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**Signaling and Entry Deterrence:  
A Multi-Dimensional Analysis**

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# Signaling and Entry Deterrence: A Multi-dimensional Analysis

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## 1. Introduction

This paper considers a long-standing question in the field of Industrial Organization: Can an incumbent firm price and advertise so as to deter entry that otherwise would be profitable? For the most part, the first economists to consider this question give affirmative answers.<sup>1</sup> Braithwaite (1928) and Robinson (1933) offer early informal remarks in support of the view that advertising has an entry-deterrence effect. Bain (1949) provides an early argument that an incumbent may deter entry by limit pricing (i.e., by pricing below the monopoly price), and Williamson (1963) later develops an analogous argument that an incumbent can deter entry by committing to a low price and a high advertising level. Important early empirical contributions by Bain (1956) and Comanor and Wilson (1967, 1974) offer inter-industry evidence that is broadly consistent with the hypothesis that advertising by established firms generates an entry barrier.

Subsequent work, however, suggests that early economists exaggerate the entry-deterrence effects of incumbent pricing and advertising selections. As Needham (1976) argues, the incumbent's pre-entry behavior deters entry only if some link exists between this behavior and the potential entrant's expected post-entry profit. A link is clearly present if the incumbent can commit to maintain its pre-entry price and advertising in the event of entry, but an assumption that the incumbent can make such a commitment seems implausible. Further, as Demsetz (1973, 1974), Nelson (1974) and others argue, the early empirical efforts suffer from fundamental endogeneity and measurement problems.

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<sup>1</sup>See Bagwell (2005) for a comprehensive survey of the economic analysis of advertising.

Recent work uses game-theoretic models to reconsider whether limit pricing and advertising may deter entry. One set of work emphasizes that pre-entry advertising by the incumbent may play a commitment role if it provides consumers with durable information concerning, e.g., the incumbent's existence or location. Schmalensee (1983), Fudenberg and Tirole (1984) and Ishigaki (2000) develop models of this kind. The surprising conclusion is that, if an incumbent can deter entry with a distortion in its pre-entry advertising, then it does so by distorting its pre-entry advertising *downward*.

A second set of work proposes an informational link between the incumbent's pre-entry behavior and the entrant's expected post-entry profit. In a classic paper, Milgrom and Roberts (1982) assume that the incumbent has private information about its costs of production and show that a separating equilibrium may exist in which the incumbent limit prices and thereby signals that its costs are low. The potential entrant then infers the incumbent's cost type and enters exactly when entry would be profitable under complete information. A striking implication is thus that profitable entry is not deterred. Bagwell and Ramey (1988) extend the model to allow that the incumbent has two signals: price and advertising.<sup>2</sup> In their model, the incumbent is privately informed as to whether its costs are low or high, the potential entrant's costs are commonly known, and entry is profitable if and only if the incumbent has high costs. In the refined separating equilibrium, the low-cost incumbent engages in a "cost-reducing distortion," in the sense that it adopts the same price and advertising selection as it would were it, hypothetically, an uncontested monopoly with costs that were even lower. The low-cost incumbent thus limit prices and distorts its demand-enhancing advertising upward. Dissipative advertising is not used. Once again, due to signaling, profitable entry is not deterred.

As summarized above, the recent game-theoretic work does not support the hypothesis that an incumbent can deter profitable entry by distorting its pre-entry advertising upward. But in fact this hypothesis does find some support in the second set of work, once pooling equilibria are considered. In particular, for some parameter regions, Bagwell and Ramey (1988) show that refined pooling equilibria exist in which the high-cost incumbent uses limit pricing and an upward distortion in advertising to deter entry that would be profitable under complete information. An important feature of their model, however, is that the incumbent faces entry with probability one (zero) when the potential entrant's belief that the incumbent has high costs rises above (falls below) a threshold level. The analysis thus leaves open whether such equilibria would exist if the potential entrant's

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<sup>2</sup>For other extensions, see Albaek and Overgaard (1992a,b), Bagwell (1992a), Bagwell and Ramey (1990, 1991), Harrington (1986, 1987) and Linnemer (1998).

response were “smoother,” so that higher beliefs were everywhere associated with a greater probability of entry. A smooth response would arise, e.g., if the potential entrant were privately informed of its fixed cost of entry.

In this paper, I follow Milgrom and Roberts (1982) and Bagwell and Ramey (1988) and posit an informational link. The analysis contributes at three levels. First, I extend the model of Bagwell and Ramey (1988) to consider whether refined pooling equilibria exist when the potential entrant’s response is smooth. I refer to the associated game as the benchmark game. Following standard practice, I study refined equilibria in the sense that I characterize the Perfect Bayesian Equilibria that satisfy the Cho-Kreps (1987) intuitive criterion. For the benchmark game, my main finding is that intuitive pooling equilibria do not exist; thus, there exists a unique intuitive equilibrium outcome, in which separation occurs and the low-cost incumbent undertakes a cost-reducing distortion.<sup>3</sup> This finding resolves an open issue in the existing literature and thereby provides a solid foundation for the conclusion that the recent game-theoretic literature provides little support for the “traditional” view that advertising has an entry-deterrence effect.

The second contribution is more fundamental. With the analysis of the benchmark game at hand, I next analyze a more general game in which the incumbent has two dimensions of private information. Specifically, the incumbent is privately informed as to its cost type *and* its level of patience, where the incumbent’s cost type may be high or low and the incumbent’s patience level also may be high or low. I motivate the latter dimension with reference to the corporate finance literature, which suggests that a firm’s management may be impatient (i.e., exhibit “short-termism”) if capital markets are imperfect (Shleifer and Vishney, 1990; Stein, 1989), managerial career concerns direct attention to short-term performance measures (Narayanan, 1985) or the threat of takeover is significant (Stein, 1988). It is plausible that the incumbent has private information about the magnitude of such influences. The incumbent selects price and advertising in the pre-entry period; thus, the proposed model entails both multiple signals and multiple dimensions of private information. The potential entrant finds entry more profitable when the incumbent has high costs, but the potential entrant’s ability to infer the incumbent’s cost type is now confounded by the fact that the potential entrant also lacks information as to the incumbent’s level of patience.

In the multi-dimensional model, the patient, low-cost incumbent is the strongest type. A low-cost incumbent finds cost-reducing distortions less costly than does a high-cost incumbent; furthermore, a patient incumbent values entry deterrence

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<sup>3</sup>For related themes in models in which an incumbent uses pricing and advertising to signal one-dimensional private information about demand, see Albaek and Overgaard (1992a) and Bagwell and Ramey (1990).

more than does an impatient incumbent. At the other extreme, a high-cost, impatient incumbent is the weakest type. The incumbent also may be an impatient, low-cost type or a patient, high-cost type. A key insight is that an impatient, low-cost incumbent may be unable to separate from a patient, high-cost incumbent, since the latter discounts less the future gains associated with entry deterrence.

Several interesting findings emerge. One group of findings offers support for predictions for separating equilibria that are featured in work by Milgrom and Roberts (1982) and Bagwell and Ramey (1988). In any intuitive equilibrium, the patient, low-cost incumbent separates from the high-cost types, and separation is achieved through a cost-reducing distortion. In other words, in any intuitive equilibrium, the patient, low-cost incumbent limit prices, distorts demand-enhancing advertising upward and does not employ dissipative advertising; in turn, this behavior enables the potential entrant to infer the incumbent's cost, so that profitable entry is not deterred when this type of incumbent exists. Also, I identify three kinds of intuitive equilibria, and in one kind (Kind B) the patient and impatient low-cost incumbent types make one selection, while the patient and impatient high-cost incumbent types make another selection. As in the separating equilibria featured in the earlier models with one-dimensional private information, equilibrium behavior is then driven entirely by differences in cost types.

At the same time, a second group of findings redirects attention to pooling behavior and provides support for themes that are emphasized in the pre-game-theoretic literature. In the remaining two kinds of intuitive equilibria (Kinds A and C), the predictions of Bain (1949) and Williamson (1963) can be associated with the behavior of the patient, high-cost incumbent. In these equilibria, the patient, high-cost incumbent pools with the impatient, low-cost incumbent. The impatient, high-cost incumbent may also pool (Kind A) or may make its monopoly selection and reveal itself (Kind C). Importantly, the potential entrant is then unable to infer the incumbent's cost level when the pooled selection is observed. Further, I construct intuitive equilibria in which the pooled selection entails a cost-reducing distortion by the patient, high-cost incumbent. The patient, high-cost incumbent then limit prices and distorts upward its demand-enhancing advertising and thereby confounds the potential entrant. As a result, an equilibrium foundation is provided for the "traditional" hypothesis that limit pricing and aggressive advertising by an incumbent may deter profitable entry.

Finally, the analysis generates a novel finding that is not represented in the earlier literature. I find that intuitive equilibria (of Kind A) may exist in which the impatient, low-cost incumbent plays "soft" in the pre-entry period, by pricing above its monopoly level and distorting its demand-enhancing advertising downward, and thus sometimes induces unprofitable entry. This occurs when all types

other than the patient, low-cost type of incumbent pool, and the impatient, low-cost incumbent then refrains from deviating since it does not want to induce a less favorable belief (e.g., that it has high costs with certainty).

The third contribution of the paper is methodological. The model analyzed here has multiple signals and two-dimensional private information. Even in this multi-dimensional setting, I find that the Cho-Kreps (1987) intuitive criterion has considerable force in selecting among equilibria. As well, the analysis reveals that the existence of (pure-strategy) intuitive equilibria may be more problematic than in standard signaling models with one-dimensional private information.

The literature on signaling games with multiple dimensions of private information is quite undeveloped.<sup>4</sup> Quinzii and Rochet (1985) and Engers (1987) consider whether standard signaling results for models with one-dimensional private information extend to the multi-dimensional setting. They thus develop sufficient conditions for the existence of separating equilibria in signaling models with multi-dimensional private information. Chen (1997) presents a model in which a firm is privately informed as to its cost and the demand function, and the firm's diversification and financing decisions are possible signals of this information. He provides conditions under which an unique separating equilibrium outcome exists. In Chen's model, each signal is a "natural signal" for one kind of private information, and signals are drawn from binary sets. The model developed here, by contrast, has a different structure and allows each signal to be drawn from the space of non-negative real numbers. Finally, in their analysis of product-quality signaling, Bagwell and Riordan (1991) briefly consider an extended model, in which a monopolist is privately informed as to its quality of product and the number of informed consumers. Consumers observe the monopolist's price and attempt to infer the former; however, their inferences are confounded by their ignorance as to the latter. Bagwell and Riordan derive necessary conditions for an intuitive equilibrium. The present paper allows for multiple signals and offers a comprehensive analysis of a related structure, in that the potential entrant lacks information as to two variables (costs, patience) but is directly interested only in the value of one of these variables (cost).

The paper is organized as follows. In Section 2, I present the general model, define the equilibrium concept and develop useful properties. In Section 3, I examine the benchmark game in which the incumbent is privately informed only as to its cost level. Returning to the general model with two-dimensional private information, I characterize in Section 4 the necessary features of intuitive equilibria.

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<sup>4</sup>The literature on multi-dimensional screening is more advanced. Adams and Yellen (1976) and Armstrong (1996, 1999) offer important contributions, and valuable overviews of the literature are provided by Bolton and Dewatripont (2005, Chapter 6) and Rochet and Stole (2003).

ria. I find that an intuitive equilibrium must be of Kind A, B or C. In Sections 5 through 7, I then analyze the existence properties of intuitive equilibria of Kind A, B and C, respectively. Section 8 considers the special case in which an impatient incumbent is completely impatient. This case is of special interest and also serves to illustrate a non-existence result and a mixed-strategy resolution. Section 9 concludes. Longer proofs are collected in the Appendix.

## 2. The Model

In this section, I define the game and equilibrium concept. Next, I introduce some additional structure and derive some key properties. These properties are used in subsequent sections to derive the main findings of the paper.

### 2.1. Game and Equilibrium Concept

Consider a market with one incumbent firm and one potential entrant. The firms interact over two periods. In the pre-entry period, the incumbent selects a pre-entry price,  $P \geq 0$ , and advertising level,  $A \geq 0$ . When the incumbent makes these selections, it is privately informed as to its cost type,  $t$ , and its patience level,  $\delta$ , where  $t \in \{L, H\}$ ,  $\delta \in \{\lambda, \eta\}$ ,  $H > L > 0$  and  $\eta > \lambda \geq 0$ . The incumbent thus has two dimensions of private information: it may have low or high costs, and it also may have low or high patience. The potential entrant observes  $P$  and  $A$  but not  $t$  or  $\delta$ . Based on these observations, the potential entrant forms some belief as to the incumbent's private information and makes a decision to enter or not. In the post-entry period, the incumbent is a monopolist if the potential entrant chooses not to enter; however, if the potential entrant decides to enter, then the two firms compete as duopolists.

The incumbent's two-dimensional type,  $\{t, \delta\}$ , is determined by Nature. For simplicity, assume that Nature selects  $t = H$  with probability  $b_o$  and  $\delta = \eta$  with probability  $\beta_o$ , where  $t$  and  $\delta$  are independently selected. For most of the paper, I assume that  $b_o \in (0, 1)$  and  $\beta_o \in (0, 1)$ ; however, in Section 3, I assume  $\beta_o = 1$  and thereby examine the benchmark model in which the incumbent has private information only about its cost type. After observing the incumbent's pre-entry selections, the potential entrant forms a posterior belief as to the incumbent's private information. Let  $b = b(P, A)$  denote the potential entrant's posterior belief as to the likelihood that the incumbent has a high cost type.

In the pre-entry period, the incumbent confronts a demand function,  $D(P, A)$ . I assume only that  $D$  is continuous, nonnegative, strictly decreasing in  $P$  and weakly increasing in  $A$ . The case in which  $D$  is strictly increasing in  $A$  is the

case of *demand-enhancing advertising*. The case in which  $D$  is independent of  $A$  is the case of *dissipative advertising*. The pre-entry period profit function for an incumbent with cost type  $t$  is represented as

$$\Pi(P, A | t) \equiv (P - t)D(P, A) - A.$$

I assume that  $\Pi(P, A | t)$  has a unique maximizer,  $(P_m(t), A_m(t))$ , and that  $\Pi(P_m(t), A_m(t) | t) > 0$ .

I assume that the potential entrant's expected profit from entry is determined by its fixed costs of entry and the belief it holds as to the likelihood that the incumbent has a high cost type. In particular, the expected profit from entry is strictly increasing in the probability  $b$  that the incumbent has a high cost type. I further assume that the potential entrant's expected profit from entry is independent of any belief that it may hold as to the incumbent's level of patience. These assumptions are appropriate, e.g., if the potential entrant learns the incumbent's cost type upon entering the market and post-entry competition corresponds to the Nash equilibrium of a standard, static oligopoly game, such as the Cournot game.<sup>5</sup> At the same time, these assumptions are not appropriate for all forms of post-entry competition. For instance, if the incumbent's private information remains private after the act of entry and if post-entry competition transpires over multiple periods, then the incumbent's post-entry play might signal its costs. The incumbent's play - and thus the potential entrant's expected profit from entry - might then depend upon the incumbent's level of patience.

In the post-entry period, the incumbent naturally prefers no entry to entry. Under the assumptions given in the preceding paragraph, if the potential entrant is privately informed as to its fixed cost of entry, where the fixed cost is drawn from an interval of possible fixed costs, then the incumbent anticipates that the probability of entry is strictly increasing in  $b$ . Thus, when  $b$  is higher, the incumbent is expected to earn less in the post-entry period, since it is then more likely to receive duopoly rather than monopoly profit. It is thus possible to capture the entrant's behavior in a general way by assuming simply that the incumbent's expected post-entry profit is strictly decreasing in  $b$ . In particular, let  $\tilde{\pi}_t(b)$  denote the expected post-entry profit for an incumbent with cost type  $t$  and assume for simplicity that  $\tilde{\pi}_t(b)$  is differentiable. I assume that  $\tilde{\pi}_t(b) > 0 > \tilde{\pi}'_t(b)$ .

Consider now the discounted payoff over the game for an incumbent with cost type  $t$  and patience level  $\delta$ , when the incumbent makes pre-entry selections  $P$

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<sup>5</sup>Milgrom and Roberts (1982) and most subsequent analyses also assume that the incumbent's private cost information becomes public upon the act of entry. When firms adopt the Nash strategies for a static oligopoly game, the incumbent's post-entry play is independent of its discount factor; thus, it is then unimportant whether the incumbent's patience level remains private information after entry.



and  $A$  and thus induces the belief  $b = b(P, A)$ . This payoff is given as

$$\Pi(P, A | t) + \delta\tilde{\pi}_t(b),$$

where  $t \in \{L, H\}$  and  $\delta \in \{\lambda, \eta\}$ .

The next task is to define a Perfect Bayesian Equilibrium (PBE). The triplet  $\{P(t, \delta), A(t, \delta), b(P, A)\}$  forms an *equilibrium* if and only if

(E1). For all  $t \in \{L, H\}$  and  $\delta \in \{\lambda, \eta\}$ ,

$$(P(t, \delta), A(t, \delta)) \in \operatorname{argmax}\{\Pi(P, A | t) + \delta\tilde{\pi}_t(b(P, A))\}, \text{ and}$$

(E2). Whenever possible,  $b(P, A)$  is consistent with Bayes' rule.<sup>6</sup>

Thus, whatever its two-dimensional type, the incumbent selects its price and advertising levels as a best response to the potential entrant's belief function. In turn, the potential entrant's beliefs must be derived by application of Bayes' rule for price and advertising selections that are on the equilibrium path. For selections that fall off the equilibrium path (i.e., for  $(P, A) \notin \{P(t, \delta), A(t, \delta)\}$ ), however, no restriction is placed on the potential entrant's beliefs. As in familiar one-dimensional signaling models, this freedom in specifying off-equilibrium-path beliefs is a source of multiple equilibria.

For signaling models with one-dimensional private information, it is standard practice to follow Cho and Kreps (1987) and place structure on off-equilibrium-path beliefs with the requirement that players never believe that "equilibrium-dominated" strategies are selected. A similar practice can be followed in the present setting with two dimensions of private information. To formalize this requirement, fix a particular equilibrium satisfying (E1) and (E2). Let  $\hat{\Pi}(t, \delta)$  denote the equilibrium discounted game payoff for an incumbent with the two-dimensional type  $(t, \delta)$ . Formally, this value is defined by

$$\hat{\Pi}(t, \delta) \equiv \Pi(P(t, \delta), A(t, \delta) | t) + \delta\tilde{\pi}_t(b_{t\delta}),$$

where  $b_{t\delta} \equiv b(P(t, \delta), A(t, \delta))$ . Consider now a deviant selection,  $(P, A)$ , where  $(P, A) \neq (P(t, \delta), A(t, \delta))$  for any  $t$  and  $\delta$ . This deviation is *equilibrium dominated* for an incumbent with type  $(t, \delta)$  if and only if

$$\Pi(P, A | t) + \delta\tilde{\pi}_t(0) < \hat{\Pi}(t, \delta).$$

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<sup>6</sup>Formally, the definition of a PBE should also include the requirement that the potential entrant's belief as to the incumbent's level of patience is formed in a manner that is consistent with Bayes' rule. Under the assumptions adopted above, however, the potential entrant's expected profit from entry is independent of any belief it holds with regard to the incumbent's level of patience, and so this belief may be ignored without loss of generality.

