

PRICE COMPETITION AND THE IMPACT OF
SERVICE ATTRIBUTES: STRUCTURAL
ESTIMATION AND ANALYTICAL
CHARACTERIZATIONS OF EQUILIBRIUM BEHAVIOR

MARGARET PIERSON

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ABSTRACT

Price Competition and the Impact of Service Attributes: structural estimation and analytical characterizations of equilibrium behavior

Margaret Pierson

This dissertation addresses a number of outstanding, fundamental questions in operations management and industrial organization literature. Operations management literature has a long history of studying the competitive impact of operational, firm-level strategic decisions within oligopoly markets.

The first essay reports on an empirical study of an important industry, the drive-thru fast-food industry. We estimate a competition model, derived from an underlying Mixed MultiNomial Logit (MNML) consumer choice model, using detailed empirical data. The main goal is to measure to what extent waiting time performance, along with price levels, brand attributes, geographical and demographic factors, impacts competing firms' market shares.

The primary goal of our second essay is to characterize the equilibrium behavior of price competition models with Mixed Multinomial Logit (MMNL) demand functions under affine cost structures. In spite of the huge popularity of MMNL models in both the theoretical and empirical literature, it is not known, in general, whether a Nash equilibrium (in pure strategies) of prices exists, and whether the equilibria can be uniquely characterized as the solutions to the system of First Order Condition (FOC) equations.

The third essay, which is the most general in its context, we establish that in the absence of cost efficiencies resulting from a merger, aggregate profits of the merging firms increase as do equilibrium prices for general *price* competition models with general non-linear demand and cost functions as long as the models are supermodular, with two additional structural conditions: (i) each firm's profit function is strictly quasi-concave in its own price(s), and (ii) markets are competitive, *i.e.*, in the pre-merger industry,

each firm's profits increase when any of his competitors increases his price, unilaterally. Even the equilibrium profits of the *remaining* firms in the industry increase, while the consumer ends up holding the bag, *i.e.*, consumer welfare declines. As demonstrated by this essay, the answers to these sorts of strategy questions have implications not only for the firms and customers but also the policy makers policing these markets.

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*“I can be changed by what happens to me
but I refuse to be reduced by it.”*

- Maya Angelou

Dedicated to the Survivors

Chapter 1

Introduction and Overview

1.1 Introduction

Operations management literature has a long history of studying the competitive impact of operational, firm-level strategic decisions within oligopoly markets. The literature on competition in the service industry, the context of our first essay, dates back to the late 1970s when Luski (1976) and Levhari and Luski (1978) were the first to model competition between service providers. Other strategic decisions often studied at the market level include inventory, assortment, and quality decisions (see, e.g., Olivares and Cachon (2009) and Gans (2002)). As demonstrated by our third essay, the answers to these sorts of strategy questions have implications not only for the firms and customers but also the policy makers policing these markets. Modeling and estimation techniques have been evolving since the inception of this area of research. Of particular import to the methods used in the first two essays were the introduction of an empirical method for estimating consumer choice models and cost structures in oligopolistic markets with differentiated goods by Berry, Levinsohn, and Pakes (1995) and the extension of this work to the case where market shares are not observed by Feenstra and Levinsohn (1995), and the seminal paper, Caplin and Nalebuff (1991), which establishes sufficient conditions for the existence of a price equilibrium when the demand functions are based on a broad

class of MMNL models used in the so called BLP method. The third essay, which is the most general in its context, contributes to a stream of literature which seeks to respond to the seemingly paradoxical results of Salant, Switzer, and Reynolds (1983). In addition to our contributions to the study and practice of operations management we add to the economics literature on empirical industrial organization and coalition formation in oligopoly settings.

1.2 Overview

Through a series of essays, this dissertation examines three topics in industrial organization for oligopoly markets with differentiated products. A variety of modeling techniques ranging from stylized models to empirical investigations are employed in each problem. The first essay focuses on competition in service industries. In many such industries, companies compete with each other on the basis of the waiting time their customers' experience, along with other strategic instruments such as the price they charge for their service. The objective of this essay is to conduct an empirical study of an important industry to measure to what extent waiting time performance impacts different firms' market shares and price decisions. We report on a large scale empirical industrial organization study in which the demand equations for fast-food drive-thru restaurants in Cook County are estimated based on so-called structural estimation methods. Our results confirm the belief, expressed by industry experts, that in the fast-food drive-thru industry customers trade off price with waiting time. More interestingly, our estimates indicate that consumers attribute a very high cost to the time they spend waiting. In the second essay, we support the estimation method in our first essay by addressing an open question in the economics literature. We postulate a general class of price competition models with Mixed Multinomial Logit demand functions under affine cost functions. We first characterize the equilibrium behavior of this class of models in the case where each product in the market is sold by a separate, independent firm and customers share

a common income level. We identify a simple and very broadly satisfied condition under which a Nash equilibrium exists while the set of Nash equilibria coincides with the solutions of the system of First Order Condition equations, a property of essential importance to empirical studies. This condition specifies that in every market segment, each firm captures less than 50% of the potential customer population when pricing at a level which, under the condition, can be shown to be an upper bound for a rational price choice for the firm irrespective of the prices chosen by its competitors. We show that under a somewhat stronger, but still broadly satisfied version of the above condition, a unique equilibrium exists. We complete the picture, establishing the existence of a Nash equilibrium, indeed a unique Nash equilibrium, for markets with an arbitrary degree of concentration; under sufficiently tight price bounds. We then discuss three extensions of our model: unequal customer income, a continuum of customer types, and the case of multi-product firms. The essay concludes with a discussion of implications for structural estimation methods. Fundamental to our understanding of competitive dynamics in an oligopolistic industry is the question of what impact mergers and acquisitions have on key equilibrium performance measures. In the third and final essay, we address these questions in the context of price competition models with differentiated goods, allowing for general non-linear demand and cost functions merely assuming that both the pre- and post-merger competition games are supermodular along with two minor technical conditions. We show that, in the absence of cost synergies, post-merger equilibrium prices exceed their pre-merger levels. Moreover, the post-merger equilibrium profit of the merged firms exceeds the pre-merger aggregate of the equilibrium profits of the merging firms. Finally, we show that the equilibrium profit of the non-merging firms increases as well. We establish our results, at first, for settings where each firm in the industry offers a single product; we then generalize them to industries with general multi-product firms. We also derive conditions under which cost synergies by themselves result in lower equilibrium prices and discuss how the combined effect of increased market concentration and cost synergies can be assessed efficiently.

1.3 How much is a reduction of your customers' wait worth? An empirical study of the fast-food drive-thru industry based on structural estimation methods.

This essay reports on an empirical study of an important industry, the drive-thru fast-food industry. We estimate a competition model, derived from an underlying Mixed MultiNomial Logit (MNML) consumer choice model, using detailed empirical data. The main goal is to measure to what extent waiting time performance, along with price levels, brand attributes, geographical and demographic factors, impacts competing firms' market shares. In the literature it is commonly assumed that customers attribute a cost rate to their waiting time that can be proxied by an earnings rate, for example the disposable per capita income in the market (see e.g. Mueller (1985)). Our results demonstrate that this may result in questionable policy analyses as we find that customers attribute an implicit value to their wait time, which is many times the average wage in the US. We also characterize how the market's price equilibrium responds to changes in the waiting time standards. Based on this market analysis, we show that the trend to continuously improve waiting times and service levels can be explained on game-theoretical grounds, creating a valuable framework for future market dynamics studies in various industries. While our empirical study is focused on the drive-thru fast-food industry, we apply a methodology based on structural estimation methods frequently used in the Industrial Organization literature which, *mutatis mutandis*, can be employed to establish the impact service attributes have on market shares in other industries.

In many service industries, companies compete with each other on the basis of the waiting time (or other service quality attributes) their customers experience, along with other strategic instruments such as their price. Executives realize that time is money for the consumer but it is unclear how much money, how the exchange rate differs in

different industries, and how it varies with other factors such as location, brand etcetera. Often, specific waiting time standards or guarantees are advertised. For example, in 2002 Ameritrade increased its market share in the online discount brokerage market by “guaranteeing” that equity trades take no more than 10 seconds to be executed; the guarantee is backed up with a commission waiver if the time limit is violated. This led most major online brokerage firms (E-trade, Fidelity) to offer and aggressively advertise even more ambitious waiting time standards. Various call centers promise that the customer will be helped within one hour, say, possibly by a callback. In other industries, average waiting times are monitored by independent organizations. For example, in the airline industry independent government agencies as well as Internet travel services report, on a flight by flight basis, the average delay and percentage of flights arriving within 15 minutes of schedule. See Allon and Federgruen (2007) for a longer list of examples.

A fundamental premise of the by now extensive theoretical literature on service competition is the belief that waiting times have a major impact on consumer choices and market shares, similar to or perhaps even in excess of price differentials. However, this premise has rarely been substantiated by empirical field studies. In the fast-food industry, almost all outlets are owned by independent franchisees who select their own prices. In contrast, chains set national waiting time standards by prescribing a uniform operational process to their franchisees along with specific recipes for their standard menu items. These processes include standard customer greetings, order taking, the maximum number of burgers on a grill and amount of time they may be cooking, the relationship between number of drive-thru lanes and demand volume etcetera, all of which determine the chain’s waiting time standard; see Garber (2005) and Jargon (2006). DeHoratius et al. (2010) describe how tightly the McDonald’s chain standardizes and engineers the service operations process of its outlets. Since chains implement a national uniform waiting time (distribution), we were able to obtain these distributions from a national Drive-Thru Time Study Database, which we purchased from the industry organization QSR.

Chains invest heavily to shave seconds off their average waiting times, clearly believing

that their market shares are very sensitive to the relative waiting times experienced. Hughlett (Nov 28, 2008) attributes the following statement to the president of one of the main technology vendors serving the fast-food industry: “There is an industry maxim that for every seven-second reduction in total service time, sales will increase by 1% over time”¹. It is the belief expressed in this maxim which underlies the chains’ continuous strategic focus on waiting time reductions in their outlets, via technological and process improvements. An estimate of the expected consumer response to reductions in waiting time standards, such as that generated by our study, would be of high value to the industry when evaluating the potential profitability of investments of this type. We therefore look to support these beliefs via an empirical study of a large fast-food drive-thru market, focusing on the following series of research questions, of equal interest to the academic community and the competing chains in the (fast-food) industry.

1. Does the customer’s waiting time at the fast-food drive-thru lane represent a significant determinant of consumer choices and resulting market shares, as believed by the industry and operations management literature alike?
2. When comparing the demand sensitivity to waiting time and price differentials, is the implied value of time of the same order of magnitude as the average wage or income earned per hour? If not, is it of a larger or smaller order of magnitude?
3. Can the above stated industry maxim be substantiated by empirical estimates? In particular, when taking into account that the various outlets are likely to adjust their prices in response to reduced waiting time standards, adopting a new price equilibrium, does the maxim hold?
4. Furthermore, this maxim expresses the belief that a given waiting time reduction is equally valuable for all chains in terms of resulting increases in market shares and sales. Plausible consumer choice models may imply that these benefits, in fact, vary

¹We interpret the 1% increase in sales to refer to a 1% increase in market share, rather than observed sales dollars. As discussed below, we verify that, with this interpretation, the industry maxim is on average correct. See Section 3.3 for estimates of the impact on both measures

with the initial market shares of the chains, either in a robustly predictable way or one that depends on the specific parameter estimates. Either way, it is important to understand to what extent these benefits vary with the size of the chain and other chain attributes. In addition, do the increases in market shares accrue primarily from customers switching between chains or from the acquisition of new customers to the fast-food market?

Our empirical research thus follows a slight variant of the standard paradigm as discussed, for example, by Fisher (2007), see Figure 5 *ibid.* Our starting point is a series of premises, maxims and questions which arise from the theoretical operations management literature on service competition as well as practitioner discussions and surveys in the fast-food industry, reported in such outlets as the industry organization's main publication, the QSR magazine, the Nation's Restaurant News, and the general press.

As mentioned, almost all contributions to the literature on service competition have been theoretical, with numerical investigations confined to small hypothetical examples. Indeed, we believe ours to be one of the first market-wide empirical studies to complement the theoretical service competition literature. There are several reasons for the paucity of empirical studies. It is very difficult to access data regarding customer waiting times, in particular when seeking to quantify the waiting time experience at all competing service providers. While absolute waiting times at a given firm might explain the firm's demand volume in a monopoly setting, it is the relative waiting times at various competing providers which, along with the firms' other strategic choices, explain ultimate consumer choices and hence, realized market shares. Similarly, it is typically very hard, if not impossible, to collect data on sales volumes or market shares of the competing outlets. Although such data are sometimes accessible for consumer products, in the service industry it is rare that sales volumes can be gathered by outsiders. Firms are reluctant to provide the information, considering it of the highest strategic value. Indeed, sales volumes were unavailable in our context. Instead, we infer them by estimating the parameters in the system of equations characterizing the unique equilibrium in a compe-

tition game resulting from a detailed consumer choice model and an outlet cost structure reflecting a broad category of queueing systems. In other words, the demand function parameters are backed out from the equilibrium conditions, with the help of the observed equilibrium. This technique has been applied in a number of economics studies, e.g., Feenstra and Levinsohn (1995) and Thomadsen (2005a) but, to our knowledge, not in the operations management literature.

More specifically, we accommodate the absence of demand data with three assumptions: (1) Consumers attribute a utility level to each potential outlet which depends stochastically on price, waiting time, the distance to the outlet and various chain characteristics. Similarly, consumers assign a utility level to the no-purchase option, which depends stochastically on the consumer's gender, race, age bracket and occupational status. (2) Outlets encounter a cost structure which is affine in the sales volume, with random noise terms for the marginal costs; this cost structure applies to many queueing models used to describe the service process such as M/M/1 systems or open Jackson networks. (3) Outlets adopt a pure Nash equilibrium in the price competition model which results from the above consumer choice model and the outlets' cost structure.

The first assumption is used to derive the relationships between prices, service levels, and sales quantities. Based on the second and third assumptions, these relationships are subsequently used to derive the firms' Nash equilibrium conditions to jointly estimate the parameters of the indirect utility functions of the consumers as well as the parameters of the outlets' cost structure. Our estimation method is a Generalized Method of Moments (GMM) technique, as opposed to more standard maximum likelihood estimators for systems of non-linear equations, for reasons explained in Section 5.

In summary, the main contribution of this essay is that, to our knowledge, it is one of the first to estimate, for the benefit of market observers and the firms alike, how sales volumes for a service organization depend on the prices and waiting times of all competing providers within a given region, their location, as well as other attributes (e.g., brand-specific characteristics). In particular, we conclude that consumers attribute a

value to their waiting time which is many times the average wage level. We use counterfactual studies to confirm that a seven-second reduction by a single chain results, on “average”, in a 1% market share increase for that chain. However, for a large chain like McDonald’s, the increase is more than 3%, showing that the industry’s “7 second rule” needs to be qualified. The increased market share results primarily from the acquisition of new customers who were previously opting for the outside good as opposed to customers switching between chains. Our model explains the continuing trend of all chains investing heavily to reduce their waiting time standards. We show, in addition, that neglecting to include any waiting time measure in the consumer choice model results in significantly over-estimated price sensitivities. This validates our belief that overlooking service as a competitive instrument in the model specification results in distorted managerial insights. Not accounting for the waiting time as an attribute also distorts the estimated value of the no-purchase option, as well as the importance of the number of chain outlets, as a proxy for the consumers’ perceived quality of the chain.

1.4 Price Competition under Multinomial Logit Demand Functions with Random Coefficients

Our primary goal in this essay is to characterize the equilibrium behavior of price competition models with Mixed Multinomial Logit (MMNL) demand functions under affine cost structures. In such models, the market is partitioned into a finite set or a continuous spectrum of customer segments, differentiated by, for example, demographic attributes, income level, and/or geographic location. In each market segment, the firms’ sales volumes are given by a Multinomial Logit Model (MNL). In spite of the huge popularity of MMNL models in both the theoretical and empirical literature, it is not known, in general, whether a Nash equilibrium (in pure strategies²) of prices exists, and whether the equilibria can be uniquely characterized as the solutions to the system of First

²Henceforth, ‘equilibrium’ will refer to pure strategy equilibrium unless otherwise stated.

Order Condition (FOC) equations. (This system of equations is obtained by specifying that all firms' marginal profit values equal zero.) Indeed, as elaborated on in the next section, there are many elementary price competition models in which either *no* or a *multiplicity* of Nash equilibrium exist.

Consider, for example, the seminal paper by Berry et al. (1995) studying market shares in the United States automobile industry which introduced, at least in the empirical industrial organization literature, a new estimation methodology to circumvent the problem that prices, as explanatory variables of sales volumes, are typically endogenously determined. The paper postulates a MMNL model for the industry. One of the empirical methods developed in the paper is based on estimating the model parameters as those under which the observed price vector satisfies the FOC equations. The authors acknowledge (in their footnote 12) that it is unclear whether their model possesses an equilibrium, let alone a unique equilibrium. Even if these questions can be answered in the affirmative, so that the observed price vector can be viewed as the unique price equilibrium, it is unclear whether it is necessarily identified by the FOC equations which the estimation method relies on³.

In a more recent example, Thomadsen (2005b) pointed out that in many empirical studies the distance between the consumer and each of the competing product outlets or service providers is naturally and essentially added to the specification of the utility value. (Examples following this practice include Manuszak (2000), Dube et al. (2002), Bradlow et al. (2005), Thomadsen (2005a), Davis (2006) and Chapter 2 of this dissertation.) Distance attributes depend jointly on the firm and the consumer. Such geography-dependent utility functions can be cast as special cases of the general model in Caplin and Nalebuff

³In their footnote 12, Berry et al. (1995) wrote “We assume that a Nash equilibrium to this pricing game exists, and that the equilibrium prices are in the interior of the firms' strategy sets (the positive orthant). While Caplin and Nalebuff (1991) provide a set of conditions for the existence of equilibrium for related models of single product firms, their theorems do not easily generalize to the multi-product case. However, we are able to check numerically whether our final estimates are consistent with the existence of an equilibrium. Note that none of the properties of the estimates require uniqueness of equilibrium, although without uniqueness it is not clear how to use our estimates to examine the effects of policy and environmental changes.”. We explain several of the reasons why the conditions in Caplin and Nalebuff (1991) fail to apply to the BLP model, beyond its multi-product feature in Section 3.5.1 footnote 10.

(1991) the most frequently employed foundation for the existence of an equilibrium. However, Thomadsen (2005b) points out that the conditions in Caplin and Nalebuff (1991) that guarantee the existence of an equilibrium do not apply to such specifications, except for very restrictive geographical distributions of the (potential) consumer base. Similar difficulties in the application of the Caplin-Nalebuff existence conditions arise when the utility functions involve other attributes that depend jointly on the firm/customer type combination, for example brand loyalty characteristics; see Section 3.

We identify a simple and very broadly satisfied condition under which a Nash equilibrium exists and that the set of Nash equilibria coincides with the solutions of the system of FOC equations, a property of essential importance to empirical studies. This condition specifies that in every market segment, each firm captures less than 50% of the *potential* customer population when pricing at a level which, under the condition, can be shown to be an upper bound for a rational price choice for the firm irrespective of the prices chosen by its competitors. Moreover, we show that under a somewhat stronger, but still broadly satisfied version of the above condition, a *unique* equilibrium exists.

Finally, while the above results characterize the equilibrium behavior for all but heavily concentrated markets we complete the picture, giving a condition for the existence of a Nash equilibrium, indeed a *unique* Nash equilibrium, for markets with an *arbitrary* degree of concentration: The condition specifies that, the maximum feasible price vector falls below a given upper bound. In other words, to guarantee that a market with an arbitrary degree of concentration has a (unique) Nash equilibrium, sufficiently tight exogenous price limits must prevail while no such limits are needed when (one of) the above market concentration condition(s) applies. Another important distinction is that under the price limit condition, the equilibrium may reside at the boundary of the feasible price region and therefore fails to satisfy the FOC equations. A counterexample shows that if neither a very high level of market concentration can be excluded, nor the feasible price region sufficiently confined, *no* Nash equilibrium may exist for the price competition model. We also discuss the implications of these results for econometricians

both in settings where a specific price vector is *observed* and *assumed* to be the (or an) equilibrium, and those where neither the model parameters nor a price equilibrium is observed.

Our class of MMNL models generalizes the class of models treated in the seminal paper by Caplin and Nalebuff (1991), itself a generalization of many existing models in the industrial organization literature. In particular, along with a similar utility measure for the outside option, our MMNL model is based on postulating a utility function for each product and market segment which consists of three parts: the first component is an arbitrary function of the product's non-price attributes and the non-income related customer characteristics in the given market segment. The second term captures the joint impact of the product's price and the customer's income level via a general product dependent price-income sensitivity function of both, merely assumed to be concave decreasing in the price variable. The third and final term denotes a random utility component with an extreme value distribution as in standard Multinomial Logit models. Our structure generalizes that in Caplin and Nalebuff (1991) in two ways: First, Caplin and Nalebuff specify the first term in the utility functions as a weighted average of the non-price related product attributes, with each customer type or market segment characterized by a unique vector of weights. Second, their price-income sensitivity function is specified as a concave function of the difference of the customer's income and the product's price, as opposed to our general function of income and price. Perhaps most importantly, Caplin and Nalebuff require that the distribution of population sizes across the different customer types satisfies specific (ρ -concavity) properties which are violated in many applications. We impose *no* restrictions on this distribution.

As in Caplin and Nalebuff (1991), we start with the assumption that each firm sells a single product and that all customers share the same income level. Settings with an arbitrary income distribution and those where the firms offer an arbitrary number of products are covered in our Extensions Section 3.5. (As far as the former is concerned, we show that our results continue to apply as long as the price-income sensitivity functions

are separable into an income and a price-dependent function.)

Characterization of the equilibrium behavior in price competition models with MMNL demand functions has remained a formidable challenge because the firms' profit functions fail, in general, to have any of the standard structural properties under which the existence of an equilibrium can be established. For example, the profit functions fail to be quasi-concave. (When firms offer multiple products, this quasi-concavity property is absent, even in a *pure* rather than a *mixed* MNL model, as shown by Hanson and Martin (1996), exhibiting a counterexample in a 3-product monopolist model with logit demand functions.)

Our approach is to (i) identify a compact region in the feasible price space on which the profit functions *are* quasi-concave in the firm's own price(s), or one in which they possess the so-called single point crossing property, discussed in Section 3.3; this guarantees the existence of an equilibrium in the restricted price region. We then establish that (ii) the equilibria identified with respect to the restricted region continue to be equilibria in the full price region and (iii) that no equilibria exist outside the identified restricted price region.

1.5 The impact of horizontal mergers and acquisitions in price competition models

Fundamental to our understanding of competitive dynamics in an oligopolistic industry is the question of what impact mergers and acquisitions have on key equilibrium performance measures. These include the aggregate profits of the merging firms, those of the other firms in the industry (hereafter referred to as the "remaining firms"), equilibrium prices, and consumer welfare.

Early strategy works, for example Steiner (1975), postulate that mergers should increase aggregate profits of the merging firms, even in the absence of any cost efficiencies resulting from economies of scope or scale. It was also conjectured that horizontal merg-

ers should result in an increase in equilibrium prices, for *all* of the products offered by the industry. This was assumed, for example, in the classical paper by Williamson (1968). However, early attempts to substantiate these conjectures on the basis of industrial organization models failed. For example, Szidarovszky and Yakowitz (1982), Salant et al. (1983), and Davidson and Deneckere (1984) all concluded from their analyses that aggregate profits of merging firms actually *decline*, unless accompanied with significant cost efficiencies due to synergies or economies of scope. At first sight, this appears counterintuitive, since the merged firm always has the option to maintain the (quantity) decisions pertaining to the pre-merger equilibrium and improve on these to achieve higher aggregate profits. However, the dynamics in the competition models analyzed by the above authors are such that the new post-merger equilibrium is associated with *lower* aggregate profits.

A seminal step toward resolving this enigma was provided by Deneckere and Davidson (1985). These authors explained that the counterintuitive findings in the prior literature were the result of analyzing the question in the context of Cournot competition models, in which firms select sales quantities or targets as opposed to prices. The authors proceeded to show that, under price (Bertrand) competition, the anticipated effects can be demonstrated: In the absence of cost efficiencies resulting from a merger, aggregate profits of the merging firms increase as do equilibrium prices. Even the equilibrium profits of the *remaining* firms in the industry increase, while the consumer ends up holding the bag, *i.e.*, consumer welfare declines. Their analysis is based on a model with completely symmetric firms and linear demand and cost functions. This model was first proposed by Shubik and Levitan (1980). In their appendix, Deneckere and Davidson extend the results to competition models with non-linear demand functions satisfying five assumptions, the most important of which is that the industry is *symmetrically* differentiated, *i.e.*, all firms share the same constant marginal procurement cost rate and the demand for any pair of products is identical when both its own price and those of the competitors are the same.

In this essay, we establish the above conjectures for general *price* competition models with general non-linear demand and cost functions as long as the models are supermodular, with two additional structural conditions: (i) each firm's profit function is strictly quasi-concave in its own price(s), and (ii) markets are competitive, *i.e.*, in the pre-merger industry, each firm's profits increase when any of his competitors increases his price, unilaterally. In particular, we show that in the absence of cost synergies, both the component-wise smallest and largest post-merger price equilibrium are larger than their pre-merger counterparts, implying reduced consumer welfare. (The existence of a component-wise smallest and largest equilibrium follows from the fact that both the pre-merger and post-merger competition models are supermodular.) In addition, the post-merger equilibrium profit of the merged firm exceeds the pre-merger aggregate equilibrium profits of the merging firms. Perhaps most surprisingly, even the equilibrium profits of the remaining firms increase. (In the case of multiple equilibria, all of these comparison results pertain both to the largest and smallest equilibrium.) These results have been conjectured to hold for general supermodular price competition models, see for example the influential survey chapters by Whinston (2006, 2007).

We establish our results, at first, for settings where each firm in the industry offers a single product; we then generalize them to industries with general multi-product firms. We also derive conditions under which cost synergies, by themselves, result in *lower* equilibrium prices and discuss how the combined effect of increased market concentration and cost synergies can be assessed efficiently.

The results in this essay are of interest to individual firms competing in an oligopoly and either considering a merger or wishing to evaluate the consequences of a potential merger by others. They are also of interest to government agencies such as the Antitrust Division of the Department of Justice (DOJ) and the Federal Trade Commission (FTC). As discussed in detail, in Section 5, these agencies are charged with the task of evaluating all proposed mergers and acquisitions of firms with a capital value (or so-called "transaction value") exceeding an annually adjusted threshold, more than a thousand

proposals annually. The DOJ and FTC initially focused on relatively simple market concentration measures such as the (post-merger) Herfindahl-Hirschman Index (HHI) to assess the potential for a “substantial lessening of competition in the industry,” the standard prescribed by the Clayton and Hart-Scott-Rodino Act. However, economists have pointed out, repeatedly, that, in particular in the case of differentiated products, these market concentration measures are poor surrogates for the actual changes in equilibrium prices, consumer welfare, and firm profits. As a consequence, the DOJ and FTC have, since the early nineties, conducted *merger simulations* when mergers in differentiated product markets are proposed. Here, an oligopoly model for the industry is estimated and a pre- and post-merger equilibrium computed, see Werden and Froeb (1994, 1996), and Werden (1997). The merger simulation approach was also widely adopted by other economists (, outside of the DOJ and FTC,) for example Baker and Bresnahan (1985), Berry and Pakes (1993), Hausman et al. (1994), Hausman and Leonard (1999), Nevo (2000a), Dube (2005), and Thomadsen (2005a).

The results in our essay provide important support for these merger simulations. In the above described class of supermodular price competition models, we are able to show that, if the pre-merger industry has a unique equilibrium, the *post-merger* smallest equilibrium can be computed by applying a simple *tatônnement* scheme with the *pre-merger* price vector as the starting point. Moreover, this scheme generates an *increasing* sequence of price vectors which converges to the post-merger equilibrium. (The post-merger smallest and largest equilibrium can also be computed by applying a *tatônnement* scheme, with the smallest and largest feasible price vector as its starting point, respectively; these schemes are monotone as well.)

Chapter 2

How much is a reduction of your customers' wait worth? An empirical study of the fast-food drive-thru industry based on structural estimation methods.

We refer to Section 1.3 for an introduction and summary of the models and estimation techniques used in this chapter. Recall the following series of research questions:

1. Does the customer's waiting time at the fast-food drive-thru lane represent a significant determinant of consumer choices and resulting market shares, as believed by the industry and operations management literature alike?
2. When comparing the demand sensitivity to waiting time and price differentials, is the implied value of time of the same order of magnitude as the average wage or income earned per hour? If not, is it of a larger or smaller order of magnitude?
3. Can the above stated industry maxim be substantiated by empirical estimates? In

particular, when taking into account that the various outlets are likely to adjust their prices in response to reduced waiting time standards, adopting a new price equilibrium, does the maxim hold?

4. Furthermore, this maxim expresses the belief that a given waiting time reduction is equally valuable for all chains in terms of resulting increases in market shares and sales. Plausible consumer choice models may imply that these benefits, in fact, vary with the initial market shares of the chains, either in a robustly predictable way or one that depends on the specific parameter estimates. Either way, it is important to understand to what extent these benefits vary with the size of the chain and other chain attributes. In addition, do the increases in market shares accrue primarily from customers switching between chains or from the acquisition of new customers to the fast-food market?

This chapter is organized as follows: Section 2.1 provides a review of the relevant literature. Section 2.2 develops our consumer choice and competition model. Section 2.3 describes the many data sources employed and the approach we adopted to collect the data. Section 2.4 is devoted to a description of the GMM estimation technique as applied to our model. Section 2.5 describes the estimation results and counterfactual studies. Finally, Section 2.6 completes the essay with a discussion of possible extensions.

