Monopolistic Competition and Optimum Product Diversity: Reply

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Our model of monopolistic competition and product diversity (Dixit and Stiglitz, 1977) was intended for the large-group case. Therefore we used an approximation neglecting from the own and cross elasticities terms of order $1/n$, where n is the equilibrium number of firms. Xiaokai Yang and Ben J. Heijdra (1993) have generalized the solution for a special case of our model, retaining some of these terms but assuming a unitary elasticity of substitution between the monopolistic group and the numeraire good. We will discuss their model by placing it in an even more general framework.

The underlying monopolistically competitive equilibrium of a symmetric group is governed by two conditions: each firm's maximization equates marginal revenue and marginal cost, and free entry equates average revenue and average cost. Let n denote the number of products or firms in the group, x the output of each such firm, and p the price of each product in the group relative to the competitive numeraire. Let $C(x)$ be each firm's cost of production, and let $\varepsilon(p,n)$ be the elasticity of demand perceived by each firm. Then the two conditions can be written

$$(1) \qquad p\left[1-\frac{1}{\varepsilon(p,n)}\right]=C'(x)$$

$$(2) \qquad p=C(x)/x.$$

Further, let the demand relation in equilibrium be

$$(3) \qquad p=D(x,n).$$

These three equations determine p, x, and n. Unless one wants a closed-form solution, the various functions, including the elasticity $\varepsilon(p,n)$, can have any functional form (restricted only by minor requirements for existence and the relevant second-order conditions). This formulation subsumes not only Yang and Heijdra's model, but also other widely different models including the one based on Hotelling-like spatial product differentiation. Indeed, Elhanan Helpman and Paul Krugman (1985 Chapters 6–8 [especially section 7.1]) develop the monopolistic competition model for their international trade applications in just such a general way.

Our original (1977) model started with a CES (constant elasticity of substitution) utility function and then employed an approximation ignoring some terms of order $1/n$ to make ε constant. Yang and Heijdra dispense with this approximation, using instead

$$(4) \qquad \varepsilon(p,n)=\sigma-\frac{\sigma-1+\theta(n^{1/\rho}p)}{n}$$

where σ, required to be greater than 1, is the elasticity of substitution between any pair of goods within the monopolistically competitive group, $\rho=(\sigma-1)/\sigma$. Finally, θ is the share of the consumer's expenditure on the monopolistically competitive group and is a function of the group price index $(n^{1/\rho}p)$. Since Yang and Heijdra insist on a closed-form solution, they have to assume that θ is constant, which amounts to requiring a unitary elasticity of substitution in utility between the monopolistically competitive group and the perfectly competitive numeraire good.

We believe that Yang and Heijdra's misplaced emphasis on closed-form solutions

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has led them astray. The desirable features they claim to have added can be attained in far better and more general ways in the formulation (1)–(3) above. To the extent that closed forms are needed for simple applications in macroeconomics and international trade, for example, Yang and Heijdra's formulation adds a minor wrinkle at one point but incurs a large cost in reduced generality elsewhere.

I. Justification for the Approximation

Yang and Heijdra (1993) argue that since the equilibrium number n_e of firms is endogenous, an assumption neglecting $1/n_e$ needs to be justified in terms of the underlying parameters. This is true, but not at all hard to do. If one insists on a closed form, one must resort to the unitary-elasticity case used by Yang and Heijdra. There, n_e is large if and only if

$$\gamma l(1-\rho)/a \gg 1 \tag{5}$$

where γ is the share of the monopolistic group in expenditure, l is the economy's labor endowment, ρ measures substitution within the monopolistic group and a is the fixed cost of producing each variety in this group. However, without a closed-form solution, we can get a qualitative idea of which parameter combinations are conducive to a large n_e by examining the total differential of the system (1)–(3) above. In rough terms, we want low fixed costs and imperfect substitution between products in the group.

We should also point out that Yang and Heijdra destroy the force of their own criticism when they assume that cross elasticities are negligible. Here they see no need to express the condition in terms of exogenous parameters: "in any application of this model one must ascertain *ex post* that ε_{ij} is indeed close to zero *in equilibrium*" (Yang and Heijdra, 1993 p. 296 [italics in the original]).

