Exploring Algebra Based Problem Solving Methods and Strategies

of Spanish-speaking High School Students

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This dissertation analyzes differences found in Spanish-speaking middle school and high school students in algebra-based problem solving. It identifies the accuracy differences between word problems presented in English, Spanish and numerically based problems. The study also explores accuracy differences between each subgroup of Spanish-speaking students in each category. It identifies specific strategies used by successful students when solving algebra problems. The study also sought to identify factors that could serve to predict Spanish-speaking students’ ability to accurately solve algebra word problems presented in English and Spanish.

A heterogeneous urban sample composed of one hundred and fifty two middle school and high school students were given an assessment composed of pre-approved algebra-based problems and a biographical information sheet. Specific students were then chosen for individual interviews in which researcher sought to gain more in depth information about student’s reaction to assessment. The study found that the average accuracy rate for Hispanics non-ELL and non-Hispanic students was significantly higher for numerically based problems than Spanish word problems. Similarly, the average accuracy rate for Hispanics non-ELL and non-Hispanic students was significantly higher in English word problems that in Spanish word problems. Results showed that there was a significant difference in the overall performance of the assessment between Hispanic ELL and Hispanic non-ELL students. On one particular set, set C (Spanish word problems), findings showed that Hispanic ELL students performed better than
Hispanic non-ELL students and non-Hispanic students. All other subgroup comparisons did not show a significant difference.

The study found that students who were most successful in the assessment: (a) used previous linguistics knowledge and memory of previously seen mathematical problems properly; (b) highlighted the question being asked; (c) used key words to identify mathematical principles and to aid in the translation process; (d) used diagrams, tables and graphs to organize data; (e) showed work and had all processes laid out clearly; and (f) displayed a clear verification process for their answer as strategies for successfully answering the problems. As it was evident through the study, the diversity in the Spanish speaking population and their needs exposes the need for teaching methods, which are inclusive of all populations. Schools must be sensitive to the diversity in which students learn and aim to individualize the teaching for every student. As Hispanics become the largest minority in the United States, understanding the diverse needs of Spanish speaking students in the classroom will be necessary for the development of a better educated society.
Table of Contents

LIST OF ILLUSTRATIONS .................................................................................................................. IV
LIST OF TABLES ................................................................................................................................. V
ACKNOWLEDGMENTS .......................................................................................................................... VI
DEDICATION .......................................................................................................................................... IX

CHAPTER I: INTRODUCTION .............................................................................................................. 1
Need for the Study ................................................................................................................................. 1
Procedures of the Study ......................................................................................................................... 6
Materials............................................................................................................................................... 7
Data Analysis ......................................................................................................................................... 9

CHAPTER II: LITERATURE REVIEW ................................................................................................... 11
Does Language Play a Major Role in Mathematics Problem Solving? ............................................. 11
Everyday English Versus Academic English ....................................................................................... 13
Reading Mathematics .......................................................................................................................... 16
Effect of Language on Representation of Numbers ............................................................................... 20
Effect of Language on Mathematics Discourse .................................................................................. 23
Effect of Multiculturalism on Language Acquisition in Mathematics ............................................... 25
Effect of Language in Mathematics Problem Solving ........................................................................ 26
Effect of Language in Mathematical Understanding ........................................................................... 29
Methodology Research ....................................................................................................................... 31

CHAPTER III: METHODS .................................................................................................................... 34
Research Purpose and Questions ........................................................................................................ 34
Community ........................................................................................................................................... 36
School .................................................................................................................................................. 36

Table 3.1 The Chosen School Demographics ....................................................................................... 37

Setting .................................................................................................................................................. 38
Individual Interviews ........................................................................................................................... 42
Subjects ................................................................................................................................................ 44
Quantitative Study Procedure ............................................................................................................ 45
Qualitative Study Procedure ............................................................................................................... 47
Additional Instruments .......................................................................................................................... 48
Data Analysis ......................................................................................................................................... 49

CHAPTER IV: RESULTS & DISCUSSION ............................................................................................. 51
Quantitative Analysis Overview: Accuracy of Problem Solving ........................................................ 52

Table 4.0 Question Description ........................................................................................................... 53
Problem 1 Accuracy Analysis .............................................................................................................. 55

Figure 4.1.1 Problem 1 Correct Solution ............................................................................................. 56

Table 4.1 Problem 1 Accuracy Analysis ............................................................................................... 57
Problem 2 Accuracy Analysis .............................................................................................................. 59

Table 4.2 Problem 2 Accuracy Analysis ............................................................................................... 59
REFERENCES

CHAPTER V: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

SUMMARY

CONCLUSIONS

RECOMMENDATIONS

Limitations

Benefits of the Study

Future Implications for Educational Practice and Research

REFERENCES

APPENDIX

APPENDIX I- SAMPLE PROBLEMS FOR ALGEBRA ASSESSMENT

APPENDIX II- STUDENT QUESTIONNAIRE

APPENDIX III PROBLEM CHOICE ANALYSIS

Problem 1 Frequency Analysis

Problem 2 Frequency Analysis

Problem 3 Frequency Analysis

Problem 4 Frequency Analysis

Problem 5 Frequency Analysis

Problem 6 Frequency Analysis

APPENDIX

APPENDIX I- SAMPLE PROBLEMS FOR ALGEBRA ASSESSMENT

APPENDIX II- STUDENT QUESTIONNAIRE

APPENDIX III PROBLEM CHOICE ANALYSIS

Problem 1 Frequency Analysis

Problem 2 Frequency Analysis

Problem 3 Frequency Analysis

Problem 4 Frequency Analysis

Problem 5 Frequency Analysis

Problem 6 Frequency Analysis

REFERENCES
Problem 7 Frequency Analysis ................................................................. 134
Frequency Comparison for All Problems .................................................. 137
APPENDIX IV: NATIONAL SPANISH EXAMINATION DESCRIPTION ...................... 142
APPENDIX V: STUDENTS CONSENT (SPANISH) ........................................... 144
APPENDIX VI: SCHOOL APPROVAL LETTER ................................................. 151
LIST OF ILLUSTRATIONS

Figure 4.1.1 Problem 1 Correct Solution

Figure 4.1.2 Problem 1 Incorrect Solution: Interpretation

Figure 4.3.1 Problem 3 Correct Solution (Teacher Provided)

Figure 4.3.2 Problem 3 Partial Credit – Non-Hispanic (Arithmetic Error)

Figure 4.3.3 Problem 3 Partial Credit – Hispanic ELL (Conceptual Error)

Figure 4.3.4 Problem 3 Partial Credit -Non-Hispanic (Conceptual Error)

Figure 4.3.5 Problem 3 No Credit– Non-Hispanic (Multiple Errors)

Figure 4.3.6 Problem 3 No Credit - Hispanic non-ELL (Multiple Errors)

Figure 4.4.1 Problem 4 Partially Correct (Hispanic non-ELL)

Figure 4.4.2 Problem 4 Most Common Incorrect Answer (Hispanic non-ELL)

Figure 4.4.3 Problem 4 Incorrect Answer (Hispanic ELL)

Figure 4.4.4 Problem 4 Incorrect Answer (non-Hispanic)

Figure 4.5.1 Problem 5 Non-Hispanic Student

Figure 4.7.1 Student’s work on Problem 7
LIST OF TABLES

Table 3.1 The Chosen School Demographics

Table 4.0 Question Description

Table 4.1 Problem 1 Accuracy Analysis
Table 4.2 Problem 2 Accuracy Analysis
Table 4.3 Problem 3 Accuracy Analysis
Table 4.4 Problem 4 Accuracy Analysis
Table 4.5 Problem 5 Accuracy Analysis
Table 4.6 Problem 6 Accuracy Analysis
Table 4.7 Problem 7 Accuracy Analysis

Table 4.8 Average Accuracy Rate Comparison Hispanic ELL and Hispanic non-ELL
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Los Angeles, CA April 2013

Andrea Duhon
DEDICATION

This PhD project is dedicated to all of the women, specifically women of color who have a dream to have a PhD one day. I salute you and I encourage you to pursue your dreams. Remember that the greatest success in life is not what we do for ourselves but rather that which we do for those who follow our footsteps.
CHAPTER I: INTRODUCTION

Need for the Study

Many states across the United States have nearly doubled their Hispanic population in the first decade of the twenty first century. A Hispanic by definition is a Spanish-speaking person living in the United States, especially one of Spanish or Latin American descent. In 2004, the U.S. Census Bureau projected an increase of 115% in the Hispanic population between 2010 and 2050 (U.S Census Bureau, Population Division, 2004). The 2010 Census reported that of 308.7 million residents of the United States on April 1, 2010, 50.5 million (or 16%) were of Hispanic origin. The Hispanic population increased by 15.2 million between 2000 and 2010, accounting for over half of the 27.3 million increase in the total population of the country. Between 2000 and 2010, the Hispanic population grew by 43%, four times the 10% growth in the total population (U.S Census Bureau, Population Division, 2010). The Hispanic population grew in every region, most significantly in the South and Midwest. In North Carolina alone, the Hispanic population more than doubled between 2000 and 2010. According to the U.S. Census, Hispanics are projected to be the largest minority group in the United States by 2015 (U.S Census Bureau, Population Division, 2010).

Given the growth of the Hispanic population in the United States, the number of Hispanic students enrolled in public school systems has increased rapidly. This has required school systems to tailor their methods to the needs of this emerging population of learners, known for decades to educators as English Language Learners (ELL students), but early in the twenty first century increasingly referred to as “emergent bilinguals” (Garcia, Kleifgen & Falchi, 2008)

One issue is the mathematics achievement of Hispanic students. According to Thomas
Rivera of the Policy Institute of the University of Southern California, Hispanics are poorly represented in the Science, Technology, Engineering and Mathematics (STEM) professions. A study by the National Science Foundation (2007) found that in 2001 only 728 (4%) of STEM PhD’s were awarded to Hispanics. In 2003, Hispanics composed only 6% of STEM bachelor degrees awarded, and only 1% of Master degrees. Furthermore, of the 1% of master degrees awarded to Hispanics that year, 43% were awarded to international students, which does not include students raised and educated in the United States. The same level of underrepresentation presents itself at the undergraduate level. Although the numbers of Hispanic students interested in STEM fields increased by 33% between 1995 and 2004, a significant number of freshman STEM students drop out of school or leave their STEM fields of study after their first year of college. Today’s limited job market places increased importance on higher education and on STEM. With Hispanics composing a large portion of the minority population in the United States, there is a societal concern about the percentage of higher education degrees awarded to Hispanics in STEM fields. To address the needs of this new population of mathematics students, the National Council of Teachers of Mathematics (NCTM) has proposed measures to help increase the number of Hispanics in mathematics and science careers.

Jorge Chapa of the Indiana University Policy Evaluation & Research Center (2006) describes a leaky pipeline model, where each stage represents a new level of higher education. In the model, demographic and educational trends confirm that the percentage of Hispanic students decreases considerably at each level of higher education (Policy Evaluation & Research Center, 2006). The deficiency in numbers at a collegiate level could be attributed to the decrease in achievement among Hispanic students as they move through the educational system, beginning as early as upper elementary school. In 2008, the Center for Educational Policy found that the
percentage of proficient Hispanic students based on standardized State Mathematics Tests decreased from 67% in fourth grade, to 55% in eighth grade, to 50% in high school (Center for Educational Policy, 2010). One possible explanation for the deficiency in numbers that exist in STEM fields among higher education institutions is this decrease in mathematic proficiency.

One issue relevant to this deficiency in mathematics proficiency is language comprehension. However, language has not been proven to be the sole cause of mathematical comprehension differences among students. While lack of language competency does play a role in the success of students, particularly ELL students, with mathematics problem solving there is not sufficient evidence to determine if such a difference exists between ELL students and their same grade-level counterparts, in isolation of other variables such as cultural relevance, socio-economic status, and parent education among others. Additionally, understanding differences in accuracy and identifying successful strategies used by ELL students of Hispanic descent does not guarantee a solution for the deficiencies that currently exist in terms of numbers of Hispanics in the STEM field.

Understanding the approach used by successful students, however, would begin the process for the creation of appropriate interventions that will aid in improving the current state of mathematics competence among Hispanic students classified as ELL (Khisty, 1997). Hernandez & Nietfield (2004) conducted a study to examine the differences in native Spanish-speaking college students in higher-level mathematics problem solving. In particular, the study sought to answer the following questions:
1. Do significant differences exist in performance, latency and the ability to accurately monitor oneself when solving mathematics problems with a numerical versus verbal component for Hispanic students whose first language is Spanish?

2. What types of learning variables and strategies assist Hispanic English Language Learners in overcoming language barriers to achieve a higher performance in mathematics?

They found that students performed better, monitored more accurately, and showed less “overconfidence” on numerically based problems than on word problems. For one of the problems, there was no significant difference in accuracy between problems presented in numerical form and problems written in English, even after a student had the opportunity to read the problem initially in Spanish. The results prompted the need for an extension in this research. Specifically, further exploration of students' mathematical accuracy based on written language presentation, with a focus on Spanish versus English, which could help to explain the difference in student performance, was needed.

The students’ diverse college majors and daily exposure to mathematics in the classroom provided certain limitations to the study, and prompted the researchers to inquire about a possible extension of the pilot study. The researchers hoped to expand their focus to include grades from middle and high school education.

The current study serves as an extension of the work of Hernandez & Nietfield (2004). The researcher hoped to enhance and extend the previous findings by examining the difference in problem solving accuracy between problems presented in written form, in both Spanish and English, and those presented as algorithms, among a younger sample of students. The researcher
aimed to explore how students’ language familiarity was related to their ability to solve mathematics word problems in both Spanish and English.

Purpose of the Study

The purpose of this research study is to compare students’ accuracy when solving algebra-based problems presented as English word problems, Spanish word problems, and numerically based problems. The study sought to investigate how the different linguistic representations affect an individual student’s accuracy in algebra-based problem solving, specifically focusing on ELL students of Hispanic descent. Simultaneously, the study aims to identify specific strategies used by Spanish-speaking students who successfully solve algebra-based mathematics problems, which could help decrease the deficits in accuracy—if any exist—among the three types of mathematics problems. The aim was to answer the following research questions:

1. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in English with accuracy comparable to numerically based problems?

2. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in Spanish with accuracy comparable to numerically based problems?

3. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in English with accuracy comparable to word problems presented in Spanish?
4. Are there significant accuracy differences in algebra-based problem solving between Hispanic ELL, Hispanic non-ELL and non-Hispanic students when presented with English word problems, Spanish word problems and numerically based problems?

5. What specific linguistic strategies do successful Spanish-speaking students use to solve algebra-based problems accurately?

**Procedures of the Study**

The study uses quantitative and qualitative methods to investigate student performance in mathematics. The quantitative portion of the study tested the accuracy level of students’ performance in problem solving, while the qualitative portion aimed to explore the strategies used by students who were both successful and unsuccessful in solving word problems.

**Subjects**

One hundred fifty two students from six different classes participated in the study. Three categories were created into which students were grouped: Category 1 (Hispanic ELL), Category 2 (Hispanic non-ELL), and Category 3 (non-Hispanic). Students in Category 1 were of Hispanic descent, born outside of the United States, with Spanish as their first language and English as their second language. Students in Category 2 were of Hispanic descent, born in the United States, with English as their first language and Spanish as their second language. All students in Category 2 had at least one parent who spoke fluent Spanish and Spanish was the primary language spoken at home. Students in Category 3 were Non-Hispanic students born in the United States, with English as their first language, who were currently enrolled in a Spanish language program. Some students in this non-Hispanic, native English-speakers category were bilingual in
languages other than Spanish. There was an equal distribution of female and male participants in all categories. No native country preference was given in selecting Hispanic participants.

All participants had completed a first year Integrated algebra course, the first course in the new three-year course curriculum implemented by New York State in 2008. In Integrated algebra I students learn to how write, solve, and graph equations and inequalities. They learn to solve systems of equations, and to simplify exponents, quadratic equations, exponential functions, polynomials, radicals, and rational expressions. A basic introduction of probability and statistics was also included. At the conclusion of the one-year course, students were required to take the New York State Regents Exam (Sibol & Watson, 2012). Integrated algebra is the first course designated for ninth grade students; however, advanced students often take it in the 8th grade. No student participating in the study was enrolled in a class higher than Foundations of Integrated algebra II & Trigonometry in order to avoid a mathematical knowledge gap. Foundations of Integrated algebra II & Trigonometry is a course for students who need to improve their algebra skills and are interested in preparing for the Regents level course in algebra 2 and Trigonometry and/or the mathematics portions of the SAT or ACT examinations. Topics include triangles and trigonometry; coordinate geometry, polynomials, complex numbers, functions, transformations, logarithms and familiarization with SAT problem solving. Student subjects ranged from eighth to tenth grade.

Materials

For the quantitative portion of the study, which sought to answer Research Questions 1-4, students were administered an assessment developed by the researcher and approved by the mathematics assistant principal. The researcher drew upon the pilot study, Examining Individual Differences in Native Spanish-speaking College Students in Higher Level Math Problem-
Solving, conducted by Hernandez & Nietfield in 2004, but was modified based on the difference in age and educational background between their subjects and the subjects of the current research. The quantitative portion of the assessment included a series of high school level algebra problems displayed in three different ways: English word problems, Spanish word problems, and numerically based problems. All three types of problems tested the same concept, but used different numbers to prevent students from memorizing results. The question bank included seven different problems for each set. Students were required to solve only three problems in each of the sets. Students were given scrap paper, and were instructed to show as much of their work as possible. The assessment and scrap paper were collected at the end of the assessment period. A secondary assessment included a survey of students' biographical information, mathematical background, and comfort with mathematics (Appendix II). Both the New York State Spanish Comprehension Exam and the New York State English Comprehension Exam were used to determine each student’s level of language comprehension.

For the qualitative portion of the study, five students were chosen for in-person interviews, two from Category 1 (Hispanic ELL), two from Category 2 (Hispanic non-ELL), and one from Category 3 (non-Hispanic). The following criteria were used to select the two students in each Hispanic category: Student 1 was chosen for having performed with comparable accuracy in all parts of the assessment, and Student 2 for having shown significant gaps in accuracy between Spanish and English mathematics word problems. The student in the non-Hispanic category was chosen for having performed with comparable accuracy in English and Spanish word problems.

The in-person, one-hour interview was digitally recorded for data collection purposes. During the interview, the investigator discussed the student’s biographical information and
written work on the assessment. The interviewer took hand-written notes on the interviewee's responses and attitude toward both the assessment and mathematics in general.

**Data Analysis**

The analysis of data was divided into two portions. For the quantitative assessment, the research was centered on the accuracy of answers given. The range score for accuracy was 0-2 for each problem (2 for a completely correct answer; 1 for a partially correct answer; 0 for an incorrect answer). Since students had a choice of which problems to attempt, an analysis of the question choice is presented in Appendix III.

In an effort to answer Research Questions 1-3, statistical analysis using ANOVA were carried out in order to identify accuracy difference patterns within each of the three categories (Hispanic ELL, Hispanic non-ELL, and non-Hispanic) between sets. Similarly, in order to answer Research Question 4, statistical analysis using t-test were used to identify accuracy difference patterns within each of the three sets (English word problem, Spanish word problem and numerically based problems) between categories. Patterns associated with gender and grade level differences were also explored where applicable. Analysis of the interview portion of the assessment was used to answer Research Question 5. The interviews sought to supply knowledge about specific problem solving strategies for successful Spanish-speaking students. In addition, and in an effort to better understand each student, the mathematical history of each student, the students’ comfort level with mathematics, their general motivation in their studies, and their reaction to the assessment administered was noted.
All the interviews were digitally recorded, which allowed for the development of code patterns among the subjects. Using this coded pattern system, the researcher sought factors that could help predict Spanish-speaking students’ ability to solve algebra-based word problems successfully. During each interview, the students discussed their thinking as they solved each of the problems in the assessment. Students were asked a series of questions based on their individual written work and their level of accuracy. Through this series of interviews, the researcher aimed to discover a possible relationship among the student’s mathematical ability, mathematical understanding, and language comprehension.
CHAPTER II: LITERATURE REVIEW

This chapter is a review of the literature in mathematics education that establishes a need for the study. The chapter begins by exposing the reader to two particular claims in answering the question: Does language play a major role in mathematics problem solving? That discussion is followed by one offering insight on everyday versus academic language, an exploration of what it means to read mathematics, and the effects of language on: (a) Representation of numbers, (b) Mathematical Discourse, (c) Multiculturalism on Language Acquisition in Mathematics, (d) Mathematics Problem Solving and (e) Mathematical Understanding.

Does Language Play a Major Role in Mathematics Problem Solving?

Research in mathematics education has made strong arguments for and against the claim that language plays a major role in the learning of mathematics (Boulet, 2007). Since the 1990s, there has been an increase in the number of studies that focus on the language of mathematics. According to Cobb (1997), studies on the role of language in the teaching and learning of mathematics have traditionally focused primarily on mathematical discourse: “The current reform movement in mathematics education places considerable emphasis on the role that classroom discourse can play in supporting students’ conceptual development” (p. 258). Some researchers have focused on the development of methodological frameworks to analyze mathematics discourse in the classroom (Cobb et al., 1997; Krussel, Springer & Edwards, 2004; Ryve, 2006; Sfard, 2001), while others are focused on the development of approaches that aim to facilitate the reading and the writing of mathematics (Barwell, Leung, Morgan & Street, 2002; Esty, 1992; Adams, 2003; Usiskin, 1996). Researchers in mathematics education are in agreement that communication is essential to the learning of mathematics (Ryve, 2004).
However, as Ryve (2004) cautions, the issue is not whether or not mathematics is learned through communicating, but rather the establishment of means that will help promote productive discourses. As Kaput (1988) states: “The language of mathematics is both a means of communication and an instrument of thought” (p. 167). A student learns a second language best when he can learn it in an authentic and interactive environment (Radford, Netten, & Duquette, 1997).

The idea that mathematics is a universal language has also been discussed in the literature (Whiteford, 2010). While Whiteford sees the reasoning behind the statement “Mathematics is a universal language”, he reminds us that it is important to keep in mind that the phrase is a play on ambiguity. He describes how notations in which we write mathematics are not universal, nor are the terms attached to the notations, nor the way in which we explain them. Moreover, things that seem natural to one culture (through long practice) may seem to be merely complicated “work-arounds” to others.

“The play on words involved in saying that mathematics is a universal language lies in this: mathematics is universal, and mathematics is a language; but it is not universal as a language, nor is it a language insofar as it is universal. The underlying principles, the things discussed, are universal; quantities and structures of various kinds and the logic, so to speak, of how they can relate to each other. But we human beings do not have immediate intellectual access to these things, so we build up to an intellectual understanding of them by efforts of the imagination -- cognitive processes leading to expression in talking, writing, and drawing” (Clemares, Sanchidria & Gervas, 2013). As an educator, mathematician and ELL student, I have experienced the need for understanding of language to understand mathematics. As I transitioned between elementary mathematics into pre-algebra and algebra courses, I noticed a shift in the
complexity of language in mathematics word problems. This language complexity quickly became a barrier in my level of comprehension of the mathematics displayed through word problems.

Language becomes an essential part of understanding mathematics as a student reaches curricula with more advanced topics (Moschkovich, 2000). Researchers in the field often draw upon the experience of English Language Learners, and their progress in mathematics learning, in order to study the role that language plays in the learning of mathematics. The question researchers aim to answer is whether or not language is the sole factor in learning numerical algorithmic patterns, arithmetic word problems, logical thinking progression, mathematical discourse, or some combination of the included factors (Barwell, 2005). The issue of linguistic proficiency and vocabulary comprehension is also important when collecting data and measuring mathematics skills. Vocabulary comprehension has been identified as a major variable in the understanding of mathematics concepts (Kemp & Partyka, 2009). Computational concepts, algorithms, numerical concepts, measurement concepts and the structure of word problems are not necessarily universal (Secada, 1983). According to the New York State Board of Education, three major variables should be considered when assessing and planning appropriate instruction for ELL students: (a) language (literacy and orally in both native and second languages), (b) culture, and (c) educational history.

