Optically Probing Emergent Phases of Electrons in the Second Landau Level

Antonio Luis Levy

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Abstract

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In this dissertation, I present optical emission and light scattering studies on ultraclean two-dimensional electron systems. These studies focus on emerging phases in the second Landau level.

I report for the excitation spectrum for fractional quantum Hall states at filling factors $\nu = 2 + 1/3$, $\nu = 2 + 3/8$, and $\nu = 2 + 2/5$ through resonant inelastic light scattering. Resonant Rayleigh scattering is used to demonstrate that these fractional quantum Hall states are anisotropic. This work provides new insights into the nature of quasiparticle interactions of these states. It also sets the stage for the subsequent discussions about competing and coexistent phases.
I present studies of emergent phases in the filling factor range $2 \leq \nu \leq 3$ using weak optical emission from the second Landau level and resonant inelastic light scattering by spin wave excitations. A multiplet of optical emission peaks observed that exhibit striking filling factor dependence manifest phase competition in the second Landau level. A correlation of emission peaks in the multiplet with anomalies observed in the spin wave spectrum uncover major impact of the spin degree of freedom on the emergent phases in the second Landau level. These experiments demonstrate the promise of optical emission from excited Landau levels as a probe of emergent phases.

Results from optical emission and resonant inelastic light scattering studies of the second Landau level conducted at higher temperatures ($T \approx 1$ K) are also presented. Evidence that many phases observed at these higher temperatures are shown to be the same as those at lower ($T \approx 40$ mK) temperatures. Striking and anomalous temperature-dependence of optical emission experiments is used to gain further insight into the nature of these competing phases.
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Chapter 1

Introduction

Ultra-clean two-dimensional electron systems under high perpendicular magnetic fields and at low temperatures have proven a source of fascinating physics resulting from strong electron-electron interactions in reduced phase space. In 1980, the discovery by K. von Klitzing of the integer quantum Hall effect (IQHE) revealed that the Hall conductivity of a two-dimensional electron system (2DES) displays plateaus at integer values of the Landau level filling factor $\nu$ that are quantized at exact values $\sigma_{xy} = \nu e^2/h$, with simultaneous vanishing longitudinal magneto-conductivity $\sigma_{xx}$ [1]. This quantization of the Hall conductance represented a sharp contrast with the expected linear behavior observed in classical Hall systems. The exact quantization of the Hall conductivity was brilliantly interpreted by R. B. Laughlin, who showed that gauge invariance makes this effect robust against disorder [2].

In 1982, D. C. Tsui, H. L. Stormer, and A. C. Gossard discovered that under perpendicular magnetic fields high enough to confine the entire 2DES
to the lowest (N=0) spin-split Landau level, the Hall resistance becomes exactly quantized at values of $\rho_{xy} = \frac{1}{\nu e^2}$ when the Landau level filling factor takes the values $\nu = \frac{1}{3}$ and $\frac{2}{3}$ [3]. In another inspired breakthrough, R. B. Laughlin constructed many-body wavefunctions for these fractional quantum Hall effect (FQHE) states that supported fractionally-charged excitations [4, 5]. Subsequently, numerous further FQHE states at odd-denominator filling factors were discovered in both spin-branches of the lowest Landau level (LLL) [6].

The discoveries of the IQHE and FQHE had an enormous impact on physics. The exact quantization of the Hall effect showed the formation of a many-electron quantum fluid that is robust against impurities and other disorder, and that does not depend on the material in which the 2DES resides. The reduction in dimensionality in conjunction with high magnetic field that lead to the FQHE demonstrate striking electron correlation effects governed solely by Coulomb interactions [5, 6].

The fractional quantum Hall fluid has no weakly-interacting analogue from which it is derived. These electron quantum fluids are linked to the emergence of new quasi-particles. A major breakthrough has been the introduction of composite fermion quasiparticles (loosely speaking, electrons bound to an even number of magnetic flux quanta) [7]. Composite fermions currently interpret many of the observed FQHE states in the LLL [6, 7, 8]. Research in quantum Hall physics remains a benchmark in studies of correlated electron quantum fluids and serves as a major inspiration to fields of great current interest such as topological quantum computation and topo-
logical insulators [9, 10].

Coulomb interactions in the second Landau level (SLL) are fundamentally distinct from those in the LLL. These differences manifest in intriguing even-denominator FQHE states and in striking reentrant IQHE states at non-integer filling factors [11, 12, 13, 14]. Clearly, phases other than conventional odd-denominator FQHE liquids can serve here as overlapping ground states. This makes the SLL home to striking competition between quantum phases [15].

The interplay of competing phases in the SLL has been studied by introduction of in-plane magnetic fields [13, 16, 17, 18, 19]. These experiments provide evidence that possibly smectic- or nematic-like anisotropic phases that break full rotational invariance coexist with FQHE liquids[20, 21, 22]. The large anisotropy in magneto-transport induced in the system at the FQHE states at $\nu = 5/2$ and $\nu = 7/3$ by relatively small in-plane magnetic fields [16, 17, 18] supports interpretations in terms of a new state of electron matter with FQHE states that occur in the environment of a nematic stripe phase [20, 21, 22].

While many of the fascinating aspects of quantum Hall physics are made manifest in transport, the character of FQHE liquids in the SLL is still under intense investigation. Studies of low-lying collective excitation spectra from competing phases in the SLL should provide crucial insights to these states’ fascinating physics.

This dissertation focuses on optical studies of phases of electrons that emerge in the SLL. My work seeks novel insights into the competition and
coexistence of these emergent phases. We use resonant inelastic light scattering, resonant Rayleigh scattering, and optical emission as probes of these systems.

These optical spectroscopy tools methods have proven useful in studying FQHE systems. Resonant inelastic light scattering is a powerful probe of both single-particle and collective modes of 2DES’s [23, 24, 25]. This method has been introduced as a unique tool in probing low-lying excitations of FQHE systems [26, 27]. Optical emission has also proven a powerful indicator and probe of fractional quantum Hall phases in the LLL [28, 29, 30]. Resonant Rayleigh scattering has recently proven very useful in studying quantum Hall fluids by detecting the localization and delocalization of states [31, 32, 33].

Chapter 2 begins with an overview of the properties of the 2D electron systems we study. I first describe the heterostructures that house our 2DES’s, and discuss some basic properties under zero and finite magnetic fields. The remainder of the chapter summarizes the IQHE, reentrant IQHE, and FQHE. Here, I briefly describe composite fermion theory and introduce concepts of ordered electron phases.

Chapter 3 introduces the physics and applications of the spectroscopic techniques we use. It begins with a description of the experimental setup for optical mili-Kelvin spectroscopy. This is followed by a discussion of optical emission spectroscopy of 2D electron systems in GaAs quantum wells under zero and finite magnetic fields. The remainder of the chapter focuses on resonant Rayleigh scattering and on resonant inelastic light scattering by single-particle and collective excitations. Selection rules of scattering from
different and their application to utilizing inelastic light scattering as a spectroscopic technique are discussed in some detail. The discussion of collective excitations includes an overview of the modes seen and expected to result from the fractional quantum Hall states and the other electron phases mentioned in Chapter 2.

Chapter 4 describes inelastic and elastic light scattering experiments, performed on the $\nu = \frac{7}{3}$, $\nu = \frac{12}{5}$, and $\nu = \frac{19}{8}$ fractional quantum Hall states and presented in Ref. [34]. These modes exhibit pronounced temperature- and filling factor-dependence; changes in filling factor as small as $\Delta \nu = 0.005$ (approximately the step-size of these states’ quantum Hall plateaus), result in the disappearance of these energy gaps, providing strong evidence that the excitations observed arise from the FQHE states themselves.

Detailed analysis of the observed modes reveals a sharp mode with energy slightly below the Zeeman energy and low-energy modes whose energy is approximately 70 $\mu$eV, which are interpreted as spin wave and neutral gap modes respectively. The neutral mode spectrum of the FQHE state at $\nu = \frac{7}{3}$ shows strong similarities to that of its analogous counterpart in the first Landau level at $\nu = \frac{1}{3}$ [34], suggesting that the neutral excitation spectrum of the $\nu = \frac{7}{3}$ state could exhibit similar magnetoroton minima. The striking polarization-dependence of resonant Rayleigh and resonant inelastic scattering spectra presents promising evidence suggesting that the FQHE states observed in these experiments are anisotropic when subjected to small in-plane magnetic fields. This opens the discussion for our work on the phases themselves in the following chapter.
Chapter 5 discusses our work using optical emission spectroscopy to probe phases of electrons in the second Landau level [35]. While optical emission across the GaAs band gap from the first Landau level had been used to probe fractional quantum Hall states [28, 29, 30] this was the first time that the weak photoluminescence from the second Landau level was used to analyze emergent phases of electrons. Resonant Rayleigh and inelastic light scattering spectra provide definite evidence that these optical emission spectra manifest emergent phases in the second Landau level. Measurements of the spin wave by resonant inelastic light scattering are used to probe spin-rotational invariance in the phases by optical emission spectroscopy [36].

Chapter 6 presents further studies of emergent phases discussed in the previous chapter at higher temperature ($T \gtrsim 1$ °K). Results from optical emission across the GaAs bandgap and resonant inelastic light scattering discussed in this chapter suggest that many of the emergent SLL phases observed at mili-Kelvin temperatures are also observed at temperatures as high as 1.35K. The SLL photoluminescence spectra display striking temperature dependence for $T > 1$ °K, providing further valuable insight into the nature of the phases introduced in the previous chapter.

In Chapter 7, I present some summarizing remarks and briefly discuss possible further studies. Appendices A and B are included to provide further background into the physics of stripe and bubble phases and lineshape analysis respectively.
Chapter 2

General Physics of
Two-Dimensional Electron Systems

“Be yourself; everyone else is taken.”

- Oscar Wilde

2.1 Overview

This chapter provides a background for understanding the striking physics of 2DES under high magnetic fields. This background includes the physics of the heterostructure that houses the 2DES, the physics of 2DES under high magnetic fields, and an introduction to quantum Hall physics.

The chapter begins with a description of symmetrically-doped quantum wells’ subband structures, providing a brief discussion of the properties of a
2DES under zero magnetic field.

A description of the basic physics of 2DES’s under finite magnetic fields follows. This discussion introduces the concepts of Landau levels (LL’s) and filling factor $\nu$.

I then describe the integer quantum Hall effect (IQHE). I discuss what made the IQHE so groundbreaking, and provide a summary of Laughlin’s brilliant argument explaining it.

The following section covers the fractional quantum Hall effect and discusses the implications of Laughlin’s wavefunction for the $\nu = \frac{1}{3}$ state. Here, I introduce the theory of composite fermions and its implications for excitations of FQHE states.

This is followed by a brief overview of ordered phases of electrons, where I introduce the concepts of stripe phases, bubble phases, and Wigner crystals.

The chapter ends with a discussion of the second Landau level, where both ordered phases of electrons and fractional quantum Hall states are observed. Here, I give a brief overview of competition and coexistence between phases.

## 2.2 Our Samples

### 2.2.1 Introduction: Design

The 2DES’s we study reside in a state-of-the-art modulation-doped GaAs-Al$_x$Ga$_{1-x}$As quantum well heterostructure. The layer sequence and the variation of the conduction band edge of the samples are shown schematically in Fig. 2.1. The confining potential comes from the difference in GaAs and
Al$_x$Ga$_{1-x}$As conduction band energies [37]. Due to the nearly perfect lattice match of GaAs and AlGaAs [38], state-of-the-art hetero-layers with negligible mismatch strain are grown by molecular beam epitaxy (MBE) [39, 40].

As shown in Fig. 2.1, free carriers in the quantum well are introduced by modulation doping with Si donors in the AlGaAs barrier layers. A spacer layer separating the donors in the barrier from the quantum well ensures high electron mobility in the heterostructure. Mobility is hindered by ionized impurity scattering from the donors in the barriers; separating the donors from the carriers in the quantum well minimizes this problem. In our samples, Si-doping layers are placed on both sides of the quantum well, a doping method we call symmetrically doping (as opposed to one-sided doping). The samples were grown along the $\langle 001 \rangle$ direction (which we will call the $z$-direction). The creation of high-mobility GaAs quantum wells is an art that requires exquisite fine tuning and is a topic at the forefront of condensed matter science [39, 40].
Figure 2.1: Schematic of symmetrically modulation-doped quantum well heterostructures (not to scale). The Silicon doping layers are separated from the quantum well by a distance $s$ of approximately 100 nm. The left shows a schematic of the conduction band energy ($E_c$). Electrons are confined to the GaAs quantum well, as shown by the fact that the chemical potential $\mu$ is below the conduction band everywhere else in the sample. In addition to allowing for higher electron densities in the quantum well, symmetric doping ensures a symmetric potential in the quantum well. The growth direction is marked as the $z$-direction in this dissertation.
2.2.2 Inside the GaAs Quantum Well

The quantum well depicted in Fig. 2.1 is the host of the two dimensional electron fluids I research. The one-electron states in a GaAs quantum well like ours can be described by the effective-mass and envelope-function approximations [37, 41, 42]. In this framework, the electrons introduced by doping populate the k-space near the Γ-point, with isotropic conduction band effective mass \( m^* \). Our wave function takes the form:

\[
\psi_{c,j,k}(\vec{r}, z) = u_{c,k}(\vec{r})\chi_{\xi}(z)e^{i\vec{k}\cdot\vec{r}}
\]  

(2.1)

where \( z \) is the direction of confinement, \( \vec{r} \) is the in-plane component of the position vector, \( \chi_{\xi}(z) \) corresponds to an eigenstate of the quantum well indexed by \( \xi \), and \( u_{c,k}(\vec{r}) \) denotes the component of the Bloch function periodic in the crystal lattice. For zero magnetic field, we apply the Hamiltonian:

\[
\hat{H} = \frac{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2}{2m^*} + V(z)
\]  

(2.2)

where we approximate \( V(z) \) as

\[
V(z) = \begin{cases} 
\Delta E_c & |z| \geq \frac{w}{2} \\
0 & |z| < \frac{w}{2}
\end{cases}
\]  

(2.3)

where \( w \) is the well width. \( \Delta E_c \) is the difference in the conduction band edges at the Al\(_x\)Ga\(_{1-x}\)As/GaAs interface. The quantum well potential is shown schematically in Fig. 2.2. Typically, \( \Delta E_c \) is greater than 200 meV for
2.2.2. Inside the GaAs Quantum Well

Figure 2.2: Schematic of the quantum well’s potential. The red lines represent the bottom of the Al$_x$Ga$_{1-x}$As conduction band. The depth of the quantum well is determined by the difference in the conduction band energies. The lowest two eigenstates of the quantum well are indexed by $j$. The energy of these subbands is depicted by a dashed line and the wave function is depicted by a solid line. The $k$-dispersion of the electrons in each subband at zero magnetic field is shown on the right.

our quantum wells [43, 44, 45]. The presence of the quantum well results in the creation of discretized energy levels $E_{\xi}$ ($\xi = 0, 1, 2, \ldots$) for motion normal to the quantum well. The energies of single-particle states of electrons in the modulation-doped quantum well can be written as:

$$E_{\xi}(\vec{k}) = \frac{\hbar^2 \vec{k}^2}{2m^*} + E_{\xi}$$

(2.4)

where $\vec{k}$ is the two-dimensional wave vector. As we see in Fig 2.2, the energy levels due to the quantum well results in the presence of energy 'subbands'.
2.3 2D Electron Systems under High Magnetic Field

2.3.1 Landau Levels in a 2DES

The primary focus of the research reported here is on the 2DES under high magnetic fields. When subjected to a finite magnetic field $\vec{B}$, the electron momentum is written $\vec{p} + \frac{e}{c} \vec{A}$, where $\vec{A}$ is a vector potential satisfying $\vec{B} = \nabla \times \vec{A}$. The single-electron Hamiltonian becomes [46]:

$$H = \frac{1}{2m^*} \left[ \vec{p} + \frac{e}{c} \vec{A} \right]^2 + E_Z$$

(2.5)

where $E_Z = \pm g^* \mu_B S B$ is the Zeeman energy, where $g^*$ is the effective Lande factor, $S$ is the electron spin, and the sign depends on whether the electron spin points parallel or anti-parallel to the magnetic field. For now, let us ignore $E_Z$ and assume that the magnetic field lies purely in the $z$-direction.

In the Landau gauge, the vector potential is $\langle A_x, A_y, A_z \rangle = \langle 0, Bx, 0 \rangle$ for a magnetic field normal to the 2DES, and the Hamiltonian is:

$$H = \frac{1}{2m^*} \left[ p_x^2 + \left( p_y + \frac{e}{c} Bx \right)^2 \right]$$

(2.6)

For this Hamiltonian, eigenfunctions take the form [41]:

$$\psi(x, y) = X(x)Y(y) = X(x)e^{ik_y y}$$

(2.7)

We assume that the 2D system is in a large but finite system of area
2.3.1. Landau Levels in a 2DES

![Diagram of Landau Levels](image)

Figure 2.3: **Left:** Density of states of a 2DES in a quantum well under zero magnetic field. **Center:** When we apply a non-zero magnetic field, the continuum of the 0-field 2DES is replaced with discrete Landau Levels (LL’s), separated in the cyclotron energy \( \hbar \omega_c = \hbar \frac{eB}{m^* c} \). The red and black bands represent the density of states for the two spin orientations of electrons in each LL. **Right:** Representation of LL’s in the presence of random disorder.

\( A = L^2 \). Our wave function in the \( y \)-direction is constructed to obey the cyclic boundary condition: \( \psi(x, y + L) = \psi(x, y) \) or, equivalently, \( \psi(y + L) = \psi(y) \). \( k_y \) takes the form \( k_y = \frac{2\pi}{L} \zeta \), where \( \zeta \) takes integer values. Schrödinger equation for the Hamiltonian in Equation (2.6) is:

\[
\left[ \frac{p_x^2}{2m^*} + \frac{1}{2} m^* \omega_c^2 \left( x + l_B^{-2}k_y \right)^2 \right] X(x) = E_N X(x) \tag{2.8}
\]

where \( \omega_c = \frac{eB}{m^* c} \) is the cyclotron frequency, and \( l_B = \sqrt{\frac{\hbar c}{eB}} \) is the magnetic length. Equation (2.8) is the Schrödinger equation for a simple harmonic oscillator centered at \( x_c = -k_y l_B^2 = -\frac{2\pi}{L} \zeta l_B^2 \) with frequency \( \omega_c \). Therefore, the energy levels are:

\[
E_N = \left( N + \frac{1}{2} \right) \hbar \omega_c \tag{2.9}
\]
where $N$ takes nonnegative integer values. These quantized energy levels are the famous Landau levels (LL’s). The wave functions take the form:

$$
\psi(x, y) = e^{ik_y y} H_N \left( \frac{x + k_y l_B^2}{l_B} \right) e^{-\frac{(x + k_y l_B^2)^2}{2l_B^2}}
$$

(2.10)

where $H_N$ are Hermite polynomials.

