Minorities and Storable Votes*

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ABSTRACT

The paper studies a simple voting system that can increase the power of minorities without sacrificing aggregate efficiency or treating voters asymmetrically. *Storable votes* grant each voter a stock of votes to spend as desired over a series of binary decisions and thus elicit voters' strength of preferences. The potential of the mechanism is particularly clear in the presence of systematic minorities: by accumulating votes on issues that it deems most important, the minority can win occasionally. But because the majority typically can outvote it, the minority wins only if its strength of preference is high and the majority's strength of preference is low. The result is that the minority's preferences are represented, while aggregate efficiency either falls little or in fact rises, relative to simple majority voting. The theoretical predictions of our model are confirmed by a series of experiments: the frequency of minority victories, the relative payoff of the minority versus the majority, and the aggregate payoffs all match the theory.

Recent decades have witnessed historic efforts at designing democratic institutions, at many levels. New constitutions were created in much of Eastern Europe and the former Soviet Republics. International organizations such as the European Union and the World Trade Organization have been evolving rapidly, and many developing countries

have moved from autocratic regimes to regimes based on elected representation with majoritarian principles.

While majoritarian principles provide a solid foundation for democracy, there are imperfections. This paper focuses on one particular imperfection that has presented a challenge to designers of democratic institutions for centuries: the *tyranny of the majority*, or the risk of excluding minority groups from representation. At least since Madison, Mill, and Tocqueville, political thinkers have argued that a necessary condition for the legitimacy of a democratic system is for no group with socially acceptable goals to be disenfranchised. In the history of US constitutional law, ensuring fair representation to each group is seen as the crucial second step in the evolution of democratic institutions, after granting the franchise: once all individuals are guaranteed the right to participate in the political process, should separate weights be given to each group's political interest? The core of the difficulty is that the two goals seem inherently contradictory.

The 1965 Voting Rights Act and the debate that continues to accompany its implementation focus on the need to guarantee that minorities, in particular racial minorities, have some direct representation. The obstacle is the possibility that their vote be *de facto diluted* by their minority status in all districts. In this paper, we study a related but different problem: the respect of minority preferences not in the choice of representatives, but in the very act of decision-making. We argue for it not only on the basis of fairness and legitimacy, but also on grounds of aggregate efficiency. Chwe (1999) took a similar perspective and proposed granting special voting power to the minority to ensure its participation when voting aggregates diffuse information. The voting system we analyze treats everyone identically, and we base our analysis on private value considerations — voting in our model aggregates divergent preferences, not diffuse information. But the efficiency rationale remains. A simple example illustrates why.

Suppose there are just two groups in a polity comprised of 100 citizens. Group A has 55 members and group B has 45 members. There are 3 proposals on the table. All citizens in group A have identical preferences and strictly prefer to pass all proposals; all citizens in group B have identical preferences and strictly prefer the *status quo* on all 3 issues. The table below gives a specific utility function for each member on each issue, and preferences are assumed to be additive. For each citizen, the utility of the less preferred option is normalized to 0.

Issue	U_A (pass)	U_A (sq)	U_B (pass)	U_B (sq)
1	3	0	0	1
2	2	0	0	2
3	1	0	0	3

Note that the *intensity* of preferences varies across the issues, and on a given issue the preference intensity for a group A member may be different from the intensity of a group B member. Issue 1 is important to group A but not to group B, and issue 3 is important to group B but not to group A.

Now consider what would happen with simple majority rule when issues are decided independently: since group A has a majority, all three proposals pass. Indeed, even if there were a million different issues, group A would always have a majority on all issues, so the B citizens are effectively disenfranchised — the outcome is exactly the same as it would be in a political system where only A citizens were allowed to vote.

Why is this outcome undesirable? First, equity considerations demand that the minority be able to win on at least some issues. But in addition, from a purely utilitarian standpoint, there are plausible welfare criteria according to which the outcome is socially inefficient. In our example, if each individual is treated equally and decisions are evaluated *ex ante*, before membership into the groups is known, the *status quo* should prevail on issue 3. Thus, the tyranny of the majority imposes costs both in terms of equity and in terms of efficiency. The equity problem stems from the existence of a smaller group whose preferences are systematically in the opposite *direction* of the larger group's preferences. The efficiency problem stems from differences in the *strength* of preferences of the two groups. Nothing fundamental depends on all citizens in a group having the same intensity of preferences on every issue, a simplification adopted only to keep the example transparent.[1]

How can the tyranny of the majority problem be solved, or at least mitigated? Any solution must deviate from issue-by-issue simple majority voting. An immediate possibility might be vote trading or some corresponding log-rolling scheme: members of one group could trade their vote on one issue in exchange for votes on other issues. But, in the simple example we constructed above, there are no gains to trading across groups, because every A citizen is already winning on all issues. Any system that allows the minority group to win on even one issue will make all A citizens worse off, and thus would not emerge spontaneously. With the perfect correlation of preferences we have posited above, an explicit institution re-enfranchising the minority is necessary.

Consider then endowing every voter with an initial stock of votes, and rather than requiring voters to cast exactly one vote on each issue, allowing them to lump their votes together, casting heavier votes on some issues and lighter votes on other issues. It is this voting mechanism, called *storable votes*, that we study in this paper. Even if the initial stock of votes is identical for all voters, storable votes allow the minority to win some of the time, and in particular, to win when its preferences are most intense. But because the majority generally holds more votes, it is in a position to overrule the minority if it cares to do so: the minority can win only those issues over which its strength of preferences is high *and*, at the same time, the majority's preference intensity is weak. These are exactly the issues where the minority *should* win from an efficiency viewpoint: the equity gains resulting from the possibility of occasional minority's victory need not come at a cost to aggregate efficiency.

In most of the specifications of the environment that we study in this paper, we find that standard economic measures of aggregate efficiency rise with storable votes. The main contribution of this paper then is not to suggest a new reason to increase minority's

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[1]   The central idea also does not depend on the direction of preferences within the group being *perfectly* correlated either — there may be some conflicting preferences within groups.

representation but to propose a specific voting scheme with the potential to achieve this goal even in the case of a systematic minority, when other voting mechanisms would fail, and to do so without violating the equal treatment of all voters.

The topic of minorities is felt so intensely, and the terms are so emotionally loaded, that there is a need to be scrupulously clear in terminology. As the example makes clear, we define a minority as a clearly identifiable group characterized by two features: first, a relatively small numerical size; second, preferences that are systematically different from the preferences of the rest of the polity. Thus, a minority in this paper is a political minority, which may, but need not, correspond to a minority according to racial, ethnic, religious or any other type of considerations. In terms of political decisions, what matters in the present context are the coherent and idiosyncratic preferences of the group, as opposed to the specific source of its identity.

The use of storable votes was initially proposed in Casella (2005), in a model that ignored systematic minorities. The desirable efficiency properties of storable votes remain true there, because the basic principle of bunching one's votes on more salient decisions continues to apply, with the implication that the probability of obtaining the desired outcome shifts away from decisions that matter little and toward decisions that matter more, with positive welfare effects. Storable votes are a particularly natural application of the idea that preferences can be elicited by linking independent decisions through a common budget constraint, an idea that can be exploited quite generally, as shown by Jackson and Sonnenschein (2007).[2] From a practical point of view, storable votes seem particularly well-suited to the protection of minority interests, where they have the potential to increase efficiency while improving equity at the same time.

A voting system similar to storable votes is *cumulative voting*, a mechanism used in single multi-candidate elections. It grants each voter a budget of votes, with the proviso that the votes can spread or concentrated on as many or as few of the candidates as the voter wishes. Cumulative voting has been advocated for the protection of minority rights (Guinier 1994) and has been recommended by the courts to redress violations of fair representation in local elections (Issacharoff *et al.* 2002). There is theoretical (Cox 1990), experimental (Gerber *et al.* 1998), and empirical (Pildes and Donoghue 1995; Bowler *et al.* 2003) evidence that cumulative voting does indeed help minorities. The general motivation behind the storable votes mechanism is similar to cumulative voting, but storable votes apply to a sequence of independent binary decisions, a substantively different strategic problem, with different applications. In addition, we explicitly study the efficiency properties of the mechanism, as well as its distributional effects on minorities.

The desirable properties of storable votes are features of the equilibrium of the resulting voting game — they emerge if every voter chooses the correct number of votes,

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[2]   Jackson and Sonnenschein propose a specific mechanism that converges to the first best allocation as the number of decisions grows large. The mechanism allows individuals to assign different priority to different actions but constrains their choices in a tightly specified manner. The design of the correct menu of choices offered to the agents is complex, but the mechanism achieves the first best. Storable votes are simple but in general do not achieve the first best. A mechanism very similar to storable votes was developed independently in Horta-Vallve (2006).

given what he rationally expects others to do. In practice there is a need to consider the robustness of the mechanism. Could the outcome be much worse if voters made mistakes? This is an appropriate concern here because the storable votes game is quite complex: voters need to trade-off the different probabilities of casting the pivotal vote along the full logical tree of possible scenarios, a task further complicated by coordination problems within the two groups, and multiple equilibria. If actual voters were confronted with the problem, what type of decisions would they make?

The second part of the paper presents the results of a set of experiments showing that under storable votes, the minority does indeed win on a significant number of issues. Both the minority payoff and the aggregate efficiency of the mechanism match the theoretical predictions, indicating that the equity gains accrue with little or no loss of efficiency. Voters use responsive strategies, consistently casting more votes when valuations are higher, a behavior that appears sufficient to take them most of the way toward their equilibrium payoffs, even when the number of votes they cast differs from the theoretical equilibrium. Previous experiments with storable votes in symmetric environments (Casella *et al.* 2006) had found a similar robustness of efficiency properties to strategic mistakes. Here the introduction of minorities complicates the game very significantly, and the robustness we observe in the experiments is a by the different cost of mistakes faced by majority members, who are likely to win anyway, and minority members, whose deviations are particularly costly (and rarer in the data). Whether because of the inherent robustness of storable votes, or because the minority made few mistakes, we see the minority's success in appropriating a significant share of the surplus with little if any aggregate cost as an encouraging sign of the practical viability of the mechanism.

THE MODEL

A committee with n members meets for T periods to vote over a series of binary proposals $\{P_1, \ldots, P_T\}$, each of which can either pass or fail. Voter i's preferences over proposal P_t are summarized by a valuation $v_{it} \in \mathbb{R}$. A positive valuation means that the voter is in favor of the proposal, a negative valuation means that the voter is against, and voter i's payoff from each proposal is given by $|v_{it}| \equiv v_{it}$ if the outcome of the vote is as he desires, and 0 otherwise. Thus voter i's utility function has the form:

$$U_i(P_1, \ldots, P_T) = \sum_{t-1}^{T} u_{it}(P_t),$$

where

$$u_{it}(P_t) = v_{it} \text{ if } \begin{cases} v_{it} > 0 & \text{and} \quad P_t \text{ passes} \\ v_{it} < 0 & \text{and} \quad P_t \text{ fails} \end{cases}$$
$$= 0 \text{ otherwise.}$$

The magnitude of the valuation, v_{it}, is called the preference *intensity* of voter i on proposal t. The profile of valuations, $v = (v_{11}, \ldots, v_{1T}, \ldots, v_{n1}, \ldots, v_{nT})$, is a random variable that is distributed according to the commonly known distribution $\Gamma(v)$, satisfying the assumptions we detail below.

The committee is composed of two *groups*, the *Majority group* **M**, with M members and the *Minority group* **m**, with $m < M$ members. The two groups differ systematically in their preferences: members of **m** are in favor of all proposals, and members of **M** are against. For all t:

$$v_{it} > 0 \quad \text{if } i \in \mathbf{m}$$
$$v_{it} < 0 \quad \text{if } i \in \mathbf{M}.$$

All members of the minority have valuations drawn from a distribution G_m with support $[0, 1]$, identical across proposals, while all members of the majority have valuations drawn from a distribution G_M with support $[-1, 0]$, again identical across proposals. We assume symmetry in the distributions across the two groups and call $G'_M = G_m \equiv F$ defined over the support $[0, 1]$ the distribution of intensities for each group. F is common knowledge.

Intensities are always drawn independently across proposals and across the two groups. With respect to the correlation of the intensities *within* each group, we consider two polar cases. In the first case (case B), intensities are drawn independently for each member of a group; in the second case (case C) intensities are identical for all members within a group. Hence, although all members of a group always agree on the preferred outcome, in the B case they may have conflicting priorities, while they do not in the C case. The correlation of within group intensities (or lack thereof) is common knowledge, as is the independence of intensities across proposals and groups.

