Black Markets and Optimal Evadable Taxation

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Abstract

A simple model of indirect taxation, evasion, and enforcement is presented in which some surprising results emerge. Because of a 'market-thinning' effect of high prices, high taxes lead to multiple equilibria (low-price black markets and high-price legal markets), and black-market comparative statics tend to give the 'wrong' sign. Further, the incentive compatibility constraint for tax compliance is shown to confer a kind of concavity on enforcement costs. As a result, enforcement costs may be minimized by tax rates that vary dramatically between sectors. This consideration is dominant for relatively ineffective tax administrations, so for them the optimal tax system follows a 'cash cow' pattern, with one sector bearing all of the tax; but for relatively effective administrations, the optimum follows a slightly modified Ramsey rule. This bifurcation of the parameter space provides one approach to a positive theory of tax systems.
1. Introduction.

In a black market a good is traded illegally by way of evasion of a tax or regulation. Where indirect taxes are potentially evadable, the pattern and prevalence of black markets is likely to be much affected by the pattern of tax rates; thus, potentially, the overall pattern of optimal taxes might be very much affected by consideration of the enforcement problem. This paper proposes a simple model for studying this problem.

This question is of some practical importance in most tax jurisdictions, since indirect taxes have an almost ubiquitous presence. For example, interjurisdictional differences in excise rates on cigarettes have led to a long history of smuggling between U.S. states\(^1\) and from the U.S. into Canada\(^2\). In addition, rising Canadian reliance on a federal sales tax (the GST) has led to new forms of evasion in that country. For example, the existence of a black market for kielbasa in Montreal, circulating in evasion of the GST, has recently been documented \((Montreal Gazette, December 23, 1995, p.A1)\).

However, it is likely to be much more of an issue in the public finance of medium and low-income economies, since poorer countries lack the means for effective direct taxation\(^3\), and

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\(^1\)One study estimated that in 1975 16 U.S. states lost 8% or more of their cigarette tax revenue through smuggling, and 8 states had 100 million or more packs smuggled into them. Simon and White (1982, pp. 34, 36).


\(^3\)See Tanzi (1987), and Due (1988, ch.2) for documentation of the much greater reliance of poorer economies on indirect taxes. As of 1985, countries with per capita GNP below $351 relied on indirect taxes for 61% of total tax revenue on average, while for countries with income above $8,000 the figure was 36.4%. Due (1988, p. 22).
since it is in the poorer countries that black markets are most in evidence\textsuperscript{4}. So far, however, accounts of optimal taxation theory for the Third World have usually assumed costless enforcement (see Newbery and Stern (1987), for an overview and many contributions). This practice has also carried over into empirical exercises in optimal taxation (e.g., Ray (1989), Newbery (1990)) and tax reform (Ahmad and Stern (1991)). This seems like a glaring lack, precisely because the administrative difficulties that make indirect taxation so important in poorer countries, making it difficult to get at incomes for taxation, are the very difficulties that make enforcement costs so crucial. A shortage of skilled cadres is key in both\textsuperscript{5}.

It would also be wrong to presume that an enforcement infrastructure, however costly, will not affect optimal tax rates because it will simply constitute a fixed cost like any bureaucratic overhead. This argument ignores the incentives created by taxes for evasion. This is because it should stand to reason that an increase in a given tax rate also increases the profits to be made from evading the tax, and thus increases the enforcement effort needed to keep the tax in effect. There is thus a marginal enforcement cost to commodity taxation, which as will be seen later can qualitatively change the structure of optimal taxes.

In this paper a simple model of legal and illegal trade is presented in which the legality

\textsuperscript{4}Examples abound. For example, Mehta (1990, p. 181) estimates that 39\% of the sales tax in Rajasthan state, India, are evaded, and 73\% of the state excise duties, along with 75-92\% of the excise on ghee and butter (pp. 121-2). Street vending in Mexico city, beyond the hand of the sales tax, are studied in Cross (1995). Polyconomics, Inc. (1990) concluded that between 25\% and 50\% of the Mexican economy is in the “informal” sector (pp. 65-6), and that in Mexico “tax avoidance [is] the \textit{raison d’etre} of the informal economy” (p. 70).

\textsuperscript{5}McLaren (1996) reviews some recent work on the evasion problem in LDC’s. Incidentally, these issues are certain to survive the waves of fiscal reform ongoing in many of these countries, because indirect taxes will continue to be very important, and in many cases have become more important with reform.
of trade is endogenized and the effect of this on optimal taxation is analyzed. The intention is to set up the simplest possible model of indirect tax evasion in order to get through the complexities of characterizing the optimal tax system, in which both the set of taxes and the enforcement system are chosen by the government. Some novel results emerging from this model are: (i) For high enough taxes, there are always multiple equilibria, some with tax compliance and some without\(^6\). This comes from a "market thinning effect" of tax compliance: Tax compliance raises the price of the good in question, discouraging consumption and making the market easier to police. (ii) In a black market for good \(i\), a rise in the price of a substitute will \textit{lower} the price of good \(i\), while the rise in the price of a complement will raise it. (iii) Even in the presence of exogenously bounded fines for evasion and costly enforcement, it is always possible, and \textit{optimal}, to design the tax system so that no evasion occurs. (iv) The enforcement cost of the tax system can be written as a function of the vector of commodity taxes, and it is always non-convex. As a result, enforcement costs will in some cases be minimized by tax rates that vary drastically from sector to sector. For this reason, in a simple parametric model, it turns out that if the enforcement technology is poor enough, a 'cash-cow' system of taxation, in which only one sector is taxed, becomes optimal -- even if all sectors are \textit{ex ante} symmetric. This can give rise to a theory of changing tax structure over the course of economic development\(^7\).

\(^6\)A related argument is made by Cowell (1990, chapter 6), who discusses the possibility of 'evasion epidemics.' In that model, this arises from the assumption that a taxpayer's taste for evasion is increasing in the amount of evasion conducted by others, possibly because of a reduced stigma.