The centers of orbit $x_c$ are limited by the size of the system: $-\frac{L}{2} \leq x_c \leq \frac{L}{2}$.

This limits the possible values of $\zeta$ to:

$$
\left| \frac{2\pi hc}{eBL} \zeta \right| \leq \frac{L}{2}
$$

$$
|\zeta| \leq \frac{L^2 eB}{2hc}
$$

(2.11)

The degeneracy $D$ of a spin-split LL is given by the number of possible $\zeta$-values:

$$
D = \frac{e}{hc} L^2 B = \frac{AB}{\phi_0}
$$

(2.12)

where $\phi_0 = \frac{hc}{e}$ is the magnetic flux quantum. We note that the degeneracy of a spin-split LL is given by the ratio of the magnetic flux ($AB$) to the magnetic flux quantum.

An important parameter for quantum Hall systems is the filling factor $\nu$, which denotes the number of occupied spin-split LL’s in the ground state. For areal density $n$, the filling factor is given by:

$$
\nu = \frac{n}{D/A} = \frac{n}{B/\phi_0} = 2\pi n l_B^2
$$

(2.13)

Quantum Hall physics is very sensitive to changes in the filling factor.
2.4 Quantum Hall Effects

2.4.1 Background: the Hall Effect

In Hall effect experiments, the 2DES metal is subjected to a magnetic field, and a voltage $V_{xx}$ is applied to create a current $I_x$, as indicated in Fig. 2.4. For a current in the $x$-direction and the magnetic field in the $z$-direction, electrons experience a Lorentz force in the $y$-direction $\mathbf{F} = -e\mathbf{v} \times \mathbf{B}$. The Hall resistivity $\rho_H = \rho_{xy} = \rho_{yx}$ is given by the off-diagonal elements of the resistivity tensor:

$$\rho_H = \frac{E_y}{J_x} = \frac{B}{nec}$$  \hspace{1cm} (2.14)

The classical Hall resistivity is therefore directly proportional to the magnetic field and inversely proportional to the 2D electron density.

Figure 2.4: Transport geometry for the Hall experiment. The magnetic field is perpendicular to the conducting slab (normal to the page).
2.4.2 The Integer Quantum Hall Effect

While studying magneto-transport on an ultraclean 2DES in a Si MOSFET at low temperatures, von Klitzing, Dorda, and Pepper made an extraordinary discovery [1]. This experiment showed that near integer filling factor $\nu$, the Hall resistance $R_H$ ($R_H = \rho_H$ in 2D) takes quantized values $R_H = \frac{h}{e^2\nu}$ and the longitudinal magneto-resistance $R_{xx}$ vanishes. This is the integer quantum Hall effect (IQHE). The groundbreaking results of this experiment are shown in Fig. 2.5.

The IQHE is remarkable, in part, due to the Hall resistance’s exact quantization and dependence only on fundamental constants. The IQHE does not depend on the geometry or the material of the device in which it is observed [1]; it is a truly fundamental effect!

To understand the IQHE, it is important to understand the critical role played by edge states in conducting current near integer filling factors [2]. At integer filling factors, the Fermi energy lies in the gap between two sequential spin-split LL’s, resulting in an insulating state with a gap in the excitation spectrum of the bulk. Edge states, which arise from the finite size of the sample, are conducting.

Edge states are formed due to band bending near the junction of a solid and vacuum, as depicted in Fig. 2.6. Under non-zero magnetic fields, edge states are chiral (as depicted in Fig. 2.6) and do not backscatter [2, 47, 48].

The following is a summary of the argument explaining the IQHE presented in Ref. [2].

Begin by considering a Hall bar with perpendicular magnetic field $B$
Figure 2.5: Results from the experiment that discovered the IQHE[1]. A schematic is shown in the upper right hand corner. The sample was gated so that the density of the 2DES could altered by applying a gate voltage $V_g$. A constant current of 1 $\mu$A flows from the source to the drain probes. $\nu$ is altered by changing the gate voltage (which controls $n$). The Hall voltage $U_H = IR_H$ corresponds to the potential difference perpendicular to current flow. $U_{PP} = IR_{xx}$ is the potential difference measured parallel to the current flow. The plateaus in the $U_H$ and minima of $U_{PP}$ occur at integer filling factors. Extracted from Ref. [1].
2.4.2. The Integer Quantum Hall Effect

Figure 2.6: **Top:** Energy of LL's as a function of position. The energy increases near the edges. The Fermi energy $E_F$ resides between LL's. The red open circles denote current-carrying edge states. **Bottom:** Cartoon of electron states at $\nu = 1$. Chiral edge states, which are responsible for the quantum Hall effect depicted in red and a localized bulk states is depicted in dark blue. Adapted from Refs. [47] and [49] respectively.

with voltage $V_x$ applied across the edges in the x-direction. We impose cyclic boundary conditions in the y-direction, requiring [48]:

$$\psi(x, y) = \psi(x, y + L_y)$$  \hspace{1cm} (2.15)

These boundary conditions allow the system to be represented by ribbon as shown in Fig. 2.7 [2, 48]. To determine the Hall conductivity of this system, the total current $I$ must be related to the potential difference between the
edges $V_x$. This current in this system is given by the following (adiabatic) derivative [2]:

$$I = c \frac{\partial U}{\partial \phi}$$  \hspace{1cm} (2.16)

where $U$ is the energy of the system and $\phi$ is the magnetic flux [2]. The Hamiltonian for this system can be written:

$$H = \frac{1}{2m^*} \left[ \vec{p} + \frac{e}{c} \vec{A} \right]^2 + eEx$$  \hspace{1cm} (2.17)

where $E = V_x/L_x$ is the constant electric field in the x-direction. Given the invariance of the system along the y-direction, it is most convenient to use the Landau gauge $\vec{A} = xB\hat{e}_y$, which allows us to express the Hamiltonian as:
\[ H = \frac{1}{2m^*} \left[ p_x^2 + m^*\omega_c^2 \left( x - \frac{\hbar c}{eB} k_y \right)^2 \right] + e\frac{V_x}{L_x} x \] (2.18)

The eigenfunctions of this Hamiltonian take the form:

\[ \psi(x, y) = e^{ik_y y} \phi_N \left( x - \frac{\hbar c}{eB} k_y + \frac{c^2 m^* E}{B^2} \right) \] (2.19)

where \( \phi_N(x) \) is the wavefunction of the \( N^{th} \)-excited eigenstate of a simple harmonic oscillator with frequency \( \omega_c \). The cyclic boundary conditions of Equation (2.15) require that \( k_y \) take the form \( k_y = \frac{2\pi}{L_y} \zeta \) where \( \zeta \) takes integer values.

Due to the geometry of the system, adiabatically adding a quantum of flux results in an increase of \( k_y \) by \( \frac{2\pi}{L_y} \) for all states. This change in \( k_y \) with the flux quantum corresponds to the \( k_y \)-level spacing [47, 48]. Adding a quantum of flux is equivalent to moving the edge states which cross the Fermi level on one edge in each occupied spin-LL to the other edge of the system (above the Fermi level) as depicted in Fig. 2.8.

Because of the difference in the edge’s potential \( V_x \), adding a quantum of flux to a system with \( n \) filled spin-LL’s changes the energy of the system \( \Delta U = neV_x \) [47]. The Hall current is therefore given by [2]:

\[ I_y = \frac{\partial U}{\partial \phi} = \frac{\Delta U}{\Delta \phi} = n\frac{e^2}{\hbar} V_x \] (2.20)

This equation demonstrates that the Hall resistance \( R_{xy} \) (which is the same
2.4.2. The Integer Quantum Hall Effect

Figure 2.8: Energy of LL’s as a function of position for $\nu = 2$. Occupied edge states, depicted by the black circles, before and the after insertion of a flux quantum. Adapted from Ref. [50].

as the Hall resistivity in 2D) is given by $R_{xy} = \frac{h}{ne^2}$.

This value of $R_{xy}$ depends on the number of filled spin-LL’s and the derivation assumed only that the system had an insulating bulk. The presence of finite disorder results in an insulating bulk for filling factors away from exact integer values, resulting in a constant value of $R_{xy}$ for a finite range of filling factors [47], as shown by the plateaus in the Hall voltage shown in Fig. 2.5. Because the Hall current $I_y$ for integer quantum Hall states is determined only by the number of filled spin-LL’s and the voltage $V_x$, Laughlin showed that $I_y$ is unaffected by ”edge effects” such as defects despite being conducted by edge states [2]. The integer quantum Hall effect manifests topological bulk-edge correspondence!
2.4.3 Fractional Quantum Hall Effects

In 1982, Tsui, Stormer and Gossard discovered the quantization of the Hall effect at fractional filling factors: the fractional quantum Hall effect (FQHE) [3]. The groundbreaking results of this experiment are shown in Fig. 2.9.

The single-particle picture used for the IQHE cannot be used to understand the FQHE; understanding the FQHE required a completely new interpretation. This explanation came in another brilliant paper published the following year by R. Laughlin [4], which is summarized below.

Laughlin began with the single-particle eigenstates of the LLL in the symmetric gauge $\vec{A} = \frac{B}{2}(-y\hat{c}_x + x\hat{c}_y)$, which are also eigenfunctions of angular momentum. Laughlin proposed the ansatz that the many-body state take the form:

$$\Psi(z) = \prod_{a<b}(z_a - z_b)^m \exp \left[ -\sum_{c=1}^{N} \frac{|z_c|^2}{4l_B^2} \right] \quad (2.21)$$

where $z = x - iy$, and $a$, $b$, and $c$ index the electrons. The term $(z_a - z_b)^m$ accounts for the Coulomb interaction between electrons which is given by $V_{ab}^C = \frac{e^2}{4\pi\epsilon |z_a - z_b|}$ for any two electrons $a$ and $b$. Because of fermions’ anti-symmetry under particle exchange, the term $(z_a - z_b)^m$ must be odd; $m$ can only take odd-integer values. This term was chosen to be a monomial to satisfy conservation of angular momentum. Laughlin proved that this wave function corresponds to a FQHE state at filling factor $\nu = \frac{1}{m}$ [4].

From complex analysis, we know that $z_a = re^{-i\theta}$ is a function with a vortex at the origin (due to its angular dependence). The term $(z_a - z_b)^m$ in Laughlin’s wave function generates a phase of $2\pi \times m$ when the electron
2.4.3. Fractional Quantum Hall Effects

Figure 2.9: Results from the experiment by Tsui, Stormer and Gossard that discovered the FQHE. **Top:** Measurements of the Hall resistivity with filling factor. Plateaus are seen at values of $\rho_{xy} = \frac{h}{ye^2}$ for filling factors $\nu = \frac{1}{3}$ and $\nu = \frac{2}{3}$. **Bottom:** Measurements of longitudinal magneto-resistivity, which displays strong minima for fractional filling factors $\nu = \frac{1}{3}$ and $\nu = \frac{2}{3}$. Extracted from Ref. [3].
centered at $z_a$ executes a closed loop around the electron centered at $z_b$; the term $(z_a - z_b)^m$ effectively adds $m$ quantized vortices to each electron [51, 52]. Because the Aharonov-Bohm phase of a closed path around $m$ quantized vortices is equal to that of a closed path around $m$ flux quanta, it is useful to interpret the wave function in Equation (2.21) as associating $m$ flux quanta with every electron [51, 52].

To explore excitations of the FQHE states, one can adiabatically add a flux quantum at some point $z_q$ [4]. Laughlin showed the charge of this excitation to be $\frac{e}{m}$.

### 2.4.4 Composite Fermions

Laughlin's inspired wave function, while powerful, described only FQHE states with filling factor $\nu = \frac{1}{m}$ where $m$ is an odd integer. A more general theory is needed to describe the FQHE states observed at filling factors such as $\nu = \frac{2}{5}$ and $\nu = \frac{3}{5}$. Here, we discuss one such model; the composite fermion theory, proposed by J. Jain in 1989 [7].

The composite fermion (CF) model describes the FQHE in the N=0 LL as an integer quantum Hall effect of composite fermion quasiparticles, defined as electrons associated with an even number $(m - 1)$ of flux quanta. The CF model explained more FQHE states and predicted the filling factors at which additional FQHE states should appear. The following is a summary of the CF model, described in greater detail in Ref. [7, 8].

To understand the CF model of the FQHE, it is convenient to use the Laughlin wave function corresponding to filling factor $\nu = 1$ (recall that
2.4.4. Composite Fermions

\[ m = \frac{1}{\nu} \]

\[
\Psi_{\nu=1}(z) = \prod_{a<\beta} (z_a - z_\beta)^1 \exp \left[ -\sum_{c=1}^{N} \frac{|z_c|^2}{4l_B^2} \right]
\]

(2.22)

One can now rewrite the Laughlin wave function at \( \nu = \frac{1}{m} \) as \([7]\):

\[
\Psi_{\nu=\frac{1}{m}}(z) = \prod_{a<\beta} (z_a - z_\beta)^{m-1} \Psi_{\nu=1}(z)
\]

(2.23)

This wave function now resembles a \( \nu = 1 \) state of quasiparticles consisting of electrons associated with \( m-1 \) vortices (flux quanta): composite fermions. Because these wave functions admit fractionally-charge excitations \([4]\), CF’s are interpreted as having fractional charge \( \frac{\nu}{m} \) \([8]\).

Because each CF effectively "absorbs" \( m - 1 \) flux quanta, the effective magnetic field \( B^* \) of the CF system is reduced. For electrons of density \( n \), we find the effective magnetic field to be given by

\[
B^* = B - n(m - 1)\phi_0
\]

(2.24)

Finite \( B^* \) are associated with a filling factor \( \nu^* \) given by \( \nu^* = \frac{n\phi_0}{|B^*|} \) which is also given by:

\[
\nu = \frac{n\phi_0}{|B|} = \frac{\nu^*}{\nu^*(m - 1) \pm 1}
\]

(2.25)

Here the sign \( \pm \) is determined by the sign of \( B^* \) \([7, 8]\). A schematic representation of the CF model from Jain’s book "Composite Fermions" is shown in Fig. 2.10.

In the presence of a finite effective magnetic field \( B^* \), CF’s, are expected to form LL-like bands called A-levels; the effective filling factor \( \nu^* \) is the A-
Figure 2.10: **Left:** Filled LL’s responsible for the IQHE. **Right:** Representation of filled CF A-levels thought to be responsible for the FQHE. The energy splitting between the A-levels is given by the effective cyclotron energy. Extracted from Ref.[8].
level filling factor. The CF model predicts FQHE states for integer values of \( \nu^* \). The sequence of corresponding \( \nu \) is known as Jain’s sequence.

To generalize the wavefunction in Equation (2.23) to include FQHE states with \( \nu^* > 1 \), Jain suggested the following trial wavefunction:

\[
\Psi_{\frac{\nu}{\nu^*(m-1)+1}}(z) = \hat{P}_{LLL} \prod_{a<b} (z_a - z_b)^{m-1} \Psi_{\frac{\nu}{\nu^*}}(z)
\]  \hspace{1cm} (2.26)

where \( \hat{P}_{LLL} \), the projection operator to the LLL, ensures that the electron states remain in the LLL \([7, 8]\).

### 2.4.5 Phases of Electrons

Effects such as the reentrant integer quantum Hall effect (RIQHE) and transport anisotropy have been linked to the formation of ordered, charge density wave-like phases of electrons \([13, 53, 54, 55, 56, 57, 58]\). These ordered phases of electrons are the ground states predicted by Hartree-Fock calculations \([59, 60, 54, 61, 57]\). These phases are shown schematically in Fig. 2.11. The black dots in this figure represent the electron guiding centers (the centers of cyclotron orbit). The following is a brief overview of these phases. I include a more extensive discussion of these phases and their links to the RIQHE in Appendix A.

Near integer filling factors, the ground state is predicted to be Wigner crystal (WC), a triangular lattice of electrons or holes in the highest LL \([59]\). As the filling factor moves away from integer values, the population of electrons (or holes) in the highest occupied spin-split LL increases, resulting
2.4.5. Phases of Electrons

Figure 2.11: Left: Representations of the stripe phase, multi-electron bubble phase, and WC. Extracted from Ref.[54]. Right: Schematic of the 2DES at different partial filling factors for a higher LL. Extracted from Ref.[61]. As partial filling factor increases, the bubble phases develop more electrons. As the filling factor approaches half-integer, the phases lose their rotational symmetry. The black dots represent guiding centers of the wavefunctions in ground states with more than one electron per lattice site [59, 60, 54, 61, 57]. These are many-electron bubble phases.

As the filling factor approaches half-integer filling factors, the bubble phases become unstable against anisotropy, and an anisotropic “stripe” phase is expected to form [62, 63, 64]. Pronounced transport anisotropy in at half-integer filling factors in higher LL’s has been linked to this phase [65, 58, 66]. Here, stripes are thought to form along the ⟨110⟩ due to weak anisotropy in GaAs.

2.4.5.1 Competing Phases in the Second Landau Level

We now turn our attention to the N=1 LL. The unique electron interactions in the N=1 LL result in FQHE states that display remarkable deviations from those in the N=0 LL, including FQHE states in the N=1 LL with no counterparts in the N=0 LL such as the FQHE states at half-integer filling
2.4.5. Phases of Electrons

factors $\nu = \frac{5}{3}$ and $\nu = \frac{7}{2}$ [12, 67]. From Equation (2.24), half-integer filling factors are tantamount to a Fermi sea of 2-vortex CF’s with $B^* = 0$. This has drawn extensive interest to the FQHE observed at this filling factor, with CF-theory describing them as BCS superconducting states[68, 69]. These states, as well as others including the FQHE state at $\nu = \frac{7}{3}$ (the N=1 LL equivalent of the FQHE state at $\nu = \frac{1}{3}$), are thought to exhibit a variety of exotic statistical properties [68, 69, 70, 71].

The N=1 LL is also the only LL for which the FQHE, the RIQHE, and transport anisotropy are all observed [65, 58, 66, 53]. Competition and coexistence of the FQHE states and ordered phases of electrons as ground states results in unexpected many-body physics. The most striking example is the observation of N=1 LL anisotropic FQHE states in the presence of finite in-plane magnetic fields [72].

As seen in the previous section, the quantum Hall effects were largely due to edge channels conducting the current: the bulk was isotropic. Having partially-ordered FQHE states potentially suggests that other ordered phases of quasi-particles may also serve as ground states [21, 73].
Chapter 3

Optical Spectroscopy of 2D Electron Systems

"Before I speak, I have something important to say"

- Groucho Marx

3.1 Overview

Optical spectroscopy has proven useful in studying a wide variety of condensed matter systems. In this chapter I discuss optical probes - bandgap optical emission in photoluminescence (PL), resonant Rayleigh scattering (RRS), and resonant inelastic light scattering (RILS) - and their use in studying 2DES’s under zero and finite magnetic field. This chapter provides a framework for understanding the results of our experiments.

