Development of a MEMS Sensor for sub-kPa Shear Stress Measurements

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Abstract

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This dissertation discusses the development of MEMS sensors for measuring sub-kPa (<1000Pa) wall shear stresses in high-speed turbulent flows. Wall shear stress is an important flow quantity that is used to characterize flows that can be found in aerospace, automotive, and biomedical applications. Sensors that can measure this quantity could have many uses ranging from pure turbulence research to flow control of vehicles.

MEMS fabrication techniques allow for the creation of micro-scale sensors that are small enough to accurately measure fluctuating turbulent shear stress. Utilizing a direct-shear stress measurement with a floating element allows the sensors to be calibrated in a well-known shear flow before being installed in an unknown flow environment. The sensors use a differential capacitance measurement scheme combined with non-intrusive backside sensor connections, allowing measurements in recirculating and separating flows.

As part of the sensor design process, 36 different sensor designs were created with varying feature sizes and performance ranges. This was done to mitigate the risks inherent in MEMS fabrication processes and to increase the chances of developing working sensors which could operate in the desired shear stress range (1 – 1000Pa). The sensors were fabricated with the floating element in the top layer of an SOI wafer, with thru-wafer electrical interconnects (vias) created to connect the frontside of the sensor to the backside of the chip.
Post-fabrication, the sensors were characterized electrically and mechanically under a microscope probe station. Sensors were then installed in a custom-made package which integrated off-the-shelf capacitance measuring circuitry with the MEMS sensor.

Using a subsonic duct flow setup, sensors were calibrated in compressible turbulent air flow up to a mean shear stress of 335Pa and a friction velocity Re value of 200,000. After numerical temperature compensation was implemented (required due to temperature-dependent material properties) the sensor gain was calculated as 0.16mV/Pa. The mean shear stress calibration was then used to analyze the turbulence fluctuations inherent in the sensor signal. Turbulence measurements (including intensity, spectral density, and probability density function) indicated that the sensors were responding to shear stress fluctuations, but not detecting the entire turbulent energy spectrum due to low-pass filtering caused by the electrical circuitry.

Experimental measurement of wall shear stress in compressible turbulent flows is an area that has not been fully explored due to previous limitations in available measurement technology. Once a better understanding is gained of the operation and limitations of the MEMS wall shear stress sensors, there lies a great potential to increase our understanding of turbulent flows with unprecedented measurements.
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## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$A_c$</td>
<td>capacitor area</td>
</tr>
<tr>
<td>ACT</td>
<td>actuation</td>
</tr>
<tr>
<td>$A_s$</td>
<td>shear stress surface area</td>
</tr>
<tr>
<td>$b$</td>
<td>duct height</td>
</tr>
<tr>
<td>$b_f$</td>
<td>spring length</td>
</tr>
<tr>
<td>$B_i$</td>
<td>Biot number</td>
</tr>
<tr>
<td>$b_k$</td>
<td>spring width</td>
</tr>
<tr>
<td>$B_W$</td>
<td>bandwidth</td>
</tr>
<tr>
<td>$C$</td>
<td>capacitance (T = top, B = bottom, V = via, sub = substrate)</td>
</tr>
<tr>
<td>$C_{\text{sub}.23}$</td>
<td>capacitance from substrate</td>
</tr>
<tr>
<td>$c_c$</td>
<td>coefficient of critical damping</td>
</tr>
<tr>
<td>$C_f$</td>
<td>skin friction coefficient</td>
</tr>
<tr>
<td>CF</td>
<td>MS3110 feedback capacitor</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat</td>
</tr>
<tr>
<td>CP2, CP1</td>
<td>capacitance from package</td>
</tr>
<tr>
<td>CS2, CS1</td>
<td>sense capacitors</td>
</tr>
<tr>
<td>CS2$<em>{\text{eff}}$, CS1$</em>{\text{eff}}$</td>
<td>effective capacitance for CS2 and CS1</td>
</tr>
<tr>
<td>CS2trim, CS1trim</td>
<td>MS3110 trim capacitors</td>
</tr>
<tr>
<td>CSCOM</td>
<td>MS3110 pin connection</td>
</tr>
<tr>
<td>CTE</td>
<td>coefficient of thermal expansion</td>
</tr>
<tr>
<td>$D_{AB}$</td>
<td>Diffusion coefficient for species A and B</td>
</tr>
<tr>
<td>$d_0$</td>
<td>finger gap</td>
</tr>
</tbody>
</table>
dp/dx pressure gradient in streamwise direction
E Young’s modulus
Fe electrostatic force
F_{shear} force from shear stress
F_{spring} force from spring
F_{testsignal} force from test signal
f frequency
f^+ non-dimensional frequency
G gain
H gap between floating element and substrate
h_c heat transfer coefficient
h_m mass transfer coefficient
h thickness of device layer
I current
k spring constant
L length
L_f sensing finger length
L_k spring length
L_s floating element length
m mass
mdot mass flow rate
Na number of actuation fingers per side
N_f number of sensing fingers per side
Nh  number of holes in floating element
p  non-dimensional electrostatic force
P0  total pressure
Pw  normal pressure at wall
q  non-dimensional shear force
qLOAD  distributed load
R  resistance (B = backside, F = frontside, P = parallel)
Re  Reynolds number
RMS  root-mean-square
s  fabrication offset
S  sensitivity
Sh  Sherwood number
SOFF  MS3110 voltage offset
St  Stanton number
t  time
T0  total temperature
u  streamwise velocity
\bar{u}  mean turbulent velocity
u'(t)  fluctuating velocity
u_c  friction velocity
u^+  velocity scaled by wall variables
U_e  freestream velocity
v  Kolmogorov velocity scale
V    voltage
V_b  bias voltage
V_{ref}  reference voltage
V_s  test signal voltage
W    width
w    displacement
x, y, z  streamwise, transverse, and out-of-plane directions
x_0  finger overlap
y^+  distance from wall scaled by wall variable
\alpha  distance amplification factor
\beta_T  dielectric coefficient of temperature
\delta C  differential capacitance
\delta  boundary layer thickness
\Delta V, \Delta T, etc.  change in quantity
\epsilon  dielectric constant
\zeta  damping coefficient
\eta  Kolmogorov microscale
\kappa  von Kármán constant
\lambda  length scale of large eddies
\Lambda  substrate capacitance mismatch coefficient
\mu  dynamic viscosity
\nu  kinematic viscosity
\theta  azimuthal angle
\( \rho \)  
Density

\( \tau_w \)  
Wall shear stress

\( \tau'_w \)  
Wall shear stress fluctuation

\( \overline{\tau_w} \)  
Mean wall shear stress

\( \tau \)  
Kolmogorov time scale

\( \Phi^+ \)  
Non-dimensional spectral density

\( \Phi \)  
Spectral density

\( \omega_d \)  
Damped natural frequency.

\( \omega_n \)  
Undamped natural frequency

Subscripts

0  
Initial value

eff  
Effective

eq  
Equivalent

f  
Film

fb  
Fingerbend

lam  
Laminar

neg  
Negative

pos  
Positive

sub  
Substrate

theory  
Theoretical value

turb  
Turbulent

w  
Wall
1 Introduction

1.1 Motivation

In simple terms, wall shear stress can be described as the viscous drag force caused by a fluid passing over a solid body or a wall. This same flow property is referred to as “skin friction” in the aerospace industry, or “drag” in the geophysical and hydraulic industries. Shear stress plays a major role in a variety of industries including but not limited to aerospace, automobile, naval, manufacturing, chemical, and buildings energy. Additionally, shear stress is an important quantity in physical processes such as convective heat transfer, turbulent flows, geophysical flows/climatology, and chemical reactions.

In aerodynamics and automobile design, the shear stress is a viscous drag on vehicles (the other effect being a pressure drag), which opposes the vehicle motion. Combustion engines have internal drag from the air passing through them, the effect of which has been shown to reduce combustion efficiency significantly. Estimates have shown that even 5 – 10% decreases in viscous drag would result in the savings of half billion dollars per year spent on fuel in the airline industry alone [1]. Keep in mind that this does not include the savings that could be generated from similar drag reductions on ships, barges, trains, tractor trailers, and smaller automobiles. Shear stress can also be used to determine points of boundary layer separation on a vehicle’s body, which occurs when the shear stress is equal to zero. Separation is known to greatly increase the drag of vehicles and can create difficulty in controlling vehicle stability.

1 Throughout this dissertation, the term “shear stress” will often be used for brevity instead of “wall shear stress.” This is not to be confused with the “Reynolds shear stress” found in turbulent flows.
In the biomedical engineering field, shear stress is studied due to its effects on a multitude of biological processes such as cell adhesion [2] and tissue response [3]. Additionally, there is conclusive evidence that atherosclerosis, the leading cause of death in the developed world, is caused by atherosclerotic lesions (plaque) which are deposited in the low shear stress regions of blood vessels [4].

In turbulent heat transfer, shear stress is an important quantity as it can be directly related to convective heat transfer via the Reynolds Analogy [5]:

\[
\frac{C_f}{2} \sim S_t
\]

where \( C_f \) is the skin friction coefficient and \( S_t \) is the Stanton number, defined as:

\[
C_f = \frac{\tau_w}{\frac{1}{2} \rho U^2} \quad \text{St} = \frac{h_c}{\rho U c_p}
\]

where \( \rho \) is the density, \( U \) is the velocity, \( \tau_w \) is the wall shear stress, \( h_c \) is the convective heat transfer coefficient, and \( c_p \) is the fluid specific heat. Eq. (1) can be further reduced to:

\[
\tau_w = \frac{h_c}{U c_p}
\]

which illustrates the direct relationship between shear stress and convective heat transfer. This is an important concept utilized in heating and cooling applications to make them more effective.

The analogous behavior of heat diffusion and mass diffusion can be used to develop a similar relationship between mass and momentum transfer:

\[
\frac{C_f}{2} \sim \frac{Sh}{Re}
\]

where \( Sh \) is the Sherwood number and \( Re \) is the Reynolds number, defined as:

\[
Sh = \frac{h_w L}{D_{AB}} \quad \text{Re} = \frac{\rho U L}{\mu}
\]
where $h_m =$ convective mass transfer coefficient, $D_{AB} =$ diffusion coefficient for species A and B, $L =$ characteristic length scale, and $\mu =$ dynamic viscosity. Rearranging the previous three equations results in:

$$\tau_w \sim \frac{\mu U h_m}{D_{AB}}$$  \hspace{1cm} (8)

which illustrates how the shear stress is directly related to the convective mass transfer. This connection is utilized in chemical engineering and manufacturing applications.

In steady incompressible laminar flow (for a Newtonian fluid), the shear stress is constant in time and equal to the product of the fluid dynamic viscosity and the velocity gradient at the wall:

$$\tau_w = \mu \frac{du}{dy}$$  \hspace{1cm} (9)

where $u$ is the velocity in the x (streamwise) direction, and $y$ is the direction normal to the wall. Provided that the velocity profile and viscosity can be determined, it is relatively straightforward to calculate the shear stress. Unfortunately, turbulent flow environments are considerably more common in the real-world and much more complex. In these flows velocities will vary with time and can be decomposed into a fluctuating quantity $u'(t)$ and a time-averaged mean quantity $\overline{u}(t)$:

$$u(t) = \overline{u} + u'(t)$$  \hspace{1cm} (10)

For example, in turbulent flow through a pipe at a constant rate, the mean flow could be considered to be at a steady constant velocity, but the turbulent components will always be changing in time. Similarly to velocity decomposition, turbulent wall shear stress can be split into mean and fluctuating components:

$$\tau_w(t) = \overline{\tau_w} + \overline{\tau'_w(t)}$$  \hspace{1cm} (11)
where the mean quantity is assumed to be constant in time and the turbulent component is the fluctuating quantity. The wall shear stress can also be decomposed in the following manner:

\[ \tau_w(t) = \tau_{w,\text{lam}} + \tau_{w,\text{turb}} \]  

(12)

where \( \tau_{w,\text{lam}} \) is the laminar shear stress, and \( \tau_{w,\text{turb}} \) is the turbulent shear stress. Alternately, in later portions of this thesis (Section 6.1.3) this decomposition is written (for brevity) as:

\[ \tau_w = \tau_{\text{mean}} + \tau_{\text{turb}} \]  

(13)

where \( \tau_{\text{mean}} \) is the mean shear stress and \( \tau_{\text{turb}} \) is the turbulent shear stress.

This description of turbulent shear stress does not immediately lend itself as challenging, however in practice, there is nothing simple about predicting or measuring turbulent shear stress. These difficulties stem from the fact that turbulent flows are incredibly varied and complicated to analyze. One reason for this complexity is that turbulent flows have multiple length and time scales which cover multiple orders of magnitude. An example to consider is the fluid flow of a hurricane, where there is a large length scale roughly the size of a continent, and a small length scale approximately the diameter of a pin [6].

If a researcher today was interested in purchasing an “off-the-shelf” shear stress sensor to make fluctuating turbulent shear stress measurements, their options would be very limited compared to the sensors available if they wanted to measure pressure, temperature, or other flow properties. Furthermore, if the sensor was to be installed in an unknown flow regime, for example installed on the wing of an airplane through the duration of a flight, the options would be even further reduced. This is despite the fact that the engineers have been trying to make accurate wall shear stress sensors since the 1950s.
On the theoretical side, if a researcher needed to analytically predict the skin friction of compressible turbulent flow over a flat plate, often considered one of the simplest flow geometries, they would likely use the Van Driest II correlation [7]. This equation is still widely used today and became popular in 1964 when Spalding and Chi [8] showed that it was the theory with the best fit (within 11%) to the available experimental data at the time. It is a real testament to the challenges of turbulent flows that the Van Driest II equation was first formulated in 1956!

Wall shear stress plays so many roles in turbulent flows that it is difficult to overstate its importance. For example, in wall-bounded incompressible turbulent flows, the shear stress acts as an important quantity, defining the friction velocity ($u_\tau$) [9]:

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

where $\rho$ is the fluid density. Turbulence research has shown that the friction velocity is an important scaling parameter in both the mean flow properties and fluctuating properties.

In turbulent wall-bounded flows, the boundary layer is often split into different regions, a near-wall, inner layer (called the viscous sublayer) where viscous wall shear stress dominates, and an outer layer where the turbulent shear stress dominates [9].

In the viscous sublayer, the velocity profile is linear when scaled with inner wall variables and can be written as:

$$u^+ = y^+$$

where $u^+$ and $y^+$ are non-dimensional quantities called “wall units”:

$$u^+ = \frac{u}{u_\tau} \quad y^+ = \frac{yu_\tau}{\nu}$$

(16), (17)
where $\bar{u}$ is the mean velocity, and $\nu$ is the kinematic viscosity at the wall. The linear velocity profile is generally considered to be valid for $y^+ < 6$. Inside the viscous sublayer of the boundary layer, $\tau_{w,\text{lam}}$ is the dominant shear stress component and can be related to the velocity gradient by:

$$
\tau_w(t) \approx \tau_{\text{lam}} \approx \mu \frac{du(t)}{dy}
$$

(18)

In the Outer Layer, the velocity profile is not linear and instead the scaling is given by the Velocity Defect Law:

$$
\frac{U_e - \bar{u}}{u\tau} = g \left( \frac{y}{\delta}, \frac{u\tau}{U_e} \right)
$$

(19)

where $U_e$ is the freestream velocity, $\delta$ is the boundary layer thickness, and ‘$g$’ is a function relating the non-dimensional quantities in the equation.

The region between the inner layer and outer layer is an overlapping regime (also called the logarithmic region) where the inner-law and outer-laws are both valid (viscous and turbulent shear are important) with a velocity profile given by:

$$
u^+ = \frac{1}{\kappa} \ln y^+ + B
$$

(20)

where $\kappa = 0.41$ and $B = 5.0$ are generally accepted values for the constants [9].

Note the prevalence of $u\tau$ (and hence $\tau_w$) in nearly all of the equations presented above. There is also evidence that $u\tau$ can be used in the scaling of turbulent statistics. These include quantities such as fluctuating velocities, Reynolds’s Stress, and spectral density which will be discussed later in Section 1.5. Overall, there is a strong argument that wall shear stress is one of the most important turbulent flow quantities, and yet even after years of efforts, it remains one of the most difficult to measure.
1.2 MEMS Sensors in Shear Stress Measurements

It was not until the development of MEMS (Micro-Electro-Mechanical Systems) sensor technologies that sensors could be fabricated that are physically small enough and can respond fast enough to accurately measure fluctuating shear stress. This is because turbulent flows have time scales $O(\mu S)$ and length scales $O(10\mu m)$ which require miniature sensors with very fast frequency responses. If the sensor response is too slow, it will act as a low-pass filter for the fluctuating shear-stress measurement, and if the size is too large, it will serve to average the measurement over the area of the sensor.

MEMS fabrication techniques allows for the creation of sensors with $O(\mu m)$ features by using the same techniques used in IC processing. Often, MEMS sensors will have free-standing moving parts which would be too small to create using machine-shop fabrication tools. Sensors typically benefit from these small parts because they have small masses, necessary for responding quickly to inertial forces and temperature changes. MEMS also offers the benefit of batch fabrication techniques where multiple sensor designs can be created on the same silicon wafer. Once a “recipe” for making a MEMS sensor is established, it is easy to create 10 or 100 copies of the same sensor at one time.

Despite the progress made in the field of MEMS shear stress sensors, a 2004 paper by Scott [10] puts the current state of the field in perspective. The paper outlines the available techniques for generating a known shear stress to calibrate a sensor against, as well as the difficulties inherent in designing a shear stress sensor. He discusses the fact that there currently is no calibration standard for a shear stress sensor as there is for pressure or temperature sensors. The final line in the paper succinctly summarizes the challenge: “The unique combination of applying the difficult field of MEMS design to the equally complex shear stress measurement
challenge can be both a rewarding and frustrating endeavor and will most likely not be solved tomorrow.”

### 1.3 Shear Stress Measurement Methods

The earliest recorded attempts at quantitatively measuring shear stress were most likely conducted by Froude [11] in 1872. He studied the skin friction experienced by planks dragged across water for the purpose of understanding the drag forces on ships traveling through water. Since those experiments nearly 140 years ago, researchers have attempted to measure shear stress using a variety of measurement techniques and analytical methods. Much of this effort was driven by the aerospace and naval industries, and their efforts at reducing drag on rockets, submarines, and other vehicles. More recently however, there has been an additional emphasis on measuring turbulent fluctuating shear stress as part of an effort to understand turbulent flow phenomenon.

Rather than offer a detailed description of the history of shear stress measurements, this section will primarily discuss different types of shear stress sensors. For a thorough overview of the many methods utilized to measure shear stress, the reader is referred to the following measurement reviews: Winter [12], Hanratty and Campbell [13], Fernholz et al. [14], Naughton and Sheplak [15], Kornilov [16], and Sheplak et al. [17]. The next section is a brief description of some of the more common shear stress sensor designs and their benefits and challenges. Discussion of specific sensor performance will be discussed in Sec. 1.3.3.

Typically shear stress measurements are divided up into direct and indirect methods. Direct methods are named as such because the shear stress is “directly” measured, in contrast
with “indirect” methods where shear stress is calculated through a relationship with another measured quantity such as heat or mass transfer.

1.3.1 Direct Method Sensors

1.3.1.1 Floating Element Sensor Designs

The most widely used method of directly measuring shear stress is by using a floating element. This is a small platform that “feels” the force of the shear stress, and is attached to a spring (or multiple springs) so that it deflects a small amount when placed in the flow (Figure 1). This deflection can be sensed in many different ways by using optical, capacitive, piezoresistive, or piezoelectric methods. Closed-loop setups can also be utilized where the floating element is held in place by feedback force, and the shear stress force is calculated by assuming that it is equal to the necessary feedback force. The main benefit of the floating element over other methods is that theoretically, one could flush-mount a sensor inside a wall, and as long as the boundary layer is not disturbed significantly, the shear stress can be measured without any prior knowledge of the flow. In a real flow situation, the sensor would also have to be designed with the proper measurement range and resolution, as well as be able to survive the temperatures and flow forces that it would be exposed to.
Floating element shear stress sensors have been used since the mid-1900s, with various levels of success. Early sensors, commonly referred to as “skin friction gages” were made from macro-scale components $O(10^{-2} \text{m})$. These sensors were mainly used for aerospace and naval applications and could only measure mean shear stress. Typical designs included a cylindrical element with a small clearance inside of a plug flush-mounted in a wall. The element was displaced when exposed to the shear stress force and the deflection was commonly measured using a LDVT (Linear Differential Voltage Transformer) or strain gauges.

Figure 2 is a cross-section of the macro-scale floating element sensor from Brown and Joubert [18] with a sensing element 1.9cm (0.75in.) in diameter and a displacement that was measured using a LDVT. This sensor was developed in 1969 and tested in a low-speed wind tunnel with an adverse pressure gradient. Figures 3 and 4 show later designs from Allen [19] in 1980 and Goyne and Stalker [20] in 2003, respectively, which have improved sensing techniques, but ultimately retain the same overall design.
Figure 2 (L): Floating element sensor of Brown and Joubert [18] which has a sensing element diameter of 1.9cm (0.75in.) and uses an LDVT to sense displacement.
The majority of recent floating element designs have utilized MEMS fabrication techniques. MEMS technologies allow for the fabrication of sensor features as small as 1\(\mu\)m, and can include multiple stacked layers of different metal and semiconductor materials. The most popular design consists of a rectangular floating element, with springs formed from long, thin beams fabricated in the same material layer. The material underneath the floating element and springs are removed by chemical etching, “releasing” the sensor and allowing it to move when exposed to a shear flow.

Because there are so many different designs for floating element MEMS sensors, only a few are described here to illustrate the breadth of design variations. Figure 5 is a sensor design from Barlian et al. [21], which utilizes implanted piezoresistors in the springs. When the springs bend, the stress changes the piezoresistance which is measured using a Wheatstone bridge. Hyman et al. [22] used a floating element comb drive (Figure 6) where displacement is sensed by
measuring a differential capacitance change. Long fingers attached to the floating element have matching stationary fingers which form capacitors with air acting as the dielectric. When the floating element moves, the capacitance will change depending on the direction of motion. Multiple sets of fingers are used to increase the overall capacitance and sensitivity. Zhe et al. [23] used a differential capacitive sensing scheme (Figure 7) with a cantilever spring design, and the floating element acting as one side of a parallel-plate capacitor.

In addition to different shear sensing architecture, floating element designs are custom-made depending on the intended application. For example, a sensor designed for a high-temperature laminar industrial flow will likely have a much different design than one designed for high-speed turbulent external flow. Furthermore researchers have utilized different ways to characterize sensors (both with and without flow) that are custom-tailored to the specific sensor design. This will be discussed further in Section 1.3.3.

Figure 5: Floating element sensor of Barlian et al. [21] utilizing piezoresistive implants in the spring.
Figure 6: Floating element sensor of Pan et al. [22] utilizing capacitive sensing and a multiple finger comb drive design.

Figure 7: Floating element sensor of Zhe et al. [23] using a cantilever spring design and differential capacitive sensing.
The main benefits of floating-element designs are that a properly designed sensor would be non-intrusive to the flow and able to measure shear stress in variety of flows without much prior knowledge of the flow (in contrast to indirect measurement methods). However, there are significant design challenges to making accurate floating element sensors which have hindered development efforts. Winter [12] developed a list of these challenges:

(1) …the compromise between the requirement to measure local properties and the necessity of having an element of sufficient size that the force on it can be measured accurately.

(2) The effect of the necessary gaps around the floating element.

(3) The effects of misalignment of the floating element.

(4) Forces arising from pressure gradients.

(5) The effects of gravity or of acceleration if the balance is to be used in a moving vehicle.

(6) Effects of temperature changes.

(7) Effects of heat transfer.

(8) Use with boundary-layer injection or suction.

(9) Effects of leaks.

(10) Protection of the measuring system against transient normal forces during starting and stopping if the balance is to be used in a supersonic tunnel.
The miniaturization of floating element sensors by using MEMS technology has mitigated many of these effects, especially (2)-(5), (9), and (10) which are the direct results of the macroscale components used in many earlier skin friction gauges. However, the use of MEMS sensors adds additional challenges:

(11) Effect of the gaps in the package.
(12) Effects of misalignment of the package.
(13) Susceptibility of the floating element to dust and oil particles.
(14) Possible sensitivity to EMR or light.
(15) Effects of electrostatic forces needed for electrical sensing methods.
(16) External equipment needed for optical sensing of deflection.
(17) May be sensitive to properties of flow media (e.g. electrical, thermal, etc.)

Ultimately MEMS sensors offer many more benefits than the traditional macro-scale floating element sensors which is why they represent the majority of new floating element shear sensors currently being developed.

1.3.1.2 Oil-Film Interferometry

This method of measuring shear stress does not use a sensor, however it is included as it is the most common direct method of measurement that does not use a floating element sensor. Interferometry [15] is used to determine film thickness by reflecting light off of it, and viewing the interference patterns (Figure 8). This can be used to measure the wall shear stress on a surface by applying a thin layer of oil to the surface, and recording the interference patterns with
a camera as they change with the flow. The transient film thickness can be directly related to the shear stress using fluid mechanics equations. Measurements can be single-point, line or consist of an entire surface. The primary challenges with this measurement method are that the oil needs to be reapplied for continuous testing; it requires an external light source and camera, making it not well-suited for internal flows.

**Figure 8 (L):** Diagram [15] showing interferometry on an oil film. The top image is constructive interference, and bottom image is destructive interference.  
**Figure 8 (R):** Schematic of typical oil-film interferometry set-up [15].

### 1.3.2 Indirect Methods

Many sensors have been developed that use indirect methods of measurement where the shear stress is “indirectly” related to another quantity that is measured. Generally indirect sensors rely on three different methods: 1) using a relationship between a measured velocity profile and shear stress, 2) using a relationship between heat transfer (or mass flux) and shear stress, or 3) generating a pressure change in the viscous sublayer which can be related to shear stress.
1.3.2.1 Thermal / Hot-Wire Sensors

Hot-wire anemometry (HWA) is a method of measuring fluid velocity by heating a thin wire (resistor) and determining the voltage needed to keep the wire at a constant temperature. This method is implemented by measuring the sensor resistance which can easily be related to the temperature. HWA works well for measuring freestream velocities but is considered too inaccurate to resolve the velocity gradient in the viscous sublayer [24]. In lieu of this, wall-mounted hot wires are utilized where the heat transfer from the sensor can be related to the shear stress via the Reynolds Analogy (eq. 1). Wall hot-wires can be slightly set above the wall, remaining inside the viscous sublayer, or flush-mounted on the wall [25]. A similar design is the hot-film which uses the same methodology but are typically made using thin-film MEMS fabrication techniques (Figure 9).

Hot-wire and hot-film sensors must be calibrated against a known shear stress in a similar flow situation. For example a separate calibration is required for laminar and turbulent flows due the difference in the heat transfer mechanism in the flows. A major challenge with thermal sensors is heat conduction to the substrate. This has limited the applicability of thermal sensors to flows with low thermally conducting fluid (e.g. air) as more heat will be transferred to the substrate than the flow. This limits the static sensitivity and the dynamic frequency response of the sensor due to the transient nature of the process. To mitigate this effect, recent sensors are fabricated with a vacuum cavity underneath the heating element (Figure 9). Additional limitations found with hot-film and hot-wire sensors are their inability to measure flow recirculation and difficulties in designing a calibration method that can be used in non-isothermal flows [17].
Wall Pulsed Wire probe is a hybrid of hot-wire and hot-film composed of three similar sized wires flush-mounted in the wall [27] [28]. The center wire heats up the immediate fluid which then moves with the flow as a “tracer”. The tracer can go up or downstream, allowing for the measurement of flow in two directions, useful in flows with separation or recirculation. Similar to other thermal sensors, the wall pulsed wire probe requires calibration in a known shear stress; typically a Preston tube has been used. A major limitation of this sensor type is that it suffers from a low frequency response, caused by the travel time of the heated fluid from the center wire to an adjacent wire. Typical developed sensors have frequency responses that are $O(\text{Hz})$, which makes them incapable of measuring turbulent fluctuations in high-speed flows [14].

1.3.2.2 Laser Doppler Velocimetry

Laser Doppler Velocimetry (LDV) is a well-established method for measuring fluid velocity by scattering laser light off of small particles in the flow and using the resulting Doppler shift to calculate the fluid velocity at different points [29]. Durst et al. [30] used this method to
determine the wall shear stress in turbulent pipe and channel flows. This is a very useful method because it is non-intrusive to the flow, however two major drawbacks are that it requires flow seeding and a large amount of external equipment. Recently similar technology has been condensed into a small thru-wall sensor [26] which uses diverging fringe patterns projected into the flow to determine the velocity gradient (Figures 10 and 11).

![Figure 10 (L): Schematic of laser doppler velocimetry concept [26].](image1)
![Figure 11 (R): Working model of diverging fringe thru-wall LDV sensor [26].](image2)

**1.3.2.3 Micro-PIV**

Particle Image Velocimetry (PIV) is similar to LDV in that the flow is seeded with small particles in order to measure the velocity profile. The main difference, however, is that PIV uses a camera to record the flow and capture velocity by analyzing successive images. Recently efforts have been aimed at developing Micro-PIV sensors, with the ability to resolve near-wall velocity components. Readers are referred to Depardon et al. [31] and Kähler et al. [32] for more discussion on this topic.
1.3.2.4 Micro-Cantilevers and Micro-Pillars

Various sensors have been developed around the deflection of a miniature cantilever or vertical cylinder (pillar) mounted on the wall inside a boundary layer [33], [34], [35], and [36]. Deflection can be related to the flow velocity, and then to pressure or shear stress. Sensor heights typically range from 10 - 1000μm depending on the flow conditions being measured.

The Micro-Pillar wall shear stress sensor (MPS³) is a vertical cylindrical pillar made from PDMS [37]. The pillar is mounted on the wall, inside the viscous sublayer, and deflects due to the drag of the flow around it. The flow is in the regime of Stoke’s flow, and a relationship between the pillar tip deflection and the wall shear stress is developed using linear bending theory. The deflection is captured using a high-speed camera viewing the pillar tip from above. Arrays of MPS³ can be fabricated, allowing for the measurement of fluctuating shear stress at multiple points. The main negative quality of this sensor design is that it requires an externally mounted camera which limits the applications of this sensor. For more information about the Micro-Cantilevers and Micro-Pillars, the reader is referred to the dissertation of Große [38].

Figure 12 (L): Conceptual illustration of Micro-pillar sensor [22].
Figure 13 (R): Schematic from Tihon et al. [40] of mass transfer probe downstream of backwards-facing step.
1.3.2.5 Mass Transfer / Electrochemical Sensors

Indirect shear stress sensors have been developed that utilize the similarities between fluid flow and mass diffusion, described in Section 1.1, (eqs. 5–8). These are also referred to as “electrochemical” or “electrodiffusion” sensors. They operate similar to constant temperature hot-wires and hot-films in that the concentration of the diffusing species is held constant on the sensor surface. The current is proportional to the rate of mass transfer to the electrode which can then be related to the velocity gradient and the wall shear stress [39].

Successful electrochemical probes have been developed to be used in low-speed turbulent flows. For example, Tihon et al. [40] used a flush-mounted probe to measure turbulent recirculating flow downstream of a backward facing step (Figure 13). However, this type of probe suffers from similar challenges to thermal sensors, notably a low frequency response O(10^2 Hz) [41], and the requirement to calibrate the sensor in a known, similar shear flow. Additional information on mass transfer sensors can be found in the shear stress review of Winter [12].

1.3.2.6 Pressure Drop Methods

The Stanton tube, Preston tube, and surface fence all operate by the principle of obstructing a flow and measuring the resulting pressure drop. The Preston tube is a flattened Pitot tube placed on the wall, the Stanton tube is a razor blade partially covered a pressure tap in the wall, and the surface fence is a thin rectangular wall the protrudes vertically into the flow (Figure 14). When the obstruction is contained within the viscous sublayer, the pressure drop and the shear stress can be related to flow properties by creating a function, ‘g’, based on non-dimensional variables [42]:
\[ \frac{\Delta P}{\tau_w} = g \left( \frac{Ud}{v} \right) \]  \hfill (21)

where \( \Delta P \) is the differential pressure across the obstruction, \( U \) is the freestream velocity, \( v \) is the kinematic viscosity, and \( d \) is the diameter of the Pitot or Stanton tube, or \( d \) is the height of the surface fence. Calculation of the relationship (‘\( g \)’) requires flow testing and calibration against a known shear stress, and will be specific to the flow regime.

Due to the simplicity of the designs, pressure drop methods were the most commonly used shear stress measurement methods other than macroscale floating element skin-friction sensors. Preston Tubes were often utilized in high-speed aerodynamic flows, and are generally considered to have an accuracy of within 5% [32]. The Preston Tube has also been applied to other flow regimes (e.g. pressure gradient, heat transfer, tube falls outside viscous sublayer, etc.) but has come under much criticism because eq. (21) is not functionally correct outside of the viscous sublayer[43]. More recently Schober et al. [44] have developed a MEMS surface fence which uses a piezoresistive measurement rather than a pressure differential, but still operates using the same basic principal, eq. (21). Further discussion of pressure drop methods can be found in the previously mentioned measurement reviews.

Figure 14: Diagram of Pitot Tube (Left), Stanton Tube (Center), and Wall Fence (Right).
1.3.3 Relevant Floating Element Sensor Designs

Due to the large number of shear stress sensor developed over the years, discussion of specific sensor performance will place an emphasis on MEMS floating element designs that have been flow-tested. There are many sensors that have been designed and characterized, yet never made it to the flow testing stage. These sensors are considered “relevant” to this project, because they share similarities in either sensor design or in the flow testing regime. A summary of the major features of these designs is shown in Table 1.

In 1988, Schmidt et al.[45] developed the first MEMS floating element shear stress sensor. This design had a polyimide floating element that was 500μm x 500μm in size. The sensor used an on-chip differential capacitive measurement and was calibrated in a laminar flow cell up at 12Pa. Although designed for turbulence measurements, the sensor was never flow tested in that manner. The authors reported that the sensor suffered from a weak output signal that was susceptible to EMI, as well as electrical drift from water-vapor absorption of the polyimide.

Shajii et al. [46] developed floating element sensors for polymer extrusion applications that could measure high shear stresses of 1 to100kPa. The sensing element was 120μm x 140μm and calibrated using liquid rotating in cone flow up to 11.5kPa. Later, Goldberg et al. [47] created a larger sensor for the same application (500μm x 500μm floating element) that utilized backside connections to reduce flow disturbances. Both sensors used standard piezoresistive methods of measurement, and survived harsh industrial processing conditions. The sensitivity of the sensors was too low for them to be used for turbulence measurements.

Hyman et al.[22] used a typical floating element comb-drive design (Figure 6) to measure laminar parallel plate shear stress up to 50Pa. Sensor displacement was measured using a
differential capacitance setup as well as by optical measurements, similar to the methods used in this thesis.

Zhe et al. [23] used a cantilever floating design (Figure 7) to measure shear stress in laminar parallel plate flow from 0.04Pa to 0.16Pa. This sensor also used the same differential capacitive sensing chip (MS3110 IC) that will be implemented in this thesis. This sensor should have been able to measure reverse flow (due to the frontside contacts being off to the side of the flow), although the author did not report any test data of that kind.

Padmanabhan et al. [48] developed a floating element design that utilized photo-diodes in order to sense displacement. The sensor was statically calibrated over a range of laminar shear stress from 0.0014Pa to 10Pa. This sensor was also dynamically tested up to 10kHz using an acoustic plane wave tube (PWT) which created an oscillating shear stress. Further testing in a wind tunnel showed satisfactory turbulent flow measurements [49]. A major impediment associated with this sensor design is that it requires an external light source, which severely limits the eventual applications of this sensor design.

Recently, Li et al. [50] and Chandrasekharan et al. [51] [52] have been developing floating-element designs that use piezoresistive and capacitive sensing, respectively. Both sensors have been tested in turbulent flows up to approximately 2Pa, and have been characterized dynamically up to 6.7kHz using the PWT method described in [48].

Table 1 illustrates a broad range of MEMS floating element sensor designs over the past 21 years. The only sensor with backside pads that the author is aware of is the one of Goldberg et al. [47]. The most common spring type is the tensile-beam design, although cantilever and folded-beam designs are also utilized (see Section 2.2.1 for an explanation on different spring types). All of the devices on the list (except for Hyman, Pan, Reshotko, Mehregany [22]) use a
single Si device layer for operation, with other layers only used for insulation and the electrode connections. Hyman et al.’s device requires three connected layers of Si, due to the arrangement of the sensing electrodes, which significantly increases the fabrication challenges. The sensor effort described in this thesis is a blend of previous efforts, but is unique in that it will use backside pads with a single layer design and two different spring types, while being designed to measure turbulent shear stress measurements at a sub-kPa level (1-1000Pa).
Table 1: Summary of relevant floating element shear stress sensors.

<table>
<thead>
<tr>
<th>Authors (year)</th>
<th>Detection Scheme</th>
<th>Package Connection</th>
<th>Test Setup</th>
<th>Spring Type</th>
<th>Device Layers</th>
<th>Shear Meas. Range</th>
<th>Can Measure Reverse Flow?</th>
<th>Dynamic Meas.?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shajii, Ng, Schmidt [46] (1992)</td>
<td>Piezoresistive</td>
<td>F</td>
<td>Liquid Rotating Cone Flow</td>
<td>Tensile-Beam</td>
<td>Single Layer</td>
<td>Up to ~11.5k Pa</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Goldberg, Breuer, Schmidt [47] (1994)</td>
<td>Piezoresistive</td>
<td>B</td>
<td>Liquid Polymer Extruder</td>
<td>Tensile-Beam</td>
<td>Single Layer</td>
<td>1kPa to 100kPa</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Hyman, Pan, Reshotko, Mehregany [22] (1999)</td>
<td>Capacitive, Optical</td>
<td>F</td>
<td>Channel Flow</td>
<td>Folded-Beam</td>
<td>Multilayer</td>
<td>Up to 50Pa</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Zhe, Modi, Farmer [23] (2005)</td>
<td>Capacitive</td>
<td>F</td>
<td>Parallel Plate Flow</td>
<td>Cantilever</td>
<td>Single Layer</td>
<td>0.04 to 0.16Pa</td>
<td>Y</td>
<td>N</td>
</tr>
<tr>
<td>Padmanabhan, Sheplak, Breuer, Schmidt [48] (1997)</td>
<td>Optical</td>
<td>F</td>
<td>Parallel Plate and Wind Tunnel</td>
<td>Tensile-Beam</td>
<td>Single Layer</td>
<td>0.0014 to 10Pa</td>
<td>N</td>
<td>Y</td>
</tr>
</tbody>
</table>

Key – Package Connection: B = Backside, F = Frontside, Device Layers: Single Layer = Only one Si device layer, Multilayer = More than one Si device layer needed for operation Can Measure Reverse Flow?: Y = Can measure reverse flow, N = Cannot measure reverse flow Dynamic Meas.?: Y = Paper reports fluctuating measurement or dynamic testing, N = Does not report
1.4 Previous Research on Misalignment of Shear Stress Sensors

A major area of concern for wall shear stress sensors is the effect of sensor misalignment on the measurement. “Misalignment” is a broad term used to refer to any steps, protrusions, or gaps that are present in the wall as a result of the sensor. Shear stress sensors are typically thru-wall mounted packages or they are thin layers which are directly attached to the surface of measurement. In either case there will be boundary layer perturbation, raising the question of whether or not the local shear stress, and hence the measurement, has been affected by the presence of the sensor.