\(^7\)See Gardner and Kimbrough (1992) for a different model of evolution of the tax system. In that paper, exogenously specified collection costs lead to a movement away from tariffs, through a combined tariff and excise system, to a pure income tax system as the government grows. Income tax collection costs are assumed to have a kind of fixed cost nature, so when an
The model of a black market presented here joins a long tradition. Following the seminal Bhagwati and Hansen (1973) (BH), the great bulk of existing literature has taken policy as given and asked the question, “Does smuggling raise or lower welfare,” through either a model with an exogenously specified smuggling cost function (for example, BH, Lovely (1994), Connolly, Devereux, and Cortes, (1995)), or assumptions about the risk of detection (Kemp (1976), Pitt (1981), Sheikh (1989), Virmani (1989)). The program of this paper is different. Here we take the possibility of smuggling as given and ask how the government may optimally design the system of taxes together with the enforcement system, taking that possibility into account.

A similar question has been posed in a number of earlier papers. Bhagwati and Srinivasan (1974) study the effect of smuggling on optimal tariff setting, finding the effect ambiguous. Kaplow (1990) studies optimal taxation and enforcement in a simple model of tax administration with and without costly evasion. However, the fact that by assumption only one good is taxable means that the question of the optimal tax structure, which is central to this paper, is absent. Further, Kaplow assumes an interior optimum and studies the first order condition, whereas a major point made here is that the enforcement problem can make the first order condition irrelevant. Nevertheless, Kaplow’s integration of the taxation and enforcement problems is an important contribution. In a similar vein, Lovely (1995) studies the first order conditions for the optimal tax vector in a model with costly evasion.

In related work, Kemp (1976) addresses the question of optimal tariffs in the face of smuggling with a model of neoclassical trade in a small open economy. Importers may evade income tax is introduced, it suddenly replaces all other taxes. In the present paper, collection costs are endogenous, and the non-convexity in them is derived from the incentive constraints underlying the problem.
the tariff at the expense of fines, which are then distributed in a lump-sum manner to the general public. There are no resource costs to illegal trade. Confining interest to the case in which legal trade and smuggling coexist, he shows that any equilibrium without smuggling is equivalent to one with smuggling, so that smuggling is irrelevant to the optimal tariff. The current paper is very different in that we do not require black and legal markets to coexist, and we take account of the real resource cost of enforcement. Finally, Lovely (1994) approaches the spirit of this paper, by: (i) Considering the BH question under the assumption of pre-smuggling Ramsey-optimal taxes; (ii) Considering the welfare effects of tightening enforcement.

One limit to the model we present is the assumption of perfect competition in trade, regardless of its legality. This leads to some strong results, including naturally the absence of rents to black market activity. In many markets this surely is realistic. To anyone who has had the experience of standing in one spot in an African city and being approached by a unending stream of young men, all offering illegal currency sales, the supply of traders can seem elastic indeed. Illegal trade does not necessarily require significant capital investment; all that most West African smugglers need, for example, is a bicycle, moped or canoe to join a large, organized market for smuggled goods (Deardorff and Stolper, 1990, p.133). However, there are likely to be black markets into which entry is not free and rents are important. For example, the lucrative illegal importation of cigarettes into Nigeria from Niger has been said to be the domain of exactly fourteen traders (Azam, 1990). The mob violence that victimized some of the “market women” in Accra at the time of the first Rawlings coup seems to have been motivated partly by a public perception that those merchants had accumulated rents from their practice of dealing at illegal prices (Hettne, 1980, pp. 184-5). These issues of market structure lie outside of the scope.
of this paper, but are surely important in some settings\(^8\).

A second key assumption made here is that the law of one price holds even in a black market. A well-recognized price emerges which agents take as given, and any deviations could be arbitrated away. Some observers argue that this is not the way real world black markets work; Bevan, Collier and Gunning (1989), for example, argue that the inability to advertise illegal prices makes price search costly, eliminating arbitrage and possibly market clearing. They suggest that this view is consistent with Tanzanian survey data. Other case studies, however, forcefully reaffirm the law of one price in illegal transactions: Morris and Newman (1989) in Senegalese cereal markets, Azam (1990) in the market for Nairas, and Wade (1985) in various bureaucratic bribes in India. Which view is a better approximation will likely depend on the market and country in question but the assumption made here has the virtue of enormously simplifying the analysis.

Section 2 studies the black markets of the model in partial equilibrium, taking policy as given. Section 3 sets up the government’s social choice problem given the nature of black markets in general equilibrium, and provides two propositions which hugely simplify optimization. The modified Ramsey rule is derived and its differences from the conventional one discussed; the non-convexity of enforcement costs is demonstrated. A simple parametric example is presented. Section 4 summarizes.

\(^{8}\)Gordon (1990) provides a nice example of how the analysis can be affected by monopoly power. He shows that if sellers of a taxed good are price setters, they may choose to offer two prices for the same good, one for legal transactions and the other for illegal ones in which the tax is retained. Consumers choose which to pay, thereby revealing something of themselves. Thus the excise tax can give a monopolist an opportunity for imperfect price discrimination. This is qualitatively different from anything in the present model.
2. Black Markets.


Consider a market for a good called i which is sold perfectly elastically by producers or foreigners at the price $p_i$, but on which the government has imposed a tax of $\tau_i$. Call $q_i^o = p_i + \tau_i$ the "official price" of the good. It is legal for anyone to trade the good and there is free entry into trade, with no transport costs, and all private agents take prices as given. Now, any trader, having bought the good at the world price, has the option of selling it, trying to hide the transaction from the government\(^9\), and keeping the tax revenues, but if she is caught doing so her profit on the transaction is confiscated and she must pay a fine of $K_i$ as well\(^10\). There are $L_i$ tax inspectors in the market, checking for irregularities, and each can audit $\alpha_i$ transactions. Aggregate consumption of the good is denoted by $x_i = X_i(q)$, where $q$ is the price paid by

\(^9\)We do not allow here for real resources consumed by attempts to conceal illegal transactions. That is the heart of Bhagwati and Hansen (1973), but it is not clear that such costs are always important in real world black markets. Indeed, Deardorff and Stolper (1990) argue that in African economies illegal trade tends to consume fewer real resources than legal trade, because it is free from the heavy regulation that burdens legal traders. However, it is indisputable that real resources are consumed in the enforcement of taxes, and we do allow for that.