2.1 Literature Review

The literature on competition in service industries dates back to the late 1970s. Luski (1976) and Levhari and Luski (1978) were the first to model competition between service providers. The latter paper addresses a *duopoly* where each of the firms acts as an M/M/1 system, with exogenous and identical service rates. In this model, customers select their service provider strictly on the basis of the *full price*, defined as the direct price plus the expected steady state waiting time multiplied with the waiting time cost rate. The question whether a price equilibrium exists in this model remained an open question,

until it was recently resolved in the affirmative by Chen and Wan (2003), albeit for the basic model with a uniform cost rate. These authors show, however, that the Nash equilibrium may fail to be unique. More recent variants of the Levhari and Luski models include Li and Lee (1994), Armony and Haviv (2001) and Wang and Olsen (2008).

Cachon and Harker (2002) and So (2000) analyzed the first models in which customers consider criteria beyond the lowest full price when choosing a service provider (*e.g.*, quality). Both confined themselves, again, to M/M/1 service providers. Allon and Federgruen (2007, 2006) treat the price and waiting time standard as completely independent firm attributes which different customers may trade off in different ways. Nevertheless, Allon and Federgruen (2007) confines itself to systems of demand rates that are linear in the prices and to M/M/1 service providers, while Allon and Federgruen (2006) studies more general demand models, such as attraction models, and allows for more general queueing facilities. We refer to these two papers as well as Hassin and Haviv (2003) for additional references of service competition models. Another stream of papers, in particular Hall and Porteus (2000) and Gans (2002), models the competition between services providers selecting a distribution for the (non-congestion related) quality of service, based on specific consumer choice models. Gans et al. (2007) describes an empirical study to test these models with laboratory experiments, as opposed to econometric field studies.

Many service processes are provided via call centers. Here, customers are known to be very sensitive to their waiting times, which is why such centers are designed and staffed to meet specific service level agreements (SLAs), see Hasija et al. (2007) for a recent survey of such agreements. However, virtually all planning models in the vast literature related to call centers assume that demand processes are exogenous inputs, or, at best, dependent on service charges. We refer to Gans et al. (2003) for an excellent tutorial on call center management. When describing future challenges in this area, the authors emphasize “a better understanding of customer behavior” (§7.3) and the need to model and estimate “multiple levels of equilibria”. Beyond these levels, we suggest the desirability of models incorporating the competitive effect of service levels provided by

the call centers of competing service providers.

The above reviewed literature is based on the observation that firms compete along the service level dimension as well as anecdotal and empirical evidence that customers value waiting time when making decisions regarding their preferred service provider. One example in the fast-food industry is F and Vollmann (1990) examining consumer choice criteria with a sample of 723 customers who were asked to rank their satisfaction with various aspects of the delivery process. The authors established that the satisfaction scores were highly correlated with the experienced waiting time. Time of day, store location, and whether the customer was at work or school were important factors determining the strength of the waiting time sensitivity. Day of week and participation activities other than work around the meal (*e.g.*, shopping, visiting friends) were not significant.

This empirical study complements the earlier quoted plethora of trade literature documenting the centrality of waiting times in this industry. Our study complements the theoretical literature on competition models by estimating the parameters used and assumed by these models. There have been very few other works which attempt to estimate these parameters. The two empirical studies investigating questions closest to our own are Deacon and Sonstelie (1985) and Png and Reitman (1994). The former appears to have been the first to estimate the impact price differentials and average waiting times have on sales volumes; however, the setting is one where prices are exogenously determined by government price controls, avoiding the endogeneity challenge inherent in most studies including our own. The selected estimation method is based on a probit model, applicable in the case of two firms only. The model does not apply to settings with price selecting firms or those where customer choices depend on factors other than the full price. Png and Reitman (1994) describes the peak hour sales in a market of 1501 gas stations in four Massachusetts counties via a system of demand equations. These equations are not derived from an underlying consumer choice model as, for example, in Deacon and Sonstelie (1985) or our essay. In the absence of actual observations of the waiting times or the peak hour sales volumes, the authors specify the logarithm of a firm's (peak

hour) sales volume as a linear function of the logarithm of the firm's own price, that of the average of the prices of the "nearby" stations, a vector of station attributes, and a proxy for the average waiting time. The latter is postulated as the ratio of the peak hour sales volume and a predetermined power of the number of pumps, while the peak hour sales volume is assumed to be given by the aggregate weekly sales divided by a given power of the number of operating hours. The coefficients in this model are assumed to be homogeneous constants which are estimated via a Least Squares Regression method. The authors address the problem of the explanatory prices and capacity variables being endogenous to the system, by the use of a two stage least squares method, invoking instrumental variables claimed to be uncorrelated with the error terms.

The transportation research literature has often specified the demand for alternative transportation modes as arising from a mixed multinomial logit model with prices and travel times as explanatory variables, similar to our essay. However, the maximum likelihood estimations typically employed are challenged by the above mentioned endogeneity problems; see Hess et al. (2005) for a recent example, estimating the implicit cost associated with travel time to be in excess of \$100/hr. Finally, an earlier economics paper by De Vany et al. (1983) estimated the effect waiting times have on patient volumes in dentist offices; ignoring the impact of competition and employing OLS, the authors obtain a statistically significant positive value for the waiting time sensitivity, perhaps because in their setting demand is relatively inelastic with respect to waiting times while capacity is inflexible. In contrast, we estimate the impact of waiting times, prices, geographic dispersion, chain attributes, and demographic factors on demand. Our approach follows the work by Bresnahan (1987), Berry (1994), Berry et al. (1995). These authors demonstrate how to estimate consumer choice models and cost structures in oligopolistic markets with differentiated goods using aggregate consumer level data and structural models of competition. The general approach posits a distribution of consumer preferences for the competing goods based on their attributes. The preferences are aggregated into a market level demand system that, when combined with assumptions on cost and

price-setting behavior, allows one to estimate the parameters.

In the above papers, market shares are observed. Feenstra and Levinsohn (1995) were the first to demonstrate how this estimation framework can be used in the absence of quantity data. As mentioned in the introduction, we face the same challenge since in the fast-food industry, sales data are not reported and are treated as strategic and proprietary information.

More recent work by Davis (2006) and Thomadsen (2005a) incorporated geography in the BLP framework. Thomadsen (2005a) studies the impact of ownership structure on prices in the fast-food industry. The author uses this method to establish that the impact of mergers in such an industry can be large, but the impact of mergers decreases as the merging outlets are further apart. Our consumer choice model adds the waiting time measure to the set of outlet and chain-dependent explanatory variables employed in Thomadsen (2005a) but does not include ownership structure. For reasons explained in the model section, we incorporate other chain attributes that act as indicators of perceived quality, instead of the chain dummy variables employed in Thomadsen (2005a)

Our study is also related to the recent empirical literature in operations management. To our knowledge, most of this literature focuses on consumer products rather than services. See Olivares and Cachon (2009) and Musalem et al. (2009) for surveys of this literature. Notable exceptions include Olivares et al. (2008) and Diwas and Terwiesch (2009a,b) which focus on the health care industry.

2.2 The Model

In this section we develop the competition model representing the competitive interdependencies and interactions among the outlets in our geographic region. The model combines two sub-models: (a) a *consumer choice* model which determines how many of the region's residents and commuters choose, for any given lunch or dinner meal, to go to a fast-food establishment and, among those, how many select a specific outlet, and (b) a model to

represent the *variable cost* structure of the different outlets as a function of their sales volume and service level (*i.e.*, its waiting time standard). Combining the two sub-models permits us to derive the outlets' profit functions. As explained, in the fast-food industry, waiting time standards are selected by the chains. However, price decisions are relegated to the independent outlets, if for no other reason than to avoid illegal forms of price fixing. As franchising became popular in the sixties, the US courts began to limit the types of pricing restrictions chains can impose on their franchises. Only maximum retail prices have become legal, under certain conditions, based on the Supreme Court ruling in *State Oil vs. Khan* (1997). (In the prior thirty years, even maximum price levels had been illegal, see *Albrecht vs. Herald* (1968).)

When collecting data, we called the *chains* for price recommendations that they may give to their franchisees. Consistent with the Supreme Court rulings, we were told that the practice of *suggesting* prices to the outlets is illegal. Indeed, we have observed significant price differences among outlets of the same chain, see Table 2.2 in Section 2.3. Thus, waiting time standards are selected centrally by the chains but prices are chosen by the individual outlets. We can, therefore, assume that the prices observed in the market represent the equilibrium in a price competition model, under given waiting time standards specified by the chains operating in the selected geographical region. We show that this price equilibrium model has an equilibrium which is a solution of a non-linear system of equations. It is this system of equations which permits us to estimate the parameters that describe the consumer choice model and associated demand functions, as well as the parameters in the cost structure.

2.2.1 The Consumer Choice Model

Demand for fast-food meals at each outlet is specified by a discrete choice model. Consumers choose either to purchase a specific lunch or dinner meal from one of the fast-food outlets or to consume an outside good. Consumers assign a utility value to each outlet, as well as to the no-purchase option, specified as a linear function of the price, waiting

time, distance, chain identity of the outlet, and various demographic factors including the consumer's gender, race, age bracket and occupational status. Each of these utility equations contains an additional random noise term. It is natural to assume that customers make their choices in two stages: (i) they first decide whether to dine at a fast-food outlet as opposed to alternatives, such as eating at home or a different type of restaurant, and (ii) assuming the first question is answered in the affirmative, which of the various outlets in the region to patronize. We model the two stage choice process by assuming that the (potential) customer attributes a utility value to the no-purchase option which depends on her demographic attributes. The customer also assigns a utility value to each of the outlets in the region that depends on attributes of both the outlet and the chain it belongs to. The customer purchases a meal at one of the fast-food outlets if and only if the highest of the outlets' utility values is in excess of that of the no-purchase option; in this case the meal is consumed at the outlet with the highest utility value.

Formally, the conditional indirect utility of consumer i from fast-food outlet j is specified as follows:

$$U_{i,j} = \beta + X'_{k(j)}\zeta - \delta D_{ij} - \gamma P_j - \alpha W_{k(j)} + \eta_{ij}, \quad (2.1)$$

where $k(j)$ denotes the chain k to which outlet j belongs, $X_{k(j)}$ is a column vector of observed properties of the chain to which outlet j belongs, D_{ij} is the distance between consumer i and outlet j , P_j is the price of a (standard) meal at outlet j , $W_{k(j)}$ is the waiting time standard (= average steady-state waiting time in system) of chain $k(j)$ associated with outlet j , η_{ij} is the portion of the utility of individual i at outlet j which is unobserved by the modeler, and $(\alpha, \beta, \gamma, \delta, \zeta)$ represents a parameter string with ζ an array of the same dimension as X .

Our estimation of waiting time sensitivity is based on three assumptions: (i) consumers make purchasing decisions based on the steady state waiting time distribution at an outlet, not on the prevailing queue length (the only varying observable characteristic)

at the time of arrival (ii) consumers characterize the steady state waiting time distribution by its average and (iii) all outlets belonging to the same chain share the same waiting time distribution. The first assumption is based on our understanding that in most cases consumers make their selection before traveling to any specific outlet based on the “average” experience. The second assumption is not inherent to our approach and could easily be replaced by other characteristics of the steady state waiting time distribution such as the 95th percentile. As for the final assumption, we explained earlier that in the fast-food industry the chains select and announce, to their franchisees, a common waiting time standard for all of their outlets, implemented with tight process prescription and control. Indeed, chains achieve remarkably uniform average waiting times at their franchises¹. Industry trade organizations such as Quick Service Restaurant (QSR), publicize yearly surveys of the average waiting time experienced at the various fast-food chains. Outlet specific samples in our waiting time data set are too small to make our own empirical verifications of assumption (iii). Furthermore, although there are empirical papers which have individual waiting time observations for a single outlet or chain in a service industry, such a data set is extremely difficult to obtain for industry-wide studies.

Since all outlets belonging to the same chain share the same waiting time standard, it is important to include in the individual utility functions (2.1) any other observable chain-wide attributes which (i) are correlated with the waiting time standard and (ii) may plausibly serve as a quality indicator for the chain. The only such attributes we were able to identify are the density of the chain network (, as measured by the number of outlets,) in the county and the intensity of the chain’s advertising efforts, as quantified by its aggregate national advertising spending. We do not use chain identity indicator variables, as is frequently done, because its inclusion among the explanatory variables in (2.1) results in an identification problem ².

¹In 2006 Jim Hyatt, Burger King’s VP and COO, stated that the average waiting time at its 6,900 domestic franchises cover a very narrow range from 165 to 170 seconds, while this average waiting time was reduced by 22 seconds, compared to the previous year. (See Jargon (2006))

²Normalizing the coefficient of one of the chain indicator variables removes collinearity in the utility function; however market shares remain invariant to a common additive shift in the coefficients of the indicator variables and that of the waiting time standard.

A recent QSR-commissioned study, Frank N Magid Associates (2010), reports on a survey among 1,120 drive-thru customers, in which each respondent was asked to list which of ten attributes makes the drive-thru experience the best. The most frequently cited attribute was “speed - wait time of drive-thru service” (22% of respondents), followed by “price” and “order accuracy” (12% each). Location was listed almost as frequently as “price” (and order accuracy), *i.e.*, by 11% of respondents. Our consumer choice model represents all of these attributes with the exception of “order accuracy”; the latter should be included in future studies, in particular as it may shed light on an optimal balance between “speed of service” and “order accuracy”.

We assume that for every outlet j , the random components of $\{\eta_{ij}\}$ represent non-systematic unobservable variations in the perceived utility of the outlet among potential customers of the same demographic type residing or working in the same location. We therefore assume that the $\{\eta_{ij}\}$ variables are i.i.d. Random utility models of type (2.1) often contain an additional outlet specific component $\xi_j, j = 1, \dots, N$, to address systematic attributes of the firm (outlet), known to the firms and customers but not to the modeler. As argued in Thomadsen (2005a), in the case of the fast-food drive-thru industry, this term may be omitted because other than through price and location, different outlets belonging to the same chain offer close to identical attributes. At the same time, as explained above, all relevant chain specific attributes are captured by the chain variables X in the first term in (2.1)³, along with the waiting time standard W . The indirect utility associated with the no-purchase option is given by

$$U_{i,0} = \beta_0 + M_i\pi + \eta_{i,0}. \quad (2.2)$$

Here, M_i is a row vector specifying the consumer’s age, gender, race, and whether they

³Thomadsen (2005a, p915) states: “not only is the food identical, but the chains also try to make the experiences at each of their outlets identical. For example, their outlets have a uniform appearance, their menu boards look very similar, and their workers wear similar uniforms.” See Anonymous (2006) for one of many industry publications reporting on the same uniformity in the customer experience. “Wendy’s ensures that customers who go to a Wendy’s in Wisconsin will have the same experience as those who visit a store in Shanghai.”

are making the decision as a commuter or resident (i.e., people are allowed to have a different preference for the outside good when they are at work versus at home). If the age distribution is characterized by A age classes, the M_j vector is a binary vector of dimension $(A+2)$: for $l = 1, \dots, A - 1$, $M_{il} = 1$ if consumer i belongs to the l th age bracket and 0 otherwise, similarly, for $l = A, A + 1$, and $A + 2$, $M_{il} = 1(0)$ if the consumer is female (male), African American (white), and a resident (worker), respectively. β_0 and π represent another set of parameters to be estimated and η_{i0} denotes the unknown portion of the utility of individual i for the non-purchase option. Once again, the random components $\{\eta_{i0}\}$ are i.i.d.

We consider a limited number of age brackets. Therefore, there is a finite list of $\{1, \dots, M\}$ of consumer-types, combining age, gender, race and occupational status. In view of the importance of the distances between the consumer and the various outlets, we partition our geographic region into a grid of very small sub-areas $\mathcal{B} = \{1, \dots, B\}$ and assume all consumers residing in a sub-area are located at the sub area's centroid. (In our study, we use tracts, as defined by the U.S. Census, with an average area of 1.2 square miles in Cook county.) Thus, all potential consumers residing in a given sub-area $b \in \mathcal{B}$ and belonging to a given demographic group $m \in M$, share the same *mean* utility value for all outlets and the no-purchase option.

Assuming the distributions of the random noise terms, $\{\eta_{ij} : j = 0, \dots, J\}$, are Gumbel (or doubly exponential) with common scale parameter μ , and assuming every consumer selects the alternative with the highest utility value, this gives rise to the following multinomial logit model in which each outlet's market share for each tract and demographic group is given by the following expression:

$$S_{j,b,m}(P, W, X|\beta, \zeta, \delta, \gamma, \alpha, \pi) = \frac{e^{(\beta + X'_{k(j)}\zeta - D_{bj}\delta - P_j\gamma - W_{k(j)}\alpha)/\mu}}{e^{(\pi M + \beta_0)/\mu} + \sum_{t=1}^J e^{(\beta + X'_{k(t)}\zeta - D_{bt}\delta - P_t\gamma - W_{k(t)}\alpha)/\mu}},$$

$$j = 1, \dots, J; b = 1, \dots, B; m = 1, \dots, M. \quad (2.3)$$

Without loss of generality, we express the utility levels in units such that the scale

parameter $\mu = 1$. Also, the consumer choices only depend on the *relative* ranking of the utility values for the different outlets and the no-purchase option; they are therefore invariant to a common additive shift. This permits us to normalize the intercept β in the utility function (2.1) to $\beta = 0$.

Multiplying the market shares with $h(b, m)$, the number of consumers of demographic group m , residing in or commuting to geographic region b allows us to specify expected aggregate sales in an outlet as a function of the various parameters $\theta \equiv \{\zeta, \delta, \gamma, \alpha, \pi\}$ in the utility equations:

$$Q_j(P, W, X|\zeta, \delta, \gamma, \pi) = \sum_b \sum_m h(b, m) S_{j,b,m}(P, W, X|\zeta, \delta, \gamma, \pi,). \quad (2.4)$$

2.2.2 The Outlets' Cost Structure

When assessing the impact of operational measures, it is important to specify a cost structure which is rigorously substantiated by an adequate operational model. We have selected a structure, in which an outlet's costs, expressed as a function of its expected sales volume, is affine with an intercept that is proportional with the reciprocal of the waiting time standard:

$$C_j(Q_j) = \bar{c}_j Q_j + \bar{d}_j / W_{k(j)} = (c_{k(j)} + \epsilon_j) Q_j + [d_{k(j)} + u_j] / W_{k(j)}, \quad j = 1, \dots, J. \quad (2.5)$$

Here, for every outlet $j = 1, \dots, J$ and chain $k = 1, \dots, K$:

- J_k = the set of outlets belonging to chain k , *i.e.*, $J_k = \{j : k(j) = k\}$
- $c_{k(j)}$ = the average variable food, labor and equipment cost rate per customer for an outlet of chain k ,
- $d_{k(j)}$ = the average variable capacity cost rate for an outlet of chain k ,
- ϵ_j = a noise term, denoting the difference between outlet j 's variable cost rate \bar{c}_j and the norm or average for this chain $c_{k(j)}$,
- u_j = a noise term, denoting the difference between outlet j 's variable capacity cost rate \bar{d}_j and the norm or average for this chain $d_{k(j)}$.

Each outlet's marginal cost rate, as well as the capacity cost rate, is equal to a common chain-specific cost plus a zero-mean, unobserved outlet-specific component. This specification is supported by the franchisers' effort to create a uniform customer experience across their outlets, via standardization of the equipment, as well as the preparation process and food components used at each of its outlets. The unobserved shock to the cost rate comes from outlet specific conditions (*e.g.*, deficiencies in labor productivity, management efficiency, or smaller kitchens creating crowding and reduced efficiency).

The affine cost structure in (2.5) arises in several queueing models which may describe the service process of an outlet. For example, the structure in (2.5) arises in an $M/M/1$ system, where the waiting time standard W denotes the expected total sojourn time in the drive-thru queue and the variable capacity cost is assumed to be proportional with the service rate. More realistically, a fast-food service process could be represented as a Jackson (queueing) network. A food order may travel along a path of service stages, from order taking to the cooking of the hamburgers, assembly of the cooked burgers with the side dish and required drink and back to the drive-thru counter. Allon and Federgruen (2006) have shown that the cost structure in (2.5) applies to a general Jackson network,

assuming the variable capacity costs are proportional with the service rates installed at the various nodes of the network. Alternatively, the service process may be best described as a $GI/GI/s$ system, with an arbitrary renewal arrival process, arbitrary service time distribution and a team of s parallel servers. If the consumer is particularly focused on the *delay* experienced in the drive-thru queue and if W denotes a given fractile of the delay distribution, then the cost structure in (2.5) arises as a close approximation, see Allon and Federgruen (2006). This identity is, in fact, exact, rather than an asymptotically correct approximation when the service time distribution is exponential, i.e. in the case of a $GI/M/s$ system.

We refer to Allon and Federgruen (2006) for additional queueing models resulting in affine cost structures of type (2.5). These authors also show that an even larger set of queueing models give rise to a more complex family of cost functions. Our estimation method, which fits the model parameters to the FOC of the underlying competition model, can be adapted to this more general cost structure, see section 2.2.3 for more discussion. However, Allon and Gurvich (2010) show that approximating a more complex capacity function by an affine function results in only minor discrepancies in the price equilibrium. Thus, disregarding higher order terms does not significantly alter the outcomes of the market.

2.2.3 The Price Competition Model

We are now ready to analyze the price competition model which arises when all waiting time standards have been specified. We assume that every outlet is independently owned. However, our methodology is readily adapted if various outlets are jointly managed by the same franchisee, see below. In view of (2.5),

$$\pi_j(P, W, X, \theta) = (P_j - \bar{c}_j)Q_j(P, W, X|\theta) - \bar{d}_j/W_{k(j)}, \quad j = 1, \dots, J \quad (2.6)$$

denotes firms j 's profit level as a function of all prices charged by the various outlets. Each firm j selects its price within a given range $[\bar{c}_j, p_{j,max}]$. It is a long standing conjecture that a price competition model with a mixed multinomial logit demand function and an affine cost structure has a *unique* interior point equilibrium which is the unique solution of the system of equations given by the First Order Conditions (FOC):

$$Q_j(P, W, X|\theta) + (P_j - c_{k(j)} - \epsilon_j) \frac{\partial Q_j(P, W, X|\theta)}{\partial P_j} = 0, \quad j = 1, \dots, J. \quad (2.7)$$

This conjecture underlies almost all structural estimation methods in models with demand equations of this type. Indeed, the essence of these estimation methods is to find parameter combinations under which the FOC equations (2.7) are satisfied as closely as possible since the competing firms are assumed to have adopted the observed price vector as the (a) Nash equilibrium.

Unfortunately, little was known about whether or when the above conjecture holds, see e.g., Berry et al. (1995)⁴. In Chapter 3 we show that a Nash equilibrium exists and that the set of equilibria corresponds with the set of solutions to (2.7), provided once can ensure that no single firm attains an excessively large share of the market when pricing at a specific level which, under the condition, is shown to be an upper bound for a rational price choice. More specifically the authors introduce the following parameterized condition:

C(μ) Each firm j captures, in each market segment, i.e., each tract/demographic group combination (b,m) less than a fraction of the market when pricing at the level $\bar{p}_j = \bar{c}_j + 1/(1 - \mu)\gamma, j = 1, \dots, N$.

⁴For example, in their classical paper Berry et al. (1995) note: "We assume that a Nash equilibrium to this pricing game exists, and that the equilibrium prices are in the interior of the firms' strategy sets (the positive orthant). While Caplin and Nalebuff (1991) provide a set of conditions for the existence of an equilibrium for related models of single product firms, their theorems do not easily generalize to the multi product case. However, we are able to check numerically whether our final estimates are consistent with the existence of an equilibrium. Note that none of the properties of the estimates require uniqueness of equilibrium, although without uniqueness it is not clear how to use our estimates to examine the effects of policy and environmental changes." Indeed in Chapter 3 we show that an equilibrium may fail to exist in the general model without *any* parameter restrictions.

This condition is easily satisfied by checking for every firm j and market segment (b, m) that:

$$\frac{e^{[X'_{k(j)}\zeta - D_{b,j}\delta - \bar{p}_j\gamma - W_{k(j)}\alpha]}}{e^{(\pi M - \beta_0)} + e^{[X'_{k(j)}\zeta - D_{b,j}\delta - \bar{p}_j\gamma - W_{k(j)}\alpha]} + \sum_{t \neq j} e^{[X'_{k(t)}\zeta - D_{b,t}\delta - p_t^{max}\gamma - W_{k(t)}\alpha]}} \leq \mu. \quad (2.8)$$

A firm's market share is, of course, maximized when its competitors adopt their maximum price levels. (See Chapter 3 for sufficient conditions of $C(\mu)$ that are independent of the choice of the $\{p_j^{max}\}$ -values.) Under condition $C(\mu)$, Chapter 3 Lemma 3.4.1 shows, in fact, that, for all $j = 1, \dots, N$ the price level $\bar{p}_j = \bar{c}_j + 1/(1 - \mu)\gamma$ arises as an upper bound for firm j 's price level. Of particular importance are conditions $C(1/2)$ and $C(1/3)$ which ensure that no outlet captures more than 50% or 33% of the market, respectively (under the above mentioned price levels). Conditions $C(1/2)$ and $C(1/3)$ are easily satisfied in most industries and the fast-food industry, in particular. (See §2.5 for a verification). Theorems 3.4.2 and 3.4.3 in Chapter 3, applied to our model, imply:

- Theorem 2.2.1 (a)** *Under condition $C(1/2)$, the price competition model has an equilibrium which is an interior point of the price cube $X_{j=1}^N[\bar{c}_j, \bar{c}_j + 2/\gamma]$*
- (b)** *Under condition $C(1/2)$ every Nash equilibrium is a solution to the FOC (2.7), and, vice, versa, every solution to (2.7) is a Nash equilibrium.*
- (c)** *Under condition $C(1/3)$, the price competition model has a unique equilibrium which is an interior point of the price cube $X_{j=1}^N[\bar{c}_j, \bar{c}_j + 1.5/\gamma]$; this equilibrium is a solution to the FOC (2.7).*

Given the affine cost structure (2.5) as substantiated in section 2.2.2 on the basis of underlying queueing models, the waiting time measures impact only via the demand model, *i.e.*, via the marginal price sensitivities $\{\partial Q_j(P, W, X|\theta)\partial p_j : j = 1, \dots, J\}$, see (3), (4). As discussed in section 2.2.2 under more general queueing models, the marginal costs become a function of the waiting time measures as well, in which case the vector W impacts the structure of the FOC equations both via the demand and the supply model.

2.3 Data

We have studied the hamburger drive-thru fast-food industry in Cook County, Illinois. We have chosen this industry both because of the availability of data and because this is an industry that has historically placed a premium on competing via its service levels. The QSR magazine 2007 Drive-Thru Time Study notes that in 2007 all quick-service chains made major efforts to improve speed-of-service in their drive-thrus, see Nuckolls (2007). Examples of new technology improving speed-of-service include timer systems that allow in-store managers as well as regional and national offices to monitor waiting times at outlets and the outsourcing of drive-thru order taking. The 2008 QSR Drive-Thru study reports that this trend is continuing, with the fastest chain, Wendy's, shaving off an additional seven seconds from the average waiting time in the previous year. There is a plethora of anecdotal evidence that the industry is reacting to consumer expectations regarding waiting times. For example, the same 2007 QSR Drive-Thru study reported that 70% of surveyed customers said speed is an important factor in the drive-thru experience.

We believe that for our purposes, Cook County is representative of all urban/suburban counties in the country. The propensity to consume hamburger fast-food meals as opposed to alternatives may differ on different parts of the country. However, we see no reason why within urban/suburban areas, the relative trade offs between price, waiting times, geography, and other chain attributes among those interested in a fast-food meal would vary significantly.

We use as our data set, all fast-food outlets belonging to chains selling hamburgers and with a presence of more than five outlets in the county. We consider only outlets with drive-thru windows because outlets without drive-thru windows tend to be located in places such as malls and airports where consumers are facing a different set of considerations. This results in a total of 388 outlets belonging to McDonald's (173), Burger King (92), Wendy's (62), White Castle (42), Dairy Queen (10), and Steak 'n Shake (9).

Our consumer choice model does not differentiate among the various items on the outlets' menus. Our model choice is based on the assumption that consumers when trading off different outlets (, as well as the no purchase option,) consider a *general* price assessment about each restaurant rather than a complete comparison of all fully itemized menus, information they are unlikely to possess let alone be able to aggregate in a comprehensive trade-off among alternative outlets. We have demonstrated that this trade-off requires the consideration of waiting time, geography and chain attributes along with the general price level. As a proxy for the general price level of a hamburger drive-thru restaurant we have computed the price of a "standard meal" consisting of: the franchise's signature burger, a small fries order, and a small soft-drink. (We gathered prices by calling each location.) The type of burger selected was standardized by weight and in the case of White Castle, which sells small burgers, we use the price for four sliders. As noted in the introduction to Section 2.2, we have observed very significant price differences among outlets belonging to the same chain, with the most expensive McDonald's or Burger King outlet being about 50% more expensive than the cheapest outlet of that chain in the county, see Table 2.2.

Recall that, in the absence of data on joint ownership among franchises, we assume each outlet is owned independently, or, at least, operating as an independent profit center⁵. The only exception among the six chains in our study is White Castle all of whose outlets are owned by the chain. Since all of the outlets sell similar products, i.e. hamburgers, and use the same means of service, i.e. drive thru facilities, we assume that all of these outlets compete with each other as alternative producers in the same market.