Intuitively, an incumbent with type  $(t, \delta)$  would never select  $(P, A)$ , if in the best case ( $b = 0$ ) the deviant selection offers the incumbent a lower payoff than it could obtain by following the equilibrium (i.e., by selecting  $(P(t, \delta), A(t, \delta))$ ).

An *intuitive equilibrium* is now defined as a triplet  $\{P(t, \delta), A(t, \delta), b(P, A)\}$  that satisfies (E1), (E2) and

(E3). If  $(P, A)$  is equilibrium dominated for  $(H, \lambda)$  and  $(H, \eta)$  but not for  $(L, \lambda)$  or  $(L, \eta)$ , then  $b(P, A) = 0$ . If  $(P, A)$  is equilibrium dominated for  $(L, \lambda)$  and  $(L, \eta)$  but not for  $(H, \lambda)$  or  $(H, \eta)$ , then  $b(P, A) = 1$ .

Thus, in an intuitive equilibrium, if a deviation is equilibrium dominated for an incumbent of one cost type whatever that incumbent's level of patience and if the deviation is not equilibrium dominated for an incumbent with the other cost type for at least one level of patience, then the potential entrant must believe that the incumbent has the latter cost type.

## 2.2. Additional Structure and Key Properties

As shown by Bagwell and Ramey (1988), signaling models with multiple signals often can be easily analyzed once an extended profit function is defined.<sup>7</sup> I extend here this approach to a setting with multiple dimensions of private information. The extended profit function is defined as

$$\Pi(P, A | c) \equiv (P - c)D(P, A) - A,$$

where  $c$  is drawn from the set of positive real numbers. Thus, even though the incumbent's cost type is either  $t = L$  or  $t = H$ , it is still possible to extend the definition of the incumbent's pre-entry profit function to cover the hypothetical case in which the incumbent has an arbitrary cost level,  $c$ . Next, I assume that  $\Pi(P, A | c)$  has a unique maximizer,  $\psi(c)$ , which is continuous and defined as

$$\psi(c) \equiv (P(c), A(c)) = \operatorname{argmax}\{\Pi(P, A | c)\}.$$

Observe that  $\psi(H) = (P_m(H), A_m(H))$  and  $\psi(L) = (P_m(L), A_m(L))$ . Thus,  $\psi(c)$  gives the monopoly price and advertising selections for a firm with cost level  $c$ . Given that  $D$  is strictly decreasing in  $P$  and that  $\Pi(P, A | c)$  has a unique maximizer, it is straightforward to confirm that  $P(c)$  is strictly increasing. I assume that  $A(c)$  is strictly decreasing in the case of demand-enhancing advertising. In the case of dissipative advertising,  $A(c)$  is constant at zero.

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<sup>7</sup>See also Albaek and Overgaard (1992a,b), Bagwell (1992b), Bagwell and Ramey (1990), Overgaard (1991) and Ramey (1996).

The extended profit function satisfies a simple ranking property. Fix any  $(P_1, A_1)$  and  $(P_2, A_2)$ , where  $(P_1, A_1) \neq (P_2, A_2)$ . Define

$$\Delta(c) \equiv \Pi(P_2, A_2 | c) - \Pi(P_1, A_1 | c),$$

and observe that  $\Delta(c)$  is differentiable and indeed linear in  $c$ . In particular, it is immediate that  $\Delta'(c) = D(P_1, A_1) - D(P_2, A_2)$ . It is now convenient to record the following ranking property.

**Lemma 2.1.** *Fix any  $(P_1, A_1)$  and  $(P_2, A_2)$ , where  $(P_1, A_1) \neq (P_2, A_2)$ . Then  $\Delta'(c) = D(P_1, A_1) - D(P_2, A_2)$ , and so  $\Delta(c)$  is either constant, strictly increasing or strictly decreasing in  $c$ .*

To see how Lemma 2.1 might be used, suppose it were known that  $\Delta(H) = 0 < \Delta(c)$  for some  $c < H$ . In this situation, a high-cost incumbent's pre-entry profit is the same whether  $(P_1, A_1)$  or  $(P_2, A_2)$  is selected; however, a hypothetical incumbent with cost  $c < H$  enjoys higher pre-entry profit under the latter selection. Lemma 2.1 then guarantees that  $\Delta(c)$  is strictly decreasing, and this in turn ensures that  $\Delta(L) > 0$ . In this way, Lemma 2.1 points to a single-dimensional ranking statistic,  $c$ , for comparisons that involve multiple signals.

The ranking property yields a valuable monotonicity property.

**Lemma 2.2.** *Fix  $t \in \{L, H\}$ . Then  $\Pi(\psi(c) | t)$  is continuous in  $c$ , strictly decreases in  $c$  for  $c > t$ , and strictly increases in  $c$  for  $c < t$ .*

**Proof:** Continuity is immediate, since  $\Pi$  is continuous in  $P$  and  $A$  while  $\psi(c)$  is continuous in  $c$ . To establish monotonicity, pick  $c_2 > c_1$ . Let  $(P_2, A_2) = \psi(c_2)$  and  $(P_1, A_1) = \psi(c_1)$ . Then  $\Delta(c_2) > 0 > \Delta(c_1)$ , and so  $\Delta(c)$  is strictly increasing by Lemma 2.1. Thus, if  $c_2 > c_1 > t$ , then  $\Delta(t) = \Pi(\psi(c_2) | t) - \Pi(\psi(c_1) | t) < 0$ . Likewise, if  $t > c_2 > c_1$ , then  $\Delta(t) = \Pi(\psi(c_2) | t) - \Pi(\psi(c_1) | t) > 0$ . ■

I now add some additional structure to the model with the following three maintained assumptions.

**Assumption 1** (Boundary Conditions)

- (i) There exists  $\underline{c} < H$  such that  $\Pi(\psi(\underline{c}) | H) + \eta\tilde{\pi}_H(0) < \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1)$ .
- (ii) There exists  $\bar{c} > H$  such that  $\Pi(\psi(\bar{c}) | H) + \eta\tilde{\pi}_H(0) < \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1)$ .

**Assumption 2** (The Value of Deterrence and Cost Types)

$$\frac{d}{db}[\tilde{\pi}_L(b) - \tilde{\pi}_H(b)] \leq 0.$$

**Assumption 3** (Distortion Required)

$$\Pi(\psi(L) | H) + \eta\tilde{\pi}_H(0) > \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1).$$

Assumption 1 is a technical condition, the role of which is clarified below. Assumption 2 indicates that the benefit of a lower belief is weakly greater for an incumbent with low costs than an incumbent with high costs. Finally, Assumption 3 indicates that a patient, high-cost incumbent would rather mimic the low-cost monopoly selection and be perceived to have low costs than to make the high-cost monopoly selection and be recognized as a high-cost incumbent. Under this assumption, in any equilibrium in which the high-cost incumbent makes its monopoly selection and is revealed to have high costs, the low-cost incumbent must distort its selection away from its monopoly selection. All three assumptions are standard; e.g., Bagwell and Ramey (1988) impose analogous assumptions.

Together, the ranking property in Lemma 2.1 and Assumption 2 suggest the possibility of a “single-crossing property” for payoffs in the two-period game. The following lemmas offer different expressions of this idea. Both lemmas are established for any  $\delta \in \{\lambda, \eta\}$ .

**Lemma 2.3.** *Consider any  $(P_1, A_1, b_1)$  and  $(P_2, A_2, b_2)$  and  $c < H$  such that*

$$\begin{aligned} \Pi(P_2, A_2 | c) + \delta\tilde{\pi}_H(b_2) &> \Pi(P_1, A_1 | c) + \delta\tilde{\pi}_H(b_1), \text{ and} \\ \Pi(P_2, A_2 | H) + \delta\tilde{\pi}_H(b_2) &= \Pi(P_1, A_1 | H) + \delta\tilde{\pi}_H(b_1). \end{aligned}$$

*If  $b_2 \leq b_1$ , then*

$$\Pi(P_2, A_2 | L) + \delta\tilde{\pi}_L(b_2) > \Pi(P_1, A_1 | L) + \delta\tilde{\pi}_L(b_1).$$

**Proof:** Under the given assumptions,  $\Delta(c) > \delta[\tilde{\pi}_H(b_1) - \tilde{\pi}_H(b_2)] = \Delta(H)$ ; thus, by Lemma 2.1  $\Delta(c)$  is strictly decreasing. Hence,  $\Delta(L) > \Delta(H) = \delta[\tilde{\pi}_H(b_1) - \tilde{\pi}_H(b_2)] \geq \delta[\tilde{\pi}_L(b_1) - \tilde{\pi}_L(b_2)]$ , where the final inequality follows from Assumption 2 and  $b_2 \leq b_1$ . Thus,  $\Delta(L) > \delta[\tilde{\pi}_L(b_1) - \tilde{\pi}_L(b_2)]$ . ■

According to Lemma 2.3, if an incumbent with a high cost type is indifferent between  $(P_1, A_1, b_1)$  and  $(P_2, A_2, b_2)$  where  $b_2 \leq b_1$ , and yet the incumbent would prefer the latter option were its cost level at some lower level  $c$ , then the incumbent must also prefer the latter option when its cost type is low.

The second lemma captures a related implication.

**Lemma 2.4.** *Consider any  $(P_1, A_1, b_1)$  and  $(P_2, A_2, b_2)$  and  $c < L$  such that*

$$\begin{aligned} \Pi(P_2, A_2 | c) + \delta\tilde{\pi}_H(b_2) &\geq \Pi(P_1, A_1 | c) + \delta\tilde{\pi}_H(b_1), \text{ and} \\ \Pi(P_2, A_2 | H) + \delta\tilde{\pi}_H(b_2) &\geq \Pi(P_1, A_1 | H) + \delta\tilde{\pi}_H(b_1), \end{aligned}$$

with at least one inequality strict. If  $b_2 \leq b_1$ , then

$$\Pi(P_2, A_2 | L) + \delta\tilde{\pi}_L(b_2) > \Pi(P_1, A_1 | L) + \delta\tilde{\pi}_L(b_1).$$

**Proof:** Under the given assumptions,  $\Delta(c) \geq \delta[\tilde{\pi}_H(b_1) - \tilde{\pi}_H(b_2)]$  and  $\Delta(H) \geq \delta[\tilde{\pi}_H(b_1) - \tilde{\pi}_H(b_2)]$ , with at least one inequality strict; thus, using Lemma 2.1 and  $L \in (c, H)$ , it follows that  $\Delta(L) > \delta[\tilde{\pi}_H(b_1) - \tilde{\pi}_H(b_2)] \geq \delta[\tilde{\pi}_L(b_1) - \tilde{\pi}_L(b_2)]$ , where the final inequality follows from Assumption 2 and  $b_2 \leq b_1$ . Thus,  $\Delta(L) > \delta[\tilde{\pi}_L(b_1) - \tilde{\pi}_L(b_2)]$ . ■

Lemma 2.4 allows that the incumbent with a high cost type prefers the triplet  $(P_2, A_2, b_2)$  to the triplet  $(P_1, A_1, b_1)$  where  $b_2 \leq b_1$ ; however, this lemma requires that the fictitious cost level  $c$  lies strictly below  $L$ .

I conclude this section by establishing some properties of the iso-payoff curve for a patient incumbent with a high cost type. In particular, suppose that a patient, high-cost incumbent selects a pair  $(P, A)$  and that this selection “fools” the potential entrant and induces the belief that the incumbent has the low cost type. When would such selections yield the same payoff for the patient, high-cost incumbent as it would make were it instead to make its monopoly selection,  $\psi(H)$ , and induce the (correct) belief that it has the high cost type? This question motivates consideration of the iso-payoff curve defined by

$$\Pi(P, A | H) + \eta\tilde{\pi}_H(0) = \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1).$$

Using Assumptions 1 and 3, it is evident that

$$\Pi(\psi(\underline{c}) | H) + \eta\tilde{\pi}_H(0) < \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1) < \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(0).$$

Thus, by Lemma 2.2, there exists a unique cost level  $c_o \in (\underline{c}, L)$  such that

$$\Pi(\psi(c_o) | H) + \eta\tilde{\pi}_H(0) = \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1). \quad (2.1)$$

Likewise, Assumption 1 and  $\tilde{\pi}'_H(b) < 0$  also imply that

$$\Pi(\psi(\bar{c}) | H) + \eta\tilde{\pi}_H(0) < \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1) < \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(0).$$

Thus, by Lemma 2.2, there exists a unique cost level  $c_{oo} \in (H, \bar{c})$  such that

$$\Pi(\psi(c_{oo}) | H) + \eta\tilde{\pi}_H(0) = \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1). \quad (2.2)$$

The iso-payoff curve and associated values  $c_o$  and  $c_{oo}$  are depicted in Figure 1.

Using (2.1) and (2.2), it is now possible to offer a first characterization of equilibrium behavior.

**Lemma 2.5.** *In any equilibrium, there exists a unique cost level  $c_e \in [c_o, H]$  and a unique cost level  $c_{ee} \in (H, c_{oo}]$  such that*

$$\widehat{\Pi}(H, \eta) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0) = \Pi(\psi(c_{ee}) | H) + \eta\tilde{\pi}_H(0).$$

**Proof:** Fix any equilibrium. Consider an incumbent with the two-dimensional type  $(H, \eta)$ . This incumbent must weakly prefer its equilibrium selection to the option of selecting its monopoly selection,  $\psi(H)$ . Thus,  $\widehat{\Pi}(H, \eta) \geq \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(b(\psi(H))) \geq \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1) = \Pi(\psi(c_o) | H) + \eta\tilde{\pi}_H(0) = \Pi(\psi(c_{oo}) | H) + \eta\tilde{\pi}_H(0)$ . Of course, this incumbent's payoff is maximized when it makes its monopoly selection and yet induces the belief  $b = 0$ . In any equilibrium, however, (E2) ensures that  $b_{H\eta} > 0$ . Thus,  $\Pi(\psi(H) | H) + \eta\tilde{\pi}_H(0) > \widehat{\Pi}(H, \eta)$ . The proof is now completed through the application of Lemma 2.2. ■

Figure 1 also illustrates possible values for  $c_e$  and  $c_{ee}$ . Notice that  $c_e > L$  is possible, even under Assumption 3, if  $\widehat{\Pi}(H, \eta)$  exceeds  $\Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1)$  to a sufficient degree.

### 3. Benchmark: One-Dimensional Private Information

In this section, I consider a benchmark model in which the incumbent's patience level is public information. Thus, in the benchmark model, the incumbent's pre-entry selections signal only its cost type,  $t$ . A characterization of the intuitive equilibria of the benchmark model is useful, since it is then possible to identify those predictions in the model with two dimensions of private information that are attributable to the extra dimension of hidden information.

The benchmark model can be analyzed with minor modifications of the structure presented above. In particular, suppose now that  $\beta_o = 1$ , so that the incumbent is known to have the patience level  $\delta \equiv \eta$ . With this modification in mind, I may represent the incumbent's strategy as  $(P(t, \eta), A(t, \eta))$ . The definition of an equilibrium is unaltered, with the understanding that (E1) is now required only when  $\delta \equiv \eta$ . For the benchmark model, a deviation is *equilibrium dominated* for an incumbent of type  $t$  if and only if  $\Pi(P, A | t) + \eta\tilde{\pi}_t(0) < \widehat{\Pi}(t, \eta)$ . Finally, an intuitive equilibrium is defined as before, once we modify (E3) in the following fashion: If  $(P, A)$  is equilibrium dominated for  $t = H$  but not for  $t = L$ , then  $b(P, A) = 0$ ; likewise, if  $(P, A)$  is equilibrium dominated for  $t = L$  but not for  $t = H$ , then  $b(P, A) = 1$ . The additional structure and key properties developed above apply to the benchmark model without further modification.

It is convenient now to distinguish between the two kinds of equilibria that are possible in the benchmark game. For the benchmark game, a *separating equilibrium* occurs if  $(P(H, \eta), A(H, \eta)) \neq (P(L, \eta), A(L, \eta))$ . In such an equilibrium,

the potential entrant can infer the incumbent's cost type, and so (E2) requires that  $b(P(H, \eta), A(H, \eta)) = 1 > 0 = b(P(L, \eta), A(L, \eta))$ . By contrast, in a *pooling equilibrium* of the benchmark game,  $(P(H, \eta), A(H, \eta)) = (P(L, \eta), A(L, \eta))$  and then (E2) requires that  $b(P(H, \eta), A(H, \eta)) = b_o$ .

I begin with the following lemma.

**Lemma 3.1.** *In any separating equilibrium of the benchmark game,  $(P(H, \eta), A(H, \eta)) = \psi(H)$ .*

**Proof:** Assume to the contrary that a separating equilibrium exists in which  $(P(H, \eta), A(H, \eta)) \neq \psi(H)$ . Then under (E2)  $b(P(H, \eta), A(H, \eta)) = 1$ , and so  $\hat{\Pi}(H, \eta) = \Pi(P(H, \eta), A(H, \eta) \mid H) + \eta\tilde{\pi}_H(1) < \Pi(\psi(H) \mid H) + \eta\tilde{\pi}_H(1) \leq \Pi(\psi(H) \mid H) + \eta\tilde{\pi}_H(b(\psi(H)))$ . Thus, (E1) fails: the incumbent with cost type  $H$  would gain by deviating to  $\psi(H)$ . ■

Intuitively, in a separating equilibrium of the benchmark model, the incumbent with the high cost type is “found out” and faces the maximum probability of entry. If  $(P(H, \eta), A(H, \eta)) \neq \psi(H)$ , then this incumbent would deviate to  $\psi(H)$ , as it then gains in the pre-entry period and does no worse in the post-entry period.

I next characterize the set of intuitive separating equilibria.

**Proposition 3.2.** *In the benchmark game, there exists at most one intuitive separating equilibrium outcome, and in it  $(P(L, \eta), A(L, \eta)) = \psi(c_o)$ , where  $c_o < L$  is defined by (2.1).*

**Proof:** The existence of  $c_o < L$  is established above in (2.1). Consider now the following program:  $\max_{P, A} \Pi(P, A \mid L) + \eta\tilde{\pi}_L(0)$  subject to  $\Pi(P, A \mid H) + \eta\tilde{\pi}_H(0) \leq \Pi(\psi(H) \mid H) + \eta\tilde{\pi}_H(1)$ . The claim is that  $\psi(c_o)$  is the unique solution to this program. Observe that  $\psi(c_o) \equiv (P_2, A_2)$  satisfies the program's constraint. Pick any  $(P_1, A_1) \neq \psi(c_o)$  that also satisfies this constraint. Note that  $\Delta(c_o) > 0$ ; further,  $\Delta(H) \geq 0$ , since  $\Pi(\psi(c_o) \mid H) + \eta\tilde{\pi}_H(0) = \Pi(\psi(H) \mid H) + \eta\tilde{\pi}_H(1)$ . Given  $c_o < L$ , Lemma 2.4 may be applied (with  $b_1 = b_2 = 0$ ) to yield  $\Delta(L) > 0$ , and the claim is established. Now, assume to the contrary that  $(P(L, \eta), A(L, \eta)) \neq \psi(c_o)$  in an intuitive separating equilibrium of the benchmark model. By Lemma 3.1,  $\hat{\Pi}(H, \eta) = \Pi(\psi(H) \mid H) + \eta\tilde{\pi}_H(1)$ ; thus, (E1) is satisfied when  $t = H$  only if  $(P(L, \eta), A(L, \eta))$  satisfies the constraint of the program just analyzed. The established claim thus implies that  $\Pi(\psi(c_o) \mid L) + \eta\tilde{\pi}_L(0) > \Pi(P(L, \eta), A(L, \eta) \mid L) + \eta\tilde{\pi}_L(0)$ . Consider a deviation,  $\psi(c_o - \varepsilon)$ , where  $\varepsilon > 0$  is small. Clearly, this deviation is not equilibrium dominated for type  $t = L$ . But, by Lemma 2.2, the deviation is equilibrium dominated for type  $t = H$ . Thus, under (E3),

$b(\psi(c_o - \varepsilon)) = 0$ . But then the incumbent with cost type  $L$  would deviate, and a contradiction is thus established. ■

As confirmed in the proof of this proposition, in any intuitive separating equilibrium for the benchmark model, the “least-cost” separating equilibrium is characterized by  $(P(L, \eta), A(L, \eta)) = \psi(c_o)$  and must obtain.