\(^10\) The penalty process as described by a street vendor in Mexico city: "...there was no permission to sell, so since the (street inspector) trucks arrived, we had to find a way of working early. We worked from 5 to 7:30 in the morning because the trucks arrived and removed us. At first they just took the people, and the things were left thrown on the ground. But later, since the government saw that some kept selling, they changed their system again. Then they came back and they took all the merchandise. They let us go free, but they took your merchandise. Everything, they took everything, whether it was money or the merchandise, everything there was..." (Cross, 1996, p. 7.)
consumers, and each unit consumed requires a separate transaction, so the probability of being caught at evasion is \( e_i = \alpha_i L_i / x_i \). Assume that \( X_i(q) \) is continuous and decreasing, that \( X_i(0) > \alpha_i L_i \), so that monitoring is imperfect, and that \( X_i(q) \to 0 \) as \( q \to \infty \).

Equilibrium entails a price \( q \) at which the good is sold to consumers together with a decision on whether or not to evade such that traders are maximizing expected profits and potential traders are indifferent between entering and not entering. Profits from trading legally are necessarily zero, so each trader will be willing to evade the tax if and only if \((1-e_i)(q-p_i) - e_i K_i \geq 0\). Thus, if \((1-e_i)\tau_i = (1-e_i)(q^*_i-p_i) \leq e_i K_i\), then legality will be an equilibrium, while if \( \tau_i > K_i e_i / (1-e_i) \) all traders will evade in equilibrium. But then \( q^*_i \) will not be the equilibrium price because that would give traders positive expected profits and lead to entry. We call the equilibrium price the "street price," and denote it \( q^*_i \). It must satisfy \( q^*_i = p_i + K_i e_i / (1-e_i) \), so that evading traders will break even. In a black market equilibrium, that is, one in which all traders evade the tax, the price paid by consumers must be \( q^*_i \), and this must be below the official price, or a vendor could enter offering a price between the official and street prices, remit the tax required, and make a risk-free profit.

In a black market equilibrium, then, all active traders "hawk" the good, or sell it below the official price. Anyone trying to comply with the tax in such an equilibrium will be undersold and driven out of business\(^{11}\). The hawker themselves will at times appear to earn rents, as they pocket the difference between the street price and the producer price, but on average the fines they pay periodically will cancel those gains out. Their incomes from trade will thus be as they

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\(^{11}\)Thus, legal and illegal trade cannot coexist unless \( q^*_i = q^*_i \). This contrasts with Pitt (1981) and Virmani (1989), who allow for the important possibility that a given trader can use legal sales to camouflage illegal ones.
would have been under legal trade, that is, zero\(^{12}\).

Now we reach the first substantive point. The calculation of the street price is not straightforward, owing to the endogeneity of the enforcement rate \(e_i\). This comes from the dependence of \(e_i\) on \(x_i\). Demand for good \(i\) partly determines its price here because for a given enforcement system the probability of getting caught in an illegal transaction is greater the smaller the number of total transactions in the market, so in a real sense the cost of hawking a less popular good is higher than that of hawking one more popular. We will call this the "market thinning effect". The key thing to note here is that one of the relevant consumer prices for determining \(x_i\) is \(q_i^*\), which is being determined. Thus, \(q_i^*\) depends on \(e_i\), which depends

\[\text{Figure 1: The determination of the street price.}\]

\(^{12}\)In the real world, of course, trade consumes real resources, including the trader's time and effort, which would receive an equilibrium return and add to the street price. Adding this to the model would be immaterial to the points being made here -- indeed, if the resource cost of a unit of trade is \(u\) hours of labor and \(w\) is the going wage, then that would simply add \(wu\) to every equilibrium price. We might as well let \(p_i\) include such costs rather than worry about them. What is a significant assumption here is that trade is competitive whether it takes place legally or not, so that high taxes do not generate monopoly profits for a handful of traders who can evade successfully.
in turn on $q_i^*$.

Define the function $\phi(q) = q_i^* - \frac{\alpha_i L_i K_i}{[X_i(q_i^*) - \alpha_i L_i]} = q_i^* - K_i e_i / (1 - e_i)$. Any $q \leq q_i^*$ which satisfies $\phi(q) = p_i$ is the street price for a black market equilibrium. This is illustrated in Figure 1. Further, if $\phi(q_i^*) \leq p_i$ then legality is an equilibrium, because then we will have $(1 - e_i)(q_i^*-p_i) \leq e_i K_i$, so that at the official price evasion is unprofitable. Finally, since $x_i \to 0$ as $q_i^* \to \infty$, there is a point at which $\phi_i$ approaches minus infinity. These together show, with the help of Figure 1, that $\phi_i$ has a maximum, which we denote $\bar{p}$.

If the producer price $p_i$ is less than $\bar{p}$, then there are at least two solutions\(^{13}\) to $\phi(q_i^*) = p_i$, say $q'$ and $q''$. If the official price is below $q'$, then only legal trade is an equilibrium. If the official price is above $q'$, then $q'$ is a black market equilibrium. If it is above $q'$ and $q''$, then they are both black market equilibria. But in this case there must be at least one more equilibrium: if $q_i^* > q'' > q'$ and $\phi(q_i^*) \leq p_i$, then legal trade is once again an equilibrium, and if $\phi(q_i^*) \geq p_i$, then there must be a third solution $q'''$ to $\phi(q''') = p_i$, with $q''' < q_i^*$, so that there are three black market equilibria. These observations can be summarized thus:

**Proposition 1.** If good $i$ is characterized by imperfect monitoring and if there is any street price that would allow hawkers to break even (i.e., $\phi_i > 0$ anywhere), then there is a threshold value $\bar{p}$ for the producer price such that:

(i) If $p_i > \bar{p}$, there can be no black market for good $i$.

(ii) If $p_i < \bar{p}$, there will be zero, one, or at least two black market equilibria depending on

\(^{13}\)For simplicity, the Figure is drawn as if there are only two, but of course in principle there could be more.
whether the official price is low, moderate, or high. When there are at least two, there is always another equilibrium, which may be a legal one; indeed, for a high enough official price there will always be at least two different black market equilibria and one legal one.

