Transient-Eddy Feedbacks Derived from Linear Theory and Observations

ANDREW W. ROBERTSON AND WERNER METZ

Meteorologisches Institut, Universität München, München, FRG

(Manuscript received 18 January 1990, in final form 3 May 1990)

ABSTRACT

Linear baroclinic instability theory is used to investigate the subweekly time scale transient eddies (TEs) and their feedbacks associated with three-dimensional basic flows on the Northern Hemisphere, in terms of a two-layer quasi-geostrophic model. We consider an eight-winter time-mean flow as well as four composites of North Atlantic large-scale quasi-stationary patterns.

The structures of the two fastest-growing normal modes associated with the eight-winter climatology are found to compare very well in many aspects with the leading complex empirical orthogonal functions (CEOFs) of the observed bandpass filtered flow, with pattern correlations up to 0.65; although the normal modes are less localized than the CEOFs. The barotropic feedback implied by the linear modes is also found to compare quite reasonably with the observations, especially over the west Atlantic, but the baroclinic (negative) feedback is less well represented.

Composites corresponding to blocking (BL), zonal (ZO), Greenland anticyclone (GA), and Atlantic ridge (AR) weather regimes are next used to define basic states and composite maps of TE feedback. In all four cases the principal displacements of TE activity over the North Atlantic are captured by the fastest-growing Atlantic cyclogenesis modes. The structure of the barotropic feedback associated with the quasi-stationary anomalies is also reasonably simulated by the linear modes in many respects in the BL and ZO cases, but in the GA and AR cases the linear model is less successful.

1. Introduction

The idea that extratropical atmospheric variability on intraseasonal time scales is characterized by several "weather regimes" separated by periods of transition, owes its recent popularity to the works of Charney and DeVore (1979) and Rheinhold and Pierrehumbert (1982). The latter advanced the concept of the "weather regime," characterized by a quasi-stationary large-scale flow field in equilibrium with the feedbacks of its attendant synoptic scale baroclinic eddies. Building on the work of Charney and DeVore, they suggested that stochastic transient eddy surges could be responsible for transitions to and from quasi-stationary regimes, and demonstrated the feasibility of their hypotheses using a very low-order model with two waves. In the meantime it has become clear that transports of momentum associated with the subweekly time scale transient eddies (TEs hereafter) play an important role in maintaining blocking patterns (Illari and Marshall 1983; Shutts 1986; Metz 1986; Mullen 1987), whilst Hoskins et al. (1983) and Lau and Holopainen (1984) have shown the importance of TE momentum and heat transports for the time-mean winter climatological flow.

Vautard and Legras (1988) have lent considerable weight to the concept of weather regimes. Taking TE feedbacks explicitly into account, they have identified equilibrated zonal- and blocking-type weather regimes in a long quasi-geostrophic model integration with moderate resolution and no topographic forcing. Recently, Vautard (1990) has extended this study in a simplified form to real atmospheric data over the North Atlantic area. Searching a lowpass filtered 37-winter dataset for quasi-stationary large-scale patterns, he was able to identify four distinct Atlantic patterns, including blocking.

The concept of a persistent quasi-stationary large-scale flow regime, in which the synoptic scales are in statistical equilibrium with the large-scale flow, suggests that the average synoptic scale activity and its feedbacks onto the large scale, might be accounted for reasonably by linear baroclinic instability theory. A constant basic state about which to linearize is naturally defined. This idea was also tested by Rheinhold and Pierrehumbert (1982) and found to be quite successful for their simple model case. Indeed they were able to use a linear instability analysis at each time step in their model, to parameterize the synoptic scale wave in terms of the large-scale wave. Vautard and Legras (1988) have also examined the fastest-growing linear unstable mode associated with their (more sophisticated) model's zonal- and blocking-regimes mentioned above. They found that in the zonal case, linear theory gave a good account

Corresponding author address: Dr. Andrew Robertson, Meteorologisches Institut, Universität München, Theresienstrasse 37, D-8000 München 2, F.R.G..

of the TE feedback, whereas in the blocking case, the linear result was less satisfactory.

The ability of linear theory to account for observed TE fluxes of heat and momentum in the climatological case has been investigated by Frederiksen (1982). His results demonstrate that the fastest-growing modes of a two-layer quasi-geostrophic model with sufficient spatial resolution, linearized about a winter climatology are able to account quite well for the observed positions of the maxima in heat and momentum fluxes associated with the "storm tracks." However, it was found that the ratio of the $\overline{u'v'}$ -eddy momentum flux to the poleward eddy heat flux was generally smaller than observed, and that poleward fluxes of momentum were too weak compared with equatorial fluxes.

In a previous article (Robertson and Metz 1989, RM in the following), the authors demonstrated the ability of a similar linear model to reproduce quite reasonably the displacement of the stormtrack associated with large-scale persistent anomalies, similar to those of Dole (1986). Linearizing about three-dimensional basic states consisting of a climatological winter mean to which large-scale persistent anomalies were added, a marked steering of the modes by the large-scale flow anomalies over the Atlantic was found to be generally in qualitative agreement with observed data; although the model consistently underestimated the amplitude of the Atlantic storm track. It was suggested that the weakness of the cyclogenesis modes in the Atlantic region, was due to the quasi- β -plane approximations of the so-called "B-model" employed in the study; a problem not encountered in the fully spherical "P-model" used by Frederiksen (1982).

The aim of the present study is to examine the linear baroclinic instability of observed time-mean flows and to investigate the TE feedbacks implied by the fastest-growing modes. Because of the problems associated with the instability characteristics of the B-model used by RM, we will use a two-layer quasi-geostrophic P-model. We consider both an eight-winter Northern Hemisphere climatology and the quasi-stationary North Atlantic large-scale flow regimes derived by Vautard (1990), with the objective of extending Rheinhold and Pierrehumbert's (1982) linear instability analysis of weather regimes in a simple model to a more sophisticated analysis of weather regimes in the real atmosphere.

The two-layer quasi-geostrophic model, apart from providing the simplest model of linear baroclinic instability of a general zonally asymmetric flow, also provides a simple framework for evaluating the feedbacks (barotropic and baroclinic) of baroclinic eddies onto the time-mean flow. As a bridge between the structures of the fastest-growing linear modes and their implied barotropic forcing of the basic state, we will make use of the *E*-vector of Hoskins et al. (1983). A divergence of *E* implies a westerly forcing of the time-

mean zonal wind, whilst *E* itself gives information on eddy shape and relative group velocity.

Before tackling the problem of weather regimes, we will test the model's ability to simulate the TEs and their feedbacks associated with an eight-winter climatological mean flow. Although the time-mean flow is not strictly quasi-stationary, the time scale separation between the mean flow and the TEs is ideal. A major emphasis will be given to the use of complex empirical orthogonal functions (CEOFs) of the bandpass filtered observations as a bench mark for interpreting the fastest-growing linear modes and their implied feedbacks. Metz and Lu (1990) have recently demonstrated the lucidity with which the leading CEOFs of the bandpass 500 mb geopotential height describe the storm-track eddies and their transports, and remarked upon the striking similarity between their CEOF structures, and those of the cyclogenesis-type linear modes of Frederiksen (1982). We repeat the CEOF analysis of Metz and Lu for the two-layer model geostrophic stream-functions.

We then turn our attention to Vautard's (1990) quasi-stationary flows, which comprise flows characterized by a European blocking dipole, an enhanced Atlantic zonal flow, a positive anomaly over Greenland, and a ridge over the east Atlantic. Events of these Atlantic patterns have been identified by Vautard in his dataset via pattern correlation of daily maps with each pattern respectively. For our basic states, we construct composites of each type for our eight-winter dataset. We then compute the fastest-growing linear unstable modes and their implied barotropic feedbacks. Bandpass eddy statistics for each event composite, including TE-feedback and *E*, are constructed to interpret the linear mode quantities.

In section 2, we present the P-model used in the linear instability analysis, together with the database. Section 3 presents the instability analysis of the winter climatological flow, and compares the results with a CEOF analysis of the bandpass filtered observed data. In section 4 we discuss the barotropic eddy feedbacks associated with the quasi-stationary Atlantic flow regimes, and investigate the eddy forcing implied by the three-dimensional linear stability analysis. We conclude with a discussion and summary in section 5.

