Commodity-Price Destabilizing
Commodity Price Stabilization

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I have benefitted from a conversation with Timothy Besley, but I alone am responsible for any errors.
Abstract

In buffer stocks established by International Commodity Agreements, typically the buffer stock manager (BSM) is given: (i) inadequate resources to defend the price floor on her own; (ii) considerable discretion in day-to-day operations. These two features are shown to imply that the buffer stock can as easily add volatility to the market as remove it. Fact (i) implies that the BSM needs to use speculators: the floor will be defended only if speculators "bet" on the buffer stock's success. Fact (ii) implies that their expectations about the buffer stock's success can become self-fulfilling prophecies. This opens the way for the price to fluctuate in response to "sunspots", or intrinsically irrelevant phenomena, which would have been impossible without the buffer stock. The theory is given anecdotal support from the markets for tin and natural rubber.
1. Introduction.

A buffer stock is a public organ which buys, sells and stores a commodity in order to control its price. It is a common feature of the many International Commodity Agreements which have been struck in many cases by governments to try to reduce the volatility in world prices of primary commodities. However, historical buffer stocks have a dismal record on this score. Table I shows various crude measures\(^1\) of price volatility for tin, cocoa and rubber, the three commodities with the most important international postwar buffer stock programs\(^2\). By all three measures, the most volatile periods by far for cocoa and for tin were under the influence of their buffer stocks. Only in the case of rubber was the buffer stock period less volatile. Thus, there has been no marked and obvious tendency for the prices of these commodities to be more stable under the action of buffer stocks than at other times.

One might ascribe high volatility under a buffer stock to bad luck or incompetence. This paper offers a different interpretation. It will be shown that it is quite possible for a realistic commodity price management scheme to *induce* volatility in equilibrium even though it is administered by an infinitely competent and farsighted manager in the presence of perfectly informed and rational speculators. The example given is of a buffer stock established to defend a price floor, but the principle is a simple one which is likely to be relevant in many other cases.

\(^{1}\)These measures are the standard deviation of the first difference, the average absolute first difference, and the estimated standard error from a first order linear autoregression of the price with intercept.

\(^{2}\)These are based on monthly IMF data from the *International Financial Statistics*. Tin and cocoa are deflated by the UK CPI, rubber by the US CPI. January 1980 is the base period. The tin data run from January 1957 to July 1992; rubber, January 1957 to August 1992; and cocoa, January 1959 to July 1988. The endpoints for the periods studied are 1/57-10/85 and 11/85-7/92 for tin; 1/57-9/80 and 10/80-8/92 for rubber; and 1/59-4/73, 5/73-3/80, and 8/81-7/88 for cocoa.
as well.

Specifically, in the likely event that the buffer stock is insufficiently endowed with resources to be able to take over storage entirely from the private sector, the buffer stock manager may need the help of private speculators in meeting the objectives with which she has been charged. For this to occur, the speculators will at times need to store more of the good themselves than they would under *laissez faire*. They will be willing to do so only if they expect future prices to be higher than they would be under *laissez faire*. This gives rise to a situation in which if speculators expect the buffer stock to succeed, it will, because by betting on it they will help it along; but if they do not, it will not. The private sector’s expectations then can be self-fulfilling, and random fluctuations in “confidence” or “mood” have an effect on price which would have been inconsistent with rational behaviour in the absence of the buffer stock. The equilibrium with the buffer stock has a source of volatility that would have been absent without it.

This argument contrasts with the orthodox interpretation of buffer stocks as stabilizing institutions which buy when the price is low, forcing the price up, and sell when the price is high, forcing the price back down. We may include in this class of analyses Salant’s (1983) model of speculative attacks on buffer stocks, since the discrete jumps that occur during an attack in that model occur in the distribution of stocks between the public and private sectors, not in

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3See Gustafson (1958) for the pioneering study, and Newbery and Stiglitz (1981, Ch. 29, 30) for an analysis along similar lines. These authors for the most part assume away private storage in analyzing buffer stocks. Salant’s (1983) breakthrough was to show how the same analysis can accommodate risk-neutral speculators with rational expectations who understand the policy the buffer stock follows; a key result is periodic speculative attacks. See Williams and Wright (1991) for further variations on the theme.
prices. See his Figure 7. The attacks in his model put a limit on the degree to which a buffer stock can stabilize the price, while here it is argued that in some cases the buffer stock can destabilize a price. This argument also contrasts with other models of destabilizing effects of speculation. For example, Hart and Kreps (1983) show that rational commodity speculation under laissez-faire can be destabilizing in a wide range of conditions. By contrast, the novelty of the example of this paper is that in it speculation is destabilizing only in the presence of policy.

The argument also bears a relation to the theory of sunspots. In a wide class of models, overlapping generations induce a natural form of missing markets under which irrelevant random variables can have an effect on equilibrium (“sunspots can matter”)\(^4\). In the example of this paper, by contrast, “sunspots matter” only in the presence of certain types of policy. In that respect, the example is much closer to the spirit of Obstfeld (1986), in which “sunspots matter” in the market for foreign exchange under a fixed exchange rate if the private sector makes certain conjectures about how government will respond to a run on the central bank. In the example of this paper, (rational) conjectures about the response of the buffer stock to various kinds of speculative behaviour are crucial.

The basic argument of this paper is presented in the next section, along with the stylized facts on which it is founded. Section 2 presents a simple mathematical model of a commodity market; in section 3, a buffer stock, constructed according to the stylized facts, is added to the model, and the excess volatility result is proven. Section 4 is a discussion.

\(^4\)See Cass and Shell (1985) for the seminal general equilibrium examples, and Jackson and Peck (1991) for a model more closely related to commodity speculation.
1.A. The argument.

The most natural assumption to make about the way a buffer stock works is that it buys during a glut when the price is very low, forcing the price up to the floor price, and sells during periods of scarcity when the price is very high, pushing the price back down. We will call this a "buy low, sell high" mechanism and argue that it is a misleading interpretation of actual buffer stocks. The argument made here is based on two stylized facts not incorporated in existing models and which are documented in section 1.B. First:

(S1) The storage capacity of actual buffer stocks is typically well below normal levels of private sector storage.

The immediate consequence of (S1) is to make it impossible for the buffer stock manager (BSM) to defend the price floor on her own through the "buy low, sell high" mechanism. To see this, say that the buffer stock "bites" in some state of nature if in that state it stores more than the private sector would have stored in its absence. If the buffer stock never bites, then it will have no effect on the rational expectations equilibrium behavior of prices and will simply substitute for some portion of private sector storage⁵. However, in order for the BSM to defend the floor on her own by buying low and selling high it would be necessary for the buffer stock to bite during gluts, when the price is in the lower tail of its distribution; but this is precisely

⁵See McLaren (1992), where this result is proven and an analogy is explained between it and the Modigliani-Miller theorem.
when private storage is in the upper tail of its distribution. (S1) implies that in such periods, private storage in the absence of the buffer stock would dwarf the buffer stock's capacity. In other words, the buffer stock can bite only when the commodity is scarce, and this is when the price is high, not low. Thus, a buffer stock with a “buy low, sell high” policy would likely have no effect on prices.6

Since the buffer stock cannot bite during gluts, the only way the BSM can affect the price at such times is to convince the private sector that it will bite later. Thus, a successful BSM must convince private traders to store more during gluts than they otherwise would by promising that they will have higher prices when they eventually sell than they otherwise would. If she can commit to this behavior, which amounts to protecting traders speculative profits ex post if they bet on the buffer stock ex ante, then the goals of the buffer stock may be realized despite the constraint represented by (S1). However, the second stylized fact makes such commitment difficult:

(S2) In practice, managers of buffer stocks have considerable discretion in day-to-day operations.

