form by fax or photocopy.

One hundred and sixty-nine adults from South Korea properly replied to the questionnaire and were participants in the study. All were considered healthy adults, without any mental or physical health problems or conditions. A total of 190 participants originally responded to the questionnaire, but not all replied appropriately. Twenty-one of the 190 who received abacus education for six months or less skipped questions in the questionnaire, and their results were excluded in the final data analysis. The remaining 169 participants were included in the study. This study assumed a similar abacus training for all of the participants. The study population includes Korean adults aged from 25 to 65 who had experience in using a physical abacus for more than one year. With the advice of the Korean National Abacus Mathematics Education Association (KNAMEA), the study participants were categorized into one of three levels of expertise in abacus skill according to grades established by the Korean National Abacus Mathematics Education Association (KNAMEA): advanced level, intermediate level, and beginner level. Participants who had used the abacus for more than 10 years were categorized as an advanced level group, those who had more than three years of experience in learning and using abacus skills but less than 10 years were categorized as an intermediate level group; and these with less than 3 years of experience in learning and using abacus skills were categorized as a novice level group. According to the Korean National Abacus Mathematics Education Association (KNAMEA), the period of physical abacus use is positively associated with the level of the abacus skill and mental abacus and mental computation skill (Kim & Gang, 2006; Shin, 2012). Each of the above three levels corresponds to a level of expertise in a physical abacus skill in South Korea.

Instruments and Data Collection Procedures

Questionnaires. All 169 Participants who agreed to participate answered a total of six survey questions (Appendix B) regarding basic background information about their abacus training as well as their current use of the abacus for arithmetic computations.

To address Research Question 1a, two questions were asked of all participants in the questionnaire, appendix B: "Please write down your age." and "Please provide your length of abacus training." In order to determine the association between the length of abacus training and practice, the numbers of years of abacus training along with years of use were elicited in the open-ended question of the survey. Related to that, Question 1 and 5 (Appendix B) were asked to the study participants in order to address Research Question 1b. These two questions also could provide evidence of the prevalence of Korean adults' use of the physical abacus and its influence of prevalence on Korean mental abacus as related to Research Question 3. Information for Research Question 2a was obtained from Survey Questions 1 and 5. They addressed how frequently the participants use a physical abacus today. Additionally, Question 5 inquired about the types of computational tools that are used by the participant. To address Research Question 4a, the participants were questioned how confident they were about their mental computation. In order to seek evidence for Research Questions 4b and c, an arithmetic mental assessment test was used (Appendix C).

After evidence was obtained using the Questionnaire Questions # 1 and # 5, additional questions, formatted as Likert-type items, were examined to address Research Questions 2 and 3. They addressed whether participants regularly use the abacus or not (Likert item 1). The participants were asked whether they use a mental image of an abacus when they do mental computations (Likert item 2). Likert item 3 asked if they feel comfortable when performing a mental computation. Finally, Likert item 4 inquired "How confident are you about your answer when you use a mental abacus?" These questions (except for the first question) were formatted as a five-point, Likert scale. The Likert response options to each question were:

1= never, 2 = rarely, 3 = sometimes, 4 = usually, 5 = always.

The final question (item 5) in the questionnaire addressed what kind of computation tool the respondent preferred from among this list:

Do you use a calculator?

Do you use an abacus?

Do you need to have a pen and paper for your calculation?

Or, do you not need to have any other tool for your calculation?

Computation skill test. Research Question 4 was addressed using a basic arithmetic competency test containing a set of eight basic computation problems (Appendix C). The problem set consists of four basic arithmetic calculations: addition, subtraction, multiplication and division. Each computation set contains two questions, containing eight items in total. Participants were assessed on two digit- and three digit- number calculations in each of all four parts. Problem sets were generated by the investigator. The participants were asked to compute the results to decimal places, not round off to the nearest whole number. The correct answers of the assessment set are natural numbers or rational numbers with one decimal place. All numbers of the problem sets are whole numbers and correct answers are all positive natural or rational numbers. There is no problem for which the answer is a negative integer or no fraction.
The eight problem sets were presented to each subject in the same consecutive order, one through eight.

Each data collection, consisting of two sessions, was completed in one day. All participants, including experts, intermediates and novices, answered all six items on the questionnaire and all eight of the problems with their available tools. All problems were presented using a paper survey form. The procedure was easily managed by all subjects.

Test scoring and preparation for data analysis. In order to address Research Question 1a, all responses from the first open-ended question were entered into Excel. Age groups were categorized into four groups, aged from 25-34, 35-44, 45-54 and 55-65.

To examine Research Question 2 and 3, Each respondent's responses to the Likert scale were scored by taking the total sum of the response values (1, 2, 3, 4, or 5) to all of the survey items. Additionally, all responses on each question were counted and reported as total frequency of each response value and percent of each response since all the response values are ordinal variables. All raw data from Excel were imported into SPSS Statistics, 22.0.0, and the total frequency and the percent of the frequency were obtained by SPSS Statistics, 22.0.0.

For the competency test items (Appendix C), items 1 to 3 were scored by assigning a score of 0 if incorrect or a 1 if correct, but for item 4 on division, each sub-question was scored as 0, 0.5, or 1, depending on how accurately the respondent completed the computation: zero for incorrect, and 0.5 or 1 depending on how accurately they reported the number of decimal places.

Data Analysis and Statistical Tests

The quantitative data were analyzed using a Microsoft Office Excel 2010 and a SPSS statistical package to address Research Questions 1a 1b and 1c, 2a, 3a, and 4 as explained more fully below. Once the data were collected, the data were placed into an Excel file. All data were imported into and analyzed by the most recent updated version of SPSS Statistics 22.0.0.

To address Research Question 1a, means and standard deviations for Years of abacus learning and Years of abacus training within each age group were obtained using Excel and SPSS. Additionally, a Wilcoxon matched pair test was used to determine if the data for the two extreme age groups (1 and 4) were statistically significant for each set of data: "Years of abacus learning" and "Years of abacus use;" because, the variances for the two groups were not comparable."

A Pearson correlation (SPSS) was obtained between the period of abacus training and its use for research question 1b. Also, a multiple Pearson correlation analysis was used to examine relationships among pairs of computational tool use (Appendix B. survey question #5). To address question 1c, multiple correlation analysis (SPSS) was used to examine the relationships among mental abacus use, mental computation confidence level, and confidence of accuracy in use of abacus.

To answer Research Question 2a, the frequency reported and the percentage of current manual abacus use were obtained using an SPSS Statistic program to describe the characteristics of the users.

For Research Question 3a, Pearson pairwise correlation coefficients were calculated using SPSS for relations among: 1) mental computation vs. frequency of abacus usages; 2) frequency of abacus usage vs. level of comfort with mental computation using the abacus; and 3) frequency of abacus usage vs. confidence of the user's mental computation accuracy.

For Research Question 4a, the responses to questionnaire item 4 in Appendix B were analyzed using descriptive statistics (frequency, percent, and cumulative percent). For the second part of question 4a, the number of participants who obtained a perfect score, also expressed as a percent, was obtained for each of three groups: 1) All calculator users, 2) All abacus users, and 3) All other tool users. For Question 4b, the mean and standard deviation of the scores for each question (1-4) were obtained using SPSS. For Research Question 4c, a nonparametric, Wilcoxon signed ranks test was used to examine if accuracy of performance on each arithmetic operation task (addition, subtraction, multiplication and division) differed significantly from one another, because the data distribution did not meet the assumptions of a parametric test.

Qualitative Phase

Research Questions

The research questions addressed in the qualitative phase are as follows:

2b. To what extent do Korean adults report that the physical abacus was an important tool, with respect to their responses to interview questions intended to elicit their reflective and analytical thinking about their use of the abacus?

3b. To what extent do Korean adults report use of a “mental abacus” when performing mental calculations, with respect to their response to interview questions intended to draw out if mental abacus image use is associated to duration of learning or the way that the physical abacus is utilized?

5. To what extent do Korean adults report that mental abacus use influences their thoughts and tasks when responding to interviews?

6. What is the status of Stigler’s claims for the mental abacus in view of the reported experience of Korean adults in this study in response to interviews?

The qualitative design was a case study. A case study implies that the data collection focuses on one phenomenon (McMillan, Schumacher, & Singh, 1997). In this study, the phenomenon investigated was the implementation of the abacus curriculum on Korean adults during school curriculum and its influences in their lifetime. Further, “case studies are particularly valuable when the evaluation aims to capture individual differences or unique variations from one program setting to another or from one program experience to another” (Patton, 1990, p. 54). In particular, the investigator in this study investigated the effects the abacus training had on the study participants’ mathematics abilities and mathematics strategies in South Korea. In Partton’s (1990) study, the investigation included data collection in the classroom for a one-month period, and the researcher used a combination of data and data collection devices in order to strengthen the study (Patton, 1990). In this study, the researcher triangulated the results using three sources of data: 1) participants' interviews, 2) close observations of mental abacus test performance by a Korean grandmaster of abacus and also by abacus teachers who belonged to the Korean National Abacus Mathematics Education Association (KNAMEA), and 3) document analysis. As Patton states, "Combinations of interviewing, observation, and document analysis are expected in much social science fieldwork" (Patton, 1990, p. 187).

In order to explore the research question 3 and 4 in depth, participants in this study were from various places in South Korea who had period of learning and use physical abacus at least three years. According to several pervious abacus studies in Korea (Lim, 2006; Kim & Gang, 2006; Sung, 2008; Shin, 2012), those who had learned and practiced more than two years are more likely to draw upon use of the mental abacus image. Based on the data, 59 adults from different locations were chosen from among the 169 original study population to participate in the qualitative interview. Participants were chosen within the 25- to 65-age range, without consideration of age, or gender, or occupation. Korean adult subjects were chosen by the investigator. The investigator used particular criteria to select adults who participated as follows: the participant must have at least three years of abacus training and their abacus practices, and also high scores on their assessment test. The assessment score from the qualitative test (Appendix C) is one of the positive indicators for abacus performance and influences of abacus education on abacus skill, and also served as a selection criterion for participation in the qualitative study. Based on previous research by Stigler (1983) and Hatano's study (1977), high performance evidence on arithmetic computation while using the mental abacus image was also documented. Therefore, assessment scores from the quantitative study was one of the appropriate selection criteria for participants in the qualitative phase of Korean adults' use of the mental abacus.

Participant Interviews

Fifty-nine adults were involved in this part of the qualitative portion of the study. They were selected by the investigator from 169 survey participants. The investigator selected Korean adults who had over one year of abacus training, over three years of abacus practice and perfect scores on the quantitative test. At first 107 participants (48 male and 59 female) who were trained in or used the abacus for over 3 years were selected from a total of 169 participants. Then, 65 (35 male and 30 female) participants were chosen from those 107 participants based on maintaining a balance in the ratio of genders. Finally, 59 interviewees were selected including two extensive abacus experts by the researcher according to their age, occupation, and the length of abacus use. The investigator conducted an interview with each adult (the interview questions are included in Appendix D). During the course of each interview, adults completed written and oral tests following the interview questions. The purpose of these interviews was to investigate the extent of the "mental abacus" influence on their qualitative thoughts and tasks. The written tasks in the selected participant interview process were focused on the prevalence of mental abacus skills and their effects on the subjects' lives. These tests were written by the investigator and they are included in Appendix D.

Once the participant interviews were completed and transcribed, the researcher sought out differences by comparing responses to each question and documented the responses. For example, the associations between their occupations and their thoughts about the mental abacus use were examined. Also, participants who intensively use the mental abacus were examined in depth regarding their strategies and mental computation task performance by recording the accuracy of the assessment results and speed of performance as was done in previous studies (Stigler, 1984; Hatano, Miyake and Bink, 1977).

Summary

In the quantitative design, Korean adults' perceptions about their use of, and their arithmetic computation ability with, the abacus was obtained by questionnaire evidence and a competency test. The researcher investigated the extent of physical abacus uses and other tools for basic mathematical competence tasks and the participants' preferences for mathematical computation tools. The extent of mental abacus use for the same tasks was examined. This served as a general context for the more in-depth analysis of their perceptions of, and thinking about, the use of the abacus in arithmetic operations obtained in the qualitative phase involving a case study. In the qualitative study, participants who use the mental abacus were selected by the researcher from among all of the participants in the quantitative phase. In the case study, the participants performed written work and interviews. During individual interviews, they were examined regarding the extent of their mental abacus use and its effect on their lives. By using both quantitative and qualitative methods, the researcher sought information regarding the prevalence of mental abacus use as well as physical abacus use and its effect on Korean adults. From this work, it was expected that the study would provide some fundamental information concerning the power of Stigler's mental abacus in mathematics and how it relates to Korean adults' daily life.