**Everyday English Versus Academic English**

As a student demonstrates when reading a problem or developing an algorithm, language connects ideas to symbols (Barwell, 2005). Experts in academia would agree that academic English is different from everyday English (Barwell, 2005). Mastery in everyday English does
not imply mastery of academic English (Moschkovich, 1999). According to Thomas & Collier (2002), English as an Additional Language (EAL) learners become proficient in conversational English long before they do so in academic English. Thomas & Collier (2002) found that the most significant variable in how long it takes to learn English is the amount of formal schooling students have received in their first language.

Both of these language conventions, everyday language and academic language, are experienced in the classroom daily. Everyday language conventions are highly contextual, and ELL students are able to infer meaning and interpret visual cues and body language (Jarrett, 1999). Meanings in social discourse are built collaboratively. Jarrett (1999) argues that academic language is more abstract, and common words can take on specialized meanings. In academic discourse, students are often individually responsible for constructing meanings, and must rely on their own understanding of both the language and concepts involved. Both “languages” are important to students’ learning and social development; while students can be relatively proficient in every day language, they must be explicitly taught to use academic language (Kang & Pham, 1995; Laplante, 1997; Lee & Fradd, 1996).

Much debate has centered on which language should be used as the primary language of instruction, English or the child’s home language. Research shows that students’ home languages can play an important role in their mathematics learning, whether or not the teacher speaks these languages (Jarrett, 1999). When students are allowed to use their home language in the classroom, their academic performance and their English-language development often improves (Kang & Pham, 1995; Latham, 1998). Using the home language in academic learning can be especially helpful to younger students, because it can enable them to build a foundation of mathematics concepts before entering higher grades, where language becomes more “de-
contextualized and cognitively demanding” (Cummins, 1992, as cited by Rupp, 1992). According to James Crawford (1995) “skills in content areas like mathematics, once learned in the first language, are retained when instruction shifts to the second language”.

Thomas and Collier (1997) studied a group of Asian and Hispanic students from an affluent suburban school district receiving 1-3 hours second language support per day in a well-regarded English as a Second Language (ESL) program. These students generally exited the ESL program after two years. All of the students were at or above grade level in native language literacy. The study found that students who were between 8-11 years old and had 2-3 years of native language education took 5-7 years to test at grade level in English, students with little or no formal schooling who arrived before the age of eight took 7-10 years to reach grade level in English language literacy, and students who were below grade level in native language literacy took 7-10 years to reach the 50th percentile. This was true regardless of the home language, country of origin, and socioeconomic status (Thomas & Collier, 1997). English Language Learners (ELL students) are, therefore, faced with a challenge: they must confront the content of academic courses with a less-than-mastery level of spoken English. According to recent language acquisition research, this may help to explain why ELL students are lagging behind in mathematics education (Moschkovich, 2000).

As ELL students progress through their academic careers, they may be placed into English as a Second Language (ESL) courses (Miller, & Endo, 2004). Such courses intend to alleviate the language deficiency by aiding the transition process from a student’s experience to his present enrollment in academic courses such as science, social studies and mathematics (New York State Board of Education). Although they may be able to communicate in spoken English, many ELL students are not prepared to face academic English presented in the various subject
areas (Barwell, 2005). This limited knowledge of academic English negatively affects their mathematics achievement.

As classes begin to require a more academic and complex understanding of English, students begin to struggle. At the high school and college level, when the language of mathematics becomes increasingly challenging and complex, ELL students are barely catching up to an appropriate level of academic English (Thomas & Collier, 1997). In such cases, students are left trying to bridge the ever-widening gap between ordinary language and mathematical language (Moschkovich, 2000).

**Reading Mathematics**

In the study of mathematics, language is an integral part of the learning process. The level of connection a student has to reading the language directly affects his ability to understand mathematics (Wallace & Clark, n.d.). According to Wallace & Clark (n.d), the importance of language mastery in an academic setting becomes more apparent as we explore the difference between a literal reading of each word problem and a deeper comprehension when reading mathematics.

Wallace and Clark (n.d.) use three different stances to describe reading mathematics. The first, “Stance I: Reading Problems”, describes the reading of mathematics as the reading of problems: literally, transmitting words into mathematics and highlighting the scope and sequence. Here, the reader aims to solve an immediate problem. Language mastery at this level of reading can be minimal, since there is no necessary inference.

“Stance II: Reading Mathematics”, broadens the meaning of reading mathematics to include understanding the language of mathematics. This stance expands and diversifies the texts
that are used, while focusing on a much broader concept of how mathematics should be used. In such a case, the reader must move beyond the common language to investigate the meaning and implications of language. It is no longer sufficient to take the words of a mathematics problem verbatim to define its algebraic representation. The reader must transition from a procedural understanding to a conceptual understanding of the material.

“Stance III: Reading Life” focuses on the application of concepts, and expands the understanding of mathematics to include the understanding of sociopolitical issues that have a connection with the social context of a student’s life (Wallace & Clark, n.d.). In this reading stance, it is important for the reader not only to understand the language used, but also the implications and cultural context in which the problems are defined. According to Moschkovich (1999), a learner must have the ability to read the language and interpret it, while making inferences based on the language. This process requires a complex use of language not always mastered by English as an Additional Language (EAL) learners (Moschkovich, 1999).

Research about reading mathematics attempts to demonstrate the different levels of mathematical interaction with reading. Such research implies that an ELL student must work beyond the initial reading of a problem in order to determine an answer. Students must be mathematically coherent and able to use language—not only in the translation of meaning but as application to their lives—in order to reach the third level in reading mathematics (Wallace & Clark, n.d.).

According to Wallace & Clark (n.d), reading stances demonstrate why it is critical for students to be able to speak the language, and to read and comprehend academic linguistics, which affects a student’s progress in subject areas other than mathematics. If students can merely
“defend themselves” in understanding everyday English, the only mathematical understanding that can be achieved is that of Stance I: Reading Problems (Wallace & Clark, n.d.). Rather than isolating language comprehension as an element unrelated to mathematics problem solving, educators should encourage students to take an active role in mathematical literacy, because it is deeply embedded in society.

The most useful tool for understanding reading mathematics is the student’s ability to connect mathematics to life (Moschkovich, 1999). Thus, when evaluating students’ achievement in the area of mathematics, we must consider that, in order to perform at or above the level that native English-speaking students do, ELL students must possess the same comprehension level for academic English that native English-speaking students do. In the area of mathematics, a learner’s ability to read “simple” language is not sufficient; rather, he must communicate through language patterns that assimilate mathematics to society (Wallace & Clark, n.d.).

A study carried out in the United Kingdom (UK) examined the way in which students simultaneously learn mathematics and English as an Additional Language. Barwell (2005), who conducted the investigation over a three-year span, investigated how EAL students made sense of arithmetic word problems. The research drew upon ideas from discursive psychology to explore different patterns of attention to genre, mathematical structure, narrative experiences, and written form (Barwell, 2005). The study displayed a notable relationship between mathematics education and bilingual education (Barwell, 2005), namely that language skills are the vehicles through which students learn, apply, and are tested on mathematics concepts and skills, and therefore proficiency in language aids in building proficiency in mathematics. “Our measurement of arithmetic is a measure of two things: sheer mathematical knowledge on the one hand, and acquaintance with language on the other (Thorndike, 1912).” Many students learning
English in the UK study were learning the language alongside a different language. Therefore, EAL students were used in the study instead of ESL students. English as a Second Language (ESL) implies that the students are learning English as the second language; this was not necessarily the case for many of the students in the study (Barwell, 2005). Thus, while ELL students may attain an oral proficiency in 3 to 5 years, academic English proficiency can take 4 to 7 years to master (Hahta, Butler & Witt, 2000).

According to Lemke (2002), language is in many ways the most complex of the known semiotic resource systems. He defines it as “a process which enables students to make, always and simultaneously in every linguistic sign, three kinds of meanings: (a) Presentational meanings, which are presentations of states-of-affairs, of relations among (abstract) "participants" (or "actants") and processes (doings and happenings) involving such participants; (b) Orientation meanings, which index the stance that the meaning-maker is taking to real and potential audiences and interlocutors, and to the presentational "content" (e.g. indications of speaker evaluations of its desirability, importance etc.); and (c) Organizational meanings, which define relations of whole-part and part-part on multiple scales of organization in the linguistic ‘text’ ” (Lemke, 2002). While research in bilingual education helps in some ways to explain why language affects the learning of mathematics, it does not address issues that are specific to the mathematics classroom, nor does it explore how such issues may affect student learning.

Other studies in mathematics learning and the effect of language in learning mathematics include the work of Clarkson and Dawe (1997), who investigated how EAL students’ level of proficiency in English reflected their ability to solve arithmetic word problems in English. Clarkson grouped students using the level of their linguistic proficiency, and tested the students using mathematics problems written in both English and each student’s native language. The
results of his study indicated that students’ low proficiency in two languages implies lower success in mathematics (Barwell, 2005). There was not enough evidence, however, to draw a conclusion from students who had proficiency in only one language (Barwell, 2005). Linguistic proficiency is likely just one of many factors involved in mathematics problem solving. The results of this study, therefore, must be treated with caution when used to make more general conclusions about whether or not language acquisition is a sole determinant in successful mathematics problem solving.

**Effect of Language on Representation of Numbers**

A student’s understanding of number representation greatly influences his ability to understand basic mathematics. In the study *Language Influence on Children’s Cognitive Number Representation*, Alsawaie (2004) examined the way countries with the highest achievement on international tests introduce the concept of counting numbers to young students. The sample used in the study was a group of 90 Arabic children with a mean age of 80 months. The study explored how the representation of numbers might affect the way that students understand mathematics.

The results of the study revealed that language plays an important role in children’s cognitive number representation; further, the presentation of instructions for solving numerical analysis problems affects how the children performed the given task (Alsawaie, 2004). Lee (2007) reiterates, “traditional algorithms can only be meaningfully taught if students have opportunities to engage in conceptually sound activities and to appreciate the meaning of algorithms at the early stage, instead of relying on the mechanical memorization” (p. 48). “When the procedural aspect of computation is overemphasized without clear conceptual understanding
of the place value system, students tend not to think about the meaning of what they are doing and simply parrot someone else’s directions in order to perform calculations” (Lee, 2007, p.48).

Why do Asian children consistently outperform other nations’ children in international tests? Explanations include quantity and quality of mathematics, quality of pre-school education, style of interaction between student and teachers, quality of mathematics textbooks, and attitude of parents and children. One explanation that is often ignored, however, is the theory of linguistic relativity, studied by Miura and Okamoto (1989). Linguistic relativity is the idea that language parallels mathematics in a way that helps to connect mathematical ideas through language (Alsawaie, 2004). Language supports a student’s understanding of mathematics rather than hindering it. The concept of linguistic relativity greatly influences how students develop their mathematical ability, which forms very early in their mathematics careers.

The counting system is one example of how language parallels mathematics. In most eastern cultures, language parallels base-ten counting (Miura, Kim, Chang & Okamoto, 1988). A study involving Asian children examined how having a language that parallels the counting system can raise the performance level of eastern countries. The study showed that the sample group of children from Asian countries had an easier time interpreting base ten number systems (Alsawaie, 2004). Asian students recognized that 23 ones was equivalent to two tens and three ones, while American children struggled to make this connection.

Researchers hypothesized that Asian students were able to make this connection because of the linguistic pattern that exists in Asian languages such as Chinese and the base-ten counting system, where numbers higher than 10 follow a linguistic pattern. For example, rather than using a non-related word such as “twenty” to represent “20,” Asian languages link the word used for
20 to having “two tens” or “two sets of ten.” In the English language, numbers such as 11 and 12 have no connection to the base-ten counting system. In many Asian languages, however, such numbers are expressed as “one ten and one one” and “one ten and two ones,” respectively (Alsawaie, 2004). This connection helped students’ link language representation in mathematics to the numerical patterns of the base-ten counting number system.

Numerical representation is essential to a student’s understanding of mathematics, as stated in the representation standard of the *Principles and Standards for School Mathematics*, published in 2001 by the National Council of Teachers in Mathematics. A disconnect to language exists in American culture, which may help to explain common troubles faced by elementary school children in understanding the counting system. The same challenge may appear in addition and subtraction problems, where students are asked to apply their understanding of the base-ten counting system.

Asian children succeed in transitioning between ones, tens, and hundreds because they use a linguistic pattern that follows a logical order (Alsawaie, 2004). Asia’s linguistic counting system demonstrates the way in which mathematical patterns follow linguistic patterns: the number “ten” is followed by the number “ten-and-one,” followed by the number “ten-and-two,” followed by the number “ten-and-three,” and so on. Thus, high numbers are split into groups and the counting system is organized in the same pattern as the language system. Children who have been taught a language that follows a pattern of grouping under base-ten are better equipped to understand that 23 ones is the same as two tens and three ones (Alsawaie, 2004). This correlation does not explain, however, why Asian children continue to perform better on international tests. Mathematics’ extension of the meaning resources of natural languages begins with the growth of the system of natural numbers (the counting integers) from the first few, to the first few dozen, to
hundreds, thousands, and higher (Lemke, 2002). Nonetheless, it does help to provide a potential area of linguistic influence in early mathematical understanding.

**Effect of Language on Mathematics Discourse**

Learning vocabulary requires learning definitions. According to Morgan (2005), “the choice of a particular definition is presented both in relation to general community values (transparency) and as a personal or contextual matter, related to ‘our intended applications’” (p. 110). This does not mean that a particular concept changes depending on the context. It simply means that different perspectives or descriptions of the same concept serve different purposes at different times. For example, consider the definition of a square. At times, it could be more useful to define it in terms of its properties, e.g., opposite sides are parallel and of the same length and all four angles are 90°, and, at other times, in terms of its classification, e.g., a rectangle whose sides are all the same length. In any case, the same concept—the square—is being defined. According to Morgan (2005), this important but subtle distinction between the manner in which a concept is defined and the substance of the concept can make a difference in how teachers interpret and use definitions in the classroom.

Language plays a key role in the mathematics classroom. In fact, “fluency in it provides access to the whole world of mathematics” (Esty, 1992, p. 32). Much of the attention to mathematical discourse focuses on students’ ability to communicate by clarifying and justifying their ideas and procedures (National Council of Teachers of Mathematics, 1991). Yet the teacher’s role in fostering productive mathematical discourse in the classroom is central. In addition to being responsible for creating the opportunities for students to engage in discussion, exploration, negotiation, and the sharing of knowledge (Manouchehri & Enderson, 1999), the teacher’s own use of language in the mathematics classroom serves as an important example of
effective communication. For example, in saying *two hundred and three-thousandths*, we could be speaking about either 200.003 or 0.203. However, according to Byers (2007), “ambiguous situations contain the potential for change; they are dynamic and can be creative”.

As students progress in their mathematical careers and begin to develop mathematical instinct, their future success depends heavily on their ability to participate in mathematical discourse. Mathematical discourse includes the ability to communicate mathematics, both verbally and in writing (Krussel, Springer & Edwards, 2004). Such an ability is closely related to the ability to use academic language. Students, who are unable to properly communicate with mathematical expressions, may encounter barriers and limitations in the level of mathematics to which they are exposed (Krussel, Springer & Edwards, 2004). This is especially true in higher-level mathematics, where it is often necessary for students to express mathematical concepts in forms other than simple algorithms. Mastery of language at the appropriate level helps learners to achieve mastery in written and verbal expression (Moschkovich, 1999).

Mathematics requires more than a basic ability to set up arithmetic problems and arrive at an answer. Mathematical discourse is important in evaluating a student’s understanding of mathematics, as it helps to determine whether or not he is able to demonstrate understanding in both verbal and written form. Students who are initially challenged by language can consequently struggle to express proper mathematical thinking (Krussel, Springer, & Edwards, 2004). Learners who lack the appropriate level of language mastery may incorrectly display their mathematical understanding of the concepts being evaluated (Krussel, Springer, & Edwards, 2004). This occurrence may help to explain why ELL students are often lagging behind non-ELL students when asked to display their mathematical ability for standardized tests. This is
specifically the case for students in higher education mathematics courses, where discourse is a fundamental element used to demonstrate mastery of a topic.

Effect of Multiculturalism on Language Acquisition in Mathematics

Research in language acquisition also discusses how culture may affect an EAL student’s ability to perform at the same level as a student who has mastered academic English (Moschkovich, 1999). Special attention should be given to cases involving students from different cultural contexts. How does culture affect the development and understanding of mathematics word problems? Are the problems students are asked to solve culturally applicable for more than just the mainstream student, so that students can understand the content of the problem conceptually and focus on its numerically based representation? Those investigating mathematics and language acquisition should pay close attention to the ways in which “real-world” problems relate to the student’s own experience of the world. Researchers should not ignore the way in which teachers mediate the connection between mathematics and the context of these “real world” problems; such knowledge can be a key factor in explaining the effects of language acquisition.

Culturally responsive teaching means that the student’s prior experiences, including fund of knowledge (González, Moll, Floyd-Tenery, Rivera, Rendón, Gonzales, & Amanti, February 1994), home language background, and socio-cultural background are considered. A review of the student’s socio-cultural background should address culturally and linguistically-based issues of motivation and the student’s prior knowledge of the material being learned or studied. For example, students with different cultural backgrounds may be motivated to a greater degree by rewards for collaborative, group efforts than for individual efforts. All of these variables help to determine how the student learns best, in what settings, and under what teaching direction. In
some cases, a student may not benefit from a specific learning strategy simply because he needs a different learning or teaching approach, not because he cannot comprehend the content of the lesson.

Teaching is also a key factor in demonstrating how to negotiate the understanding of a word problem, relate the context of the word problem to the student’s own experience, and negotiate their relationship with one another. Findings suggest that EAL learners draw on their own experiences to contribute to a supportive linguistic context, within which they are able to work on their mathematical tasks (Moschkovich, 2000). The ways in which social interaction within a classroom contributes to the student’s development in mathematics significantly affects how students work together (Moschkovich, 1999). Teachers must adjust their style of presentation and their choice of context to make word problems relevant and sensitive to all cultures. Appropriate instruction includes instruction that is linguistically and culturally responsive. This means that instruction and interventions must consider and build upon a student’s cultural background and experiences as well as their linguistic proficiency (in both English and the native language) (Esparza Brown & Doolittle: NCCREST, 2008; Gutierrez, 2002).

**Effect of Language in Mathematics Problem Solving**

Reitman (1965) defined a problem being given the description of something without having anything that satisfies that description. Reitman's discussion described a problem solver as a person perceiving and accepting a goal without an immediate means of reaching the goal. Henderson & Pingry (1953) wrote that in order to have problem solving there must be a goal, a
blocking of that goal for the individual, and acceptance of that goal by the individual. What one student considers a problem may not be a problem for another -- either because there is no blocking or no acceptance of the goal. Schoenfeld (1985) also pointed out that defining what is a problem is always relative to the individual student.

The basis for a considerable part of mathematics problem solving research for secondary school students in the decades 1980-2010 can be found in the writings of Polya (1973,1962,1965), the field of cognitive psychology, and specifically in cognitive science. Cognitive psychologists and cognitive scientists seek to create or validate theories of human learning (Frederiksen, 1984), whereas mathematics educators seek to understand how students interact with mathematics (Schoenfeld, 1985; Silver, 1987). The area of cognitive science has relied in large part on computer simulations of problem solving (Newell & Simon, 1972; Wilson, 1967). When computer programs generate a sequence of behaviors similar to the sequence for human subjects, the program becomes a theoretical model for the behavior. Newell and Simon (1972), Larkin (1980), and Bobrow (1964) have provided simulations of mathematical problem solving. These simulations may be used to understand mathematics problem solving.

In recent years, constructivist theories have received notable acceptance in mathematics education. In the constructivist perspective, learners are actively involved in the construction of one's own knowledge rather than passively receiving external knowledge. Thus, arranging situations and contexts within which the learner constructs appropriate knowledge is the responsibility of the educator (Steffe, & Wood, 1990; von Glasersfeld, 1989). Constructivism is consistent with current cognitive views of problem solving involving exploration, pattern finding, and mathematical thinking (Schoenfeld, 1988; Kaput, 1979; Lochhead, 1979); thus it is essential that teachers and teacher educators become familiar with constructivist views and
evaluate such views often in order to restructure their own approach to teaching, learning, and research dealing with problem solving.

Good problem solvers in mathematics must develop a base of mathematics knowledge and become effective in organizing that knowledge, which also contributes to successful problem solving (Hadaway, Wilson & Fernandez, 1993). Kantowski (1974) found that students with a good knowledge base were more likely to use the heuristics in geometry instruction. Schoenfeld and Herrmann (1982) found that novices attended to surface features of problems, whereas experts categorized problems on the basis of the fundamental mathematical principles involved. Silver (1979) found that successful problem solvers were more likely to categorize mathematics problems on the basis of their underlying similarities in mathematical structure. Wilson (1967) found that general heuristics were utilized only when preceded by task-specific heuristics.

One of the most important parts of problem solving is checking the solution. It is the set of activities that provides the primary opportunity for students to learn from the problem. The phase was identified by Polya (1973) by such activities as checking the result, checking the argument, deriving the result differently, using the result, or method, for some other problem, reinterpreting the problem, interpreting the result, or stating a new problem to solve. Teachers and researchers report, however, that developing the disposition to look back is very hard to accomplish with students. Kantowski (1977) found little evidence of students checking their work even though the instruction had stressed it.

Problem solving strategies used by successful Spanish-speaking students can help shape a necessary approach to help Hispanic ELL students be more successful in the mathematics classroom (Virginia Board of Education, 2004). Specifically, there is a distinct need to identify
how Hispanic ELL students solve algebra-based mathematics problems (Roberts & Flores, 2009). According to The Final Report of the National Mathematics Advisory Panel (2006) algebra was a major concern in mathematics achievement. There is a sharp falloff in mathematics that begins in middle school, where most students start to deal with algebraic concepts.

**Effect of Language in Mathematical Understanding**

The inevitable connection of meaning and concept may explain why language plays a role in understanding mathematics (Moschkovich, 2000). A comparison of students in the United States and Asia suggests that language can both help and hinder students’ comprehension when learning mathematics. While Han’s study focuses on primary education and basic definitions, one can infer that the alignment to language continues through the secondary mathematics curriculum (Lescault, 2002). According to Han (1998), language plays a major role in the understanding of complex mathematics. What kind of meanings are made with mathematics, and how people learn and can be more effectively taught to create such meanings, is essentially what educators want to know (Lemke, 2002)

In Han’s study, students unfamiliar with the English language easily misinterpreted mathematical terminology. Certain words used in mathematics to express particular concepts are also used in the English language in a multitude of disciplines with different connotations (Khisty, 1995). In fact, mathematical symbolism originated almost entirely as abbreviations for Greek, Latin, and modern European words and phrases (Cajori 1928), which makes it difficult for an ELL student to understand the intended meaning of a word in the context in which it is being used. Mathematics becomes complex as we identify the kinds of meanings it makes:
meanings about addition, subtraction, multiplication, and division; about numerical difference and equality; about geometrical relationships of parallelism, similarity, congruence, tangency, etc., and many more in mathematical jargon (Lemke, 2002). English Language Learners may be forced to translate multiple times before arriving at an appropriate definition (Virginia Board of Education, 2004). The correct translation becomes particularly important when students encounter language-intensive problems as they are introduced to in algebra class.

At the secondary level, students require more than a basic understanding of English—they are expected to understand academic language as well. The language competency is particularly important for Hispanic ELL students who face an additional step in translation when attempting to make sense of mathematical concepts (LaCelle-Peterson & Rivera, 1994). Understanding the deficiencies and strengths among the Hispanic population provides insight on how various strategies can help ELL students excel in mathematics courses during higher education (Khisty & Chaval, 2002).

