Speculatively Constrained Optimal Commodity Policies

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Speculatively Constrained Optimal Commodity Policies*

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Abstract. Governments store primary commodities for redistributive reasons, or to deal with missing risk markets. For either reason, interactions between public and private storage make it difficult to compute an optimal storage policy: Any announced policy rule will induce equilibrium private responses, in general, at each date; accounting for this makes searching for an optimal policy prohibitively complex. Here, a method is developed that permits computation of the optimum in a wide range of cases, by mapping the problem into a far simpler one, a form of dynamic program, without loss of generality. Among the findings are that for a non-trivial portion of the parameter space, the optimal policy pegs the price to a function of the current harvest alone; and that commodity price stabilization per se is more attractive as a way of assisting producers, the less risk averse the producers are.

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Public sector commodity storage policies, or buffer stocks, have a long history, as international price management schemes,\(^1\) as national 'strategic reserves,'\(^2\) as instruments of exporting-country monopoly power,\(^3\) and as food stocks kept as government farm policy or to prevent consumption disruptions.\(^4\) The rationale for such policies varies; in most cases a transfer motive appears to be important,\(^5\) but a risk-spreading motive, implicitly a response to presumed

\(^1\)Currently, natural rubber is the subject of an international buffer stock scheme, and the producer country arrangement for coffee may be thought of as a *per se* buffer stock policy in some respects. Historically, tin, wheat, and cocoa have also been involved in such programs. In the 1970’s, UNCTAD attempted to create buffer stocks for several ‘key’ commodities as an instrument of international development policy. See Gilbert (1987) and Finlayson and Zacher (1988) for surveys.

\(^2\)US strategic reserves of petroleum and tin are examples. The latter still have a capacity of three quarters of a billion barrels, and played a conspicuous role moderating price swings during the Gulf War. See Blumstein and Komor (1996); Wright and Williams (1982) provide a theoretical analysis.

\(^3\)See Krasner (1973) and Newbery (1984) for the case of coffee in Brazil, and Bardsley (1994) for the case of wool in Australia.

\(^4\)See Ravallion (1987) and Osmani (1991), for example, for discussions of the Bangladesh rice buffer stock intended to prevent consumption shortfalls. See Ellis (1990, 1993) for the Indonesian buffer stock system, which has a capacity of 3.5 million tonnes (Ellis, 1993, p. 106). Faruqee, Coleman, and Scott (1997) discuss wheat price stabilization policies in Pakistan, which include an important element of government storage. It should be emphasized that domestic buffer stocks for this purpose can have a role even for a traded commodity, because transport costs allow for large fluctuations in local prices apart from world prices. See Ravallion (1987) for extensive evidence.

\(^5\)For example, US support for some international commodity programs in the 1960’s was explicitly a form of aid within the Alliance for Progress (see Finlayson and Zacher (1988)). Naturally, this rationale implicitly assumes that lump-sum transfers are difficult or impossible, which is often a reasonable assumption in this context.
missing risk markets, is often discussed as well.\(^6\)

If we grant some role for public sector storage, can we say anything about the *optimal* buffer stock policy? In this paper it will be argued that the existing writing on this question has come short of an answer, despite the rich literature on the welfare economics of buffer stocks.\(^7\) The key difficulty is the interaction of the public storage policy with private storage. Since a fully optimal policy would make use of public commitment to future storage policies, it would not only affect welfare and prices directly, by the act of government storage; it would also affect expectations within the market about future prices, thereby affecting the storage behaviour of private agents, and thus affecting welfare and prices indirectly as well. This second class of effects, acting through the expectations of private agents, makes the optimal policy problem an order of magnitude more difficult than a conventional dynamic programming problem. In Section 3 it will be argued that existing normative writing on buffer stock economics has in each case fallen short of identifying an optimal policy, largely because this interaction has remained a formidable technical obstacle.

In this paper, a technique is developed that can, in a well-defined class of models, overcome that obstacle. Under a series of assumptions developed in the text, it is shown that all buffer stock policies can be mapped into a narrow class in which the government takes over all storage from the private sector (‘simple preemption policies’). Once this has been done, the problem reduces to a modified version of a familiar dynamic programming problem. This makes the full optimum in the


presence of private storage computationally feasible for the first time. The problem in this form is well-behaved in the sense that it exhibits a contraction mapping property, so that one can find the optimal policy through familiar iterative techniques (although it is badly behaved in the sense that there is no guarantee of a concave value function). Using these techniques, optimal policies are exhibited for a variety of numerical examples. A key result is that commodity price stabilization *per se* can at times be optimal, but it is much more likely to be attractive as a way of assisting the producers of the commodity, the less risk averse they are. This contrasts sharply with conclusions arrived at by some other authors, who have used a non-optimizing approach.

The approach pursued in this paper has a number of important limitations. It is based on a model of private storage, involving risk-neutral, price-taking speculators with rational expectations, which has a strong tradition in the literature but is not universally accepted empirically. It is not clear to what extent the approach can be generalized to incorporate risk-averse speculators, imperfect competition, or even non-constant marginal storage costs. Further, there is no role in the model as here presented for futures markets, which have been extensively discussed elsewhere as an alternative way of helping producers deal with risk. The goal here is to take the simplest

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8 This model is developed in Samuelson (1971), Salant (1983), Scheinkman and Schechtman (1983), Wright and Williams (1988), and Deaton and Laroque (1992), for example.

9 For example, Deaton and Laroque (1996) have questioned the consistency of the model with price autocorrelation patterns in the data. See Williams and Wright (1991) for a survey.

10 In the ‘convenience yield’ literature it is argued that sharply rising (negative) marginal storage costs at low levels of storage are important empirically in some commodity markets. See Pindyck (1994) for evidence.

11 This is discussed at length by Newbery and Stiglitz (1981, ch. 13) and by Kletzer, Newbery and Wright (1992), who also discuss a wide variety of other contractual solutions. It should be noted that each of these offers a second-best outcome, leaving some room for buffer stock policies in...
canonical model of storage and show how a solution can be obtained there for storage policies alone, with extensions to these other concerns left for the future.

Nonetheless, the calculation of a full optimum in this canonical model, taking full account of the interaction with private storage, is a question of some interest, and the solution suggested here appears to be new. In this light, the importance of this interaction to the buffer stock problem should be underlined. Adnan (1972) emphasized the importance of speculators (whom he regarded as a nuisance) in his experience as the manager of the buffer stock for tin. In discussing the 1974 famine in Bangladesh, Osmani (1991, pp. 329-31), and Ravallion (1987) emphasize the action of speculators in frustrating the stabilizing goals of the rice buffer stock. Hallwood (1977) points out that private storage in the commodities in question dwarfed even the ambitious buffer stocks UNCTAD had proposed, and argued that failure to account for interactions between the two would doom the proposed policies. The interaction has found dramatic expression in Salant's (1983) theory of 'speculative attacks' on buffer stocks. As a result of the importance of private storage to the analysis of public storage, any analysis of the latter that ignores the former would appear to be seriously remiss.

The following section presents the canonical optimal buffer stock problem, shows that a variety of important instances in practice can be seen as special cases, and shows why the optimization problem is so difficult. Section 3 reviews some well-known attempts to analyze this problem or a variant of it, and suggests that they all come short of solving the full optimization problem. Section 4 shows how the problem can be mapped into a much simpler equivalent problem. Section 5 discusses basic properties of the optimization problem, including the 'amnesia' conjunction with these other instruments. See Lapan and Moschini (1996).
property that follows when a solution is fully interior; and section 6 discusses some numerical examples of optimal policies. Section 7 briefly comments on comparisons between the optimal policies generated by this model and policies observed in practice.

2. The Problem.

Consider a market for a storable commodity with price-taking producers, consumers, and speculators. Harvests at each date are an identically and independently distributed random variable, and denoted by $h_t$ (it would make no important difference if we had deterministic production but a similar random shock to demand). Consumers have a downward-sloping demand curve $P(Q)$, where $Q$ is the quantity they consume; its inverse is $P^{-1}$. The realized price of the commodity at time $t$ is denoted $p_t$. Storage can be undertaken at any time either by the government or by speculators. Both have the same technology. Beginning-of-period government inventories are $I_t$ (a state variable); end-of-period government storage is $g_t$ (a choice variable). Beginning-of-period speculative stocks are denoted by $J_t$. Storage incurs a constant marginal cost $k$ and constant depreciation rate $\delta$ in either the public or private sector.

Government is able to commit fully and publicly to a policy, but in order to focus on the optimal buffer stock problem, we will assume that the only instrument government has at its disposal is storage.