The Limitations of the Study

One limitation of this study was the voluntary nature of the participants in the sample. Although the sample consisted of adults in many locations, the participants volunteered to be in this study and they were given the choice of being in the study group. Therefore, this study was not random.

Recruiting differences are another possible limitation of this study. It is possible that there were differences in the ability levels of the study group. Most likely, there were environmental factors associated with the use of the abacus by the participants such as their occupations. Such factors could influence their willingness to volunteer for the study, and their mathematics calculation competency. It might have affected their assessment results and, therefore, the selection of the interviewee group.

"Population validity" concerns the extent to which the results of an experiment can be generalized from the specific sample that was studied to a larger group of subjects (Borg & Gall, 1983). The present research was done in several different locations—banks, universities, and offices. These recruiting locations were spread over the country and focused on the major cities where most people live. In previous study from Borg and Gall (1983), they make the following statement regarding population validity from a study done with 125 high school students: "One can validly generalize the research findings from participating students to all students in the school district" (p. 639). Borg and Gall (1983) recruited and studied with limited aged groups and schools and they rationalized the validity of their study. By limitation on aged group and recruiting places, Borg and Gall (1983) compromised and obtained the study population validity. Compared to their study, the present study would gain the validity of study and generalize in prospective of that participants were from an extended group from age 25 to 65 in South Korea, and involved several recruited spots over the country who had trained abacus at least one year. At the same time, the participants were not very limited on the age group and recruited places in South Korea; therefore, this study would be limited for population validity. In Chapter IV, the study results in the quantitative and qualitative experiments will be presented and discussed.

Chapter IV

FINDINGS AND RESULTS

In the subsequent sections of this chapter, the relevant research questions are answered and interpreted in sequential order for each subsection: Quantitative Analysis and Qualitative Analysis.

Quantitative Analysis

Research question 1a: What relationship exists if any between abacus training and reported use of the abacus for each of the age groups?

Among age groups there are clear associations with both years of learning abacus and those of use. Table 1 illustrates that older people are more likely to learn and use the abacus as an arithmetic computational tools than younger people. Table 1 shows the average years of abacus learning in each age group. This might result from the introduction and emphasis of new tools such as calculators or computers in Korean mathematics education. After calculators or computers were introduced in Korea in the 1980s, the Korean Government encouraged teachers and students to use them. This change might affect the recognition or preference of abacus training and use and reflect on the age group in years of its use.

	Years of abacus learning		Years of abacus use	
Age group	Mean (Years)	SD	Mean (Years)	SD
1 (25-34)	1.5	1.08	1.42	0.95
2 (35-44)	2.58	1.59	6.45	5.87
3 (45-54)	5.12	5.96	8.21	7.50
4 (55-64)	4.52	13.82	18.19	16.32

Table 1. The average years of abacus learning and use in each group.

Average years of abacus learning are less than 10 years for most age groups. Among those below age 44, participants showed less mean years of learning compared to those over age 45. Particularly, the majority of those with age under 34 (67%) had experience in learning the abacus for about 1 year. This reflects on the years of abacus use for the same age group. On the other hand, older groups aged over 45 have more years of abacus learning and abacus use than younger groups. To examine the two extreme groups (1 and 4 in Table 1), there was a significant difference in the total time of learning the abacus ($Z = -4.12$, $p < 0.001$), and for the total years of use ($Z = -3.82$, $p < 0.001$). As study participants have experience in abacus learning, they have a tendency to use the abacus more.

Theses results are probably a reflection of two mathematics education policy changes in Korean Mathematics Education History from the 1950s. After the Korean War, the Korean government redesigned mathematics education standards. Since Korean standard mathematics education emphasized essential arithmetic calculation in the early modern mathematics standard education policy, those in the age group 4 had trained with a focus on developing accuracy and speed of addition, substitution, multiplication and division. Unlike the Japanese mathematical standard educational policy, Korea did not include abacus training in the schools.

Research question 1b: What relationship exists if any between abacus training and reported use of the abacus, specifically for the correlations among the reported preference for use of arithmetic operation tools?

In this study, years of learning the abacus show a similar pattern with years of abacus use. Years of learning the abacus were positively associated with the years of abacus use in each age group (Table 2).

The amount of abacus training time in the age groups is different from one to the other. The amount of time learning and using the abacus is associated with the age group. There is a tendency that older groups spend more time learning and using the abacus, while younger groups spend less time learning and using it.

		Duration for abacus learning	Duration for abacus use
Duration for abacus learning	Pearson Correlation	1	.62**
	Sig. (2-tailed)		0
	N	168	168
** *Correlation is significant at the 0.01 level (2-tailed).*			

Table 2. Correlation coefficient for years of abacus learning and its use.

The correlation between groups of calculator users and abacus users, ($r = -0.55$) is significant ($p < 0.01$), accounting for approximately 36% of the variance. Between groups of abacus users and mental computations the correlation is significant but weak in value ($r = 0.29$), accounting for approximately 9 % of the variance (Table 3). The negative correlation between abacus users and calculator users is understandable given the strong differences in mind set and skills used between the two.

		Arithmetic operation tool 1: calculator use	Arithmetic operation tool 2: abacus use	Arithmetic operation tool 3: pen and pencil use	Arithmetic operation tool 4: mental computation
Arithmetic operation tool 1: calculator use	Pearson Correlation	1	- .55**	0.05	- .55**
	Sig. (2-tailed)		0	0.484	0
	N		169	169	168
Arithmetic operation tool 2: abacus use	Pearson Correlation		1	- 0.06	.29**
	Sig. (2-tailed)			0.44	0
	N			169	168
Arithmetic operation tool 3: pen and paper use	Pearson Correlation			1	0.05
	Sig. (2-tailed)				0.55
	N				168
Arithmetic operation tool 4: mental computation	Pearson Correlation				1
** Correlation is significant at the 0.01 level (2-tailed).					

Table 3. Multiple correlation coefficients for each pair of computation tools user group.

Research question 1c: What relationship exists if any between abacus training reported use of the abacus, in relation to correlations among mental abacus use, mental computation confidence level, and confidence of accuracy in use of abacus?

From the analysis of the inter-correlation matrix, there is a strong correlation (r = 0.8) between how comfortable subjects find mental computation and how confident they are about the accuracy of their results accounting 64% of the variance (Table 4).

Inter-Item Correlation Matrix				
	Frequency of abacus use	Mental computation and abacus image	Mental computation confidence	Confidence of accuracy in use of abacus
Frequency of abacus use	1	0.5	0.4	0.4
Mental computation and abacus image		1	0.6	0.5
Mental computation confidence			1	0.8*
* Correlation is significant at the 0.01 level (2-tailed).				

Table 4. Inter-item correlation matrix.

Research question 2a: To what extent do Korean adults report that the physical abacus was an important tool, with respect to their percepts of their experiences and uses of it?

For the question of frequency of abacus use, even though 82% of participants in this study reported using the abacus never or sometimes (S.D = 1.18), 6% indicated that they always use an abacus as their arithmetic operation tool (Table 5).

Apparently, although computation tools such as calculators and computers are widely available and convenient to use, the abacus is still used as one of the arithmetic tools by about 20% of Korean adults. Considering the fact that the Korean standard mathematics education curriculum has never included abacus training, the rate of learning the abacus and the period and frequency of its use tell us that abacus skill still could affect the basic mathematics competency of some Koreans given the reported usage.

Frequency of abacus use? l: never, 2: sometimes, 3:often, 4: usually, 5:always		Frequency	Percent	Cumulative Percent
Valid	1	120	71	71
	2	19	11.2	82.2
	3	11	6.5	88.8
	4	9	5.3	94.1
	5	10	5.9	100
	Total	169	100	

Table 5. The frequency of abacus use for Korean adults.

Research question 3a: To what extent do Korean adults report use of a "mental abacus" when performing mental calculations with respect to their responses of their duration of learning and use of it?

Seventy-five (44%) of 169 total study participants had never used mental computations with an abacus image for their mathematic operations (Table 6). In contrast, 71 (42%) survey responders reported that they are able to do arithmetic operations mentally and use an abacus image (code categories 3-5). These data show that Korean adults who have been educated in abacus use could have the competency of mental computation and the ability to develop a mental abacus image depending on their period or frequency of abacus use.

Mental abacus image during computation		Frequency	Percent	Cumulative Percent
Valid	1	75	44.4	44.4
	2	23	13.6	58
	3	31	18.3	76.3
	4	13	7.7	84
	5	27	16	100
	Total	169	100	

Table 6. Frequency of mental abacus image use during computation for Korean adults. See Table 5 for explanation of the numerical codes 1-5.

Most participants in this study felt comfortable during mental computation (Table 7). All totaled, 62.9% of participants reported that their feelings toward mental computation are more or less comfortable (code categories 3-5). Twenty-one of them (22%) expressed strong confidence in doing arithmetic calculation. Only 22 responders out of a total of 169 (13%) were not comfortable at all when they do mental computation. These results are consistent with the results about how confident people feel concerning their accuracy of mental calculation. Eighty percent of Korean participants in the study reported that they are confident in their results of mental calculation. About 35% of them are strongly confident in their mental arithmetic calculation competency and accuracy.

How comfortable in mental computation		Frequency	Percent	Cumulative Percent
Valid	1	22	13	13
	2	41	24.3	37.3
	3	59	34.9	72.2
	4	26	15.4	87.6
	5	21	12.4	100
	Total	169	100	

Table 7. Frequency of comfort level in mental computation for Korean adults.
See Table 5 for explanation of the numerical codes 1-5.

The above data indicate that more than 50% of the Korean participants have positive views of their comfort with mental calculation. It seems that even those who are not so comfortable in doing mental computation have felt more assured of their competency in producing accurate results as reported in Table 8. Only 35% of the respondents reported feeling comfortable (code 3, Table 7); however, 45% of the responses reflected reasonable confidence in accuracy of answers from mental computation (code 3, Table 8). These results suggest that Koreans believe they are more certain about their answers from mathematical calculation, even if they are less comfortable about doing mental computation (Table 8).

How confident that your answer is correct during mental computation		Frequency	Percent	Cumulative Percent
Valid	1	16	9.5	9.5
	2	18	10.7	20.1
	3	76	45	65.1
	4	45	26.6	91.7
	5	14	8.3	100
	Total	169	100	

Table 8. Frequency of confidence in accuracy of their answer.
See Table 5 for explanation of the numerical codes 1-5.

Research question 4a: With respect to arithmetic competencies, do Koreans who learned to use the physical abacus have arithmetic competency?

In order to examine Koreans' current preference toward tools for arithmetic calculation, participants were asked questions regarding arithmetic operation tools (Tables 9, 10, 11, and 12). In Table 9, the data show that most 138 (82%) of the 169 Koreans in this study utilize calculators. In contrast, data in Table 10 show that the abacus was used by 13 out of a total 169 responders (8%). These data indicate that the calculator is a major computational tool for those subjects who have been trained in the use of a physical abacus. This might explain why most of these participants were not likely to do mental computation and come up with a mental abacus, and were less comfortable with mental computation. This also is consistent with the negative correlation in Table 3 indicating an inverse relationship between those who prefer mental computation and those who use calculators.

Arithmetic operation tool 1: calculator use (Yes: 1, No: 0)		Frequency	Percent	Cumulative Percent
Valid	0	31	18.3	18.3
	1	138	81.7	100
	Total	169	100	

Table 9. Korean adults' preference for calculator use.

Arithmetic operation tool 2: abacus use (Yes: 1, No: 0)		Frequency	Percent	Cumulative Percent
Valid	0	156	92.3	92.3
	1	13	7.7	100
	Total	169	100	

Table 10. Korean adults' preference for abacus use.

Arithmetic operation tool 3: pen and paper use (Yes: 1, No: 0)		Frequency	Percent	Cumulative Percent
Valid	0	128	75.7	75.7
	1	41	24.3	100
	Total	169	100	

Table 11. Korean adults' preference for pen and paper use.