Misalignment has not played a large role in the research of MEMS shear stress sensors. The majority of MEMS sensors have been utilized in low-speed, low-shear flows, which have a viscous sublayer ($y^+=6$) height that is typically $O(10^{-4}m)$ or greater. The MEMS sensors have small feature sizes $O(10^{-6}$ to $10^{-5}m)$, so that gaps and protrusions will typically fall within the viscous sublayer, and are considered to have a negligible effect on the flow structure[9]. This is also referred to as the flow being “hydraulically smooth.” However, in high-speed, high-shear flows (expected for the current sensor design), the viscous sublayer height can be $O(10^{-6})$ or smaller, resulting in sensor features that are not considered hydraulically smooth and can affect the wall shear stress that is “felt” by the sensor.

1.4.1 Macro-scale Sensor Misalignment

Thorough investigations quantifying sensor misalignment were conducted by O’Donnell [53] and Allen [54] [55] between the mid-1960s and the early 1980s. This research focused on the misalignment in macro-scale floating element skin friction sensors, including protrusion and
recession, and variations in gap size. The major conclusions of this research were that: 1) larger
gaps helped to reduce the effect of misalignment, 2) Re has only a minor effect on misalignment,
and that 3) there is no preference for sensor protrusion or recession. In these studies the main
cause of the error in the measurement was an unbalanced pressure gradient (dp/dx) across the
floating element.

More recently, MacLean [56] conducted CFD analysis on a macro-scale floating element
sensor in a variety of flow situations (e.g. zero dp/dx, favorable dp/dx, adverse dp/dx) to evaluate
the effect of misalignment on the measured shear stress. The CFD results demonstrated similar
trends as the experimental data of Allen, and MacLean suggested that with proper design many
of the misalignment effects could be minimized.

However, these results are not particularly relevant to modern micro-scale shear stress
sensors because in the earlier research, the main driver was the unbalanced pressure-gradient. In
microscale sensors, this dp/dx force is negligible compared to other forces, as will be discussed
later in Section 2.5.2.

1.4.2 Micro-scale Sensor Misalignment

We were unable to locate any studies of misalignment affects in MEMS sensors other
than those of Spazzini et al. [57]. They developed a hot-wire based probe mounted in a
cylindrical package consisting of two hot-wires mounted over a cavity. The wake generated from
the flow over the first wire would affect the heat transfer for the second hot-wire. The difference
in the heat transfer rates from the wires was then calibrated with shear stress in a turbulent
channel flow from 0.05Pa to 0.9Pa. To evaluate the effect of non-flush wall mounting, the
package was recessed at 0.4mm and 0.8mm below the channel wall which correspond to 2% and
4% of the channel height, respectively. The recess distance was 10% and 20% of the distance from the step to the sensing element. Figure 15 shows the sensor voltage output normalized by the reference output (no recess), which indicate a large effect on the sensor output. For the smaller recession of 0.4mm, the output quickly reduces by approximately 30% for most of the shear stress range, while for 0.8mm recess, the output drops below 10% at lower shear stress values and then seems to approach a constant value of 56%. The authors noted that the second data set may not be accurate as the sensing element may fall within a region of reverse flow from being located downstream of a step. These results illustrate the drastic effects that can be caused by incorrect package installation of MEMS shear stress sensors.

Figure 15: Results from Spazzini [57] for sensor recession (misalignment) testing.

Many researchers have tested their sensors in low-speed flows where it is valid to make the “hydraulically smooth” assumption (i.e. all sensor features are contained within the viscous
sublayer). However, in high speed turbulent flows, \( \mu \text{-scale sensor features will not always be contained within the viscous sublayer. We can analyze the relationship between the height of the viscous sublayer and the wall shear stress by using eq. (17) and solving the inequality } y^+ < 6 \text{ to receive:}

\[
y < \frac{6 \nu \sqrt{\rho}}{\sqrt{\tau_w}}
\]

(22)

If we insert the properties of air at standard conditions: \( \nu \sim 10^{-5} \text{ [m}^2/\text{s}] \), \( \rho \sim 1 \text{ [kg} / \text{m}^3] \), the result is:

\[
y < \frac{6 \times 10^{-5}}{\sqrt{\tau_w}}
\]

(23)

and we can see that the height of the viscous sublayer is related to the inverse square root of the shear stress. This means that if the shear stress is 1Pa, vertical features should be less than 60\( \mu \)m to not perturb the boundary layer, and if the shear stress is 100Pa (typical of our design range), vertical features should be less than 6\( \mu \)m. It is clear that as the shear stress increases, the walls will not be hydraulically smooth, and the effect that this has on the sensor measurement should be evaluated.

Furthermore there can be additional gaps or protrusions caused by the sensor mounting inside the package and from the package being installed in the flow setup. These features will likely be macro-scale unless care is taken to minimize them. Analysis of these scenarios would likely include modeling of shear stress downstream of forward-facing or backward-facing steps, or downstream of a cavity (or a slot). While much research has been done on these three types of flow, the majority of it did not involve shear stress measurements, due the challenges of creating reliable sensors.
1.5 Previous Research on Fluctuating Shear Stress

The fluctuating wall shear stress in turbulent flows has been determined using a variety of sensor methods, such as floating element, thermal, electrochemical, and micro-pillar sensors. Computational methods such as DNS (Direct Numerical Simulation), which finds exact solutions for turbulent flows, have also been utilized to analyze low Re flows ($<10^4$) [58]. There are many different quantities associated with fluctuating shear stress [59], but we will focus primarily on the following four areas for the streamwise flow:

1) Turbulent Shear Stress Fluctuations
2) PDF (Probability Density Function)
3) Spectral Density
4) Pre-multiplied Spectral Density

1.5.1 Turbulent Shear Stress Fluctuations

In a turbulent wall-bounded flow, shear stress fluctuations are calculated by subtracting the mean shear stress from the time-varying shear stress:

$$\tau_{w,x}'(t) = \tau_{w,x}(t) - \overline{\tau_{w,x}}$$

(24)

The intensity of the turbulent fluctuations is calculated by taking the root-mean-square of the shear stress fluctuations, which averages the absolute intensity of the fluctuations over a period of time:

$$\tau_{x,RMS} = \sqrt{\left(\tau_{w,x}'(t)\right)^2}$$

(25)
The turbulent fluctuations are normalized by the mean shear stress, in order to compare different flows. Reported values for this ratio commonly range between 0.3 and 0.4, and there is currently no general consensus on whether or not this ratio is affected by the Reynolds Number (Re) of the flow.

In 1988, Alfredsson et al. [60], utilized hot-film measurements in air, oil, and water flows to determine turbulence intensity. They conducted tests in both channel flows and boundary layer flows and measured a constant ratio of 0.4, independent of Re. Prior to Alfredsson, the literature included reports of constant values ranging from 0.05 to 0.4 and independent of Re. In 1994, Wietrzak et al. [61] tested a hot-wire probe mounted on a transverse cylinder in 10m/s wind tunnel and measured a constant value of 0.32, also independent of Re. More recently, in 2003, Colella and Keith [62], conducted measurements using an array of flush-mounted hot-wall shear sensors attached to a plate dragged in a tow-tank facility. They reported values that decreased from 0.36 to 0.25 as Re increased, and theorized that sensors were attenuating measurements as Re increased. This was because shear stress fluctuations become smaller as Re increases (this will be discussed more in Section 2.6), so the sensor ends up spatially averaging multiple measurements. Große [63] used a micro-pillar sensor to take shear stress measurements in water flowing through a pipe. His measurements decreased from 0.39 to 0.34 as Re increased, and he theorized that this decrease may have been caused by spatial averaging from the micro-pillar protruding further into the near-wall region as Re increased (the height of the viscous sublayer decreases with Re). Große also installed the same sensor on a flat plate in a high-Re wind tunnel but reported only one data point at 0.37, noting that the value decreased slightly with Re. Abe and Kawamura [64] and Hu et al. [65] both conducted DNS simulations on turbulent channel
flow data, and found shear stress intensities that increased slightly with $Re$, with good agreement among their results.

The turbulent fluctuation intensity data that has been discussed so far is plotted in Figure 16. Individual data points are connected with lines to emphasize trends in the data. The $x$-axis is $Re_\tau$, the Reynolds Number based on the friction velocity ($U_\tau$), which has been more commonly reported in recent years:

$$Re_\tau = \frac{\rho U_\tau L}{\mu} \quad (26)$$

$Re_\tau$ is used because it is a scaling based on the near-wall velocity of the flow rather than large-scale features such as the duct height or momentum thickness. Also, since previous researchers used $Re$ definitions based on different length scales to report their data, $Re_\tau$ acts as a more universal representation of turbulent flows. In the graph, the data of Alfredsson and Wietrzak are drawn as straight lines since they measured constant values and did not provide enough information to calculate the measured $Re_\tau$ values.

Looking at the data as a whole, there is a lot of scatter and not much agreement among the trends. A positive result is that nearly all of the data points lie between 0.3 and 0.4. The largest $Re_\tau$ value that we were able to locate was 3105, for the flat plate measurements of Große. Since two of the groups voiced concerns about sensor attenuation and averaging, there needs to be more emphasis on creating sensors that do not suffer from these limitations. This remains an area of active investigation where much research remains to be done.
1.5.2 PDF (Probability Density Function)

For turbulent shear stress fluctuations, the probability density function (PDF) is used to represent the strength of the fluctuations compared to the average intensity, as well as their likelihood of occurrence. The PDF is calculated by taking a shear stress measurement in time, and determining how often a fluctuation is between a range of values. The curve is normalized such that its integral is 1. Typical results are represented by the data plotted in Figure 17, referring to the researchers described in the previous section. A PDF of a Gaussian curve is also included for comparison.

There is good agreement among researchers on the general shape of the PDF. The fluctuation strength with the maximum probability of occurring lies between -0.55 and -0.8,
implying that the most common fluctuations are negative (i.e. their magnitude is lower than the mean value). The tail on the positive side of the curve is longer than the negative side. This is where the strongest but least frequent occurring fluctuations lie. Additionally, there is agreement that Re dependence is either minimal or non-existent.

![Figure 17: PDF results for turbulent shear stress fluctuations](image)

### 1.5.3 Spectral Density

Spectral density calculations take a time-varying signal (in this case fluctuating shear stress) and convert it to the frequency domain using Fourier transforms. This is done so that we can analyze the different frequency components of a signal. The spectral density of the signal ($\Phi$) is calculated and normalized as:

$$\int_{0}^{F} \Phi(f)df = \left(\tau_{RMS}'(t)\right)^2$$  \hspace{1cm} (27)
where $f$ is frequency, and $F_s$ is the Nyquist frequency ($1/2$ the sampling frequency). To non-
dimensionalize $\Phi$ and $f$ we use an inner time-scale ($t_i$) given by:

$$t_i = \frac{\nu}{u_\tau}$$

(28)
to create two non-dimensional variables, $\Phi^+$ and $f^+$:

$$\Phi^+ = \frac{\Phi}{(\tau_{x,RMS})^2} \cdot \left(\frac{1}{t_i}\right) = \frac{\Phi}{(\tau_{x,RMS})^2} \cdot \left(\frac{u_\tau^2}{\nu}\right)$$

(29)

$$f^+ = f \cdot (t_i) = f \cdot \left(\frac{\nu}{u_\tau^2}\right)$$

(30)

Figure 18 shows spectral density results for four sensors that were described in the previous two
sections. There is generally good agreement among the results in that the curves have similar
shapes over most of their frequency values ($f^+$), and even contain some common values. In
general, $\Phi^+$ approaches zero as $f^+$ increases, which can be caused by the dissipation of higher
frequencies in the flow, or from the limitations of sensor response to the higher frequencies of
the flow. Furthermore, for the four sensors shown, the domain of $f^+$ spans 3-4 orders of
magnitude and $\Phi^+$ has a range of 5-6 orders of magnitude. This data is useful in that it provides a
comparison for the expected spectral density values of a sensor measuring turbulent fluctuations,
as well as the approximate shape of the curve of the graphed data.
1.5.4 Pre-multiplied Spectral Density

The pre-multiplied spectral density is calculated by multiplying the non-dimensionalized spectral density ($\Phi^+$) and the non-dimensionalized frequency ($f^+$). When the data is plotted, it is normalized so that the area under the curve is equal to 1. The pre-multiplied spectral density allows for visualization of the highest energy containing frequencies of the sensor signal. Figure 19 shows data for three of the sensor described in the previous two sections. We can see that they all have energy peaks centered near an $f^+$ value of 0.01, where most of the turbulent energy is located. At higher and lower frequencies, the values approach zero. This is useful in that it allows for determination of whether or not the entire turbulence spectrum is being captured.
Figure 19: Normalized and non-dimensionalized pre-multiplied spectral density.

1.5.5 Turbulent Wall Pressure Fluctuations

A MEMS sensor that is measuring the fluctuating shear stress ($\tau_w'$) near the wall of a turbulent flow will also be exposed to a fluctuating wall pressure ($P_w'$). This wall pressure will result in a force that is normal to the floating element surface and can create motion in the out-of-plane direction, affecting the sensor measurement. Hu et al. [65] calculated both the streamwise shear stress fluctuations and wall pressure fluctuations using DNS (Figure 20), and found that wall pressure fluctuations are larger across the entire turbulence spectrum. At lower $f^+$ values ($f^+ < 0.05$), $P_w'$ is less than 10 times larger than $\tau_w'$ however, at certain frequencies, $P_w'$ can be as much as two orders-of-magnitude larger than shear stress fluctuation (near $f^+ \sim 0.5$).

MEMS shear stress sensors are designed to be less sensitive to forces in the out-of-plane direction, however analysis is required to quantify the amount that the pressure fluctuations may
be affecting the sensor measurement. The effect that the wall pressure has on the sensors developed for this thesis will be discussed further in Section 2.6.2.

![Normalized and non-dimensionalized pre-multiplied spectral density for shear stress fluctuations (dashed line) and wall pressure fluctuations (solid line) [65]. Note that both the x and y axes are log scales.](image)

Figure 20: Normalized and non-dimensionalized pre-multiplied spectral density for shear stress fluctuations (dashed line) and wall pressure fluctuations (solid line) [65]. Note that both the x and y axes are log scales.

## 1.6 Potential Applications

### 1.6.1 Flow Control

MEMS shear stress sensors can be utilized in flow control systems aimed at reducing drag. Flow control systems generally are divided into two main categories: passive control and active control. Passive control generally involves changing the geometry of the flow using riblets, or large-eddy break-up devices (LEBUs) without requiring any specific control effort. Active control schemes involve utilizing energy to directly affect flows and can further be
separated into pre-determined and reactive systems. Pre-determined setups implement a control effort regardless of the specific state of the flow, while reactive systems implement a control effort based on measured sensor quantities [66].

Figure 21 (L): Diagram showing setup for flow control of air over bluff body. The spinning cylinder at the rear delays boundary layer separation reducing drag [67].
Figure 22 (R): Diagram showing near wall vortex counteracted by blowing and suction at the wall (Lee et al. [68]); streamwise flow direction (x) is into page.

MEMS shear stress sensors are well-suited for a reactive control system where $\tau_w$ is measured at one or more locations, and a control effort is applied using an actuator (macro or micro-scale). Beaudoin et al. [67] used adaptive control to reduce drag over a bluff body (Figure 22). Drag was measured using strain gauges attached to the bottom of the bluff body, and a spinning cylinder was attached at the rear of the body to delay flow separation, reducing drag. Control was implemented by varying the rotational speed of the cylinder, and was optimized to reduce the cost, defined as the sum of the aerodynamic power (drag x freestream velocity) and the electric power (voltage x current). In a similar manner, a MEMS shear sensor mounted on a bluff body can be utilized in attempts to reduce the drag with control being implemented via micro-flaps, micro-jets, wall suction or other methods.

Early flow control schemes generally focused on pre-determined wall suction to prevent the boundary layer from transitioning from laminar to turbulent flow. Recently it has been
demonstrated that greater drag reduction can be acquired applying sporadic blowing/suction to turbulent flow. Sporadic suction involves locating low-speed near-wall vortices and counter-acting them using a combination of blowing and suction (Figure 21). Numerical studies [68] have shown that using closely-spaced wall-mounted sensors with blowing and suction in a checkerboard layout can result in a 6% decrease in drag.

MEMS sensors are well-suited for applications requiring multiple sensors and actuators to be located in a small flow area. Once batch fabrication techniques are refined for a particular sensor/actuator pairing, a yield of 50 or more sensor/actuator pairs per wafer is very realistic. These qualities allow for flush-mounted MEMS wall shear stress sensors to offer a practical method for attempting to reduce drag by focusing on the near-wall vortices in turbulent flow.

1.6.2 Computational Fluid Dynamics (CFD) Modeling

Computational fluid dynamics (CFD) modeling is used to calculate fluid flows where analytical solutions are not available. Applying CFD to turbulent flows, except in the case of DNS (Direct Numerical Simulation), requires making assumptions about the turbulence structure using closure models. Currently, DNS models are limited to low Re flows (~10^4) due to the large computational requirements, which scale with Re^3 [58]. Therefore, it is necessary to use 1st and 2nd order turbulence models, which were developed specifically for different flows (e.g. separated, recirculating, transitioning flow etc.). A MEMS shear stress sensor would offer the ability to experimentally verify and quantify the separation, recirculation, or transition in these flows, which are necessary inputs for the current CFD codes, but also could be used for validation of developing CFD codes.
When utilizing CFD codes, a fundamental quantity necessary for properly setting up the fluid mechanics model is the friction velocity, \( u_\tau \), which is a function of the wall shear stress \( \tau_w \) and the flow density \( \rho \):}

\[
    u_\tau = \sqrt{\frac{\tau_w}{\rho}}
\]  

The friction velocity controls the viscous length scales and the height of the viscous sublayer. When developing the equations in the near-wall region, generally, two methods are used, the wall function approach and the near-wall modeling approach (Figure 23). The wall function approach represents the viscous sublayer using semi-empirical equations, such as the law-of-the-wall:

\[
    u^+ = \frac{U}{u_\tau} = \frac{1}{\kappa} \ln y^+ + B
\]  

where \( \kappa = 0.41 \) and \( B = 5.0 \) are flow constants [9], and \( y^+ \) is the viscous length scale, also referred to as a “wall unit”:
\begin{equation}
y^+ = \frac{y u^*}{v} = \frac{y}{v} \sqrt{\frac{\tau_w}{\rho}}
\end{equation}

where \(v\) is the fluid’s kinematic viscosity and \(y\) is the distance from the wall.

The second method uses a non-uniform grid spacing, with the mesh becoming finer as the wall is approached. The grid size and spacing are closely related to \(y^+\), and should be small enough to resolve velocity gradients present in the viscous sublayer.

Implementing turbulent CFD codes requires knowledge about \(y^+\) in order to develop and run the model. To ensure accuracy of solutions, post-analysis is conducted to verify solution grid-independence and proper resolution of the near-wall region. Codes are often run multiple times, which can lead to unnecessary costs in both time and money. The MEMS shear stress sensor would be used to measure the wall shear stress in flows to aid in properly designing and applying the CFD codes. Additionally, it would be of great value in validating results of newly developed CFD codes.

1.7 Thesis Objectives and Scope

The main effort of this thesis is to develop a MEMS capacitive floating element sensor capable of measuring shear stress in the sub-kPa range (1 – 1000Pa) in a high-speed subsonic turbulent flow environment. This thesis was an offspring of collaboration between Columbia University, the Université de Sherbrooke (Quebec, Canada), and ATK-GASL (Ronkonkoma, Long Island, NY) in an effort to develop a MEMS Silicon Carbide shear sensor to measure shear stresses up to 10000Pa in high-temperature environments. Publications related to this work are listed as references [70] – [75].
The overall thesis objectives are described as the following:

- Electromechanical Design and Analysis of Sensor
- Design and Fabrication of Packaging
- Characterization of Sensor
- Mean Shear Stress Calibration for Sensor
- Fluctuating Shear Stress (Turbulence) Measurements

The remaining chapters of this thesis will discuss the efforts to accomplish those objectives. The chapters of the dissertation are described here for your convenience.

Chapter 2 describes the theoretical sensor design and performance using analytical and FEA modeling. Chapter 3 describes the sensor fabrication and the challenges encountered. The bulk of this work was conducted at the Université de Sherbrooke by Ronan Larger and Jean-Philippe Desbiens under the advisement of Prof. Luc Fréchette. Chapter 4 describes the sensor backside packaging as well as an alternate frontside packaging scheme that was used for initial flow testing. Chapter 5 describes electrical modeling of the sensor in terms of absolute and differential capacitance, and parasitic capacitances. Calculations are included to account for expected capacitance changes resulting from temperature variations in sensor physical properties. Chapter 6 describes the capacitance sensing circuitry and the effects of parallel resistance. A Pspice model is developed that will simulate sensor behavior when the parallel resistance changes due to temperature. Chapter 7 discusses sensor characterization that is non-flow based. This includes capacitance and resistance measurements, V-I measurements, mechanical actuation, and voltage
actuation tests. Chapter 0 discusses the fluid mechanics, the experimental setups, and the results for the wall jet flow testing and the duct flow testing.

Chapter 9.1 discusses conclusions from the measured data, contributions to the field, and suggested future work on this topic.

The main contribution of this thesis is the development of a MEMS floating element shear stress sensor with backside connections that can be used for high-speed turbulent flow measurements. Previous research efforts either did not have backside connections, or did not attempt to measure fluctuating shear stress in high-speed flows. This sensor effort is unique in that it accomplishes both. Measurements are also reported of the fluctuating shear stress intensity at Reynolds numbers (up to \(\text{Re}_r = 2 \times 10^4\)) much higher than any other data that we are aware of. More generally, this thesis also contributes to the general knowledge about designing capacitive MEMS sensors and testing them in high-speed flow environments.
2 MEMS Shear Stress Sensor Design and Analysis

2.1 Sensor Goals and Innovative Characteristics

The main goal of this thesis is to develop a MEMS shear stress sensor for application in high-speed turbulent flow environments. The specific sensor goals are as follows:

1) Operates in sub-kPa shear stress range (1-1000Pa)
2) Operates in high-speed turbulent compressible flow environment
3) Bidirectional sensor operation (can measure in +x and –x directions)
4) Non-intrusive to flow
5) Utilizes backside sensor connections
6) Utilizes a MEMS floating element design for direct shear stress measurement
7) Uses a differential capacitive sensing scheme and off-the-shelf electronics
8) Fast dynamic response O(10-100kHz)

This sensor design offers innovation primarily in the fact that it utilizes backside connections and is being operated in a sub-kPa shear stress range in a turbulent compressible environment. Although there have been many floating-element capacitive sensors developed in the past, as described in the previous chapter, the author is unaware of any sensors that have all of these features and have been utilized in a similar flow environment. Additionally, we will be flow testing at much higher Reynolds Numbers than previous efforts that we are aware of (Figure 24), and attempting to measure turbulent fluctuations at those high-speeds.
Turbulent flow is a challenging environment to measure shear stress in as it becomes necessary to analyze the signal in an attempt to separate turbulent shear stress from electrical noise. In addition, the nature of the compressible flow setup will result in temperature changes that will require sensor temperature compensation.

![Graph showing shear stress vs. Reynolds number for different shear stress sensors reporting turbulence measurements.](image)

**Figure 24:** Graph showing shear stress vs. Reynolds number for different shear stress sensors reporting turbulence measurements.

### 2.1.1 Sensor Theory of Operation

The MEMS shear stress sensor that is being developed utilizes the floating-element method of measurement. The sensor is flush-mounted in a shear flow and exposed to the wall shear stress ($\tau_w$). This creates a shear force ($F_{\text{shear}}$) on the top surface of the floating element given by:

$$F_{\text{shear}} = \tau_w A_s$$  \hspace{1cm} (31)
where $A_s$ is the shear stress sensing area. The floating element is connected to two springs in parallel (Figure 25), with spring constants of $k/2$, for an overall spring stiffness of $k$. $F_{\text{shear}}$ deflects the floating element an amount, $x$, so that it is balanced by the spring force, $F_{\text{spring}}$:

$$F_{\text{spring}} = kx$$  \hspace{1cm} (32)

$\tau_w$ and $x$ can be related by using the fact that $F_{\text{shear}} = F_{\text{spring}}$ and combining the previous two equations:

$$\tau_w = \frac{kx}{A_s}$$  \hspace{1cm} (33)

Figure 25: Schematic of the sensing element of the MEMS shear stress sensor. The floating element moves in the flow direction ($x$) when $F_{\text{shear}}$ is applied, changing the differential capacitance ($\delta C = CS2-CS1$).
Figures 25 and 26 are schematics illustrating the sensor concept. The floating element has \( N_f \) number of sensing fingers with length \( (L_f) \) and width \( (b_f) \) along both of its sides. Sensing fingers are interdigitated with stationary fingers that are attached to anchors. Every sensing finger has a “near” finger at a gap distance \( d_0 \), and a “far” finger at a gap distance \( \alpha d_0 \) (\( \alpha \) is nominally 4). This finger layout is used to mitigate fabrication challenges as it only requires a single device layer, reducing the number of steps in the sensor fabrication process.

Each pair of fingers forms a parallel plate capacitor with air as the dielectric. The pairs of fingers on each side of the floating element are electrically connected in parallel, and their capacitances will add up to create a larger overall capacitance. The sets of fingers on the right and left sides of the sensor will form the sense capacitors, CS2 and CS1, respectively. The absolute capacitance of CS2 is dependent on the sensor displacement (x), and is determined using an empirical relationship from Nishiyama [76]:

\[
CS2(x) = \frac{\varepsilon A_s}{d_0 - x} \left( 1 + 1.9861 \left( \frac{d_0 - x}{h} \right)^{0.8258} \right) + \frac{\varepsilon A_s}{\alpha d_0 + x} \left( 1 + 1.9861 \left( \frac{\alpha d_0 + x}{h} \right)^{0.8258} \right)
\] (34)
where $\varepsilon$ = dielectric permittivity, where $\alpha$ = distance amplification factor, $h$ = sensing finger thickness, $A_c$ = capacitance area, and $d_0$ = gap space between capacitor plates. The capacitance area ($A_c$) is calculated by multiplying the capacitance area of single pair of fingers by $N_f$:

$$A_c = N_f \left( x_0 h \right)$$  \hspace{1cm} (35)

where $N_f$ = number of sensing fingers per side and $x_0$ = sensing/stationary finger overlap. The capacitance equation for the other sensing capacitor, CS1, is given by:

$$CS1(x) = \frac{\varepsilon A_c}{d_0 + x} \left( 1 + 1.9861 \left( \frac{d_0 + x}{h} \right)^{0.8258} \right) + \frac{\varepsilon A_c}{\alpha d_0 - x} \left( 1 + 1.9861 \left( \frac{\alpha d_0 - x}{h} \right)^{0.8258} \right)$$  \hspace{1cm} (36)

The Nishiyama equation is used because it accounts for the presence of parallel plate fringing, which is found in the additional terms in the parentheses in equations (34) and (36). As discussed in [76], fringing effects are only negligible when the parallel plate capacitor has an aspect ratio ($AR = h/d_0$) larger than 50, a constraint not satisfied by the sensing fingers which have a maximum AR of 4. The equation for CS2 without fringing (and similarly CS1) is given by:

$$CS2(x) = \frac{\varepsilon A_c}{d_0 - x} + \frac{\varepsilon A_c}{\alpha d_0 + x}$$  \hspace{1cm} (37)

however, it is only used in this thesis when simplified equations are necessary for analytical purposes (e.g. Capacitive Test Signal Force – Section 2.5.2.1, Lateral Finger Bending - 2.5.2.2, etc.). Equation (34) is more accurate than equation (37), because it accounts for fringing effects, however it does not include the presence of the substrate underneath the sensor which can act as a ground plane. The author was unable to locate an equation in the literature which accounts for both fringing and a ground plane, although such an equation would be considered a more accurate representation of the sensor capacitance.
When there is an applied shear stress in the +x direction, the floating element displaces, and CS2 will increase while CS1 will decrease. To increase the overall capacitance change (and sensor output), a differential capacitance measurement is used for the sensor:

$$\delta C(x) = CS2(x) - CS1(x)$$  \hspace{1cm} (38)

This results in a larger magnitude measurement change than a single capacitor (CS2 or CS1) alone. For flow in the opposite direction, the roles of CS2 and CS1 will be reversed and $$\delta C$$ will become larger in the negative direction. When fringing is ignored, the differential capacitance is given by:

$$\delta C(x) = CS2(x) - CS1(x) = \varepsilon A \left[ \left( \frac{1}{d_0 - x} + \frac{1}{\alpha d_0 + x} \right) - \left( \frac{1}{d_0 + x} + \frac{1}{\alpha d_0 - x} \right) \right]$$  \hspace{1cm} (39)

The sensor sensitivity $$S(x)$$ is defined as the expected capacitance change for an applied shear stress:

$$S(x) = \frac{d\delta C(x)}{d\tau_w} = \frac{d\delta C(x)}{dx} \frac{dx}{d\tau_w}$$  \hspace{1cm} (40)

For a differential capacitance setup, $$S(x)$$ will be non-linear and is calculated numerically.

A typical sensor design and its dimensions are represented by sensor 3F2-E ($$d_0 = 2\mu m$$, $$bf = 2\mu m$$, $$L_k = 2\mu m$$, $$h = 8\mu m$$, $$L_f = 120\mu m$$, $$N_f = 15$$, and $$W = 70\mu m$$). Figure 27 shows theoretical performance data for the sensor using differential capacitive sensing ($$\delta C$$). For a positive shear stress (defined as along the streamwise flow direction (+x), $$\delta C$$ changes 400fF ($$4 \times 10^{-15} F$$) for $$\tau_w$$ of 600Pa. As the shear stress increases and the finger gap closes (and $$x$$ approaches $$d_0$$), $$\delta C$$ increases dramatically due to the denominators of terms in equation 39 approaching zero. When the flow changes its orientation by 180° (denoted as negative shear stress in Figure 27), utilizing a $$\delta C$$ configuration gives a measurement which is the negative of the capacitance change.
for the positive shear stress. The sensitivity $S(x)$ increases with $\tau_w$, with a value of approximately 0.1 fF/Pa at 100 Pa which then dramatically increases (as $x$ approaches $d_0$) to approximately 6 fF/Pa at 600 Pa.

![Graph showing theoretical sensor (3F2-E) performance. The solid line is differential capacitance change ($\delta C$) vs. shear stress ($\tau_w$), and the dashed line is the sensitivity ($S$) vs. shear stress ($\tau_w$). Negative shear stress denotes a 180° change in the flow direction.](image)

Differential capacitance offers the following benefits over a measurement with a single capacitor (referred to as single-ended): (1) increased sensitivity; (2) bidirectional shear stress sensing (operation in (+x) and (-x) directions); and (3) capacitance changes (e.g. due to parasitics) affecting CS2 and CS1 in the same manner (common-mode) will be cancelled out.

During operations, the MEMS sensor will generate a capacitance change when it is exposed to a shear flow. This $\delta C$ will be measured with circuitry that will convert the output to a voltage change, $\Delta V_0$. If the sensor is being used to determine an unknown shear stress, we can take the measured $\Delta V_0$, and work backwards using the equations above to determine the
unknown $\tau_w$. This will be discussed further in Chapter 8 - Sensor Cold Flow Characterization Testing.

### 2.2 Design Variations and Risk Mitigation

Multiple sensor designs were created for the purpose of generating a broad range of sensors with varying performance specifications. This also served to mitigate some of the risks inherent with the sensor fabrication processes. As discussed in [70] and [72], previous efforts at manufacturing sensors encountered large residual stress gradients, misalignment between layers, increased/decreased feature size and gap spacing. These fabrication issues adversely affected sensor performance often rendering the sensors completely immobile and inoperable.

Sensor design analysis began with the goal of minimizing the aforementioned effects of fabrication limitations on sensor performance. In taking this approach, a single device layer design was used in contrast with previous multi-device layer designs. This served to reduce the challenges in fabricating multiple layers that have to line up geometrically to make the sensors operate properly. It also makes sensor release (chemical etching) easier as there are fewer areas that are hidden from view, which helps with troubleshooting. Additionally, utilizing a single layer significantly decreased the time and cost required for device fabrication because a common SOI (Silicon–on–insulator) wafer could be used for the devices.

In creating the sensor designs, various parameters were analyzed to determine the appropriate combinations for reducing fabrication risk while satisfying the sensor requirements. The following items were evaluated in sensor design:

- two different types of springs: folded springs and tensile beam springs
• parallel versus perpendicular finger layout
• 3 pad versus 4 pad layout
• minimum feature size, i.e. 2μm finger width and finger gap vs. 3μm
• number of sensing fingers ($N_f$)
• component size, i.e. floating element length ($L_s$) and width ($W$), finger length ($L_f$), spring length ($L_k$)

The following subsections (Sensor Springs 2.2.1 - 2.2.5) will describe in more detail the variations included in the different sensor designs.

### 2.2.1 Sensor Springs

Sensor designs included two common spring configurations for MEMS devices, which are the folded-beam design and the tensile-beam designs (Figure 28). The folded-beam design maximizes spring compliance while minimizing the spring length in the direction of displacement. However, the long beams are susceptible to out-of-plane bending from residual stress gradients in the same manner that the fingers are. The tensile-beam design is less sensitive to stress gradients however, “spring-stiffening” can occur from the residual axial stress in the beam structures [77].

The spring constants ($k$) for single folded-spring and single tensile-spring designs are derived using small-deflection theory, and assuming compliant springs attached to a rigid floating element that does not deform [78]:

$$k_{\text{folded, single}} = \frac{12EI_x}{L_k^3} = \frac{Ehb_k^3}{L_k^3}$$

(41)
and

\[ k_{\text{tensile, single}} = \frac{2AEI_x}{L_k^3} = \frac{2Ehb_k^3}{L_k^3} \quad (42) \]

where \( E \) = Young's Modulus, \( h \) = Thickness, \( b_k \) = Spring Width, \( L_k \) = Spring Length, and \( I_x \) = Area Moment of Inertia in x-direction \((\frac{1}{12}h b_k^3)\).

While the small deflection assumption is not technically correct, because the deflection of the floating element can be as large as the spring width, it is used here for simplicity and the spring constants are viewed as estimates. Later (Section 2.5.1), the spring constant will be determined using FEM analysis, and those results are considered to be more accurate than the calculations using the analytical results described here.

All designs have springs on both sides of the floating element; with the device spring constant taken as twice the stiffness of a single spring (i.e. springs are in parallel).

\[ k_{\text{folded, device}} = \frac{2Ehb_k^3}{L_k^3} \quad (43) \]

and

\[ k_{\text{tensile, device}} = \frac{4Ehb_k^3}{L_k^3} \quad (44) \]

Throughout the remainder of this thesis, discussion of sensor springs and usage of the variable ‘\( k \)’ will refer to the overall spring constants of the sensors (eqs. 43 and 44), not the individual springs.
Figure 28: Diagrams of folded-spring design (L) and tensile spring-design (R) (spring length = \(L_k\) and spring width = \(b_k\)). The shaded portions represent the anchor region of the spring.

The sensor springs were designed to be much more compliant in the streamwise (x) direction than in the transverse (y) and out-of-plane (z) directions. Stiffness in the y-direction results in the sensor only responding to the flow vector in the x-direction, necessary for measuring in an unknown flow environment. Stiffness in the z-direction is required to make the floating element insensitive to out-of-plane forces which would result from a misaligned sensor, non-tangential flow, or from turbulent wall pressure fluctuations (Section 2.6.2).

Due to the complicated spring geometry, there is no simple analytical solution for \(k_y\), the spring constant in the transverse direction. However, FEA analysis is conducted (Section 2.5.1) which shows that \(k_y\) typically has a value that is at least 100 times greater than \(k_x\). The increased stiffness in the y-direction is sufficient to ensure that sensor motion in that direction is minimal compared to motion in the x-direction.

The spring constant in the out-of-plane direction, \(k_z\), can be calculated by assuming that the springs will bend in a similar manner as in the z-direction, but \(I_z\) is used rather than \(I_x\) because the neutral axis has changed:

\[
k_{z,\text{folded}} = \frac{24EI_z}{L_k^3} = \frac{2Ebh_k^3}{L_k^3}
\]  

(45)
The ratio of the $k_z$ to $k_x$ for both types of springs is then given by:

$$\frac{k_z}{k_x} = \frac{h^2}{b_k^2}$$  \hspace{1cm} (47)

For typical design values of $h = 8\mu m$ and $b_k = 2\mu m$, the sensor will be 16 times stiffer in the out-of-plane direction. The ratio of $k_z$ to $k_x$ stiffness is crucial to sensor operation because the floating element will be exposed to forces in both the x and z-directions and motion in either direction will affect the differential capacitance of the sensor. This ratio should be maximized so that sensor motion is primarily caused by the in-plane shear stress. The sensitivity of the MEMS sensors to out-of-plane forces will be discussed further in section 2.6.2 – Fluctuating Pressure in Turbulent Flows.