Now, we make some assumptions about the preferences of the four classes of agents. First, assume that all agents have time-separable, Von Neumann preferences and share a discount factor $\beta$. Next, assume that this commodity is a small part of the consumers' budget sets (and there are
no income or taste shocks), so that we can treat their marginal utility of income as constant and focus on consumers' surplus as a measure of per period consumer utility.\footnote{12}{Further, assume that the buffer stock operation is a sufficiently small portion of the government's budget, and the fiscal risks sufficiently diversifiable, that we can treat the taxpayers' marginal utility of income as constant as well, and focus on taxpayers' surplus as a measure of per period taxpayer utility. By contrast, producers will be assumed to have a weakly concave Von-Neuman-Morgenstern utility function $v$, to allow for producer risk aversion. Third, assume that producers cannot save or borrow. This is a \textit{per se} unsatisfying assumption, maintained principally because it removes one level of complexity from the problem. However, for certain crops and certain countries it may not be an outlandish approximation; the question of binding liquidity constraints facing Third World farmers has generated an enormous literature.\footnote{13}{It is worth mentioning that the assumption does not matter at all in the event that farmers are risk neutral. Finally, as noted above, assume that the speculators are all risk neutral.}}

These assumptions mean that the utility of the various agents can be represented for our purposes as a function of the anticipated time path of the 'surplus' of the various agents, as follows:

Taxpayers: $E[\sum_{t=0}^{\infty} \beta^t TS_t]$; Consumers: $E[\sum_{t=0}^{\infty} \beta^t CS_t]$; Speculators: $E[\sum_{t=0}^{\infty} \beta^t SS_t]$;

\footnote{12}{This assumption can be relaxed very easily without changing materially the approach taken in this paper. It is made here for simplicity and for consistency with the majority of the literature.}

\footnote{13}{See the symposium by Case et. al (1995) for a review. Further, it is possible that the presumed lack of access by producers to financial markets is one of the market failures the buffer stock is intended to address, in a second-best manner. It is likely to be possible to allow for endogenous farmer saving behaviour through techniques similar to the techniques for dealing with private storage in the text, but this is beyond the scope of the paper.}
Producers: $E[\sum_{t=0}^{\infty} \beta^t v(PS_t)]$,

where 'E' denotes expectations with respect to $\{h_t\}_{t=0}^{\infty}$, and where:

$$CS_t = U(Q) - Q P(Q) = \int_0^Q P(q) dq - Q P(Q);$$

$$PS_t = p_t h_t;$$

$$TS_t = p_t(I_t - g_t) - kg_t;$$ and

$$SS_t = p_t(J_t - J_{t+1}/(1-\delta)) - kJ_{t+1}/(1-\delta).$$

For simplicity, we assume away production costs and storage by producers, and thus treat producers’ income as simply the revenue received from sale of the crop. The quantity $(I_t - g_t)$ is the government’s current net sale of the commodity. If this is positive, it is a source of revenue to the government, which may provide tax relief elsewhere or additional public goods; if it is negative, it is a current fiscal burden. Combining this with the government’s storage cost $kg_t$ gives the full fiscal effect, $TS_t$, of the policy in period $t$. The speculators’ surplus is similar; since by definition of $J_t$, end-of-period-$t$ storage must equal $J_{t+1}/(1-\delta)$; the period-$t$ net sale of speculators is equal to $(J_t - J_{t+1}/(1-\delta))$; and $kJ_{t+1}/(1-\delta)$ is their current storage cost.

Next, we need an assumption about the preferences of the government. Assume that the government is utilitarian, in the sense that it is not interested in anything beyond the well-being of the four classes of agent; thus its objective function is a (weakly) increasing function of the utilities of the four agents as just derived. Thus, within the class of policies from which it has to choose (i.e., within the class of pure storage policies), its optimal choice will always be Pareto efficient.
More specifically, assume that this objective function is a linear combination of these utilities. This provides the problem with an enormously useful additional structure.

With all of these assumptions, the objective function can be written as:

$$E[\sum_{t=0}^{\infty} \beta^t \{a_c [U(Q_t) - Q_t P(Q_t)] + a_p \nu(p, h_t) + a_T [p_t (I_t - g_t) - k g_t] + a_S [p_t (J_t - J_{t+1}/(1-\delta)) - k J_{t+1}/(1-\delta)]\}].$$

An important state variable is total beginning-of-period stocks, $S_t$:

1. $S_t = I_t + J_t$,

and there are two key laws of motion:

2. $J_{t+1} = (1-\delta)(S_t + h_t - g_t - Q_t)$;
3. $I_{t+1} = (1-\delta)g_t$; $I_t = 0$.

In general, the government may wish to condition its behaviour at a future date on the full history that has elapsed up to that point. To allow for this possibility, we must define some history variables. Define $S_t = \{S_t\}_{t=0}^{\infty}$ as the history of total stocks to date $t$, and similarly define $I_t$ and $h_t$. Then a buffer stock rule is a sequence of functions $f_t = \{f_t\}_{t=0}^{\infty}$ giving government storage $g_t =$
\( \gamma_t(S_t, I_t, h_t) \) as a function of the current state and history of the market.\(^{14}\) Allowing for the policy to depend on the full history may at first glance appear to be excessive generality, but in fact it merely reflects the fact that the rule is being chosen under full commitment.\(^{15}\) In committing itself publicly to a certain pattern of action at a future date, say, \( t = 10 \), the government is directly affecting outcomes at date 10, but also indirectly affecting outcomes at date 9 by affecting the behaviour of speculators at that date. The type of influence the government may wish to have on the speculators at date 9 may well depend on conditions at date 9; thus, we cannot rule out the possibility that the government will wish to commit itself to different date 10 actions depending on conditions at date 9. The same logic applies to periods 1 through 8, so in general they must be included in the history variable as well.\(^{16}\) In addition to being required by the full commitment in the problem, the dependence of the rule on the complete history has a huge benefit later on, because it makes proof of the representation theorem of section 4, which simplifies the problem, enormously simpler than it would be with some arbitrary truncation of the permissible history. Restricting the generality of the rule at this point would make things much more difficult later.

There are two key constraints on government action. First, any buffer stock rule, to be feasible, must satisfy the physical constraints:

\(^{14}\)For the first period, assume that the government commits itself to \( y_0 \) at the beginning of the period, before it learns the realization of \( h_0 \). This is simply a convenient timing convention.

\(^{15}\)The analytics of the public sector storage problem under discretion are qualitatively different. McLaren (1995) offers a simple example.

\(^{16}\)This dependence of optimal policy on history is well-known in the dynamic public finance literature. See, for example, Chari, Christiano, and Kehoe (1994).
The second constraint comes from the action of the market. The heart of the problem is the interaction between the buffer stock rule and speculators. To capture this, for any buffer stock rule \( I \), we must seek an 'equilibrium:' a sequence of price functions \( \Psi = \{ \psi_t \}_{t=0}^\infty \) giving the price \( p_t = \psi_t(S_t, I_t, h_t) \) as a function of the current state and history. Such an equilibrium must satisfy the market-clearing conditions:

\[
\begin{align*}
 p_t &> \beta(1-\delta)E_{t+1}p_{t+1} - k; \\
p_t &= \beta(1-\delta)E_{t+1}p_{t+1} - k \text{ whenever } p_t > P(S_t + h_t - g_t).
\end{align*}
\]

Here and throughout, '\( E_{t+1} \)' denotes an expectation with respect to the harvests at dates \( t + 1 \) and later, conditional on all history to that point\(^{17}\) (even though in this equation only \( h_{t+1} \) is relevant). The first condition in (5) says that speculators never make positive expected profits per unit stored; if they did, their demand for the commodity would be infinite. The second condition says that if speculators store a strictly positive amount, they must break even in expectation; otherwise they would have chosen to store nothing. This can all be summarized in a Deaton and Laroque-type (1992) functional equation as follows:

\[
\psi_t(S_t, I_t, h_t) = \max\{ P(S_t + h_t - \gamma_t(S_t, I_t, h_t)), \beta(1-\delta)E_{t+1} [\psi_{t+1}(S_{t+1}, I_{t+1}, h_{t+1})] - k \}.
\]

\(^{17}\)It is convenient for this problem to think of \( h_t \) as being realized at the 'end' of period \( t \), with \( E_t \) being evaluated at the 'beginning' of period \( t \). Thus, '\( E_0 \)' denotes an expectation with respect to all harvests, from \( h_0 \) on after.
for all $t \geq 0$ and for all possible histories, where $S_{t+1}$, $I_{t+1}$, and $h_{t+1}$ are derived from the period-$t$ values and the period-$(t + 1)$ harvest by (1), (2), and (3). Any policy/price pair $(\Gamma, \Psi)$ satisfying the physical constraints (4) and the market-clearing conditions (6) will be called 'feasible.'

