Chapter 2

Interaction between rotating tearing modes and non-axisymmetric fields in EXTRAP T2R

The work in this chapter was born out of an idea by F. Volpe to use the frequency modulation of a naturally rotating tearing mode to detect error fields. The experiments were conducted during a two week visit to EXTRAP T2R by the author, and under the direction of L. Frassinetti, F. Volpe, and P. Brunsell. Fourier decomposition codes developed by L. Frassinetti are used to Fourier decompose the magnetics. Sensor analysis by P. Brunsell is used to map from the measured phase of a Fourier mode to its position in real space. The author used ordering arguments to simplify the model in [15] to explain the experimental results. The signal processing and error analysis techniques were developed by the author, as well as the methods to independently validate the error field. The comparison to theory and extrapolation to ITER were written by the author, with input from all coauthors (F. Volpe, L. Frassinetti, P. Brunsell, and R. Fridstöm). This chapter is published in Plasma Physics and Controlled Fusion [81], and is also available in the arXiv [82].
In this chapter, the first demonstration of error field identification using naturally rotating tearing modes is presented. Error fields are a concern for existing and future magnetic confinement fusion plasmas, and this work provides an additional tool by which error fields might be identified. This technique can be deployed in high performance plasmas, and has the potential to operate in real-time.

2.1 Introduction

Error fields are non-axisymmetric fields that result from mis-shaped or misaligned coils and their current feeds. Error fields (EFs) with low-order toroidal harmonics \( (n) \) and of small relative amplitudes, of the order of \( 10^{-4} \) of the equilibrium field, are found to cause deleterious effects to tokamak plasmas [83, 84, 19]. In reversed field pinches, these field errors are observed to modify the plasma rotation [85] and can cause rotating tearing modes to lock [9].

Various techniques have been demonstrated to measure components of EFs, each with a different set of strengths and weaknesses. Perhaps the most routinely used in tokamaks to date is the low density locked mode onset technique commonly referred to as the 'compass scan' technique [83, 86]. Error fields are identified in a low density plasmas by measuring the critical applied \( n = 1 \) field that drives an error field penetration induced locked mode at three or more phases of the applied field. The intrinsic error field is then determined from the location and magnitude of the asymmetry in the critical field as a function of toroidal phase. This method is effective at measuring EFs in low density Ohmic discharges, and has improved stability in high beta H-modes in tokamaks. Vacuum measurements of non-axisymmetric fields can also be made using an in vessel apparatus to accurately measure fields at the location of the plasma, usually during a vessel vent as has been done in DIII-D [87, 88]. This method is effective at measuring vacuum EFs, but it does not account for deviations of the equilibrium coils due to thermal expansion and \( \mathbf{j} \times \mathbf{B} \) forces resulting from the plasma current.

The experiments in this chapter were conducted on the EXTRAP T2R reversed
field pinch (RFP) [14]. Consistent with the RFP literature, perturbations are Fourier analyzed using the form $e^{i(m\theta + n\phi)}$, which leads to the resonance condition when the safety factor takes on a value $q(r) \equiv -m/n$. In these discharges, $1 \gg q(0) > 0$ on axis and decreases monotonically passing through 0 as the toroidal field $B_T$ reverses at $r/a \approx 0.85$, where $a$ is the plasma minor radius, and attains negative values in the range $0 > q(a) \gg -1$ at the plasma edge. The reversal of the toroidal field can be seen in figure 2-1, where the core field points in the counter-clockwise direction in the core, and in the clockwise direction at the edge. The reversal surface is where the field is purely poloidal. As a result of $|q| \ll 1$ everywhere, the main resonant harmonics have $m = 1$ and $|n| \sim 10$. Taking $m$ always positive, the toroidal harmonics are therefore such that $n < 0$ and $n > 0$ inside and outside of the field reversal surface respectively. For typical EXTRAP T2R plasma configurations, $m = 1$ and $n \leq -12$ are TMs inside the reversal surface, $-12 < n < n_{edge}$ are RWMs, and $n > n_{edge}$ are TMs outside the reversal surface [89]. The value of $n$ at the edge is given by $n_{edge} = -1/q_{edge}$ (note that since $q_{edge} < 0$, $n_{edge} > 0$). The most internal resonance is the $m/n = 1/ -12$ which hosts the TM that will be studied in this chapter.

The magnetohydrodynamic (MHD) modes that are often driven by field errors may also be used to diagnose them. In EXTRAP T2R, a stable kink was driven using a resonant magnetic perturbation (RMP) rotating at 50 Hz and the non-uniform
rotation and amplitude of the kink was used to diagnose the resonant component of the intrinsic EF [90]. In the DIII-D tokamak, both EF penetration locked modes and saturated locked modes were rotated with RMPs and the EF characterized by their rotation dynamics [91, 92, 80].

Like the stable kink, EF penetration induced locked mode, and saturated locked mode used for EF identification, naturally rotating saturated tearing modes (TMs) in EXTRAP T2R are also observed to interact with static resonant fields. The velocity and amplitude of the naturally rotating $m/n = 1/ -12$ TM in EXTRAP T2R is observed to modulate in the presence of a resonant field that is $\sim 10^{-3}$ smaller than the equilibrium field [15].

Here we investigate the use of the modulations in the $m/n = 1/ -12$ Fourier coefficient of the poloidal field $b_{g}^{1-12}$ of these naturally rotating TMs to measure externally applied resonant fields at the rational surface (details on the magnetic diagnostics and analyses used are presented in section 2.2). The use of saturated TMs allows probing of field components $(m, n)$ that are not accessible to the driven external kink technique mentioned above [90]. This technique is suitable for real-time EF correction (EFC) in plasmas where rotating TMs are present, either because they are unsuppressed or because they are "intentional", as is the case in the tokamak 'hybrid' scenario [93, 94]. When TMs are undesirable (as is often the case), this method is intended to validate, or improve the error field correction only when a TM appears. The ability to directly measure resonant fields in H-mode would remove the need to extrapolate measurements made in L-mode.

Although the perturbed TM velocity data appear less sensitive to the external fields than the perturbed amplitude data in these experiments, it should be noted that these are an additional source of external field information. Future EF identification algorithms similar to this one might use perturbed velocity data in place of poloidal field data as cylindrical modeling suggests that this approach might be more applicable to ITER (see section 2.7.2).

This chapter is organized as follows. In section 2.2, the experimental setup is described. In section 2.3, an analytic first-order model describing the time varying width
of a naturally rotating TM in the presence of an external resonant field is derived. In section 2.4, the methods used to analyze the experimental data are described. In section 2.5, the phase identification technique is validated by measuring the known phase of an applied RMP. In section 2.6, the technique is used to measure the amplitude and phase of the unknown \( m/n = 1/ - 12 \) intrinsic EF of EXTRAP T2R. In section 2.7, limitations of the model and the analysis are discussed, and separately the potential of this and similar techniques for use in ITER are briefly investigated. Appendix B is dedicated to relating the two coordinate systems used in this chapter, and Appendix C to consolidating all angle parameters in a single table with brief definitions. The reader is encouraged to refer to Appendix C as needed.
2.2 Experimental setup

EXTRAP T2R is a reversed field pinch with major and minor radii $R = 1.24$ m and $r = 0.183$ m. Magnetic feedback on unstable modes in EXTRAP T2R allows plasma discharges of duration $\approx 70 - 90$ ms. In the experiments presented here we have $I_p \approx 90$ kA, $n_e \approx (0.5 - 1.0) \times 10^{19}$ m$^{-3}$, and $T_e \approx 200 - 400$ eV. The stainless steel vacuum vessel is located at $r_v = 0.192$ m with a resistive diffusion time constant for the $m/n = 1/ - 12$ mode of $\tau_v \approx 58$ µs as reported in [95]. Outside of the vacuum vessel at $r_w = 0.198$ m is a concentric copper shell consisting of two layers with a total thickness of $\delta_w = 1$ mm and a resistive diffusion time constant of $\tau_w \approx 2 \times 6.3 \approx 13$ ms [14], which is in good agreement with the theoretical value $\tau_w = \mu_0 \sigma_w r_w \delta_w \approx 13.8$ ms.

![Diagram of a poloidal slice of EXTRAP T2R showing the radius of the limiters $a$, the vacuum vessel at $r_v$, the copper shells located at $r_w$, the radial and poloidal field sensors at $r_{sensor}$ and $r_{ls}$, and the active control coils at $r_c$. Figure taken from reference [9]. ©IOP publishing. Reproduced with permission. All rights reserved.](image)

Tearing mode dynamics are measured by $4$ (poloidal) $\times$ $64$ (toroidal) magnetic
probes [96, 15] measuring the local poloidal field $b_\theta(\mathbf{x}, t)$ at position $\mathbf{x}$ and time $t$. The probes are positioned between the vacuum vessel and the copper shell, as shown in figure 2-2 (labeled as “local sensors for TMs”). The rotation frequencies of interest are such that $\omega^{1,-12} \sim \tau_v^{-1}$ and therefore compensation of vessel eddy currents is important. To decouple the effect of eddy currents, the vessel is assumed to be a first-order low-pass filter with transfer function $H(f) = 1 + \frac{if}{f_c}$ for signals with frequency $f$ and where $f_c = 1/2\pi \tau_v = 2.7$ kHz [95] is the cutoff frequency (see reference [96] for details). The magnetics are compensated by applying an inverse filter. An additional array of $4 \times 32$ saddle loops located inside the copper shells, shown in blue in figure 2-3, are used to measure radial magnetic fields on slow timescales (less than or equal to $\tau_w^{-1}$), which are suppressed by feedback in normal operation by a complementary array of $4 \times 32$ actuator coils outside the shells, shown in red. The revised intelligent shell algorithm [39, 40], a proportional-integral-derivative based algorithms, takes the radial field from the saddle loops as input and can be programmed to suppress all harmonics, or to fix chosen harmonics to a given set-point, as will be done here with the $m/n = 1/ -12$ harmonic. Choosing a set amplitude and phase in this algorithm for the $m/n = 1/ -12$ field is different from applying a constant $m/n = 1/ -12$ field with the actuator coils as the feedback takes into account the plasma response since it measures the total field, driving the total field (RMP and plasma response) toward the requested set-point.
2.2.1 Notations for rotation frequencies and toroidal phases of TMs

When working with TMs with $|n| \neq 1$, two frequencies and corresponding phases may be used to describe their toroidal motion. A magnetic sensor fixed to the vessel will measure a field that oscillates like $\sin(\delta - n\omega^{1,-12}t)$ where $\omega^{1,-12}$ is the toroidal plasma rotation frequency at the rational surface, assuming the TM is entrained in the plasma flow, and $\delta$ is an arbitrary offset depending on the sensor position and the initial position of the TM. A second frequency can be defined which treats one sinusoidal oscillation in the magnetics as one period, and is thus defined as $\omega \equiv n\omega^{1,-12}$. Correspondingly, we can time integrate this relationship to find $\phi \equiv n\phi^{1,-12}$. All references to rotation frequencies $\omega$ and toroidal angles $\phi$ in following sections will refer to $\omega$ and $\phi$ as defined here (i.e. $\omega^{1,-12}$ and $\phi^{1,-12}$ will not be used outside of section 2.2). Note that this applies to all angles and frequencies, including the TM, the applied RMP, and the intrinsic EF.

The $\phi$ position of a continuous field is ambiguous, so here we will explicitly define what the $\phi$ position of all $m/n = 1/ - 12$ fields in this chapter means. Assuming the existence of some $m/n = 1/ - 12$ field that we call $A$, we take the toroidal angle where the radial field is maximally directed outward at the outboard mid-plane to be $\phi_A$. Note that although this point is twelve times degenerate in the $\phi^{1,-12}$ coordinate system on the domain $0 \leq \phi^{1,-12} < 2\pi$, it is unique in the $\phi$ coordinate system on the domain $0 \leq \phi < 2\pi$ (recall that $\phi \equiv n\phi^{1,-12}$).

Finally, all field magnitudes in what follows will refer to the $m/n = 1/ - 12$ Fourier coefficient. Superscripts 1,-12 will appear in few places for clarity, but the reader should interpret all field magnitudes without superscripts as the $m/n = 1/ - 12$ Fourier coefficient.

2.2.2 Discharge design

By $t = 20$ ms, the transients associated with startup have decayed and the plasma has settled into an approximately constant equilibrium. At this time, we program
Figure 2-4: Time traces of $B_{RMP}$ with $m = 1$ and various $n$ harmonics measured by the saddle loops. The purple trace shows the programmed $m/n = 1/12$ field which increases to finite amplitude at 20 ms, and decreases again at 40 ms. All other unstable $n$ harmonics are suppressed by the feedback.

the feedback to apply a static $m/n = 1/12$ RMP of constant amplitude $B_{RMP}$, as shown in figure 2-4. The RMP is applied between $t = 20 - 40$ ms, and is the time window over which all of the following analysis is done.

The phase of the applied RMP $\phi_{RMP}$ is changed between shots providing 20 ms of interaction between the TM and RMP for each $\phi_{RMP}$. A scan of 9 phases spanning 0 to $2\pi$ was completed with an amplitude of $B_{RMP} = 2$ G, and a scan of 7 phases spanning 0 to $2\pi$ was completed for a 50% stronger RMP with $B_{RMP} = 3$ G. The results of these two scans are detailed in section 2.5 to demonstrate phase identification of a known RMP. To motivate the technique that is used, we now introduce a simple analytic model to describe the expected TM behavior.
2.3 Model of fast TM/EF interaction

While in reference [15] the Modified Rutherford Equation (MRE) and equation of motion are solved numerically, here we will model only the island width behavior using the MRE and seek an analytic first-order perturbation expansion solution. Although we will see that the equation of motion and the MRE are coupled, the effect of the coupling is expected to be second order and is omitted from this model.

The estimated TM width $W_0$ is found to be comparable with the linear layer width. This implies that the TM is weakly nonlinear as parameterized by $\lambda \sim (s/W_0)^{3/2} \sim 0.3$ (see equation 101 in reference[64]). Despite being weakly nonlinear, the Modified Rutherford Equation is used to describe the observed TM amplitude dynamics.

2.3.1 Modified Rutherford Equation

Similar to [15], we take the following form for the Modified Rutherford Equation (MRE) not including the bootstrap current term (that is, describing classical TMs, not neoclassical TMs),

$$\frac{\tau_R}{r_s} \frac{dW}{dt} = \Delta'(W)r_s + \Gamma \frac{W^2_v}{W^2} e^{i\Delta\phi(t)} \tag{2.1}$$

where $\tau_R = \mu_0 r_s^2/\eta$ is the resistive diffusion timescale, $r_s$ is the minor radius of the rational surface, $W$ is the island width, $\Delta'(W)$ is the classical stability index which depends on the island width, $\Gamma$ is a function that depends on the geometry of the fields and the boundary conditions at conducting surfaces (see Appendix A in reference [15]), $W_v$ is the vacuum island width driven by an external resonant field, and $\Delta\phi(t)$ is the toroidal angle between O-points of the vacuum island and plasma island (the vacuum island is found by superimposing the external field on the equilibrium field in vacuum). The neoclassical bootstrap term and the stabilizing effect of pressure-curvature [42] often included in modeling of high beta devices is considered negligible here as the pressure is relatively low and the inverse aspect ratio is small.

Complex notation is used in equation 2.1 though only the real part has physi-
cal meaning, and throughout this chapter we will only consider the real part of all equations and quantities, including the island width $W$.

As in [97, 64], we express $\Delta'(W)$ as a constant plus a linear term in the island width,

$$\Delta'(W) = C_0 - C_1 \frac{W}{r_s}$$

(2.2)

where $C_0$ and $C_1$ are dimensionless constants. We will be concerned with saturated rotating islands and relatively weak external fields such that $W > W_v$. We will therefore take $\Gamma(W_v/W)^2 \sim \epsilon$, where $\epsilon$ is a small quantity used for ordering. The zeroth order saturated island width is given by,

$$W_0 = C_0 r_s$$

(2.3)

We are now interested in small perturbations about $W_0$. Substituting $W = W_0 + \epsilon W_1$ in equation 2.1, and retaining only terms of order $\epsilon$, we find,

$$\frac{\tau_R}{r} \frac{dW_1}{dt} = -C_1 \frac{W_1}{r} + \Gamma \frac{W_v^2}{W_0^2} \epsilon \Delta \phi(t)$$

(2.4)

where we have used the assumption that $\Gamma(W_v/W)^2 \sim \epsilon$, and kept only the zeroth order Taylor expansion of this term.

The external field term involving $\Delta \phi(t)$ is responsible for coupling the MRE and the equation of motion in this high-frequency regime where $\omega_o \gg \tau_w^{-1}$ (at lower frequencies, couplings also occur due to resistive eddy currents in the wall [98]). Although the rotation of the TM is not uniform in the presence of an external field, the perturbations to $\phi(t)$ are observed to be relatively small. As we have already taken the second term on the right-hand-side of equation 2.4 to be order $\epsilon$, the effect of this small perturbation in $\Delta \phi(t)$ is order $\epsilon^2$ and therefore omitted. That is to say that in this model for the perturbed island width, we assume uniform rotation. Taking $\Delta \phi(t) \approx \omega_o t$, where the arbitrary initial phase is chosen such that $\Delta \phi(0) = 0$ (the TM and EF are aligned at $t = 0$), we find,
\[
\frac{W_1(t)}{r} = \left(\frac{W_v}{W_0}\right)^2 \frac{\Gamma}{\tau_R} \left[ \frac{(C_1/\tau_R) - i\omega_0}{\omega_0^2 + (C_1/\tau_R)^2} \right] e^{i\omega_0 t}
\]  

(2.5)

From equation 2.5 we see that the oscillation in island width maximizes on the domain \(\Delta \phi = [0, \pi/2]\) (recall that \(\Delta \phi(t)\) and \(\omega_s t\) are approximately interchangeable). The exact phase, referred to as \(\Delta \phi_{max}\), depends on the relative sizes of \(C_1/\tau_R\) and \(\omega_o\).

