Do Converters Facilitate the Transition to a New Incompatible Technology?:
A Dynamic Analysis of Converters

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Abstract

This paper analyzes the process of transition in standards between incompatible technologies when converters are available. Contrary to a common presumption that converters facilitate the transition from an old technology to an otherwise incompatible new technology, I find circumstances in which the possibility of transition is blockaded by the existence of converters. When there are changes in the adoption behavior due to the existence of converters, I also analyze the welfare consequences of the changes in the adoption regimes. In the welfare analysis of converters, a distinction is made between ex ante and ex post efficiency effects.

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I. Introduction

This paper analyzes the process of dynamic transition in standards between incompatible technologies when *converters are available*. In particular, I evaluate the validity of a commonly held belief that the existence of converters facilitates the transition from an old technology to an otherwise incompatible new technology. The rationale for this belief is based on the observation that the use of converters can alleviate the cost of incompatibilities for the early adopters of the new technology who otherwise bear a disproportionate share of incompatibility costs. It turns out that the validity of the proposition above is not always upheld. Even though there is some truth in it, I also find circumstances in which the possibility of transition to a new technology is blockaded by the existence of converters. The reason is that the availability of converters enhances not only the value of adopting the new technology but also that of joining the installed base since the threat of being stranded is also weakened.

The analysis of dynamic transition process in the presence of converters is important for the following reasons, which motivate this paper. First of all, in many high-tech industries, such as telecommunications and computer, compatibility plays an increasingly critical role in harnessing potential demand-side scale economies, now known as network externalities.¹ Given the rapid developments of new technologies and innovations in those industries, one crucial determinant of market performance is how a newly emerging technology will perform against the pre-installed base of an old technology. As recognized by Farrell and Saloner (1986), however, the existence of an incompatible installed base can adversely affect the adoption of a new technology when compatibility is important. This has serious implications not only for an individual

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¹Compatibility can be of strategic importance for different manufacturers even in the absence of network externalities if complementary components are required to assemble a "system". See Economides (1989) and Matutes and Regibeau (1988) for an analysis of the firms' incentives to make their products compatible with other firms' complementary products.
industry performance but also for an economy-wide economic growth because in the market economy the fate of technologies will be, to some extent, determined by the way consumers respond in the face of potentially incompatible technologies. Second, most analyses in the literature on network externalities and compatibility have been carried out on the assumption of all-or-nothing compatibility. With this assumption standardization has been considered the predominant mode to achieve compatibility among the products adopted; by choosing the same interface or sharing the same specification in key components, products from different manufacturers can be combined and used in a harmonious way to exploit network externalities. However, the reality tells us that compatibility is a matter of degree [Berg(1988)]. Moreover, it has been recognized that standardization is not the only way to exploit the network externalities arising from compatibility. An alternative way to achieve compatibility, which has received relatively little attention in the literature, is through the development of converters that allow consumers of one network to utilize the network benefits of another [Katz and Shapiro (1985), David and Bunn (1988), and Farrell and Saloner (1992)]. Third, the transition process between the two incompatible technologies are inherently dynamic and requires a truly dynamic model. However, the literature on converters is exclusively static and is not adequate to analyze the inherently dynamic process of technology transition.

My model builds on the previous work of Farrell and Saloner (1986) who envision the following scenario. Potential users arrive gradually over time and continue to adopt only one technology available before some time T, at which a new incompatible technology becomes available unexpectedly. In this framework, Farrell and Saloner ask what is the pattern of adoption behavior of newly arriving consumers after time T? They show that there are two possibilities: everyone from T onwards adopts the new technology or nobody adopts the new technology. They call the first outcome adoption equilibrium and the second, nonadoption.

I modify their model to allow for the availability of converters. I consider two
types of converters; "one-way" converters and (symmetric) "two-way" converters.\(^2\) One-way converters confer the benefit of partial compatibility only on its owners while symmetric two-way converters confer the same degree of partial compatibility benefit not only on its owners but also on users of the rival technology.\(^3\) I then analyze the effect of converters on the equilibrium technology adoption patterns. If there are changes in the adoption behavior due to the existence of converters, I also compare the welfare consequences of the changes in the adoption regimes.