We use two chain attributes, in addition to the chains' waiting time standard, as potential indicators of the consumer's perception of chain quality: (1) the density of the chain network measured by the number of outlets in Cook County, and (2) the "intensity"

⁵Kalnins and Lafontaine (2004) have documented that in the state of Texas, there is a limited amount of joint ownership among franchisees. The median number of outlets that a given outlet shares ownership with is 5 for McDonald's and 7 for Burger King. In addition, the authors document that the larger chains "allow multiple unit ownership to arise only via purchases of individual units. In this case, franchisees must satisfy the chain that they can efficiently run their current set of units before being granted the 'right to expand'."

of the chain’s advertising efforts, as measured by the national advertising spend. The advertising spend was taken from the AdSpender database using the “Total Ad Spend” figure for the “General Promotion” category for each chain in 2005. All media outlets were included (e.g., television, US Internet, radio, and print). As discussed, all chains select and strive for a common waiting time standard among all of their outlets. In addition, customers often frequent more than a single outlet of a chain and expect to experience a similar service level, irrespective of the specific outlet they visit. We have selected the average steady-state waiting time, defined as the time spent in the drive-thru queue plus the service time, as the waiting time standard used in the consumer choice model of Subsection 2.2.1. To arrive at the average waiting time standards for the different chains, we have employed the QSR magazine’s 2005 Drive-Thru Time Study Database, which we purchased from QSR. The database contains, for a national sample of outlets, two random observations at lunch and at dinner time. We obtained each chain’s average waiting time by averaging the recorded observations over all outlets that belong to the relevant chains, nationwide. These national average waiting times vary significantly across chains, with the worst performer being close to twice as slow as the best performer, see Table 2.1 below. The chain-wide waiting time standards of the six chains in our study have a mean of 225.92 seconds, a standard deviation of 38.21, and a range of [173.34,269.45]. Using a two-sample t-test assuming unequal variances on all

Table 2.1: Average Waiting Time as Determined from 2005 QSR Drive-Thru Study

Chain	Mean Wait (sec)	# Outlets	Nat’l Advertising Spend (’05)
WENDY’S	173.34	62	\$360MM
BURGER KING	192.29	92	\$265MM
MCDONALD’S	224.27	173	\$638MM
DAIRY QUEEN	231.85	10	\$56MM
STEAK’N SHAKE	264.3	9	\$12.5MM
WHITE CASTLE	269.45	42	\$12.5MM

the national waiting time observations we have verified that waiting time observations for

the six different chains were indeed drawn from different distributions with the exception of two pairs: McDonald's & Dairy Queen and White Castle & Steak 'n Shake. (Note from Table 2.1, that the mean waiting times are nearly identical within each of these two pairs as well). This confirms that different chains offer systematically different waiting time experiences to the consumer. The results of this analysis can be seen in Table A.1 in Appendix A.1.

Demographic and geographic information was gathered with a very fine granularity at the so-called tract level. Tracts are geographic areas defined by the U.S. Census Bureau to contain 2,500 to 8,000 people. Cook County consists of 1,343 tracts with an average area of only 1.2 square miles. In urban areas a tract corresponds with a few city blocks. The next smallest geographic area recognized by the U.S. Census, the so-called block groups, are so small that some demographic data, such as race, cannot be reported without revealing the exact household being discussed and hence are not available to the public. We have considered the following age brackets: 0-9, 10-19, 20-39, 40-59, and 60+. We considered African American and white consumers only because these are the racial groups for which we had the necessary data for employing the macro moments discussed in Section 6. As mentioned in Section 2.2, consumers are also differentiated based on whether they are at work or home. As far as the residents in a tract are concerned, we collected the number of people of each age bracket, race, and gender combination from the 2000 U.S. Census data. As to the population working in each of the tracts, the Bureau of Transportation Statistics reports the number of people commuting between every tract pair. We aggregated the flow of workers into each tract in Cook County from any originating tract (whether or not the originating tract was within Cook County). Unlike the U.S. Census data, the Bureau of Transportation Statistics data are not broken down by age, gender and race combinations, so, we estimated the population size for each triplet combination by assuming the three demographic attributes are independent. If a person lives and works in Cook County they are counted as two consumers. We do this because such consumers have the potential to consume one meal (*e.g.*, lunch) while at

work and another meal (*e.g.* dinner) while at home. Distinguishing between commuters and residents, two genders and two racial groups, as well as among five age brackets, we have thus divided the population into 40 different demographic groups. The distance from the consumer to each outlet is calculated as the distance between the restaurant and the centroid of the tract in which the consumer is located. To compute these distances we employed the ArcView Geographic Information System modeling and mapping software.

In addition to the independent variables, we collected data for the so-called instruments used in the estimation method. As discussed in the next section, these are outlet specific variables that we argue are correlated with one or more of the independent variables but not with the noise terms $\{\epsilon_j : j = 1, \dots, J\}$ in the cost rates, i.e., the outlet specific shock on chain-wide marginal cost. Following the recommendation in Thomadsen (2005a), we have selected the following instrumental variables: V_{1j} = the distance from outlet j to the nearest outlet, V_{2j} = the number of outlets within two miles of outlet j , V_{3j} = the population density in the tract to which outlet j belongs, and V_{4j} = the worker density in this tract. Table 2.2 shows summary statistics for these instruments as well as the price variables.

Table 2.2: **Summary Statistics for Outlet Specific Data**

Variable Name	Mean	Standard Deviation	Min	Max
McDonald's Price (\$)	4.96	0.25	4.20	6.09
Burger King Price (\$)	4.85	0.28	3.63	5.39
Wendy's Price (\$)	4.75	0.20	4.27	5.24
White Castle Price (\$)	4.46	0.09	4.23	4.78
Dairy Queen Price (\$)	5.66	0.26	5.07	6.07
Steak n' Shake Price (\$)	4.99	0.36	4.67	5.84
Distance to Nearest Outlet (mi)	0.55	0.48	0.00	2.52
No. Outlets within 2 mi	5.93	2.54	1	14
Population Density (100K/sq mi)	0.09	0.09	1.71E-04	0.80
Worker Density (100K/sq mi)	0.04	0.05	1.33E-03	0.36

2.4 Estimation

As mentioned in the introduction, the major hurdle when estimating the parameters of the demand functions and the firms' cost structure, is the lack of available demand data. As explained, this challenge is not unique to the fast-food industry, but presents itself in almost all service industries. Because of the unavailability of sales data, we employ a technique that estimates the parameters on the basis of the system of (FOC) equations (2.7) the solutions of which, by Theorem 2.2.1, coincide with the Nash equilibria of the price competition model.

The equilibrium conditions (2.7) represent a system of equations which involve only the *observed* price vector P , waiting time standards W , outlet attribute matrix X , and distances $\{D_{j,b} | j = 1, \dots, J, b = 1, \dots, B\}$, as well as the *unknown* parameter string and cost rate residuals. (In particular, the system of equations does not involve the unobservable sales volumes.) The system of equations (2.7) allows us to determine the cost rate residual as closed form functions of the observed explanatory variables and unknown parameters. It is easily verified that

$$\frac{\partial Q_j(P, W, X|\theta)}{\partial P_j} = -\gamma \sum_{b=1}^B \sum_{m=1}^M h(b, m) \left(1 - \frac{S_{j,b,m}(P, W, X|\theta)}{h(b, m)} \right) S_{j,b,m}(P, W, X|\theta). \quad (2.9)$$

In matrix notation, the equilibrium conditions (2.7) can be stated as:

$$Q(P, X, W) + \Omega(P - \bar{c}) = 0, \quad (2.10)$$

where Ω is a diagonal $J \times J$ matrix whose j -th diagonal element $\Omega_{j,j} = \frac{\partial Q_j}{\partial P_j}$ ⁶. For any choice of the parameters $\theta' = (\zeta', \gamma', \delta', \pi', \alpha')$ the corresponding vector of cost rates can thus be determined in closed form:

$$\bar{c} = P + \Omega(P, X, W|\theta)^{-1} Q(P, X, W|\theta). \quad (2.11)$$

⁶When multiple outlets are owned by the same franchisee, the first order conditions continue to be of the form (2.11), however with a non-diagonal matrix Ω . See Thomadsen (2005a) for details.

The cost rate residuals ϵ can then be determined for each outlet as the difference of the outlet's total cost rate and the average cost rate of the chain to which it belongs, *i.e.*,

$$c_{k(j)} = \frac{1}{|J_k|} \sum_{j' \in J_k} \bar{c}_{j'}, \quad \epsilon_j = \bar{c}_j - c_{k(j)}, \quad \forall j = 1 \dots J. \quad (2.12)$$

One might be tempted to estimate the unknown parameters from (2.11) with the help of standard maximum likelihood methods. However, such methods require a choice of the specific unconditional distributions for the cost rate noise terms ϵ . Moreover, because of the endogeneity of the price vector P , these variables are correlated with the noise terms, so that all conditional distributions $[\epsilon_j | P_l : 1 \leq j, l \leq J]$ need to be pre-specified as well. Incorrect guesses for these various distributions, result in biased inferences, see Hall (2005). The GMM technique overcomes both difficulties. See Nevo (2000b) and Hall (2005) for clear expositions. It employs a vector of so-called instrument variables Z_j which are correlated with (some of) the explanatory variables $\{P, X, W, D\}$, but uncorrelated with the cost rate noise terms ϵ , *i.e.*, $E[Z_j \epsilon_j] = 0$ for all cost rates and all outlets.

Our instruments are based on the four instrumental variables $V_{1j}, V_{2j}, V_{3j}, V_{4j}$ defined in Section 2.3. In order to account for asymmetries in the way that different chains are affected by these instrumental variables, we interact these variables with the chain indicator vectors, I^k , $k = 1, \dots, K$, to arrive at a total of 24 instruments: for all $j = 1, \dots, J$, Z_j is a (24x1) vector defined by $Z_j \equiv \{Z_{l,k,j} = V_{lj} \cdot I_{kj}, l = 1, \dots, 4, k = 1, \dots, K\}$. Intuitively, these instruments affect demand by altering the strength of competition and the size of the potential market. Moreover, they appear to be uncorrelated with the cost rate differential an outlet is experiencing vis-a-vis the chain norm. Note that all of Cook County is urban or suburban with a total land area of only 946 square miles. It therefore faces a labor market with fairly uniform labor rates and skills. This implies that it is very unlikely that any cost efficiencies or inefficiencies of any outlet compared to the chain norm can be attributed to the population or worker density in its area. Had the study been conducted on a nationwide level, the assumption of independence would be

more questionable. Other than labor cost, the remaining variable costs in this industry are associated with food, energy, and other process inputs which are tightly prescribed by the chains. Outlets may face different real estate costs depending upon whether they are in downtown Chicago as opposed to suburban areas: however, these differences affect the outlets' fixed costs rather than their variable costs. In other words, it is reasonable to assume that the above stated orthogonality conditions apply. In view of the population moment conditions we must have, for the proper parameter vector θ , the sample average, $G(\theta) = \frac{1}{J} \sum_{j=1}^J Z_j \epsilon_j(\theta)$, of the vectors of random variables $\{Z_j \epsilon_j, j = 1, \dots, J\}$ as close to zero as possible. The GMM estimator computes a parameter vector $\hat{\theta}$ which minimizes a quadratic function of this sample average; more specifically, for a given weighting matrix A

$$\hat{\theta} = \arg \min_{\theta} G(\theta)' A G(\theta). \quad (2.13)$$

The optimal weighting matrix for the GMM estimator has been shown to be the inverse of the asymptotic variance-covariance matrix of the moment conditions. However, as this matrix is not available a-priori, we follow the commonly used two-step estimation procedure: in the first step, we use the GMM with weighting matrix $A_1 = I$ to get a consistent initial estimator $\hat{\theta}_1$ from (2.13). We then use $\hat{\theta}_1$ to estimate the asymptotic variance-covariance matrix of the moment conditions, $(E[G(\hat{\theta}_1)G(\hat{\theta}_1)'])$, and solve the optimization problem (2.13) a second time with $A_2 = (E[G(\hat{\theta}_1)G(\hat{\theta}_1)'])^{-1}$ as the weighting matrix⁷.

There are well documented technical difficulties associated with optimization problem (2.13). Its objective function has many local optima. In addition, there are large regions where this function is close to flat, creating formidable difficulties for standard gradient methods. As a consequence, we designed a specific optimization method, described in Appendix A.2, and ran this algorithm with 20 different starting points.

While there are asymptotically accurate approximations for the variances of the pa-

⁷For a nice discussion of the two step method and a proof that the inverse of the asymptotic variance-covariance matrix is the optimal weighting matrix, see Hall (2005).

parameter estimates, see e.g. Hall (2005), these are often known to perform badly (See Brown and Newey (2002) and the 1996 special issue of the Journal of Business and Economic Statistics quoted therein). Therefore, in order to validate the statistical significance of these estimates, we have constructed confidence intervals using a bootstrapping procedure. This procedure is advocated when no sample data are available beyond those used to obtain the estimate, see e.g. Brown and Newey (2002). The idea is to use subsets of the sample and calculate the value of the estimators in each subset in order to estimate the variance. To that end, we selected 80 random subsets of the tracts and ran the second stage of the above algorithm on each subset, for all 20 starting points, resulting in a total of 80 parameter vector estimates. Each subset has 134 tracts (10% of total number of tracts). Eighty subsets were used because randomly selecting 80 out of 120 such subsets consistently yielded similar confidence intervals. We used the empirical distribution to construct the confidence intervals for each parameter.

We undertook two additional robustness tests for our estimates. To attempt to improve the efficiency of our estimates, we supplemented the twenty-four micro-moments, introduced in §5, with additional so-called macro-moments. Imbens and Lancaster (1994) suggest supplementing micro-moments with macro-moments to increase the efficiency of the estimates. This approach has been used in industrial organization studies by Petrin (2002) and Davis (2006). See Appendix A.3 for a specification of the macro-moments. In our second robustness check we ran the estimation using only subsets of the six chains included in our base model. We ran the estimation once assuming that only the three largest chains (McDonald's, Wendy's, and Burger King) have a presence in the market and a second time excluding the largest chain (McDonald's) from the market.

2.5 Results

In this section, we report the results of the estimation process and robustness checks. We use either the chain's number of county outlets (OUT), the chain's national advertising

spend (ADV), or neither of these, as an additional chain attribute X in the utility function (2.1). (The sensitivity to the ADV attribute was found to be statistically insignificant when this attribute was added as the single X -variable. We have therefore omitted the specification with both OUT and ADV as additional chain attributes.) We focus on the key parameters of interest, emphasizing those that are statistically significant. Table 2.3 reports the estimated value of each of the main demand coefficients: price, waiting time and distance sensitivity (γ, δ, α) , as well as the sensitivity to the additional chain attributes, if applicable. We report the estimates obtained by averaging all 20 two-stage optimization solutions as well as their 95% confidence intervals⁸. The global optimum among all 20 two-stage solutions is consistently close to the averages; for example, for the preferred specification with OUT as an additional chain attribute, the gap is never larger than 50%.

Table 2.3: Estimates of Consumer Sensitivity to Key Attributes Under 3 Model Specifications^b

Model	Price Sens. (\$)	Wait Time Sens. (sec.)	Distance Sens. (mi.)	Chain Proxy Sens.
No Brand Proxy	4.86E-01** (0.450, 0.497)	2.64E-02** (2.65E-02, 2.93E-02)	1.73E-01** (0.127, 0.222)	NA NA
Num. Outlets	4.92E-01** (0.405, 0.500)	2.37E-02** (2.05E-02, 2.95E-02)	8.24E-01** (0.811, 1.26)	1.39E-02** (1.37E-02, 2.59E-02)
Advertising Sp.	4.92E-01** (0.412, 0.502)	2.34E-02** (1.65E-02, 2.99E-02)	9.15E-01** (0.869, 1.29)	-5.55E-04 (-1.24E-02, 3.01E-03)

^{a**} Indicates significance at the 99% confidence level. Significance level determined via a two-tail test.

^{b**} Indicates significance at the 99% confidence level. Significance level determined via a two-tail test.

We first note that the estimates of the main parameters of interest, i.e., the sensitivity parameters for price, waiting time, and distance (α, γ, δ) , are remarkably consistent across all three model specifications. This applies both to the point estimates and their 95%

⁸Three starting points yielded solutions with negative waiting time coefficients in all specifications aside from that without a brand proxy so those three points were excluded when calculating the average solution but not the bootstrapping procedure used to generate the confidence intervals.

confidence intervals. (The only exception is the estimate of the distance sensitivity in the model without the additional chain attribute X , which is approximately five times smaller than the estimate in the other two specifications.) The best fit is obtained for the specification with OUT. Its coefficient is significantly positive at the 99% level, as opposed to that of the ADV variable which is statistically insignificant. In view of the consistency of the estimates across the three model specifications and the relative superiority of the OUT specification, we focus on this specification in the remaining base model discussion as well as the robustness tests.

To ensure that the observed price vector is a Nash equilibrium under the estimated parameter values, we have verified that condition (C) is satisfied⁹. Indeed, as proven under this condition in Theorem 2.2.1, we observed that $p_j < \bar{c}_j + 2/\gamma$ for all $j = 1, \dots, J$. In fact we found that $p_j < \tilde{p}_j = \bar{c}_j + 1/\gamma$, so that the observed price vector is in fact the *unique* equilibrium as long as \tilde{p} is chosen as the maximum price vector, see the discussion at the end of Section 3 and Allon et al. (2009, Thm 4.4).

Table 2.3 shows that all of the parameters $(\alpha, \delta, \gamma, \zeta)$ are significant at a 99% confidence level. Our estimates indicate that consumers attribute a very high cost to the time they spend waiting. Both the price and waiting time parameters have a significant impact on the consumer's decision. These results confirm our initial conjecture, as well as the belief expressed by industry experts, that in the fast-food drive-thru industry customers trade off price and waiting time. In particular, to overcome an additional *second* of waiting time, an outlet will need to compensate an average customer by as much as \$0.05 ($= 0.0237/0.492$) in a meal whose typical price ranges from \$2.25 to \$6. This corresponds with an hourly cost rate of approximately *ten* times the (pre-tax) average wage of \$18/hour and nearly 30 times the (pre-tax) minimum wage in Illinois in 2005 (\$6.50/hr). Even when comparing (opposite) extreme values of the 95% confidence intervals, the av-

⁹To verify (C) we first checked that $\forall j = 1, \dots, N \quad p_j < \bar{p}_j = \bar{c}_j + 2/\gamma$ for our estimates of \bar{c}_j and γ . The maximum aggregate share (for the hamburger drive-thru industry relative to the potential consumer population) observed for any tract/demographic group pair with these estimates was 26%. If any one firm j raises his price to \bar{p}_j and all others raised their prices to p^{max} , the aggregate market share and, a fortiori, that of outlet j by itself, is even lower. Therefore, (C) is satisfied.

average consumer assigns a cost to waiting which corresponds with a rate of at least \$0.04 per second. Since price differences in this industry, as in many others, are rather modest, this valuation implies that in the drive-thru market waiting time plays a more significant role than pricing in explaining sales volumes. Moreover, these results seem to justify the continuing trend of chains making substantial investments to improve their waiting time.

It is also interesting to compare waiting time and distance sensitivity. After all, the disutility associated with the distance factor arises mainly from the associated time loss. Assuming, for example, an average velocity of 30 miles/hr, the estimate of δ implies that every additional second spent driving to the outlet reduces the utility measure by $0.824 \times 30 / 3600 = 0.69E^{-2}$. Thus the disutility of time spent waiting in the drive-thru line is at least three times that associated with the traveling time. This finding is consistent with the literature, see e.g. Kahneman and Tversky (1984) and Larson (1987), which reveal that individuals value time very differently, depending on the context and the degree to which time is spent is pleasurable or not: most people mind time spent driving far less than time waiting idly; some even enjoy the ride.

There are some limitations to be noted with the estimation of the contribution of travel time to the overall utility value. First, we do not have an exact measure of the road distance between the consumer's residence and the outlets. Even if we did, this distance is not the best possible measure for the additional effort and time she needs to expend to travel to the outlet. After all, many consumers stop in a drive-thru on the way from one point to the other, so that the disutility associated with travel time is not perfectly measured by the distance between the consumer's residence or work place and the outlet. Finally, it is not clear that a consumer's disutility from travel varies linearly with the distance driven.

When omitting waiting time as an explanatory variable, the resulting price sensitivity estimate is 0.543, a 10.5% increase compared with the estimate we obtain with the full model. In fact, this estimate lies outside the confidence interval obtained using the full model. Indeed, when service level attributes such as waiting time are disregarded, any

reasonable estimation method can be expected to attribute a greater weight to price differentials to explain differences in sales volumes and market shares. Furthermore, estimating the elasticity of demand with respect to price using the model without waiting time results in overestimating this price elasticity by more than 10%. Not accounting for the waiting time as a strategic attribute also overstates the utility value of the outside good and disguises the importance of the chains' number of outlets in the county, the best identified indicator of the consumers' perception of chain quality. These various distortions contribute to suboptimal pricing decisions when ignoring waiting time as an explanatory variable in the consumer choice model.

It is also of interest to compare our results with those of Thomadsen (2005a) who employed a similar model to estimate market share equations in the hamburger fast-food industry in Santa Clara, California. Thomadsen's consumer choice model disregards differences in service attributes as explanatory variables but includes the co-ownership structure (i.e., multi-outlet owners) and brand dummies. Consistent with our findings regarding the impact of omitted service attributes, Thomadsen's estimate for the price sensitivity parameter is systematically larger than (, in his case approximately double,) the value we obtain. Along with considering all national chains with five or more outlets in the county, we represent the fast-food market as more competitive than Thomadsen does, in that we disregard the fact that a certain percentage of franchise owners own multiple outlets. (As mentioned, we lacked information about common ownership.) Ignoring the limited co-ownership phenomenon, see Footnote 10, results in underestimated equilibrium price sensitivity estimates, for given observed price levels. Note, that even if the price sensitivity parameters were double our estimate, the estimated cost of waiting time would be approximately \$90/hr. Finally, Thomadsen reports only a 90% confidence interval on the price estimate, (.14,1.68), which has a margin of error nearly 20 times that of our 90% interval. Indeed, our confidence interval is entirely contained in his.

2.5.1 Robustness Testing

We have conducted various tests to confirm the robustness of our estimates beyond the consistency of the parameter estimates across the three model specifications, as well as the relatively narrow confidence intervals. These additional robustness tests consist of (i) adding the macro moments discussed in Section 2.4 and listed in Appendix A.3 to the GMM estimation procedure¹⁰; (ii) repeating the estimation procedure under the assumption that only the three largest chains - McDonald's, Burger King, and Wendy's - have a presence in the county and (iii) repeating the estimation under the assumption that the largest chain, i.e., McDonald's is absent in the county, thereby reducing the number of outlets by 44%. All of the additional robustness tests employ the specification with the OUT attribute. Table 2.4 reports the estimates under the above three alternatives, while restating the estimates of the base model. (As before, the numbers within parentheses denote a 95% confidence interval.) Once again, we have remarkable consistency in all of the parameter estimates among the different variants of the model/estimation procedure.

Table 2.4: **Parameter Estimates Under 3 Alternatives to the Base Model**

	Base Model	with MM	{McD, BK, WN}	w/out McD
Price (\$)	4.92E-01 (4.05E-01, 5.00E-01)	5.03E-01	5.08E-01	5.14E-01
Waiting Time (sec)	2.37E-02 (2.05E-02, 2.95E-02)	2.01E-02	2.05E-02	2.13E-02
Distance (mi)	8.24E-01 (8.11E-01, 1.26E+00)	7.70E-01	8.53E-01	7.16E-01
Brand Proxy (Num. Outlets)	1.39E-02 (1.37E-02, 2.59E-02)	9.07E-03	1.34E-02	2.47E-02

¹⁰The estimates with macro-moments are based on a single stage of estimation. We were unable to perform the second estimation stage since the moment variance-covariance matrices at most candidate optima were close to singular.

2.5.2 Counterfactuals

How much, then, is it worth to reduce the waiting time standards? We mentioned the industry maxim that a seven-second reduction in waiting times increases a chain's market share by 1%. We have therefore investigated the impact of a *single* chain reducing its waiting time standard by seven seconds, allowing all outlets to adjust their prices to the new price equilibrium. The results of this experiment can be seen in Table 2.5. In the "first row" section (i.e., the two rows with 'Initial' in the title), we give the estimated daily demand and market share of each chain at the current waiting time standards and prices. The second row section, titled 'McD', shows the change in every chain's market share and demand volume when McDonald's reduces its waiting time by seven seconds. The following five row sections contain the results of the same experiment for the remaining five chains. The percentage of the total market captured at the current waiting time standards closely matches the results in Paeratakul et al. (2003), providing further validation of our estimates.

Our results confirm that the industry maxim is, on "average", correct. However, the absolute change in market share ranges from 3% at McDonald's (the market leader) to 0.04% at Dairy Queen, with Wendy's, the chain with the fastest service in 2007 and 2008, experiencing an increase by 1.33%. (The *percentage* increase in market share ranges between 4% at McDonald's and 20% at Dairy Queen.) Here, a chain's market share is defined as the chains' sales as a percentage of the total sales in the hamburger drive-thru industry. Even more importantly, an unmatched reduction of McDonald's waiting time standard by seven seconds results in an increase of its sales volume by approximately 15%. Note that the increase in demand comes primarily from attracting new customers to the market. The percentage of the potential fast-food market captured by all the chains grows by more than 1% when any of the three large players lower their waiting time. As further discussed in Section 2.6, any chain's unilateral waiting time reduction is likely to induce waiting time changes by the competing firms. Indeed, between 2005 and 2008, almost all chains gradually reduced their waiting time standards, McDonald's from 224 to 158

Table 2.5: Change In Market Share Following 7-Second Wait Standard Reduction at a Chain

	McD	BK	WN	Wh. C	DQ	S 'n S	%Tot Mkt
Initial Demand	5.58E+5	2.26E+5	1.33E+5	1.83E+4	1.95E+3	1.35E+3	14.2%
Initial Mkt Share	59.47%	24.06%	14.16%	1.95%	0.21%	0.14%	NA
McD (Δ Dem) (Δ Mkt Sh)	6.76E+4 3.02%	-2.90E+3 -1.80%	-1.57E+3 -1.05%	-2.32E+2 -0.15%	-2.04E+1 -0.02%	-1.32E+1 -0.01%	15.2%
BK	-2.93E+3 -1.88%	2.93E+4 2.40%	-6.36E+2 -0.44%	-9.67E+1 -0.06%	-8.01E+0 -0.01%	-5.10E+0 0.00%	14.6%
WN	-1.58E+3 -1.12%	-6.38E+2 -0.45%	1.76E+04 1.62%	-5.34E+1 -0.04%	-5.66E+0 0.00%	-3.68E+0 0.00%	14.5%
WC	-2.36E+2 -0.16%	-9.73E+1 -0.06%	-5.36E+1 -0.04%	2.48E+3 0.26%	-6.47E-1 0.00%	-2.43E-1 0.00%	14.3%
DQ	-2.07E+1 -0.02%	-8.06E+0 -0.01%	-5.69E+0 0.00%	-6.48E-1 0.00%	2.64E+2 0.03%	-7.12E-2 0.00%	14.2%
SS	-1.33E+1 -0.01%	-5.13E+0 0.00%	-3.69E+0 0.00%	-2.43E-1 0.00%	-7.12E-2 0.00%	1.84E+2 0.02%	14.2%
All	6.23E+4 -0.01%	2.52E+4 -0.01%	1.51E+4 0.02%	2.05E+3 0.00%	2.25E+2 0.00%	1.59E+2 0.00%	15.8%

seconds and Wendy's from 173 to 131. Therefore, in the last row section of Table 2.5 we report the impact of a simultaneous seven-second reduction of the average waiting time by *all* chains. This simultaneous service improvement results in the six chains capturing an additional 1.5% of the potential market. Relative market share changes are small, with Wendy's the prime beneficiary in relative terms, perhaps because, for it, the seven-second reduction is the largest relative service improvement among all six chains.

2.6 Conclusions and Extensions

In this essay, we have proposed an approach to estimate how sales volumes for a service organization depend on all prices and waiting times of the various service providers in the region, along with other relevant attributes. We have applied this approach to the

drive-thru fast-food industry in Cook County, IL. Here, consumers assign an implicit value to waiting in the drive-thru queue which amounts to many times the pre-tax U.S. wage, thus answering the first two of the four main research questions raised in the Introduction. Most importantly, chains can improve their absolute and relative market shares very significantly by relatively modest reductions in waiting time, which explains why all chains make continuous efforts to shave off seconds from their consumer waiting time: reducing waiting time standards pays off handsomely in the fast-food industry. A seven-second reduction, the magnitude of Wendy's improvement from 2007 to 2008, implies an "average" increase of a chain's market share by approximately one percentage point which confirms the above industry maxim and answers the third research question. However, for a large chain like McDonald's, it would result in an increase by more than 3% while the increase is 0.04% for a small chain like Dairy Queen, thus providing an answer to the fourth and final research question. The competitive dynamics are such that, to the extent feasible via incremental process and technological improvements, it is in all chains' interests to reduce their waiting times; this occurs to a large extent because such service improvements result in more potential consumers selecting the fast-food option.

Several important extensions of our study and underlying model would be valuable. First, it is not clear whether the waiting time experience is best characterized by the average alone, or (, additionally,) by other measures such as the standard deviation and/or a percentile (say the 90-th percentile) of the waiting time distribution. Even if the average waiting time is the best proxy, it is conceivable that the consumer's utility level diminishes in a non-linear way with it. A similar non-linear dependence on the distance variable may be explored as well. In addition, other service attributes such as the accuracy of the order filling process and the clarity of the speaker and menu board could be included as explanatory variables in the random utility model (2.1).

Studying the impact of finer segmentations of the population, including past patronage of specific outlets, so as to estimate the impact of loyalty/inertia would be of interest. However, without sales volumes for the period of interest, this fine level of segmentation

is not feasible. Nevertheless, in the modified specification of the utility functions, we have added explanatory variables that reflect brand penetration and awareness in Cook County.

While, as explained in Section 2.3, we believe that a consumer choice model with a *single* price indicator per outlet is most appropriate for this industry, it could be worthwhile to test a far more detailed model that considers a menu of items to be purchased at every outlet and consumers choosing an outlet/menu item combination. It goes without saying that the data gathering and estimation challenges associated with such a detailed model are formidable, especially in the absence of sales data.