### 2.2.2 Finger Layout

Two types of finger layouts were considered: parallel and perpendicular. In the perpendicular layout, the fingers are perpendicular to the flow and in the parallel layout, they are in line with the flow (Figures 29 and 30). Because we are using a single device layer, all of the stationary fingers on each side of the floating element are electrically connected to a single electrode. This contrasts the many multi-layer designs where each moving finger is between two stationary fingers that are connected to two separate electrodes [22], [70].

For the perpendicular design this necessitates that the stationary fingers are staggered with respect to the sensing fingers in order for a nonzero differential capacitance to occur between the capacitors. Each sensing finger is affected by the “near” electrode which is located
at a distance $d_0$ (typically 2um), and the “far” electrode located at distance $\alpha d_0$ ($\alpha$ is taken as 4). The result is a less compact design; however using a perpendicular layout is very beneficial as it provides sensitivity (differential capacitance / sensor displacement) an order of magnitude higher than for a similar parallel layout [70]. As described in Section 2.1.1, the differential capacitance for this layout is approximated by:

$$\delta C(x) = \varepsilon N_f x_0 h \left[ \left( \frac{1}{d_0 - x} + \frac{1}{\alpha d_0 + x} \right) - \left( \frac{1}{d_0 + x} + \frac{1}{\alpha d_0 - x} \right) \right]$$

(48)

where fringing effects have been ignored. $\delta C(x)$ is non-linear which results in a sensitivity that varies with displacements, which is not a desirable property for a sensor.

For a parallel design, each sensing finger will have two “near” electrodes located at a distance $d_0$. When the sensor displaces, the finger overlap ($x_0$) of the sensors will vary by the displacement amount ($x$), changing the capacitive area as:

$$\delta C(x) = C_2(x) - C_1(x) = \frac{\varepsilon [2N_f (x_0 + x)h]}{d_0} - \frac{\varepsilon [2N_f (x_0 - x)h]}{d_0} = \frac{4N_f \varepsilon rh}{d_0}$$

(49)

Parallel designs have the benefit of behaving more “linearly” as the differential capacitance is proportional to the floating element displacement. However, their sensitivities, $S(x)$, are much lower than for a similar-sized perpendicular designs. This is because parallel plate capacitors (represented by the pairs of sensing fingers) are more sensitive to changes in dielectric thickness (represented by $d_0$) than they are to plate overlap (represented by $x_0$). Ultimately the majority of the devices were fabricated with perpendicular fingers. This design selection was driven by a combination of the sensor design requirements (Section 2.1) and the performance of the capacitance measurement electronics that were utilized (discussed further in Chapter 6). Additionally, as will be described in Section 3.3, nearly all of the parallel designs were destroyed during fabrication as they had large floating elements to compensate for reduced sensitivity,
which resulted in sensors not being fully released. Therefore, all experimental data discussed in this thesis will result from perpendicular designs only.

Figure 29 (top): Diagram of perpendicular finger layout design.  
Figure 30 (bottom): Diagram of parallel finger layout design.

2.2.3 3 Pad vs. 4 Pad

Previous attempts by the team [70] [72], of designing shear stress sensors using off-the-shelf electronics, showed that it was very difficult to acquire a capacitive meter that could measure differential capacitance while simultaneously applying a large bias voltage across the same electrodes. Irvine Sensors Corp., manufacturers of the MS3110 Universal Capacitive
Readout (used in this thesis), and Analog Devices, manufacturers of the AD7152/AD7153 Capacitance to Digital Converter, both warned that a voltage larger than 5 volts placed across the IC sensing electrodes could disturb the capacitive measurement signal, or destroy the IC chip.

Previous MEMS devices have been developed which utilize differential capacitance and electrostatic actuation using three electrode pads, however these have contained custom-made electronics which is outside the scope of this project. In an attempt to circumvent this problem, “4 pad designs” were included that allow for capacitive sensing in two directions, and electrostatic actuation in one direction. The standard design is the “3 pad” design, referred to as such, because it only has three electrically independent pads: CS2, CS1, and CSCOM, the floating element (Figure 31). In contrast, the 4 pad designs have CS2, CS1, CSCOM, and an additional pad, ACT, which connects to two sets of actuation fingers (Figure 32).

Ultimately, electrostatic actuation was not used because the MS3110 would not operate when a voltage bias was applied via external amplifier. This was because CSCOM is connected to both the MS3110 and the external amplifier, and interfered with the charge integrating amplifier built into the MS3110. In the cases where 4 pad designs were tested, they were operated in an open-loop manner, without any connections to the actuation pad, ACT.
Figure 31 (Top): Diagram of 3 pad layout design.
Figure 32 (Bottom): Diagram of 4 pad layout design.

2.2.4 Feature Size: 2 μm vs. 3 μm

Previous attempts [70] at fabricating MEMS shear stress sensors with 2.5μm features and 2μm gaps, led to sensors with wider or narrower fingers than initially designed. In extreme cases,
this led to a finger width increase (during micro-fabrication) from 1μm–2μm for 2.5μm fingers, sometimes resulting in complete closure of finger gaps, rendering the sensors useless. Other times, sensors were over-etched resulting in smaller finger widths and larger gaps, completely changing the design characteristics of the sensors.

To minimize the chances of all sensors being destroyed or not in the proper shear stress range, designs were included for sensors with 2μm features and 2μm gaps, and also 3μm features and 3μm gaps.

2.2.5 Component Size

For further risk mitigation, additional design variations included differences in the sensors’ floating element sizes (lengths ($L_s$) and widths ($W$), different finger lengths ($L_f$) and spring lengths ($L_k$), and differences in the number of fingers per side ($N_f$). Since the floating element size is related to the shear stress force, and the number and size of the sensing fingers affects the sensor capacitance, this resulted in sensors with a broad range of shear stress ranges, and shear sensing resolutions (See Section 2.4).

2.3 Device Layout

The MEMS shear stress sensors are fabricated on a square silicon substrate (5mm x 5mm) with a thickness of approximately 0.5mm. Figure 33 presents the location of the main features on the frontside of the sensors. The arrow micro-fabricated in the southwest corner, designates the direction of the shear flow. Electrical paths connect the sensing element (made up of the floating element, springs, and stationary fingers) to 8 frontside electrode pads in a circular
pattern around the sensor. The electrode pads are 800\(\mu\)m x 800\(\mu\)m and are used to interface the MEMS shear stress sensor with the macroscale world. Redundant sets of pads (1 and 2, 3 and 7, 4 and 5) were included for risk mitigation and sensor continuity testing. Pads 6 and 8 were included for 4 pad sensor designs which would implement closed-loop feedback control via electrostatic actuation. Ultimately, only open-loop sensor operation was utilized because the capacitive sensing electronics were unable to be integrated with the closed-loop sensor operation without designing a custom-circuit. Note that pads 3-7 connect to the floating element, pads 1-2 connect to CS2 and pads 4-5 connect to CS1.

In addition to the 8 frontside pads there are 8 backside pads on the bottom side of the device in the same pattern. They are electrically connected to the frontside pads by metallized vias through the thickness of the substrate, as shown in Figure 34.
Figure 33: Schematic of frontside of the MEMS shear stress sensor. Black lines represent electrically connected electrode pads. The arrow in the southwest corner designates the primary flow direction. The sensing element is located in the center and represented as two comb-drive capacitors (CS2 and CS1).

Figure 34: SEM image of through-substrate electrical interconnects (vias) [72].
2.4 Summary of Designs

The team chose 36 shear sensor designs, covering a broad cross-section of the various parameters mentioned in the previous section. The choices reflect the inherent balance between optimum design and the risks associated with fabrication. A “doublet sensor” design was also included which has two copies of the 4F2-B design, orientated for sensing in both x and y in-plane flow directions. Table 2 broadly categorizes the included designs by Feature Size, # of Pads, Finger Layout, Spring Type, while Table 3 includes the dimensions and components specific to each of the 36 designs.

The following designations are used when referring to the specific sensor designs:

1st digit: 4 = 4 pad, 3 = 3 pad

2nd digit: F = Folded spring, T = Tensile beam spring

3rd digit: 2 = 2μm feature/gap, 3 = 3μm feature/gap

4th digit: Letter identifying design features

Table 2: Inventory of sensor element designs chosen for fabrication.

<table>
<thead>
<tr>
<th>Feature / Gap Size</th>
<th># of Pads</th>
<th>Finger Layout</th>
<th>Spring Type</th>
<th># of Designs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2μm</td>
<td>3</td>
<td>Perpendicular</td>
<td>Folded</td>
<td>10</td>
</tr>
<tr>
<td>2μm</td>
<td>3</td>
<td>Perpendicular</td>
<td>Tensile</td>
<td>2</td>
</tr>
<tr>
<td>2μm</td>
<td>3</td>
<td>Parallel</td>
<td>Folded</td>
<td>2</td>
</tr>
<tr>
<td>2μm</td>
<td>3</td>
<td>Parallel</td>
<td>Tensile</td>
<td>1</td>
</tr>
<tr>
<td>2μm</td>
<td>4</td>
<td>Perpendicular</td>
<td>Folded</td>
<td>8</td>
</tr>
<tr>
<td>2μm</td>
<td>4</td>
<td>Perpendicular</td>
<td>Tensile</td>
<td>5</td>
</tr>
<tr>
<td>3μm</td>
<td>3</td>
<td>Perpendicular</td>
<td>Folded</td>
<td>6</td>
</tr>
<tr>
<td>3μm</td>
<td>3</td>
<td>Perpendicular</td>
<td>Tensile</td>
<td>1</td>
</tr>
<tr>
<td>2μm</td>
<td>Doublet Design</td>
<td>Perpendicular</td>
<td>Folded</td>
<td>1</td>
</tr>
<tr>
<td>Feature</td>
<td>Name [Units]</td>
<td>3F2-A</td>
<td>3F2-B</td>
<td>3F2-C</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>--------------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>Number of Pads</td>
<td></td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Spring Type</td>
<td>Folded</td>
<td>Folded</td>
<td>Folded</td>
<td>Folded</td>
</tr>
<tr>
<td>Gap Space (d_0 [\mu m])</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Finger Length (Lf [\mu m])</td>
<td>120</td>
<td>90</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Finger Overlap (x_0 [\mu m])</td>
<td>100</td>
<td>70</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Finger Width (b_f [\mu m])</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td># of Sense Fingers Per Side</td>
<td>(N_f)</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td># of Actuation Fingers Per Side</td>
<td>(N_a)</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Element Width (W [\mu m])</td>
<td>35</td>
<td>46</td>
<td>46</td>
<td>52</td>
</tr>
<tr>
<td>Total Element Length (L_e [\mu m])</td>
<td>156</td>
<td>156</td>
<td>156</td>
<td>226</td>
</tr>
<tr>
<td>Number of Holes in Element</td>
<td>(N_h)</td>
<td>24</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>Feature</td>
<td>Name [Units]</td>
<td>3T2-A</td>
<td>3T2-B</td>
<td>3F3-A</td>
</tr>
<tr>
<td>---------------------------------</td>
<td>--------------</td>
<td>-------</td>
<td>-------</td>
<td>-------</td>
</tr>
<tr>
<td>Number of Pads</td>
<td></td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Spring Type</td>
<td>Tensile</td>
<td>Tensile</td>
<td>Folded</td>
<td>Folded</td>
</tr>
<tr>
<td>Gap Space $d_0 [\mu m]$</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Finger Length $L_f [\mu m]$</td>
<td>120</td>
<td>60</td>
<td>120</td>
<td>120</td>
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<tr>
<td>Finger Overlap $x_0 [\mu m]$</td>
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<td>40</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Finger Width $b_f [\mu m]$</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td># of Sense Fingers Per Side $N_f$</td>
<td>30</td>
<td>50</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td># of Actuation Fingers Per Side $N_a$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Element Width $W [\mu m]$</td>
<td>84</td>
<td>140</td>
<td>70</td>
<td>48</td>
</tr>
<tr>
<td>Total Element Length $L_s [\mu m]$</td>
<td>436</td>
<td>716</td>
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<td>339</td>
</tr>
<tr>
<td>Number of Holes in Element $N_h$</td>
<td>172</td>
<td>462</td>
<td>55</td>
<td>116</td>
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</table>

Table 3 (cont’d): Inventory of Sensor Designs
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<tr>
<th>Feature</th>
<th>Name [Units]</th>
<th>4F2-B</th>
<th>4F2-C</th>
<th>4F2-D</th>
<th>4F2-E</th>
<th>4F2-F</th>
<th>4F2-G</th>
<th>4F2-H</th>
<th>4T2-A</th>
<th>4T2-B</th>
<th>4T2-C</th>
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</thead>
<tbody>
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<td>Number of Pads</td>
<td></td>
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<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
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<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Spring Type</td>
<td></td>
<td>Folded</td>
<td>Folded</td>
<td>Folded</td>
<td>Folded</td>
<td>Folded</td>
<td>Folded</td>
<td>Tensile</td>
<td>Tensile</td>
<td>Tensile</td>
<td>Tensile</td>
</tr>
<tr>
<td>Gap Space [μm]</td>
<td>$d_0$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Finger Length [μm]</td>
<td>$L_f$</td>
<td>120</td>
<td>90</td>
<td>120</td>
<td>90</td>
<td>120</td>
<td>120</td>
<td>100</td>
<td>120</td>
<td>120</td>
<td>120</td>
</tr>
<tr>
<td>Finger Overlap [μm]</td>
<td>$x_0$</td>
<td>115</td>
<td>85</td>
<td>115</td>
<td>85</td>
<td>115</td>
<td>115</td>
<td>95</td>
<td>115</td>
<td>115</td>
<td>115</td>
</tr>
<tr>
<td>Finger Width [μm]</td>
<td>$b_f$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td># of Sense Fingers Per Side</td>
<td>$N_f$</td>
<td>20</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>36</td>
<td>55</td>
<td>15</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td># of Actuation Fingers Per Side</td>
<td>$N_a$</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Element Width [μm]</td>
<td>$W$</td>
<td>93</td>
<td>88</td>
<td>88</td>
<td>116</td>
<td>116</td>
<td>98</td>
<td>154</td>
<td>52</td>
<td>70</td>
<td>88</td>
</tr>
<tr>
<td>Total Element Length [μm]</td>
<td>$L_s$</td>
<td>366</td>
<td>436</td>
<td>436</td>
<td>436</td>
<td>436</td>
<td>576</td>
<td>856</td>
<td>296</td>
<td>366</td>
<td>436</td>
</tr>
<tr>
<td>Number of Holes in Element</td>
<td>$N_h$</td>
<td>144</td>
<td>172</td>
<td>172</td>
<td>237</td>
<td>237</td>
<td>257</td>
<td>638</td>
<td>28</td>
<td>108</td>
<td>172</td>
</tr>
</tbody>
</table>
Table 3(cont’d): Inventory of Sensors

<table>
<thead>
<tr>
<th>Feature</th>
<th>Name [Units]</th>
<th>4T2-D</th>
<th>4T2-E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pads</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Spring Type</td>
<td>Tensile</td>
<td>Tensile</td>
<td></td>
</tr>
<tr>
<td>Gap Space</td>
<td>$d_0 , [\mu\text{m}]$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Finger Length</td>
<td>$L_f , [\mu\text{m}]$</td>
<td>120</td>
<td>100</td>
</tr>
<tr>
<td>Finger Overlap</td>
<td>$x_0 , [\mu\text{m}]$</td>
<td>1155</td>
<td>95</td>
</tr>
<tr>
<td>Finger Width</td>
<td>$b_f , [\mu\text{m}]$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td># of Sense Fingers Per Side</td>
<td>$N_f$</td>
<td>36</td>
<td>55</td>
</tr>
<tr>
<td># of Actuation Fingers Per Side</td>
<td>$N_a$</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Element Width</td>
<td>$W , [\mu\text{m}]$</td>
<td>98</td>
<td>154</td>
</tr>
<tr>
<td>Total Element Length</td>
<td>$L_s , [\mu\text{m}]$</td>
<td>576</td>
<td>856</td>
</tr>
<tr>
<td>Number of Holes in Element</td>
<td>$N_h$</td>
<td>257</td>
<td>638</td>
</tr>
</tbody>
</table>
Figure 35: Shear resolution and maximum operating ranges for the 3 pad MEMS sensor designs.

Figure 36: Shear resolution and maximum operating ranges for the 4 pad MEMS sensor designs.

Figure 35 and Figure 36 show design variations for the maximum open-loop shear stress, maximum closed-loop shear stress, and the minimum shear stress resolution (Shear Res.), assuming a minimum detectable capacitance ($\delta C_{min}$) of 1fF. Both the 3 Pad and 4 Pad designs have increased shear stress sensing when closed-loop feedback is implemented, although the 3 Pad designs generally have higher sensitivity due to the sensing fingers along the entire length of
the floating element. The main purpose of the 4 Pad designs is to illustrate closed-loop sensor feedback utilizing COTS electronics.

2.5 Static Modeling of Sensor

2.5.1 Finite Element Model of Sensor Displacement

Finite element models (FEM) of different sensor designs were created using the COMSOL software package, in order to verify the mechanical behavior of the sensors. The floating elements of the sensors were modeled in 3-D and the sensor deformation equations were solved using the steady-state stress-strain solver with the large deformation option. The boundary conditions were set as follows: 1) anchors were fixed in space, 2) distributed shear stress force on the top surface of sensor including floating element, sensing fingers, and springs, and 3) all other sections were unloaded and free to move. The direction of the distributed force was dependent on whether the streamwise (x), transverse (y), or out-of-plane (z) motion was being analyzed. A typical COMSOL sensor design is show in Figure 37 where the red shaded portions on the top surface represent the areas where the shear force was applied.
Figure 37: COMSOL model of sensor used for stress-strain analysis. The red shaded portions represent the boundaries where the shear forces were applied.

Figure 38: COMSOL model showing sensor displaced in the positive-x direction. The red shaded portions represent the boundaries where the shear forces were applied.
Table 4: Comparison of analytical and FEA spring calculations.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Spring Constants</th>
<th>Analytical</th>
<th>FEA</th>
<th>Analytical</th>
<th>FEA</th>
</tr>
</thead>
<tbody>
<tr>
<td>3F2-B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>As-designed dimensions</td>
<td>h = 8.0um, d0 = 2.0um</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD3F2-A</td>
<td>As-built dimensions</td>
<td>h = 6.8um, d0 = 2.3um</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Streamwise</td>
<td>k_x</td>
<td>28.1</td>
<td>28.9</td>
<td>6.2</td>
<td>6.61</td>
</tr>
<tr>
<td>Transverse</td>
<td>k_y</td>
<td>N/A</td>
<td>3535</td>
<td>N/A</td>
<td>1286</td>
</tr>
<tr>
<td>Out-of-Plane</td>
<td>k_z</td>
<td>450</td>
<td>348</td>
<td>99</td>
<td>88</td>
</tr>
<tr>
<td>Ratio</td>
<td>k_y / k_x</td>
<td>N/A</td>
<td>122</td>
<td>N/A</td>
<td>195</td>
</tr>
<tr>
<td>Ratio</td>
<td>k_z / k_x</td>
<td>16.0</td>
<td>12.1</td>
<td>16.0</td>
<td>13.3</td>
</tr>
</tbody>
</table>

Table 4 is a comparison of the analytical and FEA results for two folded-beam sensors, 3F2-B and CD3F2-A. The sensor spring constants $k_x$, $k_y$, and $k_z$ were calculated by multiplying the distributed force by the sensor surface area ($A_s$), and dividing by the displacement of the floating element in the relevant direction.

Calculating $k_x$ for two sensor designs, the FEA results are within 7% of the analytical values, indicating that the models are in good agreement. We do not have an analytical calculation for $k_y$ however the sensors were designed to be much stiffer in the transverse direction, so the $k_y / k_x$ ratios of 122 and 195 are positive results.

The FEA results for $k_z$ were 22.5% and 11.5% lower than the analytical results for the two sensors resulting in stiffness ratios $(k_z / k_x)$ of 12.1 and 13.3 rather than the expected ratio of 16. The FEA results validate the accuracy of the analytical spring equations for $k_x$, and indicate that the equations for $k_z$ are within an acceptable degree of accuracy. Furthermore, the FEA results confirm that the sensor is much stiffer in the y and z-directions than it is in the flow direction (x).
2.5.2 Secondary Forces in Flowing Environment

When operating in a real flow environment, the MEMS sensors will be subjected to forces in the streamwise direction that are secondary to the wall shear stress being measured. Secondary forces include uneven pressure forces from the pressure gradient \( \frac{dp}{dx} \) in the duct, viscous forces from flow underneath the floating element, and electrostatic forces from the capacitive test signal. This section will focus only on static secondary forces, while transient sensor forces (e.g. inertial, viscous, and turbulent pressure fluctuations) will be discussed in the dynamic modeling part of this thesis (Section 2.6).

It is important to evaluate the magnitude of these secondary static forces to determine if the sensor output must be compensated for non-shear stress forces. An order of magnitude analysis on the various forces interacting with the shear stress sensor is presented below, using the assumptions of: adiabatic, fully-developed, steady-state, incompressible flow. In addition, note that \( F_{\text{shear, sub}} \) is a steady state shear force caused by Poiseuille flow underneath the floating element which is a result of \( \frac{dp}{dx} \).

Figure 39 is a side-view schematic of a simplified floating element with one finger, indicating the direction and location of streamwise forces. In addition to the force from the wall shear stress, the following forces are expected to be present in a flow with a favorable pressure gradient [79], [22]:

1) \( F_{\text{shear-top}} \) - Shear force from flow on top side of sensor, assuming an adiabatic incompressible duct solution [80]:

\[
F_{\text{shear}} = \tau w A_s = \frac{b}{4} \left( -\frac{dP}{dx} \right) A_s
\]  

(50)
where \( A_s = \) shear sensing area, \( b = \) duct height, \( \frac{-dP}{dx} = \) streamwise pressure gradient

2) \( F_{\text{shear,sub}} - \) Shear force from channel flow underneath floating element:

\[
F_{\text{shear,sub}} = \frac{H}{2} \left( \frac{-dP}{dx} \right) A_s
\]  
\( (51) \)

where \( H = \) gap between underside of sensor and substrate

3) \( F_{dpdx\_finger} - \) Pressure gradient force across fingers:

\[
F_{dpdx\_finger} = 2N_f b_f h L_f \left( \frac{-dP}{dx} \right)
\]  
\( (52) \)

where \( b_f = \) sensing finger width, \( L_f = \) sensing finger length, \( h = \) sensing finger thickness, and \( N_f = \) number of fingers per side of sensor

4) \( F_{dpdx\_element} - \) Pressure gradient force across floating element:

\[
F_{dpdx\_element} = w h L_s \left( \frac{-dP}{dx} \right)
\]  
\( (53) \)

where \( L_s = \) floating element length, \( w = \) floating element width; \( h = \) floating element thickness.

5) \( F_{\text{spring}} - \) Mechanical force from sensor spring:

\[
F_{\text{spring}} = k x
\]  
\( (54) \)

where \( k = \) spring constant, \( x = \) sensor streamwise displacement

6) \( F_{\text{testsignal}}: \) Electrostatic force from capacitive test signal:
where $x =$ sensor displacement, $V_s =$ effective test signal voltage, $CS2(x) =$ sense capacitor $CS2$, and $CSI(x) =$ sense capacitor $CS1$. $F_{test\text{signal}}$ is an electrostatic force which is a function of both the capacitance sensing signal and the sensor displacement, and is therefore a function of shear stress. Analysis of the effect of the sensor test signal will be discussed separately in Section 2.5.2.1.

![Diagram](attachment:image.png)

**Figure 39:** Side view of floating element with finger showing streamwise forces on shear stress sensor in favorable pressure-gradient flow; $k_z >> k_x$.

A balance of all of the streamwise forces (without $F_{test\text{signal}}$) gives the following equation:

$$F_{spring} = F_{shear} + F_{shear\ _sub} + F_{dpdx\ _finger} + F_{dpdx\ _element} \quad (56)$$

An effective shear stress ($\tau_{eff}$), which is the equivalent shear stress which actuates the sensing element, (composed of the fluidic shear stress plus additional streamwise stresses, as shown in Figure 39), is defined as:

$$\tau_{eff} = \frac{F_{spring}}{A_s} \quad (57)$$
We can rewrite eqs. (56) and (57) as:

\[
\tau_{\text{eff}} = \frac{F_{\text{shear}}}{A_s} + \frac{F_{\text{shear}_{\text{sub}}}}{A_s} + \frac{F_{dpdx_{\text{finger}}}}{A_s} + \frac{F_{dpdx_{\text{element}}}}{A_s}
\] (58)

Substituting in equations (50-53):

\[
\tau_{\text{eff}} = \frac{b}{4} \left( \frac{-dP}{dx} \right) + \frac{H}{2} \left( \frac{-dP}{dx} \right) + \frac{2 N_f b_f h L_f}{A_s} \left( \frac{-dP}{dx} \right) + \frac{w h L_s}{A_s} \left( \frac{-dP}{dx} \right)
\] (59)

Factoring out \(-dp/dx\) and using the following order-of-magnitude quantities:

\[b_f \sim 10^{-6} [m], L_f \sim 10^{-4} [m], h \sim 10^{-6} [m], L_s \sim 10^{-4} [m], H \sim 10^{-6} [m], b \sim 10^{-2} [m],\]
\[N_f \sim 10^4, A_s \sim 10^{-8} [m^2], w \sim 10^{-4} [m]\]

gives us:

\[
\tau_{\text{eff}} = \left[ (10^{-2}) + (10^{-6}) + (10^{-7}) + (10^{-6}) \right] \left( \frac{-dP}{dx} \right)
\] (60)

The first term on the right-hand side represents \(\tau_w\) and is clearly the dominant term in the shear stress measurement irrespective of dp/dx. Therefore, additional forces present on the floating element can be neglected, and we can assume:

\[
\tau_{\text{eff}} = \tau_w
\] (61)

The next section will examine the effect of \(F_{\text{testsignal}}\) on the MEMS shear sensor measurement.

### 2.5.2.1 Capacitive Test Signal Force

When measuring capacitance, a high-frequency AC test signal is applied across the circuit to determine the impedance, resulting in an electrostatic force. The purpose of this section is to illustrate the effects caused by the test signal force, and how they affect sensor operation. The effective electrostatic force, \(F_{\text{testsignal}}\), is given by:
\[ F_{\text{testsignal}} = \frac{V_s^2}{2} \left[ \frac{d}{dx} CS2(x) + \frac{d}{dx} CSI(x) \right] \]  \hspace{1cm} (62)

where \( V_s \) is the effective voltage, or the RMS (Root-Mean-Square) voltage of the testsignal. This analysis is considered to be steady-state because the capacitive test signal frequency is 100kHz or greater, which is much higher than the sensors’ resonant frequencies (Section 2.6.1). Since the sensor cannot physically move at the same high-frequency as the AC testsignal, the RMS voltage is used to model an “averaged” steady-state voltage on the sensor.

To allow for non-dimensionalization, the capacitance equation neglecting fringe capacitance will be used. Calculations show that making this assumption will decrease \( F_{\text{testsignal}} \) by approximately 10\%, however the trends of the sensor behavior are the same as when fringe effects are included. Neglecting fringe capacitance, \( F_{\text{testsignal}} \) is given by:

\[ F_{\text{testsignal}} = \frac{\varepsilon A V_s^2}{2} \left( \frac{1}{(d_0 - x)^2} - \frac{1}{(\alpha d_0 + x)^2} - \frac{1}{(d + x)^2} + \frac{1}{(\alpha d_0 - x)^2} \right) \]  \hspace{1cm} (63)

\( F_{\text{testsignal}} \) is an attractive force between the sensing fingers and stationary fingers that increases with the inverse square of the distance. As the sensing fingers are pushed closer together by the applied shear flow, \( F_{\text{testsignal}} \) increases the total force on the sensing element, displacing it further than the shear stress alone. This effect can be substantial depending on the applied voltage of the test signal and the specific characteristics of the sensor design, and may require measurement compensation which will be detailed below.

Both \( F_{\text{testsignal}} \) and the spring restoring force, \( F_{\text{spring}} \), will increase with displacement (x). After a certain displacement, \( F_{\text{testsignal}} \), which varies with \( x^{(2)} \), begins to increase faster than \( F_{\text{spring}} \), which varies with x. This results in a mechanical instability called “snapdown,” causing the floating element to be pulled in the direction of the electrostatic force, and closing the gaps between the fingers [77]. Often removing the shear flow and the applied voltage will not separate
the fingers. Removing the applied charge by shorting the sensor connections together through the packaging will occasionally separate the fingers. If the previous methods do not work, the sensor requires shear flow in the opposite direction or in some cases; a mechanical probe can be used while viewing the sensor under a microscope to manually separate the sensing fingers. Figure 40 illustrates the snapdown phenomenon, with a sensor in the normal condition displayed on the left, and a snapped-down sensor shown on the right. The floating element has moved as far as it can to the left, and requires a mechanical force to return it to the normal condition.

The challenges resulting from snapdown make it important to avoid the shear stress regime which may result in snapdown. This effect is entirely a result of the electronics and in future work could be avoided by proper electronics design. Additionally, a modified sensor design including mechanical stops on each finger would avoid this problem.

![Diagram](image)

**Figure 40:** (Left) Sensor in normal condition. (Right) Sensor after snapping down (floating element moved to left).

For the MEMS shear stress sensors, two sets of electronics were used for measuring capacitance, to check for the electrostatic test signal affects. The Keithley 590CV applies a 15mV RMS 100kHz sine wave which generates a negligible electrostatic force, while the MS3110 IC applies a 2.25V 100kHz square wave which can lead to non-negligible electrostatic force at certain shear stress levels.
To analyze the effect of the electrostatic force \(F_{test\text{signal}}\), we start with a force balance of all of the secondary static forces that were described in Section 2.5.2.

\[
F_{spring} = F_{shear} + F_{shear\_sub} + F_{dpdx\_finger} + F_{dpdx\_element} + F_{test\text{signal}} \quad (64)
\]

Based on the analysis in Section 2.5.2, we are justified in dropping the terms \(F_{shear\_sub}, F_{dpdx\_finger},\) and \(F_{dpdx\_element}\) as they are negligible compared to the shear stress on the top of the sensor, \(F_{shear}:

\[
F_{spring} = F_{shear} + F_{test\text{signal}} \quad (65)
\]

substituting eqs. (32 and 63):

\[
kx = F_{shear} + \frac{eA}{2}V_s^2 \left( \frac{1}{(d_0 - x)^2} - \frac{1}{(\alpha d_0 + x)^2} - \frac{1}{(d + x)^2} + \frac{1}{(\alpha d_0 - x)^2} \right) \quad (66)
\]

where \(x\) = sensor displacement, \(d_0\) = sensor finger gap, \(\alpha\) = sensor distance amplification factor, \(V_s\) = average test signal voltage, \(k\) = sensor spring constant, \(F_{shear}\) = applied shear stress \(\epsilon\) = dielectric permittivity of air, and \(A_c\) = capacitance area.

Defining the following quantities:

\[
x = \frac{x}{d_0}, \quad F_e = \frac{eA_c}{2}V_s^2 \quad (67), (68)
\]

we can rewrite the previous equation as:

\[
kx d_0 = F_{shear} + F_e \left( \frac{1}{1-x} - \frac{1}{\alpha + x} - \frac{1}{1+x} + \frac{1}{\alpha - x} \right) \quad (69)
\]

and then defining the following quantities:

\[
p = \frac{F_e}{kd_0}, \quad q = \frac{F_{shear}}{kd_0} \quad (70), (71)
\]

and solving for \(q\) gives:
\[ q = x - p \left( \frac{1}{(1-x)^2} - \frac{1}{\alpha + x} + \frac{1}{1+x} \right) \] (72)

The dimensionless quantity \( p \) is the ratio between the electrostatic force at zero displacement, and the force on the sensor when displacing the entire gap between the sensing fingers, \( d_0 \) (i.e. measuring the maximum designed shear stress). The dimensionless quantity \( q \) is the ratio between the shear stress force, and the maximum designed shear stress force for the sensor. Figure 41 shows \( q \) plotted vs. \( x/d_0 \) for different values of \( p \). The specific curves shown are for a 3 pad design with \( \alpha = 4 \) and \( d_0 = 2 \mu m \), which are typical design values for the shear stress sensors. During sensor operation, as the flow shear stress (or mdot) is increased, the sensor will displace a specific amount \( (x/d_0) \) which is a function of both \( q \) (the shear stress force), and \( p \) (the electrostatic force from the test signal). The line \( p = 0 \) represents the ideal case when there is no test signal force and the displacement is linear with the shear force.

For \( p = 0.0001 \), the line closely follows \( p = 0 \) until about \( x/d_0 = 0.9 \), where it starts to split off as \( F_{test signal} \) becomes large enough to increase sensor displacement. The line \( p = 0.0001 \) peaks around \( q = 0.93 \) near \( x/d_0 = 0.95 \) indicating the stability limit. Increasing \( q \) further, which represents \( F_{shear} \), will further reduce the gap and then cause the sensor to snapdown as described in the previous section. As \( p \) increases, the peak on each curve is lower, while the curves deviate more dramatically from the ideal case, \( p = 0 \). Figure 42 is a graph showing the maximum value of \( q \) (qmax) vs. \( p \) for typical sensor designs. For a sensor to operate over 90% of the shear stress range without snapping down, we need \( q_{max} > 0.9 \) which requires \( p < 0.0001 \).

\( F_{test signal} \) is considered to be an “averaged” steady-state force because the sensor cannot mechanically respond fast enough to the high-frequency AC voltage signal. This electrostatic
force will result in the sensor measuring a larger shear stress than is present if compensation is not implemented. For example (Figure 41), at \( q = 0.6 \), the sensor is displaced \( x/d_0 \sim 0.6 \) for \( p = 0 \), while it is displaced approximately 15% further \( (x/d_0 \sim 0.7) \) for \( p = 0.01 \). The measured shear stress of the sensor is proportional to the sensor displacement eq. (33); therefore we will overestimate the shear stress by 15% if the electrostatic force from the test signal is not included.

![Figure 41: \( q \) vs. normalized distance \((x/d_0)\) for different values of \( p \). Calculated for 3 pad sensors with \( \alpha = 4 \) and \( d_0 = 2 \mu m \).](image)

![Figure 42: Maximum \( q = F_{shear}/kd_0 \) vs. \( p = F_{e0}/kd_0 \). Calculated for 3 pad sensors with \( \alpha = 4 \) and \( d_0 = 2 \mu m \).](image)
Reworking eq. (66), in terms of shear stress, by dividing both sides of the equation by the sensing area \((A_s)\) leads to:

\[
\frac{kx}{A_s} = F_{\text{shear}} + \frac{eA_sV_s^2}{2A_s} \left( \frac{1}{(d_0 - x)^2} - \frac{1}{(c d_0 + x)^2} - \frac{1}{(c d_0 - x)^2} \right) \quad (73)
\]

\[
\tau_{\text{eff}} = \tau_w + \tau_{\text{testsignal}} \quad (74)
\]

where: \(\tau_{\text{eff}} = \) effective shear stress, \(\tau_w = \) actual shear stress, \(\tau_{\text{testsignal}} = \) artificial shear stress, due to test signal and \(A_s = \) shear stress sensing area.

**Figure 43: Shear stress range showing effect of test signal force for sensor design CB-3F2E using as-built dimensions and MS3110 IC for differential capacitive sensing.**

Figure 43 shows the effects of \(F_{\text{testsignal}}\) on a typical MEMS shear stress sensor utilizing the MS3110IC for differential capacitive sensing. In this case the data is plotted for sensor design CB-3F2E with the equations modified for the as-built sensor dimensions. The solid straight line is \(\tau_{\text{eff}}\), the “effective-shear stress” which is directly measured by the MEMS sensor. \(\tau_{\text{eff}}\) is
calculated by: (1) converting the measured sensor output ($\Delta V$) to a differential capacitance change ($\delta C$); (2) converting the calculated $\delta C$ to sensor displacement ($x$), eq. (38); and (3) converting displacement to $\tau_{\text{eff}}$. The “actual shear stress” ($\tau_w$) is calculated by subtracting the “test signal shear stress” ($\tau_{\text{test signal}}$) from $\tau_{\text{eff}}$, eq. (74). Note that the plotted data does include the additional electrostatic force contributed by the fringe capacitance.

The short-dashed line represents $\tau_w$ with the peak representing the onset of snapdown. For each sensor design, a safe shear stress ($\tau_{\text{safe}}$) is defined (illustrated in Figure 43 by the dash-dot line) at 50% of the shear stress which would result in snapdown. During sensor flow testing, attempts are made to keep the applied $\tau_w$ below $\tau_{\text{safe}}$ to avoid snapping down the sensor and limiting its operating lifespan.

In addition to requiring compensation in the calculation of $\tau_w$, the effect of the MS3110 IC test signal serves to limit the operational range of the sensors. Figure 44 shows maximum shear stress values for 3 pad sensors with and without the MS3110 test signal force. The effect of the MS3110 sensing force reduces the maximum shear stress of a MEMS sensor by 10 – 30% depending on the design and its dimensions.
Figure 44: Maximum shear stress values for 3 pad designs illustrating the effect of the MS3110 IC test signal force on the sensor operation. The maximum values are reduced by 10 – 30% due to the electrostatic force, artificially increasing the active fluidic shear stress.

Figure 45: Shear stress range vs. duct mass flow rate showing effect of test signal force. Data is for sensor design CB-3F2E using as-built dimensions and MS3110 IC for differential capacitive sensing. Maximum mdot in tunnel is 0.33 lbm/sec.

Figure 45 is the same data as Figure 43, except that the x-axis values have been replaced with mdot (mass flow rate) values from the calibrated subsonic duct (See Section 8.3). The duct
has a maximum \(\text{mdot} \) of 0.33 lbm/sec (lbm = pound-mass) which will not cause snapdown for this particular sensor. At a duct shear stress of 400Pa, the sensor will have an effective shear stress of 406, resulting in an error of 1.5\%. During sensor flow testing, \(\text{mdot} \) would be kept below 0.20 lbm/sec to stay below \(\tau_{\text{safe}} \) and to avoid snapping down the sensor. A similar figure with separate curves was developed for each sensor design to guide in the selection of the appropriate flow testing shear stress.