2. Linear stability analysis

a. The P-model

We use the two-layer quasi-geostrophic P-model (Lorenz 1960; Frederiksen 1982), whose adiabatic nondimensional equations may be written

$$\partial\theta/\partial t = -J(\psi, \theta) + \sigma_0 \nabla^2 \chi \quad (1)$$

$$\partial \nabla^2 \psi / \partial t = -J(\psi, \nabla^2 \psi + 2\mu) - J(\tau, \nabla^2 \tau) \quad (2)$$

$$\begin{aligned} \partial \nabla^2 \tau / \partial t &= -J(\psi, \nabla^2 \tau) \\ &\quad - J(\tau, \nabla^2 \psi + 2\mu) + \nabla \cdot 2\mu \nabla \chi \quad (3) \\ \nabla^2 \theta &= \nabla \cdot 2\mu \nabla \tau \quad (4) \end{aligned}$$

where

$$\psi \equiv 1/2(\psi_{250 \text{ mb}} + \psi_{750 \text{ mb}}) \quad \text{and}$$

$$\tau \equiv 1/2(\psi_{250 \text{ mb}} - \psi_{750 \text{ mb}})$$

define the barotropic and baroclinic modes of the streamfunction respectively, θ is the vertically averaged potential temperature defined at the layer interface (500 mb), and

$$\sigma_0 = bc_p \Delta \theta_0 / 2a^2 \Omega^2$$

is the dimensionless static stability parameter where $\Delta \theta_0$ is a constant defining the mean difference in potential temperature between the layers. Also χ is the velocity potential in the lower layer, $\mu = \sin(\text{latitude})$, $b = 0.124$ (Lorenz 1960), a is the earth's radius, and Ω its angular velocity.

We eliminate χ and τ from the spectral transform of (1)–(4), and linearize the resulting equations for ψ and τ about a three-dimensional time-mean basic state $(\bar{\psi}, \bar{\tau})$. The equations for the perturbations $(\psi' = \psi - \bar{\psi}, \tau' = \tau - \bar{\tau})$ constitute an eigenvalue problem for the complex angular frequencies $\omega^{(k)}$, together with the eigenvectors $\hat{\mathbf{x}}^{(k)} = (\hat{\psi}, \hat{\tau})^{(k)}$. Details of the method of solution are given in the Appendix. Following Frederiksen (1982), the basic states are assumed to be stationary solutions of (1)–(4) (with the appropriate forcing terms), and the perturbations are taken to be adiabatic. It was shown in RM that the effect of surface friction on the normal modes is of secondary importance.

As in RM, we choose $\Delta \theta_0$ such that the fastest-growing modes compare optimally with observed time-mean baroclinic eddy statistics. A value of $\Delta \theta_0 = 45$ K proves to be appropriate at T21 truncation.

b. The dataset

Eight winters of Northern Hemisphere winter geopotential height data (850, 700, 500, 300 and 200 mb) from the European Centre for Medium Range Weather Forecasts (ECMWF) for the period 1979/80 to 1986/87 [in terms of spherical harmonics truncated triangularly at total wavenumber 21 (T21)], were available for use in this study. Each winter consists of 110 days of 0000 UTC daily analyses starting on 20 November. To separate out the high-frequency transient eddies (TEs) primarily associated with baroclinic instability, we use a bandpass filter retaining oscillations in the 2.5–6 day band, whilst a 10-day lowpass filter is used to separate out the low-frequency oscillations. Both filters are from Blackmon and Lau (1980), and require an extra 10 days at the end of each winter, so that in

section 4, we have effectively only 90-day winters, starting 30 November, at our disposal. The geostrophic streamfunction is derived from the geopotential height by solving the linear balance equation.

3. The eight-winter climatology

a. Linear instability analysis

Here we present the results of linearizing the P-model about the eight-winter time-mean flow, using $\Delta \theta_0 = 45$ K. As in RM, we take 850 and 300 mb to be representative of the lower and upper layers in the two-layer model, respectively, so that $\psi \equiv 1/2(\psi_{300 \text{ mb}} + \psi_{850 \text{ mb}})$ and $\tau \equiv 1/2(\psi_{300 \text{ mb}} - \psi_{850 \text{ mb}})$, in order to include low-level baroclinity to some extent. Both the perturbation and the basic state are truncated at T21. Figure 1 illustrates the climatology of the barotropic component of the flow, in terms of isotachs of the geostrophic zonal wind $\bar{u}_g$, and regions of large bandpass $(\psi'^2)^{1/2}$ (hatching). One sees the two principal jets over the western oceans and their associated “storm tracks” (regions of large bandpass rms ψ) situated on the northern flanks of the jets, downstream of the jet maxima.

The eigenvalues of the 10 fastest-growing normal modes are tabulated in Table 1. Both growth rates and periods compare quite well with those of a similar study by Frederiksen (1982, 1986) though he uses a value of $\Delta \theta_0 = 23$ K. The fastest-growing modes (especially growth rates and modal ordering) are sensitive to the static stability (RM; Frederiksen 1982), and although

TABLE 1. Eigenvalues of the ten fastest-growing modes for the eight-winter mean basic state.

Mode k	Angular frequency $ \omega_r $ (°/day)	Period T (days)	Growth rate ω_i (1/days)
1	106.5	3.4	0.43
2	127.8	2.8	0.41
3	84.4	4.3	0.38
4	151.0	2.4	0.36
5	92.2	3.9	0.36
6	65.9	5.5	0.31
7	106.5	3.4	0.30
8	81.4	4.4	0.28
9	47.9	7.5	0.27
10	55.1	6.5	0.26

Frederiksen uses a different eight-winter basic state and truncation, this difference is rather puzzling. The B-model with $\Delta\theta_0 = 36$ K yielded a fastest-growing mode with a similar growth rate to Table 1 and modal periods of the same order, although these tended to be less a function of growth rate than the P-model periods (Table 1 and Fig. 4 below). Even in the P-model, the somewhat slower-growing modes have a broad frequency spectrum.

Figure 2 shows the real phase of $\hat{\psi}$ and $\hat{\tau}$ for the two fastest-growing modes in gridpoint space (the quadrature phase has a very similar structure). The fastest-growing mode has the structure of a cyclogenesis mode (cf. Frederiksen 1982), characterized by a synoptic scale, regionally confined zonal wavetrain. Both modes have maxima over the Pacific and the west Atlantic consistent with the locations of the bandpass rms maxima in Fig. 1; although the linear modes are rather too spread out, especially mode 1 over the west Pacific. The modal variances, computed by averaging over phase following Frederiksen (1982) (not shown), confirm that in both cases, the Atlantic bandpass variance maximum is well reproduced; whilst over the Pacific the modal variance (mode 1) extends too far west. Mode 2 compares quite closely with the fastest-growing mode of Frederiksen (1982). However, the results contrast with those of the B-model of RM, where the fastest-growing mode does not have the characteristics of a cyclogenesis mode. The B-model's leading cyclogenesis modes tend to be Pacific modes that are often highly localized, but with rather small spatial scale, and situated somewhat too far south.¹ The P-model seems to capture the local nature of the Atlantic storm track better than the Pacific one. Viewing the ψ - and τ -structures together, it is clear that the modes evolve along the storm tracks from a highly baroclinic state to rather

equivalent barotropic as the modes propagate eastward. This behavior was not discernible in the B-model modes (RM Fig. 1).

The eight-winter time mean was split into two consecutive four-winter means, and the stability analysis repeated for each case respectively. In both cases the periods and structure of the three fastest-growing modes remained very similar, though their ordering (within the first three) changed in both cases (3, 1, 2 and 2, 1, 3, respectively). The value of $\Delta\theta_0$ was found to affect primarily the growth rates and the ordering of the modes. A higher static stability gives slower-growing modes, with the modes with larger periods (and larger spatial scales) being relatively faster growing, as also discussed by Frederiksen (1982). The experiment was repeated with a parallelogram-type truncation [basic state truncated at T18, perturbation at rhomboidal 15 (R15) with 10 meridional modes retained ($M = 15$, $N = M + 20$)], similar to that employed by Frederiksen (1982), with very little change in the results. There is a much greater sensitivity to the vertical structure of the basic state. Taking the time-mean flow at 750 and 250 mb as the basic state, and so defining $\bar{\psi} \equiv 1/12(2(\bar{\psi}_{850 \text{ mb}} + 2\bar{\psi}_{700 \text{ mb}}) + 3(\bar{\psi}_{300 \text{ mb}} + \bar{\psi}_{200 \text{ mb}}))$, and $\bar{\tau} \equiv 1/12(-2(\bar{\psi}_{850 \text{ mb}} + 2\bar{\psi}_{700 \text{ mb}}) + 3(\bar{\psi}_{300 \text{ mb}} + \bar{\psi}_{200 \text{ mb}}))$ from our dataset, where the overbar denotes a time average, gave a fastest-growing mode with growth rate of only 0.28 day^{-1} . In this case choosing $\Delta\theta_0 = 35$ K gave comparable growth rates to those of our control experiment, but modal periods were reduced, and modal structures were smaller-scale in the Atlantic region. Since the main effect of taking the time-mean flow at 850 mb and 300 mb in the basic state is to include the effect of low-level baroclinity somewhat, the model appears to be quite sensitive to the strength of the basic-state baroclinity.

b. Complex empirical orthogonal function (CEOF) analysis

In order to qualify and quantify the extent to which the normal modes reflect the wintertime TEs, we now compare them with a CEOF analysis of the eight-winter bandpass filtered ψ and τ . A CEOF analysis for the combined complex state vector $\mathbf{X}(t) = (\psi(t), \tau(t))$, made up of the complex spherical harmonic coefficients of ψ together with those of τ , was performed as described in Metz and Lu (1990), by evaluating the complex covariance matrix $\mathbf{X}\mathbf{X}^*$ and then finding its (real) eigenvalues and (complex unitary) eigenvectors. As usual, the modes are ranked by explained variance, in decreasing order.