We will begin in Section 3.2 with a discussion of the experimental setup that enabled us to perform optical experiments on quantum Hall systems.
Here, I discuss both the dilution refrigerator and the optical setup used to probe the samples.

The next section briefly discusses the conduction and valence bands in GaAs. Here, I introduce the light and heavy hole bands, and discuss some of the bands' properties that prove very important in determining selection rules for optical transitions.

The discussion of optical spectroscopy begins in Section 3.4 with a look at optical emission. This section discusses selection rules for optical transitions in semiconductors that host 2DES's under zero and finite magnetic fields.

Section 3.5 is devoted to a discussion of resonant Rayleigh scattering (RRS). This discussion provides a framework for understanding RRS and introduces concepts important to understanding resonant inelastic light scattering (RILS), discussed in the following section.

Chapter 3 ends with a discussion of RILS by 2DES in quantum wells. RILS is a uniquely powerful tool in probing collective excitations. In this section, I briefly discuss the selection rules and how they allow us to isolate signatures from charge and spin density excitations. I also discuss higher-order scattering processes that exploit the breakdown of wavevector conservation to enable experimentalists to probe excitations at relevant large wavevectors.
3.2 Experimental Considerations

Our measurements are performed on ultrahigh-mobility, symmetrically doped 24 nm-30 nm wide single GaAs quantum wells of density between $2.9 \times 10^{11}$ cm$^{-2}$ and $4 \times 10^{11}$ cm$^{-2}$. The samples are grown by MBE with a sequence of layers described in Fig. 2.1. This relatively high density range is useful for studying the second Landau level (LL). State-of-the-art samples with high mobility, typically $\gtrsim 20 \times 10^6$ cm$^2$ V/s, are needed to observe the exotic phases that have made the second Landau level the subject of intense study.

The suitability of these samples for optical experiments at high magnetic fields is determined by optical probes conducted in an $^4$He optical cryostat at zero magnetic field and temperature in the range 2°K - 10°K. The samples are mounted on a copper cold finger. Only part of the samples are adhered to the cold fingers in order to minimize the impact of strain.

For experiments at lower temperatures ($T < 1.5$K), samples are attached to the cold finger of an Oxford Instruments Kelvinox 400 dilution refrigerator with windows for optical access. The cooling power is 400μW at temperatures of 100 mK. Base temperature close to 25 mK. When the windows are blocked, the base temperature is less than 9 mK. Samples are attached to a cold finger at the end of a copper rod with copper-loaded vacuum grease to ensure thermal contact. Gold wires are attached from the GaAs quantum well to the copper rod to further ensure thermal contact between the cold finger and the 2DES. For high magnetic field experiments, the dilution refrigerator is inserted into the bore of a 16T superconducting magnet, as shown in Fig. 3.1.
Four parallel windows grant optical access to the samples in the dilution refrigerator. The window at the bottom of the dilution refrigerator is mated to the $^4$He window, as shown in Fig. 3.1. These windows are extremely close, resulting in the formation of a vapor lock that prevents boiling $^4$He liquid from obscuring the optical path. The next window is thermally anchored to the Nitrogen jacket at 77K. The outer-most window is attached to the outer wall of the cryostat. The optical windows are made of Spectrosil B, which transmits light in the wavelength range of interest ($\lambda \approx 800nm$) and blocks radiation with $\lambda > 4\mu m$ (blackbody radiation for 300K peaks near $\lambda = 10\mu m$).

For the excitation optics, we use a tunable Coherent MBR-110 Ti:sapphire laser which is optically pumped by a Coherent Verdi diode-pumped solid state laser. The Ti:sapphire-laser emits ultrasharp lines, tuned to operate at wavelengths in the range 720 – 940 nm. The polarization direction of the linearly-polarized laser light is controlled by a polarization rotator. The laser is focused on the samples by a spherical lens to a 200$\mu m$-spot with power density below $10^{-4}$ W/cm$^2$. This low power density prevents the 2DES from heating significantly at base temperature.

The scattering geometry is shown in Fig. 3.2. $\theta$ is the angle between the normal of the 2DES’s plane and the incident photon’s direction of propagation. It should be noted that a finite angle results in a finite in-plane magnetic, the effects of which were touched upon in Chapter 2.

From Fig. 3.2, one can show that momentum conservation requires that
Figure 3.1: Schematic of the optical spectroscopy experiments performed on samples in the dilution refrigerator. As the upper left corner shows, the geometry of the experiment results in a non-zero in-plane component of the magnetic field.
Figure 3.2: (a) Scattering geometry for our experiments. (b) Scattering takes place within the 2DES.
the in-plane wavevector $k$ transferred to the 2DES be approximated by:

$$k = 2k_L \sin(\theta)$$  \hfill (3.1)

where $k_L = \omega_L/c \approx 7.5 \times 10^4 \text{ cm}^{-1}$ is the wave vector of the incident photon. In our experiments, $k \sim 10^5 \text{ cm}^{-1}$. While the wave vector transferred from photons is significantly less than $1/l_B \sim 10^6 \text{ cm}^{-1}$ in our experiments, we are able to exploit the breakdown of wave vector conservation due to random, weak fluctuations in potential in order to probe critical points in the excitation spectrum that occur at wave vectors greater than $1/l_B$ [74]. Probing high wave vector modes by resonant inelastic light scattering is discussed later in this chapter.

The optical setup is optimized for collection of light. In the dilution refrigerator, the f-number of 4. The focal lengths of the lenses are determined by the desired magnification of the collection path and the f-number’s of the dilution refrigerator, the spectrometer, and the collection optics. The finite solid angle of collection determines the range of the wave vector transferred to the 2DES by the photons.

Scattered light is dispersed by a T64000 triple grating spectrometer, which has holographic master gratings with 1800 lines per millimeter that minimize stray light. Photons are detected in multi-channel mode by a charge coupled device (CCD) with 13\(\mu\)m pixels. The combined resolution of the system is 30 \(\mu\)eV when the entrance slits of the spectrometer are set to 50 \(\mu\)m. The response of the spectrometer is linearly polarized, allowing us to measure
3.3 Valence and Conduction States of GaAs Quantum Wells

Table 3.1: CB and VB’s for III-V Zinblende structures like GaAs near the Γ-point according to the Kane model. The CB states is an s-orbital. X, Y, and Z refer to the p-orbital basis of the heavy hole (HH), light hole (LH), and split-off (SO) valence bands. The quantum numbers of these states are the total angular momentum J and its z-component J_z.

The signal of scattered photons with linear polarization parallel (polarized) or perpendicular (depolarized) to the incident photons’ polarization.

### 3.3 Valence and Conduction States of GaAs Quantum Wells

Optical transitions of interest to us occur between valence and conduction band states of the GaAs quantum wells that host the 2DES in our research. The Kane model provides an accurate picture of valence and conduction band states for III-V semiconductors with a zincblende lattice structure (like GaAs) near the Γ-point [75]. The VB and CB states that incorporates spin-
orbit coupling according to the Kane model are shown in Table 3.1. The conduction band states are s-like (S ↑ and S ↓, where ↑ and ↓ are spins). The valence band (VB) states' basis functions have p-like symmetry, and are denoted as X ↑, X ↓, Y ↑, Y ↓, Z ↑ and Z ↓. VB states are combinations of these basis functions with well-defined total angular momentum \( \vec{J} = \vec{L} + \vec{S} \). The dispersion of VB states with wave vector is shown in Fig. 3.3.

Figure 3.3: Band structure for GaAs. The presence of the quantum well creates splitting between the LH and HH bands in our heterostructures (after Ref. [76, 77]).

In the quantum well (QW) all the band states near the \( \Gamma \)-pint become
3.4. Optical Emission

The optical emission process is represented schematically in Fig. 3.4. In photoluminescence (PL) experiments, electron-hole pairs are excited into the system by the incident photons, as shown in Fig. 3.4(a). Upon creation, these photo-excited electrons and holes are out of equilibrium and need to relax into their ground states. The particles’ excess energy is transferred to the 2DES by Coulomb interactions and dissipated by phonon emission, as shown in Fig. 3.4(b). To study optical emission, we will focus on the final step in this process: optical recombination.

3.4.1 Optical Transitions

The intensity $I$ of the signal from an optical transition between states $i$ and $f$ is proportional to:

$$I \propto \Gamma_{i\rightarrow f} f(E_i) (1 - f(E_f))$$  \hspace{1cm} (3.2)

where $E_i$ and $E_f$ are the energies of the initial and final states respectively, $f(E)$ is the Fermi-Dirac distribution at energy $E$, and $\Gamma_{i\rightarrow f}$ is the optical
Figure 3.4: Optical Emission. The horizontal and vertical axes correspond to wave vector and energy respectively. (a) Excitation: photons create electron-hole pairs. (b) Thermalization: out-of-equilibrium photoexcited electrons and holes thermalize, coupling to the 2DES through single-particle and collective excitations of the 2DES (represented by the gray arrow). (c) Emission electrons recombine with the thermal distribution of holes (shown in red), releasing a photon with energy $\hbar \omega_L$. Cooling of the 2DES takes place by phonon emission through the electron-phonon interaction (not shown).

transition rate, given by Fermi’s golden rule [78]:

$$\Gamma_{i \rightarrow f} = \frac{2\pi}{\hbar} \left| \left\langle \psi_f | \hat{H} | \psi_i \right\rangle \right|^2$$  \hspace{1cm} (3.3)

The Hamiltonian of the electron in the presence of photons is given by:

$$H = \sum_{j=1}^{N} \left[ \frac{1}{2m^*} \left( \vec{p}_j - \frac{e}{c} \vec{A}(\vec{r}_j) \right)^2 + \frac{1}{2} \sum_{k \neq j} V(\vec{r}_k - \vec{r}_j) \right]$$ \hspace{1cm} (3.4)

Where $\vec{A}$ is the vector potential of the electromagnetic field, $\vec{p}$ is the momentum operator, and $V(\vec{r}_k - \vec{r}_j)$ is the Coulomb interaction between electrons
at positions $\vec{r}_k$ and $\vec{r}_j$. It is useful to rewrite $H$ as:

$$H = H_e + H^{(1)} + H^{(2)}$$

$$H_e = \sum_{j=1}^{N} \left[ \frac{\vec{p}_j^2}{2m^*} + \frac{1}{2} \sum_{k \neq j} V(\vec{r}_k - \vec{r}_j) \right]$$

$$H^{(1)} = \sum_{j=1}^{N} \frac{1}{2m^*} \left[ \vec{p}_j \cdot \frac{e}{c} \vec{A}(\vec{r}_j) + \frac{e}{c} \vec{A}(\vec{r}_j) \cdot \vec{p}_j \right]$$

$$H^{(2)} = \sum_{j=1}^{N} \frac{1}{2m^*} \left[ \frac{e\vec{A}(\vec{r})}{c} \right]^2$$

where $H_e$ is the electron Hamiltonian unperturbed by electron-photon interactions. The term $H^{(2)}$ involves two photons and can be ignored in discussions of optical emission.

The spatial dependence of $H^{(1)}$ can be neglected in our experiments because the length scale associated with the in-plane component of the photon’s wavevector is significantly larger than that of electrons in the 2DES [42]:

$$H^{(1)} = \sum_{j=1}^{N} \frac{ieF \xi e^{-i\omega t}}{m^* c^2 \omega} \vec{p}_j \cdot \hat{e}$$

where $\hat{e}$ refers to the polarization direction, $\omega$ is the frequency of the photon, and $F$ is the magnitude of the electric field.

Using the wave functions of GaAs bands in Table 3.1 and Equation (2.1), we can rewrite the wavefunctions of electrons under zero magnetic field as:

$$\psi_{n,J_s,J_z,s,\vec{k}}(\vec{r}, z, t) = u_{n,J_s,J_z}(\vec{r}, z) \chi_{\xi}(z) e^{i\vec{k} \cdot \vec{r}} e^{-i\vec{k}^t t}$$

(3.7)
where $J$ and $J^z$ are the total and $z$-component of the total spin and orbital angular momentum, $E$ is the energy of the state, and $n$ indexes states in the CB or VB (see Table 3.1).

It is not difficult to show that for CB-VB transitions \[42\]:

$$\Gamma_{i\rightarrow f} \propto \left| \langle \chi_f | \chi_i \rangle \langle e^{\tilde{k}_f \cdot \tilde{r}} | e^{\tilde{k}_i \cdot \tilde{r}} \rangle \delta(\Delta E - \hbar \omega) \right|^2 \quad (3.8)$$

where $\Delta E = |E_i - E_f|$. Application of the Wigner-Eckart theorem to the first term on the right-hand side of the equation dictates the allowed polarizations of photons emitted by CB-HH, CB-LH, and CB-SO transitions. The transition rate for the allowed polarization is therefore proportional to the overlap integrals $|\langle \chi_f | \chi_i \rangle|^2$ and $|\langle \tilde{k}_f | \tilde{k}_i \rangle|^2$. These overlap integrals are proportional to delta functions in $\xi$ and $\tilde{k}$ respectively: these quantities must be conserved for CB-VB transitions. Note that when the finite transition rate between states is taken into account, the term $|\delta(E_i - E_f - \hbar \omega)|^2$ is Lorentz-broadened, as discussed in Appendix B. This broadening plays a key role in determining the lineshape of optical emission from 2DES under finite magnetic fields.

Fig. 3.5 shows an optical emission spectrum for a 2DES under zero magnetic field. The Fermi edge results from the Fermi-Dirac distribution of electrons in the CB. The difference in energy between the Fermi Edge and bandgap is given by $E_F \left(1 + \frac{m^*}{m_h} \right)$, where $m^*_e$ and $m^*_h$ are the electron and heavy hole effective masses [79]. This property enables optical emission spectra to probe density of 2DES's. When performing characterization
Figure 3.5: Optical emission spectrum for a 2D electron system at 0 magnetic field. The Fermi edge results from the Fermi-Dirac distribution of electrons in the CB, and the exponential decay in intensity with energy results from the thermal distribution of photoexcited holes. The energy difference between the energies of the Fermi edge and the bandgap ($E_g$) is given by $E_F \left(1 + \frac{m_e^*}{m_h^*}\right)$ where $E_F$ is the Fermi energy, and $m_e^*$ and $m_h^*$ denote the electron and heavy hole effective masses respectively.

Experiments on our samples, it is useful to know whether or not the density of electrons in the quantum well is sensitive to laser power. Samples whose density changes with laser power are undesirable for experiments that probe quantum Hall fluids, for which uncertainty in filling factor is highly problematic.

Optical emission spectra also contain information about the temperature of the 2DES. The spectrum intensity’s exponential decay results from thermal distribution of photo-excited holes in the VB and is described by
Figure 3.6: (a) Optical emission spectrum from a 2DES at filling factor $\nu = 3$. (b) Optical emission from the spin-split LL’s in the CB to the spin-split LL’s in the HH VB. The emitted light is circularly polarized with polarization determined by the difference in total angular momentum between the CB or HH VB state. The red-shifted N=1 emission is the subject of Chapter 5.
\[ I \propto e^{-\beta E_{\text{mag}}/k_B T} \] where \( I \) is the intensity and \( \beta = \frac{1}{k_B T} \) \([79, 80]\). Optical emission spectra at zero magnetic field can therefore measure the temperature of the 2DES when it is out of equilibrium with the lattice (due to the photoexcited electrons and holes).

### 3.4.2 Magneto Luminescence

We now discuss optical emission of 2DES’s under high magnetic fields. As shown in Fig. 3.6, optical emission spectra clearly manifest the formation of LL’s.

An optical emission spectrum from a 2DES at filling factor \( \nu = 3 \) is shown in Fig. 3.6(a). In our experiments, holes are only photoexcited into the HH VB. Selection rules permit the following CB-HH transitions: \( |\frac{1}{2}, +\frac{1}{2}\rangle \rightarrow |\frac{3}{2}, +\frac{3}{2}\rangle \) and \( |\frac{1}{2}, -\frac{1}{2}\rangle \rightarrow |\frac{3}{2}, -\frac{3}{2}\rangle \) \([81]\), as depicted in Fig. 3.6(b). Because the response of the spectrometer is linearly polarized, emitted photons with both circular polarizations (\( \sigma^+ \) and \( \sigma^- \)) are measured.

### 3.5 Resonant Rayleigh Scattering

The remainder of this chapter focuses on light scattering. In these processes, incoming photons with frequency, wave vector, and polarization \( (\omega_L, \vec{k}_L, \hat{e}_L) \) are annihilated by a medium, resulting in the production of photons with frequency, wave, vector, and polarization \( (\omega_S, \vec{k}_S, \hat{e}_S) \).

This section focuses on Rayleigh scattering, for which \( \omega_S = \omega_L \), as depicted Fig 3.7. As Fig. 3.7 shows, Resonant Rayleigh scattering (RRS) is
a two-step, second-order process that probes transitions from occupied VB states to vacant CB states.

The intensity $I$ of Rayleigh scattering can be shown to be proportional to [78]:

$$I(\omega_L) \propto \sum_{I,F} \left| \frac{\langle I | H^{(1)}_L | F \rangle \langle F | H^{(1)}_S | I \rangle}{\omega_{IF} - \omega_L + i\Gamma(\omega_L)} + \frac{\langle I | H^{(1)}_L | F \rangle \langle F | H^{(1)}_S | I \rangle}{\omega_{IF} + \omega_L + i\Gamma(-\omega_L)} \right|^2$$

(3.9)

Figure 3.7: Depiction of elastic scattering. (a) Photons are annihilated, creating electron-hole pairs. (b) The electrons and holes recombine, emitting a photon at the same energy as the incident photon.

Here, $\hbar \omega_{IF}$ is the difference in energy between the initial VB state $I$ and final vacant CB state $F$, and $\Gamma(\omega)$ is the transition rate, which is taken to be small relative to $\omega$ and $\omega_{IF}$. Because we are discussing resonant elastic light
scattering, we will neglect the non-resonant second term in Equation (3.9).

Taking into account the fact that $H^{(1)}$ corresponds to dipole transitions, Equation (3.9) can be rewritten [78]:

$$I(\omega_L) \propto \sum_{I,F} \left| \frac{\left( \vec{d}_{FI} \cdot \hat{e}_S \right) \left( \vec{d}_{IF} \cdot \hat{e}_L \right)}{\omega_{IF} - \omega_L + i\Gamma(\omega_L)} \right|^2$$  \hspace{1cm} (3.10)

Here, $\vec{d}$ is the dipole operator and the unit vector $\hat{e}$ denotes the polarization direction of the incident or scattered light. For 2DES with isotropic susceptibility anisotropy in the 2DES susceptibility can lead to increases in the intensity for processes in which $\hat{e}_L \perp \hat{e}_S$ (cross-polarized RRS) [82]. The polarization dependence of RRS can be used to detect anisotropic phases in the 2DES.

