Low-Frequency Variability in a Baroclinic β Channel with Land–Sea Contrast*

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ABSTRACT

Atmospheric low-frequency variability (LFV) is studied in a two-layer quasigeostrophic model. The model geometry is a periodic β channel with flat bottom and zonally inhomogeneous thermal forcing. As a result of the idealized land–sea contrast, the model produces a zonally modulated climatological jet with realistic amplitude. The model's LFV is equivalent barotropic; principal component analysis reveals that it consists of (i) dominant stationary patterns with red-noise-like temporal behavior and (ii) propagating waves with periods of 37 and 50 days superimposed on the former.

The vorticity forcing due to synoptic eddies is dominated by self-interaction of high-pass filtered model fields. Applying a phase-randomized, stochastic analog of this forcing to a version of the full model in which fast baroclinic instability and, therefore, synoptic eddies are suppressed, produces a climatology and LFV that are very similar to those in the full model. Synoptic eddies are solely represented in the simplified model version by means of stochastic forcing that is independent of the low-frequency flow. It follows that, while fast synoptic eddies are modulated in the full model by the LFV, this modulation is fairly passive: anomalous generation of the synoptic eddies in the course of the full system's low-frequency evolution, the so-called synoptic-eddy feedback, is not essential in selecting the system's low-frequency modes; the main role of synoptic eddies is to supply energy to these modes.

Further analysis indicates that the LFV in this thermally driven model originates from the barotropic mode's dynamics. The baroclinic mode passively follows, to first order, the low-frequency changes in the barotropic mode. The latter changes are due to stochastically excited, weakly damped linear eigenmodes of the barotropic-mode equation. Two distinct stationary eigenmodes, as well as two pairs of propagating modes with periods of 27 and 36 days, respectively, dominate the low-frequency behavior. The leading empirical orthogonal functions in this model are associated with these six particular eigenmodes. The latter are not well separated, however, from the other eigenmodes in terms of damping time scale, and it is the barotropic nonlinearity that selects the six dynamically important modes over the others. Interactions between these six modes also result in the occurrence of probability density maxima in two-dimensional subspaces of the model's phase space.

1. Introduction

a. Motivation

In this paper, we study the effects of zonally inhomogeneous thermal forcing on the dynamics of atmospheric low-frequency variability (LFV) in the extratropics. LFV refers to time scales longer than that of

the synoptic eddies and is known to be predominantly equivalent barotropic (Wallace 1983). It influences, however, regional weather (e.g., Cai and Van den Dool 1991, 1992), which is dominated by the fast processes associated with baroclinic instability. It is important, therefore, to understand LFV for predicting intraseasonal weather modulations.

LFV has been shown to possess both stationary and traveling wave patterns with very low zonal wavenumbers (Branstator 1987; Kushnir 1987; Ghil and Mo 1991a,b). It has been described in episodic, intermittent terms via multiple regimes, as well as in quasi-periodic, oscillatory terms via intraseasonal oscillations (Ghil 1987; Plaut and Vautard 1994; Ghil and Robertson 2000).

Zonal inhomogeneities in lower boundary conditions affect to various degrees the climatology and variability of Northern Hemisphere (NH) flows. The relative role of topography (Charney and Eliassen 1949) and of land–sea contrast, or “thermal topography” (Smagorinsky 1953; Döös 1962), in producing quasi-stationary, geo-

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graphically fixed waves in the NH is still a matter of some debate (Held 1983). Even more so is their relative role in affecting NH LFV.

The role of topography in NH LFV has been explored in a sequence of papers (Charney and DeVore 1979; Pedlosky 1981; Legras and Ghil 1985; Ghil and Robertson 2000, and references therein). Vautard et al. (1988) have introduced a zonally inhomogeneous jet forcing to represent, very indirectly, the effects of thermal land–sea contrast on the climatological wind field. Given recent interest in ocean–atmosphere–land interactions (e.g., Martinson et al. 1995), we introduce here more explicitly—although still in an idealized manner—lower boundary conditions that discriminate between the effects of land and ocean on atmospheric flows.

b. Background

1) LFV OBSERVATIONS

The North Atlantic Oscillation (NAO; e.g., Hurrell 1995) and the so-called zonal flow vacillation (e.g., Yu and Hartmann 1993; Koo et al. 2002) in the Southern Hemisphere (SH) are well-known examples of extratropical LFV. The Arctic oscillation (AO) appears to be strongly related to the NAO (Deser 2000; Wallace 2000). Robertson (2001) argues that the AO, defined statistically in terms of principal component analysis, is a combination of stationary and propagating dynamical patterns.

An important example of extratropical oscillatory signal is the 20–25-day wave of Branstator (1987) and Kushnir (1987). Plaut and Vautard (1994) identified two distinct low-frequency patterns active in the North Atlantic region with periods of 70 and 30–35 days. The first one involves changes in both intensity and position of the Atlantic jet, the second consists of a propagating dipole pattern. Both oscillations were reproduced in the semiempirical model of Da Costa and Vautard (1997) and confirmed by the observational study of Zhang et al. (1997). Keppen et al. (2000) computed the power spectra of an NAO index and identified oscillations with periods of 18, 25, 35, and 65–70 days, in agreement with the previously mentioned studies and with their own two-layer, shallow-water model results.

2) LFV THEORIES

(i) Linear theories

The equivalent-barotropic character of observed LFV has motivated the use of barotropic models in a number of LFV studies. Simmons et al. (1983) have shown that the leading eigenmodes of such a model, linearized about the observed climatological flow, exhibit certain characteristics of observed LFV patterns (see also Branstator 1992; Metz 1994; Da Costa and Vautard 1997). Zonal asymmetries in the basic state of linear models or in the forcing of nonlinear models have been found to be important for the model-generated LFV (Legras

and Ghil 1985; Branstator 1992; Metz 1994; Yamane and Yoden 1998; Keppen et al. 2000; Swanson 2000).

The leading barotropic eigenmodes are nearly neutral or damped for realistic values of friction (e.g., Metz 1994). Purely linear, barotropic dynamics cannot, therefore, account fully for the observed LFV. Branstator (1992) showed that time-dependent forcing by high-frequency synoptic transients is an important contributor to the low-frequency, equivalent-barotropic vorticity balance. LFV may thus be due to damped linear barotropic eigenmodes excited by a spatially coherent forcing with higher frequencies and shorter wavelengths, at least to some extent. This forcing can be derived either from more complete baroclinic models (Metz 1994) or from observations (Branstator 1992; Da Costa and Vautard 1997). Nonlinear interactions between linear low-frequency modes have also been shown to play a role in generating LFV (Metz 1994; Bladé 1996; Newman et al. 1997).

(ii) Nonlinear theories

The nonlinear interaction between storm tracks and low-frequency patterns in the extratropics is a longstanding topic of investigation. Synoptic eddies can be shown to maintain LFV patterns and may, in turn, be modified by LFV (Namias 1953; Shutts 1983; Illari 1984; Dole and Gordon 1983; Robertson and Metz 1989, 1990; Robinson 1991; Yu and Hartmann 1993; Branstator 1995; Cuff and Cai 1995; Koo et al. 2002). A feedback between the two might involve modification of the storm track due to LFV, which in turn amplifies the LFV pattern, leading to a self-maintaining structure. Cai and Van den Dool (1994) showed, however, that synoptic-eddy feedback is actually one of the smallest contributors to the total low-frequency tendency; still, they argued that it is vital for the occurrence of LFV patterns. Synoptic-eddy feedback is at the heart of the original concept of *weather regimes* (Reinhold and Pierrehumbert 1982). These authors argued that interaction between the synoptic eddies and low-frequency flow results in the occurrence of flow patterns that persist longer than the lifetime of an individual synoptic eddy. A more detailed discussion of synoptic-eddy feedback appears in section 4.

Another possibility is that LFV is a separate dynamical entity that involves nonlinear, self-sustained low-frequency oscillations coexisting with multiple quasi-stationary states (e.g., Legras and Ghil 1985; Da Costa and Vautard 1997; Itoh and Kimoto 1999; Ghil and Robertson 2002). This possibility has been explored in the framework of dynamical systems theory and shown to give rise to *persistent planetary flow regimes* (Legras and Ghil 1985; Ghil and Childress 1987, Chapter 6).