This paper is an addition to a small but growing number of papers dealing with the implications of gateway technologies in the compatibility literature. In a detailed case study of electric supply systems, David and Bunn (1988) document how the discovery of a converter (or gateway technology in their terminology) can tip the balance in the battle for de facto standards. In a recent paper, Farrell and Saloner (1992) explore the implications of converters in the adoption of technology when there is a conflict between variety and compatibility benefits. Contrary to the common beliefs that converters make possible the joint benefit of variety and network externalities, they show that the existence of converters can actually lead to less compatibility than would prevail in the absence of converters. To make things worse, this phenomenon occurs in the circumstances when the benefit of compatibility is greatest. Their model, however, is essentially static and is not adequate to analyze the dynamic transition from one technology to another. Their model is also limited in its ability to determine who actually supplies converters when two incompatible

\(^2\)See Farrell and Saloner (1992) for the discussion of various types of converters.

\(^3\)In the case of two-way converters, each user group of incompatible technologies will prefer to free-ride on the converters provided by the other group if converters are costly. In that case, due to the public good characteristic of two-way converters, we first have to answer the question of who provide converters in order to analyze the (possible) transition process. In Choi (1993), assuming that the newly arriving consumers decide to adopt a new incompatible technology, I derive the equilibrium behavior in the provision of converters and compare it to the socially optimal outcome. I find that there can be two types of market inefficiencies in the provision of converters. First, converters can be supplied by the "wrong" group. Second, the timing of provision may be too late since the providers of converters ignore the positive externality that their patronage would confer on the users of the rival technology.
technologies are coexistent. In their model, any equilibrium in which converters are purchased is asymmetric despite the assumption of ex ante symmetric technologies. Therefore, they have to rely on an arbitrary assumption of coordination to break symmetry. My model provides a more systematic answer to the question since it, by virtue of being dynamic, analyzes competition between naturally asymmetric technologies: one with a pre-installed base and the other without one. However, these two papers should be viewed as complementary since the foci of the papers are rather different.

The paper is organized in the following way. Section II sets up a basic framework to analyze the effect of converters in the transition process from an old technology to a new, but incompatible, technology. In Section III, I analyze the technology adoption process in the presence of converters. Then, the equilibrium behavior in the technology adoption is compared with the one in the absence of converters. Welfare implications of the availability of converters are also derived. Concluding remarks follow.

II. The Setup

I consider a variant of Farrell and Saloner (1986) with the exception of the availability of converters. As emphasized in David and Bunn (1988) and modeled by Farrell and Saloner (1992) in a static context, compatibility and standardization is a matter of degree and the extent of compatibility can be reduced by the availability of converters at a cost. I investigate how the availability of converters can affect the dynamic choice of technology.

I examine competition between two technologies, the old and the new denoted by U and V, respectively. I consider a continuous time framework in which consumers arrive sequentially over time. They are assumed to be infinitesimal. The timing is normalized by assuming that only technology U is available before time zero, at which point a new technology V becomes available unexpectedly. The assumption of unexpected availability of the new technology spares us the need to analyze the waiting option for the consumers
before time zero. I assume that these two technologies are supplied competitively or unsponsored in the terminology of Arthur (1989). The marginal costs of these two technologies are assumed to be the same and constant over time, which implies that prices are also constant over time and allows me to ignore prices in the comparison of purchasing decisions. I further assume that the price of each technology is sufficiently high to not justify scrapping the old unit and buy a new one. This assumption is meant to capture the fact that "new users have an option that was unavailable to previous users, and moreover, those (the installed base) who had previously adopted the old technology may be at least somewhat committed (Farrell and Saloner, 1986, italics original)."