2.5.2.2 Lateral Bending of Sensing Fingers

The shear force being measured will be applied to the top surface of the floating element as well as to the sensing fingers. Because the sensing fingers are similar dimensions to the springs, they will deflect slightly, causing a capacitance change larger than expected by only the floating element displacement. Figure 46 shows diagrams of a non-displaced sensor (left) and a displaced sensor (right). The displaced sensor shows exaggerated bending of both the sensing and stationary fingers from the shear force that occurs on their surfaces. Note that while the sensing fingers on CS2 bend towards their “near” stationary fingers, which in turn bend away. The opposite effect occurs on CS1, which complicates the calculation. The reader should refer to Figure 25 in Section 2.1.1 for the specific nomenclature used here.
Figure 46: Schematics showing sensor before displacement (left), and after displacement (right) with exaggerated finger bending.

The shear force on an individual finger can be calculated in the same manner as a distributed load on a cantilever beam. The distributed load \( q_{LOAD} \) is given by the shear stress:

\[
q_{LOAD} = \tau_w b_f
\]

where \( \tau_w \) is the shear stress, and \( b_f \) is the width of the sensing finger.

If the cantilever is fixed on the left end (at \( y = 0 \)), the displacement \( w_{pos}(y) \) along the finger length is:

\[
w_{pos}(y) = \frac{q_{LOAD} y^2}{24EI} \left( y^2 + 6L_f^2 - 4L_f y \right) = \frac{\tau_w b_f y^2}{24E} \left( y^2 + 6L_f^2 - 4L_f y \right)
\]

where \( I \) is the bending moment of inertia, \( h \) is the thickness of the finger, \( E \) is the Young’s Modulus, and \( L_f \) is length of the sensing finger. This quantity represents bending of a sensing finger in the +x direction as a function of \( y \) [77]. If the cantilever is fixed on the right end (at \( y = 2L_f - x_0 \)), we first define a new length:

\[
\tilde{L} = 2L_f - x_0
\]

and rewrite the displacement equation with the new length:
\[ w_{\text{neg}}(y) = \frac{\tau_w b_f \left( L - y \right)^2}{24E \frac{1}{12} hb_f^3 \left( L - y \right)^2 + 6L_f^2 - 4L_f \left( L - y \right)} \]  

(78)

In addition to the individual bending, the entire sensor (floating element) and fingers, will displace an amount \( x \) due to the spring bending:

\[ x = \frac{\tau_w A_s}{k} \]  

(79)

The capacitance for the \( N_f \) fingers on CS2 is calculated by finding the capacitance for a differential length of finger, and then integrating along the length. In this analysis fringing effects are being ignored, as they are not expected to significantly affect the results:

\[ dCS_{2FB}(y) = \frac{N_f \varepsilon h}{d_0 - w_{\text{pos}}(y) + w_{\text{neg}}(y) - \frac{\tau_w A_s}{k}} dy + ... \\
\]

\[ ... + \frac{N_f \varepsilon h}{\alpha d_0 + w_{\text{pos}}(y) - w_{\text{neg}}(y) + \frac{\tau_w A_s}{k}} dy \]  

(80)

where the first fraction represents the capacitance between the sets of near fingers, and the second fraction represents the fractions between the sets of far fingers. The signs of the terms in the denominator represent whether a specific deflection increases the gap or not. The overall capacitance change of \( CS_{2FB} \) can be found by taking the definite integral of eq. (80) between the limits of \( L_f - x_0 \) and \( L_f \), which is the length where the fingers overlap.

\[ CS_{2FB}(x) = N_f \varepsilon h \int_{L_f - x_0}^{L_f} \frac{1}{d_0 - w_{\text{pos}}(y) + w_{\text{neg}}(y) - x} dy + ... \]

\[ ... + N_f \varepsilon h \int_{L_f - x_0}^{L_f} \frac{1}{\alpha d_0 + w_{\text{pos}}(y) - w_{\text{neg}}(y) + x} dy \]  

(81)
The solution is written as $CS_{2FB}(x)$ to emphasize that this capacitance is a function of the floating element displacement ($x$), which is also proportional to the shear stress.

The other half of the comb drive, $CS_{1FB}(x)$ can be calculated in the same manner by switching the signs of all of the displacements in the denominator, and integrating between the same limits. The overall differential capacitance is then given by

$$\delta C(x)^{FB} = CS2(x)^{FB} - CS1(x)^{FB}$$  \hspace{1cm} (82)\]

![Graph showing the effect of finger bending on shear stress vs. differential capacitance. Sensor 3F2A with 120μm long fingers. Note that the data in the graph is for the “higher” shear range of the sensor.

Figure 47: Effect of finger bending on shear stress vs. differential capacitance. Sensor 3F2A with 120μm long fingers. Note that the data in the graph is for the “higher” shear range of the sensor.

Figure 47 represents typical results for a sensor (3F2-A) with the longest sensing fingers (120μm). For approximately 80% of the shear range, the capacitance change will vary by less than 1% when individual finger bending is taken into account. At shear values above this 80%, it still remains within 5%. This is considered to be an acceptable value for these sensor designs. This specific sensor was chosen for analysis because it has longer fingers; designs with similar
length fingers will have similar results, and designs with shorter fingers will exhibit even less of an effect when finger bending is considered in the analysis.

### 2.5.3 Performance Variation Resulting from Fabrication

#### 2.5.3.1 Offset of sensing fingers

Due to precision limitations inherent in MEMS processing, it is possible to have a sensor with a floating element that is offset in either the $+x$ or $-x$ direction. This will change the performance characteristics of the sensors, notably the relationship between shear stress and differential capacitance. To analyze this effect, we will assume sensors that have their floating element displaced by an amount $(s)$, which represents $+10\%$, $+20\%$, $-10\%$, and $-20\%$ of the gap distance $(d_0)$. The differential capacitance equation (ignoring fringing), eq.(39) can be modified to account for this offset as:

$$\partial C(x) = \varepsilon A \left[ \left( \frac{1}{(d_0 - s) - x} + \frac{1}{(\alpha d_0 + s) + x} \right) - \left( \frac{1}{(d_0 + s) + x} + \frac{1}{(\alpha d_0 - s) - x} \right) \right]$$ (83)

Typical sensor results are shown in Figure 48 for sensor design 3F2-B with a nominal $d_0$ value of $2.0\mu$m, and a maximum shear stress of 5000Pa. The red dashed data represents a sensor with no fabrication offset, while the solid lines represent different S values. Error bars of +/-10% are shown to help estimate the variations in the curves. For an S value of +/- 0.1$d_0$ the relationship between shear stress and differential capacitance is unchanged by more than +/- 10% for more than half of the shear stress range (0 to 3000Pa), but the error grows after that. For values of +/- 0.2$d_0$ difference in the curves is closer to +/- 20% for the first half of the shear range.
In general these are acceptable modifications to the sensors’ performance, based on fabrication effects which are difficult to control. However, to reduce this effect, it is recommended that each fabricated sensor sensors has its own calibration curve developed based on the “as-built” dimensions.

![Shear stress vs. differential capacitance for offset in floating element due to fabrication effects (Design 3F2-B). Error bars are +/- 10% of shear stress.](image)

**Figure 48:** Shear stress vs. differential capacitance for offset in floating element due to fabrication effects (Design 3F2-B). Error bars are +/- 10% of shear stress.

### 2.5.3.2 Mismatch of sensing fingers

Due to non-uniformities in the MEMS etching and deposition processes, it is possible to encounter situations where the two sets of sensing capacitors (CS2 and CS1) do not have same dimensions as designed. For example, one set of fingers could be etched more than another, increasing the gap size ($d_0$), or changing the thickness of the fingers ($h$). To determine the effects of these fabrication defects, we again used eq.(39), and input sensors with different mismatched dimensions. The cases that were looked at were for $h_2/h_1 = (0.9, 0.95, 1, 1.05, 1.1)$ and $d_{02}/d_{01} = (0.9, 0.95, 1, 1.05, 1.1)$. The cases $h_2/h_1$ represent when CS2 and CS1 have different finger...
thicknesses (or even overlap (x₀) values), and the cases d₀₂/d₀₁ represent when the gap spacing is different.

Figure 49 illustrates typical results for variations in h₂/h₁ for sensor design 3F2-B. Error bars of +/-5% are plotted on the h₂/h₁ = 1 case which represents no mismatch. The error bars encompass nearly all of the data points indicating that this effect is minimal. Figure 50 shows similar results for d₀₂/d₀₁ variations for the same sensor design, 3F2-B. Similar results are expected for the remaining designs, so we can conclude that this type of effect will be minor and likely can be neglected.

![Graph showing shear stress vs. differential capacitance for variations in capacitor mismatch (h₂/h₁) for design 3F2-B. Error bars are +/-5% of shear stress.](image-url)
Figure 50: Shear stress vs. differential capacitance for variations in capacitor mismatch \((d_{02}/d_{01})\) for design 3F2-B. Error bars are +/-5% of shear stress.

2.6 Dynamic Modeling of Sensor

The MEMS shear stress sensor, placed in a turbulent flow environment, will be subjected to an unsteady shear stress \(\tau_w(t)\) with both laminar and turbulent components:

\[
\tau_w(t) = \tau_{w,\text{lam}} + \tau_{w,\text{turb}}
\]

where \(\tau_{w,\text{lam}}\) is the laminar shear stress, and \(\tau_{w,\text{turb}}\) is the turbulent shear stress[9]. The sensor is designed to measure the unsteady shear stress at the wall \(\tau_w\), and is flush-mounted with the wall, to lie within the viscous sublayer of the boundary layer where the laminar viscous shear stress dominates and can be related to the velocity gradient by:

\[
\tau_w(t) \approx \tau_{w,\text{lam}} \approx \mu \frac{du(t)}{dz}
\]
where $\mu$ is the dynamic viscosity of the fluid at the wall, $u(t)$ is the streamwise velocity at the wall, and $z$ is the direction normal to the wall.

The height of the viscous sublayer is given by $z^+ = 6$ where $z^+$ is a viscous wall unit [80] defined as:

$$z^+ = \frac{z u_t}{\nu} = \frac{z}{\nu} \sqrt{\frac{\tau_w}{\rho}} \tag{85}$$

where $u_t$ is the friction velocity, $\rho$ is the fluid density at the wall, and $\nu$ is the kinematic viscosity at the wall.

To approximate the maximum heights of the sensor that will fall within the viscous sublayer, we assume the following order-of-magnitude quantities for supersonic air flow: $\nu \sim 10^{-5}$ $[m^2/s]$, $\rho \sim 1$ $[kg/m^3]$, $\tau_w \sim 10^2$ $[N/m^2]$. Inserting these values into eq. (85) the height of the viscous sublayer ($z^+ = 6$) is approximated as $\sim 100 \mu m$, although it can be as small as $\sim 1 \mu m$ for supersonic flows. The MEMS shear stress sensor is fabricated with maximum feature heights of $8 \mu m$, indicating that it will lie within the viscous sublayer for many turbulent flows.

Inside the viscous sublayer, velocity fluctuations are given by the Kolmogorov microscale ($\eta$) which estimates the size of the smallest-scale eddies expected to be present in the flow. The wall shear stress is proportional to the velocity gradient, so that the velocity fluctuations result in shear stress fluctuations. Using the same analysis as detailed in Tennekes and Lumley [81] and Chandrasekaran [82], the temporal scale for the velocity fluctuations is given by the Kolmogorov time scale ($\tau$):

$$\tau \sim \frac{\lambda}{u}(\frac{u\lambda}{\nu})^{-1/2} \tag{86}$$

where $\lambda$ = length scale of large eddies (e.g. boundary layer thickness, duct height, etc.).
\( \nu = \) kinematic viscosity, and \( u = \) large eddy velocity scale. The length scale for the velocity fluctuations is given by the Kolmogorov microscale (\( \eta \)):

\[
\eta \sim \frac{\lambda(u\lambda/\nu)^{3/4}}{}
\]  (87)

\( \eta \) and \( \tau \) are combined to give the Kolmogorov velocity scale (\( \nu \)) for the turbulent fluctuations:

\[
\nu \sim \frac{\eta}{\tau} - u(u\lambda/\nu)^{1/4}
\]  (88)

Using the approximation of \( u/U \sim 10^{-2} \) from [81], and defining the Reynolds number as:

\[
\text{Re} = \frac{U\lambda}{\nu}
\]  (89)

eqs. (86-88) can be re-written as:

\[
\tau \sim 10^3 \frac{\lambda}{U} \left(\text{Re}^{-1/2}\right)
\]  (90)

\[
\eta \sim 10^{3/2} \lambda \left(\text{Re}^{-3/4}\right)
\]  (91)

\[
\nu \sim 10^{-3/2} U \left(\text{Re}^{-1/4}\right)
\]  (92)

to clearly indicate that the Kolmogorov scales have a decreasing trend as Re increases. In general, the shear stress will also increase with Re, meaning that sensors measuring larger shear stresses will need to be smaller in size (to resolve the length scales) and respond quicker in time (to resolve the time scales) than at lower shear stresses.

To accurately measure the turbulent fluctuating shear stress, the MEMS sensor needs to have a response time smaller than the Kolmogorov time scale (\( \tau \)) in order to detect the fluctuations. Additionally, the sensor floating element should have dimensions smaller than (\( \eta \)) to measure the shear stress from a single fluctuation as opposed to measuring the average shear stress from multiple fluctuations. To determine the MEMS shear stress sensor response time to turbulent fluctuating shear stress, the length and time scales of the expected turbulent
fluctuations are estimated by using the following order-of-magnitude quantities for supersonic air flow: $v \sim 10^{-5} \text{[m}^2\text{/s]},$ $\lambda \sim 10^{-2} \text{[m]},$ $U$ (the boundary layer edge velocity) $\sim 10^2 \text{[m/s]},$ and $u/U \sim 10^{-2}$ from [81].

Using the approximate values listed above with eq. (86), typical flows will have a Kolmogorov time scale of $\tau \sim 10^{-1} \text{[ms]}$ or a frequency scale of $f = 1 / \tau \sim 10\text{kHz}$. The MEMS shear stress sensors require a frequency response on the order of $f$ to accurately measure the fluctuating turbulent shear stress. As shown later in Section 2.6.1.2, sensor designs have theoretical damped frequency responses ranging from 15 – 55kHz, indicating they should be able to mechanically respond fast enough to accurately measure the fluctuating shear stress in some turbulent flows.

Inserting the above values into eq. (87) gives $\eta \sim 5 \times 10^{-5} \text{[m]}$ or 50 [$\mu\text{m}$] representing the maximum length of the floating element required to avoid spatial averaging of the measurement. Gad-el-Hak [66] suggests that sensing element lengths only need to be on the order of $\eta$ making the size of many of the MEMS shear stress sensor designs (with floating element sizes ranging from 35 – 1000[$\mu\text{m}$]) sufficient for detecting the shear stress fluctuations in supersonic air flows.

The length and time scales of the turbulent fluctuations are highly-dependent on the actual flow environment and need to be analyzed on a case-by-case basis. However, we conclude that the MEMS shear sensor designs are capable of mechanically responding to the fluctuating shear stress in some turbulent environments. Measurement limitations arising from sensing electronics will be discussed in Section 6.1.3.
2.6.1 Theoretical Frequency Response

2.6.1.1 Undamped Frequency Response

Undamped natural frequencies are important because they are an upper bound on how fast an object can respond to a force in a particular direction. An object cannot be excited at a frequency higher than its natural frequency. They differ from “damped” natural frequencies in that there is an assumption of no viscous damping (fluidic friction). Damped frequencies will be discussed in Section 2.6.1.2.

To determine the theoretical undamped frequency response of the sensor, it is modeled as an unforced 1-D mass-spring system moving in the x-direction:

\[
m \frac{d^2x}{dt^2} + kx = 0
\]  
(93)

with a natural frequency given by:

\[
f_x = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]  
(94)

where \(m\) is the mass of the floating element (assuming a density of 2331kg/m\(^3\) for Silicon). This model also assumes 1-D motion (i.e. there is no coupling between motion in the x and y directions, etc.). Similar equations can be developed in the y and z-directions to calculate \(f_y\) and \(f_z\).

Undamped natural frequencies in the streamwise direction (\(f_x\)) for all of the sensors were calculated using the analytical spring constants (eqs. 43 and 44), and the masses of each sensor design. The values are shown in Figure 56, and plotted next to the corresponding damped natural frequencies. As discussed in Section 2.6 it is important that \(f_x\) values are large enough, O(10kHz), so that the sensors can respond to the turbulent fluctuations. All of the undamped
values are correct order-of-magnitude meaning that the sensors should be able to mechanically respond to the shear stress fluctuations in the streamwise direction.

The undamped frequencies were also calculated using FEA results from COMSOL. The first method used the calculated FEA spring constants (Table 4) in their respective direction. The second method used the COMSOL Eigenvalue solver, which is a built-in program that can determine the eigenfrequencies of a structure, which are the same as the undamped natural frequencies. There are different Eigenfrequencies corresponding to the in-plane motion in the x, y, and z directions, as well as eigenfrequencies related to torsional motion (rotation) in xy, yz, and zx directions.

Table 5: Inventory of sensor element designs chosen for FEA analysis.

<table>
<thead>
<tr>
<th>Method:</th>
<th>Sensor 3F2B</th>
<th>Sensor CD-3F2A</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>As-designed dimensions</td>
<td>As-built dimensions</td>
</tr>
<tr>
<td></td>
<td>( h = 8.0\text{um}, \ d_0 = 2.0\text{um} )</td>
<td>( h = 6.8\text{um}, \ d_0 = 2.3\text{um} )</td>
</tr>
<tr>
<td>Method:</td>
<td>Analytical</td>
<td>FEA</td>
</tr>
<tr>
<td>Calculation:</td>
<td>( \sqrt{k/m} )</td>
<td>( \sqrt{k_{FEA}/m} )</td>
</tr>
<tr>
<td>Units:</td>
<td>[kHz]</td>
<td>[kHz]</td>
</tr>
<tr>
<td>( f_x ) – Streamwise</td>
<td>59.5</td>
<td>55.7</td>
</tr>
<tr>
<td>( f_y ) - Transverse</td>
<td>N/A</td>
<td>615.3</td>
</tr>
<tr>
<td>( f_z ) – Out of Plane</td>
<td>238.6</td>
<td>193.2</td>
</tr>
</tbody>
</table>

Figures 51 - 54 show the first four modes of vibration for sensor CD-3F2A, which are typical folded-beam sensor designs. In each of the figures, the displacement is exaggerated for visualization purposes, which results in some non-physical motions. For example in Figure 51, the floating element has such a large displacement that parts of the spring (see the top part) are shown to pass through solid objects, which is not physically possible.
The first vibrational mode, occurring at 31.4kHz, is the streamwise sensor displacement. This value matches well with the prediction of 31.0kHz given by the natural frequency calculated using the analytical spring equations. The $2^{nd}$, $3^{rd}$, and $4^{th}$ modes are y-normal, xy-torsion, and z-normal, which occur at 105kHz, 110.3kHz, and 115kHz, respectively. There are additional torsional modes at higher frequencies, however these are not expected to play a role in the sensor operation as it will be predominantly subjected to normal forces.

Table 5 is a summary of the undamped natural frequencies for the two FEA sensor models. There is good agreement among the three different methods for finding $f_x$, the maximum sensor frequency in the x-direction. For $f_y$, the two FEA methods do not match up well. This is because when $k_{y,FEA}$ was calculated, it was based on displacement of the sensing fingers in the transverse (y) direction only, while the Eigenfrequency solution is based on a coupled x-y motion where the springs move more than the fingers (Figure 52). Essentially, there is no natural frequency that is purely in the y-direction, and so the Eigenfrequency solution is considered to be a more accurate representation of the natural frequency of the device. The $f_z$ values from the two different FEA methods match within 5%, but are approximately 13-20% lower than the analytical models. This is representative of the results from Section 2.5.1 where the FEA gave a lower out-of-plane stiffness ($k_z$).
Figure 51: Sensor CD3F2A - 1\textsuperscript{st} mode of vibration \((f = 31.4\text{kHz})\) – floating element displacement in x-direction (displacement is enlarged for visualization purposes).

Figure 52: Sensor CD3F2A - 2\textsuperscript{nd} mode of vibration \((f = 105\text{kHz})\) – floating element displacement in y-direction.
Figure 53: Sensor CD3F2A - 3rd mode of vibration ($f = 110.3$kHz) – floating element torsion in xy-plane.

Figure 54: Sensor CD3F2A - 4th mode of vibration ($f = 115$kHz) – floating element displacement in z-direction.
2.6.1.2 Damped Frequency Response

The frequency response of the sensing element in the streamwise (x) direction dictates the sensor response time to variations in the shear stress. The damped natural frequency is calculated while accounting for the viscous effects of the sensor moving through the fluid. Viscous damping is caused by a combination of Couette flow and squeezed-film damping around sensor features [77]. Couette flow results from the sensor floating element moving and creating a shear layer between the bottom of the floating element, and the sensor substrate. Squeezed film damping results from the sensing fingers moving towards the stationary fingers and pushing the air out of the gap.

The motion of the floating element can be modeled as a 1-D spring-mass-damper system in the x-direction:

\[ m \frac{d^2 x}{dt^2} + c \frac{dx}{dt} + kx = 0 \]  \hspace{1cm} (95)

where \( c \) is the coefficient of damping. The equations for the damping frequency \( (\omega_d) \) are given in eqs. (96-99).

\[ c_{total} = c_{couette} + c_{squeezedfilm} = \frac{\mu A_s}{H} + \frac{96\mu x_0 h^3}{\pi^4 d_0^6} N_f \]  \hspace{1cm} (96)

\[ \zeta = \frac{c_{total}}{c_c} = \frac{2m \omega_n}{c_{total}} \]  \hspace{1cm} (97)

\[ \omega_n = \sqrt{\frac{k}{m}} \]  \hspace{1cm} (98)

\[ \omega_d = \sqrt{1 - \zeta^2} \omega_n = \sqrt{1 - \zeta^2} \sqrt{\frac{k}{m}} \]  \hspace{1cm} (99)
where: $c_c =$ coefficient of critical damping, $\zeta =$ damping coefficient, $\omega_n =$ undamped natural frequency, $N_f =$ number of fingers per side of floating element, $H =$ gap between floating element and substrate, and $\omega_d$ is the damping frequency.

Sensor viscous damping was evaluated assuming air properties at standard conditions (20°C, 14.7psia). Figure 55 shows that for 3 pad designs, the damping ratio ranges from 0.005 – 0.319, resulting in an underdamped response (since $\zeta < 1$). Figure 56 shows the undamped and damped natural frequencies for all 3 pad designs. Undamped natural frequencies range from approximately 17 – 60kHz and are dependent on the sensor design. Damping changes the natural frequencies insignificantly, with an average frequency decrease of 1.3%. Therefore, viscous damping in air (at standard conditions) has only a minor effect on the sensor response time. However, looking at eqs. (96) - (99), $\omega_d$ scales with $\mu$ of the fluid, meaning that if a fluid with a much different viscosity is used, or $\mu$ changes appreciably due to temperature, this would affect the sensor response time.

Turbulent fluctuations are expected to occur on a frequency scale O(10kHz), which is satisfied for all of the sensor designs, even with damping taken into account. Therefore the sensor designs should prove adequate for mechanically responding to the turbulence. However, excitations occurring close to the sensor natural frequencies can lead to resonance since the floating element motion is underdamped.
2.6.2 Fluctuating Pressure in Turbulent Flows

As discussed previously (Section 1.5.5), experimental studies have shown that in turbulent boundary layers, pressure fluctuations normal to the wall (out-of-plane) can be as much as 2 orders of magnitude larger than the in-plane(x) shear stress fluctuations [65], [51]. To
evaluate sensor performance, it is necessary to determine the effect that this will have on the sensor output, which is designed for measuring in-plane shear stress.

As previously described, the sensors are designed so that they are stiffer in the out-of-plane (z) direction than in the streamwise (x) direction. Additionally, any z-motion is cancelled out to first order by the differential capacitance measurement. However, z-displacement sensitivity will be increased due to fabrication mismatch between CS2 and CS1. Additionally, when the sensor displaces in x, CS2 and CS1 become more mismatched (because the gap changes size) causing the z-sensitivity to increase. This analysis represents the case when there are forces in both the x and z directions simultaneously.

To determine the effect of the normal-pressure fluctuations, the sensor mismatch is varied and the effect on the sensor output is analyzed. The normalized cross-axis sensitivity is defined by:

\[ S(zx) = \frac{S(z)}{S(x)} = \frac{\frac{d\delta Cz}{dP_w}}{\frac{d\delta Cx}{d\tau_w}} \]  

where \( S(z) \) = sensitivity in z-direction, \( S(x) \) = sensitivity in x-direction, and \( P_w \) = normal pressure. \( \delta Cx \) and \( \delta Cz \) are capacitance changes caused by motion in x and z, respectively, and \( \delta Cz = CS2(z) - CS1(z) \), and \( \delta Cx = CS2(x) - CS1(x) \). \( S(zx) \) is a function of sensor displacement, and the sensor mismatch:

\[ S(zx) = f(x/d_0, h_2/h_1, d_{02}/d_{01}) \]  

where \( h_2 \) = thickness of CS2 finger, \( h_1 \) = thickness of CS1 finger, \( d_{02} \) = CS2 finger gap, and \( d_{01} \) = CS1 finger gap.

Figures 57 and 58 show the results for a typical folded-beam sensor design, 3F2-B, with \( h = 8\mu m \) and \( d_0 = 2\mu m \). The ratios \( h_2/h_1 \) and \( d_{02}/d_{01} \) are varied from 0.9 to 1.1 in steps of 0.05 to
mimic sensor designs with large fabrication defects. The ratio $h_2/h_1$ represents a sensor that does not have same capacitance area ($A_c$) for both CS2 and CS1. This could be a result of fingers bending out-of-plane, non-uniform fabrication, or a floating element that has been tilted to one side. The ratio $d_{02}/d_{01}$ represents the case if the sensor was initially offset towards CS2 or CS1 which would increase $d_{02}$ and decrease $d_{01}$ or vice versa.

To determine $S_{zx}$ as the sensor deflects, $x/d_0$ is varied in steps of 0.01, and $S_x$ and $S_z$ are calculated at each step using a 2\textsuperscript{nd} order central difference formula. For this particular sensor, the maximum magnitude is $-6.88 \times 10^{-3}$ which occurs at $x/d_0 = 0.47$ for $d_{02}/d_{01} = 0.9$. In other words, when comparing the sensitivity in the z and x directions, in the worst case scenario, the z sensitivity would be only 0.688% of the x-sensitivity. The out-of-plane force would be negligible if it is the same order-of-magnitude as the streamwise force.

![Figure 57: Normalized cross-axis sensitivity ($S_{zx}$) for varied thickness mismatch.](image-url)
To determine how the fluctuating wall pressure ($P_w'$) will affect the sensor signal, it is necessary to account for the fact that in the worst-case scenario, $P_w'$ can be up to 2 orders of magnitude larger than the fluctuating shear stress ($\tau_w'$)[65]:

$$P_w' \sim (10^2)\tau_w'$$  \hspace{1cm} (102)

The following ratio relates the capacitance changes due to pressure fluctuations (in the $z$-direction) and the shear stress fluctuations (in the $x$-direction):

$$C_{zx} = \frac{\delta C_z}{\delta C_x} = \frac{dC_z/dP_w}{dC_x/d\tau_w} = \frac{P_w'}{\tau_w'\delta} = S_{zx}(10^2)$$  \hspace{1cm} (103)
Figure 59: Ratio of theoretical capacitance changes (Czx) caused by pressure fluctuations normalized by shear stress fluctuations for varied thickness mismatch.

Figure 60: Ratio of theoretical capacitance changes (Czx) caused by pressure fluctuations normalized by shear stress fluctuations for varied gap mismatch.
For the 10 fabrication situations considered, the maximum magnitude of $C_{xz}$ is $-0.69$, again occurring at $x/d_0 = 0.47$ for $d_{02}/d_{01} = 1.1$. These results are interpreted as that in the worst-case scenario, the capacitance change caused by the out-of-plane fluctuation could be as large as 69% of the in-plane fluctuation. This possibility is unacceptably large as it can easily interfere with the streamwise turbulent fluctuation measurements. However, at lower frequencies ($f^* < 0.05$) $P_{w'}$ is at most 10 times larger than $\tau_{w'}$ resulting in an out-of-plane capacitance change that is less than 7% of the in-plane capacitance change which is an acceptable amount of error in measurement. In general, the error introduced by the wall pressure is frequency-dependent and should be analyzed based on the specific flow conditions. This will be discussed further with regards to sensor test results in Section 8.3.8.

This effect is driven primarily by the fact that as the sensor displaces in the x-direction, it becomes much more sensitive to motion in the z-direction. Due to the large effect that that cross-sensitivity can have on the sensor measurement, this is an area of research that certainly should be considered in future efforts (Section 9.2).
3 Sensor Fabrication

The MEMS sensors were fabricated by Ronan Larger at the Université de Sherbrooke utilizing AutoCAD drawings made by Jean-Philippe Desbiens. The sensor designs and layouts were developed through collaboration between Columbia University, the Université de Sherbrooke, and ATK-GASL. Specific designs were chosen to reflect a broad range of sensor performance ranges and to offer risk mitigation from manufacturing errors (see Section 2.2). Sensor fabrication is described only briefly in this chapter as the focus of this thesis is on sensor design, characterization, packaging, and testing. For additional information on sensor fabrication, the reader is referred to the thesis of Ronan Larger [75].

3.1 Mask Generation and Wafer Layout

The 36 shear sensor designs were drawn up in AutoCAD, with a 5mm x 5mm space allocated for each design. Each sensor has 8 electrode pads, including redundant connections to allow for electrical checkout, 2 electrode pads for connecting the substrate to ground, and appropriate test structures for measuring capacitance. Also, a “doublet sensor” design was included which has two copies of the 4F2-B design, oriented for sensing in both ‘x’ and ‘y’ in-plane flow directions. Linear transmission line structures have been included to measure contact resistance to for material characterization. All of this fits into one quadrant of a 4in. diameter silicon-on-insulator (SOI) wafer, providing four copies of each sensor design per wafer. Figure 61 shows an AutoCAD drawing of the shear stress sensor designs, with zoomed in images of a 3 pad design, a 4 pad design, and the doublet design.
Fabrication of finger widths exactly as designed is not feasible due to manufacturer limitations; however finger width is critical to sensor performance. For optical photomasks, the smallest features available from typical laser photomask fabrication procedures are 2μm with a tolerance of +/- 0.5μm. This tolerance is unacceptably risky to us since every sensor design hinges on the correct size of design features. Therefore, a more expensive electron beam lithography process was utilized to fabricate the design layer mask with a tolerance of +/- 0.1μm. Stainless steel shadow masks necessary for metallization of electrode pads were also purchased.

Figure 61: Sensor element layout on a 4-inch wafer.
3.2 Process Flow

The shear stress transducer is micromachined in the device layer of a Silicon-on-Insulator (SOI) wafer. The device layer is highly doped O(10^{20}) atoms/cm³ n-type Si with a thickness ranging from 6.8μm to 8.0μm depending on the specific wafer. The SiO₂ (oxide) thickness is roughly 2μm, The Si substrate is low conductivity, with a resistivity greater than 1000Ω·cm, to reduce current leakage between the capacitors.

The major steps in process flow of the sensor are shown in Figure 62. First, the through-substrate vias are created by etching from the backside using Deep-Reactive-Ion-Etching (DRIE), contacting the buried surface of the device layer. Next, the vias are insulated (with SiO₂) using thermal oxidation, and RIE from the backside is used to remove oxide at the bottom of the vias. A lithographic polymer dry film (MX5020 from Dupont) is laminated and utilized as a mask during the RIE. The polymer film is patterned to overhang the Via edges, which is necessary to avoid etching through the oxide on the top corners of the vias. A metal layer in the vias is deposited using a physical mask to form electrical contact between the devices and the backside of the wafer. Sensing elements are etched by DRIE on the frontside of the sensor, and released by a wet etching of the oxide underneath the device layer. The floating elements have 5μm diameter holes in them to aid in completely under-etching (and fully-releasing) the structure and allowing for it to move when exposed to a shear force. Additional technical details of the sensor microfabrication can be found in Larger and Fréchette [71] and the thesis of Ronan Larger [75].
These novel through-substrate interconnects allow flush-mounting of the sensor without perturbing the flow and they also protect the electrical connections from the flow environment.
(only Si and SiO$_2$ are exposed). There was a great deal of challenge in fabricating the Vias, and often they were not electrically connected to both the top and bottom pads. In order to complete the electrical path it was necessary to apply a small amount of conductive silver epoxy by hand. Figure 63 shows Scanning Electron Microscope (SEM) photos of a successfully fabricated sensor, which was cleaved through Vias for visualization purposes.

**Figure 63:** SEM photos of fabricated sensor. This is a side-view of a fabricated sensor with cleaved vias in the foreground, and the sensing element located in the center of the chip.
3.3 Sensor Yield

The MEMS sensors were fabricated by Ronan Larger at the Université de Sherbrooke, and evaluated to check for release of floating elements and electrically connected backside pads. They were then shipped to Columbia University for characterization, packaging, and testing. Each MEMS shear stress sensor is designated by a 6 or 7 digit code for inventory purposes. The first 2 or 3 digits (e.g. CD, BA, ECA, etc.) are used to designate which silicon wafer that the MEMS sensor was fabricated on, as well as the date on which it was received at Columbia University. Sensors from different wafers will typically have different values for $d_0$ (gap distance) and $h$ (device layer thickness) due to some variation in fabrication processes. The fabrication values for the sensor mentioned in this report are listed in Table 6. As previously mentioned, the last four digits in the sensor name designate specific sensor design features (Table 3).

<table>
<thead>
<tr>
<th>Wafer</th>
<th>$h$ [μm]</th>
<th>$d_0$ [μm]</th>
<th>$b_f$ [μm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BA</td>
<td>8.0</td>
<td>2.8</td>
<td>1.2</td>
</tr>
<tr>
<td>CB</td>
<td>6.85</td>
<td>2.25</td>
<td>1.75</td>
</tr>
<tr>
<td>CD</td>
<td>6.85</td>
<td>2.25</td>
<td>1.75</td>
</tr>
<tr>
<td>CA</td>
<td>8.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>ECA</td>
<td>8.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

During the initial etching process it was discovered that many of the frontside electrical paths were undercut so much that they could not survive the forces occurring on them during the flow testing. Therefore, new masks were developed which allowed for protection of these paths while allowing the floating element, fingers, and springs to be fully-etched and released.
Additional difficulties were encountered regarding the release of the floating elements. During the etching, wafer pieces with multiple sensors were processed, which resulted in sensors with different sized floating elements being etched simultaneously. Unfortunately this meant that if a larger floating element was to be released, it was likely that a nearby smaller floating element would be overetched and possibly destroyed. Similarly if a smaller floating element was etched, that often resulted in underetching of larger sensors. Attempts were made to re-etch individual sensors that were not fully-released; however this was generally not successful.
4 Sensor Packaging

4.1 Backside Packaging

The MEMS sensor “packaging” or “package” is the receptacle that holds the sensor and interfaces the micro-scale sensor with the macro-scale world. Due to its importance, the team began considering different package designs early on in the sensor design process. The team at ATK-GASL contributed the initial effort on developing a package, and generated a design and mounting method which inspired the final product and mounting process that were used for sensor testing at Columbia University.

For the MEMS capacitive shear stress sensor, the package serves two main purposes: (1) ensuring that the sensor is secure so that it can be installed flushed-mounted with the tunnel wall; and (2) interfacing the sensor backside with the macro-sized electrical connections needed for the capacitance measurements. Furthermore, this electrical connection needs to be short in distance (to reduce parasitic capacitance), and fully grounded and shielded (to reduce electrical noise).

One of the main objectives of this sensor design was backside electrical connections, so that flow could be measured forwards and backwards without wires perturbing the flow. Currently, the majority of shear stress sensors utilize frontside package connections made by wire bonding from electrode pads on the sensor to pads on the package. Despite the sensor benefits, backside connections are not typically used due to the additional MEMS fabrication complexities, coupled with the challenges of creating a package to interface with those backside connections.
In a typical frontside packaging scheme, wire bonds are located downstream of the sensing element to not perturb the measured flow, and can be coated in epoxy resin or other materials to protect them from the flow environment [51]. This method is effective, however it limits the directionality of the sensor, because flow from the reverse direction would be obstructed. There also is a significant risk of damaging the sensor when liquids are directly applied directly onto the surface. The small features on the sensors can act as channels with capillary forces wicking excess liquid onto the sensor which can destroy the sensor.

The primary package design for this MEMS sensor will utilize backside sensor connections. However, separately, a frontside package design (Section 4.1.3) will be used during the initial stages of sensor development since it is much simpler to design and fabricate.

Figure 64: Schematic illustrating the procedure for securing a sensor in the cold flow testing package. Conductive epoxy is applied to the top of the connector pins. The sensor is then placed on a suction tube which is lowered so that connector pins match up with backside pads. The epoxy provides both the electrical and mechanical connections.
Figure 65: Diagram of a packaged MEMS sensor installed in flow testing environment. The aluminum package is then flush-mounted with the walls to reduce flow perturbations.

4.1.1 Description of Packaging and Mounting Procedure

The packaging consists of a 0.375in. diameter aluminum cylinder with a 1.1mm diameter hole for each of the 8 connector pins; which mate with the contact pads. The holes are in a pattern matching the electrode pads on the backside of the sensor as shown in Figure 33. The connector pins are made from insulated wire (0.59mm ID / 1.0mm OD) with the insulation at the ends stripped off. The pins are aligned to the same height and secured in place at the bottom end by soldering to a custom-made, close coupled PC Board (PCB). The MEMS sensor has a thickness of approximately 0.5mm, and when placed on top of the pins will have its sensing element flush with the top surface of the package.
The following procedure is followed to mount a MEMS shear stress sensor in the package (Figure 64 provides an illustrative overview of the mounting process):

1) A small amount of conductive epoxy is applied to the eight connector pins using a fine wire. Trials were performed to determine the appropriate amount of epoxy to apply so that all 8 pads were connected to the sensor pads while not shorting to one another.

2) A 1/16 in. OD piece of aluminum tubing connected to a vacuum pump is placed through the center of the package, to hold the sensor in place.