Arithmetic operation tool 4: mental computation (Yes: 1, No: 0)		Frequency	Percent	Cumulative Percent
Valid	0	117	69.2	69.6
	1	51	30.2	100
	Total	168	99.4	
Missing		1	0.6	
Total		169	100	

Table 12. Korean adults' preference for mental computation.

In answer to the question regarding computational tool use, some of the participants recorded two or more tools. The group of calculator users reported that they mainly utilize calculators; however, sometimes they use pen and paper or mental computation skills as well. Since the majority of the study population is in the group of calculator users, a Wilcoxon matched pair test was used to explore whether the calculator group's assessment results are significantly different from each of the other tool users as reported in the latter part of the next section for Research Question 4b.

Research question 4b: With respect to arithmetic competencies, to what extent do Korean abacus learners perform accurate arithmetic computations? If so, which arithmetic operation, if any, is performed the best?

Tables 13 and 14 show the assessment results of analyzing competencies of the abacus learners for all four arithmetic calculations. Table 13 shows the overall means and the standard deviations for addition, subtraction, multiplication and division regardless of the type of computation tool user. Overall, addition ranked as the highest mean value for accuracy (M = 0.97, S.D = 0.12) among the participants. Table 14, shows the frequency of perfect scores for each assessment category and for each type of computation tool user. The pattern of highest perfect scores in each category is rather consistent with that of Table 13. It is likely that Korean adults in the study were able to compute addition test problems more correctly than division test problems. It is shown that most Koreans (despite preferential use of computation tools) are more confident and accurate in addition than subtraction, multiplication, and division. These results are consistent with many other previous studies regarding arithmetic performance in addition, subtraction, multiplication, and division (Hatano, Miyake, & Binks, 1977; Stigler, 1984; Frank & Barner, 2011).

The results of different tool users' accuracy in all four arithmetic computations appear in the tables. Even though the number of abacus users is fairly small compared to all calculator users or other tool users such as pen and paper, abacus users' accuracy percentages for most operations are higher than those of others. In particular, all abacus users obtained perfect scores on the arithmetic addition assessment. In the divisional operation assessment, abacus users received higher scores than all other tool users. From this result, arithmetic operational performance might vary among the categories depending on the operational aids.

From all statistical results obtained in this study, most Korean participants aged 25 to 65 (regardless of gender) preferred to use calculators for their basic mathematics operation. About 82 percent of participants reported that they preferred calculator use and 13 percent of the total participants reported they used the physical abacus as a computational aid. Interestingly, 55.5% of the study population reported that they experienced use of a mental abacus during mental computation, even thought they do not use a physical abacus. This may result from their physical abacus training or the duration of their abacus use over the years.

Score of test answer (score range 0 to 1.0)		Addition	Subtraction	Multiplication	Division
N	Valid	169	169	169	169
Mean		0.97	0.96	0.87	0.83
Standard Deviation		0.12	0.16	0.31	0.31

Table 13. Overall assessment results for accuracy in computation of each of the four operations. A perfect score would be 1, thus mean values are equivalent to the proportion of possible perfect score achieved (See the Methods Section for the scoring procedure).

	All calculator users (Total: 138)		All abacus users (Total: 12)		All other tool users (Total: 19)	
Assessment category	No. of perfect scores	%	No. of perfect scores	%	No. of perfect scores	%
Addition	128	92.8	12	100	19	100
Subtraction	129	93.5	11	91.7	16	84.2
Multiplication	115	83.3	10	83.3	16	84.2
Division	102	73.9	9	75	12	63.2

Table 14. Assessment results for each group of different computational tools.

Research question 4c: With respect to arithmetic competencies, is there any influence on computation assessment results by current preference for computation tool?

The research design included 4 groups of participants for all calculator users ($N = 138$): calculator use only ($N = 89$), calculator and pen use ($N = 24$), calculator and mental computation use ($N = 13$), and all uses ($N = 12$). In an effort to test whether each arithmetic operation performance differed significantly in each group of users (according to tool preference cited above), a Wilcoxon matched pair test was used to compare each of the pairs of the tasks: addition, subtraction, multiplication, and division.

In the group of calculator use only, most mean assessment scores for each operation varied significantly from each other. The addition scores were significantly different from the division scores ($Z = -4.34$, $p < 0.001$), and subtraction scores were significantly different from division ($Z = -4.55$, $p < 0.001$). These results suggest that the mean score of the division competency assessment among those who use only calculators is significantly less than the mean score for addition and subtraction.

Similar to the results for those who use only calculators, the mean score of the participants who use both calculator and a pen, the addition task mean score was significantly higher than that of multiplication or division. A Wilcoxon signed ranks test was used to compare assessment results for the differences between addition and multiplication mean scores ($Z = -2.64$, $p = 0.008$). For the difference between addition and division the Z score is - 2.65 ($p = 0.008$).

Furthermore in Table 14, by contrast to the two groups of survey participants referenced above, there is no significant mean difference ($p > 0.05$) between all four arithmetic assessment results among the group of subjects who currently use both calculator and mental calculation for arithmetic operations. The results of the study also confirmed that among those who use calculator, pen and mental calculation at the same time, the "all task" scores did not differ significantly ($p > 0.05$) from each other. This result suggests that individuals who use both calculators and mental mathematics achieve consistent arithmetic computation scores over all operations than people who use both calculator only and calculator and pen.

Interestingly, for all calculator users in the study, there was no significant mean difference between mean addition score and the subtraction task score (Table 14). These results suggest that the assessment score of addition tasks does not differ from that of subtraction tasks for the calculator users with/without any other computational tools. In other words, it indicates that all calculator users (regardless of other tools' image) are likely to perform consistently in the operations of addition and subtraction.

Based on the statistical test results showing significant differences among the calculator only users and those who use mental mathematics, there are more likely to be inconsistent assessment scores in division compared to the scores in addition or subtraction. This might be interpreted as the division task being prone to inaccuracy or being a more challenging than addition or subtraction.

Qualitative Analysis

This section deals entirely with the responses given during interviews (Appendix D). Almost every individual response is discussed. All trends and common responses are emphasized as well. The research questions addressed here are: 2b, 3b, 5 and 6. The interview data with a smaller group of interviewees (59) provide more in-depth insights into abacus users' thinking and opinions. Some of the data also provide evidence of triangulation in relation to the data gathered by quantitative means in the preceding section.

Research question 2b: To what extent do Korean adults report that the physical abacus is an important tool, with respect to their responses to interview questions intended to elicit their reflective and analytical thinking about their use of the abacus? (Answers obtained from survey questions 2, 3, and 4)

(Interview question 2): Do you still use an abacus for your computation?

For the purpose of a systemic examination of whether Korean adults document an abacus as an important tool, it is essential to ask whether they still use an abacus or not, as well as their reasons. Interview question 2 investigates how many Korean adults in this study population use an abacus. Most interviewees described themselves as only occasionally using an abacus. Table 15 shows that 15.3% of the total study population responded that they perform arithmetic computation by using an abacus.

Two interviewees use the abacus only when it is around them at the moment. This is one of the main reasons why Korean adults do not employ an abacus nowadays. They cannot carry the abacus all the time. Korean adults consider carrying the abacus itself inconvenient. This idea is reflected in the statements from those who think of the abacus as an important tool and who said so in response to interview question 3.

	Number of Responses
I currently use an abacus	7
I don't currently use an abacus	50
I use an abacus if it is available	2
Total	59

Table 15. Response to "Do you still use an abacus for your computation?"

(Interview question 3) What is your opinion of using an abacus to do computation? Is it helpful for your arithmetic calculation? Please explain your reasons.

Using interview question 3, the researcher explored how Korean adults think about abacus use for computation. Interviewees responded with mixed attitudes on the use of the abacus. Their opinions are complex and cannot be divided simply into two groups. Some believe that the abacus is not an important tool for many reasons, saying that the abacus is outdated and that calculators or computers have replaced its role. Others believe that the abacus is still an important tool for them because it is useful for not only computational purposes but also educational purposes. Interestingly, among the responses from those who find that the abacus is not an important tool for calculation nowadays, there was mention that it might be a great tool for other purposes such as brain exercises. Overall, depending on the interviewee's perspective on the abacus's use, many believe that the abacus might be a great tool for Korean adults.

Table 16 illustrates various responses concerning the usefulness of the abacus. 33.9% of the total responses were negative on abacus usage for calculation in daily life. The interviewees believe that the abacus is not an essential tool for reasons mainly concerning its functional usage for arithmetic computation. Three main reasons are its popularity, availability and accuracy and speed in calculation.

(1) Popularity

Eight of the total 59 interviewees stated that the abacus is not an important tool nowadays because calculators or other electronic devices for calculation have become popular. They all agreed that the abacus is one of the old types of calculating tools. Some of them described those who use it as a group who lag far behind the rest of society and said that abacus use is old-fashioned. Six interviewees mentioned that the abacus used to be a good device for arithmetic computation; however, it is not anymore since calculators have become widespread. Most of them favored the calculator's ease of operation. Calculators are easy to use and simple to carry. Two interviewees mentioned the added convenience of other computational tools such as calculator functions on cellular phones.

(2) Availability

Calculators or computers (or even calculator functions on cellular phones) are both easy to use and convenient for Korean adults. Because the devices are around them all the time, they do not feel the need to carry an outmoded tool such as the abacus. Eight interview participants answered that the abacus is cumbersome to carry. They expressed that the abacus would be a good device if it were easy to carry. One of them told the researcher that he has forgotten how to operate the abacus because of its scarcity nowadays.

(3) Accuracy and Speed

Two interviewees said that results from abacus calculation might be less accurate than those from other methods. They mentioned that during multiplication or division, the computational results are prone to errors and inaccuracy. They believed that calculators or electronic devices are more accurate than abacus. Also, some interviewees attested that the abacus was slower in computation than calculators. They stated that the speed of moving fingers on the abacus is slower than touching the keypad on the calculator. Hence, it would be less efficient where speedy calculations are required.

Considering the calculator's popularity, operational ease, availability and computational accuracy and speed, abacus use is less efficient according to the interviewees of people who questioned the role of the abacus in this study. They all cast doubts on the abacus as an important tool, because they mainly view it as having only a computational purpose.

In contrast to those who were skeptical about the abacus as an important tool, most interviewees in this study reported that the abacus is helpful for varied reasons. Over 65% of the total interviewees reported positive roles for the abacus, not only for computation but also for other purposes. They responded three main reasons: accuracy and speed in computations, mental computations, and brain activity.

1) Accuracy and speed in computations

Eleven of the total of 59 Korean adult participants who learned abacus operations endorse the abacus as an important tool with precise computational results and fast performance. This is completely opposite to the position that the abacus yields less accurate results and slow performance on arithmetic computation compared to the electronic devices such as calculators or computers. Some believe that the abacus is helpful for a variety of mathematical calculations. Three interviewees expressed the belief that the abacus is very useful to perform all four arithmetic operations. They believed that the computation results are very precise for simple digit computation involving large numbers. Along with those opinions, four interviewees said that an abacus skill is good for partial or simple computation, especially addition or subtraction. They asserted that for lower digit computation it is speedier and more accurate than a calculator. Many interviewees even stated that for simple arithmetic computation the abacus is easy and quick, and its result is more accurate than a calculator. This point also runs contrary to the skeptical opinion others have expressed about the abacus' primary use. The other explanation was that by using the abacus, multiple computations might be easier while still producing precise and correct calculations.

2) Mental computation

Mental computation skills might be acquired in the process of abacus training, which requires the trainees to perform computations repeatedly. While they express the numbers with finger movements, they repeat numbers and their representations in their minds. As a result, they naturally obtain the mental computation skills.

Some interviewees reported that based on their experience, abacus training is important to gain mental computation competency. Nine interviewees stressed that abacus education and practice with it allows the learners to form mental abacus images. They added that these mental computational competencies are automatically acquired by repeated practice. The mental abacus helps them to compute in their minds. They described that once this "mental abacus" is obtained, it helps precise calculation. Four survey participants said that abacus skill is helpful once they are well experienced in abacus use.