We now write the full time dependent island evolution as,

\[
W(t) = W_0 + W_1(t)
\]  

(2.6)

We know that \(W \propto \sqrt{b_\theta}\) where \(b_\theta\) here is the \(m/n = 1/ - 12\) Fourier coefficient [64]. Thus, taking the square of equation 2.6 and omitting terms of order \(W_1^2\),

\[
b_\theta(t) \approx W_0^2 + 2W_0W_1(t) = b_{\theta 0} + b_{\theta 1}
\]  

(2.7)

We now have a model for the time dependent poloidal field of the TM. In summary, in this section we have seen that \(b_\theta\) oscillates once per rotation period, and the phase at which \(b_\theta\) is maximum contains information on the toroidal phase of the EF in the lab frame.
2.4 Methods

2.4.1 Data filtering

First, the raw $dB/dt$ data are time integrated and compensated for wall eddy currents as discussed in section 2.2. From the model developed in section 2.3, we expect the amplitude of the $m/n = 1/ - 12$ TM to oscillate with frequency $\omega_o$. The amplitude and phase of the TM are also affected by other mechanisms, in particular the sawtooth oscillations. The sawtooth instability in the RFP is thought to be due to interaction of core $m = 1$ tearing modes, and might be responsible for the toroidal magnetic flux generation [99]. The TM dynamics associated with the sawtooth oscillations are not of interest here, and therefore the data are filtered to remove these transient events (the sawtooth oscillation time in EXTRAP T2R is typically 0.3-0.4 ms [15]). A high-pass filter is implemented by simply subtracting from the time-integrated magnetics, a copy of the signals smoothed over the characteristic rotation period time. A second high-pass filter is then implemented by Fourier transforming into the frequency domain, zeroing the spectrum below 30 kHz, and performing an inverse transform on the truncated spectrum. At this point the data are Fourier analyzed in space, and all following analysis will refer to the $m/n = 1/ - 12$ TM.

![Figure 2-5: The amplitude of the 1/-12 TM $b_\theta$ (green) prior to filtering. Several periods of rotation are shown, as indicated by the TM toroidal phase $\phi$ in blue. Note that the frequency filtering for all following analysis is done prior to spatial Fourier fitting, unlike the unfiltered 1/-12 amplitude trace shown here. A single period of the post-filtered data is shown in figure 2-7 in the following section.](image)

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The integrated $m/n = 1/ - 12$ field amplitude is now low-pass filtered with a
cutoff frequency of 110 kHz to remove high-frequency noise. Next, a rotation filter is
applied, which is based upon the observed TM rotation, as measured by the magnetics.
The data must satisfy the following two conditions to be included in the analysis;
(1) $\phi$ must complete full rotations from 0 to $2\pi$, and (2) the time derivative of $\phi$
must remain above a specified threshold. Condition (1) removes times when the
amplitude modulation becomes larger than the unperturbed amplitude, where the
mode repeatedly appears and disappears [15]. Condition (2) ensures that the mode
is not locked. Further, conditions (1) and (2) must be satisfied for a duration $\tau \geq 0.1$
ms, which ensures that the majority of the rotation data are temporally isolated from
nearby locking events. Figure 2-6 shows periods of suitable rotation data, marked
with red bars, and periods where these conditions are not met (the regions between
the red bars). After the high-pass and rotation filters, we notate the resulting data
$b_\phi$ and $\phi$. Note that we have dropped the superscripts denoting $1,-12$ on $b_\phi$.

Figure 2-6: The phase of the $1/-12$ TM over many periods of rotation. The TM is
considered to make a full toroidal rotation if the phase intersects each of the dashed
lines in succession. Periods of suitable rotation for analysis are marked with red
horizontal bars. Periods not marked with red bars are excluded from the analysis.
2.4.2 Feature extraction

We now proceed with the physical analysis of these signals. A semi-empirical algorithm is developed to process the $b_\theta$ and $\phi$ data, returning an estimate of the external resonant field phase (e.g. the error field phase $\phi_{EF}$, the RMP phase $\phi_{RMP}$, or their superposition $\phi_{EF+RMP}$) with which the TM interacts. The algorithm is based on the premise that $b_\theta$ is expected to reach a single maxima during each period of rotation (see equation 2.7).

Motivated by this discussion, we look for the phase of the TM at which $b_\theta$ is maximized in each rotation period. Figure 2-7 shows $b_{\theta 1}$ (i.e. only the time-varying portion of the TM Fourier coefficient) and $\phi$ during one rotation period in the presence of a $B_r = 2$ G RMP applied with static phase. The vertical dashed line intersects both the maximum of $b_{\theta 1}$, and the phase $\phi$ at which this maximum is achieved. We refer to this phase in rotation period $i$ as $\phi_{\text{max}}^i$. The process of identifying $\phi_{\text{max}}^i$ is then repeated for all rotation periods, providing $\sim 1,500$ values of $\phi_{\text{max}}^i$ for a given phase of the external resonant field (a typical shot contains 1 to 2 thousand rotation periods).

All estimates of $\phi_{\text{max}}^i$ for a given external field phase are now binned in toroidal angle. These histograms are mapped onto a polar plot, as shown in figure 2-8. Note that this polar plot spans 30 degrees in real space, such that an $n = 12$ perturbation will appear $n = 1$ on this plot. Each bin in figure 2-8 is represented by a red point. The counts in each bin are normalized by the total number of counts across all bins,
thus representing the fraction of counts $f_c$. The line segments intersecting each point correspond to the Poisson counting error. The value of $f_c^j$ for bin $j$ determines the radial distance of the red point from the origin. A TM that behaved exactly according to our model (equation 2.7) would produce a distribution where all $\phi_{\text{max}}^i$ fall within a single bin $j$ with $f_c^j = 1$. The distribution in figure 2-8 is clearly much broader than this expected distribution, suggesting that our model is too simple to capture all of the physics here. Possible explanations of the broadened distribution will be discussed in section 2.7.

However, although simplistic, the model in section 2.3 might explain why the distribution of $\phi_{\text{max}}^i$ does peak at a specific $\Delta \phi$ (i.e. $\Delta \phi_{\text{max}}$). As our physical model does not describe the shapes of these polar histograms, we employ a simple approach to extract the external field phase from the histogram data. We attribute equal 'mass' to each data point in figure 2-8 and calculate the center of mass in the polar plane. The phasor that points to it will be called the 'centroid phasor'\(^1\) (the red phasor in figure 2-8 is the centroid phasor scaled by four for visual purposes). The toroidal angle of this centroid phasor is then used as an estimate of the external field phase. The black phasor in figure 2-8 shows the phase of the applied RMP. The magnitude of the black phasor is arbitrary and therefore should not be compared with the magnitude

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\(^1\)If the points were small coins of equal mass, the distribution would balance if suspended from the end of the centroid phasor.
of the red phasor.

2.4.3 Statistical uncertainty

A Monte-Carlo technique is used to quantify the uncertainty in the centroid phase: each bin value is varied about the measured value according to a normal distribution. The standard deviation of such distribution is given by the Poisson counting error, shown by the line segments intersecting the red histogram data in figure 2-8. The histograms are perturbed in this way 200 times, and the phase of the centroid recalculated for each perturbed distribution. The standard deviation of the 200 centroid phases is then used to provide an uncertainty in the measured centroid phase. This is not shown in figure 2-8, but will appear as an error bar in figures 2-10 and 2-12.
Figure 2-9: Polar histograms plotted for an 9 shot scan (8 shown for visual purposes) of the phase of an applied RMP with amplitude $B_r = 2$ G. See figure 2-8 for a detailed explanation of an enlarged version of figure 2-9e. The center-of-mass centroids in these subfigures (red arrow) are scaled by 4 for visual purposes. (a) Polar histogram for $\phi_{RMP} = 0.375 \pi \equiv \phi_o$ (note that this corresponds to $\Phi_{RMP} = 0$; see appendix B and section 2.2). (b-h) Polar histograms for each $\pi/4$ increment of the toroidal position of the RMP completing a $2\pi$ scan. Note that the polar plots here span $30^\circ$ in real space (i.e. what appears $n = 1$ here is $n = 12$ in real space).

2.5 Passive EF phase identification

Here the object of the measurement - the external field - is the resultant of the residual $m/n = 1/ - 12$ EF and of applied RMPs of the same $m$ and $n$. The measurement is passive in the sense that it is not necessary to actively probe the system with applied RMPs. For the sake of validation, we will apply known RMP fields and measure them with this technique. In this section we report the results from two sets of discharges where the toroidal phase of a $m/n = 1/ - 12$ RMP is varied between discharges, and the amplitude of the RMP is changed between the two sets. In each discharge, the RMP is applied between $t = 20$ and 40 ms with static phase. All other harmonics of the slowly varying radial field are suppressed by the feedback.

The results of the $B_r = 2$ G RMP phase scan are shown in figure 2-9. The applied RMP starts at $\phi_{RMP} = \phi_o \approx 0.33 \pi$ and is incremented in steps of $\pi/4$ until
Figure 2-10: (a) The measured phase of the centroids $\phi_{meas}$ from figure 2-9 are plotted as a function of the applied RMP phase. The dashed line shows where $\phi_{meas} = \phi_{RMP}$. Error bars are not shown here to better visualize the overlapping data points, but these data points correspond to the data points in (b) where the error bars are shown. (b) The data from (a) are shifted down by $\Delta \phi_{max} = 0.1\pi$ (referred to as $\phi_{res}$ after this shift), and are observed to oscillate about the dashed line $\phi_{res} = \phi_{RMP}$. Both datasets are fit to equation 2.8 shown by the solid curves. These fits provide estimates of the EF amplitude and phase which are reported in figure 2-11.

Completing a full scan (note $\phi_o$ corresponds to $\Phi = 0$ in the active coil coordinate system used by the feedback; see appendix B). Recall that a complete scan of $\phi$ from 0 to $2\pi$ here corresponds to a span of $2\pi/12$ in real space. The red centroid phasors, shown by the red arrows in the subpanels of figure 2-9, are observed to lead the black RMP phasors by up to $\Delta \phi \approx \pi/4$. Note the systematic increase of $\Delta \phi$ from subpanel (c) to (h), followed by a decrease from (h) to (a), (b), and (c).

In figure 2-10a, the measured centroid phase is plotted as a function of the RMP phase for the $B_{RMP} = 2$ G scan and $B_{RMP} = 3$ G scan (open blue and red triangles). It is clear that the measured phase is tracking the phase of the RMP up to a small constant offset (seen by the general vertical shift of the data above the dashed line) and a systematically varying offset (seen by the approximately sinusoidal variation of the data). The vertical shift is the quantity $\Delta \phi_{max}$ introduced in section 2.3, and is not expected to depend on the magnitude of the RMP. We will show in the following section that $\Delta \phi_{max}$ can be measured once with a given RMP, and then
used for discharges with stronger or weaker RMPs. Further, we will also show that the sinusoidal deviation is the result of an intrinsic EF in EXTRAP T2R. Prior to quantifying $\Delta \phi_{\text{max}}$ and identifying the intrinsic EF, it is difficult to deduce how accurately this passive technique identifies the phase of the applied RMP. In the following section we account for both $\Delta \phi_{\text{max}}$ and the intrinsic EF, and show that the phase resolution of the active technique is $\pm 0.08 \pi$ radians. It happens that this is also the phase resolution of the passive technique, since the phase resolution of the active technique is constrained by discharges without an applied RMP (see the magenta contours in figure 2-11). To conclude, we have shown here that the passive EF identification method follows the phase of the applied RMP well, and we will show in the following section that the phase resolution is $\pm 0.08 \pi$ radians. Recall that this passive technique can only identify the phase of the EF (or the superposition of the intrinsic EF and the applied RMP in this case), but not the amplitude.
2.6 Active amplitude and phase identification of intrinsic EF

Here, we consider the amplitude and phase of the RMP to be known, and instead search for the existence of an unknown resonant field (e.g., an EF). In addition to identifying the phase of the unknown field as in section 2.5, the presence of a known RMP will allow us to also measure the amplitude of the unknown EF. The two sets of shots from section 2.5 ($B_{RMP} = 2, 3$ G) will be used again here (now considering the applied RMPs as known), as well as three additional sets with $B_{RMP} = 0, 0.5,$ and $1$ G. The discharge with no applied RMP measures only the phase of the intrinsic EF for the reasons just discussed (a mathematical justification will be given later in this section).

Recall that all analysis reported in this chapter refers to the time-interval between $t = 20$ and $40$ ms in the shot cycle, where we expect the plasma to be in equilibrium. Assuming that the currents in the equilibrium coils and in the plasma are unchanging during this time, we might also expect that the intrinsic EF is static.

The superposition of the RMP and intrinsic EF phasors produces a resultant phasor, with toroidal phase given by,

$$\phi_{res} = \text{arg}\left\{ B_{RMP}e^{i\phi_{RMP}} + B_{EF}e^{i\phi_{EF}} \right\} \tag{2.8}$$

where $B_{EF}$ and $\phi_{EF}$ are the magnitude and phase of the unknown intrinsic EF, and $\text{arg}\{\}$ is the argument function that returns the angle between the positive real axis and the phasor in the complex plane. In general, the measured phase of the TM maximum, $\phi_{meas}$, will differ from the just defined $\phi_{res}$ by an amount $\Delta\phi_{max}$:

$$\phi_{meas} = \phi_{res} + \Delta\phi_{max} \tag{2.9}$$

This model is fit to the $B_{RMP} = 2$ G data in figure 2-10a, where $B_{EF}$, $\phi_{EF}$, and $\Delta\phi_{max}$ are free fitting parameters. From this fit, we estimate $\Delta\phi_{max} \approx 0.1\pi$. This value of $\Delta\phi_{max}$ is used to convert all $\phi_{meas}$ measurements to $\phi_{res}$ in the remainder of
Figure 2-11: (a) The best fit EF amplitude and phase from the $B_{RMP} = 1$ G shot scan are shown by the black square. The black contour bounds the one sigma confidence region within which the EF amplitude and phase are expected to exist. The horizontal black dashed line shows the amplitude of the RMP. Note that the black contour is bounded from above by this dashed line. (b) Five scans of $\phi_{RMP}$ at a given $B_{RMP}$ (red 3 G, blue 2 G, black 1 G [same as (a)], cyan 0.5 G, and magenta 0 G). The magenta square is replaced by a magenta vertical dash-dotted line as the measurement with $B_{RMP} = 0$ is not sensitive to the EF amplitude (see equation 2.8). The gray region shows where the error field is predicted in $B_{EF}$ vs. $\phi_{EF}$ space. The vertical dotted black line shows the position of the RMPs in figure 2-13.

this chapter. The data are shifted downward by $\Delta \phi_{max}$ and the model fits are shown in figure 2-10b.

Equation 2.5 shows that the value of $\Delta \phi_{max}$ has implications on the relative sizes of $C_1/\tau_R$ and $\omega_p$. Although interesting for validation of theory, these implications are not important to the present analysis but will be discussed later in section 2.7.

For redundancy, equation 2.9 is fit to five different shot scans (with $B_{RMP} = 0$, 0.5, 1, 2, and 3 G), providing five independent estimates of the intrinsic EF amplitude and phase (note that $\Delta \phi_{max}$ is now known, and no longer a fit parameter). The results of this analysis are shown in figure 2-11 where squares show each individual estimate. Such estimates are the loci, in the $\phi_{EF}, B_{EF}$ plane, where the reduced chi-square $\chi_r^2$ is minimum ($\chi_r^2 = \chi_{r,min}^2$). The contours in figure 2-11, on the other hand, correspond to $\chi_r^2 = \chi_{r,min}^2 + 1$. Hence, they bound the regions where the fit-parameters are known to within one standard deviation, $\sigma$ (see reference [100] for details).

The set of discharges in which no RMPs are applied (magenta) show the highest
sensitivity to $\phi_{EF}$, constraining the phase to $\phi_{EF} = (1.28 \pm 0.08)\pi$ radians. Despite the high phase sensitivity, the amplitude of the EF cannot be deduced when $B_{RMP} = 0$, as it can be recognized from equation 2.8 that $B_{EF}$ scales both components of the complex phasor, and therefore does not change $\phi_{res}$. For this reason, a magenta vertical dotted line marks the phase of $\chi_{r,\text{min}}^2$ for the no RMP scan. The $B_{RMP} = 3$ G scan (red) has the lowest sensitivity to $\phi_{EF}$ as within one sigma, it cannot constrain $\phi_{EF}$ at all.

Figure 2-12: (a) Four shot scan of a 0.5 G RMP, varying $\phi_{RMP}$ shot-to-shot. The blue curve is a fit of equation 2.9, where the minimum $\chi_{r}^2$ occurs for $B_{EF} \approx 0.6$ G and $\phi_{EF} \approx 1.3\pi$. (b) Five shot scan of $\phi_{RMP}$ with a constant $B_r = 1$ G. The black curve is a fit of equation 2.9, where the minimum $\chi_{r}^2$ occurs for $B_{EF} \approx 0.9$ G and $\phi_{EF} \approx 1.25\pi$. The dashed black line in both (a) and (b) shows where $\phi_{meas} = \phi_{RMP}$.

Using the estimated parameters from fitting equation 2.9 to a single toroidal scan provides at best $\pm 0.4$ G EF amplitude resolution, as shown by the black contour for the $B_{RMP} = 1$ G scan. Nonetheless, an important observation can be made from figure 2-11 that increases the amplitude resolution: when the applied RMP is smaller than the unknown field, the resulting measured data span a range of less than $\pi$ radians. Therefore, we can conclude from the range of the measured data whether the unknown field is larger than, or smaller than the RMP. For example, the $\phi_{res}$ data from the 0.5 G phase scan span less than $\pi$ radians, as seen in figure 2-12a. This suggests that $B_{EF} > 0.5$ G. Corroborating this, the $1 \cdot \sigma$ contour shown in cyan in figure 2-11 is lower-bounded by $B_{EF} \approx 0.5$ G. The $\phi_{res}$ data from the 1 G phase
scan span more than $\pi$ radians as seen in figure 2-12b, suggesting that $B_{RMP} > B_{EF}$.

Again, the corresponding black contour in figure 2-11 corroborates this observation as we see that the dashed black horizontal line bounds the contour from above, meaning that this fit for $B_{RMP} = 1 \text{ G}$ is not consistent with $B_{EF} > 1 \text{ G}$.