I now consider the existence of an intuitive separating equilibrium.

**Proposition 3.3.** *In the benchmark game, there exists a unique intuitive separating equilibrium outcome.*

**Proof:** Let  $(P(H, \eta), A(H, \eta)) = \psi(H)$  and  $(P(L, \eta), A(L, \eta)) = \psi(c_o)$  and set  $b(\psi(H)) = 1 > 0 = b(\psi(c_o))$ . I first derive a key inequality for the payoff of  $t = L$ . Let  $(P_1, A_1, b_1) = (\psi(L), 1)$  and  $(P_2, A_2, b_2) = (\psi(c_o), 0)$ . By construction, type  $H$  is indifferent between  $(P_2, A_2, b_2)$  and  $(\psi(H), 1)$ ; consequently, type  $H$  strictly prefers  $(P_2, A_2, b_2)$  to  $(P_1, A_1, b_1)$ . Also, if the incumbent’s cost level were  $c_o$ , then it would strictly prefer  $(P_2, A_2, b_2)$  to  $(P_1, A_1, b_1)$ . Since  $b_2 < b_1$  and  $c_o < L$ , it thus follows from Lemma 2.4 that  $\Pi(\psi(c_o) | L) + \eta\tilde{\pi}_L(0) > \Pi(\psi(L) | L) + \eta\tilde{\pi}_L(1)$ . The second step is to specify beliefs when deviations are observed. For  $(P, A) \notin \{\psi(c_o), \psi(H)\}$ , beliefs are specified as follows: if  $\Pi(P, A | H) + \eta\tilde{\pi}_H(0) < \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1)$ , then  $b(P, A) = 0$ ; and if  $\Pi(P, A | H) + \eta\tilde{\pi}_H(0) \geq \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1)$ , then  $b(P, A) = 1$ . Notice that, since  $\hat{\Pi}(H, \eta) = \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1)$ ,  $(P, A)$  in the first set are equilibrium dominated for  $t = H$  while  $(P, A)$  in the second set are not equilibrium dominated for  $t = H$ . The third step is to verify that (E1)-(E3) are satisfied. Given the belief specification, it is immediate that (E1) is satisfied for  $t = H$ . (E1) is also satisfied for  $t = L$ : the best deviation over the range for which  $b(P, A) = 1$  is  $\psi(L)$ , and the first step above establishes that this deviation strictly lowers the payoff for  $t = L$ ; furthermore, as shown in the proof of Proposition 3.2, the equilibrium selection,  $\psi(c_o)$ , uniquely maximizes the payoff for  $t = L$  over the range for which  $b(P, A) = 0$ . (E2) is clearly satisfied. Finally, (E3) is sure to hold, since  $b(P, A) = 0$  when  $(P, A)$  is equilibrium dominated for  $t = H$ , and  $b(P, A) = 1$  when  $(P, A)$  is not equilibrium dominated for  $t = H$ . ■

The final task is to consider the possibility of intuitive pooling equilibria.

**Proposition 3.4.** *In the benchmark game, there does not exist an intuitive pooling equilibrium.*

**Proof:** Suppose a pooling equilibrium exists in the benchmark game, and let  $(P_p, A_p) \equiv (P(H, \eta), A(H, \eta)) = (P(L, \eta), A(L, \eta))$ . By (E2),  $b(P_p, A_p) = b_o$ , and



so  $\widehat{\Pi}(H, \eta) = \Pi(P_p, A_p \mid H) + \eta \widetilde{\pi}_H(b_o)$ . By Lemma 2.5, there exists a unique cost level  $c_e \in [c_o, H)$  such that  $\Pi(\psi(c_e) \mid H) + \eta \widetilde{\pi}_H(0) = \widehat{\Pi}(H, \eta)$ . Now, let  $(P_1, A_1, b_1) = (P_p, A_p, b_o)$  and  $(P_2, A_2, b_2) = (\psi(c_e), 0)$ . By construction, type  $H$  is indifferent between the two options. Also, if the incumbent's type were  $c_e$ , then it would strictly prefer  $(P_2, A_2, b_2)$  to  $(P_1, A_1, b_1)$ . Since  $c_e < H$  and  $b_2 < b_1$ , it follows from Lemma 2.3 that type  $L$  strictly prefers  $(P_2, A_2, b_2)$  to  $(P_1, A_1, b_1)$ , or equivalently,  $\Pi(\psi(c_e) \mid L) + \delta \widetilde{\pi}_L(0) > \Pi(P_p, A_p \mid L) + \delta \widetilde{\pi}_L(b_o)$ . Now consider a deviation,  $\psi(c_e - \varepsilon)$ , where  $\varepsilon > 0$  is small. Clearly, this deviation is not equilibrium dominated for type  $t = L$ . Given  $c_e < H$ , it follows from Lemma 2.2 that the deviation is equilibrium dominated for type  $t = H$ . Thus, under (E3),  $b(\psi(c_e - \varepsilon)) = 0$ . But then (E1) fails, since the incumbent with cost type  $L$  would deviate to  $\psi(c_e - \varepsilon)$ . This establishes that no intuitive pooling equilibrium exists. ■

The following corollary summarizes the findings reported above.

**Corollary 3.5.** *In the benchmark game, there exists a unique intuitive equilibrium outcome, and in it  $(P(H, \eta), A(H, \eta)) = \psi(H)$  and  $(P(L, \eta), A(L, \eta)) = \psi(c_o)$ , where  $c_o < L$  is defined by (2.1).*

In effect, the low-cost incumbent engages in a “cost-reducing” distortion. It acts as if its costs were lower than they truly are and then simply selects the corresponding monopoly selection. The low-cost incumbent thus limit prices:  $P(c_o) < P(L) \equiv P_m(L)$ . Moreover, when advertising is demand enhancing, the low-cost incumbent also distorts upward its level of advertising:  $A(c_o) > A(L) \equiv A_m(L)$ . On the other hand, when advertising is dissipative, the monopoly level of advertising is zero, and so the low-cost incumbent does not use dissipative advertising:  $A(c_o) = 0 = A(L) \equiv A_m(L)$ . With these selections, the low-cost incumbent achieves separation and thus deters entry to the maximal extent possible. Notice, though, that limit pricing and upward distortions in demand-enhancing advertising do not deter profitable entry; instead, the signaling behavior of the low-cost incumbent enables the potential entrant to accurately infer the incumbent's cost level, so that entry can occur exactly when it is profitable.

The analysis presented in the section captures the central theme of the classic limit pricing paper by Milgrom and Roberts (1982), in that limit pricing occurs and yet profitable entry is never deterred. The analysis presented here goes further, however, and characterizes the “refined” equilibrium outcome for a model in which both pre-entry pricing and advertising may serve as signals. Bagwell and Ramey (1988) present a related model. The main difference between that paper and the present effort is that post-entry incumbent profit is modeled here as a

smooth function of the potential entrant’s beliefs, whereas Bagwell and Ramey (1988) assume the probability of entry jumps from zero to one once the belief rises above a critical value. Their formulation derives from a model in which the potential entrant’s fixed cost of entry takes a single value and is public information. As they show, intuitive pooling equilibria then may exist. The approach adopted in the present paper has the virtue of capturing the realistic possibility that the incumbent is uninformed as to the potential entrant’s fixed cost of entry and thus always has some uncertainty as to whether entry is profitable. In this “smoother” formulation, intuitive pooling equilibria fail to exist, and the focal outcome is thus the unique intuitive separating equilibrium outcome.

#### 4. Intuitive Equilibria: Necessary Properties

With the benchmark model now analyzed, I return to the general model with two dimensions of private information. This model is fully described in Section 2, under the assumption that  $\beta_o \in (0, 1)$ . I maintain this assumption henceforth. In the present section, I derive necessary properties of intuitive equilibria. These properties direct attention to the possible kinds of intuitive equilibrium behavior, and the corresponding equilibria are constructed in the next section.

##### 4.1. The Patient, Low-Cost incumbent

I focus first on the behavior of the patient, low-cost incumbent (i.e., the incumbent of type  $(L, \eta)$ ). I show that the intuitive criterion has significant power even in the model with two-dimensional private information and, indeed, that the patient, low-cost incumbent’s behavior is similar to that which is predicted in the intuitive equilibrium of the benchmark model for the low-cost incumbent. Thus, in a broad sense, the predictions of the earlier literature (Milgrom and Roberts, 1982; Bagwell and Ramey, 1988) carry over to the setting with two dimensions of private information, with regard to the behavior of the patient, low-cost incumbent.

This broad point is developed through a series of lemmas.

**Lemma 4.1.** *In any intuitive equilibrium,  $(P(L, \eta), A(L, \eta)) \neq (P(H, \eta), A(H, \eta))$ .*

**Proof:** Assume to the contrary that  $(P(L, \eta), A(L, \eta)) = (P(H, \eta), A(H, \eta))$  in an intuitive equilibrium. Let  $b_{H\eta} \in (0, 1)$  denote the corresponding belief. Using Lemma 2.5, there exists a unique cost level  $c_e \in [c_o, H)$  such that  $\widehat{\Pi}(H, \eta) \equiv \Pi(P(H, \eta), A(H, \eta) | H) + \eta\tilde{\pi}_H(b_{H\eta}) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ . Thus, type  $(H, \eta)$  is indifferent between  $(P_1, A_1, b_1) \equiv (P(H, \eta), A(H, \eta), b_{H\eta})$  and  $(P_2, A_2, b_2) \equiv (\psi(c_e), 0)$ . Further, an incumbent with cost level  $c_e$  strictly prefers the latter

option. Given that  $b_2 = 0 < b_1 = b_{H\eta}$  and  $c_e < H$ , Lemma 2.3 now ensures that type  $(L, \eta)$  strictly prefers the latter option:  $\widehat{\Pi}(L, \eta) \equiv \Pi(P(H, \eta), A(H, \eta) \mid L) + \eta\tilde{\pi}_L(b_{H\eta}) < \Pi(\psi(c_e) \mid L) + \eta\tilde{\pi}_L(0)$ . Finally, consider type  $(H, \lambda)$ . This type weakly prefers its equilibrium selection to  $(P_1, A_1, b_1)$  and strictly prefers  $(P_1, A_1, b_1)$  to  $(P_2, A_2, b_2)$ , since

$$\begin{aligned}\widehat{\Pi}(H, \lambda) &\geq \Pi(P(H, \eta), A(H, \eta) \mid H) + \lambda\tilde{\pi}_H(b_{H\eta}) \\ &= \Pi(\psi(c_e) \mid H) + \lambda\tilde{\pi}_H(0) + (\eta - \lambda)(\tilde{\pi}_H(0) - \tilde{\pi}_H(b_{H\eta})) \\ &> \Pi(\psi(c_e) \mid H) + \lambda\tilde{\pi}_H(0).\end{aligned}$$

By Lemma 2.2, for  $\varepsilon > 0$  and sufficiently small,  $\psi(c_e - \varepsilon)$  is equilibrium dominated for  $(H, \eta)$  and  $(H, \lambda)$  but not for  $(L, \eta)$ . Under (E3),  $b(\psi(c_e - \varepsilon)) = 0$ . But then type  $(L, \eta)$  would deviate to  $\psi(c_e - \varepsilon)$ . A contradiction is thus obtained. ■

Intuitively, the two-dimensional type  $(L, \eta)$  is the “strong, strong” type, since it both has low costs and discounts less the benefit from entry deterrence. If this type were to pool with type  $(H, \eta)$ , then it thus would be able to utilize the intuitive criterion of Cho and Kreps (1987) and find attractive deviations that are equilibrium dominated for types  $(H, \eta)$  and  $(H, \lambda)$ .

I show next that type  $(L, \eta)$  is also unwilling to pool with type  $(H, \lambda)$ .

**Lemma 4.2.** *In any intuitive equilibrium,  $(P(L, \eta), A(L, \eta)) \neq (P(H, \lambda), A(H, \lambda))$ .*

**Proof:** Assume to the contrary that  $(P(L, \eta), A(L, \eta)) = (P(H, \lambda), A(H, \lambda))$  in an intuitive equilibrium. By Lemma 4.1,  $(P(L, \eta), A(L, \eta)) \neq (P(H, \eta), A(H, \eta))$ . A first observation is that  $b_{H\eta} \leq b_{L\eta} = b_{H\lambda}$  is necessary. To see this, note that the following equilibrium conditions must hold for types  $(H, \eta)$  and  $(H, \lambda)$  :

$$\begin{aligned}\Pi(P(H, \eta), A(H, \eta) \mid H) + \eta\tilde{\pi}_H(b_{H\eta}) &\geq \Pi(P(L, \eta), A(L, \eta) \mid H) + \eta\tilde{\pi}_H(b_{L\eta}) \\ \Pi(P(L, \eta), A(L, \eta) \mid H) + \lambda\tilde{\pi}_H(b_{L\eta}) &\geq \Pi(P(H, \eta), A(H, \eta) \mid H) + \lambda\tilde{\pi}_H(b_{H\eta}).\end{aligned}$$

Adding yields  $[\eta - \lambda][\tilde{\pi}_H(b_{H\eta}) - \tilde{\pi}_H(b_{L\eta})] \geq 0$ , and so the first observation now follows. A second observation is that  $(P(H, \eta), A(H, \eta)) = (P(L, \lambda), A(L, \lambda))$  is also necessary. Otherwise,  $b_{H\eta} = 1 > b_{L\eta}$ , which contradicts the first observation. A third observation is that  $b_{H\eta} = b_{L\lambda} = b_{L\eta} = b_{H\lambda}$  is then necessary. To see this, consider the equilibrium conditions for types  $(L, \eta)$  and  $(L, \lambda)$  :

$$\begin{aligned}\Pi(P(L, \eta), A(L, \eta) \mid L) + \eta\tilde{\pi}_L(b_{L\eta}) &\geq \Pi(P(H, \eta), A(H, \eta) \mid L) + \eta\tilde{\pi}_L(b_{H\eta}) \\ \Pi(P(H, \eta), A(H, \eta) \mid L) + \lambda\tilde{\pi}_L(b_{H\eta}) &\geq \Pi(P(L, \eta), A(L, \eta) \mid L) + \lambda\tilde{\pi}_L(b_{L\eta}).\end{aligned}$$

Adding yields  $[\eta - \lambda][\tilde{\pi}_L(b_{L\eta}) - \tilde{\pi}_L(b_{H\eta})] \geq 0$ , and so  $b_{L\eta} = b_{H\lambda} \leq b_{H\eta} = b_{L\lambda}$ . But the first two observations establish the opposite weak inequality; thus, the third observation follows.<sup>8</sup> Fourth, it follows from the third observation that, for all  $\delta$  and  $t$ ,  $\delta\tilde{\pi}_t(b_{H\eta}) = \delta\tilde{\pi}_t(b_{L\eta})$ ; thus, the equilibrium conditions set out above must bind and  $\Pi(P(H, \eta), A(H, \eta) | t) = \Pi(P(L, \eta), A(L, \eta) | t)$ .

By Lemma 2.5, there exists  $c_e \in [c_o, H)$  such that  $\widehat{\Pi}(H, \eta) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ . Thus, type  $(H, \eta)$  is indifferent between  $(P_1, A_1, b_1) \equiv (P(H, \eta), A(H, \eta), b_{H\eta})$  and  $(P_2, A_2, b_2) \equiv (\psi(c_e), 0)$ . An incumbent with cost level  $c_e$  strictly prefers the latter option. Given that  $b_2 = 0 < b_1 = b_{H\eta}$  and  $c_e < H$ , Lemma 2.3 now ensures that type  $(L, \eta)$  strictly prefers the latter option. Using the fourth observation above and this strict preference, it thus follows that

$$\begin{aligned}\widehat{\Pi}(L, \eta) &\equiv \Pi(P(L, \eta), A(L, \eta) | L) + \eta\tilde{\pi}_L(b_{L\eta}) \\ &= \Pi(P(H, \eta), A(H, \eta) | L) + \eta\tilde{\pi}_L(b_{H\eta}) \\ &< \Pi(\psi(c_e) | L) + \eta\tilde{\pi}_L(0).\end{aligned}$$

Finally, consider type  $(H, \lambda)$ . Using the fourth observation above, this type is indifferent between its equilibrium selection and  $(P_1, A_1, b_1)$  and strictly prefers  $(P_1, A_1, b_1)$  to  $(P_2, A_2, b_2)$ , since

$$\begin{aligned}\widehat{\Pi}(H, \lambda) &= \Pi(P(H, \eta), A(H, \eta) | H) + \lambda\tilde{\pi}_H(b_{H\eta}) \\ &= \Pi(\psi(c_e) | H) + \lambda\tilde{\pi}_H(0) + (\eta - \lambda)(\tilde{\pi}_H(0) - \tilde{\pi}_H(b_{H\eta})) \\ &> \Pi(\psi(c_e) | H) + \lambda\tilde{\pi}_H(0).\end{aligned}$$

By Lemma 2.2, for  $\varepsilon > 0$  and sufficiently small,  $\psi(c_e - \varepsilon)$  is equilibrium dominated for  $(H, \eta)$  and  $(H, \lambda)$  but not for  $(L, \eta)$ . Under (E3),  $b(\psi(c_e - \varepsilon)) = 0$ . But then type  $(L, \eta)$  would deviate to  $\psi(c_e - \varepsilon)$ . A contradiction is thus obtained. ■

The proof of Lemma 4.2 is similar to the proof of Lemma 4.1, once it is established that type  $(L, \eta)$  earns the same payoff by following its equilibrium strategies as it would by following those of type  $(H, \eta)$ . This payoff relationship follows immediately in the proof of Lemma 4.1, since it is posited there that these types pool. In the proof of Lemma 4.2, the hypothesis is instead that types  $(L, \eta)$  and  $(H, \lambda)$  pool, and the payoff relationship is established through a series of observations about the consequent equilibrium behavior.

Together, Lemmas 4.1 and 4.2 establish that, in any intuitive equilibrium, the patient, low-cost incumbent must separate from both the patient and impatient

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<sup>8</sup>It is now evident that such an equilibrium is possible only for a non-generic specification of prior probabilities,  $b_o$  and  $\beta_o$ .

high-cost incumbents. It remains possible that the patient, low-cost incumbent pools with the impatient, low-cost incumbent. In any event, an immediate implication of the two lemmas is that the potential entrant infers that the incumbent has low costs, upon observing the pre-entry selection of the patient, low-cost incumbent; that is, in any intuitive equilibrium,  $b_{L\eta} = 0$ .