The consequence of (S2) is that the BSM must maintain a reputation for protecting the ex post profits of traders. But once we have thus reinterpreted the way the buffer stock works,

6Strictly speaking, this argument depends on two conditions which may or may not hold in the real world. First, it requires speculators to face no binding capacity constraint. Second, it requires speculators to be price-takers. That these two assumptions are good approximations for the markets of interest here seems plausible, but is a tricky empirical question for which we certainly have no proof. See McLaren (1992) for details.
we must reinterpret the whole nature of the equilibrium. The "reputation" of the BSM becomes crucial to the behaviour of prices in the market, and yet it is not based on any fundamentals and could indeed fluctuate for apparently irrelevant reasons. The two stylized facts listed above thus give rise to a situation in which if speculators expect the buffer stock to succeed, it will, because by betting on it they will together help it along; but if they do not, it will not. The private sector's expectations then can be self-fulfilling, and random fluctuations in "confidence" or "mood" may have an effect on price which would have been inconsistent with rational behaviour in the absence of the buffer stock. The equilibrium with the buffer stock has a source of volatility that would have been absent without it. Thus, the richer theory of buffer stocks gives a striking result, formalized and proven in the body of the paper:

(Result.) In the absence of a buffer stock, the market has a single equilibrium, which is affected only by fundamentals of supply and demand. However, with a buffer stock satisfying (S1) and (S2), there are many equilibria, including equilibria in which price fluctuates randomly for no good reason.

In the jargon of one literature, "sunspots cannot matter" without a buffer stock but they can with one (Cass and Shell (1985)). This is completely inimical to the orthodox theory of buffer stocks; Salant (1983) proves the uniqueness of equilibrium in his model, and McLaren (1992) does so in a much broader class of models. Some anecdotal evidence of this sort of volatility is discussed in section 4.

Clearly, this argument depends on the two stylized facts, S1 and S2. Before proceeding,
we will document them.

1.C. The Stylized Facts.

Historical buffer stocks since the Second World War have tended to have a capacity much smaller than the level of normal speculative storage. For example, the tin buffer stock was long dogged by complaints that its capacity was much too small for it to do its job\(^7\). In fact, the financial endowment of the stock was sufficient to acquire 26,800 tonnes under the first International Tin Agreement (ITA), 20,300 tonnes under the second and third ITA’s, and 20,000 tonnes under the fourth and fifth ITA’s (Gilbert, 1977, p. 111); during this period private world stocks averaged 49,000 tonnes with a minimum of 35,700 and a maximum of 73,700\(^8,9\). Similarly, the first two International Cocoa Agreements, initially signed in 1972, provided for a buffer stock with a maximum capacity of 250,000 metric tons, and the third, signed in 1980, of only 100,000 metric tons (Gilbert, 1987, p. 601). By contrast, from 1972 to 1980, end-of-year world stocks of cocoa averaged 461,000 metric tons, with a minimum of 315,000 and a maximum of 730,000 (Gill and Duffus, 1981, p. 18). Finally, the buffer stock for natural rubber was

\(^7\)See Fox (1974, p. 401), Gordon-Ashworth, 1984, pp. 124-5, and Gilbert (1977, p. 111). The United States also frequently harped upon the inadequacy of buffer stock resources in negotiations and used it as a reason to stay out of the tin agreements (Finlayson and Zacher, 1988, pp. 95, 101).

\(^8\)Fox (1974, p. 109). The figures are from 1950 to 1972. They are an underestimate of private stocks, since they result from subtracting the buffer stock from total world stocks, but only part of the buffer stock is included in the total world stocks. They also omit the US Strategic Stockpile.

\(^9\)This changed dramatically in the 1980’s, in a way which will be discussed in section 4.
endowed with a capacity of 400,000 tonnes for normal operations and 150,000 tonnes of additional "contingency stocks" in case of emergencies (Gilbert, 1987, pp. 606-7). However, between April 1986 and December 1992, world stocks of natural rubber averaged 1.521 million tonnes, with a minimum of 1.290 million and a maximum of 1.950 million\textsuperscript{10}.

Even the ambitious buffer stock scheme in the Integrated Program for Commodities proposed by UNCTAD in the late 1970's had proposed levels of capacity that were well below customary levels of private sector storage. Hallwood (1977, p. 350) points out that UNCTAD had proposed buffer stocks with capacities of 8% of annual consumption at most, but at the end of 1973, private stocks for coffee, cotton, sugar, wool, cocoa, copper and wheat were respectively 48%, 45%, 26%, 25%, 20%, 10% and 9% of annual consumption. Hallwood claims that these are rather lower than usual figures for these markets. Of course, the UNCTAD plan foundered because consuming countries found it too ambitious and expensive.

Thus, (S1) is abundantly supported by facts. The second stylized fact is that the BSM is given considerable discretion. Discretion on the part of the buffer stock manager is an inevitable feature of buffer stock schemes because it is not possible to envision all possible contingencies at the time the international agreement is signed. Considerable constraints are typically placed on managers, and considerable pressure to achieve results, but most day to day decisions are left up to them. For example, the International Tin Agreement and the International Natural Rubber Agreement both specified price ranges within which the BSM was obliged to sell, permitted to sell, or obliged not to sell, and similarly for buying (Gilbert, 1987, p. 593). In middle ranges the

\textsuperscript{10}Economist Intelligence Unit, various issues. Up to January 1989 the figures are monthly, and after that they are quarterly; the average accordingly gives triple weight to each of the latter figures. This barely affects the final figure.
BSM had discretion. As a former Secretary of the ITA explained:

In the lower sector the first agreement gave the Manager authority to buy at the market price "if he considers it necessary to prevent the market price from falling too steeply". This provision in the agreements gave the Manager in the falling lower sector and in the rising upper sector -- covering between them about two-thirds of the whole range between floor and ceiling -- a very wide nominal freedom of action in which he was fettered solely by his own opinion. (Fox, 1974, p. 270).

BSM's have sometimes even made a virtue of unpredictability and have kept market participants guessing about their next move. As an extreme example, the BSM for tin was permitted to operate on forward markets, but after the breakdown of the tin agreement in 1985 member governments claimed not to know exactly what the BSM had been doing on them (Gilbert, 1987, p. 612), and many believed them.

Thus, the two generalizations on which the above argument turns can fairly be called stylized facts. Now we incorporate them into a simple stylized model.

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11 "Some suggest that he may start selling when the five-day moving average reaches 245 cents while others think that he will wait until the price moves higher." The guessers are rubber traders, the subject of their curiosity the BSM for natural rubber. *Natural Rubber News*, March 1984, p. 10.

12 "...did the left hand really know, or care, what the right hand was doing?... One may .. concede, that, if they are true, the charges against the BSM that he did not consistently keep ITC [International Tin Council] delegates up to date with his activities, are charges which anybody in his position would probably gladly face rather than report (daily?) to such a diverse group of 22 members." Gibson-Jarvie (1986, pp. 30, 31).
2. The model under laissez-faire.

Divide the year up into two seasons of equal length, indexed by \( n = 1, 2 \). Consider a market for a good with a harvest of one unit in the first season of every year and no harvest the rest of the year. Thus, we can call season 1 the “harvest” season and season 2 the “planting” season. This is done to produce in the simplest possible way alternating periods of glut and scarcity; similar arguments to those made below could be made in a model of i.i.d. random harvests, or (as would better fit a mineral commodity) a constant level of production but a demand curve shifting back and forth with world business cycles. The words “season” and “period” will be used interchangeably.

Thus, denoting the harvest by \( h_t \), \( h_t = 1 \) if \( n_t = 1 \) and \( h_t = 0 \) otherwise. Consumer demand for the good is described by the demand curve:

\[
(2.1) \quad p_t = \frac{1}{Q_t},
\]

where \( p_t \) is the price in period \( t \) and \( Q_t \) is the quantity consumed in period \( t \).