II. Internal Consistency of the Approximation

Having decided to treat $1/n$ as small, we do so with consistency throughout our paper. Yang and Heijdra (1993) emphasize the terms of order $1/n$ in the own elasticities ε_i but ignore them in the cross elasticities ε_{ij}. This is particularly strange in view of the fact that the error terms in treating $\varepsilon_i \approx -\sigma$ and $\varepsilon_{ij} \approx 0$ are *exactly the same*, namely, $[\sigma - 1 + \theta(q)]/n$. It does not seem logically consistent to emphasize the error in one place and to ignore it in another in the same model.

If Yang and Heijdra want to consider a group sufficiently small that $1/n$ is not negligible, they should do so consistently. When ε_{ij} is significantly different from zero, it raises serious doubts concerning the validity of the price–Nash game stipulated in the model, and alternatives including tacit collusion and entry deterrence should be explored.

III. Passage to a Competitive Limit

Yang and Heijdra (1993) argue that there is merit in letting elasticities increase with n. This is true in some applications, although in others (e.g., Helpman and Krugman, 1985 Chapters 7–8) it makes no difference to the qualitative results. In any case, a correction of order $1/n$ does not capture such dependence in a satisfactory way. In Yang and Heijdra's equation [see (4), above], as n goes to infinity the own elasticity will rise in numerical value to σ. If one wants a perfectly competitive limit of the model, as many would, the own and cross elasticities will have to rise to *infinity*, not to a finite limit.

IV. Optimality of the Equilibrium

While the limiting case of perfect competition and full optimality may be of some interest, we found the equilibrium based on our approximation to be second-best (break-even constrained) optimal. This result is sensitive to the specification, as we pointed out. What is really needed is a constant ratio of average and marginal utilities (or consumer surplus and revenue), and a constant elasticity of demand allows that. However, Yang and Heijdra's (1993) modification can only shift the equilibrium in a small way when n is large; the numbers and prices will be close to those in our approximation, and therefore their equilibrium will be close to being constrained optimal. Serious objections to the optimality result must again be based on more substantial departures from our specification. For a spatial-differentiation example in which the dimension of the attribute space makes a difference, see Stiglitz (1986).

V. Existence of Equilibrium

Yang and Heijdra (1993) point out that if ε is constant in (1) and increasing returns take the power form, the system (1)–(3) is in general inconsistent. Write the cost function as $C(x)=x^{\beta}$ with $\beta<1$; then $C'(x)=\beta C(x)/x$, and dividing (1) by (2) gives

$$1-1/\varepsilon=\beta$$

which is generally impossible when ε and β are separately and exogenously specified. In Dixit and Stiglitz (1977) the cost function had a fixed component with constant marginal cost, so the problem did not arise. Yang and Heijdra propose to rescue matters for the power cost function by letting ε vary endogenously as in (4) above. However, for large n the variation of ε is very slow, and the equilibrium shifts dramatically when some parameter changes by a small amount. The problem of such "fragile" equilibria is in practice not much better than nonexistence (see e.g., Lawrence Summers, 1991). Again we think that Yang and Heijdra's aim is much better accomplished in the general setting of (1)–(3) above.

VI. Price Paid for Abandoning the Approximation

To introduce a term of order $1/n$ Yang and Heijdra (1993) have restricted the model to the case of a unitary elasticity of substitution between the monopolistic group and the numeraire good. We judge the trade-off to be quite unfavorable; the drastic reduction in parameter space of the model buys a correction that in practice is of minor significance.

This case has been used in some applications to make some broad thematic points about the nature of trade or the possibility of underemployment equilibrium, where the exact value of the elasticity was irrelevant. However, the presence or absence of $1/n$ terms in elasticities was equally irrelevant there.

Finally, we should note that Yang and Heijdra are not the first to add the $1/n$ terms to our model, although they appear to have been the first to be published doing so. To record priority, we should mention an unpublished note by Kelvin Lancaster (1980); doubtless there are others unknown to us.

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