It would also be desirable to investigate how the chains in the industry select their waiting time standards and how the costs associated with waiting time reductions compare with the resulting revenue enhancements. To this end, it appears natural to view the price competition model in this essay as the second stage in a two-stage game preceded by a first stage in which the chains as competing players select waiting time standards to maximize their profits. Specification and estimation of such a first stage model meets with various challenges. First, Lafontaine and Shaw (1999) document that franchises typically pay a fixed periodic fee to the chain, along with a percentage of the revenues. However, it is unclear how these parameters are set as a function of the desired waiting time standard. It is also unclear how investments and operational costs depend on this strategic choice. Moreover, since chains select national standards, a two stage game is needed with all US outlets participating in the second stage price competition game. It is also unclear whether all chains face the same potential lower limits for the waiting time standard. A final challenge is to verify whether the first stage competition model has a (unique) pure strategy equilibrium.

More broadly, the modeling approach and estimation technique of our study could be applied in other service industries in which consumers make purchasing decisions based on a steady state service measure as opposed to the one prevailing at the time they consider entering the service system. In most other service industries, one may expect that the

service level measures vary by individual service provider. This greatly simplifies the identification challenges in the specification of the consumer choice model but increases the data collection effort as one needs direct observations for every provider in the chosen market. In addition, in our study we are able to estimate the model without knowledge of sales volumes or marginal cost rates because of the absence of unobservable firm (i.e., outlet)-specific attributes that can be argued to have a consistent impact on consumers' purchasing decisions. In industries where such an argument is not valid, the estimation approach can only be applied if either sales volume or variable cost rate data can be assessed.

Chapter 3

Price Competition under

Multinomial Logit Demand

Functions with Random Coefficients

We refer to Section 1.4 for an introduction and summary of the models used and results obtained in this chapter. This chapter is organized as follows: Section 3.1 provides a review of the relevant literature. Section 3.2 introduces the consumer choice model. Our equilibrium existence and uniqueness results are presented in Section 3.3. Section 3.4 develops the example showing that a Nash equilibrium may fail to exist in the absence of any conditions precluding highly concentrated markets or, alternatively, enforcing sufficiently tight price limits. Section 3.5 discusses extensions of our base model which allow for a continuous specification of customer types, firms offering multiple products, and settings with a general income distribution. Our final section 3.6 describes the implications of these results for the econometrician attempting to estimate the model parameters.

3.1 Literature Review

There has been a plethora of price competition models for industries with differentiated products or services, beginning with the seminal paper by Bertrand (1883). One important class of such competition models employs demand functions based on a MNL discrete choice model. This model was proposed by McFadden (1976), a contribution later awarded with the 2000 Nobel Prize in Economics. As explained in the Introduction, the model may be derived from an underlying random utility model, see (3.1) in Section 3.2, with homogeneous coefficients, *i.e.*, the special case where the customer population does not need to be segmented. Luce and Suppes (1965) attribute this derivation to an unpublished manuscript by Holman and Marley. The MNL model has been widely used in the economics, marketing, and operations management literature, among many other fields, see, for example Ben-Akiva and Lerman (1993), Anderson et al. (2001), and Talluri and Van Ryzin (2005). The MNL model satisfies the so-called Independence of Irrelevant Alternatives (IIA) axiom according to which the *ratio* of any pair of firms' market shares is independent of the set of other alternatives that are offered to the consumers. This axiom was first postulated by Luce (1959) but Debreu (1960) pointed out that the IIA property is highly restrictive, as illustrated by his famous red bus-blue bus example: the relative market share of an alternative is, in general, significantly affected if a close substitute to this alternative is added to the choice set.

To remedy this problem, Ben-Akiva (1973) introduced the so-called *nested logit* model, where the choice process is modeled as a two-stage nested process: the consumer first selects among broad classes of alternatives (, *e.g.*, air versus ground transportation) and subsequently a specific variant among the selected class of alternatives (*e.g.*, a specific flight). This approach still ignores systematic differences in the way different customer segments trade off relevant attributes of the various products or services. To address the issue of systematic customer heterogeneity, the *mixed multinomial logit model* (MMNL) was introduced, apparently first by Boyd and Mellman (1980) and Cardell and Dunbar

(1980); earlier papers in the seventies, for example Westin (1974), had derived a similar model by treating, in a single segment model, the attribute vector as random with a given distribution. The properties of the MMNL model have been extensively studied in the economics and marketing literature, see *e.g.*, Train et al. (1987), Steckel and Vanhonacker (1988), Gonul and Srinivasan (1993), Berry (1994), Jain et al. (1994). More recently, McFadden and Train (2000) show that, under mild conditions, any discrete choice model derived from random utility maximization generates choice probabilities that can be approximated, arbitrarily closely, by a MMNL model. Moreover, these authors show that MMNL models enjoy numerical and estimation advantages beyond other discrete choice models. (It would be of considerable interest to extend our results to the general class of choice models considered by McFadden and Train (2000).)

Whether or not a Nash equilibrium exists in a Bertrand price competition model depends fundamentally on the structure of the demand functions as well as the cost structure. The same applies to the uniqueness of the equilibrium. Milgrom and Roberts (1990) and Topkis (1998) identified broad classes of demand functions under which the resulting price-competition model is supermodular, a property guaranteeing the existence of a Nash equilibrium.

More specifically, for the *pure* MNL model with a cost structure that is affine in the sales volume, Anderson et al. (2001) established the existence of a (unique) Nash equilibrium in the special case where all firms are symmetric, *i.e.*, have identical characteristics. Bernstein and Federgruen (2004) extended this result for the case of general asymmetric firms, and a generalization of MNL models referred to as attraction models. For the same model, Gallego et al. (2006) provide sufficient conditions for the existence of a unique equilibrium, under cost structures which depend on the firm's sales volume according to an increasing convex function. Konovalov and Sándor (2009) recently showed that the existence of a unique equilibrium can be guaranteed in the multi-product generalization of a pure MNL-price competition model.

Seemingly minor variants of the pure MNL model may result in a fundamentally

different equilibrium behavior of the associated price competition model. For example, Cachon and Harker (2003) report that under a simple piecewise linear transformation of the MNL demand functions, and a cost function that is proportional to the square root of the sales volume, the model may have no, one or multiple equilibria as a single parameter is varied. (This is demonstrated with an example involving two symmetric firms.) Similar erratic behavior was demonstrated by Chen and Wan (2003) for what is, arguably, the seminal price competition model for service competition, due to Luski (1976) and Levhari and Luski (1978).

For price competition models with nested logit demand functions, Liu (2006) recently established the existence of a unique Nash equilibrium. As mentioned in the Introduction, Caplin and Nalebuff (1991) is the seminal paper establishing sufficient conditions for the existence of a price equilibrium when the demand functions are based on a broad class of MMNL models. They also show that, under these conditions, a unique price equilibrium exists in the case of a *duopoly* or when products are characterized by their price and a *single*, one-dimensional attribute, while the density of the customer type distribution is log-concave (See Dierker (1991) for an alternative treatment). As mentioned by many authors, *e.g.*, Berry et al. (1995) and Thomadsen (2005b), these sufficient conditions are often not satisfied in many industry-based models.

Peitz (2000, 2002) have shown that a price equilibrium exists in certain variants of the Caplin and Nalebuff model, allowing for settings where customers maximize their utility functions subject to a budget constraint or when they may purchase an arbitrary amount of each of the products in the market, as opposed to a single unit. Unfortunately, the utility functions in Peitz do not depend on the product prices, so that the firms' incentive to mitigate price levels arises purely from the customers' budget constraints. Mizuno (2003) establishes the existence of a unique price equilibrium for certain classes of models (*e.g.*, logit, nested logit) in which the demand functions *are* log-supermodular. As we show at the end of Section 3.3 this property fails to apply in general MMNL models.

As explained in the Introduction, our model assumptions generalize those made in

Caplin and Nalebuff (1991). Our essay also builds on results in Thomadsen (2005b) which provide a sufficient condition for the existence of a price equilibrium - but not its uniqueness - when the demand functions arise from a general MMNL model; his condition relates the firm's variable cost rate to the value of the non-price related variables in the utility measures, see (3.1) below. It is difficult to assess how widely applicable the condition is.

3.2 The Price Competition Model

Consider an industry with J competing single-product firms each selling a specific good or service. The firms differentiate themselves via an arbitrary series of observable product characteristics as well as their price. Each firm faces a cost structure which is affine in the expected sales volume. Customers are assumed to purchase only one unit and can be segmented into K distinct groups, each with a known population size. (In Section 3.5, we discuss models with a continuum of customer types or market segments. All of the results obtained in Section 3.3, for the case of a finite set of market segments, continue to apply there.) If the potential buyers in the model represent consumers, the different segments may, for example, represent different geographical areas, in combination with socioeconomic attributes, such as age, gender, race, income level, number of years of formal education, occupational and marital status, etc. In the case of Business to Business (B2B) markets, the different segments may again represent different geographical regions, industry sub-sectors (government agencies, educational institutions, for profit companies) and firm size levels¹. When modeling, for example, an industry of automobile part suppliers, each automobile manufacturer may represent a segment by itself. The chosen segmentation should reflect the various observable factors which may impact how different product attributes are traded off by the potential buyers. We use the following notation

¹Firm size may, for example, be defined as the firm's annual revenues or its capital value.

for all firms $j = 1, \dots, J$ and customer segments $k = 1, \dots, K$:

- x_j = an L -dimensional vector of observable *non-price* attributes for firm j ;
- c_j = the variable cost rate for firm j ;
- p_j = the price selected by firm j ; $p_j \in [p_j^{min}, p_j^{max}]$ with $0 \leq p_j^{min} \leq c_j \leq p_j^{max}$;
- h_k = the population size of customer segment k ;
- S_{jk} = expected sales volume for firm j among customers in segment k ;
- S_j = expected aggregate sales volume for firm j across all customer segments;
- π_{jk} = expected profit for firm j derived from sales to customers in segment k ;
- π_j = expected aggregate profits for firm j .

We thus assume that each firm selects its price from a given closed interval of feasible prices. To our knowledge, compact feasible price ranges are required for any of the known approaches to establish the existence of a Nash equilibrium². At the same time, the restriction is without loss of essential generality. Consider first p_j^{min} . In the absence of other considerations, we may set $p_j^{min} = 0$ ³. As for p_j^{max} , price limits may result from a variety of sources, *e.g.*, government regulation, maximum price levels specified by suppliers or franchisers, limits set by industry organizations, or branding considerations. In other settings, where no such exogenous price limits prevail, one can always select unrestrictive upper bounds for p_j^{max} which are well above reasonable price choices. (For example, no fast food meal will be priced beyond \$20 and no subcompact car beyond the \$25,000 level.) Moreover, we will show that under a widely applicable condition and p_j^{max} sufficiently large, the choice of p_j^{max} has no impact on the price equilibrium.

Market shares within each customer segment may be derived from a standard random

²Caplin and Nalebuff (1991), for example, assume that prices are selected from a closed interval $[p_j^{min}, p_j^{max}]$ with $p_j^{min} = c_j$ and $p_j^{max} = Y$, the consumer's income level. We make no upfront specification for these limits, allowing $0 \leq p_j^{min} < c_j$ and $p_j^{max} \neq Y$. Indeed, for certain durable or investment goods and certain income levels, p_j may be in excess of Y .

³We assume $p_j^{min} \leq c$ to ensure, under our existence conditions, that any Nash equilibrium $p^* > p_j^{min}$, see Lemma 3.3.1 below.

utility model, as follows. First, let

$$u_{ijk} = U_{jk}(x_j) + G_j(Y_i, p_j) + \epsilon_{ijk}, j = 1, \dots, J; k = 1, \dots, K; \text{ and } i = 1, 2, \dots, \quad (3.1)$$

denote the utility attributed to product j by the i th customer in segment k , with income or firm size Y_i . Recall that x_j is a vector of *observable* product attributes. Conversely, ϵ_{ijk} denotes a random *unobserved* component of customer utility. The functions $\{U_{jk}, j = 0, \dots, J\}$ are completely general. The second term $G_j(\cdot, p_j)$ which we refer to as the *price-income sensitivity function* reflects how the utility of each product depends on its price, where marginal price sensitivity may vary with income level. Similarly, the utility associated with the no-purchase option is given by

$$u_{i0k} = U_{0k}(x_1, \dots, x_J) + \epsilon_{i0k}, k = 1, \dots, K; i = 1, 2, \dots \quad (3.2)$$

As we exclude Veblen goods, $G_j(\cdot, p_j)$ is decreasing in the price level p_j . We assume that the G_j functions in (3.1) are twice differentiable and concave in p_j , with $\lim_{p_j \uparrow \infty} G_j(\cdot, p_j) = -\infty$. Thus, let

$$g_j(\cdot, p_j) = \left| \frac{\partial G_j(\cdot, p_j)}{\partial p_j} \right| = \frac{-\partial G_j(\cdot, p_j)}{\partial p_j} > 0, j = 1, \dots, J \text{ and } k = 1, \dots, K, \quad (3.3)$$

denote the (absolute value of the) marginal change in the utility value of product j due to a marginal change in its price. (Below, we discuss an alternative interpretation of the g_j -functions.) In view of the concavity of the G_j -functions, we have $\forall j = 1, \dots, J$ and $k = 1, \dots, K$

$$g_j(\cdot, p_j) \text{ is increasing in the price level } p_j. \quad (3.4)$$

Many model specifications in the literature employ a $G(\cdot, \cdot)$ function *common* to all products $j = 1, \dots, J$, see, *e.g.*, the various models listed in Section 3 of Caplin and Nalebuff (1991). However, in some applications, even the marginal utility shift due to

a price increase may differ among the different competing products⁴. No assumptions are needed with respect to the dependence of the price sensitivity function $g_j(Y_i, p_j)$ on Y_i , even though one would typically have that it is decreasing in Y_i . As in Caplin and Nalebuff (1991), we initially assume that all consumers share the same income level or firm size Y (hereafter referred to as ‘income’ alone). Extensions to the general model, with varying income levels, are developed in Section 3.5.

To complete the specification of utility functions (3.1) and (3.2), $\{\epsilon_{ijk}, j = 0, 1, \dots, N\}$ is an i.i.d. sequence of random variables, for all $i = 1, 2, \dots$ and $k = 1, \dots, K$. We further assume that the random components ϵ_{ijk} follow a type 1 - extreme value or Gumbel distribution:

$$Pr[\epsilon_{ijk} \leq z] = e^{-e^{-(z/\delta + \gamma)}}, j = 0, \dots, J; k = 1, \dots, K; i = 1, 2, \dots, \quad (3.5)$$

where γ is Euler’s constant (0.5772) and δ is a scale parameter. The mean and variance of the random term $\{\epsilon_{ijk}\}$ are $E[\epsilon_{ijk}] = 0$ and $var[\epsilon_{ijk}] = \delta^2 \pi^2 / 6$. Without loss of generality, we scale, for each customer segment $k=1, \dots, K$, the units in which the utility values are measured such that $\delta = 1$. This random utility model results in the well known MNL model for demand for product j among customers of segment k .

$$S_{jk} = h_k \frac{e^{U_{jk}(x_j) + G_j(Y, p_j)}}{\left[e^{U_{0k}(x_1, \dots, x_J)} + \sum_{m=1}^J e^{U_{mk}(x_m) + G_m(Y, p_m)} \right]}; j = 1, \dots, J; k = 1, \dots, K. \quad (3.6)$$

Aggregating the sales volume for individual customer segments in (3.6) over all segments results in the following expected sales functions for each product:

$$S_j = \sum_{k=1}^K S_{jk} = \sum_{k=1}^K h_k \frac{e^{[U_{jk}(x_j) + G_j(Y, p_j)]}}{\left[e^{U_{0k}(x_1, \dots, x_J)} + \sum_{m=1}^J e^{U_{mk}(x_m) + G_m(Y, p_m)} \right]}. \quad (3.7)$$

⁴Caplin and Nalebuff (1991) already recognized the value of allowing for product-dependent price-income sensitivity functions. As explained below, see (3.15), they confine themselves to the case when these functions differ by a proportionality constant only, thus assuming that for any pair of products, the *ratio* of the marginal utility changes due to a \$1 price increase remains *constant*, irrespective of the products’ price levels.

An alternative foundation for the sales volume formula (3.6) is to assume that among potential customers in segment k , each firm j and the no-purchase option have a so-called attraction value given by:

$$a_{jk} = e^{U_{jk}(x_j) + G_j(Y, p_j)}, \quad j = 1, \dots, J, \quad k = 1, \dots, K, \quad (3.8)$$

$$a_{0k} = e^{U_{0k}(x_1, \dots, x_J)}, \quad k = 1, \dots, K. \quad (3.9)$$

Under the four intuitive axioms specified in Bell et al. (1975), this uniquely gives rise to the demand volumes specified in (3.6).

The above consumer choice model thus distinguishes between two types of customer heterogeneity: (i) heterogeneity that is attributable to observable customer attributes such as their geographical location or socio-economic profile, and (ii) intrinsic heterogeneity not explained by any systematic or observable customer attributes. This model specification covers most random utility models in the literature. As an example, consider the following general specification used in Berry (1994).

$$u_{ij} = x_j \beta_i + \xi_j - \alpha p_j + \epsilon_{ij}, \quad j = 1, \dots, J; \quad i = 1, 2, \dots \quad (3.10)$$

$$\beta_{il} = \beta_l + \sigma_l \zeta_{il}, \quad l = 1, \dots, L \quad \text{and} \quad i = 1, 2, \dots, \quad (3.11)$$

Here, $\{\epsilon_{ij}\}$ is again a sequence of unobservable random noise terms which is i.i.d. The vector $[\alpha, \beta, \sigma]$ is a $2L+1$ dimensional string of parameters. Finally, the sequences $\{\zeta_{il}\}$ and $\{\xi_j\}$ are random sequences with zero mean, which may, or may not, be observable. Often, ξ_j is used to represent an *unobservable* utility component which reflects attributes of firm j unobserved by the modeler but with common value among the customers.

To verify that the general structure in Berry (1994) can be treated as a special case of (3.1) - (3.5), assume the $\{\zeta_{il}\}$ distributions are discrete and segment the customer population such that all customers in any segment k , share the same ζ_{il} -value for each of the L observable product attributes, i.e., $\zeta_{il} = \hat{\zeta}_{kl}$ for all customers i in segment k . Specifying

$U_{jk}(x_j) = \xi_j + \sum_{l=1}^L x_{jl}[\beta_l + \theta\hat{\zeta}_{kl}]$ and $G_j(Y_i, p_j) = -\alpha p_j$, we note that the general Berry-model arises as a special linear specification of our structure. A restriction inherent in the Berry model is the assumption that α , the marginal disutility for firm j 's product due to a marginal price increase, is uniform across all products and all price and income levels. In many practical applications, price sensitivity may vary significantly along any one of these dimensions.

Other MMNL consumer choice models employ one or more measurable attributes which depend on the specific firm and customer segment combination. For example, if the customer segmentation is in part based on the customer's geographic location, a measure d_{jk} for the distance between customer segment k and firm j may be added to the specification in (3.1) as follows:

$$U_{jk}(x_j) = F_j(x_j, d_{jk}) + \xi_{jk}, \quad (3.12)$$

with ξ_{jk} , again, an unobservable component in firm j 's utility measure that is common among all customers of segment k . See, for example, the discussion of Thomadsen (2005a) below.

In other applications, the distance measure d_{jk} refers to a measure of a-priori affinity. For example, if, on the basis of nationalistic sentiments, customers have a propensity to buy from a domestic provider, this may be modeled by basing the segmentation in part on the consumer's nationality and defining the distance $d_{jk} = 0$ if segment k represents the same nationality as firm j , and $d_{jk} > 0$, otherwise. Alternatively, the a-priori affinity may be based on past purchasing behavior. Both the economics and the marketing literature have addressed the fact that customers tend to be inert or firm/brand loyal; *i.e.*, because of explicit or psychological switching costs, customers tend to stay with their current provider or brand, even if they would otherwise be more attracted by a competitor. Dube et al. (2008b), for example, models this as a MMNL model, segmenting customers, in part, on the basis of the firm most recently patronized; a distance measure d_{jk} is added to the utility measure where $d_{jk} = 0$ if customers of segment k used to buy from firm j

and $d_{jk} = 1$ otherwise.

Another general model was introduced in the seminal paper by Caplin and Nalebuff (1991) with the specific objective of establishing the existence of a price equilibrium for a broad class of consumer choice models. This general model assumes that each potential customer i is characterized by a weight vector $\alpha_i \in \Re^L$ as well as an income level Y , such that

$$u_{ij} = \sum_{l=1}^L \alpha_{il} h_l(x_{jl}) + \beta_j \Gamma(Y - p_j), \quad j = 1, \dots, J \quad \text{and} \quad i = 1, \dots, N, \quad (3.13)$$

for given functions $\Gamma(\cdot)$ and $h_l(\cdot)$, with $\Gamma(\cdot)$ concave and increasing, and for given constants $\beta_j > 0$, $j = 1, \dots, J$ ⁵. In other words, the Caplin-Nalebuff model assumes that customers characterize each product j in terms of a transformed attribute vector x'_j , the l -th component of which is given by $x'_{jl} \equiv h_l(x_{jl})$, $j = 1, \dots, J$ and $l = 1, \dots, L$. Customers then aggregate the (transformed) attribute values via a linear aggregate measure with different customers applying a different weight vector α to the attribute values. Assuming the distribution of α is discrete we obtain the Caplin-Nalebuff structure as a special case of our random utility model (3.1) - (3.5), as follows: segment the customer population into segments such that all customers in a segment share the same α values. In other words, for all customers in segment k , $\alpha_{il} = \alpha_i^{(k)}$. The Caplin-Nalebuff model (3.13) thus arises as a special case of our model with

$$U_{kj}(x_j) = \sum_{l=1}^L \alpha_l^{(k)} h_l(x_{jl}), \quad \forall j = 1, \dots, J \quad \text{and} \quad k = 1, \dots, K; \quad (3.14)$$

$$G_j(Y_i, P_j) = \beta_j \Gamma(Y_i - p_j), \quad (3.15)$$

while the scale parameter δ of the $\{\epsilon_{jk}\}$ -variables are chosen such that $\delta = 0$ ⁶. Alter-

⁵Caplin and Nalebuff (1991) consider, in addition, a generalization of (3.13) in which the L -dimensional vector of product attributes x is first transformed into a L' -dimensional vector of utility benefits $t(x)$. Instead of (3.13), the utility value of firm j for customer i is then specified as $u_{ij} = \sum_{l=1}^{L'} \alpha_{il} t_l(x_j) + \beta_j g(Y_i - p_j)$. This specification can also be shown to be a special case of our model. The authors state, however, that in most applications, preferences take the simpler form of (3.13).

⁶Caplin and Nalebuff (1991) represent the proportionality constant β_j as the $(n+1)$ -st utility benefit

natively, the L -dimensional attribute vector x_j may be partitioned into an observable and an unobservable part: $x = [x', x'']$ with x' an L' -dimensional vector of observable attributes and x'' a J -dimensional vector of product indicator variables, *i.e.*, $x_{j,L'+j} = 1$ and $x_{j,L'+m} = 0 \forall m \neq j$. If the weights $\{\alpha_l : l = L' + 1, \dots, L' + J\}$ follow independent Gumbel distributions, denoting (unobserved) utility components while each point $(\alpha_1, \dots, \alpha_{L'})$ constitutes a separate market segment, we retrieve a (specific type of) MMNL models where the mixture is over the given distribution of $(\alpha_1, \dots, \alpha_{L'})$ only. To obtain the existence of a Nash equilibrium in this price competition model, the authors assume, further, that the probability density function $f(\alpha)$ of the consumer attribute vector α is ρ -concave for a specific value of ρ , *i.e.* for any pair of points $\alpha^{(0)}$ and $\alpha^{(1)}$ in the convex support of the distributions, and any scalar $0 < \lambda < 1$:

$$f(\lambda\alpha^{(0)} + (1 - \lambda)\alpha^{(1)}) \geq [\lambda f(\alpha^{(0)})^\rho + (1 - \lambda)f(\alpha^{(1)})^\rho]^{1/\rho} \quad \text{and} \quad \rho = -1/(L + 1). \quad (3.16)$$

Thomadsen (2005a) has shown that geographic distance measures can be incorporated in this specification by appending an indicator vector for each of the J firm locations. However, the author also shows that the requirement of a ρ -concave probability density function for the customer attribute vector α precludes all but the most restrictive geographic customer distributions. In addition, under the Caplin-Nalebuff model, the price income sensitivity function G_j for the different products $j = 1, \dots, J$ differ from each other only in the proportionality constant β_j . Moreover, the customer's income and the product's price impact the product's utility value only via their difference. This represents a significant restriction, in particular when dealing with items or services, the unit price of which constitutes a negligible fraction of a typical customer's income.

Caplin and Nalebuff (1991) showed that many of the existing consumer choice models arise as a special case of their model, including the classical models by Hotelling (1929) and Lancaster Kelvin (1966), Perloff and Salop (1985), Jaskold Gabszewicz and Thisse (1979),

measure associated with the product, *i.e.*, $\beta_j = t_{n+1}(x_j)$.

Shaked and Sutton (1982), Economides (1989), Christensen et al. (1975), and Anderson et al. (2001). All of these models specify $\Gamma(Y_i - p_j) = Y_i - p_j$ or $\Gamma(Y_i - p_j) = \log(Y_i - p_j)$. This includes the consumer choice model in the later, seminal Berry et al. (1995) paper; where $G_j(Y_i, p_j) = \beta\Gamma(Y_i - p_j) = \beta\log(Y_i - p_j)$.

We conclude this section with a few preliminary results related to our model. It is easily verified that, in each market segment, the price sensitivity of each firm's demand with respect to its own price is given by

$$\frac{\partial S_{jk}}{\partial p_j} = -g_j(Y, p_j)S_{jk}\left(1 - \frac{S_{jk}}{h_k}\right), \quad j = 1, \dots, J; k = 1, \dots, K, \quad \text{so that} \quad (3.17)$$

$$g_j(Y, p_j) = \frac{-\partial \log S_{jk}}{\partial p_j} / \left(1 - \frac{S_{jk}}{h_k}\right), \quad j = 1, \dots, J; k = 1, \dots, K. \quad (3.18)$$

In other words, $g_j(Y, p_j)$ may be interpreted as the percentage increase in firm j 's market share, due to a unit price decrease, expressed as a fraction of the percentage of market segment k , not yet captured by the firm. We therefore refer to $g_j(\cdot, \cdot)$ as the *price penetration rate*. Similarly, the price sensitivity of firm j 's demand with respect to the competitor's price is given by

$$\frac{\partial S_{jk}}{\partial p_m} = g_m(Y, p_m)S_{mk}S_{jk}/h_k, \quad m \neq j. \quad (3.19)$$

We assume, without loss of essential generality, that for all market segments $k = 1, \dots, K$:

$$\left| \frac{\partial S_{jk}}{\partial p_j} \right| > \sum_{m \neq j} \frac{\partial S_{jk}}{\partial p_m}, \quad j = 1, \dots, J. \quad (3.20)$$

This condition is a classical dominant-diagonal condition (see e.g. Vives (2001)) and merely precludes that a uniform price *increase* by *all* J firms would result in an *increase* of any of the firms' expected sales volume.

3.3 The equilibrium behavior in the price competition model

In this section, we provide a sufficient condition under which the price competition model permits a Nash equilibrium and a second, somewhat stronger, condition under which this equilibrium is unique. These conditions merely preclude a very high degree of market concentration and are easily verified on the basis of the model primitives only. We conclude the section with a sufficient condition for a (unique) Nash equilibrium that applies to markets with an arbitrary degree of market concentration. Unlike, for example, the existence conditions in Caplin and Nalebuff (1991), our conditions allow for arbitrary distributions of the population sizes $\{h_k : k = 1, \dots, K\}$ in the various customer segments.

Recall that, for any of the K market segments, $g_j(Y, p_j)$ may be interpreted as the percentage increase in firm j 's market share - expressed as a function of the percentage of the market segment not yet captured by the firm - due to a *unit* decrease in the firm's prices. Similarly, let

$$\omega_j(Y, p_j) = (p_j - c_j)g_j(Y, p_j), \quad (3.21)$$

denote a *dimensionless* elasticity, *i.e.*, for any of the K market segments the *percentage* increase in firm j 's market share - expressed as a function of the percentage of the market not yet captured by the firm - due to a one *percent* decrease in the *variable profit margin*. As the product of two continuous functions $\omega_j(Y, p_j)$ is continuous, with $\omega_j(Y, c_j) = 0$ and $\lim_{p_j \uparrow \infty} \omega_j(Y, p_j) = \infty$. By the intermediate value theorem, we conclude that, for any critical elasticity level $\eta > 0$ there exists a price level $\bar{p}_j(\eta) > c_j$, with $\omega_j(Y, \bar{p}_j(\eta)) = \eta$. Moreover, ω_j is strictly increasing as the product of an increasing and a strictly increasing function, implying the existence of a *unique* price level $\bar{p}_j(\eta)$ such that for all (p_j^1, p_j^2) with $p_j^1 \leq \bar{p}_j(\eta) \leq p_j^2$,

$$\omega_j(Y, p_j^1) \leq \omega_j(Y, \bar{p}_j(\eta)) = (\bar{p}_j(\eta) - c_j)g_j(Y, \bar{p}_j(\eta)) = \eta \leq \omega_j(Y, p_j^2). \quad (3.22)$$

Our main condition for the existence of a Nash equilibrium in the interior of the price region, or even a *unique* such equilibrium, consists of excluding the possibility of *excessive market concentration*. In particular, existence of a Nash equilibrium can be guaranteed if any single firm captures less than 50% of the potential market in any customer segment when pricing at a level which, under the condition, will be shown to be an *upper bound* for the firm's equilibrium price choice. Similarly, if every single firm captures less than *one third* of the potential market in each segment (- again when pricing at a level which, under the condition, is shown to be an upper bound for his price choice -), a *unique* Nash equilibrium can be guaranteed. Thus, for any maximum market share $0 < \mu < 1$ among all *potential* customers in each segment, we define the following condition:

C(μ) In each market segment $k = 1, \dots, K$, each firm j captures less than μ of the market among all potential customers when pricing at the level $\bar{p}_j((1 - \mu)^{-1})(j = 1, \dots, J)$, (irrespective of what prices the competitors choose within the feasible price range).