This is quite a surprising result, since this market would trivially have only one equilibrium without the tax, while with a high tax it necessarily has at least three. This multiplicity arises from the market thinning effect: at \( q^* \), if all hawkers simultaneously offered a lower price, demand would be stronger, the market thicker, and hence transactions less risky. Thus they could all break even at the lower price; but one trader lowering the price alone would make losses.

Further, we can easily imagine two identical countries with identical tax systems, one riddled with black markets and the other law abiding. Stranger still, if transport costs within the same country were high we could observe one city in which taxes are paid and another, a kind of fiscal Gomorrah, in which each citizen evaded simply because everyone else in town did the same. Both would be equilibria. Many observers of tax systems insist that 'psychological' and 'social' pressures play a great role in aggregate tax compliance, independently of parameters of the tax system\(^\text{14}\); Proposition 1 may be thought of as a formalization of that suspicion through an economic mechanism. Similarly, some seasoned observers of Third World tax systems have

\(^{14}\)For example, Mehta (1990, p.22) maintains that "Psychological factors" leading to evasion include "... the social environment, in which the tax payer lives. It may change the behaviour and attitude of tax payers... thus, when a tax payer comes to know that many people in his society are not paying taxes, then his determination in favor of tax compliance becomes feeble."
long had a hunch that compliance *en masse* has a kind of self-reinforcing character. Finally, we can at times find a belief in this sort of multiple equilibria in the testimony of hawkers themselves. In recent years Mexico City has accrued a large population of street vendors, who sell a large variety of goods, particularly in the ‘Historic District,’ without paying any sales tax. The government’s relationship with them has been varied and ambivalent, but from 1993 to 1994 the Mexican government attempted to put an end to street vending in the Historic District by building formal markets and requiring the street vendors to set up in them (Cross, 1995). Once set up in a stall in the market, a vendor would be easily regulated by government inspectors (with the help of government-sanctioned street vendor associations) and required to pay all sales taxes. An analysis offered by one vendor then working within the government market system is instructive.

A leader of a small association noted that, while he was loyal to the PRI, “People are thinking ... if after the elections they don’t offer us a solution for the markets they are going to go back to the streets--whatever opportunity they see they’ll take.” But they had to wait for a large association to take the first step: “If I had the number of people (the large associations) have I would do it. If I did it now they would squash me with one hand. But I don’t know--sometimes its worth it to take a risk,” he added. Furthermore, as this leader pointed out, the competitiveness among leaders meant that as soon as one association started to reinvade the streets, the others would be compelled to follow to prevent other groups of vendors from taking over the streets that over the years they have considered as “theirs”. (Cross, 1995, p. 14.)

The situation is complicated by the presence of the vendors’ associations, the politics of the

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15“*No matter how ideal the tax structure may be in terms of its objectives, the latter will not be attained unless the tax is collected with a high degree of efficiency. Evasion by some firms breeds evasion by others, who might have to evade to remain competitive. Evasion leads to general disregard for the law. Loss of revenue may result in increasing tax rates, further increasing the incentive to evade.*” Due (1988, p. 194.)
situation, and other factors. However, the basic point is the same as the point of Proposition 1. The vendor would like to return to illegality but it would be excessively dangerous if he did so alone. If the bulk of vendors did so, on the other hand, not only would it be safe enough to be possible for him to do so; competitive pressure would demand it. Thus, there are (at least) two equilibria.

Having noted the odd effects of a very high tax, we will from here on in restrict attention to small taxes, in other words, assume that there is only one solution $q^*_i$ to $\phi_i(q^*_i)=p$, with $q^*_i \leq q^*$. Thus necessarily $\phi'_i(q^*_i) > 0$ (see Figure 1). With a unique equilibrium, we can speak of the street price as a function of all exogenous variables. Some properties of this function emerge from applying the Implicit Function Theorem to $\phi_i(q^*_i)-p_i=0$, which can also be read as shifting the $\phi_i$ function in Figure 1. It is easy to verify that $\partial \phi_i/\partial L_i^c < 0$, so that $\partial q^*_i/\partial L_i^c > 0$. Thus, as one might guess, more vigorous policing of hawkers makes the good more expensive. Similarly, $\partial q^*_i/\partial K_i > 0$. More surprisingly, if $q_j$ is the price of some other good denoted by $j$, since $\partial \phi_i/\partial q_j = \alpha_i L_i [x_i - \alpha_i L_i]^2 \partial X_i/\partial q_j$, we have $\partial q^*_i/\partial q_j < 0$ if $i$ and $j$ are gross substitutes and $> 0$ if they are gross complements. This is exactly the reverse of the result one gets in partial equilibrium analysis of a legal market. The reason is that raising the price of a gross substitute to $i$ brings consumers into the market for $i$, thickening the market and thus making sales of $i$ safer.

A. Objectives, Constraints, and Two Simplifications.

We have looked in some detail at the individual black markets in the model. Now they can be assembled into general equilibrium, and the optimal tax system considered. Suppose that the economy consists of a continuum of identical households of unit measure, each with an indirect utility function \( u \), whose arguments are the consumer prices \( q \) of the \( n \) private goods available, the quantity \( x_g \) of a pure public good, and income \( I \). Assume that \( u \) is differentiable and the derivative with respect to the last two arguments is strictly positive everywhere; that it is additively separable in \( x_g \); and that the commodity demands, \( x(\cdot) \), have uniformly bounded derivatives. Each household is endowed with a unit of labor, which it sells inelastically, and which has a constant marginal value product of \( w \). There are no other factors of production. The government wishes to maximize the utility of the representative citizen\(^\text{16}\), \( u(q, x_g, w) \).

The economy is small and open. For focus and brevity, we assume that there are no taxes possible on imports, exports or production, merely on consumption. Extensions to those other policies are obvious. We thus have (following the tradition of the optimal tax literature) a set of exogenous producer prices \( p_1, \ldots, p_n \). We further assume that lump-sum taxes are unavailable. The government can produce the public good with labor by the technology \( x_g = f(L_g) \), where \( L_g \) is the amount of labor hired by the government for this purpose, and \( f'(L) > 0 \) and \( f''(L) < 0 \) \( \forall L \).

