Learning about Ability and the Effects of Pay Incentives

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ABSTRACT
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This dissertation studies how pay incentives interact with learning about ability and labor turnover to shape the employment dynamics at a US call center. The first chapter provides an introduction to my work and summarizes my main results. The second chapter offers a descriptive analysis of the work environment, the production process, and the effects of pay incentives. The third chapter introduces learning about ability and turnover in a model of effort choice under moral hazard. This model is then used to evaluate the effects of changing pay incentives at the call center. The effect of incentives on effort is significant but small. The results indicate that turnover is a major channel through which incentives affect average performance. Simulating the estimated model shows that neglecting learning and turnover makes estimates of the effect of incentives on effort twice as big as they should be. The fourth chapter investigates how considerations about the quality mix shape pay policy and profits. Building on the estimation approach in chapter 3, the fourth chapter presents a two-step procedure that is used to estimate a fully structural version of the model introduced in the previous chapter. The results provide the basis for counterfactual policy analysis. The optimal policy, in the class of linear contracts in output, not only induces employees to exert effort but also acts as a selection mechanism that helps the firm build a workforce of high match quality over time. The results show that turnover is the major channel through which pay incentives affect profits.
# Table of Contents

Table of Contents  
List of Tables  
List of Figures  
Acknowledgements  
1 Introduction  
2 The Employment Dynamics at a Firm Offering Short Term Jobs  
   2.1 Introduction  
   2.2 Data and Descriptive Analysis  
      2.2.1 Work Environment and Pay Policies  
      2.2.2 Data  
      2.2.3 Descriptive Analysis: Calendar Time  
      2.2.4 Descriptive Analysis: Tenure  
      2.2.5 Quality of Service  
      2.2.6 Separations  
      2.2.7 Signal Technology  
   2.3 Regression Analysis  
   2.4 Conclusion
### 3 Estimating the Effects of Incentives When Workers Learn about Their Ability

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>46</td>
</tr>
<tr>
<td>3.2</td>
<td>Bayesian Learning and Nonrandom Attrition</td>
<td>52</td>
</tr>
<tr>
<td>3.2.1</td>
<td>Model</td>
<td>53</td>
</tr>
<tr>
<td>3.2.2</td>
<td>Estimating the Effects of Tenure and of Incentives</td>
<td>56</td>
</tr>
<tr>
<td>3.3</td>
<td>Data</td>
<td>61</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Context</td>
<td>61</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Descriptive Analysis</td>
<td>63</td>
</tr>
<tr>
<td>3.4</td>
<td>Estimation</td>
<td>65</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Performance and Attrition Equations</td>
<td>65</td>
</tr>
<tr>
<td>3.4.2</td>
<td>MLE</td>
<td>68</td>
</tr>
<tr>
<td>3.5</td>
<td>Results</td>
<td>69</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Results for the Attrition Model</td>
<td>70</td>
</tr>
<tr>
<td>3.5.2</td>
<td>The Fixed Effects Estimator</td>
<td>82</td>
</tr>
<tr>
<td>3.5.3</td>
<td>Simulations</td>
<td>85</td>
</tr>
<tr>
<td>3.5.4</td>
<td>Robustness Checks</td>
<td>87</td>
</tr>
<tr>
<td>3.6</td>
<td>Conclusion and Related Research</td>
<td>94</td>
</tr>
<tr>
<td>3.7</td>
<td>Appendix A</td>
<td>96</td>
</tr>
<tr>
<td>3.8</td>
<td>Appendix B</td>
<td>101</td>
</tr>
</tbody>
</table>

### 4 Incentives to Work or Incentives to Quit?

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Introduction</td>
<td>104</td>
</tr>
<tr>
<td>4.2</td>
<td>Model</td>
<td>109</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Worker’s Problem</td>
<td>109</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Firm’s Problem</td>
<td>112</td>
</tr>
<tr>
<td>4.3</td>
<td>Data</td>
<td>114</td>
</tr>
</tbody>
</table>
List of Tables

2.1 Summary statistics for main variables. ................................. 21
2.2 Variance before and after switching from regime 1 to regime 3. 35
2.3 Mean and standard deviation of performance for workers who stay at
least 9 months. ................................................................. 36
2.4 Mann-Whitney test for identical distributions of demeaned performance
across regimes at t=1. ....................................................... 37
2.5 Mann-Whitney test for conditions 1 and 2 with a threshold of 0.5 . . . 38
2.6 Estimates for the observational equation in months 7 to 9 using fixed
and random effects for the subsample of employees who stay at least 9
months. ................................................................. 39
2.7 Estimates for the observational equation using fixed and random effects
for the subsample of employees who stay at least 9 months. . . . . 40
2.8 Estimates for the observational equation using fixed effects for the sub-
sample of employees who stay at least 9 months. . . . . . . . . . 41
3.1 Summary statistics for the first 6 months for workers who start and work
under the same regime. ................................................................. 64
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>Estimates for the performance equation in the attrition model when ability is known, when workers learn about it, and when the hypothesis of learning is tested.</td>
<td>75</td>
</tr>
<tr>
<td>3.3</td>
<td>Estimates for the separation equation in the attrition model when ability is known, when workers learn about it, and when the hypothesis of learning is tested.</td>
<td>76</td>
</tr>
<tr>
<td>3.4</td>
<td>Estimates for ability and other structural parameters when ability is known, when workers learn about it, and when the hypothesis of learning is tested.</td>
<td>77</td>
</tr>
<tr>
<td>3.5</td>
<td>Estimates based on the fixed effects estimator.</td>
<td>83</td>
</tr>
<tr>
<td>3.6</td>
<td>Simulation results showing the presence of attrition bias.</td>
<td>85</td>
</tr>
<tr>
<td>3.7</td>
<td>Estimates for the performance equation under alternative specifications of the attrition model.</td>
<td>90</td>
</tr>
<tr>
<td>3.8</td>
<td>Estimates for the separation equation under alternative specifications of the attrition model.</td>
<td>91</td>
</tr>
<tr>
<td>3.9</td>
<td>Estimates for ability and learning under alternative specifications of the attrition model.</td>
<td>92</td>
</tr>
<tr>
<td>4.1</td>
<td>Estimates for the performance equation in the attrition model.</td>
<td>126</td>
</tr>
<tr>
<td>4.2</td>
<td>Estimates for the separation equation in the attrition model.</td>
<td>126</td>
</tr>
<tr>
<td>4.3</td>
<td>Estimates of parameters related to ability and learning in the attrition model.</td>
<td>127</td>
</tr>
<tr>
<td>4.4</td>
<td>Estimates of the structural parameters of the model.</td>
<td>129</td>
</tr>
<tr>
<td>4.5</td>
<td>Profits, effort, ability and tenure per workstation under different regimes.</td>
<td>136</td>
</tr>
<tr>
<td>4.6</td>
<td>Effects of different pay regimes relative to hourly wage.</td>
<td>137</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Average performance by regime of hiring, January 2005 - May 2006 . . . 23
2.2 Separation rates by regime of hiring, January 2005 - May 2006 . . . . 25
2.3 Average performance in months 2 to 5 of workers who stay at least 5
months, conditional on performance quartile in first month. . . . . . . . 26
2.4 Separation rate for workers hired and operating under the same regime:
by tenure. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27
2.5 Average performance for employees hired and working under same regime:
by monthly tenure . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 27
2.6 Average past performance of stayers and quitters in the month of deci-
sion: by tenure. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 28
2.7 Average performance of stayers and quitters in the month prior to deci-
sion: by tenure. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 29
2.8 Separation rate for workers hired and working under regime 1: by tenure. 34
2.9 Predicted performance - tenure profile for an employee of average ability
using the fixed effects approach for those employees who stay at least 9
months. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 43
3.1 Distribution of \( \theta \) at \( t \) under regime 1, conditional on staying at least \( t \)
months. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 71
3.2 Distribution of $\theta$ under regimes 1, 2, and 3 at $t = 3, 6$, conditional on staying at least $t$ months. ................................................. 71

3.3 Increase in the importance of observed signals relative to the initial prior when workers decide to stay or quit. ................................................. 72

3.4 Expected performance among stayers at different tenure horizons under regime 1-3. ................................................................. 74

3.5 Predicted performance - tenure profile for $\theta_i = 0$ under regime 1, 2, and 3. 78

3.6 Comparison between the expected performance for stayers and the tenure - performance for an employee of average ability. ................................................. 79

3.7 Probability of staying for $\theta_i = 0$, and $\theta_i = \pm \sigma_\theta$ under regimes 1 and 2. . 80

3.8 Probability of quitting in $t = 6$ by posterior mean $\mu_{i6}$ and pay regime. . 81

3.9 Performance-tenure profile at entry for $\theta_i = 0$ under regime 1 and 2: comparison between the estimates of FE approach and learning specification of the attrition model. ................................................. 84

3.10 Attrition bias in the estimated effect of tenure on performance when using the FE approach. ................................................................. 86

4.1 Ability under regime 1, 2, and the optimal regimes when the turnover cost is $750$ and when it is the industry average of $8800$. ............................... 134

4.2 Comparison between profits under regime 1, the optimal regime when turnover cost is $750$ and when it is $8800$. Turnover cost is $750$. . . . 135

4.3 Gains from switching to the optimal regime from hourly wage. Turnover cost is $750$. ....................................................................................... 138

4.4 Comparison between profits under regime 1, the optimal regime when turnover cost is $750$ and the optimal regime when it is $8800$. Turnover cost of $8800$. ....................................................................................... 138
4.5 Gains from switching to the optimal regime from hourly wage. Turnover cost is $8800. ................................................................. 140

4.6 Ability at entry under the optimal regimes when workers know match quality, when they learn about it, and when the latter is applied to an environment in which workers know their match quality. Turnover cost is $750. ................................................................. 142

4.7 Comparison between profits under the optimal regimes when workers know match quality at entry, when they learn about it, and when the latter is applied to an environment in which workers know it. Turnover cost of $8800. ................................................................. 143
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To My Parents and Grandparents
Chapter 1

Introduction

My dissertation studies how pay incentives, worker’s ability, experience, and turnover shape the dynamics and outcomes of employment relations. I depart from the previous literature that studies labor turnover and pay incentives separately for two reasons. First, it is not possible to fully characterize the production process and work environment without explicitly modeling turnover. Second, pay incentives affect profits and worker’s welfare not only though effort but also through the composition of the work force at different tenure horizons.

In my empirical work, I use the personnel records of a US call center, which are almost ideally suited for the purposes of estimating the effect of incentives on performance. The data set contains a clean measure of individual performance (defined as output per hour), known compensation policies based on an hourly rate plus a bonus rate proportionate to performance, and several changes in the compensation policies that allow for the identification of the effect of pay incentives on performance through moral hazard and selection. Chapter 2 provides a detailed description of the organization of the production process, the technology, and the work environment. It also presents an overview of the data set used throughout my work. A brief descriptive
overview of performance and turnover from January 2005 to May 2006, the duration of
the observational period, shows that the employment dynamics is complicated and could
not be explained by a simple model of moral hazard. Then, I cut the data by monthly
tenure to provide an alternative descriptive analysis of performance and turnover. De-
spite some noise in the data, I conclude that pay incentives have a significant effect on
performance that is consistent with the predictions in the literature on moral hazard.
Furthermore, labor turnover is nonrandom: workers with higher past performance are
more likely to stay. I also find evidence for unobserved heterogeneity among workers
that leads to persistent differences in performance. Interestingly, performance does not
"fan out" as tenure increases. Finally, I observe that workers accumulate experience
during the first 6 months that leads to an increase in their performance.

Next, I review the effect of changes in the pay incentives on the quality of the
service provided by the call center. The monthly summary statistics indicate that the
total number of hours that employees work at the company does not vary with the
compensation policy. Moreover, employees record high scores on tests of the quality
of service; there is no bunching around the threshold of permissible quality and most
employees consistently score well above that threshold. Thus, I find no evidence for a
trade-off between quality and quantity in the context of the studied data. If anything,
there is a slight positive relation between number of successful calls and quality of
service.

A combination of nonparametric tests and some popular techniques in the related
literature allow me to identify some key features of the technology that generates the
performance signal. On the basis of these findings, I estimate the observational equation
for performance. The results show that changing pay incentives has a significant effect
of effort choice, that workers accumulate considerable experience during the first five to
six months on the job, and that there is unobserved heterogeneity that is not correlated
with observed characteristics of the employees. These results are achieved without explicitly modeling turnover and for this reason they are of little help in the analysis of the unobserved variable(s) that generate persistent differences in performance at different tenure horizons. Without knowledge of the technology and the process that generates turnover, one could say little about the effect of changes in pay incentives on profits and individual welfare.

Chapter 3 addresses these issues by modeling turnover explicitly. I consider a variation on the standard model of search by experience, first introduced in Jovanovic (1979). Each period, the workers choose not only whether to stay or quit, but also how much effort to exert. The crucial element in the model is an ability parameter that represents the quality of the match between the employer and the employee. The parameter is unknown at the time of hiring and the employee learns about its value over time through a sequence of noisy performance signals. At the beginning of each period, the worker draws an outside offer from a known distribution and decides to stay if the value of continued employment is greater than the outside offer. If the worker decides to stay, she chooses effort that is not observable or verifiable by the firm. Then she observes a performance signal used for the update of her beliefs and is paid according to a linear contract that depends on performance.

I observe that steeper incentives are associated with higher performance and that persistent differences in individual performance are driven by differences in the quality of the employer-employee match. Moreover, I find evidence that employees learn about the quality of the match in the course of the employment relation. Their posterior beliefs are largely responsible for their decision to stay or quit. Researchers have recognized that in such circumstances the distribution of characteristics in the workforce differ from one period to the next. The solution to the resulting econometric challenges, it has been suggested, is to limit empirical work to the subsample of employees who stay
for the duration of the study. If one does so, the set of unobserved productivity effects remains the same over time and the effect of changes to pay incentives is related to changes in performance within each worker’s performance series.

One of my main results is that this solution, which I refer to as "the fixed effects estimator," yields biased estimates when workers learn about their ability. At the core of this finding lies the realization that employees decide to stay only if their posterior beliefs are sufficiently 'optimistic' which in turn implies that they must have been sufficiently lucky in the realized signals. Then, the decision to stay provides information about the noisy performance signals in the past. As an example, consider two workers, Alice and Bob, who have identical ability but Alice receives a good signal in the first period and then a bad signal in the second period, while Bob receives a bad signal in the first period and then a good signal in the second period. When Alice and Bob know their ability, the noisy signal does not affect their decision to stay or quit at the end of the first period. However, when they learn about their ability, all other things equal, Alice is more likely to stay than Bob because she entertains more "optimistic" beliefs. As a result, the econometrician observes more Alices than Bobs, conditional on staying for at least two periods. That is, the decision to stay imposes conditions on the performance signal that relate performance noise in one period to performance noise in another.

The analysis of Chapter 2 shows that the data are consistent with a stochastic technology that is additively separable in effort, tenure, and individual productivity. For such a technology, I show how one can identify the effect of Bayesian learning on performance and turnover. The additivity of the technology plays a crucial role in building and estimating a model of moral hazard and nonrandom attrition that nests as a special case the hypothesis of Bayesian learning. The results indicate the presence of learning about match quality but the proposed estimation approach allows me to obtain valid estimates of the effect of incentives on performance, recover the distribution of ability
at different tenure horizons and characterize how these vary with pay incentives. They also show that there is selective turnover, driven by pay incentives and learning about ability. Furthermore, the effect of incentives on effort is significant but small. Most importantly, the results show that turnover is a major channel through which pay incentives affect average performance and in turn profits. I also recover the distribution of match quality at different tenure horizons across regimes, and characterize the dynamics of learning. Controlling for pay incentives, I find that the employees of high ability or those who believe to be of high ability are more likely to stay. As a result, mean ability among those who stay increases with tenure. These findings provide evidence that ability is firm-specific and that the firm enjoys some monopsony power when setting its compensation schedule. Finally, I simulate the model and find that when the fixed effects estimator is applied to the simulated data it overestimates the effect of incentives on effort by a factor of three and the effect of tenure on performance by a factor of two.

Structural models of learning about ability are difficult to estimate because the econometrician needs to solve the dynamic program of each worker at each period for each set of Bayesian beliefs. To reach the results in Chapter 3, I approximate the value of continued employment rather than solve the set of functional equations within each step of the MLE optimization algorithm. While simplifying the estimation, this approach also makes it possible to identify only changes in effort that result from changes in pay incentives. As a result, one cannot determine how much of the profits under a given regime are due to effort and how much to ability. Moreover, the results in Chapter 3 do not provide the basis for counterfactual policy analysis. Chapter 4 proposes a simple two-step procedure to estimate a structural model of pay incentives, Bayesian learning, and labor turnover. The first step is based on the approach presented in the preceding Chapter 3: I estimate a semi-structural attrition model and recover the
stochastic technology up to a constant, as well as a scaled version of the value of continued employment. I use these estimates in the second step to estimate the remaining structural parameters using the method of moments. The results allow me to perform counterfactual policy analysis and find the optimal linear contract in performance. I limit my search to linear contracts for two reasons. First, the firm whose personnel records I use itself implemented such linear contracts and one of my objectives is to characterize the profitability of the firm’s compensation policies. Second, firms often apply simple compensation policies based on such linear contracts and the problem of finding and characterizing the optimal linear contract is of interest on its own.

The results show that most of the increase in profits from switching to the optimal linear contract from hourly wage can be traced back to the effect of incentives on the quality mix. The optimal contract gives incentives to workers of high performance to stay longer than workers of low performance and in this way it helps the firm improve the quality mix of its workforce over time. This effect more than offsets the loss associated with replacing an experienced worker with a newly hired one of no experience and unknown ability. Furthermore, the employer exploits the firm-specific nature of match quality to capture most of the surplus generated by the employment relation. To achieve that, the firm offers pay incentives that induces little effort, so high level of effort and low turnover are not necessarily attributes of the profit-maximizing pay policy. However, an exercise in comparative statics shows that as turnover costs grow, the firm increases compensation to induce lower turnover by offering much steeper incentives. Given the strong evidence of high turnover costs in some industries, this finding cautions that empirical studies of job mobility in the vein of Keane and Wolpin (1997) should incorporate turnover costs as a crucial ingredient. Finally, another counterfactual experiment shows that the firm’s profits would have been more than 25% higher if match quality was known to the employees at the time of hiring due to self-select into
the firm.

However, my work also suffers from some limitations. First, the personnel records of the firm do not contain information about the employment history of workers before and after their spell at the call center. For this reason, I could not evaluate the effect of their work at the call center on their career prospects and wage dynamics. Furthermore, in Chapters 3 and 4 I make strong distributional assumptions of normality and restrict the stochastic technology only to functions that are additive in ability, effort, and experience. The restrictions on the stochastic technology are motivated by the analysis in Chapter 2 and the testable implications of the considered model in Chapter 3. Still, the appropriateness of the distributional assumptions can be judged only on the basis of postestimation tests, if at all. Finally, the theoretical and empirical problem of finding the optimal contract in a model of moral hazard, labor mobility, and learning about ability remains a topic for future research. In this context, the properties of the optimal contracts from Chapter 4 likely depend heavily on the linearity of the considered contracts.

The results reported here relate to several strands of the literature. Farber (1999) provides a set of stylized facts about the US labor market. He points out that long-term employment relations are common and that the probability of a job change declines with tenure. At the same time, most new jobs end early and are the result of a job-to-job transition, even sometimes with intervening spells of non-employment, by workers in short-term jobs. The literature on labor mobility has almost exclusively focused on long-term jobs, largely because researchers are restricted to using annual data while short-term jobs last rarely more than a few years. Baker, Gibbs, and Holmstrom (1994) describe employment dynamics at a large institutions offering long-term jobs. Chapter 2 aims at doing the same for an institution offering transitional employment.

My data set has two crucial advantages: it contains an individual-level objective
measure of performance data and known compensation policies. In this respect it is similar to the data sets used in Lazear (2000) and Bandiera, Barankay, and Rasul (2005) and faces similar challenges: in particular, how to distinguish between moral hazard and adverse selection. To control for the effect of nonrandom attrition on the characteristics of the workforce at different tenure horizons, researchers have employed the fixed effects estimator discussed above. My work shows that the results reported in them may not be valid when labor turnover is driven by learning about ability. Moreover, in Chapter 3 I show how one can estimate the effect of pay incentives while controlling for learning for a family of stochastic technologies. In the process of estimating the model, I recover the tenure-performance profile across regimes. This result relates to a large body of literature starting with the empirical findings in the late 1970s and early 1980s. Yet, Abraham and Farber (1987) caution that the empirically observed strong relation between tenure and earning in many cases is a statistical artifact due to the positive relation between seniority and an omitted variable representing the quality of the employer-employee match, job, or the worker. Since then a number of papers have tried to identify the true effect of tenure, match quality, and individual ability. I also recover the distribution of ability and characterize the dynamics of learning which are topics of a literature that can be traced back to the theoretical work on search by inspection and wage rigidity in Jovanovic (1979) and Harris and Holmstrom (1982). As with testing for moral hazard, the availability only of compensation data has posited a major challenge to empirical work in the area. Chiappori, Salanié, and Valentin (1999) overcome this challenge by exploring the testable implications of Bayesian learning and downward rigidity on the dynamics of compensation series. Since turnover is close to nonexistent in their data, their estimates do not suffer from the econometric problems discussed above. Yet, even the dynamics of compensation series are of limited help in distinguishing between learning about match quality and learning-by-doing: Mortensen
(1988) shows that learning about match quality and learning-by-doing impose the same testable implications on the dynamics of compensation data, leading to insurmountable identification problems to empirical work. For this reason, Gibbons, Katz, Lemieux, and Parent (2005) are forced to assume away tenure effects in order to estimate the quality of industry-specific matching. Moreover, they do not take into account the structure in the noise series that is introduced by conditioning on staying in the same firm or industry for a certain period of time. In contrast, the availability of performance data allows me to identify the functional form of the stochastic technology up to a constant, which is crucial to distinguishing between tenure effects of and learning about match quality. To my knowledge, this is the first work using observed productivity signals to provide evidence for learning about match quality and characterize its dynamics, as well as the distribution of ability at different tenure horizons.

Moral hazard, on one hand, and Bayesian learning and labor turnover, on the other, are subjects that are usually analyzed separately in the literature on structural estimation. For example, Shearer (2004) and Shearer and Paarsch (2009) study the effect of incentives on performance and conduct a related policy analysis but the context of their study allows them to assume away issues related to labor turnover. The presence of turnover complicates the problem of finding the optimal contract. While in the standard moral hazard problem and in Shearer and Paarsch (2009) the base pay ensures participation, in the presence of outside offers an increase in the base pay increases the chances of staying but decreases profits conditional on staying. Similarly, in a model of learning about ability and turnover, steeper incentives induce more effort and increase the probability of staying but also cut in the rent extracted by the employer from the firm-specific ability of the employees. Still, both moral hazard and labor turnover are defining features of the analytical environment at most workplaces and their interaction shapes employment outcomes and through them profits and individual welfare. The
contribution, relative to Shearer and Paarsch (2009), is that optimal pay incentives are allowed to affect the composition of the workforce at different tenure horizons. Thus, the chapter extends the work in Lazear (1998, 2000) on the effect of incentives on the quality mix by studying how turnover shapes the properties of the optimal pay policy. The results show that turnover was the primary channel through which pay incentives affected profits at the call center. They also caution that models of equilibrium labor mobility should take into account the effect of turnover costs on equilibrium policy and profits.
Chapter 2

The Employment Dynamics at a Firm Offering Short Term Jobs

2.1 Introduction

The descriptive literature on labor mobility distinguishes two types of employment in the US: long term jobs and short term jobs. Farber (1999) conducts a descriptive analysis of the US labor markets on the basis of the annual NLSY, CPS, and PSID data and offers a number of stylized facts about labor mobility in the US. In contrast to the prevailing opinion, he points out that long term employment relations are common. Furthermore, compensation grows with tenure, while the probability of separation declines with it. At the same time, the American labor market is characterized by a high level of job creation and job destruction. Farber (1999) argues that the two sets of observations are not mutually exclusive because of three interrelated facts. First, most new jobs end early within one to two years after which the probability of changing jobs declines sharply. Second, most labor turnover is driven mainly by job-to-job transitions and sometimes even by transitions to and from the pool of non-employment. Third,
a large group of workers migrate from one short-term job to another short-term job. Descriptive studies, such as Houseman (2001) and Abraham (1988) show that, depending on the industry, between 19% and 41% of all transitions from unemployment or non-employment to employment follow a temporary to permanent job pattern. Autor and Houseman (2005) document that temporary job agencies play an important role in this type of transition. However, both temporary jobs and the role of temporary job agencies, as Autor (2008) show are understudied and both the empirical and theoretical mechanisms are little understood.

Despite their importance for understanding the dynamics of US labor markets, the employment dynamics at firms offering short term jobs is not well studied. In particular, the literature on labor mobility has focused on long-term jobs because most data are annual while short-term jobs last rarely more than a few years. Baker, Gibbs, and Holmstrom (1994) describe the employment dynamics and internal labor markets at a large institution offering long-term jobs. Yet, to my knowledge, there is no similar study of the employment dynamics at an institution that offers short term jobs. This chapter tries to fill in this gap by describing the organization, production process, compensation, and turnover dynamics at a US call center. I relate my work to two prominent frameworks for the analysis of employment relations. Their stylized predictions are summarized below.

Within the context of the basic model of effort choice in Lazear (1995 and 2000), Lazear and Oyer (2010) present evidence for the empirical validity of the theoretical predictions that (1) steeper incentives induce more effort, leading to an increase in observed performance; and that (2) steeper incentives lead to an increase in the variance of performance. With respect to the literature on search by inspection, Ericson and Pakes (1999) develop a model of Bayesian learning under some very general assumptions. The testable implications of the model, translated to the context of this
The call center collects debt on behalf of a major US cable TV company. It has a simple hierarchical structure and the nature of its operations allows the management to measure objectively both the productivity and the quality of service provided by the employees. The firm does not use promotions or layoffs to motivate or shape the quality of its workforce. Instead, it relies on its compensation policy to do so. The compensation policy itself is a simple type of a linear contract that depends only on present performance. By rewarding high performance, it encourages top performers to stay and low performers to quit. I also find evidence for unobserved heterogeneity among workers that leads to persistent differences in performance. Furthermore, I find that labor turnover depends on past performance: workers with higher past performance are more likely to stay. Interestingly, performance does not "fan out" as tenure increases. In addition, I show that workers accumulate experience during the first 6 months that leads to an increase in their performance.

The data set also allows me to study how changes in pay incentives affect the quality of service provided by the employees. Employees score high on tests of the quality of service, and there is no bunching around the permissible level of quality. Most importantly the majority of the employees consistently score well above that threshold. Thus, I find no evidence for a trade-off between quality and quantity in the context of the studied data. If anything, I observe a slightly positive relation between the number of successful calls and the quality of service.

A combination of nonparametric tests and some popular techniques in the related literature allow me to identify key features of the technology that generate the perfor-
mance signal. On the basis of these findings, I estimate the observational equation for performance. The results show that changing pay incentives has a significant effect of effort choice, that workers accumulate considerable experience during the first five to six months on the job, and that there is unobserved heterogeneity that is not correlated with the observed characteristics of the employees. These results are achieved without explicitly modeling turnover. Precisely because of that, however, I also could not characterize the unobserved variable(s) that generate persistent differences in performance at different tenure horizons. Without knowledge of the technology and the process that generates turnover, one could say little about the effect of changes in pay incentives on profits and individual welfare.

The rest of this chapter is organized as follows. Section 2.2 describes the employment environment and pay policy. After presenting the data set used in my work, it offers an overview of the operations of the call center. Section 2.2 also considers the effect of incentives on the quality of calls and presents some nonparametric tests for the performance series. Section 2.3 presents some preliminary regression analysis that helps me test for the effect of incentives on effort, for the presence of unobserved heterogeneity, and the accumulation of experience. Section 2.4 concludes by relating my findings to the literature on moral hazard and search by inspection.

2.2 Data and Descriptive Analysis

The data come from a call center in a large metropolitan area in the US with approximately 250 workstations. The call center collects debt on behalf of a major cable TV company which places with the call center account information about customers with outstanding balance prior to disconnecting their service. The operators both receive inbound calls and make outbound calls to the customers. The records of the Bureau
of Labor Statistics show that there is no other call center engaged in debt collection in the metropolitan statistical area. The ownership of the call center changed in 2004 and, according to the new management, at the time of purchase, its operations were characterized by very high turnover and low productivity (performance, from now on) relative to the industry averages. As a result, the new management decided to switch from hourly wage to a compensation policy that rewards high performance in order to reduce turnover and improve average performance. In addition, the management wanted to evaluate the effects of pay incentives at one of its US call centers in order to decide whether similar pay policy should be implemented at all of its call centers.

The rest of this section is organized as follows. The first subsection provides a detailed review of the work environment, pay policies and production process. The second subsection presents the main variables in the data set that I use in my work. The third and fourth subsections offer descriptive analysis of the data by calendar month and by tenure, respectively. The fifth subsection shifts the focus on the relation between quality of service and the number of calls that end with collection of the outstanding debt. The sixth subsection presents briefly the separation and hiring policies of the firm. Finally, the seventh subsection presents some descriptive statistics and nonparametric tests that impose restrictions on possible candidate specifications for the stochastic technology that generates the noisy performance signals.