As mentioned, the critical maximal market shares μ of importance in the results below are $\mu = 1/2$ and $\mu = 1/3$.

The following lemma shows that, under $C(\mu)$, any firm j 's relevant price region may be restricted to $[c_j, \bar{p}_j((1 - \mu)^{-1})]$.

Lemma 3.3.1 *Fix $\mu > 0$. Under condition $C(\mu)$, the best response of any firm j to any given feasible price vector p_{-j} is a price $c_j \leq p_j^*(p_{-j}) \leq \bar{p}_j((1 - \mu)^{-1})$.*

Proof: Fix $j = 1, \dots, J$. Clearly, $\pi_j(p_j, p_{-j}) < 0 = \pi_j(c_j, p_{-j})$ for any $p_j < c_j$ so that $c_j \leq p_j^*(p_{-j})$. Note that $\pi_j = \sum_{k=1}^K \pi_{jk} = \sum_{k=1}^K (p_j - c_j)S_{jk}$. Invoking (3.17) we obtain

$$\frac{\partial \pi_j}{\partial p_j} = \sum_{k=1}^K S_{jk} \left[1 - (p_j - c_j)g_j(Y, p_j) \left(1 - \frac{S_{jk}}{h_k} \right) \right] = \sum_{k=1}^K S_{jk} \left[1 - \omega_j(Y, p_j) \left(1 - \frac{S_{jk}}{h_k} \right) \right] < 0, \quad (3.23)$$

for any price $p_j \geq \bar{p}_j((1 - \mu)^{-1})$. To verify (3.23), note from (3.22) that for $p_j \geq \bar{p}_j((1 - \mu)^{-1})$, $\omega_j(Y, p_j) \geq (1 - \mu)^{-1}$, while $(1 - \frac{S_{jk}}{h_k}) > (1 - \mu)$. The later inequality follows

from S_{jk} being decreasing in p_j , see (3.17), while $S_{jk}/h_k < \mu$ when $p_j = \bar{p}_j((1 - \mu)^{-1})$, see $C(\mu)$. ■

Thus, the market concentration test $C(\mu)$ is conducted while setting each firm's price level *above* what (, under the condition,) is rational. Therefore, as rational firms will price below $\bar{p}((1 - \mu)^{-1})$, condition $C(\mu)$ does not preclude that, *in equilibrium*, a firm captures a share *above* μ in some or all market segments. Since a firm's market share is maximized when all competitors adopt maximal prices, condition $C(\mu)$ is easily verified as follows:

$$\frac{e^{[U_{jk}(x_j)+G_j(Y,\bar{p}_j)]}}{e^{U_{0k}(x_1,\dots,x_J)} + e^{[U_{jk}(x_j)+G_j(Y,\bar{p}_j)]} + \sum_{m \neq j} e^{[U_{mk}(x_m)+G_m(Y,p_m^{max})]}} \leq \mu, \forall j = 1, \dots, J, \quad k = 1, \dots, K, \quad (3.24)$$

where \bar{p}_j , is shorthand notation for $\bar{p}_j((1 - \mu)^{-1})$. Clearly, the larger the value chosen for p^{max} , the stronger condition $C(\mu)$ becomes. Therefore, if one is unwilling to specify p^{max} upfront, there are two alternative ways to proceed. First, one may determine, $\hat{p}(\mu)$ as the smallest of the JK unique roots of the equations in the single variable p :

$$\frac{e^{[U_{jk}(x_j)+G_j(Y,\bar{p}_j)]}}{e^{U_{0k}(x_1,\dots,x_J)} + e^{[U_{jk}(x_j)+G_j(Y,\bar{p}_j)]} + \sum_{m \neq j} e^{[U_{mk}(x_m)+G_m(Y,p)]}} = \mu \quad j = 1, \dots, J \text{ and } k = 1, \dots, K. \quad (3.25)$$

$C(\mu)$ is satisfied for any $p^{max} \leq \hat{p}(\mu)$. If $\hat{p}(\mu)$ is in excess of a reasonable upper bound for the products' prices, p^{max} may be set to $\hat{p}(\mu)$ without loss of generality and $C(\mu)$ may be assumed up front. Second, the following is a much stronger version of $C(\mu)$, which is obtained by letting $p^{max} \rightarrow \infty$ and is therefore independent of the boundary of the feasible region:

C'(μ) No individual firm j has, in any of the market segments, an expected utility measure larger than that of the no-purchase option, assuming the firm's product is priced at the level $\bar{p}_j((1 - \mu)^{-1})$, *i.e.*,

$$U_{jk}(x_j) + G_j(Y, \bar{p}_j((1 - \mu)^{-1})) + \log(\mu^{-1} - 1) \leq U_{0j}(x_1, \dots, x_J) \quad \forall j = 1, \dots, J, \quad k = 1, \dots, K. \quad (3.26)$$

The implication $C'(\mu) \Rightarrow C(\mu)$ follows, after some algebra, by observing that the left hand side of (3.24) is increasing in p^{max} , and letting $p^{max} \rightarrow \infty$.

For $\mu = 1/2$, condition $C(1/2)$ is easily satisfied in the applications we are familiar with as is its stronger version $C'(1/2)$ ⁷. In these industrial organization studies, no single firm captures the majority of the *potential* market, (in particular when pricing at a most unfavorable price level). For example, in the drive-thru fast food industry studied in Chapter 2, the largest (estimated) market share obtained by the *entire industry* in any one market segment, defined in that paper as customers sharing demographic features and a geographic location, is 0.26. Furthermore, all firms in the study had prices below $\bar{p}(2)$. Therefore, if any given firm j raises its price to $\bar{p}_j(2)$, and all others raise their prices to their maximum levels, the largest market share of the entire industry, and, a fortiori, that of firm j itself, would certainly be below 26%. Therefore, $C(1/2)$ is satisfied. Even condition $C'(1/2)$ is very easily satisfied since the largest value for the left hand side of (3.24) is 0.0047 when $p^{max} = \infty$. Thomadsen (2005a) studies the drive-thru fast food industry in Santa Clara where a similar dispersion of market shares, in each of the market segments considered, can be assumed.

As a last example, consider the ready-to-eat cereal industry, which is widely characterized as one with high concentration, high price-cost margins, a quote from the opening sentence in Nevo (2001); see Schmalensee (1978) and Scherer(1982) for similar characterizations. In this industry, each of the competing manufacturers offers a series of cereals, so an adequate representation of this industry requires a multi-product competition model as in Section 3.6.3. (Indeed, Nevo(2001) has estimated such a multi-product MMNL model for the industry.) In spite of this industry being viewed as one of high concentration, the aggregate market share of Kellogg, the largest competitor, varied between 41,2% in the first quarter of 1988 and 32.6% in the last quarter of 1992, with market shares calculated among all cereal consumers as opposed to the potential consumer population.

⁷The following is another sufficient condition for $C(1/2)$: The no-purchase option captures the majority of the consumer population, in each market segment, even when all firms select p^{min} . While much stronger than $C(1/2)$, it is clearly satisfied in many industries.

We now establish that, under condition $C(1/2)$, a Nash equilibrium exists and that the set of Nash equilibria coincides exactly with the solutions to the system of FOC equations.

Theorem 3.3.2 *Assume condition $C(1/2)$ applies and $\bar{p}(2) \in [p^{min}, p^{max}]$ ⁸.*

(a) *The Price Competition Model has a Nash equilibrium.*

(b) *Every Nash equilibrium p^* is a solution to the First Order Conditions (FOC):*

$$\frac{\partial \pi_j}{\partial p_j} = \sum_{k=1}^K S_{jk} \left[1 - (p_j - c_j) g_j(Y, p_j) \left(1 - \frac{S_{jk}}{h_k} \right) \right] = 0, \quad \forall j = 1, \dots, J, \quad (3.27)$$

and has $c < p^* < \bar{p}(2)$.

(c) *Every solution to the FOC is a Nash equilibrium.*

Proof: To simplify the notation, we write \bar{p} as shorthand for $\bar{p}(2)$.

(a) In order to prove the result on the full price cube, we first establish the existence of a Nash equilibrium p^* in the *interior* of the restricted price cube $X_{j=1}^J [p_j^{min}, \bar{p}_j]$. This follows from the Nash-Debreu theorem as each firm's feasible action set $[p_j^{min}, \bar{p}_j]$ is a compact, convex set and as the profit function $\pi_j(p)$ is concave in p_j on the complete price cube $X_{j=1}^J [p_j^{min}, \bar{p}_j]$. Concavity follows by differentiating (3.23) with respect to p_j

⁸If $\bar{p}(2) \notin [p^{min}, p^{max}]$, it is still possible to establish the existence of a Nash equilibrium, however one in which some or all of the price levels are at the boundary of the feasible price region, *i.e.*, parts (b) and (c) of the theorem fail to apply in this case.

as follows:

$$\begin{aligned}
\frac{\partial^2 \pi_j}{\partial p_j^2} &= \sum_{k=1}^K \left\{ -S_{jk} g_j(Y, p_j) \left(1 - \frac{S_{jk}}{h_k}\right) \left[1 - (p_j - c_j) g_j(Y, p_j) \left(1 - \frac{S_{jk}}{h_k}\right)\right] \right. \\
&\quad \left. - S_{jk} g_j(Y, p_j) \left(1 - \frac{S_{jk}}{h_k}\right) + \frac{S_{jk}}{h_k} (p_j - c_j) g_j(Y, p_j) S_{jk} (-g_j(Y, p_j)) \left(1 - \frac{S_{jk}}{h_k}\right) \right\} \\
&\quad - \frac{\partial g_j(Y, p_j)}{\partial p_j} (p_j - c_j) \sum_{k=1}^K S_{jk} \left(1 - \frac{S_{jk}}{h_k}\right), \tag{3.28} \\
&= \sum_{k=1}^K h_k g(Y, p_j) \left(\frac{S_{jk}}{h_k}\right) \left(1 - \frac{S_{jk}}{h_k}\right) \left[-2 + g_j(Y, p_j) (p_j - c_j) \left(1 - 2 \frac{S_{jk}}{h_k}\right)\right] \\
&\quad - \frac{\partial g_j(Y, p_j)}{\partial p_j} (p_j - c_j) \sum_{k=1}^K S_{jk} \left(1 - \frac{S_{jk}}{h_k}\right) \\
&\quad < 0.
\end{aligned}$$

To verify the inequality, note that the second term to the right of (3.28) is negative since $g_j(Y, p_j)$ is increasing in p_j (see (3.4)). As to the first term, it follows from (3.22) that $g_j(Y, p_j)(p_j - c_j) < 2$ for all $p_j < \bar{p}_j$. Thus, since $S_{jk}/h_k \geq 0$,

$$\left[-2 + g_j(Y, p_j)(p_j - c_j) \left(1 - 2 \frac{S_{jk}}{h_k}\right)\right] < 0, \quad k = 1, \dots, K. \tag{3.29}$$

We have shown that a price vector p^* exists which is a Nash equilibrium on the restricted price cube $X_{j=1}^J[p_j^{min}, \bar{p}_j]$. To show that p^* is a Nash equilibrium on the full price range $X_{j=1}^J[p_j^{min}, p_j^{max}]$ as well, it suffices to show that $\pi_j(p_j, p_{-j}^*) < \pi_j(\bar{p}_j, p_{-j}^*) \leq \pi_j(p_j^*, p_{-j}^*) \forall p_j > \bar{p}_j$.

The first inequality follows from Lemma 3.3.1, while the second inequality follows from the fact that p^* is a Nash equilibrium on the price vector $X_{j=1}^J[p_j^{min}, \bar{p}_j]$.

(b) In view of Lemma 3.3.1 and since $p^{min} \leq c$, any price equilibrium $p^* \in X_{j=1}^J[p_j^{min}, \bar{p}_j]$.

To show that it is, in fact, an *interior* point of $X_{j=1}^J[c_j, \bar{p}_j]$, and hence a solution of the FOC (3.27), note that $\frac{\partial \pi_j(c_j, p_{-j}^*)}{\partial p_j} = \sum_{k=1}^K S_{jk} = S_j > 0$ while $\frac{\partial \pi_j(\bar{p}_j, p_{-j}^*)}{\partial p_j} < 0$, by Lemma 3.3.1.

(c) Consider a solution, p^* of the FOC (3.27). It follows from (3.23) that $p_j^* < \bar{p}_j \forall j = 1, \dots, J$. In view of the concavity of $\pi_j(p_j, p_{-j})$ in p_j on the price cube $X_{j=1}^J[p_j^{min}, \bar{p}_j]$, p^* is a Nash equilibrium on this price cube and, by the proof of part (a) on the full price range $X_{j=1}^J[p_j^{min}, p_j^{max}]$ as well. ■

The following theorem establishes that a *unique* Nash equilibrium can be guaranteed under the slightly stronger condition $C(1/3)$.

Theorem 3.3.3 *Assume condition $C(1/3)$ applies and $\bar{p}(3/2) \in [p^{min}, p^{max}]$.*

(a) *The price competition model has a unique Nash equilibrium $p^* < \bar{p}(3/2)$ which satisfies the FOC equations (3.27).*

(b) *The FOC equations (3.27) have p^* as their unique solution.*

Proof: Following the proof of Theorem 3.3.2, replacing $\mu = 1/2$ by $\mu = 1/3$, we obtain the existence of a Nash equilibrium $c < p^* < \bar{p}(3/2)$, which is a solution to the FOC equations (3.27), and, vice versa, every solution to this system of equations is a Nash equilibrium. Moreover, without loss of generality, the price region may be restricted to $P = X_{j=1}^J[c_j, \tilde{p}_j(3/2)]$. It therefore suffices to show that the equilibrium is unique. We establish this by showing that on the price region P :

$$\left| \frac{\partial \pi_j^2}{\partial p_j^2} \right| > \sum_{m \neq j} \left| \frac{\partial^2 \pi_j}{\partial p_j \partial p_m} \right| \quad , \quad j = 1, \dots, J. \quad (3.30)$$

This inequality is a sufficient condition for the best response function to be a contraction mapping, see Vives (2001). Fix $j = 1, \dots, J$. By the definition of $\bar{p}_j(3/2) = \bar{p}_j((1-1/3)^{-1})$ and (3.22), we have

$$g_j(Y, p_j)(p_j - c_j) < 3/2 \quad \forall p_j < \bar{p}_j(3/2), \quad \text{so that}$$

$$1/2 > 1 - g_j(Y, p_j)(p_j - c_j)(1 - 2\frac{S_{jk}}{h_k}) > -0.5, \quad \forall p_j < \bar{p}_j(\frac{3}{2}), \quad \forall k = 1, \dots, K, \quad (3.31)$$

$$2 - g_j(Y, p_j)(p_j - c_j)(1 - 2\frac{S_{jk}}{h_k}) > +0.5, \quad \forall p_j < \bar{p}_j(\frac{3}{2}), \quad \forall k = 1, \dots, K, \quad (3.32)$$

by the mere fact that $1/3 \geq S_{jk}/h_k > 0$. In particular, for all $p_j < \bar{p}_j(\frac{3}{2})$, and all $k = 1, \dots, K$:

$$2 - g_j(Y, p_j)(p_j - c_j)(1 - 2\frac{S_{jk}}{h_k}) > |1 - g_j(Y, p_j)(p_j - c_j)(1 - 2\frac{S_{jk}}{h_k})|. \quad (3.33)$$

Multiplying (3.20) with (3.33) and summing over all $k = 1, \dots, K$, we obtain:

$$\begin{aligned} \left| \frac{\partial^2 \pi_j}{\partial p_j^2} \right| &= \sum_{k=1}^K \left| \frac{\partial S_{jk}}{\partial p_j} \right| \left[2 - g_j(Y, p_j)(p_j - c_j)(1 - 2\frac{S_{jk}}{h_k}) \right] \\ &\quad + \frac{\partial g_j(Y, p_j)}{\partial p_j} (p_j - c_j) \sum_{k=1}^K S_{jk} (1 - \frac{S_{jk}}{h_k}) \\ &> \sum_{k=1}^K \left| \frac{\partial S_{jk}}{\partial p_j} \right| \left[2 - g_j(Y, p_j)(p_j - c_j)(1 - 2\frac{S_{jk}}{h_k}) \right] \\ &> \sum_{k=1}^K \sum_{m \neq j} \frac{\partial S_{jk}}{\partial p_m} \left[1 - g_j(Y, p_j)(p_j - c_j)(1 - 2\frac{S_{jk}}{h_k}) \right] \\ &> \sum_{m \neq j} \left| \sum_{k=1}^K \frac{\partial S_{jk}}{\partial p_m} \left[1 - g_j(Y, p_j)(p_j - c_j)(1 - 2\frac{S_{jk}}{h_k}) \right] \right| = \sum_{m \neq j} \left| \frac{\partial^2 \pi_j}{\partial p_j \partial p_m} \right|, \end{aligned} \quad (3.34)$$

where the first inequality follows from (3.4), thus completing the verification of (3.30). ■

The conditions needed for existence and uniqueness, $C(\mu)$, bear a remarkable relation to standard policy criteria used to define “moderately” or “highly concentrated” markets. The Department of Justice (DOJ) and the Federal Trade Commission (FTC) measure the degree of concentration in a market via the Herfindahl-Hirschman Index (HHI), defined as the sum of the squares of the market shares represented as percentages. (This index has the maximum value of 10,000 in case of a monopoly and approaches zero if the market is divided among a very large number of competitors with an equal market share.) The

DOJ-FTC 1992 Horizontal Merger Guidelines define a market with an HHI below 1,000 as “unconcentrated”, one between 1,000 and 1,800 as “moderately concentrated”, and those with an HHI above 1,800 as “highly concentrated”. Interestingly, when $C(1/3)$ is violated the minimum possible HHI equals 1,111, and 2,500 when $C(1/2)$ is violated⁹¹⁰. (Thus, while it is unclear what the cut off value of 1,800 was based on, it corresponds with the average of the minimum HHI-values when $C(1/2)$ and $C(1/3)$ are violated.) These 1992 DOJ-FTC guidelines were updated in April 2010 and the new HHI cutoff level for a “highly concentrated market” has been increased from 1,800 to 2,500, the minimal value when $C(1/2)$ is violated.

3.4 Counter Example

The following counter example demonstrates that a condition like $C(1/2)$, broadly applicable as it is, is necessary for the existence of a Nash Equilibrium. Our counter example was inspired by Su and Judd (2008) who exhibit that multiple equilibria may arise in a price competition model with 2 firms (no outside good) and 3 customer segments, and a combination of linear and Constant Elasticity of Substitution (CES) demand functions. (A similar example, showing the existence of *multiple* equilibria was, apparently, identified by Dube et al. (2008a).) Consider a market with two firms and three consumer segments (*i.e.*, $J = 2$, $K = 3$) whose consumer utility functions are defined as follows:

$$\text{Firm 1: } U_{i11} = A - p_1 + \epsilon_{i11}; \quad U_{i12} = \epsilon_{i12}; \quad U_{i13} = B - p_1 + \epsilon_{i13};$$

$$\text{Firm 2: } U_{i21} = \epsilon_{i21}; \quad U_{i22} = A - p_2 + \epsilon_{i22}; \quad U_{i23} = B - p_2 + \epsilon_{i23}.$$

⁹These minima arise when a single firm captures one third or half of the market, respectively, with the remainder of the market being divided equally among infinitely many competitors.

¹⁰The FTC calculates the HHI based on the anticipated post-merger equilibrium, measuring market shares as a percentage of aggregate sales in the industry. Our $C(\mu)$ conditions put “market concentration” in a favorable light measuring each firm’s market share as a percentage of the total *potential* customer population and under the assumption that the firm selects an above rational price level.

In this example, potential consumers in segment 1 (2) are entirely focused on firm 1 (2) and purchase the good or service as long as its price is below a consumer specific reservation value. In contrast, consumers in market segment 3 are potentially attracted by either firm. The purchase decisions of segment 3 customers are therefore based on both firms' pricing decisions. Following the derivation of (3.6) and (3.7), the demand functions for firm 1 and 2 are therefore given by

$$D_1 = N_1 \frac{e^{A-p_1}}{1+e^{A-p_1}} + N_2 \frac{1}{1+e^{A-p_2}} + N_3 \frac{e^{B-p_1}}{e^{B-p_1}+e^{B-p_2}}, \quad (3.35)$$

$$D_2 = N_1 \frac{1}{1+e^{A-p_1}} + N_2 \frac{e^{A-p_2}}{1+e^{A-p_2}} + N_3 \frac{e^{B-p_2}}{e^{B-p_1}+e^{B-p_2}}. \quad (3.36)$$

The profit for each firm is given by $\pi_j = (p_j - c_j)D_j$. The following set of parameters specify a game without a Nash equilibrium: $A = 4$, $B = 2$, $c_1 = c_2 = 1$, $N_1 = 1$, $N_2 = 2$, $N_3 = 3$ and $p_{j,max} = 10$. The following defines a cycle of best responses which is reached from any starting point in the feasible price region $[1,10] \times [1,10]$, where $br_1(p_2)$ denotes the best response of firm 1 to firm 2's price choice, p_2 , and vice versa for $br_2(p_1)$:

$$br_1(7.08) = 10; \quad br_2(10) = 8.79; \quad br_1(8.79) = 8.11; \quad br_2(8.11) = 7.08.$$

Notice that the parameters specified above violate condition $C(1/2)$, as well as the dominant diagonal condition specified in equation (3.20). However, the dominant diagonal condition, while necessary for the uniqueness of an equilibrium, is not necessary for its existence. The counterexample not only demonstrates the necessity of a condition like $C(1/2)$, but also reinforces the fact that the existence of a (unique) equilibrium can not be taken for granted. With many structural estimation models relying on the existence of a (unique) equilibrium when estimating market parameters and evaluating policies, it is important to note that without an existence guarantee for an equilibrium, these methods may result in flawed estimates.

3.5 Extensions

In this section, we discuss several generalizations of the basic model.

3.5.1 Unequal Income or Firm Size Level

Thus far, we have assumed that all potential customers share the same income level Y . Our equilibrium results carry over to the case of general income distributions provided the price-income sensitivity functions $G_j(\cdot, \cdot)$ are separable, *i.e.*,

$$G_j(Y_i, p_j) = G_j^1(Y_i) + G_j^2(p_j), \quad (3.37)$$

with G_j^2 decreasing and concave. To model income or firm size heterogeneity, design the market segmentation to be based, in part, on the income level such that all potential customers in segment $k = 1, \dots, K$ share the same income level Y_k . This case is easily handled under separable price-income sensitivity functions by replacing the term $U_{jk}(x_j)$ in (3.1) by $\bar{U}_{jk}(x_j) = U_{jk}(x_j) + G_j^1(Y_k)$, and $G_j(Y_i, p_j)$ by $\bar{G}_j(Y_i, p_j) = G_j^2(p_j)$ for all $j = 1, \dots, J$ and $k = 1, \dots, K$. Since $\bar{G}_j(\cdot, \cdot)$ does not depend on the income level, all results in Section 3.3 continue to apply. If a continuous income distribution is required, a model with a continuous rather than a finite set of customer types or segments is called for. See the next subsection for a treatment of this case.

In the presence of income heterogeneity, it would clearly be of interest to extend our results to settings where the price-income sensitivity functions fail to be separable, so that even the *marginal* utility functions $g_j(Y_j, p_j) = \frac{\partial u_{ijk}}{\partial p_j} \frac{\partial G_j(Y_i, p_j)}{\partial p_j}$ differ by income level. More generally, one would like to extend our results to settings where the price-income sensitivity functions themselves depend *both* on the firm and the customer segment in general ways, *i.e.*, not just via the customer's income level. In other words, one would like to generalize the consumer choice model (3.1) to allow for double-indexed price-income sensitivity functions $G_{jk}(Y_k, p_j)$, $j = 1, \dots, J$ and $k = 1, \dots, K$. Such

general dependencies on the customer segment have thus far failed to be tractable, see, for example Caplin and Nalebuff (1991). Indeed, in Caplin and Nalebuff's treatment of the case of income heterogeneity, *i.e.*, section 8.1, only *linear* price-income sensitivity functions are allowed, see assumption A1 *ibid*¹¹.

3.5.2 A Continuum of Customer Types

In some applications, a *continuum* of customer types need to be considered in the consumer choice model. Our model is easily respecified to allow for a continuum of customer types $\theta \in \Theta$, with a density function $h(\theta)$. As before, assume first that all potential customers share the same income level Y ; the generalization to arbitrary income distributions is handled as in Subsection 3.5.1. Let:

$$u_{ij}(\theta) = U_j(x_j|\theta) + G_j(Y, p_j) + \epsilon_{ij}(\theta), \quad j = 1, \dots, J \quad \text{and} \quad i = 1, 2, \dots, \quad (3.38)$$

$$u_{i0}(\theta) = U_0(x_1, \dots, x_J|\theta) + \epsilon_{i0}(\theta), \quad i = 1, 2, \dots \quad (3.39)$$

Here, u_{ij} denotes the utility value attributed by the i -th customer of type $\theta \in \Theta$, to product j , $j = 0, \dots, J$, and for all $j = 0, \dots, J$ and types $\theta \in \Theta$, $\{\epsilon_{ij}(\theta)\}$ represents a sequence of independent random variables with Gumbel distributions. It is easily verified that the demand functions in (3.7) need to be replaced by:

$$S_j = \int_{\theta \in \Theta} S_{j\theta} d\theta = \int_{\theta \in \Theta} h(\theta) \frac{e^{[U_j(x_j|\theta) + G_j(Y, p_j)]}}{e^{[u_0(x_1, \dots, x_J|\theta)]} + \sum_{m=1}^J e^{[U_m(x_m|\theta) + G_m(Y, p_m)]}} d\theta. \quad (3.40)$$

All of the results in Section 3.3 continue to apply.

¹¹Berry et al. (1995) appear, in the presence of income heterogeneity, to allow for a price-income sensitivity function that is non-separable, *i.e.*, $g_j(Y_i, p_j) = \alpha \log(Y_i - p_j)$, see eq. (2-7a) *ibid*. As mentioned in the Introduction, their footnote 12 suggests that only the multi-product feature of their model precludes reliance on Caplin and Nalebuff (1991). In actuality, the choice of a *non-separable* price-income sensitivity function provides a second reason why the existence results in Caplin and Nalebuff do not apply to their model.

3.5.3 The Multi-Product Case

In some settings, each firm j sells a series, say $n_j \geq 1$ of products in the market. Assuming the choice model described by equations (3.1) and (3.2) applies to each of the $\sum_{j=1}^J n_j$ products (and the no-purchase option), can simple and broadly applicable conditions, similar to condition $C(1/2)$, be identified under which the existence of a price equilibrium is guaranteed for the general “multi-product” price competition model?

To address this question, identify each product by a double index (j, r) with the first index denoting the product’s firm identity. Thus, for the r -th product of firm j , append the double index (j, r) to each relevant variable and parameter. Analogous to (3.22), define $\bar{p}_{jr}(2)$ as the unique price level for product (j, r) for which

$$(p_{jr} - c_{jr})g_j(Y, p_{jr}) = 2, \quad j = 1, \dots, J, \quad r = 1, \dots, n_j. \quad (3.41)$$

It is, again, possible to show that no firm j would choose to set *all* of its products’ prices at or above the $\{\bar{p}_{jr}(2) : r = 1, \dots, n_j\}$ levels, under a generalization of condition $C(1/2)$ which states that no firm’s *total* sales across *all* of its products exceeds 50% of the potential market in any one market segment, under such price choices. However, it is conceivable that a firm would choose *some* of its products’ prices to exceed their $\bar{p}(2)$ -levels. Therefore, the proof of Theorem 3.3.2 cannot be generalized in a direct way. At the same time, in Chapter 2 we showed that a Nash equilibrium, in fact a *unique* equilibrium, exists if the maximum prices $p^{max} \leq \bar{p}(1)$ with $\bar{p}_{jr}(1)$ the unique price level such that $(p_{jr} - c_{jr})g_j(Y, p_{jr}) = 1$, $j = 1, \dots, J$ $r = 1, \dots, n_j$, see Section 2.5.

3.6 Structural Estimation Methods

In this section, we discuss the implications of our results for the econometrician desiring to estimate the parameters of a model with MMNL demand function. Very often, empiricists implicitly or explicitly “assume” that (I) the model possesses an equilibrium

and (II) that any equilibrium, in particular the prevailing price vector (when observed), satisfies the system of FOC (3.27)¹². The problems arising due to the potential existence of multiple equilibria or no equilibrium, have been featured prominently in recent papers, as well as the fact that a solution to the system of FOC equations may fail to be an equilibrium and vice versa. See, for example, Tamer (2003), Schmedders and Judd (2005), Ferris et al. (2006), Aguirregabiria and Mira (2007)¹³, Ciliberto and Tamer (2009) and Dube et al. (2008b). Theorem 3.3.2 shows that under condition $C(1/2)$ assumptions (I) and (II) indeed apply.

In most applications, very high degrees of market concentration can be ruled out on *a priori* grounds and condition $C(1/2)$ may be assumed to hold upfront. One example is the aforementioned drive-thru fast food industry, the industry modeled with MMNL demand functions in both Thomadsen (2005a) and Chapter 2 of this dissertation: Elementary statistical studies reveal that even when aggregating across all chains, the fast food industry captures a minority of the potential market in any relevant demographic segment. Going forward, we distinguish between two types of estimation settings: estimation under an observed prices vector and estimation absent price observations.

3.6.1 Structural Estimation With a Given Observed Price Vector

As reviewed in Section 3.1, in many structural estimation studies a specific price vector p^* is observed. Assumptions (I) and (II), mentioned above, are essential for the methods to be used at all, because the structural estimation techniques rest on the assumption that the observed prices represent a price equilibrium which is the solution to the system of FOC equations (3.27). Condition $C(1/2)$ confirms both assumptions. Moreover, the model's parameters are, invariably, determined by solving a mathematical program.