\(^{16}\)It would be straightforward to introduce distributional considerations into this model by allowing for different types of household.
It should also be pointed out that the important variables $K_i$ are taken as exogenous here, because if they were allowed to be chosen in an unconstrained manner the government could almost costlessly enforce all taxes by setting astronomical penalties which would never be paid in equilibrium because no-one would ever evade in equilibrium. This is of course unrealistic, partly because if this really was possible it is hard to see why we would see real-world black markets, but partly because a threat to extract a fortune from a given guilty hawker may be quite incredible. A threat to fine a violator down to the level of subsistence may be credible if the government knows how big a fine that will have to be for a given hawker, but then that will run into exactly the same revelatory problems as make true lump-sum transfers impossible to begin with. If those problems did not exist there would be no point in messing with commodity taxes at all. Even without those difficulties, it is likely that enormous suffering inflicted for a small offense will be impossible under a real world criminal code because that would be offensive to some concepts of justice, especially given that there is always a possibility of an erroneous verdict. As difficult as it is to model the constraints on the choice of penalty, it is clear that in the real world there are severe constraints on them, so, as with the absence of lump-sum taxes, we will as a first pass simply leave them out of the set of choice variables. (For a discussion of practical difficulties enforcing penalties for tax evasion in the Third World, see Radian (1980)).

Without loss of generality, let the first $m$ goods be traded legally and the remainder hawked. The functions $\phi_i$ can be defined just as above, but now $\phi_i$ is shifted by all prices other than $q_i$ as well as by $L_i^T$ and $K_i$ (we will suppress these additional arguments for brevity). The government's budget constraint is then:
\[ \sum_{i=1}^{m}(q_i^o - p_i)x_i(q, x) + \sum_{i=m+1}^{n}[K_i+(q_i^o - p_i)] \alpha_i L_i^c \geq w \sum_{i=1}^{n} L_i^c + wL_g, \]

where the vector \( q \) contains the first \( m \) official prices, which are actually the consumer prices for those goods, followed by \( n-m \) street prices. The Kuhn-Tucker multiplier for this constraint is denoted \( \lambda_g \). The first sum represents the taxes paid, and the second fine revenue and confiscated earnings from discovered hawkers. It should be stressed that the number \( m \) is a choice variable here: by its choice of taxes and enforcement structure, the government can choose which markets will respect the tax and which will not. What is required is that for each sector \( i \), \( \phi_i(q^o_i) \leq p_i \) if \( i \) is intended for legal trade, and \( \phi_i(q^c_i) = p_i \) if \( i \) is a black market. We can call these \( n \) conditions the ‘incentive compatibility’ constraints.

In short, the government chooses \( L_g \), the set \( S \) of black markets, the vectors \( L^e = [L_1^e, ..., L_n^e] \) and \( q^e = [q_1^e, ..., q_n^e] \), and street prices \( q_i \) for \( i = m+1, ..., n \) subject to one budget constraint and \( n \) incentive compatibility constraints, to maximize welfare\(^{17}\).

The optimization problem can be simplified by two propositions. The first is akin to a “Laffer curve” proposition in Virmani (1989).

**Proposition 2.** Suppose that \( F = (L_g, S, L^e, q^e, \{q_i^c\}_{i \in S}) \) is a feasible plan of taxes, enforcement and public good provision that induces black markets. Then there exists another plan, say \( F' \).

\(^{17}\)The choice of \( n \) official prices \( q^o_i \) is of course equivalent to the choice of the \( n \) tax rates \( \tau_i \). The street prices must technically be included as choice variables to deal with the nuisance of multiple equilibria; however, we will soon be able to dispense with this since it turns out that multiple equilibria will not be an issue at the optimum.
that does not induce black markets and that yields just as high a level of social welfare as $F$
does.

**Proof:** Set $(L_g)' = L_g$; $S'$ equal to the null set; and $(L_e)' = L_e$. Then set $(q_1') = q_1$ for $i = 1,...,m$
and $(q_i') = q_i$ for $i = m+1,...,n$. This defines a plan $F'$ which satisfies the incentive compatibility
constraints trivially. $F'$ also gives the same consumer prices as $F$, hence provides the same value
for social welfare, and is feasible provided that it does not deliver any less revenue than $F$.

Revenue under plan $F'$ is given by:

$$
\sum_{i=1}^{n-1}[(q_i')-p_i]x_i = \sum_{i=1}^{m}(q_i'-p_i)x_i + \sum_{i=m+1}^{n}(q_i'-p_i)x_i = \sum_{i=1}^{m} \tau_ix_i + \sum_{i=m+1}^{n}(e/(1-e))K_ix_i,
$$

where $\tau_i = (q_i'-p_i)$. Revenue under plan $F$ is equal to:

$$
\sum_{i=1}^{m} \tau_ix_i + \sum_{i=m+1}^{n}[K_i+(q_i'-p_i)]{\{\alpha_iL_e}\}
= \sum_{i=1}^{m} \tau_ix_i + \sum_{i=m+1}^{n}[K_i+(e/(1-e))K_i]{\{e_x}\}
= \sum_{i=1}^{m} \tau_ix_i + \sum_{i=m+1}^{n}(e/(1-e))K_ix_i.
$$

Thus, revenue under the two plans is equal. **Q.E.D.**

Thus, it is never strictly better to induce black markets\(^{18}\), even with the vector of
punishments exogenously constrained. The rather striking result that black markets could always
be swept away by an optimal tax system would not hold under some of the assumptions made