Given that the patient, low-cost incumbent must separate from the patient and impatient high-cost incumbents in any intuitive equilibrium, the next step is to determine the pre-entry selections that the patient, low-cost incumbent must make. The following lemma contains an important initial finding.

**Lemma 4.3.** *In any intuitive equilibrium,  $c_e < L$ , where  $c_e$  is defined in Lemma 2.5.*

**Proof:** Fix an intuitive equilibrium. First, assume to the contrary that  $c_e > L$ . From Lemma 2.5, it then follows that  $c_e \in (L, H)$  satisfies

$$\begin{aligned}\widehat{\Pi}(H, \eta) &= \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0) \\ &> \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(0),\end{aligned}$$

where the inequality follows from Lemma 2.2. Thus,  $\psi(L)$  is equilibrium dominated for type  $(H, \eta)$ . Consider type  $(H, \lambda)$ . Observe that

$$\begin{aligned}\widehat{\Pi}(H, \lambda) &\geq \Pi(P(H, \eta), A(H, \eta) | H) + \lambda\tilde{\pi}_H(b_{H\eta}) \\ &> \Pi(P(H, \eta), A(H, \eta) | H) + \lambda\tilde{\pi}_H(b_{H\eta}) + (\eta - \lambda)[\tilde{\pi}_H(b_{H\eta}) - \tilde{\pi}_H(0)] \\ &> \Pi(\psi(L) | H) + \lambda\tilde{\pi}_H(0),\end{aligned}$$

where the first inequality follows from (E1), the second inequality arises since  $b_{H\eta} > 0$  under (E2), and the third inequality is simply a restatement of the inequality just derived for  $\widehat{\Pi}(H, \eta)$ . Thus,  $\psi(L)$  is also equilibrium dominated for type  $(H, \lambda)$ . Of course,  $\psi(L)$  is not equilibrium dominated for type  $(L, \eta)$ . Hence, under (E3), if  $(P(L, \eta), A(L, \eta)) \neq \psi(L)$ , then  $b(\psi(L)) = 0$ , and type  $(L, \eta)$  would thus deviate to  $\psi(L)$ . Thus, an intuitive equilibrium is possible only if  $(P(L, \eta), A(L, \eta)) = \psi(L)$  and hence  $b(\psi(L)) = 0$ . But in this case type  $(L, \lambda)$  would also select  $\psi(L)$ , and thus it is necessary that  $b_{H\eta} = b_{H\lambda} = 1$ . Given these beliefs, (E1) can hold for types  $(H, \eta)$  and  $(H, \lambda)$  only if  $(P(H, \eta), A(H, \eta)) = (P(H, \lambda), A(H, \lambda)) = \psi(H)$ . But this specification cannot hold under Assumption 3, since type  $(H, \eta)$  would deviate from  $\psi(H)$  to  $\psi(L)$  and enjoy thereby the belief  $b(\psi(L)) = 0$  rather than  $b(\psi(H)) = 1$ . A contradiction is thus obtained.

Second, assume to the contrary that  $c_e = L$ . If  $(P(L, \eta), A(L, \eta)) = \psi(L)$ , then as before type  $(L, \lambda)$  would also select  $\psi(L)$ . Once again, it then would be necessary that  $b_{H\eta} = b_{H\lambda} = 1$  and thus that both types of the high-cost incumbent select  $\psi(H)$ , and under Assumption 3 a contradiction would emerge. Suppose then

that  $c_e = L$  and  $(P(L, \eta), A(L, \eta)) \neq \psi(L)$ . When  $c_e = L$ , it is straightforward to modify the inequalities set off above and confirm that  $\widehat{\Pi}(H, \eta) = \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(0)$  and  $\widehat{\Pi}(H, \lambda) > \Pi(\psi(L) | H) + \lambda\tilde{\pi}_H(0)$ . Given  $(P(L, \eta), A(L, \eta)) \neq \psi(L)$ , it follows that  $\widehat{\Pi}(L, \eta) < \Pi(\psi(L) | L) + \eta\tilde{\pi}_L(0)$ . Thus, for  $\varepsilon > 0$  and sufficiently small, Lemma 2.2 implies that  $\psi(L - \varepsilon)$  is equilibrium dominated for types  $(H, \eta)$  and  $(H, \lambda)$  but not for type  $(L, \eta)$ ; hence, under (E3),  $b(\psi(L - \varepsilon)) = 0$ . But then type  $(L, \eta)$  would deviate to  $\psi(L - \varepsilon)$ , and so a contradiction is again obtained. ■

This lemma indicates that, in intuitive equilibria, there is a limit as to the payoff that a patient, high-cost incumbent can enjoy (perhaps while pooling with an impatient, low-cost incumbent). As shown in the proof, if the patient, high-cost incumbent earns too much, in the sense that  $c_e \geq L$ , then the only possible intuitive equilibria entail the patient, low-cost incumbent making its monopoly selection,  $\psi(L)$ . The impatient, low-cost incumbent would then also make this selection, in order to enjoy monopoly profit in the pre-entry period along with the belief  $b_{L\eta} = 0$ . But then the high-cost incumbent types would be revealed and thus could do no better than to select their own monopoly selection,  $\psi(H)$ . At this stage a contradiction emerges, since under Assumption 3 a patient high-cost incumbent would mimic  $\psi(L)$ .

It is now possible to precisely describe the pre-entry selection that the patient, low-cost incumbent must make in an intuitive equilibrium.

**Proposition 4.4.** *In any intuitive equilibrium,  $b_{L\eta} = 0$ ,  $c_e < L$  and  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ , where  $c_e$  is defined in Lemma 2.5.*

**Proof:** Fix an intuitive equilibrium. As noted above, Lemmas 4.1 and 4.2 imply that  $b_{L\eta} = 0$ . Lemma 4.3 establishes that  $c_e < L$ . Thus, the remaining step is to show that  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ .

First, suppose to the contrary that  $(P(L, \eta), A(L, \eta)) \equiv (P_1, A_1)$  such that  $(P_1, A_1) \neq \psi(c_e)$  and  $\Pi(P_1, A_1 | H) + \eta\tilde{\pi}_H(0) = \widehat{\Pi}(H, \eta)$ . Define  $\psi(c_e) \equiv (P_2, A_2)$  and recall from Lemma 2.5 that  $\widehat{\Pi}(H, \eta) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ . Thus, type  $(H, \eta)$  is indifferent between  $(P_1, A_1, b_1 = 0)$  and  $(P_2, A_2, b_2 = 0)$ . Clearly, with  $b_1 = b_2$ , type  $(H, \lambda)$  is also indifferent between the two options. Thus,  $\widehat{\Pi}(H, \lambda) \geq \Pi(P_1, A_1 | H) + \lambda\tilde{\pi}_H(0) = \Pi(P_2, A_2 | H) + \lambda\tilde{\pi}_H(0)$ , where the inequality reflects the equilibrium condition that type  $(H, \lambda)$  satisfy (E1). Of course, a patient incumbent with cost level  $c_e$  would strictly prefer the latter option. It follows from Lemma 2.3 that type  $(L, \eta)$  strictly prefers the latter option as well:  $\Pi(P_2, A_2 | L) + \eta\tilde{\pi}_L(0) > \Pi(P_1, A_1 | L) + \eta\tilde{\pi}_L(0) = \widehat{\Pi}(L, \eta)$ . Consider a deviation to  $\psi(c_e - \varepsilon)$ . Under Lemma 2.2, for  $\varepsilon > 0$  and sufficiently small,  $\psi(c_e - \varepsilon)$  is equilibrium dominated for types  $(H, \eta)$  and  $(H, \lambda)$  but not for type  $(L, \eta)$ ; thus, under (E3),

$b(\psi(c_e - \varepsilon)) = 0$ . But then type  $(L, \eta)$  would deviate to  $\psi(c_e - \varepsilon)$ , and so (E1) fails for this type, contradicting the supposition of an intuitive equilibrium.

Second, suppose to the contrary that  $(P(L, \eta), A(L, \eta)) \equiv (P_1, A_1)$  such that  $\Pi(P_1, A_1 | H) + \eta\tilde{\pi}_H(0) < \hat{\Pi}(H, \eta)$ . Once again, define  $\psi(c_e) \equiv (P_2, A_2)$  and recall from Lemma 2.5 that  $\hat{\Pi}(H, \eta) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ . Observe that type  $(H, \eta)$  now strictly prefers  $(P_2, A_2, b_2 = 0)$  to  $(P_1, A_1, b_1 = 0)$ ; likewise, a patient incumbent with cost level  $c_e$  would also strictly prefer  $(P_2, A_2, b_2 = 0)$  to  $(P_1, A_1, b_1 = 0)$ . Given that  $c_e < L$ , Lemma 2.4 thus implies that a patient, low-cost incumbent has the same strict preference:  $\Pi(P_2, A_2 | L) + \eta\tilde{\pi}_L(0) > \Pi(P_1, A_1 | L) + \eta\tilde{\pi}_L(0)$ . Consider now a deviation to  $\psi(c_e - \varepsilon)$ , where  $\varepsilon > 0$  is sufficiently small. Using Lemma 2.2, this deviation is equilibrium dominated for type  $(H, \eta)$ . It is also equilibrium dominated for type  $(H, \lambda)$ , since

$$\begin{aligned} \hat{\Pi}(H, \lambda) &\geq \Pi(P(H, \eta), A(H, \eta) | H) + \lambda\tilde{\pi}_H(b_{H\eta}) \\ &= \Pi(P_2, A_2 | H) + \lambda\tilde{\pi}_H(0) + (\eta - \lambda)(\tilde{\pi}_H(0) - \tilde{\pi}_H(b_{H\eta})) \\ &> \Pi(P_2, A_2 | H) + \lambda\tilde{\pi}_H(0). \end{aligned}$$

The deviation is not equilibrium dominated for type  $(L, \eta)$ ; thus, under (E3), it follows that  $b(\psi(c_e - \varepsilon)) = 0$ . But then type  $(L, \eta)$  would deviate to  $\psi(c_e - \varepsilon)$ , and a contradiction is again obtained.

Third, observe that an intuitive equilibrium cannot exist with  $(P(L, \eta), A(L, \eta)) \equiv (P_1, A_1)$  such that  $\Pi(P_1, A_1 | H) + \eta\tilde{\pi}_H(0) > \hat{\Pi}(H, \eta)$ . If this inequality were to hold, then type  $(H, \eta)$  would deviate and select  $(P_1, A_1)$  and induce the associated belief,  $b_{L\eta} = 0$ , and thereby obtain a higher payoff. The only remaining possibility is that  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ , and the proof is thus complete. ■

Like the low-cost incumbent in the benchmark model, the patient, low-cost incumbent undertakes a cost-reducing distortion. This incumbent thus distorts downward its pre-entry price and distorts upward its demand-enhancing advertising. Profitable entry is not deterred, however, since limit pricing and high demand-enhancing advertising simply communicate cost information to the potential entrant. A further implication is that the patient, low-cost incumbent does not use dissipative advertising. Finally, the finding that the patient, low-cost incumbent must select  $\psi(c_e)$  implies that a downward incentive constraint for the patient, high-cost incumbent must bind in any intuitive equilibrium; specifically, type  $(H, \eta)$  must be indifferent between following its equilibrium selection and mimicking the selection of type  $(L, \eta)$ .

## 4.2. The Impatient, Low-Cost Incumbent

I next consider the behavior of the impatient, low-cost incumbent.

**Lemma 4.5.** *In any intuitive equilibrium, either  $(P(L, \lambda), A(L, \lambda)) = (P(L, \eta), A(L, \eta))$  or  $(P(L, \lambda), A(L, \lambda)) = (P(H, \eta), A(H, \eta))$ .*

**Proof:** Fix an intuitive equilibrium. By Proposition 4.4,  $b_{L\eta} = 0, c_e < L$  and  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ . First, assume to the contrary that  $(P(L, \lambda), A(L, \lambda)) \neq (P(t, \delta), A(t, \delta))$  for all  $(t, \delta) \neq (L, \lambda)$ . Then  $b_{L\lambda} = b_{L\eta} = 0$ , and so (E1) can hold for types  $(L, \eta)$  and  $(L, \lambda)$  only if  $\Pi(P(L, \eta), A(L, \eta) \mid L) = \Pi(P(L, \lambda), A(L, \lambda) \mid L)$ . Further, by the definition of  $c_e$ ,  $\hat{\Pi}(H, \eta) = \Pi(P(L, \eta), A(L, \eta) \mid H) + \eta\tilde{\pi}_H(0)$ ; and (E1) for type  $(H, \eta)$  also requires that  $\hat{\Pi}(H, \eta) \geq \Pi(P(L, \lambda), A(L, \lambda) \mid H) + \eta\tilde{\pi}_H(0)$ . Let  $(P_1, A_1, b_1) \equiv (P(L, \lambda), A(L, \lambda), 0)$  and  $(P_2, A_2, b_2) \equiv (P(L, \eta), A(L, \eta), 0)$ . Type  $(H, \eta)$  weakly prefers the latter option, and of course a patient incumbent with cost level  $c_e$  would strictly prefer that latter option. Since  $c_e < L$ , it then follows from Lemma 2.4 that type  $(L, \eta)$  also must strictly prefer the latter option:  $\Pi(P(L, \eta), A(L, \eta) \mid L) > \Pi(P(L, \lambda), A(L, \lambda) \mid L)$ . A contradiction is thus obtained. Thus, type  $(L, \lambda)$  must pool with some type. Second, assume to the contrary that  $(P(L, \lambda), A(L, \lambda)) = (P(H, \lambda), A(H, \lambda))$  and  $(P(L, \lambda), A(L, \lambda)) \neq (P(H, \eta), A(H, \eta))$ . Since  $b_{L\eta} = 0$ , this case is possible only if  $(P(L, \lambda), A(L, \lambda)) \neq (P(L, \eta), A(L, \eta))$ . By (E2),  $b_{H\eta} = 1$  is necessary, and so (E1) implies that  $(P(H, \eta), A(H, \eta)) = \psi(H)$ . Using (E2), it is also evident that  $b_{L\lambda} = b_{H\lambda} = b_o$ . For an equilibrium of this kind to exist, (E1) for type  $(H, \eta)$  requires that  $\Pi(\psi(H) \mid H) + \eta\tilde{\pi}_H(1) \geq \Pi(P(H, \lambda), A(H, \lambda) \mid H) + \eta\tilde{\pi}_H(b_o)$ . Rewriting this inequality and using  $b_o < 1$  reveals

$$\begin{aligned} & \Pi(\psi(H) \mid H) + \lambda\tilde{\pi}_H(1) \\ & \geq \Pi(P(H, \lambda), A(H, \lambda) \mid H) + \lambda\tilde{\pi}_H(b_o) + [\eta - \lambda][\tilde{\pi}_H(b_o) - \tilde{\pi}_H(1)] \\ & > \Pi(P(H, \lambda), A(H, \lambda) \mid H) + \lambda\tilde{\pi}_H(b_o). \end{aligned}$$

But this indicates that type  $(H, \lambda)$  should deviate and select  $\psi(H)$ , which contradicts (E1) for type  $(H, \lambda)$ . ■

The impatient, low-cost incumbent thus must pool with a patient incumbent with either the low or the high cost type. In the model with two-dimensional private information, some degree of pooling is thus a necessary property of any intuitive equilibrium. This finding offers a first contrast with the characterization of intuitive equilibria for the benchmark model. As shown in Proposition 3.4, pooling never occurs in the intuitive equilibria of the benchmark model.



### 4.3. The Three Kinds of Intuitive Equilibria

The results above establish that intuitive equilibria can be of three possible kinds:<sup>9</sup>

**Kind A:** In any intuitive equilibrium of Kind A, type  $(L, \eta)$  separates and all other types pool. Thus,  $(P(L, \eta), A(L, \eta)) \neq (P(H, \eta), A(H, \eta))$  and  $(P(H, \eta), A(H, \eta)) = (P(L, \lambda), A(L, \lambda)) = (P(H, \lambda), A(H, \lambda))$ . The associated beliefs are  $b_{L\eta} = 0$  and  $b_{H\eta} = b_A \equiv \frac{b_0}{b_0 + (1 - \beta_o)(1 - b_o)}$ .

**Kind B:** In any intuitive equilibrium of Kind B, types  $(L, \eta)$  and  $(L, \lambda)$  pool at one selection and types  $(H, \eta)$  and  $(H, \lambda)$  pool at another selection. Thus,  $(P(L, \eta), A(L, \eta)) = (P(L, \lambda), A(L, \lambda))$  and  $(P(H, \eta), A(H, \eta)) = (P(H, \lambda), A(H, \lambda))$ , where  $(P(L, \eta), A(L, \eta)) \neq (P(H, \eta), A(H, \eta))$ . The associated beliefs are  $b_{L\eta} = 0$  and  $b_{H\eta} = 1$ .

**Kind C:** In any intuitive equilibrium of Kind C, type  $(L, \eta)$  separates, type  $(H, \lambda)$  separates, and types  $(H, \eta)$  and  $(L, \lambda)$  pool. Thus,  $(P(L, \eta), A(L, \eta)) \neq (P(t, \delta), A(t, \delta))$  for all  $(t, \delta) \neq (L, \eta)$ ,  $(P(H, \lambda), A(H, \lambda)) \neq (P(t, \delta), A(t, \delta))$  for all  $(t, \delta) \neq (H, \lambda)$ , and  $(P(H, \eta), A(H, \eta)) = (P(L, \lambda), A(L, \lambda))$ . The associated beliefs are  $b_{L\eta} = 0$ ,  $b_{H\lambda} = 1$  and  $b_{H\eta} = b_C \equiv \frac{b_o \beta_o}{b_o \beta_o + (1 - b_o)(1 - \beta_o)}$ .

It is useful to identify some necessary properties for the selections of high-cost incumbents in intuitive equilibria. Further results of this type are immediately available for intuitive equilibria of Kind B and C.

**Lemma 4.6.** *In any intuitive equilibrium of Kind B,  $(P(L, \eta), A(L, \eta)) = \psi(c_o)$  and  $(P(H, \delta), A(H, \delta)) = \psi(H)$  for  $\delta \in \{\eta, \lambda\}$ , where  $c_o < L$  is defined by (2.1). In any intuitive equilibrium of Kind C,  $(P(H, \lambda), A(H, \lambda)) = \psi(H)$ .*

**Proof:** Fix an intuitive equilibrium of Kind B. Given that  $b_{H\eta} = 1$ , types  $(H, \eta)$  and  $(H, \lambda)$  cannot be deterred from selecting  $\psi(H)$ . This implies that  $\widehat{\Pi}(H, \eta) = \Pi(\psi(H) | H) + \eta \tilde{\pi}_H(0)$ , and so  $c_e = c_o$ . Next, fix an intuitive equilibrium of Kind C. Given that  $b_{H\lambda} = 1$ , type  $(H, \lambda)$  cannot be deterred from selecting  $\psi(H)$ . ■

It is instructive to compare this lemma with Corollary 3.5 of the benchmark model. The comparison indicates a correspondence between intuitive equilibria

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<sup>9</sup>The categorization below excludes the possibility that types  $(L, \eta)$  and  $(L, \lambda)$  pool at one selection while types  $(H, \eta)$  and  $(H, \lambda)$  separate. This possibility cannot arise in an intuitive equilibrium, since types  $(H, \eta)$  and  $(H, \lambda)$  would then both be revealed as high-cost types:  $b_{H\eta} = b_{H\lambda} = 1$ . But then both of these types must select  $\psi(H)$ , in contradiction to the supposition that they make separate selections.

of Kind B and intuitive equilibria for the benchmark model. Thus, intuitive equilibria of Kind B represent the extension of the findings by Milgrom and Roberts (1982) and Bagwell and Ramey (1988) to the model with two-dimensional private information. This correspondence is developed further in Section 6.