The resulting eigenvalue spectrum is very similar to that found by Metz and Lu (1990) for 500 mb geopotential data from the same eight-winter dataset. The two leading modes are well separated from the rest, explaining $13.0\% \pm 1.5\%$ and $11.1\% \pm 1.3\%$ of the

¹ As discussed in RM, the B-model exaggerates instability associated with the southern flank of the Pacific jet, at the expense of the more northerly Atlantic jet.

temporal bandpass variance respectively (the third mode explains only $6.0\% \pm 0.7\%$). The error bounds are the empirical standard deviations (Horel 1984), using $T_0 = 5$ days as the time between effectively independent observations (Metz and Lu 1990). In contrast to Metz and Lu's result for the geopotential data, the error bounds on the two leading streamfunction modes overlap, so that the modes are not statistically distinct and care must be taken when considering them individually. The temporal characteristics of the am-

plitude time series of the CEOFs (i.e., the principal components), can be investigated in terms of lagged autocorrelation functions. The phase change per day of the first two modes (not shown), is very similar to that found for the geopotential height by Metz and Lu (1990). The phase speeds are nearly constant at about $90^\circ/\text{day}$ for lags up to four days, corresponding to a period of about four days.

Figure 3 illustrates the real phase of the ψ - and τ -components, respectively, of the two leading CEOFs

FIG. 2. Streamfunctions of the two fastest-growing modes for the eight-winter basic state; (a) mode 1 ψ , (b) mode 1 τ , (c) mode 2 ψ , (d) mode 2 τ . Magnitudes are arbitrary. All maps are polar-stereographic projections as in Fig. 1, with negative contours dashed and zero contour omitted.

FIG. 3. Streamfunctions ($10^6 \text{ m}^2 \text{ s}^{-1}$) of two leading EOFs for the eight-winter bandpass climatology; (a) EOF 1 ψ , (b) EOF 1 τ , (c) EOF 2 ψ , (d) EOF 2 τ . Contour interval $1 \times 10^6 \text{ m}^2 \text{ s}^{-1}$; zero contour omitted.

in gridpoint space (the quadrature phase again being very similar in each case). The geographical distribution of the modes is reversed compared with Metz and Lu's geopotential result, with the leading mode over the Pacific, but as already mentioned this is not statistically significant. Nevertheless, the modal structures resemble those of the 500 mb geopotential height very closely, and the "climatological-average" baroclinic wavetrains corresponding to the Pacific and Atlantic storm tracks (also seen in the one-point correlation

maps of Blackmon et al. 1984), are very clearly brought out. The third mode (not shown) has activity over North America, as well as a "blocking" type wavetrain skirting northwards around Western Europe, but the pattern has only half the amplitude of the two leading modes.

There is a striking similarity between the normal mode structures in Fig. 2, and those of the EOFs. The zonal-wavetrain character of the EOFs, together with their spatial scale, meridional elongation and crescent

shaping, are generally reasonably represented by the normal modes. The Atlantic storm track is well represented, although the amplitude of the normal modes is less locally confined, and the linear modes tend to have a slightly smaller zonal scale. The individual CEOFs are by definition more geographically localized than the climatological mean eddy activity, which is given by the sum over all the CEOFs. The normal modes, on the other hand, appear to represent the combined baroclinic instability over the Pacific and the Atlantic together; phenomena which in reality may possess considerable independence. The local ratio of ψ - to τ -magnitudes is generally similar, if slightly smaller in the linear modes compared to the CEOFs; although the comparison is not so good over the central Pacific where τ exceeds ψ in both linear modes. The CEOFs also exhibit a decrease in vertical tilt along the storm tracks so that the linear modes capture the basic vertical structure of the CEOFs quite well, especially over the Atlantic. The two fastest-growing normal modes have slightly smaller periods than the leading CEOFs.

c. Pattern correlation

Although specific structural features can only be compared by eye, a quantitative measure of the comparison may be obtained in terms of a complex pattern correlation between the combined complex state vectors of the normal modes $\hat{\mathbf{x}}^{(k)}$, and the CEOFs $\mathbf{X}^{(j)}$, $j = 1, 2$. Because the normal modes tend to have amplitude over both the Pacific and Atlantic, it is useful to construct a skill-score for each normal mode as in RM, defined as the sum of the squared pattern correlations of that mode with each of the two CEOFs. The results are tabulated in Table 2 for the first ten normal modes. Figure 4 illustrates the eigenvalues of the 20 fastest-growing normal modes together with their skill-scores. The modes that correlate best with the CEOFs have periods between two and four days, which is

FIG. 4. Skill scores (see text) multiplied by 10 and rounded to the nearest integer of the 20 fastest-growing modes for the eight-winter basic state, as a function of eigenvalue.

somewhat less than the CEOF value of four days. In contrast to the B-model results of RM, the two fastest-growing modes have the largest skills (up to 0.61), and these are larger than in the B-model case where a maximum skill of 0.45 was attained (correlating ψ alone). Correlating ψ - or τ -fields separately gave very similar results, though the latter case gave marginally higher correlations. Although they both have similar skills, mode 1 correlates strongly with the Pacific CEOF, whilst mode 2 correlates strongly with the Atlantic CEOF. Thus, it is meaningful to consider at least the first two normal modes in accounting for the climatological storm tracks, as is the case with the CEOFs. Indeed one would not wish to distinguish between the two leading normal modes on the grounds of their eigenvalues.

It is clear from Fig. 4 and Table 2 that the similarity between the normal modes and the CEOFs, falls off with growth rate of the former. Those of modes 3–10 with larger periods tend to have larger-scale structures (predominantly over the Pacific and east Asia) as discussed by Frederiksen (1982). Mode 4 has a similar structure to mode 2, but a slightly smaller spatial scale.

d. Eddy feedbacks onto the time-mean flow

1) THEORETICAL BACKGROUND

In this section, we express the eddy feedbacks onto the time-mean flow, associated with the normal modes and CEOFs discussed above, in the framework of the two-layer quasi-geostrophic model. However the P-model does not have a simple expression for the baroclinic feedback of the eddies because the τ -equation (A5) is implicit. In order to obtain a simple expression for the baroclinic eddy forcing, we shall make use of the B-model formulation (cf. RM).

Putting $\psi = \bar{\psi} + \psi'$ and $\tau = \bar{\tau} + \tau'$, where the overbar represents a time average, we obtain the following equations for the time-mean flow:

TABLE 2. Modulus of complex pattern correlation between the ten fastest-growing normal modes (eight-winter climatology) and the two leading CEOFs. Also "skill" defined as sum of squared correlations.

Normal mode	CEOF mode		Skill
	1	2	
1	0.63	0.44	0.59
2	0.42	0.66	0.61
3	0.44	0.27	0.27
4	0.52	0.34	0.38
5	0.52	0.25	0.33
6	0.28	0.17	0.10
7	0.33	0.20	0.15
8	0.15	0.27	0.09
9	0.23	0.03	0.05
10	0.30	0.06	0.09

$$\partial \nabla^2 \bar{\psi} / \partial t = -J(\bar{\psi}, \nabla^2 \bar{\psi} + 2\mu) - J(\bar{\tau}, \nabla^2 \bar{\tau}) + F_{\xi} \quad (5)$$

$$\begin{aligned} \partial(\nabla^2 - \Lambda^{-2})\bar{\tau} / \partial t = & -J(\bar{\psi}, (\nabla^2 - \Lambda^{-2})\bar{\tau}) \\ & - J(\bar{\tau}, \nabla^2 \bar{\psi} + 2\mu) + F_{\xi'} \quad (6) \end{aligned}$$

where

$$F_{\xi} = -\overline{J(\psi', \nabla^2 \psi')} - \overline{J(\tau', \nabla^2 \tau')}, \quad (7)$$

$$F_{\xi'} = -\overline{J(\psi', (\nabla^2 - \Lambda^{-2})\tau')} - \overline{J(\tau', \nabla^2 \psi')} \quad (8)$$

are the eddy forcings of the vertically averaged and thermal vorticities, respectively and $\Lambda \equiv (f_0^2/\sigma_0)^{-1/2}$ is the internal radius of deformation, taken to be 512 km.