\(^{18}\)Although it is worth remembering that it is a matter of indifference whether they are
permitted or not. Proposition 2 allows us to focus on the case without black markets for simplicity without losing generality.
in the smuggling literature. Thus, for example, Pitt (1981) shows an example in which legal sales, and hence tariff revenues, are greater in the presence of smuggling than in its absence, because legal sales can be used to camouflage illegal ones, and thus the two kinds of trade are in a sense joint outputs. In this case, Proposition 2 would clearly not hold. Other examples are models with a rising marginal cost to smuggling, such as Lovely (1994). Thus, Proposition 2 is not presented as a robust policy prescription, but rather a result which hugely simplifies the optimal tax problem in this model. Nonetheless, the mechanism of this proposition does have important counterparts in the practice of tax reform. The practice of lowering the tax rate in order to eliminate the smuggling is essentially the course followed by the Canadian government in February 1994 in the case of the cigarette excise; the move virtually put an end to cigarette smuggling and led to some calls for application of the policy to liquor and for imitation of it in the U.S. This was also the move taken by Paraguay in January 1990 in the case of imports for transhipment via the ‘tourist trade.’ In order to eliminate the rampant smuggling of such imports, their tariffs were unified at the rate of 7%, much lower than the previous levels. The result was an increase in tariff revenue as smuggling waned (Connolly et. al. (1995)).

From now on we will set $m=n$ since that is optimal. The second simplifying proposition concerns optimal hiring. Since the public good is assumed to have a positive marginal utility everywhere and to be additively separable in utility, it is immediate that it is never optimal to

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19 However, in a model in which there was a constant marginal resource cost to evasion, Proposition 2 would hold *a fortiori.*

hire redundant tax inspectors. Any inspectors hired for sector i once the incentive compatibility constraint for i is satisfied could be better deployed at producing the public good. Thus, we have:

**Proposition 3.** In the optimal plan the government hires tax inspectors for each market up to the point at which the incentives for tax evasion just disappear, or the street price rises until it just equals the official price. Thus, \( L_i^* \) is chosen such that \( eK_i = (1-e)\tau_i \), or equivalently,

\[
L_i^* = \frac{\tau_i x_i}{[\alpha_i (K_i + \tau_i)]} = x_i \psi_i(\tau_i) .
\]

Thus, enforcement costs are given by \( \psi \) times the size of the market, where \( \psi \) is a measure of the incentive to evade and is rising in \( \tau_i \). The usual optimal tax problem has, in effect, \( \alpha_i = \infty \) and thus \( \psi = 0 \).

**B. The Nature of the Optimum.**

Using the simplifying results above, we can now eliminate the \( L_i^* \) as choice variables, by substituting (1) in to the original optimization problem. The full first order condition for the choice of tax rate is then easy to derive:

\[
\frac{\partial \mathcal{L}}{\partial \tau_i} = \frac{\partial \mu}{\partial q_i^e} + \lambda_{g_i} [x_i + \sum_{j=i+1}^n \tau_j \frac{\partial x_j}{\partial q_i^e} - w \sum_{j=1}^n \partial q_f^e / \partial q_i] \\
= \lambda_{g_i} (1 - \sum_{j=1}^n \tau_j m_j) x_i - \mu_i x_i + \lambda_{g_i} \sum_{j=1}^n \tau_j s_{i,j} - w \lambda_{g_i} \sum_{j=1}^n \partial L_i^* / \partial \tau_i = 0 .
\]
by Roy's identity and Slutsky's Equation, where $\mu_i$ is the marginal utility of income to the representative household, $m_j$ is the marginal propensity to consume good $j$ out of income, and $s_{ij}$ is the cross-price derivative of compensated demand.

Most of this is familiar from the conventional optimal tax problem, and is a slight generalization of Ramsey-type conditions; see Stern (1987). The first term is a marginal revenue effect, the second gives the direct utility cost of a rise in the tax, and the third term is an efficiency term measuring deadweight loss. The only change the enforcement problem makes here is the addition of one more marginal cost term, namely, the last term, which records the marginal enforcement cost of raising the $i^{th}$ tax.

However, this addition has the potential to change the nature of the problem in a drastic way, because this marginal cost cannot be a globally rising cost, and indeed in many cases will be globally falling. To see this, write the marginal effect of tax $i$ on enforcement costs in this way:

\begin{equation}
\frac{\partial L_i^e}{\partial \tau_j} = \begin{cases} 
\frac{Kx_i}{\alpha_i(K_i + \tau_i)^2} + \psi_i(\tau_i) \frac{\partial x_i}{\partial q_i} & \text{if } i=j, \\
\psi_i(\tau_i) \frac{\partial x_i}{\partial q_j} & \text{else.}
\end{cases}
\end{equation}

The first term in the own-tax derivative ($i=j$) is the direct effect: raising the tax on good $i$ raises the incentive to evade the tax on $i$ (that is, $\psi_i$), thus raising the cost of enforcement. The second term is the market thinning effect: raising the tax on good $i$ reduces the number of transactions that occur in the $i^{th}$ market, thus making hawking riskier and reducing the number
of police needed to deter it. The cross-tax derivative is clearly a market-thinning or thickening effect, depending on substitutability or complementarity. What is key to observe here is, first, that the direct effect in (3) is globally decreasing in \( x \). Further, the total effect must decline to zero as \( x \to \infty \). Thus, the marginal enforcement cost of a tax cannot possibly be rising everywhere, and indeed the more usual case would be (when the direct effect dominates) that it will be falling everywhere. This is a key qualitative difference between this and the more traditional features of the optimal tax problem: enforcement costs tend to work against the second order condition.