3) Brain Activity

Some aspects of brain function development for young learners are suggested by the study interview results. Abacus training might help young students to enhance their concentration and improve brain activities as some interviewees suggested. Based on the responses from the study participants, it seems that during abacus training, learners increase their ability to concentrate. They reported this not only for the manual process of dexterity in moving fingers while computing on the abacus, but also for the cognitive skills that they gained when they had memorized numbers and focused on computational processes. They noted this was especially the case as compared to using the pen and paper method. These perspectives suggest that abacus training can lead students not only to carry out fast calculations but also to develop their brain activity, especially memorization. Therefore, some participants in this study support the notion that learning the abacus is important for children, far more important as a cognitive developmental tool than as a practical tool.

Some subjects expressed that abacus training and skill are valuable but have limited availability and portability. Three subject's responses were neutral. The three all said that the abacus is an important tool for some tasks and not for other tasks. One of the three pointed out that the abacus' usefulness depends on the circumstance of the calculation being done. Overall, the answers were disparate for many reasons, but most interviewees expressed that it is worth learning the use of the abacus.

	Yes	**No**	**N/A**
Do you think physical abacus is useful?	38	20	1
*** Reasons**			
(1) Useful			
- Quick performance and precise computation. **- Mental image is easily formed and helpful for mental computation** **- Automatically mental computation skills is acquired**			
(2) Not useful			
- It is not easy to carry **- Calculator is better tool** **- Less precise computation result and slow performance** **- Inefficient to use**			

Table 16. Responses to "What is your opinion of using an abacus to do computation? Is it helpful for your arithmetic calculation? Please explain your reasons."

(Interview question 4) When you perform an arithmetic operation such as addition, subtraction, multiplication or division, what kind of tools do you prefer? Do you use a calculator? Do you use an abacus? Do you need to have a pen and paper for your calculation? Or, do you need to have any other tool for your calculation?

Along with interview question 4, survey question 5 examines which tools Korean adults in this study prefer to use for calculation (Table 17). This is another way of investigating to what extent Korean adults who already had the experience of learning the abacus feel about the use of it. According to their responses, most of them prefer to employ only calculators (41 of the total 59). This result is consistent with the result from interview question 2. Five subjects (10%) described the abacus as a preferable tool for calculation and feel comfortable operating it. About 10% of the subjects reported that mental computation is convenient with regard to basic calculation. The study is illustrated that most Korean adults rely on calculators even if they can operate an abacus.

Among some Korean adults in this study, two or more tools were utilized together. About 17% of the interviewees answered that two or more tools are employed to compute arithmetic calculation. They choose calculators and mental computation, or abacuses and mental computation. Some indicated that they needed pen and paper along with these tools, depending on the types of arithmetic computations.

Complexity and required accuracy of arithmetic computation determine the preferences of computational tools for Korean adults who use the abacus. According to them, calculators are used with multiplication or division or multi-digit computation. In calculations involving less than three digits, they were likely to prefer mental calculation. Interestingly, all multiple tool users reported that simple computations such as addition or division of small digits are performed by mental computation. In large digit computation, calculators or abacuses are used. One interviewee mentioned that she uses a calculator for fairly large complex computations, an abacus for calculation of multi-digit numbers, and mental computation for small numbers. The number of digits or complexity and number of operations determine some Korean adults' preference for the tools.

From their responses, the subjects are confident in their skills, but they still tend to verify their computation for accuracy. One stated, "It is desirable to use two or more tools in order to increase the accuracy of results." One main tool is employed in the first step to do mathematics and the other tool is applied in the following steps in order to check computational results. Two interviewees among the users of two or more tools responded that they usually compute in their brain and check their calculation by an abacus. The main reason for the users of multiple tools such as calculator, abacus, or pen and paper is to confirm their answers or to aid them in complex multiplication or division.

Two findings from all multiple tool users attract attention. One is that they all prefer to use mental computation. Users of either calculators or abacus all feel comfortable to perform basic arithmetic in their heads. They state that the mental computation method is much faster than physical tool use for simple calculation. They are confident in using the skill and that the result is precise. The other finding is that most multiple users use either calculators or the abacus, instead of both at the same time.

	Calculator	Abacus	Pen and Paper	Mental computation	Two or more tools listed*
Which tools do you prefer to use in your arithmetic calculation?	41	3	2	5	9
*** Detailed responses for those who listed two or more tools - Calculator & mental computation: 4 - Calculator & mental computation & pen and paper: 2 - Calculator & abacus & mental computation: 1 - Abacus & mental computation: 2**					

** Those people who use two or more reported that simple and small digit mental computation is performed*

Table 17. Responses to "When you perform an arithmetic operation such as addition, subtraction, multiplication or division, what kind of tools do you prefer? Do you use a calculator? Do you use an abacus? Do you need to have a pen and paper for your calculation? Or, do you need to have any other tool for your calculation?"

Research question 3b: To what extent do Korean adults report use of a "mental abacus" when performing mental computations, with respect to their response to interview questions intended to draw out if mental abacus image use is associated to duration of learning or the way that the physical abacus utilized? (Answers collected from interview questions 5, 6, 7, and 8)

(Interview question 5) Can you do calculations in your head? If so, how would you feel about it?

Before asking the interviewees about whether they are able to draw upon a mental abacus image and its use, the researcher examined whether they were able to do mental calculation 'in their heads' without resort to any device. The researcher wished to see if mental abacus images acquired by abacus education would enable people to do mental computation. That is why it was confirmed that subjects could do mental computation, before they were asked to draw a mental abacus. Hence, interview question 5 first asked the interview participants about mental computation capability. The researcher was also interested in knowing which Korean adults with abacus education were able to do mental calculation, and how their past training effects them in their later lives. From interview question 5, most Korean adult participants in this study who could operate the abacus could do arithmetic computations in their heads. It reveals that abacus training is associated with the acquisition of mental computation ability, but not necessarily that it particularly or exclusively supports it.

In Table 18, 86.4% of the total interviewees answered that they have the ability to do mental calculation. Forty-two of the total interviewees replied "Yes" and 9 interviewees revealed that they might do either partial mental computation on simple arithmetic questions or small digit computation. For most interviewees, mental computation involves addition and subtraction. This result is consistent with previous reports showing that mental computation is useful for simple computation with a low number of digits, especially with addition or subtraction.

In contrast, 8 participants could not perform arithmetic computation mentally. Two of them mentioned that it is due to reliance upon calculators. Even though they were used to mental arithmetic, they cannot do it anymore because they are accustomed to using calculators. They are not confident in their calculations, because they have not used the skill for a long time and have lost the ability. Some Korean adults lost their skill either in abacus use or in mental computation, after they have not engaged in such activities for some time.

	Yes *	No
Can you perform mental computation?	51	8
- Partially perform	9	
-Addition & subtraction	2	
-Lower digits computation	1	

** Responses are included in those who answered 'partially perform mental calculation'.*

Table 18. Responses to “Can you do calculation in your head? Then, how do you feel about it?”

(Interview question 6) During your mental computation, do you use any mental image of an abacus?

Along with interview question 5, this question investigates whether those who are able to do mental computation in the study can draw upon a mental abacus image or not. It is known that a mental abacus image can assist people who have received training on the abacus to do mental computation. Also, it is important to figure out how many Korean adults who were involved in this survey actively use an abacus image in their mental computation process. This helps determine the existence of a relationship between abacus images and the mental computation process. In interview question 6, interviewees were asked whether they could activate an image of the abacus in their minds while they perform arithmetic calculations.

This interview evidence indicated that among Korean adults who learned the abacus, there exists a relationship between mental computational capacity and mental abacus image (Table 19). Thirty-eight of the total 59 Korean adults in the interview reported that they conjure up mental abacus images while they calculate in their heads. Twenty-one of the total answered that they do not picture a mental abacus. The nine responders who cannot perform mental computation are included in the 21. That is, 13 subjects who can calculate mentally use different methods than the mental abacus. Therefore, 74.5% of the participants who are able to do mental computation reported that they use the mental abacus image while they perform. From this result, it is concluded that mental abacus image occurrence may be associated with mental computation among Korean adults who had learned to use the abacus in the past. Previous studies (Hatano, Miyake, & Binks, 1977; Hatano & Osawa, 1983; Stigler, 1984; Stigler, Chalip & Miller, 1986) concluded that the mental abacus image might result from training people on the abacus and might affect mental computation ability.

	Number of Responses
I do use mental image of abacus	38
I do not use mental image of abacus	21
Total	59

Table 19. Responses to "During your mental computation, do you use any mental image of an abacus?"

(Interview question 7) What is the merit of using an abacus or using a mental abacus for arithmetic calculation?

In this study, one of the main questions regarding the "mental abacus" is to know to what extent Korean adults report a use of "mental abacus" when they perform mathematical computation. In previous research completed outside of Korea, many countries reported that the mental abacus image could be visualized during the intensive training of children to use the abacus. Using interview question 7, it is interesting to observe whether Korean adults feel comfortable visualizing the abacus image in their mind. This evidence shows which tool, either the physical abacus or the mental abacus, is most preferred. Responses to interview question 7 are summarized in Table 20.

Twenty-three of the total 59 interviewees responded, “Mental abacus is better to use during mathematical computation.” Considering the fact that 38 participants could come up with the mental abacus image, the researcher concludes that the mental abacus image could be useful for some Korean adults represented by this small sample, but not for all of them. It is interesting that 15 subjects felt that the mental abacus image method might not be useful for arithmetic computation.

Twenty-eight of the total 59 interviewees reported that the abacus is more appropriate for performing calculations. This result shows that among this sample of respondents, Korean adults believe that the abacus is better for arithmetic calculations than the mental abacus. It might be because of the accuracy of the results or the speed of the performance. However, it needs to be explored more fully.

Seven of the interviewees stated that neither the abacus nor the mental abacus is appropriate for calculation. It is consistent with the data shown from previous questions that some Koreans in the study firmly believe that calculators or other electronic devices are more useful for arithmetic computation than abacuses. They commented on the abacus saying that it is an inconvenient and outdated tool. One explained that the abacus is inconvenient to carry; thus, the abacus cannot be used on every occasion. An objection to using either the physical or mental abacus is that a large digit series of operations cannot be conducted, so it is better to use a calculator. All these explanations agree with the responses listed above for interview question 3.

From these responses to interview question 7, the researcher concludes that all mental abacus image occurrences may not be useful to all Korean adults in this study, for purposes of calculation. To some Koreans, the abacus is more suitable for their work for certain reasons. Further studies are needed.

	Number of Responses
I think it is better to use the mental abacus image	22
I think it is better to use an abacus	28
Neither of them	4
A simple operation is better for a mental abacus and a complicated one is better for abacus	4
No idea	1
Total	59

Table 20. Responses to "What is the merit of using an abacus or using a mental abacus for arithmetic calculation?"

(Interview question 8) Do you feel any ease or difficulty when you do a mental calculation? If yes, how comfortable or uncomfortable would you feel? Please explain your reason.

Once the subjects come up with the abacus image in their heads, they are able to do arithmetic computation without any help of physical tools. It is intriguing that, based on responses to interview questions 7 and 8, these Korean adults draw upon the mental abacus image in their heads without any difficulties. Some Korean participants in this study said that they choose the abacus rather than the mental abacus even though they are capable of coming up with the image during mental computation. This might indicate that some Koreans have difficulty conjuring the image.

Interview question 8 explores how easily Korean abacus learners in this study come up with an abacus image in their heads. Table 21 illustrates the responses. It is shown that the number of Korean adults who feel comfortable coming up with the mental abacus image (during their mathematics process) is less than those who feel uncomfortable doing so.

Twenty-three of the total interviewees reported that they can come up with the abacus image instantly without any hesitation. Some of them even said that the mental abacus image comes up immediately. As a result, they note that by use of the mental abacus image, their computation process is even faster than with the use of a calculator. Three participants stated that imaging a mental abacus is habitual and natural for them when doing arithmetic. They verbally showed their strong confidence about the mental abacus. Those who disclosed straightforwardly that they use the mental abacus commented that they repeatedly used an abacus to make a habit of utilizing the mental abacus image during simple arithmetic computation. One interviewee said, “The abacus image comes up automatically during mental calculation”.

Five interviewees among the total population in the survey reported that they could come up with the abacus image easily while doing calculations involving less than two digits. As the mathematical operation is simple, the interviewees stated, there is a tendency to come up with the abacus image immediately. This point is discussed more relative to interview question 9.