### 2.2.1 Work Environment and Pay Policies

Employees worked in teams of approximately 15 call operators with one supervisor. The supervisor had organizational and administrative duties, while the call operators communicated with the clients of the cable TV company. More than 95% of all employees worked full time on daily shifts. On average full time employees worked for 191
hours per month, which amounts to approximately 23 days of eight hour shifts every month. The teams and supervisors varied with shifts. To my knowledge, in the studied period the management did not follow any specific policy of allocating employees into teams for its monthly work schedules. Employees did not have a permanent workstation: workstations varied with shifts. The arrangement of the workstations as well as the composition of teams, remains unfortunately unknown for the purposes of my research. Only one worker handled a given call: therefore, the outcome of an interaction with the clients of the cable TV company could be attributed definitively to a specific employee. Clients were not allocated to specific call operators, but each time they were randomly matched by an automatic switchboard. The inbound calls and the planned outbound calls were added to a waiting list handled by the switchboard. Each time the call on top of the waiting list was allocated to the longest waiting operator. More than 80% of all inbound calls were answered within 45 seconds.

A call was considered to be a success if the operator managed to persuade the client to pay the outstanding balance on the account. The firm measured one’s productivity only by the number of calls that end with collection. In what follows, I will also accept that this measure provides a noisy signal about one’s productivity. The magnitude and variability of client debts are crucial to evaluating the limitations of this measure of productivity. Within this context, it is important to recall that cable TV services are usually discontinued if a customer fails to pay for more than three months which implies that the amount of money that call operators had to collect varied between $60 and $90\(^2\), rarely exceeding $100. Most accounts were placed with the call center if the client failed to pay for two consecutive months and definitely for three months, so in

\(^1\)See Mas and Moretti (2008) for an empirical investigation of the effects of learning from peers and peer pressure on productivity.

\(^2\)Most accounts were placed with the call center if the client failed to pay for two consecutive months and definitely for three months. Thus, in most cases the outstanding debt was around $60.
most cases the outstanding debt was around $60. As a result, the number of calls and the total amount of collected debt are highly correlated, so that counting the calls that end up with a payment of the balance appears to be an informative measure of one’s performance.

The firm offered a detailed script to its operators for handling conversations with clients, so that employees usually had some freedom to improvise only in the middle of a conversation. In particular, the beginning and the end of the calls were heavily scripted. For these reasons, the employee’s contribution to the outcome of a conversation was largely limited to couching her accent, diction, the pitch of her voice, speed of talking and so on to the context of her duties. The extend to which they succeed in doing so depends largely on individual characteristics and social background, which could hardly be explained by commonly observable individual characteristics, such as education, age, gender, marital status, etc. Of course, one individual may make an effort to alter a property of her speech but she needs to be motivated somehow to do so.

In January 2005, the call center switched its compensation policy from an hourly wage of approximately $9.5 to a flat hourly wage plus a bonus proportionate to performance. The new compensation policy, regime 1, stipulated a base pay of $3.8 per hour and a bonus rate of $3.3 per successful call.\footnote{Please note that in the interest of protecting the firm’s identity and trade secrets, here I report an approximation of the firm’s policy by ignoring some stipulations for workers of low performance. In practice, less than 5% of the employees under regime 1 were affected by the omitted policy stipulations.} One’s pay did not depend on the performance of others; in theory there may be competition among the employees for calls, but in practice this possibility was ruled out by a chronic shortage of workers at the call center. Concerned that the company was paying "too much," the central management implemented regime 2, a variation on regime 1, for the newly-hired employees in June 2005. Relative to regime 1, regime 2 offered both a lower base pay of $3.5
per hour and lower bonus rate of only $2.8 per hour. All previously hired employees continued to be paid according to regime 1. Worried about possible negative effects of paying by the number of successful calls on the quality of the service, the central management changed the pay regime yet again in November 2005. Under the new regime 3 all employees were paid according to the pay schedule of regime 2. Furthermore, they had to meet certain minimum quality standards of service to qualify for the bonus rate. Twenty per cent of one’s calls were randomly monitored and the quality of service was rated on a scale from 0 to 100. For the monitored calls, the operator asked the client at the end to answer some questions about the assistance offered by the operator. The supervisor applies a formula to the answers to grade the quality of provided service on a scale from 0 to 100. The threshold level for acceptable service was set to 50. If the monthly quality fell below the threshold, the worker was reduced to a flat hourly rate. Diagram 3.2 presents a time line for the implementation of the three regimes and diagram 2.2 summarizes their pay schedules. Since 99% of performance lies between 1.05 and 3.8, regimes 2 and 3 effectively lowered incentives relative to regime 1. All regime changes were made by the central management and brought down to the call center for implementation. I maintain in the rest of my work the assumption that all changes in pay regimes are unexpected from the perspective of the employee. Since the average tenure of employees hired after January 2005 is only 3.45 months, she may be aware that a regime change is likely within several years but due to the expected short stay would behave as if she was not aware.

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4 As previously discussed, here I omit stipulations of the firm for workers of low performance. In addition, in the rest of the paper, I will ignore the following curious feature of regime 2: the bonus rate was actually increased for those with performance higher than 3.8. However, since more than 95% of performance lies below the 3.8 threshold level, in the rest of the text, particularly chapter 3, I proceed under the assumption of a simple linear contract.

5 The flat hourly pay was similar to the hourly wage offered before January 2005.
2.2.2 Data

As part of its policy, the firm established a monitoring system that enabled it to record the start, end, and outcome of each conversation, along with time on the premises of the call center, actual time at one’s workstation, waiting time between calls, and time required to update the account records after the completion of a conversation. Most data that I use in this and the following chapters are based on the aggregated monthly records of the monitoring system. I combine these data with the monthly compensation records of the employees and some basic individual characteristics from the office of human resources at the company. The result is a data set that contains 3675 observations for 659 individuals who were at some time employed by the call center. It covers the period from January 2005 to May 2006. The following list presents the more important variables that are available for at least some employees on a monthly basis, along with some explanatory notes:

- **Performance**: average of the number of calls per hour that end with the collection of the outstanding balance. This variable provides a noisy signal about one’s productivity and as such is the basis for the evaluation of the effect of pay incentives. It is available for all months.

- **Idle time**: total time (in hours) spent at one’s workstation while waiting for a call. This variable shows the utilization of employees by the firm and is available for all months.

- **Handle time**: total time (in hours) that an employee spent talking with clients on the phone.

- **Wrap time**: total time spent in updating the account records after a call. Thus, wrap time provides a measure of the efficiency of the monitoring system.
• System time: system time is the sum of the preceding three types of time on the premises of the call center.

• Break time: as the name suggests, this is the total time spent away from one’s station while on the premises of the call center.

• Total time: total time is equal to the sum of break time and system time; the ratio of system time to total time gives a measure of the efficient use of time by the employee;

• Average payments: average hourly compensation in a given month that is based on the implemented pay regime, 1, 2, or 3.

• Calls per hour: average number of calls that an employee services per hour. This variable is available only for a sample of employees

• Average handle time: average handling time per hour (in seconds) that takes to an employee to service a client. As regimes changes variation in these two variables may provide evidence for perverse incentives. Again, this variable is available only for a subsample of the employees.

• Quality assurance: percentage points received on monitored calls on a scale from zero to 100. Quality assurance captures the effect of the bonus rates on the quality of service, so that it can shed some light on the trade-off between quantity and quality that the employees face. Measures of quality assurance are available only for the time when regime 3 was implemented.

• Dummies for the implemented regime in a given month, as well as dummies for the regime that was in place when an employee was originally hired.
Table 2.1: Summary statistics for main variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Overall</th>
<th>Between</th>
<th>Within</th>
<th>Obs.</th>
<th>Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>3.05</td>
<td>0.68</td>
<td>0.53</td>
<td>0.42</td>
<td>3675</td>
<td>659</td>
</tr>
<tr>
<td>Pay from regimes</td>
<td>13.13</td>
<td>2.61</td>
<td>2.20</td>
<td>1.40</td>
<td>3675</td>
<td>659</td>
</tr>
<tr>
<td>Pay, total</td>
<td>13.69</td>
<td>2.25</td>
<td>1.63</td>
<td>1.55</td>
<td>3675</td>
<td>659</td>
</tr>
<tr>
<td>Tenure</td>
<td>6.65</td>
<td>4.61</td>
<td>3.49</td>
<td>3.01</td>
<td>3675</td>
<td>659</td>
</tr>
<tr>
<td>Full Time</td>
<td>0.95</td>
<td>0.23</td>
<td>0.22</td>
<td>0.07</td>
<td>3675</td>
<td>659</td>
</tr>
<tr>
<td>Separations</td>
<td>0.11</td>
<td>0.31</td>
<td>0.21</td>
<td>0.23</td>
<td>3675</td>
<td>659</td>
</tr>
<tr>
<td>Quits</td>
<td>0.08</td>
<td>0.28</td>
<td>0.20</td>
<td>0.20</td>
<td>3675</td>
<td>659</td>
</tr>
<tr>
<td>Layoffs</td>
<td>0.02</td>
<td>0.16</td>
<td>0.11</td>
<td>0.12</td>
<td>3675</td>
<td>659</td>
</tr>
</tbody>
</table>

- Date of entry and date of exit, along with an indicator whether the employee quit or was fired.
- Individual characteristics, including age, gender, family status, race, and zip code.
- Full time employee or part time employee.
- Percentage of outbound calls.

I exclude part time workers from the analysis, who account for less than 5% of the workforce. Furthermore, for econometric reasons I drop observations associated with the second employment spell of 16 rehired employees.

Table 2.1 provides the summary statistics for the data set, defined as a panel data set with employee’s ID and monthly tenure as its dimensions. The average performance in the data set is 3.05 successful calls per hour, where successful call stands for a call that ends with debt collection. Notably, the standard deviation of 0.53 across employees contributes more to the overall standard deviation of performance than
the within standard deviation of 0.42. Total pay is the sum of pay associated with the implemented pay regimes and additional sources. The average hourly pay based only on the pay regimes is $13.13 with a within standard deviation of 1.4 and between standard deviation of 2.2. Thus, the top quartile of performers earn between $15 and $17 per hour, a not trivial amount within the set of low-skilled jobs. In contrast, the bottom quartile of performers earn between $10 and $11 per hour, so that the difference in pay of the top and bottom quartiles, assuming 40 hour work week, amounts to around $240. The average of hourly total pay, including all sources of compensation, amounts to $13.69. Thus, most of the compensation that workers receive is closely associated with the implemented pay regimes.\(^6\) For this reason, I will assume that workers behave as if their compensation is completely determined by the pay regimes. The average tenure at the firm is 6.65 months, but some of this is due to a set of grandfathered employees who were hired prior to January 2005. Again, between variation is higher than the within variation. The table also shows that the average hazard rate is 0.11 of which 0.08 is due to voluntary separations and 0.02 to layoffs. The contribution of layoffs under regime 1 is slightly above 0.02 and declines under regime 2 and 3 to below 0.02.

2.2.3 Descriptive Analysis: Calendar Time

The announcement of the introduction of regime 1 was made in November 2004. Following the news, considerable number of workers quit in November and December 2004; anecdotal evidence suggests that these workers had lower than the average productivity level. Accordingly, the set of workers who started under the pay-for-performance scheme in January 2005 was self-selected. As illustrated on Figure 2.1, from January \(^6\)Total pay includes some legally stipulated benefits. Since they do not form a large part of the employees income, I follow the related literature in abstracting away their effect on the worker's behavior.
to June 2005, the average performance increased from 2.68 to 3.22 successful calls per hour, while the monthly separation rate fluctuated between 7% and 10%. The separation rate may appear to be high, but relative to the average for 2004, approximately 25% every month, it actually declined considerably. The introduction of regime 2 in June 2005 for the newly hired complicates the descriptive analysis of the data. Figure 2.2 shows that average performance for those hired under regime 2 remained always more than 0.60 successful calls per hour below its counterpart for regime 1. However, when one compares average performance under regime 2 in the first month of its introduction and average performance under regime 1 in January 2005, it appears that the difference in performance is reduced to only less than 0.25 calls per hour. These two observations are reconciled by the slower rate of growth in average performance under the new regime than the one under regime 1. Interestingly, while average performance
for those hired under regime 3 is below the average performance for those hired under regimes 1 and 2, it always shadows the latter within only 0.1 successful calls per hour. That is, Figure 2.1 appears to suggest that relative to regime 2 the introduction of quality control did not affect the dynamics of average performance.

The dynamics of the separation rates over time, presented in Figure 2.2, indicates that factors beyond the compensation policy of the firm, such as the local labor market conditions and the transitional nature of the job of call operator, have an important effect on the decision to stay in the firm or quit. In addition, they also indicate clearly that between June 2005 and May 2006 the monthly separation rate for workers hired under regime 1 was on average only half of the monthly separation rate for workers hired under regime 2, which amounted to a rate of 18% on average. Interestingly, the introduction of regime 3 in November 2005 does not appear to affect the differences between workers hired under regime 1 and regime 2 in both performance and separation rates. Again, a comparison between the separation rate for workers hired under regime 2 and for those hired under regime 3 shows that the two regimes appear to have a very similar effect on the worker’s decision to stay or quit. Figure 2.2 indicates that separation rates across pay regimes peak in January 2005, May 2005, September 2005, January 2006, and again in May 2006. The dynamics alerts to the possibility of seasonal fluctuations in turnover and the necessity to control for the effect of cohorts or calendar time when estimating the effect of pay incentives on performance and separation decisions.

While there can be a number of theoretical explanations for the observed dynamics of performance and separations, Figures 2.1 and Figures 2.2 indicate that a simple model of moral hazard, without selective entry and turnover or accumulation of experience, cannot explain the observed pattern: such a model implies that workers hired under different pay regimes have identical economic behavior when they are subject to the same pay incentives.
2.2.4 Descriptive Analysis: Tenure

A closer look at the dynamics of performance of employees who stay at least 5 months during the first 5 months\(^7\) of their employment relation has some important implications for the unobserved heterogeneity and how it affects productivity. Figure 2.3 presents the average performance of workers staying more than 5 months for months 2 to 5, conditional on their performance quartile in period 1. If there were no persistent differences in the productivity of employees, performance in months 2 to 5 would be the same across the initial performance quartiles. This hypothesis is not supported by the data: the workers in the top initial quartile consistently have higher performance in periods 2 to 5 than their counterparts in the other three quartiles. Thus, Figure 2.3 suggests the presence of at least two unobserved "types" of employees with consistently

\(^7\)The length of the considered period is limited to 5 months, since regime 2 lasted only 6 months.
different levels of performance. Furthermore, average performance for each initial quartile increases over time: the difference in average performance in periods 1 and 5 is statistically significant at 5%. Finally, performance does not seem to be "fanning out" over time. If anything, the data appears to suggest that the average performance for the lowest initial quartile actually converges over time to the levels of performance of the other quartiles.

Figure 2.4 summarizes the separation rates under regimes 1 to 3. As expected, the separation rate under regime 1 is lowest and the separation rate under regime 3 the highest. There also appears to be a noisy downward trend in the separation rates as tenure increases. Figure 2.5 presents monthly performance for the first five months of employees hired and operating under the same regime, who stay at least 5 months. If there were only incentives and no attrition on an unobserved productivity parameter, performance under regime 1 would be consistently higher than performance
Figure 2.4: Separation rate for workers hired and operating under the same regime: by tenure.

Figure 2.5: Average performance for employees hired and working under same regime: by monthly tenure
under regimes 2 and 3. If there were only selective attrition and no effect of incentives, performance under regime 1 would have been consistently lower than the performance under regime 2 and 3: if an employee stays more than 5 months under the less generous regime 2 or 3, she must be really good at what she does. Figure 2.5 rejects both hypotheses. Thus, it appears to lend support to the view that both incentives and selective attrition are at work.

Figures 2.6 and 2.7 investigate whether there are any systematic differences in past and present performance between the stayers and quitters. Figure 2.6 presents the average past performance from period 1 to t-1 of the employees who decide to stay and of the employees who decide to quit at the beginning of period $t$. In all but one period, the average past performance of those who stay is higher than the average past performance of those who quit. This finding is consistent with the view that individuals decide to stay or quit on the basis of the value, or what they believe is the value, of a productivity parameter that remains unobserved by the econometrician. Figure 2.7
sheds some additional light on attrition. It suggests that employees who stay by the beginning of period $t + 1$ have on average higher performance in period $t$ than those who quit by the beginning of period $t + 1$. In addition to corroborating the above hypothesis, this finding suggests that there may be a negative correlation between the productivity shock and the outside offer, which also leads to nonrandom attrition.

The evidence from figures 2.1 to 2.7 is purely suggestive: few of the discussed phenomena are statistically significant. Nevertheless, it implies the following set of stylized facts about the data:

- Incentives pay has a significant effect that is consistent with the predictions in the literature on moral hazard;

- Attrition is nonrandom: workers with higher past performance are more likely to stay;

- There are at least two types of workers, high performers and low performers, but
performance does not "fan out" as tenure increases;

- Workers accumulate experience during the first 6 months that leads to an increase in their performance.

### 2.2.5 Quality of Service

The number of calls that end up with collection of the outstanding debt is an important but only one of many indicators of the production process at the call center. For example, the cable TV company may be interested not only in collecting the outstanding balance on its accounts but also in establishing rapport with its customers, so that they do not switch to another provider in the future. If this is the case, the cable TV company is likely to condition the revenue that the call center receives from processing calls on some quality standards. As a result, the management of the call center would also be interested in maintaining these standards. However, when workers are paid by a flat hourly rate and a bonus that depends only on the calls that end with collection, the interests of the management and the call operators are not aligned. This type of problems is at the core of the literature on multitasking and potentially explains why piece rates are not common in many industries.

The empirical literature on pay incentives has not studied the problem of multitasking, and in particular the relation between quality of output and quantity of output. Shearer (2004) is based on experimental data from the operations of a tree-planting company. The setting allows him to control the work environment but unfortunately he does not focus on issues related to multitasking. Even in this very simple setting there is a trade-off between the quality of planting a tree and the number of trees that are planted: in particular in relation to the depth of the hole prepared for the tree and its positioning. The problem of multitasking is even more acute in the context of
Bandiera, Barankay, and Rasul (2005, 2007): the care with which the worker picks each soft fruit is crucial to the shelf life of the product, particularly if the fruits are perishable as it appears to be the case. The authors, however, do not have access to data on how the quality of the collected fruit varies with pay incentives. To my knowledge, Lazear (2000) is the only paper that provides some anecdotal evidence for a small decrease in the quality of windscreens after switching from hourly pay to a piece rate.

In contrast, the personnel records of the call center provide an opportunity to investigate how pay incentives affect the quality of service offered in each call. Recall that the conversations are heavily scripted. For this reason, one way an operator may try to cut corner is by cutting the conversation short. An employee would be tempted to do so if it becomes clear to her that the person on the phone is not likely to pay. Another way to cut corners is by reducing attentiveness, or by covering some points slowly while omitting others altogether. Yet, after November 2005, the firm implemented a quality standard which should have been able to detect both. It is important to point out that relative to other professions, the call operators enjoyed greater flexibility of choosing their work schedule and the number of hours at work, even the full time employees. This degree of freedom is evident from the high standard deviation for the hours worked: between 86 and 96 hours for the different regimes. On average employees worked 187 hours each month under regime 1, 196 hours under regime 2, and 193 under regime 3. Thus, the changes in the compensation schedule do not appear to affect hours worked in a spectacular fashion. Most importantly, the average quality score between November 2005 and May 2006 is 85 out of 100, with a standard deviation of 16. Furthermore, employees who have survived at least six months in the firm have an average quality score of 89, with a standard deviation of 16.7, which is not significantly different from the average in the data. In fact, as tenure increases, the average quality score increases from around 82 in the first month of employment
to 88 after five months. The same growth pattern is observed even when conditioning
on staying for more than six months. However, the variance of the quality score does
not change with tenure. Moreover, these statistics indicate that most quality scores are
above the threshold level of 50. Consequently, they suggest that workers did not face a
trade off between collecting debt and being polite or following the script offered by the
firm. Thus, the pay policies of the firm, in contrast to the evidence provided in Lazear
(2000), do not appear to have negative side effects.

2.2.6 Separations

As already discussed, the hierarchy structure of the firm was very simple. Ordinary
call operators could be promoted to the position of a supervisor, but since supervisors
last for years in the company, while call operators usually stay for several months,
this is a rare event. For this reason, in the rest of the text I maintain the simplifying
assumption that workers behave as if they have no chance of becoming supervisors. The
management of the firm also does not appear to use layoffs as a selection mechanism:
the rate of layoffs is 2%. The average layoffs rate does not appear to vary with regimes:
it was 2% under regime 1 and then declined to 1.8% under regime 2 and regime 3.
Consequently, I conclude that the firm relies almost exclusively on its compensation
schedule to shape the quality mix of the workforce.

The call center hired employees after an interview with a group of supervisors and
members of the senior management. Vacant positions were advertised in the local
media. Upon entry, new employees went through a week and a half to two weeks
of training. In the first couple of days they attended lectures related to their legal
obligations, got a tour of the premises, met future coworkers, and received a briefing
on the organization of the call center, and on their main duties and rights. Then
they attended lectures whose purpose is to prepare them for their duties. During the second week, new employees shadowed some experienced colleagues and gradually started taking calls.

2.2.7 Signal Technology

In this subsection, I discuss the implications of some descriptive statistics and nonparametric tests for the functional form of the technology. Figure 2.8 plots the hazard rate for the workers who were hired and worked under regime 1 at least until nine months of tenure at the call center. It shows that after the first five to six months the hazard rate stabilizes around a rate of 6%. I take this hazard rate to represent the steady state of job destruction which settles only after all transitional dynamics ends. Next, I consider workers in periods 7 to 9 who experience a change in their pay incentives from regime 1 to regime 3. If all transitional dynamics has already ended, the only source of changes in the distribution of performance will be the unexpected change in the compensation schedule. Table 2.2 presents the means and standard deviations of performance before and after the regime change, along with the probability of equal variance of performance before and after. This probability is associated with the Levene’s test for equal variance, estimated with respect to the median. The switch from regime 1 to regime 3 induces a reduction in performance of between 0.05 calls per hour to 0.18 calls per hour. However, as the table shows, the standard deviations of performance before and after the change remain quite stable. Thus, the data do not appear to support one of the stylized predictions of the empirical literature on moral hazard: that upon a change in the compensation schedule the variance of the productivity signal also changes (in this case, expected to decline). As noted in Lazear (2000), this stylized prediction depends on the assumption that unobserved ability and effort are complements. The results of
Table 2.2 suggest that, while mean performance is sensitive to the change in pay, higher moments are not. Such a data pattern is consistent with a stochastic technology that is additive in effort and unobserved ability.

Such a technology is also consistent with the test results reported in Table 2.3. The table reports the means and standard deviations of performance under regime 1 for those workers who stayed for at least 9 months. Given the finding that workers of high past performance tend to stay longer than workers of low performance, one may conclude that those who stay for at least 9 months are top performers. This observation is confirmed by a casual examination of their average monthly performance. Conditioning on staying for at least 9 months is very similar to conditioning on having consistently high performance or high unobserved ability to the extent to which it drives persistent differences in performance. For this set of workers, the variance of performance does not appear to vary much with tenure, as indicated by the Levene’s
Table 2.2: Variance before and after switching from regime 1 to regime 3.

<table>
<thead>
<tr>
<th>Variable:</th>
<th>Regime 1</th>
<th>Regime 3</th>
<th>Prob(equal var)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perf., $t = 7$</td>
<td>3.46</td>
<td>3.32</td>
<td>0.81</td>
</tr>
<tr>
<td>Std. dev. (Perf., $t = 7$)</td>
<td>(0.61)</td>
<td>(0.59)</td>
<td></td>
</tr>
<tr>
<td>Perf., $t = 8$</td>
<td>3.42</td>
<td>3.37</td>
<td>0.78</td>
</tr>
<tr>
<td>Std. dev. (Perf., $t = 8$)</td>
<td>(0.62)</td>
<td>(0.68)</td>
<td></td>
</tr>
<tr>
<td>Perf., $t = 9$</td>
<td>3.45</td>
<td>3.29</td>
<td>0.28</td>
</tr>
<tr>
<td>Std. dev. (Perf., $t = 9$)</td>
<td>(0.55)</td>
<td>(0.61)</td>
<td></td>
</tr>
<tr>
<td>Perf., $t = 10$</td>
<td>3.48</td>
<td>3.30</td>
<td>0.58</td>
</tr>
<tr>
<td>Std. dev. (Perf., $t = 10$)</td>
<td>(0.60)</td>
<td>(0.58)</td>
<td></td>
</tr>
</tbody>
</table>

Thus, there seems to be no "fanning out" of performance with tenure.

Next, I study how performance varies with pay incentives in the early months of employment at the call center. Table 2.4 presents the Mann-Whitney test for equal distribution of demeaned performance in the first month of employment across regimes. A casual look at the standard deviations of performance in period 1 under the different regimes verifies the plausibility of the hypothesis of equal variance: the standard deviations vary between 0.45 and 0.47. This observation is confirmed by the results of the Mann-Whitney tests for equality of the demeaned distributions of performance under

---

The general formulation of the Levene’s test for equal variances is as follows. Let there be $k$ subgroups and the sample size of subgroup $i$ be $N_i$. Let $y_{ij}$ indicate the observation for individual $j$ in subgroup $i$.

- Null: $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$
- Alternative: $\sigma_i^2 \neq \sigma_j^2$ for at least one pair $(i,j)$

The test statistic is distributed as $F(k-1, N-k)$ and is defined as:

$$W = \frac{(N-k) \sum_{i=1}^{k} N_i (Z_{i\bullet} - Z_{\bullet \bullet})^2}{(k-1) \sum_{i=1}^{k} \sum_{j=1}^{N_i} (Z_{ij} - Z_{i\bullet})^2},$$

where:

$$Z_{ij} = |y_{ij} - \overline{Y}_i|$$

and $Z_{i\bullet}$ is the group mean of $Z_{ij}$ for group $i$ and $Z_{\bullet \bullet}$ is the overall mean for all $Z_{ij}$.
Table 2.3: Mean and standard deviation of performance for workers who stay at least 9 months.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.60</td>
<td>0.52</td>
</tr>
<tr>
<td>2</td>
<td>2.79</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>3.06</td>
<td>0.51</td>
</tr>
<tr>
<td>4</td>
<td>3.22</td>
<td>0.53</td>
</tr>
<tr>
<td>5</td>
<td>3.34</td>
<td>0.54</td>
</tr>
<tr>
<td>6</td>
<td>3.42</td>
<td>0.52</td>
</tr>
<tr>
<td>7</td>
<td>3.45</td>
<td>0.56</td>
</tr>
<tr>
<td>8</td>
<td>3.43</td>
<td>0.53</td>
</tr>
<tr>
<td>9</td>
<td>3.44</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Levene’s test:
Prob(equal var) 0.84

Note: Observations in each months are 162.

regimes 1, 2, and 3 in the first month: the tests fail to reject the hypothesis of equality of the demeaned distributions across regimes. As Lazear and Oyer (2010) point out, a change in pay incentives is likely to affect the type of workers who enter a firm and as a result alter the distributions of performance at entry across different regimes. Table 2.4, however, indicates that in the context of the call center such self-selection would likely be restricted only to a shift in the distribution of performance. The question then arises whether it would be possible to distinguish between selection and moral hazard discussed in some detail in the following section and Chapter 3.

Finally, I return to the dynamics of performance as tenure increases. The following table 2.5 presents the results from a Mann-Whitney test for the following two conditions, where $y^{0}_t$ stand for the demeaned performance at $t$ and $a^0$ is a threshold, such that

**Condition 1:**

$$
F \left( y^{0}_1 | s_2 = 1, y^{0}_1 > a^0 \right) \approx F \left( y^{0}_1 | y^{0}_1 > a^0 \right)
$$
Table 2.4: Mann-Whitney test for identical distributions of demeaned performance across regimes at t=1.

<table>
<thead>
<tr>
<th></th>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. Dev.</td>
<td>0.465</td>
<td>0.454</td>
<td>0.457</td>
</tr>
<tr>
<td>Regime 1 Pr</td>
<td>Pr=0.907</td>
<td>Pr=0.564</td>
<td></td>
</tr>
<tr>
<td>Regime 2 Pr</td>
<td>Pr=0.408</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Reported probability of identical distributions.

**Condition 2:**

\[ F(y_1^0 | y_1^0 > a^0) \simeq F(y_2^0 | y_1^0 > a^0) \]

Intuitively, these two conditions state that the demeaned distributions of performance in the first two months of employment must be equal to each other for the set of people who observe such a good signal that they are almost surely going to stay. For the test I consider a threshold of \( a = 0.5 \). I have performed the tests implied by the conditions for \( a = 0.3, 0.4, 0.5, 0.6, \) and \( 0.7 \). The test results for \( a = 0.3 \) reject the hypothesis for equal distribution, while for \( a = 0.4 \) the hypothesis is not rejected. The sample size at \( a = 0.5 \) is slightly above 150, but it declines below 100 for \( a = 0.6 \) and \( a = 0.7 \). Consequently, while the hypothesis for equal distribution is not rejected for \( a = 0.6 \) and \( a = 0.7 \), the small sample size cast some doubt on the results of the nonparametric tests. The F- statistics for the Mann-Whitney tests indicate that the null hypothesis of equal distributions is not rejected.
Table 2.5: Mann-Whitney test for conditions 1 and 2 with a threshold of 0.5

<table>
<thead>
<tr>
<th>Test</th>
<th>F-stat</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition 1</td>
<td>0.12</td>
<td>0.91</td>
</tr>
<tr>
<td>Condition 2, stay ≥ 2</td>
<td>0.57</td>
<td>0.56</td>
</tr>
</tbody>
</table>

2.3 Regression Analysis

Let $y_{it}$ stand for performance of individual $i$ in $t$, $\theta_i$ for an individual effect that is not observed by the econometrician (ability from now on), $l_{it}$ for effort, $h_{it}$ for accumulated experience which may depend on past effort or performance, $X_{it}$ for a vector of other observed individual characteristics, and $\varepsilon_{it}$ for identical independently distributed across tenure horizons and individuals random noise. Performance is potentially a complicated function of past and present variables. One channel for such a dependence is the accumulation of experience and another is the presence of learning about ability. As indicated in Easley and Kiefer (1988) if workers learn about $\theta_i$ in the course of the employment relation, their beliefs about ability could affect effort choice and as a result, performance in period $t$ is likely to be a complicated function of all available information up to period $t$. However, on the basis of the descriptive and nonparametric analysis of the preceding section, I consider the following reduced form stochastic technology:

$$y_{it} = \theta_i + f(R_{it}, t, X_{it}) + \varepsilon_{it}, \quad (2.1)$$

where $R_{it}$ is the pay regime assigned to worker $i$ in period $t$. Accumulated experience is modeled by orthogonal polynomials of order 2. $X_{it}$ includes percentage outbound calls, regime of hiring, race, gender, age, distance from home, and marital status. I leave the formulation of the process that governs the separation decisions for the
Table 2.6: Estimates for the observational equation in months 7 to 9 using fixed and random effects for the subsample of employees who stay at least 9 months.