Weather regimes and planetary flow regimes, although of different dynamical origin, can be derived statistically from data in a similar fashion as probability density function (PDF) maxima in the system's phase space (Ghil 1987; Mo and Ghil 1988; Vautard et al. 1988; Vautard

1990; Marshall and Molteni 1993; Michelangeli et al. 1995; Hannachi 1997; Itoh and Kimoto 1999). The two interpretations of multimodality are closely related to the question of whether the distinct PDF maxima are due to weak interactions between damped linear modes that arise due to random transient forcing, or to a strong self-sustaining nonlinearity (Molteni 2002). The latter hypothesis holds much greater promise for predictability than the former. Molteni and Corti (1998) show that random energy fluctuations may obscure or enhance multimodality in the distribution of model states for time spans as long as several decades. This adds weight to the former, stochastic, interpretation.

c. Present approach

In this paper, we concentrate on the role of synoptic eddies in extratropical LFV. This role may depend strongly on the climate system's zonal asymmetries. It is important, therefore, to include the latter in our model in a physically consistent fashion.

We study numerically the dynamics of a high-resolution, two-layer, quasigeostrophic (QG) channel model with flat bottom and zonally inhomogeneous thermal forcing, meant as the simplest dynamical setting that includes both synoptic eddies and zonal asymmetries. The model does not include topography (e.g., Legras and Ghil 1985), nor tropical effects (e.g., Inatsu et al. 2000). We will show that the simplicity of the baroclinic–barotropic coupling in the resulting model is helpful in isolating the role of synoptic eddies.

The model is formulated in section 2 and the control integration is described in detail in section 3, where the leading spatial patterns of the system's LFV are identified. In section 4, we construct a stochastically forced version of our full model, in which intrinsic synoptic variability is suppressed. This model is shown to produce LFV that is virtually identical to that of the full model. We thus argue that synoptic-eddy feedback is not essential for determining our system's LFV, and that the main role of synoptic eddies is to supply the energy to LFV modes.

Based on the insights gained in sections 3 and 4, we provide in section 5 evidence that our model's LFV is essentially generated by the interacting linear eigenmodes of the barotropic-mode equation, excited by spatially coherent, but temporally white noise. The baroclinic mode's variability is slaved to that of the barotropic mode. The main effect of nonlinearity is to select the dominant linear barotropic eigenmodes. These eigenmodes explain the leading barotropic patterns obtained via principal component analysis. Concluding remarks follow in section 6.

2. Model formulation

a. Model geometry

The model geometry is depicted in Fig. 1, with a zonal cross section through the model at the top and a

FIG. 1. Schematic diagram of model geometry. (upper) Vertical cross-section; (lower) plan view. Land areas shaded.

plan view at the bottom. To model the effects of land–sea contrast on the atmospheric circulation, an oceanic region is included in which the sea surface temperature (SST) is prescribed. This oceanic region represents a midlatitude portion of the North Atlantic basin, which is approximately 60° wide at 45°N . It extends longitudinally from $X_w = 3520$ km to $X_E = 8640$ km, and latitudinally from $Y_S = -3200$ km to $Y_N = 2400$ km (16° – 66°N), with $y = 0$ corresponding to 45°N .

Atmospheric latitudinal boundaries are situated at $Y_{a,S} = Y_S$ and $Y_{a,N} = 3200$ km, that is, at 16°N and 74°N , respectively. The longitudinal atmospheric boundaries at $X_{a,w} = 0$ and $X_{a,E} = 20\,480$ km correspond to a zonal extent that is slightly less than the length of the zonal circle at 45°N ; periodic boundary conditions are assumed in the zonal direction. The atmospheric height is $H_a = 10\,000$ m.

b. Governing equations

We use the classical two-layer QG model (e.g., Pedlosky 1987) to represent midlatitude atmospheric dynamics. The equations for the barotropic component ψ and baroclinic component τ of the streamfunction are

$$\begin{aligned} \frac{\partial q_\psi}{\partial t} + J(\psi, q_\psi) \\ = -h_1 \frac{1}{t_d} \nabla^2 \psi - \sum_{k=0}^3 \frac{1}{t_d^k} \nabla^2 \psi^k + A_H \nabla^6 \psi \end{aligned}$$

$$\begin{aligned}
& - h_1 h_2 \left[J(\tau, q_\tau) + \frac{1}{t_d} \nabla^2 \tau \right], \quad (1a) \\
& \frac{\partial q_\tau}{\partial t} + (h_2 - h_1) J(\tau, q_\tau) \\
& = \frac{f_0}{H_a} \frac{1}{h_1 h_2} F(x, y; \tau) - h_2 \frac{1}{t_d} \nabla^2 \tau \\
& - \sum_{k=0}^3 \frac{1}{t_d^k} \nabla^2 \tau^k + A_H \nabla^6 \tau \\
& - \left[J(\tau, q_\psi) + J(\psi, q_\tau) + \frac{1}{t_d} \nabla^2 \psi \right]. \quad (1b)
\end{aligned}$$

Here,

$$q_\psi = \nabla^2 \psi + \beta y, \quad q_\tau = \nabla^2 \tau - \frac{1}{R_d^2} \tau \quad (2)$$

are the barotropic and baroclinic components of the potential vorticity, respectively, while F is the forcing function; $h_1 = 0.3$ and $h_2 = 0.7$ are nondimensional thicknesses of the lower and upper atmospheric layers, $R_d = 383$ km is the Rossby radius of deformation, $f_0 = 10^{-4} \text{ s}^{-1}$ is the Coriolis parameter, $\beta = 1.87 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$ is the gradient of the Coriolis parameter at 45°N , $t_d = 4.62$ day is the bottom drag time scale, $A_H = -2 \times 10^{16} \text{ m}^4 \text{ s}^{-1}$ is the superviscosity coefficient, and $J(A, B) \equiv (\partial A / \partial x)(\partial B / \partial y) - (\partial A / \partial y)(\partial B / \partial x)$ is the Jacobian.

Additional damping terms with the characteristic time scales of $t_d^0 = 17$ day, $t_d^1 = 23$ day, $t_d^2 = 29$ day, and $t_d^3 = 37$ day are included, following Vautard et al. (1988). They act selectively on the first zonal planetary modes (0, 1, 2, 3) to prevent the excessive accumulation of energy in these modes that is inherent to rigid-lid channel models, due to the lack of meridional and vertical dispersion. These missing processes affect primarily the largest zonal atmospheric scales (Held 1983).

The forcing function $F(x, y; \tau)$ is constant in time and has an additive component that varies only with latitude y , due to solar forcing. It also incorporates the effect of land–sea contrast on the atmospheric temperature T_a by including, over the ocean only, an additive component that is proportional to the difference between the SST and T_a , while insulating lower boundary conditions are imposed over land. The SST is also a prescribed function of y only. The baroclinic streamfunction τ enters into F since T_a is an affine function of τ in our model. The radiation and heat exchange terms are described fully in Kravtsov and Robertson (2002).

Equations (1) and (2), subject to superslip and no-flow conditions on the northern and southern boundaries, as well as to mass and momentum constraints (McWilliams 1977), are discretized on a 128×41 grid with a resolution of 160 km in both x and y . They are numerically integrated using central differences in space and leapfrog time stepping, with $\Delta t = 10$ min; see Kravtsov and Robertson (2002) for further numerical details.

3. Control run and methodology

The control integration was run for $N = 36\,500$ days of physical time. The simulated fields are saved once every day. In all the plots of simulated or derived fields below, the heavy solid lines mark the contour of the ocean basin.

a. Analysis methods

We use standard principal component (PC) analysis (e.g., Preisendorfer 1988) to visualize the dominant spatial patterns of model behavior as empirical orthogonal functions (EOFs). In plotting the results of the PC analysis, each nondimensional, unit-length spatial EOF is multiplied by the standard deviation of the corresponding PC in the appropriate units; the PC itself is normalized by its standard deviation.

The spatial PCs so obtained are subjected to singular spectrum analysis (SSA) to detect oscillatory peaks in the model's solutions (Vautard and Ghil 1989; Dettinger et al. 1995; Ghil et al. 2002). SSA is particularly suitable for this task due to the nature of the model oscillations; these, as we shall see, can be represented as the sum of carrier signals with fixed frequency but modulated in amplitude.