Mathematical knowledge can be represented both linguistically and symbolically. But even using words, or abbreviations for words, mathematical sentences are about meanings that natural language has trouble articulating. The history of mathematical speaking and writing is a history of the gradual extension of the semantic reach of natural language into new domains of meaning (Lemke, 2002). An increase in nonlinguistic representations allows students to recall knowledge more effectively, and has a strong impact on student achievement (Marzano, 2001, Section 5). In classic education research, Bruner (1961) identified three modes of learning: enactive (manipulating concrete objects), iconic (using pictures or diagrams), and numerically based (working with formal equations). The iconic stage, using pictures and diagrams, is an important bridge to abstracting mathematical ideas using the symbols of an equation. Research
has also validated students’ need to see an idea in multiple representations. Lemke (2002) noted that different methods, whether writing natural language, drawing diagrams, or formulating symbolic expressions, are part of creating mathematical meaning for students.

Like Bruner and Dienes, Skemp (1993) identified the critical middle step in moving from a real-life situation to the abstraction of an equation. While students need to experience many real-life situations, they can get bogged down with the "noise" of the problem, such as names, locations, kinds of objects, and other details. These elements of linguistic meaning-making factors are part of a "system of intemperance" for mathematics (Lemke, 2002). A diagram can help students capture the numerical information in a problem, and, equally important, the relationship between the numbers. Students who use diagrams as part of their thinking process while solving a problem seem to have an easier time arriving at a final answer. They use this intermediate step as a way to organize the "noise" of the problem and shift focus to the mathematical concept being tested. Once students are comfortable with one kind of diagram, they can think about how to relate it to a new situation.

**Methodology Research**

Patten (2002) argued that research should initially conduct stratified random sampling techniques in order to increase the probability that a representative group is drawn to participate in focus groups. Yin (2003) emphasized the use of subunits as representatives of the sample, and pointed out that using data from subunits (in most cases individual students) provides the opportunity for extended analysis, which provides insight into the behavior of the subunits in any given study. Hancock & Algozzine (2006), Merriam (1998), describe the process of using qualitative methods to develop a deeper understanding of how such factors as problem set up,
language, and mathematical content influence the performance of students in a particular assessment.

Patton (2002) and Yin (2001) suggest that qualitative tools such as interviews are used to give an accurate account of the subject’s story. Patton (2002) described the benefits of qualitative research by comparing the process to the act of peeling away the outer skins of a rotted fruit to look inside and find the hidden seed. According to Patton, (2002) digging deeper than numbers through the use of interviews, particular information about the reasoning for student performance becomes more apparent. Merriam (1998) exposes us to different types of questioning techniques that can aid in conducting a great interview. Merriam (1998) describes hypothetical questions as one of the most useful types of question for research. Hypothetical questions begin with “what if” or “suppose” and provided the responder with an opportunity to imagine a particular situation. Patton (2002) and Esterberg (2001) also list four additional types of questions recommended for research: background questions, opinion and value questions, knowledge questions, and sensory questions.

Esterberg (2001) considers interviews the heart of social science research, and Patton (2002) wrote that they present an opportunity for researchers to discover what could not otherwise be observed through quantitative analysis. More akin to guided dialogues or conversations, the interviews allowed participants to express their own ideas and opinions. Hancock & Algozzine (2006) describe this method as a way to have revealed rich, personal information about the individual being interviewed. Merriam (1998) and Yin (2001), also explain the importance of listening attentively, and suggest ways to probe for more in-depth answers when necessary. They go on to warn researchers about the importance of maintaining an unbiased conversation when conducting any interview.
CHAPTER III: METHODS

As the Hispanic population of the United States grows in the early decades of the twenty-first century, public schools struggle to provide a quality education for Spanish-speaking students, specifically, English Language Learners. Middle schools and high schools face a great challenge in educating and retaining these youth; students who drop out of school may experience lost economic potential and enter the nation’s social welfare systems, draining economic resources long-term (Case Study of the English Second Language Programs at a North Carolina School District, 2010). By identifying specific strategies used by successful English Language Learners to overcome their language deficit in the classroom, the education system could help increase the likelihood that these students will succeed in school and become productive members of society. By coming to conclusions about whether instruction and assessment in mathematics in the native language is more beneficial long-term than instruction and assessment in English, the education system could retain more students.

An overview of the research design follows, which contains a description of the setting, participants, procedure, and processes that were used to gather and analyze the data. The concluding section describes the proposed data analysis, and addresses the technical adequacy of the study.

Research Purpose and Questions

The purpose of this research is to compare the accuracy a student achieves when solving algebra-based problems presented as English word problems, Spanish word problems, and numerically based problems. The study sought to investigate how the different linguistic
representations affect an individual student’s accuracy in algebra-based problem solving, specifically focusing on Spanish-speaking students of Hispanic descent. Simultaneously, the study aims to identify specific strategies used by Spanish-speaking students who successfully solve algebra-based mathematics problems, which could help decrease the deficits in accuracy—if any exist—among the three types of mathematics problems.

For the purposes of this study, linguistic representation is defined in three categories: (1) English word problems, (2) Spanish word problems, and (3) Numerically based problems. The researcher undertook an investigation to answer the following primary research questions:

1. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in English with accuracy comparable to numerically based problems?
2. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in Spanish with accuracy comparable to numerically based problems?
3. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in English with accuracy comparable to word problems presented in Spanish?
4. Are there significant accuracy differences in algebra-based problem solving between Hispanic ELL, Hispanic non-ELL and non-Hispanic students when presented with English word problems, Spanish word problems and numerically based problems?
5. What specific linguistic strategies do Hispanic ELL successful students use to solve algebra-based problems accurately?
In order to answer Research Questions 1-3, the researcher sought to determine any differences in accuracy between sets for Hispanic ELL students, Hispanic non-ELL and non-Hispanic students. In order to answer Research Question 4, the researcher sought to determine any differences in accuracy between student categories for English word problems, Spanish word problems and numerically based problems. Finally, in an effort to answer Research Question 5, the researcher aimed to investigate the strategies used by Spanish-speaking students, specifically, English Language Learners of Hispanic decent, whom maintained a high accuracy, rate for the problems they chose to solve. In order to understand students’ problem choice better, the researcher considered: Which problems were chosen with the highest frequency among Hispanic ELL, Hispanic non-ELL, and non-Hispanic students (Appendix III)?

Community

District 9, to which The Chosen School belongs, is in the South Bronx. It is part of New York’s 16th Congressional District, one of the poorest congressional districts in the United States. District 9 contains one of the largest public housing projects in the Bronx, and is composed of 83.2% economically disadvantaged families. District 9 averages a 13:2 student to teacher ratio with over 35,000 students and 2,600 teachers in the district. For ethnicity, District 9 is composed of 5.9% White (Non-Hispanic), 19.9% African American, 66.4% Hispanic, 7.4% Asian, and 0.4% American Indian peoples. Minorities, specifically those of Hispanic descent, heavily populate District 9.

School

The Chosen School is a public charter school. It is located in a low-income and
predominately Latino and African American urban community. It is a college preparatory middle and high school. The school currently serves 695 students in grades 5-12, all of whom were admitted through a lottery system. A demographic breakdown of the student population at The Chosen School:

Table 3.1 The Chosen School Demographics

<table>
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<tr>
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<th>The Chosen School</th>
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<tr>
<td>Enrollment:</td>
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<tr>
<td>Attendance:</td>
<td>95.7%</td>
</tr>
<tr>
<td>Free Lunch:</td>
<td>82%</td>
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<td>Admissions:</td>
<td>Lottery/District 9 priority</td>
</tr>
<tr>
<td>Ethnicity %:</td>
<td></td>
</tr>
<tr>
<td>0% White</td>
<td></td>
</tr>
<tr>
<td>49% African American</td>
<td></td>
</tr>
<tr>
<td>49% Hispanic</td>
<td></td>
</tr>
<tr>
<td>1% Other</td>
<td></td>
</tr>
<tr>
<td>Reading:</td>
<td>29.4%</td>
</tr>
<tr>
<td>Math:</td>
<td>47.2%</td>
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<tr>
<td>Graduation Rate:</td>
<td>70%</td>
</tr>
<tr>
<td>Graduation Rate Six Years:</td>
<td>92.5%</td>
</tr>
<tr>
<td>English Language Learners:</td>
<td>4.6%</td>
</tr>
<tr>
<td>Special Education:</td>
<td>10.8%</td>
</tr>
<tr>
<td>College Ready:</td>
<td>7.1%</td>
</tr>
<tr>
<td>College Enrollment:</td>
<td>72.5%</td>
</tr>
</tbody>
</table>

Note: The following definitions identify the statistics reported above.

1. **Attendance:** Average daily attendance
2. **Reading:** Percentages of students who met New York State standards by achieving a level 3 or 4 on the state reading exams.
3. **Math:** Percentages of students who met the New York State standards by achieving a level 3 or 4 on the state mathematics exams.
4. **College Ready:** Percentage of students who graduated in 4 years who score at least a 75 or higher on English Regents Exam and an 80 or higher on Mathematics Regents (minimum scores required to avoid remediation track at CUNY schools).
5. **College Enrollment:** Percentage of students who graduated in 4 years who enrolled in a 2 or 4 years institution or vocal program within 18 months of graduation.

The mission of the school is “to prepare underserved middle school and high school students for higher education, community involvement, and lifelong success”. Such a mission is achieved through a structured, caring environment of high academic expectations. According to the New York State Middle School Mathematics Exam, 86% of the middle school students at The Chosen School were operating at or above grade-level in mathematics in 2011. At the high
school level, however, the performances on the mathematics end of course Regents exams were not as high. In 2010, 71% of students enrolled in mathematics courses at The Chosen School passed their corresponding Regents exam. In 2011, however, only 49% of the students passed the Regents exam, compared with a 58% passing rate for the city, a significant drop for The Chosen School from the prior year. In 2012 the passing rate for students at The Chosen School went up to 79%, a rate slightly lower than the 81% passing rate for the city. It is the speculation of the researcher that the restructuring of the mathematics curriculum in New York State, which took place right before the 2010-2011 school year, could have affected the scores of The Chosen School and other New York City schools during that year.

The study used both quantitative and qualitative methods to gather knowledge about Spanish-speaking, specifically, ELL students’, algebra-based problem solving. The quantitative methods were used to find accuracy differences in problem solving among English word problems, Spanish word problems, and numerically based problems. The qualitative portion of the study explored the strategies used by both successful and unsuccessful students, particularly those who were English Language Learners, when solving algebra-based word problems.

**Setting**

A precise description of District 9’s ELL population was essential to the study. Subgroups within the population of Spanish-speaking students including English Language Learners varied academically and culturally. As recommended by Patten (2002), the researcher initially conducted a stratified random sampling to increase the probability that a representative group of ELL students was selected to participate in focus groups. The sample was randomized and a
portion of students was tested based on the participation of their mathematics teacher. Students were sorted first by grade level, second by gender, and finally by ethnicity. All students—including but not limited to English Language Learners—were tested in the teacher’s classroom. The study used two additional focus groups, therefore, as a means to compare ELL student performance in algebra-based problem solving with that of their counterparts. These two additional groups were Hispanic non-ELL students and non-Hispanic students.

All students participating in the study received a letter seeking permission for their participation in the study. The letter of consent required a signature from both the participant and his or her legal guardian. Consent forms were made available in Spanish and English for parents who had a particular language preference. Only students whose legal guardians had given permission were included in the research results. Some results were excluded from the research because consent forms were not returned. The researcher attempted to communicate via telephone or email with the parents of any student whose consent form was not return. Specific attention was paid to parents who were not fluent in English and did not understand the nature of the study.

The researcher sent e-mail invitations via the school’s e-mail service to chosen mainstream teachers. This email communication explained the purpose of the research, and presented an opportunity for the invitees to indicate their willingness to participate in the study or ask clarifying questions.

Due to a limited response from teachers, the researcher asked the Mathematics Assistant Principal to assist in identifying teachers who were willing to participate. In addition, ESL teachers assisted the researcher in identifying students who were representative of the ELL
population of the school and who were willing to participate. The students who participated in the study, therefore, were a combination of both randomly selected ELL students—chosen by means of their already assigned mathematics classrooms—and volunteers.

The teachers who were selected to participate in the study administered the assessment without supervision. Each teacher participated in a one-to-one training session with the researcher prior to administering the assessment. The test was administered to all students through their mathematics courses over two days, based on the students’ block schedules. Students were allotted only one day testing period.

Although the design of the study sought to ascertain a sample that was representative of the overall population, quantitative analysis could supply only a portion of the information necessary. The researcher used a mixed-method style case study, which included qualitative research methods. This study was conducted during the 2010-2011 school year, and was limited to the boundaries of the school district at the middle school and high school level. Although the unit of analysis was the three mathematics teachers and their six mathematics classes chosen by the assistant principal, subunit interviews of specific students occurred. As Yin (2003) pointed out, using data from subunits (individual students) provides the opportunity for an extended analysis, which enhanced insight.

The researcher used qualitative methods as described by Hancock & Algozzine (2006), Merriam (1998), and Yin (2003) to develop an understanding of how problem set-up, language, and mathematical content influence the performance of students in this assessment. After the assessment, the researcher conducted in-person, one-hour interviews. Through such interviews, the researcher hoped to understand the students’ accuracy scores on the assessment and
investigate factors that might explain the students’ success or lack of it in algebra-based problem solving. Specifically, the researcher looked to explore students’ mathematical thought processes and procedural approach. Using both the information from the interviews and the assessment results, the researcher conducted an analysis of the results acquired at The Chosen School during the study. Students were given a questionnaire that allowed the researcher to create a data classifying system into which external elements that may have influenced the results of the study—such as student background, years of study in country of origin, years of study in the United States, comfort with subject, and cultural influence—were factored.

Patton (2002) described the benefits of qualitative research by comparing the process to the act of peeling away the outer skins of a rotted fruit to look inside and find the hidden seed. Patton (2002) posits that by digging deeper than numbers through the use of interviews, particular information about the reasoning for student performance becomes apparent. Currently, the ELL population sees only marginal success in mathematics. As researchers look past empirical data to study the school setting, a more detailed and expansive picture of the realities of everyday school life for the Spanish-speaking population emerge. By carrying out qualitative research methods through personal interviews, and combining the results with quantitative data, the researcher was able to identify the strategies used by successful ELL students that allowed them to perform at or above the level of their Hispanic non-ELL and non-Hispanic counterparts when solving mathematics problems, specifically during state testing. Regarding the development of ELL students’ mathematics accuracy, qualitative methods helped to identify the gaps that might exist in quantitative methods. As recommended by Patton (2002) and Yin (2003), the researcher used qualitative tools such as interviews to give an accurate account of the Spanish-speaking student’s story.
Individual Interviews

Interviews offered an opportunity for the researcher to enter the perspective of the person being interviewed, probe for understanding, and modify questions, as the process unfolded. Esterberg (2001) viewed interviews as the heart of social science research, and Patton (2002) wrote that they presented an opportunity for researchers to discover what could not otherwise be observed through quantitative analysis. More akin to guided dialogues or conversations, the interviews allowed participants to express their own ideas and opinions. This method revealed rich personal information about the individual being interviewed (Hancock & Algozzine, 2006).

As recommended by Merriam (1998) and Yin (2001), the researcher made every effort to maintain the conversation, remain unbiased, listen attentively, and probe for more depth in answers when necessary.

The interviews were loosely structured using open-ended questions. The researcher conducted the interviews on an individual basis with students chosen based on their written assessment results. The interview guides contained hypothetical questions, one of the four types of questions recommended by Merriam (1998). The hypothetical questions began with “what if” or “suppose” and provided the responder with an opportunity to imagine a particular situation. The researcher also used four types of questions described by Patton (2002) and Esterberg (2001): background questions, opinion and value questions, knowledge questions, and sensory questions. Background questions were asked to give the interviewer insight about the participant and to assist in creating a relationship. Opinion and value questions were used to gain an understanding of the cognitive and interpretive processes of the participant. Knowledge questions were used to inquire about the actual information the participant had. Sensory questions focused on what the responder knew or perceived through the senses.
The questions were designed to seek information that might not be found in the original assessment. Many of the students displayed a limited amount of work in the assessment. It was difficult, therefore, for the researcher to identify why a student chose to solve a problem in a particular way. By interviewing the student, the researcher was able to fill the gaps in student thought created by the lack of work displayed on the scrap paper of the original assessment. Prior to beginning the assessment, students were provided a background information sheet, which then guided the researcher’s interview. Once the researcher had a good grasp on the student’s attitude towards mathematics, the researcher used both the original assessment and the student’s answers to discuss problem choice and process in solving the mathematics problem.

The researcher began the interviews by presenting the purpose of the research, reviewing the guidelines, and allowing the interviewee an opportunity to ask clarifying questions. All legal and ethical requirements were followed, and responses were confidential. All interviews were digitally recorded. Handwritten reflections of the interviews were recorded, and included both notes on verbal and nonverbal behaviors the researcher witnessed and parenthetical comments about associated thoughts or connections made by the researcher.

Because the researcher was previously employed as a teacher by The Chosen School for four years, she had easy access to the school. The researcher had also built up credibility and trust with teachers and administrators over time. While this connection offered many positive outcomes, the researcher was also aware of her ethical responsibilities. She was careful not to let personal bias or preconceived ideas enter the field. When the study was complete, all parties involved were promptly notified and letters of appreciation were sent to each individual, thanking him or her for the opportunity to use the school. At the conclusion of the study, the researcher agreed to share information including the study’s quantitative and qualitative analysis.
as deemed appropriate by all parties involved: specifically, the Mathematics AP, the principal at The Chosen School, and the Head of School.

**Subjects**

One hundred fifty two eighth and ninth grade students of three different teachers’ enrolled in six different classes participated in the study. All participants had completed 90% of a first-year Integrated Algebra course. Integrated algebra was the first course in the new three-year course curriculum implemented by New York State in 2008. Students learn to how write, solve, and graph equations and inequalities. They learn to solve systems of equations, and how to simplify exponents, quadratic equations, exponential functions, polynomials, radicals, and rational expressions. Other topics included are probability and statistics. At the conclusion of the one-year course, students took the New York State Regents Exam (Sibol & Watson, 2012). Integrated algebra is the first course designated for students to take in high school; the course is often optional to advanced students starting in the 8th grade as well.

No student participating in the study was enrolled in a class higher than Foundations of Integrated algebra II & Trigonometry. By limiting the level of mathematics that a student had been exposed to, the researcher hoped to have all students participating in the study with the same exposure to mathematics curricula at The Chosen School. Foundations of Integrated algebra II & Trigonometry is a course for students who need to improve their algebra skills and are interested in preparing for the Regents course in algebra 2 and Trigonometry and/or the mathematics portions of the SAT or ACT examinations. Topics include triangles and trigonometry; coordinate geometry, polynomials, complex numbers, functions, transformations, logarithms and familiarization with SAT problem solving.
The students were separated into the following three subgroups:

- **Category 1: Hispanic ELL** (Students of Hispanic descent, born outside of the United States, with Spanish as their first language and English as their second language).
- **Category 2: Hispanic non-ELL** (Students of Hispanic descent, born in the United States, with English as their first language and Spanish as their second language).
- **Category 3: non-Hispanic** (Students not of Hispanic descent born in the United States, with English as their first language, who are currently enrolled in a Spanish language learning program).

No native country preference was given when selecting participants.

Of the 152 students tested, teachers administering the assessment returned only 50 exams to the investigator. Of the 50 exams returned, five exams could not be used because of missing information. The remaining sample included eight students in “Category 1: Hispanic ELL,” of whom six were female and two were male; fifteen students in “Category 2: Hispanic non-ELL,” of whom six were female and nine were male; and twenty-two students in “Category 3: non-Hispanic,” of whom sixteen were female and six were male. In total, the sample was composed of 45 students: 28 females and 17 males. The three teachers chosen to administer the test were all certified mathematics teachers of New York State, currently employed by The Chosen School. No preference was given to gender. Two teachers were female and one was male. All of the teachers had been working at The Chosen School at least one full year.

**Quantitative Study Procedure**

Students were administered a written assessment (Appendix I) for the quantitative portion of the study, to allow Research Questions 1-4 to be addressed. This assessment was developed
by the investigator, and drew upon the pilot study by Hernandez & Nietfield described in Chapter 2. The quantitative portion of the assessment was comprised of a series of high school level algebra problems. The problems were displayed in three different ways, and coded (below) for statistical purposes:

- **Verbal 1**: Problems presented as an English word problem and administered to students as “Set A - English Word Problems.”

- **Numeric 1**: Problems presented using symbolic representation and administered to students as “Set B – Numerically based Problems.”

- **Verbal 2**: Problems presented as a Spanish word problem and administered to students as “Set C - Spanish Word Problems.”

All three types of problems tested the same concept, but used different numerical values in order to keep students from memorizing the final results. The question bank included seven different problems in each category, which gave the students a total of 21 problems from which to choose, seven from each category. Each problem tested a different mathematical concept, and varied in difficulty and presentation. Each problem of the seven targeted a particular skill, for which the researcher tested accuracy differences. The assessment was given in the following order: (1) English word problems, (2) numerically based problems, (3) Spanish word problems. However, students were not instructed to go in a specific order.

Students were asked to choose three of the seven problems in each category, remaining consistent for all of the sets. For example: if a student chose to solve Problems 1, 3, and 5 for Set A, he was required to answer the same problems (1, 3, and 5) for Sets B and C. All problems were previously discussed with The Chosen School Mathematics AP, who approved the material and found it to be appropriate for students who had completed at least 90% of one full year of
Integrated algebra I according to the New York state curriculum guideline. Students were given the assessment sheets and scrap paper during their check-in, and were instructed to show as much work as possible for each of the problems assigned. The students’ written work, along with the assessment, were collected at the end of the assessment period by the individual teacher proctoring the exam. Each teacher was responsible for submitting results to the investigator upon the completion of their proctoring.

**Qualitative Study Procedure**

Students were given a biographical information survey (Appendix II), which was designed by the investigator as a way to identify outside factors that may affect the results of the study. Such factors—which included a student’s gender, race, age, country of origin, and parents’ marital status—were specifically sought out in order to explore additional variables that might correlate with the results of the study. Other factors were also considered, including when the student learned English, when the student moved to the United States, and what language was spoken in the student’s home. Additionally, the survey asked questions specific to the student’s attitude towards mathematics, the number of mathematics classes the student had taken previously, and the level of student performance in his or her mathematics courses.

For the qualitative portion of the study, five total students were scheduled for an in-person, one-hour interview with the investigator. The interview was digitally recorded. During the interview, the student’s biographical information sheets and written work on the assessment were discussed. The interviewer created hand-written notes about each student interviewed. The researcher aimed to analyze students’ mathematical responses and attitude towards the assessment, as well as his or her attitude towards mathematics in general.

Of the five students chosen, two students belonged to Category 1 (Hispanic ELL:
Hispanic descent, born outside the United States, with Spanish as their first language and English as their second language); two students belonged to Category 2 (Hispanic non-ELL: Hispanic descent, born in the United States, with English as their first language and Spanish as their second language); and one student belonged to Category 3 (non-Hispanic: born in the United States, with English as their first language and in a Spanish as a second language program). The following criterion describes the method used to select the two students in each category: Student 1 was chosen for having performed with comparable accuracy in all parts of the assessment, while Student 2 was chosen based on the appearance of significant gaps in accuracy between Spanish and English algebra-based word problems. This process was repeated for each of the categories tested. The student in the non-Hispanic category was chosen for having performed with comparable accuracy in English and Spanish word problems.