Since the joint distribution of the time series $TS_t$, $CS_t$, $PS_t$, and $SS_t$ is determined by $\Gamma$ and $\Psi$, we can write the government's objective function compactly as $W(\Gamma, \Psi)$. The optimization problem then becomes:

\[
\text{(7)} \quad \text{Max}_{(\Gamma, \Psi)} \{W(\Gamma, \Psi)\} \text{ subject to (4) and (6)}. 
\]

Some examples are as follows.

(i) The objective function could be:

\[
E_0 \{\sum_{t=0}^{\infty} \beta^t [TS_t + CS_t + PS_t + SS_t]\},
\]

or the expected discounted Marshallian surplus. It is well known that an optimum to this problem involves setting $\psi_t = 0$ for all $t$, and setting $\psi_t$ equal to the unique stationary rational expectations equilibrium price function under laissez faire.\(^\text{18}\) This is no mystery, since writing the maximand in this way removes any risk-sharing motive (since it tacitly assumes away income effects) and any distributional motive (since it weights all four agents equally). Thus, with no bias, missing market

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\(^\text{18}\)Gustafson (1958), Samuelson (1971), Scheinkman and Schechtman (1983), and Deaton and Laroque (1992) deal with different aspects of this.
motive or distortion, there is no need to tamper with the market.

(ii) By contrast, if producers are risk averse and identical and have no access to risk-sharing markets, the objective function could look like this:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t [TS_t + CS_t + v(PS_i) + SS_i]\right],$$

where \(v(\cdot)\) is a strictly increasing, strictly concave function. This appears to be the simplest representation for the canonical efficiency case for buffer stock policies, but a solution to this optimization problem does not seem to be available in the existing literature.

(iii) Now, suppose that risk \textit{per se} is not an issue, but the government in question has a distributional bias. Then the objective function could look like:

$$E_0\left[\sum_{t=0}^{\infty} \beta^t [a_T TS_t + a_C CS_t + a_P PS_t + a_S SS_t]\right],$$

with \(a_i \geq 0\). The weights \(a_i\) could represent either preferences of or political pressures on a single government contemplating a buffer stock, or the relative bargaining strengths of various parties in an international negotiation. For example, we could have \(a_p > a_T = a_P = a_S\), in which case the buffer stock is really an indirect form of farm sector income support, or foreign aid with producers as the beneficiaries. As mentioned above, this has at times been an explicit goal of storage schemes. Alternatively, we could have \(a_C > a_T = a_P = a_S\), in which case the buffer stock is really aimed at keeping consumers happy and is one interpretation of cereals buffer stocks administered in India and Bangladesh. For a last example, we could have \(a_T = a_P > 0\) and \(a_C = a_S = 0\): a monopolist
exploiting its monopoly power through a buffer stock. Newbery (1984) solves a model of this sort exactly, for a particular region of the parameter space, with Brazilian coffee policy in the first half of the century as the chief example in mind.¹⁹

All of these problems can be represented in the form (7). It is now easy to see why these problems are so difficult. Typically, none of the equilibrium price functions or optimal storage rules will have closed form solutions; it is necessary to work on a computer with a particular example. To evaluate the objective function for any given storage rule $\Gamma$ requires two steps. First, a solution to the functional equation (6) must be found, in order to get an equilibrium price function. This is typically done through an iterative numerical procedure on the computer (Williams and Wright (1988, 1991) contain much detail on this). Then, with the $\Gamma$ and $\Psi$ functions in hand, $W(\Gamma, \Psi)$ must be evaluated. This is accomplished either through another iterative procedure or, more commonly, by simulations of the random harvest. Once these two cumbersome numerical procedures have been completed, one has a number for $W(\Gamma, \Psi)$; it is then necessary to search for the $\Gamma$ that gives the highest value for this. Even the simplest, stationary storage policy is a function, and hence a high dimension choice variable; even if it were quick to compute $W(\Gamma, \Psi)$ for each possible $\Gamma$, a search for the optimum could take a long time; as it is, for each policy, $W(\Gamma, \Psi)$ requires a considerable computational undertaking. Finally, even if the search is ever successful and an approximate optimum is obtained, there can be no pretense to have learned anything at all general, since the result will be pinned down to a particular set of parameter values, functional forms, and a harvest distribution.

¹⁹The trick he exploits is to find a parameter region for which (6) never binds.
3. Previous attempts.

Within the commodities literature it is possible to identify three classes of papers that have attempted to solve or at least say something useful about this problem.

(i) Attempts that assume away speculators.

A number of authors have analyzed optimal storage policy under the assumption that government alone has the power to store. Examples of this are Gustafson (1958); Gardner (1979, Ch. 3); and Newbery and Stiglitz (1981, Ch. 30). This assumption eliminates the constraint (6) and makes the price function $y(S_t, I_t, h_t) = P(S_t + h_t - \gamma(S_t, I_t, h_t))$. This has the benefit of making the problem a conventional dynamic programming problem, but it seems difficult to defend because it arbitrarily assigns powers to government that it denies to the private sector, and, as noted in the introduction, flies in the face of all evidence.

(ii) Attempts that pay close attention to speculators, but focus on the welfare effects of a limited set of policies.

Examples of this are Salant (1983); Gardner (1979, Ch. 5); Newbery and Stiglitz (1981, Ch. 29); Miranda and Helmberger (1988); and Wright and Williams (1988). This approach can provide many insights and indeed has provided some of the most rewarding papers in the commodities literature, but it does not bring us anywhere close to any statement about an optimum, or even the
qualitative features of an optimum.

(iii) Attempts that use a static model to study the value of buffer stocks.

Massell (1969) presented a static model of a commodity market with random shocks to supply and demand, in which the activity of a buffer stock was represented by the elimination of price variability. The idea appears to be that a buffer stock would, over time, transfer the commodity from states in which it is abundant to states in which it is scarce, thus removing randomness from the price, but this cannot clearly be represented in a model with no intertemporal dimension. Indeed, the mere physical feasibility of removing all price variability cannot be investigated without an intertemporal model, as is shown in Townsend (1977) and other work. Further, the model implicitly assigns to the government a function — commodity storage — that is arbitrarily denied to the private sector. This paper inspired large amounts of work along the same lines, comprising what is arguably a deeply misguided literature.

Several chapters of Newbery and Stiglitz (1981), culminating in Ch. 20, follow a similar path, in an innovative and influential attempt to evaluate the welfare effects of buffer stocks that offers much that is interesting (including a careful treatment of welfare measurement under risk aversion), but is subject to many of the same limitations as Massell (1969). These chapters implicitly attempt to evaluate public storage policies by examining the welfare effects of exogenous changes in the distribution of price in a static model with random shocks. There is no demonstration that the given change in price distribution is feasible, in the sense of being possible to generate it
with a feasible public storage program. Further, as in the earlier work, welfare measurement is purely static. It may be that the model is intended to represent the ergodic distribution of a dynamic system; but to look at only the long-run distribution of utilities rather than to look at present values would appear to require an assumption that discount rates are close to zero.

A key result from Newbery and Stiglitz (1981, Ch. 20) can be summarized as follows. In their models, stabilization of the commodity price tends to result in a moderate negative effect on farmer expected utility, composed of a large negative ‘Transfer effect’ (that is, change in expected farmer income), and a small positive ‘Risk effect’ (that is, the effect on utility due to changes in risk alone, or the total effect minus the ‘Transfer effect’). Thus, commodity-price-stabilizing buffer stocks are justified as ways to help farmers only if farmers have very high levels of risk aversion. We will revisit this finding in section 6.

(iv) Attempts that use an arbitrary objective function.

A few papers avoid the difficulties of optimizing intertemporal welfare by specifying a simplified, arbitrary ‘loss’ function as the optimand. A particularly ambitious example is the optimal storage computation of Ghosh, et. al. (1987), in which the objective function used is a weighted sum each period of the squared deviations of price, buffer stock holdings, and buffer stock net sales, respectively, from a target level (pp. 276-7). The difficulty with this is that it is not clear why any government should ever measure welfare in that way, and since the objective function is

20Indeed, some of the calculations involve a hypothetical removal of all risk from the price distribution, replacing the random price distribution with the mean of the price, again raising serious questions of feasibility.
not an aggregate of the agents' utilities, it is likely that there exist alternative buffer stock rules that would be Pareto-superior to the rule to which the computations lead. Clearly, then, the sense in which it is the 'optimal' buffer stock rule is in question.²¹

In summary, a large literature on commodities policy can be summarized as attempting to analyze some version of problem (7), but because of the difficulties involved, each has omitted a key piece of the problem, either (4) or (6), the welfare underpinnings of the problem, or its very intertemporal nature.²² The most satisfying treatments of policy and its constraints have not identified optimal policies. The reason for these problems is clearly the insurmountable technical difficulty of finding an optimum in the full problem. In the next section, a method is outlined that can produce optima without abandoning any of the problem's essential features, by mapping problem (7) into an equivalent, but much more tractable, form.