<table>
<thead>
<tr>
<th>Dependent Variable: Perf.</th>
<th>FE, ( t \in [7, 9] )</th>
<th>FE, ( t \in [7, 9] )</th>
<th>RE, ( t \in [7, 9] )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>S.E.</td>
<td>Coef.</td>
</tr>
<tr>
<td>regime 3</td>
<td>-0.15</td>
<td>0.08</td>
<td>-0.15</td>
</tr>
<tr>
<td>% outbound calls</td>
<td>-0.13</td>
<td>0.09</td>
<td>-0.12</td>
</tr>
<tr>
<td>Quit in ( t = 10 )</td>
<td>-0.02</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>3.42</td>
<td>0.55</td>
<td>3.47</td>
</tr>
<tr>
<td>Obs.</td>
<td>486</td>
<td></td>
<td>486</td>
</tr>
<tr>
<td>R(^2)</td>
<td>0.67</td>
<td></td>
<td>0.67</td>
</tr>
<tr>
<td>Hausman test</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: RE also includes: gender, age, race, distance from home, marital status.

following two chapters. Here, I will follow an estimation approach that is popular in the related literature. Specifically, I estimate a fixed effects panel data model (fixed effects estimator) for tenure between \( t_1 \) and \( t_2 \) on a subset of people who stay at least \( t_2 \) months in the firm. The performance equation includes the following explanatory variables: second degree orthogonal polynomials of tenure and third degree polynomials of calendar time, dummies for regimes of operation and regimes of hiring, as well as controls. Both regime 2 and regime 3 are allowed to interact with tenure and other observed variables.

I first estimate the performance equation from above on the sample of workers who started working under regime 1 and stayed at least 9 months in the firm. Figure 2.8 shows that the hazard rate flattens after five to six months. For this reason, I estimate the performance equation only for tenure between 7 and 9, including. The basic objective is to evaluate the effect of changing incentives from regime 1 to regime 3 on performance. I experiment with different specifications for the orthogonal polynomials of tenure but the preliminary regressions indicate that none of these specifications
Table 2.7: Estimates for the observational equation using fixed and random effects for the subsample of employees who stay at least 9 months.

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>FE, $T_i \geq 9, t \leq 9$</th>
<th>FE, $T_i \geq 9, t \leq 9$</th>
<th>RE, $T_i \geq 9, t \leq 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Performance</td>
<td>Coef.</td>
<td>S.E.</td>
<td>Coef.</td>
</tr>
<tr>
<td>$t$, orthog. pol. 1</td>
<td>-0.39</td>
<td>0.11</td>
<td>-0.37</td>
</tr>
<tr>
<td>$t$, orthog. pol. 2</td>
<td>0.23</td>
<td>0.09</td>
<td>0.27</td>
</tr>
<tr>
<td>regime 2</td>
<td>-0.24</td>
<td>0.15</td>
<td>-0.24</td>
</tr>
<tr>
<td>regime 3</td>
<td>0.15</td>
<td>0.18</td>
<td>0.15</td>
</tr>
<tr>
<td>$t$ (r 3), orthog. pol. 1</td>
<td>0.86</td>
<td>0.43</td>
<td>0.87</td>
</tr>
<tr>
<td>$t$ (r 3), orthog. pol. 2</td>
<td>0.45</td>
<td>0.21</td>
<td>0.35</td>
</tr>
<tr>
<td>% outbound calls</td>
<td>-0.13</td>
<td>0.07</td>
<td>-0.13</td>
</tr>
<tr>
<td>Quit in $t = 10$</td>
<td>-0.07</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>2.66</td>
<td>0.45</td>
<td>2.85</td>
</tr>
<tr>
<td>Obs.</td>
<td>1080</td>
<td></td>
<td>1080</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.47</td>
<td></td>
<td>0.47</td>
</tr>
<tr>
<td>Hausman test</td>
<td></td>
<td></td>
<td>Prob(equal)=0.72</td>
</tr>
</tbody>
</table>

Note: RE also includes gender, age, race, distance from home, marital status, and hiring regime.

improve upon the following restricted version of the stochastic technology:

$$y_{it} = \theta_i + \sum_k r_k I_{R_{it}=R_k} + m \left( X_{it} \right) + \varepsilon_{it}, \ t = 7, 8, 9$$

where $I_{R_{it}=R_k}$ is an indicator function equal to one when worker $i$ in period $t$ is paid according to $R_k$ and the coefficient $r_k$ indicates the effect of switching from the benchmark regime to regime $R_k$. In other words, by the seventh month on the job there is no longer accumulation of experience. The results are reported in Table 2.6. The first specification includes not only the explanatory variables for performance but also a lagged indicator of the decision of the worker to stay or quit in the tenth month of employment. This is the formulation of a popular test for nonrandom attrition, initially proposed in Nijman and Verbeek (1992). Under the null hypothesis of no attrition bias, the coefficient of this dummy variable is 0. If the null hypothesis is rejected, then the
estimates are not valid and attrition needs to be modelled explicitly. The coefficient of the lagged separation indicator is very close to zero and is not statistically significant. On the basis of this result, I conclude that the estimate of the effect of regime 3 on effort, equal to a decline of 0.16 calls per hour, is valid. Thus, I find evidence that workers respond to pay incentives. I also estimate the performance equation using random effects. The estimates are similar to those previously reported. This observation is confirmed by performing a Hausman test whose null hypothesis of no systematic difference between the fixed and random effects is not rejected at 5% significance level. This results implies that the unobserved effect $\theta_t$ is not correlated with any of the observable explanatory variables. It is important because in the following chapters I will apply an estimation approach that relies crucially on the assumption of that $\theta_t$ is not correlated with the observed variables.

Table 2.8: Estimates for the observational equation using fixed effects for the subsample of employees who stay at least 9 months.

<table>
<thead>
<tr>
<th>Dependent Variable: Performance</th>
<th>FE, $T_i \geq 6, t \leq 6$</th>
<th>FE, $T_i \geq 9, t \leq 9$</th>
<th>FE, $T_i \geq 9, t \leq 9$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef. S.E.</td>
<td>Coef. S.E.</td>
<td>Coef. S.E.</td>
</tr>
<tr>
<td>$t$, orthog. pol. 1</td>
<td>-13.78 1.52</td>
<td>-15.25 1.72</td>
<td>-0.39 0.11</td>
</tr>
<tr>
<td>$t$, orthog. pol. 2</td>
<td>-0.65 0.82</td>
<td>0.55 0.98</td>
<td>0.23 0.09</td>
</tr>
<tr>
<td>r 2</td>
<td>-0.51 0.21</td>
<td>-0.18 0.25</td>
<td>-0.24 0.15</td>
</tr>
<tr>
<td>r 3</td>
<td>0.12 0.31</td>
<td>0.08 0.22</td>
<td>0.15 0.18</td>
</tr>
<tr>
<td>$t$, (r 2), orthog. pol. 1</td>
<td>2.91 5.24</td>
<td>0.08 6.53</td>
<td></td>
</tr>
<tr>
<td>$t$, (r 2), orthog. pol. 2</td>
<td>1.56 10.35</td>
<td>0.76 12.51</td>
<td></td>
</tr>
<tr>
<td>$t$, (r 3), orthog. pol. 1</td>
<td>2.81 0.33</td>
<td>0.82 0.56</td>
<td>0.86 0.43</td>
</tr>
<tr>
<td>$t$, (r 3), orthog. pol. 2</td>
<td>0.85 0.05</td>
<td>0.53 0.25</td>
<td>0.45 0.21</td>
</tr>
<tr>
<td>% outbound calls</td>
<td>0.15 0.07</td>
<td>-0.13 0.07</td>
<td>-0.13 0.07</td>
</tr>
<tr>
<td>Quit in $t = 7$</td>
<td>-0.28 0.09</td>
<td>-0.08 0.21</td>
<td>-0.07 0.21</td>
</tr>
<tr>
<td>Quit in $t = 10$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>2.51 0.35</td>
<td>2.85 0.47</td>
<td>2.87 0.42</td>
</tr>
<tr>
<td>Obs.</td>
<td>1131</td>
<td>1080</td>
<td>1080</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.45</td>
<td>0.47</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Note: Specifications also include orthogonal polynomials of calendar time.
Unfortunately, the current specification of the technology does not allow for the estimation of the effect of tenure on performance. For this reason, I estimate the technology (2.1) for $1 \leq t \leq 9$ on those employees who stay for at least 9 months, $T_i \geq 9$, at the call center. The results from estimating this regression are reported in Table 2.7. Figure 2.9 plots the tenure–performance profile for the average entering employee conditional on staying for regimes 1 and 2. In what follows, I discuss the fixed effects specification in the middle of Table 2.7. Regime 2 is restricted only to a downward shift in performance across tenure horizons, while regime 3 is also interacted with tenure. This specification is chosen after exploring different specifications of the relation between regime 2 and the orthogonal polynomials for tenure. The results from these alternative specifications are reported in Table 2.8. They show that none of the interaction terms between regime 2 and the orthogonal polynomials is significant. The estimates indicate that the effect of changing pay from regime 1 to regime 2 on performance is negative but not significant. However, the effect of regime 2 does not differ significantly from the effect of regime 3. Switching from regime 1 to regime 2 leads to a decline in effort that translates into 0.24 fewer calls per hour, which is approximately 9% of the average initial performance under regime 1. In economic terms, the change in incentives leads to a decline in worker’s hourly pay by approximately $2.2, which is 27% of the average hourly pay in the first month of employment under regime 1. The results also report a significant improvement in performance due to the accumulation of experience: in the first 6 months of employment performance increases by approximately 1 call per hour, which translates under regime 1 into an increase in hourly pay by approximately $2.5. Similarly to above, a Hausman test indicates that there is no systematic difference in the coefficients obtained through fixed effects and

---

Note that due to selection, the distribution of the unobserved effect $\theta_i$ is very likely not to be normal and its distribution probably should be approximated by a mixture of normal distributions.
Figure 2.9: Predicted performance - tenure profile for an employee of average ability using the fixed effects approach for those employees who stay at least 9 months.

those obtained through random effects. Note that under the random effects specification the estimate for the effect of regime 2 on performance is statistically significant.

To summarize, the results above indicate the presence of an unobserved effect that leads to persistent differences in performance across individuals. They also show that pay incentives affect performance through effort choice. However, the use of a subsample of the actual data makes it impossible to characterize the unobserved heterogeneity that appears to drive most of the variation in observed performance. There is one additional issue. Table 2.8 reports the estimates under the fixed effects estimator when I condition on using only the observations of workers who stay for at least six months. For that specification, the test for nonrandom attrition rejects the null hypothesis that the coefficient of the lagged separation indicator for $t = 7$ is equal to zero. This finding is problematic. If the only potential source of attrition is some correlation between noise in the performance equation and noise and the random part of the outside offer,
then the fixed effects estimator should yield unbiased estimates for all cases reported in Table 2.8. The fact that it does not indicates that there is an alternative source of attrition bias. This is one of the topics studied in the following chapter.

2.4 Conclusion

This chapter provides descriptive and regression analysis of employment dynamics at an institution offering short term jobs. I document that the firm does not rely on a complex hierarchical structure to conduct its business. In contrast to the case of most long-term jobs, its primary output is something that could be measured and the contribution of each employee to profits established. Perhaps, in relation to these properties the firm employs a simple compensation policy based on hourly pay and a bonus proportionate to individual output. In contrast to institutions offering long-term employment, the firm does not use a system of promotions and layoffs to shape the quality mix of the workforce and motivate employees to work hard; it relies primarily on its compensation policy to achieve these objectives. The management offers steep pay incentives to achieve this type of self-selection, which leads to a considerable heterogeneity in compensation across employees. The descriptive analysis in this chapter verifies that highly productive employees stay longer in the company than employees of low productivity, which likely contributes to the high turnover rate across compensation policies.

The employment dynamics at the company conforms with the main stylized prediction in the literature on learning about match quality. Namely, expected performance among stayers increases faster than individual expected performance. In addition, the hazard rate for a given regime decreases with tenure and employees of high past performance are more likely to stay than those of low past performance. At the same time, I also find evidence that pay incentives affect performance and that performance in-
creases with tenure. Still, there is no change in the variance of performance as incentives vary, one of the two main predictions of the empirical literature on pay incentives. The availability of a measure of the quality of one’s work allows me to investigate whether a compensation policy based on piece rates has negative side effects on the quality of work at the firm. The descriptive statistics suggest that this is not the case. To summarize, the availability of detailed monthly data allows me to describe the dynamics of employment relations at a company that offers short term jobs. Nevertheless, it is impossible to evaluate the effect of pay incentives on workers’ welfare and profits without knowing how incentives affect the probability of staying in an environment of endemic turnover.
Chapter 3

Estimating the Effects of Incentives When Workers Learn about Their Ability

3.1 Introduction

Learning about ability, turnover, and unobservable effort choice are defining features of many work environments. In a departure from the existing literature, I consider a model of employment dynamics that incorporates all three. Using unique data from a call center in North Carolina, I apply the model to investigate how learning about ability affects the empirical analysis of the effect of incentives. Furthermore, the model allows me to explore the channels through which pay incentives affect average productivity (performance) and in turn profits.

The data used in this study are ideally suited for the purposes of estimating the effect of incentives on performance; they contain an objective measure of individual performance (defined as output per hour), a compensation policy based on piece rates,
and variation in the pay policy. I observe that steeper incentives are associated with higher performance and that persistent differences in individual performance are driven by differences in ability, which reflects the quality of the employer-employee match. Moreover, I find evidence that employees learn about the quality of the match in the course of the employment relation. Their posterior beliefs are largely responsible for their decision to stay or quit and the interaction between incentives and turnover appears to be crucial to evaluating the impact of incentives on individual welfare and profits.

The fact that individual performance affects labor turnover, while turnover determines what performance data are observed posits a serious econometric challenge. In the absence of learning about ability, a popular approach to address this challenge is to introduce individual fixed effects and estimate the observational equation on the subsample of employees who stay for the duration of the study. If the set of unobserved productivity effects remains fixed over time, it is then possible to estimate the time-varying elements of the observational equation. This is the approach adopted in Lazear (2000) and Bandiera, Barankay, and Rasul (2005) (hereafter BBR). I show that this approach, referred to in the rest of the paper as the fixed effects estimator, is not appropriate when workers learn about their ability. The main idea behind this result can be illustrated by an example. Consider two workers, Alice and Bob, of identical ability who observe a sequence of two identical productivity signals, a good and a bad one. The difference between them is that Alice receives the good signal first, while Bob receives the bad signal first. Their payoff is equal to the realized signal and they also can quit after the first signal and accept the realization of a random offer that is independent from the signals. When the two know their ability, their probabilities of quitting after the first signal are equal. However, when they learn about their ability, Bob is more likely to quit than Alice. Thus, there are more “Alices” than “Bobs”
among the workers who stay and changes in performance are driven by the decision to stay or quit. A failure to control for this econometric implication of learning leads to biased estimates.

Estimating the effects of pay incentives when workers learn about their ability is hard in general. However, if the stochastic technology is additively separable in effort, tenure, and individual productivity, I show how it can be done. These technology restrictions play a crucial role in my empirical work: I use them to develop and estimate a model of effort choice and turnover that nests as a special case the hypothesis of learning about ability. In this way, I obtain valid estimates of the effects of incentives. Furthermore, the estimation of the model allows me to recover the distribution of ability and trace how it evolves over time and across different pay regimes. The bias from neglecting Bayesian learning and attrition can be considerable: simulating the estimated model, I show that the fixed effects estimator overestimates the effect of incentives by a factor of two.

My work contributes to several strands of the literature, most directly to the empirical literature on pay incentives. The importance of pay incentives for performance has been recognized at least since Taylor (1911). In the last two decades, McMillan, Whalley, and Zhy (1989) provide evidence that 75% of the increase in agricultural productivity in China from 1978 to 1984 can be attributed to the introduction of a responsibility system which allows communes to retain some profits. Kahn and Sherer (1990) document the widespread use of pay incentives at white-collar office jobs and show that better evaluations are achieved by workers who face steeper incentives. Ferrie and Metcalf (1996) study how different forms of compensation affect performance among British jockeys and find that the jockeys employed on fixed compensation perform worse than those who receive prizes when they win. Finally, Lemieux, MacLeod, and Parent (2009) find that since the 1970s a growing proportion of US firms have
conditioned pay on performance and that this development contributed to a growing income inequality.

The availability of both performance data and a known compensation policy offers a number of advantages. Most importantly, researchers do not have to rely on strong assumptions to form a link between observed compensation and unobserved performance; knowing the compensation policy allows for a direct test for the effect of pay incentives on effort based on observed performance. The research potential of personnel records has been explored in a number of recent papers. Lazear (2000) considers the effect of switching from an hourly wage to a piece rate on the productivity of installers of windshields. He shows that as a result of the change, average productivity increases by 35%. However, Lazear cautions that about one third of the change can be attributed to selection at entry: the change in the pay regime attracted more qualified employees. To control for the effect of nonrandom attrition on the characteristics of the workforce at different tenure horizons, he employs the fixed effects estimator discussed above. The same estimator is also used in Lazear and Shaw (2009) that recovers monthly tenure-performance profiles. The fixed effects estimator is employed in Bandiera, Barankay, and Rasul (2005) (hereafter BBR) where the authors study the interaction between pay incentives and social preferences, BBR (2007) which considers the effect of pay incentives for managers on performance of subordinates, and BBR (2009) that analyzes the effect of pay incentives on team formation. In this context, the benefits from the experimental environment in Shearer and Paarsch (1999), Shearer (2004), and Shearer and Paarsch (2009) become evident: control over the environment allows them to focus exclusively on the effect of incentives on effort choice. Unfortunately, their framework does not allow for the study of how turnover affects profits. Moreover, economists seldom can fully control the employment environment. My work contributes to the literature on incentives effects in two ways. First, for a family of stochastic technologies,
it shows how to estimate the effect of incentives on effort in the presence of learning about ability and attrition. Second, while there is no evidence for selection at entry, the results indicate that turnover is a major channel through which pay incentives affect average performance and in turn profits.

This chapter also relates to another strand of the empirical literature on learning about match quality that can be traced back to the theoretical work on search by inspection and wage rigidity in Jovanovic (1979) and Harris and Holmstrom (1982). The availability only of compensation data has posited a major challenge to related empirical work. Chiappori, Salanié, and Valentin (1999) address this problem by exploring the testable implications of Bayesian learning and downward rigidity on the dynamics of compensation series. Since turnover is close to nonexistent in their data, their estimates do not suffer from the econometric problems discussed here. Yet, even the dynamics of compensation series are of limited help in distinguishing between learning about match quality and learning-by-doing for a large number of models: Mortensen (1988) shows that these forms of learning impose the same testable implications on the dynamics of compensation data. This property caused insurmountable identification problems to empirical work in the past. For example, in their paper Gibbons, Katz, Lemieux, and Parent (2005) are forced to assume away tenure effects in order to estimate the quality of industry-specific matching. Nagypal (2007) proposes an alternative approach to identification based on estimating a structural model of learning-by-doing and learning about match quality. However, she imposes very strong functional form restrictions and abstracts away from incentive effects. In contrast to the previous literature, the availability of performance data, along with the modelling and estimation of learning about ability and turnover, allows me to study key features of the stochastic technology. As a result, in this chapter I can test for learning about ability, characterize its

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dynamics, and explore how pay incentives affect the distribution of ability at different tenure horizons. At any tenure horizon and controlling for pay incentives, I find that employees of high ability, or employees who believe to be of high ability, are more likely to stay. Furthermore, mean ability among those who stay increases with tenure. These findings provide evidence that ability is firm-specific and knowledge about it does not affect compensation at alternative jobs. To my knowledge, this is the first work using observed productivity signals to provide evidence for learning about match quality and to characterize the quality mix at different tenure horizons.

In the process of estimating the model, I also recover the returns to months of tenure. This result relates to a large body of literature starting with the empirical findings in the late 1970s and early 1980s, Jovanovic and Mincer (1981) being one of the most cited, of a large seniority or tenure effect on earnings. Abraham and Farber (1987) caution that the empirically observed strong relation between tenure and earnings in many cases is a statistical artifact due to the positive relation between seniority and an omitted variable. Altonji and Shakotko (1987) confirm the findings in Abraham and Farber (1987). However, Topel (1991) finds that a two-step first difference approach applied to the same model and similar data yields substantially higher returns to tenure. In a later work, Altonji and Williams (2005) review the strengths and weaknesses of the applied methods and conclude that, if turnover is driven by learning about match quality, Topel (1991) does not control adequately for attrition bias. In contrast to this literature, I model explicitly the attrition process and estimate the effect of monthly tenure on performance. Furthermore, I show that learning about ability still generates a bias, as discussed in Abraham and Farber (1987), even when compensation is not a function of past performance signals but a simple deterministic rule.

Buchinsky et. al. (2009) rely purely on wage data to model explicitly turnover when it is generated through search by inspection, while preserving the structure of the observational equation as in Abraham and Farber (1987).
The rest of the chapter is organized as follows. Section 3.2 explores the implications of learning about ability and attrition on the estimation of the effect of incentives and of returns to tenure. Section 3.3 presents the data used in the empirical work. Section 3.4 demonstrates how to estimate the effects of incentives in the presence of learning and nonrandom attrition. Section 3.5 presents the estimated effects of incentives and tenure under the fixed effects estimator and under the estimation approach of section 3.4. The section continues with an investigation of the bias resulting from the inappropriate use of the fixed effects estimator. It ends with a set of robustness checks that confirm the choice of technology restrictions and the specification of the attrition process. Section 3.6 summarizes the main results and concludes with remarks on related research.

3.2 Bayesian Learning and Nonrandom Attrition

Labor turnover may depend on unobserved productivity parameters. Indeed, this is a central feature of many models, starting with Jovanovic (1979). Such models predict that workers with low values of the unobserved productivity parameters are more likely to quit. If the econometrician ignores such a process of nonrandom attrition and pools all available observations to estimate an equation for, say, performance, the estimated effects of tenure and incentives are biased. As pointed out in the introduction, past research addresses this problem by estimating the observational equation using fixed effects only on the subsample of employees who stay at the firm for the duration of the study. In what follows, I will show that this fixed effects estimator yields biased estimates when the workers engage in Bayesian learning about their ability. I discuss the econometric implications of nonrandom attrition and learning in the context of a model that incorporates both effort choice and labor turnover. A central feature of the model is the strong separability of the stochastic technology in effort, ability, and tenure. This
restriction is consistent with the properties of the data used in the empirical analysis, and it emphasizes that in the presence of Bayesian learning, even when effort choice does not depend on posterior beliefs and ability can be differenced out, the fixed effects estimator yields biased estimates of the effects of incentives.

When applied to the data generated by such a model, the fixed effects estimator yields biased estimates because there is a nonzero correlation between noise in the performance signal and the outside offer; this problem is well understood in the existing literature on selection and attrition. Here I focus on the bias generated by learning about ability and in the rest of the section maintain the assumption that noise in the performance series and the outside offer are independent. If workers learn about their ability only in the course of their employment relation, their separation decisions are based on their posterior beliefs and through them on observed noisy signals. Consequently, the decision to stay is not independent from noise in the performance series.

3.2.1 Model

The model is a variation on the standard model of search by experience, first introduced in Jovanovic (1979). Each period, workers choose not only whether to stay or quit, but also how much effort to exert. The crucial element in the model is a continuous ability parameter \( \theta_i \) with a probability density function \( f_{\theta} \). Ability is unknown at the time of hiring and both workers and the employer observe noise signals about it in the course of the employment relation. At the beginning of each period \( t \), she observes the realization of a continuous outside offer \( \xi_{it}^* \) that is independent of ability and has a probability density function \( f_{\xi^*} \). The worker decides to stay if the value of continued employment is greater than the outside offer and quits otherwise. I assume that if an
employee quits, she is never hired again.\footnote{This assumption is not restrictive to the empirical work in the second half of the paper: only five out of 675 employees are rehired after quitting. The second spells of employment are dropped out.} If the worker decides to stay, she chooses a level of effort $l_{it}$, that is not observable or verifiable by the firm.\footnote{See Malcomson and MacLeod (1992) for a discussion on the implications of these assumptions.} Then she observes a performance signal $y_{it}$ governed by the following stochastic technology:\footnote{Since both the actual and estimated performance are always greater that 0, the restriction $y_{it} \geq 0$ never binds.}

$$y_{it} = \theta_i + g(t) + l_{it} + \varepsilon_{it},$$

(3.1)

where $\varepsilon_{it}$ is continuous, independent and identically distributed over time and across individuals with a probability density function $f_\varepsilon$, $\theta_i$ is independent of the error process, and $g(t)$ represents the accumulation of firm-specific knowledge.\footnote{Jovanovic and Nyarko (1994) provide an alternative specification for the accumulation of firm-specific knowledge, which is sometimes refered to as learning-by-doing. However, the data do not support the prediction of their model that the variance of individual performance declines over time. Note that the model also assumes that the accumulation of knowledge does not depend on past or present effort.} The noise signal has two roles in the model. On its basis, the worker updates her beliefs about $\theta_i$: the belief at the beginning of $t$ is denoted as $\theta_{it}$ and depends on the initial prior $\theta_{i1}$ and the noisy signals about $\theta_i$ up to period $t - 1$ including, $\{y_{ik} - g(k) - l_{it}\}_{k=1}^{t-1}$. Beliefs are formed in a Bayesian way and $\theta_{it}$ has a probability density function $f_{\theta_{it}}$.

The performance signal also provides the basis of compensation: the worker is paid $w_{it} = \alpha_{it} + \beta_{it}y_{it}$, according to a linear compensation regime $R_{it} = (\alpha_{it}, \beta_{it})'$. For the purposes of future discussion, regime $R_{it}$ is said to be more generous than regime $R'_{it}$, $R_{it} > R'_{it}$, if both $\alpha_{it} > \alpha'_{it}$ and $\beta_{it} > \beta'_{it}$. Workers are assumed to be risk-neutral with a utility function

$$u(R_{it}, l_{it}, \theta_i, \varepsilon_{it}) = \alpha_{it} + \beta_{it}(\theta_i + g(t) + l_{it} + \varepsilon_{it}) - \psi(l_{it}).$$
where $\psi(l_{it})$ is convex in effort. The worker does not expect the compensation regime to change in the future. In this way, I abstract away the issue of forming expectations, which is outside the scope of this paper. Since $\theta_{t}$ and $l_{it}$ enter additively in the utility function, posterior beliefs do not depend on effort and optimal effort choice does not depend on beliefs and is function only of $R_{it}$, $l(R_{it})$. On this basis, the value of continued employment is defined as

$$H(\theta_{it}, R_{it}, t) = \int \int (u(R_{it}, t, \theta_{i}, \varepsilon_{it}) f_{\varepsilon}(\varepsilon_{it}) f_{\theta_{it}}(\theta_{it}) d\varepsilon_{it} d\theta_{it}$$

$$+ \delta \int \left[ \int \max[\xi_{it}^{*}, H(\theta_{it+1}, R_{it}, t+1)] f_{\xi^{*}}(\xi_{it+1}^{*}) d\xi_{it+1}^{*} \right] f(\theta_{it+1}|\theta_{it}, t) d\theta_{it+1},$$

where $f(\theta_{it+1}|\theta_{it}, t)$ is the conditional density of $\theta_{it+1}$. An employee decides to stay if

$$H(\theta_{it}, R_{it}, t) > \xi_{it}^{*} \quad \text{(3.2)}$$

and quits otherwise. Consequently, performance for period $T$ is observed only if (3.2) holds for $t = 1, \ldots, T$. The assumptions of the model, the existence of a solution to the worker’s problem and its characterization are presented in Appendix A. Under assumptions 1-6 in the appendix, if $R_{it}$ is more generous than $R_{it}'$, then optimal effort $l(R_{it}) > l(R_{it}')$ and $H(\theta_{it}, R_{it}, t) > H(\theta_{it}, R_{it}', t)$ (and as a result the probability of quitting under $R_{it}'$ is higher than the probability of quitting under $R_{it}$ across $\theta_{it}$). Furthermore, for a given regime the value of continued employment increases in $\theta_{it}$ in the sense of the likelihood ratio property.

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7 Prendergast (2002) discusses in great detail the relation between risk-aversion and optimal performance pay. However, for the purposes of this paper, the investigation the effect of a change in incentives on effort, the assumption of risk-neutrality is not restrictive.

8 While at first sight this assumption may appear restrictive, Kanemoto and MacLeod (1992) show that when firms learn about individual ability the existence of an outside option for workers disciplines firms to keep piece rates fixed even after beliefs are updated. This issue is discussed further in section 4.
Alternatively, the worker may know the value of her $\theta_i$ at the time of hiring. This is a special case of the model presented above, referred to in the rest of the paper as the case of known ability. Here the prior belief is a degenerate distribution centered at the true value. Performance in period $T$ is observed if for all $t = 1, ..., T$

$$H (\theta_i, R_{it}, t) > \xi_{it}^*$$

Again, if $R_{it}$ is more generous than $R_{it}'$, optimal effort $l (R_{it}) > l (R_{it}')$ and $H (\theta_i, R_{it}, t) > H (\theta_i, R_{it}', t)$, while $H (\theta_i, R_{it}, t)$ increases in $\theta_i$. While on the surface the two cases appear very similar, they have very different econometric implications explored below.