**Additional Instruments**

In order to determine each student’s level of linguistic ability, results from the National Spanish Examinations and the New York State English Regents Exam were used. The National Spanish Examination is an online, standardized assessment instrument for grades six through twelve that measure proficiency and achievement of students who are studying Spanish as a second language. The purpose of the National Spanish Examination is to recognize achievement in the study of the Spanish language, to promote proficiency in interpretive communication in the Spanish language, to assess the national standards as they pertain to learning Spanish, and to stimulate further interest in the teaching and learning of Spanish. In addition, this examination is often used to prepare students for other standardized tests such as AP, IB, SAT II and college placement exams. The New York State English Regents Exam is a comprehensive exam of literature review and writing skills intended to determine whether or not a student is performing
at grade level with regards to his or her English proficiency. The New York State English regents Exam is given every year in June.

**Data Analysis**

The analysis of data was broken up into two portions, qualitative and quantitative. For the quantitative portion of the assessment, the analysis focused on the accuracy of the answers to each problem. In order to grade the accuracy of the problems, the researcher made a key and assigned a score of 2, 1, or 0. A score of 2 was given for completely correct answers. A score of 1 was given for students whose work showed that the inaccuracy of their answer was due to either (a) an arithmetic error or (b) a minor conceptual error. A score of 0 was given to those whose answer was completely incorrect.

If a student did not follow instructions, he was given a DU (Did Not Understand) and his paper was not scored. For example, Problem 1 was a multiple-choice problem about identifying a type of problem to solve (Appendix I), but many students chose to solve each choice rather than identify which answer choice was solvable. The student did not understand the directions of the problem and therefore received a DU mark. Similarly, if a student chose a question as one of his three questions to answer but did not answer one of the sets for that question as instructed, he received a NA (Not Answered) for the set he did not answer. In the case where a student earns an NA mark, a student may have answered Set A and Set B for Problem 3 but did not answer Set C of Problem 3. If the problem was left blank for all sets, the student was not marked at all. The data was then analyzed statistically for patterns that existed among all students and within each of the three groups (Hispanic ELL, Hispanic non-ELL and non-Hispanic). Single and multi-variable ANOVA tests were used to find statistical significance in accuracy differences between
the three sets within each student group. Unpaired t-test were used to find statistical significance in accuracy differences between the student groups within the same set.

In an attempt to identify problem-solving strategies used by both successful and unsuccessful students, the researcher focused on the following categories: (a) the amount of written work each student displayed, (b) the number of conceptual errors that were made, and (c) how often arithmetic errors were the cause of an incorrect answer. For students who did not perform well on the assessment overall, the researcher looked for patterns in the type of errors made: whether the student made a conceptual or arithmetical error, and how often such an occurred. By determining whether the error was conceptual or arithmetic, the researcher could categorize the student responses as either completely or partially incorrect.

The second portion of the analysis, the qualitative analysis, used personal interviews in an attempt to answer Research Question 5. The interviews were used to gain further knowledge about the background of each student, and to understand the student’s comfort level with mathematics, his general motivation in his studies, and his reaction to the assessment administered. All interviews were digitally recorded, allowing the investigator to code patterns for the responses given by each of the students selected. The researcher looked for factors that could potentially help to predict native Spanish-speaking students’ ability to successfully solve mathematics word problems. During each interview, the students discussed their thinking as they solved each of the problems on the assessment. Students were asked a series of questions based on their individual work and their accuracy level (Appendix II). The student’s comfort level, previous experience, general attitude about mathematics, and the biographical questionnaire were also discussed.
CHAPTER IV: RESULTS & DISCUSSION

The results presented in this chapter are organized by research question. To answer Research Questions 1-3, the issue of mathematical accuracy is addressed. For Research Question 4 a comparison among groups for each individual category is reported. Research Question 5 addresses the identification of strategies students used during the assessment to accurately solve algebra-based problems. By addressing these issues, the researcher seeks to find (a) how students’ mathematical accuracy varied depending on the way that the problem was presented, and (b) what strategies used by students were successful in solving the problems. Additionally, the researcher hopes to (a) identify how problem choice affected the results of the study and (b) what specific factors could have prompted students to choose a particular problem.1 The researcher hopes to use findings to replicate strategies that could help future learners and educators in the field.

Appendix III discusses the effect of problem choice in the study. The researcher chose to analyze problem choice as a means of identifying how patterns in problem choice affected a student’s accuracy when solving mathematical problems. By analyzing which problems students chose to solve most often, and why, the researcher could identify potential factors that may have affected problem choice. If we can understand what type of problems students are more likely to solve, we may be able to determine if there is a relationship between the problems chosen and accuracy score achieved on the assessment.

To answer Research Question 5, results are presented based on individual interviews with specifically chosen students. The students had mastered successful strategies to deal with

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1 Results regarding problem choice can be found in Appendix III
language barriers and were able to perform at a high level, regardless of the language the problems were presented in. The researcher hoped that through her study of these particular students who were able to demonstrate mastery of mathematical concepts, educators might be able to develop better teaching and learning strategies for students who are typically unsuccessful. Gutierrez (2002) found that when students are provided with adequate support while simultaneously expressing their mathematical thinking in their most comfortable language, students who traditionally underperform could achieve success.

**Quantitative Analysis Overview: Accuracy of Problem Solving**

In an effort to answer Research Questions 1-4 a breakdown of accuracy comparisons and results follows. In the Hispanic ELL student category, students answered Problems 1-4 partially or completely correct. No student in this group received any credit for Problem 5 or 7. Problem 6 is excluded from the accuracy part of the analysis because none of the students in this group attempted the problem. In the Hispanic non-ELL student category, students who answered Problems 1-3 received partial or full credit for their responses, while no Hispanic non-ELL students received credit for their attempts in Problem 4. Problems 5, 6, and 7 are excluded from the accuracy analysis because none of the students in this category solved these problems accurately. Although five students (two Hispanic ELL students and three non-Hispanic students) attempted Problem 7, it was not solved partially or completely correctly by any of the students.
A breakdown of the type of problems in the assessment follows:

**Table 4.0 Question Description**

<table>
<thead>
<tr>
<th>Problem 1: Used multiple-choice question format, contained short sentences, had very little language complexity, and only contained whole numbers within the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1 Mathematical Content: Identifying proportions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 2: Used spoken English verbiage, only contained whole numbers within the problem, and contained context familiar and often reviewed in the algebra curriculum (such as items being sold). Problem 2 did contain additional information not relevant to solving the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 2 Mathematical Content: Addition</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 3: Contained rational numbers and limited units of measure to units used as part of the algebra curriculum, e.g. meters, centimeters, and money.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 3 Mathematical Content: Area of a square, multiplication, ratios</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 4: Used concept of time and unit measures of miles per hour. Only whole numbers were used in the problem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 4 Mathematical Content: Calculating speed, ratios, proportions, and system of equations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem 5: Involved percentages and the development of an equation. Problem 5 was written in academic context involving scientific terms.</th>
</tr>
</thead>
</table>
Problem 5 Mathematical Content: Unit conversion, multiplications, solving multi-step algebraic equations

Problem 6: Involved fraction manipulation, had complex diagrams, and required knowledge of volume (a concept taught in the algebra curriculum as an isolated topic).

Problem 6 Mathematical Content: Volume of rectangular prism, solving multi-step equations

Problem 7: Required the ability to translate a situation in the written format and identify the question being asked. Problem 7 had the most verbiage. It was the least mathematically complex problem, but it had a lot of verbal information that students were required to decipher.

Problem 7 Mathematical Content: Logic

The tables in this section present two different accuracy averages for each of the sets. Recall that for each given problem a score of 0-2 was given, 0 for an incorrect solution, 1 for a partially correct solution, and 2 for a completely correct solution. The maximum score for each set was 6 (2 points for each problem). The column labeled as “Exclusive of DU and NA” denotes the averages without consideration of students who either (a) did not understand the directions of the problem and received a DU or (b) did not answer that particular set of the problem—even though they chose that particular problem to solve—and received an NA. The column labeled as “inclusive of DU and NA” counted these students answers as completely wrong and assigned a zero value to them, as previously stated in the coding system.
Problem 1 Accuracy Analysis

Problem 1 had an unusual pattern of student responses that caused the researcher to consider two sets of data for accuracy measurement and comparison between subgroups. The accuracy level was skewed due to the number of student responses that misinterpreted the problem’s directions. In Problem 1, students were given a multiple-choice problem that asked them to identify other problems solvable by the same method as the provided problem. However, many of the students solved each of the multiple-choice answers rather than identifying the one that was solvable by the same method of the given problem. Out of the 35 students who attempted the problem, 9 of the students received a DU for misinterpreting the directions of the problem.

Most of the students who received a DU solved the multiple-choice answers correctly but did not properly identify which multiple-choice answer was solvable in the same way as the problem given. The directions of each problem were read out loud to all the students. Furthermore students were allotted 5 minutes to re-read the problems and ask any questions. It is the speculation of the researcher that students did not read the directions of the problem and assumed that the problem had multiple parts to it. This particular error in understanding the directions can be due to the already established schemas in their educational patterns when they see a problem with “a-d” choices. Although most of the students who misunderstood the directions of the problem solved each of the answer choices correctly, they did not answer the question according to the given directions. The following are some examples of student work from both those who understood the directions and those who did not understand the directions:
Figure 4.1.1 Problem 1 Correct Solution

Which of the following problems can be solved in the same way as:

3. Melons were selling three for $1. How many could Larry buy for $4?
   a) They’re selling three books for every four students. How many were there in a class of twenty students?
   b) A car travels 25 miles per hour for four hours. How far will it travel?
   c) John has 25 marbles. Sue has 12 marbles. How many more marbles does John have than Sue?
   d) If balloons cost 10 cents each and pencils cost 5 cents each, how much do three balloons and two pencils cost?

Figure 4.1.2 Problem 1 Incorrect Solution: Interpretation Error

Which of the following problems can be solved in the same way as:

1. Melons were selling three for $1. How many could Larry buy for $4?
   a) They’re selling three books for every four students. How many were there in a class of twenty students?
   b) A car travels 25 miles per hour for four hours. How far will it travel?
   c) John has 25 marbles. Sue has 12 marbles. How many more marbles does John have than Sue?
   d) If balloons cost 10 cents each and pencils cost 5 cents each, how much do three balloons and two pencils cost?
Due to this structural error, the table of accuracy for Problem 1 (below) includes two sets of accuracy data: (a) data inclusive of students who did not understand the directions (DU) and/or students who did not answer that set of the given problem (NA) but chose the problem as one of the three problems to solve and (b) data exclusive of students who did not understand the directions (DU) and/or students who did not answer that set of the given problem (NA) but chose the problem as one of the three problems to solve. For example, a student who answered all sets of Problem 1 but did not understand the directions in the problem and answered the multiple choice instead would receive a DU for that given set. Similarly, a student who chose Problems 1, 2, and 3 as their problems to solve in the assessment but did not solve Set C of Problem 1 would receive an NA for that particular set of the problem.

**Table 4.1 Problem 1 Accuracy Analysis**

<table>
<thead>
<tr>
<th>Student Category</th>
<th>Problem 1 Set A</th>
<th>Problem 1 Set A</th>
<th>Problem 1 Set B</th>
<th>Problem 1 Set B</th>
<th>Problem 1 Set C</th>
<th>Problem 1 Set C</th>
<th>Problem 1 Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exclusive of DU and NA</td>
<td>Inclusive of DU and NA</td>
<td>Exclusive of DU and NA</td>
<td>Inclusive of DU and NA</td>
<td>Exclusive of DU and NA</td>
<td>Inclusive of DU and NA</td>
<td></td>
</tr>
<tr>
<td>All Students</td>
<td>1.03</td>
<td>0.56</td>
<td>1.48</td>
<td>0.74</td>
<td>0.83</td>
<td>0.40</td>
<td>1.70</td>
</tr>
<tr>
<td>Hispanic non-ELL</td>
<td>1.00</td>
<td>0.55</td>
<td>1.62</td>
<td>0.72</td>
<td>0.66</td>
<td>0.33</td>
<td>0.61</td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>0.85</td>
<td>0.75</td>
<td>1.14</td>
<td>1.00</td>
<td>0.66</td>
<td>0.50</td>
<td>2.25</td>
</tr>
<tr>
<td>Non-Hispanic</td>
<td>1.20</td>
<td>0.50</td>
<td>1.60</td>
<td>0.66</td>
<td>1.11</td>
<td>0.41</td>
<td>1.58</td>
</tr>
</tbody>
</table>

*Note: Maximum score for each set per problem = 2 points; maximum score for all sets per problem = 6pts*

Hispanic ELL students typically performed better on Set B (Numerically-based problems) than they did on Set A (English word problems) or Set C (Spanish word problems), and they did better on Set A (English word problems) than Set C (Spanish word problems). Hispanic non-ELL students and non-Hispanic students followed similar patterns. Results did not vary when students who were given a DU or NA were excluded. There was a difference of 0.29
points in the accuracy level average between Set B (numerically-based problems) and Set A (English word problems), but a smaller difference of 0.19 existed between Set A (English word problems) and Set C (Spanish word problems). However these differences did not prove to be statistically significant with the sample size used.

Non-Hispanic students did better in Set A (English word problems) than Hispanic students did in general. Hispanic non-ELL students did better than Hispanic ELL students in Set A (English word problems). If we include the cases where students received DU and NA for misinterpreting the problem, Hispanic ELL students had the highest accuracy average followed by Hispanic non-ELL and finally by non-Hispanic students. In Set B (numerically based problems), Hispanic non-ELL students performed best by a small margin, followed by non-Hispanic students. Hispanic ELL students had the least accuracy in the problem. When cases where students received DU and NA for misinterpreting the problem are included, then Hispanic ELL students have the highest accuracy average, followed by Hispanic non-ELL and finally by non-Hispanic Students. In Set C (Spanish word problems) non-Hispanic students had the highest accuracy average, while ELL and non-ELL shared the same accuracy score just below that of non-Hispanic students. When including the cases where students received DU and NA for misinterpreting the problem, however, Hispanic ELL students had the highest accuracy average followed by non-Hispanic students and then Hispanic non-ELL students. Recall that Hispanic ELL students were the most likely of all the subgroups to choose Problem 1 to solve. Thus, Hispanic ELL students had the highest accuracy rate even thought they had the highest percentage of students choose this problem to solve.
Problem 2 Accuracy Analysis

Table 4.2 Problem 2 Accuracy Analysis

<table>
<thead>
<tr>
<th>Student Category</th>
<th>Problem 2 Set A Exclusive of DU and NA</th>
<th>Problem 2 Set A Inclusive of DU and NA</th>
<th>Problem 2 Set B Exclusive of DU and NA</th>
<th>Problem 2 Set B Inclusive of DU and NA</th>
<th>Problem 2 Set C Exclusive of DU and NA</th>
<th>Problem 2 Set C Inclusive of DU and NA</th>
<th>Problem 2 Set Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>1.62</td>
<td>1.36</td>
<td>1.63</td>
<td>1.30</td>
<td>1.24</td>
<td>0.84</td>
<td>3.5</td>
</tr>
<tr>
<td>Hispanic non-ELL</td>
<td>1.64</td>
<td>1.28</td>
<td>1.64</td>
<td>1.28</td>
<td>1.00</td>
<td>0.61</td>
<td>3.17</td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>1.43</td>
<td>1.25</td>
<td>1.57</td>
<td>1.38</td>
<td>1.71</td>
<td>1.50</td>
<td>4.13</td>
</tr>
<tr>
<td>Non-Hispanic</td>
<td>1.67</td>
<td>1.4633</td>
<td>1.63</td>
<td>1.29</td>
<td>1.19</td>
<td>0.79</td>
<td>3.54</td>
</tr>
</tbody>
</table>

For Problem 2, Hispanic non-ELL students had the same accuracy rate for Set A (English word problems) and Set B (numerically-based problems) and considerably lower accuracy for Set C (Spanish word problems). The pattern was the same when DU and NA marks were considered in the data. For Hispanic ELL students, the highest accuracy level was for Set C (English word problem), followed by Set B (numerically based problems), and finally Set A (English word problems). The pattern was the same when DU and NA marks were considered in the data. For non-Hispanic students, the accuracy level was higher for Set A (English word problems), followed by Set B (Numerically based problems) and then Set C (Spanish word problems). The difference in accuracy between Sets 1 and 2 was minimal among this group of non-Hispanic students. Again in this sub-group, the same pattern followed when DU and NA marks were considered in the data, but the difference between Sets A and B increased considerably.
Non-Hispanic students did better than Hispanic students in Set A (English word problems) for Problem 2, while non-ELL Hispanics did better than ELL Hispanics. For Set B, Hispanic non-ELL students had the highest accuracy average, followed by non-Hispanic students, and then Hispanic ELL students. However, when DU and NA were taken into account, Hispanic ELL students had the highest average, followed by non-Hispanics and then Hispanic non-ELL students. As expected for Set C, Hispanic ELL students had a considerably higher accuracy average over non-ELL Hispanics and non-Hispanics.

**Problem 3 Accuracy Analysis**

**Table 4.3 Problem 3 Accuracy Analysis**

<table>
<thead>
<tr>
<th>Student Category</th>
<th>Problem 3 Set A Exclusive of DU and NA</th>
<th>Problem 3 Set A Inclusive of DU and NA</th>
<th>Problem 3 Set B Exclusive of DU and NA</th>
<th>Problem 3 Set B Inclusive of DU and NA</th>
<th>Problem 3 Set C Exclusive of DU and NA</th>
<th>Problem 3 Set C Inclusive of DU and NA</th>
<th>Problem 3 Set Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>0.27</td>
<td>0.12</td>
<td>1.20</td>
<td>0.48</td>
<td>0.24</td>
<td>0.08</td>
<td>0.68</td>
</tr>
<tr>
<td>Hispanic Non-ELL</td>
<td>0.14</td>
<td>0.05</td>
<td>1.14</td>
<td>0.44</td>
<td>0.20</td>
<td>0.05</td>
<td>0.56</td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>0.33</td>
<td>0.13</td>
<td>1.67</td>
<td>0.63</td>
<td>0</td>
<td>0</td>
<td>0.75</td>
</tr>
<tr>
<td>Non-Hispanic</td>
<td>0.18</td>
<td>0.08</td>
<td>1.22</td>
<td>0.46</td>
<td>0.22</td>
<td>0.08</td>
<td>0.63</td>
</tr>
</tbody>
</table>

For Problem 3, similar patterns were found for all student subgroups. The highest accuracy rate was found for Set B (numerically based problems) and the lowest in Set C (Spanish word problems). There was a notable difference between Set B as compared to Sets A and C when data was exclusive of DU and NA cases. When those cases were considered, however, the gap in accuracy was notably less. It is important to note that Hispanic ELL students had very little accuracy in the English word problem and no accuracy at all in the Spanish word problem.
In general, Hispanic ELL students did better than Hispanic non-ELL and non-Hispanic students in Set A (English word problems) of Problem 3. Hispanic ELL students did considerably better than Hispanic non-ELL and non-Hispanic students in Set B (numerically based problems) of Problem 3. When data was inclusive of DU and NA entries for this problem, the gaps between each accuracy average were significantly less. It is important to point out that Hispanic ELL students did not score any points on Set C (Spanish word problem) for this problem, much of which was due to the limited amount of students who chose to solve the problem. No student in any of the subgroups received full credit for their solution. Many students did not receive any credit at all due to the amount of errors committed.

I have explored some of the student errors by analyzing student responses below. Although no student earned full credit for their response, a correct solution sample is provided as a means of comparison to student responses.

**Figure 4.3.1 Problem 3 Correct Solution (Teacher Provided)**

![Correct Solution](image)
The following response earned partial credit points for the student’s work. The student did realize the need for unit conversion, but he committed an error in calculating the area of the rectangular shaped floor by adding the measurements of the sides rather than multiplying the length and the width. The following examples of student work show similar patterns in solving method. Student responses were identical. Although students seemed to approach the problem with slight variation, there seemed to be a similarity in thought process.

**Figure 4.3.2 Problem 3 Partial Credit Non-Hispanic (Arithmetic Error)**

In the second student example for which partial credit was granted, the student did not realize the need for unit conversion in order to solve the problem. The student realized the relationship between the tiles that fit into the room and the rectangular room, but he did not seem to recognize that the units are different. It is important to notice, however, that the student highlighted both unit measurements in the problem as if to bring attention to the differences.

**Figure 4.3.3 Problem 3 Partial Credit – Hispanic ELL (Conceptual Error)**
All three partial credit examples had the same final answer, although they had slightly different approaches.

The next example reflects the error of a student who made multiple mistakes for Problem 3 and received no credit. The student did not show evidence that he or she understood the concept of area and, instead, independently tried to figure out how much each side of the rectangle would cost, given the rate stated in the problem. The student seemed to ignore the primary question and calculated the cost on a per meter basis according to the length and width measurements instead. There is little to no connection between the cost and the tiles.

In the last example student work from Set C (Spanish word problems) is analyzed. The student had a clear understanding of unit conversion, as is shown on the right side of the answer
by the handwritten scale “100cm=1m.” The student attempted to create a chart for which each meter is represented and allocates a “30.” It is unclear as to where the “30” came from in the problem. Taking in consideration that Set A of the same problem used a 30 by 30 tile as a reference, it is possible that the student maintained that reference, attempted to imitate it, and forgot to restructure the problem using the new measurements given.

**Figure 4.3.6 Problem 3 No Credit – Hispanic non-ELL (Multiple Errors)**

By analyzing the student responses for Problem 3, some patterns arise in the students’ thought process. Some major commonalities among the student responses are (a) students who got partial credit had at least some knowledge of how to work with unit conversions; (b) students who got partial credit showed work in a logical pattern and highlighted major mathematical clues in the problem; (c) students who did not receive any credit either had major conceptual errors in their mathematical understanding or did not demonstrate enough evidence for researcher to evaluate their knowledge. There is much to be explored about the thinking behind the student answers given the limitation brought by factors such as problem choice, primary language, and overall academic achievement.
Problem 4 Accuracy Analysis

Table 4.4 Problem 4 Accuracy Analysis

<table>
<thead>
<tr>
<th>Student Category</th>
<th>Problem 4 Set A Exclusive of DU and NA</th>
<th>Problem 4 Set A Inclusive of DU and NA</th>
<th>Problem 4 Set B Exclusive of DU and NA</th>
<th>Problem 4 Set B Inclusive of DU and NA</th>
<th>Problem 4 Set C Exclusive of DU and NA</th>
<th>Problem 4 Set C Inclusive of DU and NA</th>
<th>Problem 4 Set Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>0.17</td>
<td>0.06</td>
<td>0.17</td>
<td>0.04</td>
<td>0.36</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>Hispanic non-ELL</td>
<td>0.29</td>
<td>0.11</td>
<td>0.0</td>
<td>0.0</td>
<td>0.20</td>
<td>0.05</td>
<td>0.17</td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>0.20</td>
<td>0.13</td>
<td>0.0</td>
<td>0.0</td>
<td>0.67</td>
<td>0.5</td>
<td>0.63</td>
</tr>
<tr>
<td>Non-Hispanic</td>
<td>0.29</td>
<td>0.08</td>
<td>0.67</td>
<td>0.08</td>
<td>0.25</td>
<td>0.04</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Hispanic ELL students performed with more accuracy for Set C (Spanish word problems) than for the other sets. They also performed with no accuracy on Set B (Numerically based problems). Non-Hispanic students performed with more accuracy on Set B (Numerically based problems) than Set A (English word problems) and Set C (Spanish word problems). They were slightly more accurate on Set A (English word problems) than on Set C (Spanish word problems). When DU and NA cases were not considered, the accuracy level was equivalent in Set A (English word problems) and Set B (numerically based problems).