3) The MEMS sensor is placed on top of the tube using tweezers. The sensor is manually aligned so that the 8 electrode pads match up with the 8 pins in the package.

4) The vacuum tube with the MEMS sensor on it, is lowered slowly, and upon reaching the conductive epoxy, the sensor is held in place by the epoxy while the tube is removed.

5) The top of the sensor is pushed down on the corners, avoiding damaging the released structures, until the top surface of the sensor is flush with the top surface of the package. Typical height differences between the sensor surface and the package were estimated at ranging from approximately 50μm to 200μm.

6) After the sensor is successfully installed in the package, the conductive epoxy cures for 4 hours. It is very important that the sensor it is not disturbed while the conductive epoxy cures.

7) The package is then flush mounted inside the duct wall using set screws to hold it in the proper position.
Figure 65 is a schematic of a packaged sensor installed inside the duct wall for flow-testing. For cold flow testing both the mechanical and electrical connections for the MEMS sensor were provided by a conductive silver epoxy. For high-temperature applications or environments with higher flow forces, it is necessary to use high-temperature conductive materials and possibly a separate adhesive entirely for sensor adhesion. This adhesive could be applied in the remaining gaps between the MEMS sensor and the package.

4.1.2 Package Specifications and Features

Designing a working package for the MEMS shear stress sensor for cold flow testing required three iterations of working prototypes. The final design includes the following features:

1) MS3110 IC close-coupled to sensor on custom PC Board for best capacitance resolution by minimizing noise and parasitic capacitance.

2) DIP switches for connecting sensor to close-coupled MS3110 IC or to BNC connectors for an external capacitance measurement using C-V meter.

3) DIP switches to allow for disengaging individual connector pins for continuity measurements across sensor.

4) Fully-shielded metal box surrounding all components to reduce electrical noise.

5) Insulated wires used as connector pins, mounted in a shielded aluminum cylinder to reduce parasitic capacitance.

6) Ability to install sensor package in multiple flow field set-ups (e.g. duct flow, wall jet).

7) Package utilizes a ball bearing to allow the sensor to rotate 360° for azimuthal tests when installed in flow field.
8) Package allows the sensor to move up and down for protrusion / recession tests in flow field.

Figure 66 is an image of the bottom the package showing the custom-made PCB installed inside a shielded aluminum box. Arrows illustrate the locations of the MS3110 IC, sensor pins, DIP switches and connectors.

Figure 67 shows a top view of the sensor package without a sensor installed. The 8 pins are in the middle of the image, mounted inside an aluminum cylinder. This aluminum cylinder fits inside a ball bearing to allow 360° rotation for azimuthal testing. The mounting system also allows for vertical motion of the package into the duct for sensor protrusion / recession testing.
Figure 66 (Top): Bottom view of sensor package showing custom-made PCB (2in. x 2in.) and aluminum box (3.5in. x 3.5in.) with package connections.

Figure 67(Bottom): Top view of sensor package showing aluminum cylinder mounted inside ball bearing for azimuthal testing.
The MEMS package was designed to be used in multiple experimental set-ups without having to remove the sensor. Figure 68 and Figure 69 show the sensor package installed in the wall jet and duct flow test setups, respectively. Both setups will be described in detail in Section 0.

### 4.1.3 Frontside Packaging

An alternate frontside packaging method was developed for the MEMS shear stress sensors to allow for sensor testing while the team simultaneously developed a method for creating backside sensor connections. A schematic of the frontside packaging is shown in Figure 70 and Figure 71. The package includes a 0.375in. diameter aluminum cylinder with a 5.5mm x 5.5mm square recess that is the same depth as the MEMS sensor thickness (0.5mm). Three 1.1mm diameter holes are drilled in the cylinder on the downstream side of the sensor with 1.0mm diameter insulated wires mounted in them acting as the connector pins.
To mount the sensor in the package a small film of cyanoacrylate adhesive (Superglue) is applied inside the recess, and the sensor is placed on top of it. After the adhesive dries, electrical connections are made to the sensor using 100μm diameter copper wire. First, the wires are soldered to the connector pins, ensuring that the free ends of the wires are resting on the frontside pads. Next, using a microscope and probe station, conductive silver epoxy was carefully applied to connect the wires to the frontside pads. While the silver epoxy cures, it was important to keep the sensor wires away from insulating materials (e.g. glass and plastic) as they can create electrostatic forces on the wires, which can pull them off of the electrode pads.

Connecting to the MEMS shears stress sensor from the frontside was simpler from the MEMS sensor fabrication standpoint as it did not require the complex steps for fabricating through-wafer interconnects, i.e. the vias. However, a frontside sensor suffers from the following problems which are mitigated by using backside sensor connections: (1) sensor cannot measure recirculating flow as the wires will perturb the flow when it approaches the sensor from the trailing edge; (2) wires can adversely affect flow downstream of the sensor (e.g. perturbing the

Figure 70 (L): Schematic of the side view of the frontside packaging. Flow is from right to left and the connecting wires are downstream of the sensing element.  
Figure 71 (R): Schematic of the top view of the packaging. Wires are connected to pads 1, 3, and 4 for a differential capacitance measurement (CS2 – CS1).
boundary layer); and (3) wires and conductive adhesives will limit the sensor applicability in harsh environment due to their material temperature limitations.
5 Sensor Capacitance

The shear stress sensor operates by measuring a differential capacitance ($\delta C$) which varies with an applied shear stress. $\delta C$ is the difference between the two sense capacitors (CS2 and CS1), which are formed by the comb drives present on both sides of the floating element. The sense capacitors are made from the Si device layer and have air (or SiO$_2$) as the dielectric. Additionally, there are other capacitances present on the sensor resulting from frontside, backside, and “via” features having a capacitance with respect to the Si substrate, with the insulating SiO$_2$ layer as the dielectric.

Figure 72 is a schematic illustrating the sources of capacitance inherent in the MEMS sensor among Pad 2 (CS2), Pad 3 (CSCOM / Floating Element), Pad 4 (CS1), and the substrate. For simplicity, only the capacitances present among these three pads are drawn here, however similar capacitances are present at the remaining electrodes pads on the sensor. For the analysis included in this section, all of the capacitances are assumed to be at an equilibrium temperature of 20°C.
Figure 72: Schematic illustrating capacitances in MEMS shear stress sensor between pads 2, 3, 4, and substrate. Capacitors are modeled as parallel plates with the insulator (SiO₂ or air) acting as the dielectric.

The following are descriptions of the various capacitances present and their nomenclature:

CS2, CS1: Variable capacitors between the moving sensing fingers with air and the stationary fingers, as the dielectric.

C_T2, C_T3, CT4: Static capacitances between all frontside (Top ‘T’) structures (pads, paths, and anchors) and the substrate with SiO₂ as the dielectric.

C_B2, C_B3, C_B4: Static capacitances between the backside pads (Bottom, ‘B’) and the substrate with SiO₂ as the dielectric.

C_V2, C_V3, C_V4: Static capacitances between the four sides of the square metallized vias (‘V’) and the substrate with SiO₂ as the dielectric.
5.1 Overall Sensor Capacitance

To calculate the overall capacitance of the MEMS sensor, we assume multiple parallel plate capacitors in combination. For a parallel plate capacitor with an aspect ratio larger than 50:1, fringing effects can be ignored and the capacitance is given by:

$$C = \frac{\varepsilon A_c}{d}$$  \hspace{1cm} (104)$$

where $\varepsilon$ is the dielectric permittivity, $A_c$ is the capacitive area, and $d_0$ is the gap space between capacitor plates.

Eq. (104) is assumed for all of the capacitors present on the MEMS sensor except for the variable capacitors created by the sensing fingers. As discussed before (Section 2.1.1), the sense capacitors (CS2 and CS1) including fringe effects have a capacitance given by:

$$C(x) = \frac{\varepsilon A_c}{d_0 - x} \left(1 + 1.9861 \left(\frac{d_0 - x}{h}\right)^0.8258\right) + \frac{\varepsilon A_c}{\alpha d_0 + x} \left(1 + 1.9861 \left(\frac{\alpha d_0 + x}{h}\right)^0.8258\right)$$  \hspace{1cm} (105)$$

where $\alpha$ is the distance amplification factor, $h$ is the sensing finger thickness, $A_c$ is the capacitance area, $x$ is the sensor displacement, and $d_0$ is the gap space between capacitor plates.

To calculate the overall capacitance of the sensor, we start by creating an equivalent circuit for the sensor for the portion represented by Pad 2, Pad 3, and the substrate (Figure 74). In the circuit, it is assumed that all of the points connected to Pad 2 by a “conductor” are at the same potential, so they are represented by a single point. The same rule is applied to Pad 2, and the substrate.
To calculate the overall capacitance, we then create equivalent capacitances from the different capacitor combinations. For example, $C_{B2}$, $C_{V2}$, and $C_{T2}$, are all capacitances between pad 2 and the substrate. They are in parallel with each other and add up to an equivalent capacitor $C_{sub2}$:

$$C_{\text{sub}2} = C_{B2} + C_{V2} + C_{T2}$$  \hspace{1cm} (106)$$

The same equation applies to $C_{B3}$, $C_{V3}$, and $C_{T3}$:

$$C_{\text{sub}3} = C_{B3} + C_{V3} + C_{T3}$$  \hspace{1cm} (107)$$

$C_{\text{sub}2}$ and $C_{\text{sub}3}$ are capacitors in series and are combined to form the substrate capacitance between pads 2 and 3, $C_{\text{sub}23}$:

$$C_{\text{sub}23} = \frac{1}{(C_{\text{sub}2})^{-1} + (C_{\text{sub}3})^{-1}} = \frac{(C_{\text{sub}2})(C_{\text{sub}3})}{C_{\text{sub}2} + C_{\text{sub}3}}$$  \hspace{1cm} (108)$$

The capacitance measured by two probes placed at pads 2 and 3, $CS_{2\text{eff}}$, would be a combination of the sensor capacitance, $CS2$, and the substrate capacitance $C_{\text{sub}23}$, where $CS2$ is given by eq. (105):
\[ CS2_{\text{eff}} = CS2 + C_{\text{sub23}} \]  \hspace{1cm} (109)

The proceeding analysis is only for the MEMS sensor, and does not include parasitic capacitance which will be present from the packaging connections. Eq. (109) is modified to include the parasitic capacitance from the packaging (CP2) as:

\[ CS2_{\text{eff}} = CS2 + C_{\text{sub23}} + CP2 \]  \hspace{1cm} (110)

CS2\text{eff} is the value that is measured by the MS3110 IC or the Keithley 590CV. It is dependent on the packaging and needs to be evaluated for each packaging design and its specific materials. Following the same procedure, we can calculate CS1\text{eff}, the effective capacitance for CS1:

\[ CS1_{\text{eff}} = CS1_{\text{sub34}} + CP1 \]  \hspace{1cm} (111)

An equivalent circuit representation (created from Figure 72) including Pad 2, Pad 3, Pad 4, and the substrate was used to do this, and is included below:

![Equivalent circuit representation](image)

\textit{Figure 74: Equivalent circuit representation of the schematic in Figure 72 representing pads 2, 3, 4, and substrate.}
5.2 Parasitic Capacitance

The parasitic capacitance of the sensor packaging is analyzed to determine the magnitude of its contribution to the overall system capacitance. The primary backside packaging design for the MEMS shear sensors was discussed in detail in Section 4.1. It consists of eight metal connector pins embedded in a cylinder in the same circular pattern as the backside sensor pads (Figure 75). The MEMS sensor is connected to the pins using a conductive silver epoxy which serves as both the electrical and mechanical connections. The bottoms of the pins are then soldered to a PC-Board (PCB) with a thickness of 0.065in., and mounted onto the bottom side of the cylinder. The PCB (ExpressPCB, Inc.) is made from FR-4 epoxy glass, and has a capacitive sensing IC mounted on it that converts the capacitance of the MEMS sensor to an output voltage ($V_0$).

The final version of this packaging setup uses an aluminum cylinder with insulated wires (0.59mm ID / 1.0mm OD) as the connector pins. The aluminum cylinder acts as a shield in the capacitance circuit, eliminating the mutual capacitance between the connector pins along the cylinder length. An earlier design used a plexiglass cylinder with 1.0mm OD bare conductor copper wires mounted in a plexiglass cylinder. The plexiglass acts as a dielectric which increases the parasitic capacitance, and the effective capacitance of the MEMS sensor.

In analyzing the parasitic capacitance from the packaging, we focus on the capacitance between the connector pins attaching to pads 2 and 3, and apply the results to other pin configurations. The capacitance between the two pins is divided up into four different capacitances: $CP2_{_L1}$, $CP2_{_L2}$, $CP2_{_L3}$, and $CP2_{_L4}$ along the lengths, $L1$, $L2$, $L3$, and $L4$, respectively (Figure 75). The four capacitors are in parallel and can be summed to give $CP2$:

$$CP2 = CP2_{_L1} + CP2_{_L2} + CP2_{_L3} + CP2_{_L4}$$  (112)
For the aluminum cylinder, CP2_L2 and CP2_L3 will be eliminated as the metal between the pins is connected to ground to serve as a shield.

When estimating CP2, we are interested in the worst-case scenario in order to properly design the package, so we assume values that will result in the largest values of CP2. For example, the connector pins in the aluminum mount will be assumed to be 1.0 mm diameter solid conductor wires rather than 0.59mm conductors with insulation, which assumes a larger capacitance than may actually be present. The three capacitors are each modeled as two infinite wires in parallel:

\[
C = \frac{\pi \varepsilon}{\ln\left(\frac{b + \sqrt{b^2 - 4a^2}}{2a}\right)} L
\]  

(113)

where: \(a\) = wire radius, \(b\) = distance between center of wires, \(L\) = wire length, and \(\varepsilon\) = dielectric permittivity.

The calculated values of CP2 for the plexiglass and aluminum cylinders are 5.12pF and 0.65pF, respectively. There is nearly a 90% reduction in CP2 going from the first to the second design, which is a direct result of having a shielded metal cylinder instead of a plexiglass cylinder between most of the connector pin length. The dimensions of the capacitors and their calculated capacitance values are listed in Table 7.
Figure 75: Schematic of backside sensor packaging. 8 connector pins are attached to the backside pads on the sensor, and are mounted in a cylindrical mount. The cylinder is machined from plexiglass or aluminum depending on the package. The center striped portion of the cylinder is hollowed out in the plexiglass, and is solid aluminum, in the aluminum package. The bottom of the connector pins are soldered to a PCB for capacitance measurements on a mounted IC microchip.

Table 8 is a summary of the major capacitances described in Section 5.1, as well as the parasitic capacitances for the two different package designs. The major contribution to the overall capacitance in the packaged MEMS sensor is from the capacitance between the sensor features and the Si substrate (C_T2, C_B2, etc.), rather than from sensing element (CS2), or from the package parasitic capacitance (CP2). Future efforts aimed at reducing the capacitance of the MEMS sensor should focus on reducing the size of the electrode pads, or increasing the thickness of the SiO₂ dielectric in order to have the greatest effect.
Table 7: Dimensions, dielectric constants, and capacitances for two connectors in the plexiglass and aluminum packages.

<table>
<thead>
<tr>
<th>Cylinder</th>
<th>Dielectric Material</th>
<th>$\varepsilon$ [F/m]</th>
<th>L [mm]</th>
<th>b [mm]</th>
<th>A [mm]</th>
<th>C [pF]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plexiglass</td>
<td>CP2_L1 Air 8.91x10^{-12}</td>
<td>2.0</td>
<td>1.4</td>
<td>0.5</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CP2_L2 Plexiglass 3.12x10^{-11}</td>
<td>5.8</td>
<td>1.4</td>
<td>0.5</td>
<td>1.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CP2_L3 Air 8.91x10^{-12}</td>
<td>32.5</td>
<td>1.4</td>
<td>0.5</td>
<td>2.70</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CP2_L4 FR-4 3.56x10^{-11}</td>
<td>1.7</td>
<td>1.4</td>
<td>0.5</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CP2_Total</td>
<td>5.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>CP2_L1 Air 8.91x10^{-12}</td>
<td>1.2</td>
<td>1.4</td>
<td>0.5</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CP2_L2 None N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CP2_L3 None N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CP2_L4 FR-4 3.56x10^{-11}</td>
<td>1.7</td>
<td>1.4</td>
<td>0.5</td>
<td>0.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>CP2_Total</td>
<td>0.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8: Summary of expected capacitance values in MEMS sensors (between pad 2, pad 3, and substrate).

<table>
<thead>
<tr>
<th>Capacitor Name</th>
<th>Dielectric Mat. / Permittivity</th>
<th>Thickness [m]</th>
<th>Area $[m^2]$</th>
<th>Capacitance Value [pF]</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units</td>
<td>[F/m]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CS2</td>
<td>Air / 8.91x10^{-12}</td>
<td>$7 \times 10^{-6}$</td>
<td>$1 \times 10^{-8}$</td>
<td>0.047 (max displac.)</td>
<td>Sensing element (varies with $\tau$)</td>
</tr>
<tr>
<td>C_T2, C_T3</td>
<td>SiO$_2$ / 3.47x10^{-11}</td>
<td>$2 \times 10^{-6}$</td>
<td>$1.4 \times 10^{-6}$</td>
<td>24.3</td>
<td>Frontside pads and features</td>
</tr>
<tr>
<td>C_B2, C_B3</td>
<td>SiO$_2$ / 3.47x10^{-11}</td>
<td>$1.2 \times 10^{-6}$</td>
<td>$4.5 \times 10^{-7}$</td>
<td>13.0</td>
<td>Backside pads</td>
</tr>
<tr>
<td>C_V2, C_V3</td>
<td>SiO$_2$ / 3.47x10^{-11}</td>
<td>$0.5 \times 10^{-6}$</td>
<td>$3.7 \times 10^{-7}$</td>
<td>25.7</td>
<td>230$\mu m$ x 230$\mu m$ square vias 40$\mu m$ deep</td>
</tr>
<tr>
<td>C_sub2, C_sub3</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>63</td>
<td>Eq. (106) Eq. (107)</td>
</tr>
<tr>
<td>C_sub23</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>31.5</td>
<td>Eq. (108)</td>
</tr>
<tr>
<td>CP2 (plexi.)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>5.12</td>
<td>Eqs. (112,113)</td>
</tr>
<tr>
<td>CP2 (alum.)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>0.65</td>
<td>Eqs. (112,113)</td>
</tr>
<tr>
<td>CS2_eff (plexi.)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>36.7</td>
<td>Eq. (110)</td>
</tr>
<tr>
<td>CS2_eff (alum.)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>32.2</td>
<td>Eq. (110)</td>
</tr>
</tbody>
</table>
5.2.1 Capacitive-Temperature Effects

Capacitors are often temperature-dependent, and sensors implementing capacitive measurements typically require temperature-compensation during sensor operation. For the MEMS shear stress sensors, these capacitive-temperature effects must be analyzed to determine if they require compensation during sensor operation. This section develops a model to calculate the expected capacitance changes of the MEMS sensor due to temperature changes in the MEMS sensor and packaging.

The sensor capacitance values are temperature-dependent due to both thermal-expansion effects of the materials, and changes in the dielectric permittivity ($\varepsilon$) of the insulators. The permittivity of a dielectric varies with a temperature change ($\Delta T$) as:

$$\varepsilon(\Delta T) = \varepsilon_0 + \varepsilon_0(\beta_T \Delta T)$$

(114)

where $\varepsilon_0$ is the permittivity at standard conditions (20°C, 14.7Psi) and $\beta_T$ is the dielectric coefficient of temperature [83]. Due to thermal-expansion, a solid of length ($L_0$), or an area of ($A_0$) will vary with temperature as:

$$L(\Delta T) = L_0 + L_0(\alpha \Delta T)$$

(115)

$$A(\Delta T) = A_0 + A_0(1 + 2\alpha \Delta T)$$

(116)

where $\alpha$ is the coefficient of thermal expansion (CTE) of the material. Table 9 lists the materials that make up the MEMS shear stress sensors and their relevant properties.

To evaluate temperature effects on the effective capacitance ($CS_2\_eff$), we rewrite eq.(109) to indicate a reliance on a temperature change ($\Delta T$):

$$CS_2\_eff(\Delta T) = CS2(\Delta T) + C_{\_sub}23(\Delta T) + CP2(\Delta T)$$

(117)
where CS2 is the sensing capacitance (from the floating element), C_sub23 is the substrate capacitance, and CP2 is the package capacitance. In the following sections, we separately analyze each of these components.

### Table 9: Dielectric Permittivities, Dielectric Coefficients, and Coefficients of Thermal Expansion for Sensor and Package Materials [84] [85].

<table>
<thead>
<tr>
<th>Variable</th>
<th>Dielectric Permittivity</th>
<th>Dielectric Temp. Coefficient</th>
<th>Coefficient of Thermal Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Materials</td>
<td>ε</td>
<td>B_T</td>
<td>α</td>
</tr>
<tr>
<td>Units</td>
<td>[F/m]</td>
<td>[/K]</td>
<td>[/K]</td>
</tr>
<tr>
<td>Si</td>
<td>N/A</td>
<td>N/A</td>
<td>2.8 x 10^{-6}</td>
</tr>
<tr>
<td>Air</td>
<td>8.91 x 10^{-12}</td>
<td>7 x 10^{-6}</td>
<td>N/A</td>
</tr>
<tr>
<td>SiO_2</td>
<td>3.47 x 10^{-11}</td>
<td>20 x 10^{-6}</td>
<td>0.7 x 10^{-6}</td>
</tr>
<tr>
<td>Au</td>
<td>N/A</td>
<td>N/A</td>
<td>14 x 10^{-6}</td>
</tr>
<tr>
<td>Package Materials</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plexiglass</td>
<td>2.6 x 10^{11}</td>
<td>~8000 x 10^{-6}</td>
<td>10 x 10^{-6}</td>
</tr>
<tr>
<td>FR-4 (PCB)</td>
<td>4.1 x 10^{11}</td>
<td>~200 x 10^{-6}</td>
<td>15 x 10^{-6}</td>
</tr>
<tr>
<td>Aluminum</td>
<td>N/A</td>
<td>N/A</td>
<td>23 x 10^{-6}</td>
</tr>
<tr>
<td>Copper</td>
<td>N/A</td>
<td>N/A</td>
<td>17 x 10^{-6}</td>
</tr>
</tbody>
</table>

#### 5.2.1.1 Temperature Dependence of Sensing Capacitance

The sense capacitors on the MEMS sensor are formed from inter-digited fingers made of Si, which will expand in size as the temperature changes. Additionally, the permittivity of the fluid in which the sensor is being tested acts as a dielectric, and can vary with temperature. The sense capacitor, CS2, is modified for temperature changes by taking eq. (34) and inserting a \( \Delta T \) where a temperature dependence can be found:

\[
CS2(x, \Delta T) = \frac{\epsilon(\Delta T)A_c(\Delta T)}{d_o(\Delta T) - x(\Delta T)} \left[ 1 + 1.9861 \left( \frac{d_o(\Delta T) - x(\Delta T)}{h(\Delta T)} \right)^{0.8258} \right] + ...
\]

\[
+ \frac{\epsilon(\Delta T)A_c(\Delta T)}{\alpha d_o(\Delta T) + x(\Delta T)} \left[ 1 + 1.9861 \left( \frac{\alpha d_o(\Delta T) + x(\Delta T)}{h(\Delta T)} \right)^{0.8258} \right] + ...
\]

(118)
where \( \alpha = \) distance amplification factor, \( h = \) sensing finger thickness, \( A_c = \) capacitance area, \( x = \) sensor displacement, and \( d_0 = \) gap space between capacitor plates. The dielectric is assumed to be air with the variable permittivity given by:

\[
\varepsilon(\Delta T) = \varepsilon_0 \left(1 + \beta_{T_{\text{AIR}}} \Delta T\right)
\]

(119)

where \( \beta_{T_{\text{AIR}}} = \) dielectric coefficient of temperature \((7 \times 10^{-6} \text{ } / \text{K})\) and \( \varepsilon_0 = \) permittivity of air at standard conditions \((8.91 \times 10^{-12} \text{ } \text{F/ m})\). The gap space \((d_0)\) between the capacitor plates is the dielectric thickness of CS2 and will decrease due to thermal expansion of the sensing fingers as:

\[
d_0(\Delta T) = d_0 - b_f \alpha_f \Delta T
\]

(120)

where \( \alpha_f = \) coefficient of thermal expansion of the Si fingers \((2.8 \times 10^{-6} \text{ } / \text{K})\). The capacitance area and the sensing finger thickness will vary with temperature as:

\[
A_c(\Delta T) = A_c \left(1 + 2 \alpha_f \Delta T\right)
\]

(121)

and

\[
h(\Delta T) = h \left(1 + \alpha_f \Delta T\right)
\]

(122)

The sensor displacement \((x)\) has a dependence on \(\Delta T\) due to changes in the spring stiffness and the surface area of the sensor:

\[
x(\Delta T) = \frac{\tau_w A_s(\Delta T)}{k(\Delta T)}
\]

(123)

where \( A_s = \) shear stress sensing area, \( k = \) spring constant, and \( \tau_w = \) shear stress. The spring constant is temperature-dependent due to variations in the Young’s Modulus \((E)\), and thermal expansion of the spring beams. The equation for the folded-beam spring is:

\[
k = \frac{2E h_k^3}{L_k^3}
\]

(124)
where $E$ is Young’s Modulus of the spring, $L_k$ is the length of the spring beam, and $h_k$ is the thickness of the spring beam. Eq. (124) is modified for temperature-dependence as:

$$k(\Delta T) = \frac{2E(1 + \beta_E \Delta T)(h_k(1 + \alpha_f \Delta T))^3}{(L_k(1 + \alpha_f \Delta T))^3} = k \frac{(1 + \beta_E \Delta T)^3}{(1 + \alpha_f \Delta T)^3} = k(1 + \beta_E \Delta T)$$

(125)

where $\beta_E$ is the temperature coefficient of the Young’s modulus, $-6.7 \times 10^{-5}$ [1/K] for Si [86]. The springs and the sensing fingers are made of the same material, so the same value of $\alpha_f$ is used as in eq. (120). Assuming a $\Delta T$ of 100°C, then $k$ will decrease by 0.05%. The tensile-beam springs will behave similarly with temperature. Inserting eq.(125) into eq.(123) results in:

$$x(\Delta T) = \frac{\tau_w A_y(1 + 2\alpha_f \Delta T)}{k(1 + \beta_E \Delta T)} = \frac{x(1 + 2\alpha_f \Delta T)}{1 + \beta_E \Delta T}$$

(126)

Assuming a $\Delta T$ of 100°C, the sensor will displace approximately 0.1% further than it would without temperature changes. We then insert eqs. (119-123) and (125) into (118) and normalize by $C_{near}$, the value of the “near” capacitor on CS2:

$$CS2_{near} = \frac{eA_4}{d_0}$$

(127)

This is done to eliminate any dependence on $A_c$, which is specific to each sensor design. The resulting equation for the temperature-dependence of CS2 is:

$$CS2(x, \Delta T) = CS2_{near}^0 + \sum \left[ \frac{1 + \beta_{E,air} \Delta T(1 + 2\alpha_f \Delta T)}{1 - \frac{b_f \alpha_f \Delta T}{d_0} x(1 + 2\alpha_f \Delta T)} \left( \frac{d_0 - b_f \alpha_f \Delta T - x(1 + 2\alpha_f \Delta T)}{h(1 + \alpha_f \Delta T)} \right)^{0.8258} \right] + ..

+ \frac{1 + \beta_{E,air} \Delta T(1 + 2\alpha_f \Delta T)}{\alpha - \frac{\alpha b_f \alpha_f \Delta T}{d_0} x(1 + 2\alpha_f \Delta T)} \left( \frac{\alpha d_0 - \alpha b_f \alpha_f \Delta T + x(1 + 2\alpha_f \Delta T)}{h(1 + \alpha_f \Delta T)} \right)^{0.8258}$$
The temperature effect on CS2 for Si devices measured in a single-ended layout is shown in Figure 76 for different values of $x/d_0$ for $\Delta T$ ranging from 0°C to 100°C. The values from eq.(128) are normalized by their values at the same displacement ($x/d_0$) but with $\Delta T=0°C$. For example, at $\Delta T = 60°C$ and $x/d_0 = 0.8$, the value is approximately 1.004 indicating that CS2 will be 0.4% higher than at $\Delta T = 0°C$ and $x/d_0 = 0.8$. The change in the normalized capacitance increases with $x/d_0$, with a maximum value of 1.014 at $x/d_0 = 0.9$, $\Delta T=100 0°C$.

![Effect of Temperature on CS2](image)

**Figure 76: Effect of temperature changes on CS2 (sense capacitor).** Capacitance is measured using a single-ended layout and normalized by the capacitance value at $\Delta T = 0°C$, for the same value of displacement ($x/d_0$). For example, at $\Delta T = 60°C$ and $x/d_0 = 0.8$, the CS2 will be 0.4% higher than at $\Delta T = 0°C$ and $x/d_0 = 0.8$.

It should be emphasized that for the shear stress sensors, an increase in the normalized capacitance of 0.4% at $x/d_0$ does not correspond to a 0.4% error in the shear stress measurement.
At larger displacements (higher shear), the sensitivity \( \text{[fF/Pa]} \) of the MEMS sensor is greater than at smaller displacements (lower shear) of the sensor, where the sensitivity is more linear (Figure 27). For example, Figure 77 shows the effect of temperature change on a typical MEMS sensor operation with a maximum shear stress of 685 Pa when the single-ended capacitance is measured. The solid line is the theoretical shear stress versus CS2 capacitance for a MEMS sensor with no temperature changes \((\Delta T=0^\circ\text{C})\). The dashed line is for the same device assuming a temperature change of \((\Delta T=100^\circ\text{C})\). The temperature effect on the sensor measured shear stress is only significant above 500 Pa. Below 500 Pa, the \(\Delta T=100^\circ\text{C}\) curve is within 2 Pa of the \(\Delta T=0^\circ\text{C}\) curve, which is considered acceptable in terms of the sensor operation. The temperature effect does not cause more than a 2% error in the shear stress measurement for the entire range. These results are typical of all of the sensor designs, so we will assume that temperature effects on the sensing capacitors for all sensor designs are negligible.

![Temperature Effect on Measured Shear Stress](image)

**Figure 77:** Effect of temperature change of 100 °C on typical sensor operation. The sensor shown has a maximum shear stress of 685 Pa. The solid line is the theoretical curve of shear stress vs. CS2 capacitance change. The dashed line is for a device with \(\Delta T = 100^\circ\text{C}\).
5.2.1.2 Temperature Dependence of Substrate Capacitance

The substrate capacitance is represented by $C_{\text{sub}23}$, the capacitance between pads 2 and 3. $C_{\text{sub}23}$ is a combination of $C_{\text{sub}2}$ and $C_{\text{sub}3}$. $C_{\text{sub}2}$ is comprised of 3 capacitors ($C_{\text{B}2}$, $C_{\text{V}2}$, $C_{\text{T}2}$) in parallel. Each capacitor is modeled as a parallel plate (ignoring fringing effects) with the capacitance given by:

$$C = \frac{\varepsilon A_c}{d} \quad (129)$$

where $d$ = dielectric thickness, $\varepsilon$ = dielectric permittivity, and $A_c$ = capacitance area. Each capacitor is temperature-dependent due to changes in material properties given by:

$$C(\Delta T) = \frac{\varepsilon(\Delta T)A_c(\Delta T)}{d(\Delta T)} \quad (130)$$

For the three capacitors, the dielectric is SiO$_2$, and the thickness will vary with temperature as:

$$d(\Delta T) = d(1 + \alpha_t(\Delta T)) \quad (131)$$

where $\alpha_t$ = coefficient of thermal expansion (CTE) of SiO$_2$. The capacitance area of the 3 capacitors will vary with temperature as:

$$A_c(\Delta T) = A_c(1 + 2\alpha_t(\Delta T)) \quad (132)$$

with the CTE of SiO$_2$ being used rather than the CTE of the parallel plate material. This is because the CTE of SiO$_2$ is smaller than the CTE of the other parallel plate materials (Si or Au) (Table 9). As the temperature increases, both the parallel plates and the dielectric will expand laterally, however the parallel plate expansion will be constrained by the smaller expansion of the SiO$_2$ [77], so the CTE of SiO$_2$ ($0.7 \times 10^{-6} [/ K]$) is used in the calculations.

The team was unable to locate published values for the variation of the dielectric permittivity ($\varepsilon$) of SiO$_2$ with temperature. Since we are estimating maximum possible values, we
assume that the dielectric coefficient of temperature for SiO\textsubscript{2} is \(20\times10^{-6} \,[/K]\), which is considered a reasonable maximum value [85]. The dielectric permittivity of SiO\textsubscript{2} will vary as:

\[
\varepsilon(\Delta T) = \varepsilon_0 (1 + \beta_{T1}\Delta T)
\]  
(133)

where \(\varepsilon_0\) = permittivity of SiO\textsubscript{2} at standard conditions and \(\beta_{T1}\) = dielectric coefficient of temperature for SiO\textsubscript{2}. Combining eqs. (130) – (133), the capacitances for C\_B2, C\_V2, and C\_T2, are written as:

\[
C_{-}B2(\Delta T) = \frac{\varepsilon_0 (1 + \beta_{T1}\Delta T)A_{b2}(1 + 2\alpha_i\Delta T)}{d_{b2}(1 + \alpha_i\Delta T)}
\]  
(134)

\[
C_{-}V2(\Delta T) = \frac{\varepsilon_0 (1 + \beta_{T1}\Delta T)A_{v2}(1 + 2\alpha_i\Delta T)}{d_{v2}(1 + \alpha_i\Delta T)}
\]  
(135)

\[
C_{-}T2(\Delta T) = \frac{\varepsilon_0 (1 + \beta_{T1}\Delta T)A_{t2}(1 + 2\alpha_i\Delta T)}{d_{t2}(1 + \alpha_i\Delta T)}
\]  
(136)

where \(A_{b2}\) = capacitance area of bottom pads, \(d_{b2}\) = thickness of SiO\textsubscript{2} under bottom pads, \(A_{v2}\) = capacitance area of vias, \(d_{b2}\) = thickness of SiO\textsubscript{2} in vias, where \(A_{t2}\) = capacitance area of frontside pads, \(d_{t2}\) = thickness of SiO\textsubscript{2} under frontside pads. Values are provided in Table 2. The 3 capacitors above add up in parallel, so the variation of C\_sub2 with temperature dependence can be written as:

\[
C_{-}sub2(\Delta T) = \left(\frac{\varepsilon_0 A_{b2}}{d_{b2}} + \frac{\varepsilon_0 A_{v2}}{d_{v2}} + \frac{\varepsilon_0 A_{t2}}{d_{t2}}\right) \frac{(1 + \beta_{T1}\Delta T)(1 + 2\alpha_i\Delta T)}{(1 + \alpha_i\Delta T)}
\]  
(137)

The 3 terms in the parentheses add up to C\_sub2, so we can rewrite this equation as:

\[
C_{-}sub2(\Delta T) = C_{-}sub2 \frac{(1 + \beta_{T1}\Delta T)(1 + 2\alpha_i\Delta T)}{(1 + \alpha_i\Delta T)}
\]  
(138)

Similar equations are used in calculating C\_sub3, and the temperature-dependence is given by:

\[
C_{-}sub3(\Delta T) = C_{-}sub3 \frac{(1 + \beta_{T1}\Delta T)(1 + 2\alpha_i\Delta T)}{(1 + \alpha_i\Delta T)}
\]  
(139)
C\textsubscript{sub2} and C\textsubscript{sub3} combine in series to form C\textsubscript{sub23}. Using eqs. (108), (138), and (139) the temperature-dependence of C\textsubscript{sub23} is given by:

\[ C\textsubscript{sub23}(\Delta T) = \frac{(C\textsubscript{sub2}(\Delta T))(C\textsubscript{sub3}(\Delta T))}{(C\textsubscript{sub2}(\Delta T) + C\textsubscript{sub3}(\Delta T))} = C\textsubscript{sub23} \left[ \frac{(1 + \beta T_1 \Delta T)(1 + 2\alpha \Delta T)}{(1 + \alpha \Delta T)} \right] \quad (140) \]

The normalized substrate capacitance is:

\[ \frac{C\textsubscript{sub23}(\Delta T)}{C\textsubscript{sub23}} = \frac{(1 + \beta T_1 \Delta T)(1 + 2\alpha \Delta T)}{(1 + \alpha \Delta T)} \quad (141) \]

Figure 78 shows eq. (141) plotted for temperature increases up to 100°C. C\textsubscript{sub23} will increase by 0.21% for \( \Delta T = 100°C \). For a typical value of C\textsubscript{sub23} (31.5pF), this would result in a capacitance change of 66.2fF, which would affect all of the MEMS sensor designs. Note that this is for a single-ended capacitance measurement. To reduce this temperature effect, a differential capacitive sensing layout will be utilized, as discussed in Section 5.2.1.4.

![Temperature Effect on Capacitance of Substrate](image-url)

**Figure 78: Effect of temperature change of 100 °C on normalized substrate capacitance eq. (141) when using single-ended capacitance measurement.**
5.2.1.3 Temperature Dependence of Package Components

The capacitance between the connector pins for backside pads 2 and 3 is calculated by dividing each connector pin into four segments, L1, L2, L3, and L4 (Figure 75). Each segment has a capacitance (CP_L1, CP_L2, CP_L3, and CP_L4) calculated using the equation for capacitance between two parallel wires:

\[ C = \frac{\pi \varepsilon}{\ln \left( \frac{b + \sqrt{b^2 - 4a^2}}{2a} \right)} L \]  

(142)

where: \( a = \) wire radius, \( b = \) distance between center of wires, \( L = \) wire length, and \( \varepsilon = \) dielectric permittivity of the cylinder. The above equation is modified to include temperature dependence as:

\[ C(\Delta T) = \frac{\pi \varepsilon(\Delta T)}{\ln \left( \frac{b(\Delta T) + \sqrt{b(\Delta T)^2 - 4a(\Delta T)^2}}{2a(\Delta T)} \right)} L(\Delta T) \]  

(143)

The wire diameters and lengths are assumed to increase unconstrained in all four segments of the package (L1 - L4), with the radius (\( a \)) and length (\( L \)) increasing as:

\[ a(\Delta T) = a(1 + \alpha_{Cu}(\Delta T)) \]  

(144)

and

\[ L(\Delta T) = L(1 + \alpha_{Cu}(\Delta T)) \]  

(145)

where \( \alpha_{Cu} \) is the CTE of copper \((1.7 \times 10^{-5} \, [\, / K])\). The distance between the wires will increase with temperature as:

\[ b(\Delta T) = b(1 + \alpha_{Cylinder}(\Delta T)) \]  

(146)

where \( \alpha_{Cylinder} \) is the CTE of the plexiglass cylinder \((1.0 \times 10^{-5} \, [\, / K])\) or aluminum cylinder \((2.3 \times 10^{-5} \, [\, / K])\) depending on the package the sensor is mounted in. The CTE of the cylinder
is used for the package capacitance segments L1-L3 as the wires are held in place by the cylinder in those segments. In the segment L4, the CTE of the PCB material, FR-4 is used, with $\alpha_{FR-4} = 1.7 \times 10^{-5}$ [/ K].