Each sampled time series, with the sampling interval of 1 day, is embedded into a vector space of dimension $M = 100$ by considering 100 lagged copies thereof. We thus resolve oscillations with periods less than 100 days. The 100×100 lag-covariance matrix of the data is computed and its eigenvalues and eigenvectors are found. Temporal PCs (T-PCs) are obtained by projecting the time series onto each of these eigenvectors and are Fourier transformed to identify their dominant frequency.

We plot the frequency–variance plots to visualize the singular spectrum. Pairs of eigenvalues with significantly overlapping error bars could represent an oscillation. A set of ad hoc error bars is computed based on the estimated decorrelation time of the time series to locate potential “oscillatory pairs.” The pair is found to be significant if its two members have the same dominant frequency and account for a significant fraction (we used the value of 95%) of the signal variance at this frequency (Vautard et al. 1992). In addition, we check significance against a red-noise null hypothesis using the chi-square test of Allen and Smith (1996). In the graphs, we plot noise error bars that span the 2d to 97th percentiles of the noise distribution.

b. Model climate

The time-mean fields are shown in Fig. 2. In Fig. 2a, the barotropic zonal velocity is plotted in contours; the square root $E^{1/2}$ of the barotropic turbulent kinetic energy $E \equiv (1/2)[(\partial \psi' / \partial x)^2 + (\partial \psi' / \partial y)^2]$, where the prime denotes deviation from long-term climatology [see also Eq. (15) in section 3c], is plotted in grayscale. Clima-

FIG. 2. Model climatology. (upper) Dynamic fields (m s^{-1}); (lower) temperature fields ($^{\circ}\text{C}$). Side bars show gray levels; contour intervals CI are 5 units.

tological air temperature (contours) and temperature standard deviation (grayscale) are plotted in Fig. 2b.

The climatological zonal jet has an intensity of about 20 m s^{-1} , which is realistic at this model resolution, and the atmospheric temperature distribution is reasonable. The jet maximum is located over land, slightly to the west of the ocean basin's western shore (see Fig. 2c), while the model's storm track, seen in the variance of the temperature field, is located downstream of the jet maximum (see Fig. 2b); the maximum of the equivalent-barotropic LFV occurs at the exit of the storm track, to the north of the jet axis (Fig. 2a). Thus, the positions of the jet maximum, storm track, and LFV maximum are quite realistic when considered in relation to each other.

The main differences between the model climatology and the observed one are the absence of a strong stationary wave in the model and a westward shift of the variability centers. The former is probably due to the lack of topography. The latter shift was also observed in a more complete, global three-layer QG model in spherical geometry and with realistic topography (Corti et al. 1997), which might indicate that it is due to the model's QG character.

Principal component analysis was performed separately on the unfiltered barotropic (Fig. 3) and baroclinic (Fig. 4) daily streamfunction fields. The (dimensional) EOFs are plotted on the left, and the singular spectra of the (normalized) PCs on the right. The leading EOF

of the barotropic streamfunction has a pronounced zonally symmetric component and corresponds to meridional shifts of the jet that are slightly modulated in longitude. Its singular spectrum increases monotonically with decreasing frequency. EOFs 2 and 3 form a wavenumber-4 pair that exhibits spatial quadrature in the zonal direction and a spectral peak at 37 days. Shorter-period peaks at 7.5 and 8.5 days are also statistically significant. EOFs 4 and 5 have a localized structure centered about the western boundary of the ocean and possess a spectral peak with a period of 50 days. In addition, there are peaks with periods of 4–6 days, although they are barely significant.

The peaks in the SSA plots of Fig. 3 are broad, with increased variance around the main 37- and 50-day oscillatory pairs. We argue in section 5 that they are associated with propagating linear barotropic eigenmodes that interact with each other and with stationary eigenmodes that have a red-noise-type spectrum.

The EOFs of the baroclinic streamfunction are similar to the barotropic EOFs. The main difference is that the EOF that has a pronounced zonally symmetric component, EOF-6 (Fig. 4e), accounts for only 6% of the variance. EOF-6 also has significant peaks associated with the baroclinically unstable modes evident in EOFs 4 (not shown) and 5 (Fig. 4d) of τ . EOF-5 is dominated by a wavenumber-5 propagating signal with periods of about 4 and 7.5 days (Fig. 4d). The 4-day peak is probably associated with modulation of the signal by syn-

FIG. 3. EOFs based on the raw, daily values of the barotropic streamfunction ψ ; $\text{CI} = 10^6 \text{ m}^2 \text{ s}^{-1}$, negative contours dashed, zero contour omitted. (right) The corresponding singular spectra are plotted (circles); significant oscillatory pairs (bold circles). The diameter of the circles roughly corresponds to the size of ad hoc error bars of Ghil and Mo (1991a). Light solid and dashed curves represent 2d and 97th percentile of the chi-square red-noise test of Allen and Smith (1996).

optic eddies, while the 7.5-day mode may account for the peaks at a similar period in the barotropic and baroclinic EOFs that appear in Figs. 3a–c and Figs. 4a,b. We are primarily interested in behavior with longer time scales than those associated with both of these peaks and forgo, therefore, a more detailed physical explanation for them.

Aside from these differences, it can be seen that EOF-

1 and EOF-2 of τ resemble EOF-2 and EOF-3 of ψ , in both spatial pattern and power spectrum, while EOF-3 of τ resembles EOF-4 and EOF-5 of ψ in spatial pattern. The 50-day oscillation in the behavior of the barotropic EOFs is not detected in Fig. 4c.

Given the similarities between barotropic and baroclinic EOFs, we consider next in greater detail the dynamics of the barotropic mode.

FIG. 4. EOFs of the raw baroclinic streamfunction τ . Same symbols and conventions as in Fig. 3.

c. Weather regimes

In nonlinear systems, the EOFs do not necessarily correspond directly to dynamical modes (Ghil 1987; Mo and Ghil 1988), as they do in linear, self-adjoint systems (Preisendorfer 1988). A few leading EOFs, however, usually account for a significant fraction of the system's variability. It is common practice, therefore, to search for dynamically important spatial patterns by constructing composites of model fields associated with PDF maxima in a subspace spanned by the system's leading EOFs (Benzi et al. 1986; Kimoto and Ghil 1993a,b).

The patterns found in this way will be used here, furthermore, to isolate the dominant terms in the vorticity balance that is associated with the PDF maxima.

To do this, we first apply a 10-day low-pass filter of order 41 to the barotropic streamfunction field (Otnes and Enochson 1978) to concentrate on the model's low-frequency behavior. The EOFs of the low-pass filtered field (not shown) are very similar to those constructed using raw data (Fig. 3). The PDF is estimated in the phase plane spanned by EOF-1 and EOF-4 of the low-pass filtered data, by using a multivariate kernel density

FIG. 5. Weather regimes. (a) Probability density function (PDF) of the low-pass filtered (LPF) data, in the plane of the principal components (PCs) 1 and 4 of ψ (shading). Contours are 90%, 93%, and 95% confidence levels; ellipses mark areas used for compositing; the regions containing PDF maxima are denoted as I and II. The pair of univariate red-noise surrogates used in the statistical significance test is centered at the x (see text). (b),(c) Regime I composites of ψ and τ , respectively, both in $\text{m}^2 \text{s}^{-1}$; (d),(e) Regime II composites; $\text{CI} = 0.5 \times 10^6 \text{ m}^2 \text{s}^{-1}$, negative contours dashed, zero contour omitted.

estimator (Silverman 1986; Kimoto and Ghil 1993a) with a smoothing parameter equal to 0.4. The results reported later are not sensitive to variations in this parameter. Projection onto EOF-1 and EOF-4 was chosen because these are the two leading large-scale patterns that seem to be predominantly stationary; we will see that this is, in fact, not the case for EOF-4.

The central portion of the PDF so constructed is plotted in Fig. 5a. The PDF shows pronounced bimodality

along the PC-1 axis; it is strongly skewed along the PC-4 axis, as both PDF maxima contain a substantial negative PC-4 component. The bimodality, as well as the skewness, indicate that nonlinear dynamics may play an important role in the model's behavior.