## 5. Intuitive Equilibria of Kind A

The analysis to this point has directed attention to three possible kinds of intuitive equilibria. I turn now to the existence of intuitive equilibria. In this section, I focus on intuitive equilibria of Kind A. To establish conditions under which intuitive equilibria of Kind A exist, I begin by identifying conditions under which such equilibria do *not* exist.

### 5.1. Non-existence and Low $\lambda$

At a broad level, if  $\lambda$  is very low, so that an impatient incumbent puts great weight on the pre-entry period, then it is clear that intuitive equilibria of Kind A cannot exist. The idea is simply that an impatient, low-cost incumbent would then only be willing to make selections that are sufficiently close to its monopoly selection,  $\psi(L)$ , and likewise an impatient, high-cost incumbent would then only be willing to make selections that are sufficiently close to its monopoly selection,  $\psi(H)$ . But in an intuitive equilibrium of Kind A these types pool. Thus, since  $\psi(L) \neq \psi(H)$ , intuitive equilibria of Kind A cannot exist if  $\lambda$  is sufficiently low.

A formal representation of this idea is provided in the following lemma.

**Lemma 5.1.** *There exists  $\underline{\lambda}_A > 0$  such that for all  $\lambda \in [0, \underline{\lambda}_A)$  an intuitive equilibrium of Kind A does not exist.*

**Proof:** Define

$$\underline{\lambda}_A \equiv \min_{(P,A)} \max \left\{ \frac{\Pi(\psi(L) | L) - \Pi(P, A | L)}{\tilde{\pi}_L(b_A) - \tilde{\pi}_L(1)}, \frac{\Pi(\psi(H) | H) - \Pi(P, A | H)}{\tilde{\pi}_H(b_A) - \tilde{\pi}_H(1)} \right\}. \quad (5.1)$$

Notice that  $\underline{\lambda}_A > 0$ . Now suppose that  $\lambda \in [0, \underline{\lambda}_A)$  and assume to the contrary that an intuitive equilibrium of Kind A exists. Then  $\lambda < \underline{\lambda}_A \leq$

$$\max \left\{ \frac{\Pi(\psi(L) | L) - \Pi(P(L, \lambda), A(L, \lambda) | L)}{\tilde{\pi}_L(b_A) - \tilde{\pi}_L(1)}, \frac{\Pi(\psi(H) | H) - \Pi(P(L, \lambda), A(L, \lambda) | H)}{\tilde{\pi}_H(b_A) - \tilde{\pi}_H(1)} \right\}.$$

Thus, for some  $t$ ,  $\Pi(\psi(t) | t) + \lambda \tilde{\pi}_t(1) > \Pi(P(L, \lambda), A(L, \lambda) | t) + \lambda \tilde{\pi}_t(b_A)$ . It follows that some type  $(t, \lambda)$  would earn a strictly higher payoff by deviating from

$(P(L, \lambda), A(L, \lambda))$  and making instead its monopoly selection,  $\psi(t)$ , even if the deviant selection induced the worst belief,  $b = 1$ . This contradicts the hypothesis of the existence of an intuitive equilibrium of Kind A. ■

I next offer a characterization of  $\underline{\lambda}_A$ , finding that  $\underline{\lambda}_A$  is achieved by a pair  $(P, A)$  that rests on the locus  $\psi(c)$  at some point between  $\psi(L)$  and  $\psi(H)$ .

**Lemma 5.2.** *There exists  $c_m \in (L, H)$  such that*

$$\underline{\lambda}_A = \frac{\Pi(\psi(L) | L) - \Pi(\psi(c_m) | L)}{\tilde{\pi}_L(b_A) - \tilde{\pi}_L(1)} = \frac{\Pi(\psi(H) | H) - \Pi(\psi(c_m) | H)}{\tilde{\pi}_H(b_A) - \tilde{\pi}_H(1)}. \quad (5.2)$$

The proof of this lemma is located in the Appendix. The value  $c_m$  is defined as the unique value in the interval  $(L, H)$  such that the second equality in (5.2) is satisfied. The content of the lemma is then that the first and second arguments in (5.1) are equal and indeed generate the minmax value  $\underline{\lambda}_A$  when  $(P, A) = \psi(c_m)$ .

The determination of  $c_m$  and  $\underline{\lambda}_A$  is illustrated in Figure 2. As this figure shows, the determination of these values is straightforward under the monotonicity properties identified in Lemma 2.2, once it is shown that  $\underline{\lambda}_A$  must be induced by a selection that rests on the uni-dimensional locus,  $\psi(c)$ . I focus henceforth on the existence of intuitive equilibria of Kind A in which pooling occurs at  $\psi(c_m)$ . Such equilibria are maximally robust against low values for  $\lambda$ , since no other pooling selection can be part of an intuitive equilibrium of Kind A when  $\lambda = \underline{\lambda}_A$ , and no intuitive equilibria of Kind A exist when  $\lambda < \underline{\lambda}_A$ .<sup>10</sup>

## 5.2. Non-existence and Low $\eta - \lambda$

A simple logic also suggests that intuitive equilibria of Kind A cannot exist if the difference between patience and impatience,  $\eta - \lambda$ , is too small. Recall from Proposition 4.4 that type  $(H, \eta)$  must be indifferent between adopting its equilibrium selection and mimicking type  $(L, \eta)$ 's selection. Now, in an intuitive equilibrium of Kind A, type  $(H, \eta)$  pools with types  $(H, \lambda)$  and  $(L, \lambda)$ . When  $\eta - \lambda$  is small, it follows that type  $(H, \lambda)$  is almost indifferent to mimicking type  $(L, \eta)$ 's selection. Type  $(L, \lambda)$  has the same patience level as type  $(H, \lambda)$ ; however, in terms of pre-entry profit, type  $(L, \lambda)$  is more attracted to cost-reducing distortions (i.e., the selection  $\psi(c_e)$ ) than is type  $(H, \lambda)$ . Thus, when  $\eta - \lambda$  is sufficiently small, type  $(L, \lambda)$  would deviate and mimic type  $(L, \eta)$ .

A formal expression of this logic is provided in the following lemma.

<sup>10</sup>It is also possible to hold the patience levels,  $\eta$  and  $\lambda$ , fixed and provide sufficient conditions for non-existence in terms of  $b_o$  and  $\beta_o$ . Observe that  $\underline{\lambda}_A$  grows without bound as  $b_A$  goes to unity. Thus, when  $b_A$  is sufficiently close to unity,  $\underline{\lambda}_A > \eta > \lambda$  and non-existence of intuitive equilibria of Kind A is assured. In turn,  $b_A$  approaches unity when  $b_o$  or  $\beta_o$  approaches unity.

**Lemma 5.3.** *In any intuitive equilibrium of Kind A in which  $(P(L, \lambda), A(L, \lambda)) = \psi(c_m)$ , it is necessary that (i)  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ , where  $c_e$  satisfies  $c_e < L$  and  $\widehat{\Pi}(H, \eta) \equiv \Pi(\psi(c_m) \mid H) + \eta\tilde{\pi}_H(b_A) = \Pi(\psi(c_e) \mid H) + \eta\tilde{\pi}_H(0)$ , and (ii)  $\lambda \leq \bar{\lambda}_A$ , where  $\bar{\lambda}_A < \eta$  is defined by*

$$\bar{\lambda}_A \equiv \frac{\Pi(\psi(c_m) \mid L) - \Pi(\psi(c_e) \mid L)}{\tilde{\pi}_L(0) - \tilde{\pi}_L(b_A)}. \quad (5.3)$$

**Proof:** Fix an intuitive equilibrium of Kind A in which  $(P(L, \lambda), A(L, \lambda)) = \psi(c_m)$ . Since types  $(L, \lambda)$  and  $(H, \eta)$  pool, it follows that  $\widehat{\Pi}(H, \eta) \equiv \Pi(\psi(c_m) \mid H) + \eta\tilde{\pi}_H(b_A)$ . The application of Proposition 4.4 implies that  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ , where  $c_e$  satisfies  $c_e < L$  and  $\widehat{\Pi}(H, \eta) = \Pi(\psi(c_e) \mid H) + \eta\tilde{\pi}_H(0)$ . It follows that  $c_e$  is uniquely defined in any intuitive equilibrium of Kind A in which  $(P(L, \lambda), A(L, \lambda)) = \psi(c_m)$ ; thus,  $c_e$  is well-defined in (5.3). I now show that  $\lambda \leq \bar{\lambda}_A$ . The posited equilibrium can exist only if type  $(L, \lambda)$  does not gain by deviating from  $(\psi(c_m), b_A)$  to  $(\psi(c_e), 0)$ . Referring to (5.3), observe that  $\widehat{\Pi}(L, \lambda) \equiv \Pi(\psi(c_m) \mid L) + \lambda\tilde{\pi}_L(b_A) \geq \Pi(\psi(c_e) \mid L) + \lambda\tilde{\pi}_L(0)$  if and only if  $\lambda \leq \bar{\lambda}_A$ . Thus, the posited equilibrium can exist only if  $\lambda \leq \bar{\lambda}_A$ . Finally, using (5.3), observe that  $\bar{\lambda}_A < \eta$  if and only if  $\Pi(\psi(c_e) \mid L) + \eta\tilde{\pi}_L(0) > \Pi(\psi(c_m) \mid L) + \eta\tilde{\pi}_L(b_A)$ . Let  $(P_1, A_1, b_1) \equiv (\psi(c_m), b_A)$  and  $(P_2, A_2, b_2) \equiv (\psi(c_e), 0)$ . By construction, type  $(H, \eta)$  is indifferent between these two options. An incumbent with cost level  $c_e$  strictly prefers the latter option. Given that  $b_2 = 0 < b_1 = b_A$  and  $c_e < H$ , Lemma 2.3 ensures that type  $(L, \eta)$  strictly prefers the latter option. Thus,  $\bar{\lambda}_A < \eta$ . ■

Thus, by Lemmas 5.1 and 5.3, if an intuitive equilibrium of Kind A is to exist, then  $\lambda$  can neither be too low (i.e., below  $\underline{\lambda}_A$ ) nor too near  $\eta$  (i.e., above  $\bar{\lambda}_A$  and thereby near  $\eta$ ).

### 5.3. Existence

Guided by the preceding results, I now state sufficient conditions for the existence of an intuitive equilibrium of Kind A. In order to establish existence for the lowest possible values of  $\lambda$  (i.e., even when  $\lambda = \underline{\lambda}_A$ ), the equilibrium is specified so that the pooling selection is  $\psi(c_m)$ , where  $c_m \in (L, H)$  is defined in Lemma 5.2. The sufficient conditions are motivated as follows. As suggested by Lemma 5.3, a first assumption is that  $\lambda$  is bound below  $\eta$ . Consistent with Lemma 5.1, a second assumption is that  $\lambda$  is not too low. Finally, an intuitive equilibrium can exist only if  $c_e < L$ . As explained below, however, this inequality follows from the first assumption. I argue below that both assumptions hold if  $\eta$  is sufficiently large and  $\lambda$  is not too low.

Formally, the existence result is now captured in the following proposition.

**Proposition 5.4.** *Assume*

$$\lambda \leq \bar{\lambda}_A \equiv \frac{\Pi(\psi(c_m) | L) - \Pi(\psi(c_e) | L)}{\tilde{\pi}_L(0) - \tilde{\pi}_L(b_A)} \quad (5.4)$$

$$\lambda \geq \underline{\lambda}_A = \frac{\Pi(\psi(t) | t) - \Pi(\psi(c_m) | t)}{\tilde{\pi}_t(b_A) - \tilde{\pi}_t(1)}, \quad t = L, H \quad (5.5)$$

where  $c_m \in (L, H)$  is defined in Lemma 5.2 and  $c_e < c_m$  is defined by  $\Pi(\psi(c_m) | H) + \eta \tilde{\pi}_H(b_A) = \Pi(\psi(c_e) | H) + \eta \tilde{\pi}_H(0)$ . Then there exists an intuitive equilibrium of Kind A in which  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$  and  $(P(t, \delta), A(t, \delta)) = \psi(c_m)$  for  $(t, \delta) \neq (L, \eta)$ , where  $c_e < L$ .

The proof of this proposition is located in the Appendix.

Consider the ranges for  $\eta$  and  $\lambda$  over which (5.4) and (5.5) hold. Clearly, (5.5) holds if  $\lambda$  is not too small. Consider (5.4). Given the definition of  $c_e < c_m < H$ , it follows as in the proof of Lemma 5.3 that  $\eta > \bar{\lambda}_A$ . Observe that  $\bar{\lambda}_A$  is independent of  $\lambda$  and depends on  $\eta$  through  $c_e$ . Lemma 2.2 implies that  $c_e$  is strictly decreasing in  $\eta$ . When  $\eta$  is near zero,  $c_e$  is approximately  $c_m$ . As  $\eta$  rises,  $c_e$  falls toward and then below  $L$ . Referring to (5.4) and using Lemma 2.2, it thus follows that  $\bar{\lambda}_A$  is approximately zero when  $\eta$  is near zero, strictly decreases as  $\eta$  rises toward the level that induces  $c_e = L$ , and then strictly increases as  $\eta$  rises beyond this level. Given  $\lambda \geq 0$ , (5.4) requires  $\bar{\lambda}_A \geq 0$  and thus that  $c_e < L$ . In fact, (5.4) and (5.5) require that  $\eta$  is sufficiently high that  $\bar{\lambda}_A$  rises to or above  $\underline{\lambda}_A$ . I conclude that an intuitive equilibrium of Kind A exists if the impatient type of incumbent is not too impatient and the patient type of incumbent is sufficiently patient.<sup>11</sup>

The constructed intuitive equilibrium is not the only such equilibrium of Kind A. Other constructions may exist which employ different pooling selections. The intuitive equilibrium analyzed in Proposition 5.4 deserves particular attention, however, since its existence is maximally robust to lower values of  $\lambda$ .

#### 5.4. Economic Interpretation

I consider now the economic interpretation of the constructed intuitive equilibrium of Kind A. The behavior of the patient, low-cost incumbent is analogous to that of the low-cost incumbent in the benchmark model. This type of incumbent undertakes a cost-reducing distortion and does not deter profitable entry. The

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<sup>11</sup>For  $\eta$  sufficiently high, it is possible to achieve  $\bar{\lambda}_A \geq \underline{\lambda}_A$  while respecting  $c_e > 0$ , if  $\Pi(\psi(0) | L)$  is sufficiently low (and possibly negative) or if  $b_A$  is sufficiently small. It is possible that  $\eta > 1$  is then required. This possibility can be easily entertained if the post-entry stage allows for multiple periods.

other types of incumbent behave in a more novel fashion. In the constructed equilibrium, the patient and impatient high-cost incumbent types also engage in a cost-reducing distortion and thus limit price and distort demand-enhancing advertising upward. These types, however, pool with the impatient, low-cost incumbent. The potential entrant is then unable to infer the incumbent's cost type; as a consequence, when the incumbent actually has a high-cost type, entry that would be profitable under complete information may not occur in equilibrium. In other words, when the incumbent has high costs, limit pricing and high advertising may be used to deter profitable entry. The behavior of the impatient, low-cost incumbent is perhaps most unusual. This type of incumbent engages in a cost-increasing distortion, with a pre-entry price that exceeds its monopoly price and a level of demand-enhancing advertising that is below its monopoly level, and sometimes faces entry that would not have been profitable under complete information. Thus, this type of incumbent adopts a "soft" form of pre-entry behavior and sometimes ends up inducing unprofitable entry.

The constructed equilibrium offers a partial confirmation of the predictions of Milgrom and Roberts (1982) and Bagwell and Ramey (1988). This confirmation is captured in the behavior of the patient, low-cost incumbent. The equilibrium also offers support for the predictions of an earlier literature, wherein contributions by Bain (1949) and Williamson (1963) describe conditions under which limit pricing and high advertising may deter entry. The predictions of the earlier literature find representation in the behavior of the patient and impatient high-cost incumbent types. Finally, the constructed equilibrium identifies a new behavior for the type of incumbent that is impatient and has low costs. This incumbent adopts a soft pre-entry stance and sometimes induces unprofitable entry.

## 6. Intuitive Equilibria of Kind B

I now focus on intuitive equilibria of Kind B. Once again, I begin by identifying conditions under which such equilibria do *not* exist. I then provide sufficient conditions for existence.

### 6.1. Non-existence and Low $\lambda$

In general terms, it is clear that an intuitive equilibrium of Kind B cannot exist when  $\lambda$  is sufficiently small. Recall from Lemma 4.6 that an intuitive equilibrium of Kind B exists only if, regardless of the level of patience, an incumbent with a low cost type selects  $\psi(c_o)$  while an incumbent with a high cost type selects  $\psi(H)$ . If  $\lambda$  is small, however, an impatient, low-cost incumbent is unwilling to make a

selection that is not close to  $\psi(L)$ . Given that  $c_o < L$ , it is thus inevitable that an intuitive equilibrium of Kind B fails to exist when  $\lambda$  is sufficiently small.

A formal representation of this argument is provided in the following lemma.

**Lemma 6.1.** *There exists  $\underline{\lambda}_B \in (0, \eta)$  such that for all  $\lambda \in [0, \underline{\lambda}_B)$  an intuitive equilibrium of Kind B does not exist.*

**Proof:** Define

$$\underline{\lambda}_B \equiv \frac{\Pi(\psi(L) | L) - \Pi(\psi(c_o) | L)}{\tilde{\pi}_L(0) - \tilde{\pi}_L(1)},$$

where  $c_o < L$  is defined by (2.1). Thus,  $\underline{\lambda}_B > 0$ . Further,  $\underline{\lambda}_B < \eta$  if and only if  $\Pi(\psi(c_o) | L) + \eta\tilde{\pi}_L(0) > \Pi(\psi(L) | L) + \eta\tilde{\pi}_L(1)$ , and the latter inequality is established using Lemma 2.4 in the first step of the proof of Proposition 3.3. Suppose now that  $\lambda \in [0, \underline{\lambda}_B)$  and that an intuitive equilibrium of Kind B exists. In such an equilibrium, type  $(L, \lambda)$  must prefer  $(\psi(c_o), b_{L\lambda} = 0)$  to  $(\psi(L), b(\psi(L)))$ . Thus, it is necessary that

$$\Pi(\psi(c_o) | L) + \lambda\tilde{\pi}_L(0) \geq \Pi(\psi(L) | L) + \lambda\tilde{\pi}_L(b(\psi(L))) \geq \Pi(\psi(L) | L) + \lambda\tilde{\pi}_L(1).$$

But this implies that  $\lambda \geq \underline{\lambda}_B$ , and so a contradiction is obtained. ■

It is useful to emphasize the finding that  $\underline{\lambda}_B \in (0, \eta)$ . This ensures that there always exists a range of possible values for  $\lambda$  such that  $\lambda \geq \underline{\lambda}_B$  and  $\eta > \lambda$ .