The econometric implications of the interaction between learning about ability and the decision to stay or quit are investigated in the context of this model. As presented the model imposes that the outside offers are drawn from a distribution that does not change with tenure and does not depend on $\theta$. These restrictions are imposed for expositional purposes only: the econometric implications of learning about ability do not depend on them and in fact the model that is taken to the data allows the outside offer to depend on beliefs, tenure and other observed or unobserved individual characteristics.\(^9\)

### 3.2.2 Estimating the Effects of Tenure and of Incentives

Suppose for the moment that the principal interest of the econometrician lies in the estimation of returns to tenure. Assume that the observational equation is defined by (3.1) and that the piece rate remains the same across individuals and over time. In the case of known ability, $i$ knows $\theta_i$ at the time of hiring and performance is observed in

\(^9\) Mortensen (1988) discusses the additional assumptions on the process generating the outside offer that are necessary to prove the existence of a solution and to characterize it.
period $T$ if (3.3) holds for $t = 1, \ldots, T$. If $\xi_{it}^*$ is iid over time and is independent of the error process for the observational equation, the decision to stay and the errors $\{\varepsilon_{ik}\}_{k=1}^{T-1}$ are independent, so for $t = 1, \ldots, T - 1$

$$E \left[ \varepsilon_{it} \mid \min \left\{ H (\theta_i, R_{ik}, k) + \xi_{ik}^* \right\}_{k=1}^{T} > 0 \right] = 0.$$  

Thus, estimating (3.1) using fixed effects or first differences, subject to (3.3) for all $t = 1, \ldots, T$, yields unbiased estimates of the effect of tenure on performance in the first $T$ periods.

Next, consider the case of Bayesian learning, in which $i$ learns the value of $\theta_i$ over time. The expected value of the disturbance term $\varepsilon_{it}$ for $t = 1, \ldots, T - 1$

$$E \left[ \varepsilon_{it} \mid \min \left\{ H (\theta_i, R_{ik}, k) + \xi_{ik}^* \right\}_{k=1}^{T} > 0 \right]$$

is generally different from zero. Since $\theta_{it}$ is an increasing function of $\{\varepsilon_{ik}\}_{k=1}^{T-1}$, the conditions for staying define implicitly left truncations of the unconditional distribution of $\varepsilon_{it}$. To illustrate, assume that the distribution of $\varepsilon_{it}$ is log-concave. Then (3.4) is positive and increases in $\min \left\{ H (\theta_i, R_{ik}, k) + \xi_{ik}^* \right\}_{k=1}^{T}$. Intuitively, for any $\theta_i$ if employee $i$ stayed at least $T$ periods, then in each period $t$ before $T$ she could not have been excessively unlucky in her draws of $\varepsilon_{it}$. What "excessively" means depends on the structural parameters of the model, in particular on $\theta_i$. Thus, estimating the observational equation (3.1) using fixed effects or first differences, subject to (3.2) for $t = 1, \ldots, T$ yields biased estimates of the effect of tenure on performance in the first $T$ periods.

Diagram 3.1A illustrates the point that Bayesian learning imposes a left truncation

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$^{10}$More details can be found in Appendix A.
on the distribution of observed signals for $T = 2$. For example, the fact that individual $i$ stays for at least $3$ periods indicates that given $\theta_i$ she must have been sufficiently lucky in both the first and second period. However, whether the conditional expectation of noise for the first period or the second period is larger depends on the structural parameters of the model, in particular $\theta_i$. The former case is presented on Diagram 3.1B and the latter on Diagram 3.1C. Since the truncation thresholds decrease in $\theta_i$, the "learning" bias declines with $\theta_i$. If for the majority of workers the conditional expectation of noise in $t = 1$ is smaller than its counterpart in $t = 2$, as shown on Panel C, then the fixed effects estimator overestimates the effect of tenure on performance. However, if the majority of the conditional expectations trend downwards as shown on Panel B, the fixed effects estimator underestimates the effect of tenure. Consequently, the direction and magnitude of the "learning" bias depend in a complicated fashion on tenure, ability, the realized errors, and the other structural parameters.

The analysis in the preceding paragraphs assumed that the compensation regime does not vary over time or across individuals. In what follows, I relax this assumption and discuss the econometric implications of learning within an example related to the empirical context of my work. Suppose that regime 1 is more generous than regime 2, and consider the case when the pay regimes are introduced sequentially, first regime 1 and then regime 2, and suppose that $\theta_i$ is known at the time of hiring. When workers know their ability at entry, the fixed effects estimator again yields unbiased estimates of the effect of incentives, since the same set of workers are exposed to both pay regimes. However, when Bayesian learning takes place, the estimated incentives effect is biased upwards, as illustrated on Diagram 1B. Suppose that a worker switches from regime regime 1 to regime 2 in the second period and that the econometrician conditions on staying for at least two periods. Since the employee stays for more than one period, the conditional expectation of noise for $t = 1$ is positive, while for $t = 2$ it is zero. As
a result, the fixed effects estimator overestimates the effect of incentives. The same argument extends to more than two periods. Suppose that beliefs converge to the true value of $\theta_i$ quickly and that regime 1 is in place during these crucial periods. Then, the average of the conditional expectations of noise under the initial regime 1 is positive, while under regime 2 close to zero.\textsuperscript{11}

The preceding paragraphs indicate that the fixed effects estimator yields biased estimates of both tenure and incentives effect when posterior beliefs drive separation decisions. This result follows from the fact that the noise from the signals affect posterior beliefs, which in turn determine individual actions, such as the decision to stay or quit. At a higher level of generality, Bayesian learning introduces dependency on tenure in the observed series of signals through the separation decisions. These observations leave the econometrician with a choice to model attrition explicitly or to estimate the treatment and tenure effects on the set of agents who have a probability of quitting close to zero. The first approach utilizes all available data, but at the price of imposing strong assumptions on the attrition and performance processes. In what follows, I adopt the second approach and investigate the appropriateness of the associated assumptions in section 5.4.

\textsuperscript{11}Matters are complicated further by sample size and the horizon of the study. Convergence of beliefs to the true value of the unobserved parameters is achieved for the model presented in this paper. Since the issues is not central to the argument, no proofs are provided. An excellent treatment of the issue can be found in Easley and Kiefer (1988) and Aghion, Bolton, Julien and Harris (1991).
Panel A

Panel B

Panel C

Diagram 3.1: Implications of learning about ability for observed signals
3.3 Data

A detailed exposition of the work environment, the local labor market, the descriptive analysis of the data and some nonparametric tests can be found in Chapter 2. This section reviews the aspects of the work environment and the descriptive analysis that are particularly relevant to the scope of this chapter. The data set has several features that make it comparable to the data sets used by Bandiera, Barankay, and Rasul, as well as the data sets used by Lazear and Shaw. It contains a clean performance measure and three piece rates that were implemented in a way that allows to identify each one’s effect on performance. However, what makes it particularly appropriate for the study of the interaction between learning and attrition is the presence of considerable turnover, consistently above 50% during the first six months of employment. The descriptive analysis indicates that neither a pure moral hazard model nor a model of pure learning about match quality can account for the observed data patterns.

3.3.1 Context

The data are collected at a call center in North Carolina owned and operated by a multinational company. The call center collects outstanding debt and fees on behalf of cable TV companies, which ensures a stable demand for its services. An automated switchboard operator allocates inbound and outbound calls, so that the longest weighting customer is matched with the longest weighting operator. Employees rotate their work stations on a daily basis. In its recent history, the call center suffered from low average productivity and high labor turnover.

As part of its reorganization plans, the central management implemented a an hourly rate plus a piece rate (regime 1) as a pilot project to evaluate the consequences of switching from hourly wage to a piece rate across all of its call centers. This regime
change was implemented at the beginning of January 2005. The piece rate was a linear function of the performance metric, the number of calls per hour that end with collection of the outstanding debt. Importantly, one’s pay did not depend on the performance of others; in theory there may be competition among the employees for calls, but in practice this possibility is ruled out by the chronic shortage of workers at the call center. The firm experienced difficulties attracting candidates to fill in vacancies, so the management hired virtually all candidates during its monthly hiring rounds. The central management was concerned that the company was paying "too much," so it implemented a new hourly rate and piece rate for the newly-hired employees in June 2005 (regime 2). Relative to regime 1, regime 2 offered a lower base pay, decreased the slope of the piece rate for those with performance less than 3.8 call per hour, and increased the slope of the piece rate for those with performance greater than 3.8 call per hour (regime 2). All previously hired employees continued to be paid according to regime 1. Since the central management was worried about possible negative effects of the piece rate on the quality of service, it changed the pay regime yet again in November 2005. The new regime 3 had two components: all employees were paid according to the pay schedule of regime 2, but in addition employees had to meet certain minimum quality standards of service to qualify for the piece rate. Twenty per cent of one’s calls were randomly monitored and the quality of service was rated on a scale from 0 to 100. An employee who did not meet the minimum quality standard was relegated to an hourly wage equal to the base pay of the piece rate. Since 99% of performance lies between 1.05 and 3.8, regimes 2 and 3 effectively lowered incentives relative to regime 1. Diagram 3.2 shows a time line for the implementation of the three regimes and Table 2 some descriptive statistics of interest.
3.3.2 Descriptive Analysis

The call center experienced high turnover rates under all pay regimes: more than 50% of all employees under regime 1 quit within the first six months of employment, while under regimes 2 and 3 the turnover for the first six months approached 67%. There also appears to be a noisy downward trend in the separation rates as tenure increases. This noisiness is probably due to the small sample size, but it also suggests that separation decisions depend to a large extent on individual-specific factors. Table 3.1 reports the average performance for the first six months of employment across regimes. Again, as one may expect, the average performance under regime 1 is higher than its counterparts for regimes 2 and 3. Furthermore, the average performance on the subset of workers who stay for at least six months is higher than the simple average, suggesting that poor performers quit.

Figure 2.3 presents evidence for persistent differences in performance across individuals that are consistent with the existence of unobserved individual productivity effects. The figure plots average performance in periods 2 to 5 conditional on the performance
Table 3.1: Summary statistics for the first 6 months for workers who start and work under the same regime.

<table>
<thead>
<tr>
<th>Variable:</th>
<th>Regime 1</th>
<th>Regime 2</th>
<th>Regime 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ (in $$$)</td>
<td>3.3</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$\alpha$ (in $$$)</td>
<td>3.8</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>Avg. Perf. (call/hr.)</td>
<td>2.74</td>
<td>2.6</td>
<td>2.66</td>
</tr>
<tr>
<td>Std. Dev. (Perf.)</td>
<td>0.68</td>
<td>0.77</td>
<td>0.7</td>
</tr>
<tr>
<td>Avg. Perf., stay(\geq6)</td>
<td>2.91</td>
<td>2.76</td>
<td>2.71</td>
</tr>
<tr>
<td>Std. Dev. (Perf., stay(\geq6))</td>
<td>0.65</td>
<td>0.67</td>
<td>0.6</td>
</tr>
<tr>
<td>Turnover</td>
<td>0.52</td>
<td>0.68</td>
<td>0.67</td>
</tr>
<tr>
<td>Obs., stay(&gt;6)</td>
<td>113</td>
<td>59</td>
<td>9</td>
</tr>
</tbody>
</table>

quartile in the first month of employment. If there were no persistent differences in the productivity of employees, performance in months 2 to 5 would be the same across the initial performance quartiles. This hypothesis is not supported by the data: the workers in the top initial quartile have consistently higher performance in periods 2 to 5 than their counterparts in the other three quartiles. Furthermore, average performance for the employees in each initial quartile increases over time: for all quartiles, the difference between average performance in periods 1 and 5 is statistically significant at 5%. Finally, performance does not seem to be "fanning out" over time.

This evidence suggests that steep pay incentives lead to high performance; that attrition appears to be nonrandom, since workers with higher performance are more likely to stay; that individual-specific effects are present, but performance does not "fan out" over time; and finally that workers accumulate experience or knowledge in the course of their first six months of employment.
3.4 Estimation

This section starts by introducing the attrition model taken to the data. One crucial implication of the functional form restriction on the technology is that posterior beliefs do not affect effort choice which simplifies considerably the estimation of the effects of incentives. The rest of the section is devoted to presenting how to estimate the model using MLE. The validity of the technology restriction is discussed in section 5.4.

3.4.1 Performance and Attrition Equations

The estimated model is derived from the theoretical model presented in section 2 with a few modifications: performance and the separation decision are allowed to depend on \( m(X_{it}) \), a function of the individual characteristics observable at period \( t, X_{it} \):

\[
y_{it} = \theta_{it} + l(R_{it}) + g(t) + m(X_{it}) + \varepsilon_{it}
\]

Furthermore, I do not specify a utility function explicitly and approximate flexibly a normalized and scaled version of \( H(\theta_{it}, t, R_{it}) \), \( G(\theta_{it}, t, R_{it}) = \frac{1}{\sigma_{\xi^{*}}}(H(\theta_{it}, t, R_{it}) - \mu_{\xi^{*}}) \), where \( \sigma_{\xi^{*}}^2 \) is the variance of \( \xi_{it}^{*} \) and \( \mu_{\xi^{*}} \) the mean. This approach allows me to accommodate a number of variations on the basic model. For example, the outside offer may be the sum of the random component \( \xi_{it}^{*} \) and a deterministic component that varies with tenure, with some observed or unobserved individual variables or even beliefs.\(^{12}\)

Without imposing structure on the utility function, it is possible to identify only the effect of a change in pay incentives relative to the benchmark regime 1, since one cannot distinguish between effort under the initial regime 1 and the mean of the distribution of \( \theta_{i} \). Furthermore, many parameters that determine the separation decision do not

\(^{12}\)See Mortensen (1988) for a discussion of the conditions on a time-varying outside offer that ensure a well-behaved solution of the worker’s problem.
have a clear interpretation in relation to the underlying model. Nevertheless, such a specification is sufficient to test whether incentives affect performance and to recover the effect of a change in incentives on the quality mix of employees at different tenure horizons. As discussed in section 2, I also assume that employees take the piece rate as given and do not expect it to change. Given that the average tenure at the firm is around 3.5 months, an employee could have realistically expected that the same regime would last for the duration of her employment spell. Finally, let $\xi_{it} = \frac{1}{\sigma_{\varepsilon^2}} (\xi_{it}^* - \mu_{t^*})$.

The model is estimated under the following additional distributional assumption.

Assumption (i) $\varepsilon_{it} \sim N \left(0, \sigma_{\varepsilon^2}^2 \right)$ and $\xi_{it} \sim N \left(0, \sigma_{\xi^2}^2 \right)$ are iid across tenure horizons and individuals, independent from the rest of the covariates. (ii) $\theta_i \sim N \left(0, \sigma_{\theta^2}^2 \right)$ is iid across individuals and is independent from the rest of the covariates.

These assumptions impose strong restrictions: in particular, they rule out temporary but persistent health or family shocks. The plausibility of these assumptions is evaluated in section 5.4 through some simple post-estimation tests. Under the assumption above and if $\theta_{i1} \sim N \left(0, \sigma_{\theta^2}^2 \right)$, the posterior belief $\theta_{it}$ is also normally distributed, $\theta_{it} \sim N \left(\mu_{it}, \sigma_{\theta}^2 \right)$ for all $t > 1$, where

$$\mu_{it} = (1 - K_t)\mu_{i t-1} + K_t(y_{it-1} - l(R_{it-1}) - g(t - 1) - m(X_{it-1}))$$ (3.5)

$$\sigma_{\theta}^2 = \frac{\sigma_{\varepsilon^2}^2 \sigma_{\theta}^2}{\sigma_{\varepsilon^2}^2 (t - 1) + \sigma_{\theta}^2}$$

$$K_t = \frac{\sigma_{\theta}^2}{\sigma_{\varepsilon^2}^2 (t - 1) + \sigma_{\theta}^2}$$

Thus, Bayesian updating becomes quite tractable. In particular, the precision of beliefs depends only on $t$, so the average of the demeaned past signals is a sufficient statistic to characterize posterior beliefs. The formula for the posterior mean in (3.5) can be
rewritten as

\[ \mu_{it} = k(t) \cdot \left( \frac{1}{t-1} \sum_{k}^{t-1} (y_{ik} - l(R_{ik}) - g(k) - m(X_{ik})) \right). \]

The function \( k(t) \) represents the precision that the worker attaches to the average demeaned past performance as a signal about her \( \theta \). Since \( k(t) \) increases over time, she attaches greater and greater weight to the average of the demeaned past performance and less to the mean of the initial belief. Under the assumption of a common prior, this discussion implies that the average of demeaned past performance is a sufficient statistic for posterior beliefs and their effect on the decision to stay or quit.

Thus, the model taken to the data is defined by the following sets of equations:

\[ y_{it} = \theta_i + l(R_{it}) + g(t) + m(X_{it}) + \varepsilon_{it}, \]

\[ s_{ik} = 1 \left[ G \left( \lambda \frac{\mu_{ik}}{\theta_{ik}} + (1 - \lambda) \theta_i, R_{ik}, k, X_{ik} \right) - \xi_{ik} > 0 \right] \] (3.6)

where \( y_{it} \) is observed if \( s_{ik} = 1 \) for all \( k = 1, ..., t \). If \( \lambda = 1 \), Bayesian learning is present and employees share a common prior. If \( \lambda = 0 \), workers know their ability at the time of hiring. A \( \lambda \in (0, 1) \) is difficult to interpret, but probably suggests that the initial prior is correlated with ability\(^{13}\); finally \( \lambda < 0 \) is a clear rejection of the model. I estimate the model under the restriction that \( \lambda = 1 \), that \( \lambda = 0 \), and when \( \lambda \) is also estimated.

\(^{13}\)I do not pursue this avenue any further because a nonparametric test discussed later indicates that there is no self-selection at entry as regimes vary; the finding is consistent with the absence of any prior knowledge about ability.
3.4.2 MLE

The crucial difference between attrition and selection models is that \( s_{it} = 1 \) implies that observing performance in one period implies that performance is observed also in all preceding periods. Thus, \( s_{it} = 1 \) provides information about the value of \( \theta \) that affects the estimation of \( y_{it} \) across all observed periods. In contrast, selection models assume that the selection process takes place in each period independently. The literature on estimation of attrition models starts with Hausman and Wise (1979) who offer an estimation method based on a full information MLE. The MLE method in this study is similar, but also involves "integrating out" the unobserved effects\(^{14}\). The derivation of the likelihood is discussed in more detail in Appendix B. Here I provide only a summary of the main features of the MLE.

Let the observable information about individual \( i \) be \( W_i \) and \( \Theta_i \) be the vector of parameters to be estimated conditional on \( \theta_i \). The likelihood for individual \( i \) conditional on the data and \( \theta_i \) can be written in a standard way as follows:

\[
l_i (\Theta_i | \theta_i, W_i) = \prod_{t=1}^{T_i} \left( \frac{\varphi \left( \frac{y_{it} - g(t) - m(X_{it}) - l(R_{it}) - \theta_i}{\sigma} \right)}{\Phi \left( G(R_{it}, t, X_{it}, \tilde{\theta}_{it}) \right) S_{it}} \right) \left( 1 - \Phi \left( G(R_{iT_i}, T_i, X_{iT_i}, \tilde{\theta}_{iT_i}) \right) \right)^{1-S_{iT_i}}.
\]

where \( S_{it} = \prod_{k=1}^{t} s_{ik} \) and \( T_i \) is the last period in which \( i \) is observed. The interpretation of this expression is quite intuitive. The individual observes a performance signal, updates her belief, and decides whether to stay or quit. If she stays, the econometrician

\(^{14}\)Florens, Heckman, Meghir, and Vytlacil (2008) provide the basis of an alternative approach based on the use of control functions, which does not require distributional assumptions on the noise in the performance signal. However, small sample size and the fact that the MLE fits the data well, as discussed in the following section 6.2, argue in favor of the approach taken in the paper.
observes her effort choice and separation decisions in the following periods. If she quits, the econometrician does not, so the unobserved performance and separation series are "integrated out" and do not appear in the conditional ML. Since $\theta_i$ is not observed, it is integrated out to obtain

$$ l_i(\Theta|W_i) = \int l_i(\Theta_1|\theta_i, W_i) \cdot \varphi(\theta_i|W_i, \Theta_2) d\theta_i, $$

where $\Theta_2$ is a vector of parameters that define the distribution of $\theta_i$ and $\Theta$ is a vector that contains all parameters in $\Theta_1$ and $\Theta_2$. Finally, the log-likelihood is obtained by taking logs and summing over $i$:

$$ l\left(\Theta|\{W_i\}_{i=1}^N\right) = \sum_{i=1}^N \log l_i(\Theta) $$

Note that the model of attrition with Bayesian learning also implies the exclusion restriction that the average of past performances enters the attrition but not performance equations. Under alternative estimation methods, this restriction provides the basis for identification.

### 3.5 Results

This section presents the results from estimating the model from section 2 and investigates some alternative specifications. The empirical results are consistent with the presence of Bayesian learning that leads to a considerable upward bias in the estimated effect of incentives and tenure on performance when the fixed effects estimator is used. They also show that switching from regime 1 to regime 2 leads to a decline in effort but also to an improvement in expected ability among those who stay as tenure increases.
3.5.1 Results for the Attrition Model

Tables 3.2 - 3.4 reports the results from estimating the attrition model by MLE. Model 1 is estimated under the restriction of known ability of match quality, or $\lambda = 0$. Model 3 is estimated under the restriction of learning about ability or $\lambda = 1$. Model 2 nests both Models 1 and 3 as special cases and estimates $\lambda$. The performance equations under all three models are the same: the explanatory variables include second degree orthogonal polynomials of tenure and third degree polynomials of calendar time, dummies for regimes of operation and regimes of hiring, and controls. Regime 2 enters additively as implied by the theoretical model. Since regime 3 has the same pay schedule as regime 2 but conditions pay on the quality of service, the performance equation incorporates interaction terms between the tenure polynomials and regime 3. The attrition equations include third degree orthogonal polynomials interacted with regimes and, depending on the specification, $\theta_i$ or $\mu_{it}$, controls, calendar time, and regime of hiring.

The estimated $\lambda$ under Model 2 is 0.74 and is significantly different from 0, but not significantly different from 1, which is consistent with the hypothesis of learning. A likelihood ratio test fails to reject the restriction $\lambda = 1$, while rejecting the restriction $\lambda = 0$. I take these results to imply that Bayesian learning is present and that Model 3 is the correct model. Accordingly, in what follows I discuss the estimated coefficients under the restriction of Bayesian learning. The variance of initial ability is 0.48 calls per hour, significantly different from 0, and accounts for the greater part of the variance in performance in the first months of employment. The variance of the disturbance term in the performance equation is estimated at 0.17 which implies that the ratio of the variance of match quality over the variance of noise is approximately 2.6 initially. After 6 months the variance of the posterior beliefs declines to approximately 0.05 and the weight that the worker puts on observed signals when forming her beliefs, $k(t)$,
Figure 3.1: Distribution of $\theta$ at $t$ under regime 1, conditional on staying at least $t$ months.

Figure 3.2: Distribution of $\theta$ under regimes 1, 2, and 3 at $t = 3, 6$, conditional on staying at least $t$ months.
Figure 3.3: Increase in the importance of observed signals relative to the initial prior when workers decide to stay or quit.

approaches 1, as evident from Figure 3.3. The figure indicates that both the shape of the trajectory is consistent with what theory predicts and the coefficient is significantly different from zero at all tenure horizons. Furthermore, the correlation between $\varepsilon_{it}$ and $\xi_{it}$ ceases to be significantly different from zero when $\lambda$ is estimated or $\lambda$ is restricted to be 1. The absence of a correlation implies that learning about ability is the likely cause of any differences between the estimates reported here and those obtained through the fixed effects estimator.

In what follows, I discuss the two main channels through which pay incentives affect performance: effort and the quality mix of the workforce. The model that is taken to the data does not impose restrictions on the effect of beliefs on the probability of staying. Figure 3.1 presents the random truncations of the distribution of ability at
different tenure horizons under regime 1. The value of \( \theta \) is on the horizontal axis, while the vertical axis represents the proportion of agents of a certain match quality who are present in the firm at a given tenure horizon. Ability is measured in calls per hour. The figure shows that the conditional distribution of ability in a cohort of employees shifts to the right as tenure increases: by 0.62 calls per hour within the first six months on the job. Most workers in the bottom quartile of the distribution quit within the first 2 months of entry in the firm, and most workers in the bottom half of the distribution within 6 months. The figure suggests that only workers with very high match quality face a low probability of quitting. This effect of incentives is equal to almost one standard deviation of ability in the population. Thus, the results indicate that the data are consistent with an interpretation of \( \theta_i \) as the quality of the employer-employee match.

Recall that relative to regime 1, regime 2 offers less incentives to exert effort and lower base pay. Figure 3.2 explores how the change from regime 1 to regimes 2 and 3 affects expected ability among staying employees at different tenure horizons. Since regime 2 offers both less incentives to exert effort and lower base pay, the probability of quitting increases across different types of ability. Yet, this increase is not uniform: workers of low and average ability are more affected than workers of high ability. As a result, expected ability among the staying employees is 0.73 calls per hour after the first six months of employment, which is slightly more than one standard deviation of the distribution of ability in the population. The quality mix after 6 months under regime 2 is actually better than the quality mix under the more generous regime 1: this finding suggests that the call center enjoys monopsony power and can capture much of the increase in the production surplus that comes from an increase in ability. Together

\[ \text{Here truncation of } f(x) \text{ with lower limit } a \text{ is defined as } \int_a^\infty f(x) \, dx. \text{ That is, one has removed the part of the distribution less than } a \text{ but not scaled up the distribution to integrate to one over its domain.} \]
Figure 3.4: Expected performance among stayers at different tenure horizons under regime 1-3.

with the finding that changes in pay incentives do not affect the ability of entering employees, these results provide evidence that ability is firm-specific: in particular, they are consistent with the main testable implication of learning about match quality\textsuperscript{16} that across regimes as the posterior mean and ability itself increase the probability of staying increases.

Figure 3.2 also indicates that after three months at work, the difference between the distributions of ability under regimes 1 and 2 is smaller than the difference between the distributions of ability under regimes 1 and 2 after six months. This fact is consistent with the dynamics of learning presented on Figure 3.3: most learning about ability

\textsuperscript{16}See Ericson and Pakes (1999) for a model of learning about firm-specific productivity under very general assumptions. The authors derive three main testable implications, outlined in chapter 2. The results here are consistent with two of them; since I only approximate the value of continued employment, I cannot test whether, controlling for the posterior mean, the probability of quitting increases with tenure.
Table 3.2: Estimates for the performance equation in the attrition model when ability is known, when workers learn about it, and when the hypothesis of learning is tested.

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Model 1 ( \lambda = 0 )</th>
<th>Model 2 ( \lambda ) estimated</th>
<th>Model 3 ( \lambda = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Coef.</td>
<td>S.E.</td>
<td>Coef.</td>
</tr>
<tr>
<td>( t, \text{ orthog. pol. 1} )</td>
<td>-0.41</td>
<td>0.16</td>
<td>-0.56</td>
</tr>
<tr>
<td>( t, \text{ orthog. pol. 2} )</td>
<td>-0.46</td>
<td>0.09</td>
<td>-0.62</td>
</tr>
<tr>
<td>regime 1</td>
<td>-0.24</td>
<td>0.08</td>
<td>-0.19</td>
</tr>
<tr>
<td>regime 2</td>
<td>0.23</td>
<td>0.12</td>
<td>0.13</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.88</td>
<td>0.13</td>
<td>0.77</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.53</td>
<td>0.09</td>
<td>0.43</td>
</tr>
<tr>
<td>% outbound calls</td>
<td>0.11</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>Hired under regime 1</td>
<td>0.11</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>Hired under regime 2</td>
<td>-0.36</td>
<td>0.29</td>
<td>-0.34</td>
</tr>
<tr>
<td>Constant</td>
<td>3.59</td>
<td>0.11</td>
<td>3.43</td>
</tr>
</tbody>
</table>

Log-likelihood:
- Model 1: -3756.22
- Model 2: -3745.07
- Model 3: -3745.71

Notes: The specification includes calendar time, orthogonal polynomials of degree 3, and individual controls: gender, age, marriage status, distance from home, and race. Obs. = 3,675

takes place within the first six months on the job. That is, workers under both regimes 1 and 2 are willing to stay and learn about their ability during the first months of employment, but once they have learned their ability the effect of the change in pay incentives becomes larger. Figure 3.4 summarizes the differences in expected ability among stayers under regimes 1 and 2 at different tenure horizons.

Next, I discuss the estimates for the other channel through which incentives affect performance: effort choice. Figure 3.5 plots the tenure–performance profile for the average entering employee conditional on staying for regimes 1, 2, and 3. Regime 2 is restricted only to a downward shift in performance across tenure horizons, while regime 3 is also interacted with tenure. The restrictions on the way regime 2 enters in the performance equation follow directly from the restriction imposed on the stochastic technology. The fact that the estimated trajectories of performance under regime 2 and
Table 3.3: Estimates for the separation equation in the attrition model when ability is known, when workers learn about it, and when the hypothesis of learning is tested.