In contrast, 28 interviewees confessed the complexity and difficulty of evoking the mental abacus image. Seven interviewees prefer to use calculators for their computation. The most common reason for not coming up with the image is due to low frequency of abacus use. Some interviewees responded that they could not recall the mental abacus image because they currently do not perform mental computation. Not only that, they indicated that they never mastered abacus skills. Two interviewees declared that they used to draw the abacus image in their minds automatically in the past, but they could not do so now. Because they had not employed the skill, they could not use the image for their computation anymore. Adding to that, one interviewee emphasized that he could compute basic operations without any difficulty when the abacus image came up. This suggests that the mental abacus image could be lost.

Based on the responses to question 8, many Korean adults who trained on the abacus can draw the mental abacus image by themselves. Also, the interview evidence suggests that the ease of the abacus image during mental calculation is formed by how frequently or intensively the adults practiced or exercised. Previous researchers (Hatano, Miyake, & Binks, 1977; Stigler, 1984; Stigler, Chalip & Miller, 1986; Frank & Barner, 2011) asserted that mental abacus images are activated depending on the duration of learning and practice or the amount of effort to master the skill.

	Number of Responses
It is simple and easy	23
It is difficult and complicated	35
I have no idea	1
*** Reasons for the answers** **- Fairly simple and easy** **- Repeatedly practice and get accustomed** **- Depending on degree of difficulty of operations** **(Number of digits or number of series, types of operations)** **- I used to do it in the past but do not now** **- Automatic abacus image occurred** **- Fairly simple and easy** **- I do not frequently do it so the image doesn't come up**	

Table 21. Responses to "Do you feel any ease or difficulty when you do a mental calculation? If yes, how comfortable or uncomfortable would you feel? Please explain your reason."

Research question 5: To what extent do Korean adults report that mental abacus use influences their qualitative thoughts and tasks? (Answers gathered from interview questions 9, 10, 11 and 12)

(Interview question 9) On average, how long would it take for you to complete a calculation using the mental abacus?

Besides interview question 8 regarding how easy Korean abacus learners draw mental abacus images, the researcher inquired of the participants as to how long they require to come up with the image and then compute. This is aimed to investigate how Korean adults in this study report that they utilize the mental abacus skill with their computation and how confident they are in using this skill.

According to their self-estimation of the speed of mental calculation by the mental abacus method, 29 interviewees manifested that they could calculate comparatively fast, in 30 seconds (Table 22). Twenty-two of them reported, “It takes less than 10 seconds depending on the complexity of arithmetic calculation”. Six interviewees indicated that they took a long time to calculate. Those who said that they could not give an answer were 24 out of the total 59. This is attributed to the fact that some subjects might have never assessed their speed of mental computation or by the fact that they could not come up with the image during their mental computation. It was concluded that these interviewees possibly cannot perform mental calculation using the abacus.

Conclusions from this question were completely based on the responses by the interviewees. Although their reports were not tested and verified by experimental evidence using any scientific methods or standards, the responses are valuable for examining how much confidence Korean adults declare that they have in their mental computations using the mental abacus image and its application to their lives.

	Number of responses
Instantly	24
Less than 30 seconds	3
It depends on the digit number	1
It depends on the degree of difficulty in operation	3
It would take a while	4
Have no idea	24
Total	59

Table 22. Responses to “On average, how long would it take for you to complete a computation using the mental abacus?”

(Interview question 10) Do you feel any advantages or disadvantages to doing mental calculation? If yes, tell me about them with your reason.

In order to investigate to what extent Korean adults express the influences of the mental abacus, all interviewees in the study were asked about their general thoughts on the mental abacus and its computation. The responses fell into two patterns: that there are either benefits or no gain from using the mental abacus.

Seventeen interviewees asserted that use of a mental abacus is not applicable to Korean adults. They mentioned that a mental abacus is not useful for computational function. Some of these interviewees did see value, however, in the use of the abacus for computation (Table 23).

	Number of responses
1) Advantages	
- Simple to operate	10
- Convenient to use	3
- Anytime available	10
- Simple to operate	3
- Useful for brain activities	
Good for brain development	15
Good for memorization	2
Good for concentration	2
Good for reasoning	2
- Good for learning abacus	1
- High accuracy and fast speed calculation	4
- Time saving	3
2) Disadvantages	
- Use calculator	3
- Time consuming	3
- Inaccuracy	4
- Inconvenience	4
- Good but do not frequently use	1
- Good but cautious use for its purpose	1

Table 23. Responses to “Do you feel any advantages or disadvantages in doing mental calculation? If yes, tell me about it with your reason.”

Some Koreans in this study believed that mental computation is less practiced now owing to the development of technology. Because of their simple operational function, calculators are widespread among Koreans. Calculators are also easy to carry. Even cellular phones have calculator applications; therefore, most Koreans do not need to compute mentally anymore.

In addition to that, these same interviewees support the idea that calculators or other types of electronic tools are much more accurate and reliable than mental computation. During mental computation, they are not confident about their calculations, and they need to verify them by other tools such as calculators. Once they started using convenient technology, they infrequently employed the mental abacus skills that they used to have and therefore lost those skills. As a result, they are more dependent on technology for seeking accurate calculation and less confident on the mental computation.

Some interviewees believe that the use of calculators is much faster for arithmetic calculation than the use of mental computation. One of them stated "Although people can calculate mentally with speed and convenience, it is not often used in our life now". Another interviewee responded that it might be beneficial to do mental computation with simple arithmetic calculations; however, mental calculation might approach the limit of its performance when used to perform complicated operations. Others added that it may take longer to do mental computation with the mental abacus image than other methods, and may yield less accurate results. Some subjects felt that mental calculation was slow, since they feet the need to verify the result with many repetitions in order to increase the accuracy.

On the contrary, other subjects still believed that the mental abacus and its role in mental computation are beneficial to Koreans for a variety of reasons. These reasons are very similar to the benefits stated concerning the use of the abacus. Benefits for not only mathematical calculation but also for brain activities were cited.

In addition to that, abacus education and practices could help children to understand basic numbers and calculation, as well as mathematical thinking and reasoning. One of the interviewees who asserted advantages of abacus learning and mental mathematics ability stated that abacus practices led him/her to mental computation ability which was quicker and more accurate than it would otherwise have been. Another reported that mental math with a mental abacus image is a useful tool for learning mathematics with consistent practice. Other subjects who agreed with its benefits noted its convenience. Once they acquired the skills, they were able to do arithmetic computation without any tool and under any circumstance. Some said that it is much easier to operate the abacus than a calculator and that they draw accurate results with surprisingly quick speed. However, a few subjects among those who advocated the use of a mental abacus stated that the mental abacus is helpful on arithmetic calculations of more than three digits. Furthermore, one mentioned that some abacus computation is even faster compared to using calculation by computers or calculators.

One subject expressed the neutral position that mental computation is needed for appropriate use in proper conditions. The other reason that was suggested was that the mental abacus and its computational skill help young Korean children to improve their intellect. Twenty one subjects agreed that developing mental abacus skill is useful in various brain exercises such as enhancing memorization, concentration and reasoning. One subject stated that she/he could enhance memorization while he/she practiced a mental abacus training during childhood. She/he said that this skill helped with subjects other than mathematics and led to success in those classes. The other interviewee mentioned that he/she was focusing on doing mental computation by mental abacus. It was said that abacus training along with mental computation practice was useful for them to gain confidence in mathematics and help them understand basic mathematical knowledge. Based on their statements, mental abacus and computation training might have a beneficial influence to them for their lives. These statements are mainly true for young learners.

Some of the participants believed that regular mental computation using the mental abacus could stimulate brain activity in older Koreans. It is revealed that some older Korean adults do exercise mental computation by drawing upon the abacus image for health purposes. In order to prevent themselves from neurodegenerative diseases such as Alzheimer's disease, some participants regularly practice mental calculation as well as basic arithmetic computation by abacus. They firmly believed that moving fingers or mathematical computation or mental exercises would stimulate brain activity.

The abacus is an instrument used by the Chinese, Japanese and Koreans to improve mathematical skills. The improvement in mathematical skills by abacus training is said to be due to a coordinated functioning of both right and left brain hemispheres. Previous research results showed that the abacus learners at the end of one and two years had better visual and auditory memory when compared to non-abacus learners (Bhaskaran *et al.*, 2006). At the end of training, abacus learners tried to solve mathematical problems without physically using the hands and abacus beads but by visualization of those beads by the brain. It has been well documented that mathematical skills were more developed in abacus learners when compared to non-abacus learners of the same age.

The use of the abacus requires co-ordination of sight, sound and finger movement that induces an increase in the synaptic connections. The abacus learner tries to coordinate visual, auditory and sensory inputs, and then analyzes the problems and solves them. It has also been reported by Chinese workers that abacus learners coordinate right and left hemispheres to solve problems (Bhaskaran *et al*., 2006). The action of the right hand helps in developing the logical thinking and language function of the left hemisphere, and the action of the left hand help in developing creativity, imaginary and 3 dimensional skills of the right brain. Since the right and left hemispheres transmit messages to each other, the function of the whole brain is said to be developed with this system.

(Interview question 11) How confident do you feel about the accuracy of your calculation with the mental abacus?

This section discusses the accuracy of mental computation. The question asked the interviewees to indicate in the form of a percent how confident they were in the accuracy of their calculation. This question is to confirm Korean adults' confidence with regard to use of mental computation guided by the mental abacus. The number of responses in each confidence level was expressed as a percent (Table 24). Among the total 59 interviewees, 35 reported that they were confident that their mental mathematics calculations were above 90% accurate. Eight interviewees stated that they had confidence which varied depending on the complexity of the arithmetic computation required. The question was predicated on addition or subtraction of a small number of digits. In contrast to that, 9 interviewees were unsure of their performance and mentioned that they needed to check their result with calculators. Overall, it seems that the participants in this study felt confident in their calculations.

Percentage of accuracy	Number of responses
Above 90%	34
80-90%	1
70-80%	8
Below 70%	7
Reasons for not confident	9
*** Depending on the degree of difficulty in operations**	2
*** Have no idea**	1

* *Responses are included in those who answered 'reasons for not confident'*

Table 24. Responses to "How confident do you feel about the accuracy of your calculation with the mental abacus?"

(Interview question 12) Which is faster for you: an abacus calculation or a mental calculation? How long does it take for you to complete it?

In order to investigate what Korean adults perform faster by either abacus computation or mental computation, interview question12 were asked (Table 25). 14 interviewees reported that mental computation with a mental abacus image provides the quickest computation. In this way, they believe the computation results are precise within a short moment. In a number of responses, the interviewers found that two or more tools were preferable depending on the digits or complexity of the computation. Five interviewees preferred mental mathematics. Three interviewees indicated that they use mental computation and physical abacus. Thirty-three out of the total interviewees population reported use of calculators.

Even though a number of interviewees reported calculators as their main tool for computation, most responses regarding mental abacus and computations were positive. Korean adults thought that learning the abacus is helpful for mathematics education of young students.

	Number of Responses
Calculators	33
Mental abacus	14
Computer Program (Excel)	2
Pen & paper	1
Calculator & mental computation	4
Abacus & mental computation	3
Calculator, abacus, & mental computation	2

Table 25. Responses to "Which is faster for you: an abacus calculation or a mental calculation? How long does it take for you to complete it?"

Expert Interview

Research question 6: What is the status of Stigler's claims for the mental abacus in view of the reported experience of Korean adults in this study in response to interview?

In this subsection, an intensive interview with a Korean grandmaster of abacus and with a abacus teacher is discussed. Through the evidence gathered from the interview, the researcher sought to understand how the training of mental abacus may influence the thoughts and lives of expert Koreans as reflected by these master users.

Mental arithmetic experts were given various memory tasks by Hatano, Miyake, & Binks (1977), and according to these pioneers for studying abacus experts, Hatano *et al*. (1977) described such experts as follows:

> They had a much larger digit span than the average, but their span for letters of the alphabet or fruit names was not unusual. In his study, abacus experts could reproduce digits, backwards as quickly as forwards; their memory for digits was to some extent compatible with a preloaded memory for fruit names. They seemed to have a 'mental

abacus' or specific device to represent visuo-spatially a number comprising many digits just long enough for calculation.