### 3.6 Resonant Inelastic Light Scattering

We finish Chapter 3 with a discussion of resonant inelastic light scattering (RILS) by neutral excitations of the 2DES in a quantum well. Inelastic light scattering refers to scattering processes with $\omega_S \neq \omega_L$. Our work focuses on Stokes scattering, for which $\omega_S < \omega_L$ as depicted in Fig. 3.8.

Stokes shifts result from incident photons exciting single-particle or collective modes in the 2DES. Energy conservation requires that the frequency of excited modes $\omega$ be given by $\omega = \omega_L - \omega_S$. 
Figure 3.8: Schematic of light scattering spectra. The Stokes scattered peak energy is given by $\omega_S = \omega_L - \omega$, and the anti-Stokes scattered peak energy is given by $\omega_S = \omega_L + \omega$. **Above:** Light scattering spectrum in absolute energy scale. **Below:** The same light scattering spectrum in the Stokes energy shift scale, defined by $E = \hbar(\omega_S - \omega_L)$; positive shift correspond to lower energy. For elastically-scattered light, $\omega = 0$ (i.e. $\omega_S = \omega_L$).
3.6.1 Selection Rules: Single Particle Excitations

We begin our discussion of selection rules by looking differential cross section for light scattering [83, 84]:

\[
\frac{d^2 \sigma}{d\Omega d\omega} = \frac{\hbar \omega_I}{\omega_I} \left\langle \sum_F |M_{IF}|^2 \delta(E_I - E_F + \hbar \omega) \right\rangle
\]

(3.11)

where \( I \) and \( F \) are the initial and final states of the system respectively, \( M_{IF} \) denotes the relevant matrix element of the Hamiltonian, and the large brackets denote a sum over all initial states in thermal equilibrium.

As Fig. 3.9 shows, inelastic light scattering processes access neutral excitations built from neutral particle-hole pairs in the electron system. Restricting our discussion to single-particle excitations, we find that \( M_{IF} \) can be rewritten as:

\[
M_{IF} = \frac{e^2}{m^* c^2} \sum_{\alpha, \beta} \gamma_{\alpha, \beta} \left\langle F | C^\dagger_{\alpha} C_{\beta} | I \right\rangle
\]

(3.12)

where \( C^\dagger_{\alpha} \) and \( C_{\beta} \) denote creation and annihilation operators at single-particle states \( \alpha \) and \( \beta \) respectively, and \( \gamma_{\alpha, \beta} \) is given, to lowest order, by:

\[
\gamma_{\alpha, \beta} = \left\langle \alpha | e^{(\vec{k}_L - \vec{k}_S)} | \beta \right\rangle \hat{e}_L \cdot \hat{e}_S
\]

\[
+ \sum_m \left[ \frac{\left\langle \alpha | H^{(1)}_S \right| m \right\rangle \left\langle m | H^{(1)}_L \right| \beta \right\rangle}{\hbar (\omega_\beta - \omega_m + \omega_L)} + \frac{\left\langle \alpha | H^{(1)}_L \right| m \right\rangle \left\langle m | H^{(1)}_S \right| \beta \right\rangle}{\hbar (\omega_\alpha - \omega_m - \omega_L)} \right]
\]

(3.13)

Here, \( \hbar \omega_I \) and \( \hbar \omega_F \) are the energies states \( I \) and \( F \) respectively and \( H^{(1)}_L \) and \( H^{(1)}_S \) correspond to the first-order electron-photon interaction Hamiltonians.
for the incident and scattered photons respectively. Note that for excitations between quantum well subbands (intersubband excitations), the first term effectively vanishes, leaving only the resonant higher-order terms. At this order, we are only able to analyze momentum-conserving RILS by single-particle excitations, depicted in Fig. 3.9. To determine the RILS polarization selection rules from $\gamma_{\alpha\beta}$, it is convenient to look at results in 3D. These results are applicable to 2DES’s as within the effective-mass approximation, the Hamiltonian describing in-plane dynamics of 3D systems is equivalent to
that of 2DES [85]. Following the derivation from Ref. [83]:

\[
\langle \gamma \rightarrow \alpha \beta \rangle = \left\langle \alpha \left| e^{i(\mathbf{\hat{e}}_L - \mathbf{\hat{e}}_S) \cdot \hat{r}} \right| \beta \right\rangle \left[ \mathbf{\hat{e}}_S \cdot \mathbf{\hat{A}} \cdot \mathbf{\hat{e}}_L \left\langle s_{\alpha} \left| s_{\beta} \right\rangle + i \left( \mathbf{\hat{e}}_S \times \mathbf{\hat{e}}_L \right) \cdot \mathbf{\hat{B}} \cdot \left\langle s_{\alpha} \left| \hat{\sigma} \right| \left| s_{\beta} \right\rangle \right. \right]
\]

(3.14)

where \( s_{\alpha} \) and \( s_{\beta} \) denote the initial and final states’ spins, \( \hat{\sigma} \) is pauli spin matrix, and \( \mathbf{\hat{A}} \) and \( \mathbf{\hat{B}} \) are matrices determined by selection rules. For \( \mathbf{\hat{e}}_S \parallel \mathbf{\hat{e}}_L \) (polarized scattering), the scattering matrix is proportional to \( \mathbf{\hat{A}} \) and vanishes for spin-flip excitations. For \( \mathbf{\hat{e}}_S \perp \mathbf{\hat{e}}_L \) (cross-polarized scattering), the scattering matrix is proportional to \( \mathbf{\hat{B}} \) and vanishes for spin-preserving excitations.

Using the Kane model [75], \( \mathbf{\hat{A}} \) can be shown to be [83]:

\[
\mathbf{\hat{A}} = \mathbf{\hat{I}} \left( 1 + \frac{2\mathbf{P}^2}{3m} \left[ \frac{E_1^*}{E_1^* - (\hbar \omega_L)^2} + \frac{E_2^*}{E_2^* - (\hbar \omega_L)^2} + \frac{E_3^*}{E_3^* - (\hbar \omega_L)^2} \right] \right)
\]

(3.15)

where \( E_1^* \), \( E_2^* \), and \( E_3^* \) are the energy gaps to the HH, LH, and SO VB’s respectively, and \( \mathbf{\hat{I}} \) is the identity matrix. \( \mathbf{P} \) is the momentum operator applied to the \( S \) and \( P \) orbitals (\( X \), \( Y \), and \( Z \)), defined by:

\[
\mathbf{P} = \left\langle S \left| \mathbf{\hat{P}} \right| X \right\rangle \mathbf{\hat{e}}_x + \left\langle S \left| \mathbf{\hat{P}} \right| Y \right\rangle \mathbf{\hat{e}}_y + \left\langle S \left| \mathbf{\hat{P}} \right| Z \right\rangle \mathbf{\hat{e}}_z
\]

(3.16)

\( \mathbf{\hat{A}} \) describes only RILS by spin-preserving excitations. RILS by spin-flip excitations are described by:

\[
\mathbf{\hat{B}} = \mathbf{\hat{I}} \frac{2\mathbf{P}^2}{3m \hbar \omega_L} \left[ \frac{1}{E_0^* - (\hbar \omega_L)^2} - \frac{1}{(E_0^* + \Delta)^2 - (\hbar \omega_L)^2} \right]
\]

(3.17)
3.6.2 Selection Rules: Collective Excitations in Two Dimensions

Figure 3.10: Feynman diagram of RILS by collective excitations. First, a laser photon ($\gamma_L$) is absorbed, creating an electron-hole pair. The excited exciton-hole pair relaxes exciting a collective mode in the 2DES (shown in gray) before recombining and emitting a scattered photon ($\gamma_S$). The scattering matrix for RILS is of a higher order than that of PL or RRS.

where $\Delta$ is the energy shift of the spin-split VB. Note that $\vec{B}$ vanishes for vanishing $\Delta$ or $\omega_L$. The last two terms of Equation (3.17) are not relevant to our experiments, as we only excite from the III band.

3.6.2 Selection Rules: Collective Excitations in Two Dimensions

We now consider matrix elements for collective excitations, as depicted in Fig. 3.10. $M_{IF}$ for these processes is given by:

$$M_{IF}(\omega) \propto \sum_{m,l} \frac{\langle F | H^{(1)} | l \rangle \langle l | V_e | m \rangle \langle m | H^{(1)} | l \rangle}{[\omega_l - \omega_I][\omega_I - \omega_m]}$$  \hspace{1cm} (3.18)

where $V_e$ corresponds to the electrons' potential due to other other electrons and the lattice, $E_I$, $E_m$, and $E_l$ refer to the energies of the initial and virtual states respectively, and $\omega = \omega_L - \omega_S$. Equation (3.18) shows that RILS from
third-order processes has an additional resonance corresponding to:

$$\omega_I - \omega_l = \omega_L - (E_l + \omega) = \omega_s - E_l$$ \hspace{1cm} (3.19)

where $E_l$ denotes the energy transition from $l$ to $I$. This resonance corresponds to the overlap in energy of the scattered photon and optical transition: the outgoing resonance.

The intermediate states $l$ and $m$ are "virtual" states, and do not require conservation of energy.

In the presence of disorder, translational invariance is broken, resulting in a scattering term $V_q$ which breaks momentum conservation. The matrix element corresponding to $V_q$ is enhanced close to either excitation or emission resonances corresponding to a given collective excitation. This enables us to excite modes whose wave vector $|\vec{k}| >> |\vec{k}_L - \vec{k}_s|$ [74, 75, 86].

### 3.6.3 Excitations under Zero Magnetic Field

We now explore of intersubband excitations that can be probed by RILS, whose energies are depicted in Fig. 3.11. It is easy to show that the energy of single-particle excitations between lowest- and first-excited quantum well subbands of the CB is given by:

$$E_{\xi=1} - \frac{\hbar^2}{m^*}k_F q \leq E_{\xi=0} + \Delta E(\vec{q}) \leq E_{\xi=1} + \frac{\hbar^2}{m^*}k_F q$$ \hspace{1cm} (3.20)

where $\vec{q} = \vec{k}$, the in-plane wavevector transferred to the 2DES by the incident photons, as shown in Section 3.2. The spectrum of SPE is shown by the
shaded region in Fig. 3.11.

![Wave Vector (1/\ell)](image)

**Figure 3.11:** Energies of intersubband single-particle excitations (SPE), charge density excitations (CDE’s) and spin density excitations (SDE’s). After Ref. [87].

We also probe intersubband collective excitations, depicted in Fig. 3.12. These modes consist of an intersubband single-particle excitation that couples to the 2DES resulting in either collective charge density excitation (CDE) or a collective spin density excitation (SDE). Spectra of RILS by CDE’s and SDE’s are shown in Fig. 3.13. CDE’s are comprised of spin-neutral excitations which couple via the full quantum-mechanical Coulomb interaction. The CDE’s finite dipole moment results in a depolarization effect which causes the CDE energy to be blue-shifted relative to that of the SPE [41].

Because the direct coulomb interaction is spin-preserving, the SDE mode couples intersubband excitations only through the exchange interaction, resulting in the SDE having a vanishing dipole moment [88]. Because the exchange interaction is attractive, the SDE energy is red-shifted relative to
3.6.3. Excitations under Zero Magnetic Field

Figure 3.12: CDE and SDE intersubband modes shown schematically. (a) Macroscopically, the CDE consists of a charge density oscillation coupled by the full quantum mechanical Coulomb interaction. The SDE, which is spin polarized, couples through only the Coulomb-exchange interaction. (b) In the single-particle picture, these plasmons consist of coupled intersubband excitations. After Ref. [88].

that of the SPE.

The lineshape of intersubband excitations experiences homogeneous broadening due to the finite lifetime of the intermediate excitations and inhomogeneous broadening from disorder [89]. This makes spectra of RILS by CDE's a useful probe of sample cleanliness. Because the quantum Hall fluids studied in our experiments are only observed in ultraclean systems, RILS spectra of CDE's prove a necessary tool in determining which are best for our experiments.
3.7 Resonant Inelastic Light Scattering at non-zero Magnetic Field

We now turn to the inelastic light scattering in magnetic fields. As we did in previous sections, we are going to begin with our discussion with the formation of spin-split LL’s in the CB. As with 2DES’s under zero-field, RILS serves as a powerful tool for probing single-particle and collective excitations.
for 2DES’s under high magnetic fields.

Note that under magnetic fields, RILS selection rules are slightly relaxed, making both charge and spin excitations are accessible via both parallel- and cross-polarized scattering [90].

3.7.1 Neutral Excitations under Finite Magnetic Field

3.7.1.1 Inter-LL Excitations of IQHE states

We begin by exploring neutral charge density excitations of IQHE systems. The modes of interest are inter-LL excitations and spin waves (SW). In SW’s, only the spin degree of freedom changes. The dispersion of such modes was calculated for integer filling factors by Kallin and Halperin in 1984 [36]. This dispersion of intra-LL modes with change in LL quantum number $\Delta N = 1$ and no change in spin orientation displayed critical points including a relative minimum (magneto roton) at $q \approx \frac{2}{t_B}$ and a relative maximum (maxon) at $q \approx \frac{1}{t_B}$.

In 1988, Pinczuk et al. observed these critical points in the dispersion of IQHE states using RILS, as shown in Fig. 3.14. The observation of critical points at high wavevectors such as these was enabled by the breakdown of wavevector conservation by residual disorder, which makes RILS a sensitive method to states in the mode dispersions [74].

3.7.1.2 Intra-LL Excitations of FQHE States

We now consider the low-lying intra-LL modes in the fluctuations of the charge degree of freedom ("gap excitations"), which share the features shown
Figure 3.14: (a) RILS spectra of inter-LL excitations at $\nu = 1$. (b) Calculated mode dispersion at $\nu = 1$ using Hatree-Fock (after [91]). Breakdown of wavevector conservation makes large-wavevector modes accessible. Peaks in the RILS spectra result from rotons and maxons, maxima in the excitations’ density of states. After Ref. [74].
in Fig. 3.15. In 1985, Haldane and Rezayi published a paper showing that excitations of FQHE states can be studied using numerical evaluations with a finite number of particles in a spherical geometry [92]. These calculations showed that the excitation spectrum of the FQHE state at $\nu = 1/3$ is gapped and displays a roton minimum $\Delta_R$ for a wavevector in the range $q \sim 1/l_B$ [92].

![Schematic dispersion of the gap mode](image)

Figure 3.15: Schematic representation of the dispersion of the gap mode of the FQHE state at $\nu = 1/3$. The magneto roton $\Delta_R$ occurs for $q \sim 1/l_B$.

An insightful framework for these collective modes was provided by Girvin, Macdonald, and Platzman (GMP) in 1985 [93], when they extended Feynman’s single mode approximation for collective excitations of superfluid $^4$He to intra-LL excitations of Laughlin states. The single mode approximation (SMA) assumes the structure factor and oscillator strength of modes at wavevector $q'$ to be saturated by the lowest-energy mode. This approximation successfully reproduces the magneto roton dispersion for the Laughlin state at $\nu = 1/3$ uncovered in the work by Haldane and Rezayi and shows strong agreement with numerical calculations for wavevectors below the magneto
3.7.1. Neutral Excitations under Finite Magnetic Field

roton minimum [92, 93].

GMP proposed that the wave function of the excitation $\psi_q$ is approximated by [93, 94]:

$$|\psi_q\rangle = N^{-1/2} \hat{\rho}_q |\psi_0\rangle$$  \hspace{1cm} (3.21)

Here, $\psi_0$ denotes the ground state wave function and $\hat{\rho}_q$ denotes the projection of the density fluctuation operator $\hat{\rho}_q = \sum_j \exp(i\vec{q} \cdot \vec{r}_j)$ onto the lowest LL. Projection onto the lowest LL ensures that these excitations are intra-LL modes.

The excitation spectrum $\Delta(q)$ of this mode can be shown to be [93, 95, 94]:

$$\Delta(q) = \frac{\bar{f}(q)}{\bar{s}(q)}$$  \hspace{1cm} (3.22)

Here, $\bar{f}(q)$ is the oscillator strength of the excitation $\bar{f}(q) = \langle \psi_0 | \hat{\rho}_q^\dagger (H - E_0) \hat{\rho}_q | \psi_0 \rangle$, where $E_0$ is the energy of the ground state, and $\bar{s}(q)$ is the static structure function of the excitation $\bar{s}(q) = \langle \psi_0 | \hat{\rho}_q^\dagger \hat{\rho}_q | \psi_0 \rangle$ [93]. Because $\bar{f}(q)$ and $\bar{s}(q)$ vanish as $|q|^4$ as $q \to 0$, gapped excitation spectra exist within the SMA for liquid ground states in the lowest LL [93].

Within this approximation, the roton mode corresponds to a maximum in the structure factor; the energy of the roton minimum is useful to understanding electron correlation in a FQHE ground state [93]. The large structure factor of the roton mode also means an enhanced scattering cross section, indicating that RILS with breakdown of wavevector conservation could be a powerful probe of these excitations [26, 90, 96].
3.7.2 Spin Wave Excitations

![Diagram of spin wave excitations](image)

Figure 3.16: **Above:** Schematic of optical transitions that contribute to RILS by spin wave excitations. **Below:** Calculated dispersion of the SW where $\Delta E = E - E_Z$, for the IQHE state at $\nu = 1$. Because this is a rotationally invariant state, the energy gap of this mode is equal to $E_Z$ ($\Delta E = 0$). After Ref. [36].

We now discuss spin waves (SW’s) modes, with dispersion depicted in Fig 3.16 (b) for $\nu = 1$. For systems with spin rotational invariance, the energy gap of the SW occurs at vanishing wave vector (large wavelength) and is given by the Zeeman energy $E_Z = |g^*| \mu_B B_T$, where $B_T$ is the total magnetic field (see Fig. 3.1), as required by Larmor’s theorem [97, 36, 98]. Examples
of spin rotationally invariant states include odd integer IQHE states, since these states are spin-polarized and rotationally invariant. Broken full spin rotational invariance manifests in changes in the long-wavelength SW which is no longer required to be at \( E_Z \) [33, 35, 99, 100]. The use of RILS by the SW is used to probe broken spin rotational invariance is discussed further in Chapter 5.