A consumer who has technology U enjoys a flow benefit of $\alpha + \beta N_t$ at date t if the network size she belongs to at time t is $N_t$, $\alpha, \beta > 0$. I can interpret $\alpha$ as a network independent benefit (net of amortized cost of the old technology) and $\beta$ as a parameter representing the strength of the network benefit. A consumer who has technology V, which is assumed to be incompatible with U, enjoys a flow benefit of $\alpha' + \beta N_t$. The interpretation of $\alpha'$ is the same as $\alpha$ applied to the new technology. Since technologies U and V share the same parameter $\beta$, both technologies are assumed to confer the same degree of network benefit if they have the same network size. Future flow benefits are

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4In contrast, Katz and Shapiro (1992) endogenize the timing of new technology introduction in a model with technical progress. They also allow consumers to wait until the introduction of a new technology. However, in their perfect foresight equilibria consumers never exercise their option to wait since there is no uncertainty regarding the future value of the technology. See Choi (1994 a) for a model in which the waiting option is explicitly analyzed.

5Katz and Shapiro (1986) examine the dynamics of competition between incompatible technologies in a market where the sponsors of those technologies engage in strategic pricing.

6Let $p$ be the purchase price for the new technology. Then, with the notation used below, the sufficient condition for the assumption to hold can be given as follows: $0 \leq v^+ - p < u^-$. The first inequality ensures that the buyer of a new technology have nonnegative consumer surplus while the second inequality says that scrapping the old technology and buying a new one is not worthwhile. It can be easily verified that all the cases referred to later are consistent with these conditions. For an analysis of technology choice in the presence of network externalities when consumers are allowed to make repeat purchases, see Choi (1994 b) and Waldman (1993).
discounted using the constant interest rate of r. I denote the number of consumers already
having technology U at time zero by I (for installed base). The rate of continuous arrival is
normalized at n; the number of consumers arriving between time t and t + dt is simply ndt.

Typically, converters are not perfect and entail the degrading of performances. Therefore, I follow Farrell and Saloner (1992) in assuming that converters confer only a
fraction of the full compatibility benefit. If the size of the incompatible network is N, the
purchase of a converter gives rise to the network benefit of β(1-q)N in addition to the
benefit from one's own network. The parameter q represents the degree of degradation in
performance, meaning that a lower q corresponds to a better converter. It is assumed that
converters are supplied competitively at the price of c, which is constant over time.

III. Implications of Converters for Dynamic Technology Adoption

In this section, I analyze the technology adoption processes in a decentralized
market economy. In particular, our interest is in evaluating the validity of the alleged claim
that converters can facilitate the transition process to a new technology by mitigating the
pains of incompatibility for the initial adopters of the new technology. I adapt the model
of Farrell and Saloner (1986) to incorporate the availability of converters. In order to
investigate the effect of converters on the dynamic choice of technology, it is necessary to
examine the technology adoption behavior in the absence of converters.

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7 For instance, recently an alliance of Motorola, IBM and Apple Computer introduced the
Power PC, a new processor based on RISC (Reduced Instruction-Set Computer)
technology. Even though the Power PC is estimated to be faster than Intel's (currently)
most advanced Pentium chip, it cannot readily run most of the applications software that
run on Intel-based computers. A conversion technology makes it possible for a Power PC
to run existing Intel-based softwares. However, converters restrain processors and make a
Power PC run no faster than a basic Intel 486 [see The Economist, February 12th, 1994,
III.1. Technology Adoption without Converters

The equilibrium adoption behavior is presented by Farrell and Saloner (1986). For future reference and completeness, I summarize their results here. Let us define:

\[ u^- = \int_0^\infty [\alpha + \beta I] e^{-t} dt = \frac{\alpha + \beta I}{r} \] 
(1)

\[ u^+ = \int_0^\infty [\alpha + \beta I + \beta nt] e^{-t} dt = \frac{\alpha + \beta I}{r} + \frac{\beta n}{r^2} \] 
(2)

\[ v^- = \int_0^\infty \alpha' e^{-t} dt = \frac{\alpha'}{r} \] 
(3)

\[ v^+ = \int_0^\infty [\alpha' + \beta nt] e^{-t} dt = \frac{\alpha'}{r} + \frac{\beta n}{r^2} \] 
(4)

We can interpret \( u^- \) and \( v^- \) as the potential values of the old and the new technology, respectively, for a consumer arriving on the scene at time zero. They represent the discounted presented values of adopting the old or new technologies under the most optimistic assumption that all subsequent future consumers adopt the same technology chosen by her. In contrast, \( u^+ \) and \( v^+ \) are the values of each technology under the most pessimistic assumption that she is the last adopter of the corresponding technology.