4. The Key Assumptions and the Representation Theorem.

With the assumptions made to this point, it can be shown that the forbiddingly complicated problem detailed above can be reduced to a relatively simple, manageable one. Specifically, we

²¹The authors make a bold attempt to incorporate a full treatment of forward-looking speculative behaviour in the optimization problem. However, their formulation does not take advantage of recursive methods to do this (see their equation (7.29)), so a large number of compromises are forced on the authors to be able to arrive at a solution. For example, they are forced to impose a finite horizon on the problem, with an arbitrary terminal condition, and to use a log-linearization of the speculative price process that is troublesome, for several reasons that the authors acknowledge (p. 38).

²²Once again, the exception is Newbery (1984), who identifies conditions under which (6) does not bind.
show in turn that without loss of any generality we can: (i) Omit the government inventory history from the list of arguments of the policy functions; (ii) simplify the objective function; (iii) focus on policies that exclude private storage ('preemption policies'); (iv) summarize all history through the previous period's price; (v) impose stationarity on the policy after period 0; (vi) restrict the state space; and finally (vii) write the problem in the form of a modified Bellman equation.

(i) The public sector inventory history can be dropped from the argument list.

The first step is to note that without loss of generality, we can drop the $I_t$ term from the state vector on which the policy and price functions are conditioned. This is due to the following argument. Suppose that we have in hand a policy/price pair $(F_t, \psi)$ conditioned on all three state histories as above. Then necessarily $I_1 = (1 - \delta)\psi_0(S_0, 0, h_0)$, so we can write $I_1$ (and thus, trivially, $I_t$) as a function of $S_0$ and $h_0$. Now, suppose that $I_t$ can be written as a function of $S_{t-1}$ and $h_{t-1}$, say, $f(S_{t-1}, h_{t-1})$. Then $I_{t+1} = (1 - \delta)\psi_t(S_t, f(S_{t-1}, h_{t-1}), h_t)$, and is thus a function of $S_t$ and $h_t$. Thus, by induction, $I_t$ can be written as a function of $S_{t-1}$ and $h_{t-1}$ for all $t$. Thus, we may omit $I_t$ from all lists of arguments without any loss of generality whatsoever.
(ii) Simplifying the objective function.

Recall that the government’s objective function can be written as:

\[
E_0[\sum_{t=0}^{\infty} \beta^t \{ a_C [U(Q_t) - Q_t P(Q_t)] + a_s w(p_t h_t) \\
+ a_f [p_t(I_t - g_t) - kg_t] + a_s [p_t(J_t - J_{t+1}/(1-\delta)) - kJ_{t+1}/(1-\delta)]\}].
\]

Now note that the last term can be rewritten:

\[
a_s E_0[\sum_{t=0}^{\infty} \beta^t \{ p_t(J_t - J_{t+1}/(1-\delta)) - kJ_{t+1}/(1-\delta)\}]
\]

\[
= a_s E_0[p_0 J_0 + \sum_{t=0}^{\infty} \beta^t \{ \beta p_{t+1} - (p_t + k)(1-\delta)\} J_{t+1}].
\]

Consider the expectation \( z_t = E_{t+1} [\{ \beta p_{t+1} - (p_t + k)(1-\delta)\} J_{t+1} ] \) of the period-\( (t+1) \) aggregate speculative capital gains, conditional on all information up to the beginning of period \( (t+1) \). (Thus, it is conditional on \( J_{t+1} \), for example, but not on \( h_{t+1} \).) Recalling (2), if \( J_{t+1} > 0 \), then \( p_t > P(S_t + h_t - g_t) \), so by market clearing condition (5), \( z_t = 0 \). Thus, taking unconditional expectations over the whole series, the total benefit to speculators becomes:

\[
a_s E_0[p_0 J_0],
\]

and we can write the objective function more simply as:
In other words, because in equilibrium speculators make zero profits, we can ignore any effect on them after the first period, in which the announcement of the new policy will have the effect of revaluing their initial stocks.

(iii) Preemption policies.

Now, we make a major transformation to the problem. Consider a feasible policy/price pair \((T, P)\) and define a new pair by \((\tilde{T}, \tilde{P})\), where:

\[
\tilde{T}(S_t, h_t) = S_t + h_t - P^{-1}(\psi_t(S_t, h_t)) \forall t, S_t, h_t.
\]

This is simply a new policy/price pair that leaves the price (and thus consumption, and thus total storage) unchanged in each state after each history, but substitutes public sector storage for all private storage. It can easily be verified that \((\tilde{T}, \tilde{P})\) satisfies (6) and is thus a feasible pair. Further, since it leaves the price unchanged in each state, it leaves the first three sums in the objective function (the \(a_s\) term, the \(a_c\) term, and the \(a_P\) term) unchanged.

Now note that the last sum in the objective function (8) can be written:

\[
a_T E_0 \left[ \sum_{t=0}^{\infty} \beta^t \{ p_t (I_t - g_t) - kg_t \} \right] = a_T E_0 \left[ - p_0 g_0 - k g_0 + \sum_{t=1}^{\infty} \beta^t (p_t ((1-\delta)g_{t-1} - g_t) - kg_t) \right]
= a_T E_0 \left[ \sum_{t=0}^{\infty} \beta^t \{ (1-\delta) p_t - p_t - k \} g_t \right].
\]
The expression in braces is the government's net capital gain. Consider the expectation
\( E_{t+1}[\{\beta p_{t+1} - (p_t + k)/(1-\delta)\}g_t] \) of the period-(t + 1) government capital gains, conditional on all
information up to the beginning of period (t + 1). Recall (6). Either \( p_t = P(S_t + h_t - g_t) \), so that
\( \gamma_t(S_t, h_t) = S_t + h_t - P^{-1}(\psi_t(S_t, h_t)) \), in which case, by (9), \( \gamma_t(S_t, h_t) = \tilde{\gamma}_t(S_t, h_t) \), and \( g_t \) takes
the same value under the new policy as under the old policy; or \( p_t > P(S_t + h_t - g_t) \), so that \( \gamma_t(S_t, h_t) < S_t + h_t - P^{-1}(\psi_t(S_t, h_t)) \), in which case \( p_t + k = \beta(1-\delta)E_{t+1}p_{t+1} \), so expected government
capital gains are zero regardless of \( g_t \). In either case, replacing \( \gamma_t \) with \( \tilde{\gamma}_t \) has no effect on the
government's conditional expected capital gains. Thus, taking unconditional expectations over the
whole series, the objective function is unchanged.

Since the policy/price pair \((\tilde{r}, \tilde{p})\) is feasible and gives the same value to the objective function
as \((r, \varsigma)\), we can focus attention without loss of generality entirely on polices like \( \tilde{r} \) that take over
all storage from the private sector (i.e., that satisfy (9)). We will call such policies preemption
policies, and they will be the focus of the remainder of the analysis. Notice that once we make that
decision, since the price function is defined in terms of the storage rule through (9), we can speak
in terms of choosing a storage policy \( \Gamma \) instead of choosing a policy/price pair.

The problem can now be represented as the maximization of (8) by choice of \( \Gamma \) subject to
(4) and the constraint that the price functions derived from \( \Gamma \) through (9) together with \( \Gamma \) satisfy (6).

In other words:

\[
\text{Max } E[ (a_S - a_T)p_0J_0 + \sum_{t=0}^{\infty} \beta^t \{ a_C[U(S_t + h_t - g_t) - (S_t + h_t - g_t)p_t] + a_T v(p_t h_t) \\
+ a_T[p_t(S_t - g_t) - kg_t] \}].
\]
through choice of $I = \{y_t\}_{t=0}^{\infty}$, where $p_t = P(S_t + h_t - g_t)$, $S_0 = J_0$, $S_{t+1} = (1-\delta)g_t$, and $g_t = y_t(S_t, h_t)$ for all $t$, $S_t$, and $h_t$. The constraints $\{SC(t)\}_{t=0}^{\infty}$ are the equivalent of the functional equation (6) in the earlier form of the problem, and will be called the ‘speculative constraints.’ They ensure that the speculators never strictly prefer to store.23

For convenience, write the objective function as:

$$E[ (a_s - a_T)p_0J_0 + \sum_{t=0}^{\infty} \beta^t \{\pi(S_t, g_t, h_t)\}]$$

where

$$\pi(S_t, g_t, h_t) = a_c[U(S_t + h_t - g_t) - (S_t + h_t - g_t)P(S_t + h_t - g_t)]$$

$$+ a_p v(P(S_t + h_t - g_t)h_t) + a_T[P(S_t + h_t - g_t)(S_t - g_t) - kg_t]$$

is the single-period return function for the government. In addition, write the maximized value of

\[\text{23Note that the expectation in } SC(t) \text{ is taken with respect to } h_{t+1}. \text{ Thus, it is at the end of period } t \text{ that } SC(t) \text{ ensures that speculators do not strictly prefer to store.}\]
the objective function at date $r$ as $V_r(S_r, h_{r-1})$, so that:

$$V_0(S_0) = E_0 \left[ \left( a_S - a_T \right) p_0 S_0 + \sum_{t=0}^{\infty} \beta^t \{ \pi(S_t, g_t, h_t) \} \right];$$

and

$$V_r(S_r, h_{r-1}) = E_r \left[ \sum_{t=r}^{\infty} \beta^{t-r} \{ \pi(S_t, g_t, h_t) \} \right] \text{ for } r > 0,$$

where the expectation is taken with respect to $\{h_t\}_{t \geq r}$ and $I'$ has been chosen optimally.