Following Metz and Lu (1990) we can derive the effective transient eddy forcings of the zonal wind in the two-layer model:

$$\begin{aligned} F_u &= -\frac{1}{a} \frac{\partial}{\partial \phi} (\nabla^{-2} F_{\xi}), \quad \text{and} \\ F_{u'} &= -\frac{1}{a} \frac{\partial}{\partial \phi} [(\nabla^2 - \Lambda^{-2})^{-1} F_{\xi'}] \quad (9) \end{aligned}$$

where ϕ is latitude. These quantities have the advantage of being less prone to small-scale noise than the vorticity fluxes themselves, and may be interpreted directly in terms of E -vectors.

The E -vector of Hoskins et al. (1983) illustrates the relationship between the eddy shape and F_u , via the momentum fluxes themselves. From (7), F_{ξ} may be expressed approximately in cartesian coordinates (cf. Metz and Lu 1990) as

$$F_{\xi} \approx -\frac{\partial}{\partial y} (\nabla \cdot \mathbf{E}) \quad (10)$$

where

$$\mathbf{E} \equiv (\overline{v'^2 - u'^2}, -\overline{u'v'}) + (\overline{v''^2 - u''^2}, -\overline{u''v''}), \quad (11)$$

and

$$\begin{aligned} u &= -\partial \psi / \partial y, & v &= \partial \psi / \partial x, \\ u' &= -\partial \tau / \partial y, & v' &= \partial \tau / \partial x. \end{aligned}$$

Since the mean relative vorticity is $\nabla^2 \bar{\psi} = \partial \bar{v} / \partial x - \partial \bar{u} / \partial y$, the eddy vorticity flux divergence F_{ξ} occurs in (5) as if there were an eddy x -momentum flux $-\mathbf{E}$, as discussed by Hoskins et al. Where $\mathbf{E}$ is divergent, there is a forcing of the mean horizontal circulation consistent with a tendency to increase the westerly mean wind $\bar{u}$ and vice versa.

By considering the barotropic energy conversion from the transients into the time-mean flow, it can be shown assuming $\partial \bar{u} / \partial y \gg \partial \bar{v} / \partial x$, that if $\mathbf{E}$ points down the $(2-D)$ gradient of $\bar{u}$, there is a barotropic energy conversion from the transients into the time-mean flow (Wallace and Lau 1985).

The principal axis of the eddy velocity correlation tensor is aligned approximately at 90° to $\mathbf{E}$ in the case of the bandpass eddies where $\mathbf{E}$ is predominantly zonal, enabling the eddy orientation to be derived from $\mathbf{E}$ (Hoskins et al. 1983). Furthermore, the group velocity of the transient disturbances relative to the local time-mean flow subtends an angle approximately twice that of $\mathbf{E}$ with the mean flow.

Because the normal modes and CEOFs are periodic, it is necessary to average over phase, in order to construct their associated fluxes. This is done by constructing synthetic time series of ψ' and τ' , according to

$$\mathbf{x}'^{(k)}(t) = \sqrt{\lambda^{(k)}} \exp(i\varphi(t)) \hat{\mathbf{x}}^{(k)} \quad (12)$$

where the superscript k denotes the normal mode or CEOF, $\lambda^{(k)}$ is the modulus of its eigenvalue, and $\hat{\mathbf{x}}^{(k)}$ its eigenvector. In the case of the normal modes, the magnitudes are arbitrary, and we set $\lambda^{(k)} = 1$. The phase $\varphi(t)$ is computed for one complete period (i.e., $0 \leq \varphi < 2\pi$) with increments of $\pi/4$. Finally, F_u , $F_{u'}$, and $\mathbf{E}$ are then computed from these synthetic time series of ψ' and τ' via (7)–(9) and (11), respectively.

2) RESULTS

Figure 5a,b show contours of F_u together with arrows depicting $\mathbf{E}$ for the first two normal modes, respectively, and Fig. 5c shows the corresponding fields for CEOFs 1 and 2 plotted together on one map.

The F_u distributions of the CEOFs consist of zonally elongated meridional dipoles very similar to those of Metz and Lu (1990), except that the Pacific maximum is stronger than the Atlantic one, consistent with our use of streamfunction as opposed to geopotential height.

The normal mode F_u distributions also consist of zonally elongated dipoles. The location of the Atlantic CEOF dipole is well given by mode 2 (and to a reasonable extent by mode 1 also). Mode 1 simulates the region of $F_u > 0$ associated with the Pacific CEOF quite well, but the dipole structure is less well defined, and the amplitude of F_u is too large over the east Pacific and North America in both linear modes. The linear F_u distributions appear to be zonally smeared-out versions of the CEOF distributions. Nevertheless, over the northern flank of the jet exits the normal modes suggest the correct picture, implying a westerly acceleration tending to displace the jet northwards and downstream, and a deceleration to the south.

The E -vector distributions provide the key to understanding F_u in terms of eddy structure (Wallace and Lau 1985). In the CEOFs (Fig. 5c), $\mathbf{E}$ diverges out of the regions on the northern flanks of the two principal jets, in good agreement with the regions of positive F_u , with $\overline{v'^2 - u'^2} > |\overline{u'v'}|$ and $\overline{v'^2} \gg \overline{u'^2}$. Here, $\mathbf{E}$ con-

verges predominantly to the southeast and east, with equatorial propagation of wave activity via poleward $\bar{u}'v'$ becoming important. The E -vectors point down the gradient of $\bar{u}_g$ in the jet exits, implying barotropic energy convergence from the TEs into the time-mean flow. The comparison between regions of convergent E and negative F_u is less good, especially over the Pacific.

Figure 6 illustrates F_u and E calculated directly from the eight-winter bandpass statistics. Both fields are similar to those of the CEOFs except that E has appreciable amplitude over the east Pacific and North

America, and also over western Europe, and consequently the convergence is less marked over the east Pacific than in Fig. 5c. Over the east Atlantic, the climatological E is strongly convergent over a wide latitudinal band, associated with the ridge over Europe in the winter climatology. This climatological ridge might be interpreted in terms of CEOF 3 (section 3b) as being partly associated with blocking events. Analysis of the y -momentum equation by Lau (1988) indicates that the fanning out of E at the jet exit, is consistent with the forcing of a meridional diffluence in the time-mean flow causing it to split.

FIG. 5. Contours of barotropic zonal wind forcing F_u , and arrows depicting E associated with (a) linear mode 1, (b) linear mode 2, (c) CEOFs 1 and 2. Maximum arrow lengths ($|E|$): (a) and (b) 175 and 384 arbitrary units, respectively; (c) $65.5 \text{ m}^2 \text{ s}^{-2}$. Contour intervals: (a) and (b) 2.5 and 5.0 arbitrary units, respectively; (c) $0.25 \text{ m}^2 \text{ s}^{-1} \text{ d}^{-1}$.

The normal modes compare remarkably well with the CEOFs in terms of E over the jet exit regions, where the diffuence of E is well represented. Furthermore, the linear E compares even better with the bandpass climatology in Fig. 6. Nevertheless, E has generally exaggerated amplitude over North America and over the west Pacific; i.e., upstream of the observed storm tracks. Linear mode 1 exaggerates the poleward momentum flux over the west Pacific, whilst both modes overestimate the equatorward fluxes over the NW Atlantic.

To a certain extent it is the crude vertical representation of the two-layer formulation that is responsible for the successes and failures of our linear model to capture E . The upward propagation of eddy activity so crucial to the eddy strengthening of the barotropic flow, cannot be adequately represented by a linear model. Although some upward propagation of wave activity concentrated at low levels occurs in the linear stage of baroclinic growth (Edmon et al. 1980), Hoskins et al. (1985) argue that in order for an amplifying disturbance to become free to propagate vertically, the wave at low levels must become saturated. Thus, the success of the two-layer model in reproducing $\nabla \cdot E$ in the jet exits at 500 mb is substantially due to the minimal number of vertical modes in the model, in that the linear modes are always "deep." In contrast, linear models with more layers produce baroclinic waves that are too shallow (Frederiksen 1983).