There is a competitive sector of risk-neutral traders who store the good for capital gains. They face no capacity constraint and incur no marginal storage cost, but each period lose a fraction \( \delta > 0 \) of what they store to spoilage. With a discount factor of \( \beta < 1 \), market clearing thus requires that:

\[
(2.2) \quad p_t \geq \beta(1-\delta)E_p_{t+1} \quad \text{and} \quad s_t \geq 0
\]

hold complementarily, where \( s_t \) is the amount stored by traders at the end of period \( t \). The reason for this is as follows. If the first inequality failed to hold, the traders would make a positive profit on each unit stored, and thus would have an infinite demand for the good. The second
inequality is commonsensical. Finally, if both inequalities held strictly, traders would suffer
strictly negative profits on each unit stored, but would store a positive amount anyway, which
would be irrational.

The traders are all aware of some common random variable $\varepsilon$ which contains no
information relevant to the market. We may interpret this as a "sunspot", or as the "confidence"
or "mood" of the traders. We will assume that it is an i.i.d. process observed once at the
beginning of each year, before the first market of the year has opened. It can take a value of 1,
meaning "confidence is high," or 0, meaning "confidence is low."

In the absence of a buffer stock, we will define equilibrium as a function $\psi$ which gives
the price each period as a function of the current amount $x$ of the good currently available, the
season, $n$, "confidence", $\varepsilon$, and some summary of history, $H$. It does not matter for now exactly
what form $H$ takes, but it will become important to the argument of the next section. We will
require, however, that it take at most a finite number of values. We will require the equilibrium
price function to be continuous in $x$ and to be defined for all $x$ in $(0, \infty)$ for each $m$, $\varepsilon$ and $H$.
This function must satisfy:

\begin{equation}
(2.3) \quad \psi(x, n, \varepsilon, H) = \max \left\{ \frac{1}{x}, \beta(1-\delta)E_{\varepsilon}[\psi((1-\delta)(x-\psi(x, n, \varepsilon, H)^{-1}) + h', n', \varepsilon', H')] \right\},
\end{equation}

where $x$ stands for the total availability of the good, i.e., $x_t = (1-\delta)s_{t-1} + 1$ if the current season
index is 1 and $x_t = (1-\delta)s_{t-1}$ if $n=2$, and a prime on a variable indicates a next-period value. For
example, $n'$ indicates next period's "season," so $n'=2$ if $n=1$ and $n'=1$ if $n=2$, and $h'=1$ if $n=2$
and 0 if \( n=1 \). The condition (2.3) flows directly from (2.2) in the following way. The second expression within the curly brackets in (2.3) is the expected discounted next period price, since next period's total availability is given by stocks carried over to next period, \((1-\delta)(x-\psi(x, n, \varepsilon, H)^{-1})\), plus next period's harvest if any. If the current price exceeds the expected discounted next period price, traders will not store and so the current price must equal \( 1/x \). If the current price is greater than \( 1/x \), it follows that traders must be storing something and thus keeping consumption below \( x \), so that the first inequality in (2.2) must hold with equality. This is a slight adaptation of the construction found in Deaton and Laroque (1992).

**Proposition 1.** There is exactly one equilibrium (i.e., a continuous function which solves (2.3)). This equilibrium is non-increasing in \( x \) and does not depend on \( \varepsilon \) or \( H \).

Henceforth we will denote this equilibrium parsimoniously as \( \psi^*(x, n) \). The proof of this proposition is fairly standard and may be found in the appendix. This proposition is very intuitive: since \( \varepsilon \) is irrelevant to the nature of supply or demand, it has no effect whatsoever on prices. Similarly, history in this model does not affect current or future market conditions, and so should not affect equilibrium. However, we shall shortly see a slightly more complicated model in which this is not true\(^{14}\).

\(^{13}\)Note that since \( \varepsilon \) is observed only at the beginning of each year, and thus its value is constant throughout the year, \( \varepsilon' = \varepsilon \) when \( n=1 \).

\(^{14}\)Note that the irrelevance of sunspots in equilibrium holds regardless of whether traders have infinite lives or constitute an overlapping sequence of generations of finitely lived traders. This is a contrast with most of the sunspot literature, and seems to come from the requirement that the consumers are always on their demand curve, an element missing from most models of asset
It is easy to derive the path of prices in this equilibrium. Consider period $t=1$, when $n_t = n_1 = 1$. The speculators will store some positive amount $s^*_1$, so consumption will be $c^*_1 = 1 - s^*_1$. The price will be $p^*_1 = 1/c^*_1 = (1-s^*_1)^{-1}$. By (2.2), the price at time $t=2$ must satisfy $p^*_2 = p^*_1 / [(1-\delta)/(1-s^*_1)]$, and so consumption will be $c^*_2 = (1-\delta)c^*_1$. At time 2, traders will drop their stocks in anticipation of the imminent next period harvest and the consequent drop in price. Thus, the stocks carried over to the beginning of period 2 must be just enough to give a price of $p^*_2$ when they are all sold at once:

$$1-\delta)s^*_1 = 1/p^*_2 = (1-\delta)c^*_1 = (1-\delta)(1-s^*_1).$$

This is readily solved for $s^*_1$, and hence for both prices, yielding $p^*_1 = (1+\beta)$ and $p^*_2 = (1+\beta)/(1-\delta)$. Clearly, $p^*_1 = p^*_1$, since the system will go through the same cycle of storage and stock-out every year.

3. Equilibrium under a buffer stock.

3.1. The Setting.

Now suppose that a convention of governments convenes, concerned that at the trough of the cycle equilibrium prices of this good are too low. They strike an international commodity agreement and hire a buffer stock manager (BSM) who is charged with the responsibility of keeping the price above a floor, say $\bar{p} \in (p^*_1, p^*_2)$. They give the manager access to funds and a storage capacity of $K$, and leave the exact mechanics of the operation of the stock up to her. 

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prizing.
However, they give her an incentive to meet the stated price goal: the governments will pay the manager a per period salary plus a bonus every time the price stays weakly above $\bar{p}$ for a whole year\(^{15}\).

We assume that the buffer stock is modestly endowed with storage capacity:

\[(3.1) \quad K < (1-\bar{p}^{-1}).\]

If the BSM had a sufficiently large capacity, she could simply purchase $(1-\bar{p}^{-1})$ in the harvest season of every year, leaving a price of $\bar{p}$, and sell in the planting season. In this case, the price floor would be successfully defended whether or not the private sector stored anything. However, as already discussed, this is not at all the way buffer stocks have worked in the past. Indeed, \((3.1)\) is the implementation of stylized fact (S1) in the model.

The main force of \((3.1)\) is that in order for the manager to do her job, she must enlist the co-operation of the private sector. If the price is not to fall below $\bar{p}$ in the harvest period, it will be necessary for private traders to store in the harvest season, and for total storage to exceed what it would be in the absence of the buffer stock. But since $\bar{p}$ is higher than $p^*_i$, this requires that the traders expect strictly higher prices to prevail in the planting season than would have prevailed in the absence of the buffer stock. In other words, in order for the buffer stock to have

\(^{15}\)The buffer stock may make profits or losses from time to time depending on luck and the BSM's behaviour, but we will assume that this has no part in the compensation scheme. This makes sense; real world buffer stocks are not profit making enterprises, and indeed they normally make a loss most of the time. This is understood by everyone when they are set up; their avowed role is as a public good to ameliorate some ill-defined market failure. They are supported by tax payers or by an export levy in producing countries. See Adnan (1972, p. 85).
a real impact now it must be expected to have a real impact later. This can be achieved in two ways.