Multiple RMP scans where the RMP amplitude is varied above and below the unknown amplitude $B_{EF}$ constrains the amplitude better than a single scan alone. All five solid contours in figure 2-11 share a small region of intersection highlighted in gray. Note that since this region is bounded by three scans (black, cyan, and magenta), only these three scans are necessary to constrain the EF. From this region of intersection, we estimate that the EF has phase $\phi_{EF} = (1.28 \pm 0.08)\pi$ and magnitude $B_{EF} = 0.7 \pm 0.2 \text{ G}$.

### 2.6.1 Independent verification of EF estimate

A scan of RMP amplitude at a constant phase of $\phi_{RMP} = 0.33\pi$ comes close to cancellation of this predicted $m/n = 1/ - 12$ EF and is used as an independent verification of this prediction. This scan consists of $B_{RMP}$ amplitudes of -2, -1, 0, 0.5, 1.5, and 2 G. As just discussed, according to our EF estimate, the EF is believed to be located at $\phi_{EF} \approx 1.3\pi$ with an amplitude of $B_{EF} \approx 0.7 \text{ G}$. The field which would cancel this predicted EF would be located at $\phi_{RMP} = 0.3\pi$ and with an amplitude of $B_{RMP} = 0.7 \text{ G}$. Although this preferred canceling field is not included in the scan just described, the discharge with $B_{RMP} = 0.5$ and $\phi_{RMP} = 0.33\pi$ is close to the desired field.

For each of the shots in this scan we observe the frequency of TM rotation during the period when the RMP is applied. Non-axisymmetric fields in EXTRAP T2R are known to apply DC braking torques to the TM through a nonlinear interaction of the field with the amplitude modulated TM [85]. We therefore expect the TM rotation frequency to be highest when the external resonant field (i.e. the intrinsic EF plus the RMP) is smallest. Each subfigure 2-13a-f shows the TM rotation frequency for a given RMP amplitude in color plotted on top of the four other shots in black for comparison.
Figure 2-13: Tearing mode rotation frequency over a 6 shot RMP amplitude scan at $\phi_{RMP} = 0.33\pi$. (a-f) Rotation frequency of the $m/n = 1/ - 12$ TM in the presence of the RMP amplitude specified in the title and shown in color, as well as the five other frequency traces for comparison in black. All frequency data are smoothed over 2 ms. (g) Box-and-whisker plot of 2 ms smoothed frequency data from $t = 24 - 39$ ms for each amplitude of the applied RMP. Bottom and top of each box represent the $25^{th}$ and $75^{th}$ percentiles, the horizontal line inside the box is the median, and the dashed and capped lines extending from the box mark the extrema of the distribution. The vertical dashed black line at $B_{RMP} = 0$ G is a guide for the eye.
Subfigure 2-13g is a 'box-and-whisker' plot that summarizes subfigures 2-13a-f. The bottom and top of each box mark the frequency of the 25th and 75th percentiles and the horizontal line inside the box marks the median. The dashed and capped lines extending out of the top and bottom of each box mark the extreme values of each distribution.

It is evident that the box-and-whisker plot is not symmetric about $B_{RMP} = 0$ G, but rather the vertical line of symmetry appears to occur somewhere between $B_{RMP} = 0$ and 1.5 G. For example, the $B_{RMP} = -2$ and -1 G cases are characterized by significantly slower TM rotation compared with the +1.5 and +2 G cases. The fastest rotation is obtained for $B_{RMP} = 0.5$ G. These observations are consistent with the prediction that the EF lies somewhere in the intersection region of figure 2-11, and thus provides greater confidence in this prediction.

A second independent observation that corroborates the existence of an EF is the uniformity of the polar histogram where near EF correction is expected. Figures 2-14a and 2-14b show polar histograms for the cases of no RMP (i.e. no EF correction) and an RMP with $B_{RMP} = 0.5$ G at $\phi_{RMP} = 0.33\pi$. The magnitude of the center-of-mass phasor in the no-RMP case is approximately twice as large as the phasor in the $B_{RMP} = 0.5$ G case where the EF is expected to be greatly reduced. This observation is consistent with the prediction of an EF of amplitude similar to the applied RMP, and with phase anti-aligned with this RMP phase.

Figure 2-14: Polar histograms of $i_{max}^j$ for two of the shots in figure 2-13. (a) A shot with no RMP. (b) A shot with an RMP (black arrow) that cancels a field in the predicted range of the EF in figure 2-11 (i.e. $B_{RMP} = 0.5$ G and $\phi_{RMP} = 0.33\pi$). (a) and (b) here correspond to figures 2-13c and 2-13d respectively. Unlike figures 2-8 and 2-9, the $\Delta\phi_{max} = 0.1\pi$ shift has been subtracted from these data. The magnitude of the black phasor is arbitrary and cannot be compared with the magnitudes of the red phasors.
2.7 Discussion of results

Despite the success of this technique based on the model described in section 2.3, it is clear that additional physics is necessary to explain the TM dynamics. The system has been treated as consisting of a $m/n = 1/12$ TM, an applied RMP, and a resonant EF only. In reality, many other TMs and resistive wall modes are present in these discharges and can affect the $m/n = 1/12$ TM through viscous and toroidal coupling. Separately, to decouple the effect of eddy currents in the vacuum vessel from the magnetics, the vessel is assumed to be a first-order low-pass filter. This assumption is only strictly correct when $\dot{\omega}/\omega^2 \ll 1$ [101] (in other words, when $\omega$ does not change significantly within one rotation period) whereas it is observed that $\dot{\omega}/\omega^2 \leq 0.3$ (see figure 3 in reference [15]).

2.7.1 Comparison of experimental and theoretical $C_1/\tau_R$

From equation 2.5, and with the readily available experimental measurement of $\omega_o$ from the TM rotation, the phase at which the TM width is maximized (referred to as $\Delta \phi_{max}$) provides an empirical measurement of $C_1/\tau_R$. If the TM rotates quickly relative to the frequency $C_1/\tau_R$, $\Delta \phi_{max}$ goes to $\pi/2$, whereas if the TM rotates relatively slowly, $\Delta \phi_{max}$ goes to zero. Comparing this measurement with a calculated value from our model provides a check on the validity of the model. Also, predictive capability of $\Delta \phi_{max}$ would remove the need to measure it empirically. A poor prediction of $\Delta \phi_{max}$ could cause errors in the passive EF phase identification of up to $\pm \pi/2$; the effects on active EF amplitude and phase identification are not clear.

The value of $\Delta \phi_{max} \approx 0.1\pi$ measured in section 2.6 implies that $(\omega_o/C_1/\tau_R) \approx \tan(0.1\pi)$. From this and from experimental rotation frequencies $\omega = (3.1-5.7) \times 10^5$ s$^{-1}$, we arrive at an empirical measurement of $C_1/\tau_R = (0.95 - 1.8) \times 10^6$ s$^{-1}$.

Next we seek an analytic estimate of $C_1/\tau_R$. Using equation 2.3, we find the following expression for $C_1/\tau_R$:

$$\frac{C_1}{\tau_R} = \frac{C_0 \eta}{\mu_0 W_0 r_s}$$  \hspace{1cm} (2.10)
Table 2.1: Parameter ranges for EXTRAP T2R and values used for calculations on 'T2R'.

The ranges we estimate for the parameters on the right hand side of equation 2.10 are shown in the "T2R Range" column of table 2.1. Some of these parameters have very large experimental uncertainties, particularly $C_0$, $W_0$, and $r_s$. The largest value of the resistivity $\eta$ is derived from Spitzer resistivity with $T_e = 200$ eV, $Z_{eff} = 4$, and using the particle trapping correction (see appendix A in reference [97]). Choosing parameters to produce the largest estimate of $C_1/\tau_R$ consistent with geometric constraints and measured perturbed fields (see column T2R in table 2.1), we find $C_1/\tau_R = 1.1 \times 10^6$ s$^{-1}$. This value is in the experimental range $C_1/\tau_R = (0.95 - 1.8) \times 10^6$ s$^{-1}$ given by the measured $\omega_o$ and $\Delta \phi_{max}$.

2.7.2 Applicability to ITER

The theory used here for classical TMs is not directly applicable to Neoclassical TMs (NTMs) in ITER where $\Delta'(W = 0)$ is expected to be negative, and where the bootstrap drive for NTMs cannot be ignored. However, the EF is still expected to drive some width modulations of a rotating TM in ITER (the islands of interest in ITER might be the $m/n = 3/2$, 2/1, or 3/1). These modulations are expected to be small; for the 2/1 island, $\tau_R \approx 679$ s and $\omega \approx 2\pi \cdot 420$ rad/s, giving $\omega \tau_R \approx 1.8 \times 10^6$ [56]. Modeling similar to that of section 2.3 could predict the expected modulation of the NTM width, and help to determine if it will be measurable.

Cylindrical theory suggests that the TM rotation modulates in response to EFs regardless of the value of $\omega \tau_R$ [64], and therefore the modulated TM rotation might be used for EF identification in ITER. Even when considering the ideal plasma response
to a static EF (i.e. the EF goes to zero at the rational surface), the rotation of the TM still modulates due to a torque applied by EF induced currents at the rational surface (see section 4.2 of reference [64]). Modeling should be done to verify that the electromagnetic torque due to an estimated EF is sufficient to produce measurable modulation with the expected viscosity and inertia in ITER.

A simple ordering argument is presented here to support these claims regarding the likelihood of measuring a modulated TM rotation frequency \( \omega \) and a modulated TM field \( B_m \) in the presence of an expected error field \( B_{\text{vac}} \) in ITER. The following scalings are found by taking a reduced form of equation 1.34, keeping only the error field term on the right hand side, and taking the error field torque equation,

\[
\frac{\tau_R}{r_s} \frac{dw}{dt} \propto \left( \frac{B_{\text{vac}}}{B_m} \right)^2
\]

(2.11)

\[
I \frac{d\omega}{dt} \propto R_0 B_{\text{vac}} B_m
\]

(2.12)

where \( I \) is the island moment of inertia. We now take \( w \propto \sqrt{B_m} \), assume that the mass of the island is given by the plasma in a cylindrical annulus encompassing the island such that \( I \propto (nr_s w R_0) R_0^2 \) and \( w \propto r_s \), and take \( d/dt \sim \omega \) to find,

\[
\frac{dB_m}{B_m} \propto \frac{B_{\text{vac}}^2}{B_m^{3/2}} \frac{1}{\omega r_s T^{3/2}}
\]

(2.13)

\[
\frac{d\omega}{\omega} \propto \frac{B_{\text{vac}} B_m^{1/2}}{nr_s \omega^2 R_0^2}
\]

(2.14)

It is reasonable to assume \( B_{\text{vac}} \propto B_T \) and \( B_m \propto B_T \), so we have,

\[
\frac{dB_m}{B_m} \propto \frac{1}{\omega r_s \sqrt{B_T} T^{3}}
\]

(2.15)

\[
\frac{d\omega}{\omega} \propto \frac{B_T^{3/2}}{nr_s \omega^2 R_0^2}
\]

(2.16)

Next we take the following representative values for EXTRAP T2R and ITER; \( B_T = \)
[0.2, 5] T, \( \omega = 2\pi[6 \times 10^4, 10^5] \) rad/s, \( r_a = [3, 300] \) cm, \( T = [0.3, 10] \) keV, \( n = [10^{19}, 10^{20}] \) m\(^{-3}\), \( R_0 = [1.24, 6] \) m (the format here is [EXTRAP T2R value, ITER value]). Using these values, we find orders of magnitudes for the following ratios,

\[
\frac{(dB_m/B_m)_{\text{ITER}}}{(dB_m/B_m)_{\text{EXTRAP}}} \sim 10^{-2}
\]

\[
\frac{(d\omega/\omega)_{\text{ITER}}}{(d\omega/\omega)_{\text{EXTRAP}}} \sim 10
\]

These results suggest that the frequency modulation of naturally rotating TMs in ITER is expected to be \( \sim 10 \) times larger than that observed in EXTRAP T2R, while the modulation in the perturbed field is expected to be \( \sim 100 \) times smaller. This supports the earlier statement based on the \( \omega_T \) argument that modulation in the TM field at a rotation frequency of 1 kHz is not likely to be measurable in ITER. On the other hand, this scaling suggests that measuring the modulation in the rotation frequency will be possible. Viscous damping of the frequency oscillation was neglected here, which would act to reduce the modulation moderately, but is not expected to suppress it beyond measuring capability. Also, the design specifications for ITER [19] might result in a smaller relative error than that in EXTRAP T2R. This would reduce both the modulations in the island perturbed field and rotation frequency, though the former would be affected more strongly. This scaling suggests that for naturally rotating TMs, measuring a perturbed island rotation frequency in ITER might be possible, while measuring the perturbed TM field appears challenging. More advanced modeling should be done to verify this simple scaling argument.

### 2.7.3 Future work

Future EXTRAP T2R experiments might further reduce the errors in figure 2-11 by using a "binary search" technique. By this we mean using the bifurcation in the span of the resultant phase \( \phi_{\text{res}} \) (see equation 2.8) when the phase of the RMP is scanned at an amplitude above and below the EF amplitude (see figure 2-12) to more precisely determine the EF amplitude.
Fully passive EF identification should in principle be achievable using the perturbed TM amplitude or perturbed TM velocity data. Passive phase identification of the EF is demonstrated here using the TM amplitude data, assuming prior knowledge of $\Delta \phi_{\text{max}}$. The magnitude of the oscillation in the TM amplitude might also be used to identify the EF amplitude passively, though a first attempt using the data in this chapter was not successful.

Additional or alternative information could be gained from Fourier-analyzing the magnetic signals: if in the absence of an EF or RMP the TM rotates uniformly at frequency $f$, introducing an EF will make the rotation non-uniform and introduce harmonics of $f$ in the Fourier spectrum. The amplitudes and phases of these harmonics can be used to passively identify the EF. This was not attempted here because while rotation is non-uniform within a rotation period, as required for this technique, it is not sufficiently reproducible from period to period, for Fourier analysis to be applied. An analogous phase shift to $\Delta \phi_{\text{max}}$ dependent on the relative sizes of the inertial and viscous torques must be characterized before using the perturbed velocity data passively in this way.
2.8 Summary

The amplitude of naturally rotating TMs is observed to modulate at the TM rotation frequency when a static resonant EF exists in EXTRAP T2R, and the toroidal phase where the TM amplitude is maximized depends on the toroidal phase of the EF (or EF+RMP, if an RMP is applied) [15].

The present chapter describes a new EF detection technique based on this amplitude modulation. The technique is developed, validated, and used to identify the $m/n = 1/ -12$ intrinsic EF in EXTRAP T2R. A simple first-order model is derived from a Modified Rutherford Equation including classical and EF effects, and used to motivate the technique.

For validation, an RMP of approximate amplitude $10^{-3}$ relative to the equilibrium field was applied and varied in phase. The applied phases were successfully measured up to a constant offset, referred to as $\Delta \phi_{max}$, and an approximately sinusoidal deviation. The constant offset $\Delta \phi_{max}$ was easily characterized by completing a toroidal scan of a moderate amplitude RMP ($\sim 2$ to $3 \times$ larger than the intrinsic EF) and averaging the measured phases. The complex exponential in equation 2.5 is responsible for this offset $\Delta \phi_{max}$ and shows that its value is related to the TM rotation frequency, to the nonlinear correction to the classical stability index, $C_1$, and to a resistive diffusion timescale $\tau_R$. If the TM rotates slowly, $\Delta \phi_{max}$ goes to zero, whereas if the TM rotates quickly, $\Delta \phi_{max}$ goes to $\pi/2$; slow and fast here are relative to the frequency $C_1/\tau_R$. The measured $\Delta \phi_{max}$ suggests a relatively slow TM and is consistent with the theoretical calculation, though the experimental uncertainties in $C_1$ and the resistive diffusion time are large.

After accounting for $\Delta \phi_{max}$, the approximate sinusoidal deviations are then used to estimate the EF amplitude and phase (figure 2-10b). An EF of given amplitude and phase produces a unique deviation about $\Delta \phi_{max}$, as shown by equation 2.8. Three toroidal scans of an RMP, each with a constant RMP amplitude in the range $B_{RMP} = 0$ to $1$ G, constrain the EF amplitude to $B_{EF} = 0.7 \pm 0.2$ G and phase to $\phi_{EF} = (1.28 \pm 0.08)\pi$ (figure 2-11b). This EF estimate is consistent with the
highest median TM rotation frequency (figure 2-13d), and the most uniform amplitude behavior (figure 2-14b) when an approximately equal and opposite RMP is applied.

In summary, in the presence of a naturally rotating tearing mode, this technique can be used in two ways: (1) to passively (i.e. no RMP required) identify the phase of an EF (assuming $\Delta \phi_{\text{max}}$ has been characterized), or (2) to detect both the amplitude and phase of an EF by scanning a known RMP in amplitude and phase. The resolution limit of this technique has not been investigated here, but will be the focus of future work.

2.9 Interlude

When error fields become large, they can brake the rotation at resonant surfaces and drive error field penetration and locked modes. Locked modes are a serious concern for reactor scale tokamaks, as they often lead to disruptions that can damage the machine [19]. An alternate class of locked mode occurs when a rotating tearing mode, like the ones studied in EXTRAP T2R, lose angular momentum to the resistive vessel wall, eventually locking in the lab frame. A large population of these type of locked mode is studied in DIII-D, and reported in the following chapter.
Chapter 3

Statistical analysis of $m/n = 2/1$
locked and quasi-stationary modes
with rotating precursors in DIII-D

This chapter details results derived from a locked mode database developed by the author and W. Choi. F. Volpe first conceived of the locked mode database idea. The architecture of the database was planned by W. Choi and the author. Signal analysis pertaining to the rotating phase of the tearing mode including tearing mode identification and poloidal harmonic analysis, as well as analysis pertaining to disruption identification were done by W. Choi. The author developed all signal analysis pertaining to the locked phase including filtering of the magnetics, identification of locked modes, and importing of all relevant equilibrium data. A novel treatment of the magnetics owing to a low false positive rate in the identification of initially rotating locked modes was proposed by F. Volpe, and implemented by the author. The introduction was written by the author, section 3.2 was written by both W. Choi and the author, sections 3.3 and 3.4 were written by W. Choi, and all other sections and appendices were written by the author. The writer of each section also performed all the analysis presented therein. This chapter is published in Nuclear Fusion [13], and is also available on the arXiv [102].