Katz and Shapiro (1992) interpret \( u^+ - u^- \) and \( v^+ - v^- \) as the values of the confidence for each technology since they measure the benefits accruing to the adopter of each technology from attracting all future adopters to the same network. Then we have the following Proposition.

**Proposition 1** (Farrell and Saloner, 1986)

If \( u^- > v^+ \), then nonadoption is the unique subgame perfect equilibrium.

If \( v^- > u^+ \), then adoption is the unique subgame perfect equilibrium.

If \( v^+ > u^- \) and \( u^+ > v^- \) hold simultaneously, then there are multiple equilibria where both adoption and nonadoption can be sustained as subgame perfect equilibria. Which equilibrium prevails in the market depends on the expectations of consumers since the
underlying difference between the two technologies is exceeded by the value of consumer confidence.\textsuperscript{8}


III.2. Technology Adoption with Converters

In this subsection, I analyze the technology adoption process under the availability of converters. My aim is to evaluate the claim that the existence of converters can facilitate the transition to a new incompatible technology. The reasoning behind this claim is easy to understand; converters mitigate the loss from incompatibility with the installed base for the early adopters of the new technology, thereby helping the new technology gain momentum. Indeed, I find cases where the existence of converters facilitate the transition to a new technology. However, the transition is not necessarily welfare-improving. For the welfare analysis of converters, it is useful to make a distinction between \textit{ex ante} and \textit{ex post} efficiency effects. If the converters do not induce any change in the pattern of technology adoption, there will always be gains in efficiency with the availability of converters. I call this the \textit{ex post} efficiency effect since the effect is the same as when the converters become unexpectedly available \textit{after} all the adoption decisions have been made. However, when they cause \textit{ex ante} changes in the equilibrium adoption behavior, the overall welfare effect becomes ambiguous. If converters induce adoption of a new technology while there would have been excess inertia without converters, there will be further gains from the \textit{ex ante} effect of converters. However, it is also possible that converters induce excess momentum and reduce the overall welfare even after we account for the beneficial \textit{ex post} effect of converters.\textsuperscript{9}

\begin{flushleft}
\textsuperscript{8}The underlying difference between the two technologies is a measure that accounts for the difference in intrinsic values ($\alpha - \alpha'$) and initial asymmetry in the installed base ($\beta I / r$). See Katz and Shapiro (1992) for details.
\end{flushleft}

\begin{flushleft}
\textsuperscript{9}Farrell and Saloner (1986) define excess momentum and excess inertia as socially
\end{flushleft}
Perhaps more surprising and counterintuitive is the result that converters can actually introduce friction in the transition process and block the adoption of a new technology that would have been possible without converters. The reason is that the existence of converters not only enhances the value of adopting a new technology but also the value of joining the installed base since the threat of being stranded by future consumers is weakened. Interestingly in the case of the linear network benefit function, converters block adoption equilibrium only when there is excess momentum. Therefore, welfare increases when converters frustrate the transition to a new technology.

To understand the dynamics of technology adoption under the availability of (two-way) converters, I need to redefine the values of adopting each technology, $u^-, u^+, v^-$, and $v^+$, taking into account the costs and benefits of converters. For that purpose, it is prerequisite to analyze the incentive to buy converters in the case where a new incompatible technology is adopted by newly-arriving consumers. However, in order to avoid the complications of analyzing the identity of providers and the timing of provision in the redefinition of these values, I first consider a case where the cost of converters is negligible and consequently, the decision to provide converters has a minimal effect on the overall technology adoption behavior. After considering this special case, I will demonstrate that the conclusions we draw are robust to the introduction of the positive costs of converters.