Suppose that the BSM has committed publicly and credibly to a storage rule $\gamma$. The definition of equilibrium must be changed slightly from the laissez-faire case. The market clearing condition (2.3) becomes

$$(3.2) \quad \psi(x, n, e, H) = \max \{ \frac{1}{x-\gamma(x, n, e, H)},$$

$$\beta(1-\delta)E_c[\psi((1-\delta)(x-\psi(x, n, e, H)^{-1}) + h', n', e', H')] \},$$

where total availability $x$ is now the sum of buffer stock holdings and private inventories with the current harvest if any. The only difference between (3.2) and (2.3) is that the demand curve (2.1) has been replaced on the right hand side by $1/(x-\gamma(x, n, e, H))$, since this is the price that will result in the market if speculators do not store. It is easy to show that if the BSM can commit herself in this way and her resources are not too meager, a simple solution to her problem exists. She commits herself to filling her warehouse in the harvest season every year, but then selling less than the whole stock in the planting season. Her planting season hoarding will be just sufficient to drive the price up to $\bar{p} [\beta(1-\delta)]^{-1}$. Speculators, seeing this, will rush in period 1 to bid the price up to $\bar{p}$. Hence, under these conditions the BSM gets the salary bonus every single year. Formally:
Proposition 2. Define $\tilde{y}$ by:

$$
\tilde{y}(x, n, s, H) = \begin{cases} 
K & \text{if } n = 1; \\
\text{middle}\{0, K, x - (\bar{p}[\beta(1-\delta)]^{-1})^{-1}\} & \text{if } n = 2,
\end{cases}
$$

where the "middle" function takes the value of the argument whose value is in the middle of the other two.

Define

$$
\gamma_{ss} = \frac{(1-\delta)[1 - (1 + \beta)]}{1 - (1 - 2\gamma_{ss})^2} > 0.
$$

Then:

1. $K \geq \gamma_{ss}$ is consistent with (3.1) if $\delta$ is large enough or $\bar{p}$ is not too far above the laissez-faire price $(1+\beta)$.

2. Suppose that $K \geq \gamma_{ss}$ and further that at period $t=1$, $n=1$ and $x_1 \leq 1 + (1-\delta)\gamma_{ss}$.

Then if full commitment is possible, it will be optimal for the BSM to commit to $\tilde{y}$ forever, in the sense of maximizing her expected present value of income, since if she does, then she receives the salary bonus every year permanently.

Proof: See Appendix.

Here, $\gamma_{ss}$ is the steady state level of end-of-year buffer stock holdings, assuming that the
buffer stock does not run out of capacity. The condition $K \geq y_{ss}$ simply guarantees this.

The main lesson of Proposition 2 is in the form of the optimal commitment\textsuperscript{16}. The BSM promises to hang on to enough stocks in period 2, while all of the speculators are selling all they have, to drive the price up to $[\beta(1-\delta)]^{-1} p$. The ability to make this promise is of great value to the BSM, and we will see below that some equilibria in a world without commitment can be interpreted as springing from an attempt to make that promise credibly.

Full commitment is, of course, extraordinarily unlikely in the manager of a real world buffer stock. Further, in the real world any kind of legal commitment to hoard when prices are at their highest levels would meet fierce opposition from consumers, who are nearly always party to commodity agreements. Thus, we need to find another way of generating the right expectations.

3.3. Method Two: Reputation.

If the BSM cannot commit herself to future actions, then a discretionary equilibrium results that can be much worse for the BSM than the full-commitment optimum, for the same sort of reasons as make rules preferable to discretion in many other economic contexts (Kydland and Prescott, 1977). Here we modify the definition of equilibrium to take that into account.

The idea is this: Equilibrium will consist of a storage rule for the BSM and a price function such that given the BSM's storage rule, the price function will clear the market; and

\textsuperscript{16}It actually does not matter that the BSM stores to capacity in period 1. This makes the rule somewhat more realistic, but any value of first-period buffer stock storage would give the same price equilibrium.
given the price rule, the BSM's storage rule gives the optimal level of storage at each moment from the BSM's point of view. Expectations -- on both parts -- must be correct in equilibrium.

Formally, in any period, let $B \in [0, x) \cap [0, K]$ denote the amount that the BSM chooses to store. An equilibrium will be defined as a function $\gamma$ of $x$, $n$, $\varepsilon$, and $H$, and a function $\psi$ of these and $B$. $\psi$ is the price function, as before, and $\gamma$ is the storage rule followed by the BSM; in other words, at each state of the market, the choice of $B$ made in equilibrium will be given by the value of the function $\gamma$. Note that speculators are allowed to respond to current actions of the BSM, including possible deviations from the rule $\gamma$, by the dependence of $\psi$ on $B$. To be an equilibrium, any such pair of functions must satisfy two conditions. (i) The market clearing condition is:

$$\psi(x, n, \varepsilon, H, B) = \max \left\{ \frac{1}{x-B}, \beta(1-\delta)E_c[\psi(x', n', \varepsilon', H', \gamma(x', n', \varepsilon', H'))] \right\},$$

where $x' = (1-\delta)(x-\psi(x, n, \varepsilon, H, B)) + h'$.

Condition (3.3) is analogous to (2.3) and (3.2). Again, the main change is the demand curve, which has now been replaced by $1/(x-B)$, since if speculators do not store, then this will be the market price (whether $B$ is the equilibrium value of the buffer stock's storage or some one-shot deviation). (ii) The other condition is that in each state and in each period, setting $B$ equal to $\gamma(x, n, \varepsilon, H)$ maximizes the expected present discounted value of the BSM's earnings. This is the time consistency condition, and is the implementation within the model of stylized fact (S2).

Here we find as great a contrast as there could be with the laissez-faire case: there we had
a unique equilibrium, but in this setting a great many equilibria are possible. For example, there is one with no trust:

\[(3.4) \quad \gamma^0(x, n, \varepsilon, H) \equiv 0.\]
\[(3.5) \quad \psi^0(x, n, \varepsilon, H, B) = \max \{ 1/(x-B), \psi^*(x, n) \},\]

where \(\psi^*\) is the laissez faire equilibrium from section 2. In equilibrium \((\gamma^0, \psi^0)\), the buffer stock never stores and the market, knowing that, generates prices exactly as if it did not exist. If the BSM decided to store at some point, the market would simply assume it was a one-shot deviation; the market price would change only in the event that the BSM bought up all private inventories and then some, driving the price up to \(1/(x-B)\) as indicated in (3.5)\(^{17,18}\).

An equilibrium like this would occur if the BSM was unable to persuade the speculators in period 1 that she would hoard to support the price in period 2. Unlike in the full-commitment case, here she can only tell the speculators that she will do so and rely on trust. If that trust is absent, speculators correctly perceive her as having no incentive to buy at the end of the year.

\(^{17}\)It is easy to show that \(\psi^0\) satisfies (3.3). (We will drop the irrelevant \(\varepsilon\) and \(H\) from here on in discussing \(\psi^0\) and \(\gamma^0\).) Case 1. \(1/(x-B) \geq \psi^*(x, n)\). Then \(\psi^0(x, n) = 1/(x-B)\). \(1/(x-B) \geq \beta(1-\delta)\psi^*((1-\delta)(x-[\psi^*(x,n)]^{-1}) + h', n') \geq \beta(1-\delta)\psi^*(((1-\delta)(x-[\psi^0(x,n)]^{-1}) + h', n')\) since \(\psi^0 \geq \psi^*\), and this demonstrates that (3.3) holds since \(\gamma(x', n', \varepsilon', H') \equiv 0\).

Case 2. \(1/(x-B) < \psi^*(x, n, n)\). This implies that \(\psi^0(x, n) = \psi^*(x, n)\), but since \(1/x \leq 1/(x-B)\), it also implies that \(\psi^0(x, n) = \beta(1-\delta)\psi^*(((1-\delta)(x-[\psi^0(x,n)]^{-1}) + h', n')\). Thus, \(1/(x-B) < \beta(1-\delta)\psi^*(((1-\delta)(x-[\psi^0(x,n)]^{-1}) + h', n')\), which demonstrates (3.3) since \(\gamma(x', n', \varepsilon', H') \equiv 0\).