¹²See for example, the quote in the introduction of Berry et al. (1995)

¹³These authors note, for example: "The existence of multiple equilibria is a prevalent feature in most empirical games where best response functions are nonlinear in other players' actions. Models with multiple equilibria do not have a unique reduced form, and this incompleteness may pose practical and theoretical problems in the estimation of structural parameters."

(This applies both to the General Method of Moments and Maximum Likelihood Estimation techniques, see *e.g.*, Nevo (2000b).) As discussed above, high degrees of market concentration can, quite frequently, be ruled out on a priori grounds and therefore condition $C(1/2)$ may be assumed to hold upfront. In such cases, in view of Lemma 3.3.1, the following constraints can be added to the mathematical program:

$$\omega_j(Y, p_j^*) = (p_j^* - c_j)g_j(Y, p_j^*) \leq 2 = \omega_j(Y, \bar{p}_j(2)), \quad j = 1, \dots, J, \quad (3.42)$$

as they represent *necessary* conditions for an equilibrium when $C(1/2)$ holds, no less than the FOC equations (3.27) themselves¹⁴. This not only guarantees that the parameter estimates are consistent with the observed price vector being an equilibrium, it improves the estimation itself: As argued by Dube et al. (2008b) and Su and Judd (2008) for general models, and in the specific model of Chapter 2 of this dissertation, much is gained by restricting a numerically difficult search for optimal parameter values to its relevant region via the addition of known necessary conditions. As mentioned, typically, the price-income sensitivity functions $G_j(\cdot, \cdot)$ are specified within a given parametrized family of functions, *e.g.*, $G_j(Y, p_j) = g_j^1(Y) - \alpha_j p_j$, or $G_j(Y, p_j) = \alpha_j \log(Y - p_j)$. In the former case, (3.42) reduces to $\alpha_j \leq \frac{2}{(p_j^* - c_j)}$ and to $\alpha_j \leq \frac{(Y - p_j^*)}{(p_j^* - c_j)}$ in the latter. The marginal cost vector c is sometimes known and sometimes part of the parameters that need to be estimated. In the latter case, (3.42) represents a joint constraint on the parameter(s) specifying the $G_j(\cdot, \cdot)$ function and the c_j values.

Conversely, if the constraints (3.42) are not added to the mathematical program they represent a useful test for the validity of obtained estimates: If some of the inequalities in (3.42) are violated for the computed parameter estimates while condition $C(1/2)$ holds, the observed price vector *fails* to be an equilibrium under the computed parameter estimates, see Lemma 3.3.1.

¹⁴Note that we do *not* propose adding condition $C(1/2)$, via the inequalities (3.24), to the mathematical program since $C(1/2)$ represents a *sufficient* condition for existence only (albeit one that is very widely satisfied).

After the vector(s) of parameter estimates (and, if applicable, the estimate for c) are obtained, it is useful to double-check whether condition $C(1/2)$ is indeed satisfied. This test reduces to making the JK numerical comparisons in (3.24) with $\mu = 1/2$. If so, the observed price vector p^* is a (close approximation of a) Nash equilibrium under the estimated parameter vector. If positive, the same test (3.24), with $\mu = 1/2$ replaced by $\mu = 1/3$, guarantees that p^* is in fact the *unique* equilibrium.¹⁵

After the model parameters are estimated, most studies proceed to conduct counterfactual investigations. To predict changes in the price equilibrium and corresponding sales volumes resulting from a given change in one or several of the model's parameters it is important to know whether a *unique* equilibrium exists. The uniqueness conditions in Theorem 3.3.3 can again be used for this purpose: as mentioned, the former reduces to making the JK comparisons in (3.24) with $\mu = 1/3$ using the estimated parameters, while the latter reduces to the vector comparison $p^{max} \leq \bar{p}(1)$.

If condition $C(1/2)$ applies but condition $C(1/3)$ fails, one may still be able to establish that p^* is the unique equilibrium, based on an *ex post* numerical test. After all, under $C(1/2)$, in view of Theorem 3.3.2, it suffices to verify that the system of FOC (3.27) has the observed price vector p^* as its unique solution on the cube $X_{j=1}^J[p^{min}, p^{max}]$ under the parameter estimates by employing any of the known algorithms that identify all solutions to a system of equations. Thus, the characterization in parts (b) and (c) of Theorem 3.3.2 of the set of Nash equilibria as the solutions to (3.27) may be of great value in empirical studies.

An alternative *ex post* uniqueness test, under $C(1/2)$, is to verify that the *single* non-linear function given by the determinant of the Jacobian matrix associated with (3.27) has *no* root, *i.e.*,

$$\det J(p) \neq 0 \quad \forall p \in X_{j=1}^J[\hat{c}_j, \bar{p}_j], \quad (3.43)$$

¹⁵Of course, even if condition $C(1/2)$ holds for the computed parameters $\hat{\theta}$, it is conceivable that, in the absence of constraints (3.42) a different parameter vector θ' would be found with a somewhat better GMM norm or maximum likelihood value and with some of the constraints (3.42) violated. This can only happen, in the rare case where condition $C(1/2)$ is violated under θ' . In this case there is no guarantee but it is possible that the observed price vector is an equilibrium under θ' as well as under θ .

where $J(p)$ is an $J \times J$ matrix with $J(p_{mj}) = \partial^2 \pi_m / \partial p_m \partial p_j$. The validity of (3.43) follows from Kellogg (1976). (Recall, Theorem 3.3.2 (b) excludes the existence of equilibria on the boundary of the price region.)

3.6.2 Structural Estimation of the Game in the Absence of an Observed Price Vector

In other studies, the parameters of the price competition game need to be estimated in the absence of an observed price vector. This happens, for example, when estimating dynamic multi-stage games, see *e.g.*, Doraszelski and Pakes (2007). Most estimation methods consist of optimizing some objective $L(\theta, p(\theta))$ over all possible parameters vectors θ and all price vectors $p(\theta)$ that arise as a Nash equilibrium under θ . The objective may be a maximum likelihood function or pseudo-maximum likelihood function, see Aguirregabiria and Mira (2002, 2007). Alternatively it may be a (generalized) method-of-moments norm, see *e.g.* Pakes et al. (2004). The characterization of the equilibria $p(\theta)$ as the solutions to the FOC equations (3.27) helps, once again, enormously for any of these estimation methods: Traditional estimation methods, starting with Rust (1987)'s (nested) fixed point algorithmic approach, have projected the associated optimization problems onto the parameter space Θ ; solving an optimization problem of the type:

$$\min\{L(\theta, p(\theta)) \mid \theta \in \Theta \text{ and } p(\theta) \text{ is an equilibrium under } \theta\}. \quad (3.44)$$

This means that a search is conducted through the parameter space and whenever a specific trial parameter vector $\hat{\theta} \in \Theta$ is evaluated, all associated price equilibria $p(\hat{\theta})$ are computed. As pointed out, for example by Aguirregabiria and Mira (2007), this approach may be infeasible even for simple models. A further complication is that even the computation of the equilibria $p(\theta)$, for any single parameter vector θ , may be very difficult. Many have concluded that games in which multiple equilibria may exist can not be estimated, and have restricted themselves to model specifications in which uniqueness

of the equilibrium can be guaranteed, at a minimum. An example of this approach is Bresnahan and Reiss (1990) in the context of empirical games of market entry. While an “ideal” model would specify the profit function of a firm to be dependent on the specific identity of the competitors, Bresnahan and Reiss addressed a specification where it depends only on the *number* of competitors in the market, thus ensuring the existence of a *unique* equilibrium, at the expense of ignoring the impact of heterogeneity. In the context of our class of price competition models, an analogous approach would be to suppress heterogeneity among customer preferences and to assume they all belong to a single (homogeneous) market segment. Fortunately, no such model restrictions are necessary. As pointed out by Dube et al. (2008b) and Su and Judd (2008), the prevalence of multiple equilibria can comfortably be dealt with, as long as the set of equilibria can be characterized as the solutions to a (closed form) set of equations like the FOC equations (3.27). Within the context of our class of price competition models, this characterization is obtained by Theorem 3.3.2. Instead of optimizing the projected unconstrained problem (3.44), Theorem 3.3.2 permits us to estimate the parameters by solving the *constrained* optimization problem:

$$\min\{L(p, \theta) : \theta \in \Theta \text{ and (3.27)}\}. \quad (3.45)$$

As explained above, if $C(1/2)$ can be assumed on a priori grounds, in view of Lemma 3.3.1 constraints (3.42) could be added to (3.45); since these represent necessary conditions under $C(1/2)$,

$$\min\{L(p, \theta) : \theta \in \Theta, \text{ (3.27) and (3.42)}\}. \quad (3.46)$$

We refer to Section 3.6.1 for a discussion of how uniqueness of an equilibrium can be guaranteed ex ante or confirmed ex post.

3.7 Acknowledgement

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Chapter 4

The impact of horizontal mergers and acquisitions in price competition models

We refer to Section 1.5 for an introduction and summary of the models used and results obtained in this chapter. This chapter is organized as follows. In Section 4.1, we give a brief literature review. Section 4.2 presents our general model with some preliminaries. The fundamental comparison results are obtained in Section 4.3, while section 4.4 describes the relationship between our results and the current discussion of the so-called Upward Pricing Pressure (UPP) measure as a proxy for the actual changes in equilibrium prices, led by Farrell and Shapiro (2010a)¹.

4.1 Literature Review

Few topics in industrial organization economic theory have been driven as intensively by policy, legislations, and legal debates and innovation as the impact mergers and acquisitions have on the market equilibrium. As mentioned Williamson (1968) appears to have

¹Carl Shapiro is chief economist of the Antitrust Division of the Department of Justice (DOJ) and Joseph Farrell is head of the Bureau of Economics at the Federal Trade Commission (FTC).

been the first contribution to the literature in this area. The author demonstrated, with a simple model, that even if a merger results in price increases, these may be accompanied with reductions in marginal costs due to synergies. The combined effect on aggregate surplus or welfare may therefore be positive in spite of universal price increases in the industry. As elementary as this observation is in 2011, Williamson's (1968) insights directly challenged prior court criteria in the U.S., attempting to apply anti-trust laws such as the Clayton Act. For example, in the 1962 case of *Brown Shoe vs. United States*, the court refused to entertain the argument that cost efficiencies arising from the merger could result in increased welfare. In 1967, the Supreme Court went even further when evaluating Procter and Gamble's acquisition of Clorox. It argued that such cost synergies should actually be viewed as an additional argument *against* the merger, in as much as they result in additional profit and cash flow enhancements of the merged enterprise.

Prior to the eighties, the strategy literature posited that aggregate profits of merging firms should increase even in the absence of any cost synergies, see *e.g.*, Steiner (1975) chapters 2 and 3. As mentioned in the Introduction, the first attempts to establish this result in a formal oligopoly model are due to Salant et al. (1983), Szidarovszky and Yakowitz (1982), and Davidson and Deneckere (1984). However, these papers found that aggregate profits of merging firms may, in fact, decline. All three of these papers analyzed the merger effects in the context of Cournot competition for a homogeneous good. Perry and Porter (1985) countered that the enigmatic outcome in, for example, Salant et al. (1983), is due to the authors ignoring cost synergies resulting from a merger in their homogeneous Cournot model. These authors show that the aggregate profits of merging firms are guaranteed to increase, if the cost synergies are sufficiently large.

Continuing to address Cournot oligopolies with homogeneous goods, Farrell and Shapiro (1990) expanded the discussion to the impact mergers have on the *equilibrium price*. (In a model with homogeneous goods, all products are sold for the same price.) Farrell and Shapiro show that the equilibrium *price* increases under linear cost structures and in the absence of cost synergies. These authors also derive a necessary and sufficient condition

for a price increase under certain classes of *non-linear* cost functions and possible cost synergies².

Deneckere and Davidson (1985) made a seminal contribution to the discussion, showing that all of the anticipated effects can be guaranteed in specific classes of (Bertrand-) *price* competition models with differentiated goods: in the absence of cost synergies, equilibrium prices are guaranteed to increase, the equilibrium profits of a merged enterprise exceed the aggregate of the pre-merger profits of the merging firms while the equilibrium profits of all other firms increase as well. The authors established these results in a symmetric model with linear demand and cost functions. (As mentioned in our Introduction, in their appendix, the authors extend these results to non-linear demand functions, under five conditions, the most important of which is that the industry is symmetrically differentiated, see *ibid.*)

Thereafter, a few attempts have been made to generalize the Deneckere and Davidson (1985) results to more general models, allowing for asymmetry among the firms or general non-linear demand functions. Zhao and Howe (2010) generalize the Deneckere and Davidson results to models with linear demand and cost functions such that in each product's demand function the coefficient in front of the product's own price is product specific but a single uniform coefficient applies to all cross terms in all demand functions. Werden and Froeb (1994) established these results for a model with multinomial logit demands and linear costs; these authors applied the model to the U.S. market of long-distance carriers, calculating the impact of various potential mergers. Levy and Reitzes (1992) established the above results in a model where all consumers and all n firms are located on a circle: each consumer patronizes the firm whose full price, consisting of a direct *price* plus a travel cost proportional to the distance to the firm, is lowest. As mentioned in the Introduction, the influential survey chapters by Whinston (2006, 2007) *conjecture* that the results in Deneckere and Davidson should apply to general supermodular price competition models, a conjecture our essay confirms under a few additional conditions.

²Unfortunately, the necessary and sufficient condition is stated in terms of the pre-merger and post-merger equilibria outcomes rather than the primitives of the model

As discussed, merger analysis has become a standard tool to evaluate the impact of potential mergers, a trend stimulated by the development of effective structural econometric methods for oligopoly models. Here, the demand and cost functions in a price competition model are estimated. Thereafter, a counterfactual study is undertaken to estimate the price-, market share- and profit implications of a potential merger. Examples include Werden and Froeb (1994) for the market for long distance carriers, Nevo (2000a) for the ready-to-eat cereal industry, Dube (2005) for the soft drink industry and Thomadsen (2005a) for the fast-food drive thru industry in Santa Clara County. See Berry and Pakes (1993) for a general discussion of the use of the above econometric methods to enable merger simulations and Baker and Bresnahan (1985) for an early application based on more elementary estimation methods.

All of these merger simulation studies expect and confirm the above mentioned phenomena in terms of increases in equilibrium prices and profits. We refer the reader to section 4.4 for a review of the literature discussing alternatives to merger simulation, as tools to approximate the impact of mergers and acquisitions.

Finally, we show in subsection 4.2 that the post-merger equilibrium depends in a significant way on the specific structural form by which cost synergies impact the products' cost functions. As described there, after any merger plan is announced, typically, high level operational consultants are retained to characterize and quantify these synergies. In spite of the enormous impact these synergy assessment projects have, little attention has been devoted to this topic in the operations literature. Noted exceptions are Gupta and Gerchak (2002) and Iyer and Jain (2004).

4.2 The Model

We initially consider an industry with N firms, each offering a single product to the market. (In subsection 4.3 we generalize our results to industries with general multi-product firms.) The expected demand volumes for these products depend on all product

prices according to a general system of demand equations. Each firm selects its price level from a given, closed, price interval. The cost incurred by each firm depends on its sales volume according to a given, possibly nonlinear, cost function. We characterize the impact of a merger of several of the firms, without loss of generality the first I firms, with $2 \leq I \leq N$. Thus, for each firm i , $i = 1, \dots, N$, let

p_i = the price selected;

p_i^{min} (p_i^{max}) = the minimum (maximum) feasible price;

$d_i(p) = d_i(p_1, \dots, p_N)$ = the expected sales volume;

$C_i(d_i)$ = the total cost incurred by firm i , specified as a differentiable function of its sales volume.

We use the common notation, p_{-i} , to denote the $(N-1)$ -dimensional vector of prices pertaining to all but firm i 's prices. Similarly, we denote by p_{-I} , the $(N-I)$ -dimensional price vector (p_{I+1}, \dots, p_N) .

We consider fully general differentiable demand functions, merely assuming, without loss of generality, that:

$$\frac{\partial d_i}{\partial p_i} \leq 0 \quad \forall i = 1, \dots, N \quad \text{and} \quad \frac{\partial d_i}{\partial p_j} \geq 0 \quad \forall i \neq j \quad (4.1)$$

i.e., each product's demand function is downward sloping in its own price and nondecreasing in any of the competing products' prices.

As to the price bounds, $\{p_i^{min}\}$ and $\{p_i^{max}\}$ we assume that they are set loosely enough as to be non-binding whenever a firm determines the best response to a given set of choices by the competitors. We impose these bounds merely to ensure that the feasible price range for each product is a compact set.

The expected profit function of each firm when operating by itself, is thus given by

$$\pi_i(p) = p_i d_i(p) - C_i(d_i(p)), \quad i = 1, \dots, N \quad (4.2)$$

In this section, we assume that when the first I firms merge, this merger *does not* result in any cost savings, *i.e.*, the I products continue to be procured in the pre-merger way, so that the cost function of the merged firm is given by $C^m(p) = \sum_{i=1}^I C_i(d_i(p))$. The post-merger profit function for the merged firm is thus given by

$$\pi^m(p) \equiv \sum_{i=1}^I p_i d_i(p) - C^m(p) = \sum_{i=1}^I \{p_i d_i(p) - C_i(d_i(p))\} = \sum_{i=1}^I \pi_i(p). \quad (4.3)$$

We assume that the profit functions exhibit the following two properties:

- (Q) (Quasi-Concavity) Each firm i 's profit function $\pi_i(p)$ is strictly quasi-concave in its own price variable p_i , $i = 1, \dots, N$.
- (S) (Strategic Complementarity)
- (i) For all $i = 1, \dots, N$ the profit function $\pi_i(p_i, p_{-i})$ is supermodular in every price pair (p_i, p_j) with $j \neq i$.
 - (ii) The profit function of the merged firm $\pi^m(p_1, \dots, p_N)$ is supermodular in each price pair (p_i, p_j) with $i = 1, \dots, I$, $j = 1, \dots, N$, and $i \neq j$.

Condition (S) has been used, with regularity, in the literature. See, for example, Cabral and Villas-Boas (2005). Vives (1985, 1990) identified broad sufficient conditions in terms of the demand functions and cost structures which guarantee that conditions (Q) and (S.i) are satisfied simultaneously: Assume, the demand functions are *twice* differentiable and that, for example, each firm has an increasing and convex cost function $C_i(\cdot)$, $d_i(p)$ is log-concave while $\frac{\partial^2 \log d_i}{\partial p_i \partial p_j} \geq 0, \forall j \neq i$, *i.e.*, d_i is log-supermodular in every price pair (p_i, p_j) with $j \neq i$. (See remark 2 on p. 156 of Vives (2001).)

It is harder to identify sufficient conditions for property (S.ii), *i.e.*, for the supermodularity of the profit function of a merged firm in terms of simple structural properties of the individual products' cost and demand functions. However, condition (S.ii) is easily verified directly. This applies, in particular, when the profit functions are *twice* differentiable,

in which case (S.ii) is equivalent to

$$\frac{\partial^2 \pi^m}{\partial p_i \partial p_j} \geq 0 \quad \forall i = 1, \dots, I \text{ and } \forall j \neq i, \text{ on the price cube } X_{l=1}^N [p_l^{\min}, p_l^{\max}]. \quad (4.4)$$

As shown in Section 4.1, one special, but frequently applied, case in which conditions (Q) and (S) can be guaranteed upfront is when all demand and cost functions are affine.

4.3 Pre- and Post-Merger Comparison

In this section, we describe our main results. In particular we show that in the absence of cost synergies, both the component-wise smallest and largest post-merger price equilibria are larger than their pre-merger counterparts. This implies that consumer welfare declines due to the merger. In addition, all firms' equilibrium profits increase, with the understanding that we compare the profits of the newly merged firm with the aggregate of their pre-merger profits. We distinguish between the following two games, describing the competition in the industry before and after the merger:

$\Gamma^{pre} \{A_1 \dots A_I; [\pi_1, \pi_2, \dots, \pi_N]; i = 1, \dots, N\}$: This is the pre-merger N firm competition game in which each of the N firms operates as an independent competitor; firm $i = 1, \dots, N$ selects its price from the interval $A_i = [p_i^{\min}, p_i^{\max}]$ and faces the profit function π_i .

$\Gamma^{post} \{X_{l \in I} A_l, A_{I+1}, \dots, A_N; [\pi^m, \pi_{I+1}, \dots, \pi_N]; i = m, I+1, \dots, N\}$ refers to the post-merger game, with (N-I+1) players, the merged firm m and firms $I+1, \dots, N$. The merged firm selects its I-dimensional price vector from the price cube $X_{l=1}^I A_l$ and faces the profit function π^m . The remaining firms $i = I+1, \dots, N$ have the same feasible action space and profit functions as in the pre-merger game.

In addition, we define the following set of *restricted* games for any vector of prices $p_{-I}^\circ = (p_{I+1}^\circ, \dots, p_N^\circ)$, pertaining to the firms *not* involved in the merger:

$\Gamma^{res}(p_{-I}^\circ) = \{A_1, \dots, A_I; \pi_i^{res}(p_1, \dots, p_I | p_{-I}^\circ); i = 1, \dots, I\}$, where

$$\pi_i^{res}(p_1, \dots, p_I | p_{-I}^\circ) \equiv \pi_i(p_1, \dots, p_I, p_{-I}^\circ), \quad i = 1, \dots, I \quad (4.5)$$

These games have the first I firms as independent players, each with his feasible price interval as his action space and a profit function obtained from the profit function in the unrestricted pre-merger game by fixing the prices of the remaining firms $I + 1, \dots, N$ at their levels in the vector p_{-I}° . We first need the following lemma:

Lemma 4.3.1

(a) *The games $\Gamma^{pre}, \Gamma^{post}$, and $\Gamma^{res}(p_{-I}^\circ)$, for any price vector $p_{-I}^\circ \in X_{l=I+1}^N A_l$ are all supermodular and have a component-wise smallest equilibrium, which we denote by $\underline{p}^*(pre), \underline{p}^*(post)$, and $\underline{p}^*(p_{-I}^\circ)$ respectively. They also have a component-wise largest equilibrium, denoted by $\bar{p}^*(pre), \bar{p}^*(post)$, and $\bar{p}^*(p_{-I}^\circ)$.*

(b) *For each firm $i = 1, \dots, N$, there exists a unique best response $\Psi_i(p_{-i})$ for any feasible price vector $p_{-i} \in X_{l \neq i} A_l$.*

(c) *In the post-merger game, Γ^{post} , the merged firm m has a component-wise smallest [largest] best response function price vector, $\underline{\Psi}^m(p_{-I})$ [$\bar{\Psi}^m(p_{-I})$] = (p_1, \dots, p_I) , for any price vector $p_{-I} \in X_{l=I+1}^N A_l$.*

Proof: (a) All of the considered games have continuous profit functions and actions spaces that are lattices, either simple closed intervals or, for the merged firm in the game Γ^{post} , the cube $X_{l=1}^I A_l$. To establish the supermodularity of the various games, it therefore suffices to verify that the players' profit functions have the required supermodularity properties. For the games Γ^{pre} and $\Gamma^{res}(p_{-I}^\circ)$ this is immediate from condition (S.i). In the game Γ^{post} , each firm $i = I + 1, \dots, N$ has the same profit function π_i as in Γ^{pre} and this profit function is supermodular in (p_i, p_j) for all $j \neq i$. Finally, in the game Γ^{post} , the merged firm m has profit function π^m which is supermodular in (p_i, p_j) for all $i = 1, \dots, I$ and all $j = 1, \dots, N$ with $j \neq i$ by condition (S.ii). Since the games are supermodular, it follows that they have a component-wise smallest and a component-wise largest equilib-

rium, see *e.g.*, Theorem 4.2.1 in Topkis (1998).

(b) This result follows from the strict quasi-concavity of each profit function π_i in its own-price variable p_i , $i = 1, \dots, N$, see condition (Q).

(c) Part (c) follows from Lemma 4.2.2 (c) in Topkis (1998) and the supermodularity of Γ^{post} by part (a). ■

As is well known, one of the implications of a game being supermodular is that its component-wise smallest equilibrium can be computed by a simple tatônnement scheme which starts with the vector p^{min} , the component-wise smallest element of the feasible price space³. In such tatônnement schemes, the players iteratively determine best responses to choices made by their competitors in earlier iterations of the scheme. There is considerable flexibility in terms of the sequence in which best response updates are made. Topkis (1998) and Vives (2001) focus on the so-called simultaneous optimization and Round-Robin versions. In the former, all players determine (simultaneously) in each iteration, their best responses to the choices made in the prior iteration with a specific rule determining *which* best response is selected when the best response fails to be unique. In the Round-Robin version, one chooses a particular permutation of the players; following this permutation, each player is sequentially offered the opportunity to adopt his best response to the most recent choices made by all competitors.

Our first main result is to show that the component-wise smallest and largest equilibrium in the post-merger game are (component-wise) *larger* than the corresponding equilibria in the pre-merger game. Our proof is based on identifying pairs of specific tatônnement schemes one of which pertains to the post-merger game and one to the pre-merger game, such that in each iteration the price vector determined in the post-merger tatônnement scheme dominates that obtained in the pre-merger scheme, while the pre-merger scheme converges to a specific equilibrium in the pre-merger game, and the post-merger scheme converges to its counterpart in the post-merger game. This proof

³The same property applies to the component-wise largest equilibrium, starting the tatônnement scheme at the largest feasible price vector p^{max} .

technique is reminiscent of that employed in Allon and Federgruen (2007).

To show that $\underline{p}^*(post) \geq \underline{p}^*(pre)$ and $\bar{p}^*(post) \geq \bar{p}^*(pre)$, we use the following pairs of schemes, respectively:

• **Pre-Merger Increasing Scheme**

- Step 0: $p^{(0)} := p^{min}$; $k = 1$
- Step 1: For $i = 1, \dots, I$, set $(p_1^{(k)}, \dots, p_I^{(k)}) = \underline{p}^*(p_{-I}^{(k-1)})$ the smallest equilibrium of the game $\Gamma^{res}(p_{-I}^{o(k-1)})$
- For $i = I+1, I+2, \dots, N$, set $p_i^{(k)} = \Psi_i(p_{-i}^{(k-1)})$; $k = k+1$ and repeat Step 1.

• **Post-Merger Increasing Scheme**

- Step 0: $q^{(0)} := p^{min}$; $k = 1$
- Step 1: For $i = 1, \dots, I$, set $(q_1^{(k)}, \dots, q_I^{(k)}) = \underline{\Psi}^m(q_{-I}^{(k-1)})$;
- For $i = I+1, I+2, \dots, N$, set $q_i^{(k)} = \Psi(q_{-i}^{(k-1)})$; $k = k+1$ and repeat Step 1.

• **Pre-Merger Decreasing Scheme**

- Step 0: $\tilde{p}^{(0)} := p^{max}$; $k = 1$
- Step 1: For $i = 1, \dots, I$, set $(\tilde{p}_1^{(k)}, \dots, \tilde{p}_I^{(k)}) = \bar{p}^*(\tilde{p}_{-I}^{(k-1)})$ the largest equilibrium of the game $\Gamma^{res}(\tilde{p}_{-I}^{o(k-1)})$
- For $i = I+1, I+2, \dots, N$, set $\tilde{p}_i^{(k)} = \Psi_i(\tilde{p}_{-i}^{(k-1)})$; $k := k+1$ and repeat Step 1.

• **Post-Merger Decreasing Scheme**

- Step 0: $\tilde{q}^{(0)} := p^{max}$; $k = 1$
- Step 1: For $i = 1, \dots, I$, set $(\tilde{q}_1^{(k)}, \dots, \tilde{q}_I^{(k)}) = \bar{\Psi}^m(\tilde{q}_{-I}^{(k-1)})$;
- For $i = I+1, I+2, \dots, N$, set $\tilde{q}_i^{(k)} = \Psi(\tilde{q}_{-i}^{(k-1)})$; $k = k+1$ and repeat Step 1.

Lemma 4.3.2 (a) The sequence $\{p^{(k)}\}_{k=1}^{\infty}$ increases monotonically to $\underline{p}^*(pre)$.

(b) The sequence $\{q^{(k)}\}_{k=1}^{\infty}$ increases monotonically to $\underline{p}^*(post)$.

(c) The sequence $\{\tilde{p}^{(k)}\}_{k=1}^{\infty}$ decreases monotonically to $\bar{p}^*(pre)$.

(d) The sequence $\{\tilde{q}^{(k)}\}_{k=1}^{\infty}$ decreases monotonically to $\bar{p}^*(post)$.

Proof: (a) We first show, by induction, that the sequence $\{p^{(k)}\}_{k=1}^{\infty}$ is monotonically increasing. Clearly $p^{(1)} \geq p^{(0)} = p^{min}$. Assume $p^{(k-1)} \geq p^{(k-2)}$ for some $k \geq 2$. For $i = I + 1, \dots, N$, we have

$$p_i^{(k)} = \Psi_i(p_i^{(k-1)}) \geq \Psi_i(p_i^{(k-2)}) = p_i^{(k-1)} \quad (4.6)$$

where the inequality follows from the induction assumption and the fact that in a supermodular game the $\Psi_i(\cdot)$ operator is increasing, for all i , see Lemma 4.2.2 in Topkis (1998). Moreover, for the merging firms $i = 1, \dots, I$, the Pre-Merger Increasing Scheme specifies that

$$(p_1^{(k)}, \dots, p_I^{(k)}) = \underline{p}^*(p_{-I}^{(k-1)}) \geq \underline{p}^*(p_{-I}^{(k-2)}) = (p_1^{(k-1)}, \dots, p_I^{(k-1)}), \quad (4.7)$$

where the inequality follows again from the induction assumption, as well as from the fact that the smallest equilibrium in the supermodular, restricted game $\Gamma^{res}(p_{-I}^{\circ})$ is an increasing function of any of the parameters in p_{-I}° since, for all $i = 1, \dots, I$, each firm i 's payoff function in these restricted games is continuous and supermodular in (p_i, p_j) for all $j = I + 1, \dots, N$ (see Theorem 4.2.2 in Topkis (1998)).