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>Model 1 ( \lambda = 0 )</th>
<th>Model 2 ( \lambda ) estimated</th>
<th>Model 3 ( \lambda = 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision to stay</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t ), orthog. pol. 1</td>
<td>-0.44 ( \pm 0.18 )</td>
<td>-0.50 ( \pm 0.16 )</td>
<td>-0.50 ( \pm 0.17 )</td>
</tr>
<tr>
<td>( t ), orthog. pol. 2</td>
<td>-0.19 ( \pm 0.08 )</td>
<td>-0.21 ( \pm 0.08 )</td>
<td>-0.21 ( \pm 0.08 )</td>
</tr>
<tr>
<td>( t ), orthog. pol. 3</td>
<td>0.23 ( \pm 0.14 )</td>
<td>0.24 ( \pm 0.13 )</td>
<td>0.23 ( \pm 0.13 )</td>
</tr>
<tr>
<td>( t ) (regime 2), orthog. pol. 1</td>
<td>0.52 ( \pm 0.16 )</td>
<td>0.41 ( \pm 0.14 )</td>
<td>0.41 ( \pm 0.14 )</td>
</tr>
<tr>
<td>( t ) (regime 2), orthog. pol. 2</td>
<td>0.45 ( \pm 0.11 )</td>
<td>0.40 ( \pm 0.11 )</td>
<td>0.41 ( \pm 0.11 )</td>
</tr>
<tr>
<td>( t ) (regime 2), orthog. pol. 3</td>
<td>-0.68 ( \pm 1.68 )</td>
<td>-0.78 ( \pm 1.57 )</td>
<td>-0.78 ( \pm 1.57 )</td>
</tr>
<tr>
<td>( t ) (regime 3), orthog. pol. 1</td>
<td>0.41 ( \pm 0.16 )</td>
<td>0.40 ( \pm 0.16 )</td>
<td>0.40 ( \pm 0.16 )</td>
</tr>
<tr>
<td>( t ) (regime 3), orthog. pol. 2</td>
<td>0.23 ( \pm 0.08 )</td>
<td>0.22 ( \pm 0.08 )</td>
<td>0.23 ( \pm 0.08 )</td>
</tr>
<tr>
<td>( t ) (regime 3), orthog. pol. 3</td>
<td>-0.03 ( \pm 0.13 )</td>
<td>-0.03 ( \pm 0.12 )</td>
<td>-0.03 ( \pm 0.13 )</td>
</tr>
<tr>
<td>avg. % outbound calls in past</td>
<td>-0.12 ( \pm 0.09 )</td>
<td>-0.29 ( \pm 0.09 )</td>
<td>-0.31 ( \pm 0.09 )</td>
</tr>
<tr>
<td>Hired under regime 2</td>
<td>-0.02 ( \pm 0.09 )</td>
<td>-0.07 ( \pm 0.09 )</td>
<td>-0.07 ( \pm 0.09 )</td>
</tr>
<tr>
<td>Hired under regime 3</td>
<td>-0.25 ( \pm 0.14 )</td>
<td>-0.25 ( \pm 0.13 )</td>
<td>-0.25 ( \pm 0.13 )</td>
</tr>
<tr>
<td>Constant</td>
<td>0.46 ( \pm 0.10 )</td>
<td>0.56 ( \pm 0.10 )</td>
<td>0.58 ( \pm 0.10 )</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-3756.22</td>
<td>-3745.07</td>
<td>-3745.71</td>
</tr>
</tbody>
</table>

Notes: The specification includes calendar time, orthogonal polynomials of degree 3 and individual controls: gender, age, marriage status, distance from home, and race. Obs. = 3,675
Table 3.4: Estimates for ability and other structural parameters when ability is known, when workers learn about it, and when the hypothesis of learning is tested.

<table>
<thead>
<tr>
<th>Parameter or explanatory variable</th>
<th>Model 1 $\lambda = 0$</th>
<th>Model 2 $\lambda$ estimated</th>
<th>Model 3 $\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.  S.E.</td>
<td>Coef.  S.E.</td>
<td>Coef.  S.E.</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.17  0.01</td>
<td>0.17  0.01</td>
<td>0.17  0.01</td>
</tr>
<tr>
<td>$\sigma^2_\xi$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\rho(\varepsilon, \xi)$</td>
<td>0.16  0.04</td>
<td>0.05  0.04</td>
<td>0.01  0.04</td>
</tr>
<tr>
<td>$\sigma^2_\theta$</td>
<td>0.49  0.02</td>
<td>0.48  0.02</td>
<td>0.48  0.02</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.14  0.06</td>
<td>0.06  0.06</td>
<td>0.06  0.06</td>
</tr>
<tr>
<td>$t.\theta$, orthog. pol. 1</td>
<td>0.00  0.00</td>
<td>0.05  0.05</td>
<td>0.05  0.05</td>
</tr>
<tr>
<td>$t.\theta$, orthog. pol. 2</td>
<td>-0.01  0.05</td>
<td>0.00  0.04</td>
<td>0.00  0.04</td>
</tr>
<tr>
<td>$t.\theta$, orthog. pol. 3</td>
<td>0.04  0.04</td>
<td>0.00  0.03</td>
<td>0.00  0.03</td>
</tr>
<tr>
<td>$\mu_{it}$</td>
<td>0.17  0.06</td>
<td>0.18  0.05</td>
<td>0.18  0.05</td>
</tr>
<tr>
<td>$t.\mu_{it}$, orthog. pol. 1</td>
<td>0.06  0.03</td>
<td>0.06  0.03</td>
<td>0.06  0.03</td>
</tr>
<tr>
<td>$t.\mu_{it}$, orthog. pol. 2</td>
<td>0.02  0.04</td>
<td>0.02  0.04</td>
<td>0.02  0.04</td>
</tr>
<tr>
<td>$t.\mu_{it}$, orthog. pol. 3</td>
<td>0.03  0.03</td>
<td>0.03  0.03</td>
<td>0.03  0.03</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.74  0.29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Log-likelihood: -3756.22 -3745.07 -3745.71

Notes: The specification includes calendar time, orthogonal polynomials of degree 3 and individual controls: gender, age, marriage status, distance from home, and race. Obs.=3,675
3 are very similar is comforting because it suggests that workers did not practically face a trade-off between quantity of calls and quality of calls. Most importantly, the results are consistent with the theoretical predictions of section 2: the effect of changing pay from regime 1 to regime 2 on performance is negative and significant. However, the effect of regime 2 does not differ significantly from the effect of regime 3. Switching from regime 1 to regime 2 leads to a decline in effort that translates into 0.19 fewer calls per hour, which is approximately 9% of the average initial performance under regime 1. In economic terms, the change in incentives leads to a decline in worker’s hourly pay by approximately $2, which is 20% of the average hourly pay in the first month of employment under regime 1.

All models estimate a significant improvement in performance over time due to accumulation of experience: in the first 6 months of employment performance increases by approximately 0.67 calls per hour, or 23% growth in the first six months under
Figure 3.6: Comparison between the expected performance for stayers and the tenure - performance for an employee of average ability.

regime 1. Under regime 1, this growth translates in an increase in hourly pay by approximately $2.2. Finally, the dummies for regimes of hiring are not significant, which indicates that there is no self-selection into the firm on the basis of the pay regime at the time of hiring.\textsuperscript{17} In combination with the estimated effect of ability on the probability of staying, this result indicates that the data are consistent with the more restrictive model presented in section 2. Among the rest of the covariates, the percentage of outbound calls has a negative effect on performance and the average of past percentages of outbound calls has a negative and significant effect on performance. Women have on average lower performance than men, and marriage has a positive effect on performance but a negative effect on the probability of staying.

\textsuperscript{17}The test for selection at entry is the same as the one used in Lazear (2000). At least in principle, the effect of selection at entry may affect not only the mean of the distribution of $\theta$, but also other moments.
Figure 3.7: Probability of staying for $\theta_i = 0$, and $\theta_i = \pm \sigma_\theta$ under regimes 1 and 2.

Figure 3.6 presents the combined effect of change in incentives on effort and on ability among stayers. The results indicate that the two effects have different signs, so that their sum amounts to a decrease in expected performance at $t = 6$ of only 0.1 call per hour. At the same time, switching from regime 1 to regime 2 reduces compensation costs of the firm considerably. With the possibility of hiring a replacement when an employee quits, one may expect that the firm would be able to build a high quality workforce under regime 2 whose expected performance will be similar or even better than the one for $t = 6$ discussed above. This seemingly counterintuitive result is consistent with the firm-specific nature of ability: since workers cannot export their high ability to other jobs, the employer is in a position to gain much of the surplus generated by the employment relation.

What is missing from this discussion on the profitability of regimes 1 and 2 is the effect of the different regime on turnover. Figure 3.7 investigates how regimes 1 and
2 affect the probability of staying for employees of different abilities from the first to the sixth month of employment. Under both regimes 1 and 2, workers of low ability leave the firm within the first two months of employment. The greatest difference is in their effect on the workers of average and high ability. Regime 2 practically forces workers of mean ability to leave within the first six months of employment. It also reduces the number of high ability workers who stay, partly due to the effect of bad signals in the early stages of the employment relation. Figure 3.8 plots the probability of quitting at tenure $t = 6$ as a function of the posterior mean. By the sixth month of employment, workers have a much more precise beliefs about their ability than at the time of entry and the accumulation of experience has already plateaued. Thus, one may regard Figure 3.8 as an approximation to the state at which quitting behavior depends on incentives and ability only. The figure shows that the impact of regimes 1 and 2 is similar for workers in the tails of the distribution of ability and differs greatly for those
in between. Since workers of low ability would have already left, as indicated on Figure 3.7, the main impact of regime 2 relative to regime 1 is that it "weeds out" the workers of average and slightly better than average ability. Moreover, the higher turnover rate under regime 2 is also associated with greater destruction of accumulated experience than under regime 1. These effects are likely to limit the benefit from improving the ability among the long-term employees. Also, if they are high, turnover costs are likely to offset the benefits from improving the quality mix.

In short, the results indicate that not only effort choice but also turnover is a major channel through which incentives affect expected performance and in turn profits. In addition, they highlight the complexities associated with evaluating how profitable a regime is. The problem of evaluating the profitability of the implemented regimes, as well as the optimal compensation policy in the family of linear contracts in performance is left for Chapter 4.

3.5.2 The Fixed Effects Estimator

This subsection presents a set of regression results that are obtained by applying the fixed effects estimator to estimate the performance equation for the first six months of employment on the subsample of workers who stay at least six months in the firm. The specification of the performance equation is identical to the one in the attrition model from above. The estimates are very different from the ones discussed above which I attribute to the presence of learning about ability. Another limitation of the estimator is that it cannot be used to quantify the effect of incentives on the quality mix.

Table 3.5 summarizes the results; it omits the estimates of parameters that are not of direct interest to the discussion. Model 1 is estimated using fixed effects. Nijman and Verbeek (1992) propose a simple test for nonrandom attrition which in the current
Table 3.5: Estimates based on the fixed effects estimator.

<table>
<thead>
<tr>
<th>Dependent Variable: Performance</th>
<th>FE estimator</th>
<th>FE + attrition test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>S.E.</td>
</tr>
<tr>
<td>$t$, orthog. pol. 1</td>
<td>-14.78</td>
<td>0.32</td>
</tr>
<tr>
<td>$t$, orthog. pol. 2</td>
<td>-0.61</td>
<td>0.18</td>
</tr>
<tr>
<td>regime 2</td>
<td>-0.56</td>
<td>0.21</td>
</tr>
<tr>
<td>regime 3</td>
<td>0.09</td>
<td>0.31</td>
</tr>
<tr>
<td>$t$. (regime 3), orthog. pol. 1</td>
<td>2.81</td>
<td>0.33</td>
</tr>
<tr>
<td>$t$. (regime 3), orthog. pol. 2</td>
<td>0.85</td>
<td>0.05</td>
</tr>
<tr>
<td>% outbound calls</td>
<td>0.15</td>
<td>0.07</td>
</tr>
<tr>
<td>Quit in $t = 7$</td>
<td>1.71</td>
<td>0.35</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>1131</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.45</td>
<td></td>
</tr>
</tbody>
</table>

Note: Model 1 is estimated on the set of workers who stay for at least six months. Model 2 is the same as model 1 but also includes a dummy for the decision to stay or quit at $t=7$.

setting involves the inclusion of a dummy for the separation decision of the agent at the end of the sixth month. Under the null hypothesis of no attrition bias, the coefficient of this dummy variable is 0. If the null hypothesis is rejected, then the estimates are not valid and attrition needs to be modelled explicitly. The results from performing this test are reported under Model 2. The estimates under Model 1 are in line with what Lazear (2000) and Lazear and Shaw (2009) find in a similar environment. Namely, regimes 2 and 3 have a highly significant negative effect on performance relative to regime 1. However, there does not appear to be a significant difference between the levels of effort under regime 2 and 3. Finally, the coefficients of the tenure terms imply significant accumulation of experience during the first 6 months of the employment relation. The percentage of outbound calls has a positive effect on performance: this result is counterintuitive, since an employee is more likely to collect payment during an inbound than during an outbound call. Most importantly, the estimated effects of
Figure 3.9: Performance-tenure profile at entry for $\theta_i = 0$ under regime 1 and 2: comparison between the estimates of FE approach and learning specification of the attrition model.

incentives on effort and the effect of accumulated experience are considerable larger than their counterparts under the attrition model.

The dummy for quitting at the end of the sixth month has a highly significant negative effect on performance, implying that the null hypothesis of no attrition bias is soundly rejected. Intuitively, the negative coefficient of the dummy variable implies that individuals who do not stay for an extra period have lower performance than those who stay. The rejection of the hypothesis of random attrition implies that the estimated effects of incentives and tenure on performance are biased for the reasons discussed in section 2. The discrepancy between the estimates under the fixed effects estimator and the ones reported in the preceding subsection suggests that neglecting the effect of Bayesian learning on turnover may lead to considerable bias in the estimated effects of incentives. This issue is investigated formally in the following subsection.
Table 3.6: Simulation results showing the presence of attrition bias.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>regime 2</td>
<td>-0.19</td>
<td>-0.57</td>
<td>0.18</td>
</tr>
<tr>
<td>regime 3</td>
<td>0.13</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>$t$, orthog. pol. 1</td>
<td>-0.57</td>
<td>-14.89</td>
<td>0.29</td>
</tr>
<tr>
<td>$t$, orthog. pol. 2</td>
<td>-0.62</td>
<td>-0.55</td>
<td>0.16</td>
</tr>
<tr>
<td>$(t$, orthog. pol. 1)* (regime 3)</td>
<td>0.78</td>
<td>2.78</td>
<td>0.31</td>
</tr>
<tr>
<td>$(t$, orthog. pol. 2)* (regime 3)</td>
<td>0.43</td>
<td>0.85</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Note: The true parameters are equal to the estimated parameters under the attrition model with learning, Model 3. The table reports the average of the estimated coefficients by the fixed effects estimator, along with the standard deviation of the estimates.

### 3.5.3 Simulations

Section 2 establishes that if the effect of Bayesian learning on nonrandom attrition is ignored, the estimated effect of incentives on performance and the tenure-performance profile are biased. The crucial question is how large this bias is. One way to approach the question is to estimate the bias using data simulated from the estimated model. In what follows, I use this approach to evaluate the magnitude of the "learning" bias in the context of the used data.

I simulate 1,000 data sets from the explanatory variables in the original data and the estimated parameters of the model, Model 3 in Tables 3.2 - 3.4. In each simulated data set, individuals enter at the calendar time of their actual entry in the firm, but their quitting decision is endogenously determined by the simulated performance signals and the piece rate of operation. The characteristics, and duration of piece rates in the simulated data sets is exactly the same as in the original data set. For each individual I draw 1,000 error paths and a $\theta$ from the corresponding distributions, and with the help of the estimated model parameters I generate the performance and separation
If the econometrician estimates (3.1) subject to staying for at least 6 periods using the fixed effects estimator, she overestimates considerably the effect of incentives and of tenure on performance. Table 3.6 reports the mean and the variance of the estimated parameters. The results indicate that on average the fixed effects estimator overestimates the effect of switching from regime 1 to regime 2 on effort by a factor of two. The fixed effects estimator also overestimates the effect of tenure on performance by a similar magnitude. This last point is illustrated on Figure 3.10 which plots the true and the estimated tenure-performance profiles.

The channels through which Bayesian learning leads to the bias are complex. Under regime 1, the true match quality of those who survive for the first six months varies widely. Many of the survivors under regime 1 have just been lucky, since in the following months they quit. In contrast, under the less generous regime 2 and 3, only workers
with $\theta$ in the top quartile of the distribution of match quality survive. In what follows, I will discuss how the average performance error varies across regimes and at different tenure horizons, conditional on staying for at least 6 months at the firm. While the average of the noise in the performance equations, conditional on staying, is positive and increases in the first three months under both regimes, its magnitude is considerably larger under regime 1 due to the survival of individuals with low $\theta$ under that regime. Furthermore, there is a great heterogeneity in the observed performance-tenure profiles under regime 1 with low $\theta$ being associated with steep profiles. Given the additive separability of equation (3.1), the estimated effect of tenure on performance becomes the demeaned weighted average of the performance-tenure profiles across different $\theta$ and across different regimes. Since many more workers stay under regime 1 than under regime 2 and 3, the estimated effect of tenure is heavily influenced by the average of the conditional performance errors under regime 1. Thus, the differences in the tenure-performance profiles under regimes 1, 2, and 3 find their way into the estimated effect of differences in pay incentives.

### 3.5.4 Robustness Checks

The model was estimated under a number of strong assumptions. In this subsection, I perform some nonparametric test for consistency of the performance data with the imposed restrictions on the stochastic technology. Then, I move to discuss some postestimation tests of the normality and independence assumptions that underlie the MLE results. Finally, I consider several alternative specifications for the attrition model.

**Technology Tests**

The model of section 2 is based on strong distributional and technology assumptions which can be tested nonparametrically. The following observation presents one such
nonparametric test.\textsuperscript{18}

If workers start with a common prior and learn the quality of their match with the employer over time, the distribution of match quality does not vary across different pay regimes: each employee knows that the turnover after the first period will be higher under a less generous regime than under a more generous one, but at the time of hiring everyone faces the same odds of staying more than one period. Since match quality does not interact with effort, only the mean of performance in the first period varies across regimes. That is, the distributions of performance across regimes are the same up to a location parameter. Observation 1 states this argument formally.

**Observation 1.** Consider the model defined by (3.1) and (3.2) and suppose that the workers share a common prior at the time of hiring. Then the demeaned distribution of performance at $t = 1$ is the same across piece rates:

$$F(y_0^1|R) = F(y_0^1|R'),$$

for any $R$ and $R'$, where $y_0^1 = y_1 - E(y_1|R)$.

**Proof:** Since effort enters additively in the stochastic technology and optimal effort does not vary with $\theta_i$ or $\theta_{it}$ across $i$, the pay regime affects only the first moment of the conditional distribution of performance. Furthermore, under the assumption of a common prior belief at the time of hiring, the entry decision is not affected by the pay regime in place, so $F(\theta_i|R) = F(\theta)$. Therefore, $F(y_0^1|R) = F(y_0^1|R').$\textsuperscript{\textbullet}

The proof relies crucially on the assumption of common priors: if some workers had a more accurate belief about the quality of the match than others, the probability of staying more than one period will differ with beliefs leading to differences in the distribution of newly hired employees. For example, heterogeneity in priors arises

\textsuperscript{18}In what follows, the subscript $i$ is omitted where no confusion arises.
when $\theta_i$ stands for industry-specific rather than firm-specific match quality parameter. Moreover, known ability at the time of hiring is a special case of heterogeneity in priors. Thus, for single-peaked distributions with non-zero probability for every possible match quality this property is also enough to distinguish between Bayesian learning with a common prior and known ability.

Observation 1 imposes necessary restrictions on the observed performance series that are strong. Tables 2.4 presents the results from testing for the technology restrictions implied by Observation 1. A casual look at the standard deviations of performance in period 1 under the different regimes verifies the plausibility of the hypothesis of equal variance: the standard deviations vary between 0.45 and 0.47. This observation is confirmed by the results of the Mann-Whitney tests for equality of the demeaned distributions of performance under regimes 1, 2, and 3 in the first month: the tests fail to reject the hypothesis of equality of the demeaned distributions across regimes.

**Postestimation Tests and Alternative Specifications**

The estimation also relies on a number of additional assumptions; some of the more important ones are the assumptions of normality and independence of the error terms in the performance and attrition equations across individuals and tenure horizons. These considerations provide the basis for a simple Kolmogorov-Smirnov normality test for the sum of match quality and noise in the first month of employment, $y_1^0$, where the subindex indicates time, fails to reject the hypothesis of normality at the 5% significance level.

There are also a number of alternative specifications for the attrition process. I have explored these in the standard way and arrived at the attrition specification for the observables reported here; it includes all controls, the interaction terms between beliefs and tenure, as well as tenure and pay regime, but exclude second-order interactions, as
Table 3.7: Estimates for the performance equation under alternative specifications of the attrition model.

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>S.E.</td>
<td>Coef.</td>
</tr>
<tr>
<td>( t, \text{ orthog. pol. 1} )</td>
<td>-0.55</td>
<td>0.15</td>
<td>-0.55</td>
</tr>
<tr>
<td>( t, \text{ orthog. pol. 2} )</td>
<td>-0.61</td>
<td>0.09</td>
<td>-0.63</td>
</tr>
<tr>
<td>regime 2</td>
<td>-0.21</td>
<td>0.07</td>
<td>-0.19</td>
</tr>
<tr>
<td>regime 3</td>
<td>0.14</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>( t, (\text{regime 3}), \text{ orthog. pol. 1} )</td>
<td>0.76</td>
<td>0.30</td>
<td>0.69</td>
</tr>
<tr>
<td>( t, (\text{regime 3}), \text{ orthog. pol. 2} )</td>
<td>0.39</td>
<td>0.20</td>
<td>0.40</td>
</tr>
<tr>
<td>% outbound calls</td>
<td>-0.08</td>
<td>0.05</td>
<td>-0.06</td>
</tr>
<tr>
<td>Constant</td>
<td>3.73</td>
<td>0.20</td>
<td>3.85</td>
</tr>
</tbody>
</table>

Log-likelihood: \(-3745.34\), \(-3744.29\), \(-3744.52\)

Notes: The specification includes calendar time, orthogonal polynomials of degree 3 and individual controls: gender, age, marriage status, distance from home, and race. Obs.=3,675.
Table 3.8: Estimates for the separation equation under alternative specifications of the attrition model.

<table>
<thead>
<tr>
<th>Dependent Variable:</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision to stay</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Heterogeneity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in offers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coef.</td>
<td>S.E.</td>
<td>Coef.</td>
<td>S.E.</td>
</tr>
<tr>
<td>$t; $</td>
<td>orthog. pol. 1</td>
<td>-0.48</td>
<td>0.17</td>
</tr>
<tr>
<td>$t; $</td>
<td>orthog. pol. 2</td>
<td>-0.19</td>
<td>0.08</td>
</tr>
<tr>
<td>$t; $</td>
<td>orthog. pol. 3</td>
<td>0.24</td>
<td>0.13</td>
</tr>
<tr>
<td>$t; $ (regime 2); orthog. pol. 1</td>
<td>0.41</td>
<td>0.14</td>
<td>0.42</td>
</tr>
<tr>
<td>$t; $ (regime 2); orthog. pol. 2</td>
<td>0.41</td>
<td>0.11</td>
<td>0.39</td>
</tr>
<tr>
<td>$t; $ (regime 2); orthog. pol. 3</td>
<td>-0.79</td>
<td>1.57</td>
<td>-0.99</td>
</tr>
<tr>
<td>$t; $ (regime 3); orthog. pol. 1</td>
<td>0.40</td>
<td>0.16</td>
<td>0.43</td>
</tr>
<tr>
<td>$t; $ (regime 3); orthog. pol. 2</td>
<td>0.21</td>
<td>0.08</td>
<td>0.25</td>
</tr>
<tr>
<td>$t; $ (regime 3); orthog. pol. 3</td>
<td>-0.03</td>
<td>0.13</td>
<td>-0.03</td>
</tr>
<tr>
<td>avg. % outbound calls in past</td>
<td>-0.31</td>
<td>0.09</td>
<td>-0.27</td>
</tr>
<tr>
<td>Hired under regime 2</td>
<td>-0.07</td>
<td>0.09</td>
<td>-0.07</td>
</tr>
<tr>
<td>Hired under regime 3</td>
<td>-0.25</td>
<td>0.13</td>
<td>-0.25</td>
</tr>
<tr>
<td>Constant</td>
<td>1.07</td>
<td>0.33</td>
<td>1.08</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-3745.34</td>
<td>-3744.29</td>
<td>-3744.52</td>
</tr>
</tbody>
</table>

Notes: The specification includes calendar time, orthogonal polynomials of degree 3 and individual controls: gender, age, marriage status, distance from home, and race. Obs.=3,675
Table 3.9: Estimates for ability and learning under alternative specifications of the attrition model.

<table>
<thead>
<tr>
<th>Parameter or explanatory variable</th>
<th>Model 4: Heterogeneity in offers</th>
<th>Model 5: Gen. form of learning</th>
<th>Model 6: No common prior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
<td>S.E.</td>
<td>Coef.</td>
</tr>
<tr>
<td>$\sigma^2_\varepsilon$</td>
<td>0.17</td>
<td>0.01</td>
<td>0.16</td>
</tr>
<tr>
<td>$\rho (\varepsilon, \xi)$</td>
<td>0.15</td>
<td>0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td>$\sigma^2_\theta$</td>
<td>0.48</td>
<td>0.02</td>
<td>0.47</td>
</tr>
<tr>
<td>$\mu_{it}$</td>
<td>0.17</td>
<td>0.04</td>
<td>0.22</td>
</tr>
<tr>
<td>$t_{it}$, orthog. pol. 1</td>
<td>0.06</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$t_{it}$, orthog. pol. 2</td>
<td>0.02</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>$t_{it}$, orthog. pol. 3</td>
<td>0.03</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma^2_{\mu}$</td>
<td>0.00</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$\text{signal}_{t-1}$</td>
<td></td>
<td></td>
<td>0.01</td>
</tr>
<tr>
<td>$\text{signal}_{t-2}$</td>
<td></td>
<td></td>
<td>-0.06</td>
</tr>
<tr>
<td>$\text{signal}_{t-3}$</td>
<td></td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>$\text{signal}_{t-4}$</td>
<td></td>
<td></td>
<td>-0.05</td>
</tr>
<tr>
<td>$\text{signal}_{t-5}$</td>
<td></td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>$\text{signal}_{t-5}$</td>
<td></td>
<td></td>
<td>0.00</td>
</tr>
<tr>
<td>$\sigma^2_{\hat{b}_1}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_{i_{11}}$, orthog. pol. 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_{i_{11}}$, orthog. pol. 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-3745.34</td>
<td></td>
<td>-3744.29</td>
</tr>
</tbody>
</table>

Notes: The specification includes calendar time, orthogonal polynomials of degree 3 and individual controls: gender, age, marriage status, distance from home, and race. Obs.=3,675.
well as quadratic terms. With respect to the tenure-varying independent variables, I find that the average percentage outbound calls in the past has the greatest effect on attrition among all functions of the lags of the percentage outbound calls. Furthermore, I consider a number of alternatives to the proposed model of Bayesian learning. These include a model under which Bayesian learning ends within 6 or 12 months of the start of the employment relation; a more general form of dependence of the attrition equation on past signals about $\theta$, possibly adaptive learning; individual-specific heterogeneity in outside offers; and heterogeneity in prior beliefs at the beginning of the employment relation. The results from estimating each of these alternative models are reported in Tables 3.7 - 3.9.

Model 4 in Tables 3.7-3.9 incorporates an additional heterogeneity term that enters additively in the performance equation. It may represent a time-invariant individual-specific heterogeneity in the outside offers. Due to the presence of Bayesian learning, it is assumed that this additional heterogeneity is independent of $\theta$. The boundary $\chi^2$ test indicates that the variance of this term is not significantly different from zero at the 1% significance level, implying that the hypothesis of heterogeneity in outside offers is rejected. Similarly, Model 5 rejects the hypothesis that attrition depends on past signals about $\theta$ through a more general functional form than the simple average of past signals. In particular, the model implies that agents do not assign disproportionately large weight on recent signals when they decide to stay or quit. Due to computational considerations, Model 5 incorporates only the last six signals starting from $t - 2$ and their coefficients are estimated freely. The sign of the estimated coefficients varies but is

---

19 For example, workers may engage in adaptive learning or base beliefs on only the most recent signals.

20 Note that the possibility for heterogeneity in learning rates is ruled out by the nonparametric test of Observation 2 that establishes that for the sub-sample of top performers performance does not vary with tenure.

21 Such an assumption may be justified when the outside option can be decomposed into two terms: one that depends linearly on theta and one that is orthogonal to theta.
never significantly different from zero. Furthermore, the likelihood ratio test rejects the hypothesis that Model 5 is significantly different from the basic model. Finally, Model 6 allows for heterogeneity in the means of prior beliefs. This model is a special case of a more general test for heterogeneity of priors that allows for both individual-specific means and variance. The specification of Model 3 indicates that, while agents may not share the same common prior, the precision of their initial beliefs remains the same due to the firm-specific nature of the productivity parameter. In addition, I impose the restriction that this individual-specific effect is independent of $\theta$. The estimated variance of this effect is not significantly different from zero, which suggests that the assumption of a common prior is reasonable in the context of the study.

3.6 Conclusion and Related Research

This chapter demonstrates that neglecting the interaction between learning about ability and separation decisions leads to biased estimates of the effect of incentives and returns to monthly tenure. With the help of testable restrictions on the stochastic technology, I estimate the effect of incentives within a model for the employment dynamics at a call center in North Carolina that controls for the interaction between learning and attrition. The results show that incentives induce workers of low and average ability to quit which, depending on the pay regime, leads in the first 6 months of employment to 24% and 31% increase in average performance for regimes 1 and 2 respectively. Furthermore, they show that growth in performance is primarily due to quitting decisions and the accumulation of experience; the estimated effect of incentives is significant but small. Simulating the estimated model, I find that the fixed effects estimator, popular in the existing literature, overestimates the effect of incentives on effort by a factor of two and the effect of tenure on performance by a similar magnitude.
To the extent that learning about ability is likely in many environments\textsuperscript{22}, the issues discussed in this paper relate to a large body of empirical work in labor economics, applied microeconomics, and industrial organization.