For Set A (English word problems) of Problem 4, Hispanic non-ELL and non-Hispanic students were equally accurate when DU and NA cases were not considered. When such cases were considered, Hispanic ELL students had the highest average accuracy rate. Non-Hispanic students had the highest accuracy rate for Set B, but it was still considerably low at 0.08. For Set C (Spanish word problems), Hispanic-ELL students had a margin of accuracy that was considerably greater than Hispanic non-ELL students and non-Hispanic students.
It was more difficult to find patterns among the correct and incorrect responses for Problem 4 due to the limited number of students who answered the question. Seven students in the Hispanic non-ELL subgroup attempted the problem, of which only one had an accurate answer. Similarly, five students in the Hispanic ELL subgroup attempted the problem, of which no student received full credit and only one student received partial credit. A series of partially correct and incorrect answers by all student groups follow.

**Figure 4.4.1 Problem 4 Partially Correct (Hispanic non-ELL)**
Although the student above did not solve all parts of the problem correctly, he had the most success with this problem of all attempts made by the students. The student received full credit for his answer in Set A, no credit for Set B, and partial credit for Set C. The student’s layout of the verbal problems show that the student was able to use a table-like format to analyze what was happening to the cars at every hour. This format enabled the student to depict the exact time when the second car overtook the first car. One notable error was the student’s use of information from Set A (English word problems) when attempting to solve Set C (Spanish word problems). Several students misused the information as they attempted to solve Problems 4-7. The students displayed various approaches to solving the problem. Not many apparent patterns arose, although some consistencies of wrong answers appeared across all groups.

In all groups, the most common wrong answer was “1 hour,” which had no logical mathematical explanation. Below is an example of a student’s work on all three sets for which “1 hour” was the given answer. Note that the answer was the same for both Set A (English word problems) and Set C (Spanish word problems), even though the questions had different information and a different final answer. It is also interesting to see that none of the students were able to solve Set B (Numerically based problems) correctly.
Figure 4.4.2 Problem 4 Most Common Incorrect Answer (Hispanic non-ELL)

![Image of Problem 4 Non-ELL Incorrect Answer]

Figure 4.4.3 Problem 4 Incorrect Answer (Hispanic ELL)

![Image of Problem 4 ELL Incorrect Answer]
In the example above, we see that the student was able to construct a pattern of time and distance for the first problem while working to identify the car’s progression. The student seemed confused about what Set B (Numerically based problems) was asking and demonstrated no knowledge of how to solve it. On Set C (Spanish word problems), there was not enough information to make any conclusions about the student’s understanding of the problem. Although no work was shown, the final answer was given using units labeled in Spanish.

**Figure 4.4.4 Problem 4 Incorrect Answer (non-Hispanic)**
In the last example, the student shows a very different approach in solving the problem. The student attempts to identify where the first car is at 11:30 am, when the second car begins its route. The student makes some arithmetic mistakes and has clear misconceptions about how to solve the problem. Specifically, the student lacks knowledge of unit relationships and ratios. On Set C (Spanish word problems), the student attempts to solve the problem and appropriately labels the units in Spanish. The student displays a lack of linguistic mastery, however, when he confuses the quantities of miles and minutes when discussing the second car. The student demonstrated basic knowledge of how to solve Set B (Numerically based problems). Although the student showed some basic procedural errors in his attempt, his work on Set B echoed a larger pattern: on average, non-Hispanic students were more likely to have correct strategies when solving Set B (Numerically based problems) of Problem 4 as compared to the other subgroups.

**Problem 5 Accuracy Analysis**

**Table 4.5 Problem 5 Accuracy Analysis**

<table>
<thead>
<tr>
<th>Student Category</th>
<th>Problem 5 Set A Ex. of DU and NA</th>
<th>Problem 5 Set A Inc. of DU and NA</th>
<th>Problem 5 Set B Ex. of DU and NA</th>
<th>Problem 5 Set B Inc. of DU and NA</th>
<th>Problem 5 Set C Ex. of DU and NA</th>
<th>Problem 5 Set C Inc. of DU and NA</th>
<th>Problem 5 Set Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>0</td>
<td>0</td>
<td>0.50</td>
<td>0.04</td>
<td>0</td>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td>Hispanic non-ELL</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Non-Hispanic</td>
<td>0</td>
<td>0</td>
<td>0.67</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Note: (-) Denotes non-applicable result based on no participants in the category.

Problem Number 5 was not answered correctly by any of the Hispanic students. While there were very few attempts in general, the accuracy level was minimal for Hispanic students. We can speculate that this was the case because of the difficulty of the topic or the complexity of
the problem itself, which required a multi-step solving process and involved application of other academic topics. Non-Hispanic students composed the majority of cases for Problem 5 and displayed a much higher rate of accuracy for Set B (numerically based problems) than they did for Set A (English word problems) and Set C (Spanish word problems). Additionally, the accuracy level for this group was higher for the Spanish word problems than for the English word problems.

**Figure 4.5.1 Problem 5 Non-Hispanic Student**
In the example above, the student was not able to get any credit for Set A or Set C. However, it is important to note that the student had the same approach and answer for both sets, even while the sets had different contents. For Set B (numerically based problems), the student had a basic understanding of how to solve the problem but still displayed several procedural and arithmetic errors.

**Problem 6 Accuracy Analysis**

**Table 4.6 Problem 6 Accuracy Analysis**

<table>
<thead>
<tr>
<th>Student Category</th>
<th>Problem 6 Set A Excl of DU and NA</th>
<th>Problem 6 Set A Incl of DU and NA</th>
<th>Problem 6 Set B Excl of DU and NA</th>
<th>Problem 6 Set B Incl of DU and NA</th>
<th>Problem 6 Set C Excl of DU and NA</th>
<th>Problem 6 Set C Incl of DU and NA</th>
<th>Problem 6 Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>0.25</td>
<td>0.02</td>
<td>0.6</td>
<td>0.06</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
</tr>
<tr>
<td>Hispanic non-ELL</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Non-Hispanic</td>
<td>0.25</td>
<td>0.04</td>
<td>0.6</td>
<td>0.13</td>
<td>0</td>
<td>0</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Note: (-) Denotes non-applicable result based on no participants in the category.

Non-Hispanic students were the only group that attempted Problem 6. For the purposes of evaluating their accuracy, however, we will only discuss the results. Non-Hispanic students had a higher accuracy rate for Set B (Numerically based problems) than Sets A (English word problems) and C (Spanish word problems). The margins of difference between the accuracy in each were (a) substantial when DU and NA cases were excluded from averages and (b) minimal when DU and NA cases where included in the averages. The lack of accuracy among many students could be attributed to the students not having covered the topic prior to the assessment or possible intimidation due to the complex graphics presented.
Problem 7 Accuracy Analysis

Table 4.7 Problem 7 Accuracy Analysis

<table>
<thead>
<tr>
<th>Student Category</th>
<th>Problem 7 Set A Exclusive of DU and NA</th>
<th>Problem 7 Set A Inclusive of DU and NA</th>
<th>Problem 7 Set B Exclusive of DU and NA</th>
<th>Problem 7 Set B Inclusive of DU and NA</th>
<th>Problem 7 Set C Exclusive of DU and NA</th>
<th>Problem 7 Set C Inclusive of DU and NA</th>
<th>Problem 7 Set Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hispanic non-ELL</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Non-Hispanic</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: (-) Denotes non-applicable result based on no participants in the category.

While both non-Hispanic students and Hispanic ELL students attempted Problem 7, there was a 0% accuracy level for any student group. It is possible that students missed important details because the problem included substantial contextual information, was complex in vocabulary and syntax, and required the use of analysis and synthesizing. The problem did not contain a high level of mathematical complexity, but it did require that students decipher the given information in order to arrive at a conclusion. This example for all student groups supports the theory that the verbiage of a given problem determines the accuracy level for students, regardless of the level of mathematical complexity. This could be an interesting idea to explore further using a larger sample size. Students’ sample work for Problem 7 follows below.
Both of the responses above had logical wrong answers. The first response, given by an ELL student, highlights information he sees as important in order to come up with an answer. The student’s final answer is 50 miles, which is the same as the total distance between the two trains. The researcher speculates that the student assumed that the distance traveled by the hawk was the same distance between the two trains. It is clear that the student did not consider that the trains were moving towards each other. In the second example, the student adds the speed of the hawk in miles per hour to the speed of the two trains. The researcher assumes that the student perceived the common unit of measurement between the two objects and attempted to solve the problem this way, without truly understanding what the problem was asking. There was no evidence to suggest that the student attempted to draw a picture of the scenario or that he aimed to interpret the data by using a table. Because of the limited amount of work that was shown on
this problem—and the small number of students who chose to attempt it—no clear implications can be made about the student’s understanding of the problem. Too many outside factors could have influenced the student’s thought process.

**Overall Accuracy Analysis**

Student performance on each set overall can be seen in the table below. The table depicts the total average of points earned per set and the average percentage score per set for each of the student groups that were tested. The highest possible score on any set was six points, and each problem could earn the student two possible points for accuracy. Recall that a completely correct answer was given a score of 2, a partially correct answer was give a score of 1, and a completely wrong answer or unanswered item was given a score of 0. Given that students were asked to answer three problems in each set, the accuracy score per set ranged from 0 to 6. The averages were slightly skewed, however, because some students answered more or less questions than they were asked to. A score of 2.12 would therefore signify the average score on a particular set for the problem chosen by the student. The highest possible score for each problem was 6. A maximum score of 2 could be earned for complete accuracy on each set of the problem. The percentage score that follows represents the average accuracy level of the students in that given group. For example 34.4% would mean that the average accuracy rate on the given set was 34.4% out of 100%.

**Table 4.8 Accuracy Set Comparison**

<table>
<thead>
<tr>
<th>Category</th>
<th>All Sets</th>
<th>% Score</th>
<th>Set A Pt. Avg</th>
<th>Set A %</th>
<th>Set B Pt. Avg</th>
<th>Set B %</th>
<th>Set C Pt. Avg</th>
<th>Set C %</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students</td>
<td>6.20</td>
<td>34.44</td>
<td>2.12</td>
<td>35.33</td>
<td>2.66</td>
<td>44.33</td>
<td>1.42</td>
<td>23.67</td>
</tr>
<tr>
<td>Hispanic non-ELL</td>
<td>5.50</td>
<td>30.56</td>
<td>2.00</td>
<td>33.33</td>
<td>2.44</td>
<td>40.74</td>
<td>1.06</td>
<td>17.59</td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>7.75</td>
<td>43.06</td>
<td>2.25</td>
<td>37.50</td>
<td>3.00</td>
<td>50.00</td>
<td>2.50</td>
<td>41.67</td>
</tr>
<tr>
<td>Non Hispanic</td>
<td>6.21</td>
<td>34.49</td>
<td>2.17</td>
<td>36.11</td>
<td>2.71</td>
<td>45.14</td>
<td>1.33</td>
<td>22.22</td>
</tr>
</tbody>
</table>
The overall student average accuracy score on the test was 6.2 out of a possible 18 points, which was a 34.44% accuracy rate on the test. Hispanic ELL students did better overall on the test than any other group of students with an average score of 7.75 out of 18 possible points, which was a 43% average accuracy rate. Hispanic non-ELL students had the lowest score with an average score of 5.50 out of 18 possible points, which was a 30.56% average accuracy rate. Non-Hispanic students were slightly above the average with a score of 6.21 out of 18 possible points and a 34.49% average accuracy rate. For Set A, Hispanic ELL students scored the highest with a score of 2.25 out a possible 6 (37.50% accuracy rate), followed by non-Hispanic students with a score of 2.17 out of a possible 6 (36.11% accuracy rate), and finally Hispanic non-ELL students with a score of 2.0 out of a possible 6 points (33.33% accuracy rate). The .07-point difference between Hispanic ELL and non-Hispanic students was minimal, showing just a 1.39% difference in average accuracy rate. The same patterns followed for Sets 2 and 3.

Hispanic non-ELL students performed better in Set B (Numerically based problems) than Set A (English word problems) and Set C (Spanish word problems). The difference between the score on Set B (Numerically based problems) and Set C (Spanish word problems) was 1.38 points. Non-Hispanic students followed similar patterns. Hispanic ELL students performed better in Set B (Numerically based problems) than Set A (English word problems) and Set C (Spanish word problems). However, they performed better in Set C (Spanish word problems) than in Set A (English word problems). Hispanic ELL students had the highest average score in Set C (Spanish word problems). Non-Hispanic students scored the second highest in Set C (Spanish word problems), although their score was slightly below the all-student average. Non-Hispanic students may have scored below the all-student average because students whose second language
is Spanish are often exposed to a limited amount of Spanish academic language in their Spanish classes.

Results of the study are presented by research question. Additional findings that provide a clearer picture of the results will be discussed at the end of the section.

For Research Question 1, do Hispanic ELL, Hispanic non-ELL, and non-Hispanic high school students solve algebra-based word problems presented in English with accuracy comparable to numerically based problems? The results showed that Hispanic- ELL performed better in numerically based problems, with an average accuracy rate of 3 (SD=0.92), than in English word problems, with an average accuracy rate of 2.25 (SD=1.16). However, since our test statistic $f = 2.0323$ did not exceed our critical value of 4.6001, we can concluded that there was no significant difference between accuracy in English word problems and numerically based problems. Hispanic non-ELL students had a 2.44 (SD=1.88) average accuracy rate for numerically based problems and a 2 (SD=1.49) for English word problems. Since our test statistic $f = 0.9552$ did not exceed the critical value of 4.196, we can conclude that there was no significant difference between the accuracy in English word problems and numerically based problems. Non-Hispanic students also performed better in the numerically based problems with an average accuracy rate of 2.71 (SD=1.83) compared to a 2.17 (SD=1.49) on the English word problems. Since our test statistic $f = 1.5808$ is greater than our critical value of 2.0842, we can conclude that there was no significant difference between the accuracy in English word problems and numerically based problems for this population.
For Research Question 2, Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic high school students solve algebra-based word problems presented in Spanish with accuracy comparable to numerically based problems? The results showed that Hispanic ELL performed better in numerically based problems, with an average accuracy rate of 3(SD=0.92), than in Spanish word problems, with an average accuracy rate of 2.5(SD=1.41). However, since our test statistic \( f = 0.7 \) does not exceed our critical value of 4.6001, we can concluded that there was no significant difference between accuracy in Spanish word problems and numerically based problems. Hispanic non-ELL students had a 2.44(SD=1.88) average accuracy rate for Numerically based problems and a 1.05(SD=1.39) for Spanish word problems. Since our test statistic \( f = 8.5953 \) is greater than our critical value of 4.196, we can conclude that the average accuracy rate for Hispanics non-ELL was significantly higher in numerically based problems than in Spanish word problems. Non-Hispanic students also performed better in the numerically based problems, with an average accuracy rate of 2.71 (SD= 1.82) compared to a 1.33(SD=1.43) on the Spanish word problems. Since our test statistic \( f = 9.956 \) is greater than our critical value of 2.0842, we can conclude that the average accuracy rate for non-Hispanics was significantly higher in numerically based problems than in Spanish word problems.

For Research Question 3, Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic high school students solve algebra-based word problems presented in English with accuracy comparable to word problems presented in Spanish? The results showed that Hispanic ELL performed better in Spanish word problems, with an average accuracy rate of 2.5(SD=1.41), than in English word problems, with an average accuracy rate of 2.25(SD=1.16). However, since our test statistic \( f = 0.1489 \) does not exceed our critical value of4.6001, we can concluded that there was no significant difference between accuracy in Spanish word problems and numerically based
problems. Hispanic non-ELL students performed at a 2(SD=1.49) average accuracy rate for English word problems and a 1.05(SD=1.39) for Spanish word problems. Since our test statistic $f = 5.1345$ is greater than our critical value of 4.196, we can conclude that the average accuracy rate for Hispanics non-ELL was significantly higher in English word problems that in Spanish word problems. Non-Hispanic students also performed better in the English word problems, with an average accuracy rate of 2.16 (SD= 1.49), compared to a 1.33 (SD=1.46) on the Spanish word problems. Since our test statistic $f = 4.5161$ is greater than our critical value of 2.0842, we can conclude that the average accuracy rate for non-Hispanics was significantly higher in English word problems than in Spanish word problems.

In order to answer Research Question 4, we compare the performance of each subgroup among each individual problem presentation category. To accurately depict significance among results, the use of t-test was used with a $p < .05$ value. A summary of the results of comparisons among the three subgroups can be found in the table below:

### Table 4.9 Calculated t-value

<table>
<thead>
<tr>
<th>Set</th>
<th>Hispanic ELL vs. Hispanic non-ELL</th>
<th>Hispanic non-ELL vs. non-Hispanic</th>
<th>Hispanic ELL vs. Non-Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Set 1</strong></td>
<td>t-value calc</td>
<td>t-value table</td>
<td>t-value table</td>
</tr>
<tr>
<td>Hispanic ELL vs. Hispanic non-ELL</td>
<td>0.409710967</td>
<td>1.7207</td>
<td></td>
</tr>
<tr>
<td>Hispanic non-ELL vs. non-Hispanic</td>
<td>0.333070656</td>
<td>1.6896</td>
<td></td>
</tr>
<tr>
<td>Hispanic ELL vs. Non-Hispanic</td>
<td>0.142256177</td>
<td>1.7011</td>
<td></td>
</tr>
<tr>
<td><strong>Set 2</strong></td>
<td>t-value calc</td>
<td>t-value table</td>
<td>t-value table</td>
</tr>
<tr>
<td>Hispanic ELL vs. Hispanic non-ELL</td>
<td>0.778633784</td>
<td>1.7207</td>
<td></td>
</tr>
<tr>
<td>Hispanic non-ELL vs. non-Hispanic</td>
<td>0.425540458</td>
<td>1.6896</td>
<td></td>
</tr>
<tr>
<td>Hispanic ELL vs. Non-Hispanic</td>
<td>0.428050658</td>
<td>1.7011</td>
<td></td>
</tr>
<tr>
<td><strong>Set 3</strong></td>
<td>t-value calc</td>
<td>t-value table</td>
<td>t-value table</td>
</tr>
<tr>
<td>Hispanic ELL vs. Hispanic non-ELL</td>
<td>2.626768291</td>
<td>1.7207</td>
<td></td>
</tr>
<tr>
<td>Hispanic non-ELL vs. non-Hispanic</td>
<td>0.497295795</td>
<td>1.6896</td>
<td></td>
</tr>
<tr>
<td>Hispanic ELL vs. Non-Hispanic</td>
<td>1.712202634</td>
<td>1.7011</td>
<td></td>
</tr>
<tr>
<td><strong>All Sets</strong></td>
<td>t-value calc</td>
<td>t-value table</td>
<td>t-value table</td>
</tr>
<tr>
<td>Hispanic ELL vs. Hispanic non-ELL</td>
<td>3.67227943</td>
<td>1.7207</td>
<td></td>
</tr>
<tr>
<td>Hispanic non-ELL vs. non-Hispanic</td>
<td>1.49211258</td>
<td>1.6896</td>
<td></td>
</tr>
<tr>
<td>Hispanic ELL vs. Non-Hispanic</td>
<td>2.612168439</td>
<td>1.7011</td>
<td></td>
</tr>
</tbody>
</table>
Results from the t-test showed that there was a significant difference in the overall performance of the assessment between Hispanic ELL and Hispanic non-ELL. Hispanic ELL students performed better than Hispanic non-ELL students \([t(21)=3.67, p<.05]\). Similarly, Hispanic ELL students performed better than non-Hispanic students \([t(28)=2.61, p<.05]\). On one particular set, set C (Spanish word problems), findings showed that Hispanic ELL students performed better than Hispanic non-ELL students \([t(21)=2.63, p<.05]\). Similarly, Hispanic ELL students performed better than non-Hispanic students \([t(28)=1.71, p<.05]\). In set A (English word problems), Hispanic- ELL students performed with the highest accuracy at 2.25(SD=1.16) followed by non-Hispanic students at 2.16(SD=1.49) and then Hispanic non-ELL students at 2(SD=1.49). In set B (Numerically based problems), Hispanic students performed with the highest accuracy at 3(SD=0.92) followed by non-Hispanic students at 2.70(SD=1.82) and then Hispanic non-ELL students at 2.44(SD=1.88). In set C (Spanish word problems), Hispanic ELL students performed with the highest accuracy at 2.5(SD=1.41) followed by non-Hispanic students at 1.33(SD=1.43) and then Hispanic non-ELL at 1.05(SD=1.39). However no other difference in accuracy showed to be significantly different.

The results did not follow expected patterns. While the accuracy of all students was very low, Hispanic ELL students performed better in all sets that other subgroups. This result was contradictory to the hypothesis. One of the most interesting findings was between Hispanic non-ELL and non-Hispanic students. While non-Hispanic students were less exposed to the Spanish language at home, they were equally or better prepared to solve algebra-based word problems presented in Spanish than were Hispanic non-ELL students according to the study. Students classified as Hispanic non-ELL spoke both Spanish and English at home and had at
least one parent whose primary language was Spanish while students classified as non-Hispanic were enrolled in a Spanish class for at least two years.

In order to answer Research Question 5, the qualitative portion of the study will provide additional context and aid in providing answers to some remaining questions.

**Qualitative Analysis Overview: Student Interviews**

In an effort to answer Research Question 5, five selected participants were chosen for one-hour interviews to further examine the strategies they used when solving given problems. During the interviews, students were asked about their responses on the assessment as well as their attitude toward mathematical problem solving as a whole. Each student was given a background questionnaire, which included questions about student background and exposure to mathematics. The student interviews focused on specific topics such as reasons for choosing questions, strategies in translating language, and mathematical problem solving strategies.

Most students discussed how their prior test-taking habits affected how they approached the assessment. When asked about why they chose the first three problems in the sets, many of the students stated that they had begun working through the test before truly reading and listening to the directions. Students chose to continue to work on the problems they had already started rather than choosing new problems and starting over. This was also the reason why many of them answered the answer choices in Problem 1, rather than choosing an answer as instructed in the directions. One student in particular stated that when the assessment was first given, he was under the impression that the test was only the initial set of problems, which were given in English. The student began working without considering what the directions were and only realized that he only had to answer three questions after seeing a second set and returning to read
the instructions. When this same student was asked about the first problem and why he solved the problem choices rather than choosing a multiple choice answer as instructed, he answered that in most cases when he was assessed in high school, “a solvable question was never answered unless it was solved.” In essence, this particular student had a mental schema that instructed him to solve each answer choice since all of the answer choices were solvable. The student assumed that the problem directed him to solve the question rather than identify a type of problem. When asked why the student did not fix his approach for other problem sets, the student stated that he assumed the directions were the same on all sets because the questions were structured the same way. It wasn’t until after discussing the questions during the interview and taking a second look at the assessment that the student realized that the question was not asking him to solve each answer choice but, rather, was asking him to identify a type of problem. Other students in the interviews also confirmed this thought process. This particular finding could provide a limitation in the accuracy of the results, which can serve as a guide for future study structures.

Choosing Questions

In order to better understand the accuracy of students in the assessment, a central part of the interview explored how students chose the questions they would attempt in the assessment. When students were given the assessment, they were read all the directions out loud. They were then given 5 minutes to read through all of the problems first before beginning to work on any of them. Students were to choose three problems they wanted to solve; they would then solve the corresponding problems in the sets to follow. The three sets identified the way a problem was presented. Set A was composed of English word problems. Set B was composed of numerically based problems. Set C was composed of Spanish word problems. Students were to solve nine
problems total, which meant three in each set. As students were choosing questions in the assessment, they were asked to take note on why they were choosing a particular problem.

During the interview process, it was evident that many of the students did not carefully listen to the directions of the selection process. Many of the students began to work on the assessment expecting to answer every problem. Once the student realized that there was more than one set, they returned to the directions only to realize that they had overlooked some details. Four out of the five students claimed that they chose the first three problems because they were the first options they saw. Students did not consider content or layout of the problem. They just took what they considered to be “the easier route.” One student in particular stated “I found that the first few problems were solvable for me; I understood them. I found no reason to try any of the other ones.”