I denote the redefined values with converters of a negligible cost by using tildes over the corresponding values without converters. Then, we have:

$$
\tilde{u}^- = \int_0^\infty [\alpha + \beta I + \beta(1-q)n] e^{-\alpha t} dt = \frac{\alpha + \beta I}{r} + \frac{\beta(1-q)n}{r^2}
$$

inefficient adoption and nonadoption of the new technology, respectively.

10With the cost of converters negligible, there is no need to make distinction between one-way and two-way converters.

11Note that considering the effect of converters with a negligible cost is the same as analyzing the technology adoption behavior when the new technology is partially compatible with the installed base rather than completely incompatible.
I derive the condition for there to be different types of *unique* equilibrium depending on the availability of converters; without converters the unique equilibrium is for the newly arriving consumers to stick with the old technology (i.e., \( u^- - u^+ > 0 \)) while with the availability of converters, the unique equilibrium is to adopt the new technology \( (\tilde{v}^- - \tilde{u}^+ > 0) \). The condition can be written as:

\[
\beta q I + \frac{\beta q n}{r} < \Delta < \beta I - \frac{\beta n}{r}
\]  

(9)

The first and second inequalities in (9) correspond to conditions \( \tilde{v}^- - \tilde{u}^+ > 0 \) and \( u^- - v^+ > 0 \), respectively. In this case, the welfare effect of converters can be written as:

\[
dW = \int_0^\infty \left[ \left( \frac{\alpha' + \beta nt + \beta(1-q)I}{r} \right) + \frac{\beta n}{r^2} \right] e^{-\eta t} dt - \frac{\beta n I}{r^2}
\]

(10)

The integrand in equation (10) is the net gain to the new users who arrive at time \( t \) and the second term is the loss to the installed base due to the adoption of the new technology taking the effect of converters into account. The transition to the new technology with converters is socially efficient if and only if \( dW \geq 0 \), the condition for which is given by:

\[
\Delta \geq 2q \beta I
\]

(11)

When condition (11) fails, it is inefficient to adopt the new technology even after we take into account the effect of converters on mitigating the costs of incompatibility. Both possibilities (welfare-enhancing and -reducing effects of converters) are illustrated in Figure 1.
Next, I also demonstrate the possibility that the availability of converters can block the transition to a new technology which would have been possible without converters. More specifically, it is shown that there is a subgame perfect equilibrium in which the new technology is adopted without the availability of converters (ν⁺ - u⁻ > 0) but the adoption of the old technology is a dominant strategy with the availability of converters (υ⁻ - ν⁺ >0). The condition for this to happen is given by:

$$\beta I - \frac{\beta n}{r} < \Delta < \beta qI - \frac{\beta qn}{r}$$

(12)

The welfare with the availability of converters is given by

$$\int_0^t \left[ \left( \frac{\alpha + \beta nt + \beta I}{r} + \frac{\beta n}{r^2} \right) + \frac{I \left( \frac{\alpha + \beta I}{r} + \frac{\beta n}{r^2} \right)}{e^{-rT}} \right] dt$$

(13)

The first term in the integrand is the utility for the consumers arriving at time t and the second term is the utility for the old group. In contrast, without the availability of converters, there is a subgame perfect equilibrium where the new consumers adopt the new technology. The welfare in this situation is given by:

$$\int_0^t \left[ \left( \frac{\alpha + \beta nt}{r} + \frac{\beta n}{r^2} \right) + \frac{I \left( \frac{\alpha + \beta I}{r} \right)}{e^{-rT}} \right] dt$$

(14)

The changes in welfare can be written as:

$$dW = \int_0^t n \left[ \left( \frac{\alpha + \beta nt + \beta I}{r} + \frac{\beta n}{r^2} \right) - \left( \frac{\alpha' + \beta nt}{r} + \frac{\beta n}{r^2} \right) \right] e^{-rT} dt + \frac{\beta n I}{r^2}$$

$$= \frac{n(\beta I - \Delta)}{r^2} + \frac{\beta n I}{r^2}$$

(15)

Therefore, as long as Δ<2βI, the existence of converters improves the welfare. Since all Δ which satisfy condition (12) are less than 2βI, converters always increase social welfare when they block the transition to a new technology. See Figure 1.