Thus, intuitive equilibria of Kind B cannot exist if the impatient incumbent is too impatient. As shown in Lemma 5.1, a similar non-existence result applies for intuitive equilibria of Kind A. It is thus possible to state the following corollary:

**Corollary 6.2.** *Let  $\underline{\lambda} \equiv \min\{\underline{\lambda}_A, \underline{\lambda}_B\} \in (0, \eta)$ . For  $\lambda < \underline{\lambda}$ , there does not exist an intuitive equilibrium of Kind A or Kind B.*

Thus, if an impatient incumbent is sufficiently impatient, then an intuitive equilibrium can exist only if it is of Kind C.

## 6.2. Existence

I next consider the existence of an intuitive equilibrium of Kind B.

**Proposition 6.3.** *Assume*

$$\lambda \geq \underline{\lambda}_B \equiv \frac{\Pi(\psi(L) | L) - \Pi(\psi(c_o) | L)}{\tilde{\pi}_L(0) - \tilde{\pi}_L(1)},$$

where  $c_o < L$  is defined by (2.1). Then there exists an intuitive equilibrium of Kind B in which  $(P(L, \eta), A(L, \eta)) = (P(L, \lambda), A(L, \lambda)) = \psi(c_o)$  and  $(P(H, \eta), A(H, \eta)) = (P(H, \lambda), A(H, \lambda)) = \psi(H)$ .

The proof of this proposition is located in the Appendix.

Recall from Lemma 6.1 that  $\underline{\lambda}_B \in (0, \eta)$ ; thus, for any given  $\eta$  and  $\lambda$  such that  $\lambda \in [\underline{\lambda}_B, \eta)$ , an intuitive equilibrium of Kind B exists. Given  $\eta > \lambda$ , an intuitive equilibrium of Kind B thus exists if  $\lambda$  is not too small, so that  $\lambda \geq \underline{\lambda}_B$ . The sufficient condition may also be interpreted in terms of the size of  $\eta - \lambda$ . When  $\eta$  is near its lower bound, so that the inequality in Assumption 3 just holds,  $\underline{\lambda}_B$  is near zero. As  $\eta$  rises,  $\underline{\lambda}_B$  remains below  $\eta$ ; however, it follows from (2.1) that  $c_o$  strictly declines and thus by Lemma 2.2 that  $\underline{\lambda}_B$  strictly increases. Thus,  $\lambda \geq \underline{\lambda}_B$  ensures that  $\lambda$  never falls too far below  $\eta$ . The constructed intuitive equilibrium captures the only intuitive equilibrium outcome of Kind B. This outcome is thus uniquely predicted when  $\eta - \lambda$  is small, since intuitive equilibria of Kind A and C (see below) fail to exist in that case.

### 6.3. Economic Interpretation

Intuitive equilibria of Kind B capture the themes developed in the models by Milgrom and Roberts (1982) and Bagwell and Ramey (1988), wherein the incumbent has one dimension of private information. This work is represented above in the analysis of the benchmark model. In the present model, the incumbent has two dimensions of private information; yet, in intuitive equilibria of Kind B, the incumbent's behavior is identical to that predicted by the benchmark model. In particular, whether the incumbent is patient or impatient, a low-cost incumbent undertakes a cost-reducing distortion and a high-cost incumbent makes its monopoly selection and reveals its cost type. The incumbent's private information as to its patience level then does not affect the incumbent's pre-entry selections nor the potential entrant's entry decision. As noted above, however, intuitive equilibria of Kind B cannot exist if an impatient incumbent is too impatient.

## 7. Existence of Intuitive Equilibria of Kind C

I now focus on intuitive equilibria of Kind C. The existence of such equilibria is of particular interest given the finding presented in Corollary 6.2 that neither intuitive equilibria of Kind A nor Kind B exist when the impatient incumbent is too impatient. Thus, if an intuitive equilibrium is to exist in this situation, then it must be an intuitive equilibrium of Kind C. As before, I first explore conditions under which such equilibria do *not* exist, and I then provide sufficient conditions for existence.



### 7.1. Nonexistence and Low $\lambda$ or $\eta$

In any intuitive equilibrium of Kind C, type  $(L, \eta)$  separates from all other types, types  $(H, \eta)$  and  $(L, \lambda)$  pool, and type  $(H, \lambda)$  also separates from all other types. The associated beliefs are  $b_{L\eta} = 0$ ,  $b_{H\lambda} = 1$  and  $b_{H\eta} = b_C$ . Further, as Lemma 4.6 indicates, type  $(H, \lambda)$  separates by selecting  $\psi(H)$ . Finally, according to Proposition 4.4, type  $(L, \eta)$  separates by selecting  $\psi(c_e)$ , where  $c_e$  must be strictly less than  $L$  and satisfy  $\tilde{\Pi}(H, \eta) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ .

As summarized in Corollary 6.2, the existence of intuitive equilibria of Kind A or B is impossible when  $\lambda$  is sufficiently small. The key tension is that such intuitive equilibria impose requirements on the selections of impatient incumbents; however, when  $\lambda$  is sufficiently small, an impatient incumbent is only willing to make selections that are sufficiently close to its monopoly selection. In intuitive equilibria of Kind C, this tension is less pronounced. First, such equilibria require that an incumbent of type  $(H, \lambda)$  make its monopoly selection,  $\psi(H)$ . Second, such equilibria offer some flexibility with respect to the selection at which types  $(H, \eta)$  and  $(L, \lambda)$  pool. One important possibility is that these types pool at the low-cost monopoly selection,  $\psi(L)$ . The discussion here thus suggests that intuitive equilibria of Kind C may exist even when  $\lambda$  is very low (e.g., zero). Notice, though, that a related incentive issue then may arise for type  $(H, \eta)$ . If  $\eta$  is small as well, then a patient, high-cost incumbent may be unwilling to make a selection such as  $\psi(L)$  that differs from its monopoly selection,  $\psi(H)$ .

I now explore these issues at a more formal level. Suppose that an intuitive equilibrium of Kind C exists. It then must be the case that types  $(L, \lambda)$  and  $(H, \eta)$  pool at some selection  $(P(L, \lambda), A(L, \lambda))$ . The associated incentive constraints imply restrictions on  $\lambda$  and  $\eta$ .

First, it is necessary that type  $(L, \lambda)$  cannot gain by deviating to its monopoly selection. If such a deviation induces the most pessimistic belief, so that  $b(\psi(L)) = 1$ , then the relevant condition is  $\Pi(P(L, \lambda), A(L, \lambda) | L) + \lambda\tilde{\pi}_L(b_C) \geq \Pi(\psi(L) | L) + \lambda\tilde{\pi}_L(1)$ . This condition may be re-written as

$$\lambda \geq \underline{\lambda}_C \equiv \frac{\Pi(\psi(L) | L) - \Pi(P(L, \lambda), A(L, \lambda) | L)}{\tilde{\pi}_L(b_C) - \tilde{\pi}_L(1)}. \quad (7.1)$$

Using (7.1), notice that  $\underline{\lambda}_C = 0$  when  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$ ; thus, when pooling occurs at  $\psi(L)$ , the lower bound on  $\lambda$  is in fact zero.

Second, it is necessary that type  $(H, \eta)$  cannot gain by deviating to its monopoly selection. Since  $b(\psi(H)) \equiv b_{H\lambda} = 1$ , the relevant condition is  $\Pi(P(L, \lambda), A(L, \lambda) | H) + \eta\tilde{\pi}_H(b_C) \geq \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1)$ . Equivalently, this condition is given as

$$\eta \geq \underline{\eta}_C \equiv \frac{\Pi(\psi(H) | H) - \Pi(P(L, \lambda), A(L, \lambda) | H)}{\tilde{\pi}_H(b_C) - \tilde{\pi}_H(1)} \quad (7.2)$$

By (7.2),  $\underline{\eta}_C = 0$  if  $(P(L, \lambda), A(L, \lambda)) = \psi(H)$ ; however, when  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$ ,  $\underline{\eta}_C > 0$ .

As conditions (7.1) and (7.2) illustrate, if an intuitive equilibrium of Kind C specifies pooling at the low-cost monopoly selection,  $\psi(L)$ , then  $\underline{\lambda}_C = 0$  and so  $\lambda$  can assume any nonnegative value. In this case, however,  $\underline{\eta}_C > 0$ , and as a consequence it is necessary that  $\eta$  be sufficiently large. Alternatively, if an intuitive equilibrium of Kind C specifies pooling at  $\psi(H)$ , then  $\underline{\lambda}_C > 0$ , and so  $\lambda$  cannot be too small. For this specification,  $\underline{\eta}_C = 0$ ; thus, no additional restrictions arise for  $\eta$  beyond the maintained assumption that  $\eta > \lambda$ . A further possibility is that an intuitive equilibrium specifies pooling at  $\psi(c)$  for some  $c \in (L, H)$ . In that case,  $\underline{\lambda}_C > 0$  and  $\underline{\eta}_C > 0$  would be required.

As above, I emphasize the intuitive equilibrium whose existence is most robust against low values for  $\lambda$ . This focus is perhaps especially appropriate at this juncture, since Corollary 6.2 raises the question of whether any intuitive equilibrium exists when  $\lambda$  is small. Thus, in the following, I focus on the potential existence of intuitive equilibria of Kind C in which pooling occurs at the low-cost monopoly selection:  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$ . Henceforth, I thus define  $\underline{\eta}_C$  as

$$\underline{\eta}_C \equiv \frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(b_C) - \tilde{\pi}_H(1)}, \quad (7.3)$$

where  $\underline{\eta}_C > 0$ . The next lemma contains the corresponding non-existence result.

**Lemma 7.1.** *For all  $\eta < \underline{\eta}_C$  an intuitive equilibrium of Kind C in which  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$  does not exist.*

## 7.2. Non-existence and Low $\eta - \lambda$

Like intuitive equilibria of Kind A, intuitive equilibria of Kind C confront existence problems when the difference between patience and impatience,  $\eta - \lambda$ , is small. The general idea is easily related. As established in Proposition 4.4, type  $(H, \eta)$  must be indifferent between adopting its equilibrium selection and mimicking type  $(L, \eta)$ 's selection. In an intuitive equilibrium of Kind C, type  $(H, \eta)$  pools with type  $(L, \lambda)$ . When  $\eta - \lambda$  is small, type  $(L, \lambda)$  has almost the same patience level as type  $(H, \eta)$ ; however, type  $(L, \lambda)$  is more attracted to cost-reducing distortions (i.e., the selection  $\psi(c_e)$ ) than is type  $(H, \eta)$ . Thus, when  $\eta - \lambda$  is sufficiently small, type  $(L, \lambda)$  would deviate and mimic type  $(L, \eta)$ .

This idea is captured at a formal level in the following lemma.

**Lemma 7.2.** *In any intuitive equilibrium of Kind C in which  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$ , it is necessary that (i)  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ , where  $c_e$  satisfies  $c_e < L$*

and  $\widehat{\Pi}(H, \eta) \equiv \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(b_C) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ , and (ii)  $\lambda \leq \bar{\lambda}_C$ , where  $\bar{\lambda}_C \in (0, \eta)$  is defined by

$$\bar{\lambda}_C \equiv \frac{\Pi(\psi(L) | L) - \Pi(\psi(c_e) | L)}{\tilde{\pi}_L(0) - \tilde{\pi}_L(b_C)}. \quad (7.4)$$

**Proof:** Fix an intuitive equilibrium of Kind C in which  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$ . Then  $\widehat{\Pi}(H, \eta) \equiv \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(b_C)$ . By Proposition 4.4,  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ , where  $c_e$  satisfies  $c_e < L$  and  $\widehat{\Pi}(H, \eta) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ . Thus,  $c_e$  is uniquely defined for any intuitive equilibrium of Kind C in which  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$ , and so  $c_e$  is well-defined in (7.4). I now show that  $\lambda \leq \bar{\lambda}_C$ . The posited equilibrium can exist only if type  $(L, \lambda)$  does not gain by deviating from  $(\psi(L), b_C)$  to  $(\psi(c_e), 0)$ . Referring to (7.4), observe that  $\widehat{\Pi}(L, \lambda) \equiv \Pi(\psi(L) | L) + \lambda\tilde{\pi}_L(b_C) \geq \Pi(\psi(c_e) | L) + \lambda\tilde{\pi}_L(0)$  if and only if  $\bar{\lambda}_C \geq \lambda$ . Thus, the posited equilibrium can exist only if  $\lambda \leq \bar{\lambda}_C$ . Next, observe from (7.4) that  $\bar{\lambda}_C > 0$ . Finally, using (7.4) observe that  $\eta > \bar{\lambda}_C$  if and only if  $\Pi(\psi(c_e) | L) + \eta\tilde{\pi}_L(0) > \Pi(\psi(L) | L) + \eta\tilde{\pi}_L(b_C)$ . Let  $(P_1, A_1, b_1) \equiv (\psi(L), b_C)$  and  $(P_2, A_2, b_2) \equiv (\psi(c_e), 0)$ . By construction, type  $(H, \eta)$  is indifferent between these two options. An incumbent with cost level  $c_e$  strictly prefers the latter option. Given that  $b_2 = 0 < b_1 = b_C$  and  $c_e < H$ , Lemma 2.3 ensures that type  $(L, \eta)$  strictly prefers the latter option. Thus,  $\eta > \bar{\lambda}_C$ . ■

Thus, for such an equilibrium to exist,  $\lambda$  cannot be too high (i.e., above  $\bar{\lambda}_C$  and thereby near  $\eta$ ).

### 7.3. Non-existence and High $\lambda$

In an intuitive equilibrium of Kind C, an impatient, high-cost incumbent simply makes its monopoly selection and reveals its type. This type thus induces the belief  $b_{H\lambda} = 1$  and faces the maximal probability of entry. If this type is not too impatient, then it thus may be tempted to mimic the pooling selection and enjoy thereby the belief  $b_C$ . When pooling occurs at  $\psi(L)$ , an intuitive equilibrium of Kind C can thus exist only if  $\lambda$  is sufficiently low that an impatient, high-cost incumbent prefers  $(\psi(H), 1)$  to  $(\psi(L), b_C)$ .

This logic is confirmed in the following lemma:

**Lemma 7.3.** *For all  $\lambda > \underline{\eta}_C$ , an intuitive equilibrium of Kind C in which  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$  does not exist.*

**Proof:** Fix  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$  and recall that (7.3) defines  $\underline{\eta}_C > 0$ . Assume to the contrary that  $\lambda > \underline{\eta}_C$  and an intuitive equilibrium of Kind C in which  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$  exists. Under the latter assumption, (E1)

and (E2) imply that type  $(H, \lambda)$  prefers  $(\psi(H), 1)$  to  $(\psi(L), b_C)$ . Thus,  $\Pi(\psi(H) | H) + \lambda \tilde{\pi}_H(1) \geq \Pi(\psi(L) | H) + \lambda \tilde{\pi}_H(b_C)$ . But by (7.3) this is possible if and only if  $\lambda \leq \underline{\eta}_C$ , which contradicts the former assumption. ■

Together, Lemmas 7.1-7.3 suggest that an intuitive equilibrium of Kind C in which pooling occurs at  $\psi(L)$  may exist, if  $\eta$  is sufficiently large and  $\lambda$  is sufficiently small both absolutely and relative to  $\eta$ . This suggestion is explored in more detail below.

#### 7.4. Existence

Drawing on the preceding results, I now identify sufficient conditions for the existence of an intuitive equilibrium of Kind C. To establish existence for the lowest possible values of  $\lambda$  (i.e., even when  $\lambda = 0$ ), the equilibrium is specified so that the pooling selection is  $\psi(L)$ . The sufficient conditions are then expressed in three assumptions. As suggested by Lemma 7.1, the first assumption is that  $\eta$  is sufficiently large. Next, as suggested by Lemma 7.2, a second assumption is that  $\lambda$  is bound below  $\eta$ . Finally, as suggested by Lemma 7.3, a third assumption is that  $\lambda$  not be too high. As discussed further below, all of these assumptions are sure to hold if  $\eta$  is sufficiently large and  $\lambda$  is sufficiently small.

Formally, the existence result is now captured in the following proposition:

**Proposition 7.4.** *Assume*

$$\eta \geq \underline{\eta}_C \equiv \frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(b_C) - \tilde{\pi}_H(1)} \quad (7.5)$$

$$\lambda \leq \bar{\lambda}_C \equiv \frac{\Pi(\psi(L) | L) - \Pi(\psi(c_e) | L)}{\tilde{\pi}_L(0) - \tilde{\pi}_L(b_C)} \quad (7.6)$$

$$\lambda \leq \underline{\eta}_C \equiv \frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(b_C) - \tilde{\pi}_H(1)}, \quad (7.7)$$

where  $c_e < L$  is defined by  $\Pi(\psi(c_e) | H) + \eta \tilde{\pi}_H(0) = \Pi(\psi(L) | H) + \eta \tilde{\pi}_H(b_C)$ . Then there exists an intuitive equilibrium of Kind C in which  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ ,  $(P(H, \lambda), A(H, \lambda)) = \psi(H)$  and  $(P(L, \lambda), A(L, \lambda)) = (P(H, \eta), A(H, \eta)) = \psi(L)$ .

The proof of this proposition is located in the Appendix.

An intuitive equilibrium of Kind C thus exists under assumptions (7.5)-(7.7). Clearly, (7.5) holds if  $\eta$  is sufficiently large, and (7.7) holds if  $\lambda$  is sufficiently small. Consider (7.6). With  $c_e$  defined by  $\Pi(\psi(c_e) | H) + \eta \tilde{\pi}_H(0) = \Pi(\psi(L) | H) + \eta \tilde{\pi}_H(b_C)$  and  $b_C > 0$ , it follows that  $c_e < L$ . As shown in the proof of

Lemma 7.2,  $\eta > \bar{\lambda}_C > 0$  also follows. Observe that  $\bar{\lambda}_C$  is independent of  $\lambda$  and depends on  $\eta$  only through  $c_e$ . Lemma 2.2 implies that  $c_e$  is strictly decreasing in  $\eta$ . By (7.6), Lemma 2.2 thus also implies that  $\bar{\lambda}_C$  is strictly increasing in  $\eta$ . Hence, an intuitive equilibrium of Kind C exists if the impatient type of incumbent is sufficiently impatient and the patient type of incumbent is sufficiently patient.

Under appropriate assumptions, other intuitive equilibrium of Kind C can be constructed which employ different values for the pooling selection. As emphasized above, however, the constructed intuitive equilibrium deserves particular attention, since its existence is maximally robust to lower values  $\lambda$ . Indeed, the constructed equilibrium outcome is the only intuitive equilibrium outcome of Kind C that exists even when  $\lambda = 0$ .