At the beginning of period t, i privately observes v_{it} but does not observe $v_{it'}$ for $t' > t$: intensities are revealed privately and sequentially. Because draws are independent across issues, voter i's observation of v_{it} does not provide information about $v_{it'}$, and because draws are independent across groups, observation of $v_{it}, i \in \mathbf{m}$, does not provide information about $v_{jt}, j \in \mathbf{M}$ (and vice versa). Whether it provides information about the intensity of other voters in the same group, v_{jt}, with $j \in \mathbf{m}$, depends on which case we consider. In case C, members of the same group have identical preferences and observation of one's own intensity allows a voter to perfectly infer the preferences of the other members of his group. In case B, a voter's own intensity provides no information about any other voter's intensity.

The Storable Votes Mechanism

At the beginning of period 1, each voter is endowed with an account of B_0 *bonus* votes, where B_0 is an integer.[3] In the first period, the voter casts his regular vote plus as

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3   Because we want to study the effect of bonus votes *per se* in strengthening the minority's position, it seems appropriate to give the same initial allocation to all voters.

many discrete bonus votes as he wishes out of his endowment. The bonus votes cast are deducted from his endowment, which is then carried over to the next period. The current endowment of bonus votes for every voter in period t, denoted $B_t = (B_{1t}, \ldots, B_{nt})$, is common knowledge at the beginning of period t. Thus each voter i independently decides how many votes, x_{it}, to cast after observing his private intensity v_{it} and B_t, subject to $x_{it} \leq 1 + B_{it}$. The proposal passes if there are more votes in favor of the proposal than against, and fails in the opposite case. Ties are resolved randomly. In the next period, $t + 1$, voters' intensities over the new proposal are again privately observed, and voting proceeds as before, now subject to the constraint, $x_{it+1} \leq 1 + B_{it+1} = 2 + B_{it} - x_{it}$. Since $x_{it} \geq 1$, this is at least as tight a constraint as in period t. The voting continues in this fashion until the end of period T.

THEORETICAL RESULTS

Given F, m, M, B_0, T, the storable votes mechanism defines an asymmetric multistage game of incomplete information. We study the properties of the Perfect Bayesian equilibria of this game, where at each period t and for each possible intensity, v_{it}, and profile of endowments, B_t, individuals choose how many votes to cast so as to maximize expected utility, given the strategies of the other players. Because the sign of each group's preferences is common knowledge and intensities are independent over time, voting decisions cannot be used to manipulate other players' beliefs about future preferences. Assuming, in addition, that voters do not use weakly dominated strategies, the direction of each individual vote is always chosen sincerely: all the minority members' votes are cast in favor of each proposal, and all majority votes are cast against each proposal. The *state* of the game at t is defined to be the profile of bonus votes each voter has still available, B_t, and the number of remaining periods, $T - t$. We focus on strategies such that, given F, m, and M, the number of votes each individual chooses to cast each period, x_{it}, depends only on i's intensity of preferences at time t, v_{it}, and on the state of the game. We denote such strategies by $x_{it}(v_i, B_t, t)$.

The $C2$ Game

When characterizing the equilibria of our model, the correlation of intensities within each group in model C can be a source of complications. But matters can be simplified by a simple observation. Consider the following 2-player storable votes game, which we call $C2$. Voter M has M regular votes each period and a stock of MB_0 bonus votes; his valuation over each proposal is Mv_{Mt}, where v_{Mt} is independently drawn from the distribution function G_M with support $[-1, 0]$. Voter m has m regular votes each period and a stock of mB_0 bonus votes; his valuation over each proposal is mv_{mt}, where v_{mt} is independently drawn from the distribution function G_m with support $[0, 1]$. Then the following result holds:

Lemma 1 *If game C2 has an equilibrium, then the game described by model C also has an equilibrium. In addition, call $x^*_{Mt}(v_i, B_t, t)$ and $x^*_{mt}(v_i, B_t, t)$ the equilibrium strategies of voter M and voter m in game C2, and $\{x^*_{it}(v_i, B_t, t)\}$ the equilibrium strategies in C. If C2 has an equilibrium, then there exist equilibrium strategies of model C such that $\sum_{i \in m} x^*_{it}(v_i, B_t, t) = x^*_{mt}(v_i, B_t, t)$ and $\sum_{i \in M} x^*_{it}(v_i, B_t, t) = x^*_{Mt}(v_i, B_t, t)$.*

Proof: See Appendix.   ■

Lemma 1 makes a simple point. In model C voters' interests within each group are perfectly aligned; if there is an equilibrium where each group coordinates its strategy so as to maximize the group's payoff, given the aggregate strategy of the other group, then no individual voter can gain from deviating.[4] In the n-person game described by model C, we will call equilibrium *group strategies* the equilibrium individual strategies of the 2-voter game $C2$.

Equilibrium

The particular feature of storable votes is that they allow individuals to reflect the intensity of their preferences in the number of votes they cast. Lemma 1 allows us to show:

Lemma 2 *For any F, M, m, and T, both model B and model C have an equilibrium in monotone cutpoint strategies: at any state (B_t, t) and for any i with $B_i + 1$ available votes there exists a set of cutpoints $\{c_{i1}(B_t, t), c_{i2}(B_t, t), \ldots, c_{iB_i+1}(B_t, t)\}$, $0 \leq c_{ix} \leq c_{ix+1} \leq 1$, such that i will cast x votes if and only if $v_{it} \in [c_{ix}, c_{ix+1}]$. In model B, the strategies are individual equilibrium strategies and $i \in \{1, \ldots, n\}$; in model C, the strategies are group strategies and $i \in \{M, m\}$.*

Proof: See Appendix.   ■

Lemma 2 establishes that an equilibrium exists, although it does not rule out the possibility of multiple equilibria. Notice also that the lemma states that strategies may respond to valuations, as we expect intuitively, but allows for equilibria where the monotonicity is only weak — for example, possible equilibria where bonus votes are equally split among proposals, or where strategies depend on the timing of the proposals alone.

Storable votes open the possibility of minority victories. We can derive:

Theorem 1 *In both models B and C:* (i) *For any F, T, M, and $m > 1$ there is a finite $B'_0(M, m, T)$ such that for all $B_0 > B'_0$ there exist equilibria of the storable votes mechanism where the minority wins some of the time with strictly positive probability.* (ii) *If $T > M$ and $B_0 > B'_0$, then the minority wins some of the time with strictly positive probability in all equilibria of the mechanism.*

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[4]   This is the logic exploited by McLennan (1998) to show that whenever *sincere* voting is efficient in common value decision problems, then it must be a Nash equilibrium.

Proof: See Appendix.   ∎

The first part of the theorem establishes the existence of equilibria with a positive probability of minority victories, in direct contrast to the outcome with simple majority voting. The potential of storable votes to help the minority is very intuitive, although for arbitrary T the result cannot be established for all equilibria. The problem is coordination: in both models B and C (although not in $C2$, where coordination is imposed) if the other members of the minority follow a given strategy, it is difficult for a single deviating voter to be able to affect the final outcome, and thus strategies where the minority always loses can be supported in equilibrium. As an illustration, consider one possible equilibrium mentioned above, where every voter, both in the majority and in minority, distributes the bonus votes equally over all proposals: $x_i = 1 + B_0/T$ for all $i \in \{1, \ldots, n\}$. Because everyone always casts the same number of votes, the game becomes identical to simple majority voting, and the minority always loses. But unless a single minority voter deviating alone can lead to at least one proposal passing, the strategies are an equilibrium for both models B and C.[5] Notice that if $T = 2$ this equilibrium exists for all values of B_0: a deviating minority voter can shift at most $B_0/2$ votes, but over each proposal the majority is always winning by at least $1 + B_0/2$ votes (since $M \geq m + 1$). Thus, for $T = 2$ there is always at least one equilibrium where the minority always loses, regardless of the existence and of the number of bonus votes (although, as the theorem states, for appropriate values of B_0 there are also equilibria where the minority can win with positive probability).

Efficiency

Making it possible for the minority to win occasionally favors fairness and representation, but in principle could have efficiency costs because it implies that the larger group occasionally loses. However, even from a pure efficiency criterion, storable votes can be desirable. In equilibria where strategies are strictly monotonic, the minority wins when minority intensities outweigh majority intensities: the minority wins *when it should*.

We measure the efficiency of the storable votes mechanism in terms of *ex ante* efficiency: a voter's expected utility from all T proposals before any of his valuations is realized, and before knowing whether he belongs to $\mathbf{M}$ or to $\mathbf{m}$. We call our efficiency measure EV_0 and contrast it with the equivalent measure under simple majority voting, denoted by EW_0.[6]

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[5]   As mentioned, the strategies described are not equilibrium strategies for model $C2$. Lemma 1 states that the equilibria of model $C2$ are equilibria of model C; the reverse does not hold.

[6]   An important question is whether the cardinal valuations and our notion of efficiency force us into comparisons of interpersonal utilities. This is where the assumption of symmetrical distributions of intensities across all voters plays its role. The intensity draws over any specific decision should be read as normalized by a common numeraire. In our model with multiple decisions, the natural numeraire is the individual's mean intensity over the universe of all decisions that could be brought to a vote. In fact, by imposing not only the same mean but the same distribution, we are forcing the voters to adopt an equal scale and to organize the different decisions according to a fixed ordinal

The positive impact on efficiency of monotonic strategies applies to both models, but the properties of the voting mechanism are more robust and easier to characterize in model C.

Theorem 2 *In model C, for all F, M, and m > M/2, if T < $\overline{T}(M, m)$ there exists a value of B_0 and an equilibrium of the storable votes mechanism such that storable votes are ex ante superior to simple majority voting (i.e., $EV_0 > EW_0$).*

Proof: See Appendix.   ■

A few remarks will clarify the result. Note first of all that the difference in expected utility can occur only if the minority is expected to win some of the times; thus, in the equilibrium discussed in the theorem the minority itself necessarily fares better, in expected utility terms, than under simple majority voting. Note too that the minority could never win if the horizon were shorter than 2 periods; thus, again trivially, $\overline{T}(M, m) > 2$ for all M and m. The existence of an upper bound on T comes not from the logic of the mechanism but from the need to respect integer constraints: for all M and m, we require that the number of votes cast be always an integer. The proof shows that if integer constraints are ignored, $\overline{T}(M, m)$ can be made arbitrarily large for all M and m, and the result then holds for arbitrary T.

The result in the theorem requires not only that the minority be expected to win with positive probability, but also that equilibrium strategies be responsive to valuations: at least in some states strategies must be strictly monotonic. The difficulty in establishing the theorem is identifying equilibrium majority and minority cutpoints at each state such that expected minority gains and majority losses can be computed and compared for all F, M, m, and T. This is particularly true for model B, where the lack of information about the valuations of other members of one's own group makes coordination impossible. If we specialize our assumptions on F, M, m, and T the task is made much easier. The next subsection discusses the theoretical properties of the model when we restrict the set of parameter values, in line with the choices that we make in the experiment.

Theoretical Properties of the Experimental Design

In designing the experiment, the challenge is to specify a class of environments simple enough to be easily understood and replicated in the laboratory, but rich enough to preserve the main properties of the mechanism. The following specification satisfies these requirements: the total number of voters n is odd; the distribution F is Uniform; there are two consecutive proposals and each voter is endowed with two bonus votes:

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ranking, with the same proportion of decisions in any given subinterval of the support. It is this normalization that allows us to avoid interpersonal comparisons. In this model, granting individual voters different distributions would be equivalent to taking a stance on the relative intensity of their preferences.

$T = B_0 = 2$. The strategy chosen by each voter is simply the number of bonus votes to cast over the first proposal, as a function of his valuation. The proposition below characterizes equilibria for our experimental environment, where strategies are responsive to intensities and are an equilibrium not only for models B and C but also for model $C2$.[7] Its proof can be found in Casella *et al.* (2007) and in the supplementary material on the *Quarterly Journal of Political Science* web page.