Similarly, the exaggerated magnitude of E at the beginning of the storm tracks may be partly attributed to the exaggerated depth of the modes here; although as noted above, the amplitude of mode 1 is far from being as locally organized as the Pacific CEOF. Better vertical resolution may be necessary to obtain realistically local modes (Frederiksen and Bell 1987), but the associated E -vectors at upper levels are not likely to display a useful comparison with observations.

Figure 7 illustrates the TE zonal thermal wind tendency, F_u , for linear modes 1 and 2, and for CEOFs 1 and 2 together. The CEOF picture indicates a deceleration of the thermal jets in the two-layer model, extending downstream from the jet maxima. To the north and south, are small regions of positive thermal wind forcing. Whilst the leading normal modes reproduce this general picture, the distribution is rather noisy and the linear-mode $F_u < 0$ regions tend to be upstream of those in the CEOFs, and sit too far north over the Atlantic. Apparently F_u is a more difficult quantity in the two-layer model because of the very crude vertical representation. This is also reflected in the fact that it is not meaningful in the two-layer formulation to define a vertical component of the E -vector, to describe the baroclinic conversions.

Despite these difficulties, the upstream position of F_u with respect to F_u seen in the normal modes (Figs. 7a,b, and 5a,b, respectively) is consistent with the

FIG. 6. Contours of barotropic zonal wind forcing F_u , and arrows depicting E (max. $57.3 \text{ m}^2 \text{ s}^{-2}$), for bandpass filtered eight-winter climatology. Contour interval $0.25 \text{ m s}^{-1} \text{ d}^{-1}$.

structural changes of the modes, with decreasing baroclinicity from west to east along the wavetrains. The Pacific CEOF exhibits a similar but smaller phase shift, whilst over the Atlantic the CEOF indicates that the observed heat and momentum flux maxima are almost coincident in longitude; so that one might expect linear theory to be generally less applicable over the Atlantic.

4. Quasi-stationary large-scale anomalies over the North Atlantic

a. The quasi-stationary regimes

The Atlantic flow regimes discussed in this section are taken from Vautard's (1990) analysis of 37 winters of 700 mb geopotential height observations. They are defined as quasi-stationary large-scale patterns, which on statistical average are stationary, and identified using an optimization procedure. Four distinct patterns result: a typical European blocking dipole (BL), an enhanced zonal flow (ZO), an anomalous anticyclone centered over Greenland (GA), and a ridge over the east Atlantic (AR). Events of these regimes were defined by Vautard (1990) to be periods when the observed low-frequency (timescales > 10 days) 700 mb geopotential height field over the North Atlantic area has a pattern correlation with the respective quasi-stationary pattern exceeding 0.5.

For the present study, we have selected the subset of Vautard's events with durations ≥ 5 days contained in our lowpass filtered eight-winter dataset. This yielded

11 ZO, 8 GA, 6 AR and 3 BL events whose dates are given in Table 3. Not only are there only 3 BL events, but they are all rather short. In order to increase the BL sample somewhat, we have included an extra day at the beginning and end of the BL-events, giving four slightly longer events. This is justified because Vautard sampled the observations on alternate days only. Only events with durations ≥ 5 days have been included so that meaningful eddy statistics (F_u and E -vectors) can be computed for each event and then composited together. Composites of lowpass- and bandpass-filtered

streamfunction at 300 and 850 mb were formed for each of the four event-types, weighting the individual events by length. These composites are illustrated in Fig. 8 in terms of contours of zonal wind, $\bar{u}_g$ and regions of large bandpass rms ψ (hatched). Although Vautard defined his regimes in terms of large-scale data truncated at nine lowpass real EOFs, and despite our small subsets of events, our composites resemble those of Vautard quite well. Changes in the event definition had little effect on the composites, except for BL.

The BL-composite in Fig. 8a has a very localized

FIG. 7. Contours of baroclinic zonal (thermal) wind forcing F_u , associated with (a) linear mode 1, (b) linear mode 2, and (c) CEOFs 1 and 2. Contour interval in (c) is $0.25 \text{ m s}^{-1} \text{ d}^{-1}$. Magnitudes in (a) and (b) are arbitrary.

TABLE 3. Dates of quasi-stationary North Atlantic events making up composites.

Winter	BL events	ZO events	GA events	AR events
1979–80	13.1–17.1	10.12–16.12	21.1–2.2	
1980–81		14.12–26.12 1.1–5.1 15.1–19.1 4.2–8.2		26.12–15.1 31.1–10.2
1981–82			7.12–17.12 29.12–8.1	26.1–30.1
1982–83	12.2–18.2	8.12–24.12 3.1–9.1 27.1–31.1		11.1–4.2
1983–84		31.12–24.1 3.2–9.2		29.12–8.1
1984–85	2.1–4.1		4.1–8.1 18.1–22.1	
1985–86	2.2–8.2	1.1–25.1	30.11–6.12 24.12–28.12 18.2–27.2	17.1–23.1
Total	4	11	8	6

Atlantic jet, a secondary $\bar{u}_g$ maximum NE of Iceland, and a region of easterlies centered over the British Isles. Eddy activity is concentrated about these two jet maxima. Figure 9a illustrates the associated composited bandpass barotropic eddy forcing in terms of F_u and E , as defined in the previous section. The zonal-wind eddy forcing, F_u , indicates strong zonal acceleration by the eddies just upstream of the two Atlantic $\bar{u}_g$ maxima, strong deceleration east of the main jet maximum, and weak easterly forcing throughout the region of easterlies over the British Isles. It is clear that the eddies are acting to maintain the block, and also to shift it westward as also noted by Mullen (1987). The E -vectors indicate that the Atlantic stormtrack is split, the northern branch steering around the block and the southern branch impinging zonally onto it to the west. In both branches, E converges into the block (and is directed down the gradient of $\bar{u}_g$), indicating easterly forcing there, and barotropic energy conversion from the eddies into the time-mean flow. Magnitudes of both F_u and E are somewhat larger than in climatology.

The ZO-composite is characterized by an enhanced Atlantic jet centered midway over the North Atlantic. Bandpass variability is correspondingly enhanced, with a zonally elongated storm track extending right into Europe (Fig. 8b). Here, F_u forms a zonally elongated north-south dipole straddling the jet (Fig. 9b), with acceleration to the north and deceleration to the south, tending to shift the jet axis northward. Correspondingly, the E -vectors are rather zonally oriented to the north of the jet, but have an equatorward component

to the south with convergence on the southern flank of the jet. There is a clockwise turning of E around the jet exit.

In the GA-composite, the Atlantic jet and storm track are displaced southward, and are very zonally oriented (Fig. 8c). A region of westerly forcing ($F_u > 0$ in Fig. 9c) sits over the northern flank of the Atlantic jet exit (tending to shift the jet maximum northward and downstream), with regions of weak easterly forcing to the north and south (consistent with forcing of an anticyclone over Greenland). Downstream of the jet at about 30° W, there is a region of zonal deceleration, consistent with convergent E .

The AR composite (Fig. 8d) is characterized by an extended North Atlantic jet with a ridge over the east Atlantic, and a secondary jet maximum south of Iceland. As in the ZO case, the Atlantic storm track stretches right into Europe, but is arched over the east Atlantic. In terms of TE feedback (Fig. 9d), there is again acceleration to the north and deceleration to the south of the jet axis, but the main eddy tendency is *upstream* of the east Atlantic jet maximum; though over the east Atlantic F_u does support the ridge.

In the BL case, the TEs clearly act to support the quasi-stationary pattern against the time-mean flow. In the other cases the eddies generally act in the climatological manner to shift the jet axis northward, and to extend the jet maximum downstream especially in the GA case. This tendency would appear to support the ZO pattern, but the importance of the feedback for the GA and AR patterns is less clear.

b. Linear instability analysis

The procedure described in section 2 was repeated using basic states consisting of the episode composites of $(\bar{\psi}, \bar{\tau})$ for the four cases BL, ZO, GA and AR. The eigenvalues of the 10 fastest-growing modes for each case are given in Table 4. In all four cases the growth rate of the fastest-growing mode is slightly smaller than in the climatological case, but the modes are again quite bunched together. The spectrum of angular frequency is rather similar for the various basic states, but with a larger range of frequencies than in the eight-winter case.

In the following we will examine the structures and barotropic eddy feedbacks associated with the two fastest-growing cyclogenesis modes that have significant activity over the Atlantic (for each of the four cases underlined in Table 4). These tend to be modes with the largest angular frequencies. As in the eight-winter case, the lower-frequency modes have larger spatial scales and tend to be situated over the Pacific and east Asia. Although we limit our discussion to only two modes whose choice is unavoidably somewhat subjective, examination of the other leading Atlantic cyclo-

genesis modes confirmed that those to be presented are indeed representative.