\(^{18}\)It is trivial to show that \(\gamma^0\) is time-consistent. In the first period of any year \(\psi^0\) will take a value of either \(1/(x-B) \leq 1/(x-K) < \bar{p}\) by (3.1), or \(\psi^*(x, 1) \leq \psi^*(1, 1) = p_1 < \bar{p}\). Thus, in no way can the floor be defended and in no way can the BSM get a salary bonus. Thus, she is indifferent between all choices of \(B\) in each period.
The equilibrium entails an impotent buffer stock and no bonuses for the manager in any year.

We finally look at a richer equilibrium, which essentially embodies the point of this paper. First, let $H$ be a particularly simple summary of world history:

\begin{equation}
H = \begin{cases} 
1 & \text{if every time speculators have ever bought in the harvest season at a price of } \bar{p}, \text{ they have always been able to sell in the planting season at a price of at least } \left[\beta(1-\delta)\right]^{-1}\bar{p}; \\
0 & \text{otherwise.}
\end{cases}
\end{equation}

In other words, if ever the speculators have bet on the buffer stock and been burned, they remember it. Now consider the buffer stock rule:

\[ \hat{y}(x, n, \varepsilon, H) = \begin{cases} 
K & \text{if } n = 1; \\
\text{middle}\{0, K, x-(\bar{p}[\beta(1-\delta)]^{-1})^{-1}\} & \text{if } n = 2, \varepsilon = 1 \text{ and } H = 1; \\
0 & \text{otherwise.}
\end{cases} \]

This rule says that in the first period of every year, the BSM stocks up; and in the second period of every year, if confidence is high and speculators have never been burned, the BSM should store a particular quantity--the same quantity as was part of the optimal rule with commitment. However, in the second period of the year, if confidence is low or the BSM has ever deviated, she throws up her hands. Why the BSM should desire to do this will be seen shortly. The following lemma defines the price function that would be the equilibrium if it was
common knowledge that \( \gamma \) would be followed permanently by the BSM as an exogenous, immutable rule:

**Lemma.** There is a unique continuous function \( \phi \) that satisfies:

\[
\phi(x, n, \varepsilon) = \max \left\{ \frac{1}{x} - \gamma(x, n, \varepsilon, 1), \beta(1-\delta)E_u[\phi((1-\delta)(x-\gamma(x, n, \varepsilon))^{-1}) + h', n', \varepsilon'] \right\}.
\]

**Proof.** Identical to the proof of Proposition 1 in all respects, except that the demand curve \( 1/x \) is replaced by a “modified” demand curve \( 1/(x-\gamma(x, n, \varepsilon, 1)) \). Q.E.D.

Finally, we define the actual equilibrium price function:

\[
\psi(x, n, \varepsilon, H, B) = \begin{cases} 
\psi^0(x, n, B) & \text{if } H = 0 \text{ or } B \neq \gamma(x, n, c, H); \\
\phi(x, n, \varepsilon) & \text{otherwise},
\end{cases}
\]

using the \( \phi \) just defined and the price function \( \psi^0 \) from the “no trust” equilibrium. Thus, under the equilibrium \( (\gamma, \psi) \), each speculator waits to see at the beginning of each year whether or not the market is confident. If it is \( (\varepsilon=1) \), then they all assume that the BSM will intervene in the market at the end of the year to hike the price up artificially. They then bet on this, bidding the price up to \( \bar{p} \). If the market is not confident, they will doubt that she will do this and the price

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\(^{19}\text{This is not a trivial assertion. The interested reader may go through all the steps of the proof of Proposition 1 and verify that the same logic works under the assumption of the Lemma. The reason is that } \gamma \text{ is everywhere non-decreasing in } x, \text{ with a slope that never exceeds one. This is a special case of what is termed “regularity” of a buffer stock rule in McLaren (1992), where the consequences of the assumption and its absence are worked out in great detail.}\)
will remain low. If she has ever once let them down, no matter what she says or does now, they
will assume that she will do so again and will accordingly bid as if the buffer stock did not exist.

**Proposition 3.** Recall \(\gamma_{ss}\) from Proposition 2. Suppose that \(K \geq \gamma_{ss}\) and that at period \(t=1, n=1\)
and \(x_i \leq 1 + (1-\delta)\gamma_{ss}\). Then \(\hat{x}\) and \(\hat{y}\) are an equilibrium.

**Proof:** \((3.3)\) is satisfied by construction. The same calculations as in the proof of Proposition
2 can be done to show that \(x - (\bar{p}\beta(1-\delta)]^{-1})\) will be permanently below \(K\) (in fact, these
calculations hold *a fortiori* since there stocks were carried over at the end of every year, but here
they are only if \(\varepsilon=1\)). Therefore, the BSM who adheres rigourously to the equilibrium knows that she
will be able to defend the floor successfully every time \(\varepsilon=1\) and thus can look forward to a strictly
positive expected value of salary bonuses. By contrast, the BSM who deviates even once knows that from
then on \(H\) will be equal to 0, pricing will be as in the *laissez faire* equilibrium, and thus there
will be no bonuses ever again. Thus, it is always optimal for the BSM to set \(B = \hat{y}(x, n, \varepsilon, H)\),
and time consistency is proven. **Q.E.D.**

### 3.4. A Closer Look at the Equilibrium.

When this works, that is, when \(\varepsilon=H=1\), the story that unfolds is as follows. At the start
of the year, speculators, feeling confident that the buffer stock will be effective this year, bid the
price up to \(\bar{p}\), and in doing so buy up \((x-\bar{p}^{-1})\) for storage. (The BSM stocks up as well, but
because of her capacity constraint, that does not really matter.) In the second period, speculators
know that a harvest is imminent, in the first period of the next year. Thus, they know that regardless of all other factors, the price will fall next period, and therefore they will all sell their stocks now. However, if the BSM also sells her stocks, the price will now be depressed below $\tilde{p}[\beta(1-\delta)]^{-1}$ and they will suffer losses right now. They need the BSM to save them, by hoarding some of her stock to support the period 2 price.

It is feasible for the BSM to save them, despite the constraint (3.1) that made it infeasible for the BSM to defend the floor on her own in period 1. This is because the quantity that must be hoarded in the last period of the year in order to save the speculators is much smaller than $1-\tilde{p}^{-1}$, and this is what is established in demonstrating that $x-(\tilde{p}[\beta(1-\delta)]^{-1})^{-1} \leq K$ in the proof of the Proposition. The reason this works is easiest to see in the first year of operation of the system, when there are no stocks carried over into period 1. In that case, $1-\tilde{p}^{-1}$ is total storage in period 1, but then what is left over by period 2 after depreciation is a smaller quantity, and what the buffer stock must store in order to save the speculators is only part of that.

The reason it is desirable for the BSM to save the speculators, however, is found in the role of the history variable H. If she has once let them down, and failed to buy the full amount necessary to keep them from suffering losses after they have bet on the buffer stock, then they will remember. Forever after, H will have a value of zero, and speculators will only bid up to the laissez faire price every period. All-important trust will have been broken. Thus, the BSM will lose her end of year bonus permanently.

Incidentally, after having defected once, the BSM could always go ahead and hoard in the last period to drive the price up to $\tilde{p}[\beta(1-\delta)]^{-1}$, so the speculators will receive a windfall gain, but they will be unimpressed: once bitten, forever shy. They will still behave as under laissez-
faire the following year. That being the case, the BSM has no real incentive to do so. A final note to make about the equilibrium is that despite the inherently deterministic setting, equilibrium prices are random. They do depend on the irrelevant "confidence" variable ε, and will bounce up and down from year to year without good reason at all. This was impossible in the case without intervention.