Disruptions pose a serious problem to the operation of future tokamak reactors.
This work provides valuable statistical results on locked modes which are often the cause of disruptions in DIII-D. In particular, a parameter formulated by the ratio of the plasma internal inductance to the edge safety factor is found here to distinguish whether a locked mode will disrupt or not. This parameter is shown to be effective for use in disruption prediction. This parameter is related to the classical tearing stability index, and thus suggests that classical stability might determine whether a locked mode disrupts the plasma or not.

3.1 Introduction

It will be important to understand the onset, growth, saturation, and stabilization of all categories of rotating and non-rotating NTMs, in order to maintain good confinement and prevent disruptions in ITER. Here we present an extensive analysis of QSMs and LMs with rotating precursors, which we will sometimes refer to as "initially rotating locked modes", or IRLMs. The analysis was carried over approximately 22,500 DIII-D [30] plasma discharges, and restricted to poloidal/toroidal mode numbers \( m/n = 2/1 \) because these are the mode numbers that are most detrimental to plasma confinement in DIII-D and most other tokamaks [68]. QSMs and LMs of different \( m/n \) (for example 3/2, occasionally observed at DIII-D) and LMs not preceded by rotating precursors are not considered here and will be the subject of a separate work.

Classical tearing modes have been described theoretically [41, 103] and in low \( \beta \) plasmas, their stability is shown to depend on the gradient of the equilibrium current density (pressure effects also contribute at finite \( \beta \)). Observations in this work suggest that the classical stability might also determine the disruptivity of a discharge with a locked mode. When \( \beta_p \) is finite (\( \beta_p = \langle p \rangle / [B_\theta^2 / 2\mu_0] \) where \( \langle p \rangle \) is the volume averaged plasma pressure and \( B_\theta \) is the poloidal field), the resultant bootstrap current drives neoclassical [44, 68, 57] tearing modes (TMs), and it is observed here that the saturated locked island width scales with \( \beta_p \), as expected from neoclassical theory. Viscous and electromagnetic torques can influence the rotation behavior of
TMs [64], and in particular, an electromagnetic interaction with the wall referred to as the “wall torque” causes locking, supported here by the observation that faster locking events occur when the calculated wall torque is large.

In a detailed study of error-field-penetration locked modes in DIII-D [72], it is shown that locked modes occur in various regions of internal inductance $l_i$ and edge safety factor $q_{95}$ space, in contrast to the slowly rotating “quasi-stationary modes” (QSMs) in [54] which occur along a critical line in this space. In this work, a qualitatively similar line in $l_i$ and $q_{95}$ space to that found for the existence of QSMs in [54] is found to differentiate disruptive from non-disruptive locked modes. In a study of disruptions across many tokamaks [104], the disruptions that begin with an edge temperature deficiency, like the disruptions reported here, cause an increase of the current density gradient at the $q = 2$ surface, which is believed to drive the q = 2 island. It has also been observed in the JT-60U tokamak that at a critical value in $l_i$, which varies with $q_{95}$, peaking of the current profile drives tearing modes that cause disruptions [74]. Similarly, a study of disruptivity over a large set of discharges at JET [46] shows a critical limit in the parameter $l_i \times I_p/aB$, where $I_p$ is the plasma current, $a$ is the plasma minor radius, and $B$ is the toroidal field. Although attributed to low $q_{95}$ values, this parameter is closely related to $l_i/q_{95}$ and is thus qualitatively consistent with the disruption limit found here. In the NSTX tokamak, disruptivity is found to depend strongly on $l_i$, while showing less dependence on $q_{95}$ [47], though the latter might be attributable to the high values of $q_{95}$ common to NSTX operation. All of these observations are consistent with our findings that disruptive locked modes occur at high values of the ratio $l_i/q_{95}$. In addition, the growth of the $n = 1$ field prior to disruption, as observed here, is also suggestive of tearing instability.

A database study on JET, ASDEX Upgrade, and COMPASS finds that the thermal quench is onset when the measured $n = 1$ field exceeds a threshold defined by $l_i$ and $q_{95}$ [105]. Although similar to the disruption threshold on $l_i/q_{95}$ found here, it is not obvious how to compare this limit on the $n = 1$ field for thermal quench onset in [105] with the binary locked mode disruption indicator we report.

Density limit disruptions in JET are attributed to the imbalance of radiation
losses with heating [71]. In chapter 4, it will be shown on a single DIII-D locked mode discharge that the initial collapse of the thermal energy does not appear to be caused by radiative losses, suggesting that the dynamics are different that that of the density limit disruption. More statistics are needed to determine whether this observation is common, and it is not excluded that radiation might play a central role in the full locked mode thermal quench that follows the initial collapse. Radiation losses are not studied in this chapter.

Disruptivity in NSTX is observed to peak at intermediate values of normalized plasma beta \( \beta_N = \beta a B_T / I_p \) [47], in agreement with the results reported here. This work on NSTX also finds that disruptivity depends strongly on plasma shaping, which is not investigated in this work. An axisymmetric approximation of the effect of a TM on confinement is described theoretically [51], and agrees qualitatively with the significant reduction in confinement observed during the locked phase.

A disruption predictor on NSTX using the locked mode onset as a disruption indicator resulted in a significant number of false alarms [106], revealing that many locked modes do not cause disruptions, in agreement with our findings that \( \sim 30\% \) of all detected initially rotating locked modes in DIII-D do not cause a disruption. A study on MAST, also recognizing the presence of non-disruptive locked modes, finds locked modes disrupt more often in low \( q_{95} \) discharges [48], similar to findings herein. An increasing disruptivity with decreasing \( q_{95} \) has been reported for general DIII-D discharges [45] (i.e. not limited to locked mode disruptions).

A carefully validated algorithm to identify initially rotating locked modes allows the analysis of all DIII-D discharges from 2005 to 2014 in this chapter. The tendency for locked modes to disrupt under different equilibrium conditions is studied, as well as basic observations of locked modes and their effects on equilibria. Applications to disruption prediction are also briefly investigated.

During the first discharges in this database, DIII-D was equipped with one poloidal and four toroidal arrays of Mirnov probes and saddle loops [31]. During the time spanned by the database, additional sensors were added for increased 3D resolution [32]. However, a limited set of six saddle loop sensors (external to the vessel and on
the midplane, see figure 3-1) and three poloidal sensors (inside the vessel) are used for simplicity, and for consistency of the analysis across all shots, spanning the years 2005-2014.

An example of the 2/1 IRLMs considered here is illustrated in figure 3-2. The poloidal field amplitude of the rotating precursor is detected by the toroidal array of Mirnov probes around 1800 ms. The mode simultaneously grows and slows down until it locks at 1978.5 ms. Due to the finite time binning used in the Fourier analysis during the rotating phase, the rotating signal is lost at low frequency, and is instead measured by a set of large saddle loops (ESLDs: external saddle loops differenced). The response of the saddle loops increases when $\omega < \tau_w^{-1}$, where $\tau_w \sim 3$ ms is the characteristic $n = 1$ wall time for DIII-D. As shown in Fig.3-2, it is not uncommon for the amplitude of an IRLM to oscillate due to minor disruptions, and to grow prior to disruption (this will be investigated in section 3.6.2).

Three interesting results of this work are introduced now. First, it will be shown that the $m/n = 2/1$ island width cannot be used to distinguish disruptive from non-disruptive IRLMs 20 ms or more ahead of the disruption time. Similarly, the island
width shows little correlation with the IRLM survival time.

Second, the plasma internal inductance divided by the safety factor, $l_i/q_{95}$, distinguishes IRLMs that will disrupt from those that will not. The predictive capability of $l_i/q_{95}$ might be related to the energy available to drive nonlinear island growth.

Finally, a spatial parameter which couples the $q = 2$ radius and the island width, referred to as $d_{edge}$ (see section 3.5 for definition), also distinguishes disruptive from non-disruptive IRLMs well within 20 ms of the disruption. It also correlates best with the IRLM survival time. The predictive capability of $d_{edge}$ is believed to be related to the physics of the thermal quench.

This chapter is organized as follows. Section 3.2 explains the method of detection of disruptions, of rotating tearing modes, and of LMs. Section 3.3 provides some general statistics of IRLM occurrences in DIII-D. Section 3.4 quantifies the timescales of interest before locking. Section 3.5 investigates the time available to intervene before an IRLM causes a disruption. Section 3.6 discusses the width and phase behavior at locking, and the exponential growth of the $n = 1$ field before the disruption. Section
3.7 details the interdependence between IRLMs and plasma $\beta$ ($\beta = \langle p \rangle / (B^2/2\mu_0)$ where $\langle p \rangle$ is the average pressure and $B$ is the average total field strength). Section 3.8 decouples the influence of $\rho_{q2}$, $q_{95}$, and $l_i$ on IRLM disruptivity, and investigates the effectiveness of $l_i/q_{95}$, the island width, and $d_{\text{edge}}$ as disruption predictors. A discussion section follows which offers possible explanations of the physical relevance of $l_i/q_{95}$ and $d_{\text{edge}}$. Finally, two appendices (Appendix D and E) are dedicated to the mapping from radial magnetic field measurements to the perturbed island current, and from the perturbed current to an island width.
3.2 Methods

3.2.1 Detection of disruptions

To categorize disruptive and non-disruptive modes, a clear definition of disruption is needed. The plasma current decay-time is used to differentiate disruptive and non-disruptive plasma discharges. The decay-time $t_D$ is defined as the shortest interval over which 60% of the flat-top current is lost, divided by 0.6. In cases where the monotonic decrease of $I_p$ extends beyond 60%, the entire duration of the current decrease is used, with proper normalization. The disruption time is defined as the beginning of the current quench, which is usually preceded by a thermal quench, a few milliseconds prior.

The criterion $t_D < 40$ ms used to identify DIII-D disruptions was formulated as follows.

A histogram of all decay-times is shown in figure 3-3, and features of the distribution are used to define three populations. The first group peaks near $t_D=0$ and extends up to $t_D=40$ ms. These are rapid losses of $I_p$ and confinement, quicker than typical energy and particle confinement times. Discharges in this group are categorized as major disruptions, either occurring during the $I_p$ flat-top, or occurring during a partial controlled ramp-down of less than 40% of the flat-top value. The sudden loss of current during the partial ramp-down cases must be fast enough to normalize to an equivalent 40 ms or less full current quench. It is worth noting that of the 5,783 disruptions detected, 666 occurred within the first second (in the ramp-up phase), none of which were caused by a 2/1 IRLM. At the opposite limit, group iii, with $t_D > 200$ ms, contains mostly (88 ± 4%) non-disruptive discharges, in which the plasma current decays at a steady rate for at least 80% of the ramp-down. Population ii has $t_D$ in the range 40 ms < $t_D$ < 200 ms, and mostly consists of shots that disrupted during the current ramp-down with “long decay” times relative to population i disruptions.

Note that while the stringent threshold of $t_D < 40$ ms will prevent falsely categorizing non-disruptive discharges as disruptive, it may also categorize some disruptions with slightly longer decay times into group ii. However, the thresholds are chosen to
protect against false positives better than false negatives; they are chosen to compro-
mise missing a number of disruptive shots in exchange for ensuring the validity of all
disruptive discharges. In the remainder of this work, we will focus on groups $i$ and
$iii$ only.

In a manual investigation of 100 discharges in group $i$, 85 disruptions occurred
during the current flat-top, and 15 disruptions occurred during the current ramp-down
phase. Therefore, the majority of disruptions studied in this work (i.e. $85\pm4\%$) occur
during a current flat-top.

In section 3.3, disruptivity will be studied over the entire database, including shots
that did not contain IRLMs. We will refer to disruptivity in this context as *global
disruptivity* (i.e. the number of disrupted discharges divided by all discharges).

In all sections following section 3.3, disruptivity will be studied on shots that
contained IRLMs only. In most cases, we will be interested in studying what dif-
cerentiates a disruptive IRLM from a non-disruptive IRLM, and therefore we define
IRLM disruptivity as the number of disruptive IRLMs divided by the total number
of IRLMs (where the total is the sum of disruptive and non-disruptive IRLMs). In
one case, it will be useful to discuss *IRLM shot disruptivity*, which is the number
of disruptive IRLMs divided by the total number of *discharges* with IRLMs (note
that a non-disruptive discharge can have many non-disruptive IRLMs, making this
distinction non-trivial).

### 3.2.2 IRLM Disruptivity during current flat-tops

For all IRLM disruptivity studies, disruptions that occur during $I_\text{p}$ ramp-downs are
limited to $15\pm4\%$ of the studied set. $I_\text{p}$ ramp-downs are characterized by major
changes of the plasma equilibrium, and are expected to greatly impact the locked
mode evolution. Namely, key parameters such as $q_{95}$, $l_i$, and $\rho_{q2}$ evolve during an
$I_\text{p}$ ramp-down, complicating the interpretation of their effect on IRLM disruptivity.
Moreover, flat-tops will be longer and longer in ITER and DEMO, and thus dis-
ruptivity during ramp-down will become less and less important. Eventually, in a
steady-state powerplant, only flat-top disruptivity should matter.
Figure 3-3: The distribution of plasma current decay time, roughly split into three populations. Panel (a) shows the non-disruptive discharges with decay times greater than 200 ms; panel (b) further distinguishes the remaining population into major disruptions (< 40 ms, consisting of > 80% flat-top disruptions), and disruptions with longer decay times, which are predominantly disruptions during ramp-down. Note that the vertical axis on panel (a) is interrupted to better show the features in the distribution.

Out of 1,113 shots which disrupted due to an IRLM, 105 contained an additional IRLM distinct in time from the final disruptive one. As these additional IRLMs decayed or spun-up benignly, yet occurred in plasmas that ultimately disrupted, they are considered neither disruptive nor non-disruptive, and are excluded from the IRLM disruptivity studies. Similarly, a small number of discharges disrupt without an IRLM present, but contain an IRLM 100 ms before the disruption or earlier. In these cases, it is not clear whether the IRLM indirectly caused the disruption or not, and therefore these cases are also excluded.

3.2.3 Detection of rotating modes of even $m$ and $n=1$

Detection of a rotating mode is performed in two stages. For each shot, the signals from a pair of toroidally displaced outboard midplane magnetic probes are analyzed
by the newspec Fourier analysis code [31], in search of \( n=1 \) activity. Genuine \( n=1 \) magnetohydrodynamic (MHD) activity is distinguished from \( n=1 \) noise by searching for both an \( n=1 \) amplitude sustained above a chosen threshold, as well as requiring the corresponding \( n=1 \) frequency to be "smooth". An adaptive threshold is used to accept both large amplitude, short duration modes, as well as small amplitude, long duration modes. Once detected, this activity is analyzed by a simplified version of the modal analysis eigspec code, based on stochastic subspace identification [11]. Here eigspec uses 9 outboard mid-plane and 1 inboard mid-plane Mirnov probes to isolate \( n = 1 \) and determine whether \( m \) is even or odd. Selecting modes of even \( m \) and \( n = 1 \) rejects \( n = 1 \) false positives due to 1/1 sawtooth activity, and other odd \( m \) and \( n = 1 \) activity. Modes with \( n=1 \) and even \( m \) are predominantly 2/1. Provided \( q_{95} \) is sufficiently high, they might in principle be 4/1 or 6/1 modes, but these modes are rare. Manual analysis of 20 automatically detected modes of even \( m \) and \( n=1 \) found only 2/1 modes.

### 3.2.4 Detection of \( n = 1 \) locked modes

Locked modes are detected using difference pairs of the integrated external saddle loops (ESLDs). A toroidal array of six external saddle loops is available. Differenting of loops positioned 180° apart toroidally eliminates all \( n = \) even modes, including the equilibrium fields. A least squares approach is then used to fit the \( n = 1 \) and \( n = 3 \) toroidal harmonics [31]. This approach assumes that the contributions of odd \( n \geq 5 \) are negligible.

Each pair-differenced signal is compensated for pickup of the non-axisymmetric coils, using a combination of analog and digital techniques. The accuracy of the coil compensations was assessed using vacuum shots from 2011-2014. Residual coil pickup peaks at \( \sim 3 \) G, but a conservative threshold of 5 G is chosen for identification of LMs to avoid false positives. Small LMs that produce signals less than 5 G are not considered in this work.

Analog integrators are known to add linear drifts to the saddle loop signals. In addition, \( n = 1 \) asymmetries in the plasma equilibrium can also produce background...
A simple yet robust algorithm was developed to subtract this background. The algorithm works on the principle that times exist during which it is impossible for a LM to exist, and the \( n=1 \) 'locked mode signal' at those times must be zero. These times include the beginning and end of every shot, and times at which \( m/n = \text{even}/1 \) modes are known, from Mirnov probe analysis, to rotate too rapidly to be QSMs or LMs. As LMs cause a significant decrease in \( \beta_N = \beta aB/I_p \) (as will be discussed in Section 3.7), the time when \( \beta_N \) is maximized is also highly unlikely to have a coincident LM, and therefore, this time is also used. A piecewise linear function with nodes at each identified '2/1 LM free' time is fit to each ESLD signal independently and subtracted to produce a signal with minimal effects from integrator drift and non-axisymmetric equilibrium pickup.

Fifty shots automatically identified to have LMs with \( n = 1 \) ESLD signals in excess of 5 G were manually analyzed. This analysis confirmed that in most cases the automatic identification was accurate, with a percentage of false positives for LMs with rotating precursors of < 4%. The identification of locked modes without rotating precursors (born locked modes), on the other hand, exhibited a percentage of false positives > 30%. Greater accuracy is achieved for LMs with rotating precursors because the fast rotating precursor provides a LM free background subtraction just prior to locking. In addition, locked modes with rotating precursors require two subsequent events: the appearance of a rotating \( m/n = \text{even}/1 \) tearing mode followed by an \( n = 1 \) locked mode. Due to the high percentage of false positives in the identification of born LMs, they are not considered in this work but will be the topic of future work.