Proposition 1 *Suppose n odd; F Uniform, and $T = B_0 = 2$. Then*:

In model B :

a. *There is an equilibrium where*: $x_{i1} = 1$ *if $v_{i1} < 0.5$ and $x_{i1} = 3$ if $v_{i1} > 0.5$ for all i.*
   In such an equilibrium:
b. *If $M > 3m$, the majority always wins, but for all $M < 3m$ the minority wins one of the two proposals with probability $\sum_{s=k}^{m} \left[\sum_{r=0}^{m-s} \binom{M}{r}\binom{m}{r+s} 2^{-n} \right] > 0$, where $k \equiv (M - m + 1)/2$.*
   Ex ante, each of the two proposals has the same probability of a minority victory.
c. *If $M > 3m$, storable votes are identical to simple majority voting, and $EV_0 = EW_0$. But for $M < 3m$, there exist m', m'', and M' with $m'' > m'$ such that $EV_0(m', M') < EW_0(m', M')$ but $EV_0(m'', M') > EW_0(m'', M')$.*

In model C:

a. *There is an equilibrium where the minority's strategy is: $x_{m1} = m$ if $v_{m1} < 0.5$ and $x_{m1} = 3m$ if $v_{m1} > 0.5$. The majority's strategy is: if $2M > 3m$, $x_{M1} = 2M$ for all v_{M1}; if $2M \leq 3m$, $x_{M1} = \max\{M, m+3\}$ if $v_{M1} < 0.5$ and $x_{M1} = \min\{3M, 4M-(m+3)\}$ if $v_{M1} > 0.5$.*
   In such an equilibrium:
b. *If $2M > 3m$, the majority always wins, but for all $2M \leq 3m$ the minority wins one of the two proposals with probability 0.25. Ex ante, each of the two proposals has the same probability of a minority victory.*
c. *Storable votes are always ex ante weakly superior to simple majority voting: $EV_0 = EW_0$ if $2M > 3m$, and $EV_0 > EW_0$ if $2M \leq 3m$.*

Together, restricting n, F, and T allows us to identify the equilibrium cutpoints and derive stronger efficiency results than in the general case discussed in Theorem 2.

The properties of these equilibria are illustrated in Figure 1, using the case of $M = m + 1$ as an example. Efficiency is maximized when each decision is resolved in favor of the side with higher total valuation, and in the figure we compare equilibrium and efficient outcomes.

Figure 1(a) shows, for both models, the probability of a minority victory over one of the two proposals in equilibrium — the black dots — and in the first best — the grey dots. The minority can never win both proposals because the majority always

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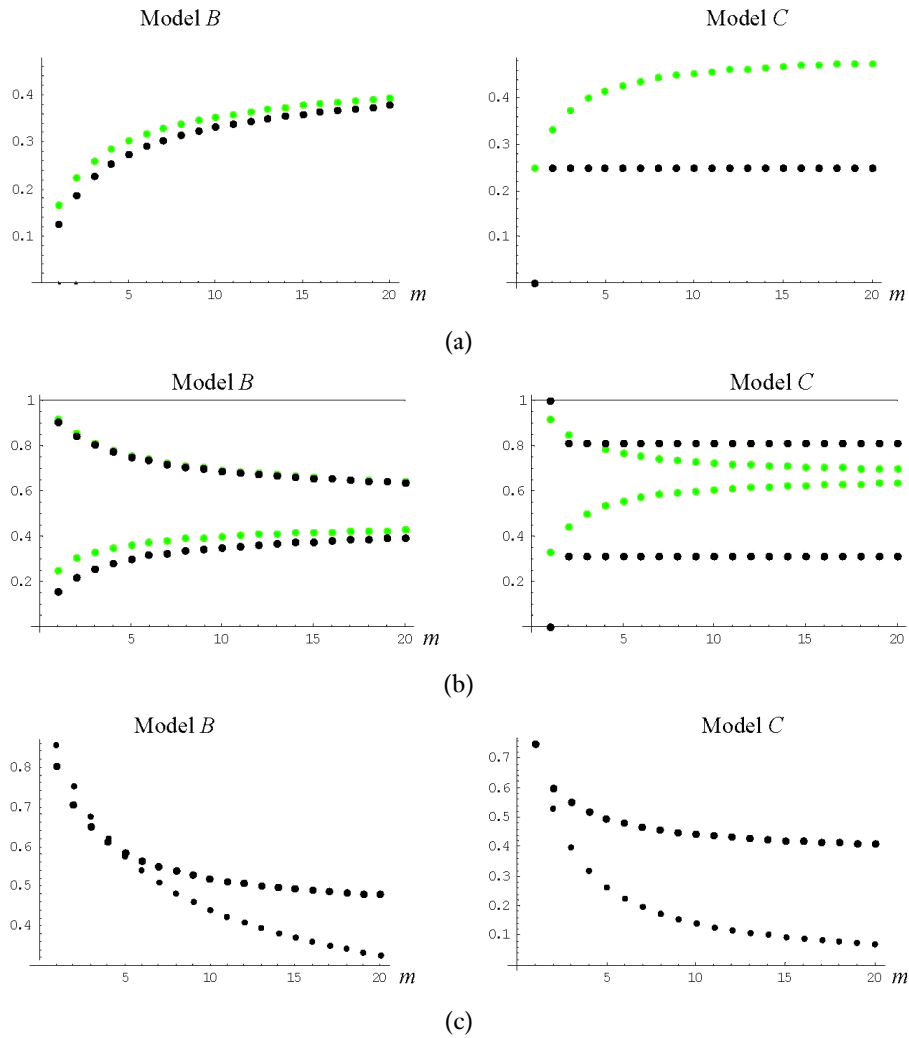[7] Recall, from earlier discussion, that there can also be nonresponsive equilibria.

Figure 1. $T = B_0 = 2$; $F(v)$ Uniform; $M = m + 1$. (a) Frequency of minority victories. (b) Expected payoff for majority and minority members (per capita). (c) Expected aggregate payoff as share of the available surplus. The large black dots plot equilibrium payoffs with storable votes; the grey dots efficient payoffs, and the small black dots payoffs with simple majority voting.

has a larger total number of votes. As m increases, the equilibrium probability of a minority victory increases. In model B, the increase is smooth, and the probability of a minority victory converges to 0.5 as the number of voters becomes large and the relative difference in size between the majority and the minority becomes negligible.

The efficient frequency of minority victories is slightly higher than the equilibrium frequency. In model C, the change in the equilibrium probability of minority victories is discontinuous, jumping from 0 to 0.25 when the majority becomes unable to guarantee itself victory on both proposals, and then remaining constant at that level. The point at which the jump occurs depends on the absolute difference between the two groups, $M - m$. The efficient frequency of minority victories on the other hand increases smoothly with the relative size of the minority and again is always higher than the equilibrium frequency.

Figure 1(b) plots the expected per capita payoff for majority and minority members. With simple majority rule, the respective values are 1 and 0 in both models. With storable votes, the expected payoffs of the two groups are closer, unless the majority can ensure itself victory, although the minority's payoff remains lower than under efficiency (the grey dots in Figure 1(b)). In model C, equilibrium per capita payoffs remain constant for each group, regardless of m, once the threshold where the majority always wins has been passed.[8]

Figure 1(c) plots a normalized measure of expected surplus for both models, where expected aggregate payoff is expressed as a share of the expected first best payoff. The figure compares storable votes and simple majority voting to each other and to first best efficiency. Because we want to measure the added value over purely random decision-making (where each proposal is equally likely to pass or fail), we normalize both numerator and denominator by the expected payoff in the random mechanism. Thus if we call EV_0^* the expected efficient aggregate payoff and R the expected payoff under random decision-making, we define the *normalized aggregate surplus* as $(EV_0 - R)/(EV_0^* - R)$ with storable votes and $(EW_0 - R)/(EV_0^* - R)$ with simple majority. Over the two proposals, $EW_0 = M$ and $R = (M + m)/2$ in both models, while EV_0 and EV_0^* can be found in the Appendix of Casella *et al.* (2007) and in the supplementary material on the *Quarterly Journal of Political Science* web page. As the figure shows, when the number of voters is small and the difference in size between the two groups relatively important, the possibility of minority victories in the storable votes mechanism is accompanied by some loss of efficiency in model B, but not in model C, where efficiency is always at least as high as under simple majority rule. The loss in model B is not large and disappears as the number of voters and the relative size of the minority increases. For most sizes of the electorate, storable votes allow voters to appropriate a larger share of the total surplus in both models.[9]

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[8]   In fact, they remain unchanged for any absolute difference between the two groups, once the threshold $3m < 2M$ has been passed. It is the threshold itself that depends on $(M - m)$.

[9]   The main difference between the two models emerges in the limit, and is not visible in the figure. In model B, the intensity draws are independent; hence, as the population becomes very large the law of large numbers guarantees that the empirical average intensity of preferences in both groups converges to the mean of the F distribution. This means that random choice, simple majority voting and storable votes all converge to first best efficiency and any efficiency-based argument for protecting the minority disappears. In model C, on the other hand, the valuation draws within each group are perfectly correlated, and the law of large numbers does not apply. As the number of voters increases, the difference in size between the two groups becomes negligible and simple

EXPERIMENTAL DESIGN

Protocol

All sessions of the experiment were run in laboratories either at the California Institute of Technology (SSEL), the University of California at Los Angeles (CASSEL), or Princeton (PLESS). Subjects were registered students, recruited through the laboratory web sites. No subject participated in more than one session. All sessions focussed on the specification just discussed: subjects voted on two consecutive proposals ($T = 2$) and were allocated 2 bonus votes ($B_0 = 2$), in addition to the regular vote they were required to cast over each proposal. With the exception of one session, committees were composed of 5 voters, divided into two groups of 3 and 2 voters with systematically opposed preferences.[10] The experiment's primary treatment variable was the correlation of intensities within each group — the distinction between model B and model C.

After entering the laboratory, the subjects were seated randomly in booths separated by partitions and assigned ID numbers corresponding to their computer terminal; when everyone was seated, the experimenter read aloud the instructions, and any question was answered publicly. The session then began.[11] Subjects were matched randomly into committees and within each committee were assigned randomly to the majority or the minority group. Each subject was then shown his valuation for the first proposal and asked to choose how many votes to cast in the first election. Valuations were restricted to integer values and were drawn by the computer, with equal probability, from the support $[-100, -1]$ for majority members, and from $[1, 100]$ for minority members. In both treatments, the valuations were drawn independently for majority and minority members.

In treatment B each member of each group was assigned a valuation drawn independently from the specified support; in treatment C all members of the same group in the same committee were assigned the same valuation (i.e., all majority members in a given committee shared the same valuation, as did all minority members in a committee). The independence of the intensities within each group in treatment B and their perfect correlation in treatment C were common knowledge. After everyone in a committee had voted, the computer screen showed to each subject the number of votes cast by each of the two groups in the subject's committee, whether the proposal had passed or not, and the subject's own payoff from that election. Valuations over the second proposal were then drawn, the remaining votes were automatically cast, and the outcome determined.

After the second proposal had been voted upon, subjects were rematched; each was assigned a new budget of bonus votes, and the game was replayed. Experimental sessions

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majority voting again converges to random choice, but random choice remains inferior to efficient decision-making and to storable votes. In very large populations, only minorities whose intensities are correlated should be protected on efficiency grounds.

[10] One session had committees of 9 voters, each divided into two opposite groups of sizes 5 and 4.

[11] A sample of the instructions can be downloaded from http://www.hss.caltech.edu/˜trp/ MINORITIES. The experiments were conducted using the Multistage Game open-source software (http://multistage.ssel.caltech.edu/).

consisted of between 15 and 30 rounds, each round a pair of consecutive proposals. In the rematching, minority members always remained minority members, and majority members always remained majority members, but the composition of each group and of each committee was randomly determined. Subjects were paid privately at the end of each session their cumulative valuations for all proposals resolved in their preferred direction, multiplied by a pre-determined exchange rate and complemented by a fixed show-up payment of $10. Average earnings were about $17 per experiment for minority subjects and about $31 for majority subjects.

Equilibrium

We found no evidence of non-responsive equilibria, and our analysis of the experimental data focuses exclusively on the equilibrium described in the previous section. Here we derive the details of the equilibrium for the specific case $M = 3$, and $m = 2$ (and for a robustness control in one experimental section, for $M = 5$, and $m = 4$). Individual equilibrium strategies in treatment B and corresponding equilibrium outcomes are in Table 1. The equilibrium cutpoints — the threshold intensities where individual voters switch from casting 0 to casting 1 bonus vote, and from casting 1 to casting 2 — are reported in row 2 of Table 1 and are denoted c_1 and c_2.[12] Rows 3 and 4 in the table report the expected frequency of minority victories in equilibrium and under efficiency, respectively. Rows 5 and 6 report the expected share of per capita payoff for a minority voter, relative to a majority voter, again in equilibrium and under efficiency. So, for example, in the $\{3, 2\}$ experiment with storable votes a minority subject is expected to win on average 26 percent of what a majority subject earns, if everybody plays the equilibrium strategy. Finally, the last two rows report the expected share of normalized aggregate surplus appropriated with storable votes (row 7) and with simple majority voting (non-storable votes, in row 8).