The dielectric permittivity of the cylinder will change with temperature as:

$$\varepsilon(\Delta T) = \varepsilon_0 (1 + \beta_T \Delta T)$$

(147)

where the value of $\beta_T$ used is consistent with the package material in each segment of the package. In the segment L1, air is the dielectric ($\beta_T = 7.0 \times 10^{-6}$ [/ K]), and in segment L4, FR-4 (the PCB material) is the dielectric ($\beta_T = 2.0 \times 10^{-4}$ [/ K]). In the plexiglass cylinder, segment L2 has plexiglass as the dielectric ($\beta_T = 8.0 \times 10^{-3}$ [/ K]), and segment L3 has air as the dielectric. In the aluminum cylinder package, L2 and L3 are made of aluminum and act as shields and not as dielectrics, contributing no capacitance to the system.

![Capacitive-Temperature Effects on Packaging](image.png)

Figure 79: Capacitance change of plexiglass and aluminum packages for a temperature range of up to $\Delta T=100 \degree C$ when using single-ended capacitance measurement.
Figure 79 shows the capacitance change caused by a temperature changes when utilizing the plexiglass and aluminum packages with a single-ended capacitance measurement. CP2_Total is the sum of the contributions from CP2_L1 – CP2_L4. For the plexiglass package, CP2_L2 is the primary component of CP2_Total, contributing more than 98% of the capacitance. The capacitance changes by approximately 1.4pF (1381fF) from $\Delta T = 1^\circ C$ to $\Delta T = 100^\circ C$. Moderate temperature changes of 10$^\circ C$ will outweigh the capacitance change from the sensor, requiring that measurements utilizing the plexiglass package will require temperature compensation at all temperatures, if a single-ended measurement is used.

The aluminum package does not have the capacitor C2_L2, which is the largest capacitance, resulting in capacitance changes which are 99% lower than for the plexiglass package. At a $\Delta T = 100^\circ C$ of the aluminum package will have a capacitance change of 12.0fF which will require temperature compensation for a single-ended capacitance measurement.

From the above calculations, the aluminum package is recommended over the plexiglass package due to the significant reduction in the capacitive-temperature effect. Note that this is for a single-ended capacitance measurement. To reduce this temperature effect, a differential capacitive sensing layout will be utilized, as discussed in the following sections.

5.2.1.4 Temperature Dependence in Differential Capacitance Measurement

A differential capacitance measurement is utilized for the MEMS sensors shear stress measurement. One of the benefits is that capacitance increases (or decreases) affecting both CS2_eff and CS1_eff will cancel out to first order. However, if the capacitors are not of equal values (i.e. mismatched), temperature changes will result in a capacitance change, affecting the
sensor shear stress measurement. This section will quantify the expected capacitance changes caused by capacitance mismatch.

The effective differential capacitance of the MEMS sensor is given by:

$$\delta C_{\text{eff}} = CS2_{\text{eff}} - CS1_{\text{eff}}$$

(148)

$$\delta C_{\text{eff}} = (CS2 - CS1) + (C_{\text{sub23}} - CS1_{\text{sub34}}) + (CP2 - CP1)$$

(149)

The overall sensor capacitance can be split up into the different components, sensing capacitance ($\delta C_{\text{device}}$), substrate capacitance ($\delta C_{\text{sub}}$), and package capacitance ($\delta C_{\text{package}}$):

$$\delta C_{\text{eff}} = \delta C_{\text{device}} + \delta C_{\text{sub}} + \delta C_{\text{package}}$$

(150)

Each of the components has a different dependence on temperature and will be analyzed separately. It was shown in Section 5.2.1.1, that capacitance changes in the device sensing capacitance are negligible. The differential version, $\delta C_{\text{device}}$, will be even smaller, so it will be considered negligible.

### 5.2.1.4.1 Effect of Capacitance Mismatch on Substrate Capacitance

The differential capacitance due to the substrate is given by:

$$\delta C_{\text{sub}} = C_{\text{sub23}} - C_{\text{sub34}}$$

(151)

where $C_{\text{sub23}}$ = substrate capacitance between pads 2 and 3 (used to connect CS2), and $C_{\text{sub34}}$ = substrate capacitance between pads 3 and 4 (used to connect CS1). The two capacitors are not perfectly matched, resulting in an initial differential capacitance offset $\delta C_{\text{sub,0}}$:

$$\delta C_{\text{sub,0}} = C_{\text{sub23}} - C_{\text{sub34}}$$

(152)

We define a substrate capacitance mismatch coefficient ($\Lambda$):
\[ \Lambda = \frac{C_{\text{sub}34}}{C_{\text{sub}23}} \quad (153) \]

Inserting \( \Lambda \) into eq. (152) gives the differential capacitance offset for no temperature change and no flow:

\[ \delta C_{\text{sub},0} = C_{\text{sub}23}(1 - \Lambda) \quad (154) \]

To evaluate temperature effects on the differential capacitance measurement, we modify eq. (151):

\[ \delta C_{\text{sub}}(\Delta T) = C_{\text{sub}23}(\Delta T) - C_{\text{sub}34}(\Delta T) \quad (155) \]

and combine with eqs. (108) and (153):

\[ \delta C_{\text{sub}}(\Delta T) = C_{\text{sub}23}(1 - \Lambda) \left( \frac{1 + \beta T_1 \Delta T}{1 + \alpha_1 \Delta T} \right)^2 \quad (156) \]

We then define the capacitance temperature dependent parameter (\( C_{\Delta T} \)):

\[ C_{\Delta T} = \frac{\delta C_{\text{sub}}(\Delta T) - \delta C_{\text{sub},0}}{C_{\text{sub}23}} = (1 - \Lambda) \left[ \frac{1 + \beta T_1 \Delta T}{1 + \alpha_1 \Delta T} \right] - 1 \quad (157) \]

\( C_{\Delta T} \) is plotted in Figure 80 for a \( \Delta T \) increase up to 100°C for \( \Lambda \) values ranging from 0.5 to 1. \( C_{\Delta T} \) increases with temperature indicating that in eq. (157), the effect of the increasing thickness of the SiO\(_2\) dielectric (which would typically reduce the capacitance), is outweighed by the increase in the permittivity and the area of the electrode pads. For the case of decreasing temperature, \( C_{\Delta T} \) will be of opposite sign and similar magnitude.

Examining an extreme case of highly mismatched capacitors, \( \Lambda = 0.5 \), and a temperature change of 100°C, we calculate \( C_{\Delta T} = 1.04 \times 10^{-3} \). \( \Lambda = 0.5 \) approximates the situation where CS2\_eff has 2 metallized vias (connected pads 1 and 2), and CS1\_eff has 1 metallized via (connected pad 3 or 4). For an expected \( C_{\text{sub}23} \) value of 63pF (2 x 31.5pF), temperature effects cause a capacitance change of 65.5fF which would need to be compensated for since the MEMS
shear stress sensors will generate capacitance changes ranging from 50-400fF depending on the specific design.

Figure 80: Capacitance temperature dependent parameter ($C_{\Delta T}$) calculated for $\Lambda$ values ranging from 0.5 – 1.0 and $\Delta T$ varying from 0 to 100°C.

For cold flow tests, the expected maximum temperature change is 10°C. In the case of highly mismatched capacitors, $\Lambda = 0.5$, the capacitance-temperature parameter is $C_{\Delta T} = 1.04 \times 10^{-4}$. If $C_{\text{sub23}}$ is again 63pF, this results in a change of 6.5fF, which may need to be compensated for depending on the specific sensor design. For further discussion on temperature compensation during flow testing, the reader is referred to Sections 6.1.2 and 8.3.7.

5.2.1.4.2 Effect of Capacitance Mismatch on Package Capacitance

The temperature effect on capacitance mismatch within the package is described in this section. The differential capacitance contribution from the package capacitance is given by:
The capacitors CP2 and CP1 are modeled as two parallel wires with a temperature dependence using eq. (143). Since this equation is non-linear, we cannot use a capacitive mismatch coefficient as done in Section 5.2.1.4.1. Instead, we will assume that the lengths of the connector pins in the package are mismatched by specific amounts. Two realistic cases were chosen, which assume that the lengths for all the segments (L1-L4) are mismatched by 5% and 10%.

\[ \Delta C_{\text{Package}}(\Delta T) = CP2(\Delta T) - CP1(\Delta T) \]  

(158)

Figure 81 shows the capacitance change caused by capacitor mismatch in the plexiglass package when utilizing a differential capacitance measurement. At \( \Delta T = 100^\circ \), the 5% and 10% mismatches will cause a capacitance change of 69.8fF and 139.5fF, respectively. The data for the single-ended measurement is included to illustrate that choosing a differential measurement over a single-ended measurement can reduce the capacitance change by more than 90%.

![Figure 81: Capacitive-temperature effects on plexiglass packaging. The dashed line is the case with a single-ended capacitance measurement. The solid line is for a differential measurement with a 5% mismatch in the lengths of the connector pins, and the dash-dot line is for a 10% mismatch.](image)
Figure 82 shows the capacitance change caused by capacitor mismatch in the aluminum package when utilizing a differential capacitance measurement. At $\Delta T = 100^\circ$, the 5% and 10% mismatches will cause a capacitance change of 0.6fF and 1.2fF, respectively. Again the single-ended measurement is plotted, showing that the differential measurement reduces the capacitance change by 90%. Overall, the aluminum package has capacitance changes two orders-of-magnitude lower than the plexiglass package, clearly making it the preferred packaging material. For further discussion on temperature compensation during flow testing, the reader is referred to Sections 6.1.2 and 8.3.7.

Figure 82: Capacitive-temperature effects on aluminum packaging. The dashed line is for the case with a single-ended capacitance measurement. The solid line is for a differential measurement with a 5% mismatch in the lengths of the connector pins, and the dash-dot line is for a 10% mismatch.
5.3 Summary of Capacitance Analysis Results

In summary, we can draw the following conclusions about capacitive-temperature effects from the results presented in Section 5:

1) The main contributors to the absolute capacitance of the sensors are the capacitances with respect to the substrate (e.g. $C_{T2}$, $C_{B2}$, $C_{V2}$, etc.). Efforts aimed at reducing the capacitance could focus on reducing the areas of the frontside and backside features, or creating a thicker Via layer.

2) The capacitive-temperature effect on the device itself ($\delta C_{device}$) is negligible when compared to the effects on the substrate capacitance ($\delta C_{sub}$), and the package capacitance ($\delta C_{package}$).

3) A differential capacitance measurement shows substantial reductions in the capacitive-temperature effect over a single-ended measurement, and should be implemented whenever possible.

4) The aluminum-type package significantly reduces the absolute capacitance and the capacitive-temperature effect when compared to the plexiglass-type package. It is not recommended that the plexiglass-type package be used in future flow-testing efforts.
6 Capacitance Sensing Circuitry

Two different pieces of equipment were used to measure the capacitance changes of the MEMS shear stress sensors, the Keithley 590 CV analyzer and the MS3110 Universal capacitive readout IC. The Keithley 590 CV Analyzer measures a capacitance in parallel with a resistance by applying a $15\text{mV}_{\text{RMS}}$ 100kHz sine wave test signal across the circuit and measuring the impedance. It measures a single-ended capacitance at a maximum sampling rate of 1kHz, with a minimum capacitance resolution of 0.1fF to 10fF depending on the range of capacitance measurement (Table 10). It is a stand-alone piece of measurement equipment (0.13mm x 0.43mm x 0.45mm) with the device under test (DUT) connected via BNC cables, as shown in Figure 66.

<table>
<thead>
<tr>
<th>Capacitance Range</th>
<th>Min. Capacitance Resolution</th>
<th>Analog Voltage Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2pF</td>
<td>0.1fF</td>
<td>0 – 200mV</td>
</tr>
<tr>
<td>0-20pF</td>
<td>1fF</td>
<td>0 – 2V</td>
</tr>
<tr>
<td>0-200pF</td>
<td>10fF</td>
<td>0 – 2V</td>
</tr>
<tr>
<td>0-2nF</td>
<td>100fF</td>
<td>0 – 2V</td>
</tr>
</tbody>
</table>

The MS3110 Universal Capacitive Readout IC (Irvine Sensors Corp.) is a microchip which can be mounted in its own evaluation board (MS3110BDPC), or in a custom-made PCB Board (PCB), as shown in Figure 66. If one were utilizing the MS3110BDPC, the sensor would then be connected to the packaging via BNC cables. Our team used a custom-made PCB in an effort to reduce parasitic capacitance by close-coupling the capacitance measurement with the sensor.
The MS3110 measures differential capacitance by applying a 2.25V, 100kHz square-wave test signal to the sense capacitors (CS2 and CS1) on the MEMS sensor. The square waves are 180° out of phase resulting in a ratiometric output when CS2 and CS1 are not equal. The MS3110 IC can also be operated in a single-ended mode with only one capacitor connected. The MS3110 bandwidth (BW) is set by the user to range from 0.5 – 8.0kHz. The manufacturer gives the capacitance resolution as a function of BW, ranging 0.09fF to 0.36fF for 500Hz and 8.0kHz, respectively [88].

| Table 11: Summary of characteristics of capacitance measurement equipment: Keithley 590CV vs. MS3110 IC. |
|-----------------------------------------|---------------------------------|-----------------|-----------------|---------------|
| Connections to sensor or package       | Measurements                    | Capacitance Measurement Type | Max. Sampling Rate | Cost |
| Keithley 590CV                        | BNC                             | Capacitance       | Single-ended     | 1 kHz        | High          |
|                                       |                                 | Parallel Resistance |                 |               |               |
| MS3110 IC                             | BNC                             | Capacitance       | Single-ended     | 8 kHz        | Low           |
|                                       | PCB                             | Differential      |                 |               |               |

Table 11 is a comparison of the different characteristics of the Keithley 590CV and the MS3110 IC. The team decided to focus efforts on interfacing the MEMS shear stress sensor with the MS3110 IC, because it offered the most feasible path for developing a compact sensor package which could be utilizing in various test articles and test tunnels. The main factor is that the Keithley 590CV can only be used with frontside packaged sensors as they have a capacitance less than 20pF, which yields a measurement resolution of 1fF (Table 10). Sensors with backside connections have capacitances greater than 20pF with a resolution of 10fF, which is too large to accurately measure the shear stress. Additionally the large size of the Keithley 590CV unit and the sole method of connecting via BNC cables make it unwieldy to work with and an unrealistic component in the design of a sensor that can be installed in a variety of flow environments.
During sensor characterization, the Keithley 590CV is periodically used for baseline sensor measurements or for measurements involving frontside packaged sensors.

### 6.1 MS3110 IC Capacitance Measurement Circuitry

The MS3110 circuit measures via a switched-capacitor charge-integrating amplifier. As this is a proprietary product, we were unable to learn all of the specific details about the operation. However, based on the MS3110 IC block diagram (Figure 83), the MS3110 IC datasheet [88], and discussions with the manufacturer, we know that the basic measurement steps are as follows:

1) Two 180° out-of-phase square waves with voltage ranges of NEG (0V) to V2P25 (2.25V) are placed across the sense capacitors (CS2IN and CS1IN) and their corresponding trim capacitors (CS2trim and CS1trim). The sense and trim capacitors are in parallel so the capacitance is additive (i.e. CS2total = CS2S + CS2trim).

2) If CS2total and CS1total are unbalanced, this results in a current, which flows towards the inverting input of the op-amp. Note that the non-inverting input is held at V2P25.

3) The feedback capacitor (CF) and the op-amp form a charge-integrating amplifier with its output at the right side of the op-amp.

4) The output of the op-amp is scaled and averaged using Sample/Hold techniques, and passed through a two-pole lowpass filter.

5) The filtered signal is scaled and trimmed, and then given a voltage offset (SOFF) of 0.5V or 2.25V, to result in the MS3110 output (V0).
Figure 83: MS3110 IC block diagram. The MEMS sensor is connected to inputs CS2IN, CS1IN and CSCOM on the left part of the figure. The MS3110 IC output is $V_0$ on the right side.

The MS3110 voltage output ($V_0$) is given by the following transfer function:

$$V_0 = \frac{5.13}{CF} (CS2_{\text{eff}} + CS2_{\text{trim}} - (CS1_{\text{eff}} + CS1_{\text{trim}})) + V_{\text{ref}} \quad (159)$$

where $CS2_{\text{eff}}$ and $CS1_{\text{eff}}$ are the capacitances from the packaged sensor, $CF$ is the MS3110 feedback capacitor, and $CS2_{\text{trim}}$ and $CS1_{\text{trim}}$ are capacitor trim settings. Table 12 is a summary of the MS3110 IC nominal settings including ranges, and steps.

<table>
<thead>
<tr>
<th>Setting</th>
<th>CF</th>
<th>CS2trim</th>
<th>CS1trim</th>
<th>$V_{\text{ref}}$</th>
<th>BW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Description</td>
<td>Feedback capacitor</td>
<td>Capacitor trim setting</td>
<td>Capacitor trim setting</td>
<td>Voltage reference</td>
<td>Bandwidth</td>
</tr>
<tr>
<td>Range</td>
<td>0 – 19.4pF</td>
<td>0 to 1.12pF</td>
<td>0 to 9.71pF</td>
<td>0.5 - 2.25V</td>
<td>0.5 - 8.0kHz</td>
</tr>
<tr>
<td>Values</td>
<td>Steps of 0.019pF</td>
<td>Steps of 0.019pF</td>
<td>Steps of 0.019pF</td>
<td>0.5 , 2.25V</td>
<td>0.5, 0.8, 1.0, 1.4, 2.0, 3.0, 4.2, 5.8, 8.0kHz</td>
</tr>
</tbody>
</table>
Figure 84: Graph showing MS3110 gain for different values of the feedback capacitor (CF). The solid line is the theoretical gain, eq. (160), and measured gain data points are calculated using eq. (161). Testing was conducted with MS3110 IC located in the MS3110BDPC with no MEMS sensor installed.

Figure 85: Graph showing the effect of parallel resistance on MS3110 measurements. A test circuit was constructed with 20pF capacitors and a variable resistor to model the MEMS shear stress sensor. The parallel resistance was varied from 100kΩ to 40MΩ for CF values of 0.513pF and 5.13pF.
The MS3110 gain (G) is changed by setting a specific value to the Feedback Capacitor (CF) on the MS3110. The gain has units of V/pF or mV/fF, and is given by:

\[
G_{\text{theory}} = \frac{5.13}{\text{CF}}
\]  

(160)

Circuit testing of the MS3110 IC shows a drop-off in the gain as it approaches its maximum value of 270mV/fF. To accurately measure capacitance changes with the MS3110 IC, tests were conducted to determine the actual gain (G). This was accomplished by varying the measured capacitance in a known manner and measuring the change in \(V_0\):

\[
G \approx \frac{\Delta V_0}{\Delta C}
\]  

(161)

The capacitance change (\(\Delta C\)) is caused by varying CS2trim or CS1trim on the MS3110 IC. Figure 84 shows G vs. \(G_{\text{theory}}\) for different CF values with no MEMS sensor installed. As CF decreases below 0.513pF, G is 10% lower than \(G_{\text{theory}}\), and becomes worse with increased G (decreased CF). In practice, CF = 0.513pF is taken as the lowest value for CF to be used in testing. When operating the sensor, it is necessary to use eq. (161) to determine the actual gain of the MS3110 rather than the values predicted by the transfer function, eq. (160).

### 6.1.1 Effect of Parallel Resistance on MS3110 Capacitance Measurement

Real-world capacitors are modeled as having a capacitance and resistance in parallel. This accounts for leakage current caused by non-idealities stemming from manufacturing processes. In the MEMS shear sensor this leakage current is caused by the Si device layer not being completely insulated from the Si substrate. Preliminary testing showed that this parallel resistance (RP) can substantially reduce the operational gain of the MS3110. This effect is a
result of the method used by the MS3110 IC to measure differential capacitance. The circuit model assumes infinite values of RP, when in fact RP measurements of the MEMS shear stress sensors indicate a range from 100kΩ to 40MΩ depending on the quality of the sensor fabrication.

To evaluate the effect of the parallel resistance on the MS3110 gain, tests were conducted using a test circuit in place of the MEMS shear stress sensor. The MS3110 IC was connected to two 20pF ceramic capacitors soldered on a PCB to model CS2S and CS1S of the sensor. A variable resistor was soldered in parallel with CS1S to model the parallel resistance. Based on the sensor design specifications, the MS3110 gain will need to range between 0.1mV/fF and 10mV/fF in order to measure the expected capacitance changes. Figure 85 shows data for CF = 0.513pF (G\text{theory} = 10mV/fF) and CF = 5.13pF (G\text{theory} = 1mV/fF). For CF = 0.513pF, changing RP from 100kΩ to 40MΩ varied the gain four orders of magnitude, from approximately 0.001mV/fF to 10mV/fF. The measured gain for CF = 5.13pF changed in a similar manner with variations in RP.

To accurately measure shear stress from the MEMS sensors, the minimum gain required is ~0.1mV/fF. Based on the measured data, the minimum requirement for RP is approximately 500kΩ. It should be noted that the circuit testing did not capture all of the possible combinations of resistances on the MEMS sensor. For instance, we did not examine the situation with finite but different values of RP1 and RP2, or the effect of RP being a function of voltage (see Section 6.1.2). Ultimately, the only method for determining the actual gain of the MS3110 when connected to a sensor is by varying the trim settings (CS2trim, CS1trim) and using eq. (161).
6.1.2 Non-linear Resistive Temperature Effects

Flow testing (Sections 8.2 and 8.3) of fabricated MEMS sensors with backside connections installed in the aluminum-type package, showed that the sensors were responding to changes in shear stress as well as temperature. The main evidence for a temperature effect was a large sensor hysteresis (higher than 50%) present during flow testing. After flow was stopped, the sensor would not return to its pre-test value for 30-60 minutes, which was highly indicative of a thermal transient effect. This temperature sensitivity was confirmed by thermocouple measurements near the sensor. The results showed that as the sensor temperature returned to the pre-test values (room temperature), the sensor output closely followed. Actual measurement data for the temperature sensitivity will be discussed in Section 8.3.7.

Due to the nature of the high-speed flow setups used for testing (Section 0), it was difficult to vary the shear stress of the flow independently from the temperature of the flow. This is because the high-speed compressible flow is generated by expanding high pressure air through a nozzle which increases the velocity (and shear stress) while decreasing the static temperature. Since we could not successfully decouple the shear stress from the temperature, it was necessary to understand in detail what was causing the temperature effect, and how this affected the MS3110 IC measurement. Once this effect was understood, we could make recommendations on the best method for compensating for it.

It was initially theorized that this temperature sensitivity stemmed from the overall sensor capacitance changing with temperature. However calculations done using the equations developed in Section 5.2.1 showed that when the aluminum package is utilized in combination with a differential capacitance measurement, the expected change from capacitive-temperature effects should be $O(1\text{fF}) - O(10\text{fF})$, much less than the observed change of $O(10^2\text{fF})$. A second
hypothesis for this cause was that the sensor parallel resistance was changing with temperature and affecting the MS3110 measurement. This is because the parallel resistance is tied to the gain of the MS3110, as discussed in Section 6.1.1.

To determine the effect of the non-linear parallel resistances on the sensor output, simulations of the MS3110 circuit were created using the computer program OrCAD Pspice (Cadence Design Systems). OrCAD was used because Voltage-Current (V-I) probe testing (Section 7.3) showed that the parallel resistance was voltage-dependent and non-linear over the MS3110 range (0 – 2.25V) making it harder to analytically evaluate these effects.

Three separate simulations in Pspice were made: Simul_1, Simul_2, and Simul_3, each design building on the previous version. The first simulation, Simul_1, illustrates the basic operation of the MS3110 with an ideal MEMS sensor composed of capacitors with no parallel resistances. The second simulation, Simul_2, looks at the effects when a parallel resistor is present on RP2, and compares it to experimental data. The last simulation, Simul_3 uses actual sensor test data to illustrate the effect of non-linear resistances that change with temperature on the MS3110. A summary of the details for the Pspice simulations is listed in Table 13.

<table>
<thead>
<tr>
<th>Simulation Name</th>
<th>Description</th>
<th>Objective</th>
<th>Parametric Variable</th>
<th>Parametric Values</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simul_1</td>
<td>Ideal Capacitors</td>
<td>Baseline of model</td>
<td>CS2</td>
<td>21.6pF to 22.4pF in steps of 0.2pF</td>
<td>Model represents MS3110 sensor operation</td>
</tr>
<tr>
<td>Simul_2</td>
<td>Constant Parallel Resistance</td>
<td>To determine effect of parallel resistance</td>
<td>RP2</td>
<td>500kΩ, 1MΩ, 2MΩ, 5MΩ, 10MΩ, 20MΩ, 50MΩ</td>
<td>Results show trends from experimental testing</td>
</tr>
<tr>
<td>Simul_3</td>
<td>Non-Linear Resistance</td>
<td>To determine effect of non-linear resistance</td>
<td>CS2, GPOLY (COEFF)(COEFF2)</td>
<td>21.6pF to 22.4pF in steps of 0.2pF, Table 14</td>
<td>Non-Linear resistance can act as offset and not effect the gain for some values of CF</td>
</tr>
</tbody>
</table>
Figure 86: Simul_1 - Pspice model of MS3110 circuit assuming ideal capacitors.

6.1.2.1 Simul_1 – Baseline model for MS3110 circuit

Figure 86 is a schematic of the PSpice model of the MS3110 with two ideal capacitors connected (i.e. no parallel resistances), denoted Simul_1. The model only represents the portion of the MS3110 IC circuit labeled as “IAMP Section” in Figure 83 as that is the part of the circuit that is most crucial to the MS3110 IC operation and is affected by the parallel resistances of the sensor.

The MS3110 test signals are represented by the pulse voltage sources VS2 and VS1. The pulses were given a rise and fall time of 0.01μS with a pulse width of 4.98μS. CS2 and CS1 represent the MEMS sensor capacitors and were given values that are typical of the actual shear stress sensors. In the case shown in the figure, CS1 is equal to 22pF and the value of CS2 is shown as “{CS2}” to indicate that it is a parametric constant in the model. Resistor R9 provides a high impedance path to ground and was a necessary addition to solve the model in Pspice and
avoid the “floating node” problem. To the right of R9 is an ideal op-amp with a feedback capacitor (CF = 5.13pF). The non-inverting input of the op-amp is held at V2P25 (2.25V). The signal output of the circuit is the op-amp output, denoted by the probe marked “V” on the right-side of the circuit.

The purpose of Simul_1 was to determine if the model was appropriately representing the actual MS3110 operation. This simulation was a parametric test with CS1 at a value of 22pF and CS2 ranging from 21.6pF to 22.4pF in steps of 0.2pF, resulting in differential capacitance values of -0.4pF to 0.4pF. CF was set to 5.13pF and 0.513pF, which give a gain of 1V/pF and 10V/pF, respectively, based on eq.(160).

The output signals for the five tests of Simul_1 for CF = 5.13pF are shown in Figure 87. For a positive \( \delta C \), there is a positive voltage change, while the opposite occurs for a negative \( \delta C \), which occurs with the MS3110. To calculate the gain for each test, we average the voltage output across one full cycle (10\( \mu \)S), and divide by the capacitance change as in eq. (161). For the four nonzero \( \delta C \) values tested, we calculate an average gain of 0.2192 +/- 0.00001V/pF. Simul_1 results with CF = 0.513pF resulted in a constant gain of 2.169 +/- 0.00001V/pF. The fact that the gain is constant for all the tested \( \delta C \) values shows that the Pspice model is behaving similarly to the MS3110.

As previously mentioned, we do not know the scaling factors implemented by the MS3110, however if we divide the theoretical gain (1V/pF) by the calculated gain (0.2192V/pF) we are left with a scaling factor of 4.56 which is roughly twice the value of V2P25 (2.25V), the on-chip voltage reference. Similarly CF = 0.513pF, with a theoretical gain of 10V/pF, had a scaling factor of 4.61. A typical IC chip would use the on-chip voltage reference for scaling, meaning that this result further validates the Pspice model developed here. Based on the limited
available information about the MS3110, we conclude that the PSpice Simul_1 model is accurately representing the IAMP section of the MS3110 IC.

![Simul_1 Output](image)

**Figure 87: Simul_1 Results for four different tests.**

### 6.1.2.2 Simul_2 – Model for MS3110 circuit with parallel resistor on CS2

A second Pspice model, Simul_2, was developed to show the effect of parallel resistance on the performance of the MS3110 IC circuit. The major difference between the models Simul_1 and Simul_2 is that Simul_2 has a resistor, RP2, in parallel with CS2 (Figure 88). RP2 is written as “\{RP2\}” because it was set as a parametric variable. Each test series consisted of solving the circuit with a \(\delta C = 10\text{fF}\) and the following values of RP2: 500\(\Omega\), 1M\(\Omega\), 2M\(\Omega\), 5M\(\Omega\), 10M\(\Omega\), 20M\(\Omega\), 50M\(\Omega\). The test series was run for two different cases, \(CF = 5.13\text{pF}\) and \(CF = 0.513\text{pF}\), and it was compared to the experimental data shown in Figure 85.
Figure 89 shows how the presence of the parallel resistance affects a single cycle (10μS) of the MS3110 IC output signal for CF = 0.513pF. Comparing Figure 89 to the ideal test signal (Figure 87) shows that the presence of a finite parallel resistance has a significant effect on the shape of the test signal. The first part of the cycle is no longer constant but becomes a ramp function with a slope that is inversely related to the resistance. The ramp function is caused by the current flowing through the parallel resistor and being integrated by the charge amplifier. As RP2 increases, the slope of the ramp decreases, and approaches a slope of zero. However, even for an RP2 value of 50MΩ (which is considered to be large for the fabricated MEMS sensors), there is still a clear affect on the shape of the test signal. A saturation limit of 5V was placed on the op-amp to imitate the non-ideal components in the MS3110 IC. 5V was chosen because that is the power supply requirement for the MS3110 IC. The second part of the cycle levels off to a constant voltage due to no current passing through the resistor or because of the 5V saturation limit.

Gain (G) is calculated using eq. (161) for the different values of RP2 with a δC of 10fF. To determine the reduction in the gain, G is normalized by G_{theory} the appropriate value of CF. The results of Simul_2 are plotted in Figure 90 to compare with the experimental MS3110 IC measurements described in Section 6.1.1. The simulation data for CF = 0.513pF closely matches the experimental data for RP2 values between 10-50MΩ, however the two curves diverge at lower resistances, although they both have a similar “S” shape. The simulation data for CF = 5.13pF follows a similar trend as the experimental data, although it decreases faster with resistance as RP2 falls below 2MΩ.

Although the experimental and simulation data curves do not match closely over their entire domain, they do have similar trends, which supports the theory that the PSpice model is
accurately representing the first stage of the MS3110 IC circuit. Unfortunately, we are unable to make a more accurate model of the MS3110 IC circuit at this time because not enough information is known about non-idealities inherent within the circuit. However, from the results of Simul_2, we are able to conclude that changing the value of RP2 significantly affects the signal output, as well as the gain of the sensor. Later results (Section 7.3), will show that RP2 can change with temperature or with a different sensor, which will have an important effect on sensor operation.

Figure 88: Simul_2 - Pspice model of MS3110 circuit with a variable resistor in parallel with CS2.
Figure 89: Signal output for Simul_2 with CF = 0.513pF and an op-amp saturation limit of 5V.

Figure 90: Normalized gains for Simul_2 results plotted against experimental data.
6.1.2.3 Simul_3 – Model for MS3110 circuit with voltage and temperature dependent parallel resistors

Experimental measurements of fabricated and packaged MEMS shear stress sensors showed a temperature effect resulting from a temperature-dependent resistance (Section 7.3). To determine the best way to compensate for this temperature effect, it was necessary to understand how this affected the MS3110 IC operation. For example it was not known if this would cause a constant offset in the sensor output, or a gain that is temperature dependent, or a combination of the two.

The Pspice model for the MS3110 with two non-linear parallel resistors, named Simul_3, is shown in Figure 91. Pspice does not have a block for a voltage dependent resistor, so the resistors (RP1 and RP2) were modeled as voltage-controlled-current-sources (VCIS). A typical parallel resistance in the shear stress sensor can be modeled in PSpice using the GPOLY block, where the current output is related to the voltage input as:

\[ I = \text{COEFF}(V) + \text{COEFF2}(V^2) \]  \quad (162)

The values for COEFF and COEFF2 were calculated in EXCEL by fitting a quadratic curve to measured V-I data from fabricated shear stress sensors as described in Section 7.3.

An example of measured V-I data from sensor ECA-3F2B is shown in Figure 92 for the conditions with no flow, and 100Psig wall jet flow over the sensor. The wall jet flow is always colder than room temperature, and it is known from multiple tests that the temperature sensitivity causes the MS3110 output to decrease with temperature. The V-I relationships for both RP1 and RP2 change with flow due to the lower temperature. A quadratic line-of-best-fit was found for each curve, with coefficients provided in Table 14. These coefficients were input into eq. (162) and Simul_3 to determine how a temperature change would affect the MS3110 IC performance.
Initially, four simulations were run with CF = 5.13pF: (1) no flow with $\delta C = 0$ pF, (2) flow with $\delta C = 0$ pF, (3) no flow with $\delta C = 0.06$ pF, and (4) flow with $\delta C = 0.06$ pF. The same conditions were then repeated with CF = 0.513pF.

Figure 93 shows one cycle of the test signal output for Simul_3 for CF = 5.13pF. The data with flow are the upper two lines (solid) while the data without flow are the lower two lines (dashed). Throughout the cycle, the slope decreases when the flow is present. This is caused by the parallel resistors (RP2 and RP1) changing with flow (temperature). Figure 94 shows a similar trend in data for CF = 0.513pF, however the data is severely affected by the 5V saturation limit imposed on the op-amp. For both sets of data, it is not readily apparent if the flow causes a constant offset, or a change in the gain, or a combination, until the data is averaged across each cycle and the gains are calculated using eq. (161).

Figure 91: Simul_3 - Pspice model of MS3110 circuit with non-linear resistors modeled as voltage controlled current sources.
Figure 92: Voltage vs. Current Curves for sensor ECA-3F2B. The curves are the quadratic lines of best fit, with coefficients given in Table 14.

Table 14: Quadratic coefficients from sensor ECA-3F2B used in Simul_3.

<table>
<thead>
<tr>
<th>Capacitor</th>
<th>No Flow COEFF</th>
<th>No Flow COEFF2</th>
<th>100Psig Wall jet Flow COEFF</th>
<th>100Psig Wall jet Flow COEFF2</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS2 (Pads 1/7)</td>
<td>$4.370 \times 10^{-8}$</td>
<td>$1.166 \times 10^{-7}$</td>
<td>$1.144 \times 10^{-9}$</td>
<td>$1.147 \times 10^{-7}$</td>
</tr>
<tr>
<td>CS1 (Pads 4/7)</td>
<td>$7.136 \times 10^{-8}$</td>
<td>$1.246 \times 10^{-7}$</td>
<td>$4.579 \times 10^{-9}$</td>
<td>$1.180 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
Figure 93: Simul_3 test signal for CF = 5.13pF with \( \delta C = 0-0.06pF \), with (solid lines) and without flow (dashed lines).

Figure 94: Simul_3 test signal for CF = 0.513pF with \( \delta C = 0-0.06pF \), with and without flow. The flat line is a result of the 5V saturation limit imposed in Simul_3.
Figures 95 and 96 show the averaged MS3110 outputs for the tested cases using Simul_3. The simulations used CF = 0.513pF and CF = 5.13pF with $\delta C$ values ranging from -0.06pF to 0.06pF in steps of 0.02pF. For CF = 5.13pF (Figure 95), both sets of data have linear fits with a slope (gain) of 0.2197V/pF, which is within 0.5% of the gain that was calculated in Simul_1. For this case we can conclude that the flow causes an offset of -0.0916V, while the gain is not affected by the presence of the flow. This is a positive result, and compensation for this offset will be described later in flow testing portion of this dissertation (Section 8.3.7).

For CF = 0.513pF (Figure 96), the flow causes an offset of -0.1028V and changes the gain from 1.418V/pF to 1.598V/pF, an increase of 12.7%. These gain values are 28 – 35% lower than the value calculated in Simul_1 (2.169V/pF). These non-linear effects are a result of the parallel resistors combining with the saturation limit imposed on the op-amp. In order to accurately measure the capacitance of the MEMS sensor, we want to avoid a temperature-dependent gain, so it is recommended to use values of CF that do not suffer from these saturation effects (i.e. CF = 5.13pF would be a good choice while CF = 0.513pF would not). Useful CF values can be determined experimentally by exposing the sensor to a temperature change and varying the CS2trim and CS1trim to measure if the gain changes with temperature.
The results from Simul_3 generally replicated trends that were encountered with the experimental testing of packaged backside sensors. Those results, to be described in more detail in Section 8.3.7, showed that the sensor output decreased with a temperature decrease. For example, a measured temperature effect was initially assumed to be capacitance decrease ranging from 200 – 500fF when a CF value of 5.13pF was used. However, if we re-examine those results based on the assumption of a voltage decrease of 0.2 – 0.5V (assuming a gain of 1V/pF) and divide by the scaling factor of 4.56, this would correspond to voltage decrease of 0.04 - 0.11V which bounds -0.0916V, the value calculated in Simul_3. We are mainly concerned with illustrating that a temperature-dependent parallel resistance is an explanation for the temperature dependence, rather than completely replicating the effect in Pspice. From the results described above, we can conclude the following about the temperature effects on the MEMS shear stress sensors:

1) The parallel resistances for the fabricated sensors have voltage and temperature dependencies which significantly affect the MS3110 signal output.