(4.6) and (4.7) together establish that $p^{(k)} \geq p^{(k-1)}$, thus completing the induction proof for the monotonicity of scheme $\{p^{(k)}\}$ which is bounded from above by p^{max} and hence converges to a limit vector p^* . By the continuity of the profit functions, p^* is a fixed point of the joint best response operator in the pre-merger game, *i.e.*, p^* is an equilibrium of the game Γ^{pre} .

It remains to be shown that $p^* = \underline{p}^*(pre)$, the component-wise *smallest* equilibrium of Γ^{pre} , *i.e.*, $p^* \leq \underline{p}^*(pre)$. To prove this inequality, consider for any precision $\epsilon > 0$, the following ϵ -approximation of the Pre-Merger Increasing Scheme:

Approximate Pre-Merger Increasing Scheme (APMIS):

- Step 0: $x^{(0)} := p^{min}$; $l:=1$
- Step 1: (Best response for firms $I+1, \dots, N$)

$$\text{For } i = 1, \dots, I, x_i^{(l)} := p_i^{min};$$

$$\text{For } i = I + 1, \dots, N, x_i^{(l)} := \Psi_i(x^{(l-1)})$$

$$l = l + 1$$

- Step 2: (Best response for firms $1, \dots, I$)

$$\text{For } i = 1, \dots, I, x_i^{(l)} := \Psi_i(x^{(l-1)});$$

$$\text{for } i = I + 1, \dots, N, x_i^{(l)} := x_i^{(l-1)}$$

$$l = l + 1$$

If $|x^{(l)} - x^{(l-1)}|_\infty \leq \epsilon$, go to Step1, otherwise, go to Step 2.

Note that when APMIS executes a batch of consecutive Step 2 iterations, an (ϵ -approximation) of the smallest equilibrium in the restricted game $\Gamma^{res}(\cdot)$ is being computed, given the most recently updated prices for the firms $I + 1, \dots, N$. Thus, modulo the ϵ -approximation in the stopping criterion of Step 2, each time APMIS reenters Step 1, a new element of the sequence $\{p^{(k)}\}$ in the Pre-Merger Increasing Scheme is being generated. Thus, the scheme $\{x^{(l)}\}_{l=1}^\infty$ converges to an ϵ -approximation $x^*(\epsilon)$ of the limit vector p^* of the scheme $\{p^{(k)}\}_{k=1}^\infty$. Moreover, by the continuity of the profit functions, $\lim_{\epsilon \downarrow 0} x^*(\epsilon) = p^*$.

To show that $p^* \leq \underline{p}^*(pre)$, it thus suffices to show that $x^*(\epsilon) \leq \underline{p}^*(pre)$. This inequality follows by comparing the sequence $\{x^{(l)}\}$ with $\{y^{(l)}\}_{l=1}^\infty$, the scheme generated by the

“simultaneous optimization” variant of the tatônnement scheme in the pre-merger game Γ^{pre} , when, like $\{x^{(l)}\}$, starting at the smallest feasible price vector p^{min} . As mentioned in the proof of part (a), since all profit functions in the pre-merger game are continuous and the firm’s feasible action sets compact, it follows from Theorem 4.3.4 in Topkis (1998) that this simultaneous optimization tatônnement scheme $\{y^{(l)}\}_{l=1}^{\infty}$ converges to $\underline{p}^*(pre)$. Moreover, $x^{(l)} \leq y^{(l)}$ for all $l = 1, 2, \dots$, since the simultaneous optimization tatônnement scheme $\{y^{(l)}\}_{l=1}^{\infty}$, executes, in each iteration, a version of Step 1 or Step 2 in which *all* firms are permitted to update their price to a *larger* best response value, as opposed to APMIS, where only firms $I + 1, \dots, N$ [1, \dots , I] in Step 1 [Step 2] are permitted to do so while the remaining firms $1, \dots, I$ [$I + 1, \dots, N$] forced to set their price level at the minimum [previous] level. Thus, the inequalities $x^{(l)} \leq y^{(l)}$, $l = 1, 2, \dots$ follow by complete induction, employing the fact that the profit functions are supermodular.

(b) The sequence $\{q^{(k)}\}_{k=1}^{\infty}$ is the sequence generated by the “simultaneous optimization” variant of the tatônnement scheme, applied to the game Γ^{post} and starting at p^{min} . By Lemma 4.3.1(a), the game Γ^{post} is supermodular. Since the payoff functions in this game are continuous and the action sets of all players compact, it follows from Theorem 4.3.4 in Topkis (1998) that the scheme $\{q^{(k)}\}_{k=1}^{\infty}$ converges to $\underline{p}^*(post)$.

(c) and (d): The proofs of parts (c) and (d) are analogous to those of parts (a) and (b), respectively. ■

In addition to the quasi-concavity and strategic complementarity conditions **(Q)** and **(S)** we need one additional assumption to allow for comparison of pre- and post-merger prices:

(MP) (Marginal Profitability for the Merged Firm) For any set of prices p_{-I}° selected by firms $I + 1, \dots, N$, the best response $\underline{\Psi}^m(p_{-I}^{\circ}) = (p_1, \dots, p_I)$ employs price levels that are larger than the products’ marginal costs, *i.e.*, $p_i \geq C'_i(d_i)$ for $i = 1, \dots, I$.

This condition is entirely innocuous when the cost functions are affine: in this case, we may, without loss of generality select $p^{min} \geq c$, the constant marginal cost rate vector.

When the cost functions are non-linear, the Marginal Profitability condition (MP) may be somewhat restrictive, but can still be argued to apply in most settings.

Indeed, the following is a frequently used sufficient condition for the Marginal Profitability condition (MP):

(CM) (Competitive Markets): For all $i = 1, \dots, N$, $\pi_i(p_i, p_{-i})$ is increasing in p_{-i} .

The (CM) condition has been postulated, for example by Milgrom and Roberts (1990) as well as Cabral and Villas-Boas (2005). The former pointed out that under the supermodularity condition (S.i), (CM) reduces to assuming that for all $i = 1, \dots, N$, firm i 's profit $\pi_i(p_i^{min}, p_{-i})$ is increasing in competitors' prices, when charging at its minimum price level. (Since π_i is supermodular in (p_i, p_j) for all $j \neq i$, it has increasing differences in every such price pair, *i.e.*, $\pi_i(p_i, p'_j) - \pi_i(p_i, p_j)$ is increasing in p_i , for any pair of prices $p_j < p'_j$. Thus $\pi_i(p_i^{min}, p'_j) - \pi_i(p_i^{min}, p_j) \geq 0 \Rightarrow \pi_i(p_i, p'_j) - \pi_i(p_i, p_j) \geq 0$ for all $p_i \geq p_i^{min}$.)

Lemma 4.3.3 *Under condition (S.i), (CM) \Rightarrow (MP).*

Proof: By (CM) we have, in every price point $p \in X_{i=1}^N A_i$, for all $i \neq j$ that $\frac{\partial \pi_i}{\partial p_j} = (p_i - C'(d_i)) \frac{\partial d_i}{\partial p_j} \geq 0$. In view of (4.1) this implies that for all $i = 1, \dots, N$ $p_i \geq C'_i(d_i)$, with $d_i = d_i(p)$ for *any* feasible price vector p , and, in particular, when the prices of the first I products are selected as best responses. ■

We now derive our first main result, *i.e.*, we show that both the largest and smallest equilibria in the post-merger game dominate, component-wise, their counterparts in the pre-merger game.

Theorem 4.3.4 (*Pre- and Post-Merger Price Comparisons*) *Assume conditions (Q), (S), and (CM) apply.*

The post-merger equilibrium $\underline{p}^(post)$ [$\bar{p}^*(post)$] is component-wise larger than the pre-merger equilibrium $\underline{p}^*(pre)$ [$\bar{p}^*(pre)$], *i.e.*, $\underline{p}^*(post) \geq \underline{p}^*(pre)$ and $\bar{p}^*(post) \geq \bar{p}^*(pre)$.*

Proof: We show that $\underline{p}^*(post) \geq \underline{p}^*(pre)$; the comparison proof for the largest equilibrium in the post- and pre-merger game is entirely analogous. In view of Lemma 4.3.2, it suffices to show that in each iteration $k = 0, 1, \dots$, $q^{(k)} \geq p^{(k)}$. We prove this by induction. The starting conditions of the two schemes have $q^{(0)} = p^{(0)}$, so that the statement holds for $k=0$. Assume it holds after the $(k-1)$ st iteration, *i.e.*, $q^{(k-1)} \geq p^{(k-1)}$. For firms $i = I + 1, \dots, N$, $q_i^{(k)} = \Psi_i(q^{(k-1)}) \geq \Psi_i(p^{(k-1)}) = p_i^{(k)}$ is immediate from the supermodularity condition (S.i), see, for example, Lemma 4.2.2c in Topkis (1998).

It thus remains to be shown that $q_i^{(k)} \geq p_i^{(k)}$ for $i = 1, \dots, I$. Since p^{min} and p^{max} are selected so as not to impact on the best response price choices in either the pre- or post-merger industry, we have that the price vector $(q_1^{(k)}, \dots, q_I^{(k)})$ is an interior point of the price space and therefore satisfies the following First Order Conditions. For all $i = 1, \dots, I$:

$$\begin{aligned} 0 &= \frac{\partial \pi^m(q_1^{(k)}, \dots, q_I^{(k)}, q_{I+1}^{(k-1)}, \dots, q_N^{(k-1)})}{\partial p_i} = \sum_{l=1}^I \frac{\partial \pi_l(q_1^{(k)}, \dots, q_I^{(k)}, q_{I+1}^{(k-1)}, \dots, q_N^{(k-1)})}{\partial p_i} \\ &= \frac{\partial \pi_i(q_1^{(k)}, \dots, q_I^{(k)}, q_{I+1}^{(k-1)}, \dots, q_N^{(k-1)})}{\partial p_i} + \sum_{l \neq i; l=1}^I [q_l^{(k)} - C'_l(d_l)] \frac{\partial d_l}{\partial p_i}, \end{aligned} \quad (4.8)$$

with $d_i \equiv d_i(q_1^{(k)}, \dots, q_I^{(k)}, q_{I+1}^{(k-1)}, \dots, q_N^{(k-1)})$. By the marginal profitability condition (MP), we have for all $l = 1, \dots, i-1, i, i+1, \dots, I$ that $q_l^{(k)} \geq C'_l(d_l)$, while $\frac{\partial d_l}{\partial p_i} \geq 0$, see (4.1). Thus, for all $i = 1, \dots, I$,

$$\frac{\partial \pi_i(q_1^{(k)}, \dots, q_I^{(k)}, q_{I+1}^{(k-1)}, \dots, q_N^{(k-1)})}{\partial p_i} \leq 0. \quad (4.9)$$

By the strict quasi-concavity of the profit functions $\{\pi_i, i = 1, \dots, I\}$ it follows that

$$\Psi_i(q_1^{(k)}, \dots, q_I^{(k)}, q_{I+1}^{(k-1)}, \dots, q_N^{(k-1)}) \leq q_i^{(k)}, \quad i = 1, \dots, I \quad (4.10)$$

Consider now the restricted game $\Gamma^{res}(q_{-I}^{(k-1)})$. This game is supermodular by Lemma 4.3.1. Let $\Psi^{res}(x)$ denote the I -dimensional joint best response vector of the I competing

firms in the game $\Gamma^{res}(q_{-I}^{o(k-1)})$ to an assumed price vector $x = (x_1, \dots, x_I)$:

$$\Psi_i^{res}(x) \equiv \Psi_i(x_1, \dots, x_I, q_{I+1}^{(k-1)}, \dots, q_N^{(k-1)}), \quad i = 1, \dots, N \quad (4.11)$$

In addition, for all $n = 1, 2, \dots$ let $\Psi^{res(n)}(\cdot)$ denote the n -fold application of the best response operator Ψ^{res} . (4.10) implies that $\Psi^{res}(q_1^{(k)}, \dots, q_I^{(k)}) \leq [q_1^{(k)}, \dots, q_I^{(k)}]$. Since the game $\Gamma^{res}(q_{-I}^{(k-1)})$ is supermodular, it follows that the best response operator $\Psi^{res}(\cdot)$ is monotonically increasing. Hence,

$$p^{min} \leq \Psi^{res(m)}(q_1^{(k)}, \dots, q_I^{(k)}) \leq \Psi^{res(m-1)}(q_1^{(k)}, \dots, q_I^{(k)}) \leq \Psi^{res}(q_1^{(k)}, \dots, q_I^{(k)}) \leq [q_1^{(k)}, \dots, q_I^{(k)}]. \quad (4.12)$$

It follows that the monotonically decreasing and bounded sequence $\{\Psi^{res(n)}(q_1^{(k)}, \dots, q_I^{(k)})\}_{n=1}^{\infty}$ converges to a limit vector q^* , with

$$[p_1^{(k)}, \dots, p_I^{(k)}] = \underline{p} * (p_{-I}^{(k-1)}) \leq \underline{p} * (q_{-I}^{(k-1)}) \leq q^* \leq [q_1^{(k)}, \dots, q_I^{(k)}], \quad (4.13)$$

thus completing the induction proof. The first inequality in (4.13) follows from the fact that the smallest equilibrium in a supermodular game is a monotonically increasing vector-function of any parameter (string) such that each player's payoff function in the game is continuous and supermodular in the player's action variable and the parameter, see Theorem 4.2.2 in Topkis (1998). This supermodularity property follows from condition (S.i).

Thus, to complete the verification of the string of inequalities in (4.13) only the second inequality remains to be substantiated. However, this inequality follows from the fact that $q^* = \lim_{n \rightarrow \infty} \Psi^{res(n)}(q_1^{(k)}, \dots, q_I^{(k)})$ is an equilibrium of the game $\Gamma^{res}(q_{-I}^{(k-1)})$ and hence dominates $\underline{p}^*(q_{-I}^{(k-1)})$ the component-wise *smallest* equilibrium of this game. The fact that q^* is an equilibrium of this game follows from Theorem 2.10 in Vives (2001) since the game $\Gamma^{res}(q_{-I}^{(k-1)})$ is supermodular with continuous payoff functions. ■

We now show that, beyond generating higher equilibrium prices, the merger also results in equilibrium profits for the merged firm that are larger than the aggregate of the pre-merger profits among the I merging firms. Moreover, and perhaps most surprisingly, the remaining $(N-I)$ firms also earn a higher expected profit *after* the merger. We establish these results under the (CM) condition, the stronger version of (MP) as shown in Lemma 4.3.3.

We show that these profit comparison results apply, both to the largest and smallest equilibria in the pre-merger and post-merger games. The comparison results are, in particular, important for the *largest* equilibrium, since it is well known from Theorem 7 in Milgrom and Roberts (1990), that under conditions (S) and (CM) the component-wise *largest* equilibrium is *simultaneously* preferred by all firms in the industry. Thus, if multiple equilibria exist, it is most plausible that the *largest* equilibrium will be adopted.

For all firms $i = I + 1, \dots, N$, let $\bar{\pi}_i(post)$ [$\underline{\pi}_i(post)$] denote the equilibrium profit in the post-merger game under the largest [smallest] equilibrium. Similarly, define for all $i = 1, \dots, N$, $\bar{\pi}_i(pre)$ [$\underline{\pi}_i(pre)$] as the corresponding equilibrium profit values in the pre-merger game. In addition, let $\bar{\pi}^m(post)$ [$\underline{\pi}^m(post)$] denote the merged firm's equilibrium profit in the post-merger game under the largest [smallest] equilibrium.

Theorem 4.3.5 (*Profit Comparison Before and After the Merger*)

Assume conditions (Q), (S), and (CM) apply.

$$(a) \quad \underline{\pi}^m(post) \geq \sum_{l=1}^I \underline{\pi}_l(pre) \tag{4.14}$$

$$\underline{\pi}_i(post) \geq \underline{\pi}_i(pre), \quad i = I + 1, \dots, N \tag{4.15}$$

$$(b) \quad \bar{\pi}^m(post) \geq \sum_{l=1}^I \bar{\pi}_l(pre) \tag{4.16}$$

$$\bar{\pi}_i(post) \geq \bar{\pi}_i(pre), \quad i = I + 1, \dots, N \tag{4.17}$$

Proof: (a) For $i = I + 1, \dots, N$: $\underline{\pi}_i(pre) = \pi_i(\underline{p}^*(pre)) \leq \pi_i(\underline{p}_i^*(pre), \underline{p}_{-i}^*(post)) \leq \pi_i(\underline{p}_i^*(post), \underline{p}_{-i}^*(post)) = \underline{\pi}_i(post)$ thus proving (4.15).

The first inequality follows from $\underline{p}_{-i}^*(post) \geq \underline{p}_{-i}^*(pre)$, see Theorem 4.3.4, and the (CM) condition; the second equality follows from $\underline{p}_{-i}^*(post)$ being a best response to $\underline{p}_{-i}^*(post)$, since $\underline{p}^*(post)$ is an equilibrium in the post-merger game.

Similarly, $\sum_{i=1}^I \pi_i(pre) = \sum_{i=1}^I \pi_i(\underline{p}^*(pre)) \leq \sum_{i=1}^I \pi_i(\underline{p}_{-i}^*(pre), \underline{p}_{-i}^*(post)) \leq \sum_{i=1}^I \pi_i(\underline{p}_{-i}^*(post), \underline{p}_{-i}^*(post)) \leq \sum_{i=1}^I \pi_i(\underline{p}^*(post)) = \pi^m(post)$, verifying (4.14). The first inequality follows again from $\underline{p}^*(post) \geq \underline{p}^*(pre)$, see Theorem 4.3.4(a), and the (CM) condition; the second inequality follows from the fact that the vector $(\underline{p}_1^*(post), \dots, \underline{p}_I^*(post))$ is a best response price vector for the merged firm to prices $\underline{p}_{-I}^*(post)$ selected by the remaining firms $I + 1, \dots, N$.

(b) The proof of part (b) is analogous to that of part (a). ■

4.3.1 Affine Demand and Cost Functions

In this subsection we apply our results to the special case where both the demand and cost functions are affine, but otherwise general, *i.e.*,

$$d_i(p) = a_i - b_i p_i + \sum_{j \neq i} \beta_{ij} p_j, \quad i = 1, \dots, N \quad (4.18)$$

$$C_i(d_i) = c_i d_i + e_i, \quad i = 1, \dots, N \quad (4.19)$$

where $\{a_i, b_i, \beta_{ij}, c_i, e_i\}$ are given parameters with $b_i, \beta_{ij} \geq 0$. This structure is used in many applications. Without loss of generality, assume each product is priced at or above its marginal cost value, *i.e.*, $p_i^{min} = c_i$ for all $i = 1, \dots, N$. It is easily verified that all three of the conditions (Q), (S), and (MC) are satisfied. This implies that both the pre-merger and post-merger games are supermodular. Assume, in addition, that the price sensitivity coefficients satisfy the well known dominant diagonality conditions:

$$(D1) \quad -\frac{\partial d_i}{\partial p_i} \geq \sum_{j \neq i} \frac{\partial d_i}{\partial p_j} \Leftrightarrow b_i \geq \sum_{j \neq i} \beta_{ij}, \quad i = 1, \dots, N$$

$$(D2) \quad \sum_{j=1}^N \frac{\partial d_j}{\partial p_i} \leq 0 \Leftrightarrow b_i \geq \sum_{j \neq i} \beta_{ji}, \quad i = 1, \dots, N$$

These conditions are very intuitive: (D) states that a uniform price increase for all firms cannot result in an increase of any product's sales volume; (D2) states that if *any* prod-

uct's price is increased, unilaterally, aggregate sales in the industry do not increase. Under the dominant diagonality conditions, we have that both the pre-merger and post-merger games have a *unique* equilibrium $p^*(pre)$ and $p^*(post)$, respectively. This follows from the fact that the Jacobian of the system of First Order Conditions is a dominant diagonal matrix, see Vives (2001) and Gabay and Moulin (1980). The following corollary is therefore immediate from Theorems 4.3.4 and 4.3.5.

Corollary 4.3.6 *Consider an industry with affine demand and cost functions (4.19, 4.18). Assume in addition that the dominant diagonality conditions (D1,D2) hold.*

(a) There exists a unique equilibrium $p^(pre)$ in the pre-merger game, and a unique equilibrium $p^*(post)$ in the post-merger game with $p^*(pre) \leq p^*(post)$.*

(b) The equilibrium profits of the merged firm exceed the aggregate equilibrium pre-merger profits of the merging firms. Similarly, the equilibrium profits of all remaining firms increase because of the merger:

$$\begin{aligned} \pi^m(post) &\equiv \pi^m(p^*(post)) \geq \sum_{i=1}^I \pi_i(p^*(pre)) \equiv \sum_{i=1}^I \pi_i(pre) \\ \pi_i(post) &\equiv \pi_i(p^*(post)) \geq \pi_i(p^*(pre)) \equiv \pi_i(pre), \quad i = I + 1, \dots, N \end{aligned}$$

4.3.2 Cost Efficiencies Resulting from the Merger

Thus far, we have assumed that the merger does not affect the cost structure of the products offered by the merging firms. Frequently, mergers result in significant cost synergies. Indeed, such synergies are often the driving force, or one of the principal impetuses, behind the merger. For example, in 2005 Proctor & Gamble announced the largest acquisition in its history, agreeing to buy Gillette in a \$57 billion stock deal. The acquisition presented P&G with the opportunity to become the leader in the household and personal care market. The merging firms had reported 2004 profits of \$6.5 billion and \$1.6 billion, respectively. In the traditional “freeze” period following the merger proposal,

AT Kearney was retained to assess the cost synergies. (Such assessments by independent consulting firms are routinely undertaken in any significant merger proposal.) The firm estimated the cost synergies at approximately \$1 billion per year; over half the size of the total pre-merger profits of Gillette.

While reduced competition results in price increases, see Theorem 4.3.4, it is generally believed that cost synergies have the opposite effect. In actuality, whether this can be guaranteed or not depends on the specific way the synergies impact on the cost functions of the products being merged. The simplest synergy model assumes that each of the *marginal* cost functions is shifted by the *same* constant $\sigma > 0$, *i.e.*,

$$C_i^{post}(d_i) = C_i(d_i) - \sigma d_i, \quad i = 1, \dots, I \quad (4.20)$$

The pre-merger and post-merger cost functions may be viewed as special cases of a parameterized set of functions $C_i(d_i, \sigma) = C_i(d_i) - \sigma d_i$, with the pre-merger [post-merger] cost function corresponding with $\sigma = 0$ [$\sigma = 1$].

Proposition 4.3.7 *Assume the merger induces synergies for the cost structures of products $i = 1, \dots, I$, as described by (4.20). Assume, in addition, that the dominant diagonal condition (D1) applies. These synergies result in price decreases for the smallest and largest post-merger equilibrium compared to their levels in the absence of any cost synergies.*

Proof: By Theorem 4.2.2 in Topkis (1998), it suffices to show that $\frac{\partial^2 \pi^m}{\partial p_i \partial \sigma} \geq 0$ for all $i = 1, \dots, I$ while for all $i = I + 1, \dots, N$ $\frac{\partial \pi_i}{\partial p_i} = 0$. Note that $\frac{\partial^2 \pi^m}{\partial p_i \partial \sigma} = - \sum_{l=1}^I \frac{\partial C_l(d_l, \sigma)}{\partial p_i \partial \sigma} = - \left\{ \frac{\partial d_i}{\partial p_i} - \sum_{l \neq i} \frac{\partial d_l}{\partial p_i} \right\} > 0$ by the dominant diagonal condition (D1). ■

Thus, a merger associated with cost synergies described by a *uniform* marginal cost reduction as in (4.20), induces two opposite effects. The “increased market concentration”, by itself, *increases* the price equilibrium; however, the cost synergies induce *decreases* in all equilibrium prices. Which of the two effects dominates, depends on the magnitude

of σ . This phenomenon was first observed by Deneckere and Davidson (1985) in their special class of Bertrand price competition models.

It should be noted that the price effects described by Proposition 4.3.7 are specific to a *uniform* reduction of the marginal cost functions of products $1, \dots, I$ by the same constant. If the marginal cost reduction is *product specific*, *i.e.*, $C_i^{post}(d_i) = C_i(d_i) - \sigma_i d_i$, or if it fails to be constant, *i.e.*, $C_i^{post}(d_i) = C_i(d_i) - \sigma(d_i)$ for some non-linear function $\sigma(\cdot)$, it does not appear to be possible to guarantee a reduction of the equilibrium prices as compared to a post-merger equilibrium without such synergies⁴.

Returning to the synergy structure (4.20), two values of interest are:

$$\sigma^+ = \min\{\sigma : p^*(post|\sigma) \leq p^*(pre)\}, \text{ and}$$

$$\sigma^- = \min\{\sigma : p_i^*(post|\sigma) < p_i^*(pre) \text{ for some } i = 1, \dots, N\} \leq \sigma^+,$$

where $p^*(post|\sigma)$ denotes the equilibrium in the post-merger game under a given marginal cost savings σ . In other words, $\sigma^+(\sigma^-)$ denotes the minimum cost savings such that all (at least one) of the equilibrium prices decreases after the merger.

In assessing whether the proposed merger is likely to “lessen competition” one may then evaluate whether the magnitude of $\sigma^-(\sigma^+)$ is a realistic possibility. (Both σ^- and σ^+ can easily be computed by embedding the tatônnement scheme in a bi-section search for the “break even” value of σ .)

Alternatively, one may assume that the merger results in a marginal cost reduction of *one* of the products of the merging firms only, and calculate σ^+ on this basis. This approach was followed, for example, by Nevo (2000a) for the ready-to-eat cereal industry. After carefully estimating the demand functions of the different ready-to-eat cereal products, Nevo (2000a) simulates various potential pairwise mergers among the six major national competitors. In his Table 5, the author reports the price increases that result from various potential mergers, assuming that the cost functions remain unaltered. Table 6 proceeds to report what marginal cost reductions for individual products would restore

⁴Farrell and Shapiro (2010a) for example assume that the cost synergies result in a uniform *percentage* reduction of the marginal costs. Their Proposition 1, confining itself to a merger of *two* firms, indeed states as an *assumption* “Suppose that the price charged by the merged firm for each product and non-decreasing in the marginal cost of the other product”.

the equilibrium prices to levels at or below the pre-merger values. These “break even” values are then discussed to evaluate whether the net effect of the merger is likely to be positive or negative.

4.3.3 Mergers of multi-product firms

In our base model, we consider a (pre-merger) industry in which each of the products is sold by an independent company. In this subsection we extend our results to the more prevalent case where some or all of the existing firms sell more than one product. (We continue to assume that each product is sold by a single firm.) We characterize the equilibrium consequences of a merger between *two* of these firms.

Assume there are n firms in the industry, numbered $i = 1, \dots, n$ with firm i offering $l_i \geq 1$ products to the market, with $N = \sum_{i=1}^n l_i$. We thus use a double index to differentiate among the various products, with product (i, j) referring to the j -th product offered by firm i , $i = 1, \dots, n$ and $j = 1, \dots, l_i$. For firm i , let $p_i = (p_{i1}, \dots, p_{il})$ denote the firm’s price vector and let p denote the N -dimensional vector containing all prices for all N products. The profit function for firm i is thus given by:

$$\pi_i(p) = \sum_{j=1}^{l_i} [p_{ij}d_{ij}(p) - c_{ij}(d_{ij}(p))]. \quad (4.21)$$

Without loss of generality, assume firms 1 and 2 merge to create a new merged firm m with profit function

$$\pi^m(p) = \pi_1(p) + \pi_2(p) \quad (4.22)$$

(As in the base model, we initially assume that the merger leaves all cost functions unaltered.) To ensure that both the pre-merger Γ^{pre} and the post-merger game Γ^{post} are supermodular we need a variant of condition (S):

(S_m) : (Strategic Complementarity)

- (i) For all $i = 1, \dots, n$, the profit function $\pi_i(p_i, p_{-i})$ is a supermodular function of

the vector p_i and has increasing differences with respect to p_{-i} .

(ii) The profit function of the merged firm, $\pi^m(p_1, \dots, p_n)$ is a supermodular function of (p_1, p_2) and has increasing differences with respect to the remaining prices $p_{-\{1,2\}} = (p_3, \dots, p_n)$.

Along with the fact that all n firms in the pre-merger game Γ^{pre} and all $n - 1$ firms in the post-merger game Γ^{post} have action spaces that are compact lattices, condition (S_m) guarantees that both games are supermodular. Similarly, we need a slight variant of the quasi-convexity condition (Q):

(Q_m) : The profit functions $\pi_i(p_1, p_2, \dots, p_n)$ are strictly quasi-concave functions of firm i 's price vector p_i , $i = 1, \dots, n$.

As in the base model, we need to consider restricted versions of the pre-merger game in which only firms 1 and 2 are able to vary their price vectors, under *given* price choices $p_{-\{1,2\}} \equiv (p_3, \dots, p_n)$ for the remaining firms. We refer to this restricted duopoly as $\Gamma^{res}(p_{-\{1,2\}})$.

In view of the strict quasi-concavity condition (Q_m) , each firm $i = 1, \dots, N$ has a *unique* best response $\Psi_i(p_{-i})$ to any given choice of prices by the remaining firms. In view of the supermodularity condition (S_m) , the merged firm has a component-wise smallest [largest] best response $\underline{\Psi}^m(p_{-\{1,2\}})$ [$\bar{\Psi}^m(p_{-\{1,2\}})$] to any given price vector $p_{-\{1,2\}}$ of the remaining firms $i = 1, \dots, N$.

As before, let $\underline{p}^*(pre)$ and $\bar{p}^*(pre)$ [$\underline{p}^*(post)$ and $\bar{p}^*(post)$] denote the component-wise smallest and largest equilibrium in the pre-merger [post-merger] game. Let $\underline{\pi}_i(pre)$, $\bar{\pi}_i(pre)$ ($i = 1, \dots, n$) and $\underline{\pi}_i(post)$, $\bar{\pi}_i(post)$ ($i = 1, \dots, n$) ($i = 3, \dots, n$) denote the associated profit values. Finally, $\underline{\pi}^m(post)$ and $\bar{\pi}^m(post)$ denote the profit values of the merged firm in these two equilibria of Γ^{post} .