As we will see in the later chapters, the magnetic field corresponding to \( \nu = 3 \) can be determined very accurately by monitoring the intensity of SW at the Zeeman energy [101, 35]. The intensity of the Zeeman mode in the N=1 LL is proportional to:

\[
I_Z(\nu) \propto \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} n_\nu
\]  

(3.23)

Here, \( N_\uparrow (N_\downarrow) \) corresponds to the number of electrons with spin parallel (anti-parallel) to the magnetic field, and \( n_\nu \) denotes the number of electrons in the highest occupied LL. At \( \nu = 3 \), \( I_Z \) is maximal due to the lower number of electrons in the N=1 LL for \( \nu < 3 \), and the reduction in spin-polarization for \( \nu > 3 \) depicted in Fig. 3.7.2 (b). As Fig. 3.7.2 (a) and (c) show, \( I_Z \) displays strong filling factor-dependence of the SW at \( \nu = 3 \). This enables us to determine the magnetic field corresponding to \( \nu = 3 \) with a high degree of precision.
3.7.2. Spin Wave Excitations

Figure 3.17: (a) Color plot of RILS spectra with relative intensity as shown in the middle panel at filling factors $\nu = 3.02$, $\nu = 3.0$, and $\nu = 2.98$ as a function of incident photon energy. Resonant enhancement of spin wave mode at the bare Zeeman energy is achieved only under extreme resonant conditions in a very narrow range of photon energies. The SW intensity is significantly reduced for small changes in filling factor away from $\nu = 3$. (b) Configuration of spins around $\nu = 3$. (c) Comparison of the resonantly enhanced spin wave modes from spectra shown in (a) (marked by the red line). The spin wave signal is significantly reduced for small deviations in filling factor of $\delta \nu = \pm 0.02$ away from $\nu = 3$. After Refs. [34] and [35].
Chapter 4

Collective Modes of FQHE States in the Second Landau Level

“Look deep into nature, and then you will understand everything better.”

-Albert Einstein

4.1 Overview

In this chapter, I report my studies using resonant inelastic light scattering (RILS) by collective charge and spin excitations of fractional quantum Hall effect (FQHE) states in the filling factor range $5/2 > \nu > 2$, in order to gain insight into the quasiparticle interactions of these FQHE states. The work presented in this chapter was published in Ref. [34].

A key property of quantum Hall fluids is their incompressibility, which is
linked to the emergence of a finite energy gap. This energy gap is why the bulk of a FQHE state is insulating and electrical current, carried by edge states, is topologically protected from scattering.

Both RILS and activated magneto-transport are used to probe the energy gaps of FQHE states. Activated magneto-transport experiments associate FQHE states’ energy gaps with the transport activation gap, conceptually linked to the $q \to \infty$ limit of the gap mode (see Fig. 3.15). The measured activation gaps are modified by residual disorder effects [102, 103, 104, 105].

The sensitivity of RILS to critical points in a state’s excitation spectra allows it to provide direct measurements of the state’s energy gap as well as to other features of its excitation spectrum such as magnetorotons [26, 90, 96], which provide valuable insight into the fundamental nature of the FQHE states currently inaccessible by other experimental methods.

In this chapter, I discuss observations of RILS signatures of gapped modes for states at filling factors $\nu = 2 + 1/3$, $\nu = 2 + 3/8$, and $\nu = 2 + 2/5$ known from magneto-transport experiments to be incompressible FQHE states [106, 107, 108]. For all of these states, three gapped modes ($E_g$, $E_S$, and $E_{DOS}$) are excited by RILS. These modes exhibit a striking dependence on filling factor (shown in Fig. 4.1) and temperature (shown in Fig. 4.3) which substantiates their link to the FQHE states at those filling factors.

Two of the modes ($E_g$ and $E_{DOS}$) are interpreted as collective charge density modes while the other ($E_S$) as a spin-wave excitation. The $E_g$ mode is interpreted as arising from low-lying, neutral excitations. Lineshape analysis of the RILS by the $E_g$ mode shown in Fig. 4.5 for $\nu = 2 + 1/3$, reveals the $E_g$
mode to be gapped with energy close to 70 $\mu$eV. Within the framework of
the breakdown of wave-vector conservation in the presence of weak residual
disorder, the $E_g$ is interpreted as arising from critical points (magnetoroton
minima) in the low-lying excitation spectum of the FQHE states. This inter-
pretation of the modes is supported by their filling factor-dependence (shown
in Fig. 4.1), temperature-dependence (shown in Fig. 4.3) and energies. The
results presented below in this chapter are the first observations of the FQHE
states' low-lying excitation spectra.

Two low-lying modes are observed in the excitation spectrum of the
FQHE state at $\nu = 2 + 1/3$, and are interpreted as magnetoroton and large
wavevector modes. The energies of these modes is approximately $0.15 \pm
0.01$ times that of similar low-lying modes of the FQHE state at $\nu = 1/3$,
the N=0 LL analogue of the $\nu = 2 + 1/3$ state. This scaling likely arises
from screened interactions between quasiparticles in the N=1 LL [109]. The
energies of these modes matches those predicted by numerical calculations
for the $\nu = 2 + 1/3$ FQHE state with a similar ratio of confinement width
to magnetic length [110]. There is also strong agreement between the energy
range for which the $E_{DOS}$ mode is observed at $\nu = 2 + 1/3$ and that of the
higher-lying continuum of states predicted by numerical calculations [110].

Strong polarization-dependence is observed for RILS and resonant Rayleigh
scattering (RRS) spectra near filling factor $\nu = 2+2/5$. This polarization de-
pendence in RILS and RRS is discussed within the framework of anisotropic
electron phases that support nematic FQHE states [111].
Figure 4.1: Filling factor-dependence of cross-polarized RILS. The peaks at $E_g$ and $E_{DOS}$ are associated with charge density modes and the peak at $E_S$ corresponds to a spin wave. These modes are only observed at filling factors very close to the FQHE states at $\nu = 2 + 2/5$, $\nu = 2 + 3/8$, and $\nu = 2 + 1/3$ in the filling factor range $2 + 1/5 < \nu < 2 + 2/5$. Adapted from Ref. [34].

4.2 Background

As we have pointed out, ultraclean two-dimensional electron systems (2DES) under high perpendicular magnetic fields are a rich source of exotic many-body physics. In the partially-populated N=0 LL, the 2DES forms odd-denominator FQHE fluids that are interpreted as liquids of weakly-interacting composite fermions (CF’s) [7, 8]. The interaction physics in higher ($N \geq 2$) LL’s leads to quantum phases referred to as bubble and stripe phases, which lead to transport anisotropy and reentrant integer quantum Hall effects [65, 58, 53]. The unique interactions in the N=1 LL manifest in odd-denominator FQHE states as well as unconventional FQHE states and competing phases similar to those observed in higher LL’s. Under finite in-plane magnetic field, a robust FQHE state with large anisotropy in longi-
tuidnal resistance is observed at filling factor $\nu = 2 + 1/3 = \frac{7}{3}$, indicating that this FQHE can be stabilized in the absence of full rotational invariance [17, 21, 22, 112]. It has been proposed that transport anisotropies in the $N=1$ LL can be explained in terms of the nematic electron liquid, a compressible metallic phase that is expected to exhibit strong polarization-dependence in light scattering experiments due to its unequal longitudinal and transverse susceptibilities $\chi_L$ and $\chi_T$ [20].

The fundamental nature of both more conventional odd-denominator $N=1$ LL FQHE states like the $\nu = 2+1/3$ state and unconventional states like the state at $\nu = 2 + 3/8$ are not yet well-understood. Similarly, the low-lying collective excitation spectra of these $N=1$ LL FQHE states, which provide valuable insights into electron correlation and quasiparticle interactions [93], had not been observed prior to our work.

Collective charge and spin modes of phases in the $N=0$ LL have been accessed by RILS methods [90, 113, 96, 114, 115], and comparisons of the measured low-lying excitations with theoretical calculations provide in-depth understanding of the physics of those emergent phases [33, 116]. The excitations of the FQHE state at $\nu = 7/3$, the $N=1$ LL analogue of the most robust FQHE state at $\nu = 1/3$, are predicted to be CF’s dressed with an exciton cloud [109]. In addition to the much-studied state at $\nu = 5/2$, interpreted as a $p$-wave paired state of composite fermions that supports non-Abelian excitations [68], the state at $\nu = 2 + 2/5$ is envisioned as a similar $p$-wave paired state [71]. It has been suggested that the weaker FQHE states at $\nu = 2 + 2/5$ and $\nu = 2 + 3/8$ exhibit even greater potential to serve as a
model system for fault-tolerant quantum computation than the FQHE state at $\nu = 5/2$. [71, 117].

4.3 Results

The ultraclean 2DES studied in this chapter is confined to a 30-nm-wide, symmetrically doped GaAs/AlGaAs quantum well. The density and mobility of the 2DES are $2.9 \times 10^{11}$ cm$^{-2}$ and $23.9 \times 10^{6}$ cm$^2$/Vs respectively. The sample is mounted on the cold finger of a $^3$He-$^4$He dilution refrigerator in the bore of a 16T superconducting magnet with bottom windows for direct optical access as shown in Fig. 3.1 in Section 3.2.

RILS and RRS spectra are excited by a tunable Ti:Sapphire laser with incident power below $10^{-4}$ W/cm$^2$. The energy of the exciting light $\hbar \omega_L$ is tuned close to optical emission from the N=1 LL to achieve resonant enhancement.

The backscattering geometry of this experiment is shown in Fig. 3.2 in Section 3.2. The sample was tilted at angles $\theta = 20^\circ$ and $\theta = 25^\circ$ in two different cooldowns. As discussed in Chapters 2 and 3, this tilt angle allows for finite wavevector transfer $k = 2 \frac{\omega_L}{c} \sin(\theta)$ and results in a non-zero in-plane magnetic field (as shown in Fig. 3.1 and in the inset of Fig.4.2(a)) that allows for well-defined FQHE states at $\nu = 5/2$ and $\nu = 2 + 1/3$ while at the same time also allowing for the formation of anisotropic phases in the N=1 LL [13, 14, 16, 17, 19].

The filling factor as a function of magnetic field is precisely determined
by the maximum of the spin-wave intensity for \( \nu = 3 \), as shown in Fig. 3.17. For the fully spin polarized state at \( \nu = 3 \), the lowest-lying collective excitation mode is a spin reversal mode, a so called spin wave (SW) mode, discussed in Subsection 3.7.2. As shown in Fig. 3.17 (b), the SW gets weaker in both directions away from \( \nu = 3 \) due to the lower number of electrons in the N=1 LL for \( \nu < 3 \) and to a reduction of spin polarization for \( \nu > 3 \). This procedure enables a precise determination of the magnetic field of filling factor \( \nu = 3 \), and the determination of \( \nu \) in the range \( 2 \leq \nu \leq 3 \).

RILS measurements at filling factors \( \nu = 3.02, 3.00, \) and 2.98, displayed in Fig. 3.17 (a), exhibit strong resonance. Resonant enhancement of spin wave mode at \( E_Z \) is achieved only under extreme resonant conditions in a very narrow range of incident photon energies. In Fig. 3.17(c), the most resonantly enhanced RILS spectra are compared for filling factors in close vicinity to \( \nu = 3 \). Minor changes of the filling factor significantly lower the intensity of the resonantly enhanced SW mode as shown in Fig. 3.17(c).

The polarization of incoming and scattered light is denoted by \( V \) for light whose polarization is parallel to \( B_\parallel \) and \( H \) for light with polarization perpendicular to \( B_\parallel \), as depicted in the inset of Fig. 4.2(a). The scattered light is analyzed with a Horiba T64000 triple grating spectrometer. The response of the spectrometer is linearly polarized so the signal recorded from light scattered with polarization \( H \) is greatly diminished relative to light with polarization \( V \). For incident photons with polarization \( H \), the scattering geometry is cross-polarized \((H,V)\) and for incident photons with polarization \( V \), the scattering geometry is parallel-polarized \((V,V)\). As shown in Fig. 4.2,
Figure 4.2: (a) Color plot of RRS (gray scale) and cross-polarized \((H,V)\) RILS intensities (color scale) for photon energies close to the optical emission from the N=1 LL at \(\nu = 2 + 1/3\) as a function of incident photon energy \(\hbar \omega_L\). Three modes are seen at energies \(E_S\), \(E_{DOS}\), and \(E_g\). Inset: The light scattering geometry and the magnetic field direction. \(H\) and \(V\) denote the linear polarization of the incident photons. Because of the spectrometer gratings, the signal from light scattered with polarization \(H\) is greatly diminished relative to light with polarization \(V\). (b) Schematic representation of resonance enhancement in RILS and RRS. (c) Comparison of RRS spectrum (red dots) with optical emission spectrum (black line). After Ref. [34].
4.3. Results

V polarized light has a finite component parallel to the in-plane magnetic field $B_{||}$, whereas $H$ is perpendicular to $B_{||}$.

Fig. 4.2 (a) also shows that the $(H, V)$ RILS spectra at $\nu = 2 + 1/3$, the filling factor of the most robust odd-denominator FQHE state in the SLL, display three features, which are interpreted as collective excitation modes of the incompressible quantum fluid are only observed close to FQHE filling factors; tuning the filling factor away from FQHE fractions by $\delta \nu = 0.02$ significantly reduces the intensity of these modes. There is a band shifts with $\omega_L$ and occurs in the energy range $0.15 \text{ meV} < E_{DOS} < 0.35 \text{ meV}$, a broad mode centered at $E_g \approx 0.08 \text{ meV}$ and a sharp mode at $E_S \approx 0.1 \text{ meV}$. These modes are surprisingly well-defined, even at the filling factors of the FQHE states at $\nu = 2 + 2/5$ and $\nu = 2 + 3/8$ as shown in Fig. 4.1, which are observed to be fragile in activated transport [106, 107, 104]. Maxima in RILS spectra are ascribed to either long-wavelength modes at $q = k$ or to maxima in the wavevector dispersion’s density of states such as rotons or maxons which are made accessible by the breakdown of wavevector conservation due to weak residual disorder.

The $E_g$ and $E_{DOS}$ modes exhibit marked temperature dependence, as shown in Fig. 4.3 for $\nu = 2 + 1/3$. As the temperature is increased from 42 mK to 250 mK, the $E_g$ mode diminishes in intensity and disappears. The $E_{DOS}$ mode displays similar sensitivity to temperature, undergoing significant spectral broadening and reduction in intensity as temperature is increased from 42 mK to 250 mK before almost vanishing as the temperature is increased to $T = 600 \text{ mK}$. While $E_S$ also displays spectral broadening and
reduced intensity as the temperature is increased to $T = 600$ mK, its temperature dependence is significantly less pronounced than that of $E_q$ and $E_{DOS}$. It is also worth noting that the $E_S$ mode, which we associate with a SW, is softened from the bare Zeeman energy in this filling factor range, which is consistent with broken spin-rotational invariance of the 2DES [118, 99, 100].

The polarization-dependence of RILS and RRS are best exemplified by the results for filling factor near $\nu = 2 + 2/5$, shown in Figs. 4.6 and 4.7 respectively. The presence of an external magnetic field causes the RILS polarization selection rules discussed in the last chapter to become relaxed due to spin coupling in valence band states: both spin and charge modes are accessible in cross- and parallel-polarized scattering [90]. RILS experiments are typically performed with $(H,V)$ geometry to suppress parasitic light at $\omega_L$ that would mask the signal from RRS and low-energy RILS modes. All the spectra in this work were obtained by careful suppression of parasitic light at $\omega_L$ to allow quantitative analyses of RILS and RRS spectra in $(H,V)$ and $(V,V)$ geometries. As Fig. 4.6(a) shows, the lowest-energy mode $E_q$ and the spin-wave mode $E_S$ are very weak in $(V,V)$ spectra. As Fig. 4.6(b) shows, the gapped modes are absent in spectra away from $\nu = 2 + 2/5$ by $\delta\nu = 0.02$ for both $(H,V)$ and $(V,V)$ spectra.

As demonstrated in Fig. 4.7, the RRS signal is remarkably stronger for $(H,V)$ polarized spectra at $\nu = 2 + 2/5$. This polarization-dependence is remarkable despite the fact that the signal from non-resonant Rayleigh scattering is typically much stronger in $(V,V)$ than $(H,V)$ geometry, because spurious signal by scattering from optical components such as windows (which
is parallel-polarized).

The resonance enhancement of RRS displays striking filling factor dependence similar to that of RILS, as shown in Fig. 4.7 for $\nu = 2 + 2/5$. Tuning the filling factor away from $\nu = 2 + 2/5$ by $\delta \nu = 0.02$ results in a significant reduction in the resonant enhancement of the RRS in both polarizations by an order of magnitude. As Fig. 4.7 shows, the resonance enhancement of the RRS signal in $(H,V)$ geometry is reduced from 5800% to 440% and in $(V,V)$ the signal is reduced from 900% to 100%.

4.4 Discussion

The $E_S$, $E_g$, and $E_{DOS}$ modes are attributed to collective excitations of FQHE states at $\nu = 2 + 1/3$, $\nu = 2 + 3/8$, and $\nu = 2 + 2/5$. This interpretation is supported by the modes' striking filling factor dependence shown and temperature dependence below $T = 600$ mK.

The $E_S$ is interpreted as the low-wavevector spin-wave excitation. For 2DES with spin-rotational invariance, this mode is observed at the bare Zeeman energy, given by $E_z = |\mu_B g B_T|$ where $\mu_B$ is the Bohr magneton, $g$ is the Lande factor, and $B_T$ is the total magnetic field [97, 99, 100]. The observation of a spin wave mode at energies below $E_z$ have been previously reported and can be interpreted as a sign of breakdown of full spin-rotational invariance [118, 99, 100].

As discussed above, Fig. $E_S$ displays weaker temperature-dependence $E_g$ and $E_{DOS}$ broadening slightly as the temperature is raised from 42 mK to
250 mK At $T = 250$ mK, $E_S$ is slightly broadened and at $T = 600$ mK it is still observable. These results are consistent with the interpretation that $E_S$ is a pure spin wave that is broadened by increased temperature, but which is not strongly affected by the melting of an incompressible fluid.

![Figure 4.3: Temperature-dependent RILS spectra at $\nu = 2 + 1/3$ for $T=42$ mK, 250 mK, and 600 mK, measured in cross-polarized $(H,V)$ geometry at $\theta = 25^\circ$. The $E_g$ band of modes is already absent at $T=250$ mK. The dispersive mode $E_{DOS}$ is significantly weakened and broadened by increasing temperature. The spin-wave mode $E_S$ is slightly weakened and broadened at $T=600$ mK. After Ref. [34].](image)

The intense $E_g$ mode in RILS spectra is regarded as a superposition of RILS and strong RRS signal, which is well-described by a single Lorentzian peak centered at $\omega_L$, as shown in Fig. 4.5(a). The subtracted spectra at $\nu = 2 + 1/3$, shown in Fig. 4.5(b), reveal that at low energy, the RILS component of the measured spectra are well-described by two Lorentzian
Figure 4.4: (a) CF excitations of the $\nu = 2 + 1/3$ in the N=1 LL. Pictorial representation of spin-wave excitations of CF’s (After Ref. [116]). (b) Pictorial representation of the CF charge excitation consisting of a CF quasiparticle ‘dressed’ with a CF excitation [109] and the . After Ref. [34].

peaks centered at $E_{g1} \approx 67$ $\mu$eV $= 7.9 \times 10^{-3} E_c$ and $E_{g2} \approx 90$ $\mu$eV $= 1.06 \times 10^{-2} E_c$ respectively where $E_c = e^2/\epsilon l_B$. The two contributions exhibit a slightly different resonant enhancement profile in RILS. The $E_{g2}$ mode is resonantly enhanced for larger incoming photon energy compared to the $E_{g1}$ mode. This verifies the interpretation that the two modes result form RILS by collective excitations.