Among other conclusions, Hatano *et al*. reported that abacus operation tends to be interiorized gradually to such a degree that many abacus masters can calculate even faster and more accurately without an abacus. They move their fingers during mental calculation as if they were actually fingering an abacus, though grandmasters manage not to do so. Their finger movements are analogous to mouthing the words while reading. Secondly, while expert practitioners are using an abacus they can carry on a conversation.

Since there have been almost no systematic, scientific studies on abacus operation, the interview study reported here was aimed at examining mental calculation and the effects of answering questions during on operation under controlled experimental conditions relying mainly upon the dual task technique. It concerned 1) whether or not most expert operators could calculate mentally, 2) how their performance would change their finger movement, or prohibited or interfered with it, and 3) whether or not they could answer a question asked during abacus operation and/or during mental operation, and 5) how the answers to 1~ 4) would depend upon their level of mastery.

The present study was aimed at investigating the question "to what extent do experts report that abacus use influences their qualitative thoughts and tasks?" based upon interviews with Korean adults who rely upon a "mental abacus", in particularly to mental abacus experts.

Two abacus experts were interviewed, focusing on how they report that mental abacus use influences their qualitative thoughts and tasks. Expert A is a 62-year old male and he holds the championship of Korean National Abacus Contest. He was a grandmaster abacus expert in the Korea national competition. Expert B is a 58-year old male and teaches abacus in the after-school mental mathematics and abacus learning institution. They had three-years training duration and have been using the abacus always for a number of years. Interviews with both of them were conducted in the office of the Korean National Abacus Mathematics Education Association (KNAMEA) in Seoul, Korea. Three questions were asked of both of them and followed by the actual experiment and their calculation performance. The first part was written in disclosure and scripted. In the last part, their performance is briefly described.

Interview Script

Kim = Interviewer, A = Korean grandmaster of abacus, B = Korean abacus teacher

Kim: How long have you been using an abacus? I am curious whether the duration of use may affect that you are able to draw it on head and do mental computation.

A: Actually I don't compute arithmetic using by drawing the image of abacus. The calculation method is the same as the one of abacus without the drawing image because I have gotten accustomed to basic calculation by abacus and always been used it. I started training abacus at my 4th grade of elementary school and achieved the first rank in the Korean national abacus contest. Since then, I have kept using the abacus calculation skill and practiced it everyday. As the result, it is a driving force to become the Grand master of Abacus contest championship and still use the skill. I am so confident on accuracy and speed of arithmetic calculation, especially addition and subtraction.

Kim: How does it evaluate abacus calculation skill and determine the rank?

B: It is established by difficulty level of question of basic arithmetic calculation. Time for answer and the number of questions and their computation accuracy determine the rank of abacus.

Kim: On what kind of effect on your life would abacus education have?

A: First of all, it affected on my career choice as a bank associate. Sometimes, I witnessed the error causing from calculators use at bank; however, my mental computation have made never mistaken. Mental computation by abacus training and skill is acquired by repetition on practices for calculation accuracy and speed. Moreover, my confidence on basic calculation in daily life always makes my day. Indeed, it is very influential on my career life of a bank associate on not only that a choice of my career, but also that last of my successful career. Furthermore, in my personal life it might be helpful on my brain function, especially memory and process speed capacity. Whenever computing large digits of numbers, it is automatically processed and retained on head. Overall abacus education have influenced throughout my life.

B: Like he mentioned, abacus education have influenced my career in my life as well. I am an abacus teacher in after-school institution. As he stated, we do not need a physical abacus because the actual image of abacus is embedded in. Although I am not a master of Korea National Abacus Contest, mental abacus image have affected on my mission, vision, and belief of my career. I am teaching the abacus operation to young children followed by my pedagogy based on my experience. Current teaching method and purpose that I designed is a quite different pedagogy and training from those that I learned in the past. At my elementary school and vocal school, I trained and acquired the skill only focusing on the part of how to operate physical abacus. The main purpose of the past training on abacus was accurate and speed calculation. After numerous repetitions of abacus training, abacus image was occurred by itself. Once the mental image had been naturally drawn, mental calculation have been performed. It is like a byproduct of result from intense abacus training. In other words, mental abacus image and its calculation was not a main goal of the past training. However, throughout my life- time experience from uses of mental abacus image for arithmetic computation, I realized the importance of its utilization in many ways such as mathematics education and enhancement of memory function. Consequently, I have developed new abacus teaching instructions and guidance. Current abacus training is more concentrated on building up mental abacus image and mental computation abilities. The instructional design is to educate children not only for the basic abacus operational method but also for mental mathematics. The teaching method is that students solve problems with instructors and they redo the same problem with the exact same finger movement while imagine the shape of physical abacus. From this exercise, the mental abacus image is introduced and formed by their accord. This pedagogic have validated by my students. It has proven that those who perform mental calculation achieved better test scores on mathematics in school. In addition to that, they have been confident on problem solving in other subjects such as science, languages, English, or social sciences. Their class performance has been improved according to their students and parents' testimonies. High performance in class allowed the students to gain self-confidence as well as to develop originality and memory retain function. These students' achievement also have influenced on my career achievement and success. Overall abacus itself has positively huge impact on my life as well as my student life.

Kim: What is your opinion on teaching mental computation by abacus?

B: Abacus is the most effective instructional tool to teach basic of mathematics, in particular number sense. It is because abacus makes children to see numbers on the rods of strings in abacus by eye and touch the rods as numbers by fingers and hands. Even abacus instructor calls numbers and then, students hear it by ear and recite the same number by mouth. These physical feeling senses and practices enhance children to learn what the number and number counting system are. I believe that mental computation ability cannot gain without knowing numbers and number system. Once the mathematic ground is shaped, next step for mental math is achieved by numerous and intensive training for a certain period. With calling numbers during abacus computation by mouth, they start imaging the abacus on mind and doing calculation. At last, they do not need any computational aids any longer. I strongly believe that mental computation is beneficial on both mathematics education and mental activity development such as memory. Not only for the children, it would be also a useful tool for prevention of adults' neurodegenerative diseases such as Alzheimer because by mental computation they continuously keep their brain active. According to many researchers, I heard that keeping our brain busy would prevent from such types of elderly diseases.

A: In addition to his opinion, I think one advantage from abacus training gain have high concentration ability. Without high focuses on arithmetic computation with busy fingers, mouths, eyes, and even ear, the trainers cannot achieve accurate results in a few seconds. That can be saying that abacus training itself is concentration training. Furthermore, number and its computational activities are conducted in our daily life. I would state that it is a part of our living activities; therefore, mental computation can aid our life to be better. I strongly believe that instructional development of abacus education would be one of the ways in order to improve quality of our life.

Kim: Can you show me how fast and accurate you compute arithmetic problems?

A&B: Sure. Why not?

After a brief interview, the two performed arithmetic problems provided by the Korean National Abacus Mathematics Education Association (KNAMEA). Person A chose the mental computation method in which he just looked at the problem on the blackboard and did it. Person A just read the problems and wrote answers on the paper. Person B solved using the physical abacus. Person B to the researcher's reading problems and solved by moving his fingers on the abacus. The subjects were given two sets of problems. The subjects solved addition and subtraction problems for one set comprised of 10 numbers of 15 digits. For multiplication and division, the other set consisted of 10 numbers of 5 digits. The problem sets were distributed by one of the Korean National Abacus Mathematics Education Association (KNAMEA) members in the office. The experiment was separately conducted on person A and person B.

The experiment results showed that mental performance of computation is likely faster and more accurate than physical performance. Person A took on average, 20-30 seconds to complete each problem. Person B, on the other hand, spent on average of 60 seconds to complete them. It is also consistent with previous researches suggested that finger and hand movement on abacus operation involves slower performance than mental abacus performance (Hatano & Osawa, 1983). Also, as to computation results, all answers obtained by the grandmaster using mental computation were correct. However, the physical abacus performer had some incorrect answers. Although there were errors in two digits-numbers, he gave nearly close to a correct answer.

In Chapter V, three main questions regarding the prevalence and aspect of Korean adults' mental abacus use will be discussed based on the results. With the quantitative analysis and qualitative analysis, the questions and further study will be explored.

Chapter V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This chapter presents a summary of the study, conclusions drawn from the study, and, based on the study's results, makes recommendations for practitioners and researchers.

Summary

The main goal of this study was to explore a concept known as a "mental abacus" and study its effect based on perceptions reported by those Koreans who have been educated in abacus use and who may have used a "mental abacus" during the course of their lives. In Korea, mental arithmetic competency is deemed necessary for learning mathematics, and intensive training for acquiring it from early ages was at one time common. One of the traditional training methods, employed less frequently today, was abacus training. Abacus training was widespread from the 1960s to the 1980s because acquiring abacus skill was considered to speed arithmetic computation. Mental abacus skill – an ability to envision and use a mental image of an abacus – could be acquired naturally during education to use the abacus or through specific training. As a result of such instruction, learners increased their accuracy and speed of arithmetic performance. However, there has been almost no examination of the long-term effects of mental abacus study.

The central investigation of this study has been to determine the prevalence of using a "mental abacus" by adults whose mathematics education in Korea included extensive use of the actual abacus as both a teaching and computational aid. Thus, this study proposed to examine the notion that Stigler (1984), drawing on earlier work by Hatano, Miyake, and Binks (1977), explored when he hypothesized the use of a "mental abacus" by Chinese children, by examining evidence for the existence and use of a mental abacus by Korean adults. The main research questions explored by the study were as follows:

1. What relationship exists if any between abacus training and reported use of the abacus:
    a. For each of the age groups?
    b. Specifically for the correlations among the reported preferences for use of arithmetic operation tools?
    c. In relation to correlations among mental abacus use, mental computation confidence level, and confidence of accuracy in use of abacus?

2. To what extent do Korean adults report that the physical abacus was an important tool:
    a. With respect to their percepts of their experiences and uses of it?
    b. With respect to their responses to interview questions intended to elicit their reflective and analytical thinking about their use of the abacus?

3. To what extent do Korean adults report use of a "mental abacus" when performing mental calculations:
    a. With respect to their responses of their duration of learning and use of it?
    b. With respect to their response to interview questions intended to draw out if mental abacus image use is associated to duration of learning or the way that the physical abacus is utilized?

4. With respect to arithmetic competencies:
    a. Do Koreans who learned to use the physical abacus have arithmetic competency?

b. To what extent do Korean abacus learners perform accurate arithmetic computations? If so, which arithmetic operation, if any, is performed the best?

c. Is there any influence on computation assessment results by current preference for computation tool?

5. To what extent do Korean adults report that mental abacus use influences their thoughts and tasks when responding to interviews?

6. What is the status of Stigler's claims for the mental abacus in view of the reported experience of Korean adults in this study in response to interviews?

Participating in the study were 169 Korean adults aged 25 to 65 who had trained with and used an abacus for a minimum of one year. The study's first four questions were answered by a quantitative analysis of questionnaires completed by the participants. The questionnaires included an assessment of participants' arithmetic computation skills. In order to explore especially the last two questions (5 and 6), 59 of the study participants were selected and interviewed regarding the extent of their mental abacus use and its effect on their lives. Interviewees were chosen based on their reported duration of physical abacus training or reported use of a mental abacus image. The purpose of the interviews was to explore the extent to which abacus training at young ages influences performance and how the physical abacus is transformed and represented for mental computations. In this way, Stigler's claim for mental abacus use was examined and its application as a teaching tool for mathematics was explored.

Conclusions

The main results for each research questions are reported in sequential order for both quantitative and qualitative phases of the research.

1. What relationship exists if any between abacus training and reported use of the abacus, for each of the age groups? Specifically, for the correlations among the reported preferences for use of arithmetic operation tools? In relation to correlations among mental abacus use, mental computation confidence level, and confidence of accuracy in use of abacus?

In this study, participants who report that they have had more experience in abacus learning, also report that they have a tendency to use the abacus more. Among the age groups, older age groups report that they have spent more time learning and using the abacus, while younger groups report that they spend less time learning and using it. This study also showed that there is a positive correlation between groups for the reported preferential use of arithmetic operation tools. In particular, between calculator users and abacus users, and between groups of abacus users and mental computations, the correlation is significant. Among mental abacus users, there is a strong correlation ($r = 0.8$) between how comfortable subjects find mental computation and how confident they are about the accuracy of their results accounting for 64% of the variance. Moreover, there is a strong correlation between mental computation confidence and accuracy among mental abacus users in this study sample. This probably is a reflection of policy changes of the Korean mathematics education after the 1950s and introduction of calculator uses by the Korean government.