Theorem 4.3.8 (*Price and Profit Comparisons for Mergers of Multi-Product Firms*)

Assume conditions (Q_m) , (S_m) , and (CM) hold.

(a) $\underline{p}^*(pre) \leq \underline{p}^*(post)$ and $\bar{p}^*(pre) \leq \bar{p}^*(post)$

$$(b) \underline{\pi}^m(post) \geq \underline{\pi}_1(pre) + \underline{\pi}_2(pre)$$

$$\bar{\pi}^m(post) \geq \bar{\pi}_1(pre) + \bar{\pi}_2(pre)$$

$$\underline{\pi}_i(post) \geq \underline{\pi}_i(pre), \quad i = 3, \dots, n$$

$$\bar{\pi}_i(post) \geq \bar{\pi}_i(pre), \quad i = 3, \dots, n$$

Proof: (a) We show $\underline{p}^*(pre) \leq \underline{p}^*(post)$, the comparison of the pair of *largest* price equilibrium being analogous. The proof goes along the lines of those of Theorem 4.3.3 and 4.3.4. Define the following pair of tailored tatônnement schemes:

Pre-Merger Increasing Scheme:

Step 0: $p^{(0)} := p^{min}$, $k = 1$

Step 1: For $i = 1, 2$ set $(p_1^{(k)}, p_2^{(k)}) = \underline{p}^*(p_{-\{1,2\}}^{(k-1)})$, the smallest equilibrium of the restricted duopoly game $\Gamma^{res}(p_{-\{1,2\}}^{(k-1)})$, under a fixed price vector $p_{-\{1,2\}}^{(k-1)}$ for the remaining firms;
For $i = 3, \dots, n$, set $p_i^{(k)} = \Psi_i(p_{-i}^{(k-1)})$; $k=k+1$.

Post-Merger Increasing Scheme:

Step 0: $q^{(0)} := p^{min}$, $k = 1$

Step 1: For $i = 1, 2$ set $(q_1^{(k)}, q_2^{(k)}) = \underline{\Psi}^m(q_{-\{1,2\}}^{(k-1)})$;
For firms $i = 3, \dots, n$, set $q_i^{(k)} = \Psi_i(q_{-i}^{(k-1)})$; $k=k+1$.

A straightforward extension of the proof of Lemma 4.3.2 establishes that, once again,

$$\begin{aligned} p^{(0)} &\leq p^{(1)} \leq \dots \leq p^{(k)} \leq \lim_{k \rightarrow \infty} p^{(k)} = \underline{p}^*(pre) \\ q^{(0)} &\leq q^{(1)} \leq \dots \leq q^{(k)} \leq \lim_{k \rightarrow \infty} q^{(k)} = \underline{p}^*(post) \end{aligned} \tag{4.23}$$

It thus suffices to prove that $q^{(k)} \geq p^{(k)}$. The proof proceeds, once again, by induction. $p^{min} = q^{(0)} \geq p^{(0)} = p^{min}$. Assume, therefore, that $q^{(k-1)} \geq p^{(k-1)}$ for some $k \geq 1$. Since p^{min} and p^{max} are selected so as not to impact on the best response choices in either the pre- or post-merger industry, we have that the price vector $(q_1^{(k)}, q_2^{(k)})$ is an interior point

of the price space and therefore satisfies the following First Order Conditions:

$$\begin{aligned}
0 &= \frac{\partial \pi^m(q_1^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})}{\partial p_{1j}} = \frac{\partial \pi^1(q_1^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})}{\partial p_{1j}} + \frac{\partial \pi^2(q_1^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})}{\partial p_{1j}} \\
&= \frac{\partial \pi^1(q_1^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})}{\partial p_{1j}} + \sum_{r=1}^{l_2} \{q_{2r}^{(k)} - c'_{2r}(d_{2r})\} \frac{\partial d_{2r}}{\partial p_{1j}}, \quad \forall j = 1, \dots, l_i. \quad (4.24)
\end{aligned}$$

$$\begin{aligned}
0 &= \frac{\partial \pi^m(q_1^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})}{\partial p_{2j}} \\
&= \frac{\partial \pi^2(q_1^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})}{\partial p_{1j}} + \sum_{r=1}^{l_1} \{q_{1r}^{(k)} - c'_{1r}(d_{1r})\} \frac{\partial d_{1r}}{\partial p_{2j}}, \quad \forall j = 1, \dots, l_j. \quad (4.25)
\end{aligned}$$

It follows from the (CM) condition and (4.1) that the second term to the far right of (4.24) [(4.25)] is non-negative so that

$$\frac{\partial \pi^i(q_1^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})}{\partial p_{ij}} \leq 0, \quad \forall i = 1, 2 \text{ and } j = 1, \dots, l_i \quad (4.26)$$

We first show that (4.26), along with conditions (Q_m) and (S_m) imply that

$$\Psi_1(q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)}) \leq q_1^{(k)}, \text{ and} \quad (4.27)$$

$$\Psi_2(q_1^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)}) \leq q_2^{(k)}. \quad (4.28)$$

Let Ψ^{res} denote the joint best response operator in the restricted duopoly $\Gamma^{res}(q_{-\{1,2\}}^{(k-1)})$, which is, again, a supermodular game by condition (S_m) . (By the strict quasi-concavity condition these Γ^{res} is uniquely defined.) Let $\Psi^{res(r)}$ denote the r -fold application of this operator, $r = 1, 2, \dots$. Since Ψ^{res} is a non-increasing operator, we obtain, by induction that

$$[p_1^{(k)}, p_2^{(k)}] = \underline{p}^*(p_{-\{1,2\}}^{(k-1)}) \leq \underline{p}^*(q_{-\{1,2\}}^{(k-1)}) \leq q^* \leq \Psi^{res(n)}([q_1^{(k)}, q_2^{(k)}]) \leq \Psi^{res(n-1)}([q_1^{(k)}, q_2^{(k)}]) \leq \dots \leq [q_1^{(k)}, q_2^{(k)}] \quad (4.29)$$

where $q^* = \lim_{n \rightarrow \infty} \Psi^{res(n)}([q_1^{(k)}, q_2^{(k)}])$ is an equilibrium of the restricted duopoly and hence component-wise larger than $\underline{p}^*(q_{-\{1,2\}}^{(k-1)})$, the smallest equilibrium in this game, thus verifying the second inequality in (4.29). The first inequality follows from the induction assumption $q^{(k-1)} \geq p^{(k-1)}$ and in particular $q_{-\{1,2\}}^{(k-1)} \geq p_{-\{1,2\}}^{(k-1)}$, as well as the fact that

the smallest equilibrium in a supermodular game is a monotonically increasing vector function of any parameter (string) such that each player's payoff function in the game is continuous and supermodular in the player's action variables and the parameters, see Theorem 4.2.2 in Topkis (1998). In addition to (4.29), we have:

$$p_i^{(k)} = \Psi_i(p^{(k-1)}) \leq \Psi_i(q^{(k-1)}) = q_i^{(k)}, \quad i = 3, \dots, n \quad (4.30)$$

where the inequality follows from the induction assumption and the fact that the best response operator Ψ_i in the supermodular pre-merger game is non-increasing. (4.29) and (4.30) together complete the induction step, *i.e.*, they verify that $q^{(k)} \geq p^{(k)}$.

It remains to be shown that (4.27) and (4.28) hold. We prove (4.27), the proof of (4.28) being analogous. Since the profit function π_1 is strictly quasi-concave in firm 1's price vector p_1 , it has a *unique* local maximum for any given price vectors p_2, \dots, p_n of the remaining firms. The (unique) best response $\Psi_1(q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})$ can be obtained as the limit of a sequence $\{p_1(t)\}_{t=1}^{\infty}$ which optimizes the individual prices $(p_{11}, p_{12}, \dots, p_{1l_1})$ of firm 1 in a Round Robin way. Thus, to prove (4.27) it suffices to show that $p_1(t) \leq q_1^{(k)}$ for all $t = 0, 1, 2, \dots$. We prove this by induction. Clearly $p_1(0) = q_1^{(k)}$. Assume $p_1(t) \leq q_1^{(k)}$ for some $t \geq 0$. Let $r \in \{1, \dots, l_1\}$ denote the index of the product whose price is being optimized in the $(t+1)$ -st iteration. Note that $\operatorname{argmax}_{p_{1r}} \{\pi_1(p_{11}(t), \dots, p_{1r}, \dots, p_{1l_1}(t), q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})\} \leq \operatorname{argmax}_{p_{1r}} \{\pi_1(q_{11}^{(k)}, \dots, p_{1r}, \dots, q_{1l_1}^{(t)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})\} \leq \operatorname{argmax}_{p_{1r}} \{\pi_1(q_{11}^{(k)}, \dots, p_{1r}, \dots, q_{1l_1}^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})\} \leq q_{1r}^{(k)}$, thus verifying $p_1(t+1) \leq q_1^{(k)}$ and completing the induction proof. (The first inequality follows from the supermodularity condition (S.i), while the second inequality follows from $\frac{\partial \pi_1(q_{11}^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})}{\partial p_{1r}} \leq 0$ (see (4.24),) and the quasi-concavity of the single variable function $\pi_1(q_{11}^{(k)}, \dots, q_{1l_1}^{(k)}, q_2^{(k)}, q_3^{(k-1)}, \dots, q_n^{(k-1)})$.)

(b) The proof of part (b) is analogous to that of Theorem 4.3.5, using part (a) of this theorem. ■

4.4 Connection with the Upward Pricing Pressure Measure

Farrell and Shapiro (2010a) discuss the difficulty government agencies such as the Antitrust Division of the DOJ and the FTC face in determining which proposed mergers are to be scrutinized to evaluate whether the merger will “substantially lessen the competition in the industry,” the phrase used in Section 7 of the 1914 Clayton Act, as modified in 1950. Since 2000, firms with a “transaction value” above \$50 million, who wish to engage in a merger, are required to notify the DOJ and FTC of their plans, an outgrowth of the 1976 Hart-Scott-Rodino Act. (The transaction value is defined as the aggregate capital value of the merging firms.) As a consequence, the DOJ and FTC reviewed, in 2008 alone, 1,726 proposed mergers and acquisitions with an aggregate value of more than \$1 trillion.

The Clayton Act requires the government agency to “prove” in court that a proposed merger would result in a “substantial lessening of the competition in the industry,” based on a comprehensive industry study. There is, therefore, a strong need for a fairly simple “pre-screening” test to identify which of the thousands of merger proposals are most likely to result in the greatest reductions of the competitive dynamics in an industry and a commensurate reduction in consumer welfare.

Traditionally, the government has used simple market concentration measures as their “litmus” test, in particular the so-called Herfindahl-Hirschman Index (HHI), defined as the sum of the squares of the (anticipated) post-merger market shares in the industry. Many economists have argued that this HHI-measure is a relatively limited predictor in the particular case of differentiated products, and have proposed alternatives instead. Building on ideas developed by O’Brien and Salop (2000) and Werden (1996), Farrell and Shapiro (2010a) have advocated the use of so-called Upward Pricing Pressure (UPP) measures instead of HHI. Based on the pre-merger equilibrium $p^*(pre)$, the UPP increase

for firms $i = 1, \dots, I$ is defined in Farrell and Shapiro (2010a, equation (0)) as follows:

$$T_i \equiv \sum_{l=1, l \neq i}^I (p_l^* - C_l) \delta_{il} > 0^5, \quad (4.31)$$

where $p^* = p^*(pre)$ and $\delta_{il} \equiv \frac{\partial d_i(p^*)}{\partial p_i}(p^*) / \left| \frac{\partial d_i(p^*)}{\partial p_i} \right|$ denotes the *diversion ratio* from product i to product l , a term coined by Farrell and Shapiro (2010a). Let ϵ_{ii} and ϵ_{il} respectively denote product i 's own and cross-price elasticity with respect to product l , measured at the pre-merger equilibrium p^* , *i.e.*,

$$\epsilon_{ii} = \left| \frac{\partial d_i(p^*)}{\partial p_i} \right| \frac{p_i^*}{d_i(p^*)} \text{ and } \epsilon_{il} = \frac{\partial d_l(p^*)}{\partial p_i} \frac{p_i^*}{d_l(p^*)} \quad (4.32)$$

Clearly,

$$\delta_{il} = \frac{\epsilon_{il} d_l(p^*)}{\epsilon_{ii} d_i(p^*)}, \quad i \neq l \quad (4.33)$$

In other words, the diversion ratio from product i to product l equals the ratio of the own- and cross-price elasticity multiplied by the ratio of product l and i 's (pre-merger) sales volume.

Note that

$$-T_i \frac{\partial d_i(p^*)}{\partial p_i} = \frac{\partial \pi^m(p^*)}{\partial p_i}, \quad i = 1, \dots, I \quad (4.34)$$

To verify this identity, recall from (4.3) that

$$\frac{\partial \pi^m(p^*)}{\partial p_i} = \frac{\partial \pi_i(p^*)}{\partial p_i} - \frac{T_i \partial d_i(p^*)}{\partial p_i} = \frac{-T_i \partial d_i(p^*)}{\partial p_i} > 0, \quad i = 1, \dots, I,$$

since $T_i > 0$ and $\frac{\partial d_i}{\partial p_i} < 0$, see (4.1). Assume, for example, that the pre-merger industry had settled on the smallest equilibrium $\underline{p}^*(pre)$. Then, the larger the UPP-measures $\{T_1, \dots, T_I\}$ are, the larger the price increases for the products of the merging firms in their best response to the pre-merger equilibrium $\underline{p}^*(pre)$, *i.e.*, the larger $|\Psi^m(\underline{p}^*(pre)) -$

⁵Farrell and Shapiro implicitly assume that the UPP-measures T_i are non-negative. As discussed above, this is guaranteed to hold under the (CM) condition.

\underline{p}^*].

While unstated in Farrell and Shapiro (2010a), the UPP-measures may thus be viewed as proxies for the ultimate measure of interest:

$$|\underline{p}^*(post) - \underline{p}^*(pre)|_\infty \geq |\underline{\Psi}^m(\underline{p}^*(pre)) - \underline{p}^*|_\infty. \quad (4.35)$$

The inequality (4.35) follows from the fact that $\underline{p}^*(pre) \leq \underline{p}^*(post)$, see Theorem 4.3.4 (a). This implies the simultaneous optimization variant of the tâtonnement scheme in the post-merger game Γ^{post} , which starts at $\underline{p}^*(pre) \leq \underline{p}^*(post)$, generates an *increasing* sequence of price vectors which converges to $\underline{p}^*(post)$:

$$\underline{p}^*(pre) \leq \underline{\Psi}^m(\underline{p}^*(pre)) \leq \underline{\Psi}^{m(n)}(\underline{p}^*(pre)) \leq \underline{\Psi}^{m(n+1)}(\underline{p}^*(pre)) \leq \underline{p}^*(post). \quad (4.36)$$

Farrell and Shapiro (2010a) describe the above iterative scheme when motivating the use of the UPP-measures, however without monotonicity or convergence proofs.

Farrell and Shapiro (2010a,b) also argue that it is considerably easier to evaluate or estimate UPP measures as compared to conducting a full blown merger simulation. Indeed, they argue that diversion ratios can often be estimated or approximated without having to estimate the industry's complete set of demand functions⁶. This has been debated by various authors such as Epstein and Rubinfeld (2010) and Schmalensee (2009).

One of Farrell and Shapiro's arguments is that, in contrast to traditional market concentration measures or a full blown merger simulation, the UPP measures do not require an upfront specification of the boundaries of the market being considered. This is always a difficult question to resolve. As an example, the DOJ, when litigating to prevent the merger between Oracle and Peoplesoft, identified the relevant market of human relations and financial management systems as consisting of these two firms and SAP. However, the court rejected the DOJ's argument, identifying other suppliers of

⁶Farrell and Shapiro (2010b) states: "for example, horizontal or documentary evidence from win/loss reports, discount approval processes, or customer switching patterns can be highly informative about the diversion ratio."

related software and faulting the DOJ for an inadequate specification of the relevant product market.

It is, of course, true that in most model specifications, estimates of own and cross-price elasticities among the products of the merging firms depend on which set of firms and products are included in the market model. However, it could be argued that these elasticities are relatively insensitive to the market boundary choice. In fact, the DOJ/FTC define the relevant market precisely by adding firms and products to the market of interest until the price elasticities of the firms are insensitive to the further addition of new products.

As to Farrell and Shapiro (2010a,b)'s argument that UPP measures are easier to evaluate than complete merger simulations, it should be noted that merger simulations may be reduced to implementing, say, the “simultaneous optimization” variant of the tâtonnement scheme in the post-merger game, starting from the current (pre-merger) equilibrium price vector, see Lemma 4.3.1 and Theorem 4.3.4.

Schmalensee (2009), while praising Farrell and Shapiro (2010a) for “having made a significant contribution that has the potential to improve merger enforcement,” takes issue with their recommendation to use the UPP measure as the indicator by which to rank different merger proposals as the “quantity is unrelated to any measure of customer harm”. Instead, Schmalensee (2009) argues for the use of an approximate estimate of post-merger *price changes* and proposes Price Change Assuming Linearity (PCAL) as an alternative to UPP. PCAL calculates the post-merger equilibrium assuming all cost-functions are linear and all demand functions for the products of the merging firms are *linear* as well. An additional major assumption is that the demand functions of the products of the merging firms do not depend on the prices of the other firms in the industry, effectively assuming that the merged firm can operate as a monopolist. The results in this essay show that the post-merger equilibrium can be calculated as the limit vector of an increasing sequence of best response price vectors to the pre-merger (observed) equilibrium. To compute this sequence, one needs to postulate a system

of demand functions. If the above linear functions - without dependence on prices of non-merging firms - is deemed adequate, this can be used to generate the price change estimates. However, if other specifications (that result in supermodular profit functions) seem more reasonable, these can be evaluated with little effort as well, on the basis of the above simple tatônnement scheme.

4.5 Conclusions and Extentions

In the preceding sections, we have provided broad conditions under which, in the absence of cost synergies, a horizontal merger with asymmetric firms and differentiated products necessarily results in an increase in prices and profits for all firms (merged and unmerged). However, we have also demonstrated that constant, marginal cost synergies for the merging firms may be sufficiently large as to reduce the price of some or all firms. These two results, as well as the additional comparative statics results used to obtain them, will be of great use to theoreticians and antitrust litigators alike. As is generally the case, there are several avenues for future investigation which would complement the work contained herein.

For instance, one might consider back-testing our results on data from historical mergers with settings appropriate to our model. Work by Werden and Froeb (1994) provides an example of this sort of empirical investigation in the merger simulation literature. It would also be of interest to identify when the marginal increase in aggregate profits due to the acquisition of a new firm is larger when this firm is acquired by a larger coalition of firms. In addition to direct extensions, we also feel that the operations literature would greatly benefit from a more comprehensive investigation of the operational cost synergies resulting from such mergers. Work modeling actual cost synergies arising in various operational settings and investigating their impact on equilibrium performance measures would be of great benefit to the OM and Economic communities. Such a literature would have the potential to impact the way firms consider mergers as well as the anti-trust

policy and litigation surrounding proposed mergers.

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Appendix A

Appendices for Chapter 2

A.1 Two Sample t-Tests

In this appendix, we report on the two-sample t-tests (assuming unequal variances) we conducted on all national waiting time observations for each of the six hamburger chains to verify whether the waiting time distributions vary by chain. The critical values for each test, with an alpha of 0.05, consistently rounded to 1.96. The t statistic is reported in the right-hand section of the table below.

Table A.1: **Two-Sample t-Test on National Chain-Wide Wait Time Observations (unit = seconds)**

Chain	Num. Obs.	Mean Wait	Std. Dev.	Mc Donald's	Burger King	Wendy's	White Castle	Dairy Queen	Steak 'n Shake
McDonald's	598	224	151	–	4.09	6.66	-3.98	-0.70	-3.86
Burger King	600	192	117	-4.09	–	2.89	-7.25	-5.15	-7.70
Wendy's	596	173	110	-6.66	-2.89	–	-9.13	-7.92	-9.93
White Castle	334	269	174	3.98	7.25	9.13	–	3.57	0.55
Dairy Queen	528	230	128	0.70	5.15	7.92	-3.57	–	-3.40
Steak 'n Shake	328	263	141	3.86	7.70	9.93	-0.55	3.40	–

A.2 The Optimization Routine

To mitigate the difficulties associated with the optimization problem (2.13), we restrict the feasible region for the parameter vector θ by imposing several reasonable constraints.

For all chains $k = 1, \dots, K$, let

$$\begin{aligned} \text{con}(\theta)_{b,m} &= \sum_{j=1}^J S_{j,b,m}(P, W, X|\theta)/h(b, m), b = 1, \dots, B; m = 1, \dots, M \\ &= \text{the fraction of the population in tract } b \text{ and socio-economic group } m \\ &\quad \text{which purchases a fast-food meal;} \end{aligned}$$

$$\overline{\text{con}}(\underline{\text{con}}) = \text{an upper (lower) bound for the fraction of the population in any geographical area and any socio-economic group to purchase a fast-food meal;}$$

$$J_k = \{j : k(j) = k\} \text{ denotes the set of outlets belonging to chain } k;$$

$$\hat{c}_k(\theta) = \text{best estimate of chain } k\text{'s standard cost rate where,}$$

$$c_k = \frac{1}{J_k} \sum_{j \in J_k} [P_j + \Omega(P, X, W|\theta)^{-1} Q(P, X, W|\theta)_j];$$

$$\underline{c}_k = 0;$$

$$\overline{c}_k = \min_{j \in J_k} P_j : k = 1, \dots, K.$$

We impose the constraints:

$$\underline{\text{con}} \leq \text{con}(\theta)_{b,m} \leq \overline{\text{con}}, \text{ for all } b = 1 \dots, B \text{ and } m = 1 \dots, M, \quad (\text{A.1})$$

$$\underline{c}_k \leq \hat{c}_k(\theta) \leq \overline{c}_k, \text{ for all } k = 1 \dots, K^1. \quad (\text{A.2})$$

Thus, instead of the unconstrained problem (2.13), we solve the constrained optimization problem: $(P) \min_{\theta} \{(2.13) \text{ s.t. } (2.14) \text{ and } (2.15)\}$. To solve the constrained optimization problem, we replaced the soft constraints (2.14) and (2.15) by penalty functions which penalize any violations of these constraints. The penalty functions are multiplied with a common multiplier Λ which, within the course of our iterative algorithm is reduced

sequentially to zero. More specifically, we have used the following perturbed objective:

$$\begin{aligned}
G'(\theta)AG(\theta) &+ \Lambda \sum_b \sum_m \{\log[\overline{con} - con(\theta)_{b,m}] + \log[con(\theta)_{b,m} - \underline{con}]\} \\
&+ \sum_k \log[\overline{c}_k - \hat{c}_k(\theta)] + \log[\hat{c}_k(\theta) - \underline{c}_k]\}.
\end{aligned} \tag{A.3}$$

We have developed a special algorithm to solve (P) via the modified objective (A.3). We begin with a large value for Λ , the weight of the penalty functions, roughly two orders of magnitude larger than the objective function value at the starting point. This is an application of the general barrier method approach for constrained non-linear optimization. The algorithm invokes a quasi-Newton search method. During this search, we restrict movement in the direction of the barriers imposed by the penalty functions so that any point within the interior of the feasible region can be reached, but points along the barrier are not approached very quickly, thus preventing the algorithm from 'trapping' itself in unfavorable points. When a stopping condition is reached, the penalty weight Λ is halved and the modified quasi-Newton search re-run. In the first iteration, when the penalty Λ is large, this generally results in the algorithm moving to a point which is quite far from the barriers. The algorithm iterates until the penalty weight is small enough to render the penalty terms insignificant compared to the regular objective function (2.13). Since, by the termination of the algorithm, the multiplier is reduced to an insignificant number, the algorithm optimizes the *true* objective function (2.13) over the feasible region described by the constraints (A.1) and (A.2).

To arrive at the reported estimates, we used a process in which, in the first stage, we took 20 starting points and ran the above algorithm with two different initial values of the penalty parameter Λ - one two orders of magnitude larger than the other - resulting in two estimates per starting point. For each of the 20 starting points we chose the estimate (of the two) that resulted in the lower objective function (excluding the penalty function), generated a weighting matrix for this estimate from the covariance matrix. In the second-stage we ran our algorithm starting with this estimate and weighting matrix,

again from both Λ values generating 40 final estimates.

A.3 The Macro-Moments

We have added macro-moments that are based on three demographic features: age, race, and gender. We use the study by Paeratakul et al. (2003), which reports the proportion of people in various demographic groups that consume fast-food over a two day period. As suggested in Thomadsen (2005a), the macro-moments are constructed based on the idea that the consumption ratio of related demographic groups in Cook County should be close to the national consumption ratios. For example, the local ratio of men to women consuming a fast-food meal should match the national ratio, *i.e.*, the percentage of women consuming fast-food in Cook County may differ from the national average but the fraction of men consuming should differ from the national average proportionally to women. The following twelve macro-moments were added to the micro-moments, based on comparisons between age brackets, one between genders and one between races).

$$G_{0-9,10-16}(\theta) = \frac{1}{J} \sum_{j=1}^J [R_{0-9} \frac{Q_{j,10-19}(\theta)}{Pop_{10-19}} - R_{10-19} \frac{Q_{j,0-9}(\theta)}{Pop_{0-9}}], \quad (\text{A.4})$$

...

$$G_{40-59,60+}(\theta) = \frac{1}{J} \sum_{j=1}^J [R_{40-59} \frac{Q_{j,60+}(\theta)}{Pop_{60+}} - R_{60+} \frac{Q_{j,40-59}(\theta)}{Pop_{40-59}}], \quad (\text{A.5})$$

$$G_J(\theta) = \frac{1}{J} \sum_{j=1}^J [R_{Male} \frac{Q_{j,Female}(\theta)}{Pop_{Female}} - R_{Female} \frac{Q_{j,Male}(\theta)}{Pop_{Male}}], \quad (\text{A.6})$$

$$G_J(\theta) = \frac{1}{J} \sum_{j=1}^J [R_{Black} \frac{Q_{j,White}(\theta)}{Pop_{White}} - R_{White} \frac{Q_{j,Black}(\theta)}{Pop_{Black}}], \quad (\text{A.7})$$

where R_{0-9} denotes the national fraction of fast-food consumers who belong to the 0-9 age bracket as estimated by the Paeratakul et al. (2003) study, Pop_{0-9} denotes the Cook County population in this age bracket, and $Q_{j,0-9}(\theta)$ denotes the demand of consumers

age 0-9 at outlet j . Similar definitions pertain to the other R_- , Pop_- , and Q_- numbers.

Appendix B

Appendices for Chapter 3

B.1 Single-Crossing Property for Segment-by-Segment Profit Functions

The single-crossing property discussed in Section 3.3 was first introduced by Milgrom and Shannon (1994) as a close variant to the preceding “Spence-Mirrlees” single-crossing condition, see Edlin and Shannon (1998). The Monotonicity Theorem (Theorem 4) in Milgrom and Shannon (1994) shows that this single crossing property is, in fact, equivalent to each of the best response functions being monotonically increasing.

Lemma B.1.1 *Fix $j = 1, \dots, N$*

For each market segment $k = 1, \dots, K$ the profit function $\pi_{jk}(p)$ has the single-crossing property in (p_j, p_{-j}) i.e., for any $p_j^1 < p_j^2$ and $p_{-j}^1 < p_{-j}^2$: $\pi_{jk}(p_j^1, p_{-j}^1) < \pi_{jk}(p_j^2, p_{-j}^1) \Rightarrow \pi_{jk}(p_j^1, p_{-j}^2) < \pi_{jk}(p_j^2, p_{-j}^2)$.

Proof: It suffices to show that

$$\frac{\partial \pi_{jk}}{\partial p_j} > 0 \Rightarrow \frac{\partial^2 \pi_{jk}}{\partial p_j \partial p_m} > 0 \quad \forall m \neq j. \quad (\text{B.1})$$

Note from (3.25) that for all $m \neq j$:

$$\frac{\partial^2 \pi_j}{\partial p_j \partial p_m} = \sum_{k=1}^K \left\{ \frac{\partial S_{jk}}{\partial p_m} - (p_j - c_j) g_j(Y, p_j) \left[\frac{\partial S_{jk}}{\partial p_m} - 2 \frac{S_{jk}}{h_k} \frac{\partial S_{jk}}{\partial p_m} \right] \right\}, \quad (\text{B.2})$$

$$= \sum_{k=1}^K \frac{\partial S_{jk}}{\partial p_m} \left[1 - (p_j - c_j) g_j(Y, p_j) \left(1 - 2 \frac{S_{jk}}{h_k} \right) \right]. \quad (\text{B.3})$$

However, by (3.23), for any $m \neq j$:

$$\begin{aligned} 0 < \frac{\partial \pi_{jk}}{\partial p_j} &= S_{jk} \left[1 - (p_j - c_j) g_j(Y, p_j) \left(1 - \frac{S_{jk}}{h_k} \right) \right] \Leftrightarrow 1 - (p_j - c_j) g_j(Y, p_j) \left(1 - \frac{S_{jk}}{h_k} \right) > 0 \\ &\Rightarrow \\ 1 - (p_j - c_j) g_j(Y, p_j) \left(1 - 2 \frac{S_{jk}}{h_k} \right) > 0 &\Leftrightarrow 0 < \frac{\partial S_{jk}}{\partial p_m} \left[1 - (p_j - c_j) g_j(Y, p_j) \left(1 - 2 \frac{S_{jk}}{h_k} \right) \right] = \frac{\partial^2 \pi_{jk}}{\partial p_j \partial p_m}. \end{aligned} \quad (\text{B.4})$$

(The second equivalence follows from $\frac{\partial S_{jk}}{\partial p_m} > 0$, see (3.19). The last identity follows from (B.3)). ■