Two of the students choose to skip Problem 3 even though they initially attempted to solve it. According to these two students, when they saw that the problem required the use of rational numbers (decimals in particular) they turned away from solving it. They felt that working with decimals was “hard” for them. Students didn’t feel as comfortable working with rational numbers. They did not have much confidence in working with rational numbers and often avoided working with such numbers in their own classroom. Students also mentioned that the diversity of units that was used in the problem alluded to more difficulty when solving the problem. This type of mathematical anxiety could have also played a role in the results of the study. Problem 3 included units such as centimeters, meters, and currency—all of which were composed of rational number values.

**Strategies in Translating Language**
Another recurring theme during the interviews was the idea of having to translate the problem before attempting it. Hispanic ELL and non-Hispanic student categories discussed how their accuracy in problem solving was a direct result of how accurate the translation process was. Students felt that if the translation process failed, then the mathematical concept was misconstrued and inaccurate results were produced. During the interviews, students described what a strenuous process it was to arrive at understanding what the problem was asking because of the level of difficulty embedded in the problem’s verbiage. Many of the words were common in an academic context—which students found difficult to decipher—as opposed to words used in spoken language. According to the students, key words played an important role in the translation process. Students could use the meaning of key words in order to understand context. In many instances, students admitted that they took a guess at what the words surrounding those key words meant. Students used knowledge of language and personal internal records of previously seen mathematical problems to develop meaning about what the problem was asking. Two of the five students stated that this translation process was another reason why they opted to solve the first three problems. These problems had less words and a more familiar mathematical setting in context overall.

In order for students to be able to solve problems in a language other than their native language, they had to go through a series of translation processes to decipher not only meaning of words, but also contextual meaning that may shift the way the question could be solved. Students—including those who had studied grammar in a second language—were relatively limited to spoken language rather than academic language. This made it more difficult for students to reach a higher level of success in problem solving during the assessment due to the multi-step translation process. First, students had to translate the assessment written at an
academic level of their second language to a spoken language version of the same problem. Then, students had to take this spoken language version of the problem in their second language and translate it to a spoken language version of the problem in their native spoken language. Finally, students had to translate this native spoken language version of the problem in their native tongue to an academic language version of the problem in their native language, so that they could maintain the context and academic rigor of the problem as it was originally stated. However, many issues arose during the later part of the process because students didn’t have a bank of academic-level vocabulary in their native tongue to translate to or from.

This lack of academic language in their native tongue created a gap in their translation process. This gap differed from one student to the next, depending on how long the student was educated in their native country and whether or not their academic language vocabulary bank was built up in their education system prior to coming to the United States and learning a different language. This idea also turned out to be true for non-Hispanic students when attempting to solve problems in Set 3 (Spanish word problems). While translation is often thought of as a one-step process, students find that there is a big difference between spoken language and academic language, which creates a multi-step translation process.

**Mathematical Problem Solving Strategies**

In order to help English Language Learner students solve mathematical word problems with more accuracy, the researcher was interested in investigating the strategies used by ELL students who were successful problem-solvers during the study. Students who did well on the assessment all showed work and had all processes laid out clearly, even for simple arithmetic problems. There was a distinct difference in the amount of work that was shown for each
problem between those students who did well and those students who did not do well. Students who performed well not only had better organization of the problem but also displayed a checking process for verifying their answer for each problem when possible.

Two particular strategies commonly used by successful students, particularly Hispanic ELL students, stood out from the rest: (a) the use of diagrams/tables to organize data and (b) the use of key mathematical terms to identify mathematical principles. Students who used these strategies accurately on the assessment were, on average, more successful than those who did not. Although not all students used both strategies consistently, those students who were in the upper 25% of the sample in accuracy level during the assessment used the strategies for the majority of the time as they were solving problems.

Using diagrams to organize information and create a visual is a mathematical strategy that has helped students get a better grasp of mathematical ideas. For this particular assessment, it was a key component of the success of students, specially the Hispanic ELL student group. Students were able to use this strategy as a way to identify what the question was asking and what information was most important in the problems given. Students laid out their work strategically, often re-stating the important facts in the problem and correlating it to a given number. Students used tables and charts to organize their information. Students showed different graphic organizing methods to help them decipher problems. Speaking about his strategies and way of displaying his work, one particular student stated that laying out the problems using a graphic organizer allowed him to decipher information that was relevant to the problem and external information given in context only. In Problem 2 (the insurance policy problem), for example, students were asked how many total policies a particular insurance agent sold. In order to solve this problem, it was not necessary to know how many total customers visited the
insurance company, given that the problem stated how many actual insurance policies of each type were sold. Students had to identify that this information was external and focus on the information that was necessary to solve the problem. Students considered that graphic organizers aided in identifying such external factors faster because they allow the student to focus on the central question and relative information, rather than external numbers used only to provide context to a real world situation.

Students also utilized key words as a way to understand the central question of the problem. This was particularly important in the translation process of problems written in a language other than the student’s native language. One particular student stated that the first thing they always do when solving a mathematics problem is identify and underline any words in the problem that they deem mathematical. For example “rectangular,” “miles per hour,” and “total cost” were terms that were often underlined by students while solving the problem. Once students had identified key mathematical terms, they turned their attention to commonly used terms that they have seen used in word problems before. For example, students looked for phrases such as “How many?” and “How much?” to help identify what the problem was asking them to do. Lastly, students highlighted the question being asked in order to better identify what the problem was asking and what information was needed in order to answer it.

Students maintained the use of these strategies even when solving problems in a language other than their own. Students’ responses included a series of highlighted words and tables to help organize the given information. Many of the students attempted to translate the problem above the given one as an effort to have full context before attempting to answer it. In most cases, students were not able to translate the entire problem but used the key words identified and translated to make meaning of what was being asked. One particular student stated that in
her translation process, she tried to identify all the words that she knew the meaning of before beginning the problem. She then identified any mathematical terms that were embedded in the context and filled in the blanks by using mental recordings of previously solved problems. No student, however, used the previous set of problems as a guide to understand the context of the problem. Most students in the interview were surprised to see that the context of the problems was the same, even though different numbers were used.

The researcher was surprised to see that students were unable to identify the similarities between problems and did not use this as a strategy to solve the questions in a language other than their native tongue. When questions were originally given to students, the researcher’s greatest concern was that students would quickly notice that the problems were all contextually the same: that the problems were in the same order and only differed in layout and language. It would be interesting to explore this idea further with a greater sample of students.
CHAPTER V: Summary, Conclusions, and Recommendations

This chapter presents a summary of the study and of the research results. The study limitations, implications for educational practice, and recommendations for future research are also presented.

Summary

This study has served as an extension of the pilot study Examining the Differences in Native Spanish-speaking College Students in Higher Level Mathematics Problem-Solving conducted at North Carolina State University in 2004. That study found that students performed better, monitored more accurately, and showed less “overconfidence” on numerically based problems than on word problems. For one particular problem, there was no significant difference in accuracy in solving that problem written in English, even after a student had the opportunity to give the problem an initial reading in Spanish. The results on that problem suggested a need for further exploration in mathematical accuracy based on written language presentation, mainly Spanish versus English. The results of the study also prompted the need to explore mathematical accuracy among a younger population of students.

The researcher in the current study hoped to enhance and extend these findings by examining the difference in algebra problem solving accuracy between English word problems, Spanish word problems, and numerically based problems, among a younger sample of students. Subjects included middle school and high school Spanish-speaking students, in which English Language Learners were included. The research explored how students’ language familiarity
was related to their ability to solve word problems in both Spanish and English. The goal was to decipher what specific challenges language placed in the accuracy level of algebra problem solving among Hispanic ELL, Hispanic non-ELL and non-Hispanic students. Hispanic non-ELL and non-Hispanic student groups were used to cross analyze results with Hispanic ELL students on algebra problem solving among students for which English was their first language. Students in the Hispanic non-ELL group spoke both English and Spanish, with Spanish being the primary language at home. Students in the non-Hispanic group were in a Spanish emersion program at The Chosen School for at least two years.

The purpose of this research was to identify the accuracy differences students exhibit when solving algebra problems presented as English word problems, Spanish word problems and numerically based problems. In addition to investigating how the different linguistic representations affect an individual student’s accuracy in algebra-based problem solving, the study aimed to identify specific strategies used by Hispanic ELL students who successfully solve algebra-based mathematics problems, which could help decrease the differences in accuracy—if any existed—among the three written formats of the problems.

The study aimed to answer the following research questions:

1. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in English with accuracy comparable to numerically based problems?

2. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in Spanish with accuracy comparable to numerically based problems?
3. Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic middle school and high school students solve algebra-based word problems presented in English with accuracy comparable to word problems presented in Spanish?

4. Are there significant accuracy differences in algebra-based problem solving between Hispanic ELL, Hispanic non-ELL and non-Hispanic students when presented with English word problems, Spanish word problems and numerically based problems?

5. What specific linguistic strategies do Hispanic ELL successful students use to accurately solve algebra-based problems?

One hundred and fifty two students were given an assessment produced by the researcher and approved by the mathematics AP at The Chosen School in an inner city area of New York City. The assessment included three sets of problems: (a) Seven word problems presented in English, (b) Seven problems presented as numerically based problems and (c) Seven word problems presented in Spanish. The mathematical content of the problems was the same, but the numerical values in the problems were changed to prevent students from copying the answers from set to set. The assessment was scored on a 2-point scale. Two points were given for a completely correct answer, one point for a partially correct answer and no points for a completely wrong answer. In addition, students were asked to fill out a biographical information sheet (Appendix II) to collect demographic information about the students as well as information about the students’ attitudes toward mathematics in general. After the assessment, five students were selected for an one-hour interview with the researcher. Students were chosen based on their ability to solve the problems in all three sets with comparable accuracy or for having notable discrepancies. The interview focused on discussing particular strategies that were
used to accurately solve the problems as well as error patterns among students who were not successful in the assessment.

Conclusions

Results of the study are presented by research question. Additional findings that provide a clearer picture of the results will be discussed at the end of the section.

Research question 1: Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic high school students solve algebra-based word problems presented in English with accuracy comparable to numerically based problems? The results showed that Hispanic ELL students performed better in numerically based problems, with an average accuracy rate of 3(SD=0.92), than in English word problems, with an average accuracy rate of 2.25(SD=1.16). However, since our test statistic $f = 2.0323$ did not exceed our critical value of 4.6001, we conclude that there was no significant difference between accuracy in English word problems and numerically based problems. Hispanic non-ELL students had a 2.44(SD=1.88) average accuracy rate for numerically based problems and a 2(SD=1.49) for English word problems. Since our test statistic $f = 0.9552$ did not exceed the critical value of 4.196, we can conclude that there was no significant difference between the accuracy in English word problems and numerically based problems. Non-Hispanic students also performed better in the numerically based problems with an average accuracy rate of 2.71 (SD=1.83) compared to a 2.17 (SD=1.49) on the English word problems. Since our test statistic $f = 1.5808$ is greater than our critical value of 2.0842, we can conclude that there was no significant difference between the accuracy in English word problems and numerically based problems for this population.

Research question 2: Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic high
school students solve algebra-based word problems presented in Spanish with accuracy comparable to numerically based problems? The results showed that Hispanic ELL students performed better in numerically based problems, with an average accuracy rate of 3(SD=0.92), than in Spanish word problems, with an average accuracy rate of 2.5(SD=1.41). However, since our test statistic \( f = 0.7 \) does not exceed our critical value of 4.6001, we can conclude that there was no significant difference between accuracy in Spanish word problems and numerically based problems. Hispanic non-ELL students had a 2.44(SD=1.88) average accuracy rate for numerically based problems and a 1.05(SD=1.39) for Spanish word problems. Since our test statistic \( f = 8.5953 \) is greater than our critical value of 4.196, we can conclude that the average accuracy rate for Hispanics non-ELL was significantly higher for numerically based problems than Spanish word problems. Non-Hispanic students also performed better in the numerically based problems, with an average accuracy rate of 2.71 (SD= 1.82) compared to a 1.33(SD=1.43) on the Spanish word problems. Since our test statistic \( f = 9.956 \) is greater than our critical value of 2.0842, we can conclude that the average accuracy rate for non-Hispanics was significantly higher in numerically based problems than in Spanish word problems.

Research question 3: Do Hispanic ELL, Hispanic non-ELL, and non-Hispanic high school students solve algebra-based word problems presented in English with accuracy comparable to word problems presented in Spanish? The results showed that Hispanic ELL students performed better in Spanish word problems, with an average accuracy rate of 2.5(SD=1.41), than in English word problems, with an average accuracy rate of 2.25(SD=1.16). However, since our test statistic \( f = 0.1489 \) does not exceed our critical value of 4.6001, we can conclude that there was no significant difference between accuracy in Spanish word problems and English word problems. Hispanic non-ELL students performed at a 2(SD=1.49) average
accuracy rate for English word problems and a 1.05 (SD=1.39) for Spanish word problems. Since our test statistic \( f = 5.1345 \) is greater than our critical value of 4.196, we can conclude that the average accuracy rate for Hispanics non-ELL was significantly higher in English word problems that in Spanish word problems. Non-Hispanic students also performed better in the English word problems, with an average accuracy rate of 2.16 (SD=1.49), compared to a 1.33 (SD=1.46) on the Spanish word problems. Since our test statistic \( f = 4.5161 \) is greater than our critical value of 2.0842, we can conclude that the average accuracy rate for non-Hispanics was significantly higher in English word problems than in Spanish word problems.

Research question 4: Are there significant accuracy differences in algebra-based problem solving between Hispanic ELL, Hispanic non-ELL and non-Hispanic students when presented with English word problems, Spanish word problems and numerically based problems? Results from the t-test with \( p>0.05 \) showed that there was a significant difference in the overall performance of the assessment between Hispanic ELL and Hispanic non-ELL students. Hispanic ELL students performed better than Hispanic non-ELL students \([t(21)=3.67, p<.05]\). Similarly, Hispanic ELL students performed better than non-Hispanic students \([t(28)=2.61, p<.05]\). On one particular set, set C (Spanish word problems), findings showed that Hispanic ELL students performed better than Hispanic non-ELL students \([t(21)=2.63, p<.05]\). Similarly, Hispanic ELL students performed better than non-Hispanic students \([t(28)=1.71, p<.05]\). All other subgroup comparisons did not have a significant difference.

Research question 5: What specific language strategies do Hispanic ELL successful students use to accurately solve algebra-based problems? Students who were most successful in the assessment used the following techniques more often than students who were not successful: (a) used previous linguistics knowledge and memory of previously seen mathematical problems
properly; (b) highlighted the question being asked; (c) used key words to identify mathematical principles and to aid in the translation process; (d) used diagrams, tables and graphs to organize data; (e) showed work and had all processes laid out clearly; and (f) displayed a clear verification process for their answer.

All students who were successful in solving the problems used at least one of the above-mentioned strategies with more consistency that those students who were less successful. Hispanic ELL students in particular used a combination of these strategies more than Hispanic non-ELL and non-Hispanic students. Hispanic ELL students in this study laid out their work strategically, often re-stating the important facts in the problem and correlating it to different algorithms through the use of graphic organizing methods such as tables, diagrams and graphs. They gave particular attention to properly identifying the question being asked. During the interviews, it became evident that Hispanic ELL students who did not perform well often provided a partially correct or completely correct answer to the wrong question. For example in Problem 2, “An insurance agent visited 585 customers. He sold 76 life insurance policies, 97 fire insurance policies and 208 auto insurance policies. How many policies did he sell in all?”, students often answered how many insurance policies were left unsold instead of answering how many total insurances were sold. A possible explanation for this could be the problem stating the amount of total policies and the students misinterpreting the question given the information provided in the problem. If a student provided “204 insurances” as their answer, they would have gotten the question incorrect. However, 204 insurances represents the number of insurances left unsold, making it evident that the student struggled with understanding what the question asked and what information was pertinent to answering the question.
It was clear that students’ confidence in choosing problems that were numerically based was greater than that for word problems in both English and Spanish. Students answered Set 2 questions (Numerically based problems) with more frequency than set 1(English word problems) and Set 3 (Spanish word problems). Similarly, students answered problems 1-4 more frequently and with more accuracy than problems 5-7. The content for problems 1-4 was more direct, less complex linguistically and shorter in length. Problems 5-7 were lengthier and used more academic language. Students seemed to decide the number of words that the problem contained or the complex visual that was displayed. It is still unclear, however, whether length of problem or complexity of terminology was the driving factor for confusion among the problems that students did not get correct.

Suarez-Orozco, Suarez-Orozco & Todorova (2008) and Cruz, (2009) believed that the longer students are in the United States, the lower the performance in mathematics they demonstrate. If this is indeed true, it could potentially help to explain the differences in accuracy between Hispanic ELL and Hispanic non-ELL students. However, based on the multiculturalism of the Hispanic population and other implicit variables not controlled, such as: years in the United States, gender, family structure, parental citizenship, use of English in the home, and parenting style; it is with great caution that we contribute the reasoning behind the differences in accuracy among Hispanic ELL and Hispanic non-ELL to Suarez-Orozco’s findings.

It is the hypothesis of the researcher that the accuracy differences among the different subgroups (Hispanic ELL, Hispanic non-ELL and non-Hispanic) within the three sets (English word problem, Spanish word problem and numerically based problem) could have been affected by these main factors:
Students could have been prompted by set B (numerically based problems) as to how to solve the problem. This could have worked for or against them in the assessment, as students could have both used the right strategy for all three sets based on the numerically based problems, or the wrong one if they made a mistake in solving the numerically based problems. There was evidence in the student work of this working against students, particularly the Hispanic non-ELL students, who sometimes assumed the answer to be the same for all sets, not realizing that the numbers, not the concept, used in the problem were changed strategically by the researcher.

Parents of Hispanic ELL students in this sample could have had more formal education and thus have been better prepare to help their children in academia. Based to the background of the Hispanic students tested being predominantly Dominican and Puerto Rican; it is possible that parents of the Hispanic ELL students in this study had higher levels of education than a typical Hispanic immigrant. Particularly those parents from Puerto Rico could have had an advantage in helping their students, given that the structure of the education system is similar to that of the United States.

Finally, the teaching at The Choice School had a strong focus on helping Hispanic ELL students adjust to the American education system. They had a particular faculty member assigned to serve as a curricular guide to help the Hispanic ELL population of students become integrated better into the school system. As part of their schedule, students had the opportunity to meet with the curricular advisor weekly during a designated period of the day to help students with their struggles in academia. Their curricular advisor had a mathematical background, which allowed students to get help with mathematics. In particular the curricular advisor worked on helping students transition between courses provided in their native language through the ESOL
program and mainstream courses provided in English. This could have provided additional guidance in how to approach problems given in different forms, as was the case during the assessment. Having this additional resource in the school could have potentially provided a “special” advantage for the Hispanic ELL students. This particular resource could have attributed to the differences accounted for in the study with this particular sample of students.

**Recommendations**

**Limitations**

This research faced some significant limitations. The first and most restraining was the time constraint. The school allotted only one class block period for the testing of students. That meant that the design of the study was constrained by the time allotted for students to take the exam. This allotted time was limited to seventy minutes, in order to provide the teachers participating in the study with ample time before and after exam for procedural issues. This time limitation to give the exam also presented issues when gathering all the necessary paperwork for student participation. Teachers focused on gathering materials only on the day they were allotted for the exam, which in turn caused students to be excluded from the sample. Consent forms and biographical sheets were lost in the shuffle, and this caused the sample size to shrink considerably.

Additionally, the study took place near the end of the school year in order for the students participating in the study to have had been taught at least 90% of the Integrated algebra I curriculum. This presented several issues for the researcher, including: (a) decrease in teacher participation, (b) decrease in complete assessments being turned in by teachers who did participate, (c) conflict with end of the year testing by the state and (d) pressure from the upcoming Regents exam for students. The students had to take a Regents exam at the end of the year, and pass it, in order to fulfill the graduation requirements at The Chosen School. The
teachers in the study indicated that having so many deadlines at the end of the year, including final exams, state exams, and school functions, resulted in failing to 1) administer assessment, 2) collecting all portions of assessment so that assessment could be valid and 3) turning in material to the researcher. Teachers expressed that it was difficult to focus on having all the materials for the research turned in when so many other assessments (which affected their own evaluations) were being given at or around the same time. It is possible that if the study took place earlier in the year, the teachers would not have been so overwhelmed and sample size would have increased. However, then the issue of curricular exposure could have been a problem.

The overall scores of the assessment were below passing level for all of the three subgroups used. This leads to the question of how effective the teaching in The Chosen School is overall, and whether there are additional external factors that have an effect on the result of the study. The students in the study were mostly low-income, minority students in an inner city area primarily composed of African American and Latino cultures. The parameters of that region could have affected the findings.

While the study provides valuable information about the subjects of the study, the low sample size may not be representative of a larger population, specifically Hispanics in the United States. The location where the study was conducted has a concentrated number of Hispanics from particular areas, mainly Dominican Republic and Puerto Rico, and could provide a skewed representation of the Hispanic population as a whole. Additionally, the subjects were in large part from low socio-economic backgrounds, and that may have had an impact on the results of the study. The small sample used for the study, for which the data has been provided, may be a representation of the situation of those students only.
**Benefits of the Study**

By understanding how Hispanic ELL students compare to Hispanic non-ELL and non-Hispanic students, educators can target the influencing factors on accurate performance for this particular population. This study provides knowledge about how students respond to algebra-based problem solving, and can help to create teaching structures for Spanish-speaking students, in particular those who are English Language Learners, based on successful problem solving strategies used by students. If ESOL programs and other programs that support Spanish-speaking students across the United States knew exactly what specific linguistic or algorithmic layouts of a problem would enhance student learning, they would inevitably be saving time and resources in teaching mathematical concepts by using such identified strategies.

Knowing what strategies were used by successful Hispanic ELL students among other Spanish-speaking students, could help the education system develop a structure of teaching methods for English Language Learners, specifically those of Hispanic descent that caters to their individual needs. Teachers may use patterns gathered by the results of the study on strategies most often utilized by successful Spanish-speaking students when solving algebra-based problems, to target specific ways to help students for whom English is not a primary language, with problem-solving. For example, teachers can focus on helping students highlight the question being asked, as a way to help them identify specific information to focus on when problem solving. Similarly, they can use key words to identify mathematical principles as they utilize them to help students in the translation process. Teachers can also focus their attention on showing students how to check their answer, a method not natural to most students, which proves in the study to have a high rate of success in supporting successful problem-solving.
amongst Spanish-speaking students.

**Future Implications for Educational Practice and Research**

This study lends itself to further research in several areas. A more extensive study could determine if the presentation of mathematical problems based on language is more beneficial for students of particular backgrounds and in particular school settings. With a diverse Hispanic population, controlling the setting could provide more accurate results. In this study, students were assigned to subgroups based on Ethnicity and whether or not they spoke English as their first language. A study with similar design could separate the “Hispanic” umbrella to specific countries of origin. In order for this study to be effective, however, the sample size must be large enough that each country of origin is sufficiently represented.

Another study could determine if students' awareness of their strengths had any effect on changes in mathematical understanding, performance, and attitudes. If we can investigate students’ attitudes about the problem before they begun to solve it, perhaps we can relate their confidence in being able to solve a problem given a particular written form to the accuracy of that given problem. Further more, an explorative investigation of factors that influence attitude towards mathematics, such as the instructor and the time of year could provide significant information about student accuracy in algebra problem solving. A comparison of student attitudes at the beginning of the year with that of the end of the year could be an interesting extension of the study.

Additionally, a study conducted to analyze the way students understand mathematical ideas opposed to the way they express them in writing could help to determine specific forms of communicating mathematical ideas which Spanish-speaking students thrive in. If the designed assessment offered an oral assessment versus a written form, perhaps we could investigate if
students had difficulty in expressing their answers in written form or if they truly did not understand the content of the problem. This study could also include an extension in which the student has autonomy to decide the method in which to provide the answer.