12Note that the statement is weaker than the previous one in that we do not require the adoption equilibrium to be unique without converters.
The availability of converters can block the transition to a new technology which would have been possible without converters.

With converters the unique equilibrium is adoption of a new technology while without converters the unique equilibrium is nonadoption.

Figure 1: The Effect of Converters on the Dynamic Choice of Technology and Its Welfare Consequences.
I have conducted our analysis by assuming a negligible cost for converters. However, it can be shown that our results are general enough to accommodate the introduction of positive costs of converters. In the Appendix, I prove the following.

**Proposition 2.** For any parameter values of \( p \), we can find the sizes of installed base \( I \) and the intrinsic advantage of the new technology \( (\Delta = \alpha' - \alpha) \) such that there is a subgame perfect equilibrium in which the new technology is adopted without the availability of converters but the adoption of the old technology is a dominant strategy with the availability of converters.\(^\text{13}\) In this sense, the existence of converters can block the transition process to a new technology. Moreover, in this case the existence of converters improves welfare.

The counterintuitive result I derived in Proposition 2 can be considered as dynamic analogue to Farrell and Saloner's (1992) static result that the availability of converters may actually lead to less compatibility in equilibrium.

I have analyzed the technology transition process assuming an exogenously given availability of converters. The logic can be applied to explore the incentive to develop and market converters by the suppliers of competing technologies. My analysis suggests that in some cases it may be better for the suppliers of the new technology *not* to make conversion technology available to get their new technology to roll on. The result is reminiscent of Choi (1994 b) and Waldman (1993) who analyzed the monopolist's incentive to make the new product incompatible with the old version of a product. In those papers, the decision to make the new product incompatible and make the old product obsolete stems from the monopolist's desire to induce the owner of the old product to make repeat purchases. In our model, repeat purchases and switching to a new technology by old users are disallowed. However, in order for the new technology to take off, complete incompatibility with the installed base may be necessary to make the threat of stranding the

\(^\text{13}\)The case of converters with a negligible cost corresponds to the case of \( p=0 \).
IV. Concluding Remarks

In this paper we have evaluated the validity of the commonly held conception that the existence of converters makes the transition path to a new technology less strenuous, and thereby promotes the development of a new technology network. We have found that it is not necessarily true. While the incompatibility can retard the transition process by conferring installed base advantage on the old technology, it can also induce new consumers to abandon the old technology if the old one is expected to be stranded. With the converters, there will be less fear of being stranded since converters can mitigate the costs of being "orphaned." Therefore, the existence of converters can forestall the transition to a new technology which would have been possible without the availability of converters. We also analyzed the welfare consequences of converters.

We have analyzed the technology adoption process assuming competitively supplied converters from the moment at which a new technology is available. In reality, however, converters are rarely available in the nascent stage of a new technology as evidenced in the case study of Bunn and David (1988). Moreover, the availability of converters may hinge crucially on the potential size of the market which is determined by the actual technology adoption. The development of conversion technology usually involves a significant amount of fixed sunk cost. Consequently, if only a small number of users adopt a technology incompatible with the main stream one, the size of the market will not justify the development cost of the converters. This suggests that the transition process and the availability of converters are jointly determined. We also expect the market for converters to be oligopolistic due to sunk development costs. The model incorporating these features awaits further research.

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14Katz and Shapiro (1992) also find that the sponsor of the new technology is biased against compatibility in their model of product introduction with network externalities.
Appendix

Proof of Proposition 2:

There are two cases to consider according to whether converters are assumed to be "one-way" or "two-way".