During the locked phase, no poloidal harmonic analysis is performed. It is assumed that the confirmed \( m/n = \text{even}/1 \) rotating mode present immediately before locking is likely a 2/1 mode, and upon locking, the mode maintains its poloidal structure. In addition, it is assumed that the locked \( n = 1 \) signal measured by the ESLDs is predominantly due to the 2/1 mode, such that this field measurement can be used to infer properties of the mode. A set of 63 disruptive IRLMs, occurring in plasmas
with $|B_T| > 2$ T, were investigated using the 40 channel electron cyclotron emission (ECE) diagnostic [36] to validate these assumptions. Only plasmas with $|B_T| > 2$ T are considered to ensure that ECE channels cover a plasma region extending from the core through the last closed flux surface on the outboard midplane. Among these 63 IRLMs, 26 exhibited QSM characteristics, making full toroidal rotations, allowing the island O-point to be observed by the toroidally localized ECE diagnostic. In all 26 cases, a flattening of the electron temperature profile is evident at the $q = 2$ surface, and no surfaces with $q > 2$ show obvious profile flattening, suggesting that higher $m$ modes are not present, or are too small to resolve with ECE channels separated by $\sim 1 - 2$ cm.

As the gradient in the electron temperature tends to approach zero near the core, the presence or absence of a $1/1$ mode is difficult to conclude. Despite the $1/1$ mode existence being unknown, island widths derived from the radial field measured by the ESLDs, where the $n = 1$ signal is assumed to be a result of the $2/1$ mode only, are calibrated to within $\pm 2$ cm with the flattened $T_e$ profiles as measured by ECE (see Appendix E). We conclude that in the majority of cases, the locked modes are $2/1$ and the inferred island widths are reasonably accurate. The minority of cases where this is not true are not expected to affect the statistical averages presented in this work.

The locked mode analysis includes a check for the existence of a $q = 2$ surface, which is a necessary condition for the existence of a $2/1$ IRLM. In 114 disruptive discharges, reconstructed equilibrium data are absent for 80% or more of the locked phase, during which time the existence of a $q = 2$ surface cannot be confirmed. The majority of these omitted discharges have locked phases lasting less than 20 ms, which is the time-resolution of equilibrium reconstructions, and therefore no equilibrium data exist after the mode locks (note that although 20 ms is a relatively short timescale, significant changes in the equilibrium are expected upon mode locking). All 114 discharges were manually analyzed, and approximately 40% were identified as vertical displacement events or operator induced disruptions, such as discharges terminated by massive gas or pellet injection. About 15% lack necessary data to man-
ually identify the cause of the disruption. The remaining 53 disruptive discharges are considered valid 2/1 IRLM disruptions. These discharges are not included in the majority of the figures and discussion herein, but will be discussed in the disruption prediction section (section 3.8.5) as they are expected to modestly decrease the performance of the predictors.

3.2.5 Perturbed currents associated with the islands

The mode amplitudes will sometimes be reported in terms of the total perturbed current carried by the island $\delta I$. $\delta I$ is a quantity that is local to the $q = 2$ surface, and its calculation accounts for toroidicity. The wire filament model used in [91] was shown to reproduce experimental magnetics signals well and was adapted for this calculation. An analytic version of this model was developed which simulates the island current perturbation with helical wire filaments that trace out a torus of circular cross-section. The torus has the major and minor radii of the $q = 2$ surface informed by experimental EFIT MHD equilibrium reconstructions [12], which use magnetics signals and Motional Stark Effect measurements [34] to constrain the reconstruction. An analytic expression is found for $\delta I$ as a function of the experimental measurement of $B_R$ from the ESLDs, and $R_0$ and $r_{q2}$ from EFIT reconstructions. The model and the resulting analytic expression are detailed in Appendix D.
3.3 Incidence of locking and global disruptivity on DIII-D

To motivate the importance of study of these $m/n = 2/1$ modes, figure 3-4 shows how often initially rotating 2/1 locked modes (IRLMs) occur in DIII-D plasmas. When considering all plasma discharges, 25% contain a 2/1 rotating NTM, 41% of which lock. Shots with IRLMs end in a major disruption 76% of the time (using only the red and green portions of figure 3-4a, and excluding the blue portion). Approximately 18% of all disruptions are a result of an IRLM, in good agreement with the $\sim 16.5\%$ reported on JET [7], and this statistic rises to 28% for shots with peak $\beta_N > 1.5$ (figure 3-4b). The correlation between high $\beta_N$ and rate of occurrence of IRLMs will be detailed in section 3.7.2.

The blue slices show the number of IRLMs excluded from the disruptivity studies, which consist of IRLMs in type $ii$ disruptions (long decay disruptions), IRLMs that terminate during a non-disruptive current ramp-down, or IRLMs that cease to exist prior to a major disruption. The “other discharges” do not contain IRLMs, and include long decay disruptions and non-disruptive discharges.

Figure 3-4: (a) Color pie chart surveying all plasma discharges, showing the fraction of discharges with disrupting and non-disrupting initially rotating locked modes (IRLMs), as well as disruptions without IRLMs. Overplotted as a hatched region are the discharges with rotating 2/1 NTMs. (b) Same pie chart as (a), but for discharges with peak $\beta_N > 1.5$. Note that there is an overlap of 23 shots between the hatched rotating NTM and the purple disruption regions.
While the vast majority of rotating NTMs lock before causing a disruption, there were approximately 23 instances of the rotating 2/1 mode growing large enough to disrupt before locking.
3.4 Timescales of locking

In this Section we present two timescales indicative of the time available for intervention before locking. These timescales are useful for disruption avoidance and mitigation techniques.

Figure 3-5 shows the duration of all rotating $m=$even, $n=1$ modes that locked. A broad peak exists between 50 and 400 ms. The rotating duration can depend on several different factors, such as the plasma rotation frequency, applied Neutral Beam Injection (NBI) torque, the island moment of inertia, and viscous torques. The spread of values gives an indication of the time available to prevent locking, if an intervention is triggered upon rotating mode detection.

The time taken for a mode rotating at 2 kHz to slow down and lock is referred to here as the slow-down time. The threshold of 2 kHz was chosen empirically, as modes that decelerate to this frequency are often observed to lock. It is probable that at this frequency, the decelerating wall torque is stronger than the accelerating viscous torque, and causes the mode to lock. Figure 3-6a shows 66% of slow-down times between 5 and 45 ms, with the peak of the distribution at $17 \pm 10$ ms; this is an indication of the time available to prevent locking if measures are taken when the mode reaches 2 kHz.

Figure 3-6b shows that modes which experience a larger wall torque generally slow down quicker than those with smaller wall torques. At low electromagnetic torque, the spread of slow-down times in figure 3-6b suggests that other effects, such as the NBI torque, also become important.

The toroidal electromagnetic torque between the rotating mode and the wall $T_{\phi,w}$ is expressed as follows [64],
Figure 3-6: (a) The time taken for a rotating $m/n = 2/1$ mode to slow from 2 kHz to locked, as measured by eigspec and ESLDs respectively. (b) A correlation is observed between the measured slow-down time and electromagnetic torque between the mode and the wall. The torques are calculated by equation 3.1, where the perturbed magnetic field is taken when the mode is rotating at 2 kHz, and $\omega \tau_w$ is set to 1, representing the maximum of the frequency dependent steady-state wall torque. The points and error bars are the mean and standard deviations of each bin respectively. Note that about 5% of the events lie beyond 300 ms, and are not plotted (in either panel).

$$T_{\phi,w} = \frac{R_0(2\pi r_s B_{rs})^2}{\mu_0 n/m} \frac{(\omega \tau_w)(r_{s+} / r_w)^{2m}}{1 + (\omega \tau_w)^2[1 - (r_{s+} / r_w)^{2m}]^2}$$  \hspace{1cm} (3.1)

where $m$ and $n$ are the poloidal and toroidal harmonics, $r_s$ is the minor radius of the $q = 2$ surface, $r_{s+} = r_s + w/2$ with $w$ being the island width, $B_{rs}$ is the perturbed radial field at the $q = 2$ surface, $r_w$ is the minor radius of the resistive wall, and $\omega$ is the rotation frequency of the NTM co-rotating with the plasma. This form of the electromagnetic torque comes from a cylindrical approximation. The maximum of this torque occurs at the rotation frequency where $\omega \tau_w = 1$, and is the quantity plotted on the horizontal axis of figure 3-6b.
3.5 Time between locking and disruption

The survival time is defined as the interval between locking and disruption (note this is only defined for disruptive IRLMs). Figure 3-7 is a histogram of the survival times of all disruptive IRLMs. Two groups can be observed, peaking at less than 60 ms and at 270 ms. The first group consists of 55 large rotating modes that lock and disrupt almost immediately, as opposed to the latter spread of LMs that reach a metastable state before disrupting. These short-lived modes, though dangerous and undesirable, could not be studied with the automated analysis as necessary equilibrium data do not exist. These discharges are the same omitted discharges mentioned previously in the end of section 3.2.4. Recall that these discharges are not included in the majority of the figures and discussion herein, but will be discussed in the disruption prediction section (section 3.8.5) as they are expected to modestly decrease the performance of the predictors.

While 75% of the population (excluding the 55 transient modes) survive between 150 to 1010 ms, the most frequent survival time is $270 \pm 60$ ms, an indication of the time available to avoid disruption when a mode locks.

Gaining predictive capability over how long a disruptive locked mode is expected to survive might guide the best course of action to take, e.g. whether to stabilize the mode, or directly deploy disruption mitigation techniques.

Figure 3-8 shows the survival time plotted against the poloidal beta $\beta_p$, the distance $d_{\text{edge}}$, and the perturbed island current $\delta I$. $d_{\text{edge}}$ is a quantity that measures the shortest distance between the island...
Figure 3-8: Survival time shows some dependence on (a) $\beta_p$ and (b) $d_{\text{edge}}$. (c) No correlation is found with $\delta I$ or similarly with the island width $w$ (see table 3.1).

Separatrix and the unperturbed plasma separatrix: $d_{\text{edge}} \equiv a - (r_{q2} + w/2)$ where $a$ is the minor radius of the unperturbed plasma separatrix, $r_{q2}$ is the minor radius of the $q = 2$ surface, and $w$ is the island width.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Correlation with $t_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{\text{edge}}$</td>
<td>0.47</td>
</tr>
<tr>
<td>$\rho_{q2}$</td>
<td>-0.42</td>
</tr>
<tr>
<td>$l_i/q_{95}$</td>
<td>-0.39</td>
</tr>
<tr>
<td>$q_{95}$</td>
<td>0.36</td>
</tr>
<tr>
<td>$\beta_p$</td>
<td>0.34</td>
</tr>
<tr>
<td>$dq/dr(r_{q2})$</td>
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</tr>
<tr>
<td>$l_i$</td>
<td>-0.11</td>
</tr>
<tr>
<td>$w$</td>
<td>0.10</td>
</tr>
<tr>
<td>$\delta I$</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

Table 3.1: Correlations of various parameters with the IRLM survival time $t_s$. The parameters are ordered in the table by the absolute value of their correlation coefficient. Negative correlation means that a linear relationship with a negative slope exists between the parameters.

In figure 3-8, the survival time shows some correlation with $\beta_p$ and $d_{\text{edge}}$, but not with $\delta I$. The correlation coefficients for these, and other parameters are listed in table 3.1. The lack of correlation between survival time and $\delta I$ is consistent with the lack of correlation with the island width $w$. This suggests that large islands will not necessarily disrupt quickly, but islands which extend near to the unperturbed separatrix (i.e. islands with small $d_{\text{edge}}$) tend to disrupt quickly. The correlation of $d_{\text{edge}}$ with survival time suggests that it is pertinent to the physics of the thermal quench. Other works have found that parameters similar to $d_{\text{edge}}$ appear to cause the
onset of the thermal quench [86, 107, 108] (see section 3.9 for details).

The best correlations in the table are considered moderate (i.e. moderate correlations are in the range \( r_c = [0.4, 0.6] \), where \( r_c \) is the correlation coefficient). The correlations in table 3.1 do not provide significant predictive capability. A macroscopic timescale like the survival time likely depends on many variables, and on the nonlinear evolution of the plasma under the influence of the locked mode. Note that 15±4% of the disruptive IRLMs in this survival time study terminate during a plasma current ramp-down, and might have survived longer, had they not been interrupted.
3.6 Mode amplitude and phase evolution

3.6.1 Distributions of IRLM toroidal phase at locking

As a rotating $n = 1$ mode is slowing down and about to lock, it tends to align with existing $n = 1$ fields. In most cases, this will be the residual error field, defined as the vector sum of the intrinsic error field and the applied $n = 1$ error field correction. Figure 3-9 shows a histogram of all locked mode phase data, normalized by mode duration and total number of modes in the given set: each mode contributes a total of $100/N$, where $N$ is the total number of disruptive or non-disruptive modes; and further normalized by the binsize of 5 degrees. A clear $n = 1$ distribution is observed in figure 3-9a (left-hand helicity discharges), with a peak at $\sim 125^\circ$ for disruptive modes and $\sim 110^\circ$ for non-disruptive modes. Figure 3-9b shows right-hand helicity discharges. The disruptive distribution shows an $n = 1$ component, though an $n = 2$ component is also clearly visible. The non-disruptive distribution shows a strong $n = 2$ component. The presence of $n = 2$ might be due to occurrences of both over and under correction of the intrinsic error field. Alternatively, the $n = 2$ distribution might arise from the presence of both locked and quasi-stationary modes. Locked modes are expected to align with the residual, whereas quasi-stationary modes are expected to move quickly past the residual, spending the most time in the anti-aligned phase. Similar analysis on subsets of these data shows consistent results, suggesting that these distributions are not specific to a certain experimental campaign.

The intrinsic $n = 1$ error field in DIII-D has been characterized by in vessel apparatuses [87, 88] and the errors attributed to poloidal field coil misalignments and ellipticity, and the toroidal field buswork. It is found that the intrinsic error is well parameterized by the plasma current $I_p$ and the toroidal field $B_T$; "standard error field correction" in the DIII-D plasma control system calculates $n = 1$ correction fields based on these. The majority of DIII-D plasmas are run with $I_p$ in the counter-clockwise direction and $B_T$ in the clockwise direction when viewed from above, referred to as the "normal" directions. Taking ranges for $I_p$ and $B_T$ expected to encompass the majority of left-hand helicity discharges (i.e. $I_p = 0.8$ to 1.5 MA and
$B_T = -1$ to -2.1 T), the standard error field correction algorithm applies correction fields between $-164^\circ$ and $-110^\circ$. The preferential locking angles shown in figure 3-9a might be due to a *residual* EF, resulting from the vector addition of the intrinsic EF and an imperfect correction field.

Figures 3-9c-d show how residual fields can arise from changing intrinsic and/or correction fields. Figures 3-9e-f illustrate how the distributions in 3-9a-b might look if the residual is reproducible, or not. In the former case, a narrow, peaked distribution is expected, whereas in the latter case, a broad, flat distribution is expected. This might explain the distributions in figures 3-9a-b. In addition, quasi-stationary modes are also present in these data, and contribute to the broadening.

### 3.6.2 Change in $n = 1$ field at locking and growth before disruption

Figure 3-10 shows the change in mode width as the NTM slows from rotating at $f = 2$ kHz to 50 ms after locking. To within the $\pm 2$ cm errors on the width estimate from magnetics, a significant growth or decay at locking is not observed for the majority of modes. Only $\approx 10\%$ of locked modes appear above the top diagonal dashed line, indicating a growth at locking beyond error.

It is observed across the database that the radial field $B_R$ measured by the ESLDs tends to grow before disruption. This growth is distinct from the dynamics of the locking process, as it often occurs hundreds of milliseconds after locking. Figure 3-11a shows the $B_R$ behavior before disruption for five randomly chosen disruptive IRLMs. A general period of growth occurs between $\sim 100$ and 5 ms prior to the disruption, followed by a sharp rise in $B_R$ within milliseconds of the current quench, marked by the transient rise in $I_p$. Although interesting in its own right, we will not investigate the details of the $B_R$ spike occurring near the current quench, as we are interested in the dynamics leading up to the thermal quench.

Histograms in radial field $B_R$ are plotted in figure 3-11b, at times before disruption, to study the average evolution. The $B_R$ field in a single case often follows
Figure 3-9: The normalized phase distribution of all locked modes. Each mode contributes a total of $100/N$ across all bins ($N$ is number of disruptive or non-disruptive IRLMs). A binsize of 5 degrees was chosen as a compromise between being large enough to have sufficient statistics in one bin and fine enough to show features of interest. The angle is where the radial field is largest and outward on the outboard midplane. (a) Left-hand helicity (i.e. normal $I_p$ and $B_T$, or both reversed) with 980 disruptive and 1029 non-disruptive IRLMs. (b) Right-hand helicity plasma discharges (i.e. either $I_p$ or $B_T$ reversed). Only 130 disruptive and 204 non-disruptive IRLMs here. (c-f) Illustrations to explain distributions. (c) The residual EF is the difference between the intrinsic EF and the applied correction. (d) Due to changes in the intrinsic and/or the correction, the residual EF can change. (e) An average residual can be defined by averaging over several shots and times. A small standard deviation in its amplitude and phase (illustrated by the small circle) are indicative of high reproducibility of the residual EF. In that case, a narrow distribution is expected in Fig.a-c. (f) In the opposite limit, the residual EF phasor can point to any quadrant, and a broad, flat distribution is expected.
a complicated trajectory, as evidenced by figure 3-11a, but these histograms reveal global trends. In Figure 3-11b, the centers of the distributions shift to higher values of $B_R$ as the disruption is approached. The median is chosen to be the representative point in the analysis that follows. The median is the point on the distribution which divides the area under the curve equally.

The median is plotted in figure 3-11c as a function of time (note the vertical axis is logarithmic). The median major radial field is about 7 G at saturation (i.e. 50 ms after locking). Later in the lifetime of the mode, and approximately 50 ms prior to the disruption, the median grows consistent with an exponential from the saturated value, reaching a final value of $>11$ G at $\sim 5$ ms before the disruption. This time is approximately coincident with the onset of the thermal quench. From the slope of the dashed line, in figure 3-11c, an exponential $e$-folding time for $B_R$ in the range $\tau_g = [80, 250]$ ms is found.