4. Discussion.

We have shown that buffer stocks as implemented in practice may have created the possibility of destabilizing speculation where no such possibility had existed before. This can be thought of as a countervailing influence to the stabilizing influence from supplementing private storage noted by other authors. Here we note two things to be said for this argument: that it accounts for some features of the experience of buffer stocks which have no place in the traditional models, and that it can encompass a fairly wide variety of BSM strategies that have been observed, quite far beyond a literal interpretation of the simple model in the paper.

One feature of the experience of buffer stocks that has already been noted is the high volatility of commodity prices even in the presence of a working buffer stock. Indeed, in the case of cocoa and tin, the market was much more volatile during the buffer stock period than in the rest of the sample. While this does not prove anything, it is odd.

More salient is the concern expressed by BSM's of the past about maintaining the right reputation with traders in the market. For example, after heavy buying in the glut of the previous two years, and with the price now near the top of the band, in March 1984 the rubber BSM was
preparing to sell for the first time ever. Before doing so, he travelled to meet with traders in Kuala Lumpur, New York and Europe, to explain to them that although he was about to sell he would be very careful to do so in a way that would not "disrupt the market" (*Natural Rubber News*, March 1984, p. 10). Now this would be mysterious in a traditional model: near the top of the band, the BSM simply sells to keep the price from crossing the ceiling; he does not care or need to care about the feelings or well-being of traders. However, the meetings make a great deal of sense in the model of this paper, in which the biggest problem the BSM faces is convincing traders that they can trust him to allow them to take normal profits in period 2, when he is selling and prices are high.

An example with the opposite flavor comes from the later years of the tin buffer stock, when the buffer stock held most of the world supply of the metal. On a number of occasions the BSM restricted physical supplies to drive up the spot price sharply for short periods. Although the buffer stock was long in futures and therefore profited by the squeeze, some seasoned observers have argued that the *harm* the action did to speculators who had sold short was as much of a motivation for the BSM's action. An interpretation, of course, is that there was a perceived value to the BSM in building a reputation for punishing traders who bet *against* the buffer stock as well as in building a reputation for rewarding traders who bet *on* it. An extension of the model in the previous sections to allow for this manipulation of futures markets would be purely mechanical. And, of course, there would be absolutely no role for "punishing"

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90Bought on credit. See the discussion of the tin BSM's credit strategy below.

91See Anderson and Gilbert (1988, p. 11). Gibson-Jarvie (1986, p. 8) reports that the squeeze technique was intended to give "a salutary lesson to the shorts."
speculators in the conventional buffer stock model.\footnote{A similar argument applies to Gilbert (1987, p. 609), in which it is reported that the tin BSM sought and won greater discretionary powers in early 1985 partly because he was “keen to put speculators into a position where they could not guarantee to avoid losses in speculating against the agreement.” Also see Prest (1986, pp. 38-9) for an account of the public-relations aspects of the squeeze.}

In general, officials overseeing buffer stocks have seemed to care quite a bit about speculators and their moods and have often expressed the view that speculative prices differ from fundamentals prices. A BSM for the Tin Agreement, R. T. Adnan, wrote that

It is well known that it is sometimes more profitable for speculators to forecast the psychology of another speculator rather than assess the real demand and supply situation. It also seems to pay to devote one’s intelligence to anticipating what average opinion expects the average opinion to be. In short, it is highly important when analyzing the market to recognize to what extent the forces of supply and demand actually exercised are a true reflection of the real supply and demand situation. (Adnan, 1972, p. 90).

The remark, of course, paraphrases Keynes; but what is significant is its source. Adnan generally speaks of speculators in admiring terms, but also mentions the existence of “unwarranted waves of optimism and pessimism” on the market (p. 90) and argues that there are periods in which speculation has an “undue influence on the market” (p. 93). William Fox, Secretary of the International Tin Council for several years, wrote of

... the problem created by the inability of [the London Metal Exchange (LME)] to arrive at prices which reflect accurately the current relationship of supply and demand for physical metal. This Exchange has also developed as a hedging market and, perhaps to an even greater extent, as a centre for purely speculative dealing. (Fox, 1974, p. 400).

Fox argues that these “massive speculative dealings” make it impossible for a buffer stock to stabilize a market completely. All of these remarks suggest that these officials believed from
their experience that “sunspots mattered” in the market; they could not possibly matter in traditional models of buffer stocks. The ironic point of this paper is that it is possible that sunspots did matter simply because of the presence of these officials.

A final feature that can be accounted for by the argument of this paper is the apparent presence of cross-market effects that do not appear to have anything to do with fundamentals. Tin and natural rubber have very little to do with each other, other than having some producer countries in common. They are not jointly produced or, to any notable degree, jointly consumed; neither is an input for the other, and they do not share important inputs. Yet the calamity in the tin market when the tin buffer stock collapsed on October 24, 1985 led to a plunge in prices on rubber markets:

One natural victim of tin’s plight is natural rubber (NR), where weak markets are causing growing disquiet among major Asian producers -- among them Sri Lanka, India, Thailand and most importantly, Malaysia, where large numbers of smallholders have mortgaged their futures to the rubber tree.

Although sluggish before the tin collapse, ... rubber prices have become the “innocent victim” (as one trader put it) of “bad sentiment spilling over from the London Metal Exchange debacle,” reaching a 10-year low on 25 November ...

(Far Eastern Economic Review, Dec. 5, 1985, p. 87)

This is inconceivable in traditional models. For example, if there was a Salant (1983) buffer stock in tin and another one in rubber and at some point the tin stock’s funds were removed so that it simply stopped functioning, this would have no effect on events in the rubber market; as already noted, in that model each market has a unique equilibrium, and this would continue to govern rubber prices. However, in the model of this paper, otherwise irrelevant events affecting
the “confidence” (ε) of rubber traders can indeed be crucial.

Thus, a lot of what has gone on under past buffer stocks can be explained by a model such as is offered in this paper but not by more traditional models. Further, it is worth noting that the idea of the model is much more general than the model itself and can accommodate a great many observed buffer stock strategies beyond those used in the model. The main idea, once again, is that (S1) implies that the BSM needs the help of traders, and (S2) means that traders are going to need to trust the BSM before they will supply that help. Therefore, any strategy that might help the BSM overcome the commitment problem is likely to be tried. Thus, it has already been noted that the use of futures market devices to punish traders for betting against the buffer stock is much more comprehensible after taking into account the limitations facing a BSM on which our model is based. Anderson and Gilbert (1988, p.11) also suggest a role as a commitment device for the long futures positions taken by the tin buffer stock itself, since if the BSM cares at least somewhat about the financial state of the buffer stock, a future sale in period 1 will give him a strong incentive to keep the spot price up in period 2. These effects could easily be built into a slightly richer model.