The PDF maxima are statistically significant against a red-noise null hypothesis. To show this we generated two sets of 1000 red-noise surrogate time series, one that has the same length, variance, and lag-1 autocor-

relation as PC-1, and the other that matches these properties for PC-4. The PDF is then computed for each of pair of univariate surrogates and recentered to have a maximum in between the two original PDF maxima, at the position marked by an x in Fig. 5a. The sample PDFs so obtained are all Gaussian to a good approximation. The percentage of these PDFs that have values smaller than the original PDF is plotted as 90%, 93%, and 95% contours in Fig. 5a. It is clear that the two PDF peaks are indeed significant at the 95% level. Very similar results were obtained by generating a set of 1000 bivariate surrogates according to the same criteria (not shown).

EOF-1 has a large zonally symmetric component (Fig. 3a). To show more explicitly that the maxima identified in Fig. 5a are not an artifact of the low-pass filtering and/or a particular choice of a phase subspace, we find the weather regimes in an alternative way using *raw* data, by performing a PC analysis on the zonally averaged barotropic field [cf. the methodology of Koo et al. (2002) for Southern Hemisphere observed fields]. The two leading EOFs of the latter field are both well separated from the others and capture 70% and 20% of the field's total variance, respectively. The PDF in the phase plane spanned by these two EOFs (not shown) is again bimodal and produces, upon compositing, regime patterns that are very similar to those obtained by analyzing the full fields that were not zonally averaged (see later).

We have also performed a separate and even longer model integration, 73 000 days long, and verified that the PDF in the phase subspace spanned by EOF-1 and EOF-4 of its barotropic *unfiltered* data is, again, significantly bimodal (not shown). In section 5, we will use yet another dynamically independent basis to estimate the system's PDF; it produces, once more, the same regimes as those obtained using the previous two methodologies.

The regime composites are constructed by averaging the fields over the data points inside the two ellipses shown in Fig. 5a; the ellipses were chosen visually, based on both the actual values of the PDF (shading), and their statistical significance (contours). The total number of days used in the composites is about 10% of the total number of days in the record, that is, it equals a few thousand days. The regime composites are shown in Figs. 5b–e in terms of anomalies with respect to model climatology. The composite anomalies are not sensitive to the size of the ellipses provided a sufficiently large number of days, on the order of several hundred at least, is contained within them. The regimes have much less zonal symmetry than EOF-1 and are equivalent barotropic, as can clearly be seen by comparing pairwise Figs. 5b,c for regime I, as well as Figs. 5d,e for regime II.

Regime I is mainly characterized by an intensified jet upstream of the ocean's western shore and a weakened jet over the ocean. Regime II consists of a northward

shift of the jet near the western shore. Each regime has a given polarity and its opposite-polarity counterpart is not encountered frequently. This points again to the importance of nonlinear dynamics in the model's behavior.

d. Barotropic vorticity budget

Since the regimes are largely barotropic, we consider next the barotropic vorticity budget. We rewrite Eq. (1a) as

$$\frac{\partial q_\psi}{\partial t} + J(\psi, q_\psi) = D_\psi\{\psi\} - h_1 h_2 J(\tau, q_\tau) + D_\tau\{\tau\} \quad (3)$$

and decompose ψ and τ as

$$\psi = \bar{\psi} + \psi', \quad \tau = \bar{\tau} + \tau', \quad (4)$$

where the overbar denotes the time mean and the prime a perturbation with respect to it, while $D_\psi\{\psi\}$ and $D_\tau\{\tau\}$ are the linear damping terms. Next, we subtract from (3) its time mean to get an equation for the perturbation barotropic vorticity:

$$\begin{aligned} \frac{\partial q'_\psi}{\partial t} + \underbrace{J(\bar{\psi}, q'_\psi)}_a + \underbrace{J(\psi', \bar{q}_\psi)}_b + \underbrace{[J(\psi', q'_\psi) - \overline{J(\psi', q'_\psi)}]}_c \\ = \underbrace{D_\psi\{\psi'\}}_c - h_1 h_2 \underbrace{[J(\bar{\tau}, q'_\tau) + J(\tau', \bar{q}_\tau)]}_d \\ - h_1 h_2 \underbrace{[J(\tau', q'_\tau) - \overline{J(\tau', q'_\tau)}]}_e + \underbrace{D_\tau\{\tau'\}}_f. \end{aligned} \quad (5)$$

Finally, we compute regime composites of each term in (5) for the two regimes identified above. The most important ones are plotted in Figs. 6a–d for regime I and Figs. 6e–h for regime II.

For each regime, the two terms that constitute (5a) are individually dominant (not shown) in amplitude compared to other terms in the equation. They correspond to the advection of the climatological vorticity by the regime anomaly and the advection of the regime vorticity by the climatological flow. However, these two terms nearly cancel, that is, their sum is less than 10% of their individual amplitudes. This cancellation is the signature of a stationary *linear* Rossby wave, and appears to be at odds with the nonlinearity suggested by the asymmetries in polarity seen in the regime composites in Fig. 5. We will return to this issue in section 5.

The terms that involve the baroclinic streamfunction τ are all individually small, and so the main vorticity balance in each regime is between the two terms that involve only the barotropic fields: the anomalous transient interactions, which tend to maintain the given anomaly (5b), and the friction (5c). Note that the composite balance, that is averaging the terms in the balance over each of the regimes, does not literally tell us how individual maps or sequences of maps get into that re-

FIG. 6. Regime composites of the anomalous eddy fluxes ($\text{m}^2 \text{s}^{-1} \text{day}^{-1}$) in the barotropic-mode equation: (a)–(d) Regime I; (e)–(h) regime II. (a),(e) Fluxes due to all transient interactions; (b),(f) fluxes due to $\psi - \psi$ interactions only; (c),(g): fluxes due to $\psi_{\text{LP}} - \psi_{\text{LP}}$ interactions only; (d),(h) fluxes due to $\psi_{\text{HP}} - \psi_{\text{HP}}$ interactions only. The subscripts LP and HP stand for low-pass and high-pass filtered fields; see text for details; $\text{CI} = 10^5 \text{ m}^2 \text{s}^{-1} \text{day}^{-1}$, negative contours dashed, zero contour omitted.

gime or stay there. Fully time-dependent analysis is necessary to determine that.

The regime-maintaining effect of the eddies is illustrated in Fig. 6. The barotropic streamfunction tendencies due to the combined (5b) and (5e) are shown in Fig. 6a for regime I and in Fig. 6e for regime II. Comparison with the regime composites in Figs. 5b and 5d, respectively, clearly shows the maintenance of each regime by the eddies. In Figs. 6b,f, the streamfunction tendency due to (5b) only is plotted. Comparison with

Figs. 6a,e demonstrates that barotropic self-interaction (5b) dominates the tendency.

We also decompose the barotropic self-interaction term into four parts by dividing the ψ -field into 10-day low-pass and high-pass filtered fields. The major contributors to it are self-interactions of the low- and high-pass filtered signals. Figures 6c,d (for regime I) and Figs. 6g,h (for regime II) show that the regime maintaining effect cannot be unambiguously attributed to either high-pass or low-pass filtered eddies alone. How-

FIG. 7. Zonally averaged regime balance. (a),(b) Zonally averaged regime composites ($\text{m}^2 \text{s}^{-1}$); (c),(d) composite zonally averaged tendency ($\text{m}^2 \text{s}^{-1} \text{day}^{-1}$) due to ψ - ψ transient interactions (heavy solid), ψ_{LP} - ψ_{LP} interaction (dashed), and ψ_{HP} - ψ_{HP} interactions (light solid). (a),(c) Regime I; and (b),(d) regime II.

ever, if we look in Fig. 7 at the zonally averaged analogs of the fields in Fig. 6, the “maintaining” effect of the fast transients is striking.

4. Role of baroclinic eddies

a. Motivation

The earlier composite analysis of zonal averages produces results that are consistent with observational studies of the zonal index (ZI) in the Southern Hemisphere

(e.g., Feldstein and Lee 1998; Lorenz and Hartmann 2001), and eddy–zonal flow feedbacks in the Northern Hemisphere (Lorenz and Hartmann 2002). Feldstein and Lee (1998) note, however, that such an analysis cannot definitively show that the eddy feedback is occurring. The tendency equation for the vertically integrated zonal-mean zonal wind u is

$$\frac{\partial \{[u]\}}{\partial t} = -\frac{\partial}{\partial y} \{[u^*v^*]\} - D; \quad (6)$$

hereafter in this section, square brackets denote zonal average, a star denotes the deviations from zonal average, curly brackets denote vertical average, v is the meridional component of velocity, and D is the bottom drag. It is clear from Eq. (6) that if a certain pattern exists for a sufficiently long time, so that the regime is quasi-stationary, the eddy momentum convergence will balance the bottom drag irrespective of whether there is an eddy feedback or not; see also Koo and Ghil (2002) and Koo et al. (2002). To demonstrate this occurrence, it would be necessary to show a mutual reinforcement between synoptic eddies and low-frequency flow.