The present analysis is not sufficient to discern between possible sources of this increased $B_R$. Out of the 26 IRLMs for which 2/1 modes can be clearly seen on ECE (see discussion in section 3.2.4), less than 10 of these provide a view of the island O-point during the disruption. One such case is shot 157247 shown in figure 3-12 which disrupted at 3723 ms. A significant flattening of the $T_e$ profile is seen at each time slice, confirming that a 2/1 island with $w > 5$ cm is present. The solid horizontal bars show the calculated island position (from EFIT) and width (derived from $\delta I$, see
Appendix E). The solid line in figure 3-12b shows the toroidal location of the island O-point on the outboard side in the midplane, and it is seen to intersect the location of the ECE diagnostic (dashed horizontal) at $t \approx 3660$ ms. The vertical dashed lines show the times of the profiles in figure 3-12a. Note that the worst toroidal alignment for O-point viewing occurs for $\phi_{LM} = -99^\circ$, where the X-point is aligned with the ECE. The horizontal bar at 3710 ms (green in online version) looks to over predict the island width by $\sim 3$ cm, though the island toroidal alignment with the ECE is intermediate between the O and X-points, where the flattened region is expected to be smaller. EFIT data do not exist for the last two timeslices.

In all shots where magnetics predict O-point alignment with ECE during the disruption, a clear flattening of the $T_e$ profile like that shown in figure 3-12 is observed. We conclude that the assumption of the presence of an $m = 2$ island during the disruption is accurate, and in the shots where ECE data are available, the predicted island widths are reasonable.

If we assume that the increase in the $n = 1$ field measured by the ESLDs is due to growth of the 2/1 island, we can estimate how much the island width would increase. We assume a constant $B_T$, $R$, and $dq/dr|_{q2}$ during the $\sim 100$ ms of growth, where the field increases from 7 to 11 G. Therefore, from equation E.8, we expect the island width to grow exponentially with a characteristic time constant $\tau_g = [80, 250]$ ms.
Figure 3-12: (a) Electron temperature $T_e$ profiles from ECE prior to an IRLM disruption show a clear flattening at the $q = 2$ surface. Horizontal bars show the automated estimation of island position and width (note the vertical position of the bars is chosen for visual purposes only). Two error bars are shown on the gray profile, and are representative of all $T_e$ measurement errors. (b) The toroidal position of the island O-point on the outboard side in the midplane. The horizontal dashed line shows the toroidal location of the ECE diagnostic, and the vertical dashed lines show the time slices from (a).

width to increase by $\sqrt{11/7} \approx 1.25$. With an average saturated width for disruptive islands of $\sim 4 - 6$ cm (this will be shown in section 3.8), a disruptive island would be expected to grow $\sim 1 - 1.5$ cm during the exponential growth. With a channel spacing of 1-2 cm for ECE measurements, and with less than 10 disruptive IRLMs whose O-points are well aligned with the ECE diagnostic during this time interval, validating this small change in island width would be difficult.

As the poloidal harmonic of this exponentially growing $n = 1$ field is unknown, in principle it is possible that the 2/1 island is unchanging, while an $m = 1$ or $m = 3$ instability is appearing and growing. Coupling of the 2/1 and 3/1 modes has been observed on ASDEX-Upgrade [109] and investigated in numerical studies [110], while coupling of the 2/1 and $m/n = 1/1$ has been observed on TEXTOR [111], the RTP Tokamak [104], and studied in simulations [108]. Although $m = 4$ might be a candidate for this growth in some shots, it will be shown in section 3.8 that more than half of the disruptive discharges have $q_{95} < 4$, so it could only explain a minority of cases. Similarly, out of 13 IRLMs that occur in plasmas with $q_{95} < 3$, at least 7 show a clear disruptive growth, in which the $m = 2$ or $m = 1$ modes are the only candidates for this growth.
Note that although we have chosen to characterize the final disruptive growth prior to the final thermal quench here, a single mode might undergo multiple growths and minor disruptions, as shown in figure 3-2.
3.7 Interdependence of locked modes and $\beta$

3.7.1 Effect of locked mode on $\beta$ and equilibrium

A common sign of the existence of a LM is a reduction in plasma $\beta$. Here we investigate $\beta_N = \beta(aB/I_p)$ as it has been shown to affect NTM onset thresholds [112] ($a$ is the plasma minor radius and $B$ is usually taken to be the toroidal field on axis).

Figure 3-13a shows $\beta_N$ at locking as a function of $\beta_N$ at mode onset. Raw data for disruptive modes are shown in red and purple. The purple disruptive modes are preceded by an earlier LM, while the red are not. The majority of the red points are observed to lie below the dashed diagonal. This observation is reiterated by the light blue points, showing the average and standard deviation also lying predominantly below the diagonal, meaning that $\beta_N$ decreases from NTM onset to locking.

Notice that a significant population of purple points lie above the dashed line for $\beta_N$ at rotating onset < 1.5. The rotating phase of these IRLMs begins with frequency $f = 0$ (locked), at which point they spin up. In these cases, $\beta_N$ has suffered a large degradation from the previous locked mode. In most of these cases, the $\beta_N$ at the following locking is greater than the degraded $\beta_N$ at the time of spin up, and therefore the points lie above the dashed line.
Figure 3-14: Both disruptive and non-disruptive modes show a linear dependence with $\Delta R_0/\Delta \beta_p \sim 4$ cm. A number of outliers are produced in the non-disruptive population by significant changes of plasma shape (e.g. diverted to wall-limited plasma).

The light blue and black averages from figure 3-13a are replotted in figures 3-13b and 3-13c in the form of percent changes in $\beta_N$ at each stage, relative to the $\beta_N$ at rotating onset. These plots show how $\beta_N$ changes from the rotating onset to locking, to $\beta_N$ saturation, and to mode termination. The time of $\beta_N$ saturation is taken to be 200 ms after locking (found empirically, in approximate agreement with twice the typical DIII-D energy confinement time, $\tau_E \approx 100$ ms), and the time of mode termination is taken to be 50 ms before disruption or disappearance of the mode. Except for the initial phase with $\beta_N$ at rotating onset $< 1.5$, a continuous decrease in $\beta_N$ is observed during successive phases of the IRLM.

On average, the disruptive modes cause a larger degradation of $\beta_N$ than non-disruptive modes during all three phases. For $\beta_N$ at rotating onset $> 1.6$, disruptive modes cause 20-50% reduction during the rotating phase, 50-70% reduction by $\beta_N$ saturation, and 50-80% reduction within 50 ms of the disruption. Of course a further, complete loss of $\beta_N$ occurs at disruption (not shown).

With the reduction in $\beta_N$ in figure 3-13, and assuming constant $I_p$ and $B_T$ (which is accurate for most modes occurring during the $I_p$ flattop), a similar reduction in $\beta_p = \langle p \rangle/(B_0^2/2\mu_0)$ is expected as well. The reduction in $\beta_p$ causes a reduction in the Shafranov shift. Figure 3-14 shows the linear dependence of $R_0$ on $\beta_p$ during the first 200 ms after locking. A linear fit to the disruptive data provides a shift ratio of $\Delta R_0/\Delta \beta_p \approx 4$ cm. Most modes reduce the Shafranov shift by 0-3 cm. This
reduction of the 1/0 shaping reduces toroidal coupling of the 1/1 and 2/1 modes, the 2/1 and 3/1 modes, and other $m/n$ and $(m + 1)/n$ mode combinations [113]. The larger scatter in the non-disruptive population can be explained at least in part by significant shape changes, such as the transition from diverted to wall limited. For this reason, the linear fit is performed only on the disruptive data.

3.7.2 Effect of $\beta$ on the saturated width, IRLM rate of occurrence, and disruptivity

As NTMs are the result of helical perturbations to the bootstrap current [57], we expect the island width to depend on terms that drive the bootstrap current. The Modified Rutherford Equation (MRE) has been shown in previous works to describe 3/2 [68, 67] as well as 2/1 [69, 70] tearing mode saturation well. The saturated modes of interest here have an average width $w \approx 5$ cm and a standard deviation $\sigma \approx 2$ cm. Being that small island terms in the MRE start to become small for $w > 2$ cm (assuming $w_{\text{pol}} \approx 2$ cm [57]), small island effects are ignored. A steady state expression of the MRE (i.e. $dw/dt \to 0$) is given as follows,

$$0 = r\Delta'(w) + \alpha e^{1/2} \frac{L_q}{L_{pe}} \beta_{pe} \frac{r}{w} + 4 \left( \frac{w_v}{w} \right)^2$$

(3.2)

where $\Delta'(w)$ is the classical stability index, $\alpha$ is an ad hoc accounting for the stabilizing effect of field curvature ($\alpha \approx 0.75$ for typical DIII-D parameters), $\epsilon = r/R$ is the local inverse aspect ratio, $L_q = q/(dq/dr)$ is the length scale of the safety factor profile, $L_{pe} = -p_e/(dp_e/dr)$ is the length scale of the electron pressure profile ($L_{pe} > 0$ as defined), $\beta_{pe}$ is the electron poloidal beta, and $w_v$ is the vacuum island width driven by the error field ($w_v \sim 1$ cm in DIII-D). Note that usually a cosine term appears on the term involving $w_v$ [114], but it is set to unity here, which assumes the most destabilizing alignment of the locked mode with the error field.

We take $\Delta'(w)$ to be of the form $\Delta'(w) = C_0/r - C_1 w/r^2$ [103]. With this definition of $\Delta'(w)$, equation 3.2 is a nontrivial cubic equation in $w$. However, an approximate solution was found by approximating $(w_v/w)^2 \approx a w_v/w - b$ (with $a = 0.45$, $b = 0.045$,
found to be accurate to within 15% over the domain $w_v/w = [0.12, 0.3]$). The resulting equation is quadratic, of straightforward solution. The approximate saturated width expression is given by,

$$\frac{2w_{sat}}{r} = \left( \frac{C_0 - 4b}{C_1} \right) + \left[ \left( \frac{C_0 - 4b}{C_1} \right)^2 + \frac{4}{C_1} \left( \alpha e^{1/2} \frac{L_p}{L_p} \beta_p + 4a \frac{w_v}{r} \right) \right]^{1/2}$$

(3.3)

The data in figure 3-15 show the normalized island width as a function of $\beta_p/(dq/dr) = \beta_p L_q/2$. The inverse aspect ratio $\epsilon$ and the GGJ factor $\alpha$ do not vary greatly across these discharges. The parameters $L_p$ and $\Delta'(w)$ (which is decomposed into $C_0$ and $C_1$ in equation 3.3) are difficult to acquire with automated analysis, and therefore are not available in the database. Therefore, a direct fitting of equation 3.3 is not appropriate, but the conclusion that $w_{sat}/r$ increases with $\beta_p L_q$ is evident. A correlation of $r_c = 0.55$ is found between $w_{sat}/r$ and $\beta_p/(dq/dr) = \beta_p L_q/2$ for plasmas with $q_{95} < 4$. Plasmas with $q_{95} > 4$ show a weaker correlation with $r_c = 0.36$. This correlation suggests that locked modes in DIII-D are driven at least in part by the bootstrap current.

A one dimensional study of IRLM frequency (or prevalence) versus $\beta_N$ appears to suggest that intermediate $\beta_N$ shots are the most prone to IRLMs, as shown in figure 3-16. The fraction of shots containing an IRLM increases with $\beta_N$ up to 2.75.

The fraction of shots with IRLMs decreases significantly for $\beta_N \geq 4.5$. A manual
investigation of the 38 shots in this bin reveals that these discharges often have (1) $q_{95} \geq 5$, or (2) high neutral beam torques of $T \approx 6$ Nm, or both. In both cases, a low occurrence of IRLMs might be expected due to a weak wall torque (see equation 3.1): (1) the $q = 2$ surface is far from the wall in discharges with high $q_{95}$, and (2) shots with high injected torque likely exhibit high plasma rotation, where the wall torque goes like $\omega^{-1}$, and is therefore relatively small. A three-dimensional analysis of IRLM frequency versus $\beta_N$, $\rho_{q2}$, and NBI torque might be more informative. These data are not populated for all shots in the database, so this analysis is reserved for future work.

The IRLM shot disruptivity as a function of $\beta_N$ and $q_{95}$ is plotted in figure 3-17. The IRLM shot disruptivity is defined as the 'number of shots with disruptive IRLMs' divided by the 'number of shots with IRLMs'. Considering IRLM shot disruptivity as a function of $\beta_N$ only, the histogram at the right of figure 3-17a shows a peaked distribution, with the highest values for $\beta_N \sim 1.5 - 3$. Similar results were obtained for the global disruptivity as a function of $\beta_N$ at NSTX [47] (the NSTX study is not limited to locked modes, and disruptivity is normalized by the amount of time spent at the given $\beta_N$ value). Reduced locked mode disruptivity at high $\beta_N$ was also observed at MAST [48].

The IRLM shot disruptivity dependence on $\beta_N$ can be explained in part by the reduced number of discharges with $q_{95} < 3.5$ when $\beta_N$ is high. To see this, we bin
Figure 3-17: (a) The raw data for the highest achieved $\beta_N$ as a function of $q_{95}$. The histogram on the right shows the one-dimensional IRLM shot disruptivity as a function of peak $\beta_N$. Windows in $\beta$ are labeled, and the binning for figures (b) and (c) are shown in gray. (b) IRLM shot disruptivity as a function of $q_{95}$ for the binned data in (a). (c) The distribution of each $\beta_N$ bin in $q_{95}$ in percent.

The data in figure 3-17a into five $\beta_N$ windows, denoted $\beta_0$, $\beta_1$, … $\beta_4$. First, note that figure 3-17b shows a general decrease in IRLM disruptivity as $q_{95}$ is increased, across all $\beta_N$ windows. Next, figure 3-17c shows the percent distribution of $q_{95}$ values in each of these $\beta_N$ windows. The two highest values of $\beta_N$ (purple and green) have the lowest percentage of discharges with $q_{95} < 3.5$, and the highest percentage of discharges with $q_{95}$ between 4.5 and 5. This distribution of $q_{95}$ reduces the number of disruptive discharges, with the net result that the higher $\beta_N$ discharges appear less disruptive.

As $q_{95}$ is shown here to affect IRLM shot disruptivity, fixing $q_{95}$ removes one variable, and allows us to more closely inspect the dependence of IRLM shot disruptivity on $\beta_N$. Figure 3-17b shows IRLM shot disruptivity across five windows in $\beta_N$ (denoted by color), for five values of $q_{95}$ (separated by vertical gray lines). In all $q_{95}$ windows, there is either no significant dependence on $\beta_N$, or a lower IRLM shot disruptivity at higher $\beta_N$, particularly in the window $3.5 < q_{95} < 4.5$. Although larger islands are expected at higher $\beta_N$, it will be shown in the next section that IRLM disruptivity depends very weakly on island width (refer to figure 3-19), which in turn might explain the weak dependence on $\beta_N$. It should be noted that only $q_{95}$ is fixed here. Other parameters that are correlated with $\beta_N$ (e.g. NBI torque, ion
and electron temperatures, and possibly others) are not fixed, and might affect the apparent IRLM disruptivity scaling.
3.8 IRLM disruptivity

3.8.1 Decoupling the effects of $\rho_{q2}$, $q_{95}$, and $l_i$ on IRLM disruptivity

Figure 3-18a shows the normalized radius of the $q = 2$ surface $\rho_{q2}$ and $q_{95}$ at the mode end, defined here to be 100 ms prior to the termination of the mode. The data are from equilibrium reconstructions constrained by both the external magnetics, and the Motional Stark Effect (MSE) diagnostic. On the right and below figure 3-18a are histograms of IRLM disruptivity as a function of $\rho_{q2}$ and $q_{95}$ respectively. The $q_{95}$ histogram shows the expected result that lower $q_{95}$ is more disruptive, as was also seen in figure 3-17b. The total disruptivity (i.e. not limited to IRLM disruptions) is observed to increase as $q_{95}$ is decreased in DIII-D [45]. This is in agreement with, but not limited to, observations from JET [46], NSTX [47], and MAST [48]. The histogram in $\rho_{q2}$ has a qualitatively similar shape, with the highest IRLM disruptivity at large $\rho_{q2}$, and the lowest at small $\rho_{q2}$.

The raw data show an expected correlation between $\rho_{q2}$ and $q_{95}$. In a circular cross-section, cylindrical plasma, $q_{95}$ can be defined as follows,

$$q_{95} = \frac{2}{\rho_{q2}^2} \left[ 1 + \frac{I_{out}}{I_{enc}} \right]^{-1}$$

where $I_{enc}$ is the total toroidal current enclosed by the $q = 2$ surface, and $I_{out}$ is the total toroidal current outside of the $q = 2$ surface. Note that the quotient $I_{out}/I_{enc}$ behaves similar to the inverse plasma internal inductance $l_i^{-1}$: when $I_{out}/I_{enc}$ is large, $l_i$ is small, and vice versa. The lack of one-to-one relationship between $\rho_{q2}$ and $q_{95}$ in the raw data of figure 3-18a may therefore be attributed in part to variations in $I_{out}/I_{enc}$. Toroidal geometry and plasma shaping may also introduce variations in the relationship between $q_{95}$ and $\rho_{q2}$.

Empirically, we find a high correlation ($r_c = 0.87$) between $l_i/q_{95}$ and $\rho_{q2}$, suggesting a relationship of the form,
\[
l_i/q_95 = \alpha \rho_{q2} + c
\]  
(3.5)

where \( \alpha = 0.67 \pm 0.01 \), and \( c = -0.23 \pm 0.01 \). This equation suggests that \( \rho_{q2} \) specifies \( l_i/q_95 \), or vice versa. We begin by studying the effects of \( q_{95} \), \( l_i \), and \( \rho_{q2} \) on IRLM disruptivity individually. Then, we investigate IRLM disruptivity as a function of \( \rho_{q2} \) and \( l_i/q_95 \), which despite the high correlation between the two, reveals that \( l_i/q_95 \) distinguishes disruptive IRLMs better than \( \rho_{q2} \).

First, to decouple the effect of \( q_{95} \) and \( \rho_{q2} \) on IRLM disruptivity, we fix one and study the dependence on the other in figure 3-18. We approximately fix \( \rho_{q2} \) by considering only data that lie in small windows of \( \rho_{q2} \) denoted \( \rho_1, \rho_2, \rho_3, \) and \( \rho_4 \) (\( \rho_4 \) covers a relatively large window in \( \rho_{q2} \) as data become sparse, but IRLM disruptivity appears constant throughout the window). IRLM disruptivity as a function of \( q_{95} \) in the \( \rho_{q2} \) windows is shown in figure 3-18b. Neither the trace corresponding to \( \rho_1 \) nor \( \rho_2 \) show a significant trend with IRLM disruptivity beyond error. Both \( \rho_3 \) and \( \rho_4 \) show a possible decrease in IRLM disruptivity for \( q_{95} > 5.5 \), but appear constant for \( q_{95} < 5.5 \).