Finally, we could easily accommodate a strategy which has had gigantic consequences in commodity markets in recent years: borrowing to extend the buffer stock’s capacity, and using the commodity itself as collateral. In the 1970’s this emerged in many observers’ minds as the solution to the tin buffer stock’s problem of inadequate resources. For example, Gilbert (1977,

\[2^{nd}\text{In a related study, Pindyck and Rotemberg (1990) found evidence that commodity prices tend to move together to a much greater degree than can be justified by fundamentals, including business cycles. One suggested interpretation was that trader “confidence” does have a role in commodity price determination apart from fundamentals and that bullish conditions in one market can spill over into high confidence in other markets.}]}
pp. 111-4) complained of the impotence of the buffer stock due to the small and falling real value of the funds with which it was permitted to buy the metal. However, there was hope, because the new ability of the BSM to borrow against his tin stock could triple the buffer stock's capacity. The BSM, Pieter de Konig, took advantage of this opportunity with a vengeance. As another commentator put it nine years later:

As the later audit of the ITC [International Tin Council] by the accountants Peat Marwick Mitchell showed, the buffer stock had entered into a complex web of deals with metal brokers, mainly on the LME. In essence these arrangements enabled the buffer stock to stretch its already thin finances, with the incidental advantage of disguising how thin they were, and take on more tin [than] would otherwise have been possible. One device was to borrow money from brokers to buy more tin, often from the same brokers who lent the money. Another was to 'borrow' tin in such a way that the buffers (sic) stock paid only interest and a premium rather than the full cost of outright purchase. (Prest, 1986, p. 39)

The striking feature of the credit strategy was its self-fulfilling quality. The buffer stock needed to borrow to support the price. If it succeeded in supporting the price, the value of its tin collateral would be high, so if it was expected to succeed, banks would be willing to lend. Thus, if it was expected to succeed, it would be able to find credit and it would succeed; if it was not expected to succeed, banks would be unwilling to lend and it would fail. Indeed, it is precisely this kind of reasoning which is used by observers who followed the market day by day to describe the rush of credit leading up to the crisis and the subsequent evaporation of credit which precipitated it.

This mechanism can be incorporated in the model of this paper. It has the two basic elements on which the model has been built: the inadequate capacity of the buffer stock on its

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own, and the enormous discretion given to the BSM. It would be straightforward to rewrite the model, replacing the speculators with creditors, who when ε is high feel ready to lend to the BSM and when it is low do not. The only difference would be the superficial one: in good years, the additional commodity would be held by the buffer stock, not by speculators. However, a quite major modification must be made if one wishes to accommodate the possibility of default, which is important since default on a calamitous scale was the final outcome. The special case of tin is explored in McLaren (1993), in which it is argued that the conditions under which the tin default can be interpreted as a self-fulfilling collapse of a market bubble in the way here described involve plausible restrictions on the elasticity along the demand curve.\footnote{The elasticity must be strictly below unity or declining in absolute value as quantity increases. Incidentally, there is (fortunately) no need for depreciation in that model, in contrast to the model of this paper.}

In conclusion, the germ of the idea of this paper can be much more widely applied than a literal reading of the model would suggest, to explain odd behaviour and volatility in quite a variety of contexts, sharing nothing but a poorly endowed buffer stock.
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Appendix.

Proof of Proposition 1.

The techniques invoked in this proof are by now fairly standard, and thus with the full proof rather intricate, much of it will be sketched. The interested reader can consult the proofs in Salant (1983), Deaton and Laroque (1992) and McLaren (1992) for rigorous treatments of all of these arguments.

Part A. Let $\psi$ be an equilibrium, that is, a continuous function that solves (2.3). Fix $\varepsilon$ and $H$. When $n=2$, $n'=1$ and $x' = (1-\delta)(x-\psi(x, n, \varepsilon, H)^{-1}) + 1 \geq 1$. Now, $\psi(x, n, \varepsilon, H) \geq 1/x$ (by (2.3)), so $\psi(x, 2, \varepsilon, H)^{-1}$ takes a limit of zero as $x \to 0$. Thus, $x' \to 1$ as $x \to 0$. Thus,

$$\beta(1-\delta)E_{\varepsilon}[\psi((1-\delta)(x-\psi(x, 2, \varepsilon, H)^{-1})+1, 1, \varepsilon', H')] \to \beta(1-\delta)E_{\varepsilon}[\psi(1, 1, \varepsilon', H')]$$

as $x \to 0$. Therefore, there exists $\bar{x}(\varepsilon, H)>0$ such that

$$x \leq \bar{x}(\varepsilon, H) \Rightarrow 1/x \geq \beta(1-\delta)E_{\varepsilon}[\psi((1-\delta)(x-\psi(x, 2, \varepsilon, H)^{-1})+1, 1, \varepsilon', H')]$$

as $x \to 0$. Therefore, there exists $\bar{x}(\varepsilon, H)>0$ such that $x \leq \bar{x}(\varepsilon, H)$ implies that $\psi(x, 2, \varepsilon, H) = 1/x$. Call this "Property A.1."

Part B. A mechanical reproduction of the proof of Theorem 4 in McLaren (1992), mutatis mutandis, would demonstrate that any equilibrium must be non-increasing in $x$. The reasoning
proceeds through several steps centered on the proof that equilibrium storage must be non-decreasing in $x$. It would be cumbersome and rather pointless to reproduce the proof here.

Part C. Parts A and B place strong conditions on any equilibrium price function in the last period of the year. We will show that these are enough to determine uniquely the whole equilibrium price function. We will do this by showing how the function for period 1 can be calculated from any candidate period 2 price function, and how the period 2 function can in turn be calculated from that. We thus get a recursive relationship characterizing the period 2 price function in terms of itself; this recursive relation will be shown to have a unique solution.

Let $\Phi$ be defined as the set of functions $\phi$ on $x, \varepsilon$ and $H$ that are continuous and non-increasing in $x$ and satisfy $\phi(x, \varepsilon, H) \geq 1/x$ for all $x$. Define $W(\phi)$ by setting $W(\phi) (x, \varepsilon, H)$ equal to the solution for $p$ of

$$p = \beta (1-\delta) \phi((1-\delta)(x-p^{-1}), H', \varepsilon).$$

This is illustrated in Figure 1. It is easy to see that if $\phi$ is in $\Phi$, then so is $W(\phi)$. Clearly, if $\phi$ is the equilibrium price function for period 2 (i.e., $\phi(x, \varepsilon, H) = \psi(x, 2, \varepsilon, H)$ for all $x, \varepsilon$, and $H$), then $W(\phi)$ gives the equilibrium price function for period 1$^{26}$. This is because with no harvest in period 2, there will always be some storage in period 1; the second term in equation (2.3) will always be the larger one. (To see this, let the value of $\psi$ in period 1 approach $1/x$; by Property A.1, the right-hand side of (2.3) approaches infinity, yielding a contradiction.)

$^{26}$Recall that since the confidence variable is revealed at the beginning of the year for the whole year, $\varepsilon'$ as of period 1 is known and equal to $\varepsilon$. See footnote 13.
For any $\phi \in \Phi$, define $U(\phi)$ by setting $U(\phi)(x, \epsilon, H)$ equal to the solution for $p$ of:

$$
\max \{ 1/x, \beta(1-\delta)E_v\{W(\phi) \left( (1-\delta)(x-p^{-1}) + 1, H', \epsilon' \right) \} \}.
$$

It is clear that if $\phi \in \Phi$, then $U(\phi) \in \Phi$. By construction, if $\phi$ is a period 2 equilibrium price function, then so must $U(\phi)$ be. Since once we have the period 2 price function we can calculate the period 1 price function from it as demonstrated, equilibrium is fully characterized by the set of fixed points of the operator $U$.

Now note that the reasoning used on the equilibrium price function in Part A can be applied to $U(\phi)$ for any $\phi \in \Phi$. Thus, $U(\phi)$ must have property A.1, and if we define $\Phi'$ as the subset of $\Phi$ whose members also satisfy property A.1, then $W$ maps $\Phi'$ into itself. Further, Part A and Part B demonstrate that any period 2 equilibrium price function must be in $\Phi'$. Now, property A.1 together with the fact that functions in $\Phi'$ are non-increasing in $x$ and the assumption that there are a finite number of values for $\epsilon$ and $H$ together imply that $p(\phi, \psi) = \sup_{(x, \epsilon, H)} \{ \phi(x, \epsilon, H) - \psi(x, \epsilon, H) \}$ is well defined on $\Phi'$.