Consider an arbitrary decomposition of Eq. (6) into low-frequency (L) and high-frequency (H) components:

$$\begin{aligned} \frac{\partial \{[u_L]\}}{\partial t} = & -\frac{\partial}{\partial y} \{[u_L^* v_L^*]\} - D_L - \frac{\partial \{[u_H]\}}{\partial t} \\ & - \frac{\partial}{\partial y} \{[u_H^* v_H^*]\} - \frac{\partial}{\partial y} \{[u_L^* v_H^* + u_H^* v_L^*]\} - D_H. \end{aligned} \quad (7a)$$

The instantaneous amplitudes of u_H and v_H are often larger than those of the low-frequency field. If we start averaging Eq. (7a) in time, however, the terms that are linear in high-frequency variables will average out first, while the quadratic high-frequency term might still be significant. Such an averaged equation will then become

$$\frac{\partial \{[u_L]\}}{\partial t} = -\frac{\partial}{\partial y} \{[u_L^* v_L^*]\} - D_L - \frac{\partial}{\partial y} \{[u_H^* v_H^*]\}. \quad (7b)$$

The balance depicted in Fig. 7 implies that the high-frequency eddies maintain the regime, while the low-frequency eddies tend to damp it, as found by Feldstein and Lee (1998) and Lorenz and Hartmann (2001, 2002). The latter authors have also shown that despite the fact that high-frequency eddies fluctuate on short time scales by construction, the spectrum of the zonal-mean momentum forcing $\partial \{[u_H^* v_H^*]\} / \partial y$ is as red, roughly, as the spectrum of the zonal-mean wind; moreover, zonal flow anomalies and synoptic-eddy momentum forcing exhibit a statistically significant cross correlation, with the former leading the latter by about 10 days. Lorenz and Hartmann thus concluded that a positive feedback of zonal-mean wind on synoptic eddies must be the cause of the redness of the last forcing term in Eq. (7b) and of the cross correlation between it and the zonal wind. Based on arguments of Robinson (2000), who considered the dynamics of a two-layer quasigeostrophic model similar to ours, they have argued that the mechanics of this synoptic-eddy feedback involves the enhancement of baroclinic wave activity in the region of anomalously strong westerlies. Their statistical analysis is consistent with this hypothesis but it does not demonstrate that such an enhancement actually happens. The redness of the synoptic-eddy forcing spectra and cross correlations between this forcing and zonal flow are also consistent with just a steering of the high-frequency field

by the low-frequency flow, without any generation of anomalous baroclinic activity.

We have repeated the analysis of Lorenz and Hartmann (2001, 2002) using the output from our model and found that the cross-spectrum of synoptic-eddy forcing and zonal flow is similar to that found in their studies. To check whether synoptic-eddy feedback, that is, the generation of anomalous synoptic-eddy activity, is involved in selecting the system's low-frequency modes, we construct a simplified model version, in which the fast baroclinic instability is suppressed, and the synoptic eddies only affect the slow modes as prescribed white-noise forcing. If such a simplified model were able to reproduce the behavior of the full model, it would demonstrate explicitly that synoptic-eddy feedback is not a primary ingredient in our model's LFV.

b. Stochastically forced model

1) CONSTRUCTION

We wish to decompose the model fields into a low-frequency "signal" ($\tilde{\psi}, \tilde{\tau}$) and higher-frequency "noise" ($\hat{\psi}, \hat{\tau}$) associated with synoptic eddies:

$$\psi = \tilde{\psi} + \hat{\psi}, \quad \tau = \tilde{\tau} + \hat{\tau}. \quad (8)$$

The synoptic-eddy forcings on the barotropic and baroclinic vorticities are computed as

$$F_\psi = \bar{F}_\psi + F'_\psi, \quad F_\tau = \bar{F}_\tau + F'_\tau, \quad (9)$$

respectively, where

$$F_\psi = -J(\hat{\psi}, \hat{q}_\psi) - h_1 h_2 J(\hat{\tau}, \hat{q}_\tau), \quad (10a)$$

$$F_\tau = -(h_2 - h_1)J(\hat{\tau}, \hat{q}_\tau) - J(\hat{\tau}, \hat{q}_\psi) - J(\hat{\psi}, \hat{q}_\tau); \quad (10b)$$

the overbar denotes the time mean over the entire time series, and a prime marks the anomaly.

We use two separate ways to obtain the desired decomposition of Eq. (8). In the first one, ($\tilde{\psi}, \tilde{\tau}$) and ($\hat{\psi}, \hat{\tau}$) are just the 10-day low-pass filtered component of (ψ, τ) and its high-pass complement, respectively. The second way is described in the appendix, and involves using our model equations as a natural nonlinear low-pass filter on model fields. The latter method turns out to produce a signal that satisfies the original equations to a good approximation, which the former does not.

The time-mean forcings and the standard deviations of the anomalous forcing are shown in Fig. 8 for the statistical decomposition, and in Fig. 9 for the dynamical one. The eddy forcing in both cases has the same qualitative structure, but there are quantitative differences. In particular, the statistically filtered eddy forcing has a larger-amplitude time mean and less variance than the dynamically filtered forcing. Using the dynamically filtered forcing in our stochastic model shown later generally produces better quantitative correspondence to the full model behavior. Qualitative behavior is, however, the same in both cases. We only show the results from

FIG. 8. Vorticity forcing (s^{-2}) due to synoptic-eddy activity based on 10-day high-pass filtered data. (a),(b) barotropic-mode equation; and (c),(d) baroclinic-mode equation. (a),(c) The mean and (b),(d) the std dev multiplied by 10^{10} . (a) $\text{CI} = 10^{-11} \text{ s}^{-2}$; (c) $\text{CI} = 2 \times 10^{-11} \text{ s}^{-2}$; negative contours dashed, zero contour omitted.

the stochastic model based on our dynamical signal versus noise decomposition.

The time-mean effect of this forcing is to maintain the climatological jet (Figs. 9a,c). Moreover, the maximum of the stochastic forcing variance (Figs. 9b,d) is located roughly in between the storm track (Fig. 2b; grayscale) and the maximum of the model's LFV (Fig. 2a; grayscale). This means that the optimal forcing of the LFV is associated with aging synoptic eddies, modified by the interaction with the low-frequency flow.

With these diagnostics in hand, we are ready to explicitly represent the equivalent stochastic forcing in the model as a phase randomized version of the residual forcings. To do so, we normalize F'_ψ and F'_τ of Eqs. (9) and (10) pointwise by their standard deviations (Figs. 9b,d) and compute the two-dimensional (2D) Fourier transforms of the resulting fields. In forcing the model, we multiply these spectra at each point in the 2D wave-number space by a Gaussian-distributed random number, transform back to the physical space and multiply back by the standard deviations. In this way, we retain

FIG. 9. Vorticity forcing (s^{-2}) due to synoptic-eddy activity based on dynamical signal vs noise decomposition. Symbols and conventions as in Fig. 8.

the spatial structure of the stochastic forcing, but fully whiten its temporal structure.

The forcings F_ψ and F_τ so constructed are added to the right-hand side of the original equations (1a), (1b). The cross terms describing the interaction of low- and high-frequency transients are not included in our stochastic forcing, and we let the stochastically forced system model their evolution explicitly. Since the deterministic part of the equations is the same as in the full system, aside from the terms representing time-mean forcing due to fast transients (Figs. 9a,c), the modified system of equations still includes the synoptic eddies associated with the baroclinic instability of the jet. To

suppress high-frequency internal dynamics in our two-layer stochastic system, the modified equation (1b) is integrated step by step, as in the full model, to produce the evolution of τ , while in advancing the modified equation (1a), the term $-h_1 h_2 [J(\tau, q_\tau) + t_d^{-1} \nabla^2 \tau]$ is only allowed to change once every 7 days, according to the updated τ -field at that time. The fast baroclinic instability is effectively suppressed by this procedure, so that the sole remaining effect of synoptic eddies resides in the stochastic forcing F_ψ and F_τ . If synoptic-eddy feedback is not crucial for model behavior, this simplified model version should accurately represent the full model's LFV. The behavior described later is not sensitive

to the exact choice of the cutoff time scale: experiments that used a 5- or 15-day cutoff gave similar results (not shown).