A.I. One-Way Converters

With one-way converters, only those who owns converters can reap the benefit of partial compatibility with the network of the rival technology. The optimal decision for users on the new network and the old network can be described as follows [see Choi (1993) for more details]:

(i) Users on the installed base buy converters at time $T^*$, where

$$
T^* = \arg\max_T \int_T^\infty [\beta(1-q)nt] e^{-rt} dt - ce^{-rT}.
$$

(ii) Newly-arriving consumers buy converters as they adopt the new technology if $I \leq \rho$, where $\rho = \frac{r}{\beta(1-q)}$. Otherwise, users on the new network never buy converters.

To simplify the analysis, let us assume that $I < \rho$. Then, I can rewrite $\tilde{u}^-$, $\tilde{u}^+$, $\tilde{v}^-$, and $\tilde{v}^+$ in the following way.

$$
\tilde{u}^- = \int_0^\infty [\alpha + \beta(1-q)n] e^{-rt} dt = \frac{\alpha + \beta n}{r} + \int_{T^*}^\infty [\beta(1-q)nt] e^{-rt} dt - ce^{-rT^*}
$$

$$
\tilde{v}^+ = \int_0^\infty [\alpha' + \beta nt + \beta(1-q)n] e^{-rt} dt - c = \frac{\alpha'}{r} + \frac{\beta n}{r^2}
$$

As in the case of negligible costs of converters, the following should be satisfied for the proposition to hold: $v^+ - u^- > 0$ and $\tilde{u}^- - \tilde{v}^+ > 0$. Note that

$$
\int_{T^*}^\infty [\beta(1-q)nt] e^{-rt} dt - ce^{-rT^*} > \int_0^\infty [\beta(1-q)nt] e^{-rt} dt - c = \frac{\beta(1-q)n}{r^2} - c
$$

Therefore, the sufficient conditions for $v^+ - u^- > 0$ and $\tilde{u}^- - \tilde{v}^+ > 0$ to hold can be rewritten as follows.
\[ \beta I - \frac{\beta n}{r} < \Delta < \beta I - \frac{\beta q n}{r} - cr \quad (A.2) \]

As long as \( \beta I - \frac{\beta q n}{r} - cr \) is larger than \( \beta I - \frac{\beta n}{r} \), we can find \( \Delta \) that satisfies \( v^+ - u^- > 0 \) and \( \tilde{v}^+ - \tilde{u}^- > 0 \) simultaneously. The condition for this is, in turn, given by:

\[ n > \frac{r^2c}{\beta(1-q)} \quad (A.3) \]

It can be easily verified that there are parameter values \( \rho, r, n \) and \( I \) such that conditions (A.3) and \( I < \rho \) are consistent.

A.II. Two-Way Converters

I first analyze the problem of who pays the costs of converters in the case where a new incompatible technology is adopted by newly-arriving consumers. Since converters are assumed to be "two-way", only users in one network need to patronize converters in order for both user groups to take advantage of the network benefits of the others. Therefore, converters will have a public good characteristic; each user group prefers the other to supply converters.

In a decentralized economy, it turns out that who purchases converters is determined by the size of the installed base \( I \) at the time of a new technology introduction [see Choi (1993) for details]:

(i) If \( I \leq \rho \left( = \frac{r c}{\beta(1-q)} \right) \), then users on the new network never buy converters and users on the installed base buy converters at time \( T^* \) defined in (A.1).