Storable votes in the B treatment are slightly less efficient from an aggregate point of view than simple majority voting, but the equilibrium efficiency loss is minor, relative to the effect of storable votes on the welfare of minorities.

Equilibrium strategies in treatment C pose a coordination problem. As described in the previous section, if the two groups are of size $\{3, 2\}$, in equilibrium the minority uses no bonus votes if its intensity is smaller than 50, and all its bonus votes if it is above; the majority casts a total of 5 votes if its intensity is smaller than 50, and 7 votes if it is larger than 50.[13] Any individual strategy compatible with these group strategies is an

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[12] Because the equilibrium cutpoints are identical for minority and majority voters, we use the symbols c_1 and c_2 for both groups.

[13] When the two groups are of size $\{3, 2\}$, the majority has other valuation-responsive equilibrium strategies, but all are payoff-equivalent and all are monotonic, and we treat them as identical when reporting the experimental results. All equilibrium strategies satisfy: cast 0, 1, or 2 bonus votes with probabilities p_0, p_1, p_2 if the absolute valuation is smaller than 50, and 4, 5, or 6 bonus votes with probabilities q_0, q_1, q_2 if the absolute valuation is larger than 50, where $p_2 \geq q_2$ and $p_1 = q_1$. The strategy described in the text corresponds to $p_0 = p_1 = 0$, and $q_1 = q_2 = 0$.

Table 1. Equilibrium strategies and outcomes

B Treatment		
M, m	3, 2	5, 4
c_1, c_2	50, 50	50, 50
% min wins, sv	19	25
% min wins, eff	22.5	28.5
% (min/maj) payoff, sv	26	36
% (min/maj) payoff, eff	35.5	45
% surplus sv	71	61
% surplus nsv	75	62

equilibrium. Hence, each minority voter has a simple symmetrical strategy that aggregates to the equilibrium group strategy: cast no bonus votes if the intensity is below 50 and cast all bonus votes if the intensity is 50 or above. But the coordination problem for majority voters is more difficult. The group strategy described above cannot be supported by *symmetric* individual strategies, and coordination on asymmetric strategies is hampered by the random rematching in our experimental design. In fact, in our experimental environment, not only is there no symmetric individual strategy that aggregates to the equilibrium group strategy, but there is no asymmetric strategy that each majority voter can adopt consistently and that would always aggregate to the equilibrium group strategy, for any possible rematching.

In practice, our basic *C* treatment is then a test of the robustness of storable votes' outcomes to coordination problems. To evaluate the role of coordination more precisely, we designed two additional treatments that replicate model *C* but where coordination problems are absent by construction.

Treatment *C2* mirrored the *C2* game: for each group, a single voter cast votes on behalf of all members of that group. Each majority group representative had 3 indivisible regular votes to cast on each of the two proposals and 6 bonus votes to cast as desired. Each minority group representative had 2 indivisible regular votes to spend on each of the two proposals and 4 bonus votes to cast as desired. Each committee then consisted of one minority and one majority representative. For each proposal, valuations were drawn independently with equal probability, from the support $[-100, -1]$ for the majority representative, and from $[1, 100]$ for the minority one. The timing of the game proceeded as described earlier. After each two-proposal round, group representatives were rematched. When we discuss experimental payoffs from this treatment, we multiply the minority representative's payoff by 2 and the majority's by 3, to make them comparable to the theoretical predictions and to the experimental payoffs for the *C* case and for the following treatment, which we call *CChat*.

In treatment *CChat* (*correlated valuations, chat option*) we replicated the *C* treatment, with each group composed of multiple individual voters rather than just two representatives. Before the vote on the first proposal, voters could exchange messages via computer

Table 2. Equilibrium group strategies and outcomes

C Treatments	
M, m	3, 2
g_L, g_H	50, 50
% min wins, sv	25
% min wins, eff	33
% (min/maj) payoff, sv	38.5
% (min/maj) payoff, eff	52
% surplus sv	60
% surplus nsv	53

with other members of the same group. Voters were instructed not to identify themselves, and the messages were anonymous but otherwise unconstrained. In particular, they allowed subjects to coordinate on their preferred group strategy. Everything else in the experiment — the stochastic properties of the valuation draws, the timing, the random re-matching — followed exactly the *C* treatment, with perfectly correlated values within a group.

Equilibrium group strategies and expected outcomes are identical in the three *C* treatments — *C*, *C2*, and *CChat*. They are reported in Table 2, where g_L and g_H denote the cutpoints where the minority switches from casting 0 bonus votes to casting 2, and from casting 2 to casting 4, and the majority from casting 2 bonus votes to casting 3, and from casting 3 to casting 4.

The outcome is more favorable to the minority in model *C* than in model *B*, both in terms of the expected frequency of minority victories and of its expected payoff, relative to the majority. In contrast to the *B* treatment, storable votes in the *C* treatment lead to efficiency gains over simple majority voting.

Table 3. Experimental design

Session	Groups size	Subject pool	# Subjects	Rounds
b1	3, 2	CIT	15	30
b2	3, 2	UCLA	20	30
b3	5, 4	UCLA	27	30
c1	3, 2	UCLA	15	30
c2	3, 2	PU	15	20
c3	3, 2	PU	10	20
c21	3, 2	CIT	12	30
c22	3, 2	UCLA	16	30
c23	3, 2	PU	12	20
CChat1	3, 2	PU	10	20
CChat2	3, 2	PU	15	15

The experimental design is summarized in Table 3. In all *b*, *c*, and *CChat* sessions the majority was formed by 3 subjects and the minority by 2, with the exception of session b_3 where the number of subjects in each group was 5 and 4, respectively. Session b_3 serves us as a control on the sensitivity of the experimental results to the size of the groups. In all *c2* sessions, a single subject represented each group, but the design was equivalent to two fully coordinated groups of 3 and 2 members, respectively.

EXPERIMENTAL RESULTS

The experiment has two principal goals. First, we want to verify whether voting *outcomes* match the theoretical predictions: are minority subjects able to win some of the votes? Are they able to do so without loss of aggregate efficiency? Second, to what extent does voting *behavior* match the theoretical predictions?

Voting Outcomes and Efficiency

How often do minority groups win?

The diagram on the left of Figure 2(a) summarizes the answer to this question. The vertical axis is the percentage of times the minority prevailed in the experimental sessions, and the horizontal axis is the percentages of times it would have prevailed if all subjects had played the equilibrium strategy, given the valuations drawn during the experiments. Different treatments are indicated by different symbols, as described in the figure's legend.

The figure can then be read in several ways. The vertical height tells us that the minority won between 22 and 26 percent of the time in *C*, *C2*, and *CChat*, with little dispersion among them; it won less frequently in the *B* sessions (around 15 percent of the time) with the exception of the one experiment of size {5, 4} where the minority won about 23 percent of the time.

Clearly, storable votes helped the minority win. The difference in this effect across treatments matches the theoretical predictions, as is evident from the way the points align along the 45-degree line. The closer to the line a point is, the closer the experiment's results are to the equilibrium predictions. If we estimate a simple regression line, the hypotheses of a unitary slope parameter and a zero constant term cannot be rejected at standard confidence values.[14] On average, the frequency of minority victories in the experiments differs from the equilibrium predictions by 3 percentage points, without clear outliers and without systematic treatment effects. We find this surprising because the complexity of the individual equilibrium strategies in the basic *C* treatment (as opposed to *C2* and *CChat*) would suggest a larger discrepancy from equilibrium predictions in that specific treatment, a discrepancy the data do not show.

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[14]   The estimated parameters are: 0.76 for the slope (standard error 0.23), and 3.4 for the constant term (standard error 5.8).

Share of minority victories
Experiments vs. equilibrium

Share of minority to majority payoff
Experiments vs. equilibrium

(a)

Aggregate surplus share
Experiments vs. equilibrium

Aggregate surplus share
Experiment vs. majority voting

Black: small: C; large: CChat
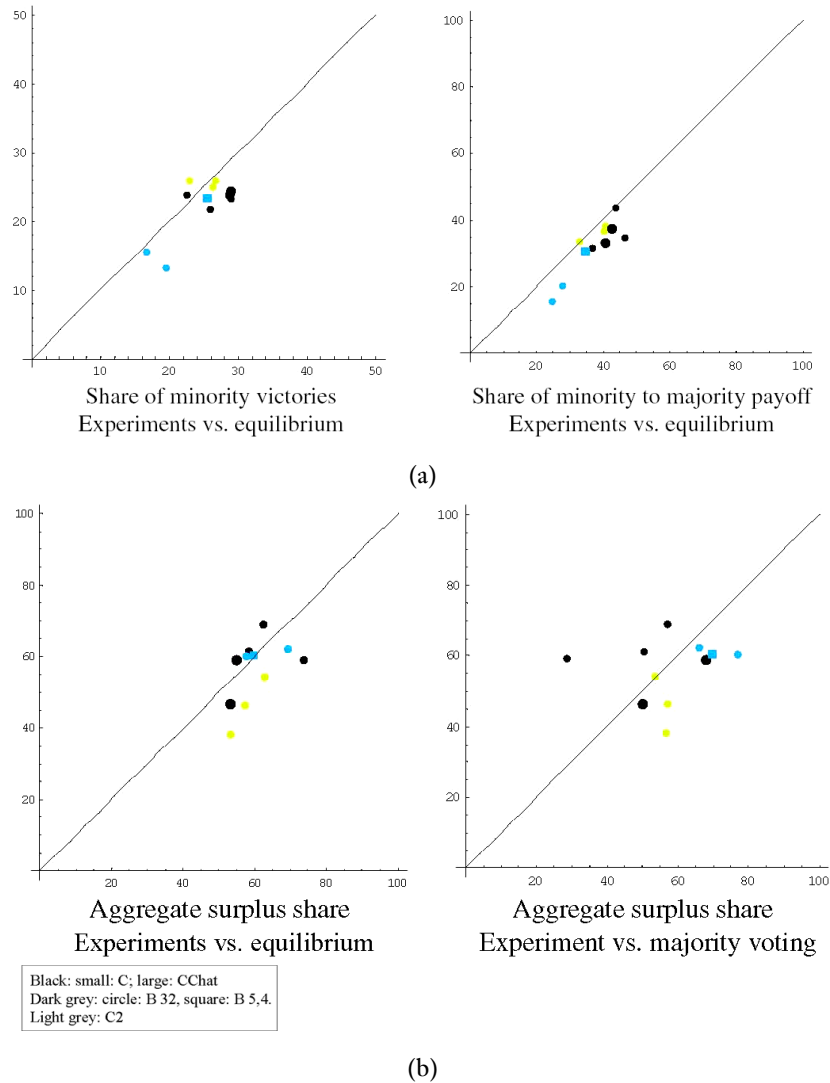Dark grey: circle: B 32, square: B 5,4.
Light grey: C2

(b)

Figure 2. Experimental outcomes. (a) Minorities' outcomes. (b) Aggregate payoff share of surplus over randomness.

Did the experimental payoff to the minority match the theoretical predictions?

The diagram on the right of Figure 2(a) plots per capita minority payoff as a percentage of per capita majority payoff in the experiments on the vertical axis, and in equilibrium Experiment vs. equilibrium on the horizontal axis, using the symbols of the previous figure to identify the different experimental sessions. In all *C*, *C*2, and *CChat* treatments

the relative minority payoff was higher than in any *B* treatments, as predicted by the theory, ranging between 32 and 44 percent of the average majority payoff, versus 16 to 20 percent in the *B* treatments of size {3, 2} and 30 percent in the *B* treatment of size {5, 4}. Again, the effect of the voting mechanism in raising the minority's payoff was significant. Out of 11 experimental sessions, all but two are below the 45-degree line, suggesting that the minority was unable to fully exploit the opportunity presented by storable votes. But the discrepancy is not large — the average distance from the 45-degree line is 5 percentage points, again without clear outliers or treatment effects, which is small in comparison to the differences across treatments.[15] Again, if we estimate a regression line, we cannot reject the hypotheses of unitary slope and zero constant.[16]

At what cost to the majority were the minority's gains? At what cost to overall efficiency?