(iv) **Summarize history through previous price.**

It turns out that most, but not all, of the history on which the policy and value functions are conditioned can be dropped without loss of generality. To see this, first note that for any date $r > 0$, the optimum must satisfy the truncated problem:

$$\text{max } E_r \left\{ \sum_{t=r}^{\infty} \beta^{t-r} \pi(S_t, g_t, h_t) \right\}$$

with respect to $\{y_t\}_{t \geq r}$, subject to $\{SC(t)\}_{t \geq 0}$ and the physical constraints (4), given $S_r$ and $h_r$. For $t < r$, neither $S_t$ nor $h_t$ enters the objective function of the truncated problem, nor affects the physical constraints relevant to it. The only role that $S_{r-1}$ and $h_{r-1}$ can play in the truncated problem is in the speculative constraints, and there only in SC($r-1$). From SC($r-1$), it is clear that the only information from $S_{r-1}$ and $h_{r-1}$ that is relevant to the truncated problem is $(S_{r-1} + h_{r-1})$ and $g_{r-1}$.

---

$^{24}$Once again, note that this is evaluated at the beginning of period $r$, before $h_r$ has been realized, consistent with the maintained timing convention. See footnote 14.
Thus, the variables \((S_{t-1} + h_{t-1})\) and \(g_{t-1}\) contain all of the pre-\(\tau\) information relevant to the truncated problem.

Thus, without loss of generality, we can write the period-\(\tau\) storage function at the optimum as a function of \((S_{t}, h_{t}, (S_{t-1} + h_{t-1}), g_{t-1})\) alone. Of course, that is equivalent to writing it as a function of \((S_{t}, h_{t}, (S_{t} + h_{t}))\) alone, since \(S_{t} = (1-\delta) g_{t-1}\). Further, since \(p_{t-1} = P(S_{t-1} + h_{t-1} - S_t/(1-\delta))\), this is equivalent to writing it as a function of \((S_{t}, h_{t}, p_{t-1})\).

We could repeat the argument for any date \(\tau > 0\), so that in general we may write our optimal policy in the form \(y_{t}(S_{t}, h_{t}, p_{t-1})\), and by extension, the value function in the form \(V_{t}(S_{t}, p_{t-1})\).

It will be convenient to use \(p_{-1}\) to refer to the previous period’s price in what follows. We will call policies of this form, namely, preemption policies that are a function of the current state and the previous period’s price alone, simple preemption policies.

(v) Stationarity.

A focus on simple preemption policies allows us once again to simplify the speculative constraint. We might call this final version the ‘parsimonious speculative constraint.’

\[
(PSC) \quad p_{t-1} \geq \beta(1-\delta)E_{t}[P(S_{t} + h_{t} - y_{t}(S_{t}, h_{t}, p_{t-1}))] - k, \quad \forall \tau > 0, p_{t-1}.
\]

Now suppose that \(I^{**} = \{y_{t}^{**}\}_{t=0}^{T}\) is an optimal simple preemption policy. Pick some \(\tau > 0\), and define an alternative policy, \(I^{*'}\), as follows. For \(t \leq \tau\), set \(y_{t}^{*'} = y_{t}^{*}\). For \(t > \tau\), set \(y_{t}^{*'} = y_{t-1}^{*}\). Thus, \(I^{*'}\) is exactly the same as \(I^{**}\), except that at date \(\tau + 1\), \(I^{*'}\) backs up one period to the period-\(\tau\) \(I^{*}\) rule, and
continues from there, permanently one period behind $I^*$. 

Now, because of the separability of the objective function across time, the choice \( \{y^*_r\}_{r=1}^\infty \) must maximize \( E_t[\sum_{i=r}^\infty \beta^{i-r} \{\pi(S, g, h_r)\}] \) given \( S_r, h_r, \) and \( p_{r-1} \) subject to the law of motion, the physical constraints, and (PSC) (possibly excepting some set of states of measure zero). But then the same sequence of policy functions must maximize \( E_t^+[\sum_{i=r}^\infty \beta^{i-r} \{\pi(S, g, h_r)\}] \) subject to the same constraints (except possibly on some set of measure zero). The fact that each \( y^*_r \) satisfies (PSC) in \( I^* \) guarantees that they still will do so in \( I^{**} \). Thus, \( I^{**} \) is a feasible policy and optimal.

By repetition of this argument we can replace \( I^{**} \) with the policy: \( y_0 = y^*_0, y_t = y^*_1 \) for \( t \geq 1 \).

(vi) Restricting the state space.

The infinite sequence of constraints imposes a number of restrictions on the domain of the value function at each date. The focus on simple preemption policies allows us to summarize all of these restrictions with a single inequality (denoted \( \text{RES}(\infty) \) below), which is indispensable for computation purposes. However, derivation of this inequality requires several steps. It is the last technical hurdle before we can state the representation theorem that is the point of the paper.

First, for any point \((S, p_{-1})\) at which the value function for date \( r > 0 \) is defined, it is necessary that the speculative constraint can be satisfied at that point, or in other words, the following restriction must hold:

\[
(\text{RES}(0)) \quad p_{-1} \geq f_0(S) = \beta(1-\delta)E_h[P(S + h)] - k.
\]
Here and henceforth, 'E_' denotes an expectation with respect to a single realization, \( h \), of the iid harvest. Condition RES(0) ensures that the speculative constraint would be satisfied if no storage were conducted in the current period at all, which is necessary and sufficient for the existence of some storage rule that would satisfy it for the current period. Clearly, \( f_0 \) is continuous and non-increasing. Further, it is necessary that the speculative constraint be able to be satisfied with a policy for the current period, say \( \gamma(S, h) \), that would ensure that RES(0) would be satisfied in the following period as well.

\[(RES(1)) \exists \gamma \exists \ (i) p_{-1} \geq \beta(1-\delta)E_h [P(S + h - \gamma(S, h)] - k.\]
\[(ii) P(S + h - \gamma(S, h)) \geq f_0((1-\delta)\gamma(S, h)) \forall h.\]

Fix \( S \). For a given \( h \), there is a minimum storage level \( \gamma(S, h) \) at which (ii) is satisfied. This is either the solution for \( g \) to:

\[P(S + h - g) = f_0((1-\delta)g)\]

if a solution exists, in which case it is unique; or it is \( \gamma(S, h) = 0 \). Call this policy function \( \gamma_{F,1}(S, h) \).

It is continuous and non-decreasing in \( S \). Clearly, RES(1) holds for \((S, p_{-1})\) if and only if:

\[p_{-1} \geq f_1(S) = \beta(1-\delta)E_h [P(S + h - \gamma_{F,1}(S, h))] - k.\]

Clearly, \( f_1 \) is continuous and non-increasing. By repeated application of this logic, we see that
(S, p_{-1}) must satisfy the recursively defined sequence of restrictions:

\[(\text{RES}(n)) \quad p_{-1} \geq f_n(S) \equiv \beta(1-\delta)E_h[P(S + h - \gamma_{F,n}(S, h)) - k],\]

for \(n = 1, \ldots, \infty\), where \(\gamma_{F,n}(S, h)\) is the unique solution for \(g\) to \(P(S + h - g) = f_{n-1}((1-\delta)g)\) if a solution exists, and zero otherwise. We have thus defined an operator mapping each \(f_n\) function into the subsequent \(f_{n+1}\) function. It is straightforward to verify that this operator is a contraction mapping,\(^{25}\) so that the \(f_n\) functions converge uniformly to a limit, which we may denote \(f_\infty\), with the corresponding minimal storage policy \(\gamma_{F,\infty}\). Noting also the monotonicity of the operator and the fact that \(f_1(S) \geq f_0(S) \ \forall S\), the sequence of \(f_n(S)\) functions is an increasing sequence, so \(\text{RES}(n+1)\) is always at least as restrictive as \(\text{RES}(n)\). Thus, a necessary and sufficient condition for satisfaction of all \(\text{RES}\) at once is:

\[(\text{RES}(\infty)) \quad p_{-1} \geq f_\infty(S).\]

A final note is that the functional operator relating subsequent functions is essentially the same as that defining equilibrium in the market with no government buffer stock (Deaton and Laroque, 1992). Thus, \(f_\infty(S)\) is the expected price in equilibrium under \textit{laissez faire} given that the inherited stock is \(S\), discounted to the previous period and net of storage costs. The previous price must not exceed this. Denote the feasible set of states thus derived \(F\):

\(^{25}\)If we denote the implicit operator \(T\), then it is clear that (i) \(f(S) \geq \gamma(S) \ \forall S\) implies that \(Tf(S) \geq T\gamma(S) \ \forall S\), and (ii) \(\kappa > 0\) implies that \(Tf + \kappa(S) \leq Tf(S) + \beta(1-\delta)\kappa\). These are sufficient for the contraction property by Blackwell's Lemma (Sargent, 1987, pp. 344-5).
The above reasoning tells us that we can focus on simple preemption policies that are stationary after the first period, say, a pair of rules \((\gamma_0, \bar{\gamma})\) for period 0 and subsequent periods, respectively. All of this can be written compactly in the following form.