1) BLOCKING CASE (BL)

Linearizing about the BL-event composite, the leading modes with Atlantic wavetrains are modes 1 and 4, whose ψ distributions are illustrated in Figs. 10a,b, respectively. Both modes appear to be "steered" to the

north of the block, and mode 4 suggests a split structure, with modal activity south of the block. Mode 5 closely resembles mode 4. High linear mode amplitude over the Atlantic coincides quite well with the bandpass variance maximum in the event composite (Fig. 8a).

As in section 3d we have constructed the eddy zonal-wind forcing associated with the normal modes in terms of F_u and E -vector distributions (Figs. 10c,d). Both modes have a region of implied westerly acceleration

FIG. 9. Event composites of bandpass barotropic zonal wind forcing F_u (contours, $0.5 \text{ m s}^{-1} \text{ d}^{-1}$), and arrows depicting E for (a) BL, (b) ZO, (c) GA, and (d) AR. Maximum arrow lengths: (a) $107 \text{ m}^2 \text{ s}^{-2}$, (b) $79.6 \text{ m}^2 \text{ s}^{-2}$, (c) $90.6 \text{ m}^2 \text{ s}^{-2}$, (d) $71.7 \text{ m}^2 \text{ s}^{-2}$.

($F_u > 0$) north of Iceland, very similar in position to that in the F_u -composite (Fig. 9a). In addition, both modes imply an easterly acceleration ($F_u < 0$) to the south over the blocking region, and mode 4 suggests strong deceleration of the jet upstream of the block, again in principal agreement with the BL-composite. Mode 4 captures the E -vector signature of the southward branch of the jet in Fig. 9a quite well. However, although the E -vectors associated with mode 1 exhibit

a clockwise turning and southward convergence north of Scotland, they hardly extend southward of 60°N .

2) ZONAL CASE (ZO)

In this case the first two modes with marked Atlantic wavetrains are modes 4 and 5 (Fig. 11). Both modes have similar structures over the Atlantic, and consistent with the zonally extended jet over the North Atlantic,

the linear wavetrains extend further eastward than in the eight-winter case. The linear "storm track" positions compare quite well with the ZO bandpass rms composite (Fig. 8b) over the Atlantic.

The modal F_u distributions over the Atlantic form zonally elongated meridional dipoles quite similar to those of the ZO-composite, which thus tend to shift the jet maximum northward and extend it downstream, although the modal dipoles are somewhat stretched out. The normal modes also have fairly realistic E -vector distributions over the NW Atlantic—capturing the SE-pointing E -vectors over the jet maximum. The clockwise turning of E around the jet exit is also captured, though it is greatly exaggerated and the linear mode E -vectors extend much too far east. It is clear from Figs. 11a,b that the southward turning of E is associated with trailing trough-like eddy structures, which become zonally elongated. This contrasts with the crescent-shaped "climatological" normal modes (Fig. 2) and their associated equatorial momentum fluxes over the NW Atlantic (Fig. 5).

3) GREENLAND ANTICYCLONE CASE (GA)

The leading Atlantic modes for the GA-composite basic state (Nos. 1 and 4) are illustrated in Fig. 12. Both modes are very similar and are characterized by small periods (Table 4), small spatial scales, and very zonally orientated wavetrains of almost hemispheric extent. The southward displacement of modal activity over the west Atlantic is correctly simulated but it extends much too far eastward (cf. Fig. 8c).

The eddy forcing F_u dipole associated with both linear modes is situated further south than in Fig. 9c, and again extends much too far eastward. However, the linear modes do indicate anticyclonic forcing over western Canada and the North Atlantic, and westerly forcing concurrent with the jet maximum. The modal E -vectors have a northward component consistent with

the easterly acceleration north of 50°N, which can be traced to a NW–SE tilt in the linear eddies. Comparing with Fig. 9c, the linear representation is actually quite reasonable around 60°W where the observed eddies are most intense, but the linear modes are much less localized up and downstream of the jet maximum.

4) ATLANTIC RIDGE CASE (AR)

In this case modes 3 and 4 (Fig. 13) are the leading linear modes with large Atlantic activity. Mode 3 (which has a very small period) is clearly steered northward around the east Atlantic ridge, whilst mode 4 has more in common with the linear modes in the ZO case. Turning to the TE feedback, both modes imply a westerly acceleration over the NE Atlantic consistent with the position of the jet maximum; although this contrasts with the upstream position of the F_u maximum in the observed composite in Fig. 9d.

To summarize the above four cases, the fastest-growing linear Atlantic modes and their implied feedbacks onto the quasi-stationary flow do seem to capture *locally* many of the main features of the corresponding composites. In the GA and AR cases, the feedbacks are less well reproduced, but are nevertheless consistent in many respect with the positions of the quasi-stationary jets. The linear modes for a particular basic state are reasonably consistent with one another but often lack locality and sometimes have unrealistically large angular frequencies.

5. Summary and discussion

The principal objective of this work has been to clarify the relevance of linear baroclinic instability theory in describing TE activity and its feedback onto the time-mean flow. We have focused our attention on a Northern Hemisphere eight-winter climatological mean flow, and on quasi-stationary large-scale flows over the North Atlantic with timescales of the order of 10–30 days.

In the eight-winter mean case, the observed leading bandpass CEOFs were used as a benchmark for interpreting the most unstable normal modes. The two fastest-growing modes (Nos. 1 and 2) were found to have spatial structures closely resembling those of the leading CEOFs, both in terms of structural features and pattern correlations. Mode 1 has a correlation of 0.63 with the Pacific CEOF, whilst mode 2 has a correlation of 0.66 with the Atlantic CEOF. Both linear modes have similar eigenvalues with periods of about three days, compared with about four days for the CEOFs. However, the two-layer P-model seems to have difficulty, especially over the west Pacific, in reproducing the regional nature of the storm tracks. Frederiksen and Bell (1987) suggest that more vertical resolution may be required.

TABLE 4. Eigenvalues of the ten fastest-growing modes for BL, ZO, GA and AR basic states. Period T (days) and growth rate ω_i (1/days). Modes considered in text are underlined.

Mode k	(T ω_i)							
	BL		ZO		GA		AR	
1	<u>3.4</u>	<u>0.38</u>	3.5	0.40	<u>2.7</u>	<u>0.41</u>	5.2	0.40
2	4.1	0.37	4.2	0.38	3.8	0.38	4.1	0.38
3	5.2	0.35	5.6	0.37	3.2	0.36	<u>2.1</u>	<u>0.37</u>
4	<u>3.1</u>	<u>0.34</u>	<u>2.4</u>	<u>0.37</u>	<u>2.4</u>	<u>0.36</u>	<u>3.5</u>	<u>0.35</u>
5	2.4	0.33	<u>2.8</u>	<u>0.37</u>	4.8	0.30	2.8	0.31
6	4.0	0.29	2.1	0.30	35.3	0.29	3.2	0.30
7	3.4	0.28	4.5	0.30	6.9	0.28	6.9	0.30
8	4.7	0.28	7.9	0.28	13.7	0.28	2.4	0.29
9	5.4	0.27	21.1	0.26	2.0	0.27	1.7	0.26
10	13.0	0.26	9.1	0.26	3.8	0.27	12.3	0.25

FIG. 10. The two fastest-growing modes with Atlantic wavetrains (modes 1 & 4), for the BL basic state. (a) and (b): respective ψ -fields, (c) and (d): respective F_u and E distributions (as Fig. 5). Maximum arrow lengths: (c) 206 and (d) 542 arbitrary units.

The CEOFs primarily represent the ensemble-average behavior of the large-amplitude stages of baroclinic eddy lifecycles. Our results appear to support the view that mature baroclinic eddies can be interpreted to a certain extent in terms of the fastest-growing linear normal modes of a simple two-layer model, which have equilibrated to finite amplitude whilst retaining their spatial structure.

Simmons and Hoskins (1979) have examined the response of a nonlinear model to a localized initial

perturbation. For a zonally symmetric baroclinically unstable atmosphere, they found that the rate of spreading (both upstream and downstream) of the region of significant disturbance was almost identical in the linear and nonlinear phases of growth. They also found that a quasi-geostrophic model gave very similar results to a primitive equation model. However, the structure of the nonlinear phase of growth showed marked asymmetries between the surface cyclones and anticyclones in the wavetrain. Although the CEOFs

are by definition symmetric with respect to the sign of the anomalies, the composite anomaly maps of Wallace et al. (1988) confirm the symmetry seen in the CEOFs. Furthermore, Wallace et al. demonstrate that superimposing a wavetrain of symmetric surface highs and lows onto a zonally varying basic state can account for the asymmetries found by Simmons and Hoskins in the nonlinear stage—nonlinear feedback onto the zonally symmetric basic state having essentially the same effect.