The two modes $E_{g1(2)}$ are interpreted as the lowest-energy collective spin-preserving excitations of the $\nu = 2 + 1/3$ FQHE state. Because such modes occur at high wavevector, we view these results as resulting from the breakdown of wavevector conservation. In this framework, modes are expected to occur at critical points in the wavevector dispersion [33, 116], such as roton minima or the limit $\Delta_\infty$ at $q \to \infty$. This interpretation of the $E_g$ modes is supported by the fact that energies of the $E_{g1}$ and $E_{g2}$ modes at $\nu = 2 + 1/3$ are the same as those predicted for such critical points in the wavevector dispersion predicted by Jolicoeur for a 2DES with a similar ratio of width to magnetic length [110]. The strong temperature-dependence of the $E_g$ mode
Figure 4.5: Mode analysis of the resonantly enhanced mode $E_g$ for $\nu = 2+1/3$ (a,b) and $\nu = 2 + 2/5$ (c) [(H,V), T= 42mK, $\theta = 20^\circ$]. (a) Individual scattering spectra at $\nu = 2+1/3$ excited by different incident photon energies $\omega_L$. The Inset shows the spectra around zero energy to highlight the RRS contribution. (b) Spectra shown after subtracting RRS signal. A mode analysis with Lorentzian peaks uncovers two resonantly enhanced low-energy modes at $E_{g1} \approx 67 \mu$eV = $7.9 \times 10^{-3} E_c$ and $E_{g2} \approx 90 \mu$eV = $1.06 \times 10^{-2} E_c$ respectively where $E_c = e^2/\epsilon_l B$. (c) In contrast, at $\nu = 2 + 2/5$, only one mode is observed at $E_g \approx 75 \mu$eV = $8.2 \times 10^{-3} E_c$. After Ref. [34].
provides further support for this interpretation.

The energy of the excitations of the $\nu = 2 + 1/3$ state are approximately $0.15 \pm 0.01$ times those of the $\nu = 1/3$ state, its analogous counterpart in the N=0 LL. This is seen as evidence of different interaction physics in the N=1 LL and is consistent with the theory that excitations of the $\nu = 2 + 1/3$ are composed of composite fermions dressed with quasiexcitons as depicted in Fig. 4.4 (b) [109].

The $E_{DOS}$ modes are interpreted as excitations into a continuum of higher-energy states [110]. This interpretation is supported by the broad energy range over which the $E_{DOS}$ is observed and the fact that at $\nu = 2 + 1/3$, the $E_{DOS}$ has a low-energy onset close to $0.15 \text{ meV} \approx 1.76 \times 10^{-2} E_c$, which is close to the onset of the continuum predicted by Jolicoeur [110].

At $\nu = 2 + 2/5$ and $\nu = 2 + 3/8$, quantitative mode analysis of the $E_g$ band uncovers only one mode, which is centered at $\Delta \lesssim 75 \mu \text{eV}$, as shown in Fig. 4.5 (c) for $\nu = 2 + 2/5$. The absence of a second mode at $\nu = 2 + 2/5$ and $\nu = 2 + 3/8$ indicates a difference in the interaction physics at these filling factors and $\nu = 2 + 1/3$. It is worth noting that at $\nu = \frac{5}{2}$, only one mode interpreted as a roton minimum has been observed at a similar energy (70 $\mu$ eV) [101]. The polarization-dependence and filling factor of RILS and RRS can be explained by the existence of anisotropic a FQHE states at $\nu = 2 + 2/5$ [119, 120, 112].

The polarization-dependence of RILS and RRS is consistent with the interpretation from transport experiments that nematic FQHE states are stabilized in the SLL at $\nu = 2 + 1/3$ [17], $\nu = 5/2$ [112], and $\nu = 2 + 2/5$.
Figure 4.6: Polarization-dependent RILS spectra in $(H, V)$ geometry (red) and $(V, V)$ geometry (blue) for (a) $\nu = 2 + 2/5$ and (b) $\nu = 2.38$ ($T=42$ mK, $\theta = 25^\circ$). After Ref. [34].
Figure 4.7: RRS spectra in \((V, V)\) geometry (blue dots) and \((H, V)\) geometry (red dots) alongside optical emission spectra (black trace) at filling factors \(\nu = 2 + 2/5\) (a) and \(\nu = 2.38\) (b). The solid line connecting the dots serve to guide the eye. The resonant conditions in RRS overlap the emission peak in the PL spectra. (a) At \(\nu = 2 + 2/5\), the peak of the RRS signal enhancement is 5800\% in \((H, V)\) geometry and 900\% in \((V, V)\) geometry. (b) At \(\nu = 2.38\), the RRS signal is enhanced by 440\% and 100\% in the \((H, V)\) and \((V, V)\) geometries respectively. The RRS is drastically reduced by tuning the filling factor away from \(\nu = 2 + 2/5\) by \(\delta \nu = 0.02\) to \(\nu = 2.38\). After Ref. [34].
[121] to anisotropic susceptibilities $\chi_\parallel$ and $\chi_\perp$ parallel and perpendicular to the in-plane component of the magnetic field $B_\parallel$, as predicted by theory [20]. This interpretation is consistent with the redshift of the spin-wave energy resulting from a reduced value of the $g$-factor due to the collapse of full rotational invariance.

4.5 Summary

Gapped low-energy modes have been observed in RILS for FQHE states at $\nu = 2 + 2/5$, $\nu = 2 + 3/8$, and $\nu = 2 + 1/3$. Three modes, interpreted as collective charge and spin modes, are clearly observable for each state. This interpretation is supported by the clear filling factor- and temperature-dependence of RILS spectra. A detailed mode analysis of the RILS spectrum of the FQHE state at $\nu = 2 + 1/3$ suggests that the energy of critical points in this state’s excitation spectrum are $0.15 \pm 0.01$ times those of the $\nu = 1/3$ FQHE state, its N=0 LL analogue. This result suggests qualitative difference between quasiparticle interactions in both states. Observations of polarization-dependence of RRS and RILS measurements at $\nu = 2 + 2/5$ are linked to anisotropic susceptibility, in agreement with the existence of nematic FQHE states. The filling factor-dependence of this polarization-dependence provides further support for this interpretation. The reported results provide in insight into the nature of the fragile and enigmatic FQHE states in the N=1 LL and can aid in distinguishing between different theoretical models.
Chapter 5

Optical Emission from Electron Phases in the Second Landau Level

"Both the man of science and the man of action live always at the edge of mystery, surrounded by it."

-J. Robert Oppenheimer

5.1 Overview

In this chapter, I report my studies using optical emission experiments to probe emergent quantum phases of the second Landau level in the filling factor range $2 < \nu < 3$. The optical recombination is from transitions across conduction to valence band states from electrons that partially populate the second (N=1) Landau level (LL). The interpretation of optical emission spec-
tra is facilitated by resonant inelastic light scattering (RILS) results from low-lying spin wave excitations of these phases, which allows us to monitor spin rotational order in these phases. The results presented in this chapter offer new insights into the quantum phases in the N=1 LL, indicating that optical methods are powerful tools for the identification and study of quantum phases in the N=1 LL. The results presented in this chapter were published in Ref. [35].

The unique interaction physics of the N=1 LL results in the presence of reentrant integer quantum Hall effect (RIQHE) states and stripe phases in addition to fractional quantum Hall effect (FQHE) fluids [13, 53, 58], resulting in striking interplay between phases [13, 58, 14] (see also Chapters 2 and 4).

As highlighted in previous chapters, ultra-clean two dimensional electron systems in the presence of high perpendicular magnetic fields $B$ are a source of unexpected and fascinating quantum many-body physics that arises from the strong electron interactions combined with a reduction in dimensionality. When $B$ is high enough for all electrons to occupy the N=0 LL, the many-electron system forms liquids of the FQHE. When $B$ is such that electrons fill states in higher ($N \geq 2$) LL’s, electrons form quantum phases referred to as stripe and bubble phases, which lead to transport anisotropy and reentrant integer quantum Hall effect (RIQHE) states [13, 58, 65]. The unique interaction physics in the N=1 LL results in the presence of RIQHE states and stripe phases in addition to even- and odd-denominator FQHE states [13, 53, 58], resulting in striking competition and coexistence of phases [13, 58, 14].
5.1. Overview

The interplay of anisotropic phases and FQHE states has been studied in transport by the introduction of in-plane magnetic fields [13, 14, 16, 17, 58, 112]. The large anisotropy induced in the system at the FQHE states at \( \nu = 5/2 \) and \( \nu = 7/3 \) by relatively small in-plane magnetic fields [16, 17, 18] supports interpretations in terms of a new state of electron matter with FQHE states that occur in the environment of a nematic or smectic stripe phase [20, 54, 122, 123, 124, 21, 22].

Competition and coexistence of phases in the \( N=1 \) LL has been studied not only by transport, but also by NMR [125, 126], microwave resonance measurements [127, 128], and light scattering methods [33, 101, 34], as well as in two-subband systems [129]. While optical emission from the \( N=0 \) LL has been used to monitor collective excitations and spin polarization of 2DES’s, [130, 131, 132, 133] this work represents the first use of optical emission from the \( N=1 \) LL to monitor phase competition in the filling factor range \( 2 \leq \nu \leq 3 \).

I discuss our optical emission experiments that probe the interplay between phases in the \( N=1 \) LL. The optical emission is from transitions across the conduction to valence band states from electrons that partially populate the \( N=1 \) LL. While this optical emission is much weaker than the one originating in the \( N=0 \) LL (see Figs. 5.1(c) and (d)), its displays a marked dependence on filling factor, which uncovers competing and overlapping quantum phases in the range \( 2 < \nu < 3 \).

Links between optical emission and emerging quantum phases are estab-
lished by comparing optical emission with the long-wavelength spin wave
obtained by resonant inelastic light scattering (RILS). Comparing the filling factor dependence of spectra from optical emission and RILS by collective spin wave excitations offers key insights into the nature of these phases. The Zeeman mode predicted by the Larmor theorem (mentioned in Subsection 3.7.2), collapses for $\nu < 3$ due to loss of full spin rotational invariance and recovers at the filling factors of certain fractional quantum Hall states. Slight softening of the spin wave from the Zeeman energy is regarded as the evidence of formation of spin textures that break the full rotational invariance of the 2D electron system due to the combined effects of Coulomb interactions and disorder [100, 33, 101].

5.2 Results

The 2D electron system is realized in two samples each with a symmetrically doped single GaAs/AlGaAs quantum wells of width 300 Å [39, 40]. The charge carrier density in the lower density sample A is $2.92 \times 10^{11}$ cm$^{-2}$, measured in transport experiments, and the carrier mobility is $23.9 \times 10^{6}$ cm$^2$/Vs (at 300 mK). The higher density sample B has a density of $3.2 \times 10^{11}$ cm$^{-2}$ and mobility of $20 \times 10^{6}$ cm$^2$/Vs (at 300 mK). The majority of the data for this work is collected from sample A.

The samples are tilted at an angle $\theta = 20^\circ$. The resulting small in-plane component of the magnetic field allows for well-defined FQHE states at $\nu = 5/2$ and $\nu = 7/3$ and also anisotropic phases in the second LL [13, 14, 16, 17].
Figure 5.1: a) Energy vs. $B_T$ observed in optical emission spectra from the N=1 LL in the filling factor range $2 \leq \nu \leq 3$ for sample A. The intensity is shown in grayscale. The band labeled X is linearly dispersed in $B_T$. L is the red-shifted optical emission that is considered in the main text. (b) Energy vs. $B_T$ plot for optical emission spectra from the N=0 LL in the range $2 \leq \nu \leq 3$ for sample A.
5.2. Results

The emission from the N=1 LL displays two major components, a singlet with linear magnetic field dependence and a red-shifted multiplet with a striking dependence on filling factor. An investigation of links between optical emission and spin waves in RLS spectra allows to link the peaks in the red-shifted optical emission to quantum phases in the N=1 LL.

Fig. 5.1 summarizes optical emission results in the range $2 \leq \nu \leq 3$. The emission doublet from the N=0 LL (Fig. 5.1(b)) is similar to those reported in previous studies [134, 135]. The focus here is on the much weaker optical recombination due to transitions that originate from partially populated states in the N=1 LL shown in Fig. 5.1(a).

The emission from the N=1 LL displays two major components, a singlet with linear magnetic field dependence and a red-shifted multiplet with a striking dependence on filling factor, labeled X and L respectively. This result is markedly different from the N=0 emission spectra in a range $\nu \leq 1$, where bands disperse linearly in $B$ and display oscillation in energy as a function of $\nu$ [134, 136, 133].

The optical emission is well represented by multiple Lorentzians and Gaussians, as shown in Fig. 5.2(b) for $\nu = 2.50$, with varying amplitude and nearly constant width (the width itself depending on the particular peak). The results of such line shape analysis in the range $2 \leq \nu \leq 3$ are summarized in Figure 5.2(b), which presents peak energies as a function of total magnetic field $B_T$. The area of each data point is proportional to the integrated intensity of the peak found from a Lorentzian fit such as shown in Fig. 5.2(a) and normalized by the electron population of the N=1 LL.
5.2. Results

Figure 5.2(a) presents a typical optical emission spectrum and resonant Rayleigh scattering (RRS) at $\nu = 2.50$. In the partially populated N=1 LL, RRS identifies the energy of the excitonic transitions between the partially populated conduction band and the valence band [33]. The energy of the singlet X band has a linear dependence on the perpendicular component of the magnetic field $B$ with a slope of $2.39 \pm 0.05$ meV/T, illustrated in Fig. 5.2(b). This value of the slope is close to that of free electrons in GaAs in the N=1 LL.

The red-shifted L emission is a multiplet structure (Fig. 5.2(a)) that exhibits a strong dependence on filling factor (Fig. 5.2(b)). The optical transitions for the L peaks are shown in Fig. 5.1(e) as red-shifted from single-particle conduction states. The RRS measurements in Fig. 5.2(a) show that the absorption edge is at the X peak, suggesting that this emission corresponds to recombination from extended states.

The interplay between the L peaks (Fig. 5.3(a)) correlates with a softening and collapse of the Zeeman mode (Fig. 5.3(b)). At $\nu = 3$, the L emission consists of a singlet peak labeled $L_0$ (Fig. 5.3(a)). Figures 5.2(b) and 5.3(a) illustrate the emergence of a new peak $L_1$ around $\nu = 2.96$, which becomes the dominant feature of the L emission for $\nu \lesssim 2.9$. Figure 5.3(b) shows a strong Zeeman mode at $\nu = 3$ that rapidly decreases in energy and collapses as the $L_1$ peak gains intensity. The correlation between the emission and spin wave spectra links the appearance of the $L_1$ band to the emergence of a new phase in the partially populated N=1 LL. The softening and collapse of the spin wave away from $\nu = 3$ indicates the presence of spin textures that
Figure 5.2: (a) RRS results overlapped with optical emission for $\nu = 2.50$ from sample A. (b) Energy of the bands in the optical emission from sample A from the $N=1$ LL as a function of total magnetic field $B_T$. The area of each data point is proportional to the integrated intensity found from a Lorentzian fit (except in the case of $L_0$ and $L_4$, which appear Gaussian) such as the green curve in (a) and normalized by the electron population of the $N=1$ LL. The black filled circles indicate low intensity emission with higher uncertainty on its energy, such as the black dashed curve in (a).
Figure 5.3: (a) Optical emission and (b) RILS spectra from sample A for filling factors close to 3. The color curves in (a) are fits with Lorentzian functions. The observed spin wave in (b) is indicated with a red arrow and compared to the Zeeman energy (blue arrow). Data shown in (b) was collected during a different cooldown of the dilution refrigerator [101], which results in a small difference in magnetic fields that achieve the same filling factors as (a).
break the full rotational invariance necessary to support long wavelength spin waves at the Zeeman energy.

Striking feature of the L multiplet is the interplay between the intensities of L₁, L₂ and L₃ peaks in the vicinity of ν = 7/3 (Fig. 5.4). As B_T increases and ν approaches 7/3, the L₁ component loses intensity and disappears from the spectra for ν ≤ 2.32. Simultaneously, the L₂ band, which becomes well-defined for ν < 5/2 (Fig. 5.2(b)), increases in intensity, as seen in Fig. 5.4 (a). A similar competition is seen in the results presented in Fig. 5.4 (c), where the intensity of L₃ increases sharply as the intensity of L₂ quickly collapses.

RILS spectra display a recovery of the long-wavelength spin waves near ν = 7/3, where the intensity of L₂ is the highest, and at ν = 2.26, where L₃ dominates the multiplet (Fig. 5.4 (b,d) and [101]). The discernible softening of the spin wave from the Zeeman energy near ν = 7/3 (Fig. 5.4(b)) and ν = 2.26 (Fig. 5.4(d)) is similar to the one observed at ν < 3 (Fig. 5.3(b)).

As Fig. 5.5 shows, the intensity of the L-peaks diminish rapidly with filling factor (relative to the X-peak) for ν < 2.2. This striking decrease in the L-peaks’ intensity. At ν = 2.12, when the L-peaks’ intensity is almost gone, a new spin wave mode is observed at the bare Zeeman energy (see Ref. [101]).

Lastly, we vary the temperature to gain additional insights into the nature of the observed quantum phases. The temperature dependence of the optical emission appears to be negligible for T < 300 mK for the entire range 2 < ν < 3 (see Fig. 5.6, where sample B is studied). For a large range of filling factors, optical emission does not exhibit a discernible temperature
Figure 5.4: (a) Optical emission and (b) RILS spectra from sample A for filling factors close to 7/3. (c) Optical emission and (d) RILS spectra from sample A for a filling factor range where peak L₃ is dominant. The color curves in (a,c) are fits with Lorentzian functions. The observed spin wave in (b,d) is indicated with a red arrow and compared to the Zeeman energy (blue arrow).
5.2. Results

Figure 5.5: Optical emission in the filling factor range $2.15 < \nu < 2$, showing the rapid decrease in optical emission intensity of the red-shifted emission multiplet as the filling factor decreases from $\nu \approx 2.2$
Figure 5.6: (a-c) Temperature dependence of PL from sample B close to $\nu = 7/3$. (d) Temperature dependence of the PL at $\nu = 2.32$ (from (b)) with fits for L peaks shown. (e) Temperature dependent optical emission spectra for $\nu = 3$ and (f) for $\nu = 2.53$. 
dependence below 650 mK, whereas there is a clear temperature dependence at certain filling factors, notably $\nu = 2.32$ (Fig. 5.6(b)), for $300 \text{ mK} < T < 650 \text{ mK}$. The fits suggest a competition between $L_2$ and $L_3$, with the $L_3$ gaining intensity and $L_2$ shrinking with increasing temperature.