It is worth clarifying the source of this non-convexity in enforcement costs: It comes from the principle that a high-tax sector must also be a high-risk sector from the point of view of hawkers. Suppose that the demand for good \( i \) is perfectly inelastic and cross-price elasticities are zero among goods with a positive tax so that only the direct effect is an issue. Consider increasing the tax on good \( i \) from some initial level \( \tau^* \) to \( \tau^* + \Delta \) and then to \( \tau^* + 2\Delta \), with \( \Delta > 0 \), each time adjusting \( L_i^c \) so that evasion is just deterred. For both increases in the tax, the level of enforcement must be raised, but the first rise in \( L_i^c \) will be larger than the second one. The reason is that when the tax is at \( \tau^* + \Delta \), the probability that a hawker is caught is higher than it is when the tax is at \( \tau^* \) (otherwise the hawker would not be indifferent at both tax rates). Thus,

\[ \text{This comes from (1), since } x_i \text{ must go to zero as } \tau_i \to \infty, \text{ and from the assumption on boundedness of demand derivatives.} \]

\[ \text{Although it is well known that the conventional optimal taxation problem can be non-concave (Diamond and Mirrlees, 1974, (part II) discusses this), there is certainly no reason to presume that it must be so; and indeed, in most parametric examples in practice, there is a tendency for the marginal benefit term to fall in } \tau_i \text{ (since the } x_i \text{ will generally fall as taxes rise) and for the marginal deadweight loss to rise (since it is roughly proportional to the tax rates). The new term, marginal enforcement cost, is the first exception to this rule of thumb. This is a marginal cost term that must fall for sufficiently high taxes. As a result, it can, in principle, change the nature of the problem entirely.} \]
although the rise in the tax rate, and thus the rise in the return to successful evasion, is the same both times, the incentive to evade rises by less the second time. Thus, a smaller increase in enforcement is needed to deter it the second time.

Thus, if the direct effect dominates, so that enforcement costs are globally concave in tax rates, there is an enforcement-cost-minimization motive to look for tax rates that vary as much as possible across sectors. It is for this reason that if enforcement costs are important enough there is now a presumption that the first order condition will identify a local minimum, and the true optimum will be a corner solution of some sort.

It is instructive to see how this works in a simple parametric example. Suppose that there are two private goods, with $p_i = p$, $x_i = a$, and $K_i = K$ for $i=1,2$. The amount of public employment required is fixed at $R$, and the utility function is CES with elasticity $\sigma$, so that indirect utility is given by:

$$v(\tau_1, \tau_2, w, \sigma) = \begin{cases} 
  w \left[ (p+\tau_1)^{1-\sigma} + (p+\tau_2)^{1-\sigma} \right]^{\frac{1}{\sigma(\sigma-1)}} & \text{if } \sigma \neq 1; \\
  \left(\frac{1}{2}\right) w [(p+\tau_1)(p+\tau_2)]^{1/2} & \text{if } \sigma = 1,
\end{cases}$$

Figure 2: The Nature of Optimal Taxes.
where we write the function in terms of tax rates for convenience.

In the absence of an enforcement problem, uniform commodity taxes would trivially be optimal in this case, and indeed, they would meet the revenue requirement with no deadweight loss. With the enforcement problem, it is immediate that uniform taxes can always solve the first order condition here because of the symmetry of the goods, and indeed, it can easily be shown that for any $\alpha$ there is a unique uniform tax rate that satisfies the budget constraint. The question is: Is this interior solution the optimum?

The full optimum can be computed as follows\textsuperscript{23}. For any $\alpha$ and $\sigma$, given any tax pair $(\tau_1, \tau_2)$, it is straightforward from Proposition 3 to see what the implied enforcement labor force is, and from that and the demand system to see whether or not the government meets its revenue needs. Thus, for any value of $\tau_2$, we can verify numerically whether or not there exists a $\tau_1$ such that the budget requirement is met; if so, we can find the smallest such $\tau_1$ and use that tax pair to calculate utility. A grid search along this interval of feasible values for $\tau_2$ then can be used to find the optimum.

The results of this procedure, for $w = p = 1$, $R = K = \frac{1}{2}$, and a range of values for $\alpha$ and $\sigma$, are summarized in Figure 2. The downward-sloping curve represents $(\sigma, \alpha)$ pairs for which the utility from a tax pair $(\tau_1, 0)$ that satisfies the budget constraint is equal to the utility from the uniform tax rate that satisfies the budget constraint. For points above the curve, the optimum is uniform taxation. For points below the curve, the optimum comes from setting one of the taxes equal to zero, thus the farthest possible departure from uniformity. The trade-off is clear: If $\alpha$ is large, the enforcement problem is not serious, and is dominated by the familiar logic of

\textsuperscript{23}Full details are available from the author on request.
deadweight loss; the optimum solves a Ramsey-type first order condition. If $\alpha$ is small, the enforcement problem is overriding, and due to the concavity of enforcement costs in the tax vector, the maximum tax spread is desired. There is nothing in between. Further, holding $\alpha$ constant, increasing $\sigma$ raises the deadweight loss from any departure from uniformity, and makes it difficult to raise the required revenue from non-uniform rates because of the inclination of consumers to flee the taxed good. Thus, it is for high values of $\alpha$ and $\sigma$ that uniform taxation is desirable, and for low values that maximally non-uniform taxes are desirable.

Thus, we see a result much unlike standard optimal tax propositions. If the administrative capability of the tax office is low or if goods from the various sectors are very poor substitutes, a 'cash cow' tax system is optimal, in which one sector is designated to be the source of all revenue\(^{24}\), even in a case in which all sectors are ex ante identical\(^{25}\). It is only for a sufficiently competent administration or high substitutability that the balancing of deadweight losses at the margin even matters. This can be used to interpret a variety of stylized facts in the history of tax systems in the Third World.

First, we may associate the 'cash cow' outcome with a narrow set of excise taxes (or possibly trade taxes) and the 'Ramsey' outcome with a broad set of indirect taxes such as a sales tax. One of the best known facts about the evolution of tax systems is a strong tendency of

\(^{24}\)In a richer model with several sectors varying in demand and administration parameters, it would not necessarily be only one sector, of course. The point is that there would be a motive, if $\alpha_\ell$'s generally were low, to create a highly non-uniform system with several sectors left out of the tax system entirely.

\(^{25}\)Please note that this occurs despite the fact that there was no assumption of a fixed cost to administering a tax. The result comes entirely from the concavity of enforcement costs, which is a consequence of the incentive compatibility constraint (1). It would clearly hold a fortiori if there were fixed costs as well -- a reasonable assumption in many settings.
excise systems with narrow effective bases to emerge first, and then after a long period of
development for the sales taxes to be introduced\textsuperscript{26}. As of 1935, only one developing country, the
Philippines, had a real sales tax\textsuperscript{27}; they all had excises, which have shown a strong tendency to
be effectively concentrated on a very small number of goods\textsuperscript{28} (although many more may be
listed as under excise officially). In many countries the government relied on a single export tax
(perhaps concealed in the 'marketing margin' of a statutory marketing board) for a large fraction
of its revenue\textsuperscript{29}. However, over time countries establish broad based sales taxes and VAT's.\textsuperscript{30}
At times, this can lead to dramatic, rapid broadenings of the tax system. For example, Zambia
had no sales tax at all until 1973, relying mainly on excises on copper and beer. As of 1983, its
sales tax accounted for 24\% of total revenue, a tremendous jump from zero. In Tanzania, the
sales tax went from 16\% of total revenue in 1974 to 54\% in 1981. (Due, 1988, p. 102.)