\( (B1) \bar{\pi}(S, p_{-1}) = \) 
\[
\max \left\{ \mathbb{E}_h[\pi(S, \bar{\gamma}(S, h, p_{-1}), h) + \beta \gamma((1-\delta)\bar{\gamma}(S, h, p_{-1}), P(S + h - \bar{\gamma}(S, h, p_{-1})))] \right\}
\]
for any \((S, p_{-1}) \in F\), subject to:

(i) \(0 < \gamma(S, h, p_{-1}) < S + h \forall h;\)

(ii) \(p_{-1} > \beta(1-\delta)\mathbb{E}_h[P(S + h - \bar{\gamma}(S, h, p_{-1}))] - k;\)

(iii) \( ((1-\delta)\bar{\gamma}(S, h, p_{-1}), P(S + h - \bar{\gamma}(S, h, p_{-1}))) \in F \forall h;\)

\( (B0) V(S_0) = \) 
\[
\max_{\gamma_0} \{ (a_s - a_T) P(S_0 + h - \gamma_0(h)) S_0 + \pi(S_0, \gamma_0(h), h) + \beta \gamma((1-\delta)\gamma_0(h), P(S_0 + h - \gamma_0(h))) \}
\]
subject to: \( (i) 0 \leq \gamma_0(h) \leq S_0 + h \forall h; \)

(ii) \( ((1-\delta)\gamma_0(h), P(S_0 + h - \gamma_0(h))) \in F \forall h. \)
(B1) is a Bellman equation that must be satisfied by the value function for period 1 and all subsequent periods. The storage rule resulting from this equation, \( \tilde{y} \), is then the storage rule for periods 1 through infinity. (B0) is then a Bellman equation that gives the corresponding value and policy function for the first period. A solution to these two equations is thus a globally optimal policy under commitment. It can easily be checked that (B1) satisfies the standard conditions for a contraction mapping (such as described in Sargent (1987, pp. 344-5)), and thus has a unique solution which can be reached by iterations on (B1) from any starting point. It is, thus, from this point of view, a very well-behaved optimization problem.

Of course, once we have computed the optimal preemption policy \( \tilde{y} \), it is straightforward to implement the same outcome through a variety of alternative policies that do not completely preempt private storage. For example, we can define the optimal ‘minimalist’ policy \( y^{\text{min}} \), where for any \( (S, h, p_{-1}) \) the value \( y^{\text{min}} (S, h, p_{-1}) \) is defined as follows. Denote \( \tilde{y} (S, h, p_{-1}) \) by \( g^* \) and \( P(S + h - g^*) \) by \( p^* \). Then if \( p^* + k = \beta (1 - \delta) E_k [P((1 - \delta)g^* + h - \tilde{y}((1 - \delta)g^* , h, p^*))] \), set \( y^{\text{min}} (S, h, p_{-1}) \) equal to zero; otherwise, set it equal to \( g^* \). It is immediate that the price process induced by \( \tilde{y} \) continues to clear the market with policy \( y^{\text{min}} \) (in other words, (6) will be satisfied), and that the value of the objective function is also unchanged, so that \( y^{\text{min}} \) is optimal. The only difference is that the minimalist policy allows private agents to do all of the storage when they are willing to do so, and takes over from private agents only when they are unwilling to store. In each applied example presented in Section 6, it will be useful to describe both the optimal preemption policy and the optimal minimalist policy, in order to show the range of policies that implement the optimal outcome.
5. Basic Features of the Problem.

(i) Concavity and convexity.

After having written it in a manageable form, the first thing to note about this problem is that there are important respects in which it is not necessarily at all well-behaved. Even if the demand curve $P$ is well-behaved and $v$ is a standard, concave utility function, there is no guarantee that the objective function is concave; and even if the objective function is concave, there is no reason to expect that the value function will be concave. For example, if $a_c > 0$ but $a_T = a_s = a_p = 0$, so that consumer surplus is all that matters, the single-period marginal benefit of storage is equal to:

$$\pi_g = a_c [S + h - g]P'(S + h - g) = a_c QP'(Q) < 0.$$ 

Naturally, increased storage in any one year hurts consumers by driving the price up. However, this marginal harm may easily be decreasing in $g$ (so that the marginal benefit is rising in $g$). A necessary and sufficient condition for this is:

$$-QP''/P' < 1.$$ 

This will be satisfied if the demand curve is concave or not too convex. Thus, for example, if the demand curve is linear and consumers are the main constituency for the buffer stock, the objective function is globally convex in the main choice variable. It is easy to find examples in which the
optimal policy that emerges is a policy of maximum instability: The government stores all it can when the harvest is low, thus driving the price up as far as it can, and then releases all of its stocks when the harvest is high, driving the price down as far as possible. The economic reason is that the consumers get the commodity practically for free in the good years and do not mind waiting through the lean years with no consumption to enjoy that. There is thus a large transfer of wealth from the producers and taxpayers to the consumers. This is closely related to the point made by Waugh (1944) that consumer surplus is a convex function of price, so that a mean-preserving spread of price \textit{ceteris paribus} benefits a risk neutral consumer.

The 'maximum instability' policy is unlikely to have any relevance in practice. The point is that even a very simple, straightforward example can quickly turn pathological.

Even if the objective function is concave, there is no guarantee that the value function will be as well. This is because the usual proof of a concave value function in dynamic programming (see Theorem 9.8 of Stokey and Lucas (1989), p. 265) does not work. Consider two levels of beginning-of-period total storage, $S$ and $S'$. Storage of $\gamma(S, h, p_-)$ is optimal with $S$ and $\gamma(S', h, p_-)$ with $S'$. If the state is $\alpha S + (1-\alpha)S'$, $\alpha \in (0, 1)$, there is no guarantee that storage of $\alpha \gamma(S, h, p_-) + (1-\alpha)\gamma(S', h, p_-)$ will be feasible; it will be physically feasible, but might violate the speculative constraint. This invalidates the usual proof of concavity of the value function, and does not leave available any obvious replacement. The result is that one should be more careful than usual in checking for optima that are local but not global, and for corner solutions, in these

\[\text{For example, suppose that the harvest follows a two-point distribution, with a harvest } H \text{ with probability } \rho \text{ each period and a harvest } L \text{ with probability } (1-\rho), H > L. \text{ Then, with a linear demand curve, it is not hard to show that the policy described is optimal if } H/L \text{ and } k \text{ are large enough. Naturally, if } k \text{ is low, speculators will greatly restrict the scope for such a policy.}\]
problems.

(ii) Fully interior optima: The 'amnesia' theorem.

Suppose that under the optimal policy, neither the physical nor speculative constraints ever binds (at any date, or at any state of nature). Call such a policy a 'fully interior' solution. Then a feasible perturbation of the optimal policy would be to store slightly more at time $\tau > 0$ and then preserve that additional amount of the commodity permanently in the silo, allowing it to depreciate naturally over the years. This would change the value of the time-$\tau$ return function by $\pi_g$ and the value of the return function for subsequent periods $\tau + n$ by $(1-\delta)^{n}[\pi_s + \pi_d] = -a_r(1-\delta)^n k$. In order for these to balance, it is necessary that:

$$\pi_g(S, g, h) = a_r k / (1 - \beta(1-\delta)) \forall \tau > 0,$$

where a subscript denotes a partial derivative. Recalling the expression for $\pi$, this can be written as:

$$-a_r P(S + h - g) - a_r k - [a_r (S - g) - a_c(S + h - g) + a_r h v'(P(S + h - g)h)]P'(S + h - g)$$

$$= -a_r p - a_r k - [a_r (P^{-1}(p) - h) - a_c P^{-1}(p) + a_r h v'(ph)]P'(P^{-1}(p))$$

$$= k / (1 - \beta(1-\delta)) \forall \tau > 0,$$

where $p = P(S + h - g)$ is the price.
Equation (10) defines a simple relationship between the current harvest, \( h \), and the current price, \( p \), that must be satisfied at all times after period 0 by a fully interior optimum. There are two useful observations to make about that. The first is that if the function \( r_g \) is at all well behaved, this will implicitly define the current price as a function of the current harvest alone. This is a very strong result indeed, since in general the optimal policy depends on the past in two ways: Indirectly, through accumulated inventories; and directly, through the previous period’s price. In the case of a fully interior solution, the price will not depend on either, and in light of this independence from the past we might call such a policy an amnesiac price peg.\(^{27}\)