5.3 Discussion

Previous applications of optical emission for $N \geq 1$ focused on probing collective interactions of 2DES’s [134], on the spin polarization of the $\nu = 5/2$ FQHE state [135], and on links of the optical emission to resonant Rayleigh scattering (RRS) [33, 34]. In contrast, our work exploits the sensitivity of optical emission to the formation of collective phases of electrons in probing the interplay of quantum phases. This work represented the first use of optical emission spectroscopy to monitor emergent phases in the $N=1$ LL [35].

In the previous section, we saw that the optical emission from the $N=1$ LL consists of two main features: a sharp peak labeled X and a multiplet of bands labeled L. The X emission is readily interpreted as an extended exciton state built from free electrons and holes. The L emission is found to have a nontrivial filling factor dependence that we link to the emergence of competing phases of electrons in the $N=1$ LL. In addition to insights gained by the temperature and filling factor dependence of the L peaks, the energy and intensity of the long wavelength SW in the RILS spectra at corresponding filling factors is used to monitor spin rotational invariance in the distinct phases, determined by departures from the prediction of the
Larmor theorem as discussed in Section 3.7.2.

The dependence of the X emission energy and intensity on magnetic field is shown in Figs. 5.1 (a) and 5.2 (b). As we have reported in the previous section, the slope of the energy with perpendicular magnetic field is $2.39 \pm 0.05$ meV/T. When interpreted in terms of the effective masses of the conduction band electrons and of the valence band holes, the value of the observed slope indicates that the X peak arises from optical transitions at energies that are close to single particle transition energies of conduction and valence LL’s. The slope is only slightly modified by excitonic interactions and weak coupling to the electron system [35]. The interpretation that the the X emission arises from excitonic transitions is further supported by its strong overlap with the RRS resonance enhancement profile (shown in Fig. 5.2 (a) for $\nu = 2.5$). The X emission’s negligible temperature dependence below 600 mK, shown in Fig. 5.6, is interpreted as evidence that the X emission is not directly linked to the emergent phases in the N=1 LL and is also consistent with this interpretation.

We continue with a brief summary of the results to be considered below. The filling factor dependence of the L peaks, summarized in Fig. 5.2 (b), is suggestive of links between the emission bands and competing N=1 LL phases. The L$_0$ peak, shown in Fig. 5.3, occurs in the IQHE state at $\nu = 3$, much like the red-shifted optical emission peak observed near $\nu = 1$, which has been attributed to shakeup by spin waves [137]. The rapid reduction of the L$_0$ intensity with filling factor departing from $\nu = 3$ (Fig. 5.3) is characteristic of the integer quantum Hall state (IQHE) state at $\nu = 3$. As
Fig. 5.3 shows, the $L_0$ emission is most prominent when the energy and intensity of the SW mode are greatest: with decreasing filling factor the collapse of $L_0$ occurs simultaneously to the softening from $E_Z$ and collapse of the long-wavelength SW.

The correlation of the $L_1$ emission with softening and collapse of the long-wavelength SW from $E_Z$ (see Fig. 5.3) links the $L_1$ to the emergence of a new phase with broken full spin rotational invariance, since such modes break the Larmor theorem (see Section 3.7.2). The fact that the onset of the $L_1$ emission corresponds to the onset of transport anisotropy at similar tilt angles [13, 14] indicates that this phase may be tied to transport anisotropy. This interpretation suggests that the $L_1$ emission could be linked to an anisotropic stripe phase.

Figure 5.7: Depiction of Skyrmion spin texture, where the electron spins are represented by arrows. After Ref. [138].

Remarkably, as the $L_1$ emission becomes most prominent, the long-wavelength SW modes' energy collapses into a broad continuum, similar to that observed in the range $2/3 < \nu < 1$ [99], which has been attributed to Skyrmions (depicted in Fig. 5.7). A theoretical formulation suggested that the formation
of Skyrmion-like spin textures in the N=1 LL is promoted by disorder [139]. Within this framework, the temperature dependence of the optical emission in the filling factor range for which L₁ is prominent (i.e. Fig. 5.6 (f) for \( \nu = 2.53 \)) can be interpreted as the emergence of a competing phase without Skyrmions, which vanish as a result of their becoming depinned with increased temperature. This interpretation is supported by the fact that previous experiments in this filling factor range have observed the recovery of the spin wave at the Zeeman energy with increased temperature [33].

The L₂ emission band correlates with the recovery of a sharp SW mode with only a small red-shift from E₂ (see Fig. 5.3 (a-b)). This red-shift arises from some broken spin rotational invariance, the precise nature of which is not yet known. The interplay of the L₁ and L₂ emission in the filling factor range 2.41 < \( \nu \) < 2.33 is therefore understood as competition between phases with distinct spin-rotational order.

The L₂ emission is prominent for 2.30 < \( \nu \) < 2.41, a filling factor range that correlates with our observing excitations of the FQIIE states at \( \nu = 2 + 1/3, 2 + 3/8, \) and \( 2 + 2/5 \) with RILS (see Ref. [34] and Chapter 4). This indicates that L₂ may be associated with a phase that supports the formation of these FQHE fluids. The interpretation that the phase associated with the L₂ emission is associated also these three FQHE states while also breaking rotational invariance is consistent with observations that the activation gap of the \( \nu = 7/3 \) FQHE state increases with in-plane magnetic field [140].

The L₃ emission correlates with the emergence of a sharp long-wavelength SW (see Figs. 5.6(c-d)) at \( \nu \approx 2 + 1/4 \): the only gapped, long-wavelength
SW we observe that is not associated with a quantum Hall state! The large intensity of this SW is interpreted as evidence that $L_3$ is likely linked to a phase with a high degree of spin-polarization. The small red-shift of the long-wavelength SW from $E_2$ suggests some breaking of spin rotational invariance which is do not yet understood. It is worth noting that the filling factor range for which $L_3$ is observed has been linked to the observation of a pinned electron solid phase, believed to be a "bubble" phase [141, 127, 142].

The vanishing intensity of the $L_3$ emission at filling factors below $\nu \approx 2.15$ (see Fig. 5.5) is associated with the emergence of a distinct phase at filling factor close to $\nu = 2$. The suppression of optical emission at low density of $N=1$ LL electrons is characteristic of an incompressible electron solid [143, 144, 145]. Furthermore, microwave experiments in the $N=1$ LL observed evidence that the phase phase associated with this filling factor range is an electron solid, interpreted as the Wigner crystal [127, 142].

5.4 Summary

I have presented optical spectroscopy experiments on phases of emergent electrons in the filling factor range $2 < \nu < 3$. Exploration of optical emission from the partially populated $N=1$ LL offers new insights into these exotic quantum phases. These results support a conceptual framework in which the bands of the L-multiplet are associated with distinct phases in the partially populated $N=1$ LL. The rapid changes that occur in the L-multiplet for filling factors near the FQHE state at $\nu = 7/3$ suggest a striking competition
between quantum ground states that are tuned by remarkably small changes in filling factor. The anomalous long-wavelength spin waves below $E_2$ linked to the presence of the $L_1$, $L_2$, and $L_3$ emission bands indicate that the corresponding phases possess broken full spin rotational invariance. The results presented in this chapter demonstrate that optical methods are a powerful tool for the identification and study of exotic quantum phases of electrons in the partially populated $N=1$ LL.
Chapter 6

Competition of Electron Phases in the $N=1$ LL at Higher Temperatures

“The most exciting phrase to hear in science, the one that heralds new discoveries, is not ‘Eureka!’ but ‘That’s funny’...”

-Isaac Asimov

6.1 Overview

In this chapter, I report the continuation of my studies using optical emission from the second ($N=1$) Landau level (LL) in the filling factor range $2 \leq \nu \leq 3$ presented in Chapter 5. These experiments focus on the temperature-dependence of these phases to gain further insights into electron interactions. The effects of anisotropy [21] and increased pinning potential [146] on
the emergent $N=1$ LL phases is studied by collecting spectra at tilt-angles $\theta = 40^\circ$ and $\theta = 5^\circ$. These experiments provide promising indications as to the identity and order of the quantum phases in the $N=1$ LL. As discussed in previous chapters, the reduced dimensionality and strong electron interactions of ultra-clean two dimensional electron systems (2DES) under high perpendicular magnetic field $B$ result in a wide variety of novel and fascinating many-body electron phases. The unique Coulomb interactions in the second ($N=1$) Landau level (LL) results in the presence of the reentrant integer quantum Hall effect (RIQHE) [141], anisotropic transport [58, 65, 13] in addition to even- and odd-denominator FQHE states. This wide variety of effects results from the rich variety of ground states in the $N=1$ LL which include both FQHE fluids and phases of electrons referred to as stripe and bubble phases. In Chapter 5, I discussed our work using optical emission spectroscopy to probe the interplay of $N=1$ LL phases. Optical emission bands were linked to emergent $N=1$ LL phases by exploring the filling factor and temperature-dependence of the optical emission. The filling factor and temperature-dependence of the phases observed through optical emission spectroscopy indicated that these phases are not FQHE or RIQHE states, but phases that coexist with them like the "stripe" or "bubble" phases [35]. Comparisons of optical emission spectra with those of RILS by the long-wavelength spin wave revealed broken full spin rotational invariance in three of the phases (see Sections 3.7.2 and 5.3). In Chapter 5, the competition and coexistence phases was studied through weak optical emission from the $N=1$ LL. Optical emission displays strong filling factor-dependence which
is attributed to competition and coexistence of these phases when considered alongside RILS and RRS spectra. While signatures of the FQHE fluids are observed in RILS, their presence appears to have no direct effect on these optical emission spectra, as shown by the spectra’s filling factor- and temperature-dependence [35].

6.2 Results

The ultraclean 2DES resides in the two samples discussed in Chapter 5 (samples A and B), each with a symmetrically doped single GaAs/AlGaAs quantum wells of width 300 Å [39, 40]. The density and mobility of the 2DES are $2.9 \times 10^{11}$ cm$^{-2}$ and $23.9 \times 10^6$ cm$^2$/V s respectively. The sample is mounted on the cold finger of a $^3$He-$^4$He dilution refrigerator in the bore of a 16T superconducting magnet with bottom windows for direct optical access as shown in Fig. 3.1 in Section 3.2.

The backscattering geometry of this experiment is shown in Fig. 3.2 in Section 3.2. In the experiments discussed in this chapter, two pieces of the sample were placed on a cold finger with with angles of $\theta = 5^\circ$ and $\theta = 40^\circ$ with respect to the total magnetic field. Results from the experiments discussed in the Chapters 4 and 5, which were performed during different cooldowns for which $\theta = 20^\circ$, are also explored.

As in the previous two chapters, the filling factor as a function of magnetic field is determined by the maximum of the spin-wave intensity of $\nu = 3$. Fig. 6.1 shows the results for the 2DES tilted at 40°. Fig. 6.2 shows the opti-
Figure 6.1: Comparison of the resonantly enhanced spin wave modes to determine the total magnetic field that corresponds to \( \nu = 3 \) for \( \theta = 40^\circ \). \( \nu = 3 \) best corresponds to \( B_T = 5.1 \) T.
6.2. Results

evaluated emission spectra for 2 < \nu \leq 3 in the temperature range 1 \text{ K} < T < 1.35 \text{ K}. The X-peak and L-emission in these spectra display a pronounced filling-factor dependence similar to that of optical emission spectra taken at \( T \lesssim 40 \text{ mK} \) discussed in Chapter 5. Individual optical emission spectra near \( \nu = 3 \)

![Image of emission spectra](image)

Figure 6.2: Optical emission spectra in the filling factor range 2 \leq \nu \leq 3 with intensity shown in grayscale for tilt angle (a) \( \theta = 5^\circ \) and (b) \( \theta = 40^\circ \).

are shown in Fig. 6.3. In this filling factor range, two competing L-emission peaks, labeled \( L'_0 \) and \( L'_1 \), are observed. As the filling factor decreases from \( \nu = 3 \), the intensity of the \( L'_1 \) emission increases while that of the \( L'_0 \) decreases; the intensity and energy of the long-wavelength spin wave peak also
diminish. As Fig. 6.4 shows, optical emission for $\nu = 3$ displays strong temperature-dependence. With increased temperature, the intensity of L$_1$ decreases significantly. Figs. 6.3 and 6.4 demonstrate the dependence on in-plane magnetic field near $\nu = 3$: in this filling factor range, the L'$_1$ emission intensity increases with in-plane magnetic fields. The L'$_1$ peak is also observed at higher temperatures for larger in-plane magnetic field. The striking temperature-dependence of the X-peak for $\nu \sim 3$ is shown in Fig. 6.4. As temperature increases, the X-peak energy and intensity decrease while the intensity of optical emission at higher energy than the X-peak increases. This temperature-dependence is most pronounced for temperatures at which the L$_1$ emission exhibits temperature-dependence. As Figs. 6.3 and 6.5 show, as the filling factor is decreased from $\nu = 3$, the L$_1$ emission becomes more prominent as the L$_0$ emission diminishes in intensity. The red-shifted optical emission is dominated by the L$_1$ emission until $\nu \approx 2.5$ at both low and high tilt angles, where the L'$_2$ emission becomes dominant, as shown in Fig. 6.5. As this figure shows, the L'$_2$ emission does not exhibit strong temperature-dependence for $T \gtrsim 1$ K. As Fig. 6.6 shows, the L'$_2$ emission intensity decreases with filling factor for $\nu < 2.45$ while the L'$_3$, L'$_4$, and L''$_4$ optical emission intensity increase. The phases responsible for the L'$_3$, L'$_4$, and L''$_4$ appear to display similar properties to those responsible for the L$_3$-L$_4$ emission at lower temperatures. Both sets of peaks appear in similar filling factor ranges and display very sharp quantum phase transitions with increasing magnetic field, as shown in Fig. 6.8. It is worth noting that the linewidths of L'$_3$ and L$_3$ are very close. The temperature-dependence of this optical emission is
Figure 6.3: Optical emission spectra for the filling factor near $\nu = 3$ for $\theta = 5^\circ$ (a) and $\theta = 40^\circ$ (b). It is worth noting that with increased angle, the intensity of the $L'_1$ emission relative to the X peak increases.
Figure 6.4: Temperature-dependence for optical emission near $\nu = 3$ for $\theta = 5^\circ$ (a) and $\theta = 40^\circ$ (b). As temperature is raised, the $L'_1$ emission decreases in intensity and the $L'_0$ emission increases in intensity. The X-peak energy decreases and spectral width increases with temperature.
Figure 6.5: Optical emission spectra for the filling factor range $2.45 < \nu < 2.7$, where the $L'_2$ emission is prominent. The linewidth of the $L'_2$ emission increases with in-plane magnetic field. The temperature-dependence of the $L'_2$ emission is of a significantly smaller magnitude than that of the $L'_1$ emission.
Figure 6.6: Optical emission spectra for the filling factor range $2.2 < \nu < 2.45$, where the $L_3', L_4'$ and $L_4''$ emission is prominent. These peaks display both marked filling factor-, temperature- and in-plane magnetic field-dependence.
Figure 6.7: Temperature-dependence of optical emission for spectra in the range $2.3 < \nu < 2.4$ at tilt angles (a) $\theta = 5^\circ$ and (b) $\theta = 40^\circ$. 
shown in Fig. 6.7. With increasing temperature, the L emission spectrum broadens and the L-peaks’ integrated intensity increase. At the same time, the L₃ and L₄ emission intensity diminishes relative to that of the other, lower-energy peaks. Fig. 6.9 shows the results of temperature-dependent optical emission spectra in chronological order. When the temperature of the system is raised above 1.3 K, the L′₃ emission diminishes significantly in intensity and a broad lower-energy peak emerges. When the temperature is subsequently lowered, the prominent and well-defined L′₃ peak does not return. Lowering the temperature further results in the near disappearance of the L emission altogether and a blueshift in the energy of the X-peak. The reduced intensity of the L emission and blueshift of the X-peak appear to
6.2. Results

\[ B_T = 6.4\text{T} \quad \nu = 2.39 \]

\[ B_T = 6.55\text{T} \quad \nu = 2.34 \]

Figure 6.9: Optical emission spectra shown in the order in which they were taken. The spectra were taken 15 minutes apart.

(a)

(b)

Figure 6.10: Optical emission spectra taken in the filling factor range \(2.25 < \nu < 2.45\) for \(T \approx 1.00\) K for tilt angle \(\theta = 40^\circ\). **Left**: Spectra taken after the temperature was lowered from \(T > 1.30\)K. **Right**: Optical emission spectra taken for a sweep in which the temperature was not raised above \(T \approx 1.00\) K.
remain indefinitely until the magnetic field is changed. Fig. 6.10 shows a comparison between optical emission spectra taken at \( T \approx 1 \) K. The spectra shown in Fig. 6.10 (a) were all taken after the temperature was first raised to \( T \approx 1.3 \) K, while the optical emission spectra in Fig. 6.10 (b) were taken without raising the temperature above \( T \approx 1 \) K. As with the optical emission measurements shown in Fig. 6.9, the optical emission spectra in this filling factor range show evidence of a hysteretic temperature-dependence. Such temperature-dependence is not observed anywhere else in the filling factor range \( 2 < \nu < 3 \).

6.3 Discussion

We reported optical emission of N=1 LL phases of electrons at temperatures for \( T \gtrsim 1 \) K. We observed emission bands whose energy and intensity display striking filling factor dependence similar to those reported in Chapter 5, which were interpreted as arising from distinct quantum phases of electrons in the N=1 LL [35]. We provide promising evidence that the emission observed in this chapter arises from the same phases observed at lower temperatures (with the possible exception of L\(_2\) and L\(_2'\)). The observation of these phases at temperatures for \( T \gtrsim 1 \) K is striking as no previous experiments has observed signatures of these phases at such high temperatures. In this section, we refer to the phases associated with the different L bands by the same name.

The effects of anisotropy and increased pinning potential on the N=1 LL phases are explored by comparing the filling-factor dependence of the
optical emission bands for small ($\theta = 5^\circ$) and large ($\theta = 40^\circ$) tilt angles. For $2.5 < \nu < 3$, we find results consistent with the $L_0$ emission arising from a $\nu = 3$ IQHE phase, and the $L_1$ emission corresponding to a phase with broken rotational order. For $2.2 < \nu < 2.4$, we see promising evidence that the $L_3$ and $L_4$ emission arise from a phase that supports an electron solid which gives rise to a RIQHE effect at lower temperatures.