If we suppress the time-dependent part of the “synoptic-eddy” forcing in the model constructed earlier, a very weak intrinsic LFV is obtained; it is only due to slow baroclinically unstable modes that are still present in the model. Fast synoptic variability is, however, suppressed entirely. The resulting climatology does not resemble that of the full model. Thus, the energy input associated with stochastic synoptic-eddy forcing is an essential part of our system’s dynamics.

2) MEAN STATE AND EOFs

Our stochastically forced runs have the same length of $N = 36\,500$ days as the control run. The difference in climatology between the two is small. The maximum difference is of about 7% in amplitude for the run with a noise cutoff time scale of 7 days; it becomes even smaller if longer cutoffs are used.

The EOFs of the barotropic streamfunction computed from the stochastically forced run are displayed in Fig. 10 and are very similar to those of the control run (see Fig. 3), except that EOF-1 is slightly stronger in the latter. Differences are also seen in the spectra. In particular, the main periodic signal detected by the SSA analysis is at 46 days (Figs. 10b,c), rather than 37 days (Figs. 3b,c), and there is some evidence of a 100-day peak in EOF-6 (Fig. 10e). These discrepancies might be associated with a rather arbitrary choice of frequency separating high-pass and low-pass filtered fields. Using a different separation frequency might result in an even more accurate representation of the noise forcing, and therefore, better climatology and time dependence. However, it is clear from the results presented here that the spatial patterns of variability and qualitative time dependence that our stochastically forced model produces resemble strikingly those of the full model.

Aside from the shift in the primary oscillation frequency, the main difference in the EOFs of the baroclinic streamfunction (cf. Fig. 11 with Fig. 4) is the absence of the 7.5-day signal; this is not surprising as the high-frequency modes are substituted by random processes in the stochastically forced system. The EOFs of the baroclinic streamfunction again resemble those of the barotropic streamfunction, even more so than in the control run. EOF-5 (not shown) is similar to EOF-3 and both resemble EOF-1 of Fig. 10a.

We conclude, therefore, that synoptic eddies are important in supplying the energy to the low-frequency modes of the system, but do not dynamically participate in their selection. The mechanics of this selection will be discussed in section 5. Note that our model results are consistent with the storm track being modified by the LFV; this modification is, however, fairly passive dynamically.

3) WEATHER REGIMES

The PDF in the stochastically forced run is significantly bimodal (see Fig. 12a), and has two maxima at the same locations as in Fig. 5a, along with an additional maximum near the climatology. The two modes turn out to be associated with spatial structures that resemble those in the control run; in fact, the composite barotropic streamfunction fields have a pattern correlation of 0.61 for regime I and 0.44 for regime II with their counterparts in the control run, and comparable amplitudes. The vorticity balance (not shown), however, gives less coherent results than for the control run. This is perhaps not surprising, given that the stochastic surrogate for the high-frequency eddy forcing has a white-noise temporal dependence.

We thus conclude that the bimodality does not depend crucially on synoptic-eddy feedback. We shall see in section 5 that, while the bimodality has its origin in the model’s physics, the relative dominance of either PDF mode can change from one sampling interval to another, even for very long intervals. We argue, therefore, that the presence of an additional peak in the PDF from the stochastically forced integration is due to sampling issues (see sections 5 and 6).

5. Role of linear barotropic eigenmodes

We have shown in sections 3 and 4 that the essence of our model’s low-frequency dynamics consists in the response of its barotropic field to the stochastic noise associated with baroclinic eddies. The baroclinic field then passively follows the low-frequency changes in the barotropic field, so that the model’s LFV is equivalent barotropic. This suggests that, to first order, the dynamically important vorticity balance is between the two terms that were jointly labeled (5a) in the barotropic equation (5) of section 3.

a. Eigenmodes of the linearized barotropic equation: Connection with EOFs

We compute, therefore, the linear eigenmodes of the operator $\mathcal{L}$ acting on the barotropic streamfunction anomaly ψ' and given by

$$\mathcal{L}\psi' = -J(\bar{\psi}, q'_\psi) - J(\psi', \bar{q}_\psi) + A_H \nabla^6 \psi'. \quad (11a)$$

These are estimated on a coarse 64×21 grid for computational convenience; the scale-selective damping is retained in the linear operator to filter out numerical modes. The stationary eigenmodes are then recomputed on the original grid by direct time integration, starting from the corresponding coarse-resolution initial state.

The most important eigenmodes of Eq. (11a) are shown in Fig. 13. Their periods and spatial structures resemble those seen in the EOF decomposition (Fig. 3). Without bottom drag, all the eigenmodes shown are nearly neutral; the first two are slightly unstable and the

FIG. 10. Leading EOFs of ψ from the stochastically forced run ($\text{m}^2 \text{s}^{-1}$); $\text{CI} = 10^6 \text{m}^2 \text{s}^{-1}$, same symbols and conventions as in Fig. 3.

others are weakly damped with decay rates on the order of years. For these eigenmodes, adding bottom drag friction to the operator, that is, considering instead the modified operator $\mathcal{L}_d$

$$\begin{aligned} \mathcal{L}_d \psi' = & -J(\bar{\psi}, q'_\psi) - J(\psi', \bar{q}_\psi) + D_\psi \{\psi'\} \\ & + A_H \nabla^6 \psi', \end{aligned} \quad (11b)$$

yields damping time scales of about 10 days, without significant changes to their spatial patterns (not shown).

The first two eigenmodes (Figs. 13a,b) are stationary and consist of zonally modulated meridional shifts of the jet. Two propagating pairs, with periods of 27 and 36 days, are shown in Figs. 13c,d and 13e,f respectively. In contrast to the stationary modes, they are characterized by strong zonal dependence. To better visualize their structure, we define the symmetric and antisymmetric components of these modes, with respect to the axis of the channel, and plot them in Fig. 14. The former have a strong zonal component with

FIG. 11. Leading EOFs of τ from the stochastically forced run ($\text{m}^2 \text{s}^{-1}$); $\text{CI} = 10^6 \text{ m}^2 \text{s}^{-1}$, same symbols and conventions as in Fig. 3.

wavenumber 4, while the latter mainly consist of dipolar and quadrupolar patterns in the vicinity of the ocean basin. These two pairs of eigenmodes clearly share many similarities with EOFs 2–5 of the barotropic streamfunction plotted in Fig. 3. The propagating linear modes described earlier thus provide a dynamical explanation of these EOFs.

We can also show that the leading EOF of Fig. 3 is explained by the two stationary barotropic eigenmodes. To do that, we first rotate the stationary ei-

genmodes 1 and 2 (Fig. 13a,b) and obtain the two orthonormal patterns shown in Figs. 15a,b. The orthogonality is necessary to subtract the two patterns from the full multivariate time series of the model's barotropic field. The barotropic field evolution from the control run is then projected onto the plane spanned by these two patterns.

We now subtract the time series associated with motion in the plane spanned by the two orthogonal stationary patterns of Figs. 15a,b from the full barotropic

FIG. 12. Weather regimes for the stochastically forced run; same layout as in Fig. 5. (a) Contours show the 93% and 95% confidence levels, and the ellipses mark the same areas as in Fig. 5a; (b)–(e) $CI = 2 \times 10^5 \text{ m}^2 \text{ s}^{-1}$, negative contours dashed, zero contour omitted.

streamfunction data set and perform PC analysis on the data set so obtained. The results are shown in Fig. 16. The first EOF of the full data (Fig. 3a) has disappeared, while the other leading EOFs are recovered. It follows that the dominant EOF of the barotropic streamfunction is associated with the stationary eigenmodes of the barotropic vorticity equation linearized about the time-mean barotropic field. The association of the statistically determined EOFs with the properties of the linearized barotropic equation adds weight to the statement that baroclinic dynamics are secondary in our model.

b. Nonlinear effects: Plausibility arguments

The non-Gaussian structure of the PDF in Fig. 5a is an indication that nonlinear effects are present in our model. To better understand the origin and nature of this nonlinearity, we compute the model PDF in the plane spanned by the two orthogonal stationary patterns of Figs. 15a,b. The PDF is computed as in section 3, but for the first half and second half of the data separately. The results are shown in Figs. 15c,d; they are less smooth than those in Figs. 5a and 12a, due to the shorter

FIG. 13. Linear eigenmodes of the nondissipative barotropic equation (11a); $CI = 1$, negative contours dashed, zero contour omitted. (a),(b) Stationary eigenmodes; (c),(d) propagating pair of modes with period 27 days; and (e),(f) propagating pair of modes with period 36 days.

record and the fact that no low-pass filtering has been applied to the patterns' time series.