### 7.5. Economic Interpretation

In the constructed equilibrium, the behavior of the patient, low-cost incumbent is again analogous to that of the low-cost incumbent in the benchmark model. Separation occurs in the form of a cost-reducing distortion, and profitable entry is not deterred. Also, the behavior of the impatient, high-cost incumbent is analogous to that of the high-cost incumbent in the benchmark model. The price-advertising selection is not distorted, and entry occurs precisely when it is profitable. The novel behavior is displayed by the impatient, low-cost and patient, high-cost types of incumbent. These types pool, and in the constructed equilibrium they pool at the low-cost monopoly selection. The potential entrant is unable to infer the incumbent's cost type upon seeing this selection; hence, when the incumbent has high costs, profitable entry is sometimes deterred. The impatient, high-cost incumbent therefore uses limit pricing and high demand-enhancing advertising so as to hide its type and, thereby, deter entry that would be profitable were its cost type known. The behavior of the impatient, low-cost incumbent is again striking. In the constructed equilibrium, this type of incumbent does not distort its price-advertising selection and sometimes faces entry that would be unprofitable were its cost type known.

The constructed intuitive equilibrium of Kind C has important similarities with the constructed equilibrium of Kind A. The key difference is that, in the former, fewer distortions occur, since the incumbent does not distort its behavior when it is impatient, regardless of its cost type. It is precisely this feature that ensures that the constructed equilibrium of Kind C can exist even when  $\lambda = 0$ .

## 8. Intuitive Equilibria and Complete Impatience

The preceding analysis is conducted under the assumption that the incumbent may be patient or impatient:  $\delta = \eta$  or  $\delta = \lambda$ , where  $\eta > \lambda$ . In the present section, I explore the extreme case in which the impatient incumbent is completely impatient:  $\lambda = 0$ . This case is of special interest, since it captures the arguably realistic possibility that the incumbent is focused exclusively on the “short term.” It is also of particular interest due to its great tractability. By analyzing the extreme case in which  $\lambda = 0$ , it is easy to illustrate the general point that intuitive equilibria may fail to exist. I conclude the section by showing that an intuitive equilibrium in mixed strategies does exist when  $\lambda = 0$ .

### 8.1. A Non-Existence Result

I assume now that  $\lambda = 0$ . By Corollary 6.2, intuitive equilibria of Kind A and Kind B do not exist. Suppose an intuitive equilibrium of Kind C exists. Recall from (7.1) that an intuitive equilibrium of Kind C can exist only if  $\lambda \geq \underline{\lambda}_C$ . Given  $\lambda = 0$ , it is thus necessary that  $\underline{\lambda}_C = 0$ . In turn, this means that types  $(H, \eta)$  and  $(L, \lambda)$  must pool at  $\psi(L)$ . Next, it is useful to remember that type  $(H, \eta)$ 's downward incentive constraint (where it mimics type  $(L, \eta)$ ) must bind in an intuitive equilibrium. Formally, with  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$ , Lemma 7.2 indicates that  $(P(L, \eta), A(L, \eta)) = \psi(c_e)$ , where  $c_e$  satisfies  $c_e < L$  and  $\Pi(\psi(L) | H) + \eta\tilde{\pi}_H(b_C) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ . Finally, as observed in Lemma 4.6, in any intuitive equilibrium of Kind C,  $(P(H, \lambda), A(H, \lambda)) = \psi(H)$ . In summary, if an intuitive equilibrium exists when  $\lambda = 0$ , then it must take the form of the intuitive equilibrium of Kind C constructed in Proposition 7.4.

Now, if an intuitive equilibrium of Kind C exists, it is also necessary that type  $(H, \eta)$  does not gain by making its monopoly selection,  $\psi(H)$ . As shown in (7.2), when  $\lambda = 0$  and thus  $(P(L, \lambda), A(L, \lambda)) = \psi(L)$ , this requirement is

$$\eta \geq \underline{\eta}_C \equiv \frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(b_C) - \tilde{\pi}_H(1)} > 0.$$

Thus, if  $\lambda = 0$  and an intuitive equilibrium of Kind C exists, then  $\eta \geq \underline{\eta}_C > 0$ .

The preceding discussion now may be summarized in a proposition.

**Proposition 8.1.** *Assume  $\lambda = 0$  and*

$$\underline{\eta}_C \equiv \frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(b_C) - \tilde{\pi}_H(1)} > \eta. \quad (8.1)$$

*Then an intuitive equilibrium does not exist.*

The assumptions made in this proposition are consistent with the maintained assumptions. In particular, Assumption 3 and (8.1) are both satisfied if

$$\frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(b_C) - \tilde{\pi}_H(1)} > \eta > \frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(0) - \tilde{\pi}_H(1)}.$$

Intuitively,  $\eta$  may be sufficiently large that type  $(H, \eta)$  prefers  $(\psi(L), 0)$  to  $(\psi(H), 1)$  and yet no so large that this type prefers  $(\psi(L), b_C)$  to  $(\psi(H), 1)$ .

This analysis reveals an important point of difference between the benchmark model with one-dimensional private information and the model with two dimensions of private information. In the benchmark model, as Corollary 3.5 confirms, an intuitive equilibrium always exists. By contrast, as Proposition 8.1 reveals, in the model with two dimensions of private information, it is possible that an intuitive equilibrium fails to exist.

## 8.2. A Mixed-Strategy Resolution

The non-existence result reported in Proposition 8.1 can be addressed by including mixed strategies. Assume  $\lambda = 0$  and that (8.1) holds. Using Assumption 3, I may now define  $\hat{b} \in (0, b_C)$  such that

$$\frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(\hat{b}) - \tilde{\pi}_H(1)} = \eta. \quad (8.2)$$

Suppose now that type  $(H, \eta)$  mixes between the selections  $\psi(L)$  and  $\psi(H)$ , while type  $(H, \lambda)$  selects  $\psi(H)$ , type  $(L, \lambda)$  selects  $\psi(L)$ , and type  $(L, \eta)$  selects  $\psi(c_e)$ , where the determination of  $c_e$  is described below. Type  $(H, \eta)$  selects  $\psi(L)$  with an appropriate probability so that the application of (E2) yields  $b(\psi(L)) = \hat{b}$ . As above, (E2) requires  $b(\psi(c_e)) = 0 < 1 = b(\psi(H))$ . When type  $(H, \eta)$  mixes in this fashion, the induced belief  $\hat{b}$  is defined by (8.2) to ensure that type  $(H, \eta)$  is indeed indifferent between  $(\psi(L), \hat{b})$  and  $(\psi(H), 1)$ . The cost level  $c_e < L$  is now determined to leave type  $(H, \eta)$  indifferent between  $(\psi(L), \hat{b})$  and  $(\psi(c_e), 0)$ . Using (2.1), it thus follows that  $c_e = c_o$ .

This discussion leads to the following proposition.

**Proposition 8.2.** *Assume  $\lambda = 0$  and that (8.1) holds. Define  $\hat{b} \in (0, b_C)$  by (8.2) and  $c_o < L$  by (2.1). Then there exists a (mixed-strategy) intuitive equilibrium in which type  $(H, \eta)$  mixes between  $\psi(L)$  and  $\psi(H)$ , type  $(H, \lambda)$  selects  $\psi(H)$ , type  $(L, \lambda)$  selects  $\psi(L)$ , and type  $(L, \eta)$  selects  $\psi(c_o)$ , where type  $(H, \eta)$  selects  $\psi(L)$  with an appropriate probability so that under Bayesian updating  $b(\psi(L)) = \hat{b}$ .*

The proposition specifies that any type  $(t, \lambda)$  selects  $\psi(t)$ . This is clearly the necessary specification when  $\lambda = 0$ . As noted above, under the construction, type  $(H, \eta)$  is indifferent between  $(\psi(L), \widehat{b})$  and  $(\psi(c_o), 0)$  and between  $(\psi(L), \widehat{b})$  and  $(\psi(H), 1)$ . Hence, this type cannot gain by altering the probability with which it selects  $\psi(L)$ ,  $\psi(H)$  or  $\psi(c_o)$ .<sup>12</sup> Using Lemma 2.3, if type  $(H, \eta)$  is indifferent between  $(\psi(L), \widehat{b})$  and  $(\psi(c_o), 0)$  where  $c_o < L$ , then type  $(L, \eta)$  must strictly prefer  $(\psi(c_o), 0)$  to  $(\psi(L), \widehat{b})$  and thus  $(\psi(H), 1)$ . From here, the proposition can be proved following steps similar to those used in the proof of Proposition 7.4.<sup>13</sup>

### 8.3. Economics Interpretation

The analysis of mixed-strategy intuitive equilibria presented here confirms that intuitive equilibria exist even if  $\lambda = 0$  and  $\eta$  is small. The constructed intuitive equilibrium can be interpreted in the same manner as the intuitive equilibrium of Kind C. The only difference here is that the incumbent of type  $(H, \eta)$  does not select  $\psi(L)$  with certainty but rather randomizes between the selection  $\psi(L)$  and the selection  $\psi(H)$ . Thus, in the constructed intuitive equilibrium, a patient, high-cost incumbent randomizes between two approaches: it either does not distort its selection and faces the maximal probability of entry, or it limit prices and distorts advertising upward and deters some profitable entry.

## 9. Conclusion

In this paper, I consider a model in which an incumbent has two dimensions of private information as well as two signals. The incumbent is privately informed as to its cost type and level of patience, and benefits if a potential entrant infers a low cost type. The potential entrant's inference, however, is confounded by the fact that it is also uninformed with regard to the incumbent's level of patience. The incumbent's two signals are its pre-entry price and advertising.

The paper offers three main contributions. First, building on earlier work by Milgrom and Roberts (1982) and Bagwell and Ramey (1988), I analyze a benchmark model in which the incumbent is privately informed only about its costs. In the unique intuitive equilibrium outcome of the benchmark model,

<sup>12</sup>Note, though, that type  $(H, \eta)$  selects  $\psi(c_o)$  with zero probability. Otherwise, type  $(L, \eta)$  would not enjoy the belief  $b_{L\eta} = 0$ , and under (E3) this type would deviate.

<sup>13</sup>More generally, the constructed mixed-strategy intuitive equilibrium exists if (8.1) holds and  $\lambda \leq [\Pi(\psi(L) | L) - \Pi(\psi(c_o) | L)] / [\widetilde{\pi}_L(0) - \widetilde{\pi}_L(\widehat{b})] \equiv \overline{\lambda}_{mix}$ . The latter condition ensures that type  $(L, \lambda)$  does not gain by deviating to  $(\psi(c_o), 0)$ . Referring to Proposition 7.4, it thus follows that a (possibly mixed-strategy) intuitive equilibrium is sure to exist, if  $\lambda$  is not too high (so that  $\lambda \leq \overline{\lambda}_{mix}$  when (8.1) holds and  $\lambda \leq \min\{\overline{\lambda}_C, \underline{\eta}_C\}$  when (8.1) fails).



the low-cost incumbent uses limit pricing and upward distortions in (demand-enhancing) advertising; however, these actions do not deter profitable entry.

Second, I characterize the intuitive equilibria of the general model with two-dimensional private information. On the one hand, the analysis provides support for the predictions in the benchmark model. In any intuitive equilibrium, a patient, low-cost incumbent limit prices and distorts demand-enhancing advertising upward, and such behavior ensures that entry occurs exactly when it is profitable. In intuitive equilibria of Kind B, the patient and impatient low-cost incumbent types make one selection while the patient and impatient high-cost incumbent types make a different selection. On the other hand, the analysis also provides support for an older literature with contributions by Bain (1949) and Williamson (1963) that emphasizes the entry-deterrence effects of limit pricing and aggressive advertising. In intuitive equilibria of Kinds A and C, the patient, high-cost incumbent pools with the impatient, low-cost incumbent (and perhaps the impatient, high-cost incumbent). In the featured intuitive equilibria, the high-cost, patient incumbent limit prices and distorts its demand-enhancing advertising upward; further, profitable entry is sometimes deterred. The analysis also offers a novel finding. Intuitive equilibria are constructed in which an impatient incumbent with low costs adopts a soft pre-entry stance - it prices above its monopoly level and distorts its demand-enhancing advertising downward - and thereby sometimes induces unprofitable entry.

Third, at a methodological level, the paper contributes by deriving predictions for intuitive equilibria in a model with multiple signals and multiple dimensions of private information. The Cho-Kreps (1987) intuitive criterion is shown to remain powerful in this setting. The analysis also suggests that the existence of (pure-strategy) intuitive equilibria may be more problematic when private information has more than one dimension.

Several important directions for future research are apparent. First, the model analyzed here is special, in that the receiver has no direct interest in one of the variables about which the sender is privately informed. This assumption seems reasonable for the application at hand and makes the analysis more tractable but is quite restrictive. Second, future work might consider alternative signals. For example, if the incumbent could commit in the pre-entry period to burn money in the post-entry period, then such a signal might provide an effective means through which the incumbent could reveal its patience level to the potential entrant. This perspective suggests that pooling might be less likely to occur, if the incumbent could also signal its information with appropriate capital-market instruments.

## 10. Appendix

**Proof of Lemma 5.2:** The two arguments in (5.1) generate associated iso-profit curves; in particular, when the argument for an incumbent with cost type  $t$  takes some value  $k_t$ , then the associated iso-profit curve for this incumbent type is  $\Pi(P, A | t) = \Pi(\psi(t) | t) - k_t(\tilde{\pi}_t(b_A) - \tilde{\pi}_t(1))$ . The iso-profit curve is centered around the point  $\psi(t)$ . Let  $(P^*, A^*)$  denote a selection that induces  $\underline{\lambda}_A$ .

A first claim is that  $\underline{\lambda}_A < [\Pi(\psi(H) | H) - \Pi(\psi(L) | H)]/[\tilde{\pi}_H(b_A) - \tilde{\pi}_H(1)]$ . Since  $\underline{\lambda}_A$  is the minimized value of the maximum function in (5.1), it is no greater than the maximum function when evaluated at  $\psi(L)$ . By Lemma 2.2, the maximum function when evaluated at  $\psi(L + \varepsilon)$  is lower than when evaluated at  $\psi(L)$ , for  $\varepsilon > 0$  and sufficiently small. Thus,  $\underline{\lambda}_A$  must be strictly less than the maximum function when evaluated at  $\psi(L)$ , and the claim is established.

A second claim is that  $(P^*, A^*) = \psi(c)$  for some  $c \in (L, H)$ . Suppose to the contrary that  $(P^*, A^*) = \psi(c)$  for some  $c \leq L$ . Referring to (5.1) and using Lemma 2.2, it is clear that a strictly lower value for  $\underline{\lambda}_A$  could be induced by using  $(P, A) = \psi(c + \varepsilon)$  for  $\varepsilon > 0$  and sufficiently small. A similar contradiction arises if  $(P^*, A^*) = \psi(c)$  for some  $c \geq H$ . Suppose next that  $(P^*, A^*) \neq \psi(c)$  for any  $c$ , and let  $k_t$  be evaluated at  $(P^*, A^*)$ . Using the first claim and the definition of  $\underline{\lambda}_A$ ,

$$\frac{\Pi(\psi(H) | H) - \Pi(\psi(L) | H)}{\tilde{\pi}_H(b_A) - \tilde{\pi}_H(1)} > \underline{\lambda}_A \geq k_H = \frac{\Pi(\psi(H) | H) - \Pi(P^*, A^* | H)}{\tilde{\pi}_H(b_A) - \tilde{\pi}_H(1)}.$$

Thus,  $\Pi(\psi(H) | H) > \Pi(P^*, A^* | H) > \Pi(\psi(L) | H)$ , and so under Lemma 2.2 there exists  $c^* \in (L, H)$  such that  $\Pi(\psi(c^*) | H) = \Pi(P^*, A^* | H)$ . Let  $(P_1, A_1) \equiv (P^*, A^*)$  and  $(P_2, A_2) \equiv \psi(c^*)$ . By Lemma 2.1,  $\Delta(c)$  is strictly decreasing, and so  $\Pi(\psi(c^*) | L) > \Pi(P^*, A^* | L)$ . Thus, for  $\varepsilon > 0$  and sufficiently small, the maximum function in (5.1) is strictly lower when evaluated at  $\psi(c^* + \varepsilon)$  than when evaluated at  $(P^*, A^*)$ , and the second claim is established.

A third claim is that  $(P^*, A^*) = \psi(c_m)$  where  $c_m \in (L, H)$  is defined in (5.2) as that value of  $c \in (L, H)$  such that  $\psi(c)$  induces  $k_L = k_H$ . Under Lemma 2.2,  $c_m$  is unique. Assume to the contrary that  $(P^*, A^*) = \psi(c)$  for some  $c \in (L, H)$  such that  $\underline{\lambda}_A = k_L > k_H$ . Then under Lemma 2.2 the maximum function in (5.1) would be strictly lower under the selection  $\psi(c - \varepsilon)$  for  $\varepsilon > 0$  and sufficiently small. Likewise, assume to the contrary that  $(P^*, A^*) = \psi(c)$  for some  $c \in (L, H)$  such that  $\underline{\lambda}_A = k_H > k_L$ . Then under Lemma 2.2 the maximum function in (5.1) would be strictly lower under the selection  $\psi(c + \varepsilon)$  for  $\varepsilon > 0$  and sufficiently small. The third claim is thus established. ■

**Proof of Proposition 5.4:** As argued in the text,  $c_e < L$  under (5.4). Further,

under (5.5),  $\eta > \lambda \geq \underline{\lambda}_A$ ; thus, for all  $\delta \in \{\eta, \lambda\}$  and  $t \in \{L, H\}$ ,

$$\Pi(\psi(c_m) | t) + \delta \tilde{\pi}_t(b_A) \geq \Pi(\psi(t) | t) + \delta \tilde{\pi}_t(1), \quad (10.1)$$

where a strict inequality holds when  $\delta = \eta$ . In establishing that the proposed specification satisfies (E1), the first step is to consider particular deviations. Specifically, I establish first that no type  $\{\delta, t\}$  could gain by deviating to (i) the equilibrium selection intended for some other type, where the belief associated with that deviation is then determined by (E2), or (ii) a monopoly selection,  $\psi(t)$ , for  $t \in \{L, H\}$ , when  $b(\psi(t)) = 1$ . Note that neither of the two monopoly selections is adopted by any type of incumbent in an intuitive equilibrium of Kind A; thus, the beliefs associated with such selections are not restricted by (E2).