6.3.1 The competing $L_0$ and $L_1$ Phases

The $L'_0$ emission appears to be linked to the $L_0$ emission discussed in Chapter 5. Both emission peaks have similar linewidths and filling factor dependence: their intensities are greatest for the filling factor range associated with the IQHE state at $\nu = 3$ and both peaks disappear as the filling factor decreases from $\nu = 3$. For this reason, we interpret the phase responsible for the $L'_0$ emission as being the same as the phase responsible for $L_0$ emission, which was attributed to a disordered phase associated with the IQHE at $\nu = 3$. Fig. 6.4 shows that the $L'_0$ emission intensity increases with temperature. This temperature-dependence is consistent with optical emission from an insulating phase of electrons pinned by weak residual disorder since depinning of electrons with increased temperature results in a larger overlap between pinned electrons and the photoexcited holes [143, 144, 145]. Within this framework, the fact that the $L'_0$ emission intensity decreases with increased in-plane magnetic field (see Figs. 6.3 and 6.4) is understood as optical emission intensity decreasing due to increased pinning potential from weak residual disorder [146]. The filling factor dependence of the $L'_1$ emission (Fig. 6.3)
suggests that it is linked to the $L_1$ emission at $T<650$ mK (Fig. 5.3). As discussed in Chapter 5, the phase associated with $L_1$ is linked to anisotropy in transport and the broken full spin-rotational invariance by spin textures. Fig. 6.3 shows that the $L'_1$ emission intensity increases with in-plane magnetic field, which is consistent with the $L'_1$ emission corresponding to an anisotropic phase, since in-plane magnetic field breaks the rotational symmetry of the 2DES, leading to transport anisotropy [13, 66, 146]. The dependence of the $L'_1$ emission intensity on in-plane magnetic field is also consistent with the corresponding phase possessing spin textures since increasing in-plane magnetic field increases the pinning potential by weak residual disorder, and this disorder is believed to promote the formation of spin textures in the $N=1$ LL [139].

6.3.2 The $L_3$-$L_4$ Phase

As Fig. 6.6 shows, the $L'_3$, $L'_4$ and $L''_4$ emission peaks emerge for $\nu \lesssim 2.4$ in this filling factor range. The $L'_3$, $L'_4$, and $L''_4$ shows striking similarities to the $L_3$-$L_4$ emission at lower temperatures. The $L'_3$ and $L_3$ peaks have similar linewidths and are accompanied by broad peaks at higher energies. While the $L'_3$ emission appears a higher filling factor range, this is consistent with the temperature displayed by the $L_3$ emission below 650 mK. Furthermore, both sets of peaks appear in similar filling factor ranges and display very sharp quantum phase transitions with increasing magnetic field, as shown in Figs. 6.8 and 5.5. Another striking similarity between the $L_3$ and $L'_3$ emission is the appearance of a spin wave excitation which correlates with both phases.
6.3.2. The $L_3$-$L_4$ Phase

As Fig. 6.8 shows, the prominence of the $L'_3$ emission (in the absence of the $L'_2$ emission) is linked to the existence of a gapped long-wavelength spin wave mode with the same effective $g$-factor as the spin wave linked to the $L_3$ emission [35]. This spin wave is observed only for filling factors in which the $L_3$-$L_4$ or $L'_3$-$L'_4$ emission is the only red-shifted optical emission in the $N=1$ LL. The existence of this spin wave mode is remarkable both at low and high temperatures: it is the only spin wave mode observed in the $N=1$ LL that is not associated with an integer or fractional quantum Hall state [101]. This temperature-dependence for optical emission is similar to that of an electron solid [143, 147, 148]. In an electron solid, the spatial overlap between localized electrons and photoexcited holes is small at lower temperature but increases as temperature is raised and collective modes of the electron lattice are excited, leading to increased rates for electron-hole recombination and a stronger final-state interaction for the electron-hole pairs [143, 148]. This leads to a spectral broadening and red-shift for optical emission from the electron solid at higher temperatures, as observed in Fig. 6.7. Figs. 6.9 and 6.10 show evidence that the $L'_3$ emission is linked to a phase whose melting results in hysteresis, indicating that the melting of this phase is a first-order phase transition. The presence of hysteresis in the $N=1$ LL has previously been interpreted as evidence of electron solids [53, 149] including multi-electron bubbles [53, 141]. As discussed in Ref. [150], sharp changes in the chemical potential in a quantum well heterostructure like ours can result in hysteresis. The formation (or melting) of electron solids results in sudden changes in chemical potential [64, 141, 151], resulting in a reduction in the
density of the 2DES in the quantum well and the formation of a metastable state [150, 152]. The reduction in the N=1 LL optical emission resulting from hysteresis shown in Figs. 6.9 and 6.10 is consistent with the interpretation of an electron solid melting. The spectral broadening is consistent with the formation of a more disordered state predicted to form in the presence of quenched disorder [152]: in our case, sudden changes in temperature [144].

6.4 Summary

In conclusion, we have reported optical emission and RILS experiments that probe phases in the filling factor range $2 \leq \nu \leq 3$ at temperatures in the range $T \gtrsim 1$ K. Spectra are collected for tilt angles $\theta = 40^\circ$ and $\theta = 5^\circ$ in order to study the effects of anisotropy and increased pinning potential on the N=1 LL phases. The results show strong evidence that many of the phases observed in both temperature ranges are, in fact, the same.

We find that the dependence of the $L_1$ emission band on temperature and in-plane magnetic field that is consistent with our interpretation presented in Chapter 5 that the $L_1$ phase arises from a phase with broken rotational invariance. We also observe the $L_3$ emission band at higher temperatures. The optical emission from the $L_3$ phase is found to display hysteretic temperature-dependence. This is striking as the observation of hysteresis has been associated with the formation of electron solids [141, 149]. This is interpreted as evidence that this phase may support RIQHE phases observed at lower temperatures.
Chapter 7

Concluding Remarks

In this work, I discuss experiments that optically probe competition and coexistence of emergent phases of the second Landau level. This work represents advances in monitoring emergent electron phases, particularly those in the second Landau level, and offers new avenues for studying phase competition in the second and higher Landau levels.

Chapters 2 and 3 provide an overview of both the physics of electrons in high magnetic fields studied and the optical emission and light scattering methods we use to study 2D electron systems. In Chapter 4, I discuss resonant inelastic light experiments in which collective spin and charge excitations at fractional quantum Hall states at $\nu = 2 + 1/3$, $\nu = 2 + 3/8$, and $\nu = 2 + 2/5$. These states’ excitation spectra provide insights into these fractional quantum Hall states and can help in determining the accuracy of different theoretical models for fractional quantum Hall states in the second Landau level. In addition, striking polarization dependence is observed at
these filling factors and is attributed to the existence of anisotropic FQHE phases that arise as a result of in-plane magnetic fields [19, 17, 112].

Chapter 5 discusses our exploration of the emergent phases in the filling factor range $2 \leq \nu \leq 3$ weak optical emission from the second Landau level. We observe a multiplet sharp emission peaks (L-peaks) whose filling factor and temperature dependence indicate a link with collective phases like the bubble or stripe phases. Broken full spin rotational invariance of these phases is probed using resonant inelastic light scattering by the spin wave [33, 99, 100, 35]. These experiments show optical emission from second Landau levels to be useful in monitoring the interplay of quantum phases in the N=1 LL.

Chapter 6 uses optical emission to explore the role of in-plane magnetic fields at higher temperatures $T \gtrsim 1 \text{ K}$ in the interplay of N=1 LL phases. The results of these experiments suggest many of the phases observed at $T \approx 40 \text{ mK}$ are observed at these higher temperatures. Optical emission experiments display a more striking temperature dependence at higher temperatures, providing further insights into the nature of the phases in the N=1 LL. Deepened understanding of the optical emission peaks’ connection to second Landau level can help develop weak optical emission into a tool capable of shedding a bright light on the mysteries of the second Landau level.
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Appendices
Appendix A

Electron Phases and the Reentrant Integer Quantum Hall Effects

In this Appendix, we discuss the formation of ordered phases and their role in the reentrant integer quantum Hall effect (RIQHE).

A.1 Bubbles and Stripes

The Coulomb potential in the N\textsuperscript{th} LL is given by\cite{50}:

\[
V_{\text{eff}}(\vec{k}) = \pi \int d\vec{k} \rho(\vec{k}) \rho(-\vec{k}) \frac{e^2}{\epsilon k}
\]

\[
V^\epsilon_{\text{eff}}(\vec{q}) = \sum_m \langle N, m|V_{\text{eff}}(\vec{k})|N, m \rangle
\]

\[
= \frac{e^2}{\epsilon q} \left[ L_N \left( \frac{q^2 B q^2}{2} \right) \right]^2 e^{-\frac{q^2 B q^2}{2}}
\]
where $L_N(x)$ is the $N^{th}$ Laguerre polynomial. Rewriting the effective potential in terms of angular momentum differences between electrons (the Haldane Pseudopotential), we find that at higher LL’s, the interaction becomes more uniform as a function of angular momentum[50].

The ground states are solved for in terms of distributions of the wave functions’ guiding centers. In higher LL’s, the ground state can be accurately determined by Hartree-Fock[59]. At small densities of electrons (or holes), solutions for the guiding center functions of the Hartree-Fock Hamiltonian gives a triangular lattice of $N_c$-electron clusters spaced a distance $\sqrt{2N} + 1l_B$ apart, known as the Wigner cyrstal (WC) [50]. This phase is shown schematically in Fig. A.1.

As the partial filling factor (the filling factor of the highest occupied LL) approaches $\frac{1}{2}$, and therefore density of electrons in the highest LL, increases, more electrons are found per bubble[55, 57, 61], as shown in Figs. A.1 and
Figure A.2: **Left:** Predicted energies of the dominant phases in the N=2 LL, determined by time-dependent Hartree-Fock calculations. As filling factor increases, the number of electrons per site is expected to increase until the phase becomes an anisotropic stripe phase. After Ref.[153]. **Right:** Predicted density of states (center) and spatial density (right) of bubble and stripe phases. After Ref. [57]

With further increases in the partial filling factor and number of electrons per bubble, the bubble phases become unstable against anisotropy, and a stripe phase is expected to form [62, 63, 64]. Pronounced transport anisotropy in at half-integer filling factors in higher LL's has been linked to this phase[65, 58, 66]. Here, stripes are thought to form along the \( \langle 110 \rangle \) due to weak anisotropy in the GaAs. As seen in Fig. A.6, this anisotropy does not have a strong effect in transport at the lower LL's: its presence is most likely due to breaking of rotational invariance.
Figure A.3: Schematic representation of the role of phase formation in the RIQHE. The plots on the left represents the system without formation of the ordered phase. On the top, the partially-filled $N^{\text{th}}$ band forms an ordered electron phase, which leaves conducting edge states from N-1 bands. On the bottom, the mostly filled LL forms a hole state, allowing for conduction from all N bands.

A.2 Bubble Phases and Transport

While bubble phases break translational invariance, they are gapped at low temperatures due to the presence of the pinning potential [154]. As we saw in Chapter 2, insulating bulks result in conduction through edge states, as depicted in Fig. A.3. Bubble phases lower the energy of the 2DES in such a way as to leave edge states only from completely filled LL’s contribute to
Figure A.4: Transport trace in the second LL that shows the RIQHE taken by Deng et al. After Ref.[15]. Note that there are multiple electron and hole RIQHE’s associated with filling factors $\nu = 2$ and $\nu = 3$.

the edge current. Thus, current is carried through edge states, and the bulk of the 2DES does not conduct, and the values of the hall conductivity taken are necessarily those of the nearest integer filling factor (see Fig. A.4). This is the RIQHE! The different RIQHE’s in the second and third LL’s shown in Fig. A.4 is interpreted as the formation of bubble phases with different numbers of electrons per lattice site [15].

A.3 Stripe Phases

The many possible stripe phases are shown schematically in Fig. A.5. The possible phases are a stripe crystal (an anisotropic WC), a smectic phase, a nematic phase, and an isotropic phase [155]. The stripe crystal displays long-
Figure A.5: Cartoon representations of the different conjectured stripe phases. The anisotropic WC is an anisotropic electron solid. It breaks translational invariance in two dimensions; adjacent stripes are "phase locked". The smectic stripe phase breaks translational invariance in only one direction (no phase locking between stripes). The nematic phase does not preserve the long-range positional order of the smectic phase, but retains the rotational order. Dislocations in the stripes caused by the pinning potential account for the differences between the smectic and nematic phases. The isotropic stripe phase results from anisotropy forming in individual bubble phases as the number of electron per bubble increases, but retains all long-range orientation and translation invariance: this phase would result in no transport anisotropy. After Ref. [155].
range translational order in both directions. The smectic phase displays long-range order in only one direction: the stripes are not "phase bound" as in the case of the stripe crystal due to pinning. The nematic phase does not display long-range translational order, due to the prevalence of dislocations caused by random fluctuations in potential. As such, it only displays rotational order. The main difference between the smectic and nematic phases is the length scale at they are considered. Finally, there is the isotropic phase, which displays no rotational or translational order and therefore has no effect on transport anisotropy; it displays only the anisotropy expected within bubbles themselves as the number of electron per bubble increases. The presence of transport anisotropy at half-integer filling factors makes this last phase highly unlikely.

It is also worth noting that while the signature of the bubble phases is a quantum Hall effect, stripes do not display any such effect [66]: stripes are conducting phases (in one direction).

As we saw earlier, the presence of an in-plane magnetic field also introduces some anisotropy in the Hamiltonian and in transport [13, 17, 18, 19, 20]. The introduction of anisotropy into a fractional quantum Hall state allows us to study the effect of anisotropy on fractional quantum Hall states, some of which (most notably the one at $\nu = \frac{7}{3}$) form new, anisotropic quantum Hall states [72].

Lastly, the two anisotropic effects discussed above are qualitatively different from one another. The crystal anisotropy results in anisotropy in the pinning potential, but the in-plane magnetic field’s produces an anisotropic
Figure A.6: **Top:** Magento-resistance measurements traces in the N=1 LL. **Left:** Transport in the absence of in-plane magnetic fields. **Right:** Transport in the presence of large in-plane magnetic field along the (110). **Bottom:** Transport traces in the higher LL’s (without in-plane magnetic field). The results at $\nu = \frac{7}{2}$ and $\nu = \frac{9}{2}$ are notable in their lack of anisotropy, due to the presence of FQHE states at those filling factors. After [58].
Hamiltonian: a nonlocal effect. This is likely why anisotropic fractional quantum Hall effects are only observed in the presence of in-plane magnetic fields: as we will see, quantum Hall states are robust against disorder.

The interaction between these anisotropic effects are visible in Fig. A.7. We see this interplay from Fig. A.7(c), where increasing the magnetic field along \((110)\) (parallel to the stripes), stripes conduct at \(\nu = \frac{9}{2}\) when the angle of the magnetic field with the 2DES at angles above \(19^\circ\). It is also worth noting that anisotropic fractional quantum Hall states' anisotropy appears to be more pronounced when the in-plane magnetic field induces anisotropy in a different direction than the crystal.
Figure A.7: Transport measurements for magnetic fields at different angles with the normal of the 2DES. In a) and b), the in-plane component of the magnetic field lies in the (1\bar{1}0)-direction (the x-direction) and in c) and d), it lies in the (1\bar{1}0)-direction (the y-direction). Magnetoresistance measurements are marked by a solid line when taken in the (1\bar{1}0) and by a dashed line when taken in the (1\bar{1}0)-direction. The anisotropic effects become pronounced at smaller angles when the in-plane magnetic field points along the (1\bar{1}0)-direction. The effects of the in-plane magnetic field are also qualitatively different for the FQHE states depending on its orientation. For example, the state at $\frac{1}{2}$ only shows anisotropy for the in-plane magnetic field pointing along the (1\bar{1}0)-direction. After [13].
Appendix B

Lineshape of Optical Emission Spectra

This appendix discusses the effects of finite transition rates and disorder on optical emission spectra.

B.1 Finite Transition Rates

We now consider this recombination’s lifetime on the lineshape. The recombinations we are currently considering are one-step processes and, as such, can be modeled as a two state system. This makes it easy to use the time-dependent Schrödinger equation to model the emission at an energy $\hbar \omega_E$ [77]. We now have to take the time-dependent term of the interaction Hamiltonian into account, giving us the interaction Hamiltonian:

$$\hat{H}' = \frac{e \hbar}{m^* \tilde{c}^2 \omega_E} E_0 \left( e^{i(\mathbf{q}_j \cdot \mathbf{r}_j - \omega_E t)} \hat{c}_\gamma \cdot \nabla \right)$$  \hspace{1cm} (B.1)
Modeling the wavefunction of our electron between the two states as \( \psi(t) = c_i(t)\psi_i + c_f(t)\psi_f \), we see that \( c_i(t) \) and \( c_f(t) \) are given by:

\[
i\hbar \begin{bmatrix} \dot{c}_i \\ \dot{c}_f \end{bmatrix} = \hbar \begin{bmatrix} \omega_i & \omega_i f e^{-i\omega_E t} \\ \omega_i f e^{i\omega_E t} & \omega_f \end{bmatrix} \begin{bmatrix} c_i \\ c_f \end{bmatrix} \quad (B.2)
\]

which, when we transform according to \( c'_a(t) = e^{-i\omega_a t} c_a(t) \), becomes:

\[
i \begin{bmatrix} \dot{c}'_i \\ \dot{c}'_f \end{bmatrix} = \begin{bmatrix} 0 & \omega_i f e^{i(\Delta\omega - \omega_E)t} \\ \omega_i f e^{-i(\Delta\omega - \omega_E)t} & 0 \end{bmatrix} \begin{bmatrix} c'_i \\ c'_f \end{bmatrix} \quad (B.3)
\]

where \( h(\Delta\omega) = E_i - E_f \). We remember that \( c_f(0) = 0 \) and \( c_i(t) \) decays with time as \( c_i(t) = c_i(0)e^{-\frac{t}{\tau_i \to f}} [77] \) to obtain the following expression for the density at the final state:

\[
c_f'(t) = -i \int_0^t dt' e^{-i(\Delta\omega - \omega_E)t'} e^{-\frac{t'}{\tau_i \to f}} \propto \frac{1}{\frac{1}{\tau_i \to f} + i(\Delta\omega - \omega_E)} \quad (B.4)
\]

This yields a Lorentzian lineshape for optical emission.

### B.1.1 Inhomogeneous (Disorder) Broadening

As we saw in Section 2.3, the presence of finite magnetic fields results in the formation of LL’s in 2DES’s. The only broadening of energy bands comes from either the formation of electron phases or from small random fluctuations in potential.

Random fluctuations in potential result in Gaussian broadening of LL
density of states. Optical emission from a LL (centered at energy $\hbar \omega_0$) is therefore given by the convolution of a Lorentzian and Gaussian function (a Voight function):

$$V(\omega - \omega_0, \Gamma, \eta) \propto \int_{-\infty}^{\infty} e^{-\frac{(\omega - \omega_0)^2}{2 \eta^2}} \frac{\Gamma^2}{\pi (\Gamma^2 + (\omega - \omega_0 - \omega')^2)} d\omega' \quad (B.5)$$

where $\eta$ is the FWHM of the Gaussian density of states. Note that optical emissions spectra from very rapid CB-VB transitions will have a predominantly Lorentzian line shape, and optical emission lineshapes due to CB-VB transitions with a long lifetime will be predominantly Gaussian.