The point is that something much like this very broad pattern would be predicted by this
model. Imagine a country with initially a very low level of literacy and information
infrastructure, a small stock of trained accountants and essentially no computers, in other words,

\textsuperscript{26}See, for example, Due (1988, Ch. 5-7).

\textsuperscript{27}Due (1988, pp. 82-3).

\textsuperscript{28}As is true of excises even now. Due (1988) lists 14 countries for whom excises on gasoline
and diesel fuel make of at least 59\% of excise revenue (p. 73); 9 countries for which excises on
alcoholic beverages make up at least 41\% of excise revenue (p. 64), and 5 for whom excises on
tobacco make up at least 44\% of excise revenue (p. 64).

\textsuperscript{29}This is extensively documented for several African states by Bates (1981). See especially
pages 12-9.

\textsuperscript{30}Although as Tanzi (1987, p. 227) points out, they are rarely really comprehensive, or as
comprehensive in practice as they appear on paper. Nonetheless, they tend to be substantially
more broad-based than the excise system.
a country with a very low $\alpha$. In this model, if that country followed the optimal tax policy it would use a very narrowly based excise tax and nothing else. On the other hand, if we assume that the level of education and infrastructure, and thus administrative effectiveness and hence $\alpha$ would rise over time, eventually the economy would cross the curve in Figure 2 and would jump to an all-inclusive sales-tax system. It is worth pointing out that one of the correlates of large-scale tax reforms in developing countries is indeed a large improvement in the overall administrative effectiveness in the tax administration (Thirsk, 1993, pp. 189-90).

Finally, the fact that Zimbabwe is the first and still one of the very few developing countries with a broad, effective sales tax at the retail level, is explained by the fact that in that country retailing is primarily carried out in urban areas, and by formal commercial establishments with literate clerks and a strong tradition of good record keeping (Due, 1988, pp. 120-1). In other words, in terms of this model it is because this is a high $\alpha$ economy.

The proposition that a more ineffective tax administration should lead to large portions of the economy being untaxed runs counter to the spirit of Ramsey-type analysis, and indeed, it is sometimes argued that in the presence of poor administration, tax uniformity may be the only practical policy (e.g., Tait (1989), Thirsk (1993, pp. 181-3). In this model, we find the opposite
result, and it roughly fits international experience.

The second empirical observation on tax reforms in practice is that they tend to be associated with a fiscal crisis in the government. This is observed, for example, by Thirsk (1993, p. 178) in his survey of recent tax reforms. The Zambian reforms coincided with the slide in copper prices in the latter 1970's, with its devastating effect on the Zambian treasury; the Mexican reforms over 1978-82 coincided with the onset of government insolvency. This also fits the current model quite well. Suppose that for reasons of increased foreign debt obligations or any other reason the amount of revenue required by the government, R, goes up. The effect on the cash-cow-Ramsey locus is shown in Figure 3. A rise in revenue requirements shifts the locus down, expanding the space in which broad-based taxation is optimal. The reason is clear: With a rise in R, the tax rate on the ‘cash-cow’ sector required to meet budget balance goes up; thus, the intersectoral distortion of this form of taxation goes up, lowering its appeal relative to the broad-based taxation. Thus, if we observe an economy at B, with R initially at ¼, and a disaster such as rising international interest rates pushes R up to ¾, if the government behaves optimally, we will see a sudden tax reform. All of the economists’ long pleas for broad-based taxation of consumption will finally be answered. However, it is not because the government has suddenly acquired wisdom; it is simply an optimal response to a changed situation. Further, the country will indeed be worse off. If misunderstanding voters perceive the reform to be the reason for their misery rather than its mitigator, politics may cause a good policy to be undone.

31Substantial broadening of the indirect tax base, including introduction of a VAT, was a key feature of these reforms (Gil Diaz, 1987). It should be noted, however, that the country slid back toward greater dependence on a narrow excise base over the 1980’s. (Polyeconomics, Inc., 1990, pp. 151-2).
4. Conclusion.

A simple model of indirect taxation, evasion, and enforcement has been presented in which some surprising results emerge. In the partial equilibrium version, we find that high taxes lead to multiple equilibria (low-price black markets and high-price legal markets), and comparative statics always give the 'wrong' sign. Both are due to the 'market-thinning' effect. In general equilibrium, we find that the optimal system is qualitatively different for relatively effective administrations compared to relatively ineffective ones. The former follow slightly modified Ramsey optima, trading off marginal deadweight loss across sectors. The latter optimally ignore such matters, pick a sector at random (in a symmetric model), and use it to raise all of the revenue. This is because the incentive compatibility constraint confers a kind of concavity on enforcement costs. This finding provides some predictive value for the evolution of tax systems.

Some small caveats may be in order. The precise mathematics are likely to differ somewhat with different assumptions about the nature of tax evasion technology, and in particular, as noted, Proposition 2 is not likely to hold with some of the assumptions popular in the literature. It may well be, however, that something similar in the behaviour of the full optimum would be likely to hold, because the arguments for why the direct effect of the tax rate on enforcement costs must fall (following (3)) seem likely to be much more general than this model. Second, there has been no attempt here to argue that actual tax systems -- in Zimbabwe or anywhere else -- are necessarily (or probably) optimal. Nonetheless, it is possible that a broad pattern in the response of the conceptual optimum to a change in the environment can give hints
as to why actual tax systems change in the way they do. Perhaps this general approach can lead to a richer and more nuanced positive theory of tax systems than that suggested here.


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