The same study is performed on \( \rho_{q2} \) by choosing windows in \( q_{95} \) (or safety factor) denoted SF1, SF2, and SF3 (figure 3-18c). The three safety factor windows agree within statistical error on an increasing linear trend, with \( <20\% \) IRLM disruptivity for \( \rho_{q2} < 0.7 \), and \( >80\% \) for \( \rho_{q2} > 0.85 \).

From equation 3.5, we see that fixing \( \rho_{q2} \) and varying \( q_{95} \) (as in figure 3-18b) implies a variation of \( l_i \) as well. The weak or absent trend in IRLM disruptivity as a function of \( q_{95} \) might be explained by competing effects of \( q_{95} \) and \( l_i \).

Similarly, equation 3.5 shows that fixing \( q_{95} \) and varying \( \rho_{q2} \) (as in figure 3-18c) also implies varying \( l_i \). The strong dependence of IRLM disruptivity on \( \rho_{q2} \) thus suggests that either \( \rho_{q2}, l_i \), or both have a strong effect on IRLM disruptivity.

To help isolate the individual effects of \( \rho_{q2}, q_{95} \), and \( l_i \), we compare how well they separate disruptive and non-disruptive IRLMs single-handedly.

In order to quantify separation of the distributions in one-dimension, we employ
Figure 3-18: (a) The relationship of disruptive and non-disruptive IRLMs in $q_{95}$ and $\rho_{q2}$ space. One-dimensional IRLM disruptivity histograms in $q_{95}$ and $\rho_{q2}$ are shown on the bottom and right. Bins in $\rho_{q2}$ and safety factor are shown in gray, for use in (b) and (c). Note that only the binning intervals are shown, and not the explicit bins (i.e. the dashed and solid gray lines are unrelated). (b) IRLM disruptivity in windows of $\rho_{q2}$ as a function of $q_{95}$. (c) IRLM disruptivity in windows of $q_{95}$ as a function of $\rho_{q2}$.

the Bhattacharyya Coefficient (BC) [115]. This coefficient was developed to measure the amount of overlap between two statistical distributions, and is commonly used in image processing, particularly for measuring overlap of color histograms for pattern recognition and target tracking [116]. For two discrete probability distributions $p$ and $q$ parameterized by $x$, the Bhattacharyya Coefficient is given by,

$$BC(p, q) = \sum_{x \in X} \sqrt{p(x)q(x)}$$

The $BC$ metric varies over the range $0 \leq BC(p, q) \leq 1$, where a value of 1 indicates that $p$ and $q$ are identical and perfectly overlapping. A value of 0 implies they are completely distinct (no overlap).

Figure 3-19 shows the 1D separation of disruptive and non-disruptive modes for six parameters at 100 ms before mode termination, and reports the $BC$ value for each. Of the three interdependent parameters (i.e. $\rho_{q2}$, $q_{95}$, and $l_i$), $\rho_{q2}$ is observed both visually, and by the $BC = 0.70$ value to best separate the two populations. The parameters $q_{95}$ and $l_i$ produce less separation with coefficients of $BC = 0.85$ and
rather than a horizontal one. For instance, choosing a threshold of

\[ l_i / q_{95} \]

plotted in the 2D space of

\[ l_i / q_{95} \]

suggested by equation 3.5. However, here we show that

\[ l_i / q_{95} \]

In the previous section, it is assumed that

\[ l_i / q_{95} \]

3.8.2 Decoupling effects of \( \rho q^2 \) and \( l_i / q_{95} \) on IRLM disruptivity

Figure 3-19: Disruptive and non-disruptive IRLM distributions are shown as a function of six parameters to reveal the dominant classifier. Amount of overlap between distributions is quantified by the Bhattacharyya Coefficient (BC). A BC value of 0 indicates no overlap, and a value of 1 indicates complete overlap. All BC values have an error bar of ±0.04. Solid curves are evaluated at 100 ms before mode termination, while the dotted red in (a) and (f) are evaluated 20 ms before the disruption, with corresponding BC\(_{20}\) values (the BC\(_{20}\) values of all other parameters, not shown, are similar to their reported BC values). Not shown, \( dq/dr|_{q^2} \) produces a value of BC=0.89.

\[ BC = 0.88 \]

respectively. All BC values reported in figure 3-19 have an error bar of ±0.04.

Disruptivity is found to scale strongly with plasma shaping in NSTX [47]. The effects of shaping on IRLM disruptivity in DIII-D will be included in a future work.

3.8.2 Decoupling effects of \( \rho q^2 \) and \( l_i / q_{95} \) on IRLM disruptivity

In the previous section, it is assumed that \( l_i / q_{95} \) and \( \rho q^2 \) are effectively equivalent, as suggested by equation 3.5. However, here we show that \( l_i / q_{95} \) has a stronger effect on IRLM disruptivity. Figure 3-20 shows all disruptive and non-disruptive IRLMs plotted in the 2D space of \( l_i / q_{95} \) and \( \rho q^2 \) at 100 ms prior to mode termination.

The data in the region where \( \rho q^2 = [0.7, 0.8] \) show a clear vertical separation, rather than a horizontal one. For instance, choosing a threshold of \( l_i / q_{95} < 0.28 \)
as the definition of a non-disruptive IRLM results in 7% mis-categorized disruptive IRLMs, and 24% mis-categorized non-disruptive IRLMs. To capture a similar number of correctly identified non-disruptive IRLMs using $\rho_{q2}$, a value of $\rho_{q2} < 0.78$ is used and results in 14% mis-categorized disruptive IRLMs, and 25% mis-categorized non-disruptive IRLMs. This confirms the result of figures 3-19b-c that $l_i/q_{95}$ categorizes disruptive and non-disruptive IRLMs better than $\rho_{q2}$.

To reduce mis-categorized disruptive modes, a threshold of $l_i/q_{95} < 0.25$ can be used which produces only 3% mis-categorized disruptive IRLMs. However, the mis-categorized non-disruptive IRLMs increase to 42%.

Similar empirical observations of a critical boundary in $l_i$ and $q_{95}$ space are reported for density limit disruptions in JET [71], for current ramp-down disruptions in JT-60U [74], and for “typical” disruptions in TFTR (see figure 6 in [117]). Theoretical interpretations of $l_i/q_{95}$ will be presented in section 3.9.

Note that $\sim$ 17 of the 53 disruptive discharges omitted due to lack of equilibrium data likely fall below the $l_i/q_{95}$ threshold, but are not included in figure 3-20, nor the categorization percentages reported in this section. The impact of these omitted discharges is assessed in section 3.8.5.
3.8.3 $d_{\text{edge}}$ discriminates disruptive IRLMs within 20 ms of disruption

In this subsection and subsection 3.8.4, it is assumed that the exponential $n = 1$ growth in $B_R$ (figure 3-11c) is due to growth of the 2/1 island. It will be shown here that $d_{\text{edge}}$ is as effective as $l_i/q_{95}$ at discriminating disruptive IRLMs within 20 ms of the disruption. A possible physical interpretation will be given in section 3.9.

Figure 3-21 shows the island half-width as a function of the distance between the $q = 2$ surface and the unperturbed plasma separatrix, $a - r_{q2}$. The shortest perpendicular distance from the black dashed line representing the unperturbed last closed flux surface (LCFS) to a given point is what we have called $d_{\text{edge}} = a - (r_{q2} + w/2)$. In other words, a point appearing near the solid black line represents an island whose radial extent reaches near the unperturbed LCFS. Due to the perturbing field of the locked mode, we expect kinking of the LCFS. This kinking of the LCFS is not accounted for in our calculation of $d_{\text{edge}}$.

$d_{\text{edge}}$ is shown to separate disruptive and non-disruptive modes as effectively as $\rho_{q2}$ at 100 ms prior to disruption in figure 3-19 (recall the ±0.04 error bar on all BC values). Within 20 ms of the disruption, $d_{\text{edge}}$ separates the two populations better with a BC value of 0.61 (compare the dotted red and solid black distributions for $d_{\text{edge}}$ in figure 3-19a). The better separation is due to the exponential growth of the $n = 1$ field that occurs in the final 50 ms before disruption: that growth of $B_R$, and thus of $w$, decreases $d_{\text{edge}}$ without affecting $\rho_{q2}$. Note that evaluating $d_{\text{edge}}$ at 20 ms prior to the disruption implicitly attributes the $n = 1$ exponential growth to a growing 2/1 island, but the validity of this assumption does not affect the discrimination ability of $d_{\text{edge}}$.

Choosing a threshold of $d_{\text{edge}} > 9$ cm evaluated 20 ms before the disruption (blue dashed line in figure 3-21), we find 8% mis-categorized disruptive IRLMs, and 28% mis-categorized non-disruptive IRLMs. This is comparable with the 7% and 24% found for $l_i/q_{95} < 0.28$, though $l_i/q_{95}$ is evaluated 80 ms earlier (these thresholds on $d_{\text{edge}}$ and $l_i/q_{95}$ correctly categorize 195 and 194 non-disruptive IRLMs respec-
tively). A more conservative threshold of $d_{\text{edge}} > 11$ cm produces 3% mis-categorized disruptive IRLMs, and 58% mis-categorized non-disruptive IRLMs.

3.8.4 Weak dependence of IRLM disruptivity on island width

The island width alone is found to be a poor discriminator of disruptive IRLMs at times 20 ms before the disruption or earlier (figure 3-19f). From the time of saturation until $\sim 20$ ms prior to the disruption, disruptive and non-disruptive island widths are similar, with a slight tendency for non-disruptive modes to be larger at the earlier times. IRLM disruptivity as a function of the island half-width is shown in the histogram on the right of figure 3-21. For the majority of islands with half-widths between 2 and 5 cm, IRLM disruptivity does not change significantly, and might be constant within statistical error.

The region below the curved dashed line in figure 3-21 shows the approximate mode detection limit due to typical signal-to-noise-ratio discussed in section 3.2.4. Modes appearing in this region were once above the detection limit, and are still measured due to the asymmetry in onset and disappearance thresholds used in the analysis. It is possible that undetected modes in this region might affect the resulting IRLM disruptivity.

3.8.5 IRLM disruption prediction

Thus far we have discussed percent mis-categorizations, separation of disruptive and non-disruptive distributions measured by the BC coefficient, and IRLM disruptivity all at specific points in time. Although useful for understanding the physics of IRLM disruptions, single time-slice analysis is not sufficient for disruption prediction. Prediction during the locked phase requires establishing a parameter threshold that will never be exceeded by a non-disruptive IRLM, but will be exceeded by a disruptive IRLM at some point before the thermal quench.

We consider $l_i/q_{95}$ and $d_{\text{edge}}$ separately as IRLM disruption predictors. These predictors are intended to be used only in the presence of a detected locked mode.
Figure 3-21: The assumption that the 2/1 island dominates the disruptive exponential growth is implicit in this figure, and therefore this figure is exploratory. The disruptive data are from 20 ms before disruption, and the non-disruptive data are from 100 ms before mode termination. The horizontal axis quantifies the distance of the $q = 2$ surface from the unperturbed last closed flux surface (LCFS). The shortest perpendicular distance from the unperturbed LCFS (black solid) to a given point is $d_{\text{edge}}$ (i.e. $d_{\text{edge}} = a - (r_{q2} + w/2)$). The blue dashed line is where $d_{\text{edge}} = 9$ cm. IRLM disruptivity as a function of island half-width is shown in the blue histogram.

Table 3.2 shows the percent missed disruptions and percent false alarms for the given thresholds, and for the given warning times. The percent missed disruptions is defined as the number of disruptive IRLMs that do not exceed the threshold within $X$ ms of the disruption, divided by the total number of disruptive IRLMs. The percent false alarms is defined as the number of shots where at least one non-disruptive IRLM exceeds the threshold at any time during its lifetime, divided by all issued alarms. Note that a single non-disruptive discharge can have multiple non-disruptive IRLMs, but only one alarm.

If all IRLMs are considered disruptive, no IRLM disruptions are missed, but the percent false alarms increases to $25 \pm 2\%$. This case is shown as the third row of table 3.2, labeled “none”, as no additional condition is required before issuing an alarm. Declaring an IRLM disruptive when it locks provides a warning time that depends on the survival time of the IRLM. The distribution of survival times, and therefore also of warning times for this disruption prediction criterion, are shown in figure 3-7.
Table 3.2: IRLM disruption prediction statistics for \( l_i/q_{95} \), \( d_{\text{edge}} \), and no prediction parameter. The two thresholds are shown graphically by the dashed blue lines in figures 3-20 and 3-21 (note that figures 3-20 and 3-21 are evaluated at single time slices, whereas these statistics are evaluated over appropriate time intervals). These prediction criteria are intended for use in the presence of a detected IRLM only. The "None" condition shows the disruption statistics for the case where all IRLMs are considered disruptive (see text for warning times for this case). Note that discharges omitted by the automated analysis might increase the missed disruption percentages by up to 3% (see text for details).

Despite the seemingly short warning timescale, 20 ms is a sufficient amount of time to deploy massive gas injection in DIII-D [118].

Out of the 53 disruptive discharges omitted due to insufficient equilibrium data (see section 3.2.4), 17 of them might contribute to increasing the missed disruption percentages associated with the \( l_i/q_{95} \) criterion reported in table 3.2 by up to 3%, with negligible effect on the percent false alarms. Also, out of the remaining 53 - 17 = 36 disruptive discharges that satisfy \( l_i/q_{95} > 0.28 \), 32 exhibit survival times around 20 ms, and thus the warning time for these modes is no longer than 20 ms. The increase in missed disruptions can be reduced to 1.5% by considering discharges with \( q_0 < 1.6 \) only, as 9 of the 17 discharges contributing to the increased percent missed disruptions satisfy this criterion (only 40 discharges with 2/1 IRLMs in the entire database satisfy this criterion). The effects of these omitted discharges on the \( d_{\text{edge}} \) prediction criterion are expected to be similar to those just discussed for the \( l_i/q_{95} \) prediction criterion.

### 3.8.6 \( \rho_{q_2} \) Evolution

The evolution of \( \rho_{q_2} \) between locking and mode end, provides insight into the evolution of both \( l_i/q_{95} \) and \( d_{\text{edge}} \). Figure 3-22a shows that \( \rho_{q_2} \) tends to increase by \( \sim 5\% \) between locking and mode end. This is noticeable by the clustering of points above
Figure 3-22: The evolution of $\rho_{q2}$ from locking to mode end (100 ms prior to mode termination) for IRLMs which terminate predominantly during the $I_p$ flat-top. The diagonal line represents unchanging $\rho_{q2}$. The horizontal line is at $\rho_{q2} = 0.75$, and marks an approximate transition from low to high IRLM disruptivity.

Figure 3-23: All disruptive and non-disruptive IRLMs shown in $l_i$ and $q_{95}$ space. The JET density limit shown in blue defines the approximate lower bound of the disruptive locked modes in DIII-D remarkably well. The cyan dashed line is the $l_i/q_{95} = 0.28$ value again, first seen in figure 3-20 to divide the populations well.

the dashed diagonal. Equation 3.5 suggests that an increase in $\rho_{q2}$ at fixed $q_{95}$ implies an increase in $l_i$. As $I_p$, $B_T$, and plasma shaping are fixed via feedback in most DIII-D discharges, the assumption of constant $q_{95}$ is very reasonable. The increase in $l_i$ during locked modes agrees with earlier works [72, 71, 104].
3.9 Discussion of locked mode database results

The physical significance of \( l_i/q_{95} \) might be related to the potential energy available for tearing growth (i.e. to the island width dependent classical stability index \( \Delta'(w) \)).

A one-dimensional simulation including both MHD evolution and transport has shown that the constraint \( q_{\text{min}} > 1 \) enforced by the sawtooth instability has a significant effect on the current profile, resulting in a steepening of the current gradient between the \( q = 1 \) and \( q = 2 \) surfaces as the edge \( q \) value is decreased [119]. A steepened current profile on the core-side of the 2/1 island is classically destabilizing, and thus, \( \Delta'(w) \) is shown to increase as the edge \( q \) decreases.

A three-dimensional simulation [120] later corroborated the classically destabilizing phenomenon found in [119]. Citing these works, a study followed [117], where \( \Delta'(w = 0) \) stability was investigated as a function of the current profile shape, with fixed axial and edge \( q \) values. In that study, monotonically decreasing current profiles with \( q_0 \sim 1 \) were varied in search of a \( \Delta' \) stable profile, and a limit in \( l_i \) and \( q_{95} \) space was found where no stable solution exists (see figure 6 of [117]). That limit is similar to the empirical IRLM disruption limit shown by the cyan dashed line in figure 3-23. These three works together [119, 120, 117] provide a theoretical basis for the hypothetical connection between \( l_i/q_{95} \) and \( \Delta' \).

\( l_i/q_{95} \) might be responsible for the pre-thermal quench growth shown in figure 3-11, and it might also influence the evolution of MHD during the thermal quench.

Separately, it is interesting that at high density, a high value of \( l_i \) is itself unstable, causing radiative contraction of the temperature and current profiles [71], which further increases \( l_i \). Since \( q_{95} \) is approximately fixed via feedback, increasing \( l_i \) implies increasing \( l_i/q_{95} \).

Alternatively, the physical significance of \( l_i/q_{95} \) might be related to an excess of radiative losses in the island, overcoming the Ohmic heating, resulting in exponential growth [121]. Figure 3-23 shows good separation of disruptive and non-disruptive IRLMs in the \( l_i \) and \( q_{95} \) space used in reference [71] to identify density limit disruptions. Although the plasmas discussed herein are expected to be far from the density...
limit [122] as a result of the degraded confinement, they are likely near the radiative tearing instability limit which is theorized as the fundamental mechanism causing the density limit [123]. This radiative tearing instability limit is expected qualitatively to scale with \( l_i/q_{95} \).