The left-hand side of Figure 2(b) plots the normalized total surplus in each session on the vertical axis, against the equilibrium predictions on the horizontal axis. The equilibrium predictions are calculated using the actual valuation draws in the experiment. Points on the 45-degree line indicate a perfect match to the theory. The mean distance from the 45-degree line is only 7 percentage points, again with little evidence of outliers, versus a mean equilibrium surplus share of 60 percent. As in the previous figures, we cannot reject a regression line with unitary slope and zero constant, although the fit is poorer.[17]

The central question is how the efficiency of storable votes compares to the efficiency of alternative voting systems — in our case to simple majority voting. In the diagram on the right of Figure 2(b), the vertical axis is again the normalized total surplus in each session, now plotted against the equivalent measure with simple majority voting calculated from the experimental valuation draws. Theory predicts that data from *C*, *C*2, and *CChat* sessions should lie above the 45-degree line, while *B* data should lie below. The prediction is confirmed by the *C* and by the *B* experiments. Surprisingly, it is the *easier* treatments with coordination, *C*2 and *CChat*, that fall short of the prediction. Two of the three most significant losses relative to non-storable votes occur in *C*2 sessions. Pooling all *C*, *C*2, and *CChat* data, the mean difference in normalized surplus is +2 percentage points, compared to the theoretical prediction of +7. Pooling all *B* data, the mean difference is −10 percentage points, compared with the theoretical prediction of −9.

The data from our experiment can be summarized in three main points. First, storable votes help minorities substantially, both in terms of the frequency with which minorities won decisions and in terms of the resulting benefits. Second, correlation of intensities

works to the advantage of the minority. Third, the efficiency costs associated with the increased representation of minority interests were small in magnitude. Without correlation, storable votes induced (small) aggregate welfare losses, but with perfectly correlated intensities, storable votes produced (small) welfare gains over simple majority voting.

Voting Behavior

We begin by studying individual behavior in the treatments that did not allow group members to coordinate their strategies (*B* and *C*). Later we turn to group behavior and discuss the effects of explicit coordination (treatments *C*2 and *CChat*).

Individual behavior

Storable votes allow voters to express intensity of preference by casting more votes, at any given state, when they have stronger preferences. Hence, *monotonicity of voting strategies is at the core of the mechanism*, and it is natural to analyze subject behavior in our experiments by studying this property first.

To obtain a measure of monotonicity of individual behavior, we estimate *monotonicity violations* and *cutpoints* for each subject. For each subject we have K pairs of observations, where K equals either 20 or 30 depending on the session.[18] Each pair consists of a first proposal intensity value and the number of votes cast for (or against) the first proposal. In treatments *B* and *C*, the number of votes cast by each subject is always 1, 2, or 3. A perfectly monotone strategy is one for which we can find two cutpoints, $c1 \leq c2$ such that whenever the subject's first period intensity was below $c1$ the subject cast 1 vote, whenever his intensity was above $c2$, the subject cast 3 votes, and for intermediate values between $c1$ and $c2$ the subject cast 2 votes. We calculate the number of monotonicity violations as the minimum number of voting choices that would have to be changed, for each subject, to make the strategy perfectly monotonic. We then identify the pair of cutpoints that is consistent with such a monotonic strategy. In some cases, multiple cutpoints are consistent with the same number of monotonicity violations; when this happens, we select the pair that is closest to the equilibrium cutpoints.

Figure 3(a) presents histograms of individual monotonicity violations in treatments *B* and *C*. The horizontal axis is divided into deciles representing the percentage of violations over the total number of voting decisions, and the vertical axis reports the fraction of subjects that belong to each decile.

In the *B* treatment, 50 percent of the subjects had 3 or fewer violations out of 30 voting decisions (10 percent). In the *C* treatment, 57 percent of subjects had violation rates less than or equal to 10 percent. As comparison, a voter choosing randomly whether to cast 0, 1, or 2 bonus votes would have a violation rate converging to 67 percent as the number of decisions becomes very large.[19] The comparison makes clear that, although there is some noise, individual choices indeed tended to be monotonic for most subjects.

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[18] With the exception of session *CChat*2, with 15 rounds.
[19] To account for the smaller number of violations that would result from the small sample and the free cutpoints, we simulated random behavior with 21 subjects and 30 rounds. We found that no

(a)

Black: share of errors ≤ 10 %
Dark grey: ≤ 20 %
Light grey: > 20 %
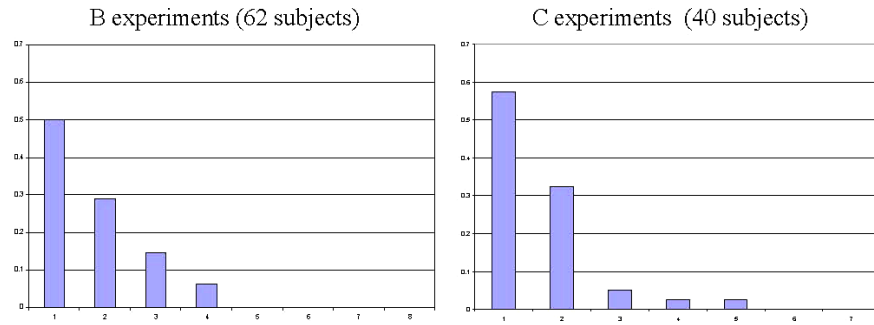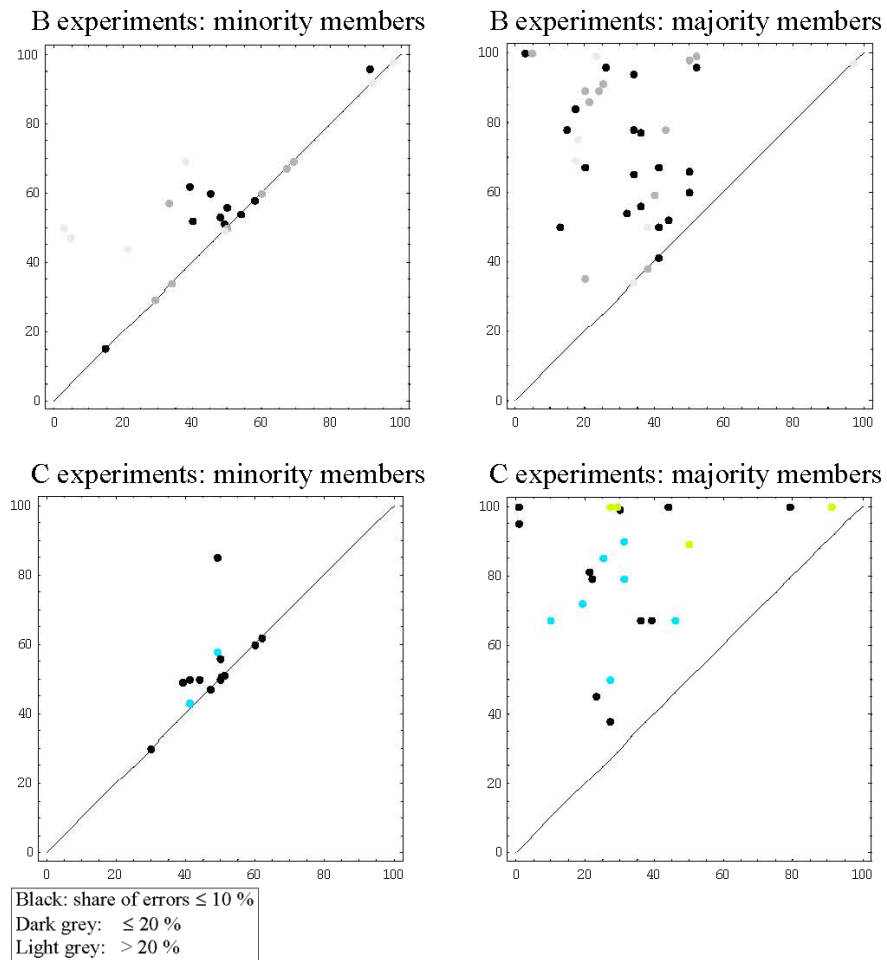
(b)

Figure 3. Individual behavior. (a) Monotonicity violations. (b) Cutpoints.

The estimated cutpoints for all individual subjects in the B and C sessions are displayed in Figure 3(b). Each point represents one subject's estimated pair of cutpoints, with $c1$ on the horizontal axis and $c2$ on the vertical axis. All cutpoints lying on the 45-degree line involve no splitting of bonus votes: always casting either both or neither of the bonus votes over the first decision. Moving to the upper left corner of the graph are cutpoints that involve more and more splitting of bonus votes, i.e., using one bonus vote in each period for a range of values that increases as one approaches the corner. The upper left corner of the graph, at $(0, 100)$ corresponds to always casting one bonus vote. Cutpoints for subjects in the minority group are in the left graph and cutpoints for the subjects in the majority group are in the right graph. The rates of monotonicity violations are indicated by shading the points, with the darkest points having the fewest monotonicity violations.

In the B treatments, the equilibrium cutpoints for both majority and minority subjects are $(50, 50)$: if everyone played the equilibrium strategies all points would be on the 45-degree line at 50. In the C treatments, $(50, 50)$ remains an equilibrium for individual minority subjects, but not for subjects in the majority, whose asymmetrical strategies are contingent on the behavior of the other members of the group and cannot be identified unambiguously in the figure.

Two features of the distribution of cutpoints appear in both treatments. First, the minority cutpoints do cluster around $(50, 50)$, and on average minority subjects whose cutpoints are closer to equilibrium have lower violation rates. Second, bonus votes are much more frequently split by majority voters, with little difference between the two treatments in spite of the different theoretical predictions. Intuitively, even in model B, majority voters have less to lose from splitting their bonus votes — their larger number implies that they are guaranteed to always win one of the two decisions, and one single vote more or less plays a smaller role than in the case of the minority. Consider the parameter values used in the experiments and a committee of size $(3, 2)$. The expected loss to a voter deviating from his equilibrium strategy and always casting one bonus vote over each proposal is 15 percent in model B and 50 percent in model C for a minority voter, versus 4 percent in model B and 8 percent in model C for a majority voter (relative to the expected equilibrium payoff).[20] The difference in the cost of splitting one's bonus votes in the two models may play some role in the more pronounced clustering of the minority cutpoints around the 45-degree line, and particularly around $(50, 50)$ in the C treatment.

Group behavior

The monotonicity of the individual strategies provides only a partial picture. Efficiency requires *group* strategies to be monotonic in the group intensity. In the B treatment the

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subject had violation rates less or equal to 30 percent; 2 subjects were in the fourth decile; 8 in the fifth, and 11 in the sixth.

[20] Supposing that all other voters play the equilibrium strategy. In model C, we consider the case where the individual majority voter's deviation causes the majority group strategy to switch from casting either 5 or 7 votes to always casting 6.

notion of *group intensity* is not clearly defined because different subjects within a group have different intensities. But we can check for *group monotonicity* in the *C* treatment, that is, we can check whether the sum of the votes by members of one group is monotone in their (common) intensity. If there is heterogeneity in behavior, monotonicity at the individual level need not imply monotonicity at the group level because individuals are continuously rematched. The problem is particularly severe for the majority whose individual equilibrium strategies are asymmetric.[21]

The histograms in the first row of Figure 4(a) illustrate the difficulty that groups had in the *C* treatment. Out of a total of 16 groups, 7 had error rates above 20 percent, compared to only 10 percent of individual subjects in the same experimental sessions (see Figure 3(a)). As expected, and as shown by the histogram on the right, most errors are associated with the majority, where 5 of the 8 groups had more than 20 percent error rates.

A comparison of these results to monotonicity violations in the *C2* and *CChat* treatments allows us to study the role of explicit coordination. According to the histograms in the second row of Figure 4, the open communication in *CChat* reduced group violations dramatically: *all* minority groups and 2 out of 5 of the majority groups had fewer than 10 percent violations. More surprising is the poor performance of the *C2* treatment, where perfect coordination is imposed by the experimental design.[22]

These results leave us with a puzzle: if the aggregate group behavior of the experimental subjects in sessions *C* often violates monotonicity, why did the outcomes of these experiments — in terms of minority victories and efficiency — still conform to the theory? Why did these sessions outperform, on average, the *C2* sessions with a comparable record of monotonicity violations. The answer comes from the underlying monotonicity of the *individual* behavior in treatment *C*. Intuitively, because individual subjects did cast their vote monotonically, the violations resulting from the uncoordinated aggregation of the votes are numerous, but not large: they tend to be concentrated around the cutpoints values. To verify this, the histograms in Figure 4(b) summarize the distribution of the average distance of *mistaken* (i.e., non-monotonic) voting choices from the cutpoints, as a percentage of the expected distance if voting choices were random.[23] The *CChat* experiments show the greatest consistency: with one outlier, all groups have error distances below 20 percent of the random case. But it is the comparison between the *C* and the *C2* treatments that is particularly revealing in explaining the differences in experimental

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[21] We identify a group by the label in the experiment (group 1, group 2, etc.), but rematching implies that the composition of each group continues to change. Note that if equilibrium strategies were symmetrical, the changing composition of the group would not matter.