Once calculators and computers became widespread in Korea in the 1980's, the Korean government encouraged teachers and students to employ them. Study results indicate the preference for tool use differs between young (under 50) and older age groups. The younger participants were more likely to choose calculators or technical aids than to choose traditional aids such as an abacus or pen and paper. Mental computation varied likewise between the two age groups. Among 59 participants who were interviewed, only four reported occasional use of mental computations.

Although changes in Korean education policies might reasonably influence use of arithmetical tools, most Korean participants in this study expressed feeling comfortable during mental computations despite showing a clear preference for calculator use rather than abacus use. Nevertheless, despite the wide availability of computation tools such as calculators and computers, the use of an abacus by Korean adults is confirmed. Considering the fact that the standard Korean national mathematics education curriculum has not included abacus training, although some commercial education institutions continued to use it, the rate of abacus learning and the reported frequency of its use indicate that abacus skill could still affect the basic mathematics competency of Koreans.

2. To what extent do Korean adults report that the physical abacus was an important tool, with respect to their percepts of their experiences and uses of it? With respect to their responses to interview questions intended to elicit their reflective and analytical thinking about their use of the abacus?

From all quantitative and qualitative analyses of the survey, most Korean adults, regardless of gender, preferred to use calculators for basic arithmetical operations; however, 20% of the participants claimed to use a physical abacus as a computational aid. Apparently, although computation tools such as calculators and computers are widely available and convenient to use, the abacus is still used as one of the arithmetic tools. It is reasonable to conclude that the prevalence of Korean abacus users is not widespread; however, the tool still functions as a computational aid in Korea for some who are most educated to use it.

Korean adults' perceptions of the physical abacus as an important tool are complex and cannot be divided simply into two groups. Some believe that the abacus is not an important tool for many reasons, saying that the abacus is outdated and that calculators or computers have replaced its role. Korean adults in the study explained that inconvenience of carrying an abacus is a main reason for not using the abacus as a arithmetic computational tool. This idea is further reflected in the interview statements from those who think of the abacus as an important tool. Others believe that the abacus is still an important tool for them because it is useful for not only computational purposes but also educational purposes. Interestingly, among the responses from those who find that the abacus is not an important tool for calculation nowadays, there was mention that it might be a great tool for other purposes such as brain exercises. Overall, depending on the interviewee's perspective on the abacus's use, many believe that the abacus might be a great tool for Korean adults.

3. To what extent do Korean adults report use of a "mental abacus" when performing mental computations, with respect to their responses of their duration of learning and use of it? With respect to their response to interview questions intended to draw out if mental abacus image use is associated to duration of learning or the way that the physical abacus is utilized?

Interestingly, 55.5% of this study's participants report experiences of employing a mental abacus during mental computations even though they do not use a physical abacus. Such experiences may derive from earlier physical abacus training and earlier physical abacus use.

This might be explained by a “reward” occurrence during the learning processes leading to the acquisition of a mental abacus. Hatano *et al*. (1987) assert that even novices may be able to imagine a mental abacus. Therefore, use of a “mental abacus” might be seen as a specialized skill not only for experts but also for novices with only a few years of abacus study. The distinction between novices or intermediate and expert mathematical calculators might depend on one’s ability with a “mental abacus”.

When used by novices or intermediates, the mental abacus, a visual-spatial representation, is thought to be of limited power in contrast with the experts’ stable, vivid and easily controllable imagery of abacus beads (Hatano, Amaiwa, & Shimizu, 1987). This phenomenon is reflected by the Korean abacus users in this study. Some reported that they would spend more time wrestling with an imaginary abacus than they would in using a physical abacus. As a result, it is much easier for them to use a physical abacus. Hatano *et al*. (1987) assert that two steps are involved in the process of developing “mental abacus” skill. First, interiorization takes place after a few years of practice (Ezaki, 1980). At this stage, learners are able to do addition and subtraction quickly without a real abacus, but the size of their mental abacus is small (two or three columns of beads). Second, extension in size occurs gradually. According to abacus educators, it usually takes one year of practice to increase each additional column beyond five. Only with a huge amount of practice and effort can a learner acquire a large-sized mental abacus.

Consistent with previous studies (Hatano, Miyake, & Binks 1977; Stigler, 1984), data from the present study confirm that after abacus training for a period of time, mental images emerged and were employed by some Korean adults. Most participants in this study expressed confidence in their results obtained by mental calculation as well as their comfort in performing such calculations.

The main goal of this study was to investigate how Korean adults who have trained on the abacus, acquire and represent an abacus image in their minds. This study reveals that intensive training with an abacus and the continuous use of an abacus influence the formation of mental images. From qualitative interviews, 74.5% of the total 59 interviewees claimed that when performing mental calculations, a mental image of an abacus was present. The reports suggest that abacus training is associated with the gaining of mental computation ability. It seems reasonable to conclude that the occurrence of a mental abacus image may be associated with mental computation among Korean adults who learned to use the abacus in the past.

In this research, one of the main questions regarding the “mental abacus” is to what extent Korean adults report a use of a “mental abacus” when they perform mathematical computations. In previous research outside Korea, many researches reported that the mental abacus image could occur while they intensively trained children to use the abacus. Using qualitative interview questions 7 and 8 of this study, it is interesting to observe whether Korean adults feel comfortable envisioning the abacus image in their minds to perform mental computations. On this issue, among the 59 Korean adults’ their responses are mixed. Even though the abacus image is mentally activated by a majority of users, 45.5% of those interviewed would rather use a physical abacus than a mental one due to the perceived accuracy of the former. Also, 11.8% of the interviewees reported that neither a physical nor a mental abacus is appropriate for mental calculation. Clearly, to some Korean adults, the abacus is more suitable for certain purposes, and not all Koreans are able to conjure a useful mental abacus image.

Despite reports from those who have difficulty using a mental abacus, many Korean adults who trained on the abacus can automatically conjure the mental abacus image and employ the skill during mental computation. Adding to that, the ease of the abacus image during mental calculation is determined by how frequently or intensively the subjects practiced or exercised. Previous researchers asserted that mental abacus images are drawn dependent on the duration of learning and practice or the amount of effort spent to master the skill. This point will be discussed later on in the context of abacus experts.

4. With respect to arithmetic competencies, do Koreans who learned to use the physical abacus have arithmetic competency? To what extent do Korean abacus learners perform accurate arithmetic computations? If so, which arithmetic operation, if any, is performed the best? Is there any influence on computation assessment results by current preference for computation tool?

In relation to the question of whether arithmetic operation performance differed significantly in each group of users according to tool preference cited above, the evidence indicates that all calculator users (regardless of other tools' image) are likely to perform consistently well in the operations of addition and subtraction. It was found that individuals who use both calculators and mental mathematics consistently achieve more accurate arithmetic computation scores over all operations than people who use both calculator only and calculator and pen.

Based on results from all four arithmetic computation assessment results in this study, even though the number of abacus users is fairly small compared to all calculator users or other tool users such as pen and paper, abacus users' accuracy percentages for most operations are higher than those of others. In particular, all abacus users obtained perfect scores on the arithmetic addition assessment. In the divisional operation assessment, abacus users received higher scores than all other tool users. From this result, arithmetic operational performance might vary among the categories depending on the operational aids they reports using.

Similar to the results for those who use only calculators, the mean score of the participants who use both calculator and a pen, the addition task mean score was significantly higher than that of multiplication or division. This result suggests that individuals who use both calculators and mental mathematics achieve consistent arithmetic computation scores over all operations than people who use both calculator only and calculator and pen.

It is likely that Korean adults in the study were able to compute addition test problems more correctly than division test problems. It is shown that most Koreans (despite preferential use of computation tools) are more confident and accurate in addition than subtraction, multiplication, and division. These results are consistent with many other previous studies regarding arithmetic performance in addition, subtraction, multiplication, and division (Hatano, Miyake, & Binks, 1977; Stigler, 1984; Frank & Barner, 2011).

5. To what extent do Korean adults report that mental abacus use influences their qualitative thoughts and tasks?

All interviewees were asked about their general thoughts concerning the mental abacus and its use. Responses fell into two categories: mental abacus use was beneficial or it was not. Some interviewees stated that mental computation is less practiced now owing to the development of technology. They saw electronic devices or aids as more reliable than a physical abacus or a mental abacus, both for accuracy and for speed of calculation. However, others strongly believed that the mental abacus is beneficial for a variety of reasons. The mental abacus was seen as a great tool not only for arithmetic computation but also for mathematics education and for "brain exercises." Some interviewees stated that mental abacus education could help children to understand basic numbers and calculations, as well as help to develop the children's mathematical thinking and reasoning. Abacus training along with mental computation practice was seen as helping young people gain both confidence and understanding in their mathematics studies.

Among those interviewed in this study were two abacus grandmasters, who displayed skills similar to those reported in previous research (Hatano, Miyake, & Binks, 1977; Hatano & Osawa, 1983; Stigler, 1984). One grand expert participating in the study completed accurate mental computations involving 10- to 15-digit numbers within 30 seconds. The acquired digit memory of abacus-derived mental arithmetic experts is attained through extensive practice (Hatano & Osawa, 1983). In this study, Expert A testified, "I started training on the abacus at my 4th grade of elementary school and achieved the first rank in the Korean national abacus contest. Since then, I have kept using the abacus calculation skill and practiced it every day." Expert B also reported, "The main purpose of past training on the abacus was accuracy and speedy calculation. After numerous repetitions of abacus training, an abacus image occurred by itself. Once the mental image had been naturally drawn, mental calculations follow. It is like a byproduct which results from intense abacus training". These statements support prior research of the Japanese grand expert study by Hatano and Osawa (1983). They asserted that extended digit memory is a by-product of practice in computing with large numbers, since their subjects had not been given any direct training to widen the mental digit span, as had been successfully done in early studies (Ericsson *et al*., 1980; Slak, 1970). These observations support Cole's

(1980) claim that people tend not only to excel at a task they have practiced extensively, but also to develop domain-specific processes or modes of processing which are functional for effectively performing the task (Hatano & Osawa, 1983). Hatano *et al.* (1977), for instance, found that Japanese abacus experts have a superior ability to memorize digits compared to most people. Their experiments demonstrated that *soroban* experts use a mental *soroban* to improve digit memory retention.

Direct consequences of abacus education and mental abacus practice accrue to Korean abacus grandmasters as well as to abacus intermediates. The qualitative interviews among the Korean adults in this study reflect varying opinions concerning the value of abacus education. One of the advantages of abacus training is thought to be the development and stimulation of brain activities. The belief is that hand tasks are connected with stimulating brain exercises; an advantage, it is thought, for both the young and the old. In fact, Hatta (1985) and Hatta and Ikeda (1988) examined the hemisphere functioning of Japanese abacus experts with a time-sharing task devised by Kinsbourne and Cook (1971), in which both sequential finger-tapping and mental arithmetic tasks were given simultaneously. The findings suggest that the right hemisphere engages more in mental calculation than the left hemisphere of *soroban* experts, while the position is reversed in non-expert subjects, perhaps because ordinary subjects who have negligible *soroban* learning experiences do not employ an imagery-based strategy.

In addition to direct consequences such as enhancing brain function, abacus experts report indirect consequences. Experts A and B credited their occupational choices to their abacus education. According to Expert A, "It affected on my career choice as a bank associate." Expert B has chosen as his vocation the teaching of abacus use to children after school. Expert B has been developing abacus instruction as an educational aid for mathematics learning, and views current abacus training as concentrated on building up the mental abacus image for purposes of mental computation abilities. The goal of instructional design is to educate children not only for basic abacus operational methods but also for mental mathematics.

6. What is the status of Stigler's claims for the mental abacus in view of the reported experience of Korean adults in this study?

This research supports Stigler's claim that calculation using a mentally imagined abacus resembles calculation using the actual abacus and that the "mental abacus" incorporates important features of the actual abacus.