Each of the two PDFs so obtained is strongly non-Gaussian, but unimodal. We compute the regime composites from the data contained in each of the circles in Figs. 15c,d. The days that belong to each of the regimes in the present classification roughly coincide with those determined previously (see Fig. 5), even though the subspace onto which the data were projected is different—selected eigenmodes here versus leading EOFs in Fig. 5—and the regimes in Fig. 15 were obtained without any filtering of the data. We then recompute the PDFs from PC-1 and PC-4 of the barotropic streamfunction as in section 3 but, again, for the first and second halves of the data separately. The results (not shown) corroborate that, indeed, regime I occurs more often in the first half of the time series and regime II in the second half.

The model's bimodality thus fluctuates on long time scales, as one regime or the other becomes more populated. This behavior is consistent with that observed by Molteni and Corti (1998), who showed that multimodality in the distribution of model states can be more

or less pronounced over time intervals as long as several decades. It is also supported by the well-known fact that trajectories can dwell for arbitrarily long times in either one of the two "regimes" of the Lorenz (1963) model. Similar phenomena have been demonstrated for full atmospheric general circulation models as well (Robertson et al. 2000).

The "stationary" part of the regimes can be determined by summing the orthonormal patterns in Figs. 15a,b with the coefficients that correspond to the coordinates of the centers of the circles in Figs. 15c,d. The results are plotted in Figs. 15e,f and approximately reproduce the linear eigenmodes in Figs. 13a,b, respectively. The amplitudes of the linear stationary eigenmodes in either regime have, however, a definite sign (Figs. 15e,f). In addition, the regime composites (Figs. 5b,d) are very different from the stationary patterns, which goes to show that the regimes do not result from the saturation of a linear instability. Subtracting these stationary patterns from the full regime composite for each regime gives a pattern similar to EOFs 3 and 4 in Fig. 16 or, equivalently, to EOFs 4 and 5 in Fig. 3. These EOFs are, in turn, similar to the antisymmetric

FIG. 14. Decomposition of propagating eigenmodes into symmetric and antisymmetric parts; same contouring as in Fig. 13.

part of the propagating linear eigenmodes of Eqs. (11a) or (11b) (Figs. 14b,d,f,h). The periods of these eigenmodes, while not equal to the periods identified using EOF analysis, are consistent with them, as explained later.

The two eastward propagating waves that determine much of the model's LFV are both dominated by a zonal

wavenumber 4 but differ by the phase of the zonal wave and the degree of localization near the model's ocean. The periods of these two waves are 27 and 36 days, and the corresponding frequencies can be written as $4f$ and $3f$, respectively, where $f = (1/108) \text{ day}^{-1}$. Due to their similar spatial structure, the two waves are likely to interact nonlinearly and produce the harmonics of f .

FIG. 15. Linear eigenmodes and regime dynamics. (a),(b) Orthonormal linear solutions of the nondissipative barotropic equation, $CI = 0.5$; (c),(d) PDF in the phase subspace spanned by the two linear patterns plotted in (a),(b) for the first and second half of the data time series, $CI = 0.01$; (e),(f) spatial patterns corresponding to PDF maxima in (c),(d) ($m^2 s^{-1}$), $CI = 0.5 \times 10^6 m^2 s^{-1}$, negative contours dashed, zero contour omitted.

An additional harmonic realized in the model is $2f = (1/54) \text{ day}^{-1}$, close to the frequency of $(1/50) \text{ day}^{-1}$ detected in Figs. 3d,e and 16c,d.

6. Discussion

We have investigated the dynamics of a two-layer, quasigeostrophic (QG), midlatitude atmospheric channel model with flat bottom, subject to zonally inhomogeneous thermal forcing. The emphasis has been on

the model's low-frequency variability (LFV), with time scales longer than that of the synoptic eddies. We have argued that this variability is associated with the weakly nonlinear interaction of damped linear eigenmodes of the barotropic vorticity equation (section 5), excited by the energy provided by the synoptic eddies.

Modifications of the synoptic-eddy activity due to LFV do not feed back significantly onto our model's LFV. This statement is supported by the results obtained with our simplified model version (section 4), in which

FIG. 16. Leading EOFs of ψ from the control run ($\text{m}^2 \text{s}^{-1}$) when the projection onto the two stationary linear eigenmodes is subtracted from the data; $\text{CI} = 10^6 \text{ m}^2 \text{s}^{-1}$, same symbols and conventions as in Fig. 3.

synoptic-eddy activity was replaced by stochastic forcing of prescribed spatial pattern but white in time. Our results in this respect are quantitatively consistent with the results of Cai and Van den Dool's (1994) observational study.

Many authors have considered the momentum balance associated with low-frequency variations of the atmospheric jet and some of them have argued that synoptic eddies are vital in maintaining LFV (e.g., Branstator 1992, 1995). Synoptic eddies are usually defined

in this context as the high-pass filtered variability. It can be shown that vorticity and momentum fluxes due to such eddies do have a low-frequency modulation that tends to maintain low-frequency anomalies (e.g., Feldstein and Lee 1998; Lorenz and Hartmann 2001, 2002). Such modulation is consistent with a feedback chain that involves anomalous generation of baroclinic eddies in the course of low-frequency evolution (Robinson 2001). In this case, LFV is dynamically dependent on the synoptic-eddy feedback in that this feedback selects

the dominant low-frequency modes (Lorenz and Hartmann 2001, 2002).

On the other hand, it is possible that the low-frequency modulation of high-pass filtered variability is due to mere steering of synoptic eddies by LFV, while the low-frequency modes arise as a result of dynamics that do not centrally involve synoptic-eddy feedback. Our results support the latter possibility, insofar as in our simplified-model version fast baroclinic modes are suppressed by allowing the barotropic and baroclinic modes to interact at low frequencies only. This simplified, stochastically forced model version does reproduce most aspects of our full model's behavior, when the random forcing has the spatial pattern of vorticity forcing due to synoptic eddies in the full model (see sections 3d and 4b).

The leading-order dynamics in our model are linear and barotropic, which is consistent with the results of Branstator (1992), Wang (1992), Metz (1994), Yamane and Yoden (1998), and Swanson (2000). Legras and Ghil (1985) and Marshall and Molteni (1993) attribute their model's LFV to multiple quasi-stationary states. Itoh and Kimoto (1999) argue that both multiple regimes and intraseasonal oscillations arise in their model due to such nonlinear dynamics. In our model two distinct linear barotropic eigenmodes dominate its LFV. Both are stationary and have a pronounced zonally symmetric component that resembles zonal-flow vacillation (Yu and Hartman 1993; Koo and Ghil 2002; Koo et al. 2002). The leading barotropic EOF in our model is dynamically explained by these two stationary eigenmodes. This state of affairs might be due to the absence of topography in our model and the resulting weakness of quasi-stationary waves in it (Swanson 2000).

Farrell and Ioannou (1995), following the work of Trefethen et al. (1993) and their own (Farrell and Ioannou 1993) on transition to turbulence in viscous shear flows, argued that atmospheric turbulence can be understood by an analysis of nonseparable barotropic–baroclinic midlatitude dynamics in terms of finite-time growth of linear perturbations around the sheared background flow. The role of nonlinearity from this point of view is just to stochastically supply the energy to the optimally growing perturbation-flow patterns. These patterns are related to singular vectors of the model's nonnormal linearized operator. In contrast, we have argued that LFV in our model is associated with ordinary linear barotropic eigenmodes, selected by nonlinear barotropic self-interaction, while baroclinic dynamics play a secondary role. These linear barotropic eigenmodes, and not the leading singular vectors, explain dynamically the leading EOFs in our model.