\( l_i/q_{95} \) and \( d_{\text{edge}} \) are both good IRLM disruption predictors and are both correlated with \( \rho_{q2} \) (the former by equation 3.5, and the latter by definition). Although this does imply some common underlying physics, \( l_i/q_{95} \) and \( d_{\text{edge}} \) differ from \( \rho_{q2} \) by capturing information on the \( q \)-profile shape and the island width respectively. Both discriminate disruptive from non-disruptive IRLMs better than \( \rho_{q2} \) alone, and do so by leveraging different physics. We conjecture that \( l_i/q_{95} \) is a proxy for \( \Delta'(w) \), as discussed earlier in this section and supported by earlier publications. The success of the \( d_{\text{edge}} \) parameter might suggest a position-dependent critical 2/1 island width, where criticality depends on proximity to the unperturbed last closed flux surface. Alternatively, if the \( n = 1 \) field is the result of multiple islands and not just the 2/1 island, as implicitly assumed in the definition of \( d_{\text{edge}} \), then island overlap might result from \( n = 1 \) islands of \( m \geq 2 \) becoming closer and larger as \( d_{\text{edge}} \) decreases. Island overlap is known to cause stochastic fields [124], and their effect on confinement might be sufficient to induce a disruption. Commonalities and differences between \( l_i/q_{95} \) and \( d_{\text{edge}} \) will be the subject of future work.

Other works have found parameters similar to \( d_{\text{edge}} \) to be relevant to the onset of the thermal quench, and are briefly listed here. In reference [107], a stable Alcator C-Mod equilibrium is used as an initial condition for a numerical simulation of massive gas injection. It is found that by uniformly distributing a high-Z gas in the edge, a 2/1 island is driven unstable and upon intersecting with the high-Z gas (i.e. when \( d_{\text{edge}} \) becomes small), the 2/1 grows rapidly followed by a 1/1 tearing mode that leads to a complete thermal quench. Similarly, a limited plasma is simulated in reference [108] with a current profile unstable to the 2/1 island, and the plasma current is ramped up to drive the 2/1 island towards the plasma edge. It is found that when the 2/1 island comes into contact with the limiter or a cold edge region (i.e. in both cases, when \( d_{\text{edge}} \) is small), a rapid growth of the 2/1 ensues followed by a 1/1 kink.
displacement of the core, ending in a disruption. Similarly, experimental results of disruptions induced by EF penetration modes on COMPASS-C \[86\] are attributed to exceeding a threshold of $w/(a - r_s) > 0.7$, where $w$ and $r_s$ are the 2/1 island width and minor radius. This observation is thought to be due to the 2/1 island interacting with a 3/1 island.

Large 3-D fields at the plasma separatrix cause radial deformations of the edge plasma, leading to edge flux-surfaces (like the $q = 2$ when $d_{\text{edge}}$ is small) intersecting the vessel \[125\]. Further, some works suggest the existence of a stochastic layer within the LCFS \[126, 127\], which might facilitate stochastization of the 2/1 island when $d_{\text{edge}}$ is sufficiently small.

Separately, the radiation drive of tearing modes is sensitive to edge proximity \[121\], causing island growth which reduces $d_{\text{edge}}$, and could then lead to the thermal quench through one of the above mechanisms. Again, recall that our interpretation of $d_{\text{edge}}$ at 20 ms prior to the disruption requires the exponential growth observed in the $n = 1 \, B_R$ signal to be due to the 2/1 island growth, which is plausible but not confirmed due to lack of poloidal harmonic analysis in the locked phase. Regardless of the theoretical interpretation, $d_{\text{edge}}$ is a useful IRLM disruption predictor within 20 ms of the disruption for DIII-D.
3.10 Summary and conclusions of the locked mode database

Approximately 22,500 DIII-D plasma discharges were automatically analyzed for the existence of initially rotating 2/1 locked modes (IRLMs). The results of this analysis permits statistical analysis of timescales, mode amplitude dynamics, effects of plasma $\beta$ and major radius $R$, and disruptivity as a function of plasma and mode properties.

Timescales investigated suggest that a rotating 2/1 NTM that will eventually lock rotates for $\sim 200$ ms, decelerates from $f = 2$ kHz to locked in $\sim 15$ ms, and survives as a locked mode for $\sim 300$ ms (these values all represent the most frequent value in their respective histograms). These timescales provide insight into how to respond to a mode locking event, whether with a disruption avoidance approach (such as fast controlled shut-down, mode spin-up, or mode stabilization) or a disruption mitigation system (such as massive gas injection), depending on the response times of different systems and approaches.

Prior to disruption, the median $n = 1$ perturbed field grows exponentially with an $e$-folding time between $\tau_g = [80, 250]$ ms, which might be due to exponential growth of the 2/1 IRLM.

The parameter $l_i/q_{95}$ is shown to have a strong ability to discriminate between disruptive and non-disruptive IRLMs, up to hundreds of milliseconds before the disruption. As an example, the criterion $l_i/q_{95} > 0.28$ in the presence of a detected IRLM misses 7% of disruptions and produces 11% false alarms, with at least 100 ms of warning time. $l_i/q_{95}$ might be related to the free energy available for tearing mode growth.

$d_{\text{edge}}$ performs comparably to $l_i/q_{95}$ in its ability to discriminate disruptive IRLMs. A threshold below which IRLMs are considered disruptive of $d_{\text{edge}} = 9$ cm produces 4% missed disruptions and 12% false alarms, with at least 20 ms of warning time. $d_{\text{edge}}$ is also observed to exhibit the best correlation with the IRLM survival time. $d_{\text{edge}}$ might be a fundamental trigger of the thermal quench, supported by similar observations by other authors [86, 108, 107, 125].
Future work will attempt to validate thermal quench onset mechanisms in plasmas with locked modes, with the goal of a fundamental understanding of locked mode disruptions, and thus how to avoid them.

3.11 Interlude

With the database of \( \geq 2000 \) IRLMs, a manual investigation of temperature, density, and rotation profiles during the locked phase followed. A transient collapse of the electron temperature at normalized radius \( \rho > 0.6 \) was observed during the lifetime of many IRLMs, ending either by full recovery of the profile, or by a minor or major disruption. This collapse, appearing to the onset of most locked mode disruptions, is studied in detail in the next chapter. We transition from large to small statistics, as the following chapter focuses on only three discharges. Nevertheless, we expect that the findings of the following chapter are common to most IRLM disruptions, though a dedicated study with large statistics is needed to validate this.
Appendix B

EXTRAP T2R coordinate transformations

Two coordinate systems are used in this work which differ only in their origin. The magnetic sensor positions are defined relative to the machine coordinate system with toroidal coordinate $\phi$. The feedback algorithm with which the RMPs are applied uses a coordinate system relative to the active coil positions with toroidal coordinate $\Phi$. In real space their origins are separated by $5.265^\circ$, but keeping with our toroidal angle convention (section 2.2.1) we have the relationship $\Phi = \phi - 5.265^\circ \times 12 = \phi - 67.5^\circ$. All reported RMP phases in this work are mapped to the $\phi$ coordinate system in order to compare with magnetic sensor data.
Appendix C

EXTRAP T2R angle definitions

Due to the large number of toroidal angle definitions in this work, the following table is provided to consolidate and provide brief descriptions. Please see text for detailed definitions.

<table>
<thead>
<tr>
<th>Param.</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^{1,-12}$</td>
<td>Position of full TM structure</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$12 \times \phi^{1,-12}$, used for model TM (Sec. 3) &amp; unfiltered TM position (Sec. 4.1)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\phi$ after filtering</td>
</tr>
<tr>
<td>$\Delta \phi$</td>
<td>$\phi - \phi_{EF+RMP}$</td>
</tr>
<tr>
<td>$\phi_{\text{max}}^i$</td>
<td>$\phi$ when amp. is max. in rotation period $i$</td>
</tr>
<tr>
<td>$\phi_{RMP}$</td>
<td>Position of applied RMP</td>
</tr>
<tr>
<td>$\phi_{EF+RMP}$</td>
<td>Position of EF + RMP</td>
</tr>
<tr>
<td>$\phi_{\text{meas}}$</td>
<td>Phase derived from single polar plot</td>
</tr>
<tr>
<td>$\phi_{\text{res}}$</td>
<td>$\phi_{\text{meas}} - \Delta \phi_{\text{max}}$</td>
</tr>
<tr>
<td>$\Delta \phi_{\text{max}}$</td>
<td>Average $\Delta \phi$ when TM amp. is max.</td>
</tr>
<tr>
<td>$\phi_{EF}$</td>
<td>Position of intrinsic EF</td>
</tr>
</tbody>
</table>

Table C.1: Brief definitions of all toroidal angle parameters. Angles are grouped by the horizontal lines; first group is fully time-resolved, second group ($\phi_{\text{max}}^i$ only) is defined on a rotation period, third group is characteristic of a single discharge, the fourth group ($\Delta \phi_{\text{max}}$ only) is characteristic of a constant-amplitude RMP phase-scan (multiple discharges), and the fifth group ($\phi_{EF}$ only) is assumed constant throughout all experiments. All angles except $\phi^{1,-12}$ span $360^\circ/12$ in real space.
Appendix D

Mapping from radial field to perturbed island current

A model was produced to map from the value of the $n = 1$ major-radial magnetic field $B_R$ at the external saddle loops to the perturbed current carried by a circular cross-section toroidal current sheet. The current sheet has major radius $R$ and minor radius $r_{q2}$, derived from equilibrium reconstructions using magnetics and Motional Stark Effect data. A shaping study was performed to test the impact of ellipticity on the $n = 1$ external saddle loop measurement. The impact was found to be relatively small, introducing corrections of less than 5%. Hence, it was concluded that circular cross-sections are sufficient for modeling external saddle loop signals for the sake of the present study.

The modeled current sheet is discretized into $N$ helical wire filaments with $2/1$ pitch giving a toroidal spacing of $360/N$ degrees. The currents are distributed among the wires to produce a $2/1$ current perturbation as shown by the colors in figure D-1a (black represents no perturbed current and red represents maximum perturbed current).

The field is calculated by numerically integrating the Biot-Savart Law along each wire. Figure D-1b shows the resulting major-radial field $B_R$ as "seen" by the external saddle loops, due to a $2/1$ current distribution. The color contour shows a clear $n = 1$ field distribution. Although the external saddle loops have complete toroidal
Figure D-1: (a) A 3D filament model of a 2/1 tearing mode used to map from the radial field at the external saddle loops $B_R$ to a perturbed island current $\delta I$. (b) The radial field at the external saddle loops is shown in the color contour for a mode carrying 3.14 kA of $n = 1$ current. The thick black lines outline the six external saddle loops. The thin black lines define cells of equal area, which are used to sample the field to compute an average over the loop.
coverage, they have only \( \sim 25\% \) poloidal coverage for the non-elongated, cylindrically
approximated plasma.

A single measurement for each saddle loop is estimated by averaging the field
at 100 sample points. Care was taken to ensure the area of each sample point is
identical, which simplifies the flux calculation. The field samples are averaged, which
is identical to calculating the total flux and dividing by saddle loop area \( A_{sl} \):

\[
\frac{\sum_{i=1}^{n} B_{R,i} a_i}{A_{sl}} = \frac{a}{na} \sum_{i=1}^{n} B_{R,i} = \frac{1}{n} \sum_{i=1}^{n} B_{R,i} \tag{D.1}
\]

where \( A_{sl} \) is the total area of one saddle loop, \( a_i \) is the constant sample area (i.e.
\( a_i = a \)), \( n \) is the number of sample regions per saddle loop, and \( B_{R,i} \) is the major-
radial field at the \( i^{th} \) sample region.

The above averaging is done for each of the six saddle loops. The simulated signals
are then pair-differenced, and a least squares fit is used to extract the amplitude of
the \( n = 1 \) component, in the same way that experimental data are analyzed in section
3.2.4.

The island current, \( \delta I \), is defined as the total current deficit, as seen in figure D-
2b. A second definition of island current is useful for mapping to a cylindrical island
width, being half the sinusoidal perturbed current \( \delta I_h \) (see figure D-2a). \( \delta I_h \) and \( \delta I \)
are related by the equation \( \delta I = \pi \delta I_h \). The current distribution in figure D-2b can be
seen as the superposition of the distribution in D-2a, plus an axisymmetric current.
The non-axisymmetric fields produced by D-2a and D-2b are identical.
The $n = 1$ $B_R$ component in the external saddle loops, $B^1_R$, was then studied as a function of $R$ and $r_{q2}$ for fixed island current $\delta I_h$ as seen in figure D-3. $B^1_R$ is found to be quadratic in $r_{q2}$ in agreement with the cylindrical approximation. As $R$ is increased, the leading order coefficient of the quadratic is seen to increase as well. Defining the leading order coefficient $\alpha(R)$ (which note is a function of $R$), we find the following relationship:

$$B^1_R = \alpha(R)\delta I_h[r_{q2}]^2 \quad (D.2)$$

where $B^1_R$ is in Gauss, $\delta I_h$ is in kA, $r_{q2}$ and $R$ are in meters, and $\alpha$ has units of G/kA/m$^2$. Evaluating the change in $\alpha$ as a function of $R$, the dependence is found to be well approximated by a second order polynomial $\alpha(R) = aR^2 + bR + c$ where $a = 19.15$ G/kA/m$^4$, $b = -50.40$ G/kA/m$^3$, and $c = 35.71$ G/kA/m$^2$.

With equation D.2, experimental $n = 1$ measurements from the external saddle loops are mapped to $\delta I_h$ by a simple inversion of the equation and with major and minor radii provided by EFIT informed by magnetics and Motional Stark Effect data:

$$\delta I_h = \frac{B^1_R}{\alpha(R)[r_{q2}]^2} \quad (D.3)$$
Appendix E

Mapping from perturbed island current to width

Although the perturbed island current $\delta I$ is an intrinsic island quantity that accounts for toroidicity, an island width was desirable for some studies. We will use $\delta I$ to map to an island width.

We solve for the radial field at the $q = 2$ surface $\tilde{B}_r$ by assuming cylindrical geometry, using the ordering $k_\theta = m/r \gg k_z = n/R$, and assuming vacuum solutions for the radial tearing eigenfunctions. The non-axisymmetric field $\tilde{B}$ produced by the island is expressed as,

$$\tilde{B} = \nabla \times \Psi \hat{z}$$  \hspace{1cm} (E.1)

where $\Psi$ is the perturbed flux function which is of the form $\Psi = \psi(r)e^{im\theta}$ (note that the toroidal wavelength is assumed infinite here). We assume the perturbed current in the island $\tilde{j}$ to be a sheet current located at the $q = 2$ surface expressed as,

$$\tilde{j} = \tilde{\sigma} \sin(2\theta)\delta(r_q - r)\hat{z}$$  \hspace{1cm} (E.2)

where $\tilde{\sigma}$ is constant with units (A/m), $\delta(...)$ represents the Dirac delta function, and the poloidal harmonic $m = 2$ is implied. Invoking Ampere’s law where the time derivative of $E$ is neglected (the characteristic velocity of the system $v \ll c$), and the
equilibrium plasma current is neglected (i.e. searching for vacuum solutions), we find

\[-\nabla^2 \Psi = \mu_0 \tilde{\sigma} \sin(2\theta) \delta(r_{q2} - r) \quad (E.3)\]

Solving the cylindrical Laplacian both inside and outside of the $q = 2$ surface, requiring the solutions be continuous across $r_{q2}$, and defining the jump in the radial derivative with the radial integral of $\tilde{j}$, we find

\[
\Psi = \frac{\mu_0 \tilde{\sigma}}{4} \sin(2\theta) \left\{ \begin{array}{ll} 
\frac{r^2}{r_{q2}} & \text{for } r < r_{q2} \\
\frac{r_{q2}^3}{r^2} & \text{for } r \geq r_{q2}
\end{array} \right. \quad (E.4)
\]

Using equation E.1, the field at the $q = 2$ surface is then given by

\[
\tilde{B}_{q2} = \frac{\mu_0 \tilde{\sigma}}{2} \left[ \cos(2\theta) \hat{r} + \sin(2\theta) \hat{\theta} \right] \quad (E.5)
\]

We now want to replace $\tilde{\sigma}$ with an expression for $\delta I$. We do so as follows (see figure D-2a),

\[
\delta I_h \equiv \int_0^{\pi/2} \int_{r_{q2}-}^{r_{q2}+} \tilde{j} \cdot d\mathbf{A} \quad (E.6)
\]

where $d\mathbf{A} = r \, dr \, d\theta \, \hat{z}$, and $r_{q2\pm} = r_{q2} \pm w/2$. After substituting equation E.2 and evaluating the integral, we find $\tilde{\sigma} = \delta I_h / r_{q2}$.

Finally, substituting this into equation E.5, we have

\[
\tilde{B}_{q2}(\delta I) = \frac{\mu_0 \delta I_h}{2r_{q2}} \left[ \cos(2\theta) \hat{r} + \sin(2\theta) \hat{\theta} \right] \quad (E.7)
\]

The cylindrical island width is given by [139],

\[
w = c \left[ \frac{16 R \tilde{B}_r q^2}{m B_T dq/dr} \right]^{1/2} \quad (E.8)
\]

where $c$ is a toroidal correction for island widths measured at the outboard midplane, $\tilde{B}_r$ is the perturbed radial field at the $q = 2$ surface, $q = 2$ is the local safety factor, $m = 2$ is the poloidal harmonic, $B_T$ is the toroidal field at the magnetic axis, and
\( dq/dr \) is the radial derivative of the safety factor evaluated at \( r_{q2} \) on the outboard midplane (the cylindrical expression for the safety factor \( q = rB_T/(RB_\theta) \) was used here). Substituting \( q = 2 \) and \( m = 2 \), and using the maximum of equation E.7 for \( \tilde{B}_r \), we have

\[
 w = c \left[ \frac{16R}{B_T dq/dr} \frac{\mu_0 \delta I_h}{r_{q2}} \right]^{1/2} \tag{E.9}
\]

This expression for the island width was validated using the electron cyclotron emission diagnostic [36] to observe the size of flattened regions in the temperature profile. Using nine islands, the toroidal correction factor is calibrated to \( c \approx 8/15 \). Using equation E.9, we expect a \( \sim 25\% \) statistical error on island width estimates. This expression for the island width is used for all plots of \( w \) and \( d_{\text{edge}} \).