This study included mathematical topics in Integrated algebra I, a wide range spectrum of initial mathematical concepts from a range of topics. Perhaps future studies could focus on specific topics in algebra in order to decrease the diversity in content and eliminate additional factors that could impact the results of the study. It may also determine what specific topics affect a student's ability to accurately solve algebra-based problems, if any exist.

Further research is necessary. This study was inconclusive on what specific factors affect the ability to solve algebra-based problems for Spanish-speaking students. It is clear that the limited number of students who participated in the study provided limited results. However, with the Hispanic population increasing steadily, identifying factors that affect their achievement in mathematics is essential in identifying best learning methods for this bilingual population. By investigating how the composition of a given mathematical problem affects the accuracy of problem solving, one can better identify and decipher where and how Spanish-speaking students, specifically those who are English Language Learners, are learning mathematics. Educators can then develop strategies that can: (1) help students decipher through vocabulary when necessary, (2) encourage students to avoid intimidation based on length of problem if any exists and (4) help students to understand struggle as part of the learning process in mathematics, in order to continue and increase Spanish-speaking student performance in algebra-based problem solving.

As the way we teach mathematics evolves and focuses on real life applications (Ellis & Berry, 2005), it is important to know and understand individual learners, in order to cater to their specific learning needs. Knowing that different students may have different learning methods,
schools must be sensitive to the diversity in which students learn and aim to individualize the teaching for every student. As Hispanics become the largest minority in the United States, understanding the diverse needs of Spanish-speaking students in the classroom will be necessary for the development of a better educated society.
REFERENCES


1. Which of the following problems can be solved in the same way as:

Melons were selling three for $1. How many could Larry buy for $4?

a) They’re selling three books for every four students. How many were there in a class of twenty students.

b) A car travels 25 miles per hour for four hours. How far will it travel?

c) John has 25 marbles. Sue has 12 marbles. How many more marbles does John have than Sue?

d) If balloons cost 10 cents each and pencils cost 5 cents each, how much do three balloons and two pencils cost?

2. An insurance agent visited 585 customers. He sold 76 life insurance policies, 97 fire insurance policies and 208 auto insurance policies. How many policies did he sell in all?

3. Floor tiles are sold in squares 30 centimeters on each side. How much would it cost to tile a rectangular room 7.2 meters long and 5.4 meters wide if the tiles cost $0.72 each?
4. A car traveling at a speed of 30 miles per hour left a certain place at 10:00am. At
11:30am, another car departed from the same place at 40 miles per hour, and traveled the
same route. In how many hours will the second car overtake the first?

5. A nurse who works in a hospital years ago mixes a 6% boric acid solution with a 12%
boric acid solution. How many pints of each are needed to make 4.5 pints of an 8% boric
acid solution?

6. A swimming pool is 30 m wide, 50 m long and 7 m deep. After an earthquake, the pool
is tilted along one edge (AB) and the water completely covers side ABCD. At this point,
3/4 of the base is covered by water. What was the water level before the earthquake?

7. Two train stations are 50 miles apart. At 1 pm on Sunday a train pulls out from each of
the stations and the trains start toward one another. Just as the trains pull out from the two
stations a hawk flies traveling the air in front of the first train and flies ahead to the front
of the second train. When the hawk reaches the second train, it turns around and flies
toward the first train. The hawk continues on his way until the trains meet. Assume that
both trains travel at the speed of 25 miles per hour and that the hawk flies at a constant
speed of 100 miles per hour.

How many miles will the hawk have flown when the trains meet?
Set B- Numerically Based Problems

1.) Which of the following problems can be solved in the same way as:

15x = y When x = 5 what does y equal?

a) 25x = y when x = 4 what is y?

b) 12y = 18 when x = 2 what is y?

c) 75 - 32 = r; Find r.

d) .65x + .35y = Z; How much do 4 chocolate bars (x) and 5 blow pops (y) cost?

2.) Simplify the following problem: 9847 + 56 + (-12) + 3746 = ?

3.) Simplify the following problem: ((9.4 * 19.3) / (0.81 * 0.67)) * 0.52 = ?

4.) Solve the following problem for R1 and R2.

27 * R1 = 38 * R2 where R1 = R2 + 4/5

5.) Solve for t.

0.075t + 0.0987(3.6 - t) = 0.7685 * 3.6

6.) Solve for d.

64 * (1/12) * (13/42) * (38) * (7) = (64) * (38) * (d)

7.) Solve the following problem:

- Two train stations are 50 miles apart.

- At 1 pm a train pulls out from each of the stations and the trains start traveling toward one another.

- Just as the trains pull out from the stations a hawk flies into the air in front of the first train and flies ahead to the front of the second train.

- When the hawk reaches the second train, it turns around and flies toward the first train.

- The hawk continues in his way until the trains meet.

- Assume that both trains travel at the speed of 25 miles per hour and that the hawk flies at a constant speed of 100 miles per hour.

- How many miles will the hawk have flown when the trains meet?
Set C- Spanish Word Problems

1. ¿Cual de los siguientes problemas, pueden ser resueltos como el siguiente problema?

   Melocotones se venden tres por $2. Cuantos melocotones puede comprar Larry con $5?

   a) Están vendiendo 4 libros por cada 5 estudiantes. Cuantos libros había en una clase de 20 estudiantes?

   b) Un carro rodaba 25 millas por hora por 4 horas. Cuanta distancia rodo?

   c) John tiene 20 canillas. Sue tiene 15 canillas. Cuantas más canillas tiene John que Sue?

   d) Si cada bomba cuesta 20 centavos y cada lápiz cuesta 25 centavos, Cuanto cuestan 4 bombas y 1 lápiz?

2. Un agente de seguros atendió a 580 visitantes. El vendió 70 seguros de carro, 90 seguros de vida, y 120 seguros de fuego. Cuantas pólizas vendió?

3. La cerámica del piso se vende en cuadrados de 25 centimetros en cada lado. Cuanto costaría por la cerámica rectangular de una alcoba de 8.2 metros de largo y 5.1 metros de ancho, si cada cerámica cuesta $0.75?

4. Un carro a 30 millas por hora se fue de un lugar a las 10 de la mañana. A las 11 de la mañana otro carro tomo es mismo curso pero a 45 millas por hora. En cuantas horas pasara el segundo carro al primer carro?
5. Una enfermera que trabaja en un hospital de Carolina del Norte mezcla Acido Boric 6\% con Acido Boric 12\%. Cuantas pintas de la solución son necesarias para hacer 4 pintas de Acido Boric de 8\%?

6. Existe una piscina de 35 m ancha, 50 m larga y 7 m onda. Después de un terremoto, la piscina se derrama por un lado (AB) de tal manera que cubre por completo el lado ABCD. Al punto de derrame 3/5 de la base esta cubierta por agua. Cual fue el nivel de agua antes del terremoto?

7. Dos paradas de tren están 40 millas aparte. A la una de la tarde el domingo un tren sale de cada estación hacia la otra. Apenas sale el tren, un águila empieza a volar al frente y vuela hasta el segundo tren. Cuando el águila alcanza al segundo tren, se devuelve y cambia la trayectoria hacia el primer tren. El águila sigue de tren en tren hasta que los dos trenes chocan. Si cada tren viaja a 20 millas por hora y el águila a 90 millas por hora, cuantas millas habrá volado el águila cuando se encuentran los dos trenes?
APPENDIX II - STUDENT QUESTIONNAIRE

Background Info

Name:

Grade:

Ethnicity:

Current Math Class (subject):

Place of birth:

Do you speak Spanish?

Is Spanish your first language? If not, what is your first language?

Do you speak other languages?
Additional Information

Do you like mathematics? Why?

Do you find that you do better in mathematic related classes than non-mathematic related classes? What may be some contributing factors?

Do you feel that language is a factor in your level of comprehension of mathematics? How?

Have you been in a situation in a mathematics class where you felt that you were unable to explain a mathematical concept verbally that you knew you understood numerically?

Do you find that the vocabulary you often choose to express a mathematical idea does not match what you truly intended to portray?

Do you feel that vocabulary is a major factor in the complete understanding of mathematics?
When you hear mathematical conversations going on around you in class, do you often understand them?

Do you find it significantly more difficult to solve mathematics word problems than problems composed of only numerical algorithms?

How does having Spanish as your first language help you to comprehend mathematics?

What do you think can be done to minimize the barriers, if any, that having English as a second language brings to the comprehension of mathematics?

At what level during your mathematics education did you feel that language became a necessary part of understanding? Did this have any effect on your success academically?
APPENDIX III PROBLEM CHOICE ANALYSIS

In order to analyze the accuracy results, an in-depth analysis of the frequency with which the problems were chosen is presented in this appendix. In order to determine the frequency, a score of 0 or 1 was assigned to each problem. A score of 1 was given to problems that a student chose to answer, and a score of 0 was given to problems that a student chose not to answer.

Several patterns surface in the way students chose problems to solve. Most of the students answered Problems 1-4. This pattern could be attributed to their location on the front side of the page. Problems 5-7 were located on the reverse side. None of the Hispanic students chose Problem 6, which asked students to find the volume of a tilted pool in a rectangular prism, given the contextual situation of an earthquake. The Problem provided a diagram with measurements in meters for each side of the rectangular prism.

The problems chosen for the assessment were all algebra 1 problems. Each of the problems varied slightly in presentation and reading difficulty. These variations allowed the researcher to assess factors that may have influenced problem choice. The frequency of student choice for each problem is described below:
Problem 1 Frequency Analysis

Problem 1 used a multiple-choice question format, contained short sentences, used very little language complexity, and only contained whole numbers. The problem aimed to identify whether or not the students knew how to group similar types of problems together. It asked students to decide which multiple-choice question could be solved in the same way as the problem given. Problem 1 for all three sets was displayed as follows:

Set A - English Word Problem

1. Which of the following problems can be solved in the same way as:

   Melons were selling three for $1. How many could Larry buy for $4?

   a) They’re selling three books for every four students. How many were there in a class of twenty students.

   b) A car travels 25 miles per hour for four hours. How far will it travel?

Set B - Numerically based Representation

1. Which of the following problems can be solved in the same way as:

   a) $25x = y$ when $x = 4$ what is $y$?

   b) $12y = 18$ when $x = 2$ what is $y$?

   c) $75 - 32 = r$; Find r.

   d) $.65x + .35y = Z$; How much do 4 chocolate bars(x) and 5 blow pops(y) cost?
Students chose to solve Problem 1 with high frequency. Out of 50 students whose results were returned to the researcher, five tests were excluded from data due to participant error. Of the 45 valid tests, 35 students chose to solve Problem 1. Table 1 below presents data on student frequency related to Problem 1.

Table #3: Frequency Averages for Students Choosing Problem 1

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Problem 1 Set A</th>
<th>Problem 1 Set A %</th>
<th>Problem 1 Set B</th>
<th>Problem 1 Set B %</th>
<th>Problem 1 Set C</th>
<th>Problem 1 Set C %</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students (n=45)</td>
<td>35</td>
<td>78%</td>
<td>34</td>
<td>76%</td>
<td>29</td>
<td>64%</td>
</tr>
<tr>
<td>Hispanic non-ELL (n=15)</td>
<td>12</td>
<td>86%</td>
<td>10</td>
<td>71%</td>
<td>10</td>
<td>71%</td>
</tr>
<tr>
<td>Hispanic ELL (n=8)</td>
<td>8</td>
<td>89%</td>
<td>8</td>
<td>89%</td>
<td>7</td>
<td>78%</td>
</tr>
<tr>
<td>Non-Hispanic (n=22)</td>
<td>15</td>
<td>68%</td>
<td>16</td>
<td>72%</td>
<td>12</td>
<td>55%</td>
</tr>
</tbody>
</table>

The frequency table for Problem 1 displays the number of students who chose to solve Problem 1 in Sets A, B and C, respectively. The table also shows the frequency percentage for
Problem 1 in each set. The student’s frequency of choice of a problem was scored by giving a score of 1 was given if the student attempted the problem, and a score of 0 was given if the student did not attempt the problem. For example, a score of 0.78 for Problem 1 in Set 1 (English word problems) for “all students” means that 35 out of 45 students—or 78% of all students—chose to attempt Problem 1 in Set 1.

Each student received one point for attempting the problem. Students were asked to solve the same problem in all three sets. For Problem 1, the student could receive a maximum score of 3 (the sum of all possible points for each of the sets, given that the student attempted one problem in each set) and a minimum score of 0. A score of 1.96 is the average of points earned by students on all sets for Problem 1. Note that students were instructed to choose only three problems and to solve the same problems in each set. Based on the discrepancies in the data, we can infer that not all students followed the directions when choosing their problems. Some students chose to solve Problem 1 only in the sets they were comfortable, while others opted to solve additional problems when possible, hoping to acquiring more points for their final score. In order to avoid this for future research it will be necessary to monitor closely if students have followed the directions by checking to see if all the questions are attempted before allowing a student to turn the assessment in.

For Problem 1, both ELL and non-ELL Hispanic students chose to answer Set A (English word problems) more frequently than Set B (numerically based problems) or Set C (Spanish word problems). Non-Hispanic students, on the other hand, chose to answer Set B (numerically based problems) more frequently than Set A (English word problems) or Set C (Spanish word problems). Although the frequency with which students chose to solve Spanish word problems for Problem 1 was comparable among ELL and non-ELL Hispanic students, there was a notable
drop in numbers for non-Hispanic students. This could be due to the non-Hispanic students’ limited exposure to reading and/or the discomfort in solving academic problems in a language they usually have to use to communicate with at a basic level. All students involved in the study had been taking Spanish classes for at least two years at The Chosen School.

**Problem 2 Frequency Analysis**

Problem 2 used only English verbiage and whole numbers, all set in a familiar context often reviewed in the algebra curriculum (such as items being sold). Problem 2 did include additional information not relevant to solving the problem. Such information was presented in a fashion familiar to the students. The New York algebra curriculum places heavy emphasis on word problems that incorporate total items being sold, tax rates, and discount. Additionally, in its numerically based representation, the problem required the identification and use of the order of operations (PEMDAS), another commonly emphasized concept in the New York algebra curriculum (http://www.p12.nysed.gov/ciai/cores.html, University of the State of New York - New York State Education Department). By the time students finish the course, they are familiar with the format of these questions and can easily identify them. This could help to explain why Problem 2 was the most commonly chosen problem among all the students. Problem 2 for all three sets was displayed as follows:

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**Set A- English Word Problem**

2. An insurance agent visited 585 customers. He sold 76 life insurance policies, 97 fire insurance policies and 208 auto insurance policies. How many policies did he sell in all?

---
Problem 2 exhibited similar patterns to Problem 1. While the frequency was the highest for all students and the subgroups, there was still a drop-off in the frequency from set to set. Only Hispanic ELL students consistently answered Problem 2 in all three sets. Hispanic non-ELL students chose to answer Set A (English word problem) and Set B (numerical representation) with equally high frequency but had a significant drop in attempting Set C (Spanish word problem). Non-Hispanic students had a decline in frequency for every set. The researcher speculates that students chose to solve fewer problems as they went through the sets. For example, there was a significant decrease in frequency for Problem 2 Set C, as compared to Set

### Table #4: Frequency Averages for Students Choosing Problem 2

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Problem 2</th>
<th>Problem 2</th>
<th>Problem 2</th>
<th>Problem 2</th>
<th>Problem 2</th>
<th>Problem 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set A</td>
<td>Set A %</td>
<td>Set B</td>
<td>Set B %</td>
<td>Set C</td>
<td>Set C %</td>
</tr>
<tr>
<td>All Students (n=45)</td>
<td>42</td>
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<td>40</td>
<td>89%</td>
<td>32</td>
<td>71%</td>
</tr>
<tr>
<td>Hispanic non-ELL (n=15)</td>
<td>13</td>
<td>93%</td>
<td>13</td>
<td>93%</td>
<td>9</td>
<td>64%</td>
</tr>
<tr>
<td>Hispanic ELL (n=8)</td>
<td>8</td>
<td>89%</td>
<td>8</td>
<td>89%</td>
<td>8</td>
<td>89%</td>
</tr>
<tr>
<td>Non-Hispanic (n=22)</td>
<td>21</td>
<td>95%</td>
<td>19</td>
<td>86%</td>
<td>15</td>
<td>68%</td>
</tr>
</tbody>
</table>

Set B – Numerically based Representation

2. Simplify the following problem: 9847+56+ (-12) +3746=?

Set C – Spanish Word Problem

2. Un agente de seguros atendió a 580 visitantes. El vendió 70 seguros de carro, 90 seguros de vida, y 120 seguros de fuego. Cuantas pólizas vendió?
A. While the frequency for Set A was 21 (95% of non-Hispanic students) choosing to solve the problem, the frequency for Set C was 15 (68% of non-Hispanic Students).

**Problem 3 Frequency Analysis**

Problem 3 introduced rational numbers, which made it unique in comparison to the other problems. In the process of choosing which problems to solve, students were able to see decimal points and units of measure in the problem. The problem limited units of measure to units often used as part of the algebra curriculum: meters, centimeters, and money for example (http://www.p12.nysed.gov/ciai/cores.html, University of the State of New York - New York State Education Department). Unit conversions were the one area that the problem addressed which could cause difficulty for students after they attempted the problem. Students were required to change the unit from meters to centimeters, or vice versa, before solving the problem. Another area that could have affected the frequency of Problem 3 was the multipart set up of the problem. Lastly, the problem did not have a diagram to facilitate understanding of what the problem was asking to do, which could have aided students in realizing that they needed to use unit conversions before beginning. In its numerically based representation, the problem only required knowledge of the order of operations. Problem 3 for all three sets was displayed as follows:

---

Set A – English Word Problem

3. Floor tiles are sold in squares 30 centimeters on each side. How much would it cost to tile a rectangular room 7.2 meters long and 5.4 meters wide if the tiles cost $0.72 each?
Table # 5: Frequency Averages for Students Choosing Problem

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Problem 3</th>
<th>Problem 3</th>
<th>Problem 3</th>
<th>Problem 3</th>
<th>Problem 3</th>
<th>Problem 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Set A</td>
<td>Set A %</td>
<td>Set B</td>
<td>Set B %</td>
<td>Set C</td>
<td>Set C %</td>
</tr>
<tr>
<td>All Students (n=45)</td>
<td>18</td>
<td>40%</td>
<td>21</td>
<td>47%</td>
<td>15</td>
<td>33%</td>
</tr>
<tr>
<td>Hispanic non-ELL (n=15)</td>
<td>7</td>
<td>50%</td>
<td>7</td>
<td>50%</td>
<td>6</td>
<td>43%</td>
</tr>
<tr>
<td>Hispanic ELL (n=8)</td>
<td>3</td>
<td>33%</td>
<td>3</td>
<td>33%</td>
<td>2</td>
<td>22%</td>
</tr>
<tr>
<td>Non-Hispanic (n=22)</td>
<td>8</td>
<td>36%</td>
<td>11</td>
<td>50%</td>
<td>7</td>
<td>32%</td>
</tr>
</tbody>
</table>

Problem 3’s frequency was considerably lower than that of Problem 1 and Problem 2. For Problem 3, only eighteen students attempted Set A, twenty-one students attempted Set B, and fifteen students attempted Set C. As in Problem 1 and 2, the ELL and non-ELL Hispanic students were the most consistent in attempting all sets; they solved English word problems, Spanish word problems, and numerically based versions of Problem 3. Non-Hispanic students, on the other hand, did not attempt all three sets of the problem they chose to solve. One possible explanation for this behavior is that this group of students may have felt intimidated by the idea of having to answer a problem in a language other than the one that is spoken at home on a regular basis. It is also possible that students of Hispanic descent were more confident in the content and were able to decipher the language with less hesitation than their counterparts.

Set B – Numerically based Representation

3. Simplify the following problem: \((9.4\times19.3)/(.81\times.67)) \times .52 = ?

Set C – Spanish Word Problem

3. La cerámica del piso se vende en cuadrados de 25 centímetros en cada lado. Cuanto costaría por la cerámica rectangular de una alcoba de 8.2 metros de largo y 5.1 metros de ancho, si cada cerámica cuesta $0.75?
Hispanic students in both categories chose Set A and Set B with equal frequency, displaying a slight drop in attempts with Set C. Non-Hispanic students had the highest frequency for Set B, followed by Set A and Set C, with very minimum difference among the two. This frequency change could be due to the use of rational numbers in the problem. It could also be due to the change of units within the problem and a student’s unfamiliarity with units in a different language. It is interesting to note that Hispanic ELL students were more likely to attempt the English word problem than the Spanish word problem, even though Spanish was their native language.

**Problem 4 Frequency Analysis**

Problem 4 was intended to test concept of time and ratios. Students needed to know what “miles per hour” meant, be able to set up a ratio, and solve a proportion. In order to avoid errors based on the number set used as opposed to the concept tested, the problem used only whole numbers. The frequency of Problem 4 was comparable to Problem 3 for most subgroups. One reason why the frequency of Problem 4 decreased from Problems 1-3 for most students could be that students had to connect their knowledge of ratios to the concept of time. Problem 4 was also the last problem on the front side of the page for the sets. This could have affected students’ problem choice. Problem 4 for all three sets was displayed as follows:

Set A- English Word Problem

4. A car traveling at a speed of 30 miles per hour left a certain place at 10:00am. At 11:30am, another car departed from the same place at 40 miles per hour and traveled the same route. In how many hours will the second car overtake the first car?
Table #6: Frequency Averages for Students Choosing Problem 4

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Problem 4 Set A</th>
<th>Problem 4 Set A %</th>
<th>Problem 4 Set B</th>
<th>Problem 4 Set B %</th>
<th>Problem 4 Set C</th>
<th>Problem 4 Set C %</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students (n=45)</td>
<td>19</td>
<td>42%</td>
<td>9</td>
<td>20%</td>
<td>14</td>
<td>31%</td>
</tr>
<tr>
<td>Hispanic non-ELL</td>
<td>6</td>
<td>43%</td>
<td>4</td>
<td>29%</td>
<td>4</td>
<td>29%</td>
</tr>
<tr>
<td>(n=15)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hispanic ELL</td>
<td>6</td>
<td>67%</td>
<td>3</td>
<td>33%</td>
<td>7</td>
<td>78%</td>
</tr>
<tr>
<td>(n=8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Hispanic</td>
<td>7</td>
<td>32%</td>
<td>2</td>
<td>9%</td>
<td>3</td>
<td>14%</td>
</tr>
<tr>
<td>(n=22)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hispanic non-ELL students had a higher frequency with Problem 4 in Set A (English word problems) than they did in Set B (Numerically-based problems) and Set C (Spanish word problems). Sets B and C were attempted with the exact same frequency for this particular subgroup. Hispanic ELL students attempted Set C with the highest frequency, followed by Set A and then Set B. Non-Hispanic students, on the other hand, and had the highest frequency with Set A. Both Hispanic ELL and non-Hispanic groups displayed the lowest frequency with Set B. This is surprising given that the frequency of Set B in all previous problems had been equal to or greater than the other sets. The frequency of Set B may have been considerably lower in this problem because it required that the student solve the system of equations using R1 and R2.
instead of X and Y. Because they are all variables, R1 and R2 behave in the same way as X and Y, but students are often less exposed to variables other than X and Y. Students may become uncomfortable with the concept when letters other than X and Y are used to represent an unknown. Not realizing that the problem was testing the same concept, several students expressed that they would have been able to solve the problem if it was written with X and Y instead of R1 and R2.