The model estimated in this chapter is semi-structural in the sense that it incorporates restrictions on the stochastic technology but does not specify a utility function. The estimation of a fully structural model that specifies the worker’s utility explicitly is the topic of the following Chapter 4. The main benefit from performing such an exercise is that the estimation of a fully structural model provides the basis for counterfactual policy analysis. I study the problem of optimal pay within a model very similar to the one considered in this paper; it incorporates effort choice, labor turnover, and learning about worker’s ability. The novelty, relative to Shearer and Paarsch (2009), is that incentives affect not only effort choice, but also the composition of the workforce. The structural model is estimated using a two-step procedure. In the first step, I estimate the attrition model as done in the present chapter and recover the stochastic technology up to a constant, as well as a scaled version of the expected utility of continued employment. I use these estimates in the second step to estimate the remaining structural parameters using the method of minimum distance estimation. The estimates are used to find and characterize the optimal linear contract in the performance signal. The main result is that switching from hourly wage to the optimal linear contract has much greater impact on profits through the effect of incentives on turnover than through the effect of incentives on effort choice. Thus, the companion paper shows that turnover is a major channel through which pay incentives affect profits.

\textsuperscript{22}A number of studies, including Pakes and Ericson (1999), Chiappori, Salanié, and Valentin (1999), and Gibbons, Katz, Lemieux, and Parent (2005), have found evidence that observed data patterns are consistent with Bayesian learning.
3.7 Appendix A

Assumption 1. (i) Suppose that ability $\theta^{23}$, $\theta \in R$, is time-invariant, continuous and has a probability density function $f_\theta$. (ii) The outside offer $\xi_i^i, \xi_i^* \in R$, is continuous, independent and identically distributed across tenure horizons $t$, where $t = 1, 2, ..., \text{independent from } \theta$, and has a probability density function $f_{\xi^*}$. (iii) The noise in the performance signal $\varepsilon_t, \varepsilon_t \in R$, is continuous, independent and identically distributed across tenure horizons $t$, independent from $\theta$, and has a probability density function $f_{\varepsilon}$.

Assumption 2. The performance signal $y_t$ is generated by the following technology:

$$y_t = \theta + g(t) + l_t + \varepsilon_t$$

where $g(t), g(t) \in R_+$, represents the accumulation of firm-specific knowledge or experience and $l_t$ is effort, $l_t \in L \subset R_+$, where $L$ is compact. $g(t)$ is increasing and continuous.

Assumption 3. The belief at the beginning of $t$ is denoted as $\theta_t$ and is formed in a Bayesian way. Let the initial prior be $\theta_1$ and suppose that it has the same distribution as $\theta$.

Assumption 4. The worker is paid $w_t = \alpha_t + \beta_t y_t$, according to a linear compensation regime $R_t = (\alpha_t, \beta_t)'$, where $\alpha_t > 0$ and $\beta_t > 0$. Regime $R_t$ is said to be more generous than regime $R_t', R_t > R_t'$, if both $\alpha_t > \alpha_t'$ and $\beta_t > \beta_t'$. The worker does not expect the compensation regime to change in the future.

Assumption 5. Workers are assumed to be risk-neutral with a utility function

$$u(R_t, l_t, t, \theta, \varepsilon_t) = \alpha_t + \beta_t (\theta + g(t) + l_t + \varepsilon_t) - \psi(l_t),$$

$^{23}$In this section, I drop the individual subscript "i"
where \( \psi(l_t) \) is strictly convex, increasing in effort, and \( \psi(0) = 0 \).

**Assumption 6.** \( \varepsilon_t \sim N(0, \sigma_{\varepsilon}^2), \xi_t^\ast \sim N(\mu_{\xi^\ast}, \sigma_{\xi^\ast}^2) \) and \( \theta_t \sim N(\mu_{\theta}, \sigma_{\theta}^2) \).

I maintain assumption 6 because normality of \( \varepsilon_t, \xi_t^\ast, \) and \( \theta_t \) is imposed on the models taken to the data in Chapters 2 and 3. However, the statement of the optimal problem of the worker, the proof of existence of a solution to the problem and its characterization can be stated more generally. Without normality, in addition to assumptions 1 to 5, one needs to impose that

**Assumption 6’** (i) \( \varepsilon_t \) and \( \theta \) are log-concave, and (ii) the sequence of noisy signals about ability \( \{y_{ik} - g(k) + l_{ik}\}_{k=1}^t \) is ordered in the sense of the likelihood ratio property.

Since \( \theta \) and \( l_t \) enter additively in the utility function, posterior beliefs do not depend on effort and optimal effort choice does not depend on beliefs and is function only of \( R_t, l(R_t) \). The assumptions on the disutility of effort imply the existence of a unique interior solution to the problem of choosing optimal effort. Furthermore, they imply that as the bonus rate \( \beta_t \) increases, optimal effort increases, too. By assumption 6 the posterior belief \( \theta_{it} \) is normally distributed for all \( t > 1 : \theta_t \sim N(\mu_t, \sigma_t^2) \), where

\[
\mu_t = (1 - K_t)\mu_{t-1} + K_t(y_{t-1} - l(R_{t-1}) - g(t - 1) - m(X_{t-1}))
\]

\[
\sigma_t^2 = \frac{\sigma_{\varepsilon}^2 \sigma_{\theta}^2}{\sigma_{\theta}^2 (t - 1) + \sigma_{\varepsilon}^2}
\]

\[
K_t = \frac{\sigma_{\theta}^2}{\sigma_{\theta}^2 (t - 1) + \sigma_{\varepsilon}^2}
\]

Note that precision of beliefs depends only on \( t \), so the average of the demeaned past signals is a sufficient statistic to characterize posterior beliefs. The posterior mean can
be rewritten as

\[ \mu = k(t) \left( \frac{1}{t - 1} \sum_{k=1}^{t-1} (y_k - l(R_k) - g(k) - m(X_k)) \right) + (1 - k(t)) \mu_0 \]

where \( k(t) = (t - 1)K_t \). By these observations, the independence of \( \theta \) from \( \varepsilon_t \) and \( \xi_t^* \), and by the additivity of \( \varepsilon_t \) in the stochastic technology, the optimal problem of the worker can be formulated as \((P)\)

\[
v(\mu_t, R_t, t) = \int \max(\xi_t, \alpha_t + \beta_t(\mu_t + g(t) + l(R_t)) - \psi(l(R_t))) \]
\[
+ \delta \int v(\mu_{t+1}, R_t, t+1) f(\mu_{t+1} | \mu_t, t) d\mu_{t+1} \]
\[
\times f_{\xi}^*(\xi_t^*) d\xi_t^*,
\]

where \( f(\mu_{t+1} | \mu_t, t) \) is the conditional density of \( \mu_{t+1} \), given \( \mu_t \) and \( t \).

Proposition 1. Under Assumptions 1 - 6:

i. The functional equation \((P)\) has a unique continuous solution \( V(\mu_t, R_t, t) \) and the optimal policy

\[ A(\mu_t, R_t, t) = \{l_t \in L \mid (P) \text{ holds.}\} \]

is a continuous function.

ii. Optimal effort \( l(R_t) > l(R_t') \) if \( R_t > R_t' \).

iii. \( V(\mu_t, R_t, t) > V(\mu_t, R_t, t) \) if \( R_t > R_t' \), and \( V(\mu_t, R_t, t) \) increases \( \mu_t \).

Proof of Proposition 1:

Part (i). The sequence of posterior beliefs is continuous which also implies that \( f(\mu_{t+1} | \mu_t, t) \) is continuous\(^{24}\), so the proof of existence is reduced to a problem which

\[^{24}\text{See Lemma 1 and 2 in Easley and Kiefer (1988) to establish proof of continuity of the transitional kernel under Assumption 6' and the more general formulation of the worker's problem in Chapter 2, Section 2.}\]
can be solved using Blackwell (1965). Define the operator $T$ by

$$(Tw)(\mu_t, R_t, t) = \int \max[\xi^*_t, (\alpha_t + \beta_t (\mu_{it} + g(t)) + l(R_t)) - \psi(l(R_t))$$

$$+ \delta \int w(\mu_{t+1}, R_t, t+1) f(\mu_{t+1}|\mu_t, t) d\mu_{t+1}] f_{\xi^*}(\xi^*_t) d\xi^*_t]$$

Let $C$ denote the set of bounded functions on $P(\Theta)$. Under the supnorm metric, $\|\cdot\|$, $C$ is a Banach space. By the contraction mapping theorem, a contraction operator $T : C \to C$ has a unique fixed point and by Blackwell's contraction mapping lemma, $T$ is a contraction if

(1). (Monotonicity) $w_1 \geq w_2$ implies $Tw_1 \geq Tw_2$ and

(2). (Discounting) there exists $\delta \in (0, 1)$, such that $T(w + c) \leq Tw + \delta c$, for any constant $c \geq 0$.

Consequently, to prove existence it is sufficient to show that (i) the operator $T$ is a contraction and that (ii) $T$ maps continuous bounded functions into the space of continuous bounded functions, $C$.

(i). This result follows by establishing that conditions (1) and (2) of the Blackwell’s contraction mapping lemma are satisfied. It is obvious that if $w_1 \geq w_2$ uniformly, then $Tw_1 \geq Tw_2$. Furthermore, for discount factor $\delta$

$$T(w + c) = \int \max[\xi^*_t, \alpha_t + \beta_t (\mu_{it} + g(t)) + l(R_t)) - \psi(l(R_t))$$

$$+ \delta \int w(\mu_{t+1}, R_t, t+1) f(\mu_{t+1}|\mu_t, t) d\mu_{t+1}] f_{\xi^*}(\xi^*_t) d\xi^*_t]$$

$$< \int \max[\xi^*_t, \alpha_t + \beta_t (\mu_{it} + g(t)) + l(R_t)) - \psi(l(R_t))$$

$$+ \delta \int w(\mu_{t+1}, R_t, t+1) f(\mu_{t+1}|\mu_t, t) d\mu_{t+1}] f_{\xi^*}(\xi^*_t) d\xi^*_t] + \delta c$$

$$= Tw + \delta c$$
(ii). As discussed above, the assumptions on the utility function and the production technology imply that there is a unique interior solution to the problem of choosing optimal effort. Again, by the assumptions of the model expected utility in the current period is continuous. Suppose that \( w(\mu_{t+1}, R_t, t + 1) \) is continuous, then

\[
(\alpha_t + \beta_t (\mu_{it} + g(t) + l(R_t)) - \psi(l(R_t)) + \delta \int w(\mu_{t+1}, R_t, t + 1) f(\mu_{t+1} | \mu_t, t) d\mu_{t+1}
\]

is also continuous. The function \( \max(a, b) \) is continuous if \( a \) and \( b \) are continuous, and the integral over \( \xi^*_t \) is also continuous if \( \xi^*_t \) is continuous. Thus, \( T \) is a contraction that maps bounded continuous functions into bounded continuous functions. The proofs of (i) and (ii) imply that a unique solution \( V(\theta_t, R_t, t) \) exists. By the theorem of the maximum, the optimal policy correspondence \( A(\theta_t, R_t, t) \) is upper-hemicontinuous, and since the utility is concave in \( l_t \), the optimal policy is a continuous function.

**Part (ii).** The absence of interaction between effort and beliefs makes the problem of choosing optimal effort static. Since the utility function obeys increasing differences in \( (\beta, l_t) \), optimal effort \( l(R_t) > l(R_t') \) if \( R_t > R_t' \).

**Part (iii).** Suppose that \( R_t > R_t' \) and \( V(\mu_{t+1}, R_t, t + 1) > V(\mu_{t+1}, R_t', t + 1) \). Since \( \xi^*_t \) does not depend on \( R_t \), and \( l(R_t) > l(R_t') \), the first part of the statement follows. Finally, suppose that \( V(\mu_{t+1}, R_t, t + 1) \) increases in \( \mu_{t+1} \); then, the integral of \( V(\mu_{t+1}, R_t, t + 1) \) over the distribution of \( \mu_{t+1} \) conditional on \( \mu_t \) and \( t \) is also increasing in \( \mu_t \) because the conditional distributions of \( \mu_{t+1} \) are ordered in the sense of the likelihood ratio property with respect to \( \mu_t \). Similarly, expected utility for the current period increases in \( \mu_t \). Thus, \( V(\mu_t, R_t, t) \) increases in \( \mu_t \) in the sense of the likelihood ratio property. ■
3.8 Appendix B

Let the joint distribution of $Y_{it} = (y_{i1}, ..., y_{it})$ and $S_{it} = (s_{i1}, ..., s_{it})$, conditional on $M_i = (M_{i1}, ..., M_{it})$, $M_{it} = (R_{it}, t, X_{it}, \mu_{it})$, and parameters $\Theta_1$, be given by $F(Y_{it}, S_{it}|M_i, \theta_i, \Theta_1)$. By the definition of conditional distribution:

$$f(Y_{it}, S_{it}|M_i, \theta_i, \Theta_1) = f_t(s_{it}|y_{it}, Y_{it-1}, S_{it-1}, M_i, \theta_i, \Theta_1) \cdot f_t(y_{it}|Y_{it-1}, S_{it-1}, M_i, \theta_i, \Theta_1) \cdot f(Y_{it-1}, S_{it-1}, M_i, \theta_i, \Theta_1)$$

Assumption 7 below plays a crucial role in deriving the likelihood and is justified by the model presented above.

**Assumption 7. Dynamic Completeness** Suppose that

$$f_t(y_{it}|Y_{it-1}, S_{it-1}, M_i, \theta_i, \Theta_1) = f_t(y_{it}|M_{it}, \theta_i, \Theta_1)$$

$$f_t(s_{it}|Y_{it}, S_{it-1}, M_i, \theta_i, \Theta_1) = f_t(s_{it}|M_{it}, \theta_i, \Theta_1).$$

By Assumption 7, the conditional density becomes:

$$f(Y_{it}, S_{it}|M_i, \theta_i, \Theta_1) = f_t(s_{it}|Y_{it}, M_{it}, \theta_i, \Theta_1) \cdot f_t(y_{it}|M_{it}, \theta_i, \Theta_1) \cdot f_{t-1}(s_{it}|Y_{it-1}, M_{it-1}, \theta_i, \Theta_1) \cdot f_{t-1}(y_{it-1}|M_{it-1}, \theta_i, \Theta_1) \cdot f(Y_{it-2}, S_{it-2}, M_i, \Theta_1) \cdot ...$$

$$= \prod_{k=1}^{t} [f(s_{ik}|Y_{ik}, M_{ik}, \theta_i, \Theta_1) f(y_{ik}|M_{ik}, \theta_i, \Theta_1)]$$
By assumptions (i) and (ii) in the text of Chapter 3, for all $k$

\[
\begin{pmatrix}
\varepsilon_{ik} \\
\xi_{ik}
\end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & \rho \\ \rho & 1 \end{pmatrix}\right)
\]

To save on notation, the MLE is developed for the case when $\rho = 0$. If $s_{it} = 1$, then

\[
f (Y_{it}, S_{it} = (1, \ldots, 1)'|M_i, \theta, \Theta_1) = \prod_{k=1}^{t} [\Pr (s_{ik} = 1|Y_{ik}, M_{ik}, \theta_i, \Theta_1) f (y_{ik}|M_{ik}, \theta_i, \Theta_1)]
\]

where

\[
\Pr (s_{ik} = 1|Y_{ik}, M_{ik}, \theta_i, \Theta_1) = \Phi (G (R_{ik}, k, X_{ik}, \lambda \mu_{ik} + (1 - \lambda) \theta_i))
\]

\[
f (y_{ik}|M_{ik}, \theta_i, \Theta_1) = \frac{1}{\sigma} \varphi \left( \frac{y_{ik} - g (W_{ik}, k) - \theta_i}{\sigma} \right)
\]

If $s_{it} = 0$, while $s_{it-1} = 1$, then the unobserved $y_{it}$ must be integrated out, leading to:

\[
f (Y_{it}, S_{it} = (1, \ldots, 1, 0)'|M_i, \theta, \Theta_1) = \Pr (s_{it} = 0|Y_{it-1}, M_{it}, \theta_i, \Theta_1) \prod_{k=1}^{t-1} [\Pr (s_{ik} = 1|Y_{it-1}, M_{ik}, \theta_i, \Theta_1) f (y_{ik}|M_{ik}, \theta_i, \Theta_1)], \ t \geq 2
\]

where

\[
\Pr (s_{it} = 0|Y_{it-1}, M_{it}, \theta_i, \Theta_1) = 1 - \Phi (G (R_{ik}, k, X_{ik}, \lambda \mu_{ik} + (1 - \lambda) \theta_i))
\]

\[
\Pr (s_{ik} = 1|Y_{ik-1}, M_{ik}, \theta_i, \Theta_1) = \Phi (G (R_{ik}, k, X_{ik}, \lambda \mu_{ik} + (1 - \lambda) \theta_i))
\]

\[
f (y_{ik}|M_{ik}, \theta_i, \Theta_1) = \frac{1}{\sigma} \varphi \left( \frac{y_{ik} - g (k) - m (X_{ik}) - l (R_{ik}) - \theta_i}{\sigma} \right)
\]
Assumption 7 allows for the use of conditional MLE, so the likelihood for individual i is:

\[ l_i (\Theta_i | \theta_i, W_i) \]

\[ = \prod_{t=1}^{T} \left( \varphi \left( \frac{y_{it} - g(t) - m(X_{it}) - l(R_{it}) - \theta_i}{\sigma} \right) \Phi \left( G(R_{it}, t, X_{it}, (1 - \lambda) \theta_i + \lambda \mu_{it}) \right) \right) \]

\[ (1 - \Phi (G(R_{it}, t, X_{it}, (1 - \lambda) \theta_i + \lambda \mu_{it})) )^{1 - \prod_{k=1}^{t} \delta_{ik}} \]

where \( T \) is the last period for which the econometrician observes performance. Next, I integrate out \( \theta_i \)

\[ l_i (\Theta|W_i) = \int_{\Theta} l_i (\Theta_1 | W_i, \theta_i) . \varphi(\theta_i|W_i, \Theta_2) d\theta_i, \]

where \( \Theta_2 \) is a vector of parameters that govern the distribution of \( \theta_i \) and \( \Theta \) is a vector of all parameters in \( \Theta_1 \) and \( \Theta_2 \). Then the log-likelihood becomes

\[ l \left( \Theta | \{W_i\}_{i=1}^{N} \right) = \sum_{i=1}^{N} \log l_i (\Theta | W_i). \]
Chapter 4

Incentives to Work or Incentives to Quit?

4.1 Introduction

Pay incentives affect profits not only through their impact on effort choice but also through their effect on the quality mix of the workforce. In this chapter, I investigate the relative importance of these two channels to maximizing profits in a structural model of employment dynamics that includes effort choice, learning about match quality, and labor turnover. My empirical analysis focuses on contracts that are linear in output: compensation is equal to the sum of a base pay and a bonus proportional to hourly output (performance from now on). I limit my attention to this class of linear contracts for two reasons. First, the firm whose personnel records I use itself implemented such linear contracts and one of the objectives of this chapter is to characterize the profitability of the firm’s compensation policies. Second, firms often apply simple compensation policies based on such linear contracts and the problem of finding and
characterizing the optimal linear contract is of interest on its own.  

The firm’s data are ideally suited for the empirical analysis of pay incentives; they come from a call center in North Carolina and contain an objective measure of individual performance (defined as output per hour), a known compensation policy based on linear contracts in performance, and a variation in the pay policies that does not depend on what the firm learns about its employees. My estimates show that steeper incentives are associated with higher performance, that persistent differences in individual performance are driven by differences in the quality of the employer-employee match, and that employees learn about the quality of the match on the job. Their posterior beliefs are largely responsible for their decision to stay or quit and the interaction between incentives and turnover appears to be crucial to evaluating the impact of pay incentives on profits. Thus, I conclude that finding the optimal pay policy requires the explicit modelling of all three: effort choice, learning about match quality, and separation decisions.

Unobserved effort, labor turnover and learning about match quality are the subject of intensive study in the structural literature but usually separately from one another. For example, Shearer and Paarsch (2009) analyze the effect of incentives on effort and conduct a related policy analysis, but the experimental design of their study does not allow them to analyze the effects of incentives on the pool of entering employees and turnover. However, Lazear (2000) points out that in the context of his study about one-third of the improvement in performance after the introduction of pay incentives can be traced back to the improvement in the quality mix of entering employees. When the quality of the match between a potential employee and the firm becomes known

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1Within a dynamic setting as the one in this paper, the firm’s flexibility in designing the contract is reduced. This general idea is first explored by Holmstrom and Milgrom (1987) who show that in some settings the optimal compensation is to provide workers with incentives that are linear in output. Besides, the cost of implementing a complicated nonlinear contract is high: see, for example, Holmstrom and Milgrom (1990) as well as Ferrall and Shearer (1999).
to the worker in the hiring process, the firm may use its pay policy not only to induce
effort but also to shape the quality mix of the newly hired employees, as discussed
in Lazear (1998); the firm also faces a trade-off between the extra revenue generated
by workers who stay and the associated increase in pay that is necessary to make
them stay. The interaction of all these considerations determine the firm’s pay policy
and the associated turnover is not necessarily low or nonexistent but depends on the
characteristics of the technology, the workforce and the alternative jobs. When the firm
and workers learn about match quality over time, an additional consideration arises.
Depending on how much starting employees know about their match, turnover becomes
the primary channel through which pay incentives affect the quality mix. In the special
case of a common prior, there is no selection at entry, and turnover is the only source
of changes in the quality mix at the workplace.

Similarly, the models in the structural literature on learning about match quality
and turnover do not incorporate effort choice. The dynamic programs in such models are
complicated enough even as they are, since they involve heterogeneity across employees
and a sequence of posterior beliefs. To my knowledge, Miller (1984) is the first paper
to estimate a model with learning about match quality. Pastorino (2009) considers
a variation of this model that incorporates correlation between ability at one job and
ability at others. Furthermore, Nagypal (2007) applies the method of indirect inference
to distinguish between learning about match quality and learning-by-doing within a
structural setting. More recently, Camargo and Pastorino (2010) estimate a structural
model of career concerns with learning-by-doing. While these models consider selection
and job mobility, they assume away potential problems of moral hazard. Furthermore,
the computational complexity associated with estimating the models usually requires
some strong assumptions about the production technology and the heterogeneity among
workers.
Here, I propose a simple two-step procedure to estimate a structural model of learning about match quality, effort choice, and turnover. In the first step, I estimate a semi-structural attrition model and recover the stochastic technology up to a constant, as well as a scaled version of the value of continued employment. Chapter 3 shows that the first step provides consistent estimates of the effect of changes in pay incentives on effort, while popular alternatives may overestimate the same effect by a factor of two. I use these estimates in the second step to recover the remaining parameters using the method of moments. That is, I use an indirect approach to estimate the value of continued employment without directly solving for the value function. The same principle underpins the literature starting with Hotz and Miller (1993) and including the recent works by Aguirregabiria and Mira (2007) and by Pesendorfer and Schmidt-Dengler (2003). Thus, from a methodological point of view, this paper adapts recent structural estimation methods to the analysis of employment relations in the presence of Bayesian learning.

The estimates of the model are then used as a basis for counterfactual policy analysis. My results suggest that firms choose their pay policy for reasons that go beyond effort choice. Most of the increase in profits from switching to the optimal policy from hourly wage can be traced back to the effect of incentives on the quality mix. The optimal linear contract induces low quality employees to quit and in this way it helps the firm build a workforce of high match quality over time. This effect more than offsets the loss associated with replacing an experienced worker with a newly hired one of no experience and unknown ability. Furthermore, the employer exploits the firm-specific nature of match quality to capture most of the surplus generated by the employment relation. To achieve that, the firm offers pay incentives that induces little effort, so high level of effort and low turnover are not necessarily attributes of the profit-maximizing pay policy. Finally, the optimal policy that I find generates only 4.8% higher profits.
than one of the actually implemented pay regimes. An exercise in comparative statics shows that as turnover costs grow, the firm increases compensation to induce lower turnover by offering much steeper incentives. The result is a decline in the quality mix of the workforce and an increase in the importance of effort to the firm’s profits. Given the strong evidence of high turnover costs in some industries, this finding cautions that models of job mobility, such as Keane and Wolpin (1997) and Hoffmann (2010), should incorporate turnover costs. Finally, another counterfactual experiment shows that the firm’s profits would have been 27% higher if match quality was known to the employees at the time of hiring. The primary reason is that workers of high match quality self-select into the firm which leads to low turnover and high level of experience.

The results indicate that incentives matter and in this way they are consistent with the literature devoted to investigating incentive effects represented by Paarsch and Shearer (2000), Lazear (2000) and Shearer (2004). More specifically, they show that workers are very responsive to changes in the slope of incentives. This is consistent with previous results obtained in Paarsch and Shearer (1999, 2009), as well as Haley (2003). The novelty, relative to Shearer and Paarsch (2009), is that optimal pay incentives are allowed to affect not only effort choice but also the composition of the workforce at different tenure horizons. Thus, the chapter extends the work in Lazear (1998, 2000) on the effect of incentives on the quality mix by studying how turnover shapes the properties of the optimal pay policy. In particular, the results show that turnover may be the primary channel through which pay incentives affect profits when workers learn about match quality on the job. The results also indicate that the considerable contribution of improved match quality to worker’s compensation that is estimated in some structural papers of job mobility, such as Hoffmann (2010), may depend on the strong assumptions of low or non-existent turnover costs.

The rest of this chapter is organized as follows. Section 4.2 presents the model and
section 4.3 the data. Section 4.4 introduces the estimation of the model. Section 4.5 discusses the estimates of the structural parameters and presents the policy analysis. Section 4.6 presents some counteractual experiments and Section 4.7 concludes with an overview of future research.

4.2 Model

4.2.1 Worker’s Problem

The model is a variation of the classical model in Jovanovic (1979) which in addition to match quality also includes effort choice. The crucial parameter in the model is match quality $\theta_i, \theta_i \in R$. Match quality is time-invariant, independent and identically distributed across workers $i$, and normally distributed, $\theta_i \sim N (\mu_\theta, \sigma_\theta^2)$, with a probability density function denoted as $f_\theta$. At the beginning of each period the worker decides whether to stay or quit by comparing the value of continued employment and the realization of an outside offer $\xi_{it}^*$. The outside offer $\xi_{it}^*, \xi_{it}^* \in R$, is normally distributed, $\xi_{it}^* \sim N (\mu_{\xi^*}, \sigma_{\xi^*}^2)$, independent and identically distributed across tenure horizons $t$ and $i$, where $t = 1, 2, ..., \text{independent from } \theta_i$, and has a probability density function denoted $f_{\xi^*}$. If the worker stays, she observes a noisy performance signal $y_{it}$. The noise in the performance signal $\varepsilon_{it}, \varepsilon_{it} \in R$, is continuous, independent and identically distributed across tenure horizons $t$ and workers $i$, independent from $\theta_i$, normally distributed, $\varepsilon_{it} \sim N (0, \sigma_\varepsilon^2)$, and has a probability density function denoted as $f_{\varepsilon}$. The performance signal $y_{it}$ is generated by the following technology$^2$:

$$y_{it} = \theta_i + g(t) + l_{it} + \varepsilon_{it}$$

$^2$Since both the actual and estimated performance are always greater that 0, the restriction $y_{it} \geq 0$ never binds.
where \( g, g(t) \in R_+ \), represents the accumulation of firm-specific knowledge or experience and \( l_{it} \) is effort, \( l_{it} \in L \subset R_+ \), where \( L \) is compact. \( g(t) \) is increasing and continuous. Chapters 1 and 2 provide evidence in support of the choice of this additive functional form for the technology. The worker is paid \( w_{it} = \alpha_{it} + \beta_{it} y_{it} \), according to a linear compensation regime \( R_{it} = (\alpha_{it}, \beta_{it})' \), where \( \alpha_{it} > 0 \) and \( \beta_{it} > 0 \). Regime \( R_{it} \) is said to be more generous than regime \( R'_{it} \), \( R_{it} > R'_{it} \), if both \( \alpha_{it} > \alpha'_{it} \) and \( \beta_{it} > \beta'_{it} \). The worker does not expect the compensation regime to change in the future. The VNM utility of worker \( i \) is:

\[
u(R_{it}, l_{it}, y_{it}) = \alpha_{it} + \beta_{it} y_{it} - \frac{\gamma}{1 + \frac{1}{\psi}} l_{it}^{1+\frac{1}{\psi}}.3
\]

This specification for the disutility of labor is popular in the related literature; for example it is used in Shearer (2004) and Paarsch and Shearer (2009). Here \( \psi \) is the elasticity of effort to its return. Since \( \theta_i \) and \( l_{it} \) enter additively in the utility function, posterior beliefs do not depend on effort and optimal effort choice does not depend on beliefs, so it is a function only of \( R_{it}, l_{it}(R_{it}) \). Optimal effort is then

\[
l_{it} = \left( \frac{\beta_{it}}{\gamma} \right)^{\psi}
\]

Intuitively, this assumption about the functional form of the utility implies that conditional on one’s ability, output is proportionate to \( \beta^\psi \).