Stigler (1984) claims that abacus training has both quantitative and qualitative effects on children's mental calculation skills, and that a "mental abacus" is used by experts. Stigler also explored the nature of the mental abacus skill to determine what aspects of the physical abacus and its operations are represented in the mental skill. His approach to the question was to compare performance on addition problems by American adults and Taiwanese students, focusing on speed and accuracy. In the present study, involving adults, rather than children, and undertaken in a different culture, participants do report using a mental abacus while performing arithmetic calculations, although no evidence appeared of actual training in envisioning a mental abacus. Successful users reported that after much practice, the abacus "just appeared". Also, subjects in the study were asked whether the existence of a mental abacus influenced their thoughts or task performance. The evidence from this study illustrates that most subjects could envision a mental abacus but not all of them could manipulate it successfully. Most Korean subjects in the study anticipate positive effects from mental abacus use, even in a few cases a life-long occupational effect.

Recommendations

The notion that physical abacus training leads to the formation of a mental abacus, and the suggestion that the physical-mental connections may play a role in learning, deserve further study and have educational implications. Reys and Barger (1994) point out the need for more research regarding the significance of general mental computation skills. The mental abacus notion seems an ideal tool for investigating the relationship between concrete materials and mental computation.

An exploration into the differences in mental abacus use between intermediates and experts might very well shed light on how a mental abacus develops and the distinctive factors operating when a mental abacus is in use. It is also interesting to ask whether the presence of a mental abacus affects the choice of problem solving strategies.

A comparison study for school children between non-abacus learners and abacus learners in the performance of other areas of mathematics such as algebra, geometry, and problem solving might also yield interesting results.

So far, only Korean and Taiwanese subjects have provided evidence of mental abacus use. Given the history of abacus use in Japan and China, an investigation into mental abacus use in those cultures might prove rewarding. The lack of any follow up studies for adults who have trained on and employed an abacus might demonstrate how abacus training is associated with the learning process and with life skills.

There may be a practical limitation to the benefits derived, if the length of time needed to fully involve the learner with the physical abacus and subsequently to develop some mental representation is excessive. That is, though there may indeed be gains in mathematical proficiency through abacus learning, if the amount of time required is very long, then the practical value may be limited in some modern societies.

The study of the mental representation used by Korean adults in mental calculation highlights the long-term effects of abacus use and acquisition of mental abacus skills. In Stigler's final statements (Stigler, 1984), he claimed, "One cannot help but wonder what kinds of results could be achieved by Americans who use a different form of representation, if they were to engage in the same amount of practice". This highlights culture-specific training: mental abacus skills are different from skills acquired in American schools. A cross-cultural study of culture-specific training in different arithmetical/mathematical processes might enrich our understanding of the educational process.

It would be useful to conduct a study to investigate the effects of the abacus curriculum on college students in the lower percentile ranges. More work needs to be done in order to make conclusive statements regarding the effects of the curriculum on this group of students. It may be determined from the results of this kind of additional studies whether:

1. The use of the abacus curriculum with college level students who are in the remedial courses can improve student achievement in some areas of mathematics.
2. The use of the abacus proves to be enjoyable for students and teachers.
3. The abacus curriculum can be incorporated into the classroom to teach a variety of mathematics topics.
4. The abacus curriculum provides a means for presenting areas of mathematics that need increased attention. In particular, to what extent such a curriculum implements mental computation strategies and can be used in the place of other concrete materials (e.g., Cuisenaire rods, etc.). Sometimes used in arithmetic learning in early grades.

5. This might lead to a good comparative cross-cultural research between Asian-American students and East Asian students, especially replicating the research reported here with Korean-American participants who also report use of the abacus.
6. The use of abacus in the mathematics classroom is a productive tool for young children who are learning by visual stimuli and kinesthetic responses.

This research study involved considerable challenges in obtaining a sufficiently large sample size, and the strategies used here may be of interest to others who contemplate studies that require soliciting participants from a broad spectrum of characteristics and/or large geographic areas. The multiple means used here to contact prospective participants proved to be quite satisfactory, and the assistance of knowledgeable professionals also promoted the final assembly of the participant pool. Hopefully, other new researchers may find some of the strategies used, in general, in this research to be informative. Among other aspects of the research design, the combined quantitative and qualitative approach provided a broader source of evidence to inform the analyses and draw conclusions. In-depth interviews along with use of data gathering instruments such as questionnaires can provide sources of complementary information to refine the interpretations of evidence relative to the specific research questions being addressed. Other information on possible further studies that can be pursued pertinent to this research topic is presented in the preceding list of recommended further studies.

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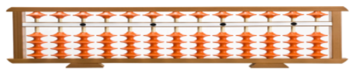
Appendix A. FLYER

HAVE YOU EVER USED AN ABACUS?



LOOKING FOR VOLUNTEERS
"STIGLER'S MENTAL ABACUS: ITS USE BY KOREAN ADULTS"
RESEARCH INVESTIGATOR: SOOMI KIM

A doctoral student at Teachers College, Columbia University is seeking volunteers for a research study.
The purpose of this research is to determine the prevalence of using a "mental abacus" by Korean adults including those who extensively use actual abacuses when performing arithmetic tasks.
There will be no direct benefits to participate in this study and your participation is completely voluntary. All information that you give will be kept private and confidential and your name will never be related to any information you provide.

Possible volunteers are:

Korean adults between the ages of 25 – 65 who have had experience using an abacus.

As a participant in this study, you would be asked to:

Recall some memories from your learning of the abacus and its use and answer questions in a two-part survey. It will take less than an hour.
* Few volunteers may also be selected for a follow-up interview after the two-part survey; it will last less than 1 hour.

If you are interested, please contact:
Soomi Kim, Investigator
soo421@gmail.com, 347-563-5801

The investigator will give you more details and discuss your possible participation in the study.
Thank you!

This study has been reviewed and approved by Teachers College, Columbia University

Appendix B. SURVEY ON EXPERIENCE WITH AND USE OF THE ABACUS

This survey is for Korean adults who were educated in using an abacus. Please return your response by DD/MM/YY. Thank you.

ID No:

In the space provided below describe how many years you have had and also how much experience you have had using the abacus (including years of teaching others to use the abacus, if any):

In the next section, for items 1-4, mark your response using the scale options:

1 = never, 2 = rarely, 3 = sometimes, 4 = usually, 5 = always

1. Do you regularly use an abacus for your computation?

   1 2 3 4 5

2. When you do mental computation, do you use a mental image of an abacus?

   1 2 3 4 5

3. Do you feel comfortable when performing a mental computation?

   1 2 3 4 5

4. How confident are you about your answer when you use a mental abacus?

   1 2 3 4 5

5. When you perform an arithmetic operation such as addition, subtraction, multiplication or division, what kind of tools would you prefer?

   Do you use a calculator? ________
   Do you use an abacus? ________
   Do you need to have a pen and paper for your calculation? ________
   Or, do you not need to have any other tool for your calculation? ________

Appendix C. ASSESSMENT TOOL

This is an example of the assessment that was used during part two of this study.
Please complete each problem. Thank you.

(Addition)

1. $22 + 15$

2. $246 + 384$

(Subtraction)

3. $59 - 33$

4. $391 - 275$

(Multiplication)

5. 75×32

6. 454×129

(Division)

7. $67 \div 5$

8. $245 \div 12$

Appendix D. INTERVIEW QUESTIONS

1. Tell me about your age/gender/occupation. When, where, and how long did you learn an abacus?

2. Do you still use an abacus for your computation?

3. What is your opinion of using an abacus to do computation? Is it helpful for your arithmetic calculation? Please explain your reasons.

4. When you perform an arithmetic operation such as addition, subtraction, multiplication or division, what kind of tools would you prefer? Do you use a calculator? Do you use an abacus? Do you need to have a pen and paper for your calculation? Or, do you need to have any other tool for your calculation?

5. Can you do calculations in your head? Then, how would you do feel about it?

6. During your mental computation, do you use any mental image of an abacus?

7. What is the merit of using an abacus or using a mental abacus for arithmetic calculation?

8. Do you feel any ease or difficulty when you do a mental calculation? If yes, how comfortable or uncomfortable would you feel? Please explain your reason.

9. How long would it take for you to complete it on average when you do computation in mental abacus?

10. Do you feel any advantage or disadvantage for doing mental calculation? If yes, tell me about it with your reason.

11. How confident do you feel about the accuracy of your calculation with the mental abacus?

12. Which is faster for you: an abacus calculation or a mental calculation? How long does it take for you to complete it?

Appendix E. PERMISSION LETTER

Dear Sir/Madam,

I am a doctoral student in Mathematics Education at Teachers College, Columbia University, in New York City. I am interested in interviewing people who may have made use of a "mental abacus" when performing arithmetic tasks. I am writing this letter to ask if you would be willing to talk to me about any such experiences you may have had.

Your participation can help me understand how people learn arithmetic and how they think and reason. Such information may help me understand how to improve the teaching of mathematics.

Your participation in my research will be completely anonymous and anything that you tell me will be kept in confidence. You can decide not to participate at any time if you so desire.
If you agree to above, please sign the contents portion of this document below.

Thank you for your time.

Sincerely,
Soomi Kim

Your name:

Signature:

Contact information (email/phone number):

Date:

Appendix F. INFORMED CONSENT DOCUMENTS

Teachers College, Columbia University
525 West 120th Street
New York NY 10027
212 678 3000
http://www.tc.edu

INFORMED CONSENT DOCUMENTS

Research Title: Stigler's Mental Abacus – Its used by Korean adults

Principal Investigator: Soomi Kim

This is a research study. The purpose of this research is to determine the prevalence of using a "mental abacus" by Korean adults including those who extensively use actual abacuses when performing arithmetic tasks. This may help me understand how people learn and reason arithmetic and how to improve the teaching of mathematics.

If you agree to participate in this study, you will be asked to complete a two-parts survey concerning your past and current experiences with your abacus skills. This will take less than 60 minutes to complete this survey. Research participants will be NOT asked to provide their names and anything that participants respond will be kept in confidence. You can withdraw from the study.

Your participation in this study is completely voluntary and you have the right to refuse to participate or leave the study at any time without any penalty. Please feel free to ask questions at any time. You can skip any question you do not feel comfortable answering.

For further information about the study, please contact Soomi Kim, sk3351@columbia.edu, (347) 563-5801. If you have any questions about the rights of research subjects, please contact the IRB administrator at Teachers College, Columbia University Institutional Review Board /IRB. The phone number for the IRB is (212) 678-4105. Or, you can write to the IRB at Teachers College, Columbia University, 525 W. 120th Street, New York, NY, 10027, Box 151.

Do you agree to participate in this survey?

___ Yes

___ No

Appendix G. PARTICIPANT'S RIGHTS

Teachers College, Columbia University
525 West 120th Street
New York NY 10027
212 678 3000
http://www.tc.edu

PARTICIPANT'S RIGHTS

Research Title: Stigler's Mental Abacus – Its used by Korean adults

Principal Investigator: Soomi Kim

I have read and discussed the purpose of the research and procedures with the researcher.

My participation in research is voluntary. I may refuse to participate or withdraw from participation at any time without jeopardy to future medical care, employment, student status or other entitlements.
The researcher may withdraw me from the research at her professional discretion.
If, during the course of the study, significant new information that has been developed becomes available which may relate to my willingness to continue to participate, the investigator will provide this information to me.

Any information derived from the research project that personally identifies me will not be voluntarily released or disclosed without my separate consent, except as specifically required by law. If at any time I have any questions regarding the research or my participation, I can contact the investigator, who will answer my questions. The investigator's phone number is (347) 563-5801.

If at any time I have comments, or concerns regarding the conduct of the research or questions about my rights as a research subject, I should contact the Teachers College, Columbia University Institutional Review Board /IRB. The phone number for the IRB is (212) 678-4105. Or, I can write to the IRB at Teachers College, Columbia University, 525 W. 120th Street, New York, NY, 10027, Box 151. I should receive a copy of the Research Description and this Participant's Rights document.

If video and/or audio taping is part of this research, I do NOT consent to being video/audio taped. The written, video and/or audio taped materials will be viewed only by the principal investigator. Written, video and/or audio taped materials may NOT be viewed in an educational setting outside the research.

My signature means that I agree to participate in this study.

Participant's signature: ________________________ Date: ____/____/____

Name: ____________________________________