[22] This appears to be the result of a single experimental session: session **c22** conducted at UCLA (where 25 percent of the subjects had a rate of violations approaching 50 percent).

[23] Following this logic, these cutpoints are estimated so as to minimize the average distance (both in the experimental data and in the theoretical random case). With a very large number of random voting choices, the two cutpoints that minimize the expected errors' distance are $(50, 50)$. The frequency of error is $2/3$, with an average distance of 25, yielding an expected distance of $50/3$. The corresponding number in the experimental data is, for a given pair of cutpoints, the sum of all errors' distances, divided by K, the number of rounds in the experiment.

C experiments. (16 groups)

C violations by group

Dark: Majority (8 groups); Light: Minority (8 groups)

CChat experiments. (10 groups)

C2 experiments. (40 subjects/groups)

(a)

C experiments. Errors' distance

CChat experiments. Errors' distance

C2 experiments. Errors' distance

C2 experiments. Errors' distance by session

Dark: Session **c2**2 (16 groups); Light: **c2**1 and **c2**3 (14 groups)
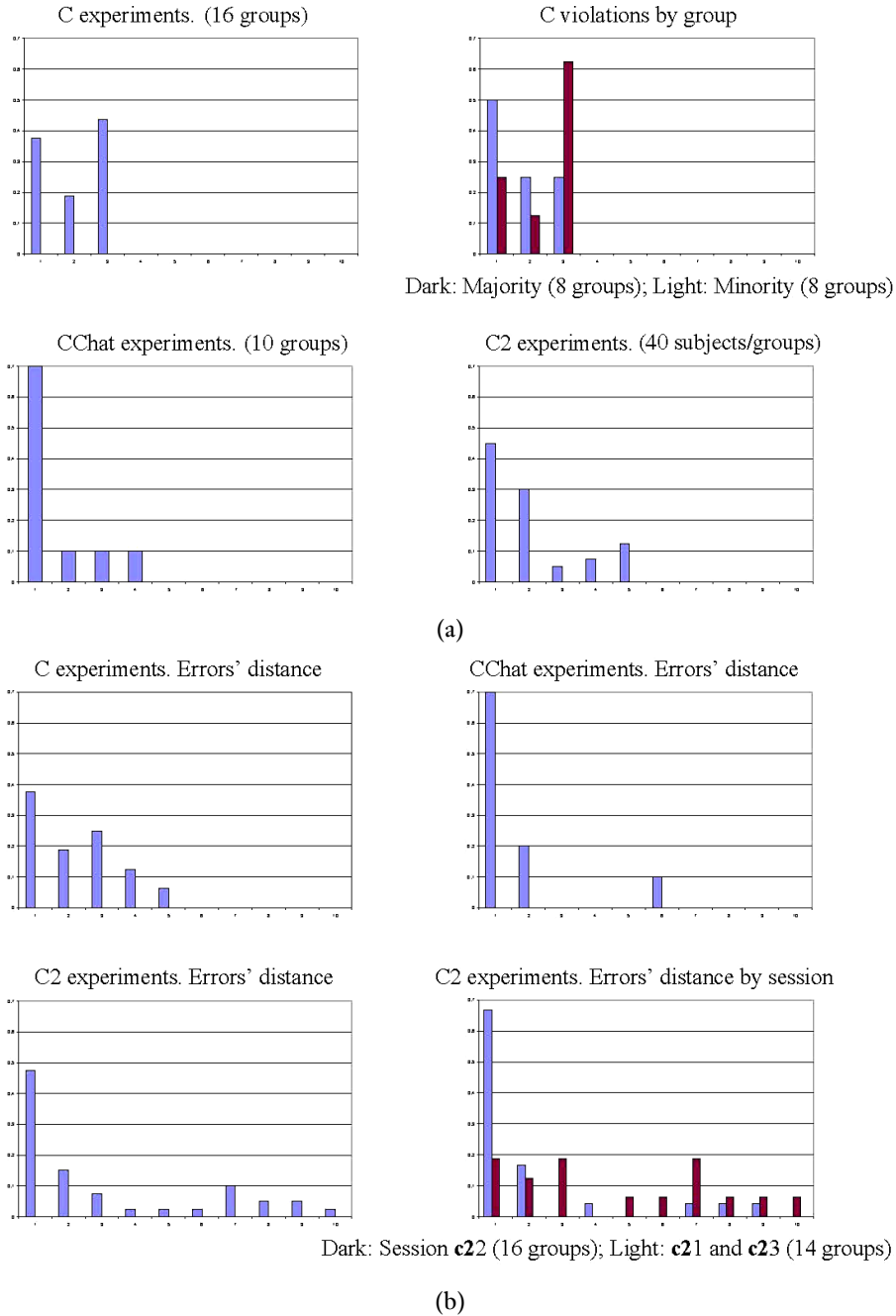
(b)

Figure 4. Group behavior — monotonicity violations. (a) Percentage of violations. (b) Average errors' distance relative to random voting.

outcomes: one-fourth of all *C*2 groups have error distances that are closer to the purely random case than *any* of the *C* groups. As mentioned, this reflects mostly one outlier session, **c22**, and how much of an outlier **c22** becomes clear in the diagram on the right, in the bottom row of Figure 4(b). The **c22** session had 16 subjects, each representing one group; of these, 7 had error distances that were closer to the purely random case than *any* of the *C* groups, and only 3 had distances that were less than 10 percent of the random case, a very different result from the other two *C*2 sessions. This explains why the aggregate experimental payoff of session **c22** falls short both of the theoretical prediction and of the payoff with simple majority. The other *C*2 sessions were much better behaved, although they too presented a few instances of almost random behavior, something we do not observe in the *C* sessions. As shown in Figure 2(b), these few cases were sufficient to exact a cost in terms of efficiency, lowering the overall performance of the *C*2 treatment. Why the treatment proved difficult to our subjects is an open question, although we can speculate that the problem may come from the larger size of the individual strategy space: each minority voter had 5 different choices of how many votes to use in the first period $(2, 3, 4, 5, 6)$, and each majority voter had 7 different choices $(3, 4, 5, 6, 7, 8, 9)$.

As in the analysis of individual behavior, the monotonicity analysis generates cutpoints estimates.[24] Group cutpoints are depicted in Figure 5, with minority cutpoints on the left and majority cutpoints on the right. In line with the equilibrium predictions, we can summarize the strategies of each group through two cutpoints, represented by a point in the diagrams and equal to $(50, 50)$ for both the minority and the majority.[25]

The first row of diagrams in Figure 5 refers to *C* treatments; the second row to *C*2 and the last to *CChat*. As in Figure 3(b), darker points indicate fewer monotonicity violations. Coordination affects the cutpoints of the minority groups: only one of the estimated cutpoints in treatments *C*2 and *CChat* lies outside the 45-degree line, as opposed to what we observe in treatment *C*. Thus in treatments *C*2 and *CChat*, in accordance with equilibrium the behavior of all minority groups is best described as voting either 2 (at lower values) or 6 (at higher values), with some dispersion around the equilibrium cutpoints $(50, 50)$. The majority's behavior, on the other hand, is best described as splitting the bonus votes for some intermediate range of values. In addition, the light shading of most points in the majority figures reflects the relatively large number of monotonicity violations for any estimate of cutpoints. The relatively greater deviation from equilibrium by the majority groups may reflect their relative low cost of such deviations. With a single coordinated strategy, the expected percentage loss to the majority from always splitting the bonus votes is about 8 percent when the minority plays the equilibrium

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[24] The cutpoints estimates that minimize the number of monotonicity violations need not be identical to those that minimize the errors' distance. In practice, they differ mostly in the case of those subjects with more random behavior. The substance of the results does not change, and we report here the cutpoints that minimize the number of violations, for consistency with the discussion of individual behavior.

[25] For the majority groups, we treat as identical all payoff-equivalent strategies, i.e., voting either 3, or 4, or 5 below g_l, and voting either 7, or 8, or 9 above g_h.
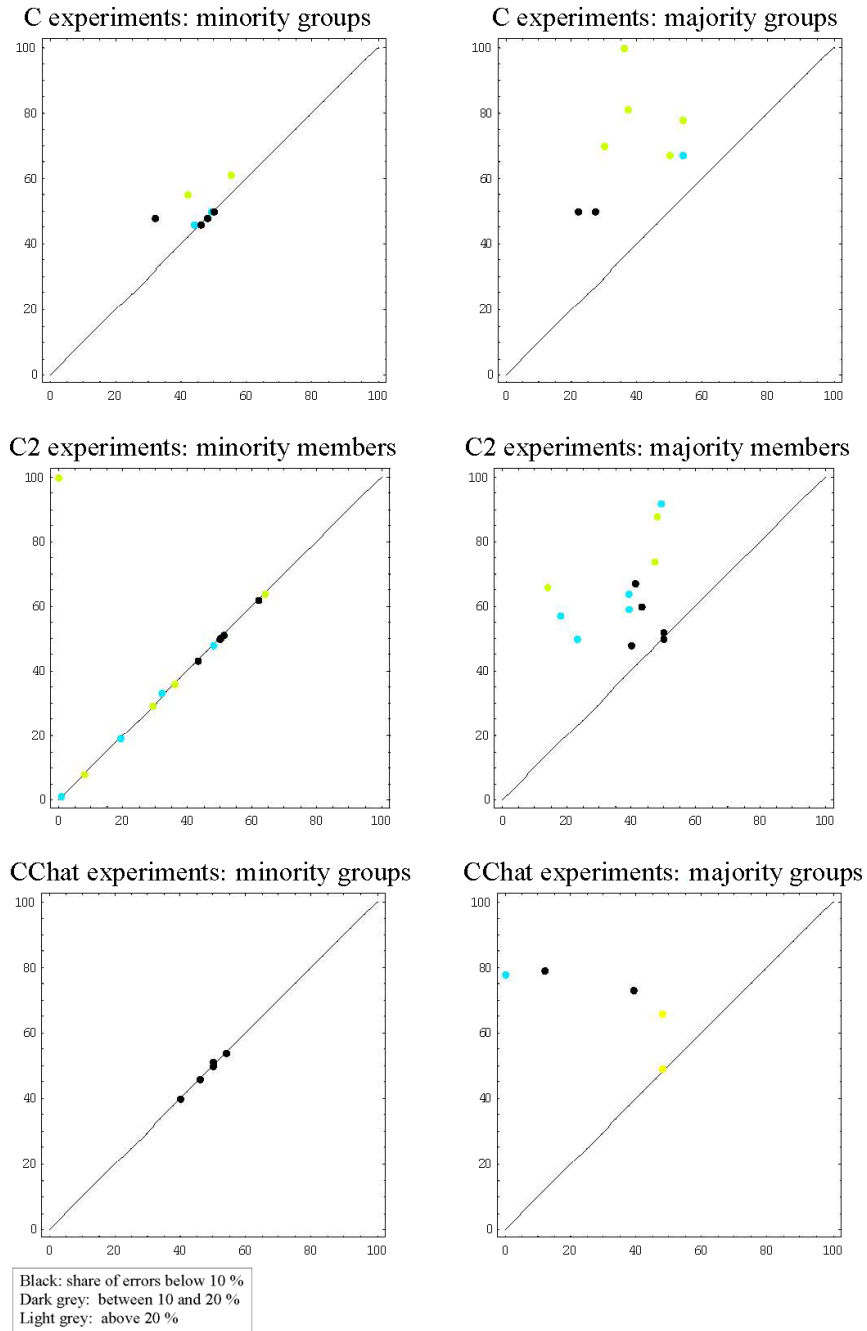
Figure 5. Group behavior — group cutpoints.

strategy.[26] For the minority, on the other hand, splitting the bonus votes can be very costly: a minority always casting 4 votes *always* loses against a majority casting 5 votes at valuations below 50 and 7 at valuations above 50.