The belief at the beginning of \( t \) is denoted as \( \theta_{it} \) and is formed in a Bayesian way. Let the initial prior be \( \theta_{i1} \) and suppose that employees share a common prior at the time of hiring - the distribution of match quality in the population of potential employees, \( \theta_{i1} \sim N(\mu_\theta, \sigma^2_\theta) \). Given the normality assumptions from above, the posterior belief \( \theta_{it} \) is normally distributed for all \( t > 1 \), \( \theta_{it} \sim N(\mu_{it}, \sigma^2_t) \), where
\[ \mu_{it} = (1 - K_t) \mu_{it-1} + K_t(y_{it-1} - l(R_{it-1}) - g(t - 1)) \]

\[ \sigma_t^2 = \frac{\sigma^2 \sigma_{\theta}^2}{\sigma^2 (t - 1) + \sigma_{\xi}^2} \]

\[ K_t = \frac{\sigma^2}{\sigma^2 (t - 1) + \sigma_{\xi}^2} \]

Note that precision of beliefs depends only on \( t \), so the average of the demeaned past signals is a sufficient statistic to characterize posterior beliefs. The posterior mean can be rewritten as

\[ \mu_{it} = k(t) \left( \frac{1}{t-1} \sum_{k=1}^{t-1} (y_{ik} - l(R_{ik}) - g(k)) \right) + (1 - k(t)) \mu_0 \]

where \( k(t) = \frac{\sigma^2 (t-1)}{\sigma^2 (t-1) + \sigma_{\xi}^2} \). Then, the expected utility from working in period \( t \) is

\[ U(\mu_{it}, R_{it}, t, \gamma, \psi) = \alpha_{it} + \beta_{it} \left( \mu_{it} + \left( \frac{\beta_{it}}{\gamma} \right)^\psi + g(t) \right) - \frac{\gamma}{1 + \frac{1}{\psi}} \left( \frac{\beta_{it}}{\gamma} \right)^{\psi+1} \]

By these observations, the independence of \( \theta_i \) from \( \varepsilon_{it} \) and \( \xi_{it}^* \), and by the additivity of \( \varepsilon_{it} \) in the stochastic technology, the optimal problem of the worker can be formulated as functional equation (P)

\[ v(\mu_{it}, R_{it}, t) = \int [\max(\xi_{it}^*, U(\mu_{it}, R_{it}, t, \gamma, \psi))] \]

\[ + \delta \int v(\mu_{it+1}, R_{it}, t + 1) f(\mu_{it+1}|\mu_{it}, t) d\mu_{it+1}] f(\xi_{it}^*) d\xi_{it}^* \]

where \( f(\mu_{it+1}|\mu_{it}, t) \) is the conditional density of \( \mu_{it+1} \), given \( \mu_{it} \) and \( t \).

**Proposition 1.** Given the specification of the model above
i. The functional equation (P) has a unique continuous solution \( V(\mu_{it}, R_{it}, t) \) and the optimal policy

\[
A(\mu_{it}, R_{it}, t) = \{l_t \in L \mid (P) \text{ holds} \}
\]

is a continuous function.

ii. Optimal effort \( l(R_{it}) > l(R'_{it}) \) if \( R_{it} > R'_{it} \).

iii. \( V(\mu_{it}, R_{it}, t) > V(\mu_{it}, R_{it}, t) \) if \( R_{it} > R'_{it} \), and \( V(\mu_{it}, R_{it}, t) \) increases \( \mu_{it} \).

The proof of Proposition 1 is presented in Appendix A. Let

\[
H(\mu_{it}, R_{it}, t) = U(\mu_{it}, R_{it}, t, \gamma, \psi)) + \delta \int \int \max\{\xi^*_{it+1}, H(\mu_{it+1}, R_{it}, t+1)\} \varphi(\mu_{it+1}|\mu_{it}, t) f_{\xi^*}(\xi^*_{it+1}) d\mu_{it+1} d\xi^*_{it+1}
\]

After the realization of the outside offer, \( i \) decides to stay if the value of continued employment is higher than the value of the outside offer

\[
H(\mu_{it}, R_{it}, t) - \xi^*_it > 0.
\]

### 4.2.2 Firm’s Problem

Based on the firm’s records, I take the revenue from a successfully processed call to be \( r = 8.5 \). This approximation is based on the firm records for average outbound and inbound calls, the reward that the firm receives from processing each type of calls, and the relation between the number of processed calls and accounts serviced by the company.\(^4\) I consider only contracts \( R(\alpha, \beta) \) that are linear in the performance signal:

\(^4\)The actual contract between the call center and the cable TV company was stated in more complicated terms. The cable TV company transferred accounts to the call center after the latter had successfully processed previously transferred calls. Thus, the call center made its profits from successfully processing accounts; the cable TV company expected more than 95% rate of collection. Yet, the contract recognized that not all attempts to contact a cable TV subscriber are successful, so it conditioned pay per account on the inbound and outbound calls that operators make to the client.
compensation $w_{it}$ is equal to a base pay $\alpha$ plus a bonus proportionate to performance: $w_{it} = \alpha + \beta y_{it}$. I assume that inbound and outbound calls, as well as the number of processed calls per account is independent from the implemented contract. Furthermore, I assume that the firm’s monthly discount factor is $\delta = 0.99$, implying an annual discount factor of just below 0.9. Quitting disrupts the production process and necessitates spending money to advertise the available job position, and train the replacement. In what follows, I incorporate turnover costs, which according to some estimates of the firm itself amount to approximately $750. Furthermore, I also allow the firm to hire a replacement immediately after a worker quits. The firm is assumed to face constant returns to scale. Finally, the profit function that I consider below abstracts away from fixed costs.

Given these assumptions, the expected profits per employee in period $t$, conditional on staying, match quality, $t$, and the pay policy, are defined as

$$\pi_{it}(\theta_i, R, t) = (r - \beta) \cdot (t_i + l(R) + g(t)) - \alpha,$$

where $r$ is the revenue per call. Let the probability of staying at least until period $t$ be $p_{it}(R, \theta_i, t, \{\xi_{ik}, \xi_{ik}^*\}_{k=1}^{t-1})$. The expected profits of the firm are:

$$\pi(R) = E_{\theta}\{\sum_{t=1}^{\infty} \delta^{t-1} [p_{it}(R, \theta_i, t, \{\xi_{ik}, \xi_{ik}^*\}_{k=1}^{t-1}) \pi_{it}(\theta_i, R, t) +
+ (1 - p_{it}(R, \theta_i, t, \{\xi_{ik}, \xi_{ik}^*\}_{k=1}^{t-1})) (\pi(R) - c)]\}.$$

The expectation operator $E_{\theta}$ indicates that the expectation is taken with respect to the initial prior belief, which coincides with the distribution of match quality in the population.

Nevertheless, the underlying factor that drives profits is the successful collection of debt because that leads to the transfer of more accounts. The approximation establishes a relation between the processed calls and serviced accounts.
tion. Each period, the employee either stays with probability \( p_{it}(R, \theta, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1}) \) and generates profits \( \pi_{it}(\theta, R, t) \) or quits and the firm hires a new employee who at entry is expected to generate exactly the same profits as the original employee, \( \pi(R) \). This equation can be solved for \( \pi(R) \)

\[
\pi(R) = E_{\theta} \left\{ \sum_{t=1}^{\infty} \frac{\delta^{-t} p_{it}(R, \theta, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1}) \pi_{it}(\theta, R, t) - (1 - p_{it}(R, \theta, t, \{\varepsilon_{ik}, \xi_{ik}^{*}\}_{k=1}^{t-1})) c}{1 - \sum_{t=1}^{\infty} \delta^{t-1} (1 - p_{it}(R, \theta, t, \{\varepsilon_{ik}, \xi_{ik}^{*}\}_{k=1}^{t-1}))} \right\}
\]

The firm chooses \((\alpha, \beta)\) to maximize \( \pi(R) \) subject to

\[
l(R) = \left( \frac{\beta}{\gamma} \right)^{\psi}.
\]

The probability of staying \( p_{it}(R, \theta, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1}) \) and the optimal effort conditions connect the firm’s problem to that of the worker presented above.

### 4.3 Data

The data set contains a clean performance measure and three known compensation regimes that were implemented in a way that allows to identify each one’s effect on performance. The data comes from a call center in North Carolina owned and operated by a multinational company. The call center collects outstanding debt and fees on behalf of cable TV companies, which ensures a stable demand for its services. An automated switchboard operator allocates inbound and outbound calls, so that the longest weighting customer is matched with the longest weighting operator. Employees rotate their work stations on a daily basis.
As part of a reorganization plan, the central management implemented a linear contract at the beginning of January 2005: a linear function of the performance metric, the number of calls per hour that end with collection of the outstanding debt. Before January 2005, compensation was based on an hourly wage of $9.5. The central management was concerned that the company was paying "too much," so it implemented a new regime for the newly-hired employees in June 2005 (regime 2). Relative to regime 1, regime 2 offered a lower base pay, decreased the slope of the piece rate for those with performance less than 3.8, and increased the slope of the piece rate for those with performance greater than 3.8 (regime 2). All previously hired employees continued to be paid according to regime 1. Since the central management was worried about possible negative effects of the piece rate on the quality of service, it changed the pay regime yet again in November 2005. The new regime 3 had two components: all employees were paid according to the pay schedule of regime 2, but in addition employees had to meet certain minimum quality standards of service to qualify for the piece rate. Twenty percent of one’s calls were randomly monitored and the quality of service was rated on a scale from 0 to 100. An employee who did not meet the minimum quality standard was relegated to an hourly wage equal to the base pay of the piece rate. Since 99% of performance lies between 1.05 and 3.8, regimes 2 and 3 effectively lowered incentives relative to regime 1. Diagram 1 shows a time line for the implementation of the three regimes.

Chapter 2 provides a detailed descriptive analysis. What follows summarizes only the most relevant pieces of this analysis. The call center experienced high turnover rates under all pay regimes: more than 50% of all employees under regime 1 quit within the first six months of employment, while under regimes 2 and 3 the turnover for the first six months approached 67%. There also appears to be a noisy downward trend in the separation rates as tenure increases. This noisiness is probably due to the small sample
size, but it also suggests that separation decisions depend to a large extent on individual-specific factors. Table 1 reports the average performance for the first six months of employment across regimes. Again, as one may expect, the average performance under regime 1 is higher than its counterparts for regimes 2 and 3. Furthermore, the average performance on the subset of stayers is higher than the simple average, suggesting that poor performers quit. This evidence suggests that steep pay incentives lead to high performance; that attrition appears to be non-random, since workers with higher performance are more likely to stay; that individual-specific effects are present; and finally that workers accumulate experience or knowledge in the course of their first six months of employment.

4.4 Estimation

Moral hazard, learning about match quality and labor mobility have been studied intensively but separately in the structural literature. Still, moral hazard and labor turnover are defining features of the analytical environment at most workplaces; their interaction shapes employment outcomes and through them profits and individual welfare. The estimation of a structural model including all these components is, however, a complicated exercise. The dynamic programs in models with Bayesian learning are quite complicated, since they involve posterior beliefs about an unobserved individual-specific parameter. As a result, estimation methods that rely on solving for the value function at each step of the optimization algorithm are computationally intensive.\(^5\)

Here, I propose a simple two-step procedure to estimate the structural model incorporating effort choice, learning about match quality, and separation decisions. In

\(^5\)See Nagypal (2007) for an application of indirect inference to the estimation of a model of learning about match quality. Smith (2003) provides a summary of the econometric challenges associated with the use of indirect inference to discrete choice problems and outlines a smoothing approach that addresses them.
principle, the structural parameters can be recovered by estimating the following model

\[ y_{it} = \theta_i + l(R_{it}) + g(t) + \varepsilon_{it} \]

\[ s_{ik} = 1 \left[ H(\mu_{ik}, R_{it}, k) - \xi_{ik} > 0 \right], \]

where \( y_{it}, t > 1 \), is observed if \( s_{ik} = 1 \) for all \( k = 1, ..., t \). Doing so, however, involves solving for the value function of each individual for each belief at each step of the optimization algorithm, which is computationally intensive. Therefore, in practice I estimate the model in two steps. In the first step, I estimate a semi-structural attrition model and recover the stochastic technology up to a constant, as well as a scaled version of the value of continued employment. I use these estimates in the second step to recover the remaining structural parameters using the method of moments. The main advantage of this two-step estimator is its computational simplicity, but it has also one important limitation. The initial flexible approximation of the value of continued employment can be imprecise in small samples and this can generate a finite sample bias. One way to investigate the magnitude of the potential problem and limit its effect is to apply a K-step procedure as presented in Aguirregabiria and Mira (2007). This issue is left for future research. Once the structural parameters are recovered, I use simulation methods to evaluate the profitability of the implemented regimes and to find the optimal linear contract. The simulation method and the optimization algorithm are discussed in the last subsection.

---

\[ ^6 \text{This is the approach taken in the literature on labor mobility, starting with Keane and Wolpin (1997).} \]
4.4.1 Worker’s Problem: Step 1

The first step is based on the same estimation method used in Chapter 3. Recall that I estimate the following semi-structural model:

\[ y_{it} = \theta_i + \ell(R_{it}) + g(t) + \varepsilon_{it} \]

\[ s_{ik} = 1 [G(\mu_{ik}, R_{it}, k) - \xi_{ik} > 0], \]

where \( y_{it} > 1 \), is observed if \( s_{ik} = 1 \) for all \( k = 1, \ldots, t \), \( \xi_{it} \sim N(0, 1) \), and

\[ G(\theta_{ik}, R_{ik}, k) = \frac{1}{\sigma_{\xi}}(H(\theta_{ik}, R_{it}, k) - \mu_{\xi}) \]

for all \( t \). Moreover, \( G(\theta_{ik}, R_{ik}, k) \) is approximated using a linear combination of orthogonal polynomials of the explanatory variables; \(^7\) I assume that the following condition holds for the approximation \( \hat{G}(\mu_{it}, R_{it}, t) \):

\[ E \left[ \hat{G}(\mu_{it}, R_{it}, t) - \frac{1}{\sigma_{\xi}}H(\mu_{it}, R_{it}, t) - \frac{\mu_{\xi}}{\sigma_{\xi}} \right] = 0 \]

This model incorporates the restrictions on the stochastic technology, but imposes no structure on the utility function. As a result, it does not impose a link between effort in the performance equation and disutility of effort in the attrition equation. By estimating the semi-structural model, I recover the stochastic technology up to a constant and \( G(\mu_{it}, R_{it}, t) \) up to a scaling parameter and an additive constant. I estimate the model using maximum likelihood as discussed in Appendix B. Here I provide a short summary.

\(^7\)Bellman, Kaleba, and Kotkin (1963) first propose the use of such a linear approximation to the value function. The approximation method remains popular in both economics and machine learning, where it is still the workhorse for approximating dynamic programs as discussed in Kveton and Hauskrecht (2004).
of the estimation method.

Let the observable information about individual \( i \) be \( W_i \) and \( \Theta_1 \) be the vector of parameters to be estimated conditional on \( \theta_i \). The likelihood for individual \( i \) conditional on the data and \( \theta_i \) can be written in a standard way as follows:

\[
\begin{align*}
l_i (\Theta_1 | \theta_i, W_i) &= \left[ \prod_{t=1}^{T_i} \left( \varphi \left( \frac{y_{it} - g(t) - l(R_{it}) - \theta_i}{\sigma} \right) \Phi \left( G(R_{it}, t, \mu_{it}) \right) \right) S_{it} \right] \\
&\quad \times (1 - \Phi \left( G(R_{iT_i}, T_i, \mu_{iT_i}) \right))^{1-S_{iT_i}}
\end{align*}
\]

where \( S_{it} = \prod_{k=1}^{t} s_{ik} \) and \( T_i \) is the last period in which \( i \) is observed. Since \( \theta_i \) is not observed, it is integrated out to obtain the individual contribution to the likelihood:

\[
l_i (\Theta | W_i) = \int l_i (\Theta_1 | \theta_i, W_i) \cdot \varphi(\theta_i | W_i, \Theta_2) d\theta_i
\]

where \( \Theta_2 \) is a vector of parameters that define the distribution of \( \theta_i \) and \( \Theta \) is a vector that contains all parameters in \( \Theta_1 \) and \( \Theta_2 \). Finally, the log-likelihood is obtained by taking logs and summing over \( i \):

\[
\log l \left( \Theta | \{ W_i \}_{i=1}^{N} \right) = \sum_{i=1}^{N} \log l_i (\Theta)
\]

These estimates are used in the second step to obtain the remaining structural parameters: the marginal disutility of one unit of effort \( \gamma \), the curvature of the disutility of effort \( \psi \), and the mean and variance of the outside offer, \( \mu_\xi \) and \( \sigma_\xi^2 \), as well as the discount factor \( \delta \).
4.4.2 Workers Problem: Step 2

Let the difference in exerted effort under regimes 1 and 2 be $\Delta l$. Then from the performance equation,

$$\Delta l = \left(\frac{1}{\gamma}\right)^\psi \left(\beta_1^\psi - \beta_2^\psi\right)$$

The first step of the estimation provides the empirical counterpart of $\Delta l$, $\Delta \hat{l}$. Thus, one can solve

$$\Delta \hat{l} = \left(\frac{1}{\gamma}\right)^\psi \left(\beta_1^\psi - \beta_2^\psi\right)$$

for $\gamma$ in terms of $\psi$ and $\Delta \hat{l}$; let the solution be $\gamma \left(\psi, \Delta \hat{l}\right)$ and substitute the solution in the expression for the disutility of labor to obtain $U \left(\mu_{it}, R_{it}, t, \Delta \hat{l}, \psi\right)$.

To save on notation, define

$$\lambda \left(G \left(\mu_{it}, R_{it}, t\right)\right)$$

$$= E_{\xi^*} \max \{\xi_{it}, G \left(\mu_{it}, R, t\right)\}$$

$$= G \left(\mu_{it}, R_{it}, t\right) \cdot \Phi \left(G \left(\mu_{it}, R_{it}, t\right)\right) + \varphi \left(G \left(\mu_{it}, R_{it}, t\right)\right)$$

Note that $V \left(\mu_{it}, R, t\right)$ can be expressed in terms of $G \left(\mu_{it}, R_{it}, t\right)$ as follows

$$V \left(\mu_{it}, R_{it}, t\right) = E_{\xi^*} \max \{\xi_{it}^*, H \left(\mu_{it}, R_{it}, t\right)\}$$

$$= \mu_{\xi^*} + \sigma_{\xi^*} E_{\xi} \max \{\xi_{it}, G \left(\mu_{it}, R, t\right)\}$$

$$= \mu_{\xi^*} + \sigma_{\xi} \left[\lambda \left(G \left(\mu_{it}, R_{it}, t\right)\right)\right]$$
From the definition of $H(\mu_{it}, R_{it}, t)$ and the above representation of $V(\mu_{it}, R_{it}, t)$,

\[
H(\mu_{it}, R_{it}, t) = \mu_{\xi^{*}} + \sigma_{\xi^{*}}G(\mu_{it}, R_{it}, t)
\]

\[
H(\mu_{it}, R_{it}, t) = U(\mu_{it}, R_{it}, t, \gamma, \psi) + \delta \left[ \mu_{\xi^{*}} + \sigma_{\xi^{*}}E_{\mu_{it+1}}(\lambda(G(\mu_{it+1}, R_{it+1}, t+1))) \right]
\]

where $E_{\mu_{it+1}}$ indicates that expectation is taken with respect to the distribution of the mean of the posterior beliefs in $t + 1$ given the information available at $t$, $\sigma_{it}^{2}$ and $\mu_{it}$. Consequently,

\[
\mu_{\xi^{*}} + \sigma_{\xi^{*}}G(\mu_{it}, R_{it}, t) = U(\mu_{it}, R_{it}, t, \gamma, \psi) + \delta \left[ \mu_{\xi^{*}} + \sigma_{\xi^{*}}E_{\mu_{it+1}}(\lambda(G(\mu_{it+1}, R_{it+1}, t+1))) \right]
\]

By this identity and the assumptions on the approximation of $G(\mu_{it}, R_{it}, t)$, the delta method implies the following conditions for $t = 1, ..., T_i$

\[
E \left( \tilde{G}(\mu_{it}, R_{it}, t) - M_{it}(\mu_{it}, R_{it}, t, \Theta_2) \right) = 0,
\]

where

\[
M_{it}(\mu_{it}, R_{it}, t, \Theta_2) = \frac{1}{\sigma_{\xi^{*}}} \left\{ U(\mu_{it}, R_{it}, t, \Delta \tilde{I}, \psi) + \delta \left[ \mu_{\xi^{*}} + \sigma_{\xi^{*}}E_{\mu_{it+1}}(\lambda(\tilde{G}(\mu_{it+1}, R_{it+1}, t+1))) \right] - \mu_{\xi^{*}} \right\},
\]

and $\Theta_2 = (\psi, \delta, \sigma_{\xi^{*}}, \mu_{\xi^{*}})$ is the vector of structural parameters recovered at the second stage. $\Theta_2$ does not contain $\gamma$ as it can be recovered from $\gamma(\psi, \Delta \tilde{I})$.

Define

\[
M_t(\Theta_2) = \sum_i M_{it}(\mu_{it}, R, t, \Theta_2) \quad \text{and} \quad \tilde{G}_t = \sum_i \tilde{G}(\mu_{it}, R, t)
\]
where the summation is over the individuals who make the decision to stay or quit in period $t$. Furthermore, let

$$M(\Theta_2) = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

and

$$\hat{G} = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

The remaining parameters $\Theta_2$ are the solution to

$$\min_{\Theta_2} \left( \hat{G} - M(\Theta_2) \right)' \Omega^{-1} \left( \hat{G} - M(\Theta_2) \right),$$

where $\Omega$ is the optimal weighing matrix. Under the specified model $\delta, \psi, \gamma, \mu_{\xi*}, \sigma_{\xi*}$ are identified; identification is discussed in Appendix C.

Let the covariance matrix of the structural parameters $\Theta_1$ estimated in the first step be $\Sigma$. By the Delta method

$$\Omega = W'\Sigma W$$

where $W = \frac{\partial G}{\partial \Theta_1}$. Finally, the covariance matrix $\Sigma$ is obtained from the first-step MLE estimates. Then, following Hansen (1982), the asymptotic covariance matrix for $\Theta_2$ is

$$(J'\Omega^{-1}J)^{-1},$$

where $J = \frac{\partial M}{\partial \Theta_2}$.

Note that the estimation of the structural parameters $\Theta_2$ is in its essence a consistency check for the estimates of the first-step attrition model: the second step can be interpreted as a search for structural parameters that generate a data process that is
consistent with the findings in the first step. The criterion function evaluated at the optimum has $\chi^2$ distribution with 12 degrees of freedom under the null hypothesis that the theoretical model is valid.

4.4.3 Firm’s Problem

For any regime, the probability of staying at tenure $t$, $p_{it} (R, \theta_i, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1})$, cannot be estimated analytically, so I resort to simulations to evaluate the profitability of pay regimes. For the set of employees who enter the firm, I draw paths of $\varepsilon_t$ and $\xi_t^{*}$ and $\theta$ to generate 1000 data sets. Using the point estimates from steps one and two, I generate the sequence of noisy performance signals and posterior beliefs. The generation of the separation indicators, $s_{ik}$, requires some care. Given a regime $R$, I solve for the value function of each individual for each posterior belief. I assume that conditional on staying for 2 years employees know the true value of their match quality and the accumulation of experience has stopped. Then, the worker’s problem becomes

$$V(\theta_i, R) = \int \max [\xi_{it}^{*}, U(\theta_i, R) + \delta V(\theta_i, R)] \, dF_{\xi^{*}},$$

where $U(\theta_i, R)$ stands for the expected utility after the individual knows her match quality $\theta_i$, there is no more experience to be gained, and $V(\theta_i, R)$ is the value of continued employment. This problem can be solved as a standard fixed-point problem using value function iteration. The starting value for the iterations is the discounted sum of expected utility, i.e.

$$V^0(\theta_i, R) = \frac{1}{1 - \delta} U(\theta_i, R)$$
Then, I solve backwards for the utility of continued employment \( V(\mu_{it}, R, t) \), using the appropriate posterior. I use the Gauss-Hermite method with 8 nodes of integration. This approach to solving for the value function is similar to the one employed in Nagypal (2007). Comparing the drawn outside offers and the values of continued employment from above generates the sequence of separation indicators. It should be noted that all workers eventually quit. For each of the simulated data sets, I find the expected profits per entering employee by averaging the discounted some of individual profits for the duration of stay. To find the expected profits per workstation, I take into account that all quits are replaced by new workers who have exactly the same expected profits at entry as the original cohort. This simulation method is used to estimate the profits of the firm under the actually implemented regimes and to evaluate the candidates for the optimal linear contract at each step of the optimization. Given the low dimension of the optimization problem, I use a version of the simplex algorithm to find the optimal linear contract.

4.5 Results

This section presents the results from estimating the structural model and then shows how they can be used to find the profit-maximizing pay policy under various assumptions about the employment environment. The policy analysis indicates that turnover is a major channel through which pay incentives affect both performance and profits.

4.5.1 Estimates of Structural Parameters

In this subsection, I present the results from estimating the structural model and characterize the employment environment. The attrition model of the first step is estimated using MLE. The results, their econometric implications, and possible alternative spec...
ifications are discussed in greater detail in Chapter 3. The explanatory variables for the performance equations include second degree orthogonal polynomials of tenure and calendar time, dummies for regimes of operation and regimes of hiring, unobserved match quality, and controls. Specifically, regime 2 enters additively as implied by the theoretical model. Since regime 3 has the same pay schedule as regime 2 but conditions pay on the quality of service, the performance equation incorporates interaction terms between the tenure polynomials and regime 3. The attrition equations include orthogonal polynomials interacted with regimes and, depending on the specification, $\theta_i$ or $\mu_{it}$, controls, calendar time, and regime of hiring. As a preliminary step, I conduct a specification search for the degrees of the orthogonal polynomials in the performance and attrition equations. I find that orthogonal polynomials of degree 2 for the performance equation and orthogonal polynomials of degree 3 for the attrition equations fit the data best. The estimates can be found in Tables 4.1-4.3. They are very similar to the ones reported in Chapter 3 for the basic attrition model. The main difference is that some not significant variables have been omitted, along with the dummies for regime of hiring. In what follows, I make a brief summary of those results that are directly related to the second step of estimation and profits. Furthermore, I measure the contribution of effort, match quality, and experience to performance in terms of successful calls per hour (just calls per hour for short from now on).

In the first step, I estimate the distribution of match quality at entry and characterize the associated dynamics of learning and turnover. The variance of match quality is 0.48 and accounts for the greater part of the variance in performance at entry under regimes 1 and 2. Moreover, it has an important effect on attrition. Figure 4.1 presents the distribution of match quality at entry and how it changes by the sixth months of employment. The value of $\theta$ is on the horizontal axis, while the vertical axis represents the proportion of agents of a certain match quality who are present in the firm at a
Table 4.1: Estimates for the performance equation in the attrition model

<table>
<thead>
<tr>
<th>Explanatory Variable</th>
<th>Coef.</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$, orthog. pol. 1</td>
<td>-0.58</td>
<td>0.15</td>
</tr>
<tr>
<td>$t$, orthog. pol. 2</td>
<td>-0.61</td>
<td>0.09</td>
</tr>
<tr>
<td>regime 2</td>
<td>-0.21</td>
<td>0.07</td>
</tr>
<tr>
<td>regime 3</td>
<td>0.13</td>
<td>0.08</td>
</tr>
<tr>
<td>$t$. (regime 3), orthog. pol. 1</td>
<td>0.78</td>
<td>0.13</td>
</tr>
<tr>
<td>$t$. (regime 3), orthog. pol. 2</td>
<td>0.43</td>
<td>0.08</td>
</tr>
<tr>
<td>% outbound calls</td>
<td>-0.16</td>
<td>0.05</td>
</tr>
<tr>
<td>Constant</td>
<td>3.43</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Log-likelihood: -3745.73

Notes: The specification also includes calendar time orthogonal polynomials of degree 2 and individual controls: gender, age, marriage status, distance from home, and race.

Table 4.2: Estimates for the separation equation in the attrition model

<table>
<thead>
<tr>
<th>Explanatory Variable:</th>
<th>Coef.</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$, orthog. pol. 1</td>
<td>-0.51</td>
<td>0.17</td>
</tr>
<tr>
<td>$t$, orthog. pol. 2</td>
<td>-0.22</td>
<td>0.08</td>
</tr>
<tr>
<td>$t$, orthog. pol. 3</td>
<td>0.23</td>
<td>0.13</td>
</tr>
<tr>
<td>$t$. (regime 2), orthog. pol. 1</td>
<td>0.41</td>
<td>0.14</td>
</tr>
<tr>
<td>$t$. (regime 2), orthog. pol. 2</td>
<td>0.42</td>
<td>0.11</td>
</tr>
<tr>
<td>$t$. (regime 2), orthog. pol. 3</td>
<td>-0.78</td>
<td>1.59</td>
</tr>
<tr>
<td>$t$. (regime 3), orthog. pol. 1</td>
<td>0.40</td>
<td>0.16</td>
</tr>
<tr>
<td>$t$. (regime 3), orthog. pol. 2</td>
<td>0.23</td>
<td>0.08</td>
</tr>
<tr>
<td>$t$. (regime 3), orthog. pol. 3</td>
<td>-0.03</td>
<td>0.13</td>
</tr>
<tr>
<td>avg. % outbound calls in past</td>
<td>-0.31</td>
<td>0.09</td>
</tr>
<tr>
<td>Constant</td>
<td>0.58</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Log-likelihood: -3745.73

Notes: The specification also includes calendar time orthogonal polynomials of degree 2 and individual controls: gender, age, marriage status, distance from home, and race.
Table 4.3: Estimates of parameters related to ability and learning in the attrition model

<table>
<thead>
<tr>
<th>Parameter or explanatory variable</th>
<th>Attrition Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coef.</td>
</tr>
<tr>
<td>$\sigma^2_\xi$</td>
<td>0.17</td>
</tr>
<tr>
<td>$\sigma_\xi$</td>
<td>1</td>
</tr>
<tr>
<td>$\rho(\varepsilon, \xi)$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\sigma^2_\theta$</td>
<td>0.48</td>
</tr>
<tr>
<td>$\mu_{it}$</td>
<td>0.18</td>
</tr>
<tr>
<td>$t.\mu_{it}$, orthog. pol. 1</td>
<td>0.06</td>
</tr>
<tr>
<td>$t.\mu_{it}$, orthog. pol. 2</td>
<td>0.02</td>
</tr>
<tr>
<td>$t.\mu_{it}$, orthog. pol. 3</td>
<td>0.03</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-3745.73</td>
</tr>
</tbody>
</table>

Notes: The specification also includes calendar time orthogonal polynomials of degree 2 and individual controls: gender, age, marriage status, distance from home, and race.