The second observation is that in the event of a fully interior solution, because of (10) the policy will generally be very easy to compute. In some cases, one will be able to calculate a full solution by hand; one such example is given in the next section. Even if we are not interested in fully interior solutions in and of themselves (and indeed, they are unlikely to be empirically important), they can help considerably in shedding some light on the parts of the parameter space for which the constraints do bind. This is because it can help map the parameter space. The statement that an optimum is fully interior is a statement that the three inequalities (i) and (ii) in (B1) hold strictly at each state. This assumption leads to (10), and a simple, easily computed policy rule. If we write down that policy rule in terms of the underlying parameters, inequalities (i) and (ii) then become conditions on those parameters, and in principle we can use those conditions to map out the region of the parameter space for which the optimum is fully interior. Each boundary of this region represents one of the three constraints for some level of the harvest; when the

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\(^{27}\) Naturally, in the case of an amnesiac price peg, there is no difference between the optimal preemption policy and the optimal minimalist policy. Both will have zero private inventories.
parameter values cross such a boundary, we move from a regime in which none of the constraints ever binds into a regime in which one of the constraints binds. Thus, examining the fully interior solution can give some insight into how features of the optimal policy are affected by changes of various directions in the parameter values. This strategy is illustrated in the next section for a simple linear-quadratic example.

6. A linear-quadratic example.

Consider the following model. The demand curve is given by \( P(Q) = a - bQ \). There are two possible values for the harvest, \( H \) and \( L \), with \( H > L \). The harvests are iid, with probability \( \rho \) of a high harvest each period. In such a market, in the absence of policy, competitive speculators follow a linear storage rule when \( h = H \) and sell everything when \( h = L \), with prices always higher in the latter state than the former one (at least if a particular inequality on parameter values is satisfied: see Newbery (1984)).

The producers' utility function is given by \( U(y) = cy - dy^2/2 \). Consumers and taxpayers are assumed to have equal weight in the government's objective function, so without loss of generality we can normalize so that \( a_c = a_r = a_p = 1 \). Thus, the single-period return function is:
\[ \pi(S_t, g_t, h_t) = \]

\[ b(S_t + h_t - g_t)^2/2 \]  

(Consumer surplus.)

\[ + c[a - b(S_t + h_t - g_t)]h_t - d\{(a - b(S_t + h_t - g_t))h_t\}^2/2 \]  

(Producer utility.)

\[ + [[a - b(S_t + h_t - g_t)](S_t - g_t) - k g_t]. \]  

(Taxpayer surplus.)

**Example 1: An Amnesiac price peg.** First, look for parameter values for which amnesiac price pegs are optimal. Figure 1 shows how the solution changes as we move through \((c, d)\) space when the other parameter values are: \(a = 100;\) \(b = 1;\) \(H = 60;\) \(L = 20;\) \(\rho = 0.75;\) \(\delta = 0.1;\) \(\beta = 0.97;\) \(k = 1.\) A movement northeast along a ray through the origin represents a rise in the weight attached to producers without changing the preferences of those producers. By contrast, a movement through the picture that places us on a lower ray represents a rise in producer risk aversion.

The solution to (10) is given by the following equation, in which \(Q_h\) denotes consumption in the event of a harvest of size \(h\).

\[ Q_h(c, d) = h - \left[ \frac{cbh - a - d(a - bh)bh^2 - \frac{k}{1 - \beta(1 - \delta)}}{db^2h^2 + b} \right]. \]

This implies a storage rule through the identity \(\gamma(S, h) = (S + h - Q_h)\). We can then define \(p_h = a - bQ_h\) for \(h = L, H\). For example, when \(c = 9\) and \(d = 0.002, p_H = 57.576\) and \(p_L = 84.514\).

Call this case Example 1; it is represented by a small circle in Figure 1. The shaded area in the figure is the region in which \(0 \leq Q_h \leq S + h\) permanently if the storage rule implied by (11) is followed; in other words, this is the region in which the amnesiac price peg satisfies the physical
constraints. The curves \( LL \) and \( HH \) give the locus of points on which the amnesiac price peg just satisfies the speculative constraints in the low harvest and the high harvest respectively. Points above and to the right of \( LL \) satisfy the constraint strictly for \( h = L \), and points below and to the left of \( HH \) for \( h = H \). Thus, the shaded region of the parameter space between \( HH \) and \( LL \) is the region within which the amnesiac price peg defined by (11) is feasible; and so (since this policy is the unconstrained optimum for the given objective function) this is the region for which it is the optimal policy.

The curve \( TT \) gives the points within this region for which \( p_H = p_L \): the locus of total price stabilization. To the right of this curve, prices are stabilized only partially, in the sense that \( p_H < p_L \), but the prices are brought close enough together that speculators do not care to store (whereas in the absence of the policy, speculators would always store when \( h = H \); again, see Newbery 1984)). To the left of \( TT \), prices are over-stabilized in the sense that \( p_H > p_L \), a reversal of the natural pattern.

Notice that as the producers become more risk averse, that is, as we move to the right within the shaded area of the diagram, price stabilization becomes less and less attractive. It is only when farmers care little about risk, that is, in the left-most section of the shaded region, that commodity price stabilization (CPS) becomes optimal (in the sense that raising \( p_H \) more than \( p_L \) can be called CPS). This is the opposite of the conclusion of Newbery and Stiglitz (1981, Ch. 20), who argued that CPS would likely make farmers poorer in expected value but reduce their risk, and thus be

\[ ^{28} \text{Strictly speaking, points where (11) satisfies the speculative constraint in the } H \text{ state are points below the } HH \text{ curve in the upper right-hand corner of the Figure, and above the curve marked ' } H' \text{ in the lower left-hand corner. The relevant inequality gives an expression for a threshold value of } c \text{ as a function of } d \text{ whose denominator changes sign at a critical value of } d. \text{ Similarly, the points satisfying the speculative constraint in the } L \text{ state are found above the } LL \text{ curve, and below the } L \text{ curve in the lower left-hand corner. Clearly, the lower curves generate no additional points satisfying all of the constraints, and can be ignored.} \]
attractive to farmers only if they were extremely risk averse. The reason this result comes about is that here we are looking at optimal policies, which means minimizing the cost to consumers and taxpayers for a given level of producer benefit. Now, the marginal fiscal cost of a rise in government purchases is \( p - (g - S)P' + k/(1 - \beta(1-\delta)) \); the marginal reduction in consumer surplus is \( -Q_hP' = -(S + h - g)P' \). The sum of these is the total marginal cost of government purchases, \( p - hP' + k/(1 - \beta(1 - \delta)) \). The marginal increase in farmers' income is \(-hP'\), and so the marginal cost of a dollar of income transferred to farmers through public storage is the ratio of the two, or:

\[
1 + \frac{p + k/(1 - \beta(1 - \delta))}{-hP'}
\]

For a given price level, this is smallest when \( h \) is high. In other words, ceteris paribus, the most efficient time to transfer income to the farmers is when the harvest is high, because then the base to which the price rise is applied (that is, \( h \)) is large relative to the distortion caused. However, a policy of raising price more aggressively when \( h \) is high than when it is low also raises the variance of the farmers' income. Thus, it would be attractive to use such a policy only if the farmers' utility function is not too concave. Thus, if we are principally trying to help out the producers, CPS is least attractive when they are quite risk averse. That fact that Newbery and Stiglitz (1981) focussed on arbitrary changes in the distribution of price, rather than on optimal storage policies,\(^{29}\) accounts for the absence of this cost-minimizing reasoning in their analysis.