(ii) If \( I > I^* \), newly-arriving consumers supply converters as they adopt the new technology, where \( I^* \) is given by:

\[ I^* = \frac{r c}{\beta(1-q)} \left( 1 - e^{-\frac{cr^2}{\beta(1-q)n}} \right) = \frac{\rho}{1 - e^{-\frac{I}{n}}} \quad (> \rho) \quad (A.4) \]

(iii) If \( \rho < I \leq I^* \), there are two pure strategy equilibria. One is in which newly-arriving consumers supply converters as they arrive and the other is the one in which users on the
old network supply at time $T^*$.\footnote{If $p < I < I^*$, there are also mixed strategy equilibria in the provision of converters since the game exhibits the payoff structure of the war of attrition. However, the two pure strategy equilibria are the most efficient ones. Any other mixed equilibria entail inefficiency due to the delay in the provision of converters. Since I am interested in demonstrating inefficiency in the market outcome, there is no loss of generality; the focus on the most efficient equilibria in the decentralized economy only makes my task harder.}

When there are multiple equilibria in the provision of converters, the values of $\tilde{u}^-$, $\tilde{u}^+$, $\tilde{v}^-$, and $\tilde{v}^+$ are not uniquely defined. To avoid this issue, suppose that $I > I^*$, i.e., the size of the installed base is large enough so that newly-arriving consumers provide converters if they adopt the new technology $V$. Then, we can unambiguously define $\tilde{u}^-$ and $\tilde{v}^+$.\footnote{If $p < I < I^*$, there are also mixed strategy equilibria in the provision of converters since the game exhibits the payoff structure of the war of attrition. However, the two pure strategy equilibria are the most efficient ones. Any other mixed equilibria entail inefficiency due to the delay in the provision of converters. Since I am interested in demonstrating inefficiency in the market outcome, there is no loss of generality; the focus on the most efficient equilibria in the decentralized economy only makes my task harder.}

$$\tilde{u}^- = \int_0^\infty \left[ \alpha + \beta I + \beta(1-q)n \right] e^{-nt} \, dt = \frac{\alpha + \beta I}{r} + \frac{\beta(1-q)n}{r^2}$$

$$\tilde{v}^+ = \int_0^\infty \left[ \alpha' + \beta nt + \beta(1-q)I \right] e^{-nt} \, dt - c = \frac{\alpha' + \beta(1-q)I}{r} + \frac{\beta n}{r^2} - c$$

As in the case of negligible costs of converters, the following should be satisfied for the proposition to hold: $\tilde{v}^+ - \tilde{u}^- > 0$ and $\tilde{v}^- - \tilde{v}^+ > 0$. The conditions can be rewritten as follows.

$$\beta I - \frac{\beta n}{r} < \Delta < \beta q I - \frac{\beta q n}{r} + cr \quad (A.5)$$

Therefore, as long as $\beta q I - \frac{\beta q n}{r} + cr$ is larger than $\beta I - \frac{\beta n}{r}$ we can find $\Delta$ that satisfies $\tilde{v}^+ - \tilde{u}^- > 0$ and $\tilde{v}^- - \tilde{v}^+ > 0$ simultaneously. The condition for this is, in turn, given by:

$$I < \frac{r c}{\beta(1-q)} + \frac{n}{r} = \rho + \frac{n}{r} \quad (A.6)$$

Recall that we derived $\tilde{v}^-$ and $\tilde{u}^-$ on the assumption that $I > I^*$. Since $\rho + \frac{n}{r}$ is always greater than $I^* = \frac{\rho}{\beta(1-q) \left[ 1 - e^{-\frac{cr^2}{\beta(1-q)n}} \right]} = \frac{\rho}{1 - e^{-\frac{I^*}{n}r}}$ for all $\rho$, $r$, $n > 0$, we can always find a suitable size for the installed base for which the proposition is valid.

I showed that there is a situation in which the existence of converters can eliminate
the adoption equilibrium in both cases. With the existence of converters, everybody sticks with the old technology and in equilibrium there is no need to purchase converters since everyone subscribes to the same network. Therefore, in the new equilibrium the converters are not actually used even though just sheer existence changes the nature of equilibrium. I do not have to calculate the purchase cost in the calculation of welfare. As a result, the welfare change will be exactly the same as the one with a negligible converter cost. Welfare improves if and only if $\Delta < 2\beta I$. Once again, it can be verified that all $\Delta$ which satisfy condition (A.2) or (A.5) are less than $2\beta I$. 
References