Given tenure horizon. The figure shows that the conditional distribution of $\theta$ shifts to the right as tenure increases under both regimes 1 and 2 and only workers with very high match quality remain employed after six months of work. As expected, the switch from regime 1 to regime 2 generates an increase in turnover at any tenure horizon. The variance of the disturbance term in the performance equation is estimated at 0.17 which implies that the signal-to-noise ratio, defined as the ratio of the variance of match quality over the variance of noise, is approximately 2.6. Consequently, within 6 months the variance of the posterior beliefs declines to approximately 0.05 and the weight on the initial belief declines to almost zero.

Furthermore, I recover the technology up to an additive constant. The estimated parameters for the performance equation are broadly consistent with the theoretical predictions. In economic terms, the switch from regime 1 to regime 2 leads to a decline in worker’s effort and in turn performance by about 0.2 calls per hour which translates into a decline in hourly pay by approximately $2. The estimates imply a significant improvement in performance over time due to the accumulation of experience: in the first 6 months of employment performance increases by approximately one successful call
per hour, or 35% growth in the first six months under regime 1. Under regime 1, this growth translates in an increase in hourly pay by approximately $3.3. Finally, I also estimate flexibly the normalized and scaled value of continued employment \( G(\mu_{it}, R, t, X_{it}) \) which provides the basis for the second step estimation.

Table 4.4 presents the estimates from the second step. Before discussing the results from the second step, I first check the validity of the overidentifying restrictions. The \( \chi^2 \)-square test with 12 degrees of freedom for the overidentifying restrictions fails to reject the null hypothesis that the restrictions are valid, since the test statistic is 5.16. Thus, I conclude that the data is consistent with the restrictions imposed by the model. The second step results allow for the characterization of optimal effort choice under regimes 1 and 2. The elasticity of effort to pay incentives \( \psi \) is estimated at 3.27 with a standard error of 0.28, implying that workers’ supply of effort is highly sensitive to changes in pay incentives. The relative benefit of effort to its subjective cost represented by \( \gamma \) is estimated at 3.9 with a standard error of 0.3, so that effort amounting to one call per hour costs to the individual $3.9. Given the estimates of \( \gamma \) and \( \psi \), the level of effort under regime 1 translates into an increase in performance by 0.59 calls per hour and under regime 2 by 0.39 calls per hour. Compared to the variation in match quality, the contribution of effort to performance is relatively small: less than one standard deviation under regime 1 and even less under regime 2. In contrast, the mean of match quality in the population of entering workers is approximately 2, i.e. independent of pay incentives an employee of average quality successfully completes two calls per hour when starting work. Furthermore, the results indicate that in the absence of selection at

\[\text{Note that the estimation of the structural parameters } \Theta_2 \text{ is a test for consistency of the first-step estimates with the specified utility in the model.}\]
Table 4.4: Estimates of the structural parameters of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>3.26</td>
<td>0.21</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.89</td>
<td>0.24</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.74</td>
<td>0.08</td>
</tr>
<tr>
<td>$\sigma^2_\xi$</td>
<td>32.75</td>
<td>0.31</td>
</tr>
<tr>
<td>$\mu_\xi$</td>
<td>45.6</td>
<td>2.36</td>
</tr>
<tr>
<td>$\Delta l$</td>
<td>-0.19</td>
<td>0.07</td>
</tr>
<tr>
<td>$\Delta \text{disutility}$</td>
<td>-0.64</td>
<td>0.07</td>
</tr>
<tr>
<td>experience by $t = 6$</td>
<td>0.99</td>
<td>0.09</td>
</tr>
<tr>
<td>$\sigma^2_\theta$</td>
<td>0.48</td>
<td>0.02</td>
</tr>
<tr>
<td>$\mu_\theta$</td>
<td>2.02</td>
<td>0.11</td>
</tr>
<tr>
<td>$\chi^2_{12}$ test stat.</td>
<td>5.06</td>
<td></td>
</tr>
</tbody>
</table>

entry and exit the optimal piece rate involves $\beta = 6.55^9$. The fact that the implemented pay regimes have $\beta$ much lower suggests that turnover has a nontrivial effect on profits.

The monthly discount factor is estimated at 0.75 with standard error of 0.08, indicating a strong preference for present to future consumption: when making decisions workers assign a weight 0.001 to consumption after two years. The mean of the distribution of outside options is $45.6 with a standard error of 2.46, where an outside option stands for the present value of hourly compensation at an alternative job. The variance of outside offers is estimated at $32.5. To give some perspective, a job with an hourly wage of $11.25 has a present value of $45, assuming that the worker has no right to leave after entry. From these characteristics of the distribution of outside offers one may guess that the workers may find alternative employment at low-skill service jobs or low-skill manufacturing jobs whose hourly wage varies between $8 and $14.

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^9Solving the profit-maximization problem of the firm subject to a participation constraint yield

$$\beta = \frac{p\psi}{\psi + 1},$$

where $p$ is the revenue generated from the employment relation and in the present context is $8.5 per call. Substituting the estimated $\psi$ gives $\beta = 6.55$
4.5.2 Profits and Policy Analysis

The estimates of the structural model provide the basis for counterfactual policy analysis. In this subsection, I start by discussing the profitability of the implemented regimes 1 and 2, as well as the contribution of effort, experience, and match quality to profits under these regimes. Then, I consider the problem of maximizing the profits of the firm. I compare the results to those previously presented for regimes 1 and 2. Finally, I consider some counterfactual changes in the firm environment and their effect on profits. In particular, I consider a higher level of turnover costs than the one reported by the firm. I also evaluate the implications for profits and the optimal compensation policy when workers learn about their match quality before deciding whether to enter the firm.

The decomposition of profits is difficult because profits depend on both performance and the probability of staying, while the latter is a highly nonlinear function of effort, beliefs about match quality and experience. I approach the problem in the following way. The additive structure of the technology allows for isolating the contribution of effort, ability, and experience to profits. I define

$$\pi_\theta (R) = E_\theta \left\{ \sum_{t=1}^{\infty} \delta^{t-1} P_{it} (R, \theta_i, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1}) (r - \beta) \theta_i \right\}$$

where

$$P_{it} (R, \theta_i, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1}) = \frac{p_{it} (R, \theta_i, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1})}{1 - \sum_{t=1}^{\infty} \delta^{t-1} (1 - p_{it} (R, \theta_i, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1}))}$$

to be the profits associated with match quality. In a similar way,

$$\pi_l (R) = E_\theta \left\{ \sum_{t=1}^{\infty} \delta^{t-1} P_{it} (R, \theta_i, t, \{\varepsilon_{ik}, \xi_{ik}\}_{k=1}^{t-1}) (r - \beta) l (R) \right\}$$
\[ \pi_t(R) = E_\theta \left\{ \sum_{t=1}^{\infty} \delta^{t-1} P_{it}(R, \theta_i, t, \{ \varepsilon_{ik}, \xi_{ik} \}_{k=1}^{t-1}) (r - \beta) g(t) \right\} \]

\( \pi_t(R) \) and \( \pi_t(R) \) stand for the contribution to profits by effort and with experience, respectively. I take hourly wage as a benchmark regime with respect to which I evaluate how total profits and the contributions of effort, experience and match quality defined above change as the pay regime changes. An alternative approach that I also apply to the study of the effect of effort on profits is to compare profits under the same pay regime when effort affects performance and stay and when it is restricted to have no effect on them: the difference in the profits provides a conservative estimate for the contribution of effort to profits.

**Implemented Regimes**

Table 4.5 presents the pay policies that I analyze, along with profits\(^{10}\), effort, average match quality and average tenure per workstation under each of them. Table 4.6 considers the channels through which pay incentives affect profits. It considers the effects on the contributions of effort, match quality, and tenure to profits when switching from the initial hourly wage to some alternative pay regimes. Under an hourly wage, employees do not exert effort. Moreover, equal hourly pay implies that workers of different match quality are equally likely to quit at any tenure horizon. I fix the hourly wage to $9.5 which was actually implemented by the firm prior to January 2005. This hourly wage is clearly quite low relative to the mean of the outside offer and leads to a very high turnover: more than 93% of the employees last at most six months in the firm.

This high turnover leads to a low level of experience in the workforce as indicated by an average tenure of 3.23. Furthermore, the failure of the hourly wage to distinguish

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\(^{10}\)Recall that from the definition of profits, total profits stands for the discounted infinite sum of hourly profits starting from the month of hiring the worker.
between workers of high and low match quality leads to an average match quality of 1.99 calls per hour. Taken together, these effects of the hourly wage lead to total profits of $19.4.\textsuperscript{11}

Switching from the hourly wage to regime 1 induces all workers to exert effort of 0.58 calls per hour but also rewards workers of high match quality more than workers of low match quality. As discussed in Chapter 3, the result is that workers of high match quality stay longer in the firm than workers of low match quality. These differences lead to an increase in average match quality to 2.88 calls per hour. The net effect of the change in the compensation policy on the separation decisions is a decline in turnover illustrated with an increase in average tenure to 11.2 months. The retention of employees of high match quality, along with the decline in their probability of quitting at any tenure horizon leads to an impressive increase in $\pi_\theta$ by $90$. The lower turnover also leads to an increase in the profits associated with experience by approximately $49$. Finally, the introduction of the bonus rate of $3.3$ per successful call induces effort that generates profits associated with effort in the amount of $31.42$.\textsuperscript{12} Total profits jump to $167$. These numbers indicate that the increase in $\pi_\theta$, followed by the increase in $\pi_t$, rather than the increase in $\pi_l$ makes the greatest contribution to the increase in profits when switching from hourly wage to regime 1. The results suggest that the firm benefits considerably from the accumulation of workers of high match quality through turnover.

Next, I consider the effect of regime 2 on profits. Recall that this regime stipulates both lower base pay and lower piece rate. This less generous compensation policy leads to a sharp increase in the probability of quitting during the first six months.

\textsuperscript{11}Under the given specification of the stochastic technology and the utility, the profits associated with match quality $\pi_\theta$ are $28.89$ and the profits associated with experience $\pi_t$ are $10.4$.

\textsuperscript{12}Recall that the firm incurs a flat hourly pay of $\alpha$ and turnover costs which must be subtracted to obtain the total profits.
which approaches the levels under the hourly wage. The result is average tenure of 6.2 months, a decrease by more than 40% relative to regime 1, which implies also lower levels of accumulated experience. While the probability of quitting increases at each tenure horizon, the firm still retains workers of very high match quality, as discussed in Chapter 3. However, average match quality under regime 2 is not higher but slightly lower than average match quality under regime 1: 2.82 calls per hour. This result indicates that the negative effect of high turnover more than offsets the effect of retaining only the workers of highest match quality. At the same time, effort declines to 0.34 calls per hour. The combined effect of these factors implies that regime 2 yields much lower profits than regime 1. Despite the fact that the piece rate declines by $0.8 calls per hour, the profits associated with match quality are still $70 higher compared to their level under the hourly wage. However, as a result of the high rate of destruction of accumulated experience, \( \pi_t \) is quite close to its level under the hourly wage: it is only $16 higher. The profits associated with effort \( \pi_t \) are approximately $10, and total profits amount to about $110. Thus, under regime 2 match quality continues to be a crucial determinant of profits and the decline relative to regime 1 is smallest in the case of \( \pi_0 \).

**Optimal Regime**

The solution of the profit-maximization problem is the optimal pay regime \( R^w \) defined by \( \alpha_w = 3.64 \) and \( \beta_w = 3.26 \). Several factors affect its properties. While steep incentives induce more effort and increase the probability of staying, they also surrender a larger proportion of the revenues to the employees. Since quitting of an employee comes with the possibility of hiring a better one in the future, the firm chooses a pay schedule that among other things, balances the benefit from continued employment of a worker and the benefit from finding one of higher quality. The results here depend crucially on the
Figure 4.1: Ability under regime 1, 2, and the optimal regimes when the turnover cost is $750 and when it is the industry average of $8800.

The firm-specific nature of the match quality parameter: in particular, the ability of the firm to extract much of the surplus from the employment relation will be limited if workers can export their match quality $\theta$ to alternative jobs. The findings also depend to some extent on the simple nature of the compensation policy: for example, the properties of the optimal pay regime will change if the firm can condition base pay $\alpha$ on posterior beliefs about match quality.

The slope and base pay of the optimal pay regime $R^w$ are very close to those implemented under regime 1. The optimal pay regime $R^w$ induces considerable turnover: only about 55% of the employees stay more than six months in the firm. Furthermore, it not only induces a similar rate of turnover but also leads to a similar quality mix at different tenure horizons as regime 1. Figure 4.1 shows that the conditional distributions of match quality after six months under regime 1 and regime $R^w$ are almost identical.
Figure 4.2: Comparison between profits under regime 1, the optimal regime when turnover cost is $750 and when it is $8800. Turnover cost is $750.

It also indicates that only workers of high match quality (match quality greater than $\theta + \sigma_\theta$) experience little or no turnover. The small slope of incentives induces effort that translates into only 0.55 calls per hour, less than one standard deviation of match quality among starting employees. The distributions of expected profits under the optimal pay regime $R^w$ and regime 1 are again very similar but some differences are also present, as evident from Figure 4.2. Regime 1 generates more income on employees of average quality while regime $R^w$ generates more profits on the top performers. This pattern is explained by the fact that under regime $R^w$ the firm captures more of the surplus from the top performers while the slightly higher slope of incentives and base pay under regime 1 induce more workers of average ability to stay and work. The net effect is that optimal pay regime $R^w$ generates approximately 4.8% higher profits than regime 1.
Relative to regime 1, the optimal regime is less generous which leads to a small increase in the probability of quitting across posterior beliefs and tenure horizons. The result is average tenure of about 10 months, slightly lower than the 11.2 months under regime 1. This finding implies that the level of accumulated experience under the optimal regime $R^w$ is just below that for regime 1. The probability of quitting increases at each tenure horizon relative to regime 1, but the firm still retains workers of very high match quality, as discussed in the previous paragraph in the context of Figure 4.2. The net effect is that the average match quality under $R^w$ is 2.91 calls per hour, slightly higher than its counterpart for regime 1 which leads also to higher profits. Furthermore, shaving off 6 cents from the bonus rate reduces effort only to 0.55 calls per hour relative to the 0.58 call per hour under regime 1. Due to the improved quality mix, the small negative effect on turnover, and the small reduction in the variable costs, the contribution of match quality to profits, $\pi_\theta$, increases by $99 relative to its level under hourly wage. The contribution of tenure to profits, $\pi_t$, increases by $44 when switching from hourly wage to regime $R^w$. The contribution of effort, $\pi_l$, declines slightly relative to regime 1 to $31. Combining all of these with the costs of turnover and base pay yields total profits of $174. The results show that switching from the benchmark hourly wage to the optimal regime $R^w$ leads to an increase in profits by more than eight times, or in absolute terms by $154. The results show that much of
Table 4.6: Effects of different pay regimes relative to hourly wage.

<table>
<thead>
<tr>
<th>Pay policy:</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\Delta \pi_1$</th>
<th>$\Delta \pi_t$</th>
<th>$\Delta \pi_\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>regime 1</td>
<td>3.8</td>
<td>3.3</td>
<td>35.06</td>
<td>49.14</td>
<td>89.85</td>
</tr>
<tr>
<td>regime 2</td>
<td>3.5</td>
<td>2.8</td>
<td>9.74</td>
<td>15.68</td>
<td>70.33</td>
</tr>
<tr>
<td>regime $R^w$</td>
<td>3.65</td>
<td>3.24</td>
<td>31.42</td>
<td>99.32</td>
<td>99.32</td>
</tr>
<tr>
<td>regime $R^w$, known ability</td>
<td>3.65</td>
<td>3.24</td>
<td>33.31</td>
<td>49.93</td>
<td>131.18</td>
</tr>
<tr>
<td>regime $R^m$, known ability</td>
<td>3.74</td>
<td>3.09</td>
<td>29.18</td>
<td>48.37</td>
<td>140.57</td>
</tr>
<tr>
<td>regime $R^h$, high costs</td>
<td>1.82</td>
<td>5.44</td>
<td>191.04</td>
<td>56.20</td>
<td>23.42</td>
</tr>
</tbody>
</table>

Note: $\Delta \pi_x = \pi_x (R) - \pi_x (\bar{w})$, $x = l, t, \theta$.

This change is due to the effect of incentives on the quality mix of the workforce rather than the effect of incentives on effort.

An alternative approach to evaluate the effect of incentives on profits through effort is to compare profits under the optimal pay regime when effort affects performance and stay and when it is restricted to have no effect on them. My analysis starts with the quality mix, assuming that effort is not a channel through which incentives affect performance or separation decisions. Figure 4.3 shows that under the hourly wage of $9.5, implemented until January 2005, the firm makes losses on some workers of below average ability and its expected profits amount only to approximately $20, due to a high quitting rate and the associated costs. The figure also indicates that the introduction of the optimal regime induces high quality employees to stay while low quality employees to quit. The firm captures 75% of the additional surplus and profits increase by more than a factor of three.

Next, I relax the restriction that incentives do not affect effort, but still maintain that effort choice has not effect on separation decisions. Figure 4.3 shows that the exerted effort leads to an additional increase in profits by 114%. Finally, I also allow effort choice to affect separation decisions, but Figure 4.3 indicates that only but a few separation decisions remain unchanged: the combined effect of effort choice and match quality for those who switch from quitting to staying accounts for a 19% increase in profits. Thus, the total effect of switching from hourly wage to the benchmark rate results in
Figure 4.3: Gains from switching to the optimal regime from hourly wage. Turnover cost is $750.

![Graph showing gains from switching to the optimal regime from hourly wage. Turnover cost is $750.](image)

Figure 4.4: Comparison between profits under regime 1, the optimal regime when turnover cost is $750 and the optimal regime when it is $8800. Turnover cost of $8800.

![Graph showing comparison between profits under regime 1, the optimal regime when turnover cost is $750 and the optimal regime when it is $8800. Turnover cost of $8800.](image)
a dramatic increase in profits, but two-thirds of the increase would have materialized even if pay incentives did not affect effort choice or the separation decisions.

To summarize, these results show that most of the increase in profits from switching to the optimal pay regime can be traced back to the effect of incentives on the quality mix. Pay incentives not only induce high quality employees to stay but also act as a selection mechanism that helps the firm build a workforce of high match quality over time. In the present context, the firm exploits the firm-specific nature of the relation to capture most of the surplus generated by the employment relation.

4.6 Counterfactual Experiments

The turnover costs of $750 reported by the firm appear very low relative to industry averages published in Superb Staff Services (2011) which vary between $4,100 and $25,000. In this subsection, I explore the effect of high turnover costs on profits under the regime $R^w$, optimal under a turnover cost of $750, and search for the optimal regime under turnover costs equal to the industry average of $8,800. Furthermore, I study the effect on profits when the worker knows her match quality before deciding to start working but the employer does not.

4.6.1 Turnover Costs

The optimal pay regime $R^h$ when turnover costs are $8,800 is defined by $\alpha_h = 1.55$ and $\beta_h = 5.42$. These slope and base pay are much different from the one’s implemented by the firm. The high-powered incentives induce little turnover, mainly in the first two months of employment, and a high level of effort resulting in 2.91 calls per hour. Figure 4.4 presents profits under regimes 1, $R^w$ and $R^h$ when turnover costs are $8,800. A comparison of profits under pay regime $R^h$ and regime $R^w$ reveals that the two have a very
Figure 4.5: Gains from switching to the optimal regime from hourly wage. Turnover cost is $8800.

similar expected profits from the top performers, while regime $R^h$ accumulates much higher profits on the employees of low and average quality. Thus, the top performers capture much of the revenue under regime $R^h$, while the firm increases its profits from the higher effort exerted by employees who would not have stayed under regimes 1 or $R^w$. Table 4.5 shows that the low turnover rate leads not only to a high average tenure of about 20 months but also to a low average match quality of 2.17 calls per hour. In contrast to the case of the optimal contract when turnover cost is only $750, the profits under optimal regime $R^h$ come mainly from high levels of effort: $\pi_l$ for this regime is $191$. Still, total profits are only $162 because of the high turnover costs.

Next, I analyze the composition of profits. I start with the quality mix, assuming that implementing regime $R^h$ does not affect performance or separation decisions through effort choice. Figure 4.5 shows that the introduction of regime $R^h$, even in the absence of any effect through effort choice, allows many employees to remain in the
firm and in turn generate revenue of which more than 67% go to the workers. Then, I relax the restriction that incentives do not affect effort but still maintain that effort is not a channel through which incentives affect separation decisions. Figure 4.5 shows that, under these new restrictions, pay incentives induce effort that increases revenues considerably in contrast to the case of the optimal regime when turnover costs are $750. Finally, I allow effort choice to affect separation decisions. Given the assumptions of the model about the utility function, the regime induces higher effort and higher utility. Thus, the introduction of effort choice changes some but not all separation decisions: some workers who would have otherwise left now decide to stay. The employees who now stay contribute to profits with their match quality and effort: as evident from Figure 4.5, the effect is not negligible.

Summing up, the total effect of switching from hourly wage to regime \( R^e \) results in an impressive increase in profits, but only 27% of this growth would have materialized if pay incentives did not affect performance and separation decisions through effort choice. Thus, this counterfactual experiment indicates the sensitivity of the solution to the profit maximization problem to turnover costs.

### 4.6.2 Workers Know the Match Quality

Table 4.5 and 4.6 report average match quality, average tenure, profits, and their decomposition when workers know their match quality before deciding to join the firm. When regime \( R^w \), optimal when workers learn their match quality, is implemented in this environment, profits increase to $216. Much of this increase can be traced back to self-selection at entry: some workers know that their match with the firm is of low quality and decide to opt out for an alternative. Figure 4.6 shows the mean of the distribution of match quality at \( t = 1 \) under \( R^w \) when workers know their match
quality is 2.35 calls per hour compared to 2.01 calls per hour when they learn about it. As a result, the firm accumulates workers of high match quality faster than when workers learn about match quality and the average match quality increases to 3.12 calls per hour, while the average tenure increases to 12.3 months. These effects lead to a considerable increase of $141 in $\pi_\theta$ relative to its level under hourly wages, which is largely responsible for the increase of total profits to $216.  

The next step is to find the optimal pay regime when workers know their match quality before deciding to enter the firm. This problem is a special case of the more general model presented above: the prior belief is a degenerate distribution centered at the true value of match quality. The solution of the profit-maximization problem is the optimal pay regime $R^n$ defined by $\alpha_n = 3.74$ and $\beta_n = 3.09$. The results for this regime are reported in tables 4.5 and 4.6. The firm offers lower incentives to exert effort
Figure 4.7: Comparison between profits under the optimal regimes when workers know match quality at entry, when they learn about it, and when the latter is applied to an environment in which workers know it. Turnover cost of $8800.

In order to capture a greater share of the profits associated with match quality which is partially offset by a modest increase in the base pay. Thus, the growth in income and the variance of the distribution of income in this environment are smaller than their counterparts when workers learn about match quality. Still, one cannot generalize too much from this result because the properties of regime $R^a$ depend considerably on the restriction to search for the optimal regime within the family of linear contracts only. The average match quality under $R^a$ is 3.17 calls per hour and the distribution of match quality among the entering employees is not much different from that under $R^w$, as shown on Figure 4.6. Average tenure is 11.6 months, compared to 12.3 months under $R^w$, while effort amounts to only 0.47 calls per hour. Total profits are $221. Figure 4.7 presents profits under regime $R^a$ when workers know their match quality, under regime $R^w$ when workers learn about their match quality, and when they know it. It
shows that the profits under $R^n$ and $R^w$ when workers know their match quality are similar. This is driven by the fact that more or less the same type of people enter the firm under both regimes and all differences arise from the fact that $R^n$ shaves off more of the revenue from top performers by decreasing the bonus rate at the expense of a slightly higher turnover. Consequently, the results indicate that employers can benefit considerably if they can introduce a technology that helps workers find out their match quality before they decide to enter the firm.

4.7 Conclusion

This chapter considers a structural model of effort choice, learning about match quality, and turnover. It shows how such a model can be estimated with a two-step procedure that borrows ideas from the literature on estimation of dynamic structural models. The results indicate that employees are very responsive to pay incentives, impatient to postpone future consumption, and face a large variety of outside options primarily selected from low-skill service and manufacturing industries. Workers accumulate experience during the first six months on the job which improves performance. Still, variability in the quality of the employer-employee match accounts for most variation in performance across individuals under a given pay regime. The chapter examines a variety of regimes to find that the firm maximizes profits by selecting and keeping the high quality employees, even at the expense of inducing low effort. It also shows that most gains from switching to the optimal pay regime from an hourly wage can be traced to the improvement in the match quality of the workforce.

In this chapter, I focused on contracts that are linear in the performance signal, both for simplicity and because such linear contracts are commonplace. In future work I will allow for contracts that are nonlinear in the performance signal. Given the presence of
learning about match quality and the accumulation of experience, it is likely that the firm would also optimally choose to have the compensation scheme vary with tenure. Moreover, the optimal compensation schedule may depend on all past performance signals, possibly through a sufficient statistic such as their average. This also calls for further work. The social surplus per workstation in a given period is the collected revenue minus disutility denominated in dollars and turnover costs; there is no reason to expect the profit-maximizing regime to be socially optimal. I plan to characterize the socially optimal scheme and compare it to the profit-maximizing scheme. Finally, my approach should allow me to explore possible gender, race, and age differences in the response to pay incentives, on which there is scant evidence so far.
4.8 Appendix C

There are 18 moment conditions and 5 parameters to be estimated. The conditions are nonlinear which complicates identification. By the assumption on the approximation of \( G(\mu_{it}, R_{it}, t) \), the minimum distance problem has a solution. The following discussion addresses the uniqueness of that solution. The discount factor is identified from variation in \( G(\mu_{it}, R_{it}, t + 1) \) with \( t \) that is associated with changes in the precision of beliefs. The results from step 1 show that by \( t = 12 \) the accumulation of experience has come to an end. Therefore, conditional on the information available at \( t, t > 12 \),

\[
U(\mu_{it+1}, R_{it}, t, \gamma, \psi) - U(\mu_{it}, R_{it}, t, \gamma, \psi) = 0.
\]

Consequently,

\[
G(\mu_{it+1}, R_{it}, t + 1) - G(\mu_{it}, R_{it}, t)
= \delta [E_{\mu_{it+1}}(\lambda (G(\mu_{it+1}, R_{it}, t + 1)) - E_{\mu_{it+2}}(\lambda (G(\mu_{it+2}, R_{it}, t + 2)))]
\]

Conditional on the information available at \( t \) and \( R_{it} = R_{it+1} \), variation in \( G(\mu_{it}, R_{it}, t) \) across periods originates from changes in the precision of posterior beliefs. Therefore, variation in the first differences on the left-hand and the right-hand side of the condition above identifies the discount factor.

Given \( \delta, \sigma_{\xi^*} \) is identified from variation in \( G(\mu_{it}, R_{it}, t + 1) \) that is associated with accumulated experience and variation in the means of the posterior beliefs. If \( R_{it} = R_{it+1} \),

\[
U(\mu_{it+1}, R_{it}, t, \gamma, \psi) - U(\mu_{it}, R_{it}, t, \gamma, \psi) = \beta_{it} (\mu_{it+1} + g(t + 1) - \mu_{it} - g(t)).
\]
Then,

$$G(\mu_{it+1}, R_{it}, t + 1) - G(\mu_{it}, R_{it}, t)$$

$$= \frac{1}{\sigma_{\xi^*}} \left( U(\mu_{it+1}, R_{it}, t, \gamma, \psi) - U(\mu_{it}, R_{it}, t, \gamma, \psi) \right)$$

$$+ \delta \left[ E_{\mu_{it+1}}(\lambda(G(\mu_{it+1}, R_{it}, t + 1))) - E_{\mu_{it+2}}(\lambda(G(\mu_{it+2}, R_{it}, t + 2))) \right]$$

Therefore, variation in the first differences of expected utility and in the first-difference in the left-hand side in the above condition identifies $\sigma_{\xi^*}$, with changes in beliefs and experience when the pay regime does not change variation in $H(\theta_{it}, R, t)$ originates from the accumulated experience and variation in beliefs, so $\sigma_{\xi}$ is identified from the ratio of the first-difference in

$$H(\theta_{it}, R, t) - \delta E_{\theta_{it+1}|\theta_{it}}(\lambda(H(\theta_{it}, R, t + 1)))$$

and the first-difference in $U(\theta_{it}, R_{it}, t)$. Given $\delta$ and $\sigma_{\xi^*}$, the structural parameters $\gamma$ and $\psi$ are identified from $\gamma = \gamma(\psi, \Delta l)$ and from variation in $G(\mu_{it}, R_{it}, t)$ with $t$ that is associated with changes in the pay regime. Given $\delta$, $\sigma_{\xi^*}, \gamma, \psi$, the mean of the outside offer is identified from variation in $G(\mu_{it}, R_{it}, t)$ and the definition of the VNM utility.
Bibliography