\(^{29}\)As noted above, optimal storage is analyzed in their Chapter 30, but for a very different problem. The object there is to analyze the optimal public sector storage policy given that the private sector is exogenously assumed not to store.
Two more examples, outside of the 'amnesiac' region of the parameter space, are summarized in the subsequent diagrams. Both retain the same values as above for all parameters except for \(c\) and \(d\). In the case illustrated in Figures 2 and 3, \(c = 2\) and \(d = 0\). Call this Example 2. It also is marked as a small circle on Figure 1. Example 2 represents the case of a pure transfer; since all agents are risk-neutral, insurance is not an issue, but farmers have twice the welfare weight given to anyone else. Figure 2 shows the price as a function of \(S\) and \(p\) under the optimal policy for Example 2, and Figure 3 shows a simulation of 200 periods of history under the optimal policy, with inventory carryin \(S\) on the left axis and the current price \(p\) on the right. Figures 4 and 5 show the same exercise for a third case, Example 3, for which \(c = 2\) and \(d = 0.001\). This can be thought of loosely as a case of a policy with a 'pure insurance' motive; the objective function does not uniformly favor the farmers over other agents because for some levels of farmer income the weighted marginal utility of farmers is below that of other agents. Thus, there is no clear transfer motive for policy, but the risk aversion of farmers does indicate a clear insurance motive. In Figures 2 and 4, the price at each state value is plotted on the vertical axis; for state values near the origin, which lie outside of \(F\) (and thus are infeasible and irrelevant, as noted in Section 4(vi)), the price is arbitrarily set equal to zero. To facilitate comparison with Example 1, in Figures 2 and 4, the prices 57.576 and 84.514 of the optimal peg in that Example are marked where appropriate. The realized harvest series in Figure 3 is exactly the same as the one in Figure 5.

\[30\] These optimal policies were generated by recursions on the modified Bellman equation (B1), using a sixth-order polynomial in \(S\) and \(p\) to approximate the value and policy functions. The Gauss program is available from the author on request.

\[31\] Strictly speaking, Example 3 departs slightly from the linear-quadratic functional form, since for the computations farmer utility was specified to be constant after the bliss point \(y = c/d\). This was to avoid negative marginal utility.
Example 2: Pure transfer motive; Preemption policy. Under the optimal policy in Example 2, note that for most of the state space, the price tends to be lower during a period of drought (Figure 2b) than in a period with a normal harvest (Figure 2a). This is explained by the cost-minimizing reasoning noted above: With the farmers risk-neutral, the most efficient form of transfer to them involves a price that is highest when the harvest is large. The behaviour this implies for the market over time is indicated in the simulation of Figure 3. In intervals of sustained good harvest, such as occurred in this simulation, for instance, during periods 69 to 81, the government supports the price aggressively, steadily accumulating stocks, and the price gradually descends to a floor of about 59. If, then, a low harvest occurs, as did here in period 82, the government takes advantage of that event to unburden itself of a large fraction of these stocks, providing a respite to both taxpayers and consumers at a time when the harm this will cause to producers is minimal. Accordingly, this drought is accompanied by a sharp drop in the price, to a value of 39. For the same reason, all of the lowest prices recorded in this simulation (all clustered around a value of 40) occur in drought periods.

On the other hand, when a fairly sustained drought occurs, as it does in periods 57 to 59, the government quickly depletes its inventories and the price rises very rapidly to a value of $P(L) = P(20) = 80$. For this reason, the highest prices in the simulation, the spikes with values rising through the high 70's, also occur in drought periods. Thus, the range of drought prices observed in equilibrium is much greater than the range observed in normal periods, as one might have guessed from a glance at Figure 2.

Example 2: Pure transfer motive; Minimalist policy. Under the optimal policy described above, the speculative constraint binds in the first period or two of a drought, and at no other times.
Accordingly, the optimal minimalist policy can approximately be described as follows. The government aggressively supports the price during periods of good harvest, with a long-run target price of around 59, and a somewhat higher target price if inventories are low. In these normal times, there will be no speculative storage. However, when a drought occurs, the government dumps all of its stocks on the market. Speculators purchase some of these released stocks, and if the next period is normal they sell them all, as government purchases resume. If, instead, the drought continues, speculators hold onto some portion of the stocks for one more period and then sell them all. In short, (in this example with a pure transfer motive) the government monopolizes storage in normal times, but in droughts it relies on the market.

Example 3: Pure insurance motive; Preemption policy. By contrast, under the optimal policy in Example 3, the price tends to be higher during a period of drought (Figure 4b) than in a period with a normal harvest (Figure 4a). Essentially, the government pulls the price up a bit during a drought, when farmers’ marginal utility is high, and pushes it down a bit when there is a normal harvest, when the farmers’ marginal utility is low. For this reason, the speculative constraint binds during normal years, and not during droughts. The government would like to release all of its stocks when there is a large harvest, but it cannot because at that point speculators would wish to store. Figure 5 shows that, with the government showing no favoritism to farmers in this case, the inventories it holds are very small, and the price is driven mainly by the value of the current harvest.

Example 3: Pure insurance motive; Minimalist policy. The foregoing discussion indicates

In interpreting Figure 2 for this purpose, it is useful to confine attention to the portion of the state space within which the system stays during the simulation, namely, the rectangle for which $S \in [0, 150]$ and $p_{-1} \in [39, 85]$. Within that range, the current price is affected much more by the current level of inventory than by the previous price.
that the optimal minimalist policy for Example 3 can be described as a policy of non-intervention and no government stocks in a normal period, and a mild price support policy during droughts. Note that the price during a drought is often slightly above 80, indicating positive accumulation to reduce the hardship of the farmers, despite the scarcity of the good in those periods. Thus, in the insurance case the behaviour of government is the opposite of what it was in the transfer case: It supports the price during a drought, and lets the market work in normal times.

These three examples are of course only a sampling, but it is hoped that they may indicate the potential usefulness of this approach and the range of government behaviours that may be optimal in different settings.

7. Concluding remark on the interpretation of observed policies.

A final word may be in order on the comparison of theoretical optimal buffer stocks with actual policies. The optimal policies discussed here have been expressed as a function of the current level of inventory, the current harvest, and a recent price. In practice, most primary commodity policies have had rules expressed as a function of none of these, but of current price (see Gilbert (1987) for several examples). This is not, however, necessarily a meaningful difference. Without making any sort of claim that any existing policy is optimal, it is nonetheless worth pointing out that these differences in and of themselves do not rule that possibility out.

First, the fact that most policy rules are conditioned on price rather than quantities is not necessarily germane. In many cases it will be possible to achieve the same outcome with a policy conditioned on quantities as with one conditioned on prices. For example, the price-peg policy
studied by Salant (1983) is a rule giving the behaviour of the buffer stock manager conditional on
the current price; however, Salant shows that the equilibrium price in that model is a function of
total availability alone, which in notation used here is given by $S_t + h_t$. Thus, total storage can also
be written as a function of total availability, and we can easily write a buffer stock rule, $\gamma(S_t + h_t)$,
that delivers exactly the same outcome. Thus, the distinction between price-based and quantity-
based rules is spurious.

Second, actual commodity storage rules in practice do not seem to condition on the current
harvest *per se* (I cannot name an example that explicitly does so). However, in the model of this
paper, that is not necessarily inconsistent with optimality. For intermediate parameter values (such
as along curve $TT'$ of Figure 1), the ‘efficient transfer’ motive discussed in section 6 balances against
farmer risk aversion, and price is independent of the current harvest *per se*. In an empirical
example, it is conceivable that the optimum could be a function of $h$ but depend on it in a weak
manner (because the parameters are near $TT$), and that in order to simplify, the framers of a buffer
stock rule would simply drop the harvest term. However, if that is the correct interpretation, it does
suggest that the parameter values must indeed be intermediate: Thus, it would imply that a
substantial risk-sharing motive has been built into these policies in the past, and that a claim that
they satisfy a pure transfer motive would be untenable.

Finally, in the same spirit, it should be clear that an actual dependence on $p_{-1}$ is not necessary
for an optimum, and, as section 4(iv) made clear, even if the optimum does depend on some element
of the past, there are several equivalent sets of conditioning variables. In this case, there actually
are empirical precedents; buffer stock rules often have trigger-price adjustment rules that depend
on the recent history of the scheme. The cocoa buffer stock rules required adjustment of the target
price range in case of buffer stock purchases or sales above a specified level over a 12 month period. The rubber buffer stock had a similar rule, and one for adjustment if the average price was outside of the target range over a 6 month period. (Ghosh, et al (1987, pp. 178-9)). Both of these are an explicit dependence of the buffer stock rule on some element of the recent past.

One question that is not easy to answer is, given that the ‘price band’ form for buffer stock rules has been the most popular, is there a situation in which that can be generated as an optimum within this model? What this would mean is a combination of demand curve, producer utility function, distribution for $h$, and welfare weights, such that the optimal preemption policy could be written as $\gamma(S + h)$, with $0 \leq \gamma' \leq 1$ and with two intervals over which $\gamma' = 1$. (This observation is a trivial generalization of the analysis of Salant (1983)). This appears to be a difficult question.
Figure 1: The parameter space, and the region of interior solutions.
Figure 2a. Example 2: Pricing behaviour in H-state.
Figure 2b. Example 2: Pricing behaviour in L-state.
Figure 3. Example 2: Simulated History.
Figure 4a. Example 3: Pricing in H-state.
Figure 4b. Example 3: Pricing behaviour in L-state.
Figure 5. Example 3: Simulated History.
Bibliography.


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