Part D. From here it is elementary to verify that $W$ satisfies Blackwell's properties on $\Phi'$ with the metric $\rho$, and hence has a unique fixed point on that set. (i) Monotonicity. If $\phi^*$ lies nowhere below $\phi$, then it is immediate that $W(\phi^*)$ lies nowhere below $W(\phi)$, and similarly for $U(\phi^*)$ and $U(\phi)$. (ii) Addition of a constant. From Figure 2, it is clear that for a constant $a > 0$, $W(\phi + a) \leq W(\phi) + \beta(1-\delta)a$, and similarly for $U$. Q.E.D.
Proof of Proposition 2.

(1) The claim, that $K > \gamma_{ss}$ is consistent with $K < (1-\bar{p}^{-1})$ if $\delta$ is large enough or $\bar{p}$ is small enough, can be verified mechanically. Rearranging $\gamma_{ss} < (1-\bar{p}^{-1})$ yields $\bar{p} < \frac{\kappa(\delta)(1+\beta)-1}{\kappa(\delta)-1}$ if $\kappa(\delta)-1 > 0$ and $\bar{p} > \frac{\kappa(\delta)(1+\beta)-1}{\kappa(\delta)-1}$ if $\kappa(\delta)-1 < 0$, where $\kappa(\delta) = (1-\delta)/(1-(1-\delta)^2)$. This, together with the constraint that $\bar{p} > p^* = (1+\beta)$, translates into the shaded region in $(\bar{p}, \delta)$ space illustrated in Figure 3.

(2) Now, for the main claims of the Proposition, first, we note that there is a unique continuous $\psi$, which we will denote $\tilde{\psi}$, which satisfies (3.2). This price function does not depend on $\varepsilon$ or $H$, so we will omit those arguments in writing it (and also in writing $\gamma$), and it has the property that $\tilde{\psi}(x, n) \to \infty$ as $x \to 0$. The proof of this is identical to the proof of Proposition 1, mutatis mutandum, as the interested reader can verify (also see footnote 19 in the text). Further, it has the property that $\tilde{\psi}(x, 2) = 1/(x-\tilde{\gamma}(x, 2))$ if $x \leq K + (\bar{p}[\beta(1-\delta)]^{-1})^{-1}$, i.e., for moderate levels of supply there is no private storage in the planting season. Call this Property A.2. This can be shown, again, using the constructions in the proof of Proposition 1, by defining $\Phi''$ as the subset of $\Phi$ satisfying Property A.2, showing that the appropriate functional operator maps $\Phi''$ into itself, and noting that the operator is a contraction on that set.

From here the proof has three parts. First, we show that if commitment to $\tilde{\gamma}$ leads to $x_i \leq K + (\bar{p}[\beta(1-\delta)]^{-1})^{-1}$ whenever $n_i = 2$, then it will guarantee that the price will always stay above the floor. Second, we show that $\tilde{\gamma}$ indeed leads to $x_i \leq K + (\bar{p}[\beta(1-\delta)]^{-1})^{-1}$ whenever $n_i = 2$ under the initial condition assumed in the Proposition. Finally, we show that these two properties guarantee the optimality of $\tilde{\gamma}$. 
(i) Under $\gamma$, if $n_t = 2$ and $x_t \leq K + (\bar{p}[\beta(1-\delta)]^{-1})^{-1}$, then

$$\gamma(x, n, \epsilon, H) = \text{middle} \{0, K, x-(\bar{p}[\beta(1-\delta)]^{-1})^{-1}\}$$

is enough to $(\bar{p}[\beta(1-\delta)]^{-1})^{-1}$,

and so by (3.2), $\psi(x, n, 2) \geq \bar{p}[\beta(1-\delta)]^{-1}$. Therefore, again by (3.2), we find that $\psi(x_{t-1}, 1) \geq \bar{p}.

Thus, we have finished the first part of the proof.

(ii) We need to show that in planting seasons $x-(\bar{p}[\beta(1-\delta)]^{-1})^{-1} \leq K$ for the largest value of $x$ that could occur in equilibrium in period 2. Assume that it is initially true. If period $t$ is a planting season and $x_t$ is the level of availability at the beginning of the period, then the amount stored is $\max\{x_t-(\bar{p}[\beta(1-\delta)]^{-1})^{-1}, 0\}$. Thus, next period's availability, $x_{t+1}$, is given by

$$x_{t+1} = \max\{(1-\delta)[x_t-(\bar{p}[\beta(1-\delta)]^{-1})^{-1}], 0\} + 1.$$  

Then the amount stored (by all agents) in period $t+1$ is $(x_{t+1}-\bar{p}^{-1})$, because the first-period price must be $\bar{p}$ (by (3.2) and Property A.2). Thus, the amount left over at the beginning of period $t+2$, which will again be a planting season, is:

$$x_{t+2} = (1-\delta)(x_{t+1}-\bar{p}^{-1}) = (1-\delta)^2 \max\{x_t-(\bar{p}[\beta(1-\delta)]^{-1})^{-1}, 0\} + (1-\delta)(1-\bar{p}^{-1}).$$

Thus, we have a stable, first order piecewise linear difference equation relating consecutive planting-season levels of availability ($x_{t+2}$ and $x_t$) to each other. This is depicted in Figure 4. The steady state solution gives the maximum value of $x$, $x_{ss}$, that could ever be seen in the second period of a year provided that the system began below that level. Plugging this into $\gamma(x, 2)$ gives a value for buffer stock storage equal to $\gamma_{ss}$, which thus is the largest possible value of $\gamma$. But $\gamma_{ss} \leq K$, by assumption, and so $x-(\bar{p}[\beta(1-\delta)]^{-1})^{-1} \leq \gamma(x, 2) \leq K$ always holds if it holds initially. Finally, the steady-state harvest-period level of availability ($x$) is clearly
(1-\delta)y_{ss+1}, which gives us the initial condition in the statement of the Proposition.

(iii). Parts (i) and (ii) together show that under the assumptions of the Proposition, commitment to \( \bar{y} \) ensures that the price never falls below \( \bar{p} \) in any period. Thus, the BSM, by using this strategy, will always receive her salary bonus. That is her first best outcome. \textbf{Q.E.D.}
Table I: Historical Volatility, with and without Buffer Stocks

<table>
<thead>
<tr>
<th></th>
<th>Std. deviation of first difference:</th>
<th>Average absolute first difference:</th>
<th>ESE from price regression:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tin:</strong></td>
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<tr>
<td>ITA Buffer Stock:</td>
<td>3.4296</td>
<td>2.1051</td>
<td>3.4202</td>
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<tr>
<td>After buffer stock:</td>
<td>2.0063</td>
<td>1.0047</td>
<td>1.6546</td>
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<td><strong>Cocoa:</strong></td>
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<tr>
<td>Before Buffer Stock:</td>
<td>4.7350</td>
<td>3.4874</td>
<td>4.6650</td>
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<tr>
<td>First and second ICCA's:</td>
<td>13.7022</td>
<td>9.9742</td>
<td>13.5557</td>
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<tr>
<td>Third ICCA:</td>
<td>3.9493</td>
<td>2.2773</td>
<td>3.8632</td>
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<td><strong>Natural Rubber:</strong></td>
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<tr>
<td>Before Buffer Stock:</td>
<td>5.071332</td>
<td>3.42412</td>
<td>5.035481</td>
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<tr>
<td>INRA buffer stock</td>
<td>2.164096</td>
<td>1.493028</td>
<td>2.054912</td>
</tr>
</tbody>
</table>
\[ \beta (1-\delta) \phi ((1-\delta)(x-p^{-1}), \varepsilon, H') \]
\[ \Delta p < \beta(1-\delta)a \]

\[ \beta (1-\delta) \phi ((1-\delta)(x-p^{-1}), \varepsilon, H') + \beta(1-\delta)a \]

**Figure 2.**
Figure 3.
Figure 4.