2) If the MS3110 signal output is not affected by non-ideal saturation effects, then the capacitive gain will be constant, and the temperature effect will cause an offset in the sensor output.

3) This offset can be subtracted to compensate for temperature if a relationship is known between temperature and the offset. This relationship is determined experimentally and will be described further in Section 8.3.7.
6.1.3 Capacitance Measurement Noise

The sensitivity (S) of a typical MEMS shear stress sensor is O(0.1fF/Pa). To measure shear stress with a resolution of O(10Pa), the typical sensor design goals, requires a minimum detectable capacitance change of O(1fF). The MS3110 IC can modify the noise of the measurement using the adjustable bandwidth (BW) settings, implemented via volatile control registers on the IC unit. The bandwidth is set by engaging a low-pass filter at the desired cut-off frequency. The available BW settings in are: 500, 800, 1000, 1400, 2000, 3000, 4200, 5800, and 8000 [Hz].

Tests were conducted on the MS3110 output signal (V₀) to determine how the noise in the measurement is affected by the different MS3110 settings available. Figure 97 shows the results of measuring the noise of the MS3110 IC output signal while installed in the MS3110BDPC evaluation board. The MS3110BDPC is mounted inside a shielded metal box with V₀ connected to a National Instruments DAQ board using a BNC cable. The terminals of the MS3110 are open, measuring the capacitance between the terminals with air as the dielectric. This represents the lowest expected noise that the MS3110 IC will realistically encounter in the laboratory and serves as a baseline for later measurements when other components are added such as packaging or MEMS sensors.

Measurements were taken at different values of CF (0.513pF, 1.026pF, 5.13pF) and BW (500Hz, 3000Hz, 8000Hz). For the values of CF used, the actual gain was determined using eq. (161) as previously described in Section 6.1.1. Voltage noise (V_{noise}) was defined as the standard deviation from the mean of the baseline signal. Figure 97 shows there is a clear trend of V_{noise} increasing with a higher gain (or lower CF). Noise typically increases with BW or stays constant in the case of CF = 5.13pF.
When the MEMS sensor is exposed to a turbulent flow environment, it will measure an unsteady wall shear stress, which can be decomposed into mean (laminar) and fluctuating (turbulent) components:

\[
\tau_w = \tau_{\text{mean}} + \tau_{\text{turb}}
\]

The vertical line at 6kHz in Figure 97 represents the expected frequency for turbulent fluctuations in the Columbia University subsonic duct (Section 8.3) occurring at a mass flow rate (mdot) of 0.08 lbm / sec. For this specific value of mdot, the MS3110 BW setting will affect the ability of the sensor to measure mean or fluctuating turbulent shear stress. At lower BW settings (0.5 < BW < 6kHz), the MS3110 output signal, \(V_0\), will represent a mean shear stress measurement (\(\tau_{\text{mean}}\)), while at higher BW settings (BW > 6kHz), \(V_0\) can include data which will represent the turbulent shear stress (\(\tau_{\text{turb}}\)); \(\tau_{\text{mean}}\) above 6kHz can be calculated by further low-pass filtering the data through either electrical or numerical schemes.

The same nine combinations of MS3110 settings tests conducted with the MS3110 IC soldered onto the custom-made PCB integrated with the packaging. The packaging is installed in the subsonic duct with no sensor attached to the eight connector pins. The test results (Figure 98) show that \(V_{\text{noise}}\) increases two orders of magnitude from the case with the MS3110 IC mounted in the M3110BDPC evaluation board. This large noise increase is due to the added components of the packaging, and requires further investigation to determine methods to reduce the noise in future sensor packaging. When analyzing the data during flow testing, the large noise requires low-pass filtering of the data, attenuating any fluctuating shear stress measurement which may be present in the signal.
Figure 97: Measured voltage noise on the MS3110 IC for 9 combinations of CF and BW settings. The vertical line at 6 kHz represents the theoretical frequency for turbulent fluctuations in the subsonic duct at $\text{mdot} = 0.08 \text{ [lbm / sec]}$ (Section 8).

Figure 98: Measured voltage noise on the MS3110 IC for 9 combinations of CF and BW settings. MS3110 is mounted on packaged PCB with no sensor installed. The vertical line at 6 kHz represents the theoretical frequency for turbulent fluctuations in the subsonic duct at $\text{mdot} = 0.08 \text{ [lbm / sec]}$ (Section 8).
When measuring capacitance with the MS3110 IC, what ultimately effects the sensor shear stress measurement is not $V_{\text{noise}}$, but rather the capacitance noise ($C_{\text{noise}}$) which is a combination of $V_{\text{noise}}$ and the MS3110 gain ($G$):

$$C_{\text{noise}} [\text{fF}] = \frac{V_{\text{noise}} [\text{mV}]}{G [\text{mV/fF}]} \quad (163)$$

Figure 99 displays the same measurement data as Figure 97 with $C_{\text{noise}}$ calculated using eq. (163). The solid line is the manufacturer’s data for the MS3110 Noise Floor which is given as $4.0 \times 10^{-18} \text{ (F/} \sqrt{\text{Hz})}$ [88]. The noise floor represents the noise inherent in the operation of the MS3110 IC. The measured data lies above the manufacturer’s data but falls below 1fF, the minimum capacitance resolution for measurement of the MEMS shear stress sensors. For each setting of $C_F$, $C_{\text{noise}}$ increases with $BW$, as higher frequencies are introduced into the measurement. There is no clear trend for how $C_{\text{noise}}$ varies with $CF$ at each $BW$ setting. In the data shown, a $CF$ setting of 1.026 at a $BW$ of 500Hz would yield the best value of $C_{\text{noise}}$.

Figure 100 shows the results of the testing with the MS3110 installed on the custom-PCB. Due to the added packaging components ($C_{\text{noise}}$) increased one to two orders-of-magnitude. The data clearly indicates that $CF = 5.13\text{pF}$ yields the lowest $C_{\text{noise}}$ at all values of $BW$ that were tested, therefore this value should be chosen during sensor testing.
Figure 99: Capacitance noise ($C_{noise}$) on MS3110 IC measurements when varying CF and BW. MS3110 Noise Floor is provided by manufacturer [88]. The vertical line at 6 kHz represents the theoretical frequency for turbulent fluctuations in the subsonic duct at $\dot{m}_{\text{dot}} = 0.08 \text{ [lbm / sec]}$ (Section 8).

Figure 100: Capacitance noise ($C_{noise}$) on MS3110 IC measurements when varying CF and BW. MS3110 is mounted on packaged PCB with no sensor installed. MS3110 Noise Floor is provided by manufacturer [88]. The vertical line at 6 kHz represents the theoretical frequency for turbulent fluctuations in the subsonic duct at $\dot{m}_{\text{dot}} = 0.08 \text{ [lbm / sec]}$. 
Each time the MEMS sensor is changed or the package is modified, it is necessary to conduct similar tests to determine the settings that yield the data with the lowest noise. Additionally, determination of the appropriate BW settings for measuring $\tau_{nub}$ or $\tau_{mean}$ will be a function of the specific flow environment, and the expected frequency of the turbulent fluctuations.
7 Sensor Characterization

Sensor characterization consists of specific tasks required to verify a packaged sensor’s response to flow actuation, and to evaluate sensor performance predictions based on “as-built” sensor dimensions. After the sensors are released via wet-etching, characterization tests are performed in a clean room (class 10,000 or better) to verify sensor operation, assess sensor performance and measure sensor as-built dimensions. Some checkout tests (such as tasks i-iv below) are required prior to testing the sensor in a flowing environment. Specific characterization tests include the following:

(i) Sensor inventory and measurement of “as-built” dimensions
(ii) Sensor capacitance / resistance measurements
(iii) Voltage / current (V-I) measurements
(iv) Mechanical and electrical actuation tests
(v) Fluidic actuation under microscope (wall jet tests)
(vi) Duct cold flow tests

7.1 Sensor Inventory and Measurement of “As-Built” Dimensions

Released MEMS shear stress sensors are examined under a light microscope in a clean-room environment to evaluate the condition of the sensor and to conduct length measurements. Digital images of “as-built” sensor features are analyzed via image processing methods, and input into sensor calculation spreadsheets to update the sensor performance models. Additionally, damaged sensor components and cleanliness are noted for inventory purposes.
Figure 101 (L): Light microscope image of MEMS shear stress sensor showing sensor feature measurements. Sensor floating element is on the left, sensor spring is near the bottom of image.

Figure 102 (R): Scanning Electron Microscope (SEM) image of MEMS shear stress sensor. Image is zoomed-out from the area shown in Figure 101.

Figure 101 is a light microscope image of a MEMS shear stress sensor taken in the Columbia University CEPSR cleanroom. The sensor floating element is on the left-side of the image, and a folded-spring is horizontal across the bottom part of the image. The bottom-left corner is an anchor-pad where the sensor spring attaches. The image shows measurements taken to determine the sensor “as-built” dimensions. This includes measuring the gap spacing ($d_0$), finger width ($b_f$), spring width ($b_k$), and finger length ($L_f$). These dimensions are critical to sensor performance as they will affect the spring constant ($k_x$) and the relationship between $\delta C$ and $\tau_w$, expressed in eqs. (33) and (38).

Figure 102 is a Scanning Electron Microscope (SEM) image of a MEMS shear sensor. The image is a zoomed out version of the same part of the sensor as shown in Figure 101. SEM images offer the benefit of allowing the sensor to be angled for evaluation of out-of-plane bending by sensor fingers. These effects can be caused by stress gradients in the device layer.
or by incomplete etching release (indicated by left-over sacrificial material underneath the sensor fingers).

Figure 103: Electrical schematic of 3 pad MEMS shear stress sensor. F1 - F8 are frontside pads. Pads B1 - B8 are backside pads. CS2 and CS1 are shear sensing capacitors. RP2 and RP1 are parallel resistances. RF1 - RF8 are resistances on frontside electrode paths. RB1 - RB8 are metallized via resistances. CS2, CS1, and CSCOM designate where the connections to the MS3110 IC are made.

7.2 Sensor Capacitance / Resistance Measurements

The MEMS sensor can be electrically modeled as a combination of resistors and parallel-plate capacitors. Resistors are formed by Si structures, or metal connections such as gold, used in the Via metallization. Capacitors are formed by either the sensing element with air as the
dielectric, or from other sensor features (e.g. vias, electrode pads, etc.), Figure 72, having capacitances with the substrate with SiO₂ as the dielectric as described earlier in Section 5.

Figure 103 is an electrical schematic of the MEMS sensor. The 8 frontside pads are designated by F1-F8, and the 8 backside pads are designated by B1-B8. These 16 pads are the locations on the overall MEMS sensor where measurements are taken via probes or package connections. For simplification, the sensor capacitances are lumped together with the electrode pad capacitances, and modeled as variable capacitors, CS2_eff and CS1_eff, as in eq. (109). Throughout this section, we are examining the sensor capacitances only, and are not including parasitic capacitance contributions from the package.

Resistors RF1 to RF8 represent electrical paths on the top side of the sensor which connect pairs of frontside pads (F1/F2, F3/F7, and F4/F5). Resistors RB1 to RB8 represent the resistance across the metallized vias that connect the sensor frontside pads to backside pads (F1/B1, F2/B2, etc.). RP2 and RP1 represent the parallel resistances which are caused by leakage current passing through the substrate as a result of structures not being completely insulated from one another. They will be discussed in the following section on voltage / current (V-I) measurements.

Capacitance and resistance measurements are used to evaluate the electrical integrity of the fabricated MEMS sensors. Repeated measurements have shown that there are expected resistances and capacitances values (Table 15) that can aid in determining which sensors will operate effectively once packaged. The effective sensor capacitance (CS2_eff or CS1_eff) will have a value typically ranging from 8-20pF for sensors connected to the frontside pads without metallized vias and therefore no backside connections. For sensors with backside connections, the values will increase to 20-50pF due to a larger capacitance area from the metallized vias and
backside pads. If the CS2\textsubscript{eff} or CS1\textsubscript{eff} capacitances are lower than 8pF this is an indication that there is an open circuit. Open circuits can result from either a physical break in a conductive path on a sensor or from the sensor not being packaged properly.

Before sensors are packaged, baseline capacitance and parallel resistance measurements are taken via probes using the Keithley 590CV. The resistances between connected frontside pads (RF1 – RF8) range from 0.5kΩ - 10kΩ, while the resistances across metallized vias (RB1 – RB8) range from 0.5kΩ - 5kΩ. Significantly larger values O(MΩ) indicate the presence of open circuits. The parallel resistances (RP2 and RP1) are dependent on the specific sensor fabrication and can range from 500kΩ - 50MΩ.

After a sensor is packaged, the measurements are repeated and compared to the baseline measurements to determine if the sensor was packaged properly. During the course of testing, capacitance measurements are periodically compared to baseline ones to determine if there were any major changes in the sensor circuitry.

<table>
<thead>
<tr>
<th>Name:</th>
<th>Description:</th>
<th>Typical Measurements for Operational Sensors:</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS2\textsubscript{eff}, CS1\textsubscript{eff}</td>
<td>Sensor total capacitance including substrate (no package)</td>
<td>8pF - 20pF for frontside pads only</td>
</tr>
<tr>
<td></td>
<td></td>
<td>20pF – 50pF for frontside and backside pads</td>
</tr>
<tr>
<td>RF1 – RF8</td>
<td>Resistance between frontside connected pads</td>
<td>0.5kΩ - 10kΩ</td>
</tr>
<tr>
<td>RB1 – RB8</td>
<td>Resistance across via between frontside and backside pads</td>
<td>0.5kΩ - 5kΩ</td>
</tr>
<tr>
<td>RP1, RP2</td>
<td>Parallel Resistance</td>
<td>500kΩ – 50MΩ</td>
</tr>
</tbody>
</table>

Table 16 shows measured absolute capacitance values of CS2\textsubscript{eff} and CS1\textsubscript{eff} for different sensors on the ECA wafer. These sensors are measured on the microscope probe station with no packaging connected to the sensor. The measured values are generally within 20-30% of
the theoretical value for the capacitance, 31.5pF, (Section 5.1), which is considered acceptable for MEMS capacitors.

Table 16: Comparison of measured capacitance versus the theoretical capacitance (31.5pF) for CS2\_eff and CS1\_eff with no packaging.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>CS2_eff [pF]</th>
<th>CS1_eff [pF]</th>
<th>Percent Error in CS2_eff</th>
<th>Percent Error in CS1_eff</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECA-3F2A</td>
<td>37.1</td>
<td>34.8</td>
<td>+17.8</td>
<td>+10.5</td>
</tr>
<tr>
<td>ECA-3F3A</td>
<td>27.9</td>
<td>27.8</td>
<td>-11.4</td>
<td>-11.8</td>
</tr>
<tr>
<td>ECA-4F2A</td>
<td>20.7</td>
<td>19.8</td>
<td>-34.3</td>
<td>-37.1</td>
</tr>
<tr>
<td>ECA-4T2A</td>
<td>20.6</td>
<td>22.0</td>
<td>-34.6</td>
<td>-30.1</td>
</tr>
<tr>
<td>ECA-3F2B</td>
<td>22.4</td>
<td>22.9</td>
<td>-28.9</td>
<td>-27.4</td>
</tr>
<tr>
<td>ECA-4T2A</td>
<td>25.9</td>
<td>26.2</td>
<td>-17.8</td>
<td>-16.8</td>
</tr>
<tr>
<td>ECA-3F2D</td>
<td>27.2</td>
<td>26.1</td>
<td>-13.6</td>
<td>-17.1</td>
</tr>
</tbody>
</table>

7.3 Voltage / Current (V-I) Measurements

Voltage-current (V-I) measurements are conducted in a cleanroom using the Keithley Semiconductor Characterization System (Keithley 4200-SCS). One purpose of V-I measurements is to evaluate the sensor electrical paths by ensuring that there are ohmic contacts between connected sensor pads (e.g. 1/2, 3/7, 4/5). Ohmic contacts are metal junctions where there is a V-I curve which is linear and symmetric (i.e. a resistance which is constant with voltage). MEMS devices contain semiconductor materials such as Silicon which do not typically have ohmic contacts when touching metal. However, through MEMS fabrication techniques, we can deposit metal electrode pads on semiconductors and create ohmic contacts [17]. If ohmic contacts are not present at the probe points of the MEMS sensor, then the capacitance test signal can be altered in magnitude and phase, resulting in an inaccurate \( \delta C \) measurement.
A secondary purpose for the V-I measurements is to evaluate the parallel resistance (RP) values of the sensor. The parallel resistances are caused by the substrate not being completely insulated from the sensor circuit. RP measurements represent the current leakage through the substrate, and are modeled as being in parallel with the sensing capacitance. The parallel resistance occurs from interaction with the Silicon substrate, a semiconductor, therefore it is non-ohmic, with a non-linear V-I relationship; this can be determined by obtaining the sensor’s VI curve.

The MS3110 IC measures capacitance by applying a 2.25V square wave as the test signal. To avoid adversely affecting the MS3110 measurement, the MEMS shear sensor needs to have both RP values, RP1 and RP2, greater than 20MΩ over the voltage range of 0 – 2.25V. Parallel resistances were measured during the sensor characterization tests (Section 7.2) using the Keithley 590CV. However, this measurement is taken at a voltage of 15mV_{RMS} and does not show the behavior of parallel resistance over the voltage range (0 – 2.25V) needed for operating the MS3110 IC.

For best performance, RP values should be as large as possible (>50MΩ), however in practice, the values of the MEMS shear stress sensor range from O(kΩ) to O(MΩ) depending on the specific sensor. RP measurements can be classified into three regimes depending on the measured values: (1) RP < 1MΩ excessive leakage; (2) 1MΩ < RP < 20MΩ requires MS3110 gain compensation; (3) RP > 20MΩ negligible leakage.

For sensors with excessive leakage (RP <1MΩ), capacitance changes from applied shear stress will be undetectable rendering the sensor inoperable. This effect is described in detail later. Sensors in the compensation range (1MΩ < RP < 20MΩ) will have a measurable capacitance change when flow tested, however the gain (G) of the MS3110 IC will be significantly lower.
than the theoretical value, eq. (160), and will need to be calculated using eq. (161). Sensors in the negligible leakage range (\( \text{RP} > 20\text{M}\Omega \)) will not have any effects from parallel resistance, and eq. (160) can be used directly in calculating \( \delta C \).

It is necessary to measure which regime the \( \text{RP} \) values of the sensor are in to determine the required MS3110 settings prior to sensor flow testing. The MS3110 trims settings must be adjusted so that \( V_0 \) changes caused by the sensor are measurable \( O(\text{mV}) \). During the test, \( V_0 \) must also fall within the bounds of the MS3110 output (0.5 V – 4.0V), or it will saturate the signal. The MS3110 must be properly adjusted prior to testing, in order to ensure acquisition of useful sensor data.

Figure 104 shows typical V-I test results for sensor ECA-3F2B tested under a walljet flow setup (Section 8.2). When there is no flow present there is a non-linear relationship between current and voltage for both \( \text{RP2} \) and \( \text{RP1} \). At lower voltages the current is \( O(10^{-8}\text{ A}) \), and increases by a factor of 10 near 2.25V. When flow is introduced, both \( \text{RP2} \) and \( \text{RP1} \) have lower currents at the given voltages.

Figure 105 uses the same data as the previous figure but the resistances are calculated using the slope at each voltage point in Figure 104. It is shown that for both \( \text{RP2} \) and \( \text{RP1} \) resistance generally decreases with voltage, from 10-30M\( \Omega \) near 0V to approximately 5M\( \Omega \) near 2.25V. When the flow is introduced, it lowers the sensor temperature and increases the resistance for both \( \text{RP2} \) and \( \text{RP1} \), with an effect that is more pronounced at lower voltages. As discussed earlier, \( \text{RP} \) values between 1M\( \Omega \) and 20M\( \Omega \) will result in working sensors, however they will require gain compensation. Furthermore, the change in the V-I relationship will require temperature compensation which will be discussed in Section 8.3.7.
Figure 104: V-I data for sensor ECA-3F2B showing RP1 and RP2 values during wall jet testing. The flow condition lowers the temperature of the sensor.

Figure 105: Resistance vs. Voltage data for sensor ECA-3F2B showing RP1 and RP2 values during wall jet testing. The flow condition lowers the temperature of the sensor.
Additionally, it was noted that the MS3110 capacitance measurement was affected by incident light on the sensor. It was verified by measurements that that V-I relationships for RP1 and RP2 were affected by light on the frontside surface of the sensor. Therefore all wall jet flow tests using capacitive data were taken with the lights off in the room. For the duct flow testing, the sensor was installed far enough inside the duct that the ambient light did not affect the sensor output.

7.4 Mechanical Actuation Tests

Mechanical actuation tests are used to determine if the sensor is fully-released and to verify a sensor output initiated by physically moving the floating element. The element is probed with a 100um diameter optical fiber attached to a micromanipulator on a Signatone Probe Station. The optical fiber is electrically insulated and was shown to not affect the capacitance measurement.

Figure 106 shows typical results from two tests on a MEMS shear stress sensor (CD-4F2A) with capacitance measured by the Keithley 590CV. During the 10 second tests, the sensing element was viewed under a light microscope while being probed back-and-forth in the direction of increasing CS2 causing an increase in the capacitance. Tests 1 and 2 attempted to displace the floating element approximately 70-90% of the gap distance \(d_0\) once per second. The variability of the probe data stems from challenges with displacing the floating element the same distance in a repeatable manner. The graph shows a maximum capacitance change of approximately 250fF caused by displacing the sensor up to 90% of \(d_0\). The data shows that the sensor capacitance is changing on the order of \(10^2\) fF as expected, solely from the motion of the floating element. The difference in the data between Tests 1 and 2 is due to difficulties in
controlling the motion of the insulated probe acting against the floating element, while displacing it.

![Mechanical Actuation Test](image)

**Figure 106:** Data from mechanical actuation (probe push) tests of frontside packaged sensor CD-4F2A. Sensor floating element is pushed with an insulated fiber optic probe causing a capacitance change which is measured via Keithley 590CV.

### 7.5 Voltage Actuation Tests

Voltage actuation testing is conducted to determine the sensor response to an applied electrostatic force. A DC voltage bias is applied across the sense capacitors (CS2 or CS1), resulting in an electrostatic force on the floating element in the streamwise direction. Measuring the sensor response to the voltage actuation is used to assess the mechanical behavior of the fabricated sensors.
To determine the sensor displacement for an applied actuation voltage the spring force and electrostatic force \((F_e)\) are balanced:

\[
F_{\text{spring}} = F_e
\]  

(164)

where \(F_{\text{spring}} = kx\) and \(F_e\) is the force from a voltage applied across one sense capacitor (CS2 or CS1). \(F_e\) is related to the derivative of the capacitance [77] by:

\[
F_e = \frac{V_B^2}{2} \frac{d}{dx} C(x)
\]  

(165)

where \(V_B\) is the applied voltage bias, and \(C(x)\) is a sensor capacitance, that accounts for electric field fringe effects, given by:

\[
C(x) = \frac{\varepsilon A_c}{d_0 - x} \left( 1 + 1.9861 \left( \frac{d_0 - x}{h} \right)^{0.8258} \right) + \frac{\varepsilon A_c}{\alpha d_0 + x} \left( 1 + 1.9861 \left( \frac{\alpha d_0 + x}{h} \right)^{0.8258} \right)
\]  

(34)

where \(\alpha = \) distance amplification factor, \(h = \) sensing finger thickness, \(A_c = \) capacitance area, \(x = \) sensor displacement, and \(d_0 = \) gap space between capacitor plates.

The derivative of \(C(x)\), \(d/dx[C(x)]\) is evaluated numerically, allowing calculation of the electrostatic force, and substituting in the spring force, given as \(kx\), we get:

\[
kx = \frac{V_B^2}{2} \frac{d}{dx} C(x)
\]  

(166)

We then define an equivalent shear stress resulting from the electrostatic force:

\[
\tau_{eq} = \frac{F_e}{A_s} = \frac{F_{\text{spring}}}{A_s} = \frac{kx}{A_s}
\]  

(167)

Figure 107 shows the typical relationship between the applied voltage bias \((V_B)\), the equivalent shear stress \((\tau_{eq})\), and the normalized displacement \((x/d_0)\). Increasing \(V_B\) results in a larger \(\tau_{eq}\) and displacement until electrostatic snapdown occurs near \(x/d_0 = 1/3\). Increasing \(V_B\) results in a larger \(\tau_{eq}\) and displacement until electrostatic snapdown occurs at 16.5V, near a
sensor displacement of \(x/d_0 = 1/3\). Snapdown is caused by a mechanical instability resulting from \(F_e\) and \(F_{spring}\) increasing with \(x\) at different rates [77]. Electrostatic snapdown should be avoided as it can destroy sensors if the sensing fingers touch and significant amounts of current are conducted.

Voltage actuation test data is shown in Figure 108 for frontside packaged sensors CD-3F2E and CD-4F2A. Displacement is measured by taking a microscope image of the sensor while a bias voltage is applied and implementing image processing methods. For both sensors, the voltage was increased in steps of 2V, with efforts made to avoid snapping down the sensor. The equivalent shear stress (\(\tau_{eq}\)) is calculated using eq. (167). The theoretical curves are calculated by combining eqs. (166) and (167). For sensor CD-3F2E, the \(\tau_{eq}\) is generally larger than predicted by theory, indicating that the sensor spring may be more compliant than expected. A different cause may be additional electrostatic effects that are not captured by eq. (166). These additional electrostatic effects may be caused by the floating element fingers and stationary fingers bending towards each other due to the attracting electrostatic force. This effect is not accounted for in the model, and would result in a larger electrostatic force being present, causing the spring to displace further than predicted, resulting in a larger \(\tau_{eq}\).

The data for sensor CD-4F2A shows a \(\tau_{eq}\) significantly smaller than predicted. During the test the voltage was increased further than predicted snapdown (15.0V) due to the small amount of observed motion. Snapdown did not occur with up to 18.5V being applied. A possible explanation for this behavior is that the sensor motion was blocked by debris, inhibiting but not preventing floating element motion and avoiding snapdown at the predicted voltage.
Figure 107: Graph showing theoretical curve for equivalent shear stress ($\tau_{eq}$) vs. actuation voltage. Secondary y-axis shows normalized sensor displacement.

Figure 108: Data from voltage actuation tests for front-side packaged sensors CD-3F2E and CD-4F2A. Displacement of sensors measured via image processing methods and converted to $\tau_{eq}$ using eq. (167). Theory curves are from combining eqs. (166) and (167).

Both sets of data, although not matching the theory, do exhibit the proper trends and are the same order-of-magnitude. This data is not used to verify the spring constants of the sensors, however it does serve to validate the overall physical model of the MEMS sensor. Additionally,
since the electrostatic force is directly related to the capacitance through eq. (166), we can conclude that the physical and electrical models for the sensor will behave in a similar manner to their designs.
8 Sensor Cold Flow Characterization Testing

8.1 Objectives of Sensor Cold Flow Characterization Testing

The main objective of the cold flow testing is to characterize and assess the performance of the MEMS shear stress sensor designs in high-speed turbulent cold flow environments. Cold flow tests were conducted in the Mechanical Engineering Project Laboratory and the MRSEC Small-Instruments Laboratory at Columbia University (NY). Cold flow testing will allow the team to: 1) assess our packaging approach; 2) test sensing and electronics; 3) verify sensor release and floating element motion; 4) characterize sensor performance under various flow conditions compared to the theoretical design performance; and (5) assess sensor survivability.

8.2 Wall Jet Cold Flow Testing

8.2.1 Flow Field Theoretical Evaluation for Wall Jet

A wall jet allows for both visual confirmation and a capacitive measurement of the sensor response to an applied turbulent shear stress. The sensor is packaged and inserted flush with the top surface of a flat plate which is underneath a high-powered light microscope (Figure 112). Compressed air is blown out of a slot forming a shear layer that fluidically actuates the MEMS shear stress sensor.
Figure 109: Schematic of wall jet flow actuation set-up under microscope.

Figure 110 (L): Diagram of 2-D wall jet indicating important flow characteristics and length scales [91].

Figure 111 (R): Variation of 2-D wall jet skin friction ($C_f$) with Reynolds Number ($Re_m$). Graph from Tachie et al. (2002) [93].
A turbulent wall jet is a hybrid of a turbulent jet and a turbulent boundary which is often studied due to its common occurrence in heating and cooling applications. The 2-D wall jet is issued from a slot in the wall of height \( b \) with a jet velocity \( U_0 \). As it spreads in the normal direction, a region of maximum velocity \( U_m \) forms at a distance \( y_m \) close to the wall (Figure 110). It has been shown by many experimentalists [89] that the length-scale for a turbulent wall jet is the jet half-width \( y_{1/2} \). The jet half-width is defined as the distance from the wall where the flow velocity is equal to one-half of \( U_m \). Details on the wall jet evolution, including scaling analysis can be found in references [90] - [93].

Correlations can be used to evaluate the expected shear stress at the sensor location with the Reynolds numbers \( (Re_m) \) and skin friction coefficient \( (C_f) \) defined by:

\[
Re_m = \frac{U_m y_m}{\nu}
\]  

and

\[
C_f = \frac{\tau_w}{y_{1/2} \rho U_m^2}
\]

Figure 111 illustrates various \( C_f \) correlations for a 2-D wall jet. Note the large variations in \( C_f \) among the different researchers. Eriksson et al. [91] determined \( C_f \) by utilizing Laser Doppler Anemometry (LDA) to directly measure the velocity gradient in the viscous sublayer down to \( y^+ = 1 \). They gave an empirical correlation for \( C_f \) as:

\[
C_f = 0.0179 \ Re_m^{-0.113}
\]

The data from Eriksson et al. is considered more accurate than previous researchers [89] [94] who had used Hot Wire Anemometry (HWA) to measure the velocity gradient, assuming a linear near-wall region extending to \( y^+ = 6 \). In [91] it was shown that the linear region only extends to \( y^+ \sim 3 \), leading to earlier measurements of \( C_f \) being inaccurate by up to 20% [92].
Therefore, any wall jet shear stress measurements should be compared to eq. (170) when evaluating the accuracy of the method.

Figure 112: Wall jet setup in MRSEC lab. Sensor package in installed underneath microscope probe station on left.

8.2.2 Description of Wall Jet Setup

The wall jet flat plate was made from polished aluminum stock, and custom fit to be placed within the confines of the probe station stage. An image of the setup is shown in Figure 112. The sensor package is attached to the bottom of the plate and the packaged sensor is inserted through a hole in the bottom of the flat plate so that it is flush-mounted with the surface. The slot is formed by a 0.125in. diameter semicircle cut into a rectangular aluminum piece. The air is supplied from a compressed dry air tank and is regulated via a hand valve.
Tests involved installing a packaged sensor in the flat plate, mounting it under the microscope and measuring a differential capacitance change (with the MS3110 IC) as a function of bottle pressure. The MS3110 IC measures a differential capacitance by connecting the sensor pads to the IC input terminals. The connection can be made via shielded BNC cables, or with an MS3110 mounted on a close-coupled PC board.

The total bottle pressure was varied via a hand valve and measured using a piezoelectric pressure transducer (Omega Engineering, Inc.). Measurements were sampled using an Agilent 54622D Mixed Signal Oscilloscope and data was saved using the built-in disk drive.

### 8.2.3 Experimental Setup and Flow Field Characterization

Experimentally determined wall jet skin friction values have typically been conducted in large volume water tanks with LDA [91] or in large-scale O(m) air flows provided by fans and utilizing HWA [24]. The wall jet setup at Columbia University (MRSEC shared facilities lab) uses a compressed air tank and a 0.125in. semicircular slot, in order to install the sensor underneath the microscope probe station.

Calculations showed that it would not be feasible to accurately determine the wall jet skin friction for this setup due to the challenges of resolving the small length scales (i.e. the $y_m$ value) necessary for using the empirical correlations described in Section 8.2.1. In order to characterize the MEMS shear stress sensors over a large portion of their operational ranges it requires wall shear stress ($\tau_w$) values on the order of 100 – 1000Pa. For a 2-D wall jet, this necessitates measuring the vertical location of the region of maximum velocity ($y_m$), which would be on the order of 10-100μm for the desired shear stress values. HWA is the most common method used for measuring velocities near a wall, however, as detailed in [24], implementing HWA near a
wall (<1mm) is very complicated, and would have required a major experimental effort outside the scope of this project.

Regardless of the applicability of the wall jet correlations to our sensors, we still conducted wall jet experiments to qualitatively verify that the sensors were moving under applied shear and not fluttering or exhibiting other adverse behavior (e.g. not surviving or not causing a capacitance change).

8.2.4 Test Operations/Procedures

The following procedures are followed for testing a packaged MEMS shear stress sensor under the wall jet:

1) Sensor package installed in wall jet setup under the microscope in the Columbia University MRSEC lab.

2) Visual inspection of sensor to ensure floating element is not broken and debris is not present on the sensor which could be blown onto the sensing element.

3) Images of sensor are taken for inventory purposes which are later compared to previous images of the same sensor to check for damage.

4) Sensor is connected to the MS3110 IC for the capacitance measurement. After this point, the lights in the room are turned off for any recorded capacitance measurements (See Section 7.3).

5) The MS3110 trim settings are changed (CS1trim and CS2trim) to make the MS3110 output as close to 2.25V as possible (0.5V for a single-ended measurement).

6) Preliminary wall jet testing is conducted. This consisted of visually monitoring the sensor and determining the appropriate bottle pressure needed to cause the sensor to displace approximately
50% of the gap distance ($d_0$). The feedback capacitor (CF) on the MS3110 is adjusted to give the largest change in $V_0$ while staying between the output range of 0.5 to 4.0V.

7) Sensor is rotated 180° to verify sensor operation is bidirectional (e.g. measures shear stress in both (+x) and (-x) directions).

8) The actual gain of the MS3110 is determined using eq. (161).

9) Wall jet testing is conducted, which consists of increasing the flow pressure in steps by varying the bottle pressure with a hand valve. Each step in bottle pressure is held for approximately 5 seconds, and is tested during the flow-down (decreasing pressure) portion of the test.

10) Additional tests included pictures of displaced sensors at specific flow pressures, and/or videos of moving sensors. The still frames were used to calculate displacement using image processing, while the videos where used to evaluate sensor motion under flow.

8.2.5 Results/Analysis of Wall Jet Cold Flow Experiments

The wall jet data is used to show that the sensor is released and responding to a shear stress. It is more qualitative in nature than the duct flow since as previously mentioned, we do not know the expected shear stress for the wall jet setup. The term “measured shear stress” refers to a shear stress calculation based on a MEMS sensor measurement. For the wall jet setup, this measurement can be capacitive (using the MS3110) or optical (using image processing). For a capacitive measurement, the measured differential capacitance is converted to a displacement using eqs. (34), and (38) in conjunction with the “as-built” sensor dimensions. This displacement is then converted to the measured shear stress using eq. (33). For an optical displacement measurement, we can calculate the measured shear stress directly by using eq. (33). In practice,
the optical displacement measurements are more repeatable and do not suffer from temperature effects.

For a wall jet test, the flow rate is increased by taking steps in the nozzle pressure at 14.8 (no flow), 21.4, 36.1, 51.2, and 66.3 Psia. At each step the pressure is held constant for approximately 5 seconds, and the flow values are averaged. Once the maximum pressure is reached the flow is stepped down, stopping at the same steps of pressure as on the steps up. Due to the large hysteresis that was observed with the MEMS sensor measurements, we will focus only on test results for “increasing flow.” Ultimately the scatter on the “decreasing flow” measurements was so large that it was hard to interpret trends.

Figure 113 shows typical wall jet results for sensor ECA-3F2A for both capacitive and optical measurement methods. Each data point is the average of three tests, and the scatter at each point is represented by the error bars, which are +/- 0.15μm and +/- 0.09μm for the optical and capacitive measurements, respectively. The horizontal error bars are 0.5Psia from the pressure measurement. The capacitive data has been corrected for temperature effects using the 90° rotation method (See Section 8.3.7). There is a clear difference in the two different data sets, although they both increase in an almost linear manner with pressure. The capacitive measurement is more prone to error since it relies on an empirical correlation (eq. 34) and also requires compensation, whereas the optical data is a direct measurement. Therefore, we will assume that the optical measurement is the actual displacement measurement, and claim that to calculate actual displacement from a capacitance measurement you can dividing the capacitance values by a constant of 1.82. This value comes from taking the ratio of the slopes of the linear regressions for the data in Figure 113. Overall, these test results are encouraging in that they
illustrate the sensor responding to fluidic actuation, with the measured shear stress increasing with the bottle pressure in a consistent manner.

![Figure 113: Typical averaged wall jet test results for sensor ECA-3F2A. Displacement is measured via optical and capacitive methods. The vertical error bars are +/- 0.15μm and +/- 0.09μm for the optical and capacitive measurements, respectively. The horizontal error bars are 0.5Psia](image)

Azimuthal sensor testing was conducted to verify that the MEMS shear stress sensor primarily responds to shear stress in the streamwise flow direction (x). The sensors were designed with the floating elements much stiffer in the transverse (y) and out-of-plane directions (z) making them relatively insensitive to shear forces not in the streamwise direction. The wall jet testing setup was designed to allow variation of the azimuthal angle by mounting the package inside of a ball bearing (Section 4.1.2).
If a shear stress force, $F_{\text{shear}}$ is applied to the sensing element that is rotated at an azimuthal angle ($\theta$), the sensor is designed to only measure the component of $F_{\text{shear}}$ that is in the direction of the sensor displacement (x). This relationship is given by:

$$F_{\text{shear}}(\theta) = F_{\text{shear}}(\theta = 0^\circ) \cos(\theta) \quad (171)$$

where $F_{\text{shear}}(\theta = 0^\circ)$ is the total shear force at angle $0^\circ$, and $F_{\text{shear}}(\theta)$ is the component of the force at angle $\theta$. By dividing the forces by the shear stress area ($A_s$), this can be rewritten in terms of shear stress as:

$$\frac{\tau_w(\theta)}{\tau_w(\theta = 0^\circ)} = \cos(\theta) \quad (172)$$

Eq. (172) is referred to as the cosine law.