CONCLUSIONS

Majoritarian principles are a fundamental ingredient of democratic institutions. But they carry with them the risk of disenfranchising minority groups and endangering the stability of the system, by violating principles of both equity and efficiency. In a well-designed democracy, a judicial system protecting the rights of minority groups needs to be supplemented by political remedies that ensure the minority a voice through the daily, ordered exercise of political rights. This paper has analyzed the potential of a simple voting system — storable votes — to fulfill this function. By granting voters a stock of votes to be divided as desired over a series of multiple binary decisions, storable votes allow the minority to cumulate votes on specific issues and to win sometime. Because the minority wins only if its strength of preferences is high, and the majority's is low, the gains in terms of equity have little, if any, cost in terms of efficiency.

We have studied two related models where two groups of different size have consistently opposite preferences. In our *correlated* model, *C*, all members of a group — whether the majority or the minority — agree not only on the direction of their preferences but also on the strength of their preferences. If we think in terms of political parties, these would be parties with strong discipline; more generally, the model is best suited to represent groups with some level of organization, sufficient to agree on the set of priorities. In our *basic* model, model *B*, on the other hand, all members of a group agree on the direction of their preferences, and the two groups have opposite preferences, but within a group the members' priorities may differ. The groups are not organized.

There are many directions for further research. We limit ourselves to mentioning two. First, it would be interesting to compare storable votes to a larger set of alternative mechanisms, both theoretically and experimentally. These alternative mechanisms should include vetoes, serial dictatorship, and even more complex systems such as the one proposed in Jackson and Sonnenschein (2007). Storable votes are more flexible but more complicated than vetoes, and less flexible and less complicated than the Jackson and Sonnenschein mechanism. Serial dictatorship requires a secondary mechanism to allocate decisions to specific individuals or groups in a somewhat efficient fashion. What can the theory tell us, and how would all compare experimentally?[27] Second, the sensitivity of storable votes to agenda manipulation is an open question. The agenda setting procedure should be part of the overall game, and voters will decide how many votes

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[26] In fact, in this model the majority's maximin strategy entails splitting the bonus votes. It corresponds to cutpoints (25, 100): cast no bonus votes for values below 25, but split the bonus votes for all values above 25.

[27] Two recent experimental analyses are Engelmann and Grimm (2006) on the Jackson–Sonnenschein mechanism, and Kagel *et al.* (2005) on veto power. Neither paper compares different mechanisms.

to cast knowing how new issues are brought to a vote. *A priori* it is not clear whether problems will arise: having multiple votes that can be shifted across proposals may make the order of the proposals more important, but also increase the ability to resist possible manipulations of this order. On the other hand, the additional consideration of political minorities may exacerbate possible problems, either because majority losses are particularly expensive in terms of efficiency or because the minority may end up unable to ever control any outcome.

APPENDIX

Proof of Lemma 1 Suppose that $x^*_{Mt}(v_i, B_t, t)$ and $x^*_{mt}(v_i, B_t, t)$ exist. Consider candidate equilibrium strategies $\{x'_{it}(v_i, B_t, t)\}$ for model C, where $\sum_{i \in m} x'_{it}(v_i, B, t) = x^*_{mt}(v_i, B, t)$ and $\sum_{i \in M} x'_{it}(v_i, B, t) = x^*_{Mt}(v_i, B, t)$. Because preferences between the two groups are always opposed, at any state only the aggregate voting choice of the opposite group affects voters' payoffs. In addition, because in model C preferences within each group are always perfectly correlated, by definition $\{x'_{it}(v_i, B, t)\}, i \in m$ maximize the expected payoff of each individual minority member, given $x^*_{Mt}(v_i, B, t)$ (and similarly for $\{x'_{it}(v_i, B, t)\}, i \in M$, given $x^*_{mt}(v_i, B, t)$). It follows that no individual deviation from the prescribed strategies can be profitable and $\{x'_{it}(v_i, B_t, t)\}$ must be equilibrium strategies. Note that in general the equilibrium will not be unique: any permutation of individual strategies that leaves the aggregate vote for the group unchanged, at given state, is an equilibrium. ∎

Proof of Lemma 2 (i) *Existence of equilibrium in pure strategies.* Milgrom and Weber (1985) discuss conditions for existence of an equilibrium in distributional strategies. In particular, conditional on a publicly observed variable, individual types are required to be independent. The publicly observed information in our case is each voter's membership in one of the two groups, and hence the support of the distribution from which valuations are drawn. Conditional on such support, individual valuations are independent in case B. The arguments in Casella (2005) remain applicable here. Hence an equilibrium in pure strategies exists for model B. Conditional on public information on the support of each distribution, valuations are independent in the two-voter version of model C. Again, the arguments in Casella (2005) apply, and an equilibrium in pure strategies exists. But since such an equilibrium must be an equilibrium of the n-voter C game, it follows that an equilibrium in pure strategies of the n-voter C game exists. (ii) *Monotonicity of the equilibrium strategies.* Call a strategy *monotonic* if, at a given state, the number of votes cast is monotonically increasing in the intensity of preferences v_{it}. Casella *et al.* (2006) shows that at any given state all individual best response strategies must be monotonic when members of each group do not play correlated strategies. Thus the argument applies immediately to equilibria of model B. It also applies to the two-voter version of model C, and hence to group strategies, as opposed to individual strategies, in the equilibrium we focus on in the n-voter C game. If, at any given state, all best response strategies must be monotonic and an equilibrium exists, it follows that equilibrium strategies must be

monotonic. Because there is a continuum of types and a finite set of strategies, then it must be that monotonic equilibrium strategies must take the form of monotone cutpoint strategies.   ∎

Proof of Theorem 1 We begin by proving the second part of the theorem. Consider any candidate equilibrium where the minority is expected to lose with probability 1 over each decision. A minority member cannot be worse off by cumulating all his bonus votes on one decision. Over all decisions, there must be at least one where with positive probability the majority casts no more than MB_0/T bonus votes, and since the minority can never cast fewer than m total votes, a deviating minority member can always find a decision where with positive probability the difference in votes cast is at most $M(1 + B_0/T) - m$. Thus with positive probability the outcome of that decision changes and deviation is profitable if $M(1 + B_0/T) \leq m + B_0$, or $B_0(1 - M/T) \geq M - m$. This condition requires $T > M$, and in this case becomes $B_0 \geq T(M - m)/(T - M)$. For all M and m, the condition is sufficient and applies to both models B and C. Since we know by Lemma 2 that an equilibrium exists for arbitrary F, T, M, and m, it must be that if $T > M$, and $B_0 \geq T(M - m)/(T - M)$ the minority is expected to win sometime with strictly positive probability in all equilibria. We now prove the first part of the theorem. Suppose $T \leq M$. Consider the following candidate equilibrium: at time $t = 1$, $x_{i1} = 1 + B_0$ for $i \in \mathbf{m}$ and $x_{j1} = 1$ for $j \in \mathbf{M}$; at all other times $t \neq 1$, $x_{it} = 1$, and $x_{jt} = 1 + B_0/(T - 1)$. If $m(1 + B_0) > M$ or $B_0 > (M - m)/m$ the minority always wins the first vote, while the majority always wins all other votes. No individual minority member can gain from deviation, for all possible realizations of his valuations, if $m + B_0 < M[1 + B_0/(T - 1)]$, or $B_0[1 - M/(T - 1)] < M - m$, a condition always satisfied when $T \leq M$. No majority member can gain from deviating, again for all possible realizations of his valuations, if $m(1 + B_0) > M + B_0$ or $B_0 > (M - m)/(m - 1)$, a threshold that is finite for all $m > 1$. Thus if $T \leq M$, $m > 1$, and $B_0 > (M - m)/(m - 1)$, the strategies described are an equilibrium, and the minority always wins the first vote.   ∎

Proof of Theorem 2 Consider the following strategies. Over the first $T - 2$ proposals, each minority member always casts only the regular vote; each majority member casts $1 + b$ votes. At $T - 1$, each minority member casts only his regular vote if $v_m < \alpha$, for a fixed $\alpha > 0$, and all bonus votes otherwise; each majority members casts $1 + b$ votes if $v_M < \alpha$ and $1 + h$ otherwise, where $b + h + (T - 2)b = B_0$. In the last election, all remaining votes are cast.

We show in step (i) that for all M and $m \geq 2$ there exist non-negative values of B_0, b, h, and T for which such strategies are equilibrium strategies, and the minority wins at $T - 1$ if $(v_{mT-1} > \alpha, v_{MT-1} < \alpha)$, and at T if $(v_{mT-1} < \alpha, v_{MT-1} > \alpha)$, but always loses otherwise. We then show in (ii) that in such an equilibrium $EV_0 > EW_0$ if $m > M/2$. Because the theorem requires $m > M/2$, it cannot apply to $m = 1$.

(i) The minority wins at $T - 1$ if $(v_{mT-1} > \alpha, v_{MT-1} < \alpha)$, and at T if $(v_{mT-1} < \alpha, v_{MT-1} > \alpha)$ if:

$$m(1 + B_0) > M(1 + b) \tag{A.1}$$

and loses in all other cases if:

$$M(1 + h) > m(1 + B_0). \tag{A.2}$$

Any unilateral deviation by a minority voter is ruled out if:

$$m + B_0 < M(1 + b). \tag{A.3}$$

Similarly, any unilateral deviation by a majority voter is ruled out if:

$$M + (M - 1)b + B_0 < m(1 + B_0). \tag{A.4}$$

If there exist values of B_0, b, h, and T for which these four inequalities are satisfied simultaneously, and the budget constraint $b + h + (T - 2)b = B_0$ holds, then the strategies are an equilibrium, delivering the outcomes described above. It is immediate that (A.4) implies (A.1). Hence, substituting the budget constraint in (A.2), three conditions must be satisfied:

$$T < 1 + \frac{(B_0 + 1)(M - m)}{Mb} \tag{A.2$'$}$$

$$B_0 < (M - m) + Mb \tag{A.3$'$}$$

$$B_0 > \frac{M - m}{m - 1} + \frac{M - m}{m - 1}b. \tag{A.4$'$}$$

If we ignore integer constraints, then for all $m \geq 2$ (A.3$'$) and (A.4$'$) are satisfied for any positive b. With b arbitrarily small, T can be arbitrarily large, and the equilibrium can be supported for any positive finite T. Integer constraints are however part of the environment, and in general impose an upper bound on T, $\overline{T}$, which depends on M and m. The following observations follow immediately from (A.2$'$), (A.3$'$) an (A.4$'$): (a) if $m > 2$, then for all $M > m$, there is an equilibrium with $b = 1$, B_0 integer $\in (2(M - m)/(m - 1), 2M - m)$, and $\overline{T}(M, m) > 2$; (b) if $m = 2$, then the only relevant case satisfying the constraint $M < 2m$ is $M = 3$. For $m = 2$ and $M = 3$, there is an equilibrium with $b = 2$, $B_0 = 6$, and $\overline{T} = 13/6 > 2$.

(ii) In any equilibrium of this type, $EV_0 > EW_0$ iff:

$$F(\alpha) \left[M \int_0^\alpha v \, dF(v) + F(\alpha)M \int_0^1 v \, dF(v) \right]$$
$$+ [1 - F(\alpha)] \left[M \int_\alpha^1 v \, dF(v) + [1 - F(\alpha)]M \int_0^1 v \, dF(v) \right]$$
$$+ F(\alpha) \left[M \int_\alpha^1 v \, dF(v) + [1 - F(\alpha)]m \int_0^1 v \, dF(v) \right]$$
$$+ F(\alpha) \left[m \int_\alpha^1 v \, dF(v) + [1 - F(\alpha)]M \int_0^1 v \, dF(v) \right] > 2M \int_0^1 v \, dF(v)$$

Simplifying:

$$F(\alpha)\,[MF(\alpha) + m(1 - F(\alpha))] \int_0^1 v\,dF(v)$$

$$+ [mF(\alpha) + M(1 - F(\alpha))] \int_\alpha^1 v\,dF(v) > M \int_0^1 v\,dF(v). \qquad \text{(A.5)}$$

Note that the left-hand side simplifies to $M \int_0^1 v\,dF(v)$ when evaluated at either $\alpha = 0$ or $\alpha = 1$, since in both cases the majority always wins (and thus $EV_0 = EW_0$). Taking the derivative of (A.5) with respect to α and evaluating it at $\alpha = 0$, we obtain:

$$\left.\frac{\partial(EV_0 - EW_0)}{\partial \alpha}\right|_{\alpha=0} = f(0) \int_0^1 v\,dF(v)(2m - M) > 0 \Leftrightarrow m > M/2$$

Thus if $m > M/2$ there exists a threshold $\alpha > 0$ such that the strategies described above lead to higher *ex ante* welfare than simple majority voting.   ∎