The barotropic eigenmodes associated with our model's LFV are not well separated from others in terms of their damping time scale. An important role of nonlinearity in the model is apparently to select relevant eigenmodes by barotropic interaction; this aspect of our

results needs further clarification, using a model version with variable topographic heights. Interactions between the stationary and propagating barotropic eigenmodes may contribute to the occurrence of PDF maxima in our system's phase space. This tentative conclusion is conceptually consistent with the observational results of Cheng and Wallace (1993), Smyth et al. (1999), and Robertson (2001), who provide some evidence that the NAO or AO may each involve several dynamical patterns, stationary as well as propagating.

The population of either regime in our system's phase space can vary on very long time scales (see also Molteni and Corti 1998; Robertson et al. 2000). Since the present model has no external, ultra-low-frequency forcing, this variation is the result of chaotic irregularity in the full model's deterministic trajectories (Lorenz 1963; Legras and Ghil 1985; Weeks et al. 1997; Tian et al. 2001).

Further nonlinear aspects of our model's behavior involve the interaction between the stationary and the propagating barotropic eigenmodes. Two eastward propagating modes are important in our model; they have periods of 27 and 36 days, respectively. Both of them consist of a zonally modulated wave in which zonal wavenumber 4 is dominant; they differ by the phase of this wavenumber and the degree of localization near the model's ocean basin. These two waves interact strongly due to their similar spatial pattern, and appear in the EOF decomposition of model fields as signals with a period of 37 and 50 days. As explained at the end of section 5, the periods of the dominant signals in our PC analysis are related to those of the eigenmode pairs as harmonics or combination tones. The associated spatial EOFs have the same wavenumber-4 dominance and their main spatial features are likewise localized near the model's ocean basin.

Similar periods and associated spatial patterns were observed by Plaut and Vautard (1994), Da Costa and Vautard (1997), and Zhang et al. (1997) in Northern Hemisphere data. Keppenne et al. (2000) identified spectral peaks at periods of 18, 25, 35, and 65–70 days in the NAO index and used a two-layer shallow-water model on the sphere, with fairly realistic topography, to explain them. Our model is highly idealized and its results cannot be compared directly with observations (Ghil and Robertson 2000). Nevertheless, the coincidence of the periods and patterns we find with those observed is intriguing.

Frisius et al. (1998) obtained a 50-day oscillation in their multilayer, thermally forced, primitive equation model and argued that this oscillation is a part of their model's storm track dynamics. In contrast, we conclude that the low-frequency oscillations in our model do not depend dynamically on synoptic eddies. Our results thus agree better with those of Keppenne et al.'s (2000) two-layer model, in which baroclinic activity is negligible.

While we believe that the regime description of LFV is an important and potentially useful concept, comple-

mentary to the intraseasonal oscillations approach, the connection between the two (Kimoto and Ghil 1993b; Plaut and Vautard 1994; Koo et al. 2002) is far from fully understood. The existence of multiple regimes by itself is still a controversial subject (Molteni 2002), although a tentative consensus is starting to emerge (Ghil and Robertson 2002).

Our model provides an illustration of the issues that arise in reliably detecting such regimes. Due to the apparently random distribution of regime visits, even over very long time intervals, straightforward PDF estimation of a multivariate histogram in a suitable phase subspace of the model may yield one, two, or three modes (see Figs. 5a, 12a, and discussion thereof). Similar issues arise when more sophisticated estimation methods, which depend on a smoothing parameter, are used on shorter time series extracted from a model run (see Figs. 12b–e and discussion). The robustness of smoothed PDF estimates in this paper is demonstrated, therefore, by computing the PDF in subspaces spanned by sets of different variables: barotropic EOFs, 1D EOFs of zonally averaged zonal wind, and stationary barotropic eigenmodes. These three sets of pairs of variables are all mutually independent, both linearly and statistically.

Our results address the issue of the equivalent-barotropic character of observed atmospheric LFV. A classical explanation relies on the turbulent energy cascade in stratified, rotating flows (Charney 1971; Rhines 1975; Salmon 1998). We provide a complementary and more detailed explanation, by showing that the sole effect of the saturated instabilities associated with fast synoptic eddies is to provide energy for LFV, which is determined by its barotropic dynamics, while the baroclinic field is slaved to barotropic evolution at low frequencies.

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APPENDIX

Nonlinear Averaging

We first note that the barotropic vorticity equation (1a) does not involve barotropic–baroclinic interactions. Therefore, if we force this equation, at every time step, with the exact history of the term $-h_1 h_2 [J(\tau, q_\tau) + t_d^{-1} \nabla^2 \tau]$ and use the resulting (ψ, q_ψ) in Eq. (1b), we will reproduce exactly the evolution of the full system. Indeed, Eq. (5), which governs barotropic-mode per-

turbations, is linearly stable when the baroclinic terms (5d), (5e) and (5f) are omitted (see sections 3b and 5); $\psi(x, y, t)$ will thus respond passively to the baroclinic forcing on Eq. (1a) and will drive, in turn, $\tau(x, y, t)$ in Eq. (1b).

We generate several synthetic evolutions of the full system by repeating the previous procedure, while updating $-h_1 h_2 [J(\tau, q_\tau) + t_d^{-1} \nabla^2 \tau]$ only every day, every 2 days, every 3 days, etc. up to every 13 days, according to the history of τ from the control run of section 3. The 14 multivariate (ψ, τ) time series so obtained, including the control run itself, are then averaged and we call the result $(\bar{\psi}, \bar{\tau})$ the low-frequency signal, since the influence of baroclinic effects has been filtered out on short time scales. We call the difference $(\tilde{\psi}, \tilde{\tau})$ between the time series given by the full control run and this signal the dynamical noise; it presumably represents the influence of the processes associated with baroclinic instability, which dominate high-frequency behavior in our model. This procedure is applied to 1/10 of the time series generated by the control run.

In Figs. A1a,b, the results of such a signal versus noise decomposition are illustrated by showing an arbitrary 500-day-long segment of the time series of the barotropic and baroclinic zonal velocities, U_ψ and U_τ , at a point near the jet maximum (left column). The signal is plotted as the bold solid line, while the light solid line shows the dynamic-noise time series. The signal time series in each panel is plotted twice, the second curve being the centered signal, with the time mean removed.

The power spectra of the signal (Fig. A1, right column) have a red-noise-type character, while the dynamic-noise spectra are white for periods longer than about 7 days. The spectra in Figs. A1a,b are computed using Welch's averaged periodogram method, by dividing the signal into 512-day-long segments, each of which is detrended, windowed, and then zero-padded to length 512; the segments overlap pairwise by one-half of their total length. The final spectrum is obtained by averaging over all the periodograms (e.g., Oppenheim and Schaffer 1989).

We now show that the signal obtained by our dynamical filtering satisfies the original equations to a good approximation. To do so, we force the barotropic vorticity equation with the term $-h_1 h_2 [J(\bar{\tau}, \bar{q}_\tau) + t_d^{-1} \nabla^2 \bar{\tau}]$, where the low-frequency signal of the baroclinic streamfunction $\bar{\tau}$, saved daily, is used. In Figs. A1c,d we replot the signal time series as a heavy solid curve. The light solid curve shows the evolution of the reconstructed signal, as described earlier. The reconstructed signal follows the original one quite closely: the correlation between the two is about 0.6 at zero lag. The velocity time series in Fig. A1 are used for the purpose of illustration. In fact, the full 3D fields of the model satisfy the original equations to a good approximation.

If a 10-day low-pass filtered time series of τ , saved

FIG. A1. Dynamical signal vs noise decomposition for the zonal velocity at a point near the jet maximum. (a),(c) Barotropic component U_ψ ; (b),(d) baroclinic component U_τ . Signal plotted in bold; (a),(b) second bold line is for the centered signal are for: (a),(b) noise time series, and (c),(d) signal reconstruction (light solid lines) (see text). (right) Corresponding spectra.

daily, is used in this procedure instead of our dynamically obtained signal, the low-pass filtered and reconstructed time series are uncorrelated (not shown). Thus, our dynamical-filtering procedure can help identify nonlinear interactions that affect the model's LFV. The fact

that the low-frequency signal satisfies the equations by itself, without reinforcement by interactions with high-frequency transients, makes it possible to argue that synoptic-eddy feedback is not crucial for our system's low-frequency variability. The stochastic-forcing ex-

periments of section 4 provide further evidence of this being true.

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