**Problem 5 Frequency Analysis**

Problem 5 involved percentage concept manipulation and the development of an equation from a real life application scenario. The problem was the first of the problems chosen that was written in academic context involving scientific terms. Students were required to apply scientific concepts in order to develop solvable algorithms to come up with a solution to the problem, which could have provided an additional translational barrier for ELL students. The numerically based representation for Problem 5 included a complicated series of rational numbers with extensive decimals. Such an appearance could have deterred students from choosing to solve this problem. Problem 5 for all three sets was displayed as follows:

<table>
<thead>
<tr>
<th>Set A- English Word Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>5. A nurse who works in a hospital in North Carolina and went to school at Duke University years ago mixes a 6% boric acid solution with a 12% boric acid solution. How many pints of each are needed to make 4.5 pints of an 8% boric acid solution?</td>
</tr>
</tbody>
</table>
Set B – Numerically based Representation

5. Solve for \( t \).
\[
0.075t + 0.0987(3.6-t) = 0.7685*3.6
\]

Set C – Spanish Word Problem

5. Una enfermera que trabaja en un hospital de Carolina del Norte mezclo Acido Boric 6% con Acido Boric 12%. Cuantas pintas de la solución son necesarias para hacer 4 pintas de Acido Boric de 8%?

Table #7: Frequency Averages for Students Choosing Problem 5

<table>
<thead>
<tr>
<th>Student Group</th>
<th>Problem 5 Set A</th>
<th>Problem 5 Set A %</th>
<th>Problem 5 Set B</th>
<th>Problem 5 Set B%</th>
<th>Problem 5 Set C</th>
<th>Problem 5 Set C%</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Students (n=45)</td>
<td>3</td>
<td>7%</td>
<td>5</td>
<td>11%</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>Hispanic non-ELL (n=15)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Hispanic ELL (n=8)</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>11%</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Non-Hispanic (n=22)</td>
<td>3</td>
<td>14%</td>
<td>4</td>
<td>18%</td>
<td>1</td>
<td>4%</td>
</tr>
</tbody>
</table>

Very few students attempted Problem 5. None of the Hispanic non-ELL students attempted any of the sets for Problem 5. Only one Hispanic ELL student attempted the problem at all; this student attempted Set B (Numerically based representation) of Problem 5. The non-Hispanic students made very few attempts for any of the sets of Problem 5. Only three students in this category attempted the problem in Set A (English word problems), four students attempted the problem in Set B (Numerically-based problems), and one student attempted the problem in Set C (Spanish word problems). More non-Hispanic students choose Problem 5 in Sets A, B, and C than did Hispanic ELL and Hispanic non-ELL students. More Hispanic ELL students chose Problem 5 in Set B than did Hispanic non-ELL.
Problem 6 Frequency Analysis

Problem 6 involved fraction manipulation, contained complex diagrams, and required knowledge of volume: a concept taught as an isolated topic in the algebra curriculum, according to the New York State algebra Standards (http://www.p12.nysed.gov/ciai/cores.html, University of the State of New York - New York State Education Department). Problem 6 for all three sets was displayed as follows:

Set A- English Word Problem

6. A swimming pool is 30 m wide, 50 m long and 7 m deep. After an earthquake, the pool is tilted along one edge (AB) and the water completely covers side ABCD. At this point, 3/4 of the base is covered by water. What was the water level before the earthquake?
Problem 6 had the least frequency among all problems for students. No Hispanic student attempted to solve Problem 6. Non-Hispanic students who attempted to solve Problem 6 most often attempted Set B (Numerically based representation). One possible reason for the decrease in frequency for Problem 6 could be that the application involved (volume) is an isolated lesson.
Problem 7 Frequency Analysis

Problem 7 involved the most verbiage and required a very low level of mathematical analysis. This problem was strategically placed in the assessment in order to evaluate a student’s ability to decipher through language. It required the ability to translate a situation in the written format and identify the question being asked. It was the least mathematically complicated problem, and it contained a lot of verbal information that students were forced to decipher. The purpose of the problem was to see if verbal complexity outweighed mathematical complexity for students when presented with a problem. Even though students were instructed to read every problem before beginning the assessment, it is possible that students never read Problem 7 because it was the last choice in the set. At this time, there is not enough evidence to conclude that the problem was not chosen with more frequency based solely on its verbal complexity.
Problem 7 for all three sets was displayed as follows:

Set A - English Word Problem

7. Two train stations are 50 miles apart. At 1 pm on Sunday a train pulls out from each of the stations and the trains start toward one another. Just as the trains pull out from the two stations a hawk flies traveling the air in front of the first train and flies ahead to the front of the second train. When the hawk reaches the second train, it turns around and flies toward the first train. The hawk continues on his way until the trains meet. Assume that both trains travel at the speed of 25 miles per hour and that the hawk flies at a constant speed of 100 miles per hour. How many miles will the hawk have flown when the trains meet?

Set B – Numerically based Representation

7. Solve the following problem:

• Two train stations are 50 miles apart.
• At 1 pm a train pulls out from each of the stations and the trains start traveling toward one another.
• Just as the trains pull out from the stations a hawk flies into the air in front of the first train and flies ahead to the front of the second train.
• When the hawk reaches the second train, it turns around and flies toward the first train.
• The hawk continues in his way until the trains meet.
• Assume that both trains travel at the speed of 25 miles per hour and that the hawk flies at a constant speed of 100 miles per hour.
• How many miles will the hawk have flown when the trains meet?

Set C – Spanish Word Problem

7. Dos paradas de tren están 40 millas aparte. A la una de la tarde el domingo un tren sale de cada estación hacia la otra. Apenas sale el tren, un águila empieza a volar al frente y vuela hasta el segundo tren. Cuando el águila alcanza al segundo tren, se devuelve y cambia la trayectoria hacia el primer tren. El águila sigue de tren en tren hasta que los dos trenes chocan. Si cada tren viaja a 20 millas por hora y el águila a 90 millas por hora, cuantas millas habrá volado el águila cuando se encuentran los dos trenes?
None of the students attempted to solve Set C (Spanish word problem) for Problem 7. Hispanic non-ELL students did not attempt to solve any sets for Problem 7. Non-Hispanic students did not attempt to solve Set B (Numerically based problems) for Problem 7. Hispanic ELL students more often attempted to solve Set B for Problem 7. Hispanic ELL students had a higher frequency rate than non-Hispanic students in regards to Set A (English word problems) and Set B (Numerically based problems).

The data appears to suggest that non-Hispanic students could have been intimidated by their first attempt at solving Problem 7. Although three of the students attempted Problem 7 in Set A, none of the students attempted the same problem in Sets B or C. It is difficult to identify the exact reason why students stopped trying to solve the problem after Set A, but it is important to note that in their work there was no obvious attempt from any student to decipher the problem. None of the tests showed erased marks, underlined words, or any other evidence to suggest that the student looked through the problem and chose not to solve it based on perceived difficulty. For Hispanic ELL students, however, evidence reflected an attempt at solving the problem along with a struggle to do so. These particular assessments show underlining of key terms, margin writings, and a clear attempt to understand. Because there was no answer provided, the students’
attempts do not count towards the frequency rate; there is clear evidence, however, that the students at least looked at the problem before deciding not to solve it. Hispanic non-ELL students, on the other hand, chose to leave this particular problem blank. Based on patterns from other problems, the researcher speculates that the Hispanic non-ELL students opted to leave problems blank when they felt any discomfort rather than attempting to solve it for some credit. Based on conversations with students during the interviews, many students in this category never looked at the problem; they had already chosen to solve a different set of problems, which in many cases was Problems 1-4.

**Frequency Comparison for All Problems**

The frequency varied from one problem to the next for each of the subgroups. For non-ELL students, however, the frequency order from highest to lowest for the first four problems was exactly the same, with Problem 2 as the most common problem. The order of frequency for problems chosen by students classified as ELL varied considerably from non-ELL students. All students had the highest frequency rate for Problem 2, but Problems 1, 3, and 4 displayed a noticeable difference in frequency. The following table describes the problems that students chose to solve most often, in order from greatest to least frequency.

<table>
<thead>
<tr>
<th>All Students</th>
<th>Hispanic (ELL)</th>
<th>Hispanic (Non-ELL)</th>
<th>Non-Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>1,3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5,6,7</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td>7</td>
</tr>
</tbody>
</table>
Based on the results displayed in the table above, all students chose Problem 2 as their top choice. Problems 1-4 had the most frequency overall, but there were some distinct differences among subgroups. Non-ELL students had the exact same order for frequency of the first four problems (2, 3, 1, 4). ELL students agreed on choosing Problem 2 most often, but they differed from non-ELL students when they chose Problem 4 over Problem 3 and gave the same importance to Problem 3 as they did Problem 1.

The table below also displays a summary of the frequency for each individual problem and represents the number of students that attempted each problem. A student was accounted for if they attempted at least one of the sets for the designated problem. For example, if a student attempted Problem 1 in Set A but didn’t attempt Problem 1 in Set B, he was still accounted for when calculating frequency. For the purpose of this study, a Hispanic student in the chart is denoted as someone who is at least partially of Hispanic descent and identifies as such. The study was made up of a total of 45 children: 8 Hispanic ELL students, 15 Hispanic non-ELL students, and 22 non-Hispanic Students. Based on procedural errors, 5 of the 50 assessments that were turned in were considered invalid.

Table #11: Student Frequency Data by Problem

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>All Students</th>
<th>Hispanic(ELL)</th>
<th>Hispanic(Non-ELL)</th>
<th>Non-Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>8</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>42</td>
<td>8</td>
<td>13</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>3</td>
<td>7</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>7</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

The table below accounts for the frequency averages that students chose to solve each of the problems on the test. It is important to note that these averages are based on the expectation
that the student solved all three sets of the same problem (English word problem, Spanish word problem and numerically based problems), which differs from the student counts in the previous table. For example, an average of “3” would imply that all the students in that category solved that problem for Sets A, B and C, which included three presentation formats of the same problem. Accuracy of the problem was not considered in the table averages.

Table #12: Frequency Average Comparison by Problem

<table>
<thead>
<tr>
<th>Problem Number</th>
<th>Hispanic ELL</th>
<th>Hispanic non-ELL</th>
<th>non-Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.88</td>
<td>0.92</td>
<td>0.95</td>
</tr>
<tr>
<td>2</td>
<td>2.66</td>
<td>2.05</td>
<td>2.29</td>
</tr>
<tr>
<td>3</td>
<td>0.88</td>
<td>1.17</td>
<td>1.08</td>
</tr>
<tr>
<td>4</td>
<td>1.78</td>
<td>0.82</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>0.11</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0.29</td>
</tr>
<tr>
<td>7</td>
<td>0.33</td>
<td>0</td>
<td>0.13</td>
</tr>
</tbody>
</table>

A possible reason as to why most students chose Problems 1-4 with higher popularity could be because they began working on those problems before really looking at the rest of the problems. Another potential reason why the students answered Problems 1-4 with more consistency than Problems 5-7 could be because Problems 1-4 were in the front side of the page as opposed to the backside. Additionally, Problems 5-7 contained longer verbal components, displayed diagrams that could have depicted complexity, and contained percentages, fractions, and an array of units within the context of the problems.

The total number of attempts in each set is based on the number of students in each category. Since there are eight students in the Hispanic ELL set, the total number of possible solved problems for all of Set A is 24. For Hispanic non-ELL students, the total amount of students is 17, and the total amount of possible solved problems for all of Set A is 45. Since there were 22 students in the non-Hispanic category, the total number of possible problems solved in
Set A is 66. By analyzing the frequencies among sets, the researcher hopes to identify patterns in which students answer questions based on the layout of the problem (i.e. whether a problem was written in an English, Spanish, or numerically-based manner).

Table #13: Frequency Count by Set

<table>
<thead>
<tr>
<th>Problem Set</th>
<th>Hispanic ELL</th>
<th>Hispanic Non-ELL</th>
<th>Non-Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set A English Word Problem</td>
<td>24</td>
<td>41</td>
<td>59</td>
</tr>
<tr>
<td>Set B Numerically based Representation</td>
<td>22</td>
<td>36</td>
<td>56</td>
</tr>
<tr>
<td>Set C Spanish Word Problem</td>
<td>21</td>
<td>32</td>
<td>39</td>
</tr>
</tbody>
</table>

The table below displays the frequency average by set for each of the subgroups. Students were asked to choose three problems. Based on the coding system used, a student should have averaged “3” for all of the sets. However, it is clear that many students opted to attempt the problems they were comfortable with and leave the sets they were uncomfortable with blank. Some students attempted more than three problems in the sets they were comfortable with and less than three problems in the sets that they were not. This was particularly true for non-Hispanic students, who tried to compensate for their lack of knowledge in Spanish and their inability to solve Set C by solving additional problems in Sets A and B.

Table #14: Frequency Average by Set

<table>
<thead>
<tr>
<th>Problem Set</th>
<th>Hispanic ELL</th>
<th>Hispanic Non-ELL</th>
<th>Non-Hispanic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set A English Word Problem</td>
<td>3</td>
<td>2.24</td>
<td>2.45</td>
</tr>
<tr>
<td>Set B Numerically based</td>
<td>2.67</td>
<td>2</td>
<td>2.33</td>
</tr>
<tr>
<td>Representation</td>
<td>2.67</td>
<td>1.71</td>
<td>1.63</td>
</tr>
<tr>
<td>Set C Spanish Word Problem</td>
<td>2.67</td>
<td>1.71</td>
<td>1.63</td>
</tr>
</tbody>
</table>
Hispanic ELL students were more likely to attempt English word problems than both numerically based problems and Spanish word problems. They were also just as likely to solve numerically based problems as Spanish word problems. Hispanic non-ELL students were more likely to attempt English word problems than numerically based problems and Spanish word problems. They were also more likely to select numerically based problems than Spanish word problems. Non-Hispanic students followed the same patterns as Hispanic non-ELL students. However, the gap between choosing numerically based problems and Spanish word problems for non-Hispanic students was notably different. Hispanic ELL students had a higher frequency rate than Hispanic non-ELL and non-Hispanic Students for Set A. Non-Hispanic students had a higher frequency rate than the Hispanic non-ELL students for the same set. Set B followed the same patterns. For Set C, Hispanic ELL students had the highest frequency rate. Similarly, the frequency rate for Hispanic non-ELL students was higher than non-Hispanic students.

Data analysis also showed that non-Hispanic students tended to be less likely to answer the problem in Spanish than Hispanic students. This could be due to non-Hispanic students’ limited exposure to the Spanish language as it relates to other subject areas such as mathematics. One should note, however, that students classified as non-Hispanic at The Chosen School had been exposed to the Spanish Language through formal Spanish classes for at least two years. In some cases, students had more years in Spanish class than ELL students had in English class. These non-Hispanic students were considered verbally fluent speakers of the Spanish language. However, due to the structural differences of the Spanish and English languages, the researcher cannot equate the time it should take to gain verbal and written proficiency in each language.
APPENDIX IV- NATIONAL SPANISH EXAMINATION DESCRIPTION

What is the National Spanish Examination?

The National Spanish Examinations are online, standardized assessment tools for Grades 6-12, which measure proficiency and achievement of students who are studying Spanish as a second language.

What is the purpose of the National Spanish Examination?

The purpose of the National Spanish Examination is

1. to recognize achievement in the study of the Spanish language
2. to promote proficiency in interpretive communication in the Spanish language
3. to assess the national standards as they pertain to learning Spanish
4. to stimulate further interest in the teaching and learning of Spanish

In addition:

* Many teachers state that they use the National Spanish Examinations to prepare students to take other standardized tests such as AP, IB, SAT II and college placement exams.
APPENDIX V STUDENTS CONSENT (SPANISH)

Teachers College, Columbia University

FORMULARIO DE CONSENTIMIENTO INFORMADO

DESCRIPCIÓN DE LA INVESTIGACIÓN:

Su hijo(a) es invitado a participar en un estudio de investigación que tiene como objetivo explorar los métodos y estrategias de resolución de problemas matemáticos para los estudiantes de ascendencia hispana / latina. El estudio se centra en la búsqueda de diferencias en el nivel de precisión entre los diferentes estudiantes hispanos / latinos en los problemas escritos presentados en español, Inglés y en forma algorítmica. Su hijo tendrá que tomar un examen de una hora en el que responderá a tres preguntas presentadas de las tres formas anteriormente mencionadas. Después de los resultados de la prueba, se seleccionará una muestra de estudiantes y se les pedirá a una entrevista personal, grabada con el investigador principal. La entrevista se enfocara en las apreciaciones que tuvo su hijo(a) sobre el examen así como su actitud personal de las matemáticas en general. Cada entrevista tendrá una duración de 1-2 horas. El estudio se llevará a cabo por la Sra. Andrea Duhon, investigador principal, conjuntamente con la Facultad y el Personal de Bronx Prep. La investigación se llevará a cabo en el colegio Bronx Preparatory Charter School durante el horario regular de la clase de matemáticas y todas las entrevistas se llevarán a cabo después del horario de la escuela.

RIESGOS Y VENTAJAS:

No hay beneficios directos para los participantes en el estudio. La participación de su hijo(a) es voluntaria y puede retirarse en cualquier momento y sin ninguna consecuencia negativa. Sin embargo, el estudio servirá como un vehículo para crear e implementar sistemas de apoyo que permitan optimizar a las insuficiencias de todos los estudiantes, con especial atención a estudiantes latinos / hispanos. El estudio puede ayudar a describir las áreas de las matemáticas con las que los estudiantes tengan mayores dificultades mientras que proporciona una visión para los maestros para entender el enfoque de los estudiantes en el momento de resolver problemas de matemáticas.
REMUNERACION:

Los estudiantes que decidan participar y sean seleccionados para una entrevista recibirán un bono de regalo por la suma de $10 (diez dólares) para la cadena de comida rápida Wendy's.

ARCHIVO DE BASE DE DATOS PARA PROTÉGÉR LA CONFIDENCIALIDAD DE LOS PARTICIPANTES:

El nombre de los estudiantes nunca es conocido. A los alumnos se le asignará un número. En su lugar, se codifica de la siguiente manera ESTUDIANTE 1, ESTUDIANTE 2, etc. Teniendo en cuenta, que el estudio está diseñado con cuatro categorías de estudiantes que se catalogarán en consecuencia.

TIEMPO DE PARTICIPACION:

La participación de su hijo tomará aproximadamente un periodo de clase. El estudio va a suceder durante su horario regular de clases de matemáticas. Una o dos horas adicionales serán requeridas solo para los estudiantes que son seleccionados para las entrevistas personales. Dichas entrevistas no superarán las 2 horas. Entrevistas personales se llevará a cabo en un horario después de la escuela.

UTILIZACION DE LOS RESULTADOS:

Los resultados del estudio serán utilizados como datos para la realización de los requisitos de tesis para el Doctorado en Filosofía en la Universidad de Columbia por la investigadora principal.
Investigador principal: Señora Andrea Duhon

Titulo de la Investigación: EXPLORAR LAS DIFERENCIAS EN LA RESOLUCIÓN DE PROBLEMAS PARA ESTUDIANTES DE SECUNDARIA DE ALGEBRA EN CUANTO A METODOLOGÍA Y ESTRATEGÍA ENTRE NATIVOS DE HABLA ESPAÑOLA

• He leído y discutido la descripción de la investigación con el investigador. He tenido la oportunidad de hacer preguntas sobre los propósitos y procedimientos relevantes a este estudio.

• Mi participación en esta investigación es voluntaria. Puedo negarme a participar o retirarme de la participación en cualquier momento, sin riesgo alguno para atención médica futura, empleo, la condición del estudiante o cualquier otro derecho.

• El investigador me puede retirar de la investigación a su discreción profesional.

• Si durante el curso del estudio, nueva información significativa desarrollada se vuelve disponible y puede ser beneficiaria para la prueba, el investigador me proporcionará la información, para decidir continuar mi participación en el mismo o no.

• Toda la información derivada del proyecto de investigación que me identifique personalmente, no será publicada o divulgada sin mi consentimiento y/o sin el consentimiento separado de mis padres, salvo que sea específicamente requerido por la ley.

• Si en algún momento tengo alguna pregunta con respecto a la investigación o de mi participación, puedo ponerme en contacto con el investigador, que deberá responder a mis preguntas. El número de teléfono del investigador es (646) 8089320.
• Si en algún momento tengo comentarios, o preocupaciones con respecto a la conducta de la investigación o preguntas acerca de mis derechos como sujeto de investigación, puedo contactar al Teachers College, Columbia University Institutional Review Board / CRI. El número de teléfono de la IRB es (212) 678 a 4105. O bien, puede escribir a la IRB en el Teachers College, Columbia University, 525 W. Calle 120, Nueva York, NY, 10027, P.O. Box 151.

• Recibiré una copia de la Descripción de Investigación y este documento en el que se especifican los Derechos del Participante.

• Doy consentimiento de video y / o audio como parte de esta investigación,

  Yo ( ) doy conocimiento de video/audio grabado
  Yo ( ) NO doy conocimiento de video/audio grabado

Sólo el investigador principal y los miembros del equipo de investigación tendrán acceso al material escrito y/ o adquirido por video o audio.

• Material escrito o grabado con video y / o audio

  ( ) Se pueden ver en un entorno educativo fuera de la investigación
  ( ) NO se pueden ver en un entorno educativo fuera de la investigación
Mi firma significa que estoy de acuerdo en participar en este estudio.

Firma del Participante: ________________________________

Fecha: ___/___/____

Nombre del Participante: _______________________________

Firma del guardián: ________________________________

Fecha: ___/___/____

Nombre del Guardian: ________________________________
Yo ________________________________ (nombre del niño) estoy de acuerdo en participar en el estudio titulado EXPLORAR LAS DIFERENCIAS EN LA RESOLUCIÓN DE PROBLEMAS PARA ESTUDIANTES DE SECUNDARIA DE ALGEBRA EN CUANTO A METODOLOGIA Y ESTRATEGIA ENTRE NATIVOS DE HABLA ESPAÑOLA.

El propósito y la naturaleza del estudio me han sido plenamente explicados por la Señora Andrea Duhon. Entiendo lo que se pide de mí, y si he de tener alguna pregunta, sé que puedo comunicarme con la Señora Andrea Duhon (investigador principal) en cualquier momento. También entiendo que puedo optar por abandonar el estudio en cualquier momento que desee.

Nombre del participante: ____________________________________________

Firma del participante: ____________________________________________

Testigo: __________________________________________________________

Fecha: ____________________________________________________________

VERIFICACION DE LA EXPLICACION POR PARTE DEL INVESTIGADOR
Yo certifico que he explicado cuidadosamente el propósito y la naturaleza de esta investigación a _________________________ (nombre del participante) en un lenguaje apropiado para su edad. El / ella ha tenido la oportunidad de discutir conmigo en detalle. He respondido a todas sus preguntas y él / ella firma la aprobación para participar en esta investigación.

Firma del Investigador: ________________________________

Fecha: ________________________________________________
APPENDIX VI SCHOOL APPROVAL LETTER

To whom it may concern,

Andrea Hernandez has my permission to conduct her study entitled Exploring differences in Problem Solving Strategies among Native Spanish-speaking High School Students in algebra-Based Problem Solving. The study will be conducted at Bronx Preparatory Charter School located at 3872 Third Ave. Bronx, NY 10456. This study, which looks to analyze the accuracy differences in problem solving among Hispanic/Latino students whose first language is not English and their counterparts, has the potential to help inform mathematics teaching and learning at our school. We have agreed that the study will not infringe on instructional time other than the one class period allotted for the quantitative testing. The qualitative portion of the study, which includes personal interviews, will be conducted after school. We have offered our site as an appropriate site for the study and will create a space determined by the head of the mathematics department that is appropriate for the study. It is our hope that we can use the results of the study to further understand our increasing population of Hispanic/Latino students, some of which fall into the category of English Language Learner. Furthermore, Bronx Preparatory Charter School is interested on the study as a potential resource guide used to replicate strategies that help in our student’s mathematical processing by creating support systems for this group of students. If you have any further questions you can contact us at 7182940841.

Sincerely,

_______________________              __________
Dr. Samona Tait                   Date
Head of School

_______________________
Jacqueline King                   Date
Principal

_______________________
Yesenia Michel                   Date
Math Department Head