SUPPLEMENTARY MATERIAL
MINORITIES AND STORABLE VOTES

PROOF OF THE PROPOSITION

Model B

(a) *Equilibrium.*   To verify that the strategy described is an equilibrium, consider the best response for voter i. If i casts x_{i1} votes in the vote over the first proposal, his expected utility over the whole game is: $EU_i|x_{i1} = v_{i1}\mathrm{prob}(W_1|x_{i1}) + E(v)\mathrm{prob}(W_2|4 - x_{i1})$, where $\mathrm{prob}(W_t|x_{it})$ is i's probability of obtaining the desired outcome in period t conditional on casting x_{it} votes, and from the symmetry of F, $E(v) = 0.5$. Since $(n - 1)$ is an even number, and every other voter is casting either 1 or 3 votes, the difference in votes between the two sides, excluding i, must be even for both proposals. Thus, when i considers the choice between casting 3, 2 or 1 votes, the only case in which the choice matters is a difference of 2 votes in his side disfavor, either over proposal 1 or proposal 2:

$$EU_i|3 > EU_i|2 \Leftrightarrow v_{i1}[\mathrm{prob}(\Delta x_{1-i} = 2)] > 0.5[\mathrm{prob}(\Delta x_{2-i} = 2)]$$

$$EU_i|2 > EU_i|1 \Leftrightarrow v_{i1}[\mathrm{prob}(\Delta x_{1-i} = 2)] > 0.5[\mathrm{prob}(\Delta x_{2-i} = 2)],$$

where Δx_{t-i} indicates the number of votes by which i's side is losing over proposal P_t, absent i's vote. Given the symmetry of F, in the candidate equilibrium the probability of any other voter casting 1 or 3 votes is identical, implying: $\mathrm{prob}(\Delta x_{1-i} = 2) = \mathrm{prob}(\Delta x_{2-i} = 2)$. Thus i's best response is to cast 1 vote if $v_{i1} < 0.5$ and 3 votes if

$v_{i1} > 0.5$; the conclusion holds for all i, and the strategy is indeed an equilibrium. If $M > 3m$, $\text{prob}(\Delta x_{1-i} = 2) = \text{prob}(\Delta x_{2-i} = 2) = 0$, and the number of votes cast is irrelevant.

(b) *Frequency of minority victories.* Write the majority size as $M = m + 2k - 1$, with $k \geq 1$ (recall than n is odd). The minority wins the first vote if there are at least k more valuations above 0.5 among the minority than the majority. Given the symmetry of F, the probability of this event is given by the formula in the lemma. The minority wins the second vote if there are at least k more valuations below 0.5 over the first proposal among the minority than the majority, an event that again, given the symmetry of F, has the probability given in the lemma. Note that the two events are mutually exclusive and that the probability can be positive only if $k < m$, implying that the majority always wins if $M > 3m$.

(c) *Expected equilibrium payoff.* With n odd and the equilibrium strategies described above, the difference in votes cast by the two groups is always an even number. In addition, the symmetry of F guarantees that the probability of any given difference in votes is equal over the two proposals. If we call $\text{prob}(W_M|x)$ the probability of obtaining the desired outcome for $i \in M$, conditional on casting x votes, then, given F Uniform, we can write the *ex ante* expected payoff of a majority member as:

$$EV_{Bi} = (3/8)\text{prob}(W_M|1) + (5/8)\text{prob}(W_M|3), \quad \forall i \in M$$

where $\text{prob}(W_M|1) = \text{prob}(x_{M-i} \geq x_m)$ and $\text{prob}(W_M|3) = \text{prob}(x_{M-i} \geq x_m - 2)$. Recall that $M = m + 2k - 1$. Given the equilibrium strategies, the symmetry of F, and the independence of the valuation draws, if we call *high* a valuation above 0.5, $\text{prob}(x_{M-i} \geq x_m)$ equals the probability that the number of high draws in the minority group is at most $k-1$ higher than for the majority group, excluding voter i:

$$\text{prob}(W_M|1) = 1 - \sum_{s=k}^{m} \left[\sum_{r=0}^{m-s} \binom{M-1}{r} \binom{m}{r+s} \right] 2^{-(M-1+m)}.$$

Similarly, $\text{prob}(x_{M-i} \geq x_m - 2)$ equals the probability that the number of high draws in the minority group is at most k higher than for the majority group, excluding voter i:

$$\text{prob}(W_M|3) = 1 - \sum_{s=k+1}^{m} \left[\sum_{r=0}^{m-s} \binom{M-1}{r} \binom{m}{r+s} \right] 2^{-(M-1+m)}.$$

Analogous calculations yield the *ex ante* expected payoff of a minority member:

$$EV_{Bj} = (3/8)\text{prob}(W_m|1) + (5/8)\text{prob}(W_m|3), \quad \forall j \in m$$

where:

$$\text{prob}(W_m|1) = \sum_{s=k}^{m-1} \left[\sum_{r=0}^{m-s-1} \binom{M}{r} \binom{m-1}{r+s} \right] 2^{-(M+m-1)}$$

and

$$\text{prob}(W_m|3) = \sum_{s=k-1}^{m-1} \left[\sum_{r=0}^{m-s-1} \binom{M}{r}\binom{m-1}{r+s} \right] 2^{-(M+m-1)}.$$

Ex ante aggregate expected payoff in equilibrium is then: $EV_B = M(EV_{Bi}) + m(EV_{Bj})$, $i \in M, j \in m$. The expressions can be simplified slightly, and after some manipulations we derive:

$$EV_B > EW_0 = M \Leftrightarrow \frac{5}{8} \frac{(M+m)}{\left(\frac{3m-M-1}{2}\right)!\left(\frac{3M-m-1}{2}\right)!}$$

$$> \sum_{s=k}^{m} \frac{M}{(m-s)!(M-1+s)!} - \sum_{s=k}^{m-1} \frac{m}{(m-s-1)!(M+s)!},$$

where $k = (M - m + 1)/2$. It is then simple to verify that for all $M = 3m - 1$ (i.e., $k = m$) or $M = 3m - 3$ (i.e., $k = m - 1$), $EV_B < EW_0$. At the same time, for M large enough it is not difficult to find values of $m = M - 1$ ($k = 1$) such that $EV_B > EW_0$, and generate examples that satisfy the statement in the lemma. $M' = 8$, $m' = 3$, and $m'' = 7$ is one such example; $M' = 6$, $m' = 3$, and $m'' = 5$ is another.

Model C

(a) *Equilibrium.* If $2M > 3m$, by setting $x_{M1} = 2M$ for all v_{M1} the majority can guarantee itself victory over both proposals. All minority strategies are equivalent, including $x_{m1} = m$ if $v_{m1} < 0.5$ and $x_{m1} = 3m$ if $v_{m1} > 0.5$. No deviation can be profitable for a member of either group, and the strategies are an equilibrium. Suppose then $2M \leq 3m$. When $x_m = m$, the minority always loses ($m < \max\{M, m + 3\} < \min\{3M, 4M - (m + 3)\}$). The only possible deviation for a minority member is to cast 2 or 3 votes when $x_{m-i} = m - 1$, but $m + 2 < \max\{M, m + 3\} < \min\{3M, 4M - (m+3)\}$: the deviation cannot be profitable. The majority always wins when casting $\min\{3M, 4M - (m + 3)\}$ votes, but loses when $x_M = \max\{M, m + 3\}$ if $x_m = 3m$. A majority member could deviate and use his bonus votes when $x_{M-i} = \max\{M - 1, m + 2\}$. But casting 2 votes cannot be profitable: with $2M \leq 3m$, $\max\{M+1, m+4\} < 3m$. And neither can casting 3: with $2M \leq 3m$, either $\max\{M + 2, m + 5\} < 3m$ and $\min\{3M - 2, 4M - (m + 5)\} > 3m$, in which case the outcomes are unchanged; or $\max\{M+2, m+5\} > 3m$ and $\min\{3M-2, 4M-(m+5)\} < 3m$, in which case the certainty of winning at $v_M > 0.5$ is traded for the certainty of winning in the future, with $E(v) = 0.5$ — a net loss in expected utility. Hence $x_{m1} = m$ if $v_{m1} < 0.5$ and $x_{m1} = 3m$ if $v_{m1} > 0.5$; and $x_{M1} = \max\{M, m + 3\}$ if $v_{M1} < 0.5$ and $x_{M1} = \min\{3M, 4M - (m + 3)\}$ if $v_{M1} > 0.5$ are equilibrium strategies.

(b) *Frequency of minority victories.* If $2M \leq 3m$ the minority wins the first vote if $(v_{m1} > 0.5 \cap v_{M1} < 0.5)$ and the second if $(v_{m1} < 0.5 \cap v_{M1} > 0.5)$ — given the symmetry of F, it wins each vote with probability 0.25.

(c) *Expected equilibrium payoff.* If $2M > 3m$, the majority always wins and the expected aggregate payoff over the two proposals equals M. If $2M \leq 3m$, the expected aggregate payoff equals: $(1/4)(M/4 + M/2) + (1/4)(3M/4 + M/2) + (1/4)(3M/4 + m/2) + (1/4)(3m/4 + M/2) = (13M + 5m)/16$ (where the first term is the expected payoff over the two proposals when $(v_{m1} < 0.5 \cap v_{M1} < 0.5)$, the second when $(v_{m1} > 0.5 \cap v_{M1} > 0.5)$, the third when $(v_{M1} > 0.5 \cap v_{m1} < 0.5)$, and the fourth when $(v_{m1} > 0.5 \cap v_{M1} < 0.5)$ — all events with probability 1/4). With simple majority voting, the majority always wins and over the two proposals $EW_0 = M$ for all M, m. In this storable votes equilibrium, $EV_0 = M$ if $2M > 3m$, but $EV_0 = (13M + 5m)/16 > M$ for all $2M \leq 3m$, establishing the result in the lemma.

CONSTRUCTION OF FIGURE 1

Model B

(a) *Efficient frequency of minority victories.* According to our efficiency criterion, the minority should win whenever the sum of its valuations is larger than the sum of the majority's valuations. Call y (z) the sum of m (M) independent random variables, each distributed Uniformly over $[0, 1]$. The efficient frequency of minority victories is then given by $\int_0^m \left(\int_z^m P_m(y)dy \right) P_M(z)dz$ where:

$$P_m(y) = \frac{1}{2(m-1)!} \sum_{s=0}^{m} (-1)^s \binom{m}{s} (y-s)^{m-1} \text{sign}(y-s) \tag{A.6}$$

(and correspondingly for $P_M(z)$).

(b) *Expected aggregate payoff under first best efficiency.* For each proposal, the *ex ante* efficient aggregate payoff EU_B^* is easily derived, given (A.6):

$$EU_B^* = \int_0^m \left(\int_z^m y P_m(y)dy \right) P_M(z)dz + \int_0^m \left(\int_y^M z P_M(z)dz \right) P_m(y)dy. \tag{A.7}$$

Over the two proposals, the *ex ante* efficient payoff is $2EU_B^*$. The first term in (A.7) corresponds to the efficient expected payoff for the minority group, and the second for the majority group. The corresponding per capita values (multiplied by 2) are plotted in Figure 1(b).

(c) *Random choice.* If each group has a 50 percent chance of winning any vote, given $E(v) = 1/2$, the aggregate expected payoff is $1/2(M/2) + 1/2(m/2)$ over each proposal, or $(M + m)/2$ for the 2-proposal game.

Model C

(a) *Efficient frequency of minority victories.* Given the perfect correlation of valuations within each group, the efficient frequency of minority victories is given by $prob(Mv_M < mv_m) = \int_0^1 \int_0^{(m/M)v_m} dv_M dv_m = m/(2M)$.

(b) *Expected aggregate payoff under first best efficiency.*   In model C, we can represent the total valuation of the minority (majority) group by a random variable y (z), Uniformly distributed over $[0, m]$ ($[0, M]$). The efficient aggregate expected payoff, per proposal, is given by

$$EU_C^* = \int_0^m \left(\int_z^m \frac{y}{m} dy \right) \frac{1}{M} dx + \int_0^m \left(\int_y^M \frac{z}{M} dz \right) \frac{1}{m} dy = \frac{m^2 + 3M^2}{6M}. \quad \text{(A.8)}$$

Over the two proposals, the *ex ante* efficient payoff is $2EU_C^*$. The first term in (A.8) corresponds to the efficient expected payoff for the minority group ($m^2/(3M)$), and the second for the majority group (($3M^2 - m^2)/6M$). The corresponding per capita values (multiplied by 2) are plotted in Figure 1(b).

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