A tough test of linear theory is its ability to simulate the climatological mean TE feedback. The barotropic feedback, in terms of effective forcing of the barotropic zonal wind (F_u), and E -vectors is quite well simulated in many respects by linear modes 1 and 2, especially in the Atlantic jet region. The F_u dipole structure is well reproduced, though somewhat smeared out. The E -vectors show the correct divergent structure in the jet exit regions, although the equatorward component of $\overline{u'v'}$ is exaggerated over the NW Atlantic. The two-

FIG. 11. The two fastest-growing modes with Atlantic wavetrains (modes 4 and 5), for the ZO basic state. (a) and (b): respective ψ -fields, (c) and (d): respective F_u and E distributions (as Fig. 5). Maximum arrow lengths: (c) 504 and (d) 334 arbitrary units.

FIG. 12. The two fastest-growing modes with Atlantic wavetrains (modes 1 and 4), for the GA basic state. (a) and (b): respective ψ -fields, (c) and (d): respective F_u and E distributions (as Fig. 5). Maximum arrow lengths: (c) 291 and (d) 381 arbitrary units.

layer model's skill at simulating the barotropic feedback is partly associated with its two-layer structure, in that the modes are always "deep," giving adequate representation of the upper-level momentum fluxes in the jet exit region, even if they are exaggerated upstream along with the modal depth. This is particularly pronounced over the west Pacific. The baroclinic (negative) feedback, which one might expect a priori to be better simulated by linear theory, is rather poorly

caught; although the maxima in the linear mode F_u distribution are upstream of the corresponding F_u maxima, consistent with the expectations of linear theory.

Simmons and Hoskins (1978) have discussed the role of nonlinearity in determining $u'v'$ in the barotropic decay stage of numerical baroclinic wave lifecycles. In their zonally symmetric background flow experiment, $u'v'$ is almost entirely poleward, and this is in-

terpreted in terms of a (nonlinear) sharpening of the zonal jet and associated changes in horizontal phase tilt. Thus, the crescent shape typical of the linear mode gives way to a NE-SW tilting phase line at all latitudes. The E -vectors associated with the CEOFs do indeed indicate smaller equatorward momentum fluxes compared with the linear modes. The climatological E -vectors in Fig. 6 compare better with the linear modes in this respect, but this may be associated with low-fre-

quency changes in storm track orientation, rather than with eddy structure. Nevertheless the use of zonal-mean formalism paints an overly pessimistic picture of the validity of linear theory. Application of the time-mean formalism of Hoskins et al. (1983) indicates that the zonal convergence of $v'^2 - u'^2$ is of similar importance to the meridional convergence of $u'v'$ in the zonal momentum balance, especially in the case of the anomalous flows. Both components appear to be quite well

FIG. 13. The two fastest-growing modes with Atlantic wavetrains (modes 3 and 4), for the AR basic state. (a) and (b): respective ψ -fields, (c) and (d): respective F_u and E distributions (as Fig. 5). Maximum arrow lengths: (c) 624 and (d) 178 arbitrary units.

represented by the linear baroclinic instability of a zonally varying basic state, although equatorward fluxes are overestimated.

Having examined the linear model's performance in the winter climatological mean case, the model was applied to quasi-stationary large-scale flows over the North Atlantic. Composites of the lowpass hemispheric flow field during large-scale quasi-stationary events over the North Atlantic were used as basic states for the linear stability analysis. The most unstable linear Atlantic cyclogenesis modes in each case exhibited marked displacements in the positions of their Atlantic baroclinic wavetrains, consistent with the presence of the large-scale anomalies. Furthermore, the structures of the relevant modes in each case showed a large degree of consistency with one another. The modal wavetrain displacements were mostly in good accordance with the bandpass rms composites for each case, especially in the region of the quasi-stationary anomaly. The barotropic eddy feedback, F_u , together with the E -vector were also evaluated for the composites in the four cases, and for their associated normal modes. Encouraging agreement was found in many respects in the blocking (BL) and zonal (ZO) cases, and to some extent in the Greenland anticyclone (GA) case, although here the linear modes were not confined enough regionally. In the Atlantic-ridge (AR) case agreement was less good. The lack of vertical resolution may be an even more serious problem in the quasi-stationary regime cases, with regard to capturing the regional nature of the instability.

RM investigated the sensitivity of the B-model to similar large-scale anomalies. In particular our GA- and ZO-composites resemble the positive and negative phases of their North Atlantic oscillation pattern respectively, and the respective blocking patterns are similar. Qualitative linear-mode displacements are similar for B- and P-models, although the P-model produces Atlantic wavetrains of much more realistic (relative) amplitude. In the GA case, the B-model gave a more realistically regional Atlantic modal maximum.

Although the two fastest-growing modes prove to be relevant to bandpass variability in the climatic case, modal selection was less straightforward in the quasi-stationary flow cases. Here, the Atlantic cyclogenesis modes were not always the fastest-growing modes. Indeed in an equilibrated regime, it is possible that the fastest-growing mode could saturate so quickly (due to nonlinear effects), that slower-growing modes could come to dominate. This highlights the problem any synoptic scale parameterization (in terms of linear modes) would have in selecting the relevant modes, even using instantaneous basic states. Our results also make it clear that whilst particular linear modes are "realistic" at one geographic location, they may be unrealistic elsewhere.

We cannot expect the linearization in the quasi-stationary flow cases to be as meaningful as in the winter climatological flow case, because we are nearing the eddy time scale, and some events in the composites are only five days long. Nevertheless, except in the BL case, the results are rather insensitive to the precise definition of the episodes. The fastest-growing linear modes have been examined for several different blocking anomalies, and the northward displacement of the linear wavetrain around the block appears to be a very robust feature.

A conceptual hurdle for normal mode theory in trying to account for the observed TEs, is that much faster transient growth may often be associated with perturbations of non-normal-mode form (Farrell 1989), particularly over the Atlantic cyclogenesis region where the upper-level remnants of Pacific depressions may be commonplace. Simmons and Hoskins (1979) found that for an initial perturbation of non-normal-mode form, growth rates were indeed significantly larger than that of the most unstable linear mode, but that phase speeds were comparable. In addition, upper-level amplitudes and downstream length scales were increased, compared with experiments using a normal mode initial perturbation. Although such features would be of paramount importance for any individual eddy development, their relevance to our highly simplified time-mean problem is questionable. Nevertheless, disturbances of non-normal-mode form may be of greater importance in the AR and GA quasi-stationary flow regimes. Normal mode theory is clearly only a first approximation to the observed ensemble-mean statistics, but it appears to be a useful one, and its use here is probably consistent with our simple two-layer approach. In the future we plan to investigate the linear instability of large-scale flows characterized by optimal correlation with TE feedbacks, derived from a regional canonical correlation analysis.

Acknowledgments. We are grateful to Robert Vautard for supplying us with the dates of his quasi-stationary events, and the reviewers for their comments. The work was supported by the German Federal Ministry of Research and Technology.

APPENDIX

Solution of the Eigenvalue Problem

Equations (1)–(4) are decomposed first in terms of complex spherical harmonics by substituting, e.g.,

$$\psi(\lambda, \mu, t) = \sum_{m=-M}^M \sum_{n=|m|}^N \psi_n^m(t) P_n^m(\mu) e^{im\lambda}$$

where (M, N) is the spectral truncation, and $P_n^m(\mu)$ are the normalized associated Legendre functions. Only

hemispheric modes are retained, so that the second summation is over components with $(n - |m|)$ odd only, yielding 121 complex coefficients for $M = N = 21$ (T21). The resulting equations are then multiplied through by $(P_n^m e^{im\lambda})^*$, and integrated over the hemisphere using the orthogonality properties of spherical harmonics. This yields the following ordinary differential equations for each spherical harmonic coefficient j , of the perturbation:

$$\frac{d}{dt} \theta_j = - \sum_{kl} i \psi_k \theta_l L_{jkl} + \sigma_0 \gamma_j \chi_j \quad (\text{A1})$$

$$\frac{d}{dt} \gamma_j \psi_j = - \sum_{kl} i (\psi_k \psi_l + \tau_k \tau_l) L_{jkl} (\gamma_l - \gamma_k) - 2im \psi_j \quad (\text{A2})$$

$$\frac{d}{dt} \gamma_j \tau_j = - \sum_{kl} i (\psi_k \tau_l + \tau_k \psi_l) L_{jkl} (\gamma_l - \gamma_k) - 2im \tau_j - 2(f_j \chi_{j-1} + f_{j+1} \chi_{j+1}) \quad (\text{A3})$$