![Variation in Azimuthal Angle](image)

Figure 114: Wall jet test results for sensor CD-3F2E. Measurements were made by measuring displacement via image processing methods. The normalized shear stress refers to “measured shear stress” values.
Figure 114 shows wall jet testing results for variation in the azimuthal angle from -60 to 60 degrees (0° is streamwise). The sensor was exposed to a pressure step of 35Psia while orientated at different angles with respect to the flow. The data is plotted against eq. (172) and generally lies within the experimental error (estimated at +/- 0.2). This indicates that the MEMS shear stress sensor is primarily responding to the shear stress in the streamwise direction; as it was designed.

### 8.3 Duct Cold Flow Testing

#### 8.3.1 Flow Field Theoretical Evaluation for Duct

In order to evaluate the performance of the MEMS shear stress sensor, it was tested in a calibrated high-speed flow environment. Flow through a long, smooth, square duct was used because it results in a well-known wall shear stress when fully-developed, turbulent, subsonic flow has been established. The packaged MEMS shear stress sensor was inserted into the bottom of this duct, and adjusted so that the sensor was flush with the bottom wall of the duct.

Measured average flow properties (e.g. temperature and pressure) and average velocity allow for estimation of skin friction using the semi-empirical correlation of Karman-Nikuradse (K-N). The K-N correlation has been shown to be valid for fully-developed adiabatic compressible duct flow [95] [96]. The aluminum duct is non-adiabatic, requiring use of the K-N correlation with modifications for heat transfer. Lowdermilk et al.[97] demonstrated that eq. (173) gives the duct skin friction \( C_f \) with sufficient accuracy when evaluating flow properties at the film temperature \( T_f \) rather than the average temperature \( T_b \). The film temperature is
defined as the average of the bulk temperature and the wall temperature \((T_w)\). The Karman-Nikuradse correlation with modifications for heat transfer is:

\[
\frac{1}{\sqrt{4C_f}} = 2.0 \log \left( \text{Re}_f \sqrt{4C_f} \right) - 0.8 \tag{173}
\]

where skin friction is defined as:

\[
C_f = \frac{\tau_w}{\frac{1}{2} \rho_f U^2} \tag{174}
\]

and the Reynolds number based on properties evaluated at \(T_f\):

\[
\text{Re}_f = \frac{bU}{\nu_f} \tag{175}
\]

where: \(U\) = duct average velocity, \(b\) = duct height, \(\nu_f\) = kinematic viscosity at film temperature, \(\rho_f\) = density at film temperature, \(\tau_w\) = wall shear stress. Figure 115 illustrates the relationship between \(Re_f\) and \(C_f\) and it is shown that the skin friction coefficient decreases with the Reynolds Number.

**Figure 115:** Relationship between \(C_f\) and \(Re_f\) (eq. 173).
The duct shear stress is calculated by using eqs. (173-175) below, which are referred to as the “K-N correlation” for the remainder of this report. Comparing the MEMS sensor output to the duct shear stress calculated using the K-N correlation provides an evaluation of the sensor performance, as well as a standard to calibrate the sensor against. Throughout this chapter, the term “duct shear stress” or “theoretical shear stress” will represent expected shear stress in the duct based on the K-N correlation.

8.3.2 Duct Design and Fabrication

The duct flow setup consists of a 7ft. long aluminum square duct (0.49in. x 0.49in. with 0.14in. thick walls) designed to provide turbulent, compressible subsonic air flow. The development length (L/b) is greater than 160 duct diameters, sufficiently long enough to result in fully-developed flow [95]. Flow measurements show that the duct Reynolds Number (Re_b) is greater than 4000 for all duct flow rates, ensuring turbulent flow at the sensor location.

Compressed dry air is supplied from three 120 gallon air tanks that are used as the air source for a supersonic blowdown nozzle experiment. Flow from the large tanks is regulated via a hand-turned ball valve. Downstream of the ball valve, the flow is connected, through 0.75in. inner diameter (ID) tubing to a venturi critical flow nozzle (FLOW-DYNE Engineering, inc.) with a throat diameter of 0.350in. for mass flow calculations. After the venturi nozzle, the flow enters a custom-made mating piece which transitions the flow from the 0.7 in. ID tubing to the 0.49in. x 0.49in. square duct. The sensor is installed near the exit of the duct via a through-hole in the bottom wall of the duct. All duct internal mating interfaces are inspected to ensure no steps to avoid perturbing the boundary layer.
Early test efforts resulted in sensors being destroyed due to a thin oil film appearing on the sensor. As the oil built up on the sensor surface, it would eventually become lodged in the sensing element. The oil film was thin enough that it was not visible with the naked eye; however the surface tension from the oil was strong enough that it would render the sensor immobile. It was determined that some of this oil was leftover coolant from the machining of the duct and package components. To fix this, every individual component was washed with isopropyl alcohol and inspected under the microscope to ensure cleanliness. The 7ft. long duct was cleaned several times from both ends with cloth soaked in isopropyl, and blow-dried with compressed air. After this initial cleaning effort, there was no evidence of any presence of oil in the duct. However, oil did periodically return during the course of testing, and the duct was cleaned as necessary. It was theorized that the oil was a result of the air compression process which mixes oil in with the air, and then filters it out. Additional filters were added to the compressed air line, and this helped to mitigate, although not completely solve this problem.
Figure 116: (Top) Diagram of duct experimental setup. Flow is from left to right. (Bottom) Close-up diagram of instrumentation near duct exit. Nomenclature: PD = pressure duct, TD = temperature duct, TE = temperature external.

### 8.3.3 Instrumentation/Data Acquisition System

The duct is instrumented to measure necessary flow properties along the duct. Figure 116 is a schematic indicating the instrumentation locations and nomenclature. Flow is from left to right, and exits into the laboratory which is at 14.9 psia and 25°C. Pressure measurements are taken at five locations with piezoelectric pressure transducers (OMEGA ENGINEERING, INC.), with ranges from 0–60 psia or 0–200 psia. Pressure measurements (PD_U2, PD_U1, PD_D1, PD_D1) are taken at static pressure taps located in the top wall of the duct. The venturi total pressure (P0) is measured using a total pressure probe upstream of the venturi nozzle.
Temperature measurements are taken at three internal locations: TD_U12, T_sensor, TD_D12 with K-type exposed junction thermocouples (OMEGA ENGINEERING, INC.). An external temperature measurement (TE_1) is taken at the outer-wall of the duct and at the same axial distance of the MEMS sensor using a surface-mount K-type thermocouple (OMEGA ENGINEERING, INC.).

Analog signals are connected to a National Instruments Connector Block (SCB-68) which feeds into a National Instruments multi-function DAQ card (NI-PCI 6221) installed inside of a PC. Tests have shown that the DAQ system acquisition rate can reach up to 1kHz when measuring 8 differential channels of pressure and thermocouple data. Decreasing the number of measured pressures can result in measurements as fast as 20kHz, which is necessary when attempting to measure turbulent fluctuations. An additional USB connected acquisition module (NI USB-9162 / NI 9211) can measure 4 differentially connected thermocouples at a rate of 2Hz.

Figure 117: Images of duct flow setup in Columbia University mechanical engineering laboratory.
All data is collected using the software program LabVIEW (National Instruments), and exported to either MS Excel or MATLAB (Mathworks) for analysis. Figure 117 are images of the duct setup with the important components labeled.

### 8.3.4 Flow Field Characterization

Flow velocity at the sensor location is calculated by using measured average flow properties (e.g. temperature and pressure), with the measured venturi mass flow rate (mdot). Using the K-N correlation eq. (173), flow velocity and properties, a theoretical shear stress inside the duct is calculated at the sensor location. Additional measurements allow calculation of the Mach numbers and shear stresses, 1in. upstream and 1in. downstream of the sensor location.

Duct flow field characterization tests were conducted by determining and measuring the axial pressure gradient along the duct, mass flow rate (mdot), average flow properties and wall shear stress ($\tau_w$). The flow rate in the duct was controlled by turning a ball valve by hand and monitoring the venturi total pressure (P0). The maximum duct air mdot measured was 0.33 lbm/s. Figure 118 shows that mdot varies with P0 in a linear manner, with a maximum value of P0 = 165psia.

Duct axial static pressure distribution is displayed in Figure 119 for different values of mdot. The magnitude of the axial pressure gradient increases with mdot as expected. The pressure gradient is within 1.5% of constant across the sensor and duct indicating fully-developed flow.

Frictional choking causes the average Mach number (M) to increase in the downstream direction and to approach a choked flow of $M = 1$ at the exit. Figure 120 shows typical mass flow data for the axial Mach number distribution versus mass flow rate. The maximum flow condition
gives a corresponding average Mach number at the sensor location (M\textsubscript{#_sensor}) of 0.85. M\textsubscript{#_U1} is the Mach number at the station U1, located 1in. upstream of the sensor, with an average value that is 1.2% lower than M\textsubscript{#_sensor}. Downstream station 1 (D1) is located 1in. downstream of the sensor, with an average Mach number (M\textsubscript{#_D1}) 1.2% higher than at the sensor. A custom-made Pitot tube was used to measure the center line velocity, and the average Mach number found to be within +/-10% of the values determined using the venturi nozzle. Due to Pitot blockage effects, and the small-size of the duct, the Mach number calculated using the venturi nozzle is considered to more accurately represent the actual flow conditions inside the duct.

The theoretical wall shear stress ($\tau_{\text{theory}}$) is calculated using eq. (173-175) with properties evaluated at the film temperature ($T_f$), which is the average of the bulk flow temperature (TD\textsubscript{sensor}), and the wall temperature ($T_w$). The wall temperature is assumed to be equivalent to the external duct wall temperature (TE\textsubscript{1}), which is measured with an external surface-mount thermocouple.

The requirement for assuming that $T_w$ and TE\textsubscript{1} are at approximately the same temperature is that the Biot Number ($Bi$) is <0.1 [5].

A small $Bi$ implies that the heat conduction inside of the wall is much faster than the convective heat transfer from the wall. $Bi$ is given by:

$$Bi = \frac{h_c L}{k} \quad (176)$$

where $h_c$ is the convective heat transfer coefficient, $L$ is the duct wall thickness, and $k$ is the thermal conductivity of the duct wall. Expected values for the duct flow setup are:

$h_c \sim 1 \text{ to } 10 \text{ [W/ m}^2\text{-K]}$, $L = 0.0035 \text{ [m]}$, $k_{aluminum} = 237 \text{ [W/ m} \cdot \text{K]}$

Inserting the above values into eq. (176) gives $Bi <0.1$ for the entire range of $h$, indicating that TE\textsubscript{1} $\sim$ $T_w$ is a valid assumption.
Figure 121 shows that the relationship between the theoretical shear stress and the mass flow rate is close to linear. At the maximum flow condition ($\text{mdot} = 0.33 \ \text{lbm/s}$), the expected value for $\tau_w$ at the sensor is approximately 400Pa.

The duct Reynolds number ($Re_b$) is calculated using average flow properties at the sensor location. $Re_b$ ranged from $1.8 \times 10^5$ to $7.1 \times 10^5$ at the installed sensor location, greater than 4000, which is commonly taken as the minimum value to ensure turbulent flow.

\[
y = 0.0022x + 0.0021 \\
R^2 = 1
\]

![Figure 118: Duct mass flow rate (mdot) as a function of venturi total pressure (P0).](image-url)
Figure 119: Static pressure distribution near sensor location (x = 0).

Figure 120: Average Mach number at sensor location and stations U1 (1 in. upstream) and D1 (1 in. downstream).
Figure 121: Theoretical wall shear stress calculated using K-N correlation with modifications for heat transfer eq. (173).

Duct testing allowed development of a curve where a shear stress can be “dialed into” the duct by opening the ball valve up to a specific total pressure. Figure 122 below shows typical test data indicating the theoretical shear stress ($\tau_{\text{theory}}$) at the sensor location for a measured venturi total pressure. $\tau_{\text{theory}}$ is calculated using eq. (173) and repeated testing showed that the theoretical shear stress typically was within +/- 3Pa of the target shear stress.

Another flow qualifying test was conducted where the MEMS sensor package was replaced with an aluminum plug to verify that the presence of the package did not adversely affect the duct pressure distribution. Testing revealed that there was no noticeable effect on the flow axial pressure distribution from the package, indicating the sensor package was non-intrusive to the flow.
Figure 122: Theoretical shear stress data plotted versus venturi total pressure. This curve is calculated from measurements and eq. (173) and is used to “dial-in” a specific shear stress for sensor testing.

Figure 123: Viscous sublayer ($y_{vs}$) height plotted against mass flow rate ($mdot$). The dotted line represents the expected height of the sensor features (8μm).

Figure 123 displays the calculated height of the viscous sublayer ($y_{vs}$) from eq. (17), where the height is assumed to be at $y^+ = 6$. The height of the vertical features on the MEMS shear stress, should be smaller than $y_{vs}$ to properly measure the shear stress at the wall ($\tau_w$).
Calculations show that vertical sensor features of 8μm, will fall within \( y_{vs} \) for \( \text{mdot} \) values less than 0.1 lbm/sec. At higher \( \text{mdot} \) values the features will extend outside the viscous sublayer, possibly affecting the boundary layer, and the wall shear stress. Additionally, there are gaps associated with the macro-scale packaging which can range from 50μm-200μm. Currently the team has not evaluated the effect of any of these gaps or protrusions into viscous sublayer, although this is an area listed in possible future investigations (Section 9.2).

![Figure 124: Kolmogorov scaling for duct flow. The frequency scale is plotted on the left y-axis, and the micro scale is the right y-axis. The dotted line at 8 kHz is the maximum MS3110 bandwidth, indicating the maximum sensing frequency.](image)

Figure 124 displays Kolmogorov scaling for duct flow. The MS3110 IC has an adjustable BW range from 0.5 to 8kHz, which is set via an integrate low-pass filter. Analysis of the expected turbulence time scales shows that for an \( \text{mdot} \) greater than 1.0 lbm/sec, the MS3110 will not be able to detect turbulent fluctuations due to the high frequency attenuation from the low-pass filter. For \( \text{mdot} \) values greater than 1.0, the MEMS shear stress sensor will be measuring the mean shear stress of the turbulent flow. To resolve the turbulent shear stress
fluctuations in the duct cold flow, it would be necessary to use another circuit for measuring capacitance with a bandwidth on the order of 20kHz, or to limit sensor testing to mdot values lower than 1.0 lbm/sec.

8.3.5 Test Operations/Procedures

The following procedures are followed to prepare a packaged shear stress sensor for high-speed cold flow testing:

1) Wall jet testing at microscope probe station to ensure that packaged sensor is responding to fluidic actuation.
2) Shakedown test of instrumentation and data acquisition system with flow through duct and no sensor installed.
3) Connection of power supply to MS3110 IC, followed by visual confirmation using a microscope that sensor is not in snap down condition.
4) Installation of sensor package into duct.
5) Determination of best capacitance resolution of MS3110 IC by adjusting capacitance trim and gain settings.
6) Determination of operational range of shear stresses for sensor and corresponding P0.

8.3.5.1 Test Series for Duct Cold Flow Testing

The main objective of the flow tests was to actuate the sensor and cause a measurable sensor output ($V_0$), while avoiding sensor snapdown of the sensor (discussed in Section 2.5.2.1).
During sensor testing, the flow was increased in pressure steps corresponding to approximately 150, 260, and 335Pa of shear stress. At each step the pressure was held constant for approximately 10 seconds to allow the flow to stabilize and approximate steady-state. All reported mean flow data comes from averaging the sampled data at each step, while the turbulence measurements use the time-varying signal at each step. Due to the large hysteresis that was observed with the MEMS sensor measurements, we will focus only on test results for “Increasing Flow.” Ultimately the scatter on the “Decreasing Flow” measurements was so large that it was hard to interpret trends. One cause for this data scatter could be a “temperature effect,” which could be sensitive to slight changes in the varied input parameter. Each sensor test was followed by a post-test without flow to evaluate sensor hysteresis after the flow had stopped. The time between concurrent tests was long (generally 10-20 minutes) because it was necessary for the duct to return to the ambient temperature where it was prior to flow testing.

### Table 17: Flow Conditions for reported sensor data. Values with an (*) next to them are for ambient conditions.

<table>
<thead>
<tr>
<th></th>
<th>Mdot [lbm/sec]</th>
<th>U_avg [m/s]</th>
<th>T_avg [K]</th>
<th>( \rho_{avg} ) [kg/m(^3)]</th>
<th>( Re_b )</th>
<th>( Re_{\tau} )</th>
<th>( \tau_w ) [Pa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>0</td>
<td>0</td>
<td>296.1*</td>
<td>1.21*</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Step 2</td>
<td>0.12</td>
<td>251.7</td>
<td>268.4</td>
<td>1.44</td>
<td>2.5x10(^5)</td>
<td>9658</td>
<td>150</td>
</tr>
<tr>
<td>Step 3</td>
<td>0.20</td>
<td>272.7</td>
<td>258.7</td>
<td>2.17</td>
<td>4.5x10(^5)</td>
<td>15322</td>
<td>260</td>
</tr>
<tr>
<td>Step 4</td>
<td>0.28</td>
<td>273.5</td>
<td>256.4</td>
<td>2.98</td>
<td>6.2x10(^5)</td>
<td>20541</td>
<td>335</td>
</tr>
</tbody>
</table>

During mean shear stress measurements the DAQ sampling rate was 100Hz for 120 seconds, and data was saved to a single spreadsheet file. When turbulent fluctuations were being measured, at each step the sensor output data was sampled at 20kHz for 0.8192 seconds (16384
samples), while the flow properties data was sampled at 100Hz for 1 second (100 samples). Different spreadsheet files were generated for each data recorded at each step in the flow.

8.3.6 Preliminary Results of Duct Cold Flow Experiments

Results for nine separate tests of sensor ECA-3F2A are shown in Figure 125. Each data point represents the average flow properties measured at steps 1 to 4. The sensor was tested three times at each orientation of 0°, 90°, and 180°. In an ideal sensor with no temperature effect, the 0° curve would be an increasing voltage, the 90° would be constant at 0V, and the 180° would be the same magnitude as the 0° data but in the negative direction. It is clear from the data that this is not occurring; however there are three distinct trends which are dependent on the orientation. The primary reason that the sensor results are not as expected is due to temperature effects which affect the sensor output. Compensation for the temperature effect will be discussed in the next section.

Figure 125: Typical duct flow test results for 3 sets of tests of sensor ECA-3F2A. Note the three distinct trends corresponding to the sensor orientation.
8.3.7 Temperature Compensation in Duct Flow

A previously discussed in Sections 6.1.2 and 7.3, the sensors demonstrated a significant “temperature effect” during flow testing. This was eventually determined to be the direct result of a changing V-I curve across the parallel resistors (RP1 and RP2), which affected the MS3110 measurement output. Based on the results from the Pspice simulation that took these effects into account (Simul_3), we are justified in the assumption that this temperature effect manifests itself as a voltage offset in the MS3110 output (provided the CF value is large enough).

In order to apply simulation results from Simul_3 to the duct flow setup, it was necessary to take V-I measurements at the specific flow conditions (Steps 1-4) within the duct. This was done by disengaging the MS3110 IC, and connecting the Keithley 4200-SCS to the sensor using the available BNC connectors (described in Section 4.1). During the V-I tests, the flow conditions were measured at each step to ensure repeatability among the tests.

The best method for decoupling the “temperature effect” from the shear stress measurement of the sensor is by testing it in the duct flow test setup while the sensor is rotated 90° azimuthally. This results in the flow hitting the sensor in the transverse direction where the sensor response is nominally zero due to its much larger spring stiffness in that direction. Thus, any change in the sensor output, would theoretically be caused only by the temperature effect.

Figure 126 shows the measured MS3110 voltage change for the 90° orientation for sensor ECA-3F2A data, compared to the Pspice model. Recall that the Pspice model was developed using the actual V-I measurements from this sensor. The error bars on the flow data are +/- 0.02V, representative of the experimental scatter at each point. There is good agreement at the first data point, and trend of both curves is similar, although they start to diverge at high shear stress values.
The objective of the Pspice simulations were to model the MS3110 behavior with changing V-I relations. Ultimately we are unable to exactly represent the MS3110 since we do not have complete knowledge of its internal circuitry. However, we can conclude that the Pspice model is a fair representation of the mechanism causing the temperature effect, and furthermore that the temperature effect can be compensated for by assuming that it is a voltage “offset” which can be subtracted out at each step (Section 6.1.2). From this point on, temperature compensation will be implemented by taking the sensor output at 90° and assuming that it is an accurate representation of the temperature effect.

Figure 126: Duct flow results for 90° orientation of sensor ECA-3F2A compared to the Pspice simulation (Simul_3) results. The error bars are +/- 0.02V, representative of the experimental scatter at each point.
8.3.8 Duct Flow Results / Analysis

Typical duct flow results for sensor ECA-3F2A are presented in Figure 127. The sensor measured shear has been calculated by taking the measured MS3110 output, compensating it for temperature by subtracting the 90° data, and calculating the shear stress using the capacitance equations, eqs. (34) and (38). The data has an average measurement error of +/- 7%, calculated from experimental scatter. The x-axis in the graph is the theoretical duct shear stress which comes from the K-N equation. The error on the duct shear stress is estimated at +/-8% and was calculated using Monte-Carlo simulations in MATLAB. There is generally a linear relationship between the two sets of data, however, the sensor overestimates the shear stress by an average factor of 5.5276.

Based on the wall jet test results, the capacitance measurement overestimates the sensor displacement by a factor of 1.8. Therefore, to compensate for this effect, the data is divided by this factor and plotted as the “compensated shear” in Figure 127. Unfortunately, even with this taken factor taken into account, the sensor is still over-estimating the shear stress by more than 200%.
Figure 127: Typical duct flow test results for sensor ECA-3F2A. Sensor Measured Shear is calculated using capacitance values. The Compensated Shear divides by a factor of 1.82 based on the wall jet results. The Theoretical Duct Shear comes from the K-N equation.

After reviewing the data from other floating-element capacitive shear stress sensors, it became clear that most researchers did not report their raw capacitance measurements. In fact we could locate only one paper, Zhe et al.[23], where the authors reported their measured capacitance values. Instead results are presented as voltage vs. theoretical shear stress, and measured capacitance is not mentioned. In lieu of this, the rest of the current results will be presented using the voltage measurements directly.

The sensor voltage output is plotted against the duct shear stress in Figure 128. A linear line of best fit gives a slope of 0.00016V/Pa or 0.16mV/Pa. The horizontal error bars are +/- 8% while the vertical error bars are +/- 0.005V and come from the data scatter.
Turbulence intensities were calculated by taking the voltage fluctuations and converting them to turbulence fluctuations ($\tau'_{w,x}$) using a sensor gain of 0.159mV/Pa. \( \tau'_{w,x} \) \( \text{RMS} \) was calculated by taking the root-mean-square of the data. The mean shear stress, \( \tau'_{w,x} \) is assumed to be the theoretical duct shear stress from the K-N equation. Values are plotted in Figure 129 against the previous data that was described in Section 1.5.1

The current data is represented by two sets, one sampled at 100Hz, and one sampled at 20kHz. There does not appear to be any significant differences between the two sets of data. A linear fit was fit to all of the current data and plotted with error bars of +/- 30%, which was calculated from the scatter. Despite the large variability in the data, the linear fit values agree well with previous researchers who theorized that the turbulent fluctuations would always be approximately 30-40% of the mean shear stress values. We were unable to locate any previous data taken at Re, values as large as the current data, so we cannot directly compare the accuracy.
of our data to others. However, our data lies within the range of previous data, so these results are welcomed.

![Figure 129: Turbulence intensity measurements for sensor ECA-3F2A. The vertical error bars are +/- 30%. Alfredsson et al. [60] and Wietrzak et al. [61] reported constant values of 0.4 and 0.32, respectively, but no Re_{τ} range.](image)

The probability density function (PDF) of the MEMS sensor is plotted in Figure 130 for Re_{b} values of 2.5x10^5, 4.5x10^5, and 6.2x10^5. The PDF was calculated in MATLAB using a bin width of 0.13. Also plotted are the data from previous researchers which was discussed in Section 1.5.2. There is no clear difference among the PDF measurements for three different Re_{b} curves, but there is generally good agreement with previous research for most of the values. One difference is that that maximum of our curves are centered around x = -0.5, while the previous researchers generally lie between -0.55 and -0.8. Also our data diverges from the previous data between x = 1 and 2 for an unknown reason. In general though, these results give confidence that we are measuring turbulent fluctuations.
The spectral density is calculated for the same Re_b values and is plotted in Figure 131 against previous researchers. The current data is located further to the left than most previous data. This is because our measurements were taken at value at lower $f^+$ values than other sensors, which corresponds to faster flows rates. The data only covers a small range of $f^+$ values due to the 8kHz frequency cut-off of the MS3110. To avoid over sampling, we can only confidently measure frequencies up to 4kHz, the Nyquist frequency. The results are encouraging in that they are in the same range as previous research, however we are unable to thoroughly compare our measurements to those of others.

As discussed in Section 1.5.5, the MEMS sensor will be subjected to additional out-of-plane forces resulting from the wall pressure fluctuations which occur inside turbulent boundary layers. Analysis from Section 2.6.2 showed that this effect on the sensor measurement is

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**Figure 130**: Probability density function (PDF) results for sensor ECA-3F2A for different Reynolds numbers

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frequency-dependent, and could be as much as approximately 70% of the signal in the worst-case scenario. However, because the measurements were taken at $f^+$ values less than 0.05, the $P_{w'}$ values will be at most 10 times larger than the $\tau_{w'}$ values. Therefore, the maximum error from the out-of-plane forces is generally going to be 7% or less which is considered an acceptable value. It should be noted that the wall pressure effect on the sensor signal is embedded in the time-varying (turbulent) part of the sensor measurement, and is not expected to affect the mean part of the shear stress measurement in a significant manner.

![Figure 131: Spectral density results for sensor ECA-3F2A.](image)

The pre-multiplied spectral density results for sensor ECA-3F2A are shown in Figure 132 and plotted against previous researchers. Our $f^+$ values only extend approximately up to 0.001, which is a direct result of the MS3110 sampling limitation of 8kHz. For visualization purposes, the current measured data is plotted against the right y-axis which has a different scale than the left y-axis. Pre-multiplied spectral results are typically normalized so that their integral is 1; when our data is normalized, (because we are missing the higher frequencies) it results in
magnitudes that are much larger than they would be if all of the higher frequencies were included. It is clear that the sensor is not measuring the entire turbulence spectrum of the flows, as our curves do not have the expected shape with the peak in the middle. To capture the entire turbulence spectrum at the measured flow rates, we would need to sample up to \( f^+ = 0.1 \) or higher, which would require a 100-fold increase in the sampling rate (since \( f^+ \) is proportional to \( f \)).

Future efforts at measuring the entire turbulence spectrum of the tested high-speed flows would require sensors and electronics that could respond at higher frequencies \( O(10^3 \text{kHz}) \) than used in this thesis. Using the current MEMS sensors and electronics, we would be limited to the application of sensors to lower-speed flows where \( f^+ = 0.1 \) corresponds to approximately 4kHz or lower.

![Figure 132: Pre-multiplied spectral density results for sensor ECA-3F2A.](image)
9 Summary and Conclusions

This dissertation describes the development of a MEMS shear stress sensor and its application in high-speed turbulent flows. The sensor was designed and fabricated as part of a larger team effort, however many of the specific tasks were completed as part of this thesis. The overall thesis objectives were presented in Section 1.7 and can be described as the following:

- Electromechanical Design and Analysis of Sensor
- Design and Fabrication of Packaging
- Characterization of Sensor
- Mean Shear Stress Calibration for Sensor
- Fluctuating Shear Stress (Turbulence) Measurements

Electromechanical Design and Analysis of Sensor

The MEMS sensor was designed with the following goals:

1) Operates in sub-kPa shear stress range (1-1000Pa)
2) Operates in high-speed turbulent compressible flow environment
3) Bidirectional sensor operation (can measure in +x and –x directions)
4) Non-intrusive to flow
5) Utilizes backside sensor connections
6) Utilizes a MEMS floating element design for direct shear stress measurement
7) Uses a differential capacitive sensing scheme and off-the-shelf electronics
8) Fast dynamic response O(10-100kHz)
All of these design goals were met and verified in the final design except for (4) and (8). An ideal floating element sensor is completely “flush-mounted” into the wall of the flow being measuring. The current MEMS sensors were fabricated with µm-sized vertical features which lie within the viscous sublayer for smaller flow rates, but extend into the overlap region at higher flow rates. Integrating the sensor with the packaging and the flow set-ups can result in even larger gaps at the wall. Efforts were made to make reduce these gaps as much as possible, however they will always remain to some extent. “Misalignment” testing was not conducted as part of this research effort, so we are unable to remark on any effect that this may have had on the sensor measurement.

Based on analytical modeling (including FEA analysis), the sensor has a dynamic frequency response in the desired range, for both damped and undamped motion. For the current sensor, measurements were limited to 8kHz due to the low-pass filterer imbedded inside the MS3110 IC. We were unable to measure the sensor response at frequencies faster than 8kHz using capacitive sensing, and therefore, the dynamic response was not verified experimentally.

Additional analysis of the sensor looked at secondary forces on the sensor, which are those that were not directly occurring on the floating element as a result of the shear stress. It was determined that the sensor was reasonably immune to these forces, the sole exceptions being the electrostatic force from the capacitive test signal and forces resulting from out-of-plane turbulent pressure fluctuations. The electrostatic force was accounted for by limiting the sensor displacement to a range where the forces could be neglected. Out-of-plane forces from turbulent pressure fluctuations can significantly affect the sensor measurement (depending on the frequency), and should be considered in future designs.
Design and Fabrication of Packaging

The sensors were successfully fabricated at the Université de Sherbrooke and sent to Columbia University where they were installed in a custom-made package that integrated the sensor with macro-scale components. The package was designed to accommodate a MEMS sensor, and the sensing electronics, while providing a means of installing the sensor in a flow testing setup. The sensor provided both the mechanical and electrical connections to the sensor. Small-sized electrical connections were used with an electrically shielded package to reduce parasitic capacitance and EMF.

Characterization of Sensor

Sensors were electrically and mechanically characterized at a microscope probe station to verify their operation. Electrical measurements included resistance, capacitance, V-I curves, and were used to determine if sensors were electrically viable. Physical characterization included mechanical actuation (probe-push test) and electrostatic actuation which were necessary for observing sensor motion. Measured V-I curves were input into a Pspice simulation to model the effect of parallel resistance on the MS3110 measurement.

Utilizing a wall jet flow setup, sensors were fluidically actuated while displacement was measured via optical and capacitive means. Comparing the two measurement types showed that calculating the displacement from the theoretical capacitance change consistently over-estimated the displacement by an average factor of 1.82. Azimuthal testing was also conducted to demonstrate that the sensors were primarily responding to flow in the streamwise (x) direction, as they were designed to.
Mean Shear Stress Calibration for Sensor

High-speed flow testing was conducted by exposing the sensor to subsonic compressible turbulent through a square duct. Sensors were tested at three values of mean shear stress (150, 260, and 335Pa) to determine a calibration equation. Due to the compressibility of the flow, there were temperature changes present which affected the sensor output. It was verified through V-I measurements, that current leakage through the substrate was temperature-dependent, and was theorized as the cause of the observed “temperature effect.”

The temperature effect was decoupled from the sensor measurement by repeating the flow tests at the same conditions while rotating it 90° azimuthally so that the sensor did not respond to shear stress. Further verification included creating a Pspice model which simulated the MS3110 operation and the temperature effect using sensor V-I measurements. The Pspice simulation and the 90° testing results matched reasonably well and indicated that the temperature effect was an offset caused by the substrate leakage, which could be subtracted out.

After the sensor output was compensated for temperature, a “sensor measured shear stress” was calculated using the theoretical capacitance equations. This calculated value overestimated the duct shear stress by more than a factor of 5. Utilizing the knowledge from the wall jet set up, that the capacitance overestimates the shear stress by a factor of 1.82, we compensated the measurements but the sensor was still found to overestimate the shear by more than a factor of 3. Ultimately a relationship between voltage and duct shear stress was used in the calibration equation, which is consistent with the majority of previous research on floating element capacitive sensors. The relationship between sensor output voltage and duct shear stress was approximately linear, and the sensor gain was found to be 0.16mV/Pa.
Fluctuating Shear Stress (Turbulence) Measurements

Turbulent fluctuations were calculated by converting the sensor output voltage to shear stress (using the calibrated gain) and subtracting the mean duct shear stress at each flow step. The resulting part of the signal is a combination of the turbulent shear stress fluctuations coupled with the wall pressure fluctuations. For the measured $f^+$ values, the wall pressure is estimated be less than 7% of the signal, which is considered an acceptable value.

The turbulence intensity measurements were found to have significant scatter, however the average of the data was consistent with the range of previous researchers at lower Reynolds numbers. We were unable to find any similar measurements in the literature for the tested Re$_\tau$ values. Calculation of the probability density function (PDF) showed results that were generally consistent with previous researchers (again at lower Re$_\tau$ values), but not over the entire measurement range.

The spectral density matched up reasonably well with previous researchers for low $f^+$ values, but indicated that we were not measuring the entire turbulence spectrum of the sensor. Ultimately the frequency range of this sensor was limited to the 8kHz cut-off of the sensing electronics, however physically the sensor can likely can respond to faster frequency fluctuations based on the design equations and FEA analysis. Utilizing the current electronics and packaging, in combination with lower-speed flow testing and sensors designed for lower shear stresses, should result in the ability to measure the entire frequency spectrum of fluctuating turbulent shear stress.
9.1 Contributions

The major contribution of this project is the development of a MEMS floating element shear stress sensor with backside connections to be used for high-speed turbulent flow measurements. Although there are many MEMS floating element designs, we were able to only locate one previous sensor which had backside connections [47]. That sensor used piezoresistive sensing, and was designed to measure liquid shear stress at much larger values than the current one. In this thesis, turbulence intensities were measured at Re, values $2 \times 10^4$, higher than previous researchers that we are aware of. This sensor also required temperature compensation due to the compressibility of the flow, which adds to the uniqueness of the development effort. Overall, this dissertation contributes to the general knowledge about MEMS floating element shear stress sensors in high-speed turbulent flows.

9.2 Future Work

Sensor Design and Fabrication

Future work on this specific MEMS sensor design could implement a variety of design changes. The overall design should be stripped down as much as possible to by discarding extra features. For example, the sensors included extra electrode pads to use in continuity testing. In practice, these extra pads ended up not being that helpful, and were detrimental in that they increased the overall capacitance of the sensor (which limited the resolution of absolute capacitance measurements).
The sensor package design should be re-designed in an effort to make it more integrated with the Silicon wafer piece, and whatever flow set-up is being utilized. Although the package design was ultimately satisfactory, there was always the risk of destroying a sensor during the packaging process. Ultimately the design was guided by the size and layout of the MS3110 and the ability to have one package that could be re-used for different sensors. Future efforts should consider integrating the electronics with the sensor during the MEMS processing rather than trying to integrate after fabrication, as this would streamline the packaging process and could result in multiple working sensors which could be tested simultaneously.

It would be preferable to focus on a few sensor designs designed for shear stress ranges that can easily be tested in a known flow. For example, it would be better to focus on more sensitive open-loop designs rather than closed-loop designs which require more complicated electronics. Also it would be recommended to have multiple copies of each sensor design so that results can be compared for repeatability.

Improvements in sensor fabrication should focus on creating the Vias without having to apply conductive epoxy by hand. Many sensors were destroyed before an application method was mastered. Effort should also be put forth at reducing the parallel resistance (i.e. substrate leakage), since this seriously affected the capacitance measurement, and also created a temperature effect. It is not clear if this leakage occurs from the conductive epoxy, however future efforts should look into this.

There was very poor agreement when a “sensor measured shear stress” was calculated using the theoretical capacitance calculations. A full FEM electrical model should be created to determine the accuracy of the utilized capacitance equations.
Many sensors were destroyed due to overetching or underetching because sensors with different-sized floating elements were next to each other on the wafers. If a longer timed etch was necessary to release a larger floating element, it would often overetch a nearby smaller floating element. It would make more sense to group similar-sized designs next to each other so that they will be released at the same time.

For this sensor effort, it was decided early on to use the MS3110 IC, a low-cost off-the-shelf capacitance meter. However, this seriously impeded sensor measurement efforts when we were trying to determine the effect of the parallel resistance. The MS3110 applies a relatively large test signal voltage (up to 2.25V) which can further compound problems with substrate leakage since measurements showed that the resistance decreased with voltage (i.e. the leakage increased). It is possible that with a mV size test signal the temperature effect would have been negligible.

More importantly the MS3110 has a bandwidth limit of 8kHz, which severely limited our ability to sense turbulent fluctuations at the tested flow rates. Future sensor efforts should utilize custom-made electronics that can measure at the required high-frequencies, unless suitable pre-made electronics are available.

Sensor Testing

Future efforts at testing should attempt to utilize a flow source that is steady-state, unlike the duct flow, and one that does not have inherent temperature changes. An ideal future flow setup would include a well-known shear stress, as well as the ability to observe the sensor motion using a microscope.
The effect of sensor misalignment was outside the scope of this thesis, but is an area that is crucial to MEMS sensor performance at high speeds. Testing sensor protrusion and recession in an area of great importance that unfortunately, was not included during the course of this research. Also, package that allowed for variation in the gaps around the sensor would be a big step in research on this topic. An additional variable that could be looked at is if there is any effect from the roughness as the flow transitions from the wall material to the sensor material (Si).

The effect of out-of-plane turbulent pressure fluctuations on floating element sensors is an area of active research by Professor Mark Sheplak’s group at the University of Florida in Gainesville [52]. It may be possible to use a plane-wave-tube to generate out-of-plane shear stresses, although this could be difficult to do at higher flow rates. Verification that the sensor is insensitive to strong out-of-plane fluctuations could also be proven by demonstrating that the sensor is orders-of-magnitude stiffer in the out-of-plane direction.
10 References


[84] ExpressPCB Manufacturing Specifications, ExpressPCB, Inc.


[88] MS3110 Datasheet, Irvine Sensors Corp.