Consider type  $(L, \eta)$ . As shown in the proof of Lemma 5.3,  $\bar{\lambda}_A < \eta$  follows from  $c_e < H$ , and so type  $(L, \eta)$  strictly prefers  $(\psi(c_e), 0)$  to  $(\psi(c_m), b_A)$ . Using this finding, (10.1) and  $\eta > \lambda$ , it thus follows that  $\hat{\Pi}(L, \eta) = \Pi(\psi(c_e) | L) + \eta \tilde{\pi}_L(0) > \Pi(\psi(c_m) | L) + \eta \tilde{\pi}_L(b_A) > \Pi(\psi(L) | L) + \eta \tilde{\pi}_L(1) > \Pi(\psi(H) | L) + \eta \tilde{\pi}_L(1)$ . Consider type  $(H, \eta)$ . This type is indifferent between selecting  $\psi(c_m)$  and deviating to  $\psi(c_e)$ . Further, by (10.1) and  $\eta > \lambda$ , this type strictly prefers  $(\psi(c_m), b_A)$  to  $(\psi(H), 1)$  and thus  $(\psi(L), 1)$ . Consider type  $(H, \lambda)$ . It is easily confirmed that

$$\begin{aligned} \hat{\Pi}(H, \lambda) &= \Pi(\psi(c_m) | H) + \lambda \tilde{\pi}_H(b_A) \\ &= \Pi(\psi(c_e) | H) + \lambda \tilde{\pi}_H(0) + (\eta - \lambda)[\tilde{\pi}_H(0) - \tilde{\pi}_H(b_A)] \\ &> \Pi(\psi(c_e) | H) + \lambda \tilde{\pi}_H(0), \end{aligned}$$

and so type  $(H, \lambda)$  strictly prefers  $(\psi(c_m), b_A)$  to  $(\psi(c_e), 0)$ . Using (10.1), it also follows that type  $(H, \lambda)$  prefers  $(\psi(c_m), b_A)$  to  $(\psi(H), 1)$  and thus  $(\psi(L), 1)$ . Finally, consider type  $(L, \lambda)$ . As in the proof of Lemma 5.3, (5.4) ensures that  $\hat{\Pi}(L, \lambda) = \Pi(\psi(c_m) | L) + \lambda \tilde{\pi}_L(b_A) \geq \Pi(\psi(c_e) | L) + \lambda \tilde{\pi}_L(0)$ . Thus, type  $(L, \lambda)$  does not gain by deviating from  $\psi(c_m)$  to  $\psi(c_e)$ . Finally, under (10.1), type  $(L, \lambda)$  also prefers  $(\psi(c_m), b_A)$  to  $(\psi(L), 1)$  and thus  $(\psi(H), 1)$ .

The next step is to specify beliefs. By (E2),  $b(\psi(c_e)) = 0 < b_A = b(\psi(c_m))$ . For  $(P, A) \notin \{\psi(c_e), \psi(c_m)\}$ , beliefs are specified as follows: if  $\Pi(P, A | H) + \eta \tilde{\pi}_H(0) < \Pi(\psi(c_m) | H) + \eta \tilde{\pi}_H(b_A)$ , then  $b(P, A) = 0$ ; and if  $\Pi(P, A | H) + \eta \tilde{\pi}_H(0) \geq \Pi(\psi(c_m) | H) + \eta \tilde{\pi}_H(b_A)$ , then  $b(P, A) = 1$ . Since  $\hat{\Pi}(H, \eta) = \Pi(\psi(c_m) | H) + \eta \tilde{\pi}_H(b_A)$ ,  $(P, A)$  in the first (second) set are (not) equilibrium dominated for type  $(H, \eta)$ . Note  $(P, A)$  in the first set are also equilibrium dominated for type  $(H, \lambda)$  :

$$\begin{aligned} \hat{\Pi}(H, \lambda) &= \Pi(\psi(c_m) | H) + \lambda \tilde{\pi}_H(b_A) \\ &> \Pi(P, A | H) + \lambda \tilde{\pi}_H(0) + (\eta - \lambda)(\tilde{\pi}_H(0) - \tilde{\pi}_H(b_A)) \\ &> \Pi(P, A | H) + \lambda \tilde{\pi}_H(0). \end{aligned}$$

It is now straightforward to confirm that this specification satisfies (E3).

The final step is to confirm that (E1) holds. Given the beliefs, the most tempting deviation for types  $(H, \eta)$  and  $(H, \lambda)$  is  $\psi(H)$ . As shown above, however, each of these types prefers  $(\psi(c_m), b_A)$  to  $(\psi(H), 1)$ . Consider type  $(L, \eta)$ . Over the range of  $(P, A)$  for which  $b(P, A) \neq 0$ , the only potentially attractive deviations are  $\psi(L)$  and  $\psi(c_m)$ ; however, as shown above, each of these options offers a strictly lower payoff than  $\psi(c_e)$ . For  $(P, A)$  such that  $b(P, A) = 0$ , type  $(H, \eta)$  strictly prefers  $(\psi(c_e), 0)$  to  $(P, A, 0)$ . A patient incumbent would have the same preference if its cost level were  $c_e$ . Given that  $c_e < L$ , it follows from Lemma 2.4 that type  $(L, \eta)$  also strictly prefers  $(\psi(c_e), 0)$  to  $(P, A, 0)$ . Last, consider type  $(L, \lambda)$ . Over the range of  $(P, A)$  for which  $b(P, A) = 1$ , the most tempting deviation is  $\psi(L)$ , which as shown above does not offer a gain. Fix  $(P, A)$  such that  $b(P, A) = 0$ . It is established above that type  $(L, \lambda)$  prefers  $(\psi(c_m), b_A)$  to  $(\psi(c_e), 0)$ . Further, as just shown, type  $(L, \eta)$  strictly prefers  $(\psi(c_e), 0)$  to  $(P, A, 0)$ ; thus,  $(L, \lambda)$  also strictly prefers  $(\psi(c_e), 0)$  to  $(P, A, 0)$ . ■

**Proof of Proposition 6.3:** I establish first that no type  $\{\delta, t\}$  could gain by deviating to (i) the equilibrium selection intended for some other type, where the belief associated with that deviation is determined by (E2), or (ii) the monopoly selection,  $\psi(L)$ , when  $b(\psi(L)) = 1$ . Note that  $\psi(H)$  is selected in an intuitive equilibrium of Kind B; thus,  $b(\psi(H))$  is determined by (E2).

Consider type  $(H, \eta)$ . By (2.1), this type is indifferent between  $(\psi(H), 1)$  and  $(\psi(c_o), 0)$ . This type strictly prefers  $(\psi(H), 1)$  to  $(\psi(L), 1)$ . Consider type  $(H, \lambda)$ :

$$\begin{aligned} \widehat{\Pi}(H, \lambda) &= \Pi(\psi(H) | H) + \lambda \tilde{\pi}_H(1) \\ &= \Pi(\psi(c_o) | H) + \lambda \tilde{\pi}_H(0) + (\eta - \lambda)[\tilde{\pi}_H(0) - \tilde{\pi}_H(1)] \\ &> \Pi(\psi(c_o) | H) + \lambda \tilde{\pi}_H(0). \end{aligned}$$

Thus, type  $(H, \lambda)$  strictly prefers  $(\psi(H), 1)$  and  $(\psi(c_o), 0)$ . This type also strictly prefers  $(\psi(H), 1)$  to  $(\psi(L), 1)$ . Consider type  $(L, \lambda)$ :

$$\Pi(\psi(c_o) | L) + \lambda \tilde{\pi}_L(0) \geq \Pi(\psi(L) | L) + \lambda \tilde{\pi}_L(1) \quad (10.2)$$

holds if and only if  $\lambda \geq \underline{\lambda}_B$ . Hence, given  $\lambda \geq \underline{\lambda}_B$ , type  $(L, \lambda)$  prefers  $(\psi(c_o), 0)$  to  $(\psi(L), 1)$  and thus  $(\psi(H), 1)$ . Finally, consider type  $(L, \eta)$ . As established using Lemma 2.4 in the first step of the proof of Proposition 3.3, type  $(L, \eta)$  strictly prefers  $(\psi(c_o), 0)$  to  $(\psi(L), 1)$  and thus  $(\psi(H), 1)$ .

The next step is to specify beliefs. By (E2),  $b(\psi(H)) = 1 > 0 = b(\psi(c_o))$ . For  $(P, A) \notin \{\psi(c_o), \psi(H)\}$ , beliefs are specified as follows: if  $\Pi(P, A | H) + \eta \tilde{\pi}_H(0) < \Pi(\psi(H) | H) + \eta \tilde{\pi}_H(1)$ , then  $b(P, A) = 0$ ; and if  $\Pi(P, A | H) + \eta \tilde{\pi}_H(0) \geq \Pi(\psi(H) | H) + \eta \tilde{\pi}_H(1)$ , then  $b(P, A) = 1$ .

$H) + \eta\tilde{\pi}_H(1)$ , then  $b(P, A) = 1$ . Since  $\widehat{\Pi}(H, \eta) = \Pi(\psi(H) | H) + \eta\tilde{\pi}_H(1)$ ,  $(P, A)$  in the first (second) set are (not) equilibrium dominated for type  $(H, \eta)$ . Observe that  $(P, A)$  in the first set are also equilibrium dominated for type  $(H, \lambda)$  :

$$\begin{aligned}\widehat{\Pi}(H, \lambda) &= \Pi(\psi(H) | H) + \lambda\tilde{\pi}_H(1) \\ &> \Pi(P, A | H) + \eta\tilde{\pi}_H(0) - (\eta - \lambda)\tilde{\pi}_H(1) \\ &> \Pi(P, A | H) + \lambda\tilde{\pi}_H(0).\end{aligned}$$

It is now straightforward to confirm that this specification satisfies (E3).

The final step is to confirm that (E1) holds. Given the beliefs, types  $(H, \eta)$  and  $(H, \lambda)$  clearly cannot gain by deviating. Consider type  $(L, \eta)$ . For  $(P, A)$  such that  $b(P, A) = 1$ , the most attractive deviation is  $\psi(L)$ . But as shown above type  $(L, \eta)$  strictly prefers  $(\psi(c_o), 0)$  to  $(\psi(L), 1)$ . For  $(P, A)$  such that  $b(P, A) = 0$ , type  $(H, \eta)$  strictly prefers  $(\psi(c_o), 0)$  to  $(P, A, 0)$ . A patient incumbent would have the same preference if its cost level were  $c_o$ . Given  $c_o < L$ , it follows from Lemma 2.4 that type  $(L, \eta)$  also strictly prefers  $(\psi(c_o), 0)$  to  $(P, A, 0)$ . Last, consider type  $(L, \lambda)$ . For  $(P, A)$  such that  $b(P, A) = 1$ , the most tempting deviation is  $\psi(L)$ , which as established in (10.2) above does not offer a gain when  $\lambda \geq \underline{\lambda}_B$ . Fix  $(P, A)$  such that  $b(P, A) = 0$ . As just shown, type  $(L, \eta)$  strictly prefers  $(\psi(c_o), 0)$  to  $(P, A, 0)$ ; thus,  $(L, \lambda)$  also strictly prefers  $(\psi(c_o), 0)$  to  $(P, A, 0)$ . ■

**Proof of Proposition 7.4:** To begin, I confirm that  $c_e < L$ . This follows immediately from Lemma 2.2, given that  $c_e$  is here defined by  $\Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0) = \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(b_C)$  and  $b_C > 0$ . To establish that the proposed specification satisfies (E1), the first step is to consider particular deviations. I establish first that no type  $\{\delta, t\}$  could gain by deviating to the equilibrium selection intended for some other type, where the associated belief is determined by (E2). Note that each of the two monopoly selections is adopted by some type of incumbent in the proposed intuitive equilibrium of Kind C.

Consider type  $(H, \eta)$ . By the definition of  $c_e$ , this type is indifferent between  $(\psi(L), b_C)$  and  $(\psi(c_e), 0)$ . Under (7.5), this type also prefers  $(\psi(L), b_C)$  and  $(\psi(H), 1)$ . Consider type  $(L, \lambda)$ . This type strictly prefers  $(\psi(L), b_C)$  to  $(\psi(H), 1)$ ; further, under (7.6), this type prefers  $(\psi(L), b_C)$  to  $(\psi(c_e), 0)$ . Consider type  $(H, \lambda)$ . Using (7.7),

$$\begin{aligned}\widehat{\Pi}(H, \lambda) &= \Pi(\psi(H) | H) + \lambda\tilde{\pi}_H(1) \\ &\geq \Pi(\psi(L) | H) + \lambda\tilde{\pi}_H(b_C) \\ &> \Pi(\psi(c_e) | H) + \lambda\tilde{\pi}_H(0).\end{aligned}$$

Thus type  $(H, \lambda)$  prefers  $(\psi(H), 1)$  to  $(\psi(L), b_C)$  and thus  $(\psi(c_e), 0)$ . Finally, consider type  $(L, \eta)$ . As shown in the proof of Lemma 7.2,  $\bar{\lambda}_C < \eta$  follows from

$c_e < H$ , and so type  $(L, \eta)$  strictly prefers  $(\psi(c_e), 0)$  to  $(\psi(L), b_C)$  and thus to  $(\psi(H), 1)$ .

The next step is to specify beliefs. By (E2),  $b(\psi(c_e)) = 0$ ,  $b(\psi(L)) = b_C$  and  $b(\psi(H)) = 1$ . For  $(P, A) \notin \{\psi(c_e), \psi(L), \psi(H)\}$ , beliefs are specified as follows: if  $\Pi(P, A | H) + \eta\tilde{\pi}_H(0) < \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(b_C)$ , then  $b(P, A) = 0$ ; and if  $\Pi(P, A | H) + \eta\tilde{\pi}_H(0) \geq \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(b_C)$ , then  $b(P, A) = 1$ . Since  $\hat{\Pi}(H, \eta) = \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(b_C)$ ,  $(P, A)$  in the first (second) set are (not) equilibrium dominated for type  $(H, \eta)$ . Observe that  $(P, A)$  in the first set are also equilibrium dominated for type  $(H, \lambda)$  :

$$\begin{aligned} \hat{\Pi}(H, \lambda) &= \Pi(\psi(H) | H) + \lambda\tilde{\pi}_H(1) \\ &\geq \Pi(\psi(L) | H) + \lambda\tilde{\pi}_H(b_C) \\ &> \Pi(P, A | H) + \eta\tilde{\pi}_H(0) - (\eta - \lambda)\tilde{\pi}_H(b_C) \\ &> \Pi(P, A | H) + \lambda\tilde{\pi}_H(0), \end{aligned}$$

where the first inequality uses (7.7). Thus, the specification satisfies (E3).

The final step is to verify that (E1) holds. Given the belief specification, if an incumbent with cost type  $t$  considers a deviation to some  $(P, A) \notin \{\psi(c_e), \psi(L), \psi(H)\}$  for which  $b(P, A) = 1$ , then the incumbent would do better by selecting  $\psi(t)$ . Types  $(H, \lambda)$  and  $(L, \lambda)$  already adopt their monopoly selections, and it is shown above that types  $(H, \eta)$  and  $(L, \eta)$  cannot gain by deviating to their respective monopoly selections. Next, fix any  $(P, A) \notin \{\psi(c_e), \psi(L), \psi(H)\}$  for which  $b(P, A) = 0$ . As established above, any such deviant selection is equilibrium dominated for types  $(H, \eta)$  and  $(H, \lambda)$ . Further,  $\Pi(P, A | H) + \eta\tilde{\pi}_H(0) < \Pi(\psi(L) | H) + \eta\tilde{\pi}_H(b_C) = \Pi(\psi(c_e) | H) + \eta\tilde{\pi}_H(0)$ . Thus, type  $(H, \eta)$  strictly prefers  $(\psi(c_e), 0)$  to  $(P, A, 0)$ . A patient incumbent with cost level  $c_e$  would also strictly prefer  $(\psi(c_e), 0)$  to  $(P, A, 0)$ . Given  $c_e < L$ , by Lemma 2.4, type  $(L, \eta)$  strictly prefers  $(\psi(c_e), 0)$  to  $(P, A, 0)$ . Consider type  $(L, \lambda)$ . It is established above that type  $(L, \lambda)$  prefers  $(\psi(L), b_C)$  to  $(\psi(c_e), 0)$ . Further, as just shown, type  $(L, \eta)$  strictly prefers  $(\psi(c_e), 0)$  to  $(P, A, 0)$ ; thus,  $(L, \lambda)$  also strictly prefers  $(\psi(c_e), 0)$  to  $(P, A, 0)$ . ■

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Figure 1: Iso-payoff curves illustrating different possible locations for  $(c_e, c_{ee})$

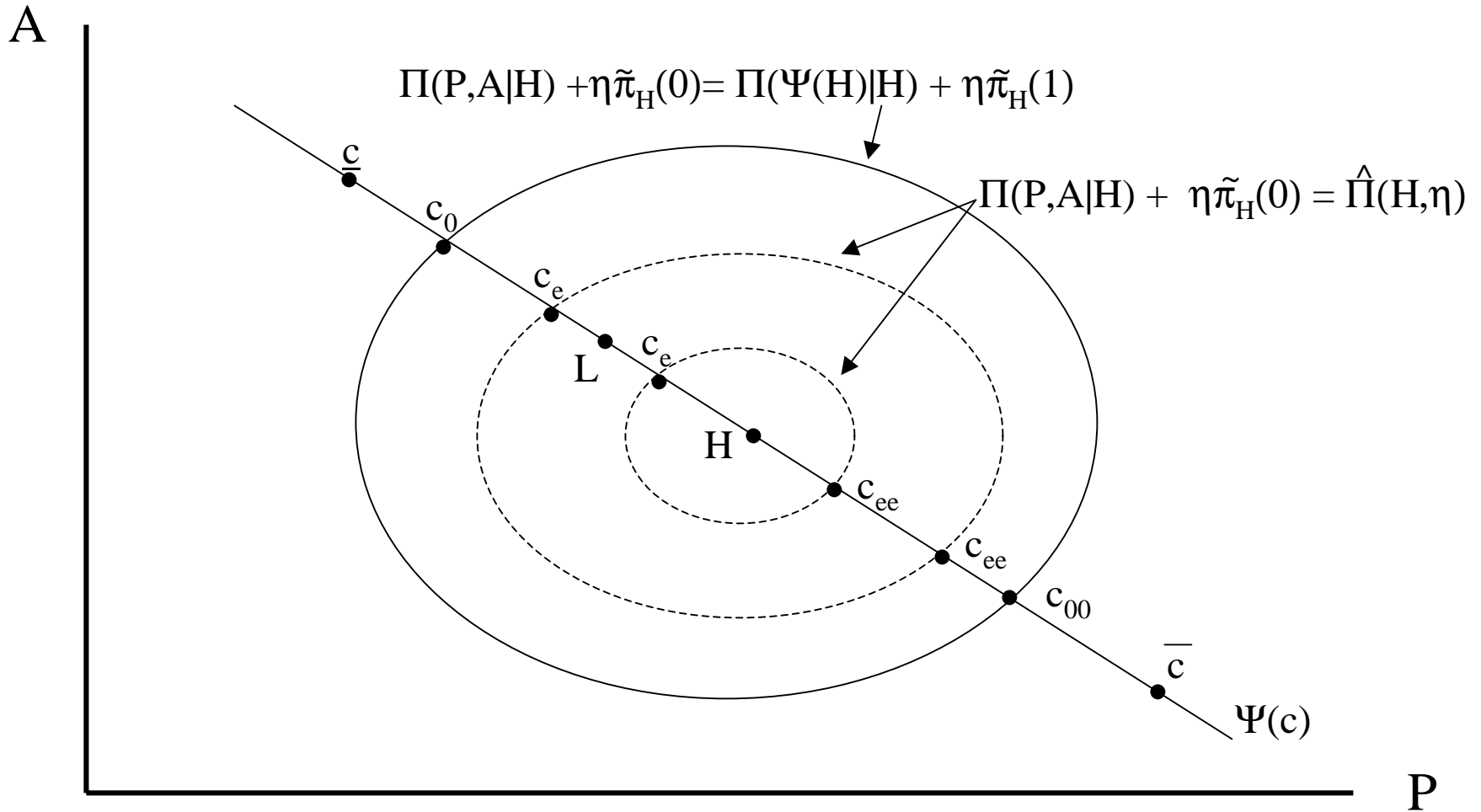


Figure 2: Determination of  $c_m$  and  $\underline{\lambda}_A$