$$\gamma_j \theta_j = -2(f_j \tau_{j-1} + f_{j+1} \tau_{j+1}) \quad (\text{A4})$$

where $\gamma_j = -n_j(n_j + 1)$, $f_j = (n_j - 1)(n_j + 1)[(n_j^2 - m_j^2)/(4n_j^2 - 1)]^{1/2}$, and L_{jkl} are the interaction coefficients. The double summations are restricted to unique combinations of k and l only. Using (A4) to eliminate θ from (A1), and eliminating χ between (A1) and (A3), equations (A1), (A3), and (A4) reduce to a single equation in ψ and τ :

$$\begin{aligned} \frac{d}{dt} (a_j \tau_{j-2} + b_j \tau_j + c_j \tau_{j+2}) \\ = - \sum_{kl} i (\psi_k \tau_l + \tau_k \psi_l) L_{jkl} (\gamma_l - \gamma_k) - 2im \tau_j \\ + \sum_{kl} \frac{2i}{\gamma_l} \psi_k (f_l \tau_{l-1} + f_{l+1} \tau_{l+1}) \\ \times (d_j L_{j-1kl} + e_j L_{j+1kl}) \quad (\text{A5}) \end{aligned}$$

where

$$\begin{aligned} a_j &= - \frac{4f_j f_{j-1}}{\sigma_0 \gamma_{j-1}^2}, \quad b_j = \gamma_j - \frac{4f_j^2}{\sigma_0 \gamma_{j-1}^2} - \frac{4f_{j+1}^2}{\sigma_0 \gamma_{j+1}^2}, \\ c_j &= - \frac{4f_{j+1} f_{j+2}}{\sigma_0 \gamma_{j+1}^2}, \quad d_j = \frac{2f_j}{\sigma_0 \gamma_{j-1}}, \quad e_j = \frac{2f_{j+1}}{\sigma_0 \gamma_{j+1}}. \end{aligned}$$

Equations (A2) and (A5) are next linearized about a three-dimensional time-mean basic state $(\bar{\psi}, \bar{\tau})$, and the equations for the perturbations $(\psi' = \psi - \bar{\psi}, \tau' = \tau - \bar{\tau})$, split into their real and imaginary parts, by substituting, for example, $\psi'_j = 1/\sqrt{2}(\psi'_{jc} - i\psi'_{js})$, where the subscripts c and s denote the cosine and sine components, respectively. Writing in matrix form then yields

$$d/dt(\mathbf{B} \cdot \mathbf{x}') = \mathbf{A} \cdot \mathbf{x}' \quad (\text{A6})$$

where the vector $\mathbf{x}$ consists of the $\psi_{jc}, \psi_{js}, \tau_{jc}, \tau_{js}$, components for all j , and the matrices $\mathbf{B}$, and $\mathbf{A}$, contain the linear terms on the left- and right-hand sides of the linearized versions of Eqs. (A2) and (A5), respectively. The solutions to the real matrix Eq. (A6) are the eigensolutions of the matrix $(\mathbf{B}^{-1} \cdot \mathbf{A})$, and have the same normal mode form as those of Eq. (8) in RM; i.e.,

$$-i\omega^{(k)} \hat{\mathbf{x}}^{(k)} = (\mathbf{B}^{-1} \cdot \mathbf{A}) \cdot \hat{\mathbf{x}}^{(k)}$$

where $-i\omega^{(k)}$ and $\hat{\mathbf{x}}^{(k)}$ are the k th complex eigenvalue and eigenvector, respectively.

The eigensolutions were verified by back substitution into the linearized Eqs. (A1)–(A4).

REFERENCES

- Blackmon, M. L., and N.-C. Lau, 1980: Regional characteristics of the Northern Hemisphere wintertime circulation: A comparison of the simulation of a GFDL general circulation model with observations. *J. Atmos. Sci.*, **37**, 479–514.
- , Y.-H. Lee and J. M. Wallace, 1984: Horizontal structure of 500 mb height fluctuations with long, intermediate and short time scales. *J. Atmos. Sci.*, **41**, 961–979.
- Charney, J. G., and J. G. DeVore, 1979: Multiple flow equilibria in the atmosphere and blocking. *J. Atmos. Sci.*, **36**, 1205–1216.
- Dole, R. M., 1986: Persistent anomalies of the extratropical Northern Hemisphere wintertime circulation: structure. *Mon. Wea. Rev.*, **114**, 178–207.
- Edmon, H. J., B. J. Hoskins and M. E. McIntyre, 1980: Eliassen-Palm cross-sections of the troposphere. *J. Atmos. Sci.*, **37**, 2600–2616.
- Farrell, B. F., 1989: Optimal excitation of baroclinic waves. *J. Atmos. Sci.*, **46**, 1193–1206.
- Frederiksen, J. S., 1982: A unified three-dimensional theory of the onset of blocking and cyclogenesis. *J. Atmos. Sci.*, **39**, 969–982.
- , 1983: Disturbances and eddy fluxes in Northern Hemisphere flows: Instability of three-dimensional January and July flows. *J. Atmos. Sci.*, **40**, 836–855.
- , 1986: Instability theory and nonlinear evolution of blocks and mature anomalies. *Adv. Geophys.*, **29**, 277–303.
- , and R. C. Bell, 1987: Teleconnection patterns and the roles of baroclinic, barotropic and topographic instability. *J. Atmos. Sci.*, **44**, 2200–2218.
- Horel, J. D., 1984: Complex principal component analysis: Theory and examples. *J. Climate Appl. Meteor.*, **23**, 1660–1673.
- Hoskins, B. J., I. N. James and G. H. White, 1983: The shape, propagation and mean-flow interaction of large-scale weather systems. *J. Atmos. Sci.*, **40**, 1595–1612.
- , M. E. McIntyre and A. W. Robertson, 1985: On the use and significance of isentropic potential vorticity maps. *Quart. J. Roy. Meteor. Soc.*, **111**, 877–946.
- Illari, L., and J. C. Marshall, 1983: On the interpretation of eddy fluxes during a blocking period. *J. Atmos. Sci.*, **40**, 2232–2242.
- Lau, N.-C., 1988: Variability of the observed midlatitude storm tracks in relation to low-frequency changes in the circulation pattern. *J. Atmos. Sci.*, **45**, 2718–2743.
- , and E. O. Holopainen, 1984: Transient eddy forcing of the time mean flow as identified by geopotential tendencies. *J. Atmos. Sci.*, **41**, 313–328.
- Lorenz, E. N., 1960: Energy and numerical weather prediction. *Tellus*, **4**, 364–373.
- Metz, W., 1986: Transient cyclone-scale vorticity forcing of blocking highs. *J. Atmos. Sci.*, **43**, 1467–1483.

- , and M.-M. Lu, 1990: Storm track eddies in the atmosphere and in a ECMWF T21 Climate Model. *Beitr. Phys. Atmos.*, **63**, 25–40.
- Mullen, S. L., 1987: Transient eddy forcing of blocking flows. *J. Atmos. Sci.*, **44**, 3–22.
- Rheinhold, B. B., and R. T. Pierrehumbert, 1982: Dynamics of weather regimes: Quasi-stationary waves and blocking. *Mon. Wea. Rev.*, **110**, 1105–1145.
- Robertson, A. W., and W. Metz, 1989: Three-dimensional instability of persistent anomalous large-scale flows. *J. Atmos. Sci.*, **46**, 2783–2801.
- Shutts, G. J., 1986: A case study of eddy forcing during an Atlantic blocking event. *Adv. Geophys.*, **29**, 135–162.
- Simmons, A. J., and B. J. Hoskins, 1978: The life cycles of some nonlinear baroclinic waves. *J. Atmos. Sci.*, **35**, 414–432.
- , and ———, 1979: The downstream and upstream development of unstable baroclinic waves. *J. Atmos. Sci.*, **36**, 1239–1254.
- Vautard, R., 1990: Multiple weather regimes over the north Atlantic. Analysis of precursors and successors. *Mon. Wea. Rev.*, **118**, 2056–2081.
- , and B. Legras, 1988: On the source of midlatitude low-frequency variability. Part II: Nonlinear equilibration of weather regimes. *J. Atmos. Sci.*, **45**, 2845–2867.
- Wallace, J. M., and N.-C. Lau, 1985: On the role of barotropic energy conversions in the general circulation. *Adv. Geophys.*, **28A**, 33–72.
- , G.-H. Lim and M. L. Blackmon, 1988: Relationship between cyclone tracks, anticyclone tracks and baroclinic waveguides. *J. Atmos. Sci.*, **45**, 439–462.